中山大学 物理与天文学院

凤凰山论坛

原子核中的赝自旋对称性与超对称量子力学

Haozhao LIANG（梁豪兆）

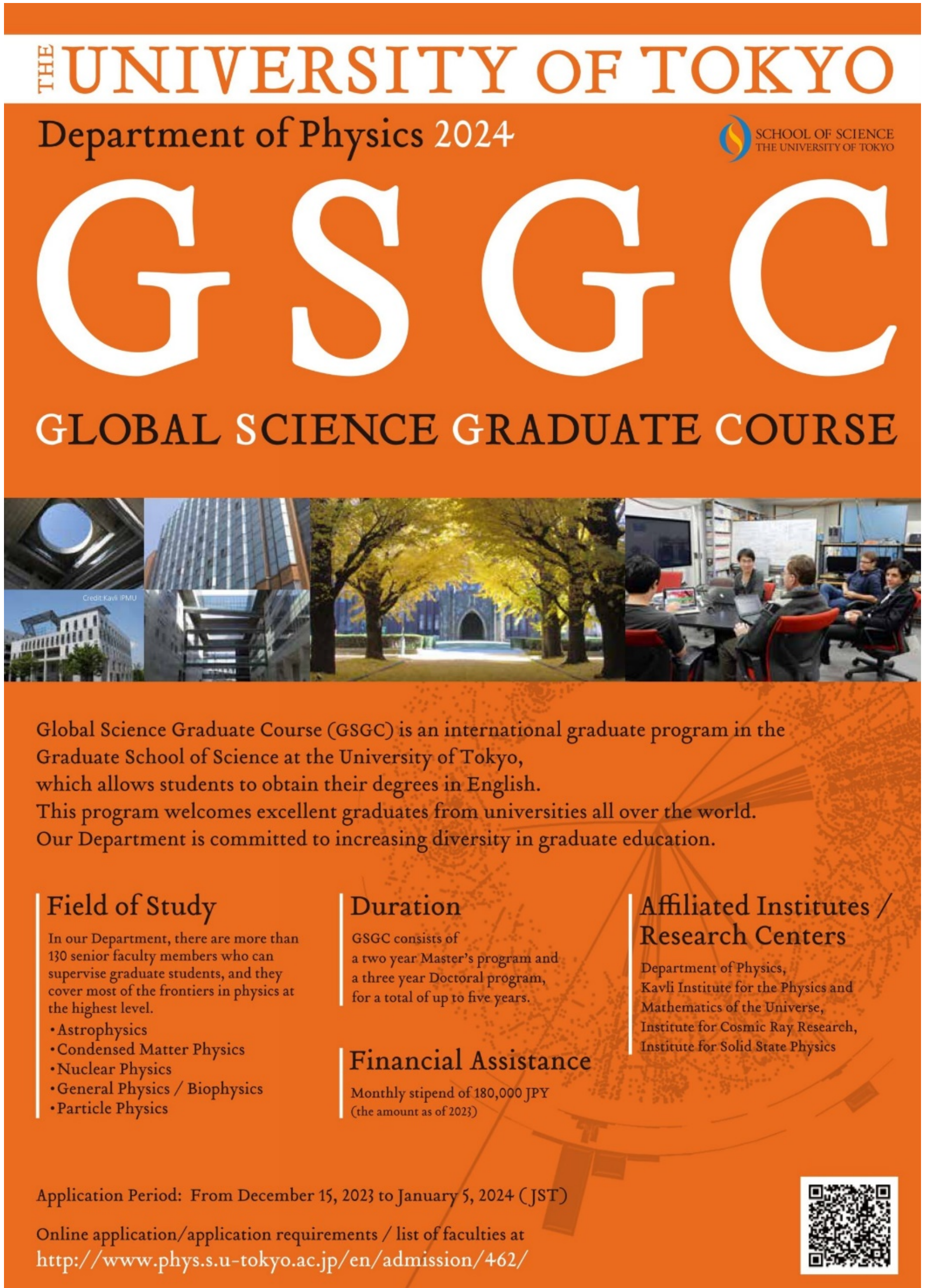
Department of Physics, The University of Tokyo, Japan

March 7, 2024

東京大学
THE UNIVERSITY OF TOKYO

Lab homepage: https://tnp.phys.s.u-tokyo.ac.jp/

(Recruiting international master and Ph.D. students every year)



https://www.phys.s.u-tokyo.ac.jp/en/admission/462/

Contents

Nuclear physics and its frontiers

- An overview of **nuclear chart**
- Superheavy elements (**SHE**) and superheavy nuclei (**SHN**)
- Exotic nuclei and their impacts on ***r*-process** nucleosynthesis

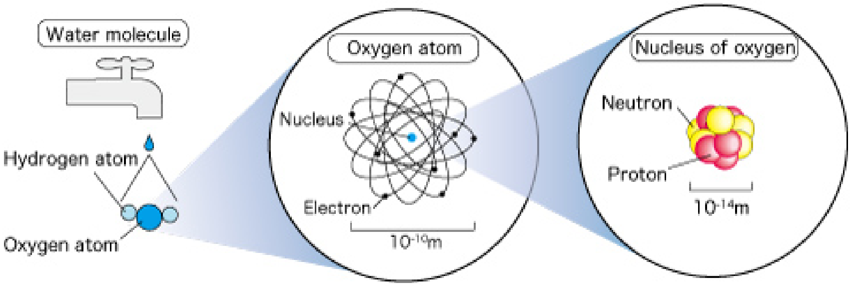
Nuclear pseudospin symmetry (PSS) and supersymmetric (SUSY) quantum mechanics

- PSS in atomic nuclei
- Conventional understandings
- Further investigations

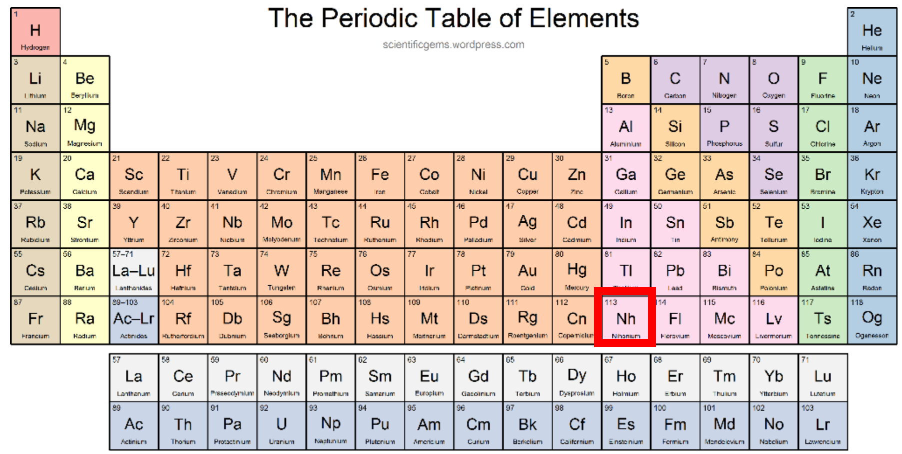
Summary and perspectives

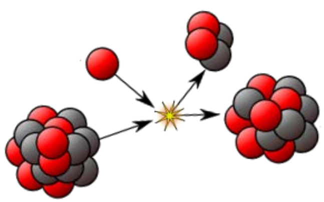
Nuclear physics research

Molecule – Atom – Nucleus



118 elements



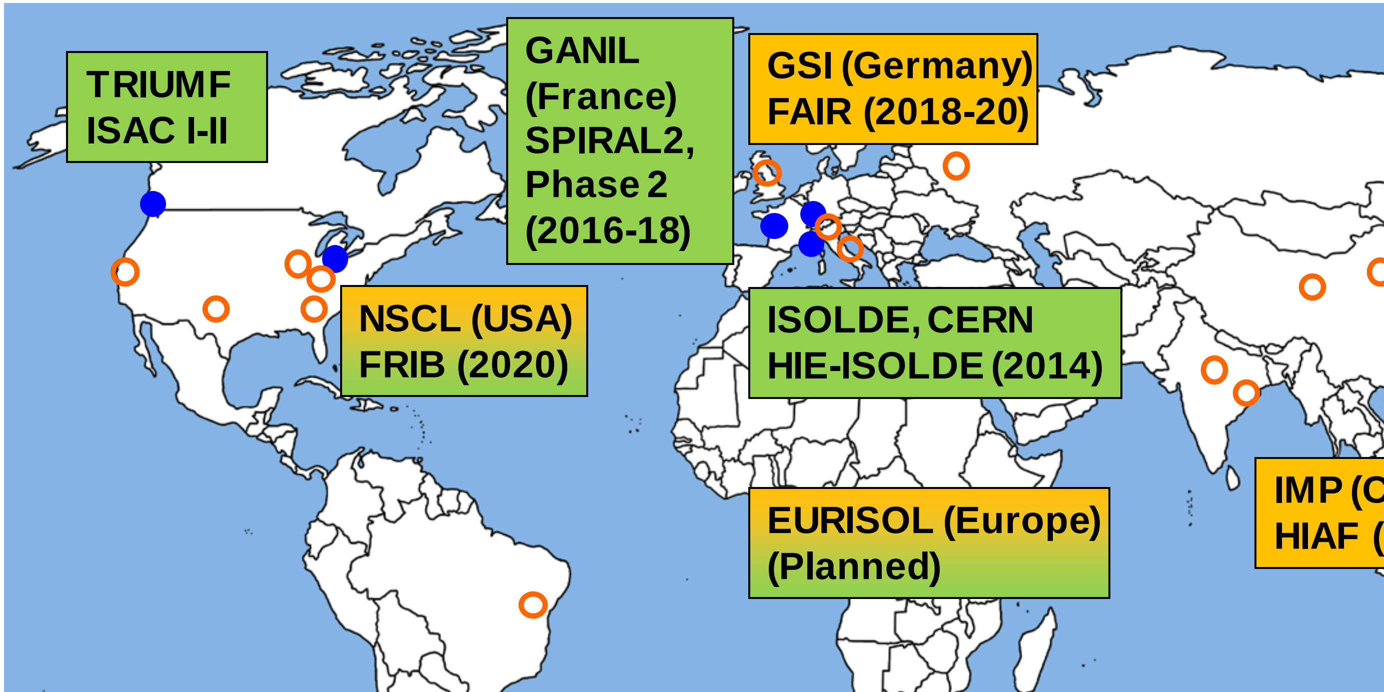


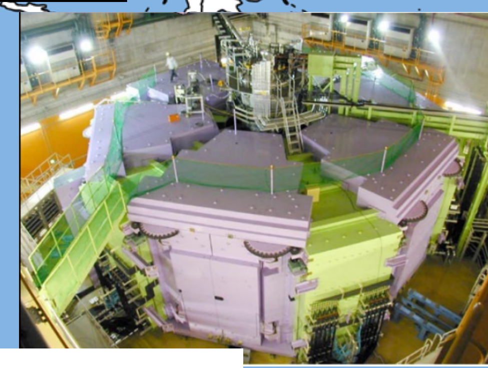
"How many kinds of nuclei exist in the Universe?" *(nuclear physics)*

"Where and how are nuclei created in the Universe?" *(nuclear physics × astrophysics)*

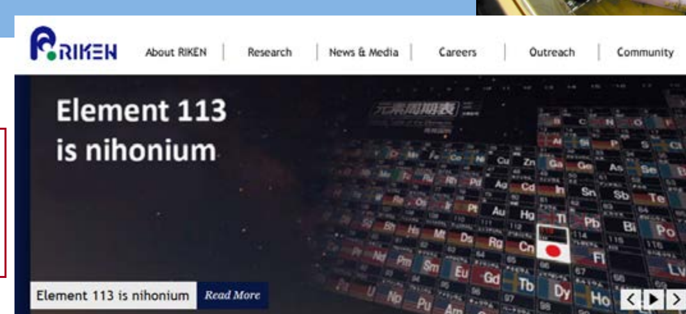
"How do we make the best and safe use of nuclei?" *(nuclear physics × technology)*

Radioactive isotope beam facilities





Best in the world
~70 % speed of light !



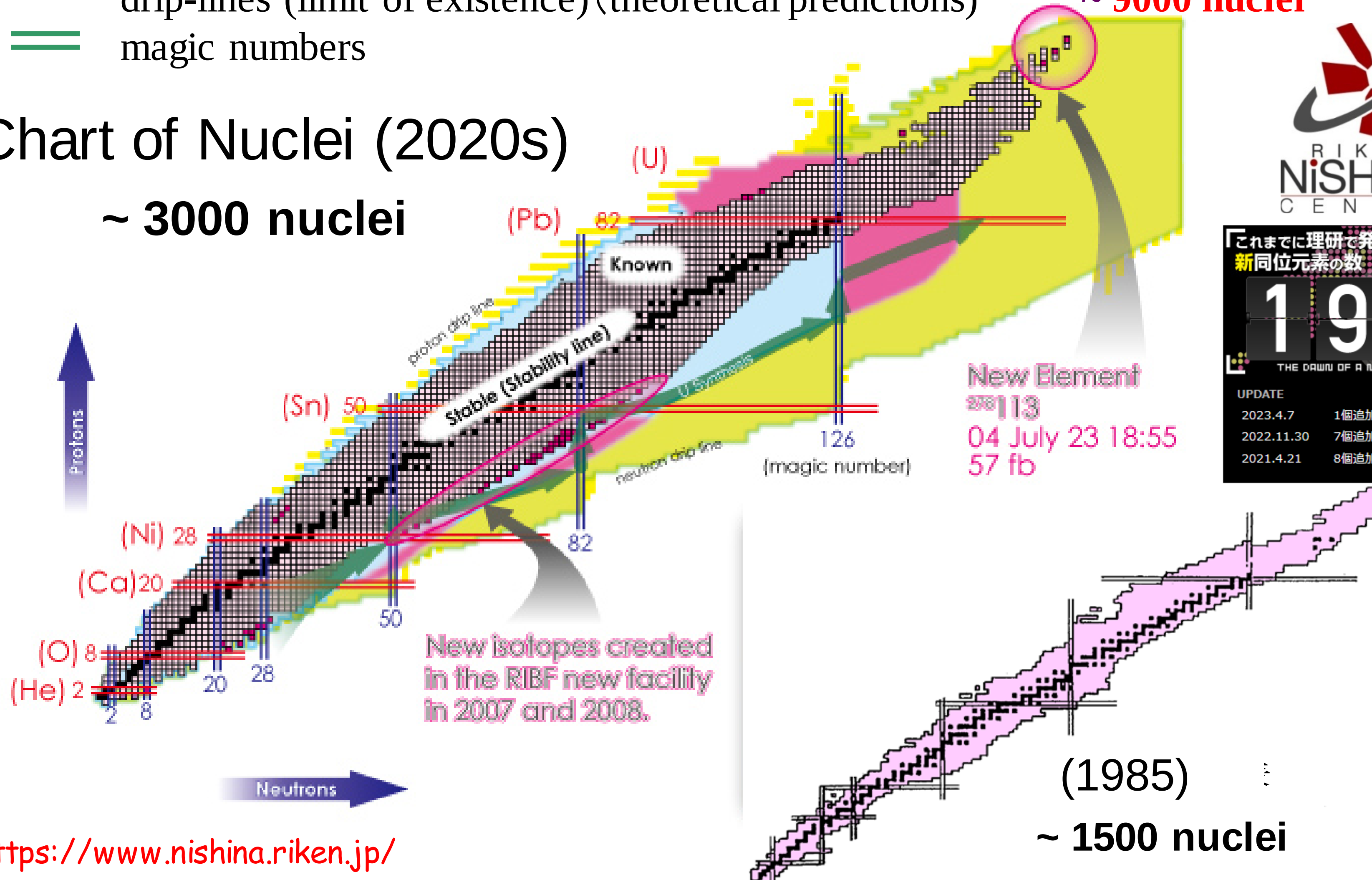
RIKEN NiSHiNA CENTER

Nuclear chart

■	stable nuclei	~ 300 nuclei
□	unstable nuclei observed so far	~ 3000 nuclei
━	drip-lines (limit of existence) (theoretical predictions)	~ 9000 nuclei
═	magic numbers	

Chart of Nuclei (2020s)

~ 3000 nuclei



(1985)

