The Hoyle state studied by NLEF

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Ab Initio Calculation of the Hoyle State

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Structure and Rotations of the Hoyle State

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Epelbaum

Bochum, Juelich & Bonn

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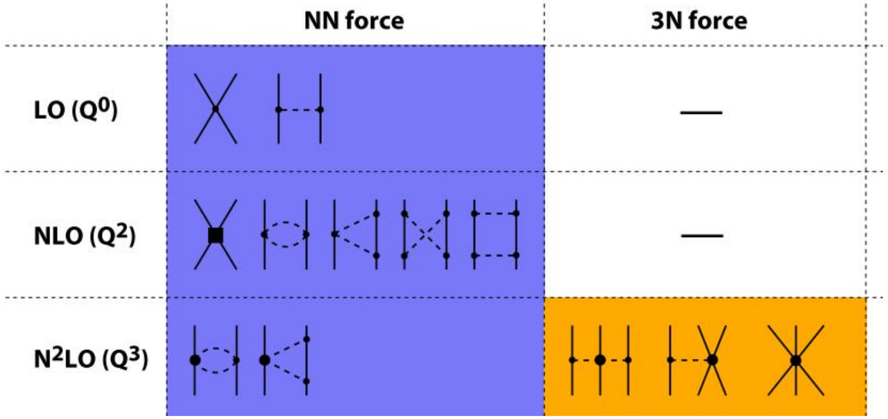
Brief introduction to NLEFT

T.Laehde,et al.,PLB732,110-115(2014).

Chiral nuclear force (NNLO)



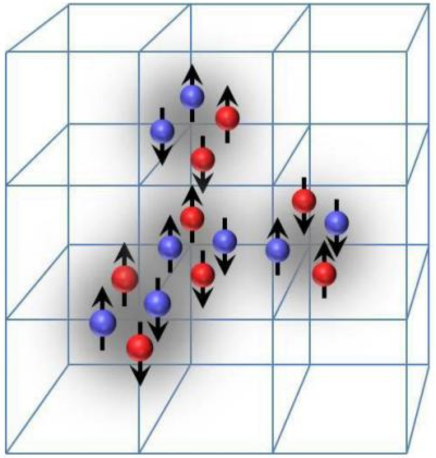
$$V_{\mathrm{LO}} = V^{(0)} + V^{\mathrm{OPEP}}$$

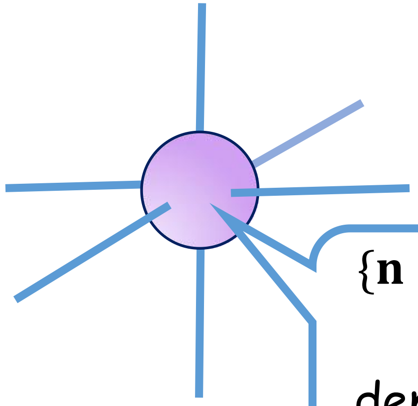
$$V_{\mathrm{NLO}} = V_{\mathrm{LO}} + \Delta V^{(0)} + V^{(2)} + V^{\mathrm{TPEP}}_{\mathrm{NLO}}.$$

$$V_{\mathrm{NNLO}} = V_{\mathrm{NLO}} + V^{(3N)}_{\mathrm{NNLO}}$$

$$H = T + V$$

Lattice calculator





$\{\mathbf{n} = (x, y, z),$
$\tau, \sigma, \mathcal{P}\}$
density: $\rho(\mathbf{n})$

kinetic term on lattice:

$$T = \frac{3}{m_N}\sum_{\mathbf{n}}\rho(\mathbf{n}) - \frac{1}{2m_N}\sum_{\mathbf{n}}\sum_{l=1}^{3}[\rho(\mathbf{n}, \mathbf{n}+e_l) + \rho(\mathbf{n}, \mathbf{n}-e_l)].$$

