

# 原子核中的赝自旋对称性与超对称量子力学

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*Department of Physics, The University of Tokyo, Japan*

March 7, 2024



Lab homepage: <https://tnp.phys.s.u-tokyo.ac.jp/>

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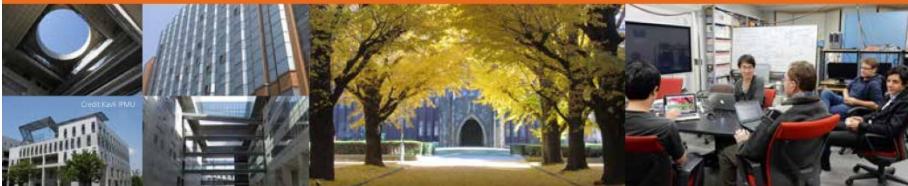
# THE UNIVERSITY OF TOKYO

Department of Physics 2024



# G S G C

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新东方前途出国 7个月前

# Contents

## Nuclear physics and its frontiers

- An overview of **nuclear chart**
- Superheavy elements (**SHE**) and superheavy nuclei (**SHN**)
- Exotic nuclei and their impacts on ***r*-process** nucleosynthesis

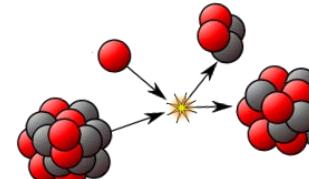
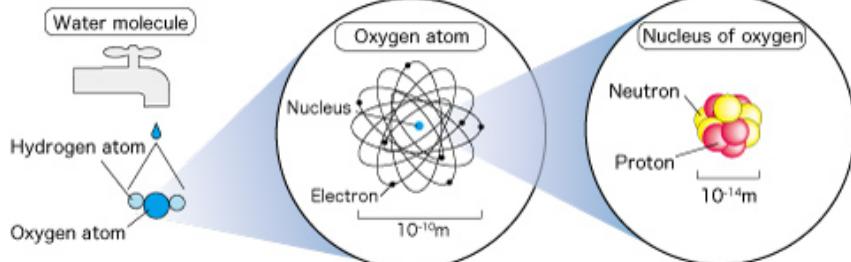
## Nuclear pseudospin symmetry (**PSS**) and supersymmetric (**SUSY**) quantum mechanics

- PSS in atomic nuclei
- Conventional understandings
- Further investigations

## Summary and perspectives

# Nuclear physics research

# Molecule – Atom – Nucleus



# 118 elements

The Periodic Table of Elements																	
scientificgems.wordpress.com																	
1 H Hydrogen																	
3 Li Lithium	4 Be Beryllium																
11 Na Sodium	12 Mg Magnesium																
19 K Potassium	20 Ca Calcium	21 Sc Scandium	22 Ti Titanium	23 V Vanadium	24 Cr Chromium	25 Mn Manganese	26 Fe Iron	27 Co Cobalt	28 Ni Nickel	29 Cu Copper	30 Zn Zinc	31 Ga Gallium	32 Ge Germanium	33 As Arsenic	34 Se Selenium	35 Br Bromine	36 Kr Krypton
37 Rb Rubidium	38 Sr Strontium	39 Y Yttrium	40 Zr Zirconium	41 Nb Niobium	42 Mo Molybdenum	43 Tc Technetium	44 Ru Ruthenium	45 Rh Rhodium	46 Pd Palladium	47 Ag Silver	48 Cd Cadmium	49 In Indium	50 Sn Tin	51 Sb Antimony	52 Te Tellurium	53 I Iodine	54 Xe Xenon
55 Cs Cesium	56 Ba Barium	57-71 La-Lu Lanthanides	72 Hf Hafnium	73 Ta Tantalum	74 W Tungsten	75 Re Rhenium	76 Os Osmium	77 Ir Iridium	78 Pt Platinum	79 Au Gold	80 Hg Mercury	81 Tl Thallium	82 Bi Bismuth	83 Po Polonium	84 At Astatine	85 Rn Radium	86 Ts Thorium
87 Fr Francium	88 Ra Radium	89-103 Ac-Lr Actinides	104 Rf Rutherfordium	105 Db Dubnium	106 Sg Seaborgium	107 Bh Bohrium	108 Hs Hassium	109 Mt Meitnerium	110 Ds Darmstadtium	111 Rg Roentgenium	112 Cn Copernicium	113 Nh Nihonium	114 Fl Flerovium	115 Mc Moscovium	116 Lv Livermorium	117 Ts Tennessine	118 Og Oganesson
57 La Lanthanum	58 Ce Cerium	59 Pr Praseodymium	60 Nd Neodymium	61 Pm Promethium	62 Sm Samarium	63 Eu Europium	64 Gd Gadolinium	65 Tb Terbium	66 Dy Dysprosium	67 Ho Holmium	68 Er Erbium	69 Tm Thulium	70 Yb Ytterbium	71 Lu Lutetium			
89 Ac Actinium	90 Th Thorium	91 Pa Protactinium	92 U Uranium	93 Np Neptunium	94 Pu Plutonium	95 Am Americium	96 Cm Curium	97 Bk Berkelium	98 Cf Californium	99 Es Einsteinium	100 Fm Fermium	101 Md Mendelevium	102 No Nobelium	103 Lr Lawrencium			

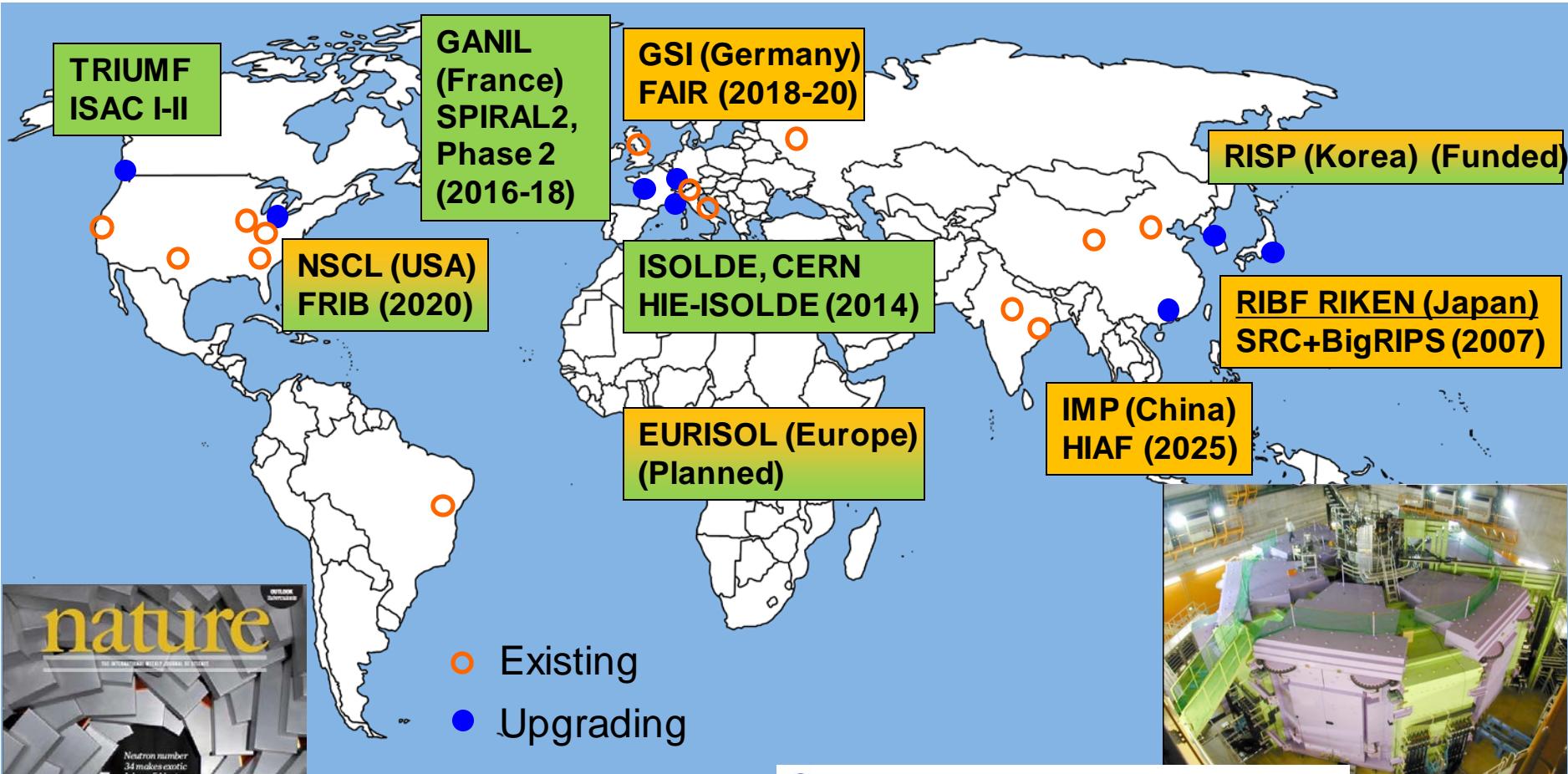
# "How many kinds of nuclei exist in the Universe?" (*nuclear physics*)

# "Where and how are nuclei created in the Universe?" (*nuclear physics × astrophysics*)

# "How do we make the best and safe use of nuclei?" (*nuclear physics* × *technology*)



# Radi oactive i sotope beam facilities



**Best in the world  
~70 % speed of light !**

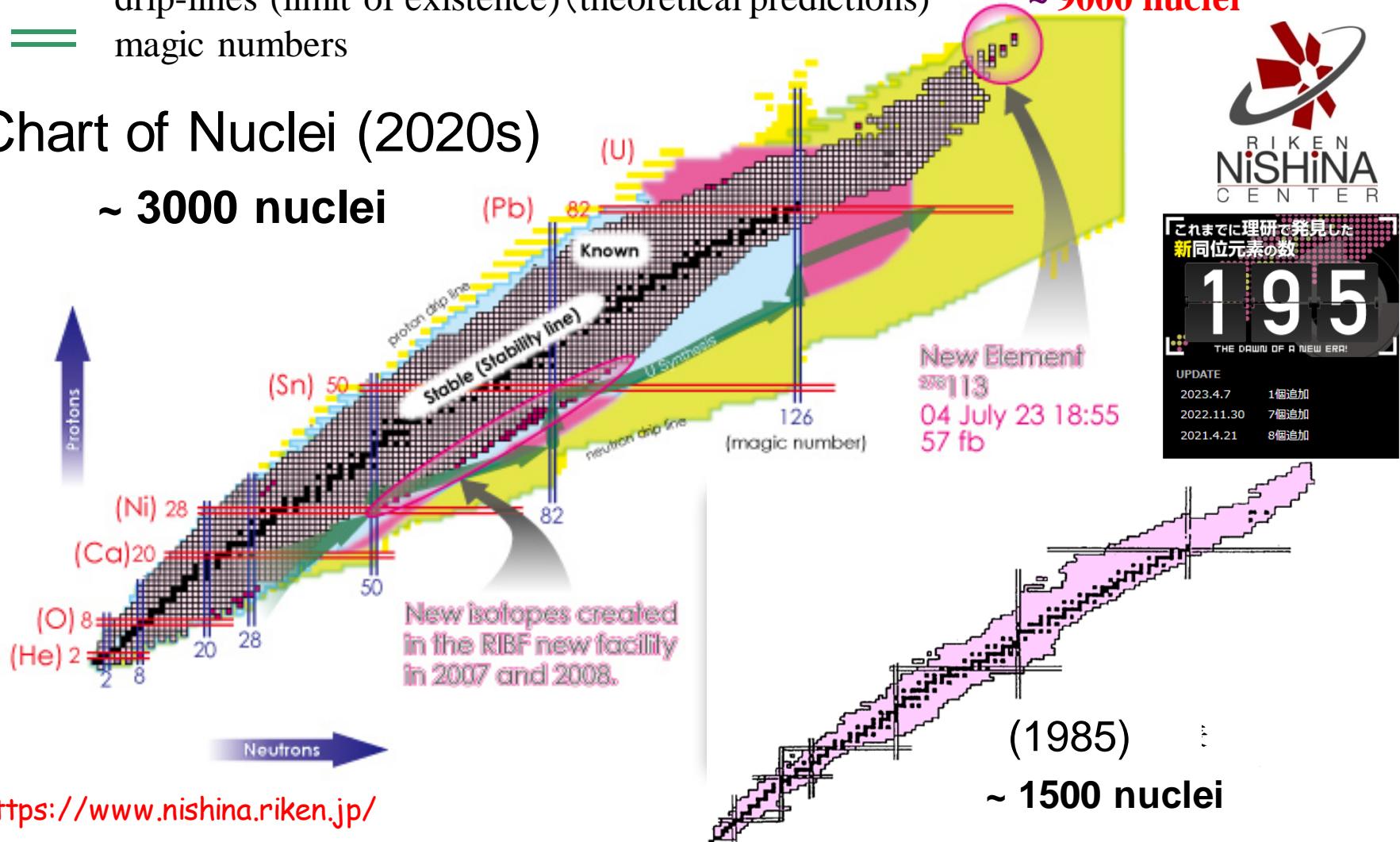


# Nuclear chart

- █ stable nuclei
  - █ unstable nuclei observed so far
  - drip-lines (limit of existence) (theoretical predictions)
  - magic numbers
- ~ 300 nuclei  
~ 3000 nuclei  
~ 9000 nuclei

## Chart of Nuclei (2020s)

~ 3000 nuclei



# Nuclear chart



stable nuclei



unstable nuclei observed so far



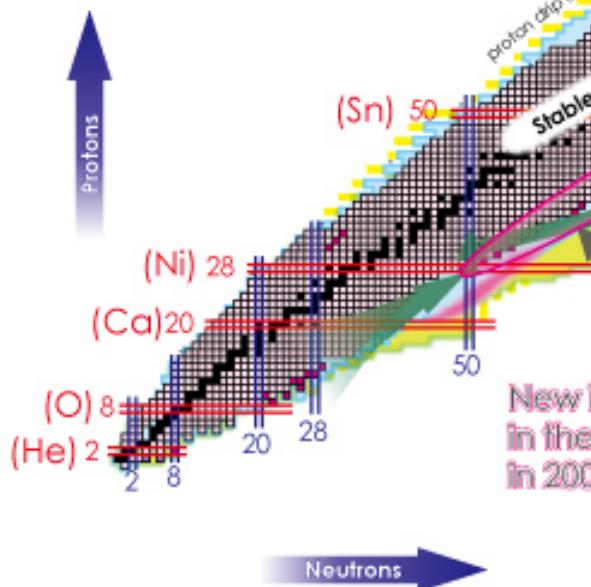
drip-lines (limit of existence) (theoretical predictions)



magic numbers

## Chart of Nuclei (2020s)

~ 3000 nuclei



## Superheavy elements

- How to synthesize more new elements?
- What are quantum tunneling properties of SHE?
- Is there island of stability? New materials?