~ 1500 nuclei

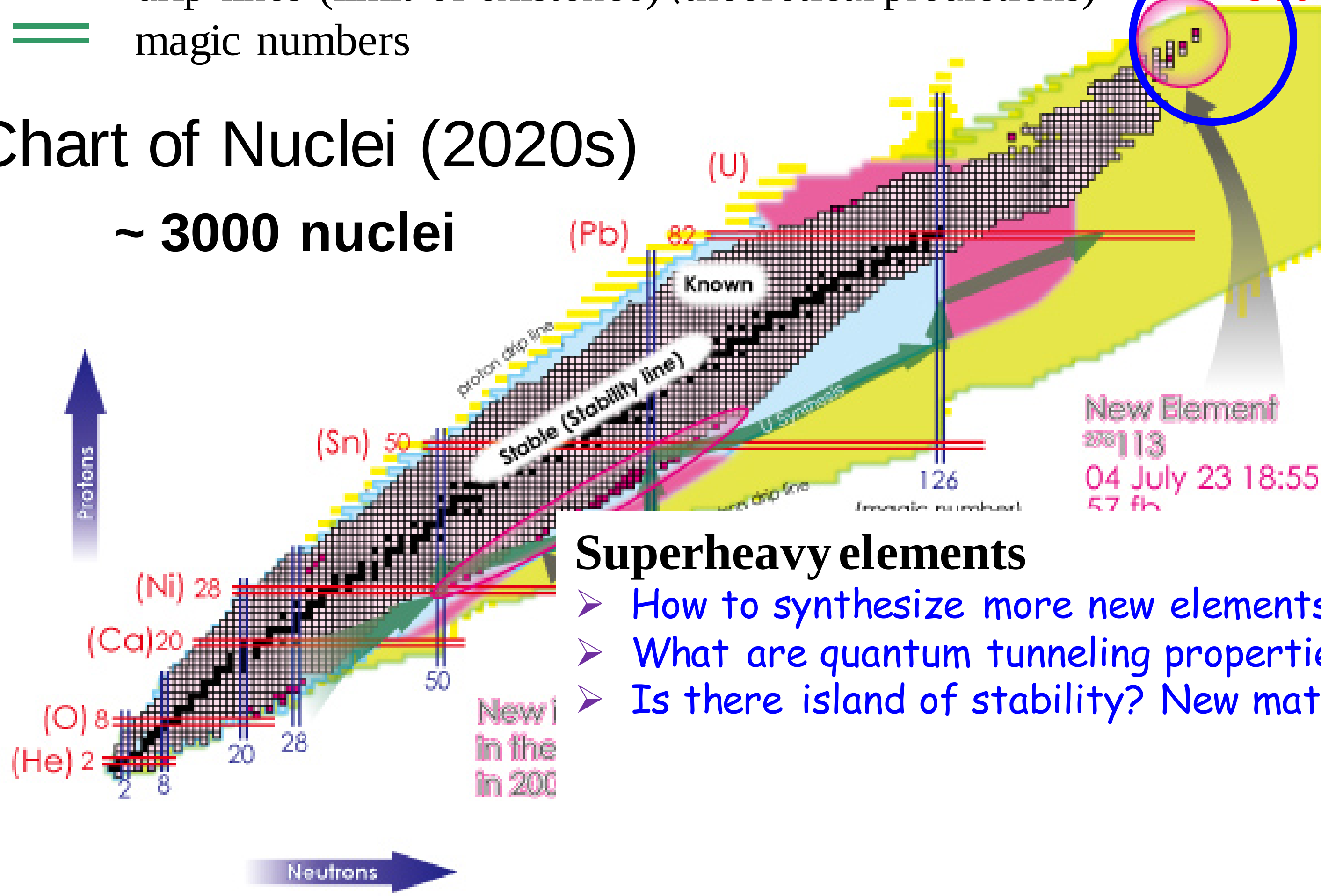
https://www.nishina.riken.jp/

Nuclear chart

■	stable nuclei	~ **300 nuclei**
□	unstable nuclei observed so far	~ **3000 nuclei**
—	drip-lines (limit of existence) (theoretical predictions)	~ **9000 nuclei**
=	magic numbers	

Chart of Nuclei (2020s)

~ 3000 nuclei



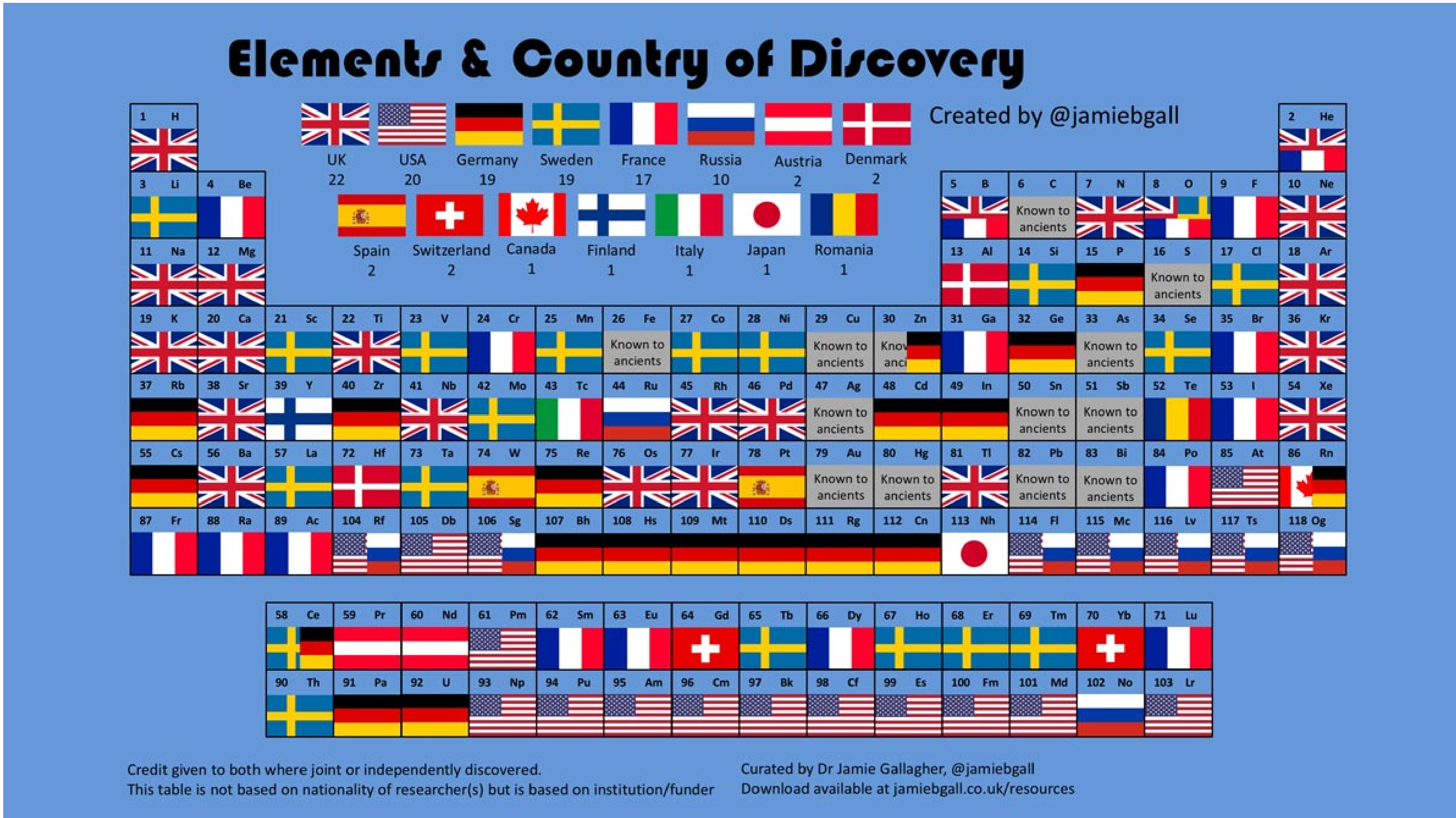
RIKEN NiSHiNA CENTER

Superheavy elements

- How to synthesize more new elements?
- What are quantum tunneling properties of SHE?
- Is there island of stability? New materials?

https://www.nishina.riken.jp/

Periodic Table with National flags



Open question:

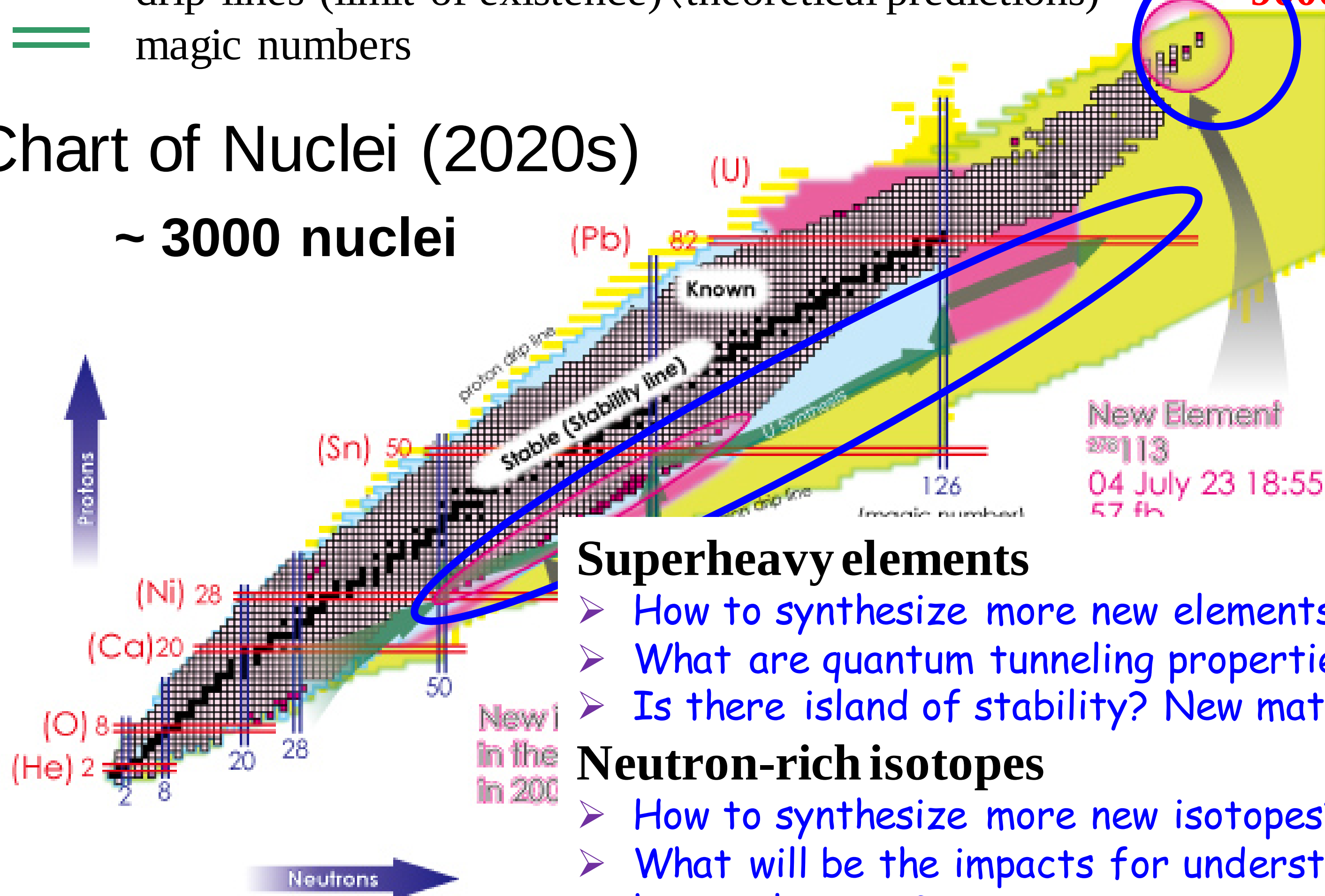
- Is there or where is **the end** of the periodic table?

Nuclear chart

■	stable nuclei	**~ 300 nuclei**
□	unstable nuclei observed so far	**~ 3000 nuclei**
—	drip-lines (limit of existence) (theoretical predictions)	**~ 9000 nuclei**
=	magic numbers	

Chart of Nuclei (2020s)

~ 3000 nuclei



RIKEN NISHINA CENTER

Superheavy elements

- How to synthesize more new elements?
- What are quantum tunneling properties of SHE?
- Is there island of stability? New materials?

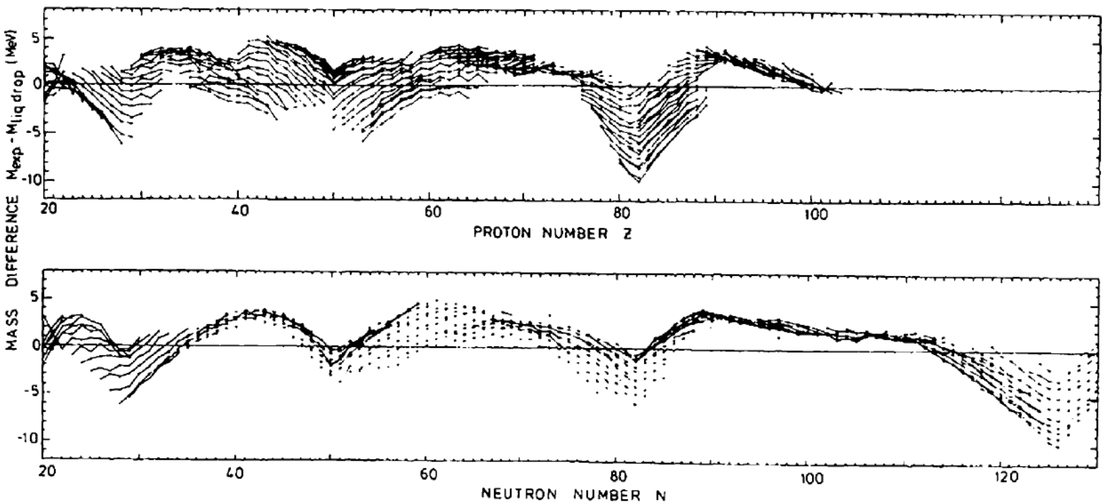
Neutron-rich isotopes

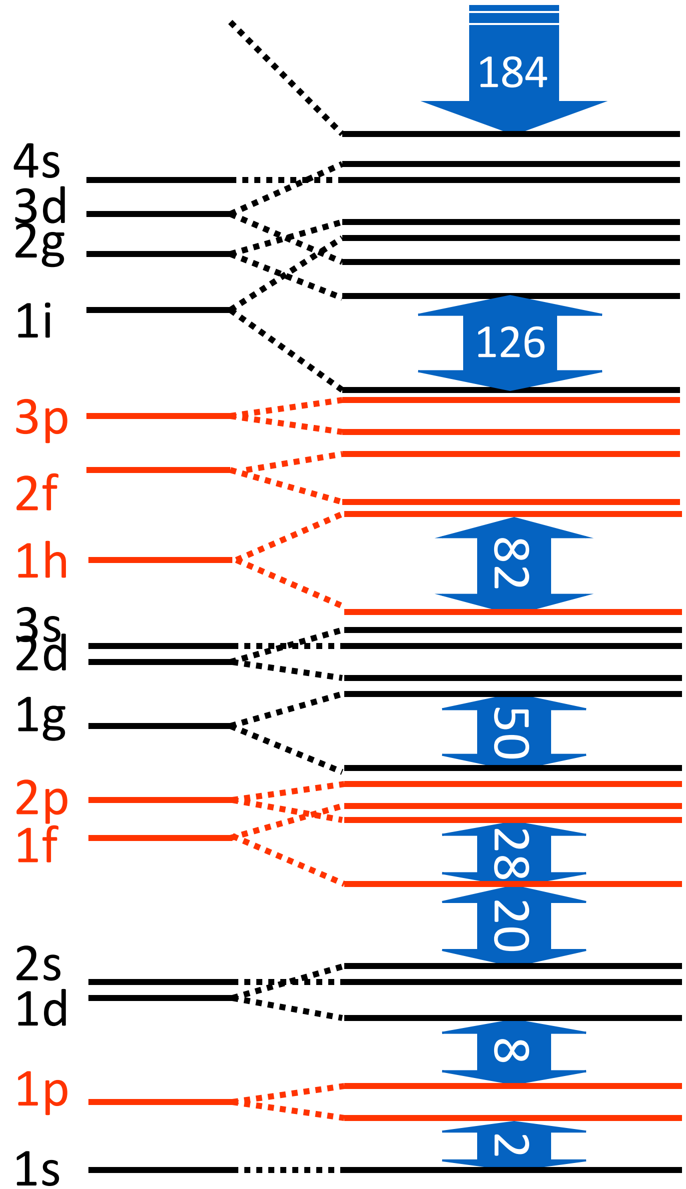
- How to synthesize more new isotopes?
- What will be the impacts for understanding origins of heavy elements?
- What will be the impacts for handling nuclear wastes?

