

Backpropagation Algorithm

1

The backpropagation algorithm consists of two phases:

1. The **forward pass** where our inputs are passed through the network and output predictions obtained (also known as the propagation phase).
2. The **backward pass** where we compute the **gradient** of the **loss function** at the final layer (i.e., predictions layer) of the network and use this gradient to recursively apply the **chain rule** to update the weights in our network (also known as the weight update phase).

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2

Search for the related information of the following concepts:

1. Forward pass
2. Backward pass
3. Gradient
4. Loss function
5. Chain rule

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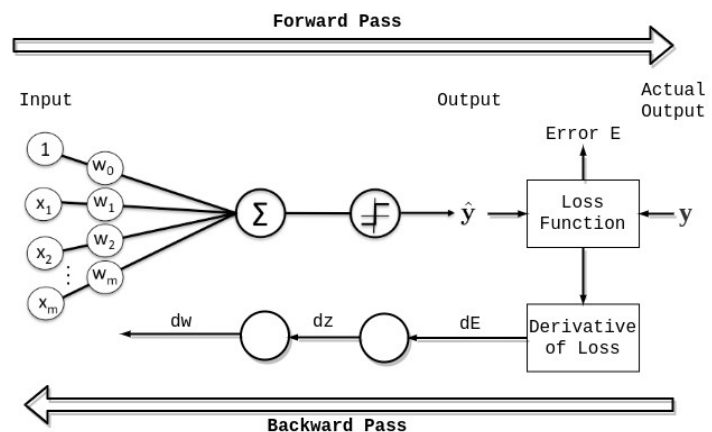
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The Forward Pass

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► The purpose of the forward pass is to propagate our inputs through the network by applying a series of dot products and activations until we reach the output layer of the network (i.e., our predictions).

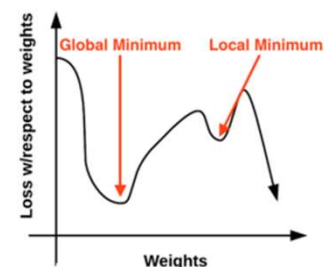


3

Gradient

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- The gradient descent method is an iterative optimization algorithm that operates over a loss landscape
- As we can see, our loss landscape has many peaks and valleys based on which values our parameters take on. Each peak is a local maximum that represents very high regions of loss – the local maximum with the largest loss across the entire loss landscape is the global maximum.
- Similarly, we also have local minimum which represents many small regions of loss



4

Gradient

5

The surface of our bowl is the loss landscape, which is a plot of the loss function. The difference between our loss landscape and your cereal bowl is that your cereal bowl only exists in three dimensions, while your loss landscape exists in many dimensions, perhaps tens, hundreds, or thousands of dimensions.

Each position along the surface of the bowl corresponds to a particular loss value given a set of parameters W (weight matrix) and b (bias vector). Our goal is to try different values of W and b , evaluate their loss, and then take a step towards more optimal values that (ideally) have lower loss.



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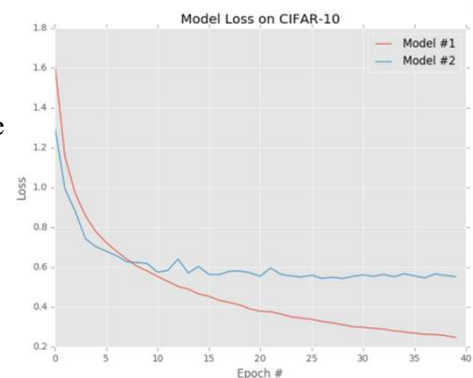
Loss function

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The loss function quantifies how “good” or “bad” of a job a given model is doing classifying data points from the dataset. Model #1 achieves considerably lower loss than Model #2.

The smaller the loss, the better a job the classifier is at modeling the relationship between the input data and output class labels.

To improve our classification accuracy, we need to tune the parameters of our weight matrix W or bias vector b . Exactly how we go about updating these parameters is an optimization problem.



