GeV Electrons via Laser Wakefield Acceleration with Pre-formed Plasma Channels

Candidacy Oral Presentation

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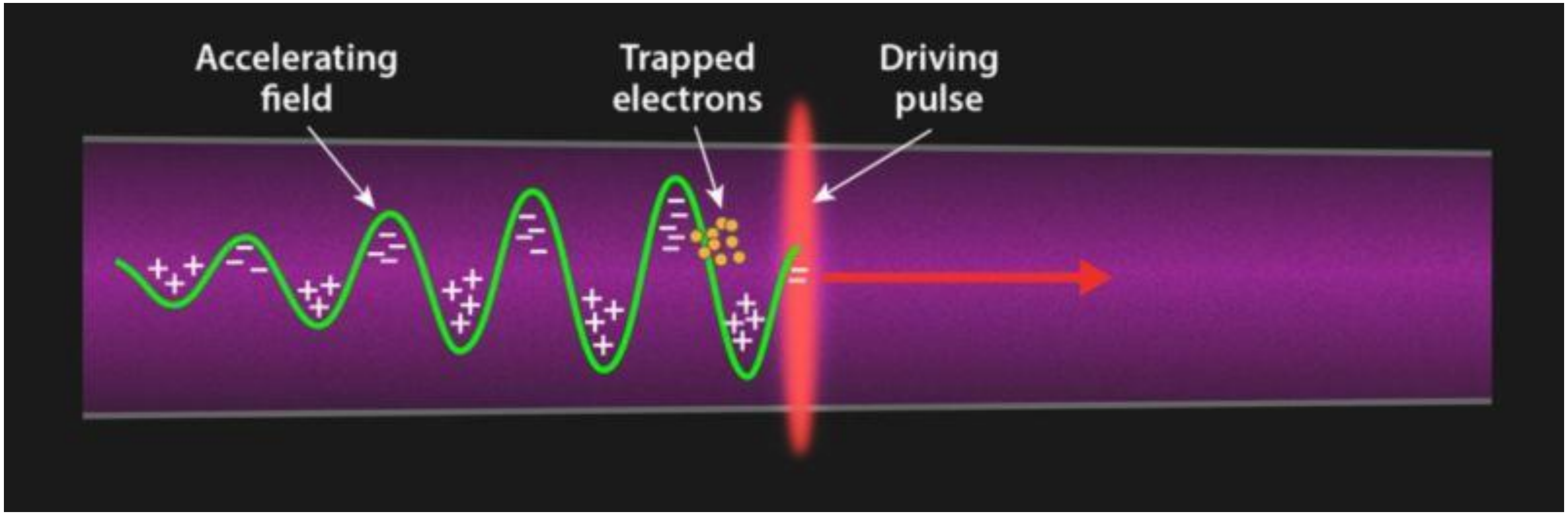
Outline

1. Laser Wakefield Acceleration
2. Limitations of LWFA
3. The Solution: Waveguiding
4. Physics of Waveguides
5. Methods of Waveguiding
    1. Relativistic Self-Guiding
    2. Capillary Discharge
    3. Optical Field Ionization

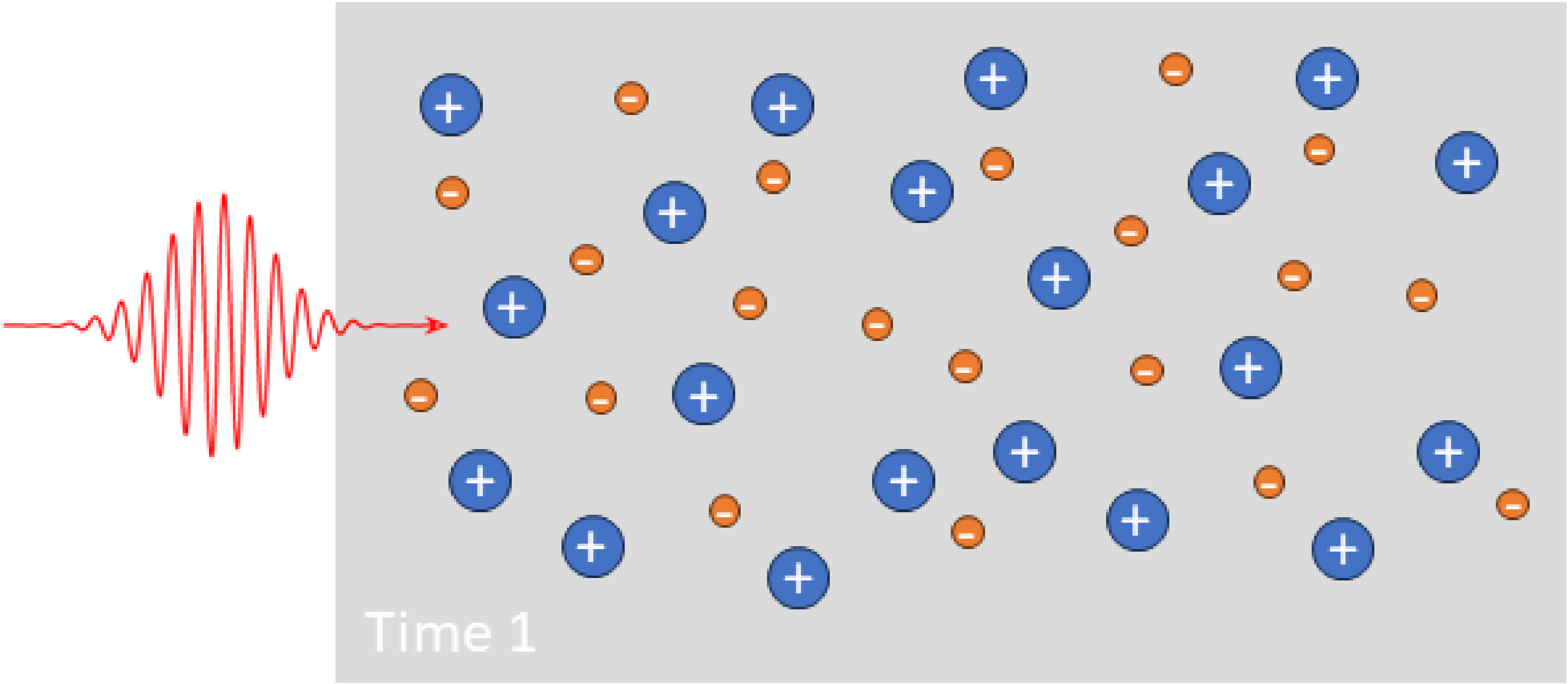
Laser Wakefield Acceleration (LWFA)

LWFA is a method of accelerating electrons in plasma via electron waves with high acceleration gradients in excess of 10-100GV/m.



