Ft values of the mirror β transitions and the weak-magnetism–induced corrections

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Abstract

Mirror β transitions with $T = \frac{1}{2}$ provide a sensitive testing ground for the **Conserved Vector Current (CVC) hypothesis** and play an important role in the precise extraction of the CKM matrix element V_{ud}.

In this work, updated $\mathcal{F}t$ values are obtained for all mirror transitions from $A = 3$ to $A = 75$, incorporating revised **radiative** and **nuclear-structure corrections**. The matrix elements determining **weak magnetism** were calculated in the nuclear **shell model** and cross-checked against experimental data, showing overall good agreement.

The updated mirror$\mathcal{F}t$ values enable a stringent **0.1% test of CVC** and contribute to an improved determination of V_{ud}, providing implications for precision tests of the Standard Model and searches for possible new physics.

Main content

Introduction

- Precision measurements in nuclear and neutron β decay have reached the **1% level** .
- At this precision, **recoil-order corrections** and **radiative effects** become significant.

Recoil-Order Corrections

- Arise from the **finite mass** of the nucleus.
- Expand β-decay matrix elements in powers of q/M:

$$M = M^{(0)} + \frac{q}{M} M^{(1)} + \cdots$$

- Include:
  - **Weak magnetism** (b)
  - Induced tensor term (d)

Radiative Effects

- Quantum electrodynamic (QED) corrections:

$\beta \to \beta + \gamma,$ β-decay loop diagram

- Two types:
  - **Outer** correction δ'_R depends on W and Z; nuclear-structure independent.
  - **Inner** correction Δ_R^V includes high-energy loops and nuclear-structure terms.

Motivation

- **Overall Goal:**
  - Test the weak interaction through high-precision measurements of mirror β decays combined with theoretical corrections.
- **Research subject:**
  - Contain both Fermi and Gamow-Teller components $\rightarrow$ allows simultaneous testing of multiple nuclear matrix elements.
  - Strongly related to nuclear structure $\rightarrow$ provides important cross-checks with theoretical models.
- **Ultimate Goal:**
  - Provide a consistent and improved Ft database for all mirror transitions, enhancing precision tests of the Standard Model and research for new physics.

Main content

β-Decay Relations

Uncorrected Partial Half-Life

$$t = \frac{K}{G_F^2\, V_{ud}^2} \cdot \frac{1}{\xi f}, \qquad \xi = g_V^2 M_F^2 + g_A^2 M_{GT}^2$$

The decay rate depends on the weak coupling constants, the Fermi and Gamow-Teller matrix elements, and the statistical phase-space factor f. V_{ud} enters directly, making precision β decay a powerful probe of the weak interaction.

Statistical Rate Function

$$f = \int_1^{W_0} pW(W_0 - W)^2 F(Z,W)\, C(Z,W)\, K(Z,W)\, dW$$

The phase-space integral f depends on the β spectrum, Coulomb interaction, nuclear structure effects, and higher-order corrections.

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Partial Half-Life and Mirror Transitions

Partial Half-Life of a Specific Transition

$$t = t_{1/2}\left(\frac{1+P_{\mathrm{EC}}}{\mathrm{BR}}\right)$$

The partial half-life t depends on three experimental quantities: the total half-life, the branching ratio of the transition, and the electron-capture fraction. These determine how often a specific decay channel occurs.

Mirror $T=\frac{1}{2}$ Transitions

$$M_F = M_F^{(0)}(1-\delta_C), \qquad M_F^{(0)} = 1$$

For mirror nuclei, the Fermi matrix element is fixed by isospin symmetry, but small Coulomb-induced isospin breaking introduces the correction δ_C. These corrections must be known precisely to extract V_{ud}.

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Isospin-Symmetry-Breaking Correction δ_C

- δ_C describes the **imperfect overlap** of proton and neutron radial wave functions.
- This arises because protons and neutrons experience **slightly different nuclear potentials** (e.g., Coulomb effect).