<https://www.nishina.riken.jp/>

~ 300 nuclei

~ 3000 nuclei

~ 9000 nuclei



# Periodic Table with National flags

## Elements & Country of Discovery

The table shows the periodic table with flags representing the countries of discovery for each element. The flags are color-coded according to the element's group. For example, Group 1 (alkali metals) includes the UK, USA, Germany, Sweden, France, Russia, Austria, Denmark, Spain, Switzerland, Canada, Finland, Italy, Japan, and Romania. Group 2 (alkaline earth metals) includes the UK, France, and the UK again. Group 17 (halogens) includes the UK, France, and the UK again. Group 18 (noble gases) includes the UK, France, and the UK again. The table also includes a section for 'Known to ancients' which includes the UK, France, and the UK again.

Created by @jamiebgall

This is a smaller section of the periodic table showing elements 58 to 118. Each element cell contains the flag of the country of its discovery. The flags are color-coded according to the element's group. For example, Group 17 (halogens) includes the UK, France, and the UK again. Group 18 (noble gases) includes the UK, France, and the UK again.

Credit given to both where joint or independently discovered.

This table is not based on nationality of researcher(s) but is based on institution/funder

Curated by Dr Jamie Gallagher, @jamiebgall

Download available at [jamiebgall.co.uk/resources](http://jamiebgall.co.uk/resources)

## Open question:

- Is there or where is **the end** of the periodic table?

# Nuclear chart



stable nuclei



unstable nuclei observed so far



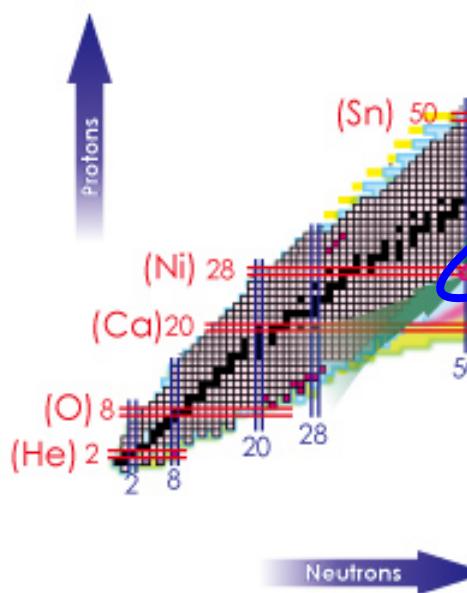
drip-lines (limit of existence) (theoretical predictions)



magic numbers

## Chart of Nuclei (2020s)

~ 3000 nuclei



<https://www.nishina.riken.jp/>

### Superheavy elements

- How to synthesize more new elements?
- What are quantum tunneling properties of SHE?
- Is there island of stability? New materials?

### Neutron-rich isotopes

- How to synthesize more new isotopes?
- What will be the impacts for understanding origins of heavy elements?
- What will be the impacts for handling nuclear wastes?

~ 300 nuclei

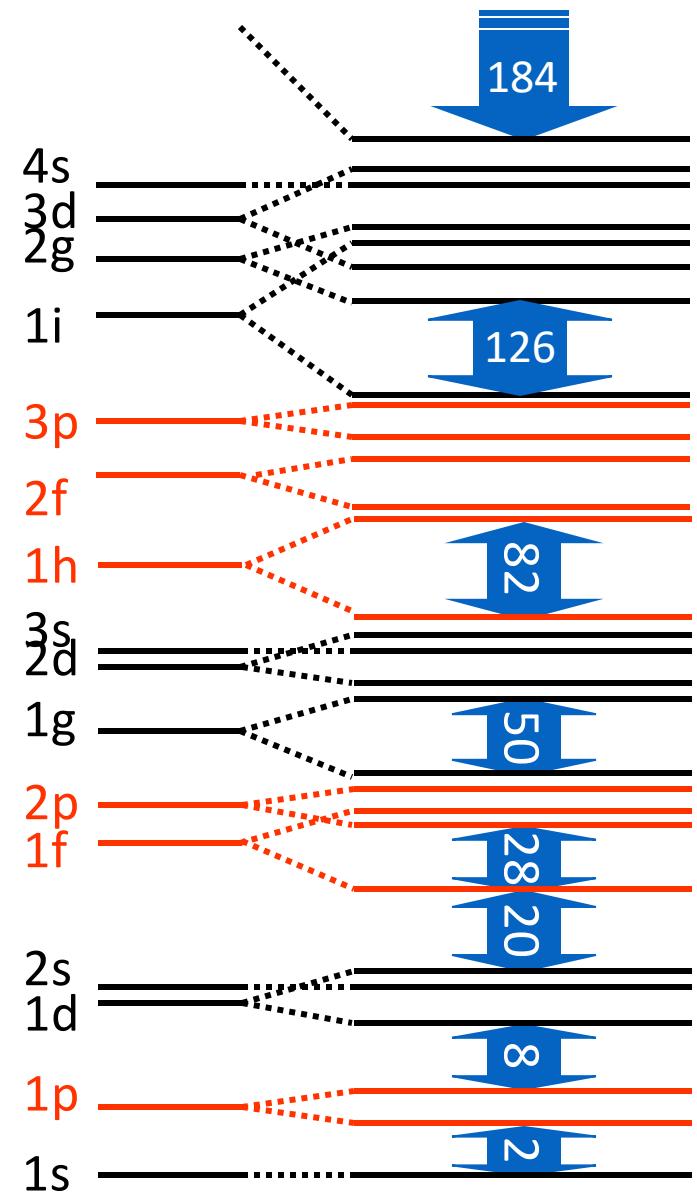
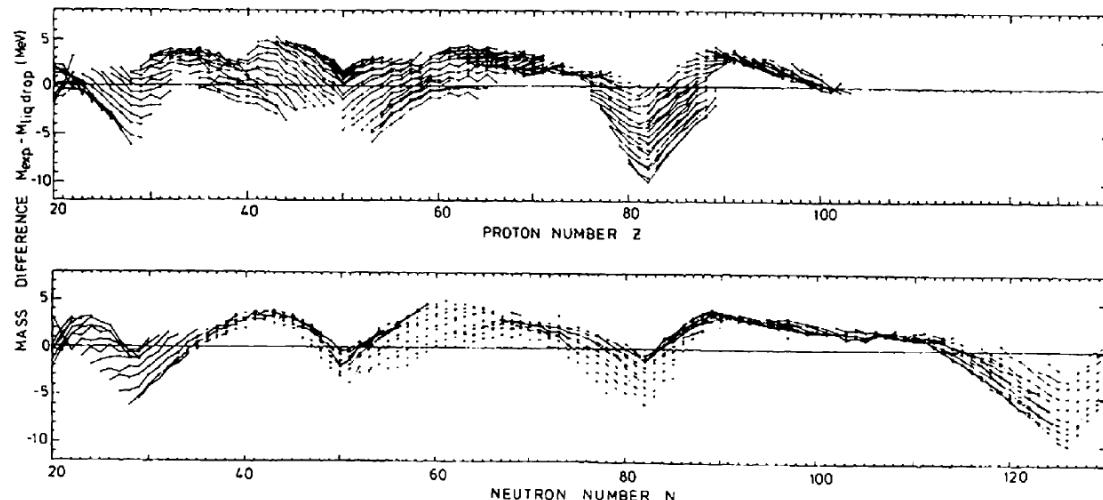
~ 3000 nuclei

~ 9000 nuclei



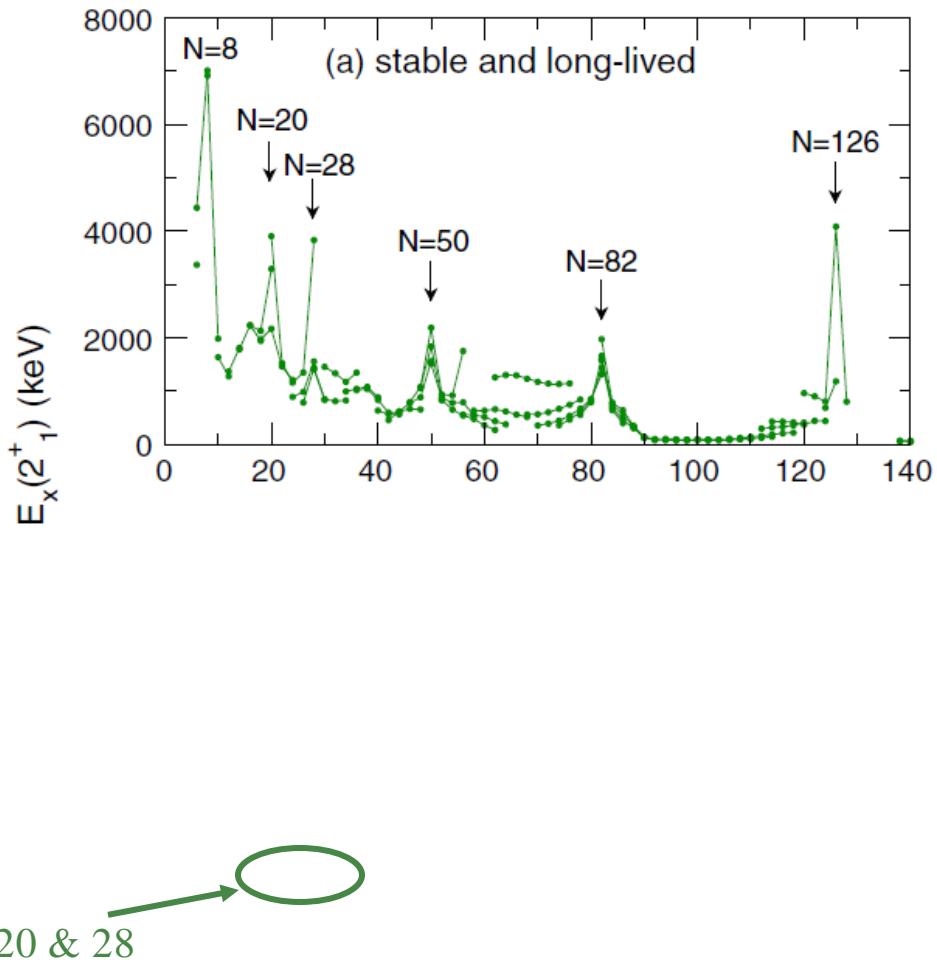
# Nuclear magnetic numbers

## ➤ Shell-correction energy

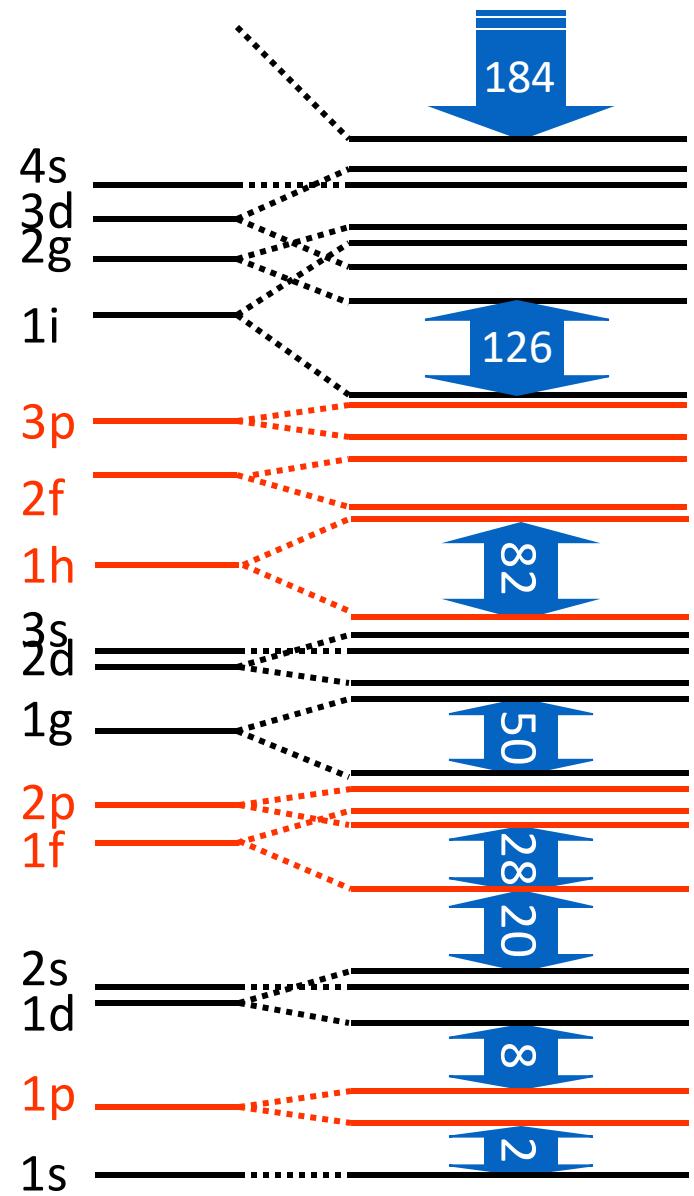


# Nuclear magnetic numbers

- Energy of the first  $2^+$  state

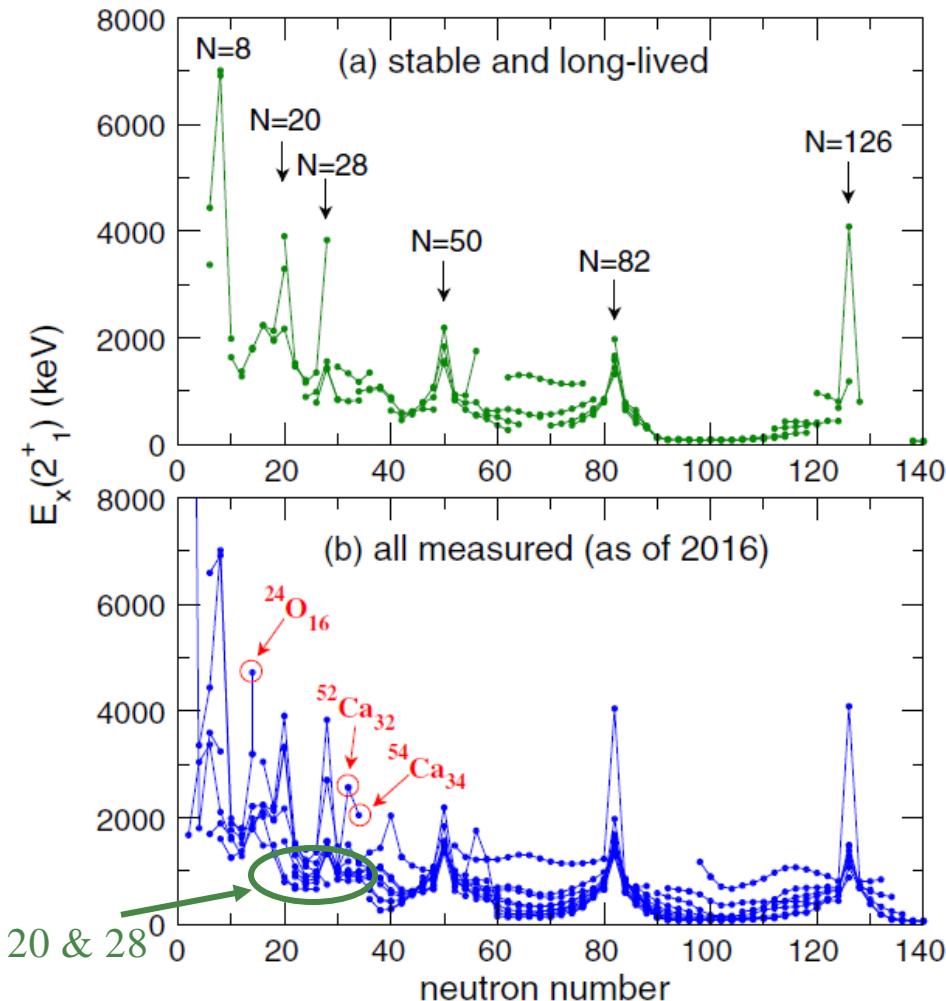


Otsuka et al., Rev. Mod. Phys. 92, 015002 (2020)



# Nuclear magic numbers

- Energy of the first  $2^+$  state



Towards neutron-rich region—  
**exotic nuclei**

- Disappearing magic numbers:  
 $\mathcal{N} = 20, 28$
- New magic numbers:  
 $\mathcal{N} = 16, 32, 34 \dots$

# Experimental progress

nature

## LETTERS

### 'Magic' nucleus $^{42}\text{Si}$

J. Fridmann<sup>1</sup>, I. Wiedenhofer<sup>1</sup>, A. Gade<sup>2</sup>, L. T. Baby<sup>1</sup>, D. Bazin<sup>2</sup>, B. A. P. D. Cottle<sup>1</sup>, E. Diffenderfer<sup>1</sup>, D.-C. Dinca<sup>2</sup>, T. Glasmacher<sup>2</sup>, P. G. Hansen<sup>2</sup>, W. F. Mueller<sup>2</sup>, H. Olliver<sup>2</sup>, E. Rodriguez-Vieitez<sup>3</sup>, J. R. Terry<sup>2</sup>, J. A.

