Neutrinoless double-beta decay transition to excited $\mathbf{0}^+$ states in MR-CDFT

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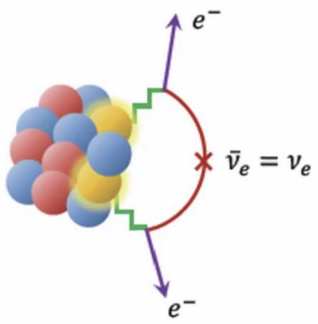
组会进展汇报

Feb. 07, 2025

Introduction of neutrinoless double beta ($0\nu\beta\beta$) decay

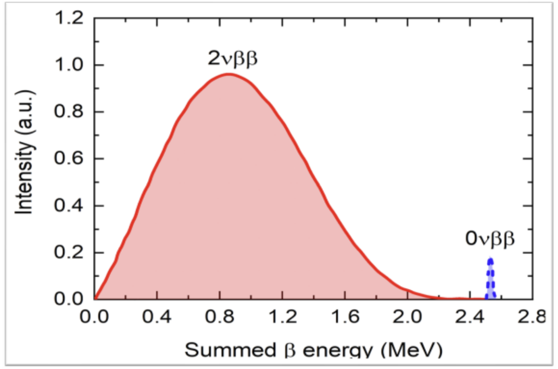
$0\nu\beta\beta$ decay is the process in which 2 neutrons decay into 2 protons and only 2 electrons emit.

$$X_Z^A \rightarrow Y_{Z+2}^A + e^- + e^-$$



- **Reveal the Majorana nature of neutrinos**
- **Determine the effective neutrino mass $\langle m_{\beta\beta} \rangle$**

$* \langle m_{\beta\beta} \rangle = |m_1 U_{e1}^2 \pm m_2 U_{e2}^2 \pm m_3 U_{e3}^2|$



In experimental research, one detect the decay signals from the emitted electron spectrum.

Decay mode	Q [keV]
${}^{48}_{20}\mathrm{Ca} \longrightarrow {}^{48}_{22}\mathrm{Ti}$	4274 ± 4 [7]
${}^{76}_{32}\mathrm{Ge} \longrightarrow {}^{76}_{34}\mathrm{Se}$	2039.04 ± 0.16 [8]
${}^{82}_{34}\mathrm{Se} \longrightarrow {}^{82}_{36}\mathrm{Kr}$	2995.5 ± 1.9 [7]
${}^{96}_{40}\mathrm{Zr} \longrightarrow {}^{96}_{42}\mathrm{Mo}$	3347.7 ± 2.2 [7]
${}^{100}_{42}\mathrm{Mo} \longrightarrow {}^{100}_{44}\mathrm{Ru}$	3034.4 ± 0.17 [8]
${}^{110}_{46}\mathrm{Pd} \longrightarrow {}^{110}_{48}\mathrm{Cd}$	2004 ± 11 [7]
${}^{116}_{48}\mathrm{Cd} \longrightarrow {}^{116}_{50}\mathrm{Sn}$	2809 ± 4 [7]
${}^{124}_{50}\mathrm{Sn} \longrightarrow {}^{124}_{52}\mathrm{Te}$	2287 ± 1.5 [7]
${}^{130}_{52}\mathrm{Te} \longrightarrow {}^{130}_{54}\mathrm{Xe}$	2527.518 ± 0.013 [9]
${}^{136}_{54}\mathrm{Xe} \longrightarrow {}^{136}_{56}\mathrm{Ba}$	2457.83 ± 0.37 [10,11]
${}^{150}_{60}\mathrm{Nd} \longrightarrow {}^{150}_{62}\mathrm{Sm}$	3371.38 ± 0.2 [12]

M. Duerr et al., PRD 84(2011)093004

Since $\frac{1}{T_{1/2}^{0\nu}} = \langle m_{\beta\beta} \rangle^2 |M^{0\nu}|^2 G^{0\nu}(Q)$ and the phase space factor $G^{0\nu}(Q)$ is related to the Q values, larger the Q values are, easier to detect the decay.

