Benchmarking nuclear matrix elements of $0\nu\beta\beta$ decay with high-energy nuclear collisions

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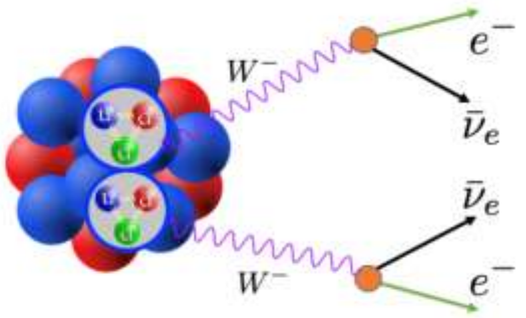
Introduction

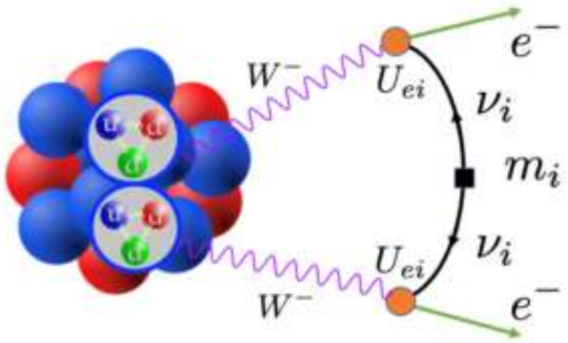
Introduction of neutrinoless double beta ($0\nu\beta\beta$) decay



- Double beta decay

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + 2\bar{\nu}_e$$

Allowed by the standard model



- Neutrinoless double beta decay

Beyond the standard model

$$(A, Z) \rightarrow (A, Z+2) + 2e^-$$

- Determine the nature of the neutrino(Dirac or Majorana)
- Demonstrate the violation of the lepton number in nature and reveal the origin of the matter-antimatter asymmetry
- Determine the effective neutrino mass $\langle m_{\beta\beta} \rangle$

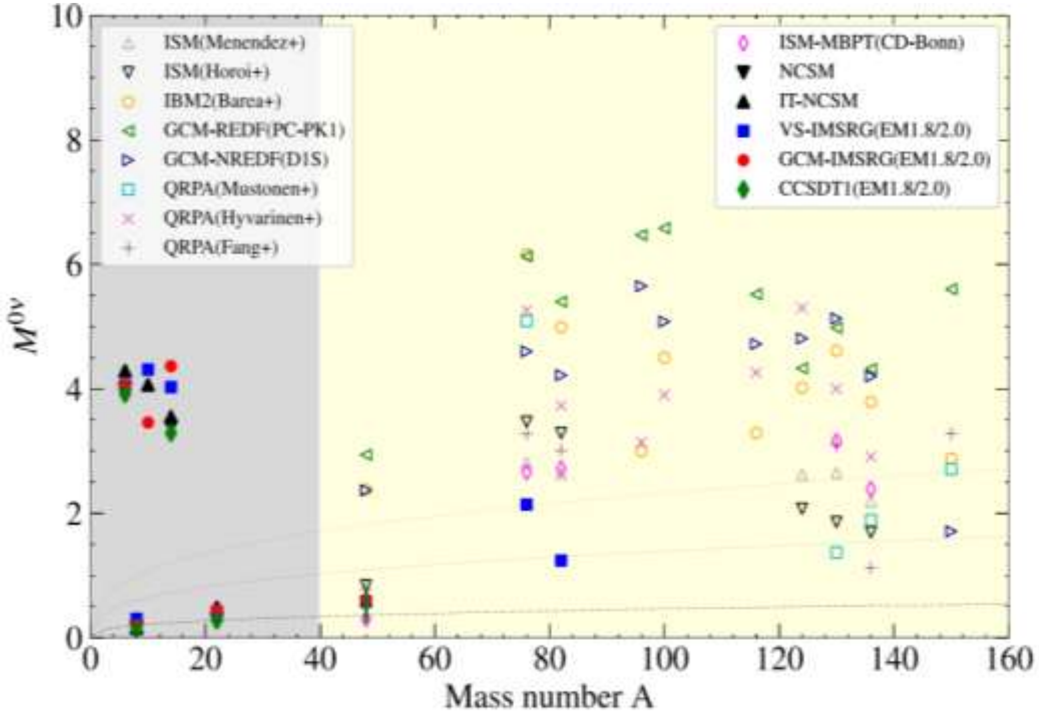
Introduction

NME of $0\nu\beta\beta$ decay

$$|\langle m_{\beta\beta}\rangle| = \left[\frac{m_e^2}{g_A^4(0)G_{0\nu}T_{1/2}^{0\nu}|M^{0\nu}|^2}\right]^{1/2}$$