6

Loss function

7

Regression Losses

Mean Bias Error	Captures average bias in prediction. But is rarely used for training.	$\mathcal{L}_{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - f(x_i))$
Mean Absolute Error	Measures absolute average bias in prediction. Also called L1 Loss.	$\mathcal{L}_{MAE} = \frac{1}{N} \sum_{i=1}^N y_i - f(x_i) $
Mean Squared Error	Average squared distance between actual and predicted. Also called L2 Loss.	$\mathcal{L}_{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - f(x_i))^2$
Root Mean Squared Error	Square root of MSE. Loss and dependent variable have same units.	$\mathcal{L}_{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - f(x_i))^2}$
Huber Loss	A combination of MSE and MAE. It is parametric loss function.	$\mathcal{L}_{Huber} = \begin{cases} \frac{1}{2}(y_i - f(x_i))^2 & : y_i - f(x_i) \leq \delta \\ \delta(y_i - f(x_i) - \frac{1}{2}\delta) & : \text{otherwise} \end{cases}$
Log Cosh Loss	Similar to Huber Loss + non-parametric. But computationally expensive.	$\mathcal{L}_{LogCosh} = \frac{1}{N} \sum_{i=1}^N \log(\cosh(f(x_i) - y_i))$

Classification Losses (Binary + Multi-class)

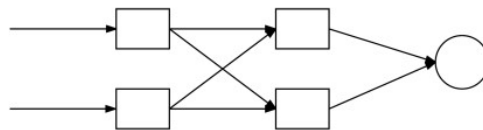
Binary Cross Entropy (BCE)	Loss function for binary classification tasks.	$\mathcal{L}_{BCE} = \frac{1}{N} \sum_{i=1}^N y_i \cdot \log(p(x_i)) + (1 - y_i) \cdot \log(1 - p(x_i))$
Hinge Loss	Penalizes wrong and right (but less confident) predictions. Commonly used in SVMs.	$\mathcal{L}_{Hinge} = \max(0, 1 - (f(x) \cdot y))$
Cross Entropy Loss	Extension of BCE loss to multi-class classification.	$\mathcal{L}_{CE} = -\frac{1}{N} \sum_{i=1}^N \sum_{j=1}^M y_{ij} \cdot \log(f(x_{ij}))$ <small>N: samples; M: classes</small>
KL Divergence	Minimizes the divergence between predicted and true probability distribution	$\mathcal{L}_{KL} = \sum_{i=1}^N y_i \cdot \log\left(\frac{y_i}{f(x_i)}\right)$

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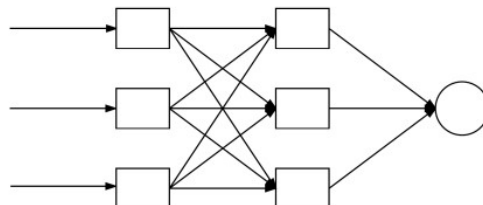
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8

2-2-1



3-3-1



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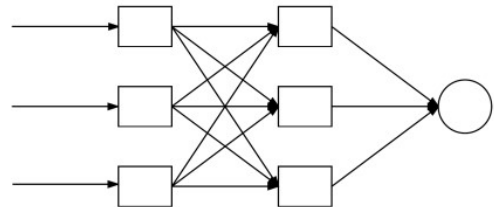
Backpropagation Algorithm

9

we present the feature vector (0,1,1) (and target output value 1 to the network). Here we can see that 0, 1, and 1 have been assigned to the three input nodes in the network.

