Chiral Effective Field Theory

—*an introduction*—

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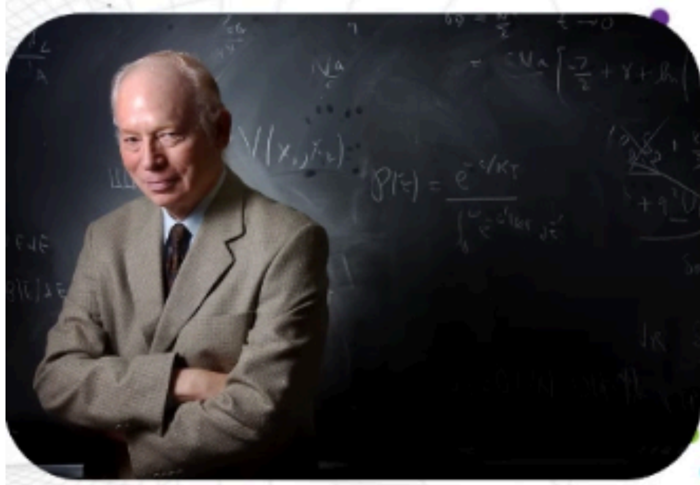
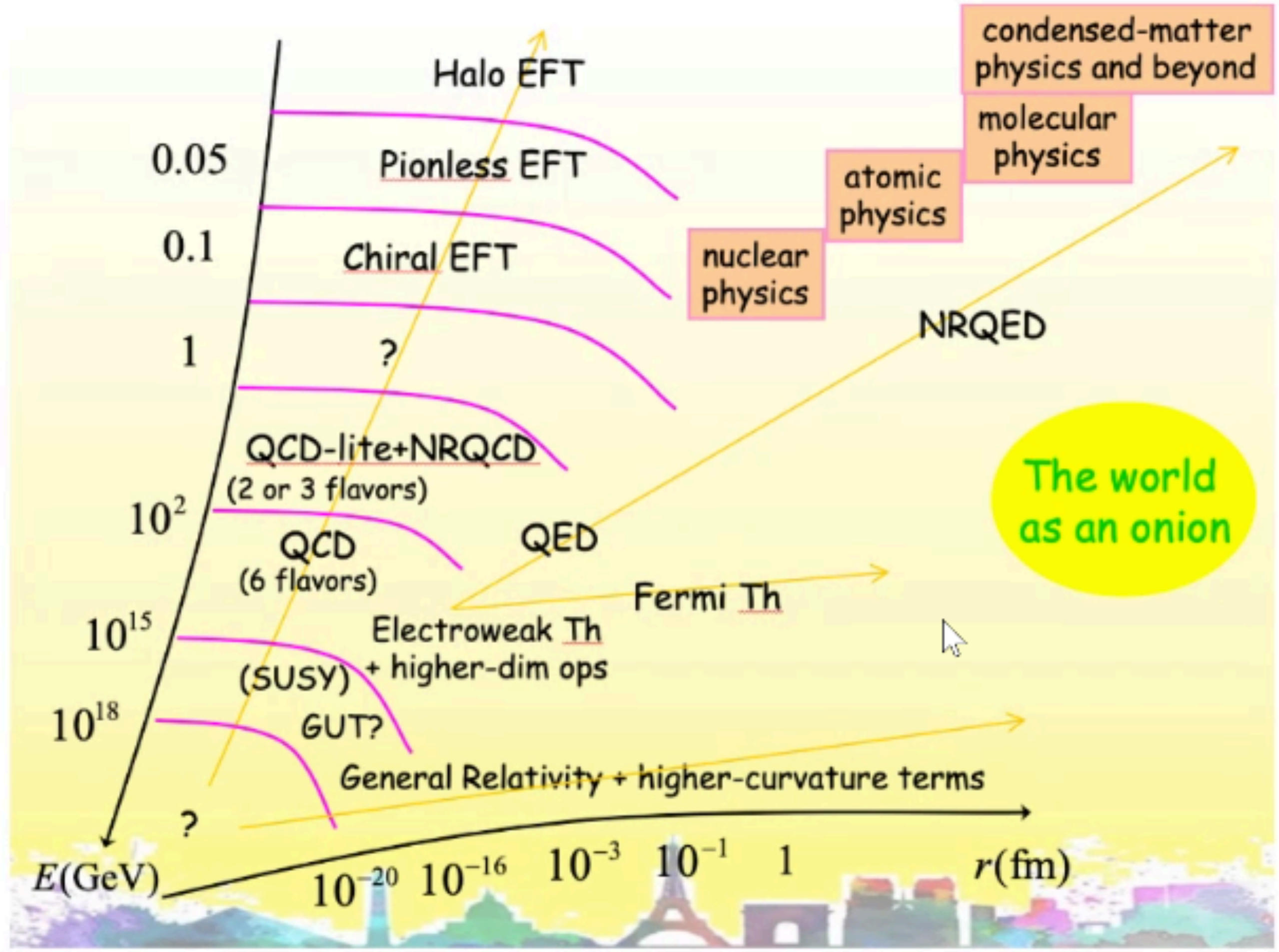
We are entering the era of EFTs





"Now, considering the fact that experiments can probe only a limited range of energies, it seems natural to take EFT as a general framework for analyzing experimental results."