## LETTER

### Evidence for a new nuclear 'magic number' from the level structure of $^{54}\text{Ca}$

D. Stepenbeck<sup>1</sup>, S. Takeuchi<sup>2</sup>, N. Aoi<sup>3</sup>, P. Doornenbal<sup>2</sup>, M. Matsushita<sup>1</sup>, H. Wang<sup>2</sup>, H. J. Lee<sup>1</sup>, K. Matsui<sup>5</sup>, S. Michimasa<sup>1</sup>, T. Motabayashi<sup>2</sup>, D. Nishimura<sup>6</sup>, T. Otsuka<sup>1,5</sup>, H. S. T. Sumikama<sup>8</sup>, H. Suzuki<sup>2</sup>, R. Taniuchi<sup>5</sup>, Y. Utsuno<sup>9</sup>, J. J. Valiente-Dobón<sup>10</sup> & K. Yoneda<sup>1</sup>



#### Article

678 | Nature | Vol 606 | 23 June 2022

### Observation of a correlated free four-neutron system

nature

Vol 465 | 27 May 2010 | doi:10.1038/nature09048

## LETTERS

### The magic nature of $^{132}\text{Sn}$ explored through the single-particle states of $^{133}\text{Sn}$

J. C. Blackmon<sup>4</sup>, K. Y. Chae<sup>1</sup>, K. A. Chippis<sup>5</sup>, J. A. Cizewski<sup>2</sup>, L. Erikson<sup>5</sup>, J. F. Liang<sup>4</sup>, R. Livesay<sup>5</sup>, Z. Ma<sup>1</sup>, B. H. Moazen<sup>1</sup>, C. D. Nesaraja<sup>4</sup>, Shapira<sup>4</sup>, J. F. Shriner Jr<sup>7</sup>, M. S. Smith<sup>4</sup>, T. P. Swan<sup>2,6</sup> & J. S. Thomas<sup>6</sup>

doi:10.1038/nature12522

## ARTICLE

<https://doi.org/10.1038/s41586-019-1155-x>

### $^{78}\text{Ni}$ revealed as a doubly magic stronghold against nuclear deformation

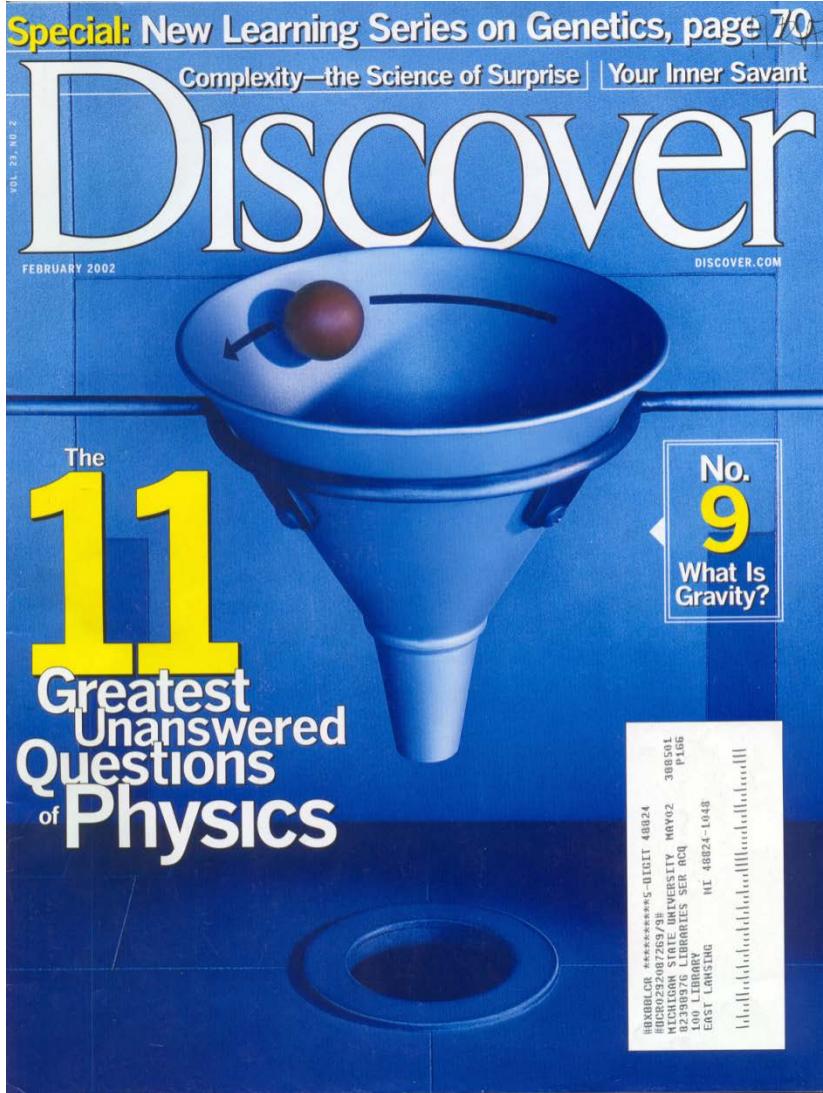
R. Taniuchi<sup>1,2</sup>, C. Santamaría<sup>2,3</sup>, P. Doornenbal<sup>2\*</sup>, A. Obertelli<sup>2,3,4</sup>, K. Yoneda<sup>2</sup>, G. Authelet<sup>3</sup>, H. Baba<sup>2</sup>, D. Calvet<sup>3</sup>, F. Château<sup>3</sup>, J. Cibert<sup>3</sup>, J. D. Holt<sup>5</sup>, T. Isobe<sup>2</sup>, V. Lapoux<sup>3</sup>, M. Matsushita<sup>6</sup>, J. Menéndez<sup>6</sup>, F. Nowacki<sup>7</sup>, K. Ogata<sup>8,9</sup>, H. Otsu<sup>2</sup>, T. Otsuka<sup>1,2,6</sup>, C. Péron<sup>3</sup>, S. Péru<sup>10</sup>, Rousse<sup>3</sup>, H. Sakurai<sup>1,2</sup>, A. Schwenk<sup>4,12,13</sup>, Y. Shiga<sup>2,14</sup>, J. Simonis<sup>4,12,15</sup>, T. Uesaka<sup>2</sup>, H. Wang<sup>2</sup>, F. Browne<sup>7</sup>, L. X. Chung<sup>18</sup>, Z. Dombradi<sup>19</sup>, S. Franchou<sup>20</sup>, Klé<sup>21</sup>, Z. Korkulu<sup>19</sup>, S. Koyama<sup>1,2</sup>, Y. Kubota<sup>2,6</sup>, J. Lee<sup>22</sup>, M. Lettmann<sup>4</sup>, C. Louchart<sup>4</sup>, Nishimura<sup>2</sup>, L. Olivier<sup>20</sup>, S. Ota<sup>6</sup>, Z. Patel<sup>24</sup>, E. Şahin<sup>21</sup>, C. Shand<sup>24</sup>, P.-A. Söderström<sup>2</sup>, D. Suzuki<sup>20</sup>, Z. Vajta<sup>19</sup>, W. Werner<sup>4</sup>, J. Wu<sup>2,26</sup> & Z. Y. Xu<sup>22</sup>

#### Article

Nature | Vol 620 | 31 August 2023 | 965

### First observation of $^{28}\text{O}$

# The 11 greatest unanswered questions of physics

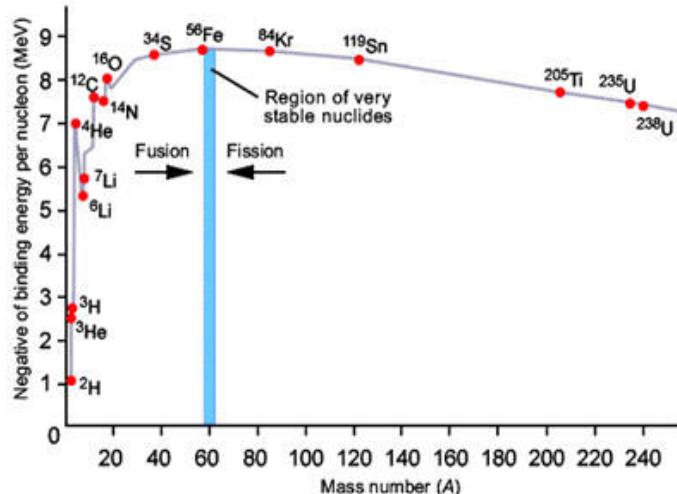


National research council's board on  
physics and astrophysics

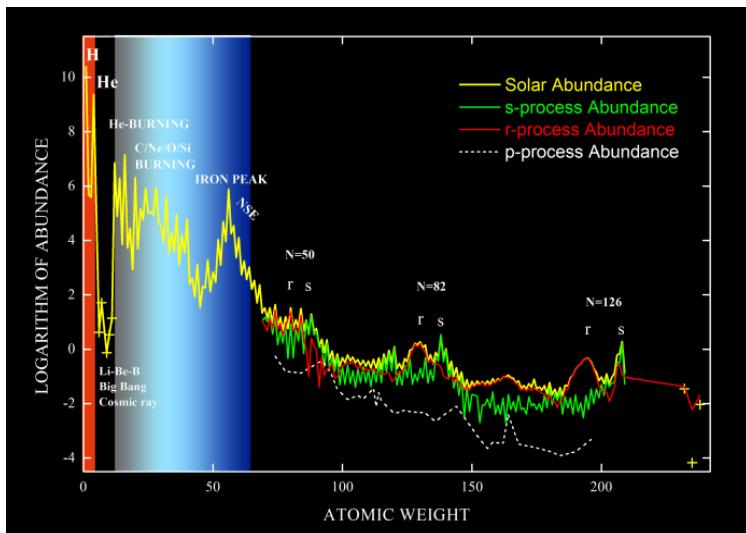
1. What is dark matter?
2. What is dark energy?
3. **How were the heavy elements from iron to uranium made?**
4. Do neutrinos have mass?
5. Where do ultrahigh-energy particles come from?
6. Is a new theory of light and matter needed to explain what happens at very high energies and temperatures?
7. Are there new states of matter at ultrahigh temperatures and densities?
8. Are protons unstable?
9. What is gravity?
10. Are there additional dimensions?
11. How did the universe begin?

# Nucl eosynthesis

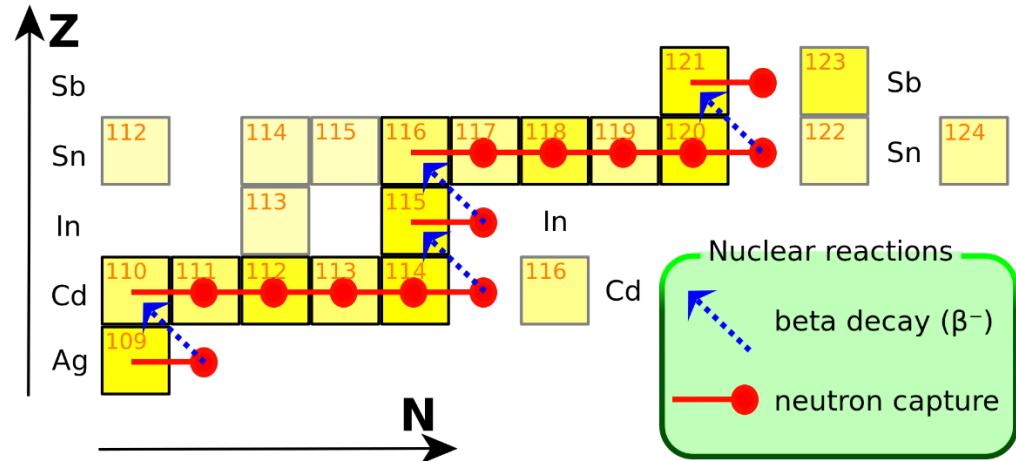
➤ Nuclear binding energy



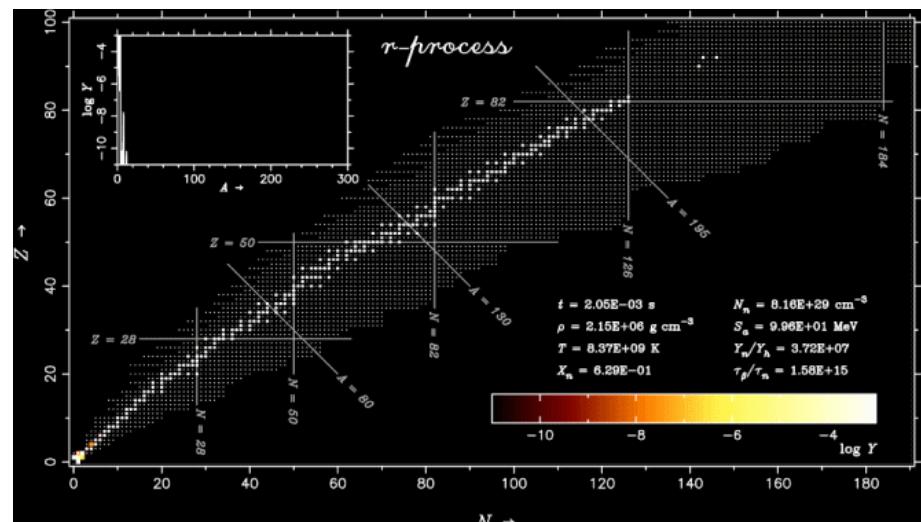
➤ Abundance



➤ Slow neutron-capture process (*s*-process)



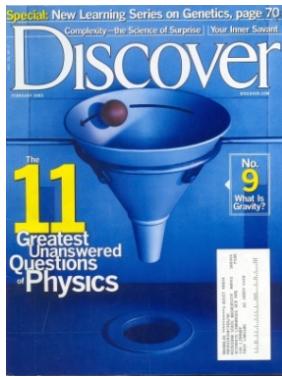
➤ Rapid neutron-capture process (*r*-process)



Courtesy of S. Wanajo

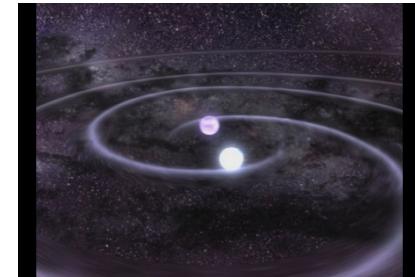
# *r*-process nucleosynthesis and nuclear inputs

## The 11 greatest unanswered questions of physics



### Question 3

How were the heavy elements from iron to uranium made?