To propagate the values through the network and obtain the final classification, we need to take the dot product between the inputs and the weight values, followed by applying an activation function (in this case, the sigmoid function, σ)

3-3-1

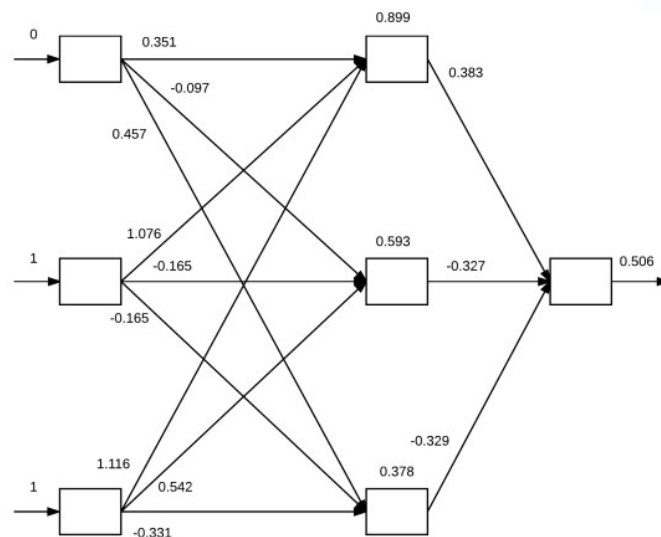


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9

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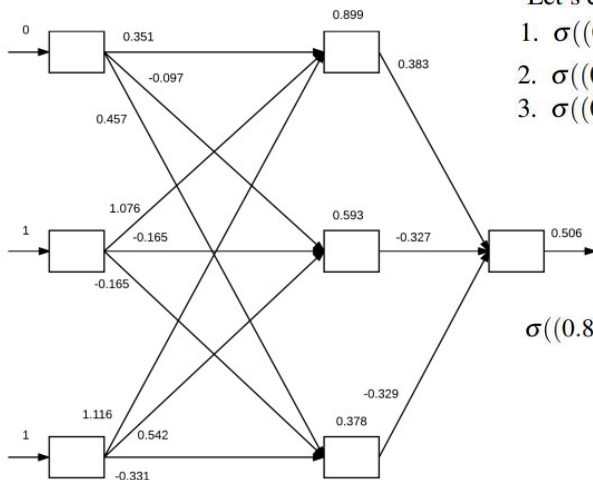


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10

11



Let's compute the inputs to the three nodes in the hidden layers:

1. $\sigma((0 \times 0.351) + (1 \times 1.076) + (1 \times 1.116)) = 0.899$
2. $\sigma((0 \times -0.097) + (1 \times -0.165) + (1 \times 0.542)) = 0.593$
3. $\sigma((0 \times 0.457) + (1 \times -0.165) + (1 \times -0.331)) = 0.378$

$$\sigma((0.899 \times 0.383) + (0.593 \times -0.327) + (0.378 \times -0.329)) = 0.506$$

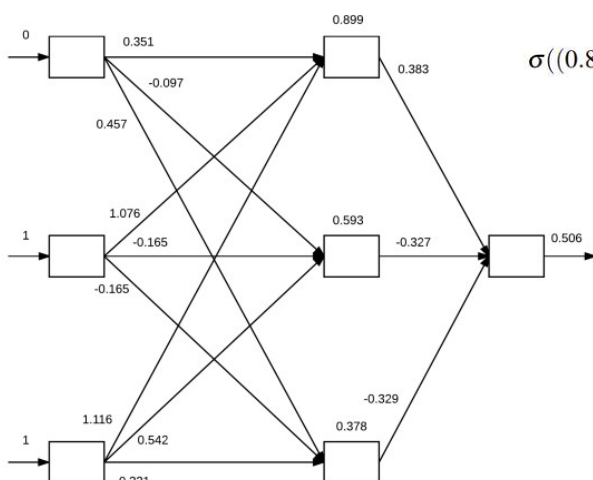
$$f(net) = \begin{cases} 1 & \text{if } net > 0 \\ 0 & \text{otherwise} \end{cases}$$

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12



$$\sigma((0.899 \times 0.383) + (0.593 \times -0.327) + (0.378 \times -0.329)) = 0.506$$

The output of the network is thus 0.506. We can apply a step function to determine if this output is the correct classification or not:

$$f(net) = \begin{cases} 1 & \text{if } net > 0 \\ 0 & \text{otherwise} \end{cases}$$

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$$f(\text{net}) = \begin{cases} 1 & \text{if } \text{net} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Applying the step function with $\text{net} = 0.506$ we see that our network predicts 1 which is, in fact, the correct class label. However, our network is not very confident in this class label – the predicted value 0.506 is very close to the threshold of the step. Ideally, this prediction should be closer to 0.98–0.99, implying that our network has truly learned the underlying pattern in the dataset. In order for our network to actually “learn”, we need to apply the backward pass.