T. Y. Cao

in *Renormalization, From Lorentz to Landau (and Beyond)*, L.M. Brown (ed.), 1993

U. van Kolck @ Beihang, 2019.03.28-04.04

- **Lecture I: QCD and its symmetries**
- **Lecture II: Chiral perturbation theory for mesons and baryons**
- **Lecture III: Chiral effective theories and their applications to low energy strong interactions**

Part I:

QCD and its symmetries

QCD Lagrangian

✦ The general QCD Lagrangian

$$\mathscr{L}_{\rm QCD} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q \underbrace{- \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a}}_{\mathscr{L}_G\text{: Gluon}} \underbrace{- \frac{1}{2\xi}\left(\partial_\mu G^{\mu a}\right)^2}_{\mathscr{L}_{GF}\text{: Gauge Fixing}} \underbrace{+ \bar{C}^a\partial^\mu D^{ab}_\mu C^b}_{\mathscr{L}_{FP}\text{: Fadeev-Popov}}$$

- **Gluon fields:** $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu$
- **Covariant derivatives in color space:** $D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}G^a_\mu\,,\quad D^{ab}_\mu = \delta^{ab}\partial_\mu - g f^{abc}G^c_\mu$
- **Quark fields and mass matrix:**

$$q = \begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix} \qquad m = \begin{pmatrix} m_u & & & \\ & m_d & & \\ & & m_s & \\ & & & \ddots \end{pmatrix}$$

- **Flavour symmetry:**

$SU(N_f)$

$N_f = 2$ **or** $N_f = 3$

$$\begin{pmatrix} m_u = (1.7\text{–}3.3)\,\text{MeV} \\ m_d = (4.1\text{–}5.8)\,\text{MeV} \\ m_s = (80\text{–}130)\,\text{MeV} \end{pmatrix} \ll 1\,\text{GeV} \le \begin{pmatrix} m_c = 1.27^{+0.07}_{-0.09}\,\text{GeV} \\ m_b = 4.19^{+0.18}_{-0.06}\,\text{GeV} \\ m_t = (172.0 \pm 0.9 \pm 1.3)\,\text{GeV} \end{pmatrix}$$

Chiral symmetry ($m_q \to 0$) **Heavy quark symmetry ($M_Q \to \infty$)**

Global Chiral Symmetry

✦ QCD Lagrangian in the chiral limit $m_q = 0$

- **Left and right-handed quark fields** $q_{L/R} = \frac{1 \mp \gamma_5}{2} q \qquad \bar{q}_{L/R} = \bar{q} \frac{1 \pm \gamma_5}{2}$

$$\mathscr{L}^0_{\mathrm{QCD}} = \bar{q}\left(i\gamma^\mu D_\mu\right) q + \mathscr{L}_G + \mathscr{L}_{GF} + \mathscr{L}_{FP}$$

$$= \left(\bar{q}_L + \bar{q}_R\right)\left(i\gamma^\mu D_\mu\right)\left(q_L + q_R\right) + \mathscr{L}_G + \mathscr{L}_{GF} + \mathscr{L}_{FP}$$

$$= \bar{q}_L\left(i\gamma^\mu D_\mu\right) q_L + \bar{q}_R\left(i\gamma^\mu D_\mu\right) q_R + \mathscr{L}_G + \mathscr{L}_{GF} + \mathscr{L}_{FP}$$

- **Global chiral symmetry**

$SU(N_f)_L$ **transformation** $\quad q_L \to e^{-i\theta^a_L T^a_L} q_L \equiv L q_L \qquad L \in SU(N_f)_L$

$SU(N_f)_R$ **transformation** $\quad q_R \to e^{-i\theta^a_R T^a_R} q_R \equiv R q_R \qquad R \in SU(N_f)_R$

$SU(N_f)_L \otimes SU(N_f)_R$

θ^a_L and θ^a_R are constants， corresponding to global transformations.

Chiral Currents

✦ Noether Theorem

Continuous global symmetries ⟷ **Conserved quantities**

- **Gell-Mann & Levy's method** $\mathscr{L} = \mathscr{L}(\Phi, \partial_\mu \Phi)$

$$\Phi_i(x) \longrightarrow \Phi'(x) = \Phi_i(x) + \delta\Phi(x) = \Phi_i(x) - i\epsilon_a(x) F_{ai}[\Phi(x)]$$

$$\delta\mathscr{L} = \mathscr{L}(\Phi', \partial\Phi') - \mathscr{L}(\Phi, \partial\Phi)$$

$$= \frac{\delta\mathscr{L}}{\delta\Phi_i}\delta\Phi_i + \frac{\delta\mathscr{L}}{\delta\partial_\mu\Phi_i}\boxed{\delta\partial_\mu\Phi_i \quad \rightarrow -i\partial_\mu\epsilon_a F_{ai} - i\epsilon_a\partial_\mu F_{ai}}$$

$$= \epsilon_a\left[-i\frac{\delta\mathscr{L}}{\delta\Phi_i}F_{ai} - i\frac{\delta\mathscr{L}}{\delta\partial_\mu\Phi_i}\partial_\mu F_{ai}\right] + \partial_\mu\epsilon_a \underbrace{\left[-i\frac{\delta\mathscr{L}}{\delta\partial_\mu\Phi_i}F_{ai}\right]}_{\equiv J_a^\mu}$$

$$\equiv \epsilon_a(\partial_\mu J_a^\mu) + (\partial_\mu\epsilon_a)J_a^\mu$$

Local →

- **Four currents**

$$J_a^\mu = \frac{\partial\delta\mathscr{L}}{\partial(\partial_\mu\epsilon_a)}$$

- **Their divergences**

$$\partial_\mu J_a^\mu = \frac{\partial\delta\mathscr{L}}{\partial\epsilon_a}$$

↓ **Global**

Conserved currents $\partial_\mu J_a^\mu = 0$ **Conserved Charge** $Q_a(t) = \int \mathrm{d}^3x J_a^0(t, \vec{x})$