- $m_e \approx 0.511\text{MeV}$ is electron mass.
- $g_A(0)$ is factorized out from the nuclear matrix element (NME) $M^{0\nu}$ (Its free-space value is around1.27).
- $G_{0\nu} \approx 10^{-14}\text{yr}^{-1}$ is the phase-space factor.
- $T_{1/2}^{0\nu}$ is the half-life of the decaying nucleus, which can be measured experimentally.
- $M^{0\nu}$ is calculated theoretically using a formula:

$$M^{0\nu} = \langle\Psi_F|O^{0\nu}|\Psi_I\rangle$$



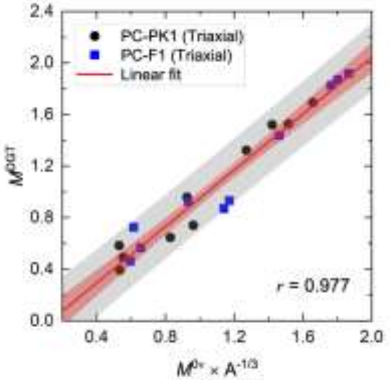
Yao J M, Meng J, Niu Y F, et al. Prog. Part. Nucl. Phys. 126, 103965 (2022)

The NME given by different models varies by 2~3 times !

Introduction

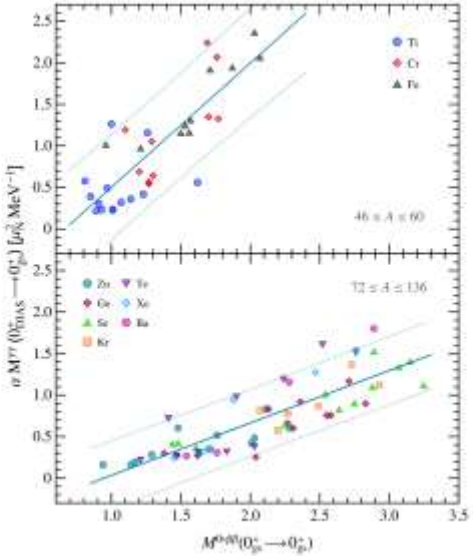
Correlations between the NMEs and observables

double Gamow-Teller transitions



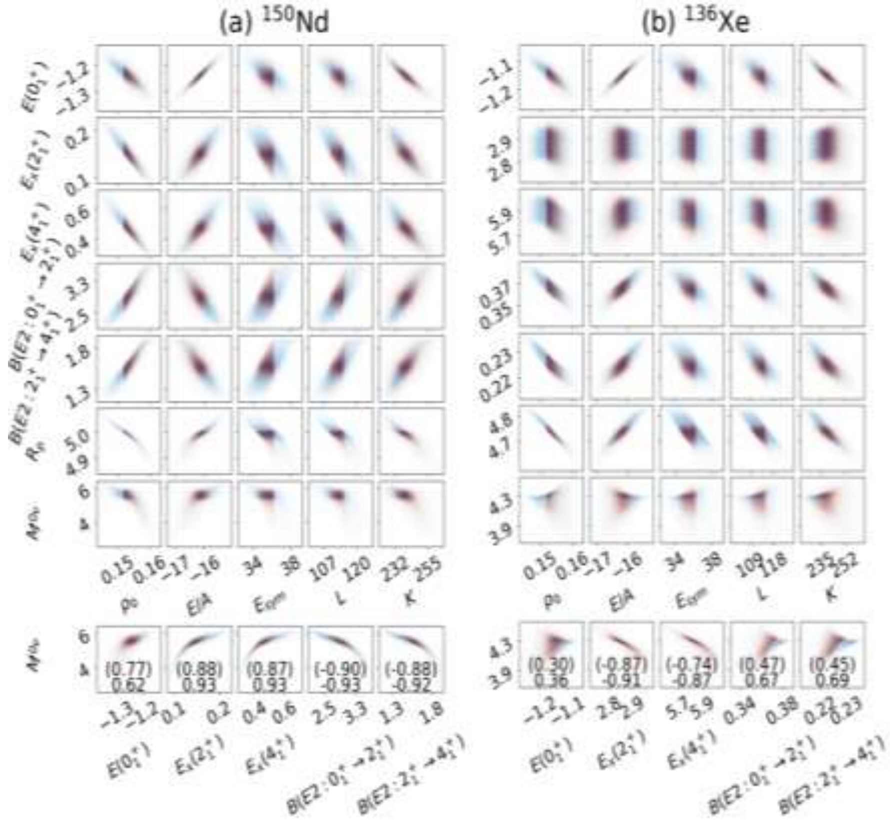
Y.K.Wang, P.W.Zhao et al, PLB 855, 138796(2024)