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14

In order to apply the backpropagation algorithm, our activation function must be *differentiable* so that we can compute the *partial derivative* of the error with respect to a given weight $w_{i,j}$, loss (E), node output o_j , and network output net_j .

$$\frac{\partial E}{\partial w_{i,j}} = \frac{\partial E}{\partial o_j} \frac{\partial o_j}{\partial \text{net}_j} \frac{\partial \text{net}_j}{\partial w_{i,j}} \quad (10.5)$$

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Open up a new file, name it `neuralnetwork.py`, store it in the `nn` submodule of `pyimagesearch`, and let's get to work:

```
1 # import the necessary packages
2 import numpy as np
3
4 class NeuralNetwork:
5     def __init__(self, layers, alpha=0.1):
6         # initialize the list of weights matrices, then store the
7         # network architecture and learning rate
8         self.W = []
9         self.layers = layers
10        self.alpha = alpha
```

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16

```
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```

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Line 5 then defines the constructor to our `NeuralNetwork` class. The constructor requires a single argument, followed by a second optional one:

- **layers**: A list of integers which represents the actual architecture of the feedforward network. For example, a value of `[2,2,1]` would imply that our first input layer has two nodes, our hidden layer has two nodes, and our final output layer has one node.
- **alpha**: Here we can specify the learning rate of our neural network. This value is applied during the weight update phase.

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17

```

1  # import the necessary packages
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5      def __init__(self, layers, alpha=0.1):
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7          # network architecture and learning rate
8          self.W = []
9          self.layers = layers
10         self.alpha = alpha

```

Line 8 initializes our list of weights for each layer, `W`. We then store layers and alpha on **Lines 9 and 10**. Our **weights** list `W` is **empty**, so let's go ahead and initialize it

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```

12         # start looping from the index of the first layer but
13         # stop before we reach the last two layers
14         for i in np.arange(0, len(layers) - 2):
15             # randomly initialize a weight matrix connecting the
16             # number of nodes in each respective layer together,
17             # adding an extra node for the bias
18             w = np.random.randn(layers[i] + 1, layers[i + 1] + 1)
19             self.W.append(w / np.sqrt(layers[i]))

```

On Line 14 we start looping over the number of layers in the network (i.e., `len(layers)`), but **we stop before the final two layer**.

Each layer in the network is randomly initialized by constructing an $M \times N$ weight matrix by sampling values from a standard, normal distribution (**Line 18**). The matrix is $M \times N$ since we wish to connect every node in current layer to every node in the next layer.

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```

We scale w by dividing by the square root of the number of nodes in the current layer, thereby normalizing the variance of each neuron's output (Line 19).

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The final code block of the constructor handles the special case where the input connections need a bias term, but the output does not:

```

21         # the last two layers are a special case where the input
22         # connections need a bias term but the output does not
23         w = np.random.randn(layers[-2] + 1, layers[-1])
24         self.W.append(w / np.sqrt(layers[-2]))

```

Again, these weight values are randomly sampled and then normalized.

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The next function we define is a Python “magic method” named `__repr__` – this function is useful for debugging:

```

26     def __repr__(self):
27         # construct and return a string that represents the network
28         # architecture
29         return "NeuralNetwork: {}".format(
30             "-".join(str(l) for l in self.layers))

```

In our case, we'll format a string for our NeuralNetwork object by concatenating the integer value of the number of nodes in each layer.