Only works with short pulse ($t \leq 1ps$), high intensity ($I \geq 10^{17} W/cm^2$) lasers and in underdense plasma ($\frac{\omega^2}{\omega_p^2} \gg 1$).

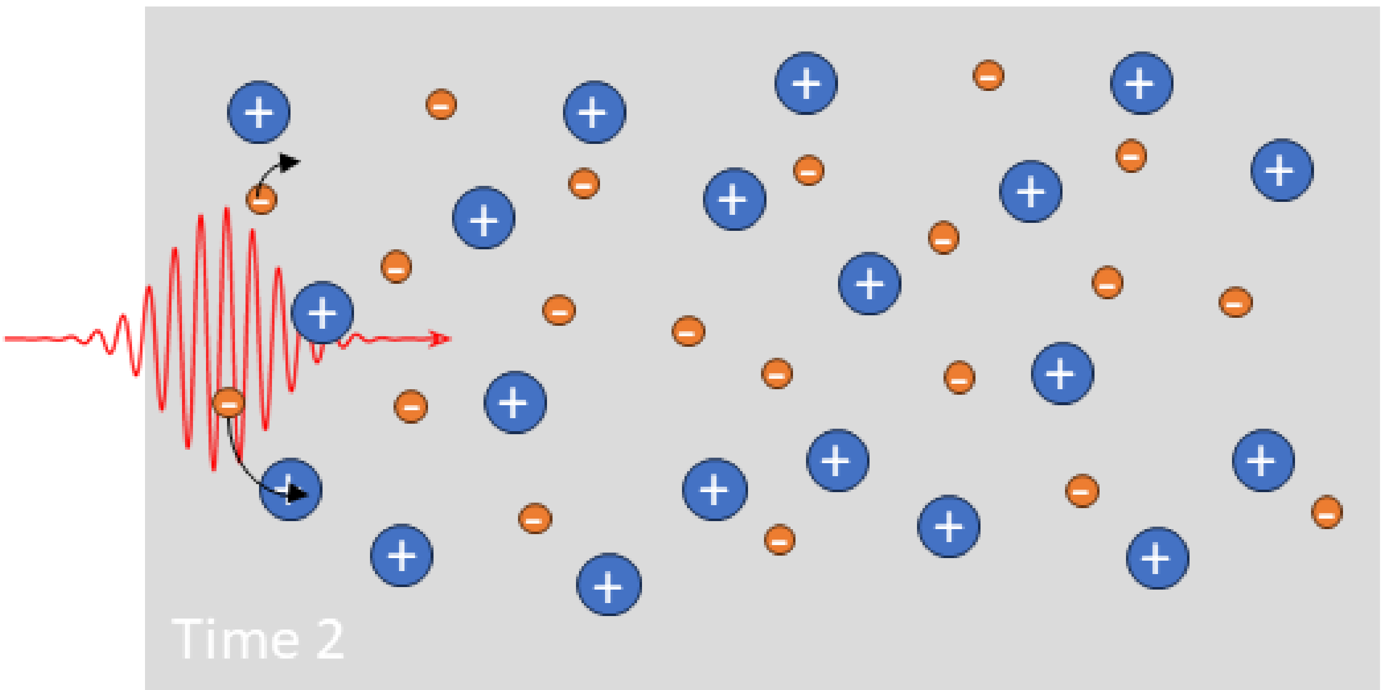
Physics of LWFA



Electrons are accelerated through the ponderomotive force.

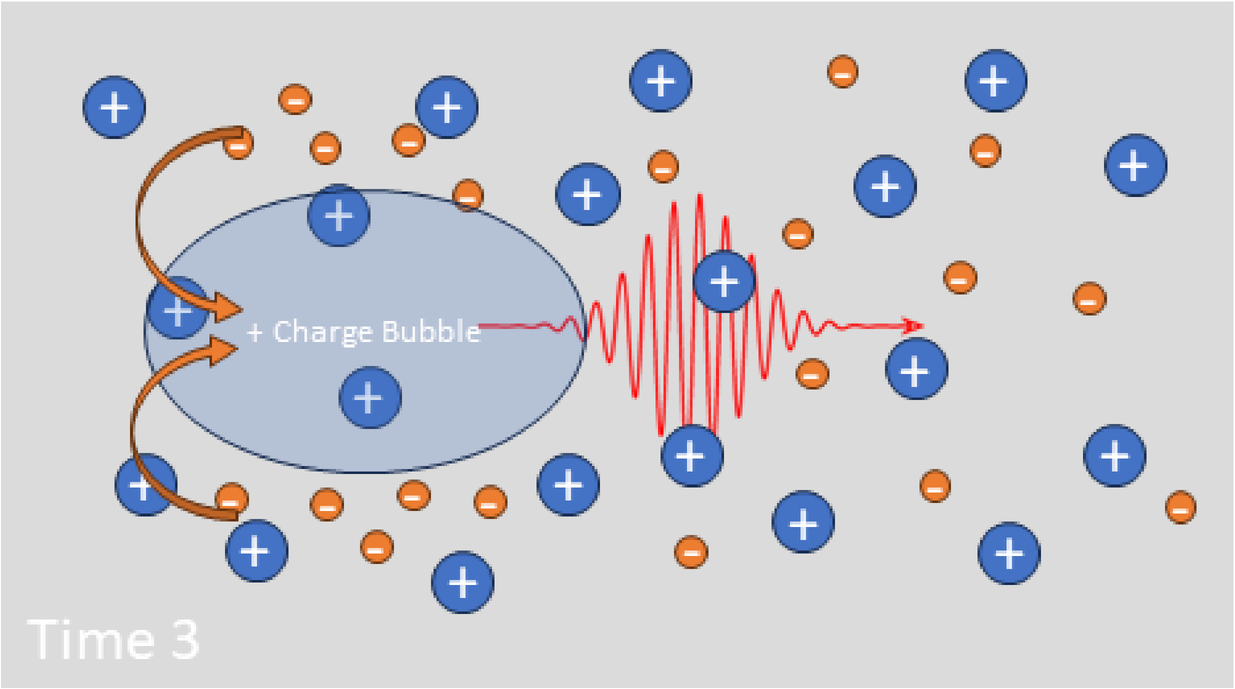
$$\vec{F}_p = -\frac{e^2}{4m\omega^2}\vec{\nabla}(E^2)$$