https://www.nishina.riken.jp/

Nuclear magic numbers

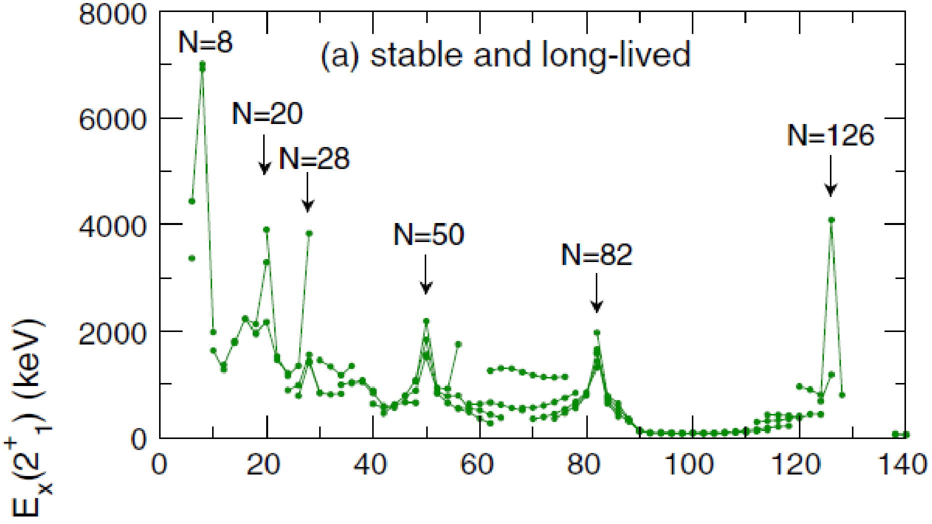
- Shell-correction energy

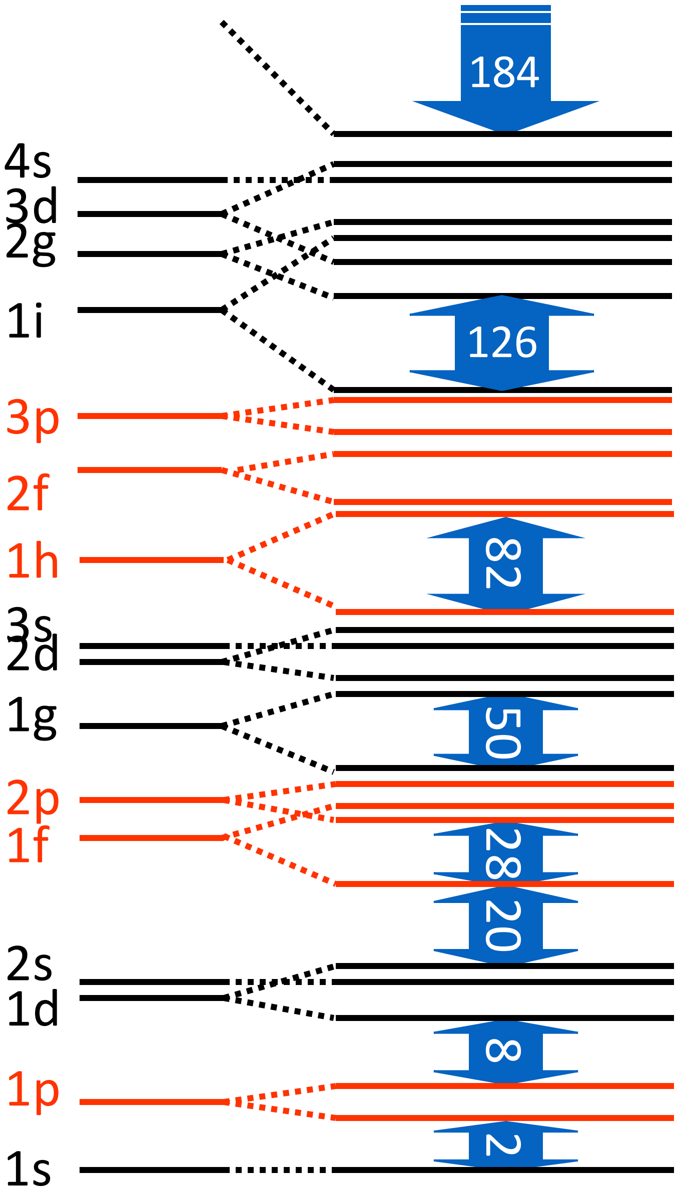




Nuclear magic numbers

- Energy of the first 2^+ state



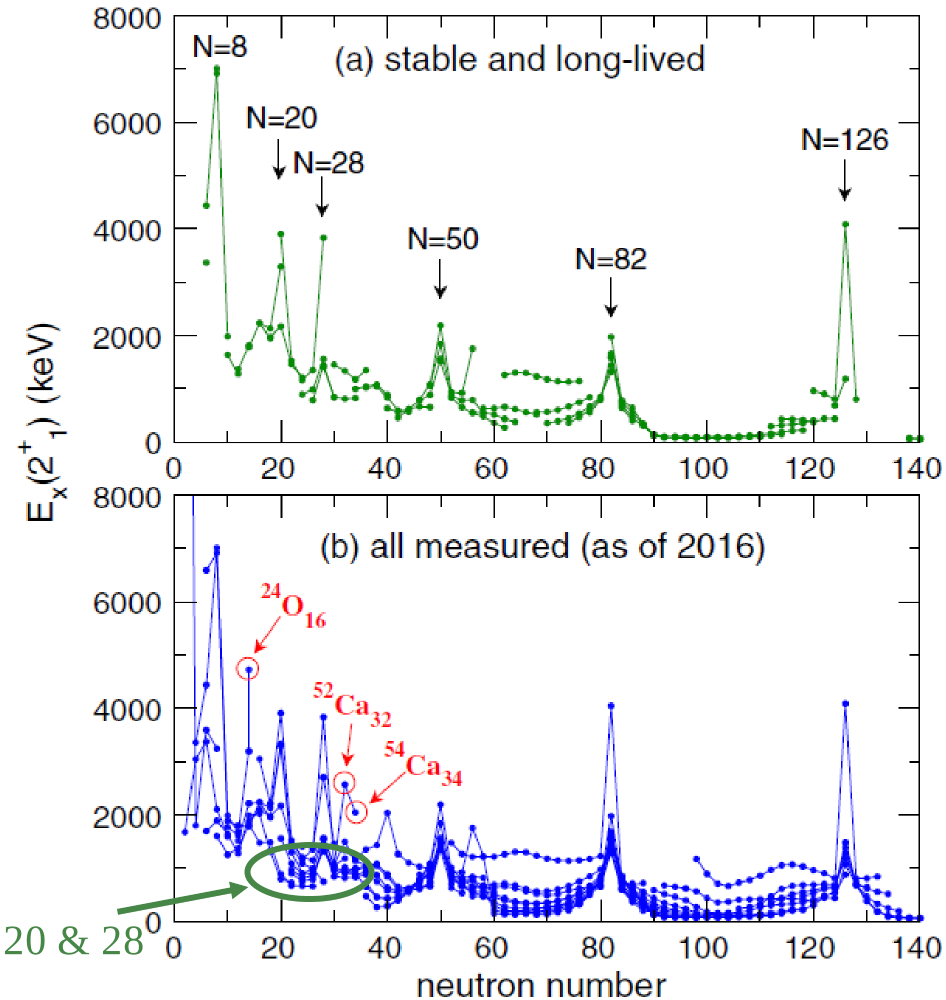


20 & 28

Otsuka et al., Rev. Mod. Phys. **92**, 015002 (2020)

Nuclear magic numbers

- Energy of the first 2^+ state



Towards neutron-rich region—**exotic nuclei**

- Disappearing magic numbers: ***N* = 20, 28**
- New magic numbers: ***N* = 16, 32, 34 …**

Otsuka *et al.*, *Rev. Mod. Phys.* **92**, 015002 (2020)

Experimental progress

nature Vol 435|16 June 2005|doi:10.1038/nature03619

LETTERS

'Magic' nucleus ^{42}Si

J. Fridmann[1], I. Wiedenhöver[1], A. Gade[2], L. T. Baby[1], D. Bazin[2], B. A
P. D. Cottle[1], E. Diffenderfer[1], D.-C. Dinca[2], T. Glasmacher[2], P. G. H
W. F. Mueller[2], H. Olliver[2], E. Rodriguez-Vieitez[3], J. R. Terry[2], J. A.

nature Vol 465|27 May 2010|doi:10.1038/nature09048

LETTERS

The magic nature of ^{132}Sn explored through the single-particle states of ^{133}Sn

, J. C. Blackmon[4], K. Y. Chae[1], K. A. Chipps[5], J. A. Cizewski[2], L. Erikson[5],
[7], J. F. Liang[4], R. Livesay[5], Z. Ma[1], B. H. Moazen[1], C. D. Nesaraja[4],
Shapira[4], J. F. Shriner Jr[7], M. S. Smith[4], T. P. Swan[2,6] & J. S. Thomas[6]

LETTER

doi:10.1038/nature12522

Evidence for a new nuclear 'magic number' from the level structure of ^{54}Ca

D. Steppenbeck[1], S. Takeuchi[2], N. Aoi[3], P. Doornenbal[2], M. Matsushita[1], H. Wang[2], H
J. Lee[2], K. Matsui[5], S. Michimasa[1], T. Motobayashi[2], D. Nishimura[6], T. Otsuka[1,5], H. S
T. Sumikama[8], H. Suzuki[2], R. Taniuchi[5], Y. Utsuno[9], J. J. Valiente-Dobón[10] & K. Yon

ARTICLE

https://doi.org/10.1038/s41586-019-1155-x

^{78}Ni revealed as a doubly magic stronghold against nuclear deformation

R. Taniuchi[1,2], C. Santamaria[2,3], P. Doornenbal[2*], A. Obertelli[2,3,4], K. Yoneda[2], G. Authelet[3], H. Baba[2], D. Calvet[3], F. Château[3],
ibert[3], J. D. Holt[5], T. Isobe[2], V. Lapoux[3], M. Matsushita[6], J. Menéndez[6],
[1], F. Nowacki[7], K. Ogata[8,9], H. Otsu[2], T. Otsuka[1,2,6], C. Péron[3], S. Péru[10],
Roussé[3], H. Sakurai[1,2], A. Schwenk[4,12,13], Y. Shiga[2,14], J. Simonis[4,12,15],
T. Uesaka[2], H. Wang[2], F. Browne[17], L. X. Chung[18], Z. Dombradi[19], S. Franchoo[20],
Kłęk[21], Z. Korkulu[19], S. Koyama[1,2], Y. Kubota[2,6], J. Lee[22], M. Lettmann[4], C. Louchart[4],
Nishimura[2], L. Olivier[20], S. Ota[6], Z. Patel[24], E. Şahin[21], C. Shand[24], P.-A. Söderström[2],
D. Suzuki[20], Z. Vajta[19], V. Werner[4], J. Wu[2,26] & Z. Y. Xu[22]

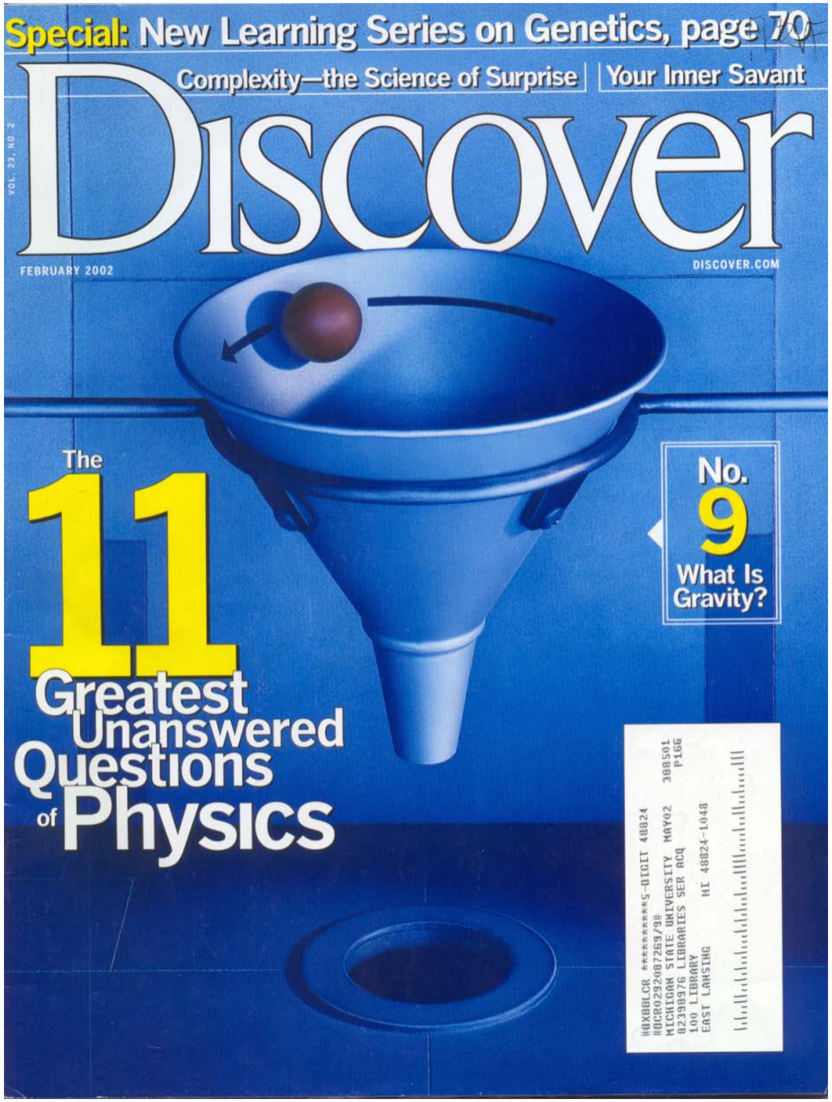
Article 678 | Nature | Vol 606 | 23 June 2022

Observation of a correlated free four-neutron system

Article Nature | Vol 620 | 31 August 2023 | 965

First observation of ^{28}O

The 11 greatest unanswered questions of physics

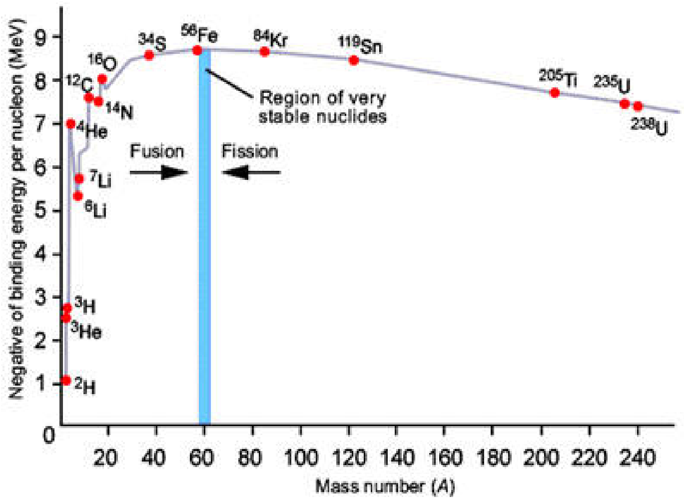


National research council's board on physics and astrophysics

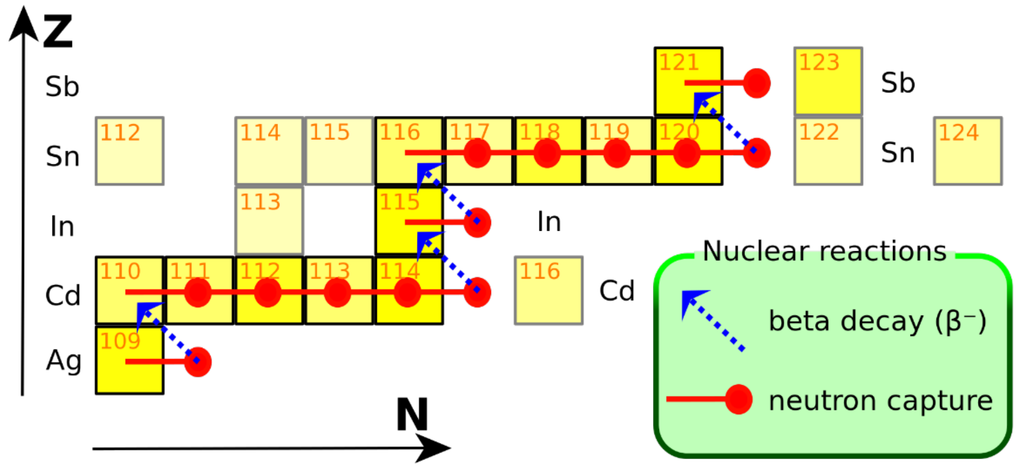
1. What is dark matter?
2. What is dark energy?
3. **How were the heavy elements from iron to uranium made?**
4. Do neutrinos have mass?
5. Where do ultrahigh-energy particles come from?
6. Is a new theory of light and matter needed to explain what happens at very high energies and temperatures?
7. Are there new states of matter at ultrahigh temperatures and densities?
8. Are protons unstable?
9. What is gravity?
10. Are there additional dimensions?
11. How did the universe begin?