Chiral Currents

✦ Chiral currents of QCD

$$\delta \mathscr{L}^0_{\rm QCD} = \bar{q}_R \left[\sum_{a=1}^{8} \partial \epsilon_{Ra} \frac{\lambda_a}{2} + \partial_\mu \epsilon_R \right] \gamma^\mu q_R + (R \to L)$$

Variation of QCD Lagrangian under global chiral transformation

$$L^\mu_a = \frac{\partial \delta \mathscr{L}^0_{\rm QCD}}{\partial \partial_\mu \epsilon_{La}} = \bar{q}_L \gamma^\mu \frac{\lambda^a}{2} q_L$$

$$R^\mu_a = \frac{\partial \delta \mathscr{L}^0_{\rm QCD}}{\partial \partial_\mu \epsilon_{Ra}} = \bar{q}_R \gamma^\mu \frac{\lambda^a}{2} q_R$$

$$L^\mu = \frac{\partial \delta \mathscr{L}^0_{\rm QCD}}{\partial \partial_\mu \epsilon_{L}} = \bar{q}_L \gamma^\mu q_L$$

$$R^\mu = \frac{\partial \delta \mathscr{L}^0_{\rm QCD}}{\partial \partial_\mu \epsilon_{R}} = \bar{q}_R \gamma^\mu q_R$$

Left- and right-handed currents

$$V^\mu_a = R^\mu_a + L^\mu_a = \bar{q} \gamma^\mu \frac{\lambda^a}{2} q$$

$$A^\mu_a = R^\mu_a - L^\mu_a = \bar{q} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} q$$

$$V^\mu = R^\mu + L^\mu = \bar{q} \gamma^\mu q$$

$$A^\mu = R^\mu - L^\mu = \bar{q} \gamma^\mu \gamma_5 q$$

Vector and axial vector currents

$$\partial_\mu V^\mu_a = 0$$

$$\partial_\mu A^\mu_a = 0$$

$$\partial_\mu V^\mu = 0$$

Conserved

$$\partial_\mu A^\mu = \frac{3 g_3^2}{32 \pi^2} \epsilon_{\mu\nu\rho\sigma} \mathscr{G}^{\mu\nu}_a \mathscr{G}^{\rho\sigma}_a$$

Chiral anomaly due to quantum correction!

Chiral algebra

✦ Chiral charges

$$Q_{La}(t) = \int \mathrm{d}^3x\, q_L^\dagger(t,\vec{x})\frac{\lambda_a}{2}q_L(t,\vec{x}) \qquad [Q_{La}(t), H^0_{\mathrm{QCD}}] = 0$$

$$Q_{Ra}(t) = \int \mathrm{d}^3x\, q_R^\dagger(t,\vec{x})\frac{\lambda_a}{2}q_R(t,\vec{x}) \qquad [Q_{Ra}(t), H^0_{\mathrm{QCD}}] = 0$$

Conserved quantities

$$Q_V(t) = \int \mathrm{d}^3x\, q^\dagger(t,\vec{x})q(t,\vec{x}) \qquad [Q_V(t), H^0_{\mathrm{QCD}}] = 0$$

✦ Chiral algebra

$$[Q_{La}, Q_{La}] = if_{abc}Q_{Lc}$$

$$[Q_{Ra}, Q_{Ra}] = if_{abc}Q_{Rc}$$

$$[Q_{La}, Q_{Ra}] = 0$$

$$[Q_{La}, Q_V] = [Q_{Va}, Q_V] = 0$$

Satisfying the commutation relations corresponding to the Lie algebra of $SU(3)_L \times SU(3)_R \times SU(3)_V$

$\Downarrow$

Generators!

Spontaneous symmetry breaking

✦ An model invariant under iso-vector rotations of $SO(3)$

- **The Lagrangian**

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V(\phi)\ , \qquad V(\phi) = \frac{m^2}{2}\phi_i\phi_i + \lambda(\phi_i\phi_i)^2\ , i = 1,2,3\ .$$

- **The transformation property of fields**

$$\phi_i \to \phi_i' = e^{iQ_k\alpha_k}\phi_i e^{-iQ_k\alpha_k} = (e^{iT_k\alpha_k})_{ij}\phi_j = [U(g)\phi]_i$$

- **Minimum of potential**

➡**Wigner-Weyl realisation ($m^2 > 0$):** $\qquad \phi_i = 0$

➡**Nambu-Goldstone realisation ($m^2 < 0$):** $\quad |\phi_0| = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2} = \left(\frac{-m^2}{4\lambda}\right)^{1/2} \equiv a$

