Problem Chosen	2022 MCM/ICM Summary	Team Control Number
B		2226129

We create a water and electricity allocation model to optimize the use of scarce water in the Colorado River for five states: Arizona, California, Colorado, New Mexico, and Wyoming. The model optimizes the water allocation for each state and remaining water levels in the reservoirs by maximizing the resource demand fulfilling rates of each state. Considering power generation processes for the two hydroelectric power plants along the river, the Glen Canyon Dam and the Hoover Dam, we maximize electricity demand fulfilling rate by optimizing water allocation. The water and electricity allocations depend on two important groups of variables. The first group of variables are the water supply to five different states, while the second group of variables are the water flow out of two dams respectively. In addition, allocation of water to one of the three usages - residential, industrial, and agricultural purposes - will be determined by our defined constants.

The model takes the form of a non-linear optimization problem, as we introduce the objective function that captures the water and energy demands of all states, with the goal of finding the optimal resource allocation while maintaining the sustainability of water level. This system is solved using the constrained optimization by linear approximation method (COBYLA).

Using the model, we predict that without precipitation and with optimal resource distribution, the Colorado River will support 70% of the demands of all stakeholders for approximately 490 days.

By applying the model to different time-constraints, we find that the optimized results for a shorter period (45 days) would lead to full fulfilling rates for water supply and electricity. For longer period (90, 180, 360 days), our model suggests that the water supply should firstly go to lower basin states instead of upper basin states due to an additional utility brought by the additional electricity generation.

Contents

1 Introduction

Since the operation of the first hydroelectric power plant in the world in 1882 [1], hydroelectric energy has become one of the most popular forms of sustainable energy. As a major component of a hydroelectric facility, a dam restricts the flow of a river to form a reservoir in the upstream and thus accumulates potential energy in the water body. Then, by controlling the dam's spillway gates, such potential energy can be converted to kinetic energy that turns the turbine blades of hydroelectric generators to generate electricity. In addition to electricity generation, dams and reservoirs are also widely used for flood control, water allocation, and recreational purposes. However, the drastically declining water level in rivers worldwide as a result of climate change directly impedes sufficient water supply and electricity generation from hydroelectric dams.

1.1 Geographical Background

In this project, we focus on electricity generation from two hydroelectric dams, the Glen Canyon Dam (GCD) and the Hoover Dam, along with water allocation from their respective reservoirs, Lake Powell and Lake Mead. Both dams are located on the Colorado River, providing resources to states along the river. In particular, the GCD divides the Colorado River Basin into the Upper Basin (Arizona, Colorado, New Mexico, Utah, and Wyoming) and the Lower Basin (Arizona, California, Nevada, New Mexico, and Utah) [2]. In our model, we only consider 5 states: Arizona (AZ), California (CA), Wyoming (WY), New Mexico (NM), and Colorado (CO).

2 Assumptions

We introduce the following assumptions:

- **Colorado, New Mexico, and Wyoming receive water only from the Glen Canyon Dam; Arizona and California only from the Hoover Dam.** Considering the flow direction of the Colorado River and cost of water transportation, we restrict water supply from the GCD only to the Upper Basin states (CO, WY, NM) and water supply from the Hoover Dam only to the Lower Basin states (AZ, CA). We do not apply this assumption to electricity assumption: both dams are electricity sources to all five states.

- **No water distribution between the GCD and the Hoover Dam.** We aim to maximize electricity generation by maximizing water level in the Hoover Dam, which has a larger capacity than the GCD. Therefore, water supplied to the Upper Basin states (CO, WY, NM) is supplied by Lake Powell, while water to the Lower basin states (AZ, CA) and Mexico is from the outflow of the Hoover Dam (after generating electricity).

- **The outflow from the GCD constitutes the sole source for the Colorado River beyond the dam, contributing 97% of water inflow to Lake Mead [3].** With this consideration, the water distribution from GCD drastically affects the water level in Lake Mead.

- **There is no surplus electricity or water supplied.** In this case, the demand from each area does not exceed the supply. This assumption is reasonable given the drought condition. Therefore, we do not consider the storage and re-usage of electricity and water.

- **The lakes satisfy the Bernoulli's principle (constant density, steady flow, and no friction) [4].** According to this principle, the flow rate at the dam only depends on the pool elevation or, in other words, the water pressure.
- **All water discharged is used in electricity generation, and this process does not reduce the amount of water.** Electricity generated can be computed from water inflow to the dam.
- **Both reservoirs take the shape of a cuboid.** We make this assumption for the ease of physical formula derivations.
- **Minimum flow rates of dams are measured as the averages of the flow rates at night and during the day.** We disregard the difference in flow rates at different time.
- **Assume the change in water level within each time unit is negligible.** For example, the water level stays constant in each day.
- **Evaporation rate is constant along the river at 3.5%** We account for evaporation of the water at 3.5% [5] and assume it to be constant.

3 Model

A non-linear optimization problem is used to model resource allocations in this project. In the following subsections, the model will be explained in detail.

3.1 Variables and Parameters

We introduce the variables and parameters in our model.