double-gamma transitions



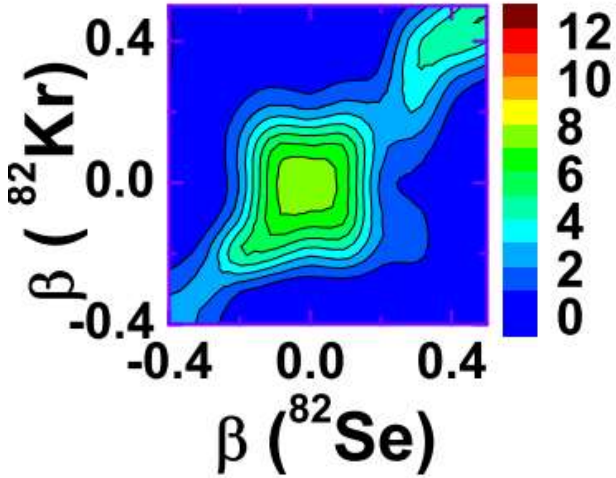
B. Romeo, D. Stramaccioni, et al, PLB 860, 139186 (2025)

low-lying states



X.Zhang et al, arXiv:2408.00691(2024)

deformation

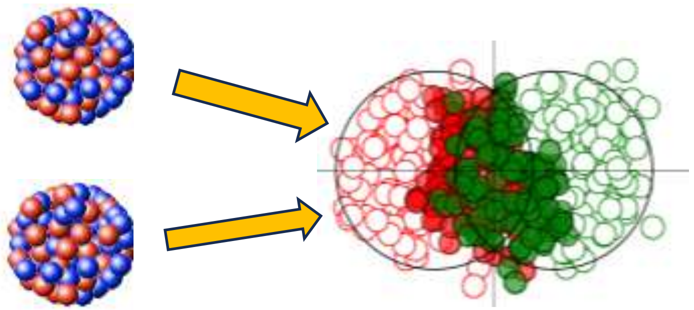


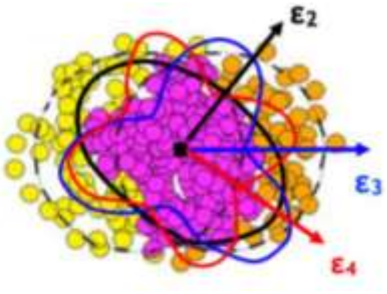
J.M.Yao etc, PRC 91, 024316(2015)

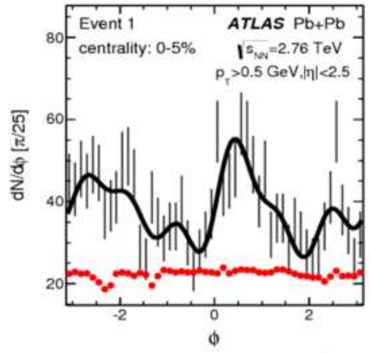
Introduction

Interfacing nuclear structure with high-energy nuclear collisions

Collision | **Initial Condition** | **Final state**







$$\frac{dN_{\mathrm{ch}}}{d^2\mathbf{p}} \propto \frac{dN_{\mathrm{ch}}}{p_T dp_T}\left(1 + \sum_{n>1} 2 v_n \cos n(\phi - \phi_n)\right)$$