Physics of LWFA



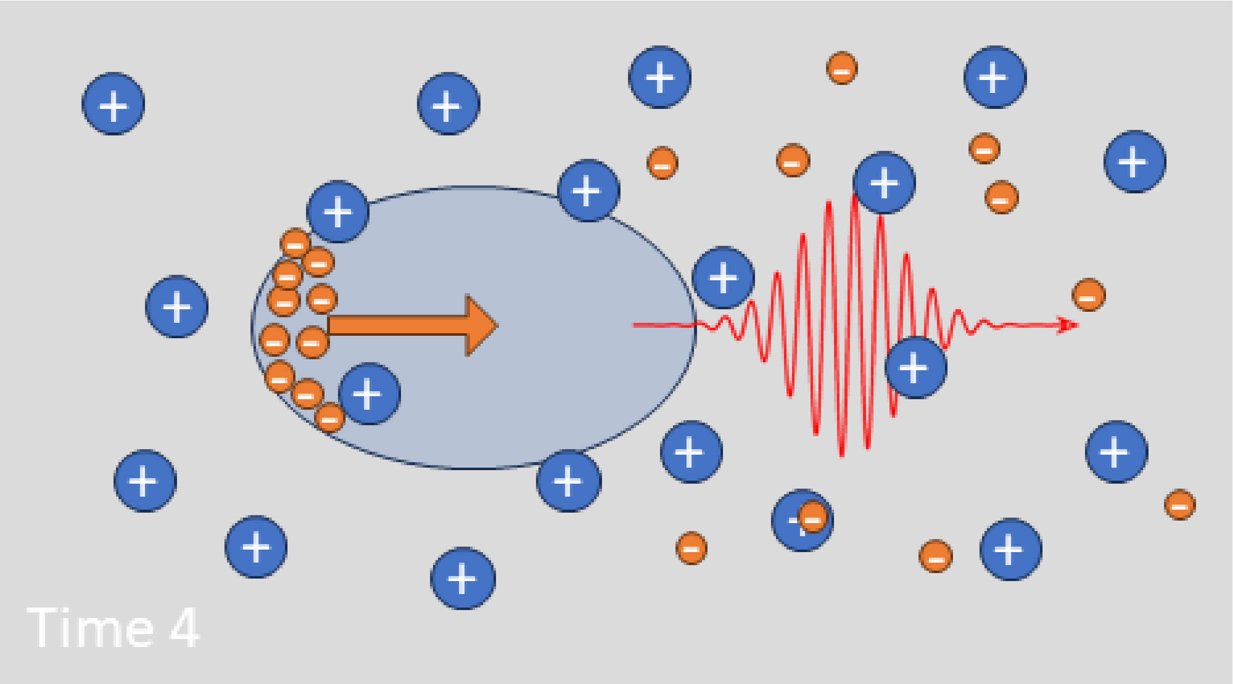
The pulse travels through the plasma, vacating electrons.

Physics of LWFA



The electrons recombine behind the pulse and are pulled along by a large electric field

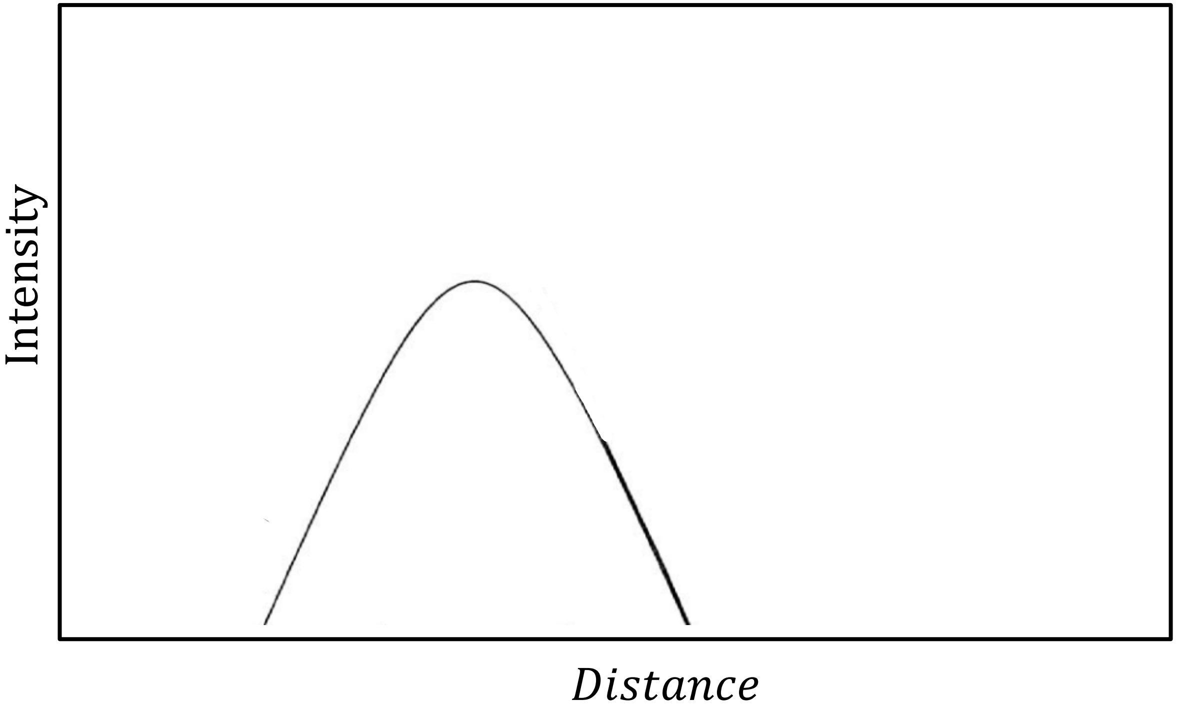
Physics of LWFA



This creates electron waves behind the laser pulse

The Problem

The goal is to get the highest possible electron beam energies from LWFA. In low density gas or vacuum, a laser can only keep LWF intensity for a few Rayleigh lengths. This limits the energy that an electron beam can reach.