For superallowed $0^+ \to 0^+$ decay:

$$M_F^0 = \sqrt{2} \quad (T = 1)$$

$$M_F = \sqrt{2}\,(1 - \delta_C)$$

- δ_C is typically small: 0.1% – 1%.

Theoretical input:

- Cannot be measured directly; must be calculated from nuclear theory.
- Different models give slightly different δ_C, contributing to the main theoretical uncertainty in V_{ud} extraction.

Corrected $\mathcal{F}t$ Value

Corrected Rate Equation

$$\frac{1}{t} = G_F^2 V_{ud}^2 \frac{K}{1+\delta_R'} \left[\frac{f_V |M_F^{(0)}|^2 (1+\delta_{NS}^V - \delta_C^V)(1+\Delta_R^V)}{g_V^2} + \frac{f_A |M_{GT}^{(0)}|^2 (1+\delta_{NS}^A - \delta_C^A)(1+\Delta_R^A)}{g_A^2} \right.$$

- G_F: Fermi weak interaction constant
- V_{ud}: CKM matrix element (u $\rightarrow$ d weak transition strength)
- K: normalization constant
- δ_R': nucleus-independent outer radiative correction
- f: statistical rate function (phase-space integral)
- δ_C: isospin-symmetry-breaking correction (proton/neutron wave function mismatch)
- δ_{NS}: nuclear-structure-dependent radiative correction
- Δ_R: nucleus-independent short-range radiative correction

This expression includes all essential electroweak and nuclear-structure corrections. Both the radiative terms (δ_R', Δ_R) and isospin breaking (δ_C) contribute at the percent level and must be treated accurately.

Mixing Ratio

$$\rho = \frac{g_A M_{GT}^{(0)}}{g_V M_F^{(0)}} \left[\frac{(1+\delta_{NS}^A - \delta_C^A)(1+\Delta_R^A)}{(1+\delta_{NS}^V - \delta_C^V)(1+\Delta_R^V)} \right]^{1/2}$$

For T = 1/2 mirror β transitions, one has

$$\rho \approx g_A M_{GT}$$

The mixing ratio ρ gives the relative size of the Gamow–Teller and Fermi components in a mirror decay. It determines the correlation coefficients and enters the corrected $\mathcal{F}t$ value.

Corrected $\mathcal{F}t$ for Mixed Fermi/Gamow–Teller Decays

$$\mathcal{F}t^{\mathrm{GT/F}} \equiv \mathcal{F}t^{\mathrm{mirror}} = f_V t(1+\delta_R')(1+\delta_{NS}^V - \delta_C^V)$$
$$= \frac{K}{G_F^2 V_{ud}^2 g_V^2 (1+\Delta_R^V)} \times \frac{1}{|M_F^0|^2 \left[1 + \frac{f_A}{f_V}\rho^2\right]}.$$

Explanation: The corrected $\mathcal{F}t$ includes radiative, nuclear-structure, and isospin-symmetry-breaking effects. The factor in brackets mixes Fermi and Gamow–Teller strengths.

Shell Model Calculates

- Model spaces and interactions
  - p shell: Cohen–Kurath sd shell: USD / USDA / USDB pf shell: KB3
- Compute nuclear matrix elements relevant to Holstein form factors:
  - Gamow–Teller matrix element:

$$M_{GT} = \langle \psi_f \| \sum_i \sigma_i \tau_i^{\pm} \| \psi_i \rangle$$

  - Orbital angular momentum matrix element:

$$M_L = \langle \psi_f \| \sum_i \mathbf{l}_i \tau_i^{\pm} \| \psi_i \rangle$$

- Computing induced form factors:

$$\frac{b}{Ac_1} = \frac{1}{g_A} \left(g_M + g_V \frac{M_L}{M_{GT}} \right)$$

- Purpose: compare theory vs. experiment for M_{GT}, M_L, and $b/(Ac)$.