Experimental researches of $0\nu\beta\beta$ decay

GERDA $({}^{76}Ge)$

$T_{1/2}^{0\nu} > 1.8 \times 10^{26}$yrs

Phys.Rev.Lett.125(2020)252502

MAJORANA $({}^{76}Ge)$

$T_{1/2}^{0\nu} > 8.3 \times 10^{25}$yrs

Phys.Rev.Lett.130(2023)062501

EXO-200 $({}^{136}Xe)$

$T_{1/2}^{0\nu} > 3.5 \times 10^{25}$yrs

Phys.Rev.Lett.123(2019)161802

CUPID-0 $({}^{82}Se)$

$T_{1/2}^{0\nu} > 2.4 \times 10^{24}$yrs

Phys.Rev.Lett.120(2018)232502

GUORE $({}^{130}Te)$

$T_{1/2}^{0\nu} > 2.2 \times 10^{25}$yrs

Nature 604(2022)53

CUPID $({}^{100}Mo)$

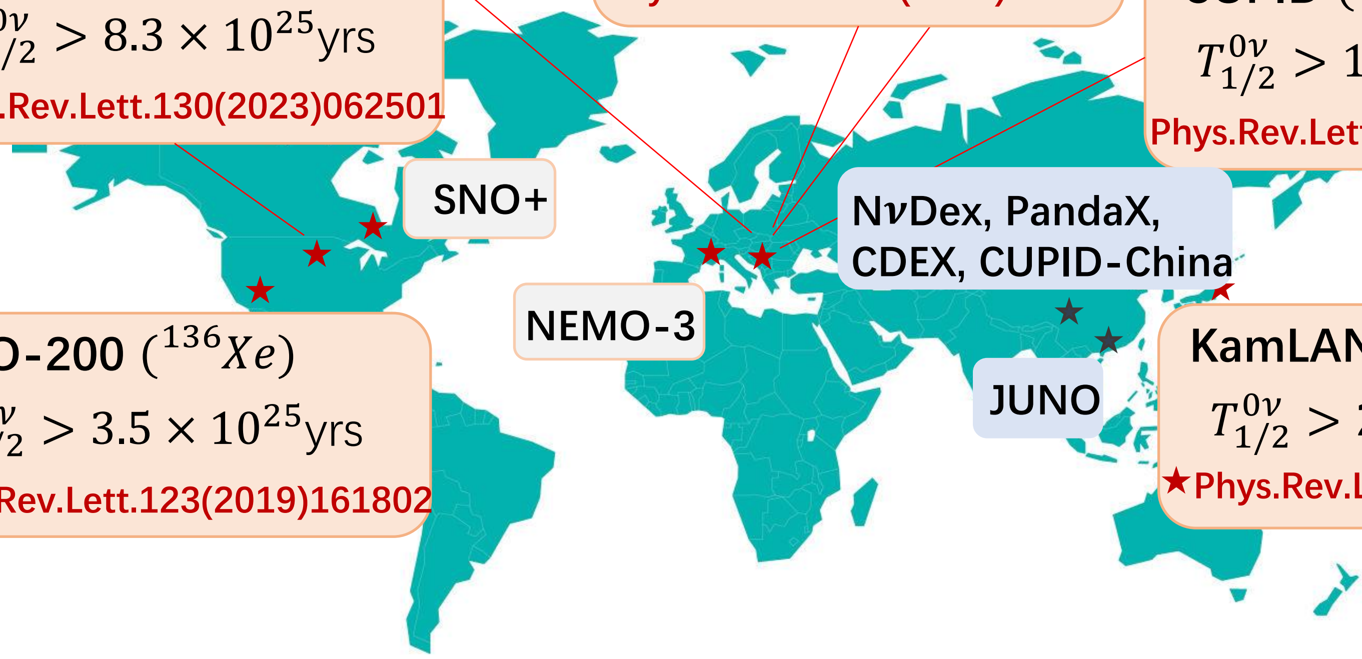
$T_{1/2}^{0\nu} > 1.5 \times 10^{24}$yrs

Phys.Rev.Lett.126(2021)181802

KamLAND-Zen $({}^{136}Xe)$

$T_{1/2}^{0\nu} > 2.3 \times 10^{26}$yrs

★Phys.Rev.Lett.130(2023)051801



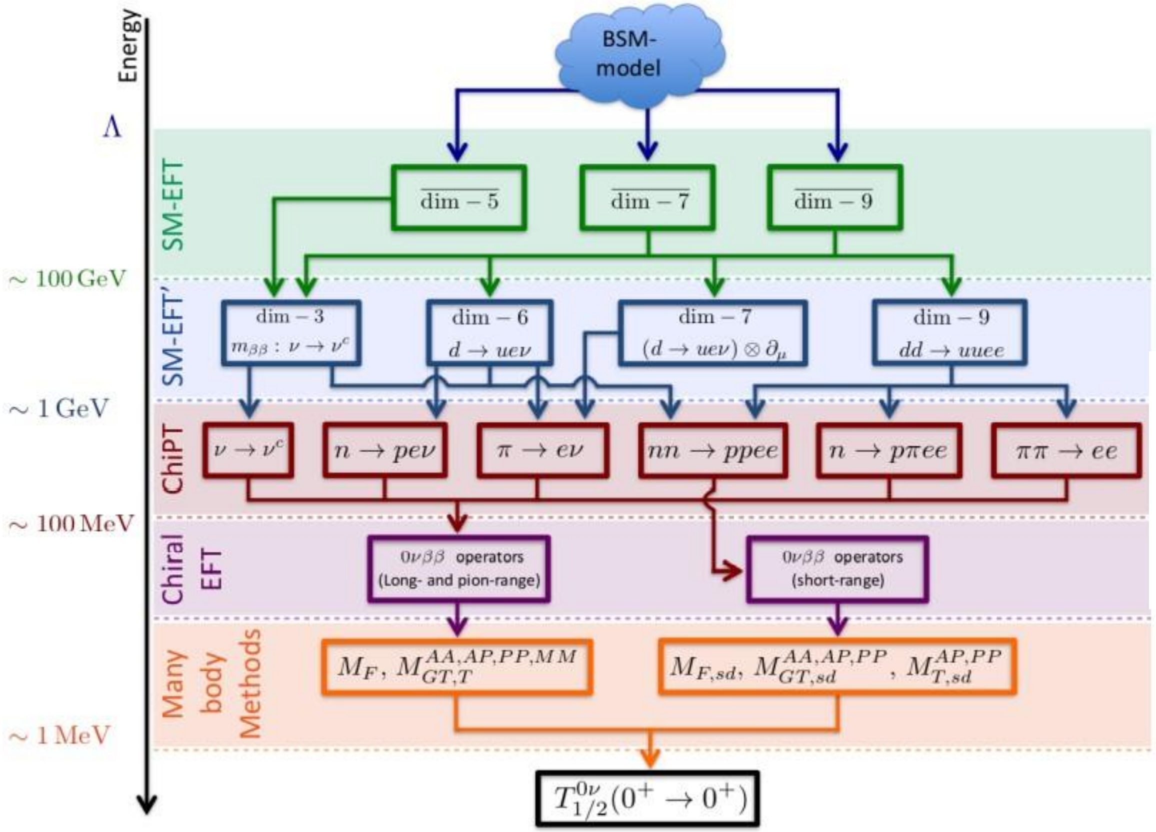
?? Why do we need so many experiments using different candidate nuclei?