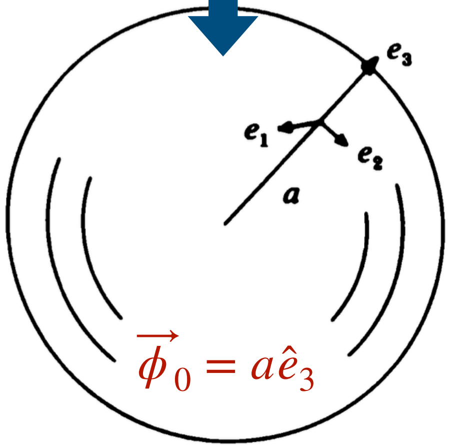
- **Spontaneous symmetry breaking** $SO(3) \to SO(2)$

Choose physics vacuum

➡**The symmetry of vacuum**

$U(h) = e^{iT_3\alpha_3}$

$$\exists g \in G:\ \phi_0' = U(g)\phi_0 \neq \phi_0 \qquad \forall h \in H < G:\ \phi_0' = U(h)\phi_0 = \phi_0$$



➡**The symmetry of potential or Lagrangian**

$$V(\phi') = V(\phi)\ , \quad \phi' = U(g)\phi\ , \quad \forall g \in G$$

Goldstone Theorem

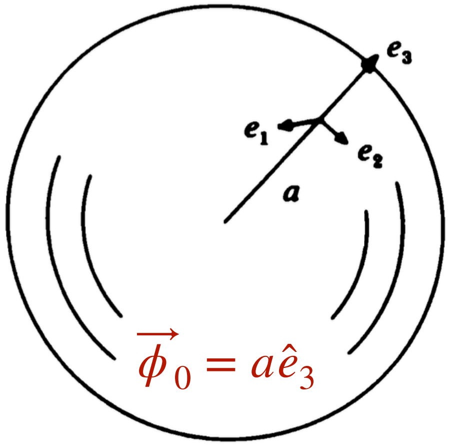
✦ The emergence of Goldstone bosons

$$V(\phi) = \frac{m^2}{2}\phi_i\phi_i + \lambda(\phi_i\phi_i)^2 \ , i = 1,2,3 \ .$$

Shift ϕ_3 field $\phi_3 = \chi + a$

$$V(\phi) = \frac{m^2}{2}\left[\phi_1^2 + \phi_2^2 + (\chi + a)^2\right] + \lambda\left[\phi_1^2 + \phi_2^2 + (\chi + a)^2\right]^2$$

$$= \boxed{4a^2\lambda\chi^2} + 4a\lambda\chi(\phi_1^2 + \phi_2^2 + \chi^2) + \lambda(\phi_1^2 + \phi_2^2 + \chi^2)^2 - \lambda a^4$$



- **Quadratic term of χ, no such terms of ϕ_1 and ϕ_2:**

$$\boxed{m_\chi^2 = 8a^2\lambda \ ,} \quad \boxed{m_{\phi_1} = m_{\phi_2} = 0}$$

One massive scalar field **Two Goldstone bosons**

$$SO(3) \overset{\text{SSB}}{\to} SO(2)$$

#GBs= $|G/H| = n_G - n_H.$

$$M^2 T_a \vec{\Phi}_{\min} = 0 \begin{cases} T_a \vec{\Phi}_{\min} = 0 \ , a = 1, \cdots, n_H & \textbf{Massive particles} \\ T_a \vec{\Phi}_{\min} \neq 0 \ , a = n_H + 1, \cdots, n_G & \to M^2 = 0 \quad \textbf{Massless Goldstone bosons} \end{cases}$$

Goldstone Theorem

✦ The emergence of Goldstone bosons

★ **Goldstone theorem**

If there is a field operator $\phi(x)$ with none-vanishing vacuum expectation value

$$\langle 0|\phi(x)|0\rangle \neq 0$$

and which is not a singlet under the transformation of some symmetry group, then massless particles must exist in the spectrum of states.

$$\phi_i \to \phi_i' = e^{iQ_k\alpha_k}\phi_i e^{-iQ_k\alpha_k} = (e^{iT_k\alpha_k})_{ij}\phi_j = [U(g)\phi]_i$$