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Given a layers value of (2, 2, 1), the output of calling this function will be:

```

1  >>> from pyimagesearch.nn import NeuralNetwork
2  >>> nn = NeuralNetwork([2, 2, 1])
3  >>> print(nn)
4  NeuralNetwork: 2-2-1

```

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23

Next, we can define our sigmoid activation function:

```

32     def sigmoid(self, x):
33         # compute and return the sigmoid activation value for a
34         # given input value
35         return 1.0 / (1 + np.exp(-x))

```

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As well as the derivative of the sigmoid which we'll use during the backward pass:

```

37     def sigmoid_deriv(self, x):
38         # compute the derivative of the sigmoid function ASSUMING
39         # that `x` has already been passed through the `sigmoid`
40         # function
41         return x * (1 - x)

```

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25

```

43 def fit(self, X, y, epochs=1000, displayUpdate=100):
44     # insert a column of 1's as the last entry in the feature
45     # matrix -- this little trick allows us to treat the bias
46     # as a trainable parameter within the weight matrix
47     X = np.c_[X, np.ones((X.shape[0]))]
48
49     # loop over the desired number of epochs
50     for epoch in np.arange(0, epochs):
51         # loop over each individual data point and train
52         # our network on it
53         for (x, target) in zip(X, y):
54             self.fit_partial(x, target)
55
56         # check to see if we should display a training update
57         if epoch == 0 or (epoch + 1) % displayUpdate == 0:
58             loss = self.calculate_loss(X, y)
59             print("[INFO] epoch={}, loss={:.7f}".format(
60                 epoch + 1, loss))

```

We'll draw inspiration from the scikit-learn library and define a **function** named **fit** which will be responsible for actually **training** our NeuralNetwork

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Implementing Backpropagation with Python

26

```

43 def fit(self, X, y, epochs=1000, displayUpdate=100):
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```

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27

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60             epoch + 1, loss))

```

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The actual heart of the backpropagation algorithm is found inside our `fit_partial` method below:

```

62     def fit_partial(self, x, y):
63         # construct our list of output activations for each layer
64         # as our data point flows through the network; the first
65         # activation is a special case -- it's just the input
66         # feature vector itself
67         A = [np.atleast_2d(x)]

```

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From here, we can start the forward propagation phase:

29

```

69     # FEEDFORWARD:
70     # loop over the layers in the network
71     for layer in np.arange(0, len(self.W)):
72         # feedforward the activation at the current layer by
73         # taking the dot product between the activation and
74         # the weight matrix -- this is called the "net input"
75         # to the current layer
76         net = A[layer].dot(self.W[layer])
77
78         # computing the "net output" is simply applying our
79         # nonlinear activation function to the net input
80         out = self.sigmoid(net)
81
82         # once we have the net output, add it to our list of
83         # activations
84         A.append(out)

```

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29

Implementing Backpropagation with Python

30

```

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82         # once we have the net output, add it to our list of
83         # activations
84         A.append(out)

```

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The final entry in A is thus the output of the last layer in our network

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30

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31

Now that the forward pass is done, we can move on to the slightly more complicated backward pass:

```

86     # BACKPROPAGATION
87     # the first phase of backpropagation is to compute the
88     # difference between our *prediction* (the final output
89     # activation in the activations list) and the true target
90     # value
91     error = A[-1] - y
92
93     # from here, we need to apply the chain rule and build our
94     # list of deltas `D`; the first entry in the deltas is
95     # simply the error of the output layer times the derivative
96     # of our activation function for the output value
97     D = [error * self.sigmoid_deriv(A[-1])]

```

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The first phase of the backward pass is to **compute our error**, or simply the difference between our predicted label and the ground-truth label (Line 91).

Since the final entry in the activations list A contains the output of the network, **we can access the output prediction via A[-1]**. The value y is the target output for the input data point x.

```

91     error = A[-1] - y
92

```

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Next, we need to start applying the **chain rule** to build our list of deltas, **D**. The deltas will **be used to update our weight matrices**, scaled by the learning rate alpha.