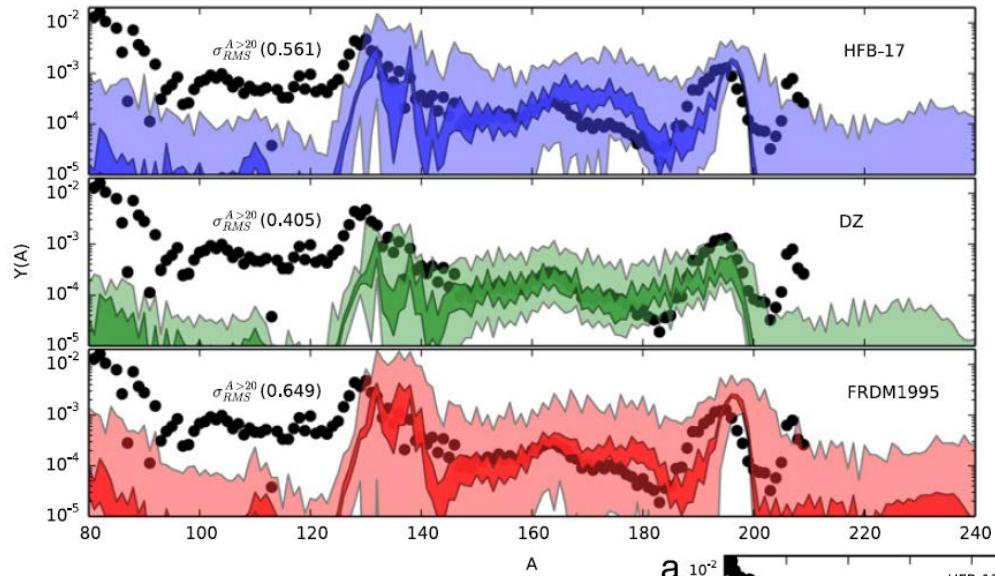


*Why are gold (金) and platinum (白金) expensive? Why is lead (铅) cheap?  
Why are rare-earth elements (稀土元素) rare?*

★ Nuclear data inputs for *r*-process

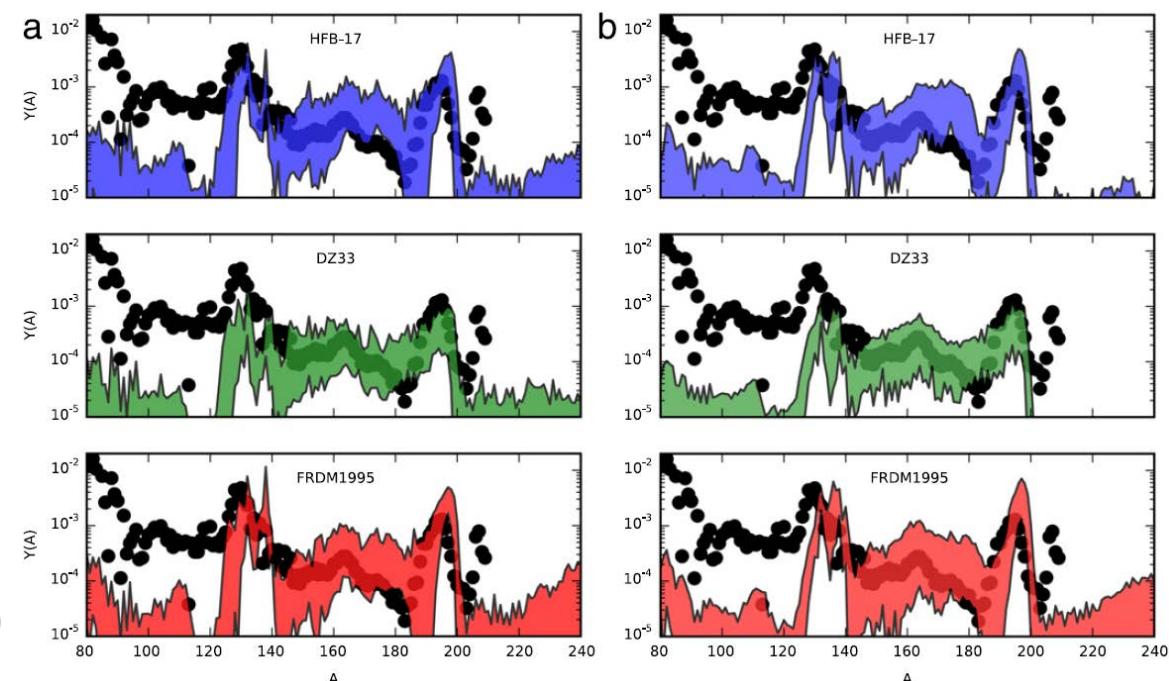
Quantity	Effects
$S_n$	neutron separation energy
$T_{1/2}$	$\beta$ -decay half-lives
$P_n$	$\beta$ -delayed $n$ -emission branchings
$Y_i$	fission (products and branchings)
$G$	partition functions
$N_A \langle \sigma \nu \rangle$	neutron capture rates

# Impacts of nuclear properties on $r$ -process



← Impacts of nuclear masses  
(~ 100 keV)

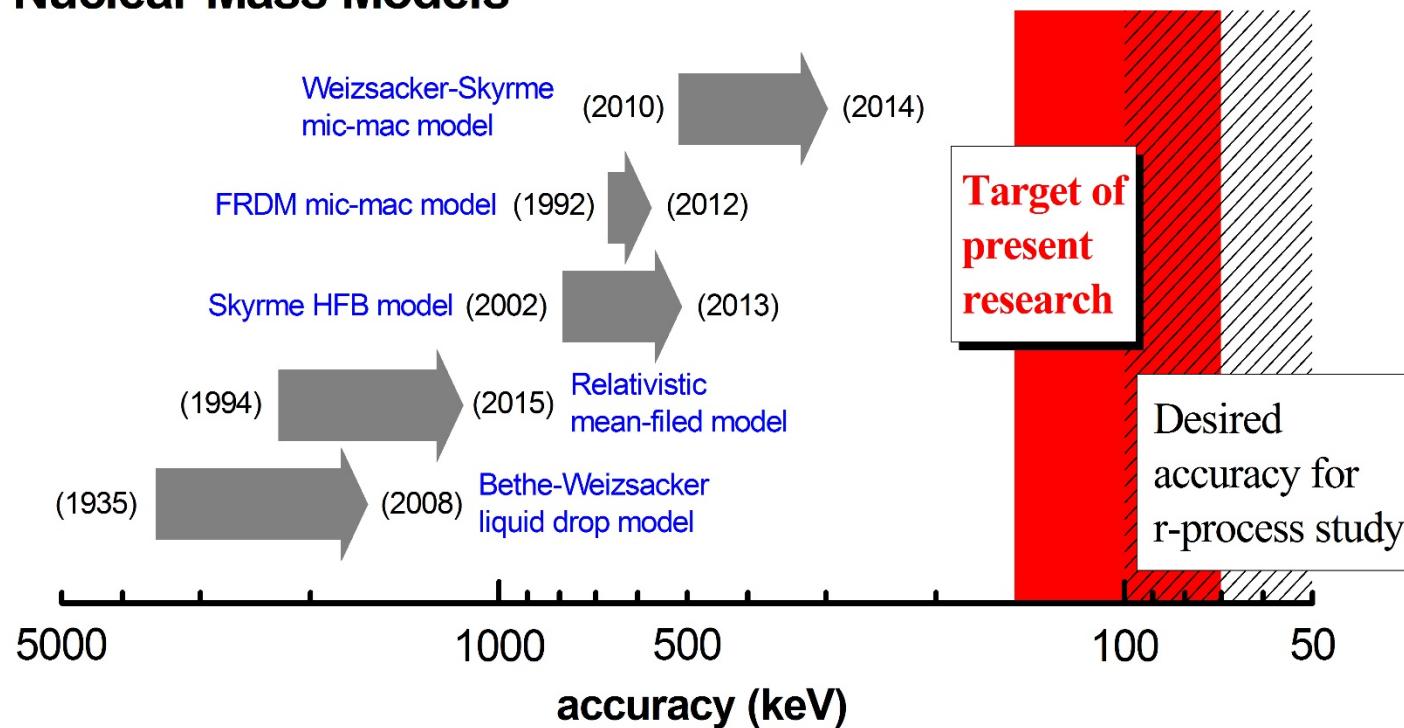
Impacts of  
nuclear  $\beta$ -decay half-lives  
(factor 10)  
neutron-capture rates  
(factor  $10^3$ )



# Nuclear mass models

- Theoretically, the development of nuclear mass model can be traced back to the early age of nuclear physics, known as **Bethe-Weizsäcker liquid drop model** in 1935.
- To take into account the nuclear shell effects: the microscopic models and the microscopic-macroscopic (mic-mac) models.

## Nuclear Mass Models



# Published database

- The first nuclear mass model with accuracy within **100 keV** is constructed. Its accuracies to  $S_*$  and  $Q_*$  are also much higher than other mass models.

Model	M	$S_n$	$S_{2n}$	$S_p$	$S_{2p}$	$S_D$	$Q_\beta$
BMM	<b>0.084</b>	<b>0.078</b>	<b>0.105</b>	<b>0.083</b>	<b>0.111</b>	<b>0.096</b>	<b>0.099</b>
HFB31	0.559	0.451	0.456	0.489	0.496	0.566	0.557
FRDM12	0.576	0.340	0.442	0.341	0.420	0.411	0.450
WS4	0.285	0.254	0.261	0.261	0.300	0.324	0.327

## Calculated results and Predictions:

- Nuclear Masses Niu and HZL, *PRC* **106**, L021303 (2022)  
<https://journals.aps.org/prc/supplemental/10.1103/PhysRevC.106.L021303>
- Nuclear  $\beta$ -decay half-lives Minato, Niu, HZL, *PRC* **106**, 024306 (2022)  
<https://journals.aps.org/prc/supplemental/10.1103/PhysRevC.106.024306>

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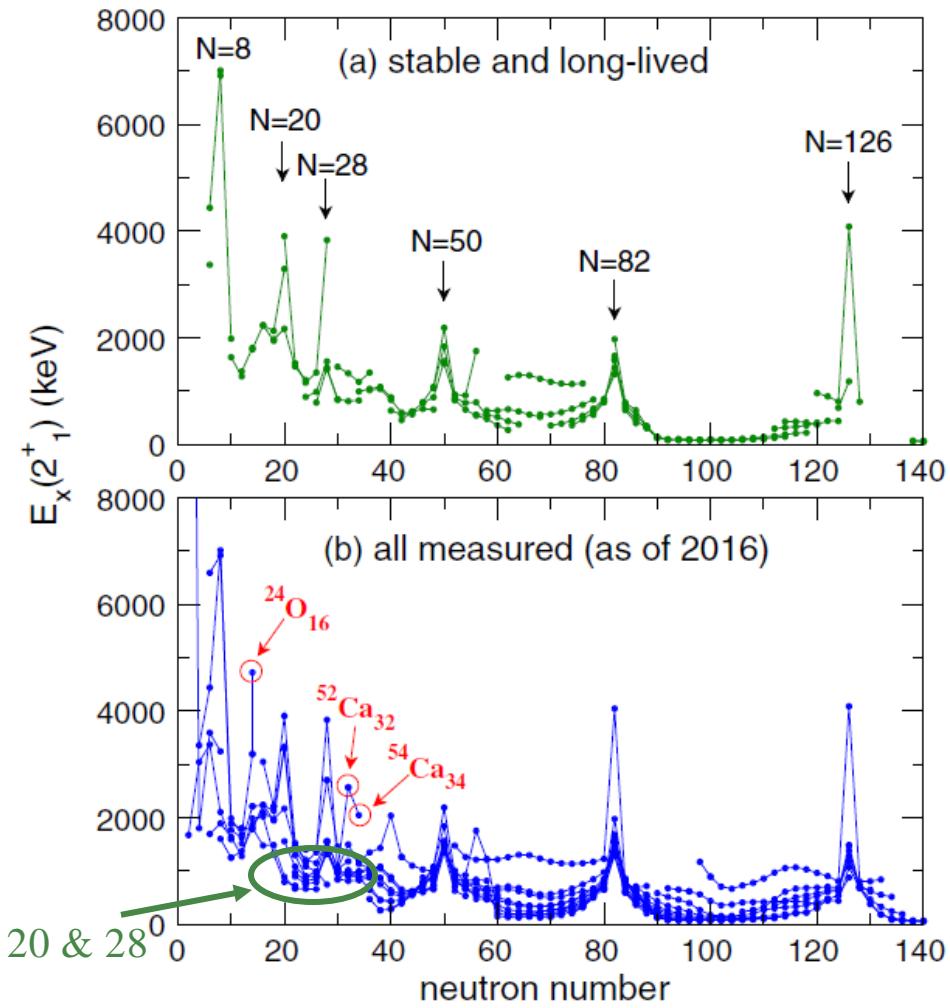
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- PSS in atomic nuclei
- Conventional understandings
- Further investigations