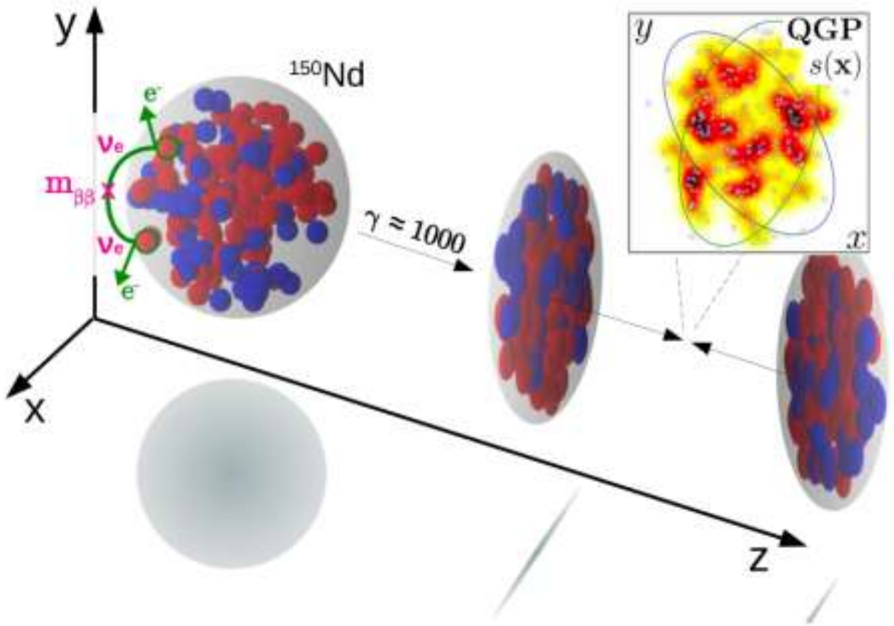
$$[p_T] = \frac{1}{N_{\mathrm{ch}}} \int_{\mathbf{p}} p_T \frac{dN}{d^2\mathbf{p}}$$

- In the early stages of the collision, a quark-gluon plasma (QGP) is generated under extreme temperature and high energy density conditions.
- After expansion and freeze-out, the QGP eventually forms observable particles that are detected by the detectors in the final state.
- The shape of the atomic nucleus can influence the geometric properties of the QGP that are experimentally accessible via collective flow measurements of final particles.

$$v_n = \kappa_n \varepsilon_n$$

Introduction

Motivation



- NMEs are sensitive to the deformation parameters of the candidate isotopes.
- Nuclear deformation information can be probed through high-energy nuclear collisions.

Benchmarking NMEs of $0\nu\beta\beta$ decay with high-energy nuclear collisions !

Theoretical Framework

MR-CDFT

SR-CDFT

- The relativistic energy density functional (EDF) are composed of the kinetic energy, the electromagnetic energy, as well as the nucleon nucleon (*NN*) interaction energy

$$E[\tau,\rho,\nabla\rho;\mathbf{C}] = \int d^3r\left[\tau(\boldsymbol{r}) + \mathcal{E}^{\mathrm{em}}(\boldsymbol{r}) + \sum_{\ell=1}^{9} c_\ell \mathcal{E}_\ell^{\mathrm{NN}}(\boldsymbol{r};\rho,\nabla\rho)\right]$$

where the parameter of NN interaction energy denoted by $\boldsymbol{C} = \{\boldsymbol{a_S}, \boldsymbol{\beta_S}, \boldsymbol{\gamma_S}, \boldsymbol{\delta_S}, \boldsymbol{\alpha_V}, \boldsymbol{\gamma_V}, \boldsymbol{\delta_V}, \boldsymbol{\alpha_{TV}}, \boldsymbol{\delta_{TV}}\}$.

- The single–particle states are determined by minimizing the EDF

$$[\gamma_\mu(i\partial^\mu - V^\mu) - (M + \Sigma_S)]\psi_k = 0.$$

- The mean-field wave functions $|\Phi(\boldsymbol{q})\rangle$ are determined through a deformation-constrained relativistic mean-field (RMF) model coupled to Bardeen–Cooper–Schrieffer (BCS) theory

$$|\Phi(\boldsymbol{q})\rangle = \prod_{k>0}\left(u_k + v_k c_k^\dagger c_{\bar{k}}^\dagger\right)|0\rangle$$

- The total number density $\rho_V(\boldsymbol{r})$ is given by the sum of the densities of each nucleon:

$$\rho_V(\boldsymbol{r}) = \sum_k v_k^2 \psi_k^\dagger(\boldsymbol{r})\psi_k(\boldsymbol{r})$$

Theoretical Framework

MR-CDFT

PNAMP

- Apply angular-momentum(J) and particle-number(N,Z) projections to mean-field wave functions $|\Phi(\boldsymbol{q})\rangle$

$$|JMNZ,\boldsymbol{q}\rangle = \hat{P}^{J}_{M0}\,\hat{P}^{N}\,\hat{P}^{Z}|\Phi(\boldsymbol{q})\rangle$$

where q denote a set of collective variables, such as deformation parameter.