Variable	**Definition**
E_G	Total electricity generated by GCD per day (MWh)
E_H	Total electricity generated by Hoover Dam per day (MWh)
W_G	Total water supplied by GCD (m^3)
W_H	Total water supplied by Hoover Dam (m^3)
O_G	Amount of outflow from GCD (m^3)
O_H	Amount of outflow from Hoover Dam (m^3)

The addition of the subscript $i \in \{1, 2, 3, 4, 5\}$ indicates the amount of resources (electricity and water) allocated to the states Colorado ($i = 1$), Wyoming ($i = 2$), New Mexico ($i = 3$), Arizona ($i = 4$), California ($i = 5$). For example, the total electricity supplied to New Mexico by Hoover Dam is expressed as $E_{H,3}$. Considering our first assumption of the geographical division of water supply (Section 2), we have the following relationships:

$$E_G = \sum_{i=1}^{5} E_{G,i}, \quad E_H = \sum_{i=1}^{5} E_{H,i} \quad W_G = \sum_{i=1}^{3} W_{G,i}, \quad W_H = \sum_{i=4}^{5} W_{H,i}$$

We introduce the following parameters to our model. Please see 7.1.1 for exact values.

Parameter	Definition
D_W	Total demand for water (m^3)
D_{AW}	Total agricultural demand for water (m^3)
D_{IW}	Total industrial demand for water (m^3)
D_{RW}	Total residential demand for water (m^3)
D_M	Amount of water for Mexico
D_E	Total electricity demand (MWh)
H_G	Remaining water level in GCD after one time unit (m)
H_H	Remaining water level in Hoover after one time unit (m)
I_G	Inflow amount to Lake Powell
α	Weight for water fulfillment
β	Weight for electricity fulfillment
γ	Weight for remaining water level

Similarly, we use the subscript $i \in \{1, 2, 3, 4, 5\}$ to denote water or electricity demand for each state. For example, the total industrial demand for water in Arizona is $D_{IW,4}$. Therefore, we have the following relationships:

$$D_{AW} = \sum_{i=1}^{5} D_{AW,i}, \quad D_{IW} = \sum_{i=1}^{5} D_{IW,i} \quad D_{RW} = \sum_{i=1}^{5} D_{RW,i}$$

and

$$D_W = \sum_{i=1}^{5} D_{W,i}, \quad D_E = \sum_{i=1}^{5} D_{E,i}$$

3.2 Definitions and Criteria

To optimize water allocation, we use demand fulfilling rate as an important measure. Given fixed resource demand for each state, (D_i) we determine resource allocations (S_i) that maximize fulfilling rates. To express mathematically, we have

$$c \sum_{i=1}^{5} \sqrt{\frac{S_i}{D_i}}$$

The weight constant, c, represents the significance of each demand fulfillment to the overall objective generation. This expression is elaborated in Section 3.3.

3.3 The Model

In our model, we aim to find the optimal water distribution that maximizes not only the demand fulfilling rates of both water and electricity for the five states but also the remaining water levels of the two reservoirs. This design lead to a non-linear optimization problem:

$$\max_{W_{G,i}, W_{H,i}, O_G, O_H} \alpha \left(\sum_{i=1}^{3} \sqrt{\frac{W_{G,i}}{D_{W,i}}} + \sum_{i=4}^{5} \sqrt{\frac{W_{G,i}}{D_{W,i}}} \right) + \beta \sum_{i=1}^{5} \sqrt{\frac{E_{G,i} + E_{H,i}}{D_{E,i}}} + \gamma \left(\frac{I_G - O_G}{\Lambda_G} + \frac{O_G/0.97 - O_H}{\Lambda_H} \right)$$

With the intuition introduced in the last section (Section 3.2), the first two terms in the objective function account for the demand fulfilling rates for water and electricity for each state. The third term accounts for the remaining water levels after one time unit. We introduce these terms in a radical form, so that the demand fulfillment for states with limited resources (low demand fulfilling rate) contributes more utility than fulfilling the demand of states with abundant resource (high demand fulfilling rate). Using the assumption that outflow of the GCD constitutes 97% of water inflow at the Hoover Dam (Section 2), we arrived at the expression above. The weights, α, β, and γ, account for the significances of water demand fulfillment, electricity fulfillment, and remaining water levels, respectively. In our model, we consider the importance of water demand over electricity and remaining water levels. Please refer to Section 7.1 for exact values of the weights.

3.4 Constraints

Based on assumptions and physical principles, we introduce the following constraints to our model.

3.4.1 Non-negative Variables

Since water and electricity distributions and water levels take non-negative values, we have

$$W_{G,i}, W_{H,i}, E_{G,i}, E_{H,i}, O_G, O_H \geq 0$$

3.4.2 Supply No More Than Demand

Since we assume the supply of water or electricity for each state does not exceed the state's demand, we apply the following constraint to our model:

$$\begin{aligned} D_{E,i} - (E_{H,i} + E_{G,i}) &\geq 0, \quad \forall i \in \{1,2,3,4,5\} \\ D_{AW,i} + D_{IW,i} + D_{RW,i} - W_{G,i} &\geq 0, \quad \forall i \in \{1,2,3\} \\ D_{AW,i} + D_{IW,i} + D_{RW,i} - W_{H,i} &\geq 0, \quad \forall i \in \{4,5\} \end{aligned}$$

3.4.3 Supply No More Than Reservoir Volume

We constrain the total water supplied to not exceed reservoir volume. Since we assume that water to CO, WY, and NM is from Lake Powell, and water to AZ and CA is from the outflow of the Hoover Dam (Section 2), this constraint only applies to water supplied by Lake Powell:

$$V_G - \sum_{i=1}^{3} W_{G,i} \geq 0,$$

where $V_G = 3.000 \times 10^{10}$ m^3.