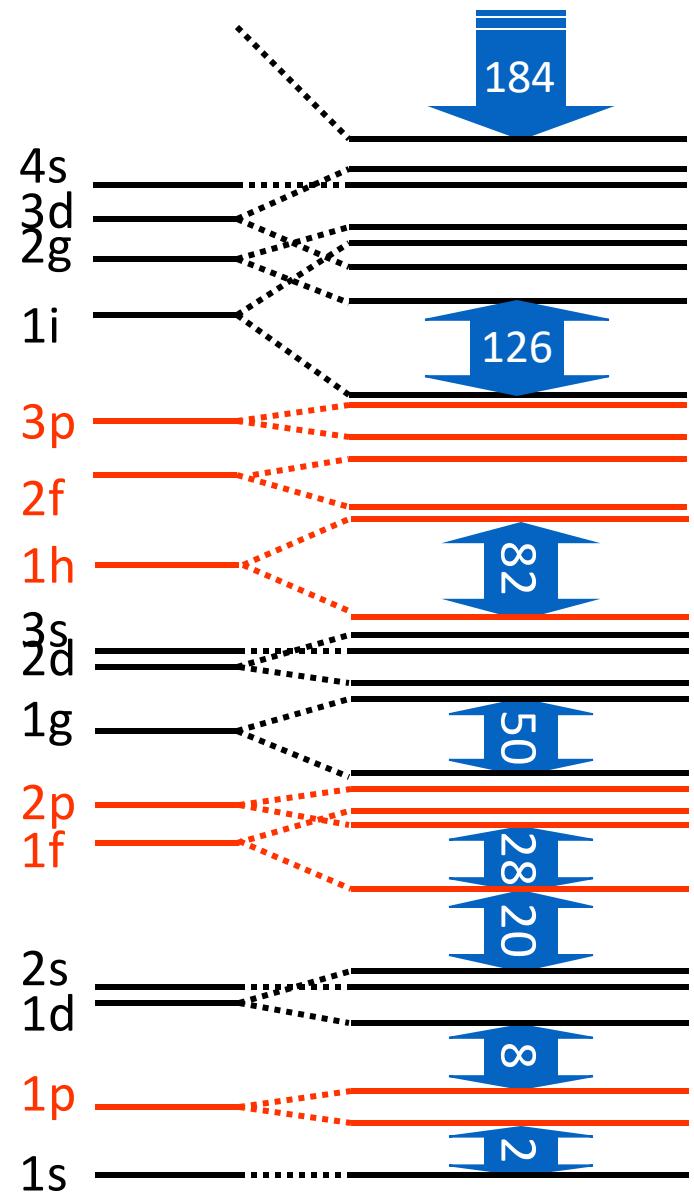
## Summary and perspectives

# Nuclear magnetic numbers

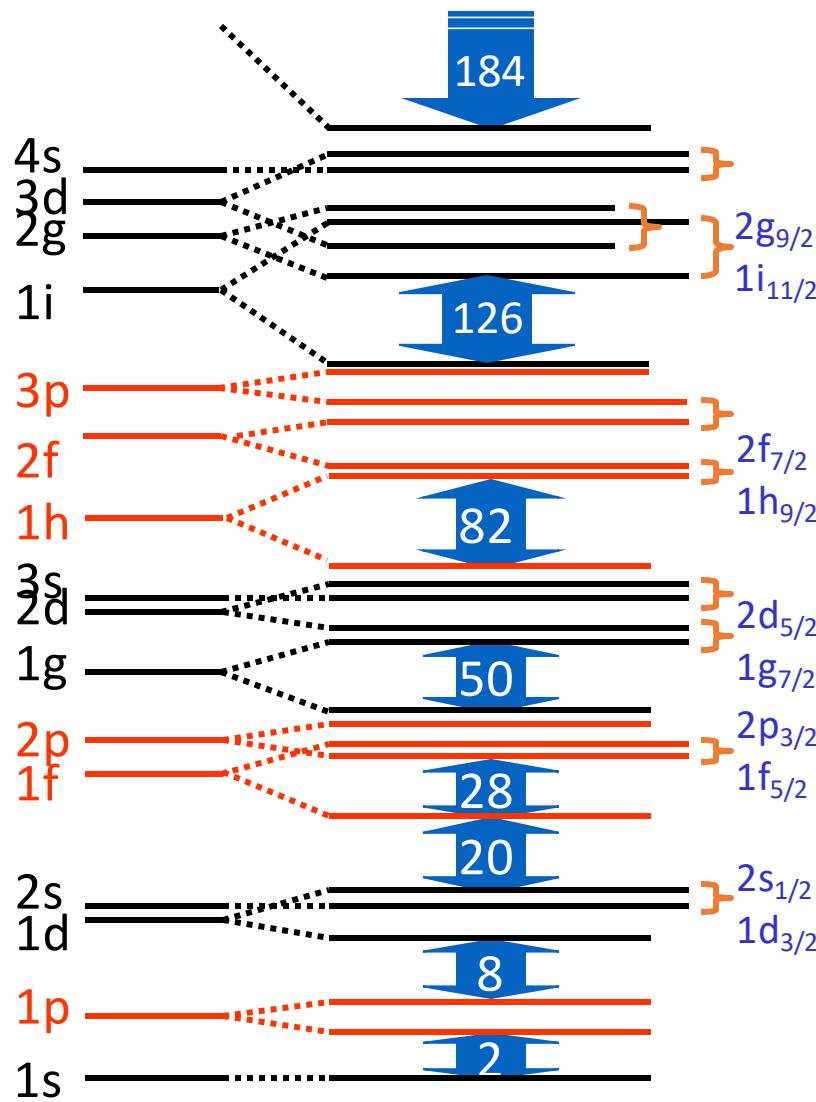
- Energy of the first  $2^+$  state



Otsuka et al., Rev. Mod. Phys. 92, 015002 (2020)



# Nuclear single particle level scheme



- Large spin-orbit splitting
- Near degeneracy of some doublets
- Pseudo-orbit angular momentum
- Accidental or of symmetry, i.e., pseudospin symmetry (**PSS**)?

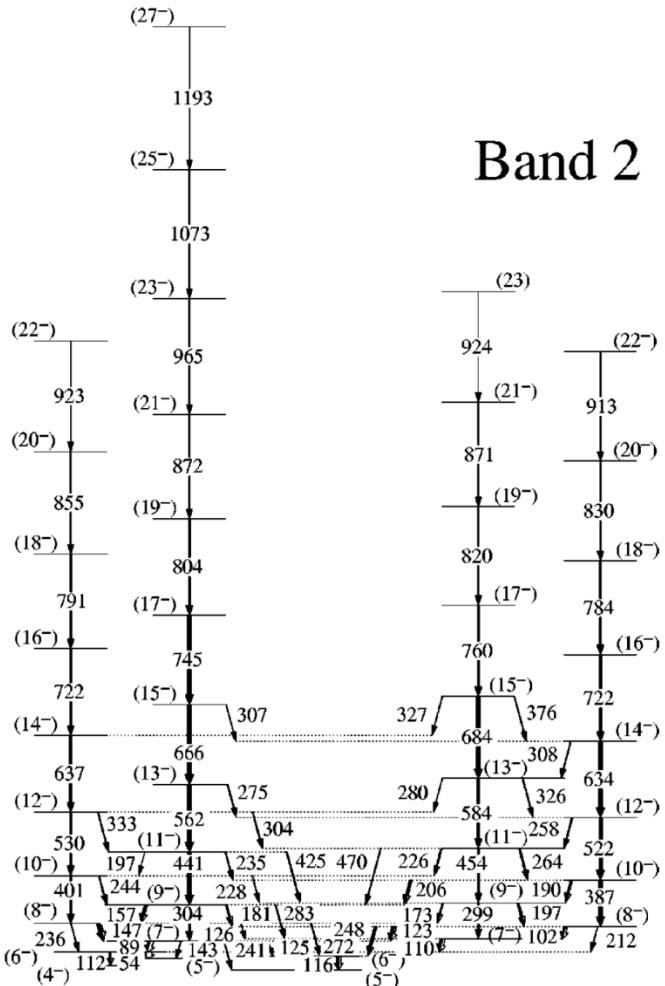
$$(n+1, l, j = l+1/2) \quad \tilde{l} = l + 1$$

$$(n, l+2, j = l+3/2) \quad \tilde{s} = 1/2$$

Arima, Harvey, Shimizu, *PLB 30*, 517 (1969)  
Hecht, Adler, *NPA 137*, 129 (1969)

# Pseudospin partner bands

Band 1



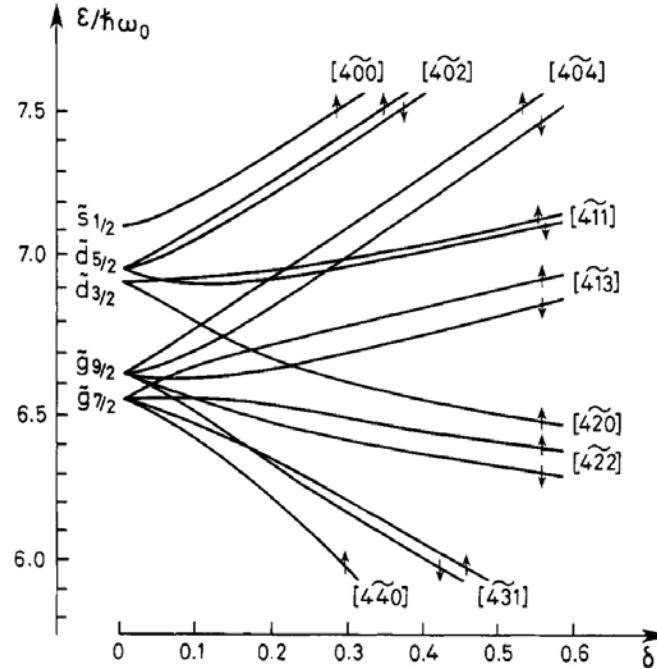
➤ Single-particle states

$$[Nn_z\Lambda]\Omega \text{ & } [Nn_z\Lambda + 2]\Omega + 1 \Rightarrow [\widetilde{Nn_z\Lambda}]$$

as pseudospin doublets

$$\tilde{N} = N - 1, \tilde{\Lambda} = \Lambda + 1, \Omega = \tilde{\Lambda} \pm 1/2$$

Band 2

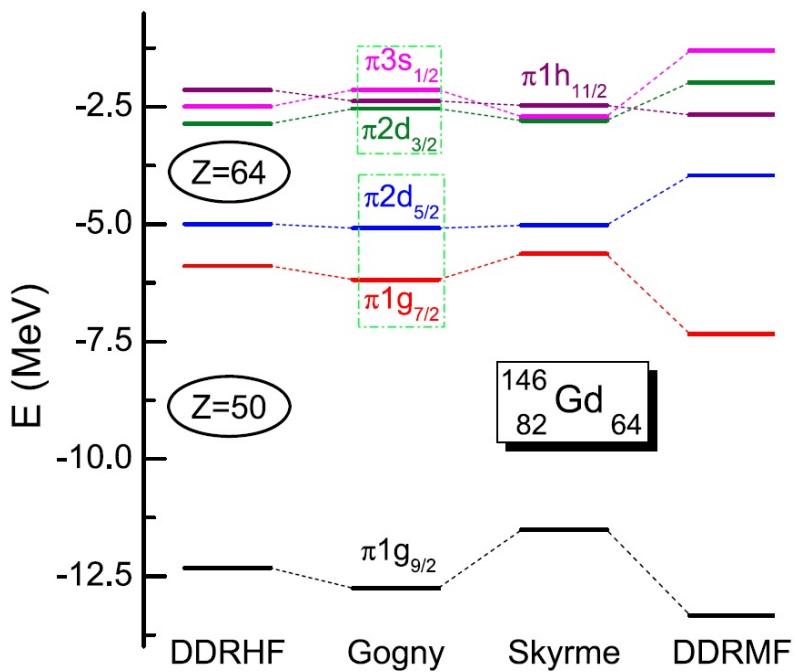


Bohr, Hamamoto, Mottelson, Phys. Scr. 26, 267 (1982)

◆ Partial level scheme of  $^{128}\text{Pr}$   
Petrache et al., PRC 65, 054324 (2002)

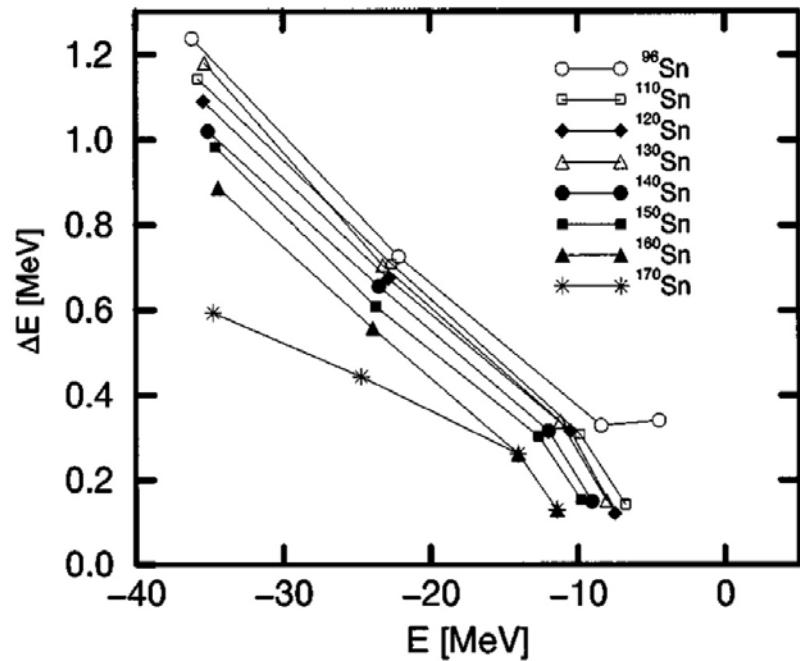
◆  $^{186}\text{Ir}$  (1997);  $^{108}\text{Tc}$  (2008);  $^{189}\text{Pt}$  (2009) .....

# PSS in shell structure evolutions



◆ Proton single-particle energies for  $^{146}\text{Gd}$   
 Long, Nakatsukasa, Sagawa, Meng, Nakada, Zhang,  
 $\text{PLB } 680$ , 428 (2009)

- Splitting of both spin and pseudospin doublets play important roles in the shell structure evolutions.



◆ Pseudospin-orbit splitting in Sn isotopes  
 Meng, Sugawara-Tanabe, Yamaji, Arima,  
 $\text{PRC } 59$ , 154 (1999)

# Pseudo quantum numbers & lower component

- Equation of motion for nucleons: Schrödinger / Dirac equations

□ What is  $\tilde{l}$  ?

- Dirac spinor:  $\psi_\alpha(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} iG_a(r) \mathcal{Y}_{j_a m_a}^{l_a}(\hat{\mathbf{r}}) \\ -F_a(r) \mathcal{Y}_{j_a m_a}^{\tilde{l}_a}(\hat{\mathbf{r}}) \end{pmatrix}$
  - Relations of quantum numbers: 
$$\begin{cases} j = l \pm 1/2 \\ \kappa = (-1)^{j+l+1/2} (j + 1/2) \\ \tilde{l} = l - \text{sign}(\kappa) \end{cases}$$
- $2s_{1/2} : \begin{pmatrix} n = 2, l = 0, j = l + 1/2 \\ \tilde{n} = 2, \tilde{l} = 1, j = \tilde{l} - 1/2 \end{pmatrix} \quad 1d_{3/2} : \begin{pmatrix} n = 1, l = 2, j = l - 1/2 \\ \tilde{n} = 2, \tilde{l} = 1, j = \tilde{l} + 1/2 \end{pmatrix}$

$$(2s_{1/2}, 1d_{3/2}) \Rightarrow (2\tilde{p}_{1/2, 3/2})$$

- $\tilde{l}$  is the orbital angular momentum of the lower component of the Dirac spinor.