Main content

Results

- The shell model iscapable in calculating M_{GT}, with the ratio differing 10% to 20% from unity and the difference being typically limited to about 0.1. Taking an unweighted average, the ratio of experimental to theoretical values to be

$$\frac{M_{GT}^{\mathrm{exp}}}{M_{GT}^{\mathrm{theo}}} \approx 0.97(8)$$

TABLE XIX. Comparison of experimental and theoretical values for the M_{GT} and M_L matrix elements (in fm units) for the mirror β transitions up to ^{45}V. Theoretical values were calculated using the shell model (see Sec. III B 3 b for details). Values for $M_{\mathrm{GT}}^{\mathrm{exp}}$ are obtained from the Gamow-Teller form factors, c, listed in Table VI. Values for M_L^{exp} were calculated from Eq. (43) using the $(b/Ac)^{\mathrm{exp}}$ values listed in Table XVIII. As to g_A, we used for the neutron the value $g_A = 1.2754(11)$ which was obtained from correlation measurements in neutron decay [56,66] and is independent of the $\mathcal{F}t^{\mathrm{mirror}}$ value, while for $A = 3$ the value $g_A = 1.27$ was used, and $g_A = 1.00$ for all other cases (see Sec. III A 3 c).

β decay	A	Shell	$M_{\mathrm{GT}}^{\mathrm{exp}}$	$M_{\mathrm{GT}}^{\mathrm{theo}}$	$\frac{M_{\mathrm{GT}}^{\mathrm{exp}}}{M_{\mathrm{GT}}^{\mathrm{theo}}}$	$M_{\mathrm{GT}}^{\mathrm{exp-theo}}$	M_L^{exp}	M_L^{theo}	$\frac{M_L^{\mathrm{exp}}}{M_L^{\mathrm{theo}}}$	$M_L^{\mathrm{exp-theo}}$	$\frac{M_L^{\mathrm{theo}}}{M_{\mathrm{GT}}^{\mathrm{theo}}}$	$\frac{M_L^{\mathrm{exp}}}{M_{\mathrm{GT}}^{\mathrm{exp}}}$
$n \to p$	1	$s_{1/2}$	+1.7335(18)	+1.732	1.001(1)	+0.002(2)	−0.0069(51)	+0.000	–	−0.007(5)	+0.000	−0.004
H → He	3		−1.6577(11)	−1.706	0.972(1)	+0.048(1)	−1.0438(58)	+0.000	–	−1.044(6)	+0.000	+0.630
C → B	11	$p_{3/2}$	−0.75442(79)	−0.789	0.956(1)	+0.035(1)	−1.1653(52)	−0.831	1.402(6)	−0.334(5)	+1.053	+1.545
N → C	13	$p_{1/2}$	−0.5596(14)	−0.568	0.985(2)	+0.008(1)	+0.8589(50)	+0.697	1.232(7)	+0.162(5)	−1.227	−1.535
O → N	15		+0.6302(16)	+0.576	1.094(3)	+0.054(2)	−1.2291(64)	−1.125	1.093(5)	−0.104(5)	−1.953	−1.950
F → O	17	$d_{5/2}$	+1.2955(11)	+1.182	1.096(1)	+0.114(1)	+1.7303(62)	+2.336	0.741(3)	−0.606(6)	+1.976	+1.336
Ne → F	19		−1.60203(92)	−1.676	0.956(1)	+0.074(1)	−0.2799(43)	−0.717	0.390(6)	+0.437(4)	+0.428	+0.175
Na → Ne	21		+0.7125(12)	+0.726	0.981(2)	−0.014(1)	+0.5823(67)	+0.943	0.617(7)	−0.361(7)	+1.299	+0.817
Mg → Na	23		−0.5541(20)	−0.588	0.942(3)	+0.034(2)	−0.948(13)	−0.763	1.242(17)	−0.185(13)	+1.298	+1.710
Al → Mg	25		+0.8084(11)	+0.781	1.035(1)	+0.027(1)	+1.5211(76)	+1.681	0.905(5)	−0.160(8)	+2.152	+1.881
Si → Al	27		−0.69659(93)	−0.769	0.906(1)	+0.072(1)	−2.0542(75)	−2.010	1.022(4)	−0.044(7)	+2.614	+2.949
P → Si	29	$s_{1/2}$	+0.5380(21)	+0.513	1.049(4)	+0.025(2)	+0.569(13)	+0.556	1.023(23)	+0.013(13)	+1.084	+1.057
S → P	31		−0.5294(15)	−0.490	1.080(3)	−0.039(2)	−0.3138(80)	−0.159	1.974(50)	−0.155(8)	+0.324	+0.593
Cl → S	33	$d_{3/2}$	−0.3142(32)	−0.328	0.958(10)	+0.014(3)	+1.622(16)	+1.611	1.007(10)	+0.011(16)	−4.912	−5.162
Ar → Cl	35		+0.2820(23)	+0.328	0.860(7)	−0.046(2)	−1.572(13)	−1.493	1.053(9)	−0.079(13)	−4.552	−5.574
K → Ar	37		−0.5779(16)	−0.624	0.926(3)	+0.046(2)	+1.5037(80)	+1.416	1.062(6)	+0.088(8)	−2.269	−2.602
Ca → K	39		+0.6606(17)	+0.764	0.865(2)	−0.103(2)	−2.2952(60)	−2.172	1.057(3)	−0.123(6)	−2.843	−3.474
Sc → Ca	41	$f_{7/2}$	+1.0743(38)	+1.116	0.963(3)	−0.042(4)	+2.910(30)	+3.307	0.880(9)	−0.397(30)	+2.963	+2.709
Ti → Sc	43		−0.810(18)	−0.989	0.819(18)	+0.179(17)	−2.29(14)	−1.794	1.279(77)	−0.50(14)	+1.814	+2.833
V → Ti	45		+0.635(20)	+0.619	1.025(33)	+0.016(20)	+1.70(19)	+2.114	0.804(88)	−0.41(19)	+3.415	+2.684