- **Benchmark the results from different experiments.**
- **Validate the accuracy of nuclear many-body calculation.**
- **Unravel the $0\nu\beta\beta$ decay mechanisms.**

L. Graf et al., PRD 106(2022)035022

Different mechanisms of $0\nu\beta\beta$ decay

⋆ $0\nu\beta\beta$ decay is a kind of a black box, any $\Delta L = 2$ process could contribute to $0\nu\beta\beta$ decay.



From the perspective of the effective field theory (EFT), after the electroweak symmetry breaking, match the LNV operators in SMEFT to the low energy scale, and then match them to chiral EFT. V. Cirigliano et al., JHEP12(2018)097

➢ Half-life master formula

$$
\begin{aligned}
\left(T_{1/2}^{0\nu}\right)^{-1} =& g_A^4 \Big\{ G_{01} \left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2\right) - 2\left(G_{01} - G_{04}\right) \mathrm{Re}\, \mathcal{A}_\nu^* \mathcal{A}_R + 4 G_{02} |\mathcal{A}_E|^2 \\
&+ 2 G_{04} \left[|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left(\mathcal{A}_{m_e}^* \left(\mathcal{A}_\nu + \mathcal{A}_R\right)\right)\right] - 2 G_{03}\, \mathrm{Re}\left[\left(\mathcal{A}_\nu + \mathcal{A}_R\right)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right] \\
&+ G_{09} |\mathcal{A}_M|^2 + G_{06}\, \mathrm{Re}\left[\left(\mathcal{A}_\nu - \mathcal{A}_R\right)\mathcal{A}_M^*\right]\Big\}
\end{aligned}
$$

The sub-amplitudes $A^\alpha = \sum_i C_i M_i^\alpha$ contain various Wilson coefficients C_i and nuclear matrix elements M_i^α.

Contain the information from high-energy physics

Q: how to distinguish the contributions from different decay mechanisms / present accurate constraints on $\langle m_{\beta\beta} \rangle$?

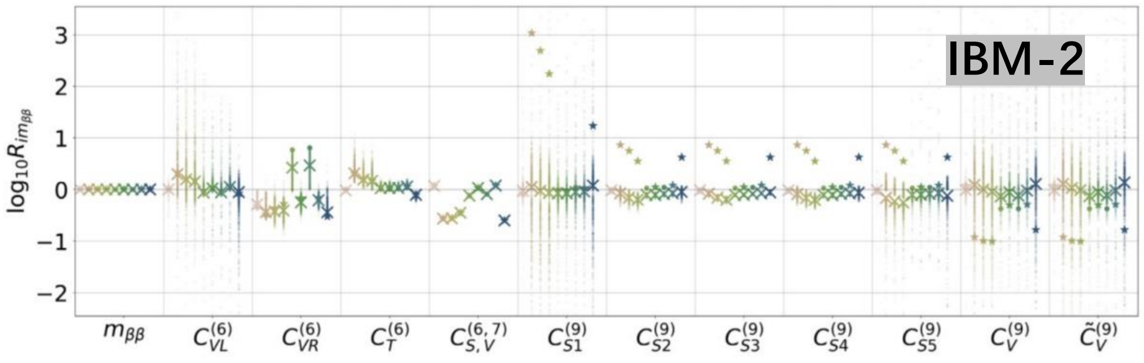
Different mechanisms of $0\nu\beta\beta$ decay

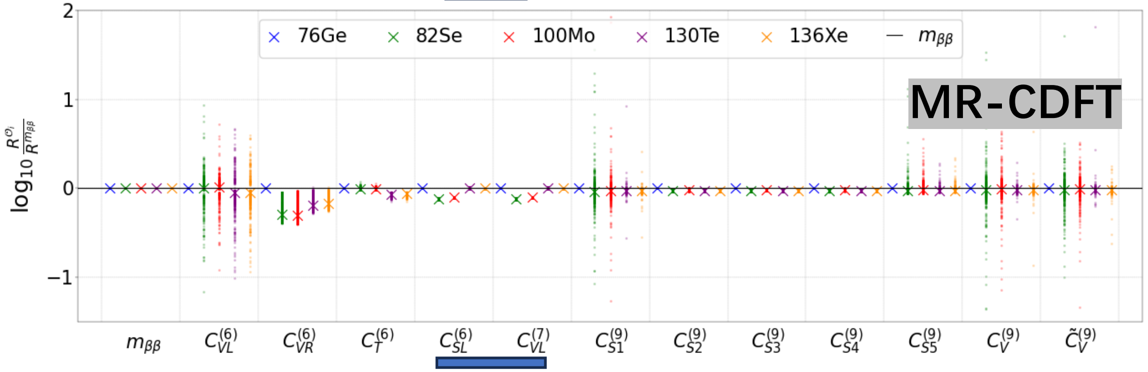
The analysis of the distinguishability of different decay mechanisms

L. Graf et al., PRD 106(2022)035022

$$R^{\mathcal{O}_i}(^AX) = \frac{T_{1/2}^{\mathcal{O}_i}(^AX)}{T_{1/2}^{\mathcal{O}_i}(^{76}Ge)} = \frac{\sum_j |\mathcal{M}_j^{\mathcal{O}_i}(^{76}Ge)|^2 G_j^{\mathcal{O}_i}(^{76}Ge)}{\sum_k |\mathcal{M}_k^{\mathcal{O}_i}(^AX)|^2 G_k^{\mathcal{O}_i}(^AX)}, \qquad R_{ij}(^AX) = \frac{R^{\mathcal{O}_i}(^AX)}{R^{\mathcal{O}_j}(^AX)}.$$

- One way: different candidate nuclei?