Transfer matrix operator :

$$M =: \exp(-\alpha_t H).$$

Brief introduction to NLEFT

Lattice calculator, cont'd

Lattice parameters:

periodic cube $L = 11.82$ fm

lattice spacing $a = 1.97$ fm

temporal spacing $a_t = 1.32$ fm

To start:

standing waves $|\Psi_A^{\text{init}}\rangle$

low-energy filter

$$H_{\text{SU}(4)} \equiv H_{\text{free}} + \frac{1}{2} C_{\text{SU}(4)} \sum_{\vec{n},\vec{n}'} :\rho(\vec{n}) f(\vec{n} - \vec{n}') \rho(\vec{n}'):$$

"trial state"

$$|\Psi_A(t')\rangle \equiv \exp(-H_{\text{SU}(4)} t') |\Psi_A^{\text{init}}\rangle$$

Euclidean-time projection amplitude

$$Z_A(t) \equiv \langle \Psi_A(t') | \exp(-H_{\text{LO}} t) | \Psi_A(t') \rangle$$

Time evolution & extracting observables:

$$E_A(t) = -\partial[\ln Z_A(t)]/\partial t$$

$$Z_A^{\mathcal{O}}(t) \equiv \langle \Psi_A(t') | \exp(-H_{\text{LO}} t/2)\, \mathcal{O} \exp(-H_{\text{LO}} t/2) | \Psi_A(t') \rangle$$

$$X_A^{\mathcal{O}}(t) = Z_A^{\mathcal{O}}(t)/Z_A(t)$$

Correlated fit procedure:

$$Z_A(t) = \int dE\, \rho_A(E) |\langle E | \Psi_A(t') \rangle|^2 \exp(-Et),$$

$$Z_A^{\mathcal{O}}(t) = \int dE\, dE'\, \rho_A(E) \rho_A(E') \exp(-(E + E')t/2),$$

$$\times \langle \Psi_A(t') | E \rangle \langle E | \mathcal{O} | E' \rangle \langle E' | \Psi_A(t') \rangle,$$

$$\rho_A(E) \approx \sum_{i=0}^{i_{\max}} c_i \delta(E - E_{A,i})$$

Ab Initio Calculation of the Hoyle state

LNEFT for light nuclei

TABLE I. Lattice results for the ground state energies for ^{4}He, ^{8}Be, and ^{12}C. For comparison we also exhibit the experimentally observed energies. All energies are in units of MeV.

	^{4}He	^{8}Be	^{12}C
LO $[O(Q^0)]$	−24.8(2)	−60.9(7)	−110(2)
NLO $[O(Q^2)]$	−24.7(2)	−60(2)	−93(3)
IB + EM $[O(Q^2)]$	−23.8(2)	−55(2)	−85(3)
NNLO $[O(Q^3)]$	−28.4(3)	−58(2)	−91(3)
Experiment	−28.30	−56.50	−92.16

Carbon-12 states: $J_n^\pi = 0_1^+$ (g.s.), 2_1^+ (lowest spin-2 state) and 0_2^+ (Hoyle state)

TABLE II. Lattice results for the low-lying excited states of ^{12}C. For comparison the experimentally observed energies are shown. All energies are in units of MeV.

	0_2^+	$2_1^+, J_z = 0$	$2_1^+, J_z = 2$
LO $[O(Q^0)]$	−94(2)	−92(2)	−89(2)
NLO $[O(Q^2)]$	−82(3)	−87(3)	−85(3)
IB + EM $[O(Q^2)]$	−74(3)	−80(3)	−78(3)
NNLO $[O(Q^3)]$	−85(3)	−88(3)	−90(4)
Experiment	−84.51	−87.72	

Extraction of excitation state (LO as example)