$$\Downarrow$$

$$[Q^a, \phi_i] = iT^a_{ij}\phi_j$$

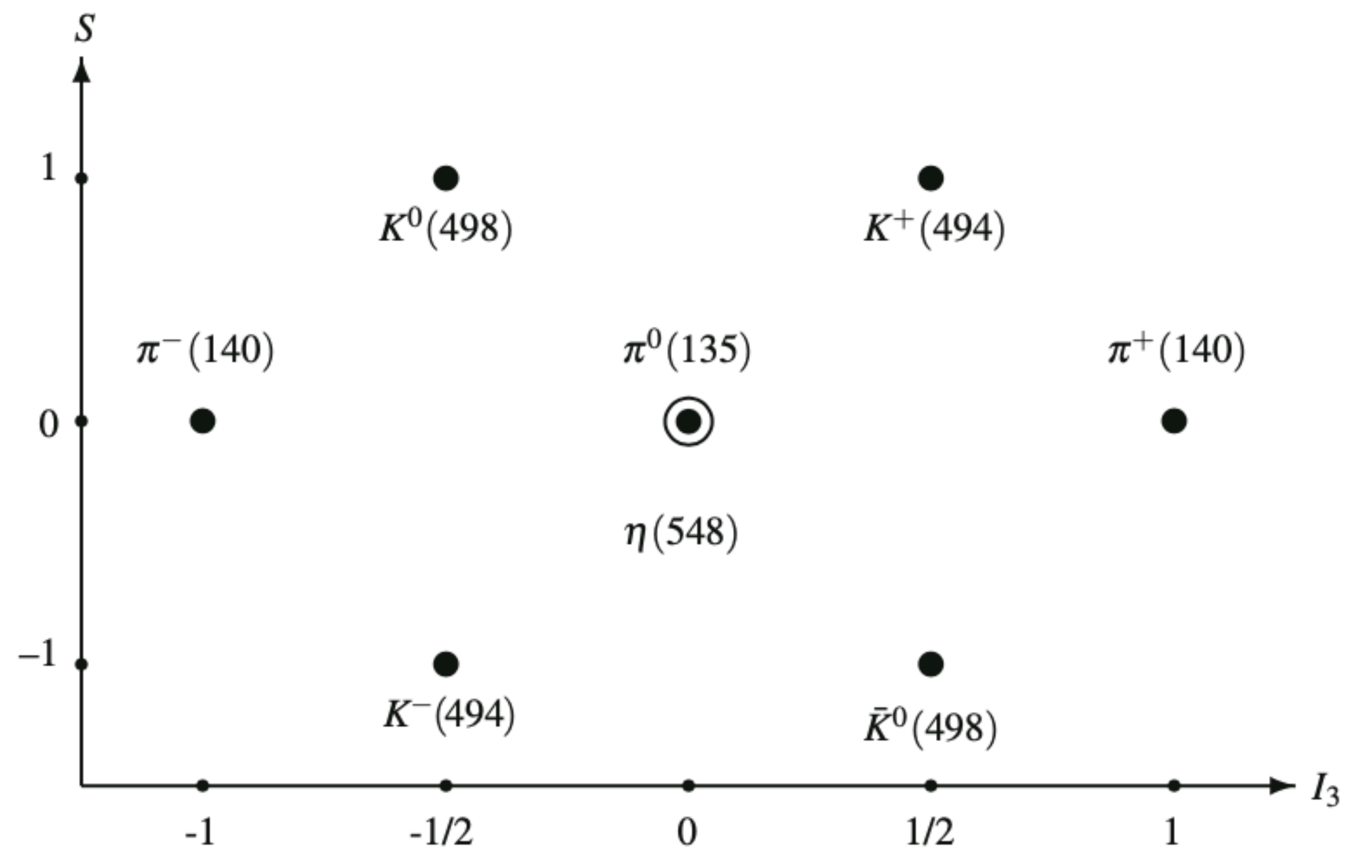
$$\langle 0|\phi_j(x)|0\rangle = 0 \quad \Longleftrightarrow \quad Q^a|0\rangle \neq 0 \quad \Longrightarrow \quad \textbf{Goldstone bosons}$$

Goldstone Theorem

✦ Goldstone bosons due to SBS chiral symmetry

- **Candidates of Goldstone bosons**

$$Q_{Va}|0\rangle = 0\ , \quad Q_{Aa}|0\rangle \neq 0$$



Small masses due to explicit chiral symmetry breaking!

$$G = SU(N_f)_L \times SU(N_f)_R \xrightarrow{\textbf{SCSB}} SU(N_f)_V = H$$

Explicit chiral symmetry breaking

✦ The mass term of quarks

$$\mathscr{L}_{\mathscr{M}} = -\bar{q}\mathscr{M}q = -(\vec{q}_R\mathscr{M}q_L + \bar{q}_L\mathscr{M}q_R)$$

$$\mathscr{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Left- and right-handed fields are mixed

$$\partial_\mu V_a^\mu = i\bar{q}\left[\mathscr{M}, \frac{\lambda_a}{2}\right]q$$

$$\partial_\mu A_a^\mu = i\bar{q}\gamma_5\left\{\mathscr{M}, \frac{\lambda_a}{2}\right\}q$$

$$\partial_\mu V^\mu = 0$$

$$\partial_\mu A^\mu = 2i\bar{q}\gamma_5\mathscr{M}q + \frac{3g_3^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathscr{G}_a^{\mu\nu}\mathscr{G}_a^{\rho\sigma} \ .$$

➡**The singlet vector current is always conserved.**
➡**If $m_u = m_d = m_s$, the 8 vector currents are conserved, which 8 axial ones not.**
➡$m_u = m_d \neq m_s$, $SU(3)$ **flavor symmetry is reduced to** $SU(2)$ **isospin symmetry.**

Local Chiral Symmetry

✦ QCD Lagrangian in the presence of external sources

$$\mathscr{L}_{\rm QCD} = \mathscr{L}^0_{\rm QCD} + \bar{q}\gamma^\mu\left(v_\mu + \gamma_5 a_\mu\right) q - \bar{q}\left(s - i\gamma_5 p\right) q$$

