Angular momentum projection in the deformed relativistic Hartree-Bogoliubov theory in continuum

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Outline

Outline

Overview

Abstract

1. The wave functions of angular momentum projected states are expanded in terms of the Dirac Woods- Saxon basis
2. Presenting the DRHBc+AMP approach to study low-lying excited states of weakly bound deformed nuclei

Conclusion

1. The calculations show that neutron-rich magnesium isotopes $^{36,38,40}\mathrm{Mg}$ are all well deformed nuclei.
2. The ground-state rotational bands of $^{36,38,40}\mathrm{Mg}$ are reproduced reasonably well with the density functional PC-F1.

Introduction

Symmetry breaking

- The wave function obtained from MF calculations is approximated by a single Slater determinant and allowed to break symmetries of the Hamiltonian. (Finding the ψ which corresponds to the Emin without any limitation,breaking the symmetry)

AMP(HO)

- Explain or predict many exotic nuclear structures connected with the nuclear collective excitation
- The BMF calculations have been performed to study the excitation of odd N(Z) nuclei
- The asymptotic behavior of the wave function in a weakly bound system cannot be described properly with this basis

Outline

The point-coupling density functionals

Under the MF and no-sea approximations, The equation of motion of the system is constructed as a functional of nucleon densities.

$$[\boldsymbol{\alpha} \cdot (\boldsymbol{p} + \boldsymbol{V}) + \beta(m + S)]\,\psi_k = \epsilon_k \psi_k \tag{1}$$

By using the Bogoliubov transformation, the MF and pairing correlations are treated self-consistently. The equation of motion for nucleons is the deformed RHB equation :

$$\begin{pmatrix} h_D - \lambda_\tau & \Delta \\ -\Delta^* & -h_D^* + \lambda_\tau \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix} \tag{2}$$

The Dirac WS basis

The quasi-particle wave function is expanded in terms of the Dirac WS basis,

$$\begin{aligned} U_k(\boldsymbol{r}s) &= \sum_{n\kappa} u^{(m)}_{k,(n\kappa)} \varphi_{n\kappa m}(\boldsymbol{r}s), \\ V_k(\boldsymbol{r}s) &= \sum_{n\kappa} v^{(m)}_{k,(n\kappa)} \bar{\varphi}_{n\kappa m}(\boldsymbol{r}s). \end{aligned} \tag{3}$$

The basis function

$$\varphi_{n\kappa m}(\boldsymbol{r}s) = \frac{1}{r} \begin{pmatrix} iG_{n\kappa}(r)\mathcal{Y}^{l}_{jm}(\Omega s) \\ -F_{n\kappa}(r)\mathcal{Y}^{\tilde{l}}_{jm}(\Omega s) \end{pmatrix} \tag{4}$$

The pairing potential

The pairing potential is written as

$$\Delta\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right)=V^{pp}\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right) \kappa\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right) \tag{5}$$

The density-dependent zero-range force

$$V^{pp}\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right)=\frac{1}{2} V_0\left(1-\hat{P}^\sigma\right) \delta\left(\boldsymbol{r}_1-\boldsymbol{r}_2\right)\left[1-\left(\frac{\rho\left(\boldsymbol{r}_1\right)}{\rho_{\text{sat}}}\right)\right] \tag{6}$$

we can find the eq. from many body

$$P^\sigma=\frac{1}{2}\left(1+2\left(S^2-s_{(1)^2}-s_{(2)^2}\right)\right)=S(S+1)-1=\begin{cases} 1 \text{ for triplet} \\ -1 \text{ for singlet.}\end{cases} \tag{7}$$

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The matrix elements

In the Dirac WS basis, the matrix elements of ρ and t can be expressed as

$$\begin{aligned} \rho^m_{n\kappa,n'\kappa'} &= \sum_{k>0} v^{(m)}_{k,n\kappa} v^{(m)}_{k,n'\kappa'}, \\ t^m_{n\kappa,n'\kappa'} &= \int d\boldsymbol{r} \varphi^\dagger_{n\kappa m}(\boldsymbol{rs})(\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta M)\varphi_{n'\kappa' m}(\boldsymbol{rs}). \end{aligned} \tag{8}$$