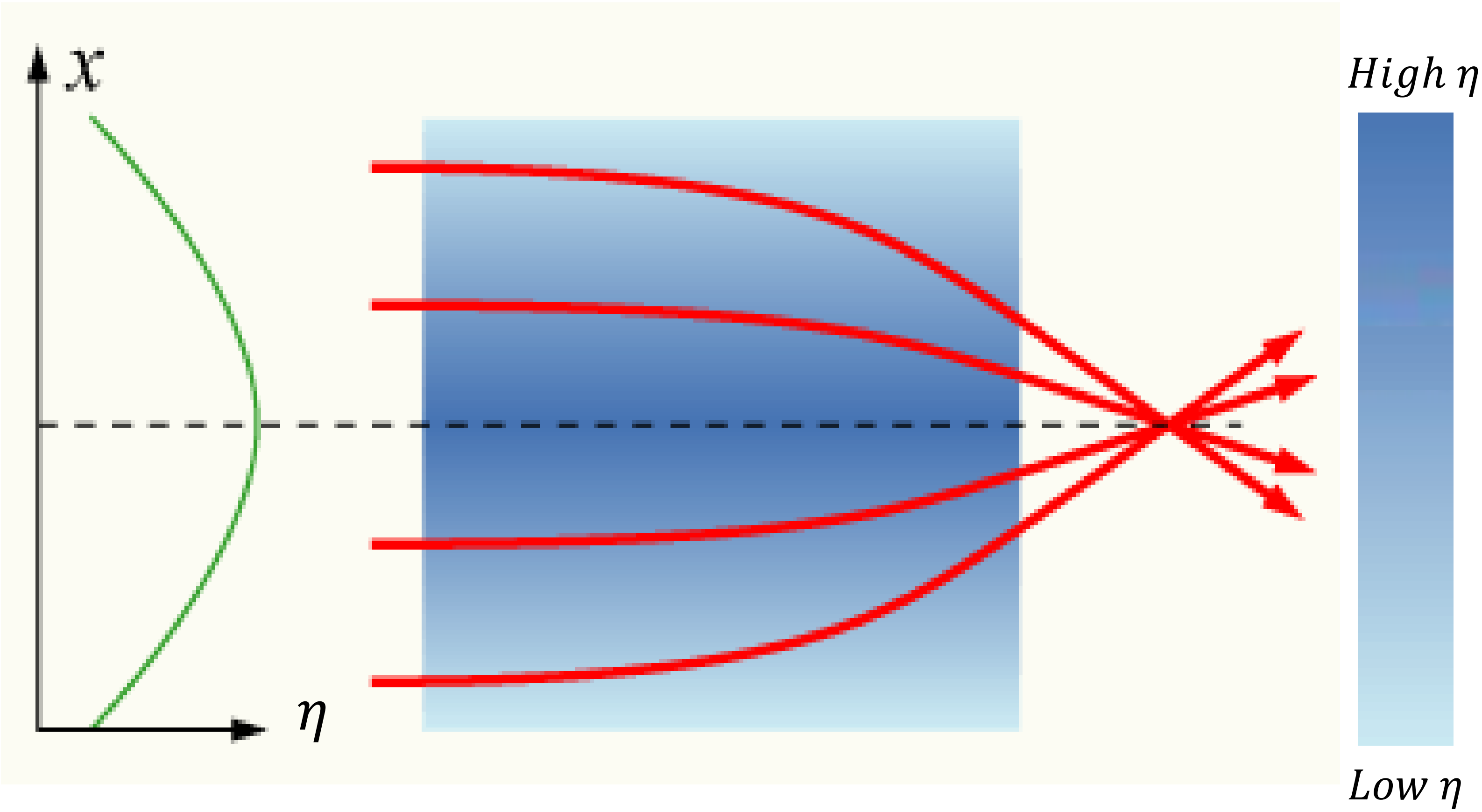


$$Z_R = \frac{\pi w_o^2 \eta}{\lambda}$$

$$I \propto \frac{I_{peak}}{1 + \frac{z^2}{Z_R^2}}$$

Laser intensity v distance traveled in vacuum.
Modified from Esarey 2009

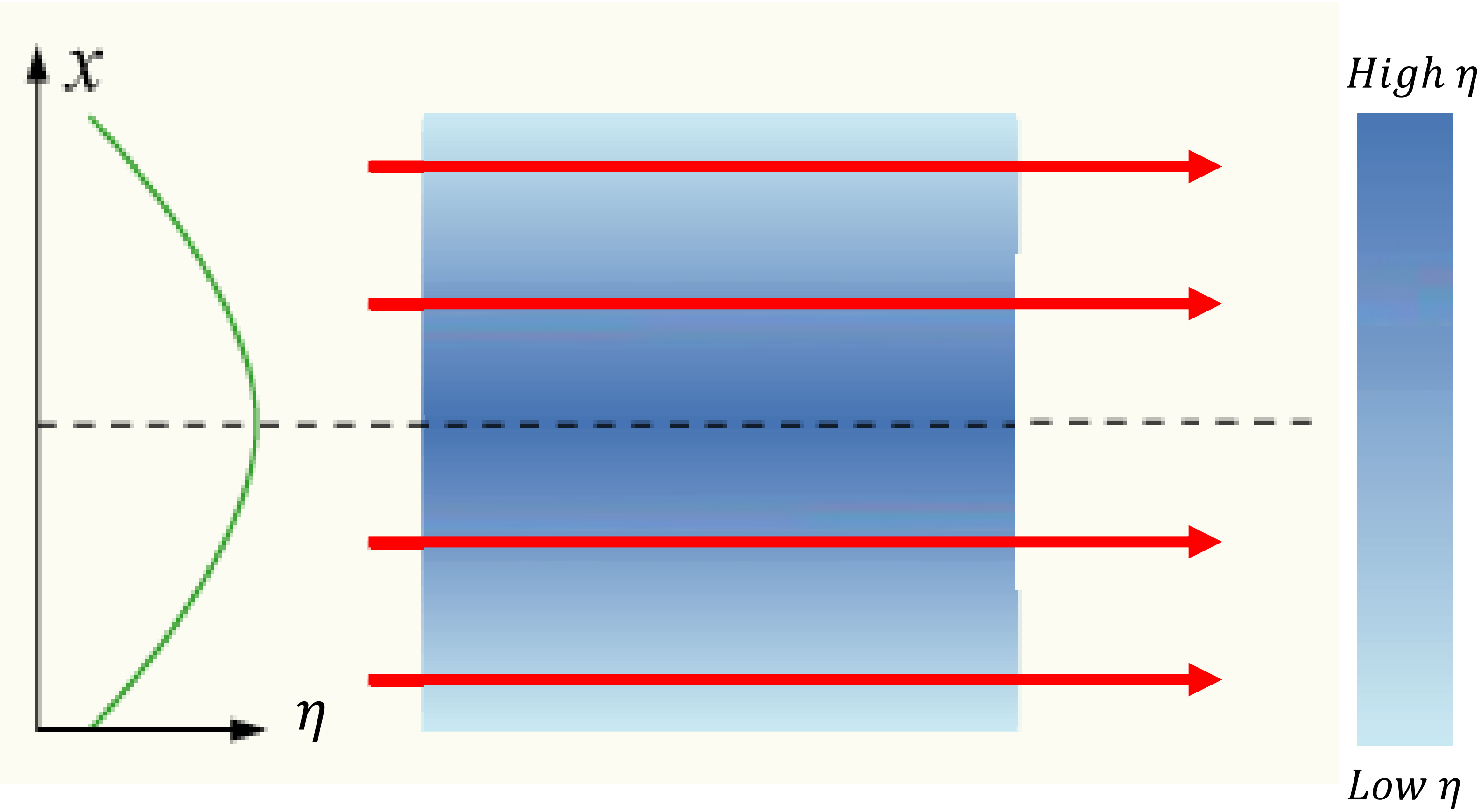
Physics of Waveguides



Index of refraction vs. distance. High on-axis index focuses laser on-axis. This is too much focusing.