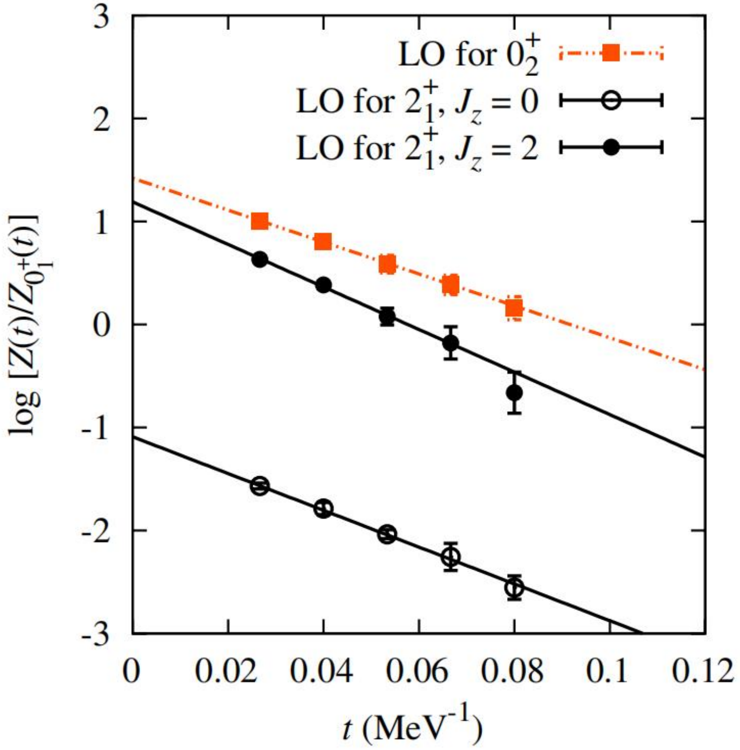


FIG. 1 (color online). Extraction of the excited states of ^{12}C from the time dependence of the projection amplitude at LO. The slope of the logarithm of $Z(t)/Z_{0_1^+}(t)$ at large t determines the energy relative to the ground state.

Ab Initio Calculation of the Hoyle state

Schematical lattice results

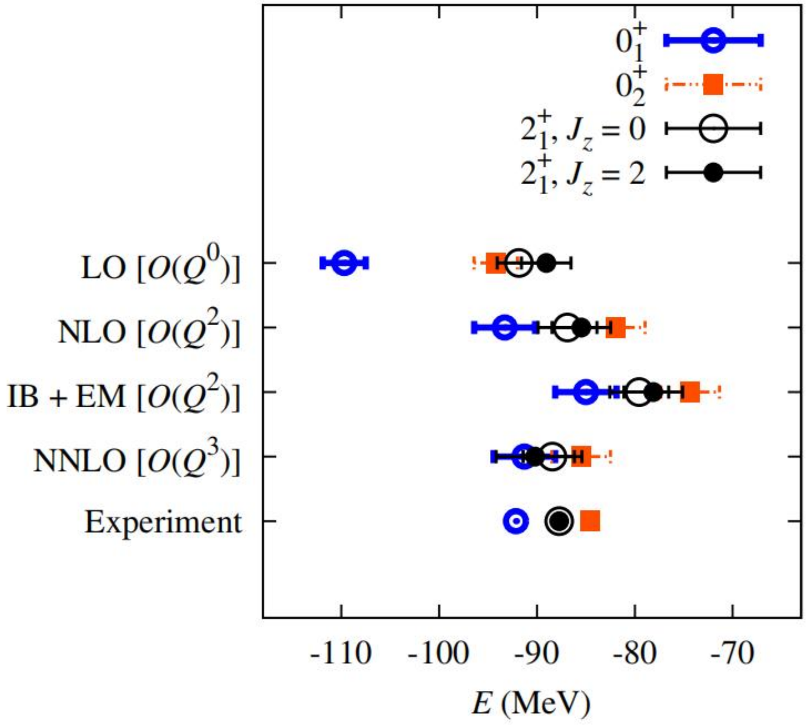


FIG. 2 (color online). Summary of lattice results for the carbon-12 spectrum and comparison with the experimental values. For each order in chiral EFT labeled on the left, results are shown for the ground state (blue circles), Hoyle state (red squares), and the $J_z = 0$ (open black circles) and $J_z = 2$ (filled black circles) projections of the spin-2 state.