Nucleosynthesis

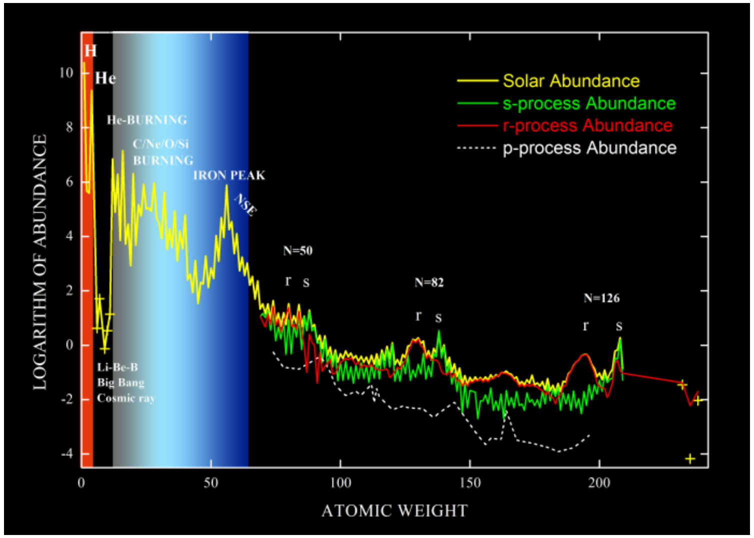
- Nuclear binding energy



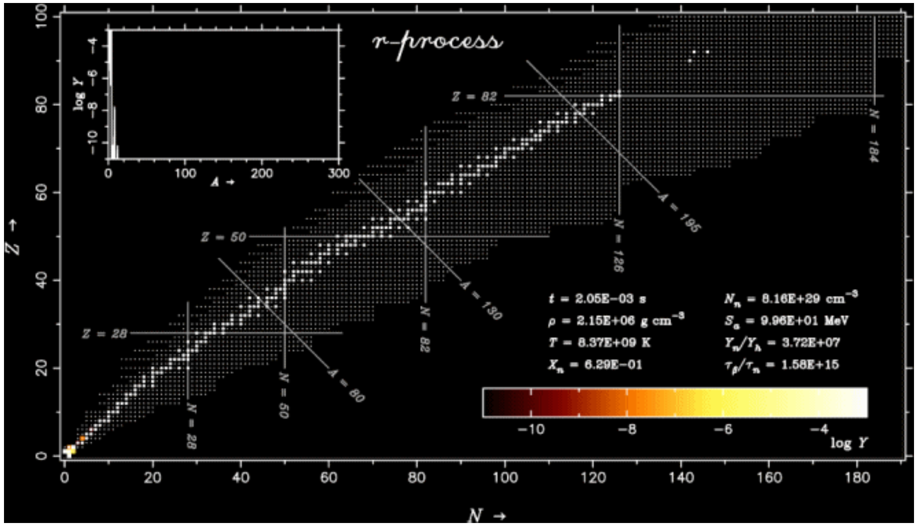
- Slow neutron-capture process (*s*-process)



- Abundance



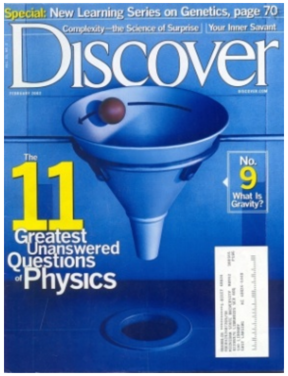
- Rapid neutron-capture process (*r*-process)



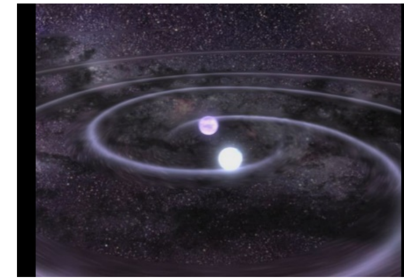
Courtesy of S. Wanajo

r-process nucleosynthesis and nuclear inputs

The 11 greatest unanswered questions of physics



Question 3
How were the heavy elements from iron to uranium made?

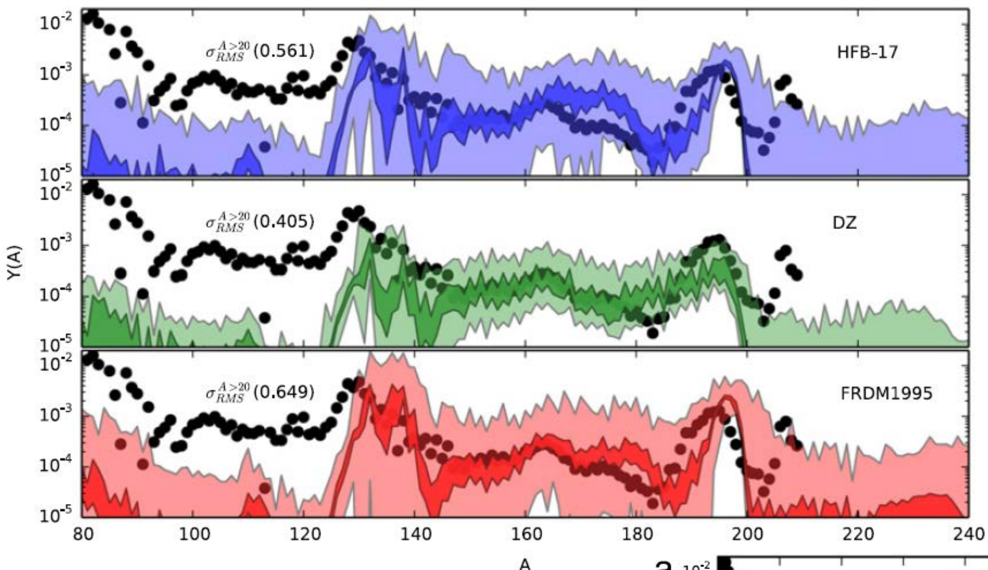


Why are gold (金) and platinum (白金) expensive? Why is lead (铅) cheap?
Why are rare-earth elements (稀土元素) rare?

★ Nuclear data inputs for *r*-process

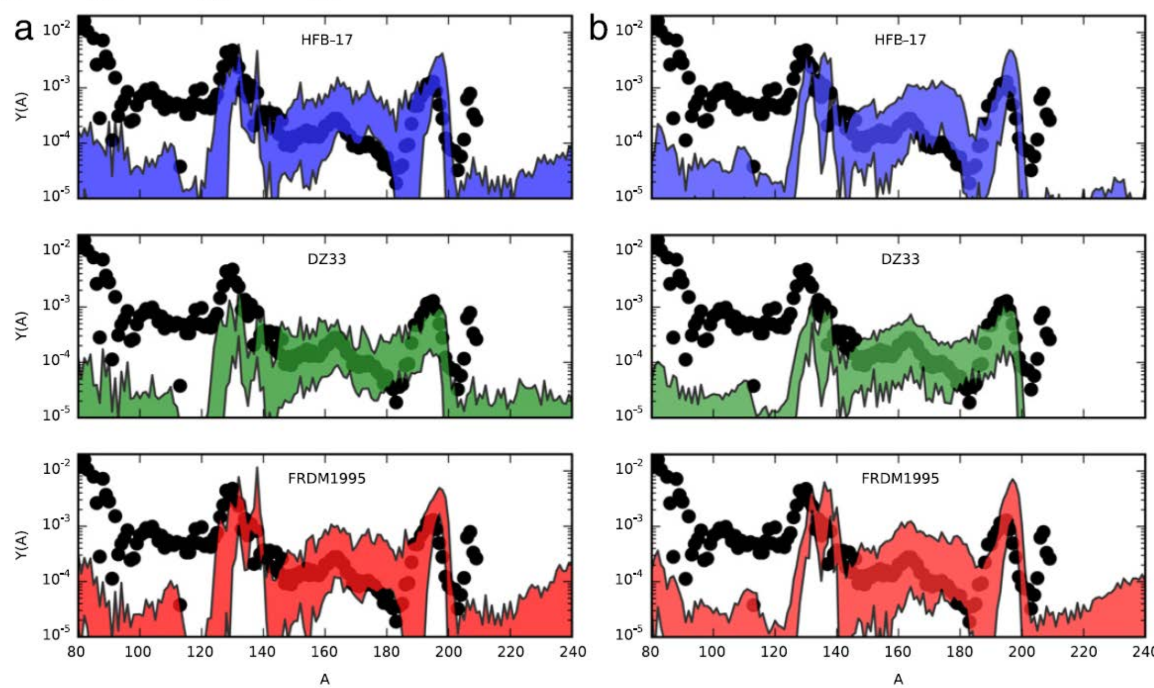
Quantity		Effects
S_n	neutron separation energy	path
$T_{1/2}$	β-decay half-lives	abundance pattern, time scale
P_n	β-delayed n-emission branchings	final abundance pattern
Y_i	fission (products and branchings)	endpoint, degree of fission cycling
		abundance pattern (?)
G	partition functions	path (very weakly)
$N_A \langle\sigma\nu\rangle$	neutron capture rates	conditions for waiting point approximation
		final abundance pattern during freezeout (?)

Impacts of nuclear properties on *r*-process



← Impacts of nuclear masses (~ 100 keV)

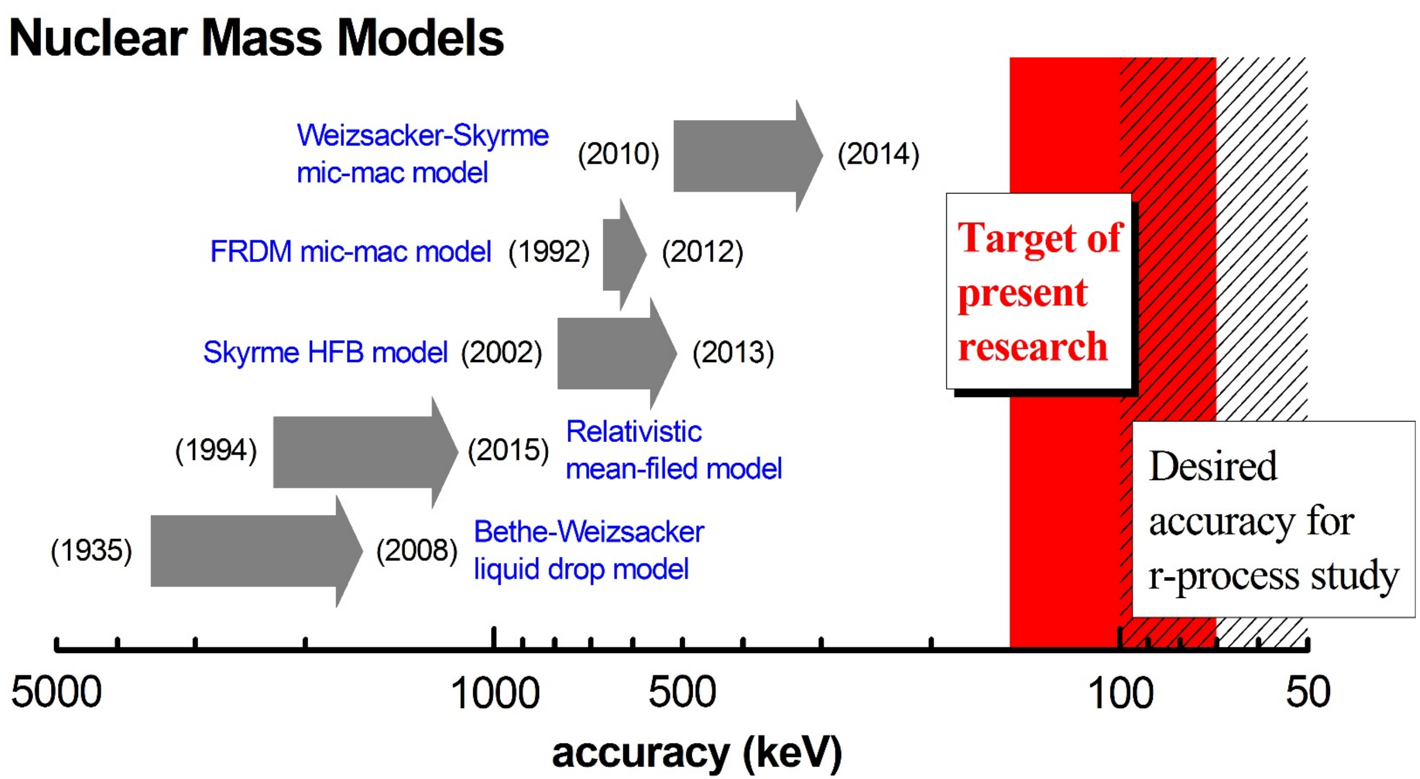
Impacts of nuclear β-decay half-lives (factor 10)
neutron-capture rates (factor 10^3) →



Mumpower et al., Prog. Part. Nucl. Phys. **86**, 86 (2016)

Nuclear mass models

- Theoretically, the development of nuclear mass model can be traced back to the early age of nuclear physics, known as Bethe-Weizsacker liquid drop model in 1935.
- To take into account the nuclear shell effects: the microscopic models and the microscopic-macroscopic (mic-mac) models.



Published database

- The first nuclear mass model with accuracy within **100 keV** is constructed. Its accuracies to S_* and Q_* are also much higher than other mass models.