3.4.4 Mexico's Rights to River

Since the Colorado River debouches into the ocean through Mexico, Mexico has legal rights to access a portion of the water resource.

> The United States and Mexico entered into a treaty on February 3, 1944, which guarantees Mexico 1,500,000 acre-feet of Colorado River water annually. This entitlement is subject to increase or decrease under circumstances provided for in the treaty [2].

Therefore, we include the following constraint to account for sufficient water supply for Mexico after allocations to AZ and CA:

$$O_H - W_{H,4} - W_{H,5} - D_M \geq 0,$$

where $D_M = 1,850,220,000 \text{ m}^3$, and its value is adjusted for different time units.

3.4.5 Outflow Rates Do Not Exceed Maxima

Since water outflow rate from either dam cannot exceed the built-in maximum amount, the total outflow amounts cannot exceed the theoretical maxima. We introduce the following constraints:

$$O_G - Q_{G\max} t_{open,G} \geq 0$$
$$O_H - Q_{H\max} t_{open,H} \geq 0,$$

where $Q_{G\max} = 708 \text{ m}^3/\text{s}$, $Q_{H\max} = 906 \text{ m}^3/\text{s}$ are maximum flow rates for the GCD and the Hoover Dam respectively, and t_{open} accounts for time unit adjustment.

3.5 Electricity Generation Approximation

In this section, we derive from physical formulas the relationship between electricity generated by dams and water stored in the reservoirs.

3.5.1 Variables and Parameters

We introduce the following variables used in the derivation of the physical formulas.

Variable	Definition
h	Absolute value of difference between water levels on both sides of the dam (m)
v	Velocity of water moving right before passing generators (m/s)
v'	Velocity of water moving after passing generators (m/s)
t_{open}	Duration of time water flowing through turbines (h)
A	Area of gates leading to the generators (m^2)
P	Electricity generating power (W)

Here, we use the subscript $j \in \{G, H\}$ (omitted in table for concision) for every variable. For example, h_G means the absolute value of the difference between water levels on both sides at Glen Canyon Dam, and h_H means the absolute value of the difference between water levels on both sides at Hoover Dam.

In the table below, we introduce the parameters used in the derivation of physical formulas.

Parameter	Value	Definition
$h_{G\max}$	174	Maximum water level difference between at GCD (m) [6]
$h_{H\max}$	180	Maximum water level difference at Hoover Dam (m) [7]
$h_{G\min}$	108	Minimum water level difference to generate electricity at GCD (m) [8]
$h_{H\min}$	93	Minimum water level difference to generate electricity at Hoover Dam (m) [7]
h_{Gdead}	78	Minimum water level difference for water to be released at GCD (m)
h_{Hdead}	77	Minimum water level difference for water to be released at Hoover Dam (m) [7]
$A_{G\max}$	12.12	Maximum area of gates leading to the generators at GCD (m^2)
$A_{H\max}$	15.25	Maximum area of gates leading to the generators at Hoover Dam (m^2)
η_G	0.9	Kinetic to potential energy transfer efficiency at GCD [9]
η_H	0.9	Kinetic to potential energy transfer efficiency at Hoover Dam [9]
$Q_{G\max}$	708	Maximum flow for power generation at GCD (m^3/s) [10]
$Q_{H\max}$	906	Maximum flow for power generation at Hoover Dam (m^3/s) [11]
C_G	1.32×10^9	Maximum capacity of GCD (W) [6]
C_H	2.074×10^9	Maximum capacity of Hoover Dam (W) [11]
Λ_G	6.531×10^8	Surface area of Lake Powell (m^2) [12]
Λ_H	6.37×10^8	Surface area of Lake Mead (m^2) [11]
g	9.8	Gravitational acceleration on Earth (m^2/s)
ρ	1000	Density of water (m^3/kg)

Please refer to Section 7.2 for the calculation of some of the variables.

3.5.2 Electricity Generation Formula

Using these variables and parameters, we measure the power of a hydropower plant as

$$P = \eta\rho Q g h$$

Then, the amount of electricity produced by the hydroelectric power plant is given by

$$E = Pt_{open} = \eta\rho A(2g^3h^3)^{1/2}t_{open}$$

The step-by-step derivation is included in Section 7.2.

4 Results

The optimization problem is solved using the constrained optimization by linear approximation (COBYLA) method. We use this method to optimize the allocation of water for different states and the generation of electricity under certain days without water inflow. Our assumption includes that the fulfilling rate of water to each state is at least 70%, which could not be satisfied after reaching 490 days. Therefore, we analyzed the cases for 45, 90, 180, and 360 days for optimal allocation strategies.

4.1 Water Supply and Allocation Demand

First, we aim to find the optimal water allocation. When the simulation only has a time-constraint of 45 days, we would see a full fulfilling rate for water demand of each state and for the electricity demand (Figure 1).