Modified from Self-Focusing Wikipedia Article

Physics of Waveguides



This is just right! Diffraction is balanced by focusing.

Modified from Self-Focusing Wikipedia Article

The Solution: Waveguiding

Waveguides work to mitigate diffraction by focusing the light back on-axis.

Method	Index	Threshold	State of Medium
Kerr Self-Focusing	$\eta = \eta_o + \eta_2 I$	$P_{cr} = \alpha \frac{\lambda^2}{4\pi\eta_o\eta_2}$	Not Ionized
Pre-formed Plasma Channel	$\eta = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$	$\Delta n_e = \frac{1}{\pi r_e r_o^2}$	Plasma - Electron Density Gradient
Relativistic Self Focusing	$\eta = \sqrt{1 - \frac{\omega_p^2}{\gamma\omega^2}}$	$P_{cr} \approx 17GW \left(\frac{\omega}{\omega_p}\right)^2$	Plasma – Constant Electron Gradient
Optical Field Ionization (OFI)	$\eta = \sqrt{1 - \frac{\omega_{p,max}^2}{\omega^2}}$	$\Delta\bar{Z} = \frac{1}{\pi n_e r_e r_o^2}$	Ionization Gradient

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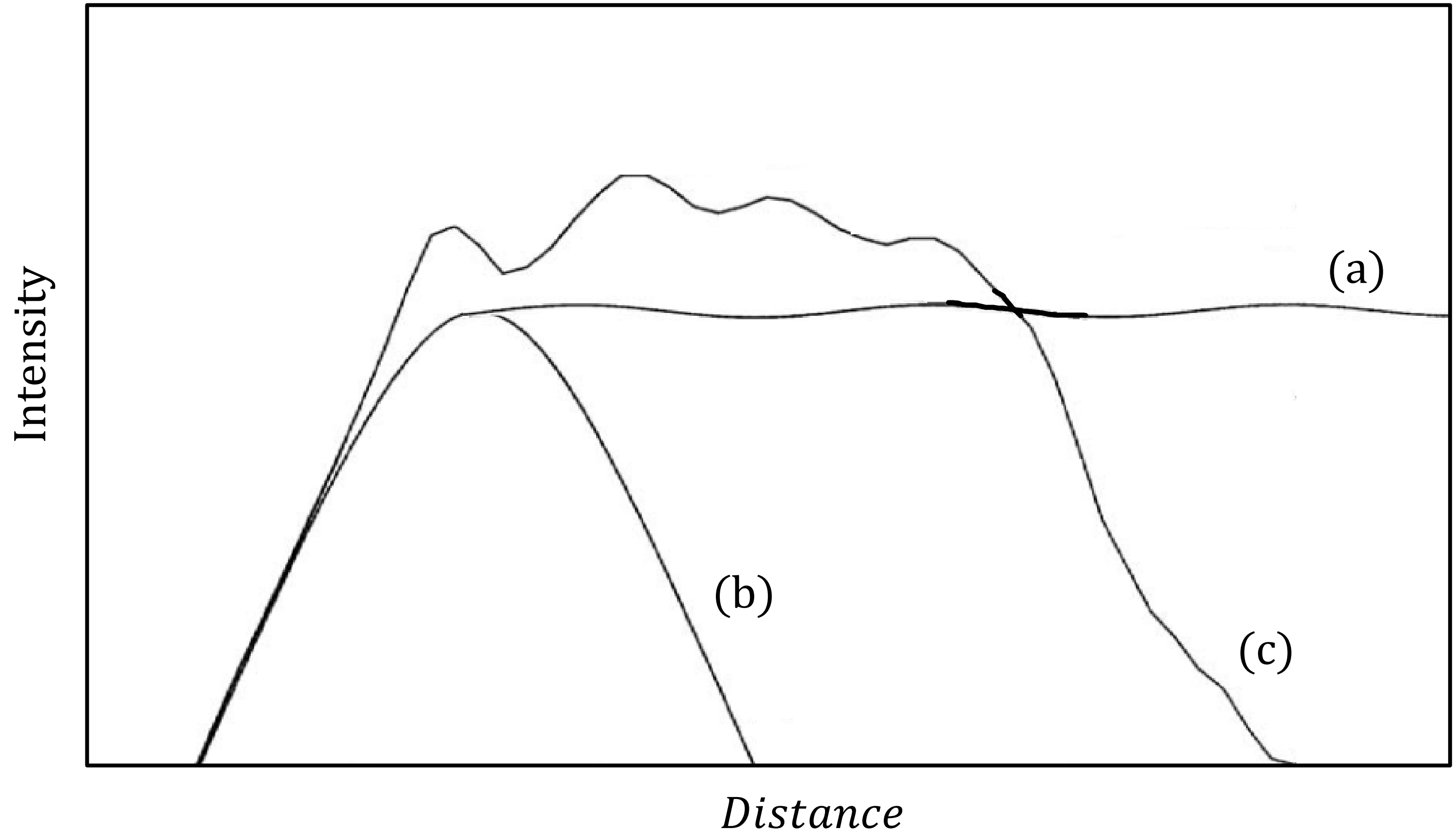
Benefits of Plasma Waveguides



Laser intensity vs propagation distance for (a) channel-guided LWFA, (b) vacuum diffraction, and (c) self-focused LWFA. Arbitrary units.

Modified from Esarey 2009

Methods of Waveguiding

Optical Guiding Method	Max Electron Beam Energy	Channel Formation Pulse Length
Relativistic Self-Guiding	2GeV	N/A
Capillary Discharge	4GeV	N/A
Discharge + Laser Heating	8GeV	~Nanosecond
OFI	5GeV	~Femtosecond

Future work is being done to combine laser heating and OFI in a 2 Bessel method.

Optical Guiding Method	Max Electron Beam Energy	Channel Formation Pulse Length
OFI + 2 Bessel	>10GeV (Simulated)	~Femtosecond

Relativistic Self-Guiding Limits

If $P > P_{cr}$, self focusing is too strong and the wave collapses. Relativistic self-guiding is limited by the relation of the electron density to the critical power and electric field.

$$P_{cr} = 17GW\,\frac{\omega^2}{\omega_p^2} = 17GW\,\frac{n_{crit}}{n_o}$$

The wave breaking equation gives a plasma wave electric field of:

$$E_o = \frac{cm_e\omega_p}{e} \approx 96\sqrt{n_o}\;[V/m]$$

Assume a limit of 10cm on the channel length. To achieve a goal of >10GeV, a density of $n_o \geq 10^{18} cm^{-3}$ is needed.