Model	M	S_n	S_{2n}	S_p	S_{2p}	S_D	Q_β
BMM	**0.084**	**0.078**	**0.105**	**0.083**	**0.111**	**0.096**	**0.099**
HFB31	0.559	0.451	0.456	0.489	0.496	0.566	0.557
FRDM12	0.576	0.340	0.442	0.341	0.420	0.411	0.450
WS4	0.285	0.254	0.261	0.261	0.300	0.324	0.327

Calculated results and Perditions:

- Nuclear Masses — Niu and HZL, *PRC* **106**, L021303 (2022)
  https://journals.aps.org/prc/supplemental/10.1103/PhysRevC.106.L021303
- Nuclear β-decay half-lives — Minato, Niu, HZL, *PRC* **106**, 024306 (2022)
  https://journals.aps.org/prc/supplemental/10.1103/PhysRevC.106.024306

PRC 106, August 2022

Contents

Nuclear physics and its frontiers

- An overview of **nuclear chart**
- Superheavy elements (**SHE**) and superheavy nuclei (**SHN**)
- Exotic nuclei and their impacts on ***r*-process** nucleosynthesis

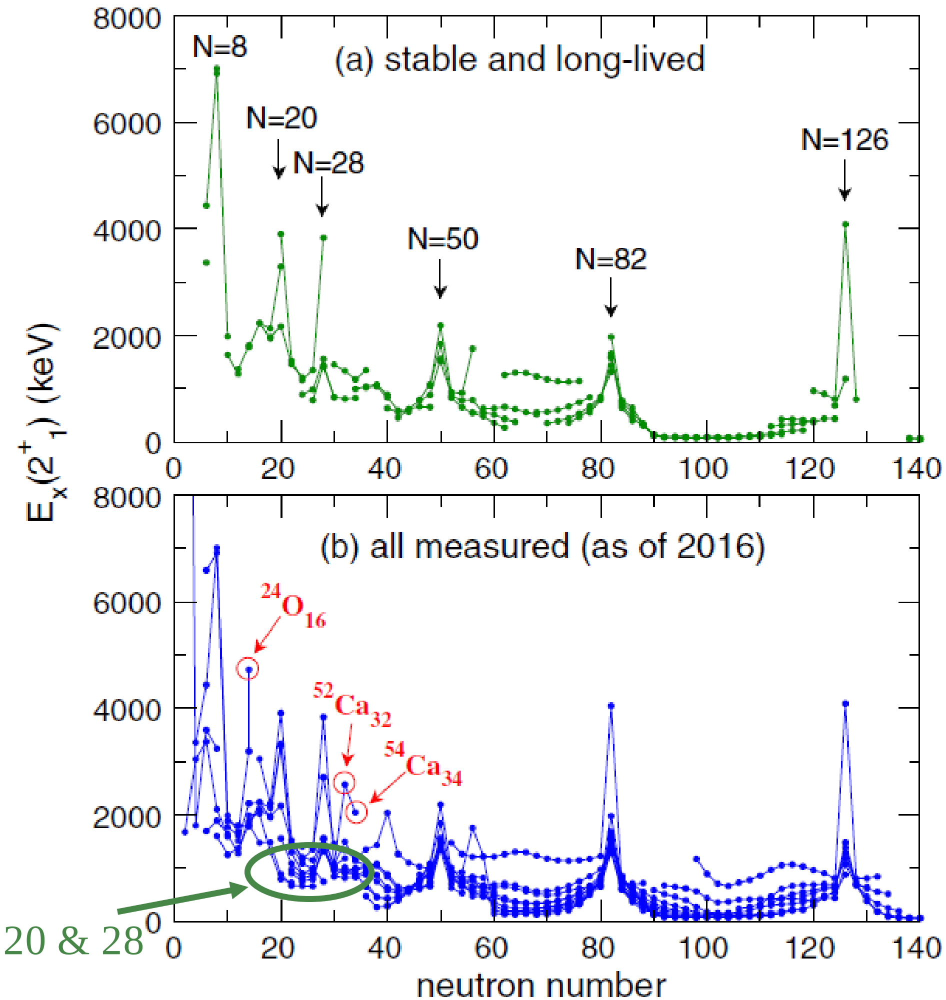
Nuclear pseudospin symmetry (PSS) and supersymmetric (SUSY) quantum mechanics

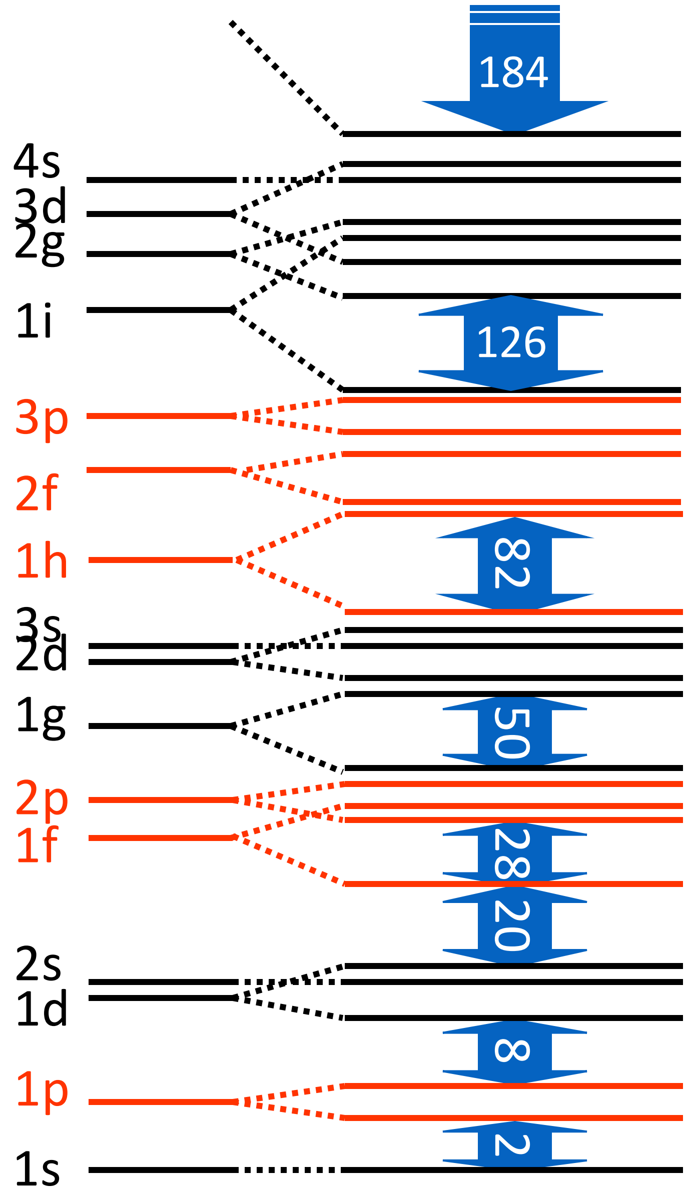
- PSS in atomic nuclei
- Conventional understandings
- Further investigations

Summary and perspectives

Nuclear magic numbers

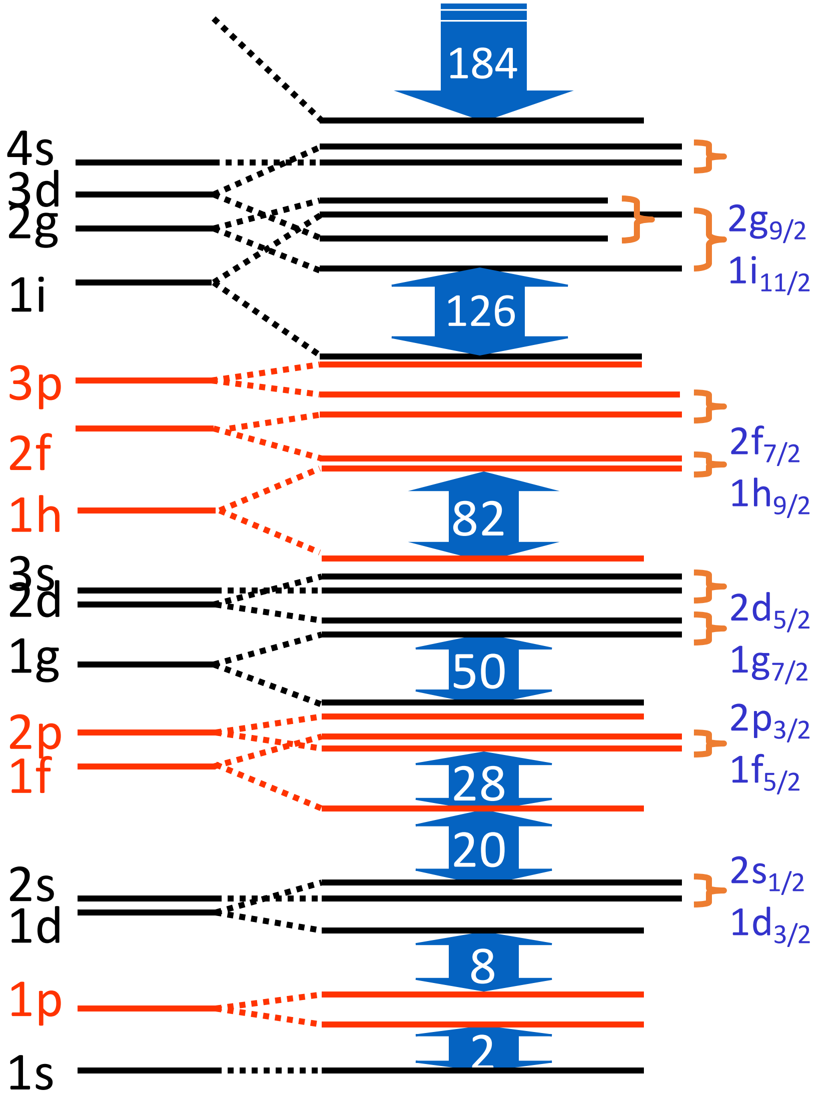
- Energy of the first 2^+ state





Otsuka *et al.*, *Rev. Mod. Phys.* **92**, 015002 (2020)

Nuclear single particle level scheme



Courtesy of S.G. Zhou

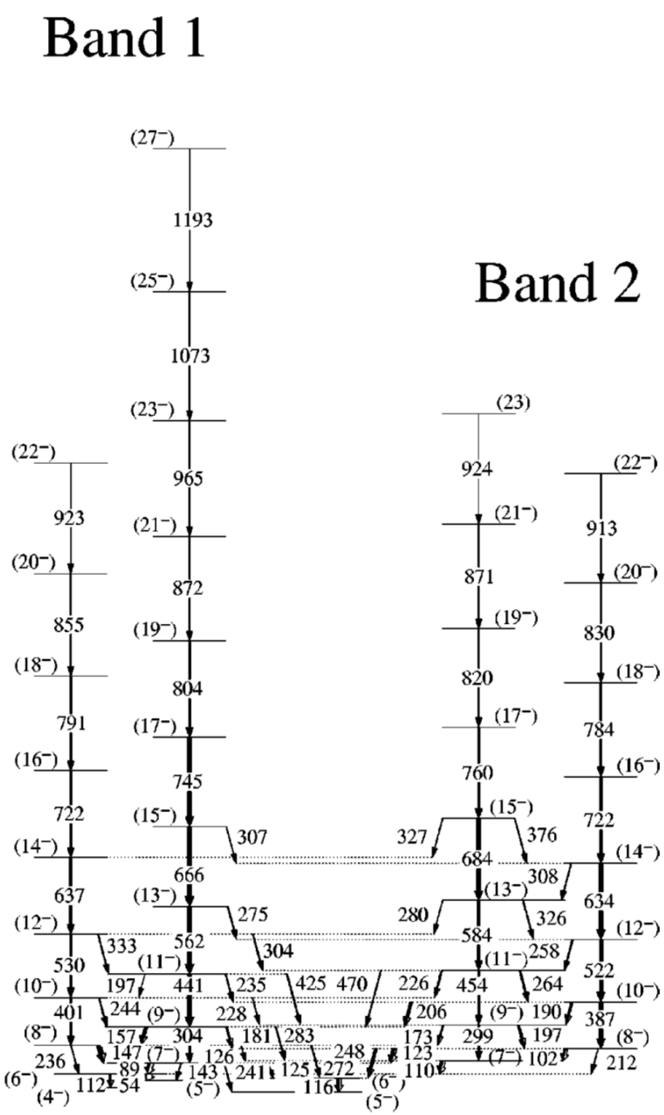
- Large spin-orbit splitting
- Near degeneracy of some doublets
- Pseudo-orbit angular momentum
- Accidental or of symmetry, i.e., pseudospin symmetry (**PSS**)?

$(n+1, l, \quad j = l+1/2)$ $\quad \tilde{l} = l + 1$

$(n \quad, l+2, j = l+3/2)$ $\quad \tilde{s} = 1/2$

Arima, Harvey, Shimizu, *PLB* **30**, 517 (1969)
Hecht, Adler, *NPA* **137**, 129 (1969)

Pseudospin partner bands

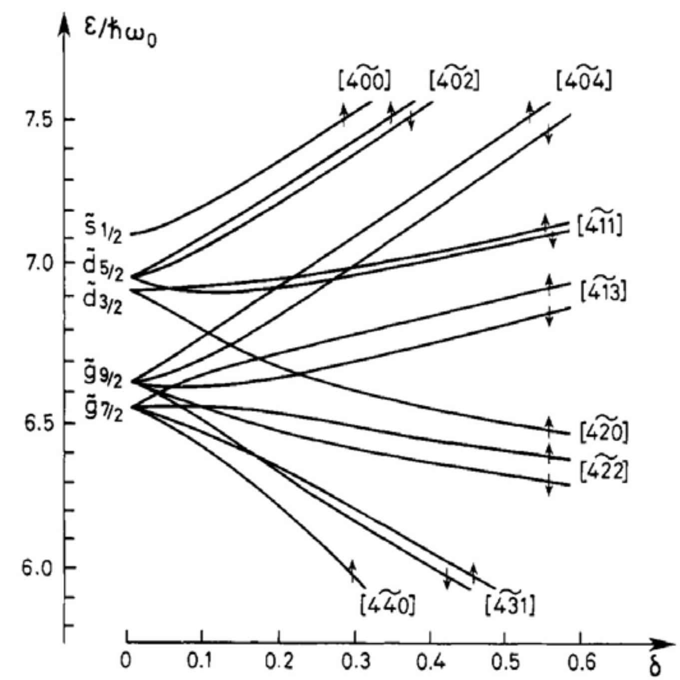


- Single-particle states

$$[Nn_z\Lambda]\Omega \ \& \ [Nn_z\Lambda+2]\Omega+1 \Rightarrow [\widetilde{Nn_z\Lambda}]$$

as pseudospin doublets

$$\tilde{N} = N-1, \tilde{\Lambda} = \Lambda+1, \Omega = \tilde{\Lambda} \pm 1/2$$



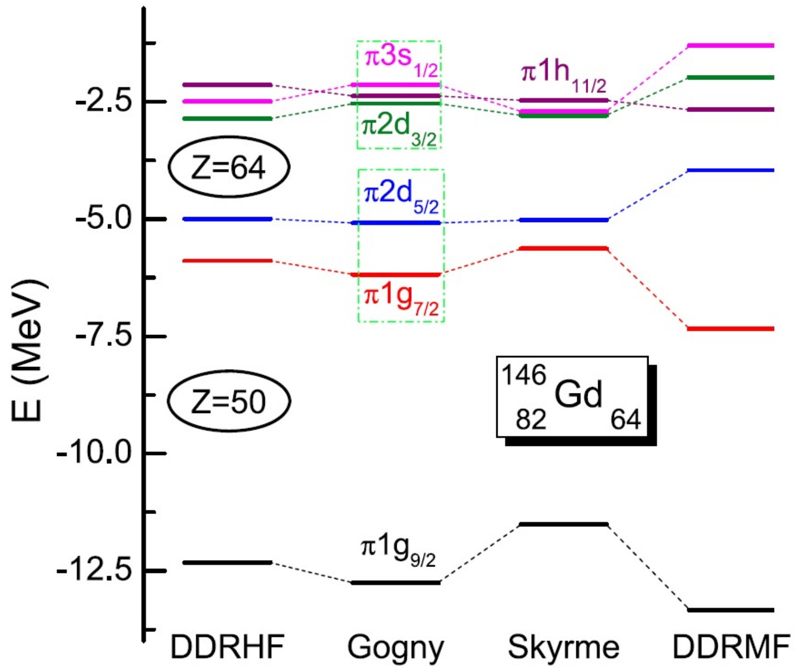
Bohr, Hamamoto, Mottelson, *Phys. Scr.* **26**, 267 (1982)

- Partial level scheme of ^{128}Pr

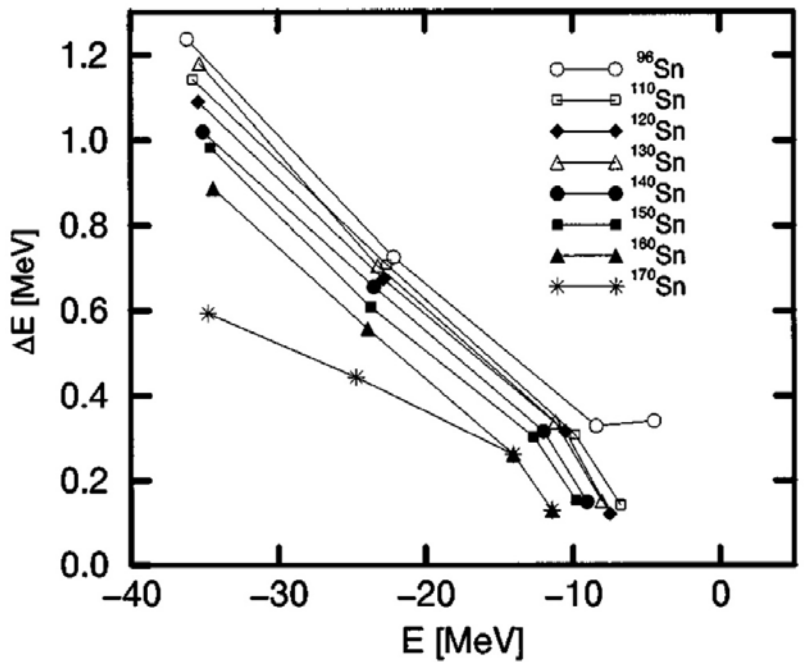
Petrache *et al.*, *PRC* **65**, 054324 (2002)

- ^{186}Ir (1997); ^{108}Tc (2008); ^{189}Pt (2009) ……

PSS in shell structure evolutions



◆ Proton single-particle energies for ^{146}Gd
Long, Nakatsukasa, Sagawa, Meng, Nakada, Zhang, *PLB* **680**, 428 (2009)



◆ Pseudospin-orbit splitting in Sn isotopes
Meng, Sugawara-Tanabe, Yamaji, Arima, *PRC* **59**, 154 (1999)

- Splitting of both spin and pseudospin doublets play important roles in the shell structure evolutions.