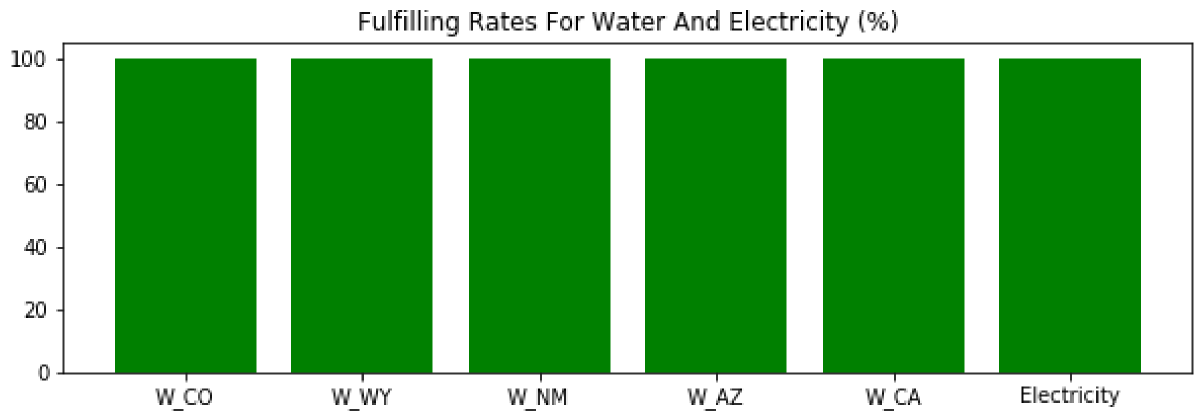

Figure 1: Water and electricity allocations and fulfilling rates for 45-day optimization

In this case, there are very slight changes in water levels of the two dams (Figure 2). We observe that the height for Glen Canyon Dam decreases faster than the height for Hoover Dam, as we assume there is no water inflow to the Glen Canyon Dam. In addition, the water level changes in a linear relationship over time, as we assume the reservoirs take cuboid geometry (Section 2).

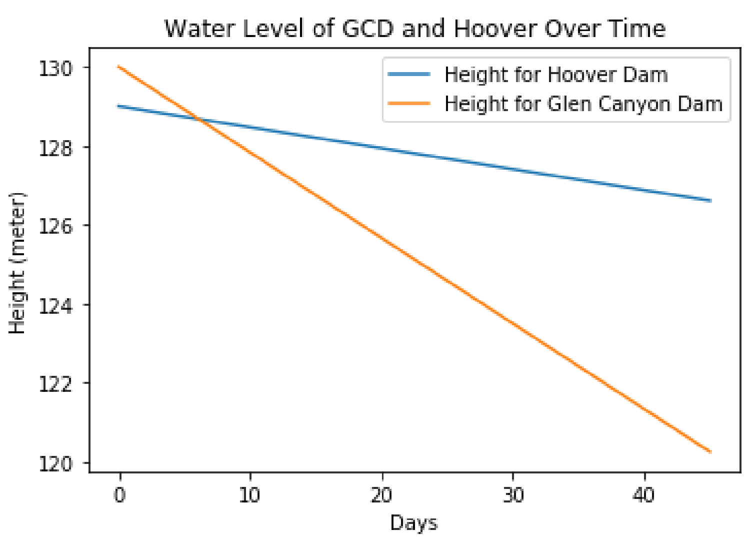

Figure 2: Water levels over 45-day optimization

Since the fulfilling rate for each state is 100% at this point, the pie chart below shows the allocation of water in the optimal case, when all the water demands are satisfied. California, with its largest population among all states in United States, demands the most water, while states with smaller populations, like Wyoming, also demand a large portion of water for their agricultural and industrial usages.

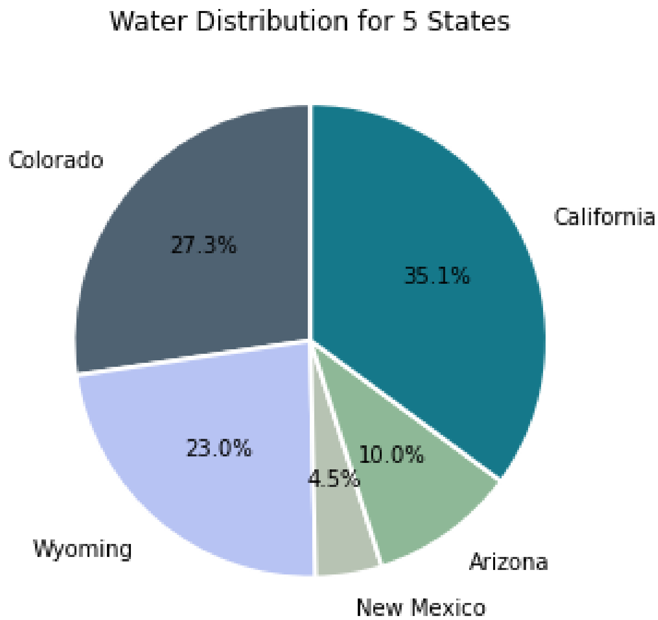

Figure 3: Water distribution for 5 states over 45-day optimization

In the case of a longer time interval, the fulfilling rates for states start to vary. Figure 4 shows the optimization results from a 90-day simulation. It is obvious that the fulfilling rate to Wyoming drastically drops, as the population weighting to each of the states start to affect the results.