Vector fields $v_\mu = v^a_\mu T^a$

Axial vector fields $a_\mu = a^a_\mu T^a$

Scalar fields $s = s^a T^a$

Pseudo-scalar fields $p = p^a T^a$

- **L-R form of the source terms**

$$\bar{q}\gamma^\mu\left(v_\mu + \gamma_5 a_\mu\right) q = \left(\bar{q}_L + \bar{q}_R\right)\gamma^\mu\left(v_\mu + \gamma_5 a_\mu\right)\left(q_L + q_R\right)$$

$$= \bar{q}_L\gamma^\mu v_\mu q_L + \bar{q}_R\gamma^\mu v_\mu q_R - \bar{q}_L\gamma^\mu a_\mu q_L + \bar{q}_R\gamma^\mu v_\mu q_R$$

$$= \bar{q}_L\gamma^\mu\left(v_\mu - a_\mu\right) q_L + \bar{q}_R\left(v_\mu + a_\mu\right) q_R$$

$$\bar{q}\left(s - i\gamma_5 p\right) q = \left(\bar{q}_L + \bar{q}_R\right)\left(s - i\gamma_5 p\right)\left(q_L + q_R\right)$$

$$= \bar{q}_L s q_R + \bar{q}_R s q_L + \bar{q}_L\left(-i\gamma_5 p\right) q_R + \bar{q}_R\left(-i\gamma_5 p\right) q_L$$

$$= \bar{q}_L\left(s - ip\right) q_R + \bar{q}_R\left(s + ip\right) q_L$$

Local Chiral Symmetry

✦ Local transformation

- **Quark kinetic terms**

$$\bar{q}_L i\gamma^\mu\partial_\mu q_L \xrightarrow{L} \left[\bar{q}_L e^{i\theta_L^a(x)T_L^a}\right] i\gamma^\mu\partial_\mu \left[e^{i\theta_L^a(x)T_L^a} q_L\right] = \bar{q}_L i\gamma^\mu\partial_\mu q_L + \bar{q}_L L^\dagger \left[-\gamma^\mu\partial_\mu\theta_L^a(x)T_L^a\right] L q_L$$

$$\bar{q}_R i\gamma^\mu\partial_\mu q_R \xrightarrow{R} \bar{q}_R i\gamma^\mu\partial_\mu q_R + \bar{q}_R R^\dagger \left[-\gamma^\mu\partial_\mu\theta_R^a(x)T_R^a\right] R q_R$$

- **External sources terms**

$$L^\dagger\left(i\partial_\mu L\right)L + L^\dagger\left(v'_\mu - a'_\mu\right)L = v_\mu - a_\mu \qquad L^\dagger\left(s' - ip'\right)R = s - ip$$

$$R^\dagger\left(i\partial_\mu R\right)R + R^\dagger\left(v'_\mu + a'_\mu\right)R = v_\mu + a_\mu \qquad R^\dagger\left(s' + ip'\right)L = s + ip$$

Cancel the new pieces from quark kinetic terms

$$(v_\mu - a_\mu)' = L(v_\mu - a_\mu - i\partial_\mu L)L^\dagger = L(v_\mu - a_\mu + i\partial_\mu)L^\dagger$$

$$(v_\mu + a_\mu)' = R(v_\mu + a_\mu - i\partial_\mu R)R^\dagger = R(v_\mu + a_\mu + i\partial_\mu)R^\dagger$$

$$(s - ip)' = L(s - ip)R^\dagger$$

$$(s + ip)' = R(s + ip)L^\dagger$$

Local $SU(N_f)_L \otimes SU(N_f)_R$

Now, $\mathscr{L}_{\mathrm{QCD}}$ is invariant under local $SU(N_f)_L \otimes SU(N_f)_R$ transformation.

From QCD to ChPT: the bridge

✦ Generating functional Z

- **Path integral representation in terms of quarks and gluons**

$$\left\langle O_{\rm out} | O_{\rm in} \right\rangle_{v,a,s,p} = e^{iZ[v,a,s,p]} = \int \mathscr{D}G_\mu \mathscr{D}q \mathscr{D}\bar{q} e^{i\int \mathrm{d}^4x \mathscr{L}(q,\bar{q},G_\mu;v,a,s,p)}$$

Vacuum to vacuum transition amplitude

- **Green functions (examples)**

$\langle 0 | T[A_a^\mu(x) P_b(y)] | 0 \rangle$ **pion decay**

$\langle 0 | T[P_a(x) J^\mu(y) P_b(z)] | 0 \rangle$ **Pion electromagnetic form factor**

$\langle 0 | T[P_a(\omega) P_b(x) P_c(y) P_d(z)] | 0 \rangle$ **Pion-pion scattering**

- **Path integral representation in terms of Goldstones**

$$\int \mathscr{D}G_\mu \mathscr{D}q \mathscr{D}\bar{q} e^{i\int \mathrm{d}^4x \mathscr{L}(q,\bar{q},G_\mu;v,a,s,p)} = e^{iZ[v,a,s,p]} = N \int \mathscr{D}U e^{i\int \mathrm{d}^4x \mathscr{L}_{\rm eff}(U;v,a,s,p)}$$

A theory of quarks and gluons **A theory of effective d.o.f of hadrons**

The same green functions can be deduced at hadronic level

Effective dofs: the Goldstone Bosons

✦ Spontaneous Chiral Symmetry Breaking

- **Goldstone bosons**

$$
\begin{array}{ccc}
G = SU(N_f)_L \times SU(N_f)_R & \xrightarrow{\textbf{SCSB}} & SU(N_f)_V = H \\
\uparrow & & \uparrow \\
n_G & & n_H
\end{array}
$$

where n_G and n_H are the numbers of generators.