Pseudo quantum numbers & lower component

- Equation of motion for nucleons: Schrödinger / Dirac equations

What is $\tilde{l}$?

- Dirac spinor: $$\psi_\alpha(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} iG_a(r)\mathscr{Y}^{l_a}_{j_a m_a}(\hat{\mathbf{r}}) \\ -F_a(r)\mathscr{Y}^{\tilde{l}_a}_{j_a m_a}(\hat{\mathbf{r}}) \end{pmatrix}$$

- Relations of quantum numbers: $$\begin{cases} j = l \pm 1/2 \\ \kappa = (-1)^{j+l+1/2}(j+1/2) \\ \tilde{l} = l - \mathrm{sign}(\kappa) \end{cases}$$

$$2s_{1/2}: \begin{pmatrix} n=2, l=0, j=l+1/2 \\ \tilde{n}=2, \tilde{l}=1, j=\tilde{l}-1/2 \end{pmatrix} \quad 1d_{3/2}: \begin{pmatrix} n=1, l=2, j=l-1/2 \\ \tilde{n}=2, \tilde{l}=1, j=\tilde{l}+1/2 \end{pmatrix}$$

$$(2s_{1/2}, 1d_{3/2}) \Rightarrow (2\tilde{p}_{1/2,3/2})$$

$\tilde{l}$ is the orbital angular momentum of the lower component of the Dirac spinor.

Ginocchio, *PRL* **78**, 436 (1997)

Dirac and Schrödinger-like equations

- Dirac equation

$$\begin{pmatrix} \Sigma(r)+M & -\frac{d}{dr}+\frac{\kappa}{r} \\ \frac{d}{dr}+\frac{\kappa}{r} & -\Delta(r)-M \end{pmatrix}\begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = E\begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

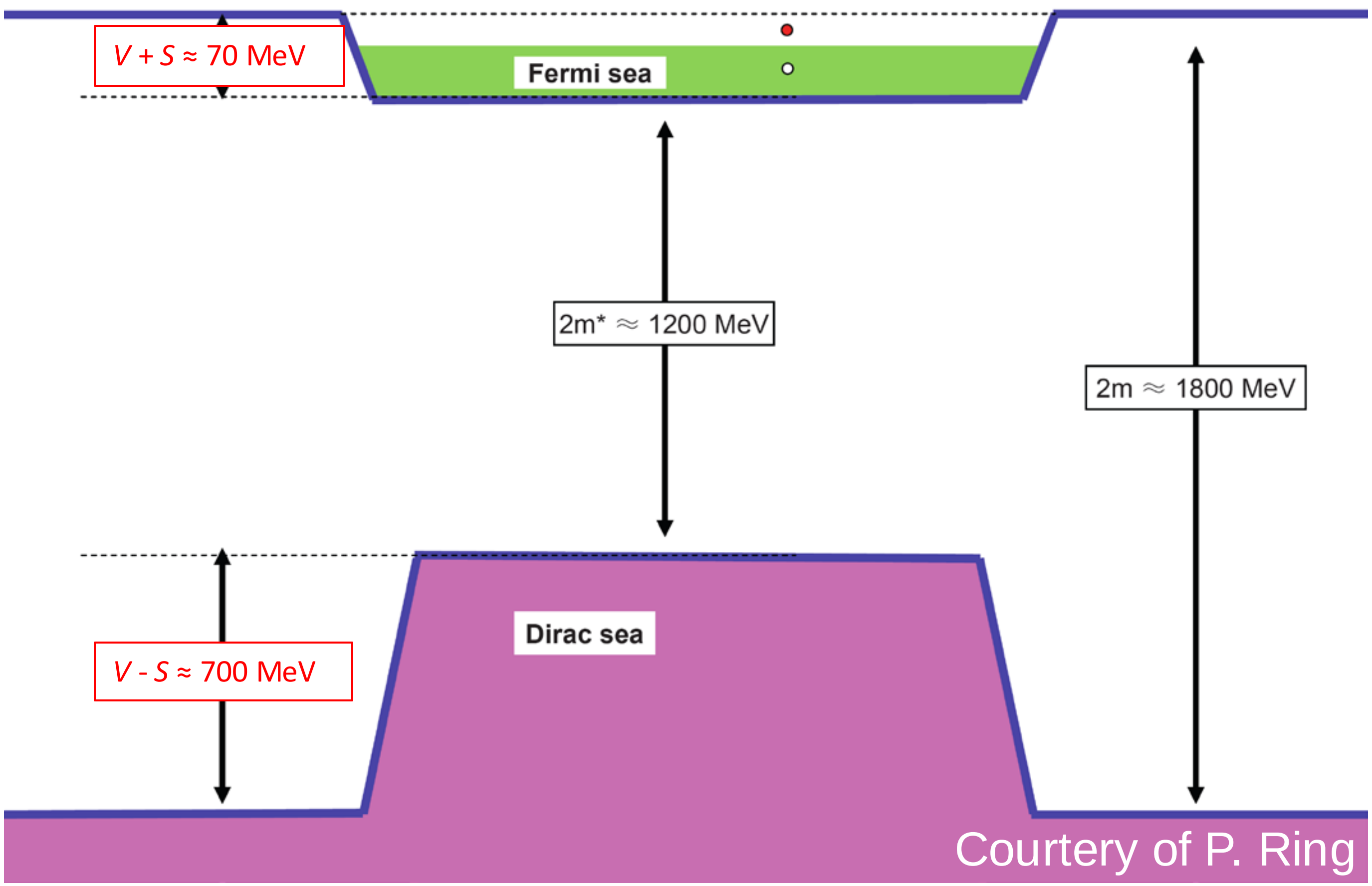
where

$$\Sigma(r)=S(r)+V(r), \qquad \Delta(r)=S(r)-V(r),$$

$$\kappa=\mp(j+1/2) \quad \text{for} \quad j_G=l_G\pm 1/2.$$

Nuclear potentials in relativistic models

- The scalar and vector potentials are both very big in amplitude, but with opposite signs: $S(r) < 0$ & $V(r) > 0$.
- This results in a shallow Fermi sea and a deep Dirac sea, the latter is responsible for the large spin-orbit coupling.



Dirac and Schrödinger-like equations

- Dirac equation

$$\begin{pmatrix} \Sigma(r)+M & -\frac{d}{dr}+\frac{\kappa}{r} \\ \frac{d}{dr}+\frac{\kappa}{r} & -\Delta(r)-M \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = E \begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

where

$$\Sigma(r) = S(r) + V(r), \qquad \Delta(r) = S(r) - V(r),$$

$$\kappa = \mp(j+1/2) \quad \text{for} \quad j_G = l_G \pm 1/2.$$

- Schrödinger-like equation for the lower component

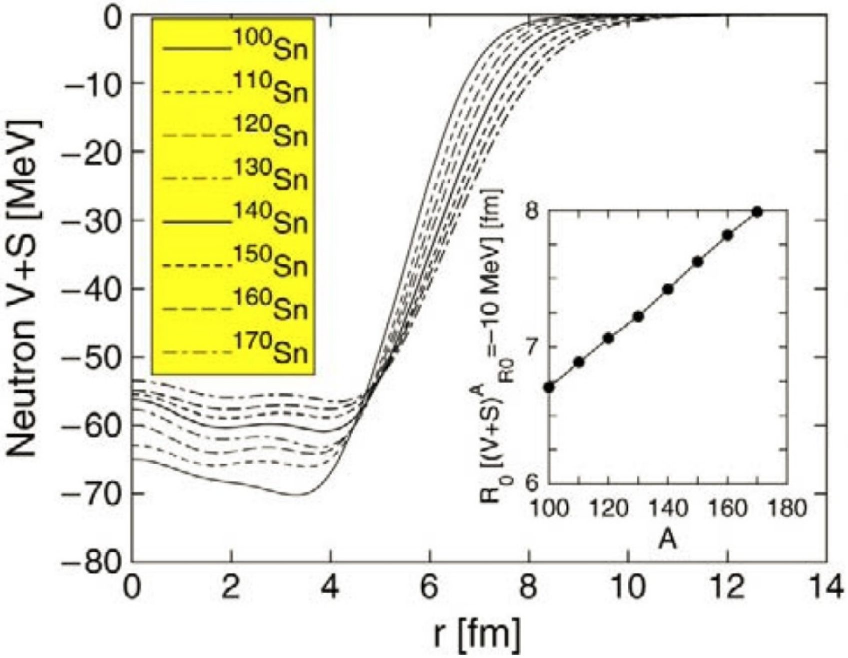
$$\left\{-\frac{1}{M_-}\frac{d^2}{dr^2} + \frac{1}{M_-^2}\frac{dM_-}{dr}\frac{d}{dr} + \left[(-M-\Delta) + \frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2} - \frac{1}{M_-^2}\frac{dM_-}{dr}\frac{\kappa}{r}\right]\right\} F = EF,$$

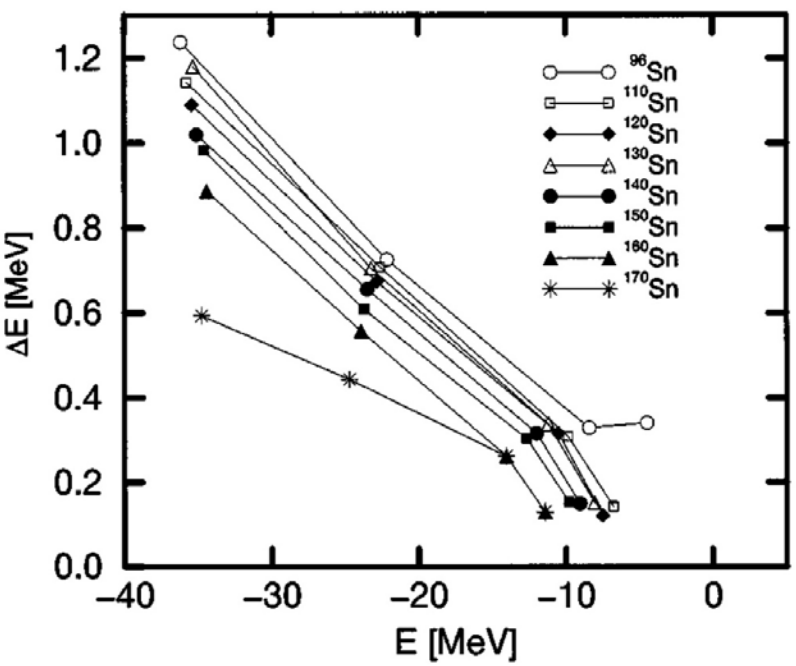
with $M_- = E - M - \Sigma$

Meng, Sugawara-Tanabe, Yamaji, Ring, Arima, *PRC* **58**, R628 (1998)

From stable nuclei to exotic nuclei

- **If $\Sigma(r)$ = 0 or $d\Sigma(r)/dr$ = 0, the PSS is exactly conserved, i.e., the PSS doublets are degenerate.**

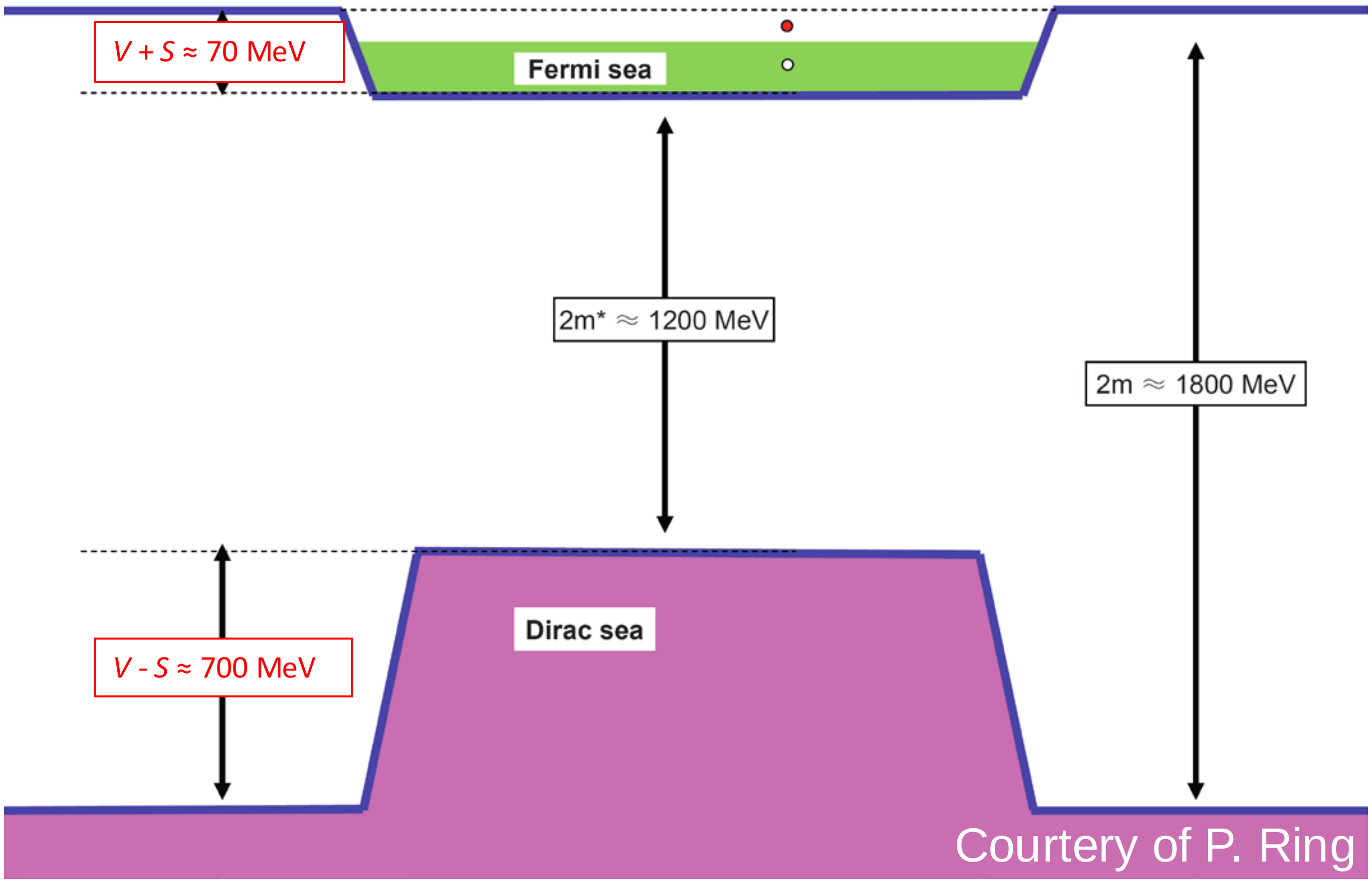




Meng, Sugawara-Tanabe, Yamaji, Ring, Arima, *PRC* **58**, R628 (1998)
Meng, Sugawara-Tanabe, Yamaji, Arima, *PRC* **59**, 154 (1999)