This is an illustration of the functionality of our weights. The higher is the demand, the less rewarding it is to supply the water, since it would be harder to increase the fulfilling rate. According to this logic, states with least demand should get fulfilled first, which does not comply with the truth. By multiplying the population weighting, we are correcting this bias. For states with larger population, they should have higher demands, making it harder to fulfill. At the same time, the rewards to fulfill the demands from these states are also higher.

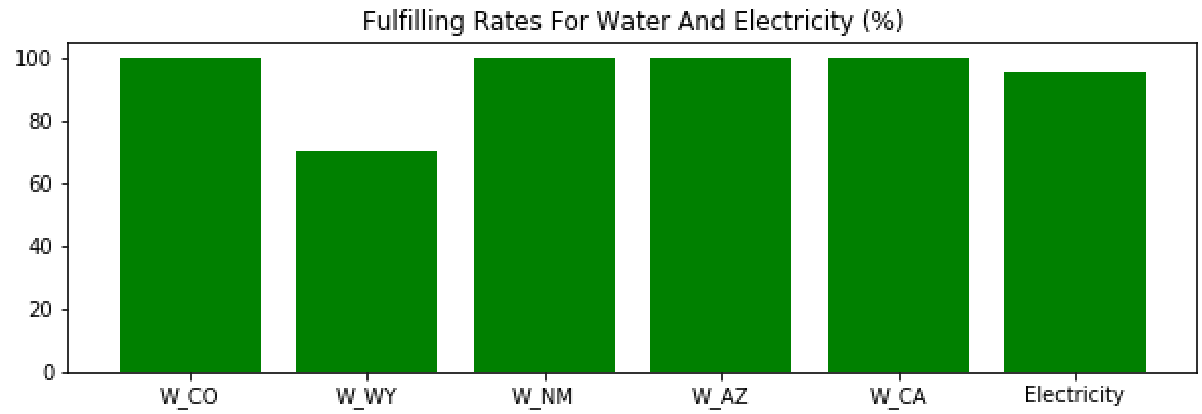

Figure 4: Fulfilling rate for 90-day optimization

The water level drops in a similar way as what we have observed in a 45-day case. Again, the height for Glen Canyon Dam decreases faster than that of Hoover Dam (Figure 5). We reward the conservation of water level in our objective function, but that of the Hoover Dam provides us with more utility, for the water supplied from the Hoover Dam to CA and AZ is also used to generate electricity.

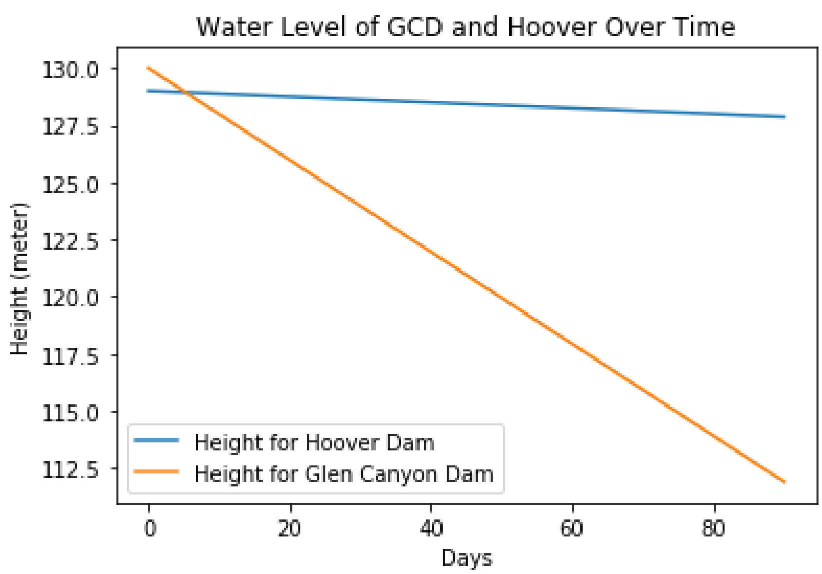

Figure 5: Water levels over 90-day optimization

In the 180-day case, one interesting situation we observe is that the states on the upper basin hit the 70% lower bound of fulfilling rates first (Figure 6). For this phenomenon, one explanation is that the water flows to lower basins states has to pass through Hoover Dam, which produces electricity and thus provides more utility to increase the objective function.

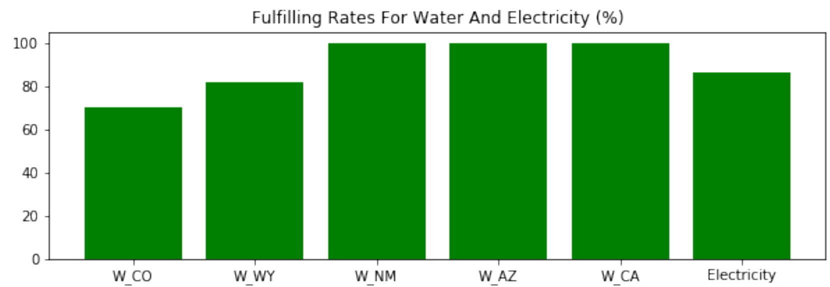

Figure 6: Fulfilling rate for 180-day optimization

In the 360-day case, the fulfilling rate for Wyoming (Figure 7) also hits the lower bound (70%) after Colorado hitting the lower bound in 180 days, while the fulfilling rate for New Mexico, Arizona, and California are still above the bound. This result agrees with our prediction that all the lower bounds will be hit in approximately 490 days.