GCM

- The collective wave functions of nuclear low-lying states within the generator coordinate method (GCM)

$$|\Psi_{I/F}(J^{+})\,\rangle = \sum_{\boldsymbol{q}} f_{\nu}^{JNZ}(\boldsymbol{q})\,|JMNZ,\boldsymbol{q}\rangle$$

where weight function $f_{\nu}^{JNZ}(\boldsymbol{q})$ is determined by solving the Hill-Wheeler-Griffin equation

$$\sum_{\mathbf{q}'}\left[\mathcal{H}(\mathbf{q},\mathbf{q}') - E_{\nu}^{JNZ}\mathcal{N}(\mathbf{q},\mathbf{q}')\right] f_{\nu}^{JNZ}(\mathbf{q}') = 0.$$

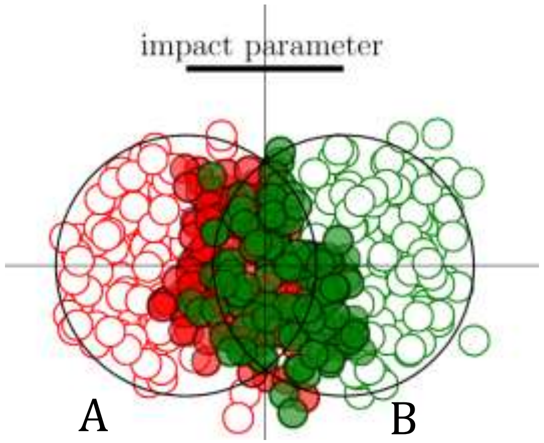
- Calculation of NME

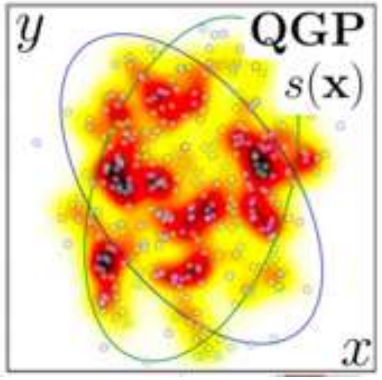
$$M^{0\nu} = \langle\Psi_F(0_1^+)\big|\hat{O}^{0\nu}\big|\Psi_I(0_1^+)\rangle$$

where $\hat{O}^{0\nu}$ is the transition operator based on the standard mechanism of exchange of light Majorana neutrinos.

Theoretical Framework

Collision simulations and observables





- The coordinates of A nucleons are sampled for each of the colliding ions using $\rho_V(\boldsymbol{r})$ as a particle density.
- A given nucleon in nucleus A is flagged as a *participant* if it lies within a radius $\sqrt{\sigma_{NN}/\pi}$ from another nucleon belonging to nucleus B, or vice versa ($\sigma_{NN} = 7\text{fm}^2$ at top LHC energy).
- The thickness functions of the nuclei $t_{A/B}$ are given by a superposition of nucleonic profiles:

$$t_{A/B}(\mathbf{x}) = \sum_{i=1}^{N_{\text{part},A/B}} \frac{\gamma_i}{2\pi w^2} \exp\left(-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2w^2}\right).$$

- The entropy of the initial state $s(\mathbf{x})$ following the T$_{\text{R}}$ENTo Ansatz