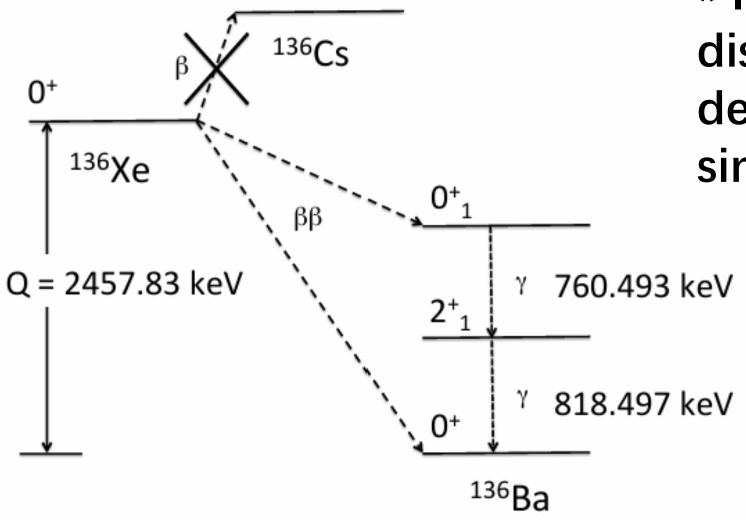




* Many-body model dependent

- Another way: different decay channels?

-- Decay to excited final 0^+ state nuclei.



* Is it possible to distinguish different decay mechanisms in single experiment?

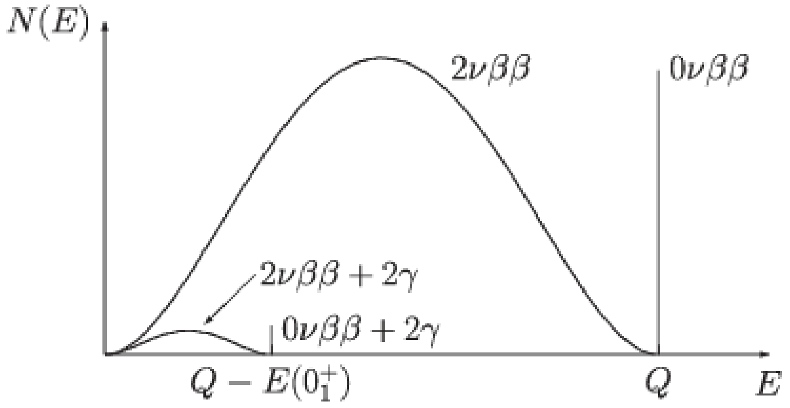
* Reduce the many-body model dependence?

J. B. Albert, et al., PRC 93(2016)035501

Search for the $0\nu\beta\beta$ decay to excited 0^+ states

* Sum energy spectrum of the emitted electrons

M. Duerr et al., PRD 84(2011)093004



The ratio between the decay rate to the excited 0^+ state and the ground state is given by

$$\frac{\Gamma_{0_1^+}}{\Gamma_{\mathrm{g.s.}}} = \boxed{\frac{(Q - E(0_1^+))^n}{Q^n}} \times \boxed{\left(\frac{\mathcal{M}^{0_1^+}}{\mathcal{M}^{\mathrm{g.s.}}}\right)^2}$$

$G^{0\nu}(Q)$

$n = 5$ for $0\nu\beta\beta$ mode

$n = 11$ for $2\nu\beta\beta$ mode

Decay mode	Q [keV]	$E(0_1^+)$ [keV]	$(Q - E(0_1^+))^5/Q^5$
$^{48}_{20}Ca \to ^{48}_{22}Ti$	4274 ± 4 [7]	2997	2.38×10^{-3}
$^{76}_{32}Ge \to ^{76}_{34}Se$	2039.04 ± 0.16 [8]	1122	1.84×10^{-2}
$^{82}_{34}Se \to ^{82}_{36}Kr$	2995.5 ± 1.9 [7]	1488	3.23×10^{-2}
$^{96}_{40}Zr \to ^{96}_{42}Mo$	3347.7 ± 2.2 [7]	1148	1.22×10^{-1}
$^{100}_{42}Mo \to ^{100}_{44}Ru$	3034.4 ± 0.17 [8]	1130	9.74×10^{-2}
$^{110}_{46}Pd \to ^{110}_{48}Cd$	2004 ± 11 [7]	1473	1.31×10^{-3}
$^{116}_{48}Cd \to ^{116}_{50}Sn$	2809 ± 4 [7]	1757	7.37×10^{-3}
$^{124}_{50}Sn \to ^{124}_{52}Te$	2287 ± 1.5 [7]	1657	1.59×10^{-3}
$^{130}_{52}Te \to ^{130}_{54}Xe$	2527.518 ± 0.013 [9]	1794	2.06×10^{-3}
$^{136}_{54}Xe \to ^{136}_{56}Ba$	2457.83 ± 0.37 [10,11]	1579	5.84×10^{-3}
$^{150}_{60}Nd \to ^{150}_{62}Sm$	3371.38 ± 0.2 [12]	740	2.9×10^{-1}