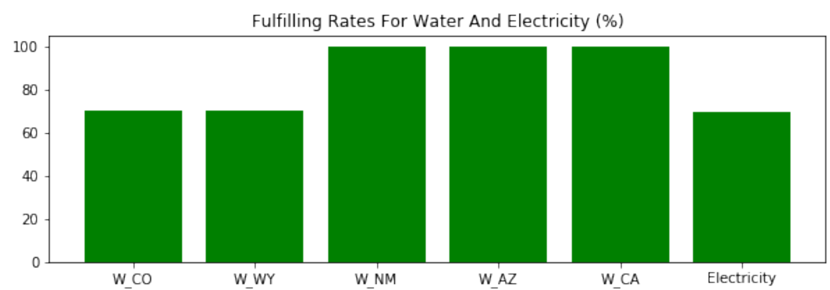

Figure 7: Fulfilling rate for 360-day optimization

4.2 Water Sustainability

We only set the demand fulfilling rate minimum for water supply and ignore that for electricity because hydroelectric power only constitutes an extremely small portion of total energy supply. In addition, we assume that there is some amount of energy generated from the water supply process.

Since all water demands are fixed, our final result indicates that the amount of water to supply every year is equivalent to sum of water demand from each state and the water demand from Mexico. Meanwhile, we also considered the water evaporation of 3.5%, which means that all demand should be multiplied by $\frac{1}{0.97}$.

In the case that there is not enough water to meet all demands, our allocation function acts as a decent method to balance the demands of different states by adding weights to the height level in both dams and parameters like population.

4.3 Competing Interest Resolution

Competing interests among different states have been incorporated into our optimization model. To consider the competing interest among different usages of water, we consider the population to be the most important part in solving the conflicts. We use $P_{w,i} = \frac{P_i}{\sum_{i=1}^{5} P_i}$ as the population weight, with P_i symbolizing the population of each state.

$$F_W = \alpha \left(\sum_{i=1}^{3} \sqrt{\frac{W_{G,i}}{D_{W,i}}} P_{w,i} + \sum_{i=4}^{5} \sqrt{\frac{W_{H,i}}{D_{W,i}}} P_{w,i} \right)$$

Here, we introduce the objective function for water demand, F_W. Our demand is not only for water, so we similarly have the objective function for electricity demand:

$$F_E = \beta \sum_{i=1}^{5} \sqrt{\frac{E_{G,i} + E_{H,i}}{D_{E,i}}} P_{w,i}$$

Before we solve our optimization model, we have the last component of our equation: the weighted height rewarding for dams' attempt to maintain the water level.

$$F_H = \gamma \left(\frac{I_G - O_G}{\Lambda_G} + \frac{O_G/0.97 - O_H}{\Lambda_H} \right)$$

Now, we need to optimize for the following equation to obtain our desired solution.

$$\max_{W_{G,i}, W_{H,i}, O_G, O_H} F_W + F_E + F_H$$

4.4 Model Scalability

We have already incorporated the population as a weighting factor to each state, which provides the convenience of future adjustment in response to population change in each states. To accommodate to the economic changes, we would expect the industrial water demand to increase proportionately, which can be described as the equation below:

$$D_{W,i,t+1} = D_{W,i,t} p_d (1 + e_g)$$

Where p_d is the percentage of demand from industrial usage, and e_g is the economy growth rate.

To consider proportion of renewable energy increases or additional water and energy conservation implemented, the model can be adapted by changing the demand for water, D_E, and the demand for electricity, D_E.

In addition, our model only accounts for the situation of evaporation by considering the evaporation rate of 3.5% per year. We assume the evaporation happening in the two lake, therefore we decreases the water volume accordingly with frequency of once per day.

Observing our fulfilling rate equation, we defined the term "not meeting demand" when we detect a significant drop in the fulfilling rate's value which suggests the shortage of water. Theoretically, this associates with our inability to meet the demand from the five states.

5 Conclusions

In this section, we discuss the model's strengths and weakness and propose directions for future improvements.

5.1 Strengths and Weaknesses

The model has the following strengths:

- The model is scalable so that resource allocation over different time frames towards a variety of categories can be easily made. Our model is feasible to deal with more variables - for example, different usages for electricity, as long as we have more constraints for these new variables.
- The model is adaptable that it works for different dams with no technical adjustment needed. The only change is the dam power performance and the reservoir's dimension. All hydraulic power plants operate in a similar manner, so our model is reusable in most cases.
- The model is competent, since it considers the the demand and supply issue for all the states at once. Our model does not target each state individually, but instead we optimize the supply for the five states.

The model also has the following weaknesses:

- The model is not yet self-generated, because it needs human assistance along the procedure. In other words, the model has not yet included all the possibilities, so there is room for errors. Some cases did not meet the mathematical requirements, so we had to reconsider it.
- The model's result will differ from the actual result. Because of numerous assumptions, the model simplifies the complexity of the issue which increases the margin of errors. For example, the volume of the reservoir will not be a perfect box, so by calculating the reservoir's volume as such will give us an answer that is not too precise.
- The model is not flexible in term of options. It will provide one option without recommending other options that similarly optimize the issue. There will definitely different ways to obtain the same result, and they have their own pros and cons. Thus, not knowing any alternative options, we may waste our resources unnoticeably.