The canonical basis can be obtained by diagonalizing the density matrix

$$\sum_{n'\kappa'} \rho^m_{n\kappa,n'\kappa'} c^i_{n'\kappa'} = v_i^2 c^i_{n\kappa} \tag{9}$$

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AMP

The basis $|\Phi(\Omega)\rangle$ is expanded by the element of symmetric Groups $\hat{R}(\Omega)$

- $\hat{H}|\Phi(\Omega)\rangle = \hat{H}\hat{R}(\Omega)|\Phi\rangle = E\hat{R}(\Omega)|\Phi\rangle$
- We can obtain the $|\Psi(\Omega)\rangle$ by $|\Phi(\Omega)\rangle$

$$|\Psi\rangle = \int d\Omega f(\Omega)|\Phi(\Omega)\rangle.$$

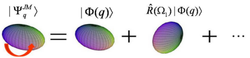

Figure 1: $|\Psi(\Omega)\rangle$ is expressed as a superposition of intrinsic wave functions with various orientations in space.

$|\Psi(\Omega)\rangle$ is invariant under transformations of the symmetry group S.

$$\hat{R}(\Omega)|\Psi\rangle = \int d\Omega' f(\Omega')\,\hat{R}(\Omega)\,|\Phi\,(\Omega')\rangle = \int d\Omega'' f(-\Omega+\Omega'')\,|\Phi\,(\Omega'')\rangle\,. \tag{10}$$

AMP

The wave function $|\Phi(\beta)\rangle$ with a certain β is not an eigenvector of $\hat{J}_z$ and $\hat{J}^2$. A low-lying excited state with good angular momentum can be constructed by performing the AMP on $|\Phi(\beta)\rangle$

$$\left|\Psi_\alpha^{JM}(\beta)\right\rangle = \sum_K f_\alpha^{JK} \hat{P}_{MK}^J |\Phi(\beta)\rangle \tag{11}$$

$$\hat{P}_{MK}^J = \frac{2J+1}{8\pi^2} \int d\Omega D_{MK}^{J*}(\Omega) \hat{R}(\Omega) \tag{12}$$

The Hill-Wheeler equation

$$\int da' \left\langle \Phi(a) | H | \Phi\left(a'\right) \right\rangle f\left(a'\right) = E \int da' \left\langle \Phi(a) \mid \Phi\left(a'\right) \right\rangle f\left(a'\right) \tag{13}$$

The energy E^J and f^{JK} of a projected state can be calculated

$$\begin{aligned} \sum_K f_\alpha^{JK} \Big[& \left\langle \Phi(\beta) \left| \hat{H} \hat{P}_{MK}^J \right| \Phi(\beta) \right\rangle \\ & - E_\alpha^J \left\langle \Phi(\beta) \left| \hat{P}_{MK}^J \right| \Phi(\beta) \right\rangle \Big] = 0. \end{aligned} \tag{14}$$

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AMP

The normal overlap kernel [106] reads

$$
\begin{aligned}
\mathcal{N}^J(\beta) \equiv & \left\langle \Phi(\beta) \left| \hat{P}^J_{00} \right| \Phi(\beta) \right\rangle \\
= & (2J+1) \int_0^{\pi/2} \sin\theta d^{J*}_{00}(\theta) \\
& \times \left\langle \Phi(\beta) \left| e^{-i\theta \hat{J}_y} \right| \Phi(\beta) \right\rangle d\theta,
\end{aligned}
\tag{15}
$$

and the Hamiltonian overlap kernel is

$$
\begin{aligned}
\mathcal{H}^J(\beta) \equiv & \left\langle \Phi(\beta) \left| \hat{H}\hat{P}^J_{00} \right| \Phi(\beta) \right\rangle \\
= & (2J+1) \int_0^{\pi/2} \sin\theta d^{J*}_{00}(\theta) \\
& \times \left\langle \Phi(\beta) \left| \hat{H} e^{-i\theta \hat{J}_y} \right| \Phi(\beta) \right\rangle d\theta
\end{aligned}
\tag{16}
$$

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AMP

Using the generalized Wick' s theorem to calculate the normal overlap kernel and Hamiltonian overlap kernel

$$\mathcal{H}^J(\beta) = (2J+1)\int_0^{\pi/2} \sin\theta d_{00}^{J*}(\theta) n(\beta;\theta)\mathcal{E}(\beta;\theta)d\theta \tag{17}$$