Figure 1: Comparison of experimental and theoretical values for the M_{GT} and M_L matrix elements (in fm units) for the mirror β transitions

Results

$$\frac{b}{Ac_1} = \frac{1}{g_A}\left(g_M + g_V\frac{M_L}{M_{GT}}\right)$$

For A =1–3, 11–31, 37–45;
18 mirror transitions:

$$\frac{(b/Ac)_{\text{exp}}}{(b/Ac)_{\text{theo}}} \approx 0.96(11)$$

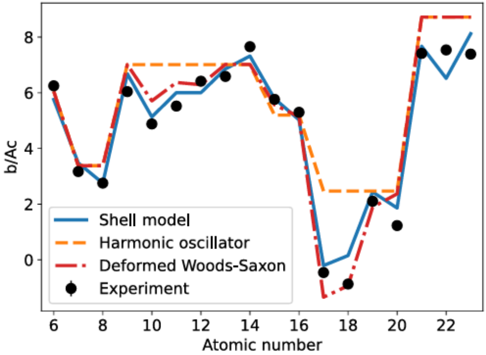

Figure 2: Comparison of different methods of calculating the weak-magnetism contribution b/Ac with experimental data using CVC

Results

$$\mathcal{F}t_0 \equiv f_V t\,(1+\delta_R')(1+\delta_{NS}^V-\delta_C^V)|M_F^0|^2\left[1+\frac{f_A}{f_V}\rho^2\right]$$

$$= \mathcal{F}t^{\text{mirror}}\left[1+\frac{f_A}{f_V}\rho^2\right] = 2\,\mathcal{F}t^{0^+\to 0^+}$$

$$= \frac{K}{G_F^2 V_{ud}^2 g_V^2 (1+\Delta_R^V)} [12pt]$$

$$\overline{\mathcal{F}t}_{mirror} = 6138.7\,\text{s}$$

Confirms the **CVC hypothesis** at the 0.1% level.Consistent with $0^+ \to 0^+$ Fermi transitions.

$$\overline{\mathcal{F}t}_{fermi} = 6144.5(37)\,\text{s}$$

Confirms the **CVC hypothesis** at the $5.2{\times}10^{-4}$ level.