Ginocchio, PRL 78, 436 (1997)

# Dirac and Schrödinger-like equations

## ➤ Dirac equation

$$\begin{pmatrix} \Sigma(r) + M & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & -\Delta(r) - M \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = E \begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

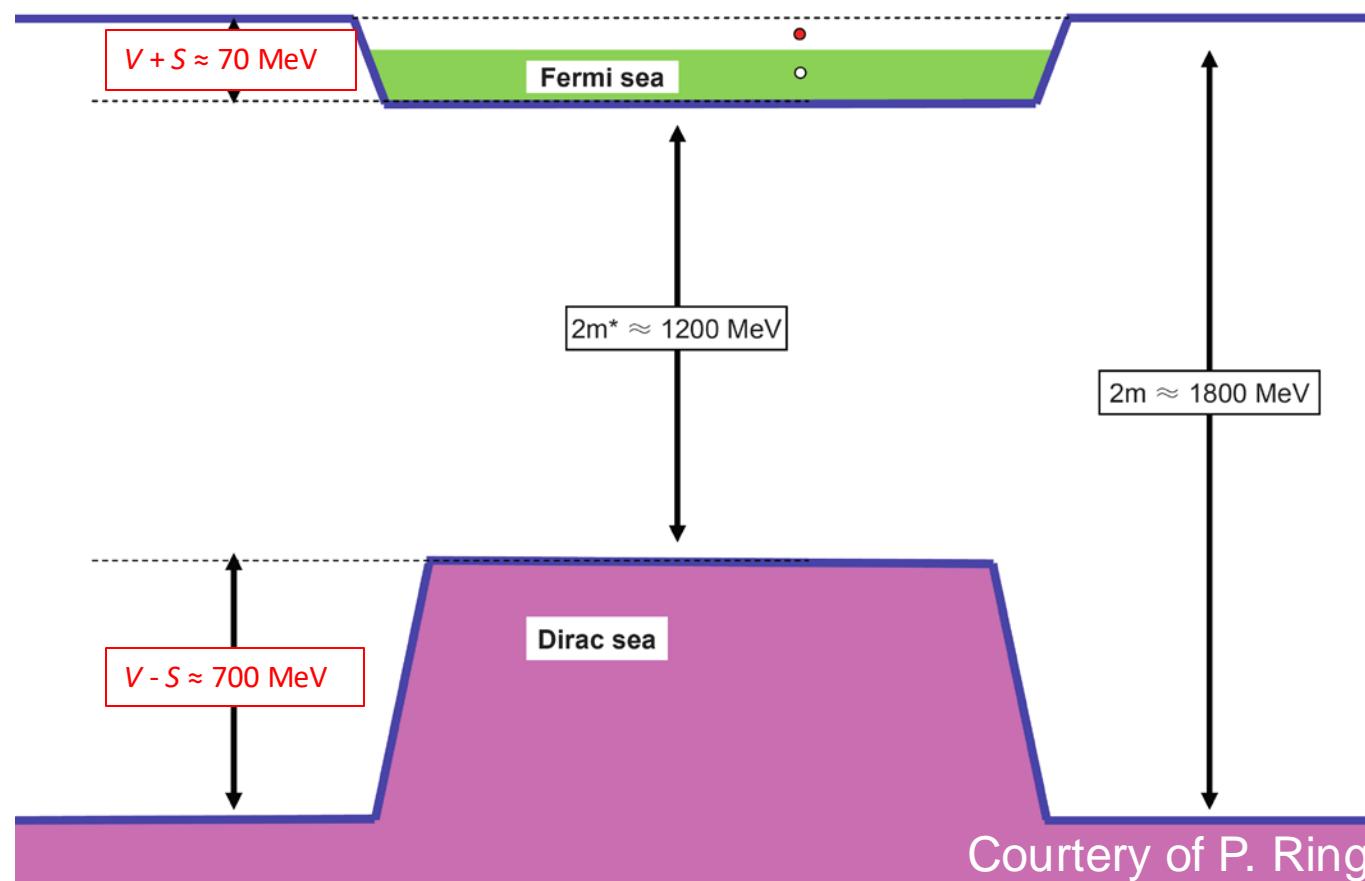
where

$$\Sigma(r) = S(r) + V(r), \quad \Delta(r) = S(r) - V(r),$$

$$\kappa = \mp(j + 1/2) \quad \text{for } j_G = l_G \pm 1/2.$$

# Nuclear potentials in relativistic models

- The scalar and vector potentials are both very big in amplitude, but with opposite signs:  $S(r) < 0$  &  $V(r) > 0$ .
- This results in a shallow Fermi sea and a deep Dirac sea, the latter is responsible for the large spin-orbit coupling.



# Dirac and Schrödinger-like equations

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$$\begin{pmatrix} \Sigma(r) + M & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & -\Delta(r) - M \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = E \begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

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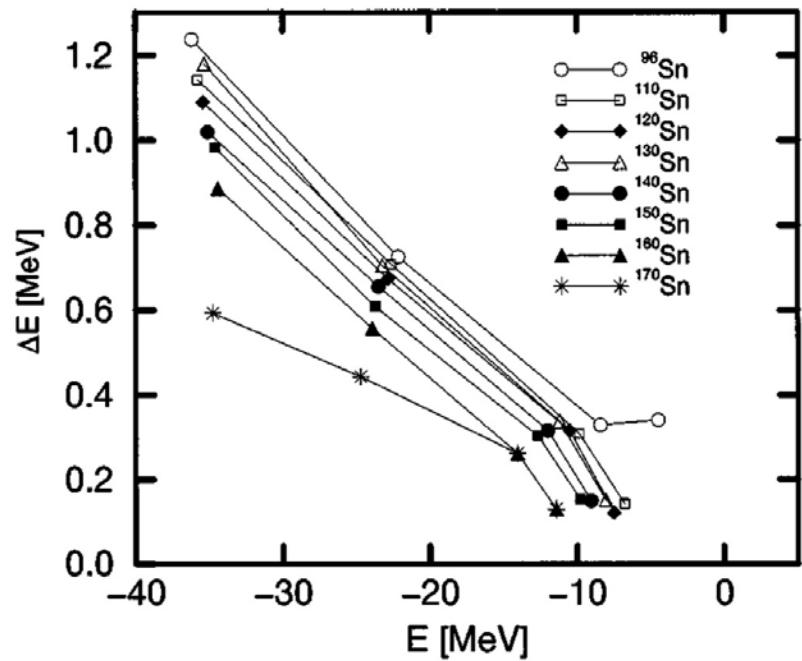
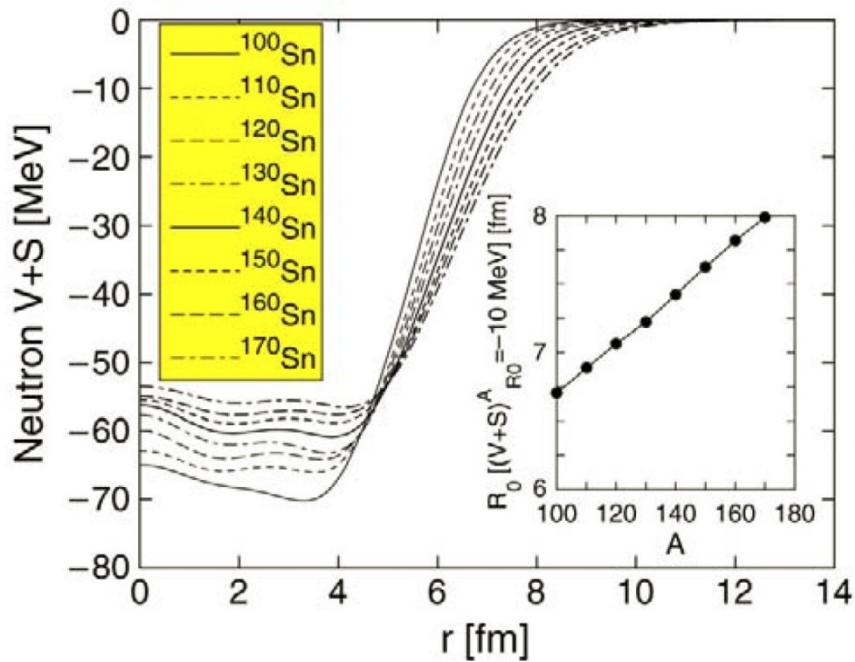
## ➤ Schrödinger-like equation for the lower component

$$\left\{ -\frac{1}{M_-} \frac{d^2}{dr^2} + \frac{1}{M_-^2} \frac{dM_-}{dr} \frac{d}{dr} + \left[ (-M - \Delta) + \frac{1}{M_-} \frac{\kappa(\kappa - 1)}{r^2} - \frac{1}{M_-^2} \frac{dM_-}{dr} \frac{\kappa}{r} \right] \right\} F = EF,$$

with  $M_- = E - M - \Sigma$

# From stable nuclei to exotic nuclei

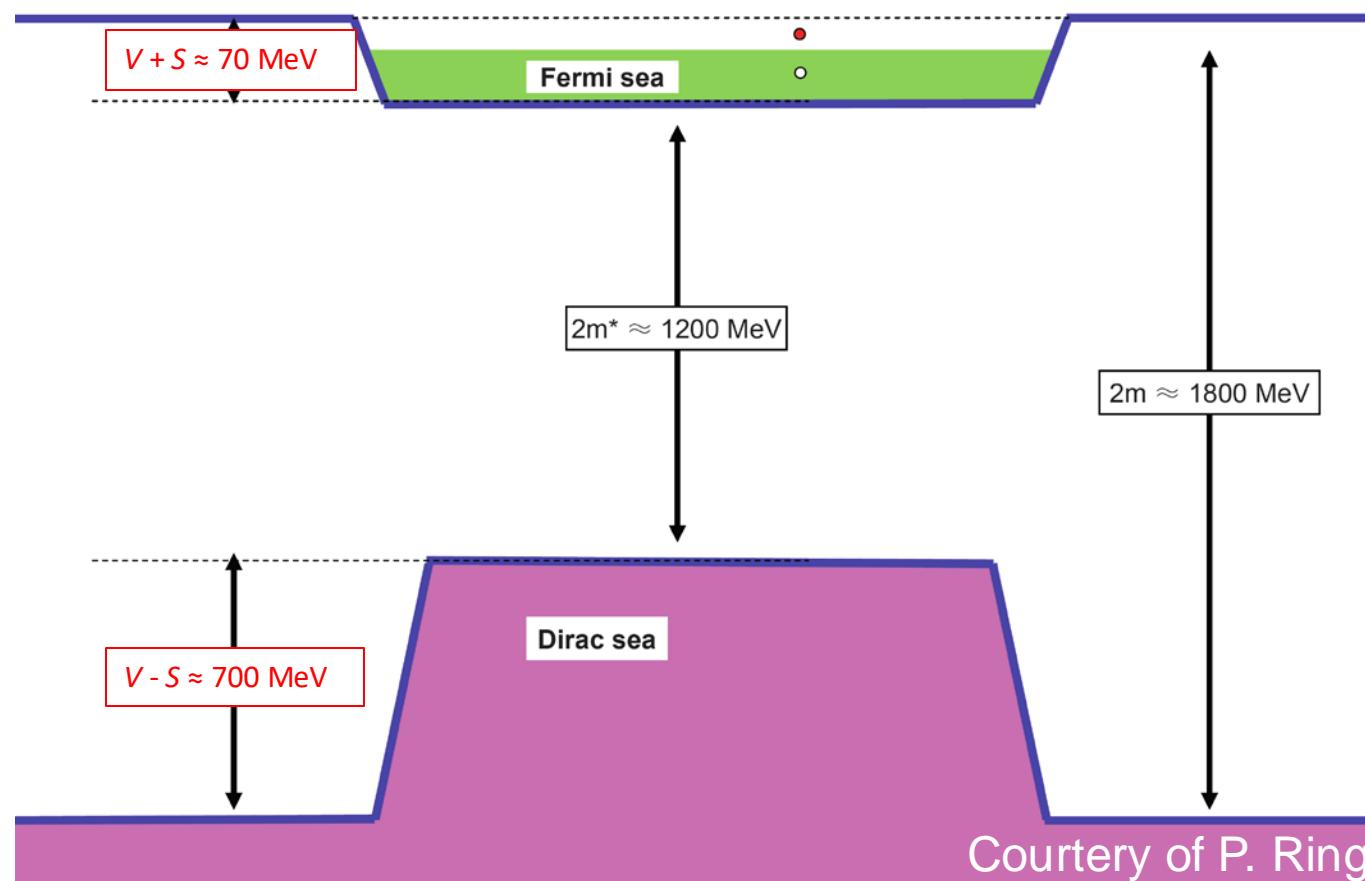
- If  $\Sigma(r) = 0$  or  $d\Sigma(r)/dr = 0$ , the PSS is exactly conserved, i.e., the PSS doublets are degenerate.



Meng, Sugawara-Tanabe, Yamaji, Ring, Arima, *PRC* **58**, R628 (1998)  
Meng, Sugawara-Tanabe, Yamaji, Arima, *PRC* **59**, 154 (1999)

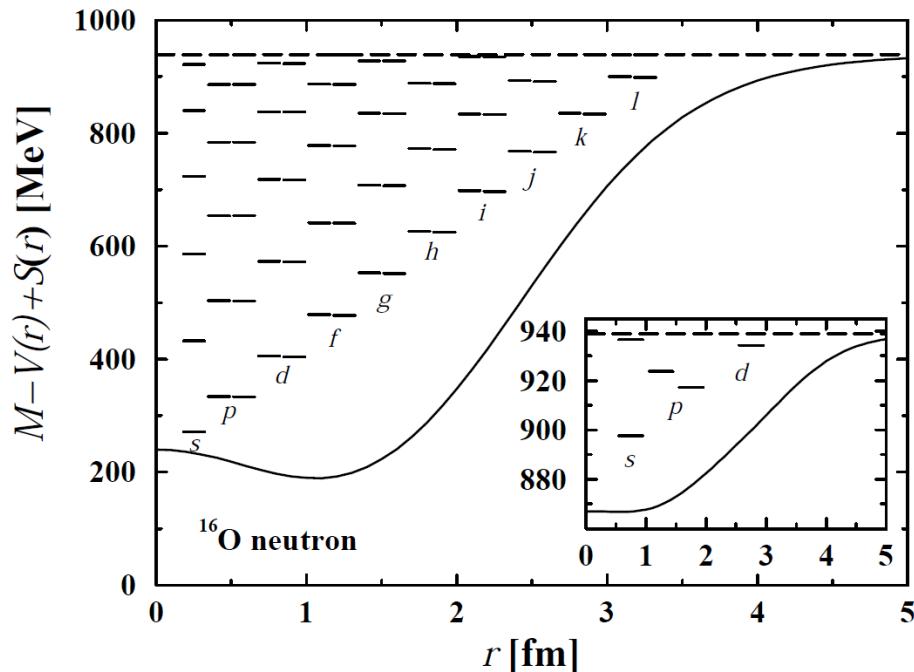
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# From nucleon spectra to anti - nucleon spectra

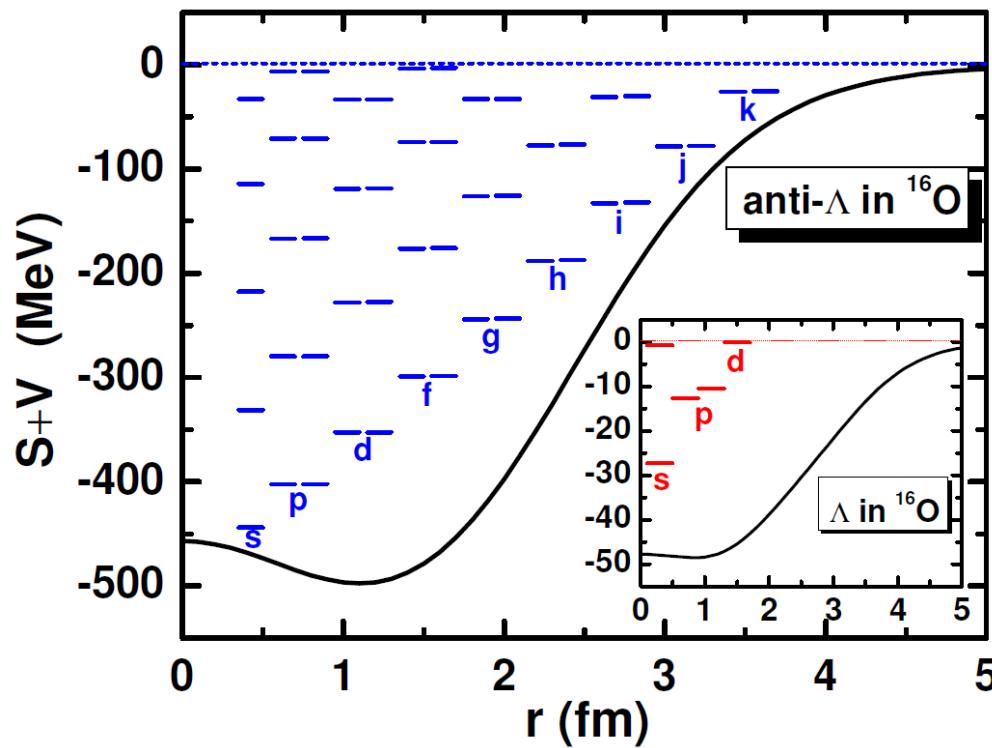
- If  $\Sigma(r) = 0$  or  $d\Sigma(r)/dr = 0$ , the spin symmetry (SS) in anti-nucleon spectra is exactly conserved.