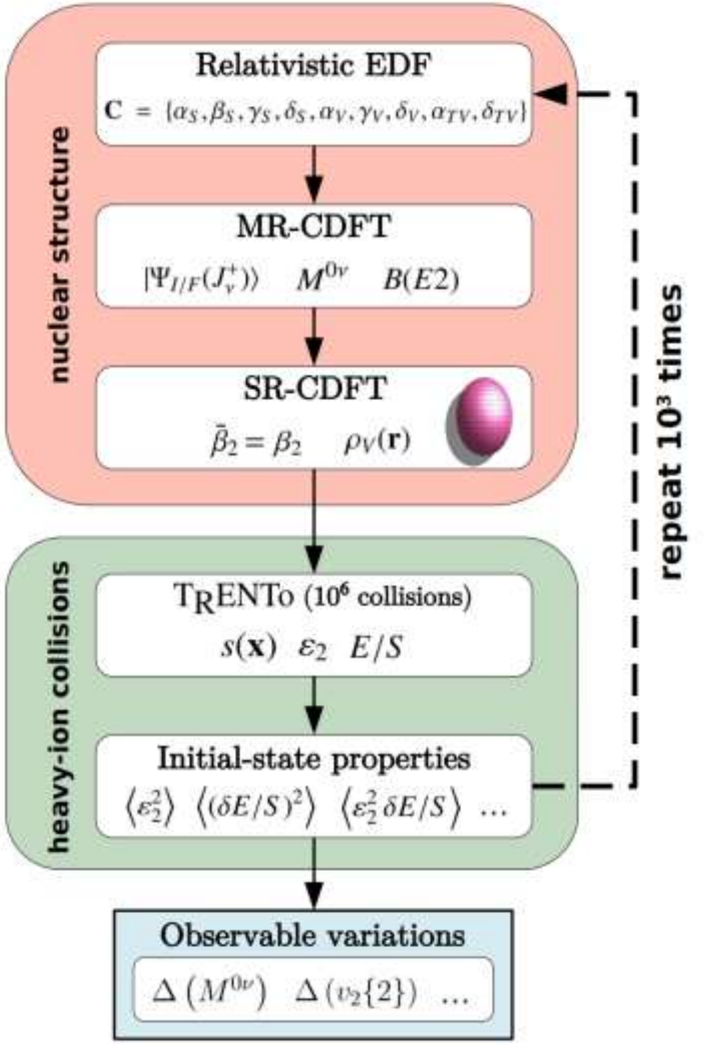
$$s(\mathbf{x}) \propto \left(\frac{t_A(\mathbf{x})^p + t_B(\mathbf{x})^p}{2}\right)^{1/p} \xrightarrow{p=0} \sqrt{t_A(\mathbf{x})t_B(\mathbf{x})}$$

- The spatial anisotropy of QGP ε_n and the energy over the entropy of the system are determined as

$$\varepsilon_n = |\mathcal{E}_n| \qquad \mathcal{E}_n = -\frac{\int_{\mathbf{x}} |\mathbf{x}|^n e^{in\phi} s(\tau_0, \mathbf{x})}{\int_{\mathbf{x}} |\mathbf{x}|^n s(\tau_0, \mathbf{x})} \qquad E/S = \frac{\int_{\mathbf{x}} e(\mathbf{x})}{\int_{\mathbf{x}} s(\mathbf{x})} \qquad e(\mathbf{x}) \propto s(\mathbf{x})^{4/3}$$

Theoretical Framework

Flow chart of framework



- Generate a set $\boldsymbol{C}$ of parameters of the nuclear EDF
- Evaluate the $M^{0\nu}$ and $BE(2)$ transition strength
- Constraint quadrupole deformation parameter $\bar{\beta}_2 = \beta_2$ by SR-CDFT and calculate the density $\rho_V(\boldsymbol{r})$, where β_2 determined as

$$\beta_2 = \frac{4\pi}{3ZR_0^2}\sqrt{B(E2; 0_1^+ \to 2_1^+)}.$$

- Simulate 10^6 times collisions and evaluate the averages

$$\langle\varepsilon_2^2\rangle,\ \langle(\delta E/S)^2\rangle,\ \langle\varepsilon_2^2\delta E/S\rangle,\ 2\langle\varepsilon_2^2\rangle^2 - \langle\varepsilon_2^4\rangle$$

- Repeat for 10^3 sets of $\boldsymbol{C}$ and calculate the variation of observables

$$\Delta(O) = \frac{O - \langle O\rangle_C}{|\langle O\rangle_C|}$$

$$\Delta(v_2\{2\}) \equiv \frac{1}{2}\Delta\left(\left\langle v_2^2\right\rangle\right),$$ elliptic flow

$$\Delta(\delta[p_T]) \equiv \frac{1}{2}\Delta\left(\left\langle(\delta[p_T])^2\right\rangle\right),$$ fluctuation of transverse momentum