Parent nucleus	$\mathcal{F}t^{\text{mirror}}$ (s)	f_A/f_V	a	A	B	ρ
n	1043.58(67)	1.0000				+2.2091(1
^{19}Ne	1721.5(10)	1.0011		−0.0391(14) [341]		−1.5995(4
^{19}Ne	1721.5(10)	1.0011		−0.03871(81) [69,342]		−1.6014(2
^{21}Na	4073.0(38)	1.0020	0.5502(60) [62]			+0.7135(7
^{29}P	4764.5(79)	1.0008		+0.681(86) [343]		+0.594(10
^{35}Ar	5694.8(60)	0.9929		+0.49(10) [344]		+0.322(75
^{35}Ar	5694.8(60)	0.9929		+0.427(23) [345]		+0.277(16
^{37}K	4611.4(55)	0.9955			−0.755(24) [74]	−0.559(27
^{37}K	4611.4(55)	0.9955		−0.5707(19) [35]		−0.5770(5

Implications for V_{ud} and Physics Beyond SM

$$|V_{ud}|^2 = \frac{K}{\mathcal{F}t_0\, G_F^2\, g_V^2 \left(1 + \Delta_R^V\right)}$$

which yields for the neutron/mirror nuclei：

$$|V_{ud}| = 0.94904(173)(mirrors)$$

$$|V_{ud}| = 0.94934(125)(neutron)$$

$$|V_{ud}| = 0.94815(60)(Fermi)$$

Parent nucleus	$f_V t$ (s)	f_A/f_V	δ'_R (%)	$\delta_C^V - \delta_{NS}^V$ (%)	$\mathcal{F}t^{\mathrm{mirror}}$ (s)	$\delta(\mathcal{F}t^{\mathrm{mirror}})$ %	$\rho = c$ [Eq. (12)]
n	1028.25 ± 0.66	1.0000	1.4902(2)[a]	N/A	1043.58 ± 0.67	0.06	+2.21086(118)
^{3}H	1113.0 ± 1.0	1.0003	1.767(1)	0.16(2)	1130.9 ± 1.0	0.09	−2.1053(14)[b]
^{11}C	3893.4 ± 1.4	0.9992	1.660(4)	1.04(3)	3916.9 ± 1.9	0.05	−0.75442(79)
^{13}N	4621.3 ± 4.7	0.9980	1.635(6)	0.33(3)	4681.3 ± 4.9	0.11	−0.5596(14)
^{15}O	4344.3 ± 5.7	0.9964	1.555(8)	0.22(3)	4402.3 ± 5.9	0.13	+0.6302(16)
^{17}F	2269.5 ± 1.7	1.0020	1.587(10)	0.62(3)	2291.2 ± 1.9	0.08	+1.2955(11)
^{19}Ne	1704.34 ± 0.63	1.0011	1.533(12)	0.52(4)	1721.5 ± 1.0	0.06	−1.60203(92)
^{21}Na	4028.8 ± 3.5	1.0020	1.513(14)	0.41(3)	4073.0 ± 3.8	0.09	+0.7125(12)
^{23}Mg	4651.9 ± 7.3	0.9994	1.476(17)	0.40(3)	4701.6 ± 7.6	0.16	−0.5541(20)
^{25}Al	3678.2 ± 2.4	1.0019	1.475(20)	0.52(5)	3713.0 ± 3.2	0.08	+0.8084(11)
^{27}Si	4095.1 ± 1.9	1.0002	1.443(23)	0.42(4)	4136.7 ± 2.7	0.07	−0.69659(93)
^{29}P	4747.0 ± 7.2	1.0008	1.453(26)	1.07(6)	4764.5 ± 7.9	0.17	+0.5380(21)
^{31}S	4770.3 ± 4.7	0.9992	1.430(29)	0.79(4)	4800.3 ± 5.3	0.11	−0.5294(15)