5.2 Future Projections

For future improvement, we want to improve some aspects of our model so that it will be more practical and useful.

- We want to include the presence of friction and other potential sources of water loss in the model. With extensive dams like the Glen Canyon and Hoover Dams, a seemingly trivial coefficient of friction will lead to an enormous amount of energy loss.
- To increase the model's applicability, we should think of the demand functions as functions with diminishing returns. The values of water and electricity to each state are not static, their values will increase depending on how much the state needs.
- We want to investigate more in the prediction of demands. Our model will vary, based on the changes in demands of each state. Thus, we need to forecast the demands accurately.

6 Article for *Drought and Thirst*

Build To Save: The Colorado River Is Drying. What Now?

Along the free-spirited flow of the Colorado River sit the nation's two biggest hydroelectric dams, the Glen Canyon Dam and the Hoover Dam. Fueled by Lake Powell and Lake Mead, these dams generate over 9 billion kilowatt-hours of electricity annually, while the reservoirs supply agricultural, industrial, and residential water to surrounding states [14][15]. The Colorado River nurtures over 40 million people, but it is draining [13].

Figure 8: Historical water levels of Lake Mead. Photo from [13].

As of January, 2022, the water level of Lake Mead reaches 1067.09 ft., in contrast to the 1134.18 ft. (a 67.09 feet drop) in Jan, 2012, and 1177.25 ft. in Jan, 1992 (a 110.16 ft. drop), yet the trend shows no sign of slowing down [16]. The drying Colorado River is yet another victim of global warming. As Arctic sea ice and glaciers melt with a startling rate, more and more of the Earth's dark surface become exposed to radiation from the Sun. As a result, the heat is absorbed rather than being reflected, leading to a warmer surface and thus more melting ice. This self-reinforcing phenomenon is know as the Albedo Effect. Wouldn't this increase of snow water from the Rocky Mountains, origin of the Colorado River, and precipitation lead to a surge in the river flow? Unfortunately, the thermally induced evapotranspiration of the river due to a warmer climate greatly exceeds the addition of freshwater. In fact, every Celsius-degree increase in temperature reduces the Colorado River's annual mean discharge by 9.3% [17].

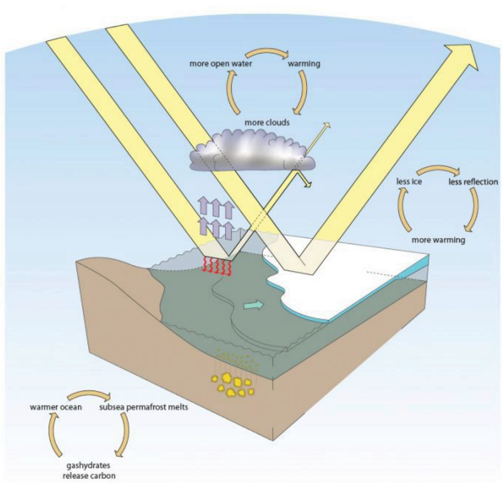

Figure 9: Albedo effect on sea ice. Picture from [18].

On Aug 16, 2021, in response to the megadrought that has swept the Southwestern United States for the past two years, the Bureau of Reclamation declared for the first time in history a water shortage on the Colorado River, implementing mandatory water cuts in Arizona, Nevada, and Mexico [19]. These restrictions will cause agonizing shifts in the status quo of the affected states' agricultural and industrial practices: farmers will need to fallow lands and grow fewer water-demanding crops, while factories will adopt water-saving production mechanisms. This declaration can be seen as a rectification of the Colorado River Compact, 1922, which allocates fixed amounts of water supply to Arizona, California, Colorado, Nevada, New Mexico, Utah, and Wyoming [20]. With a drying Colorado River, the water allocations projected in the compact are inherently flawed, for the allocated amounts far exceed the total amount of water present in the river. This century-old policy is now challenged by a declining system, calling for new strategies to accommodate water demand and power generation.

Recently, a team of undergraduate researchers explored alternative water and electricity allocation plans in response to the drying Colorado River. Their results suggest that in the extreme case of this megadrought (i.e. no precipitation of any form), it will only take 490 days for the Colorado River to be too depleted to meet at least 70% of the states' demand for water and electricity. This startling result makes it imperative to develop appropriate infrastructure policies. In fact, California has already taken measure to slow the drying of rivers by taking up innovative infrastructure plans. In 2015, 20,000 polyethylene balls were released into the Los Angeles Reservoir (Figure 10) to prevent 300 million gallons of water from evaporating annually by absorbing sunlight from the water surface [21].

Figure 10: Plastic balls released to prevent water evaporation from sunlight. Photo from [21].

Moreover, the ideal re-allocation of water suggested by the research team also calls for a drastic reduction in agricultural water usage. Considering the different demands for water and electricity for five Southwestern states (CO, WY, NM, AZ, CA), the team allocates 4.9% and 10.7% of the total water distributed to New Mexico and Arizona respectively. Like most states in the west, agriculture water usage makes up for the majority of the states' total water allocations (for NM and AZ, the numbers are 83.99% 76.98% respectively [22]). Therefore, in the recent decades, the U.S. Department of Energy has been pushing the modernization of irrigation infrastructures in hope to reduce energy and water demand from agriculture. For example, "the installation of in-conduit hydropower, coupled with pressurized piping can reduce evaporative water loss from open conduits, reduce energy use, and restore environmental habitat" [23]. This system projects to harness energy generated from water transportation towards a more sustainable water withdrawal practice. Most of these innovative infrastructure projects come with a high cost, but the imminent water depletion crisis of not only the Colorado River System but also hundreds of rivers across the world propels policymakers to use all available measures to cut water, now.