Decay mode	$\mathcal{M}^{\mathrm{g.s.}}_{0\nu}$	$\mathcal{M}^{0_1^+}_{0\nu}$
$^{48}_{20}Ca \to ^{48}_{22}Ti$	...	...
$^{76}_{32}Ge \to ^{76}_{34}Se$	5.465	2.479
$^{82}_{34}Se \to ^{82}_{36}Kr$	4.412	1.247
$^{96}_{40}Zr \to ^{96}_{42}Mo$	2.53	0.044
$^{100}_{42}Mo \to ^{100}_{44}Ru$	3.732	0.419
$^{110}_{46}Pd \to ^{110}_{48}Cd$	3.623	1.599
$^{116}_{48}Cd \to ^{116}_{50}Sn$	2.782	1.047
$^{124}_{50}Sn \to ^{124}_{52}Te$	3.532	2.721
$^{130}_{52}Te \to ^{130}_{54}Xe$	4.059	3.09
$^{136}_{54}Xe \to ^{136}_{56}Ba$	3.352	1.837
$^{150}_{60}Nd \to ^{150}_{62}Sm$	2.321	0.395

Calculated by IBM-2

J. Barea et al., PRC 79(2009)044301

NMEs of the $0\nu\beta\beta$ decay to excited 0^+ states

* QRPA: $$|0_1^+\rangle = \frac{1}{\sqrt{2}}\{\Gamma_2^{1\dagger}\otimes\Gamma_2^{1\dagger}\}^0|0_{\mathrm{g.s.}}^+\rangle,$$

$$\langle 0_1^+|\widetilde{[c_p^\dagger \tilde{c}_n]_J}|J^\pi m$$
$$=\langle 0_{RPA}^+|\frac{1}{\sqrt{2}}\{\Gamma_2\otimes\Gamma_2\}^0\{[c_p^\dagger\tilde{c}_n]_J\otimes Q_{J^\pi}^{m\dagger}\}^0|0_{RPA}^+\rangle\sqrt{2J+1}.$$

* Γ_2 is the quadrupole phonon operator

F. Simkovic et al., PRC 64(2001)035501

J. Suhonen, NPA 853(2011)36-60

* IBM-2: obtain the excited 0^+ states by s-boson excitation or d-boson pair excitation (By Deepseek)

J. Barea et al., PRC 91(2015)034304

? Similar to the QRPA excited states?

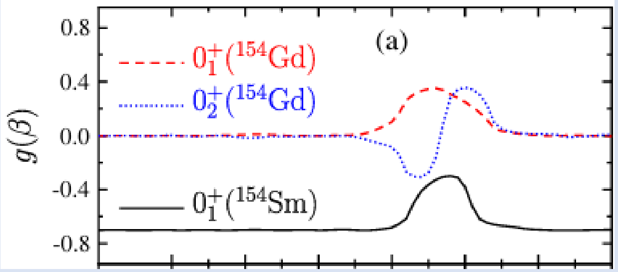
➢ *How to obtain the ground state and excited states self-consistently? --GCM*

* ISM: Diagonalize the Hamiltonian matrix in the shell model configuration space

L. Coraggio et al., arXiv: 2203.01013 J. Menendez et al., NPA 818(2009)139-151

* EDF+GCM: Diagonalize the Hamiltonian matrix in the nuclear deformation configuration space

J. Beller et al., PRL 111(2013)172501 L. S. Song et al., PRC 90(2014)054309



➢ The less pronounced change in deformation between mother ground-state nuclei and daughter excited-state nuclei may enlarge the NMEs of decay to excited states?

-- Can be tested in MR-CDFT calculations!

Theoretical framework of the MR-CDFT

- The mean-field wave function $|\boldsymbol{\Phi}(\boldsymbol{q})\rangle$ are generated from the relativistic mean-field + Bardeen-Cooper-Schrieffer (RMF+BCS) theory with constraint on the mass quadrupole moment⋯

$$\langle\Phi|\hat{H}|\Phi\rangle = \left\langle \Phi \left| \hat{H}_0 - \sum_{\tau=n,p} \lambda_\tau \hat{N} \right| \Phi \right\rangle - \frac{1}{2}\lambda_Q\left(\langle\Phi|\hat{Q}_{20}|\Phi\rangle - q_{20}\right)^2$$

and $|\boldsymbol{\Phi}\rangle$ is the BCS state $\quad |\Phi\rangle = \prod_{k>0}(u_k + v_k c_k^\dagger c_{\bar{k}}^\dagger)|0\rangle$

$$\left\langle \Phi \left| \hat{H}_0 - \sum_{\tau=n,p} \lambda_\tau \hat{N} \right| \Phi \right\rangle$$

$$= \sum_{k>0}\left[\left(2\epsilon_k - \sum_{\tau=n,p} 2\lambda_\tau\right) v_k^2 - \int d\boldsymbol{r} \sum_{\tau=n,p} V_\tau \sum_{k'>0} f_k f_{k'} u_k v_k u_{k'} v_{k'} |\varphi_k(\boldsymbol{r})|^2 |\varphi_{k'}(\boldsymbol{r})|^2\right]$$

- GCM: solving the Hill-Wheeler-Griffin (HWG) equation

$$\sum_{\boldsymbol{q}}\left[H_{00}^J(\boldsymbol{q},\boldsymbol{q}') - E_\sigma^J N_{00}^J(\boldsymbol{q},\boldsymbol{q}')\right] f_\sigma^J(\boldsymbol{q}') = 0 \quad \begin{cases} H_{00}^J(\boldsymbol{q},\boldsymbol{q}') = \langle\Phi(\boldsymbol{q})|\hat{H}\hat{P}_{00}^J\, \hat{P}^N\, \hat{P}^Z|\Phi(\boldsymbol{q}')\rangle \\ N_{00}^J(\boldsymbol{q},\boldsymbol{q}') = \langle\Phi(\boldsymbol{q})|\hat{P}_{00}^J\, \hat{P}^N\, \hat{P}^Z|\Phi(\boldsymbol{q}')\rangle \end{cases}$$

> *Three treatments:*

- Fit V_τ for all candidate nuclei and $\boldsymbol{q} \in \{q_{20}\}$.