^{33}Cl	5570.4 ± 8.6	0.9895	1.435(32)	0.93(6)	5597.8 ± 9.5	0.17	−0.3142(32)
^{35}Ar	5645.0 ± 4.9	0.9929	1.421(35)	0.53(5)	5694.8 ± 6.0	0.11	+0.2820(23)
^{37}K	4582.5 ± 4.4	0.9955	1.431(39)	0.79(6)	4611.4 ± 5.5	0.12	−0.5779(16)
^{39}Ca	4264.0 ± 4.5	0.9955	1.422(43)	0.95(8)	4283.5 ± 6.0	0.14	+0.6606(17)
^{41}Sc	2833 ± 10	1.0019	1.454(47)	0.86(7)	2849 ± 11	0.38	+1.0743(38)
^{43}Ti	3688 ± 63	0.9955	1.444(50)	0.63(11)	3718 ± 64	1.7	−0.810(18)
^{45}V	4354 ± 79	1.0042	1.438(53)	0.93(12)	4375 ± 80	1.8	+0.635(20)
^{47}Cr	4568 ± 65	1.0033	1.439(58)	0.8(2)	4596 ± 66	1.4	−0.579(17)
^{49}Mn	4739 ± 132	0.9991	1.438(61)	0.8(2)	4769 ± 133	2.8	+0.537(34)
^{51}Fe	4568 ± 77	0.9970	1.442(66)	0.8(2)	4597 ± 78	1.7	−0.581(20)
^{53}Co	4197 ± 90	1.0039	1.443(70)	0.8(2)	4224 ± 91	2.1	+0.673(23)
^{55}Ni	4199 ± 99	0.9965	1.433(73)	0.8(2)	4225 ± 100	2.4	−0.675(25)
^{57}Cu	4675 ± 45	0.9912	1.455(79)	1.5(3)	4672 ± 47	1.0	+0.564(12)
^{59}Zn	5085 ± 45	0.9856	1.440(81)	1.5(3)	5081 ± 47	0.9	−0.461(12)
^{61}Ga	4759 ± 137	0.9933	1.461(87)	1.5(3)	4756 ± 138	2.9	+0.542(35)
^{67}Se[c]	5344 ± 245	1.0184	1.461(99)	1.7(3)	5330 ± 245	4.6	−0.387(67)
^{67}Se[d]	5908 ± 289				5893 ± 288	4.9	−0.20(12)
^{71}Kr[c]	5108 ± 366	0.9976	1.474(109)	1.7(3)	5095 ± 365	7.2	+0.454(95)
^{71}Kr[d]	5991 ± 432				5976 ± 432	7.2	+0.17(22)
^{75}Sr[c]	4879 ± 590	0.9521	1.484(118)	1.7(3)	4867 ± 588	12	+0.53(15)
^{75}Sr[d]	5458 ± 662				5445 ± 661	12	+0.37(20)

Main content

4. Summary

- Provided updated, high-precision $\mathcal{F}t$ values for mirror β transitions up to $A = 75$.
- Extracted weak magnetism form factors using the CVC hypothesis.
- Found good agreement between experiment and shell-model calculations.
- Demonstrated nuclear structure dependence of weak magnetism.
- Improved basis for precise extraction of V_{ud} and for tests of new physics.

→ Strengthens the link between nuclear structure and fundamental weak interactions.

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Thank you!