**Week 1-3: Linear Algebra**

1. Let two matrices be $A = \begin{bmatrix} 4 & 3 \\ 6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 9 \\ -5 & 2 \end{bmatrix}$. What is $A + B$?

$$\begin{bmatrix} 4 & 3 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 9 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 4\text{-}2 & 3\text{+}9 \\ 6\text{-}5 & 9\text{+}2 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 12 \\ 1 & 11 \end{bmatrix}}$$

2. Let $x = \begin{bmatrix} 2 \\ 7 \\ 4 \\ 1 \end{bmatrix}$. What is $3 \times x$?

$$3 \times x = 3 \times \begin{bmatrix} 2 \\ 7 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \times 2 \\ 3 \times 7 \\ 3 \times 4 \\ 3 \times 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 \\ 21 \\ 12 \\ 3 \end{bmatrix}}$$

3. Let $u$ be a 3-dimensional vector, where specifically $u = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$. What is $u^T$?

$$\boxed{u^T = \begin{bmatrix} 8 & 1 & 4 \end{bmatrix}}$$

4. Let $u$ and $v$ be 3-dimensional vectors, where specifically $u = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$. What is $u^T v$?

$u^T = \begin{bmatrix} 3 & -5 & 4 \end{bmatrix}$. Since $u^T$ is a $1x3$ matrix and $v$ is a $3x1$ matrix, the resulting product will be a $1x1$ matrix or a scalar.

$$u^T v = u^T = \begin{bmatrix} 3 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \times 3 + 2 \times (-5) + 5 \times 4 \end{bmatrix} = \begin{bmatrix} 3 + -10 + 20 \end{bmatrix} = \boxed{[13]}$$

5. Let $A$ and $B$ be $3x3$ square matrices. Which of the following must necessarily hold true? Check all that apply.

(a) If $B$ is the $3x3$ identity matrix, then $A \times B = B \times A$.

(b) If $C = A \times B$, then C is a $3x3$ matrix.