This limits the critical power to $P_{cr} \approx 30\,TW$

If P_{cr} is substituted into E_o via ω_p, we can see that the wave breaking electric field and critical power are related by

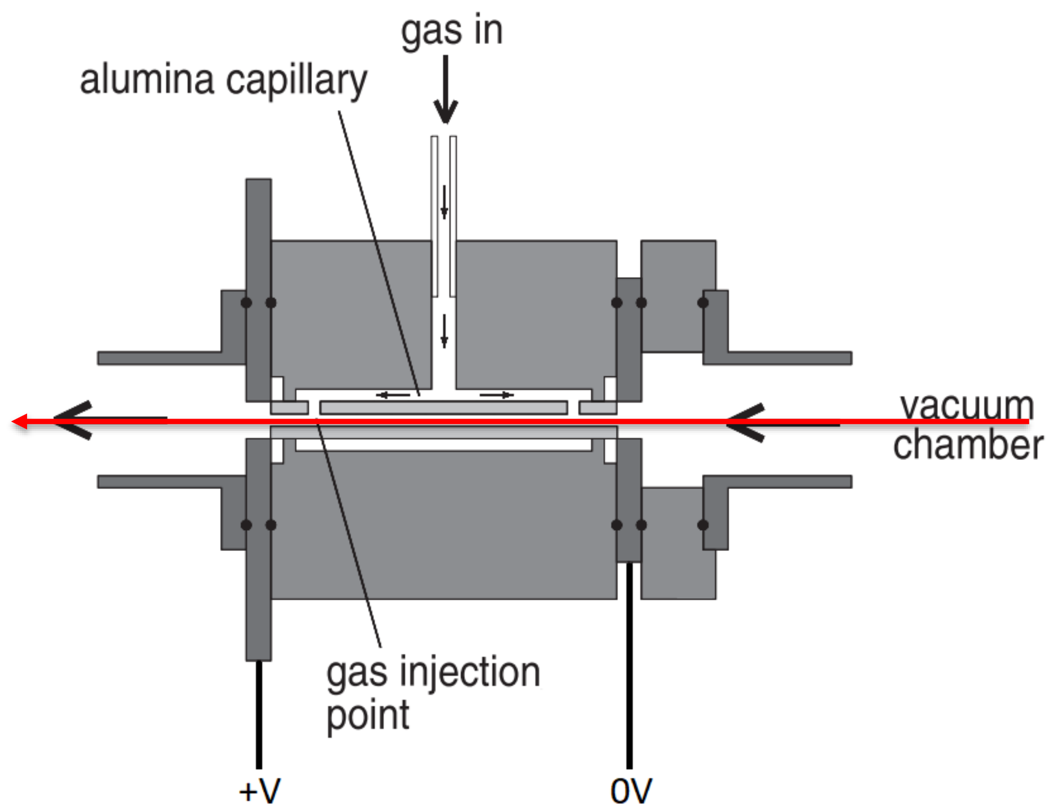
$$E_o \propto P_{cr}^{-1/2}$$

Capillary Discharge

Electrodes discharge, ionizing and heating gas chamber through Ohmic heating.

Channel walls are cooled, causing hot electrons to expand outward.

Creates low density core and high density cladding.



Modified from Spence 2001

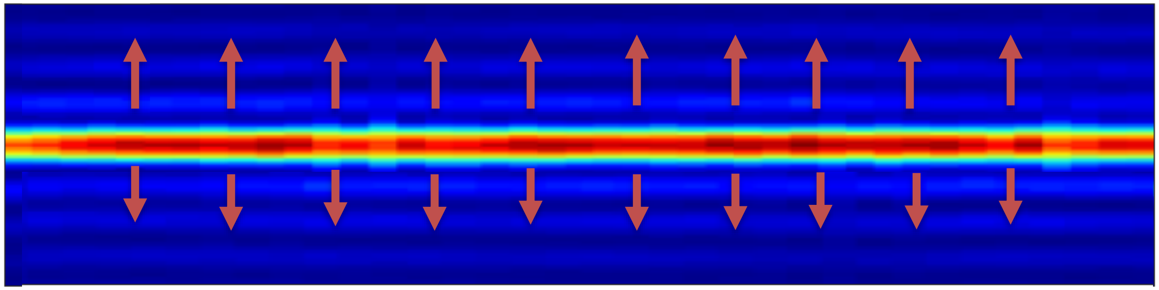
Experimental Results

	Leemans et al. 2006	Leemans et al. 2014	Gonsalves et al. 2019
Electron Energy	1GeV	4.2GeV	8GeV
Channel Length	3.3cm	9cm	20cm
Electron Density	$4.3 * 10^{18} cm^{-3}$	$7 * 10^{17} cm^{-3}$	$2.7 * 10^{17} cm^{-3}$
Laser Power	20 TW	400TW	850TW

Standard Capillary Discharge

Capillary Discharge w. Heater Pulse

Capillary Discharge with Heater Pulse



- A 0.4J, 8ns FWHM pulse follows the capillary discharge [Gonsalves 2020]
- The plasma core is heated through inverse bremsstrahlung.
- This deepens the density gradient, allowing for guiding of higher power lasers (850TW v. 400TW)
- Without the capillary formed channel, the heater pulse can only travel several Rayleigh lengths

Optical Field Ionization (OFI)

This method can produce a gradient in the index by causing the core to reach a higher ionization state.

It is beneficial because it is only dependent on laser intensity, not gas density.

Single pulse channel formation has yielded 5GeV electron beams [Miao 2022]

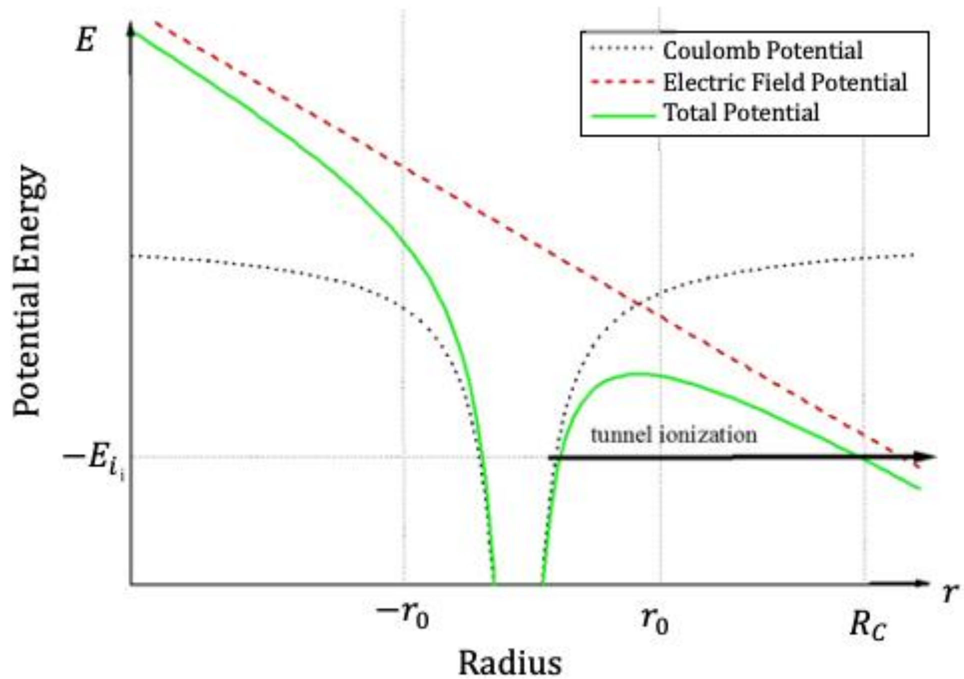


Image from Tunnel Ionization Wikipedia

2 Bessel Method

This combines the advantages of laser heating and OFI.