  --Label as GCM(PC-PK1)

- Vary V_τ for candidate nuclei and $\boldsymbol{q} \in \{q_{20}\}$.

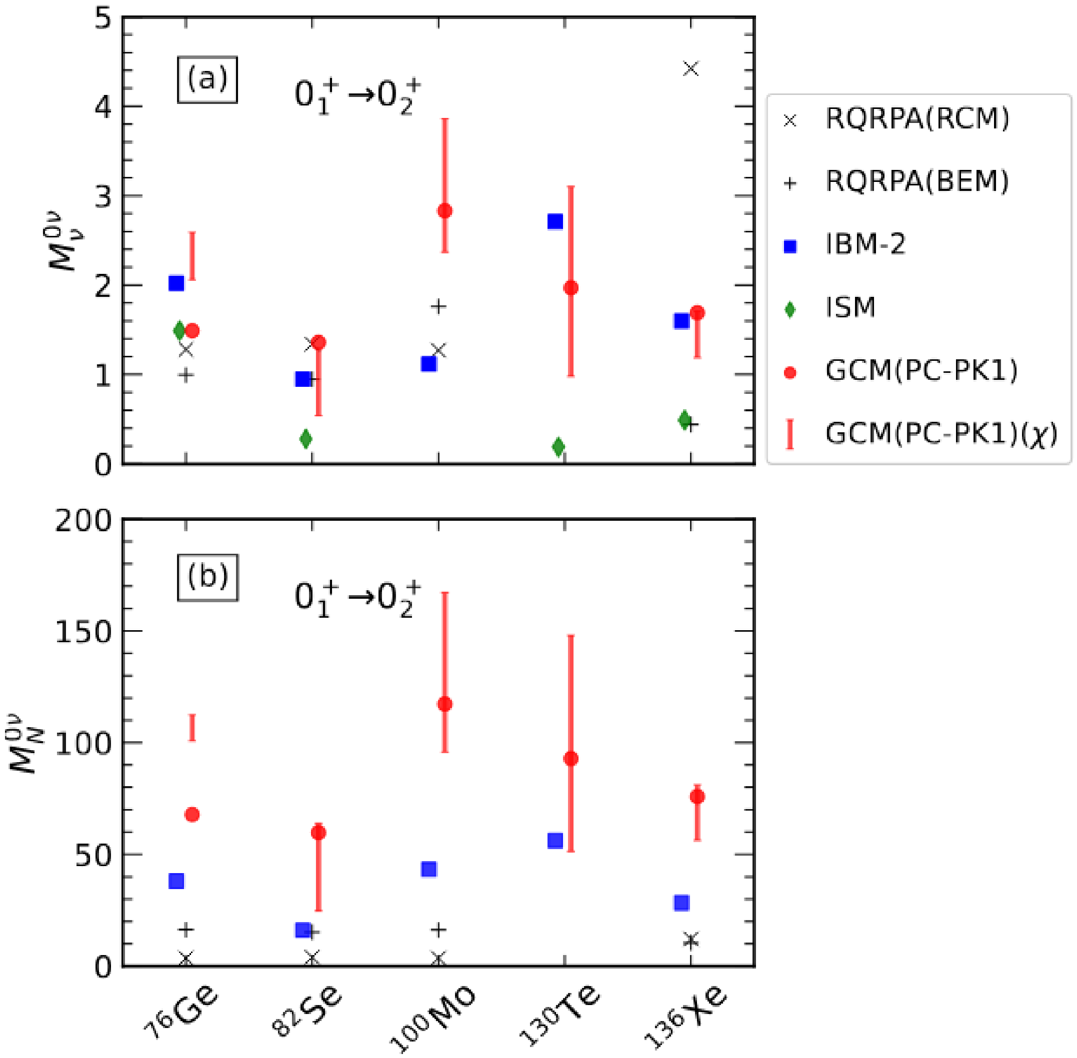
  --Label as the lower limit of GCM(PC-PK1)(χ)

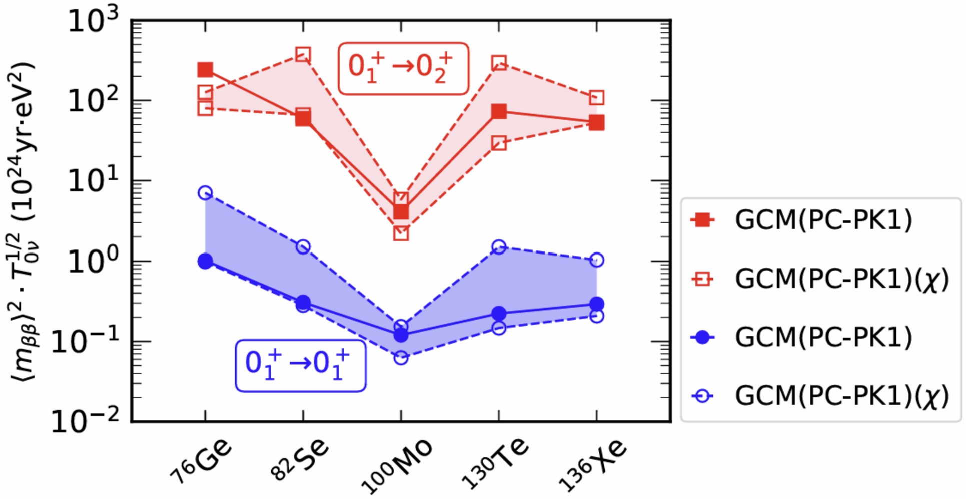
- Vary V_τ for candidate nuclei and $\boldsymbol{q} \in \{q_{20}, \Delta_{uv}\}$.