The mixed energy density has the form of

$$\mathcal{E}(\beta;\theta) = \int d^3 r \mathcal{E}[\rho(\boldsymbol{r};\beta;\theta)\kappa(\boldsymbol{r};\beta;\theta)] \tag{18}$$

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The reduced transition probability

$$B\left(E2, I_i^+ \to I_f^+\right) = \frac{e^2}{2I_i + 1} \left| \left\langle I_f \left\| \widehat{Q}_2 \right\| I_i \right\rangle \right|^2, \tag{19}$$

where the reduced matrix element of $\hat{Q}_2$ is

$$\left\langle I_f \left\| \widehat{Q}_2 \right\| I_i \right\rangle = \hat{I}_i \hat{I}_f \sum_{\mu'} \begin{pmatrix} I_i & 2 & I_f \\ -\mu' & \mu' & 0 \end{pmatrix} \\ \times \int_0^{\pi/2} d\theta \sin\theta d^{I_i^*}_{-\mu' 0}(\beta) \left\langle \Phi(\beta) \left| \widehat{Q}_{2\mu'} e^{-i\theta \hat{J}_y} \right| \Phi(\beta) \right\rangle \tag{20}$$

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DRHBc+AMP calculations

- With PC-PK1, the energy cutoff for positive energy states is Ecut = 300 MeV
- The number of mesh points in the Gaussian-Legendre quadrature :n_θ=12 in the interval $[0, \pi]$. For the mixed density and
- For the mixed density and currents ,the maximum orders are $l_\rho = 6$ and $l_j = 3$
- For determining the number of SPLs, the truncation of the occupation probability is $\xi = 10^{-7}$ and $\epsilon_{\text{cut}} = 50\mathrm{MeV}$ for SPE.
- the normal overlap can be analytically calculated by using the Gaussian overlap approximation

$$n_{\mathrm{GOA}}\left(\beta_2; \theta\right) = \exp\left[-\frac{1}{2}\left\langle \hat{J}_y^2 \right\rangle \sin^2\theta\right] \tag{21}$$

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Bulk properties

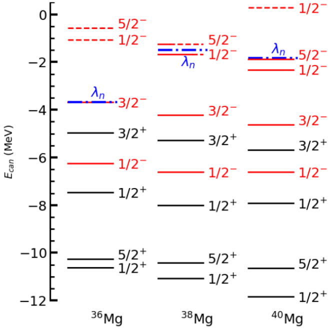

Figure 2: SPLs of neutrons around the(λ_n) of$^{36}\mathrm{Mg}$, $^{38}\mathrm{Mg}$, and $^{40}\mathrm{Mg}$ in the canonical basis. The length of the solid line is proportional to the occupation probability v^2 of each level labeled by Ω^π

- Around λn, SPLs are all fully occupied, with v2 = 1 f o r $^{36,40}\mathrm{Mg}$ and partially occupied for 38Mg, meaning the en hancement of pairing in $^{38}\mathrm{Mg}$
- the configurations of the valence neutrons for $^{38}\mathrm{Mg}$ and $^{40}\mathrm{Mg}$ all have p-wave components with considerable oc- cupation, but they are not halo nuclei .

Ground-state rotational bands

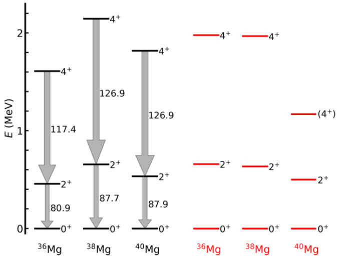

Figure 3: The ground-state rotational bands and values of B(E2)

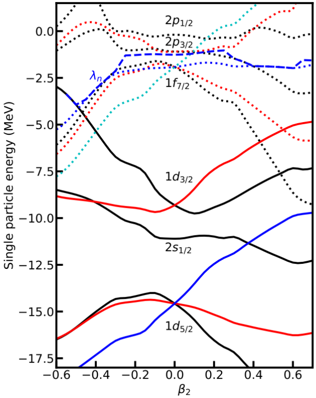

Figure 4: SPLs of neutrons for $^{40}\mathrm{Mg}$ in the canonical basis

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