Goldstone theorem

There are $n = n_G - n_H$ **Goldstone bosons** $\phi_1 , \cdots , \phi_n$ $\quad \tilde{\Phi} = \left(\phi_1 , \cdots , \phi_n\right)$

- **Non-linear realization**

$$
\begin{tikzcd}
\tilde{\Phi} \arrow[r, "g"] \arrow[d] & \tilde{\Phi}' \arrow[d] \\
\tilde{g}H \arrow[r, "g = (L , R) \in G"] & g\tilde{g}H
\end{tikzcd}
$$

Quotient G/H

An isomorphic mapping between the quotient G/H **and the Goldstone boson fields.**

- **The introduction of** U

Let $\tilde{g} \equiv \left(\tilde{L} , \tilde{R}\right) \in G$, parameterize $\tilde{g}H$ by $SU(N_f)$ matrix $U = \tilde{R}\tilde{L}^\dagger$ $[U^\dagger U = I , \det U = 1]$

$$
\tilde{g}H = \left(I , \tilde{R}\tilde{L}^\dagger\right) H , \quad g\tilde{g}H = \left(L , R\tilde{R}\tilde{L}^\dagger\right) H = \left(I , R\tilde{R}\tilde{L}L^\dagger\right)(L , L)\, H = \left(I , R\tilde{R}\tilde{L}^\dagger L^\dagger\right) H
$$

Transformation law: $\quad U = \tilde{R}L^\dagger \xrightarrow{g} R\left(\tilde{R}^\dagger\tilde{L}^\dagger\right) L^\dagger = RUL^\dagger \qquad U = e^{i\Phi}$

Effective dofs: the Goldstone Bosons

✦ Non-linear realisation of the GBs

- **Transformation property** $[U^\dagger U = 1 \;\&\; \det U = 1]$

$$U \equiv e^{i\frac{\Phi}{\sqrt{2}F}} \qquad U \xrightarrow{SU(N_f)_L \otimes SU(N_f)_R} RUL^\dagger$$

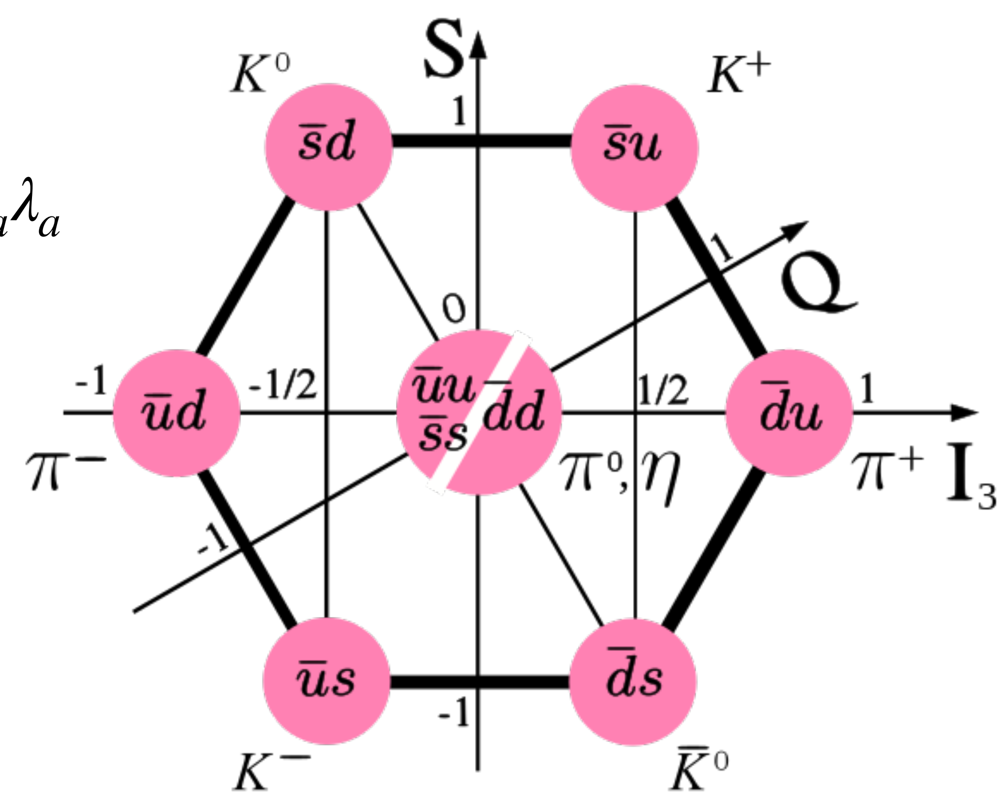
the parameter F has mass dimension

- **Goldstone bosons field are non-linearly realized.**

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{\sqrt{2}}{\sqrt{3}}\eta \end{pmatrix} = \frac{1}{\sqrt{2}}\phi_a\lambda_a$$