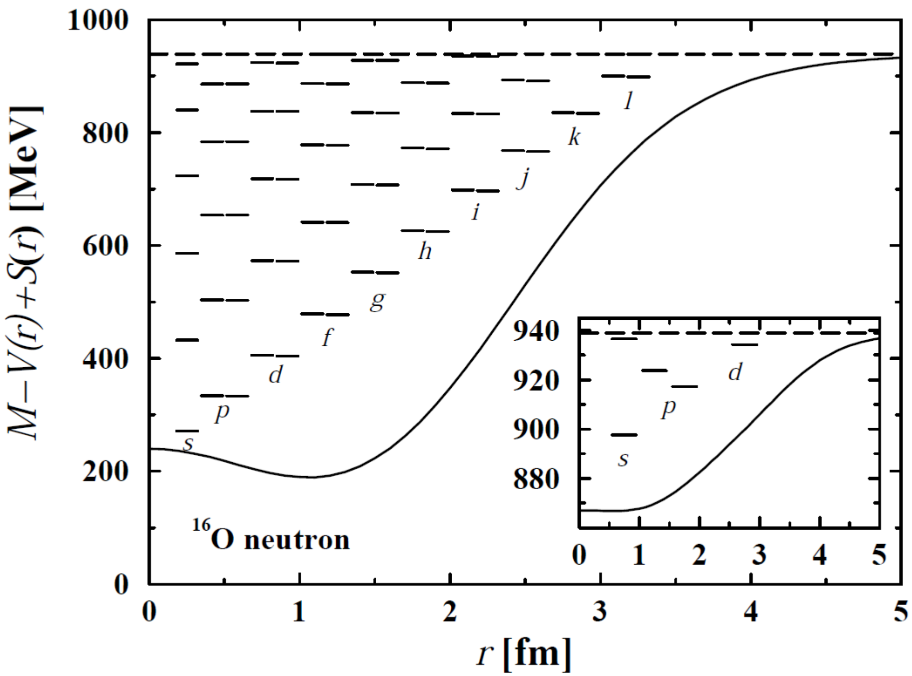
Nuclear potentials in relativistic models

- The scalar and vector potentials are both very big in amplitude, but with opposite signs: $S(r) < 0$ & $V(r) > 0$.
- This results in a shallow Fermi sea and a deep Dirac sea, the latter is responsible for the large spin-orbit coupling.



From nucleon spectra to anti-nucleon spectra

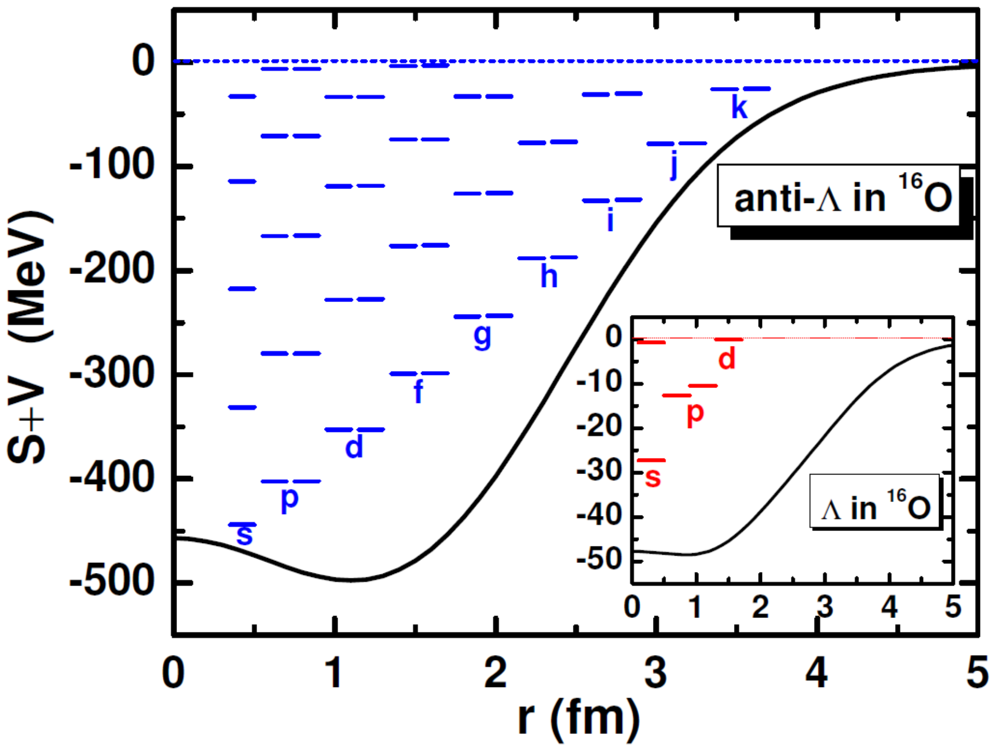
- **If $\Sigma(r) = 0$ or $d\Sigma(r)/dr = 0$, the spin symmetry (SS) in anti-nucleon spectra is exactly conserved.**



- The SS in single-anti-nucleon spectra is much better developed than that of PSS in single-nucleon spectra.

Zhou, Meng, Ring, *PRL* **91**, 262501 (2003)
He, Zhou, Meng, Zhao, Scheid, *EPJA* **28**, 265 (2006)
HZL, Long, Meng, Giai, *EPJA* **44**, 119 (2010)

From nucleon spectra to hyperon spectra



- Due to the additional strangeness degree of freedom, it is expected that the annihilation probability of an anti-hyperon in a normal nucleus is much smaller than that of an anti-nucleon.

Song, Yao, Meng, Chin. Phys. Lett. **26**, 122102 (2009)
Song, Yao, Chin. Phys. C **34**, 1425 (2010)
Song, Yao, Meng, Chin. Phys. Lett. **28**, 092101 (2011)

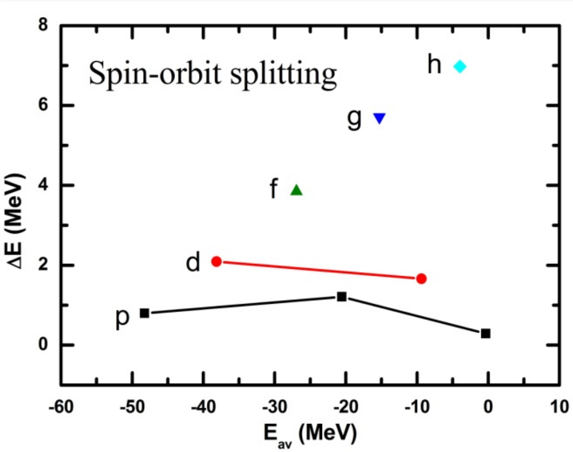
Triggers for further investigations

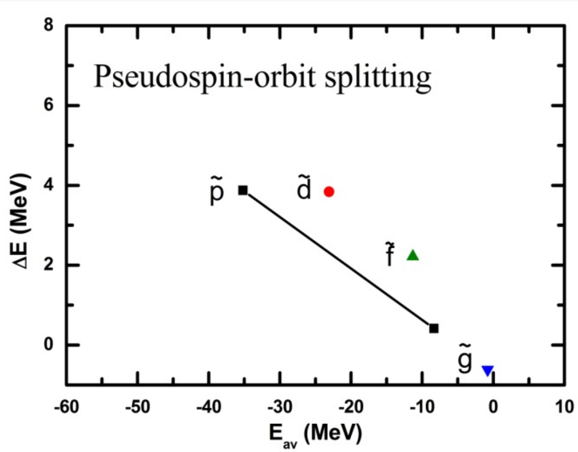
- Schrödinger-like equation for the lower component

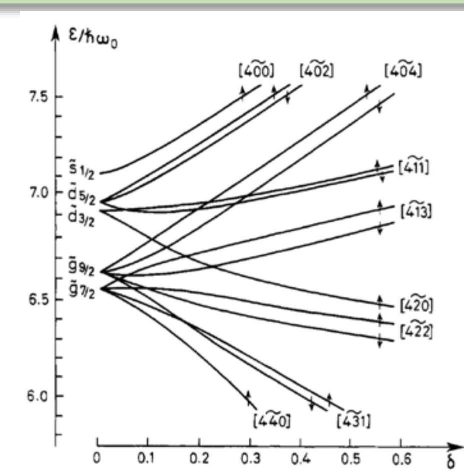
$$\left\{-\frac{1}{M_-}\frac{d^2}{dr^2}+\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{d}{dr}+\left[(-M-\Delta)+\frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2}-\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{\kappa}{r}\right]\right\}F=EF,$$

with $M_- = E - M - \Sigma$

- At the PSS limit, $\Sigma(r) = 0$ or $d\Sigma(r)/dr = 0$,
  - There are no bound states in single-nucleon spectra.
  - There are bound states only in single-anti-nucleon spectra.
- There exist singularities in $1/M_-$ for the bound states.
- Effective Hamiltonian is not Hermitian, i.e., perturbation theory can NOT be applied.







Triggers for further investigations

- Schrödinger-like equation for the lower component

$$\left\{-\frac{1}{M_-}\frac{d^2}{dr^2}+\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{d}{dr}+\left[(-M-\Delta)+\frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2}-\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{\kappa}{r}\right]\right\}F=EF,$$

with $M_- = E - M - \Sigma$

- At the PSS limit, $\Sigma(r) = 0$ or $d\Sigma(r)/dr = 0$,
  - There are no bound states in single-nucleon spectra.
  - There are bound states only in single-anti-nucleon spectra.
- There exist singularities in $1/M_-$ for the bound states.
- Effective Hamiltonian is not Hermitian, i.e., perturbation theory can NOT be applied.

Selected progress

- PSS in the single-particle resonant states Lu, Zhao, Zhou, *PRL* **109**, 072501 (2012)
- PSS with similarity renormalization group (**SRG**) Guo *et al.*, *PRL* **112**, 062502 (2014)
- PSS with SRG, supersymmetric (**SUSY**) quantum mechanics, and perturbation theory HZL *et al.*, *PRC* **83**, 041301(R) (2011); HZL *et al.*, *PRC* **87**, 014334 (2013)

i) PSS in resonant states

- Schrödinger-like equation for the lower component

$$\left\{-\frac{1}{M_-}\frac{d^2}{dr^2}+\cancel{\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{d}{dr}}+\left[(-M-\Delta)+\frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2}-\cancel{\frac{1}{M_-^2}\frac{dM_-}{dr}\frac{\kappa}{r}}\right]\right\}F=EF,$$

- At $r \rightarrow \infty$ (neutron states)

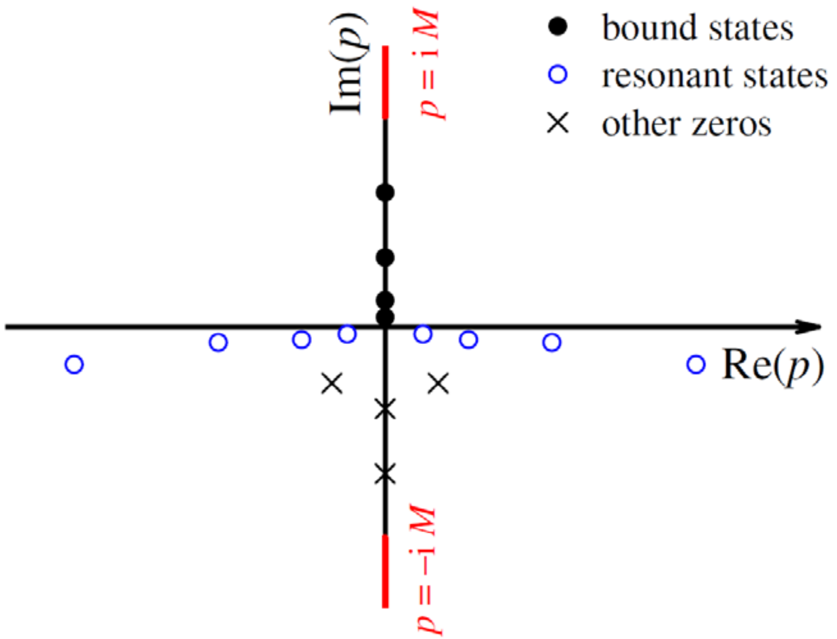
$$F(r)=\frac{i}{2}\left[\mathcal{J}_\kappa^F(p)h_{\tilde{l}}^-(pr)-\mathcal{J}_\kappa^F(p)^*h_{\tilde{l}}^+(pr)\right]$$

- **Zeros of Jost functions correspond to bound states, resonant states, ...**

$$E=E_{\rm res}-i\Gamma_{\rm res}/2=\sqrt{p^2+M^2}$$

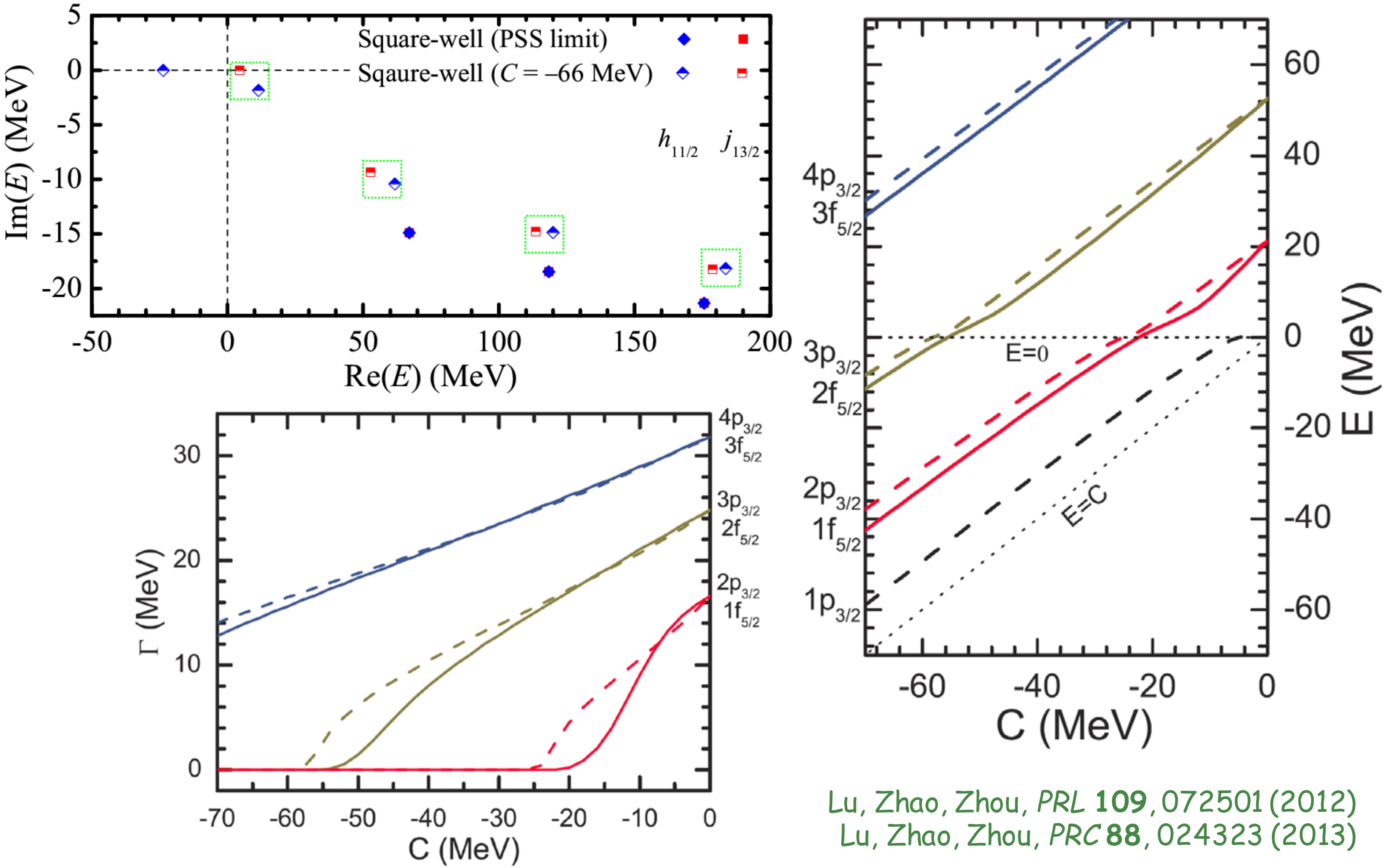
- At the PSS limit,

$$\boxed{\mathcal{J}_{\kappa_a}^F(p)=\mathcal{J}_{\kappa_b}^F(p).}$$



Lu, Zhao, Zhou, *PRL* **109**, 072501 (2012)