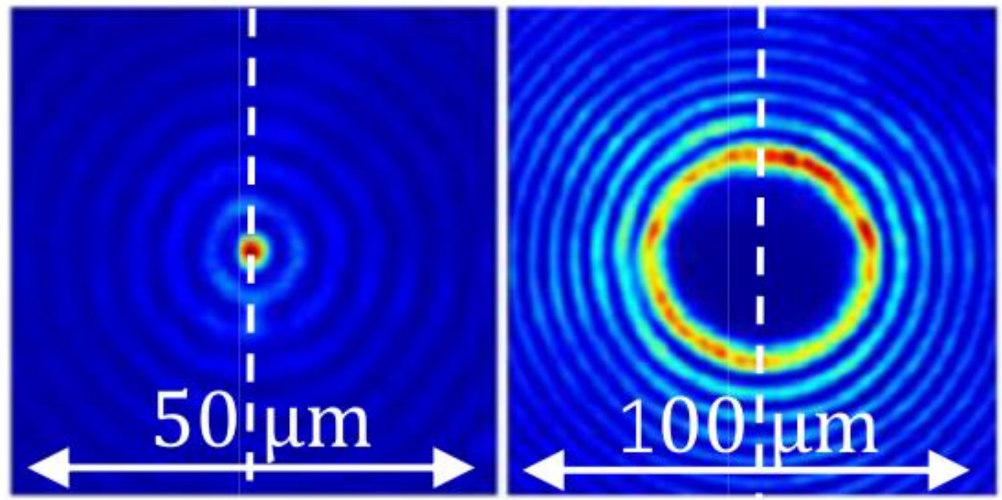
Initial J_0 pulse ionizes the core through OFI.

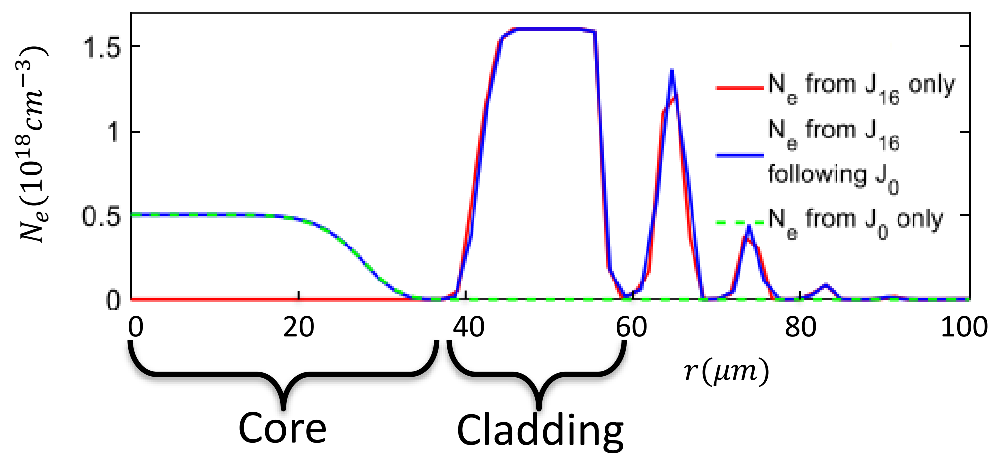
A second J_{16} pulse heats cladding through OFI

Simulations suggest that 2 pulse channel formation will yield >10GeV beams [Miao 2020]

Heating Pulse

2 Bessel Pulse





Conclusion

- LWFA without waveguiding is limited to a few Rayleigh lengths.
- Laser heating is limited by the Rayleigh length.
- Relativistic self-guiding is limited by the critical power.
- Capillary discharge has produced 8GeV electron beams. Further deepening of the plasma channel will allow higher power laser guiding, achieving higher electron beam energies.
- OFI is limited by the propagation distance of the channel forming laser. 5GeV electron beams have been produced. The 2 Bessel approach to channel formation will increase the channel depth, allowing for higher power pulses to be guided, reaching >10GeV beams.

Thank You

Supplemental Material

Waveguiding References

Method	Index	Threshold	Reference
Kerr Self-Focusing	$\eta = \eta_o + \eta_2 I(r)$	$P_{cr} = \alpha \frac{\lambda^2}{4\pi\eta_o\eta_2}$	[Wikipedia]
Pre-formed Plasma Channel	$\eta = \sqrt{1 - \frac{\omega_p(r)^2}{\omega^2}}$	$\Delta n_e = \frac{1}{\pi r_e r_o^2}$	Durfee 1993
Relativistic Self Focusing	$\eta = \sqrt{1 - \frac{\omega_p^2}{\gamma(r)\omega^2}}$	$P_{cr} \approx 17GW \left(\frac{\omega}{\omega_p}\right)^2$	Sun 1987
Optical Field Ionization (OFI)	$\eta = \sqrt{1 - \frac{\omega_{p,max}(r)^2}{\omega^2}}$	$\Delta\bar{Z} = \frac{1}{\pi n_e r_e r_o^2}$	N/A Estimated from density

Methods of Waveguiding References

Optical Guiding Method	Max Electron Beam Energy	Reference
Relativistic Self-Guiding	2GeV	Clayton 2010
Capillary Discharge	4GeV	Leemans 2014, Gonsalves 2015
Discharge + Laser Heating	8GeV	Gonsalves 2019, Gonsalves 2020
OFI	5GeV	Miao 2022

Future work is being done to combine laser heating and OFI in a 2 Bessel method.

Optical Guiding Method	Max Electron Beam Energy	Reference
OFI + 2 Bessel	>10GeV (Simulated)	Miao 2020

Physics of Waveguides

Waveguides need a high index of refraction on-axis. The index of refraction is given as (for underdense plasmas):

$$\eta = 1 - \frac{\omega_p^2}{2\omega^2}$$

Assume that the density of the channel is radially parabolic:

$$n_e(r) = n_0 + \Delta n \frac{r^2}{r_0^2}$$

This makes the index of refraction:

$$\eta(r) = 1 - \frac{\omega_{p0}^2}{2\omega^2}\left(1 + \frac{\Delta n}{n_0}\frac{r^2}{r_0^2}\right)$$

This has a maximum index on-axis.