  * Δ_{uv}: isovector pairing gap

  --Label as the upper limit of GCM(PC-PK1)(χ)

C. R. Ding et al., PRC.108(2023)054304

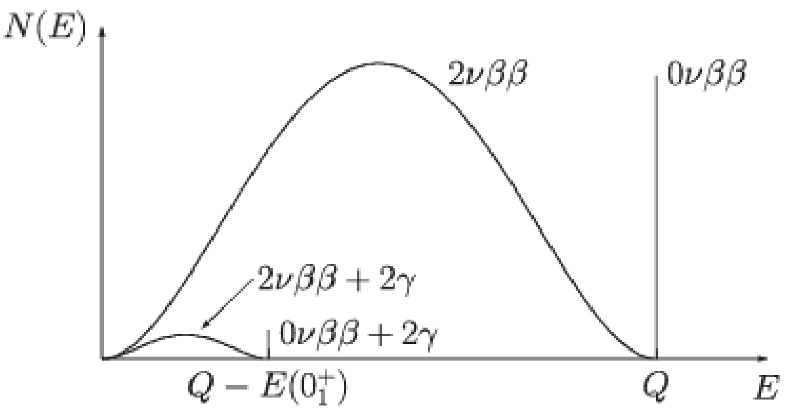




$$\frac{\left(\langle m_{\beta\beta}\rangle^2 \cdot T_{0\nu}^{1/2}\right)_{0_1^+\to 0_2^+}}{\left(\langle m_{\beta\beta}\rangle^2 \cdot T_{0\nu}^{1/2}\right)_{0_1^+\to 0_1^+}} \approx 10\sim 100$$

Is it possible to reduce the background of the decay signal to excited 0^+ states?

Implication from the perspective of experiments



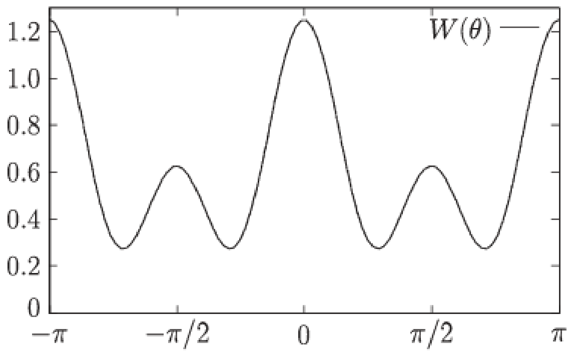
M. Duerr et al., PRD 84(2011)093004

- The expected signature for the excited 0^+ decay is 2 electrons and 2 gammas with defined energies.

A purely calorimetric approach without spatial resolution to determine the individual gammas will fail.

- ☐ From the $\gamma\gamma$ angular correlation, one can find the emitted photons at the angles 0 and π most possibly.

$$W(\theta)=\frac{5}{8}\times(1-3\cos^2\theta+4\cos^4\theta).$$



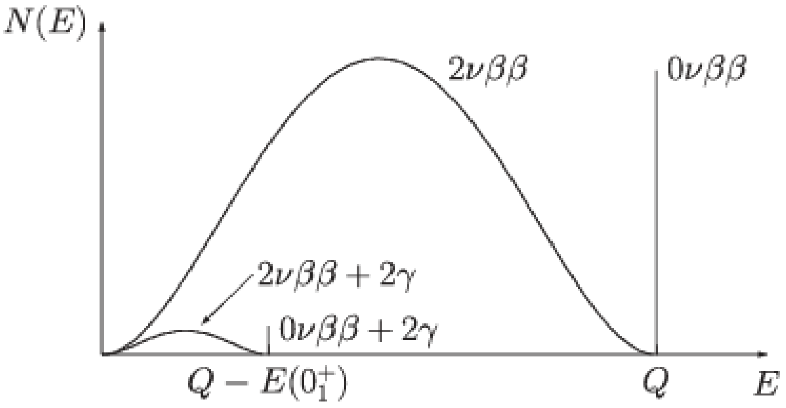
- ☐ Building a detector with tracking capabilities is essential.

The signal of a decay into excited final states will be a triple coincidence with well-defined energies. These constraints make the signal search more or less background free.

- *Major background types:*
  - * The single-beta decay to excited state from the intermediate nucleus (produced by (p,n) reactions)
    - --Depends on the energy difference between mother and intermediate nuclei…
    - --Depends on the quantum number of intermediate nucleus (allowed or forbidden beta decays)

☐ May be solved by building a detector with tracking capabilities…

Implication from the perspective of experiments



M. Duerr et al., PRD 84(2011)093004

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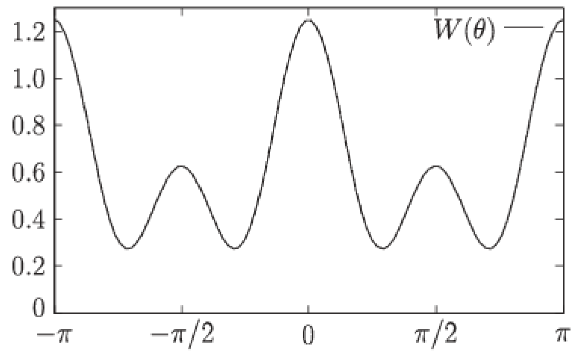


- Building a detector with tracking capabilities is essential.