The first entry in the deltas list is the **error** of our output layer **multiplied** by the **derivative of the sigmoid** for the output value (Line 97)

```
97 D = [error * self.sigmoid_deriv(A[-1])]
```

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Given the delta for the final layer in the network, we can now work backward using a for loop:

```
99 # once you understand the chain rule it becomes super easy
100 # to implement with a `for` loop -- simply loop over the
101 # layers in reverse order (ignoring the last two since we
102 # already have taken them into account)
103 for layer in np.arange(len(A) - 2, 0, -1):
104     # the delta for the current layer is equal to the delta
105     # of the *previous layer* dotted with the weight matrix
106     # of the current layer, followed by multiplying the delta
107     # by the derivative of the nonlinear activation function
108     # for the activations of the current layer
109     delta = D[-1].dot(self.W[layer].T)
110     delta = delta * self.sigmoid_deriv(A[layer])
111     D.append(delta)
```

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```

103     for layer in np.arange(len(A) - 2, 0, -1):
109         delta = D[-1].dot(self.W[layer].T)
110         delta = delta * self.sigmoid_deriv(A[layer])
111         D.append(delta)

```

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Given our deltas list D, we can move on to the weight update phase

```

115     D = D[::-1]
116
117     # WEIGHT UPDATE PHASE
118     # loop over the layers
119     for layer in np.arange(0, len(self.W)):
120         # update our weights by taking the dot product of the layer
121         # activations with their respective deltas, then multiplying
122         # this value by some small learning rate and adding to our
123         # weight matrix -- this is where the actual "learning" takes
124         # place
125         self.W[layer] += -self.alpha * A[layer].T.dot(D[layer])

```

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```

115     D = D[:,:-1]
116
117     # WEIGHT UPDATE PHASE
118     # loop over the layers
119     for layer in np.arange(0, len(self.W)):

```

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```

119     for layer in np.arange(0, len(self.W)):
120         # update our weights by taking the dot product of the layer
121         # activations with their respective deltas, then multiplying
122         # this value by some small learning rate and adding to our
123         # weight matrix -- this is where the actual "learning" takes
124         # place
125         self.W[layer] += -self.alpha * A[layer].T.dot(D[layer])

```

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Once our network is trained on a given dataset, we'll want to make predictions on the testing set, which can be accomplished via the predict method below:

```

127 def predict(self, X, addBias=True):
128     # initialize the output prediction as the input features -- this
129     # value will be (forward) propagated through the network to
130     # obtain the final prediction
131     p = np.atleast_2d(X)
132
133     # check to see if the bias column should be added
134     if addBias:
135         # insert a column of 1's as the last entry in the feature
136         # matrix (bias)
137         p = np.c_[p, np.ones((p.shape[0]))]
138
139     # loop over our layers in the network
140     for layer in np.arange(0, len(self.W)):
141         # computing the output prediction is as simple as taking
142         # the dot product between the current activation value `p`
143         # and the weight matrix associated with the current layer,
144         # then passing this value through a nonlinear activation
145         # function
146         p = self.sigmoid(np.dot(p, self.W[layer]))
147
148     # return the predicted value
149     return p

```

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```

127 def predict(self, X, addBias=True):
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```

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138

```

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42

```

139     # loop over our layers in the network
140     for layer in np.arange(0, len(self.W)):
141         # computing the output prediction is as simple as taking
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144         # then passing this value through a nonlinear activation
145         # function
146         p = self.sigmoid(np.dot(p, self.W[layer]))
147
148     # return the predicted value
149     return p

```

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The final function we'll define inside the NeuralNetwork class will be used to calculate the loss across our entire training set:

```

151     def calculate_loss(self, X, targets):
152         # make predictions for the input data points then compute
153         # the loss
154         targets = np.atleast_2d(targets)
155         predictions = self.predict(X, addBias=False)
156         loss = 0.5 * np.sum((predictions - targets) ** 2)
157
158         # return the loss
159         return loss