The laser is focused if the channel depth is:

$$\Delta n = \Delta n_c = (\pi r_e r_0^2)^{-1}$$

Critical Channel Depth

Start with a sharp difference in index of refraction for the channel

$$\eta_{out}$$

$$\eta_{in}$$

This makes Snell's Law

$$\eta_{in}\sin(\theta_c) = \eta_{out}$$

If we define the angle with respect to the boundary, not the normal:

$$\sin(\theta_c) = \sin\left(\theta_c' - \frac{\pi}{2}\right) = \cos(\theta_c{}')$$

For focusing in the channel $\theta_c' = \theta_d$

Taking $\eta \approx 1 - \frac{\omega_p^2}{2\omega^2}$ and the diffraction angle of $\theta_d = 1.22\lambda/2r_o$

$$\frac{1-\omega_{p,out}^2/2\omega^2}{1-\omega_{p,in}^2/2\omega^2} = \cos\left(\frac{1.22\lambda}{2r_0}\right)$$

Taylor expanding $\omega_p^2/\omega^2 \ll 1$ and $\theta_d \ll 1$ gives

$$\left(1-\frac{\omega_{p,out}^2}{2\omega^2}\right)\left(1+\frac{\omega_{p,in}^2}{2\omega^2}\right) = 1 - \frac{1}{2}\left(\frac{1.22\lambda}{2r_o}\right)^2$$

Ignoring higher order terms

$$\frac{\omega_{p,out}^2 - \omega_{p,in}^2}{\omega^2} = \left(\frac{1.22\lambda}{2r_o}\right)^2$$

Taking $\omega_p^2 \propto n_e$

$$\frac{\omega_{p,out}^2}{\omega^2}\left(\frac{\omega_{p,out}^2 - \omega_{p,in}^2}{\omega_{p,out}^2}\right) = \left(\frac{1.22\lambda}{2r_o}\right)^2$$

$$\frac{\Delta n_e}{n_{e,out}} = \frac{\omega^2}{\omega_{p,out}^2}\left(\frac{1.22\lambda}{2r_o}\right)^2$$

Substitute $\omega_p = \sqrt{4\pi r_e c^2 n_e}$ and $\omega = 2\pi c/\lambda$

$$\frac{\Delta n_e}{n_{e,out}} = \frac{4\pi^2 c^2}{4\pi r_e c^2 \lambda^2 n_{e,out}}\left(\frac{1.22\lambda}{2r_o}\right)^2$$

After canceling, this yields

$$\Delta n_e \propto (r_e r_o^2)^{-1}$$

Ionization Gradient

Start with the critical channel depth

$$\Delta \mathrm{n_e} = \frac{1}{\pi r_e r_o^2}$$

The ionization rate can be given by $\bar{Z} = \frac{n_e}{n_T}$ where n_T is the total density. For ionization, this is the density of the ions, so:

$$n_e = \bar{Z} n_i$$
$$\Delta n_e = \Delta \bar{Z} n_i$$

$$\Delta \bar{\mathrm{Z}} = \frac{1}{\pi n_i r_e r_o^2}$$

E-Field

Estimate from Poisson (1D)

$$\frac{dE}{dx} = \frac{\rho}{\epsilon_o}$$

For a plasma behind this E field $\rho = e(Zn_i - n_e)$. In the blowout regime, $n_e = 0$. This implies that the spot is fully ionized, meaning $Zn_i = n_{e,o}$. So, we can say

$$\frac{dE}{dx} = \frac{en_{e,o}}{\epsilon_o}$$

If we assume all the plasma electrons are oscillating with a frequency of ω_p and $k_p = \omega_p/c$, then the integral gives

$$\left(\frac{\omega_p}{c}\right) E = \frac{en_{e,o}}{\epsilon_o}$$

We know $\omega_p^2 = \frac{n_e e^2}{\epsilon_o m_e}$ so

$$\frac{n_e e}{\epsilon_o} = \frac{\omega_p^2 m_e}{e}$$

Substitution gives

$$E_o = \frac{c}{\omega_p} \frac{\omega_p^2 m_e}{e} = \frac{m_e c \omega_p}{e}$$

This is the cold nonrelativistic wave breaking equation

$$E_o \approx 100 \frac{V}{m} n_e^{1/2}$$

The relativistic correction is

$$E_r = \sqrt{2(\gamma - 1)} E_o$$

From Dawson 1959

2GeV Self-Guiding

From C. E. Clayton et al., Phys. Rev. Lett., vol. 105, p. 105003, Sep. 2010.

110 TW laser pulse guided through a 1.3cm gas cell with densities below $1.5 \times 10^{18} cm^{-3}$. This produced 1.4GeV electron beams.

110TW is well over the critical power of 20TW, but the short distance of propagation (10s of Rayleigh lengths) did not give time for catastrophic self focusing.

Leemans - 2006

Laser: 10Hz Ti:Sapph (810nm) 40fs FWHM 40TW

Focus: N=25, $r_s = 25\mu m$

Intensity ~ $10^{18} W\ cm^{-2}$

Channel Size: 190-300 μm

Leemans - 2014

Laser Energy: 16J

Focus: f = 13.5m, $w_o = 52\mu m$

Intensity: $a_o = 1.6$

Channel Size: 500 μm

Gonsalves - 2020

Discharge Current:

450A current with 400ns rise time

Channel density was $2.7 \times 10^{17} cm^{-3}$

Heater pulse:

532nm, 300mJ, 8ns pulse focused down to 84μm spot radius.

Drive Laser:

815nm, 1Hz, 31J, 850TW laser pulse focused down to a 60µm spot radius.

Laser depletion is the main limitation in this case

Miao - 2022

Channel Forming Pulse:

J0 Bessel beam pulse

0.5J, 75fs, 800nm

$$I = 8 \times 10^{15} W/cm^{-2}$$

This is much larger than the OFI threshold of hydrogen ($\sim 10^{14} W/cm^{-2}$)

Drive Pulse:

J0 Bessel beam pulse

15J, 45fs, 800nm

Focused by a f/25 OAP

Density of $3.2 \times 10^{17} cm^{-3}$

Miao – 2020: The 2 Bessel Method

1st Channel Forming Pulse:

J0 Bessel beam pulse

800nm pulse compressed to 50-100fs and focused to 3μm spot radius

2nd Channel Forming Pulse:

J8 or 16 Bessel beam pulse

800nm pulse compressed to 50-100fs and focused to a ~25μm beam radius. Follows J0 beam at a 1-3ns delay

Initial density is $\sim 10^{18} cm^{-3}$

J0 pulse reduces that by a factor of 10

J16 pulse ionizes neutral cladding, creating sharp index boundary.

Miao – 2020: The 2 Bessel Method