The signal of a decay into excited final states will be a triple coincidence with well-defined energies. These constraints make the signal search more or less background free.

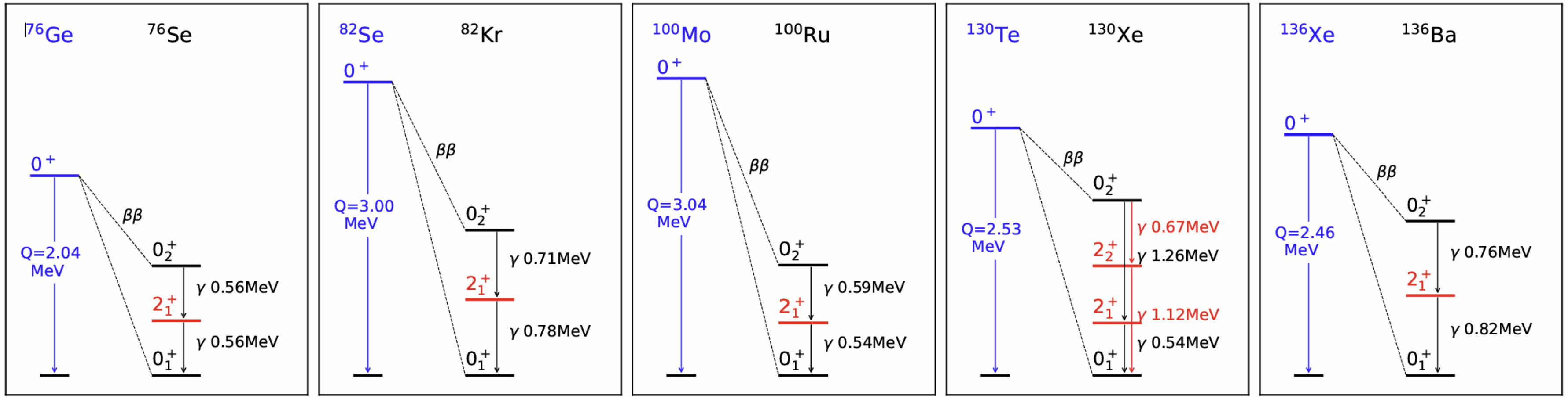
- *Major background types:*
  - * The single-beta decay to excited state from the intermediate nucleus (produced by (p,n) reactions)
  - * The $2\nu\beta\beta$ decay into the excited 0^+ state

- High energy resolution of the detector is essential.

--The background might be smaller than the case of decay to ground state, since the reduction rate of the PSF is different for $0\nu\beta\beta$ $(n=5)$ and $2\nu\beta\beta$ $(n=11)$ decay⋯

Implication from the perspective of experiments

- With the high granularity detectors, relatively small crystals in a liquid with a fair spatial resolution or tracking devices, maybe one can consider the background-free(limited) decay to excited(ground) states.



The half-life sensitivity:

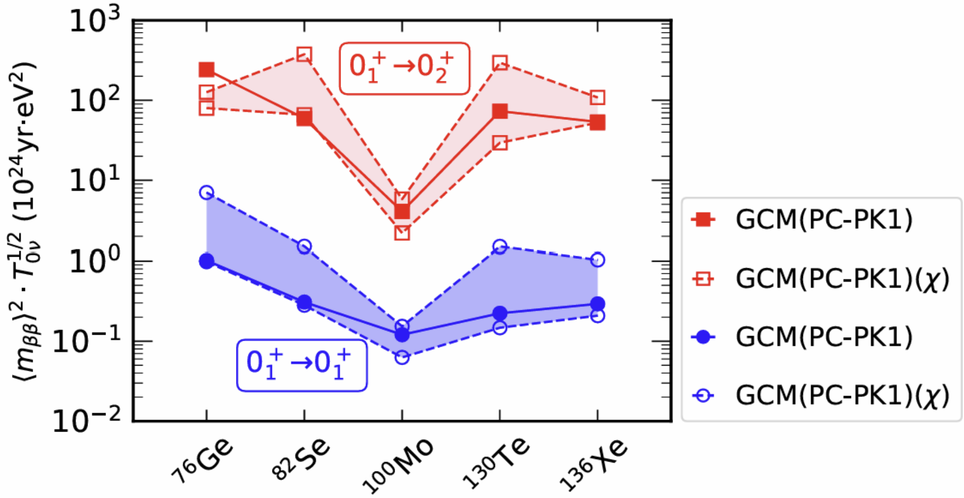
$$(T_{1/2})^{-1} \propto aM\epsilon t$$

(background free)

$$(T_{1/2})^{-1} \propto a\epsilon\sqrt{\frac{Mt}{B\Delta E}}$$

(background limited)

M. Duerr et al., PRD 84(2011)093004



- What's more, our method may allow a better distinguishability of the different decay mechanisms, since common errors in the calculations of NMEs may cancel in ratios $R_{ij}(^{A}X) = \frac{R^{O_i}(^{A}X)}{R^{O_j}(^{A}X)}$.

L. Graf et al., PRD 106(2022)035022

Thank you for your attention!