- The SS in single-anti-nucleon spectra is much better developed than that of PSS in single-nucleon spectra.

Zhou, Meng, Ring, *PRL* **91**, 262501 (2003)  
He, Zhou, Meng, Zhao, Scheid, *EPJA* **28**, 265 (2006)  
HZL, Long, Meng, Giai, *EPJA* **44**, 119 (2010)

# From nucleon spectra to hyperon spectra



- Due to the additional strangeness degree of freedom, it is expected that the annihilation probability of an anti-hyperon in a normal nucleus is much smaller than that of an anti-nucleon.

*Song, Yao, Meng, Chin. Phys. Lett. 26, 122102 (2009)*  
*Song, Yao, Chin. Phys. C 34, 1425 (2010)*  
*Song, Yao, Meng, Chin. Phys. Lett. 28, 092101 (2011)*

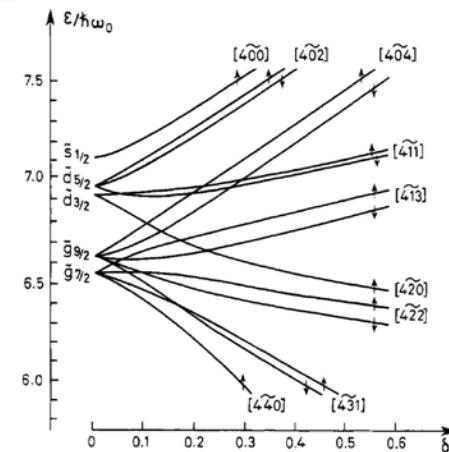
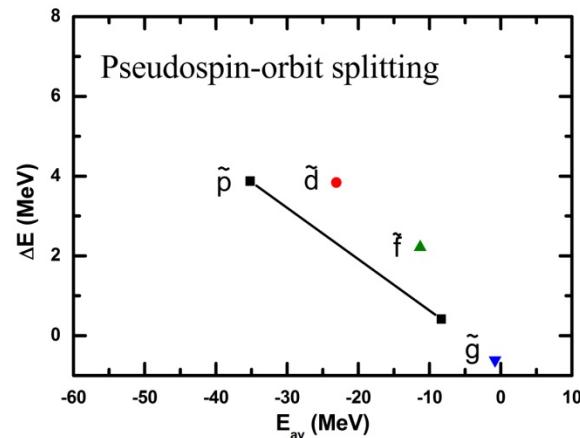
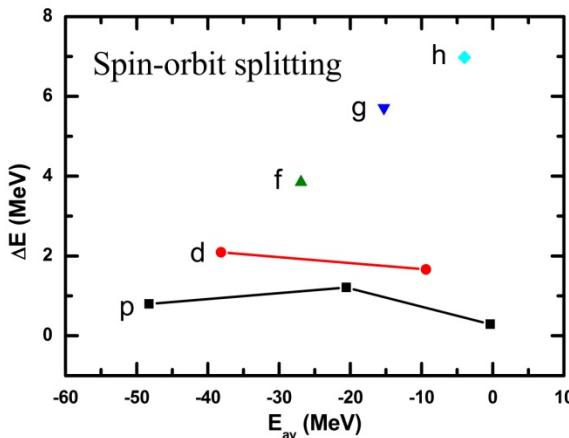
# Triggers for further investigations

- Schrödinger-like equation for the lower component

$$\left\{ -\frac{1}{M_-} \frac{d^2}{dr^2} + \frac{1}{M_-^2} \frac{dM_-}{dr} \frac{d}{dr} + \left[ (-M - \Delta) + \frac{1}{M_-} \frac{\kappa(\kappa-1)}{r^2} - \frac{1}{M_-^2} \frac{dM_-}{dr} \frac{\kappa}{r} \right] \right\} F = EF,$$

with  $M_- = E - M - \Sigma$

- At the PSS limit,  $\Sigma(r) = 0$  or  $d\Sigma(r)/dr = 0$ ,
  - There are **no bound states** in single-nucleon spectra.
  - There are bound states only in single-anti-nucleon spectra.
- There exist **singularities** in  $1/M_-$  for the bound states.
- Effective Hamiltonian is **not Hermitian**, i.e., perturbation theory can NOT be applied.



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- Effective Hamiltonian is not Hermitian, i.e., perturbation theory can NOT be applied.

## Selected progress

- PSS in the single-particle resonant states Lu, Zhao, Zhou, *PRL* **109**, 072501 (2012)
- PSS with similarity renormalization group (**SRG**) Guo et al., *PRL* **112**, 062502 (2014)
- PSS with SRG, supersymmetric (**SUSY**) quantum mechanics, and perturbation theory HZL et al., *PRC* **83**, 041301(R) (2011); HZL et al., *PRC* **87**, 014334 (2013)

## i) PSS in resonant states

- Schrödinger-like equation for the lower component

$$\left\{ -\frac{1}{M_-} \frac{d^2}{dr^2} + \cancel{\frac{1}{M_-^2} \frac{dM_-}{dr} \frac{d}{dr}} + \left[ (-M - \Delta) + \frac{1}{M_-} \frac{\kappa(\kappa - 1)}{r^2} - \cancel{\frac{1}{M_-^2} \frac{dM_-}{dr} \frac{\kappa}{r}} \right] \right\} F = EF,$$

- At  $r \rightarrow \infty$  (neutron states)

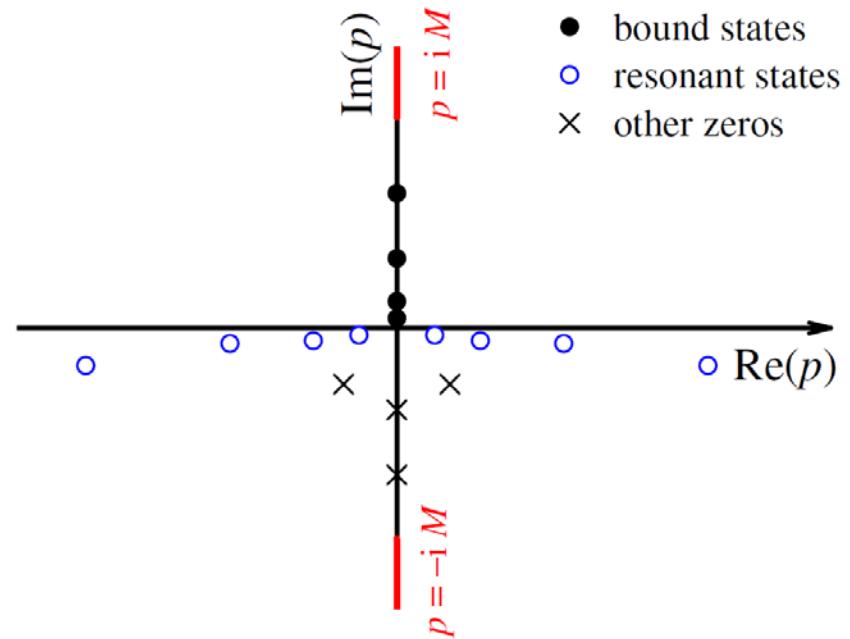
$$F(r) = \frac{i}{2} \left[ \mathcal{J}_\kappa^F(p) h_{\tilde{l}}^-(pr) - \mathcal{J}_\kappa^F(p)^* h_{\tilde{l}}^+(pr) \right]$$

- Zeros of Jost functions correspond to bound states, resonant states, ...

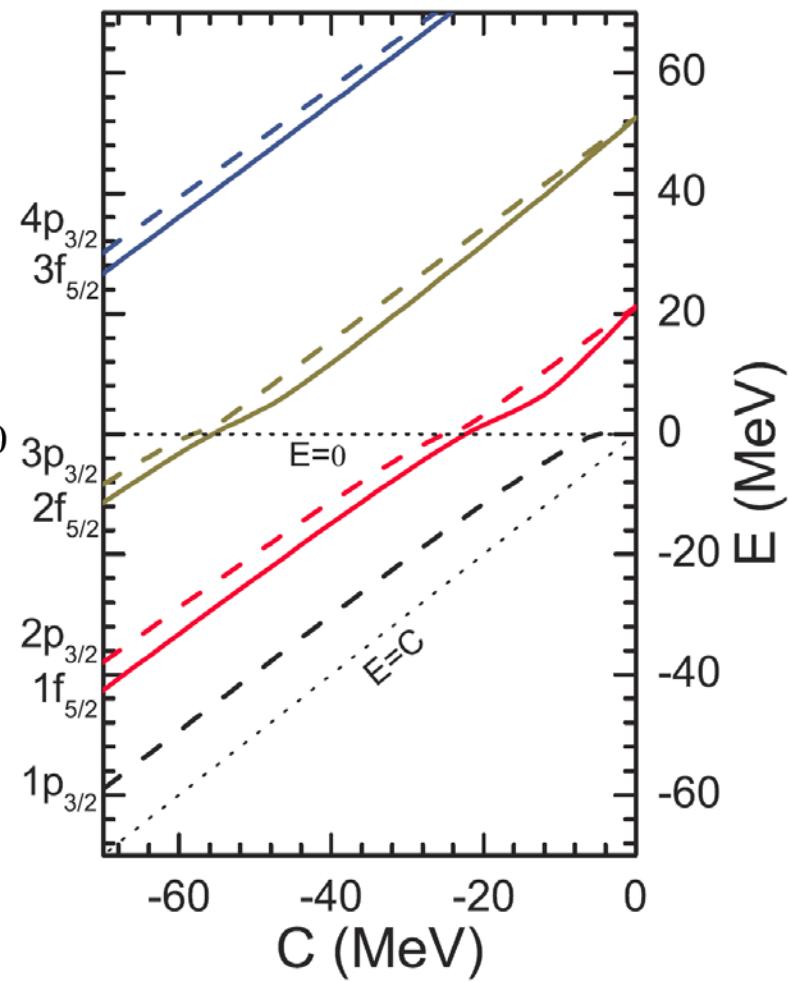
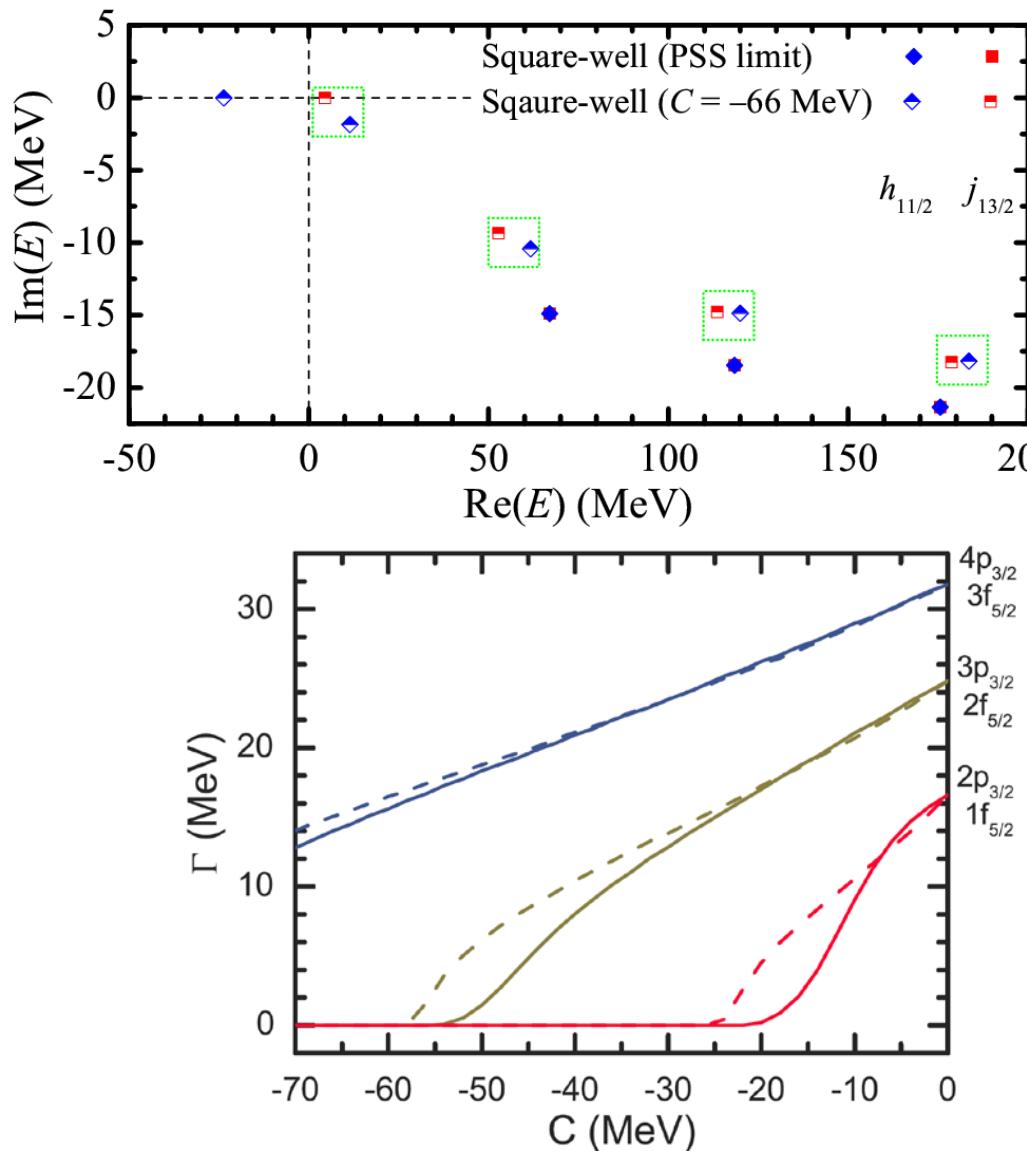
$$E = E_{\text{res}} - i\Gamma_{\text{res}}/2 = \sqrt{p^2 + M^2}$$