Deducing the structure of C12 states

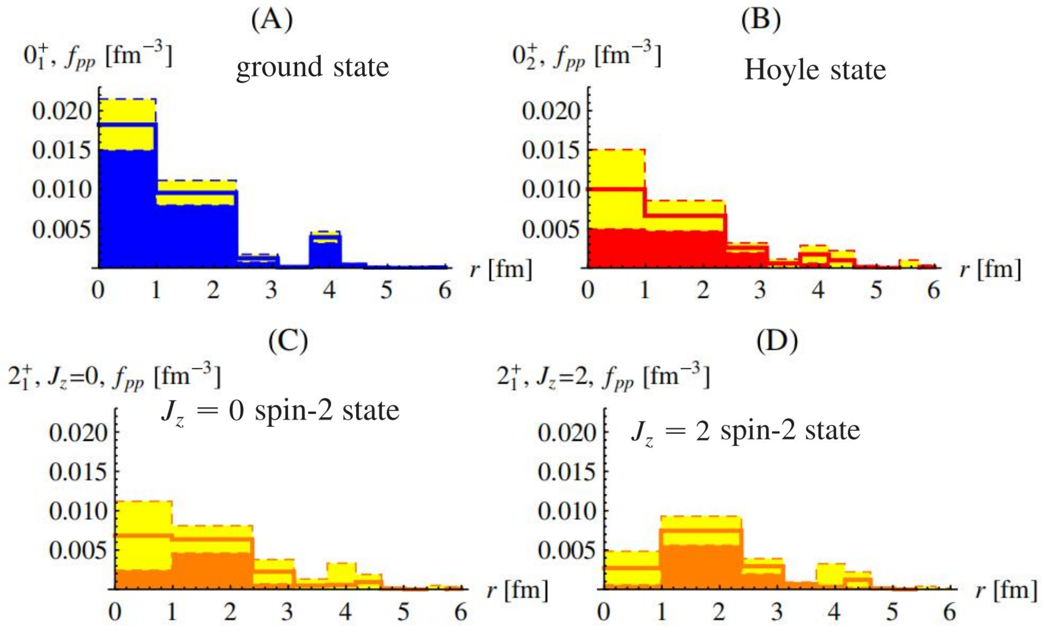


FIG. 3 (color online). The radial distribution function $f_{pp}(r)$ for the ground state (A), Hoyle state (B), and in the $J_z = 0$ (C) and $J_z = 2$ (D) projections of the spin-2 state. The yellow bands denote error bars.

Structure and Rotations of the Hoyle state

LNEFT for light nuclei, improved

TABLE I. Lattice results and experimental values for the ground state energies of ^{4}He and ^{8}Be, in units of MeV. The quoted errors are one standard deviation estimates which include both Monte Carlo statistical errors and uncertainties due to extrapolation at large Euclidean time.

	^{4}He	^{8}Be
LO [$\mathcal{O}(Q^0)$]	−28.0(3)	−57(2)
NLO [$\mathcal{O}(Q^2)$]	−24.9(5)	−47(2)
NNLO [$\mathcal{O}(Q^3)$]	−28.3(6)	−55(2)
Expt.	−28.30	−56.50

Initial configuration sensitivities

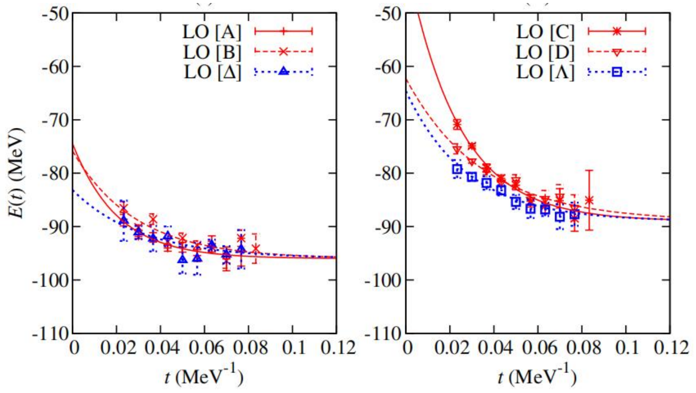


FIG. 1 (color online). Lattice results for the ^{12}C spectrum at leading order (LO). Panel I shows the results using initial states A, B, and Δ, each of which approaches the ground state energy. Panel II shows the results using initial states C, D, and Λ. These trace out an intermediate plateau at an energy $\simeq 7$ MeV above the ground state.