7 Appendix

7.1 Data for Water and Electricity Demand

7.1.1 Demand for Water

In this report, we consider the demand for water from every state to be equal to the amount of surface water used in each state reported in 2015. Since we are optimizing the allocation of water in the Colorado River, it will not be reasonable to include the use of groundwater in this case. Although the data from 2015 is outdated, we did not find a more detailed and reliable source published these recent years. We established a table that reflects the demand for residential, agricultural, and industrial uses of each state, and the table has all values in unit of million gallons per day:

State	D_{RW}	D_{AW}	D_{IW}	**Total**
Colorado	751.33	7939.91	99.11	8800
Wyoming	99.94	7122.73	135.76	7400
New Mexico	144.29	1221.42	75.32	1460
Arizona	659.08	2478.75	40.07	3220
California	2070.49	7811.91	281.24	11300

To calculate the water demand for just surface water, we assume that both groundwater and surface water will be allocated to different needs with the same ratios to the total water. For example, if residential use accounts for 30% of the total water, 30% of groundwater and 30% of surface water will be used for residential purposes.

State	**Residential**	**Agricultural**	**Industrial**	**Total**
Colorado	0.0854	0.9023	0.0113	1
Wyoming	0.0135	0.9625	0.0183	1
New Mexico	0.0988	0.8366	0.0516	1
Arizona	0.2047	0.7698	0.0124	1
California	0.1832	0.6913	0.0249	1

We also have the table in term of ratio to the total use of water in each state. This can be calculated as $\frac{D_{iW}}{Total_i}$, and we use these values as the weights to better allocate water for each purpose.

7.1.2 Demand for Electricity

Hydroelectric power made up about 7.3% of the total electricity consumed in the United States in 2020 [24]. Thus, it is reasonable to think that the predicted demands for hydroelectric power in the five states - Colorado, Wyoming, New Mexico, Arizona, and California - are equal to 7.3% of the total retail sales in 2020. Therefore, we have the below table whose values are in megawatt-hours:

State	**Total Retail Sales in 2020**[25]	**Predicted Demand**
Colorado	56050264	4091668
Wyoming	15331018	1119164
New Mexico	24777155	1808732
Arizona	81960074	5983085
California	250174672	18262751

7.2 Electricity Generation Formula Derivation

To provide a better idea how we arrive at the values for $A_{G\,\max}$ and $A_{H\,\max}$, we will derive a general formula for $A_{\max}$, the maximum area of the gates leading to the generators. Then, we can substitute our researched parameters into these formulas to obtain these values.

$$\begin{aligned} A &= Q/v \\ &= Q/(2gh)^{1/2} \end{aligned}$$

We know the maximum flow for power generation at the two dams, and this flow rate is achieved when we have the maximum velocity of water entering the turbine - the head of the water is at its highest. Since the channels to generators will be fully opened for the maximum flow, it is possible to derive at the maximum cross-section area for these channels at the two dams. We plug in $Q_{G\,\max}$, hG max at each dam into the formula above to find $A_{G\,\max}$ at Glen Canyon Dam:

$$\begin{aligned} A_{G\,\max} &= 708/(2 * 9.8 * 174)^{1/2} \\ &= 12.12 \end{aligned}$$

Similarly, for Hoover Dam, we have:

$$\begin{aligned} A_{H\,\max} &= 906/(2 * 9.8 * 180)^{1/2} \\ &= 15.25 \end{aligned}$$

Now, we are curious about η, which is the efficiency of kinetic energy transferring to electrical energy, and for most hydraulic electric power plants, η is 0.9. Thus, we think it is reasonable to have $\eta_G = \eta_H = 0.9$ in this case [9].

Our goal is to have a sense of how the amount of electricity generated at the two dams is related to the height of water inside the dam and the flux of water flowing through the dam. Through our research, the power of a hydropower plant can be calculated as [9]:

$$P = \eta\rho Q g h$$

Through this formula, we can precisely figure the power at the two dams at any given point where we know the water level and the flux's magnitude. Here, Q is equivalent to:

$$Q = A(2gh)^{1/2}$$

The flux, or the water flow rate, at any given point in the dam will have the same magnitude, and the power can be calculated as:

$$P = \eta\rho A(2g^3h^3)^{1/2}$$

As we see, P is dependent on A and h. But, what about the flux of water actually discharging from these two dams? When water passes through the turbine, its kinetic energy will partially transfer to electric energy, so the stream will come out of the dams with a much slower pace [9].

$$v' = v(1 - \eta)^{1/2}$$

However, since the volume flow rate of water will remain the same across the flow, we know the volume of water flowing out of each dam is:

$$W = Qt_{open}$$

Here, t_{open} is the amount of time that we open the gates to discharge water, and for any cubic meter of water flowing out, we generate electricity. Thus, the amount of electricity is produced:

$$E = Pt_{open} = \eta \rho A (2g^3 h^3)^{1/2} t_{open}$$

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