```

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43

Backpropagation with Python Example #1: Bitwise XOR

44

Go ahead and open up a new file, name it nn_xor.py, and insert the following code:

```

1  # import the necessary packages
2  from pyimagesearch.nn import NeuralNetwork
3  import numpy as np
4
5  # construct the XOR dataset
6  X = np.array([[0, 0], [0, 1], [1, 0], [1, 1]])
7  y = np.array([[0], [1], [1], [0]])

```

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44

Backpropagation with Python Example #1: Bitwise XOR

45

We can now define our network architecture and train it:

```

9  # define our 2-2-1 neural network and train it
10 nn = NeuralNetwork([2, 2, 1], alpha=0.5)
11 nn.fit(X, y, epochs=20000)

```

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45

Backpropagation with Python Example #1: Bitwise XOR

46

Once our network is trained, we'll loop over our XOR datasets, allow the network to predict the output for each one, and display the prediction to our screen:

```

13 # now that our network is trained, loop over the XOR data points
14 for (x, target) in zip(X, y):
15     # make a prediction on the data point and display the result
16     # to our console
17     pred = nn.predict(x)[0][0]
18     step = 1 if pred > 0.5 else 0
19     print("[INFO] data={}, ground-truth={}, pred={:.4f}, step={}".format(
20         x, target[0], pred, step))

```

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46

Backpropagation with Python Example #1: Bitwise XOR

To train our neural network using backpropagation with Python, simply execute the following command:

```
$ python nn_xor.py
[INFO] epoch=1, loss=0.5092796
[INFO] epoch=100, loss=0.4923591
[INFO] epoch=200, loss=0.4677865
...
[INFO] epoch=19800, loss=0.0002478
[INFO] epoch=19900, loss=0.0002465
[INFO] epoch=20000, loss=0.0002452
```

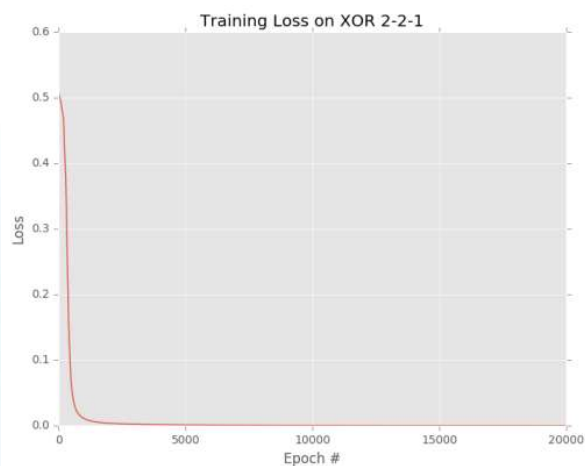
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47

Backpropagation with Python Example #1: Bitwise XOR

A plot of the squared loss is displayed below (Figure 10.11). As we can see, loss slowly decreases to approximately zero over the course of training.



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48

Backpropagation with Python Example #1: Bitwise XOR

49

Furthermore, looking at the final four lines of the output we can see our predictions:

```
[INFO] data=[0 0], ground-truth=0, pred=0.0054, step=0
[INFO] data=[0 1], ground-truth=1, pred=0.9894, step=1
[INFO] data=[1 0], ground-truth=1, pred=0.9876, step=1
[INFO] data=[1 1], ground-truth=0, pred=0.0140, step=0
```

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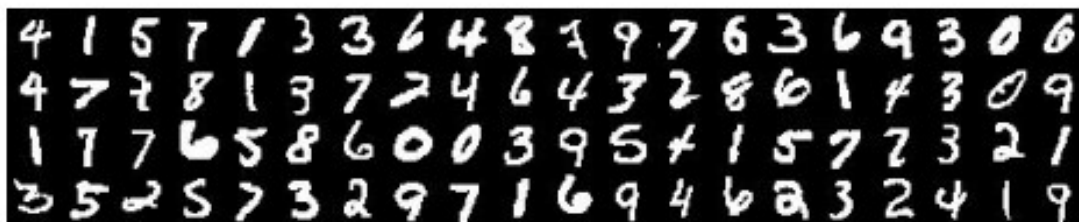
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49

Backpropagation with Python Example: MNIST Sample

50

Let's examine a subset of the MNIST dataset for handwritten digit recognition. This subset of the MNIST dataset is built-into the scikit-learn library and includes 1,797 example digits, each of which are 8×8 grayscale images (the original images are 28×28). When flattened, these images are represented by an $8 \times 8 = 64$ -dim vector.