Structure of C12, again

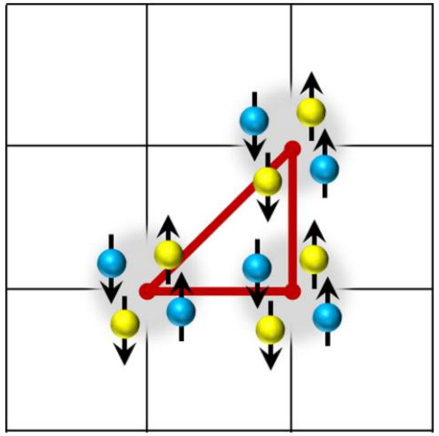


FIG. 2 (color online). Illustration of the initial state Δ. There are 12 equivalent orientations of this compact triangular configuration.

Spectrum of C12

TABLE II. Lattice and experimental results for the energies of the low-lying even-parity states of ^{12}C, in units of MeV.

	0_1^+	$2_1^+(E^+)$	0_2^+	$2_2^+(E^+)$
LO	−96(2)	−94(2)	−89(2)	−88(2)
NLO	−77(3)	−74(3)	−72(3)	−70(3)
NNLO	−92(3)	−89(3)	−85(3)	−83(3)
Expt.	−92.16	−87.72	−84.51	−82.6(1) [8,10] −81.1(3) [9] −82.32(6) [11]

Structure and Rotations of the Hoyle state

Structure of Hoyle state

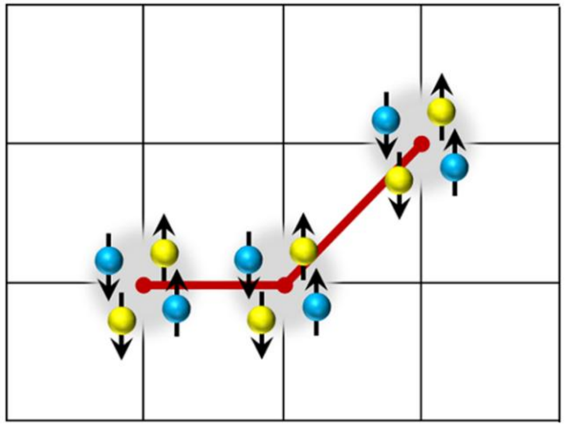


FIG. 3 (color online). Illustration of the initial state Λ. There are 24 equivalent orientations of this bent-arm or obtuse triangular configuration.

Root-mean-square radii and quadrupole moments

TABLE III. Lattice results at leading order (LO) and experimental values for the root-mean-square charge radii and quadrupole moments of the ^{12}C states.

	LO	Expt.
$r(0_1^+)$ [fm]	2.2(2)	2.47(2) [26]
$r(2_1^+)$ [fm]	2.2(2)	...
$Q(2_1^+)$ [$e\,\mathrm{fm}^2$]	6(2)	6(3) [27]
$r(0_2^+)$ [fm]	2.4(2)	...
$r(2_2^+)$ [fm]	2.4(2)	...
$Q(2_2^+)$ [$e\,\mathrm{fm}^2$]	−7(2)	...

Transitions

TABLE IV. Lattice results at leading order (LO) and experimental values for electromagnetic transitions involving the even-parity states of ^{12}C.

	LO	Expt.
$B(E2, 2_1^+ \to 0_1^+)$ [$e^2\,\mathrm{fm}^4$]	5(2)	7.6(4) [28]
$B(E2, 2_1^+ \to 0_2^+)$ [$e^2\,\mathrm{fm}^4$]	1.5(7)	2.6(4) [28]
$B(E2, 2_2^+ \to 0_1^+)$ [$e^2\,\mathrm{fm}^4$]	2(1)	...
$B(E2, 2_2^+ \to 0_2^+)$ [$e^2\,\mathrm{fm}^4$]	6(2)	...
$m(E0, 0_2^+ \to 0_1^+)$ [$e\,\mathrm{fm}^2$]	3(1)	5.5(1) [17]

Thank you for your attension!

Ending