- At the PSS limit,

$\mathcal{J}_{\kappa_a}^F(p) = \mathcal{J}_{\kappa_b}^F(p).$



# An example: Square-well potentials



Lu, Zhao, Zhou, *PRL* **109**, 072501 (2012)  
 Lu, Zhao, Zhou, *PRC* **88**, 024323 (2013)

## ii) PSS with SRG

### Similarity renormalization group (SRG)

- Start with the initial Hamiltonian  $\textcolor{red}{H}$

$$H = \alpha \cdot \mathbf{p} + \beta[M + S(\mathbf{r})] + V(\mathbf{r}).$$

- Introduce a unitary transformation

$$H(l) = U(l)H U^\dagger(l), \quad H(0) = H$$

- The flow equation with the anti-Hermitian generator

$$\frac{d}{dl}H(l) = [\eta(l), H(l)], \quad \eta(l) = \frac{dU(l)}{dl}U^\dagger(l).$$

- The off-diagonal part  $\textcolor{red}{o(l)=0}$  as  $\textcolor{red}{l \rightarrow \infty}$ , the diagonal part in a series of  $\textcolor{red}{1/M}$

$$\varepsilon(\infty) = M\varepsilon_0(\infty) + \varepsilon_1(\infty) + \frac{\varepsilon_2(\infty)}{M} + \frac{\varepsilon_3(\infty)}{M^2} + \frac{\varepsilon_4(\infty)}{M^3} + \dots$$

- Higher orders of Foldy-Wouthuysen non-relativistic reduction

Wegner, *Ann. Phys.* **506**, 77 (1994)  
*Phys. Rep.* **348**, 77 (2001)  
Bylev and Pirner, *PLB* **428**, 329 (1998)

# SRG for Dirac Hamiltonian

- Dirac Hamiltonian  $\mathbf{H}$

$$H = \alpha \cdot \mathbf{p} + \beta[M + S(\mathbf{r})] + V(\mathbf{r}).$$

Guo, PRC **85**, 021302(R) (2012)  
 Li, Chen, Guo, PRC **87**, 044311 (2013)  
 Guo et al., PRL **112**, 062502 (2014)

- After SRG,

$$H_D = \begin{pmatrix} H_P + M & 0 \\ 0 & -H_A - M \end{pmatrix}$$

with

$$H_P = H_{nr} + H_{dy} + H_{sl} + H_{km} + H_{dw}$$

$$\left\{ \begin{array}{lcl} H_{nr} & = & \Sigma(\vec{r}) + \frac{\vec{p}^2}{2M}, \\ H_{dy} & = & -\frac{1}{2M^2} (Sp^2 - \nabla S \cdot \nabla) + \frac{S}{2M^3} (Sp^2 - 2\nabla S \cdot \nabla), \\ H_{sl} & = & \frac{1}{4M^2} \left(1 - \frac{2S}{M}\right) \vec{\sigma} \cdot (\nabla \Delta \times \vec{p}), \\ H_{km} & = & -\frac{p^4}{8M^3}, \\ H_{dw} & = & \frac{1}{8M^2} \nabla^2 \Sigma - \frac{1}{16M^3} [(\nabla \Sigma)^2 - 2\nabla \Sigma \cdot \nabla \Delta + 4S \nabla^2 \Sigma]. \end{array} \right.$$

- Every operator is Hermitian.

### iii) PSS with SRG, SUSY, & perturbation

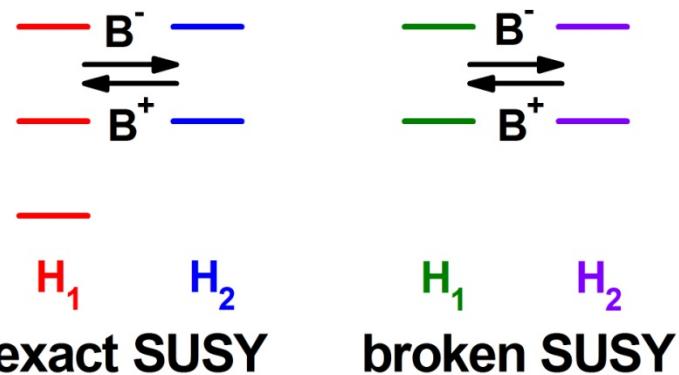
# Supersymmetric (SUSY) quantum mechanics

- ### ➤ Hamiltonian and its supersymmetric partner

$$H_1 = B^+ B^- \quad \text{and} \quad H_2 = B^- B^+.$$

- ### ➤ Their spectra and wave functions

$$\psi_2(n) = \frac{B^-}{\sqrt{E_S(n)}} \psi_1(n), \quad \psi_1(n) = \frac{B^+}{\sqrt{E_S(n)}} \psi_2(n).$$



- ## ➤ Spin and Pseudospin symmetric terms

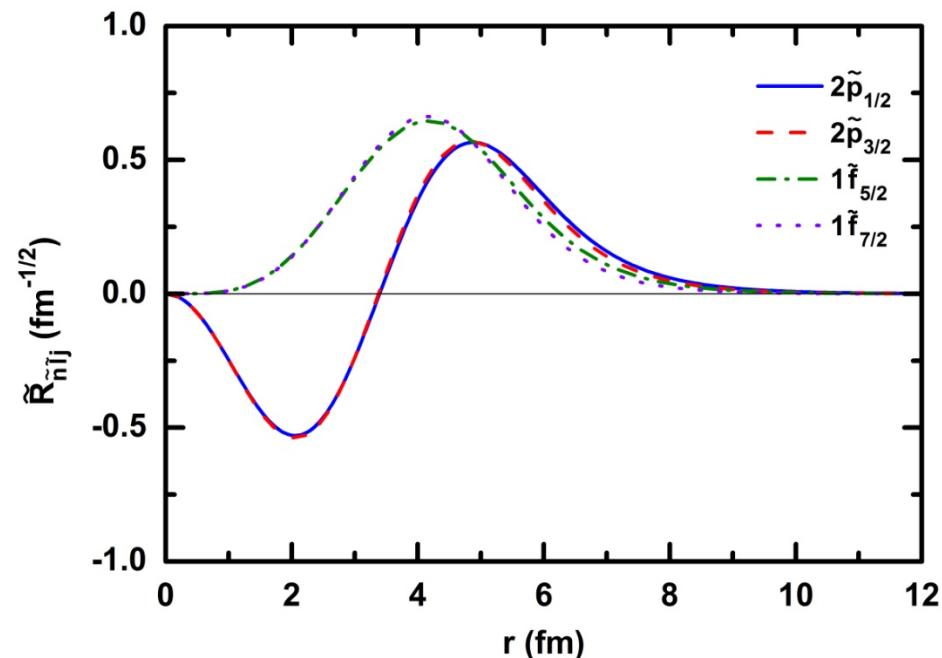
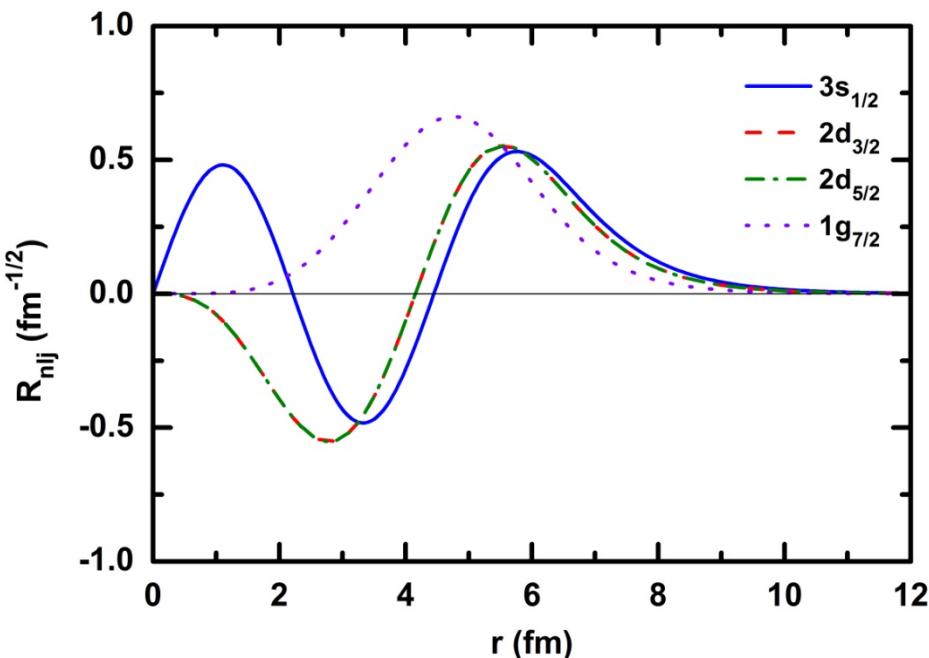
$$H_1 = B_\kappa^+ B_\kappa^- = \frac{1}{2M} \left[ -\frac{d^2}{dr^2} + \frac{\kappa(\kappa+1)}{r^2} + q_\kappa^2 + \frac{2\kappa}{r}q_\kappa - q'_\kappa \right],$$

$$H_2 = B_\kappa^- B_\kappa^+ = \frac{1}{2M} \left[ -\frac{d^2}{dr^2} + \frac{\kappa(\kappa-1)}{r^2} + q_\kappa^2 + \frac{2\kappa}{r}q_\kappa + q'_\kappa \right].$$

TypeI, NPA 806, 156 (2008)  
 HZL, Shen, Zhao, Meng, PRC 87, 014334 (2013)

# Single-particle wave functions

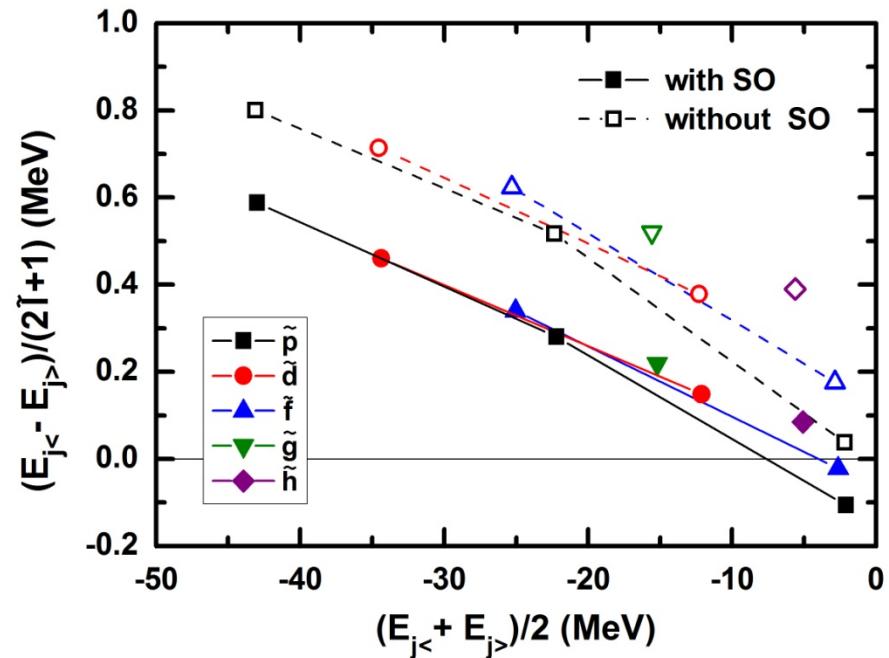
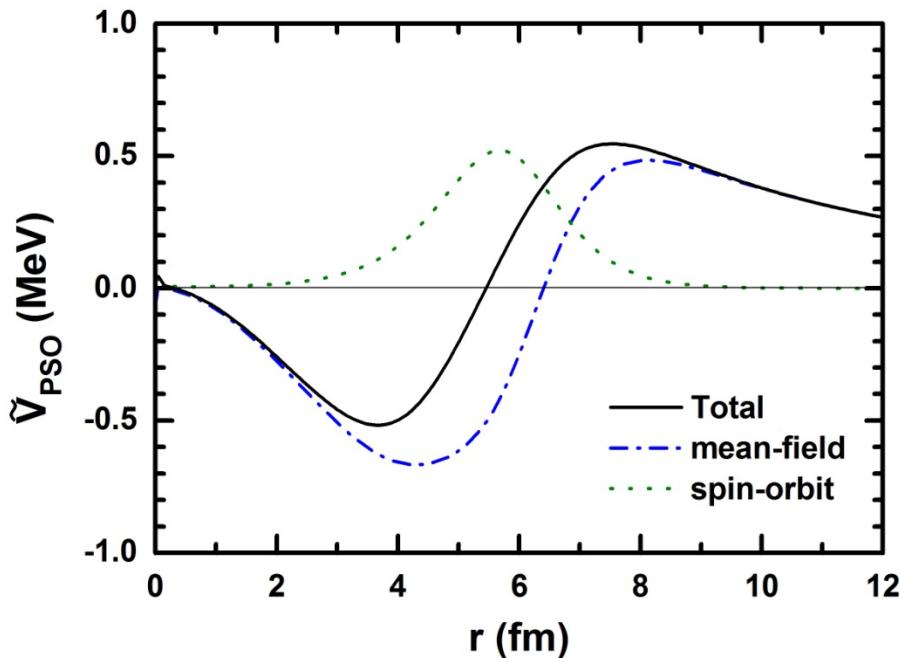
- Single-particle wave functions in  $\mathbf{H}_1$  and its SUSY partner  $\mathbf{H}_2$



- Single-particle wave functions of pseudospin doublets are very different in original Hamiltonian.
- But they are almost identical in SUSY partner Hamiltonian.

# Pseudospin-orbit potential and splitting

- Dirac to Schrödinger equations by SRG (**1/M** order + spin-orbit at **1/M<sup>2</sup>** order)



- The origin of PSS deeply hidden in  $\mathbf{H}_1$  can be traced in its SUSY partner Hamiltonian  $\mathbf{H}_2$ .
- $\Delta E_{\text{PSO}}$  become smaller with increasing  $E_{av}$  can be interpreted in an explicit and quantitative way.

# Summary and Perspectives

□ During the past 20+ years, there is various progress on the study of **PSS** and **SS** in different systems and potentials, e.g.,

- From stable to exotic nuclei
- From non-confining to confining potentials
- From local to non-local potentials
- From central to tensor potentials
- From bound to resonant states
- From nucleon to anti-nucleon spectra
- From nucleon to hyperon spectra
- From spherical to deformed nuclei
- .....

SUSY quantum mechanics might help to understand some new physics.

□ Open issues

- PSS is perturbative or not?
- The puzzle of intruder states
- PSS with SRG, SUSY, perturbation theory, .....

Various and concrete experimental evidences for PSS are highly desired!

# Acknowledgments

We would like to express our gratitude to all the collaborators and colleagues who contributed to the investigations presented, in particular to

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Hidden pseudospin and spin symmetries and their origins in atomic nuclei



Haozhao Liang<sup>a,b</sup>, Jie Meng<sup>a,c,d,\*</sup>, Shan-Gui Zhou<sup>e,f</sup>

84 pages, 58 figures, 10 tables

Thank you!