An example: Square-well potentials



Lu, Zhao, Zhou, *PRL* **109**, 072501 (2012)
Lu, Zhao, Zhou, *PRC* **88**, 024323 (2013)

ii) PSS with SRG

Similarity renormalization group (SRG)

Wegner, *Ann. Phys.* **506**, 77 (1994)
Phys. Rep. **348**, 77 (2001)
Bylev and Pirner, *PLB* **428**, 329 (1998)

- Start with the initial Hamiltonian ***H***

$$H = \alpha \cdot \mathbf{p} + \beta[M + S(\mathbf{r})] + V(\mathbf{r})\,.$$

- Introduce a unitary transformation

$$H(l) = U(l)HU^{\dagger}(l)\,, \qquad H(0) = H$$

- The flow equation with the anti-Hermitian generator

$$\frac{d}{dl}H(l) = [\eta(l), H(l)]\,, \qquad \eta(l) = \frac{dU(l)}{dl}U^{\dagger}(l)\,.$$

- The off-diagonal part $\boldsymbol{o(l) = 0}$ as $\boldsymbol{l \to \infty}$, the diagonal part in a series of $\boldsymbol{1/M}$

$$\varepsilon(\infty) = M\varepsilon_0(\infty) + \varepsilon_1(\infty) + \frac{\varepsilon_2(\infty)}{M} + \frac{\varepsilon_3(\infty)}{M^2} + \frac{\varepsilon_4(\infty)}{M^3} + \cdots$$

- Higher orders of **Foldy-Wouthuysen** non-relativistic reduction

SRG for Dirac Hamiltonian

Guo, *PRC* **85**, 021302(R) (2012)
Li, Chen, Guo, *PRC* **87**, 044311 (2013)
Guo *et al.*, *PRL* **112**, 062502 (2014)

- Dirac Hamiltonian ***H***

$$H = \boldsymbol{\alpha} \cdot \mathbf{p} + \beta[M + S(\mathbf{r})] + V(\mathbf{r})\,.$$

- After SRG,

$$H_D = \begin{pmatrix} H_P + M & 0 \\ 0 & -H_A - M \end{pmatrix}$$

with

$$H_P = H_{nr} + H_{dy} + H_{sl} + H_{km} + H_{dw}$$

$$\begin{cases} H_{nr} &= \Sigma(\vec{r}) + \frac{p^2}{2M}, \\ H_{dy} &= -\frac{1}{2M^2}\left(Sp^2 - \nabla S \cdot \nabla\right) + \frac{S}{2M^3}\left(Sp^2 - 2\nabla S \cdot \nabla\right), \\ H_{sl} &= \frac{1}{4M^2}\left(1 - \frac{2S}{M}\right)\vec{\sigma} \cdot (\nabla \Delta \times \vec{p})\,, \\ H_{km} &= -\frac{p^4}{8M^3}, \\ H_{dw} &= \frac{1}{8M^2}\nabla^2\Sigma - \frac{1}{16M^3}\left[(\nabla\Sigma)^2 - 2\nabla\Sigma \cdot \nabla\Delta + 4S\nabla^2\Sigma\right]. \end{cases}$$

- Every operator is Hermitian.

iii) PSS with SRG, SUSY, & perturbation

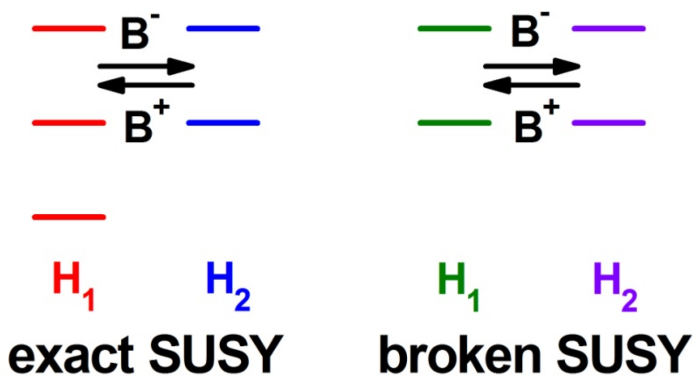
Supersymmetric (SUSY) quantum mechanics

- Hamiltonian and its supersymmetric partner

$$H_1 = B^+B^- \qquad \text{and} \qquad H_2 = B^-B^+.$$

- Their spectra and wave functions

$$\psi_2(n) = \frac{B^-}{\sqrt{E_S(n)}}\psi_1(n), \qquad \psi_1(n) = \frac{B^+}{\sqrt{E_S(n)}}\psi_2(n).$$



- **Spin and Pseudospin symmetric terms**

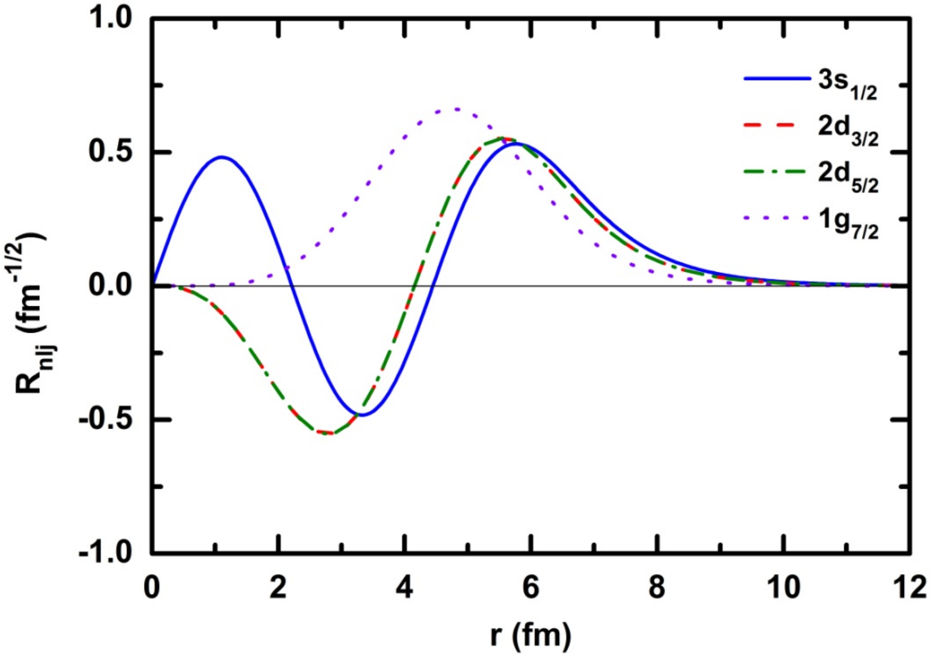
$$H_1 = B_\kappa^+ B_\kappa^- = \frac{1}{2M}\left[-\frac{d^2}{dr^2} + \frac{\kappa(\kappa+1)}{r^2} + q_\kappa^2 + \frac{2\kappa}{r}q_\kappa - q_\kappa'\right],$$

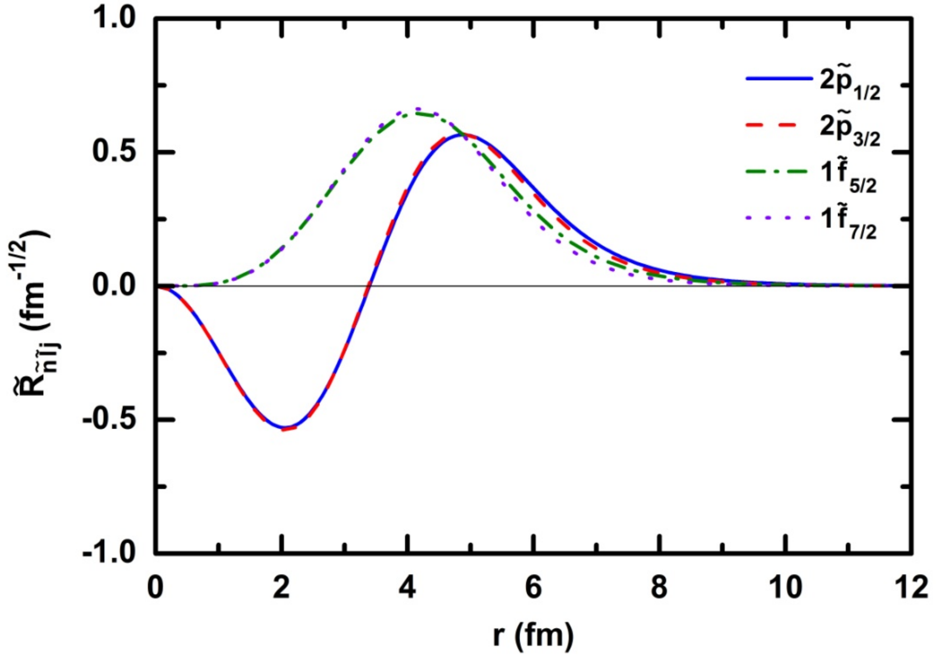
$$H_2 = B_\kappa^- B_\kappa^+ = \frac{1}{2M}\left[-\frac{d^2}{dr^2} + \frac{\kappa(\kappa-1)}{r^2} + q_\kappa^2 + \frac{2\kappa}{r}q_\kappa + q_\kappa'\right].$$

Typel, *NPA* **806**, 156 (2008)
HZL, Shen, Zhao, Meng, *PRC* **87**, 014334 (2013)

Single-particle wave functions

- Single-particle wave functions in H_1 and its SUSY partner H_2



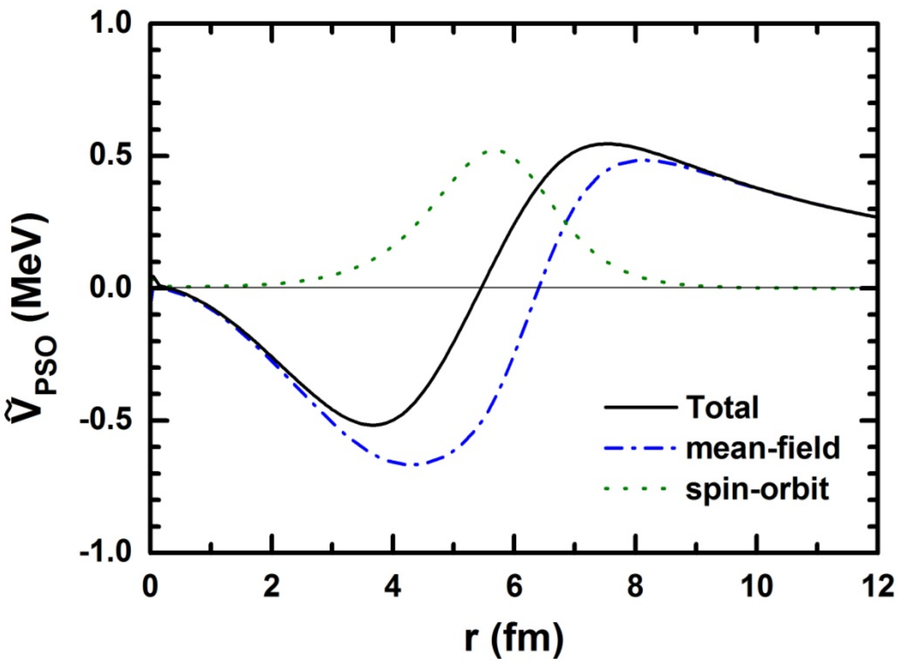


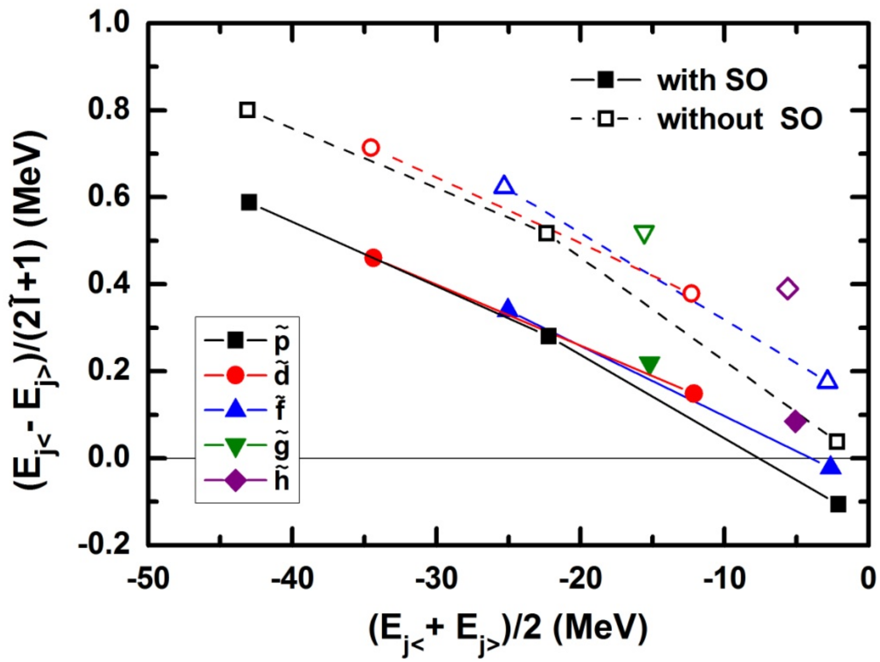
- Single-particle wave functions of pseudospin doublets are very different in original Hamiltonian.
- But they are almost identical in SUSY partner Hamiltonian.

HZL, Shen, Zhao, Meng, *PRC* **87**, 014334 (2013)

Pseudospin-orbit potential and splitting

- Dirac to Schrödinger equations by SRG (**1/*M*** order + spin-orbit at **$1/M^2$** order)





- The origin of PSS deeply hidden in H_1 can be traced in its SUSY partner Hamiltonian H_2.
- ΔE_{PSO} become smaller with increasing E_{av} can be interpreted in an explicit and quantitative way.

HZL, Shen, Zhao, Meng, *PRC* **87**, 014334 (2013); Shen, HZL, Zhao, Zhang, Meng, *PRC* **88**, 024311 (2013)

Summary and Perspectives

- During the past 20+ years, there is various progress on the study of **PSS** and **SS** in different systems and potentials, e.g.,
  - From stable to exotic nuclei
  - From non-confining to confining potentials
  - From local to non-local potentials
  - From central to tensor potentials
  - From bound to resonant states
  - From nucleon to anti-nucleon spectra
  - From nucleon to hyperon spectra
  - From spherical to deformed nuclei
  - ......
- Open issues
  - PSS is perturbative or not?
  - The puzzle of intruder states
  - PSS with SRG, SUSY, perturbation theory, ……

SUSY quantum mechanics might help to understand some new physics.

Various and concrete experimental evidences for PSS are highly desired!

Acknowledgments

We would like to express our gratitude to all the collaborators and colleagues who contributed to the investigations presented, in particular to

A. Arima, T.S. Chen, L.S. Geng, J.N. Ginocchio, J.Y. Guo, X.T. He, R.V. Jolos, A. Leviatan, F.Q. Li, W.H. Long, B.N. Lu, H.F. Lu, P. Ring, H. Sagawa, W. Scheid, S.H. Shen, C.Y. Song, K. Sugawara-Tanabe, H. Toki, N. Van Giai, S. Yamaji, S.C. Yang, J.M. Yao, S.Q. Zhang, Y. Zhang, E.G. Zhao, and P.W. Zhao.

Physics Reports 570 (2015) 1–84

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

ELSEVIER

Hidden pseudospin and spin symmetries and their origins in atomic nuclei

CrossMark

Haozhao Liang [a,b], Jie Meng [a,c,d,*], Shan-Gui Zhou [e,f]

84 pages, 58 figures, 10 tables

Thank you!