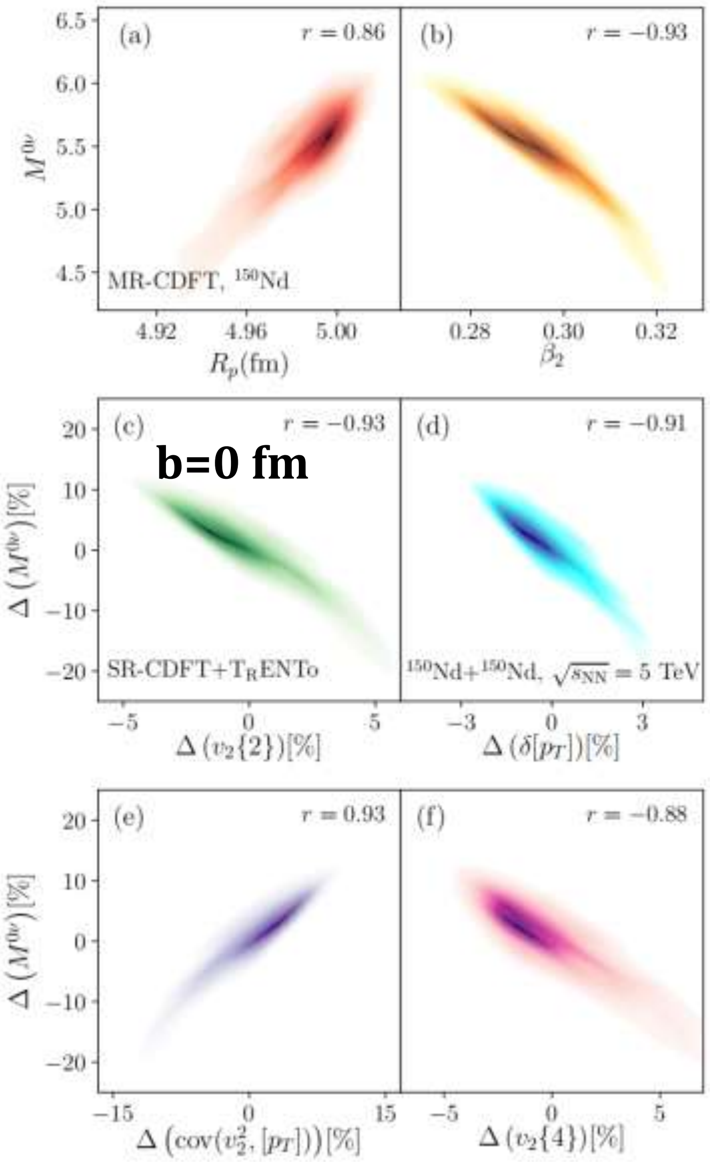
$$\Delta\left(\mathrm{cov}(v_2^2, [p_T])\right) \equiv \frac{1}{3}\Delta\left(\left\langle v_2^2\delta[p_T]\right\rangle\right),$$ covariance between v_2^2 and $\delta[p_t]$

$$\Delta(v_2\{4\}) \equiv \frac{1}{4}\Delta\left(2\left\langle v_2^2\right\rangle^2 - \left\langle v_2^4\right\rangle\right)$$ fourth-order cumulant of the elliptic flow vector

$v_2 \propto \varepsilon_2$
$[p_T] \propto E/S$

Results

Results and discussion (^{150}Nd)



- NME of is correlated with R_p and β_2. The relative systematic uncertainty on R_p is less than 0.5% , while β_2 shows variation up to 10%.

- For ultra-central $^{150}Nd + {}^{150}Nd$ collisions, v_2, $[p_T]$ fluctuation and $v_2\{4\}$ vary by serval percent. The covariance of v_2^2 and $[p_T]$ presents the strongest variation, changing by about 10%.

Summary

- All of these variations are strongly correlated with the value of the NME.
- By combining Bayesian analysis of low- and high-energy heavy-ion data would will lead to an improved determination of β_2 and thus on the NME.

Perspectives

- Include the **octupole** and the **triaxial** deformation of the candidate isotopes

- Investigate whether combining data sets from $^{150}Nd + {}^{150}Nd$ and $^{150}Sm + {}^{150}Sm$ collisions (or other pairs of candidates) gives access to *relative observables* that may present an even tighter correlation with the NME.

Thanks!