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50

51

```

1  # import the necessary packages
2  from pyimagesearch.nn import NeuralNetwork
3  from sklearn.preprocessing import LabelBinarizer
4  from sklearn.model_selection import train_test_split
5  from sklearn.metrics import classification_report
6  from sklearn import datasets

8  # load the MNIST dataset and apply min/max scaling to scale the
9  # pixel intensity values to the range [0, 1] (each image is
10 # represented by an 8 x 8 = 64-dim feature vector)
11 print("[INFO] loading MNIST (sample) dataset...")
12 digits = datasets.load_digits()
13 data = digits.data.astype("float")
14 data = (data - data.min()) / (data.max() - data.min())
15 print("[INFO] samples: {}, dim: {}".format(data.shape[0],
16      data.shape[1]))

```

We also have to prepare

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51

52

```

18 # construct the training and testing splits
19 (trainX, testX, trainY, testY) = train_test_split(data,
20     digits.target, test_size=0.25)
21
22 # convert the labels from integers to vectors
23 trainY = LabelBinarizer().fit_transform(trainY)
24 testY = LabelBinarizer().fit_transform(testY)

```

We'll also encode our class labels. We will discuss in detail how

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52

```

26 # train the network
27 print("[INFO] training network...")
28 nn = NeuralNetwork([trainX.shape[1], 32, 16, 10])
29 print("[INFO] {}".format(nn))
30 nn.fit(trainX, trainY, epochs=1000)

```

Here we can see that we are training a NeuralNetwork with a 64–32–16–10 architecture. The output layer has ten nodes due to the fact that there are ten possible output classes for the digits 0-9. We then allow our network to train for 1,000 epochs.

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53

Once our network has been trained, we can evaluate it on the testing set:

54

```

32 # evaluate the network
33 print("[INFO] evaluating network...")
34 predictions = nn.predict(testX)
35 predictions = predictions.argmax(axis=1)
36 print(classification_report(testY.argmax(axis=1), predictions))

```

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54

55

```
$ python nn_mnist.py
[INFO] loading MNIST (sample) dataset...
[INFO] samples: 1797, dim: 64
[INFO] training network...
[INFO] NeuralNetwork: 64-32-16-10
[INFO] epoch=1, loss=604.5868589
[INFO] epoch=100, loss=9.1163376
[INFO] epoch=200, loss=3.7157723
[INFO] epoch=300, loss=2.6078803
[INFO] epoch=400, loss=2.3823153
[INFO] epoch=500, loss=1.8420944
[INFO] epoch=600, loss=1.3214138
[INFO] epoch=700, loss=1.2095033
[INFO] epoch=800, loss=1.1663942
[INFO] epoch=900, loss=1.1394731
[INFO] epoch=1000, loss=1.1203779
[INFO] evaluating network...
```

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56

	precision	recall	f1-score	support
0	1.00	1.00	1.00	45
1	0.98	1.00	0.99	51
2	0.98	1.00	0.99	47
3	0.98	0.93	0.95	43
4	0.95	1.00	0.97	39
5	0.94	0.97	0.96	35
6	1.00	1.00	1.00	53
7	1.00	1.00	1.00	49
8	0.97	0.95	0.96	41
9	1.00	0.96	0.98	47
avg / total	0.98	0.98	0.98	450

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56

57



Notice how our loss starts off very high, but quickly drops during the training process. Our classification report demonstrates that we are obtaining $\approx 98\%$ classification accuracy on our testing set; however, we are having some trouble classifying digits 4 and 5 (95% and 94% accuracy, respectively).

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