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\phi_i\tau_i$$

Meson-Octet

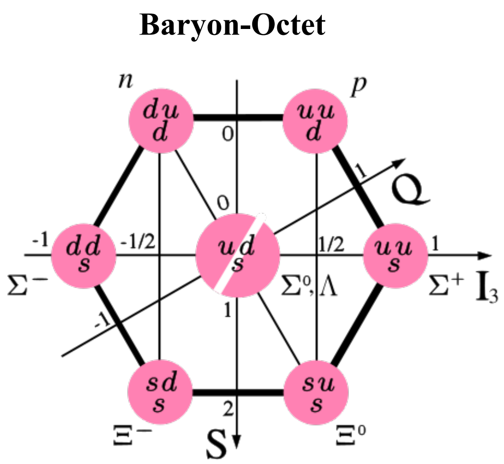


The Incorporation of Baryons

✦ Transformation properties of the Baryon fields

- **SU(3) case:** $B \overset{g}{\longmapsto} B' = K(L,R,U)BK(L,R,U)^{-1}$

$$B = \sum_{a=1}^{8} \frac{B_a\lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$



- **SU(2) case:** $\Psi \overset{g}{\longmapsto} \Psi' = K(L,R,U)\Psi \qquad \Psi = \begin{pmatrix} p \\ n \end{pmatrix}$

- **Definition of** $K(L,R,U)$

$$u^2(x) = U(x)$$

$$\sqrt{U(x)} = u(x) \overset{g}{\longmapsto} u'(x) = \sqrt{RUL^\dagger} \equiv RuK^{-1}(L,R,U) \qquad U \overset{g}{\longmapsto} RUL^\dagger$$

$$K(L,R,U) = u'^{-1}Ru = \sqrt{RUL^\dagger}^{-1} R\sqrt{U}$$

The Incorporation of Baryons

✦ Transformation properties of the Baryon fields

- **Proof: [$SU(2)$ case for example]**

1. **Define a map**

$$\varphi(g): \quad \begin{pmatrix} U \\ \Psi \end{pmatrix} \stackrel{g}{\longmapsto} \begin{pmatrix} U' \\ \Psi' \end{pmatrix} = \begin{pmatrix} RUL^\dagger \\ K(L,R,U)\Psi \end{pmatrix}$$

2. **It is a homomorphism**

$$\varphi(g_1)\varphi(g_2)\begin{pmatrix} U \\ \Psi \end{pmatrix} = \varphi(g_1)\begin{pmatrix} R_2UL_2^\dagger \\ K(L_2,R_2,U_2)\Psi \end{pmatrix} = \begin{pmatrix} R_1R_2UL_2^\dagger L_1^\dagger \\ K(L_1,R_1,R_2UL_2^\dagger)K(L_2,R_2,U)\Psi \end{pmatrix}$$

$$\overset{\text{Exercise}}{=} \begin{pmatrix} (R_1R_2)U(L_1L_2)^\dagger \\ K(L_1L_2,R_1R_2,U)\Psi \end{pmatrix} = \varphi(g_1g_2)\begin{pmatrix} U \\ \Psi \end{pmatrix}$$

- **Similar for SU(3) case**

Baryon transition amplitudes

✦ Generating functional with baryon fields

- **The effective Lagrangian with Goldstones and baryons**

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm GB} + \mathscr{L}_B + \bar{\eta}^a B^a + \bar{B}^a \eta^a \qquad\qquad \mathscr{L}_B = \bar{B}^a D^{ab} B^b$$

- **Generating functional**

$$\langle O_{\rm Out} | O_{\rm in} \rangle_{v,a,s,p;\eta\bar{\eta}} = e^{i\mathscr{Z}[v,a,s,p;\eta\bar{\eta}]} = N' \int \mathscr{D}U \mathscr{D}B \mathscr{D}\bar{B} e^{i\int {\rm d}^4x \mathscr{L}_{\rm eff}(B,\bar{B},U;v,a,s,p;\eta\bar{\eta})}$$

$$= N' \int \mathscr{D}U e^{i\int {\rm d}^4x \mathscr{L}_{\rm GB}} \int \mathscr{D}B \mathscr{D}\bar{B} e^{i\int {\rm d}^4x \left[\bar{B}^a D^{ab} B^b + \bar{\eta}^a B^a + \bar{B}^a \eta^a\right]}$$

$$= N' \int \mathscr{D}U e^{i\int {\rm d}^4x \mathscr{L}_{\rm GB}} e^{-i\int {\rm d}^4x \int {\rm d}^4y \bar{\eta}^a(x) S_{ab}(x,y) \eta^b(y)} {\rm det} D$$

- **$S^{ab}(x,y\,|\,U;v,a,s,p)$ is the baryon propagator in the presence of the meson fields and external fields.**

$$D^{ac} S^{cb} = \delta^4(x-y)\delta^{ab}$$

- **Baryon propagator in the presence of the meson fields and external fields.**

$$S^{ab}(x,y\,|\,v,a,s,p) = \frac{\delta}{i\delta\eta^a(x)} \frac{\delta}{i\delta\eta^b(y)} \mathscr{Z}\,|_{\eta=\bar{\eta}=0} = \left\langle 0\,|\,T\left[B^a(x)B^b(y)\right]|\,0\right\rangle$$

Baryon transition amplitudes

✦ Generating functional with baryon fields

- **Transfer to the momentum space**

$$\tilde{S}(p,p'|v,a,s,p) = \int \mathrm{d}^4x \int \mathrm{d}^4y e^{-ipx-ip'y} S(x,y|v,a,s,p)$$

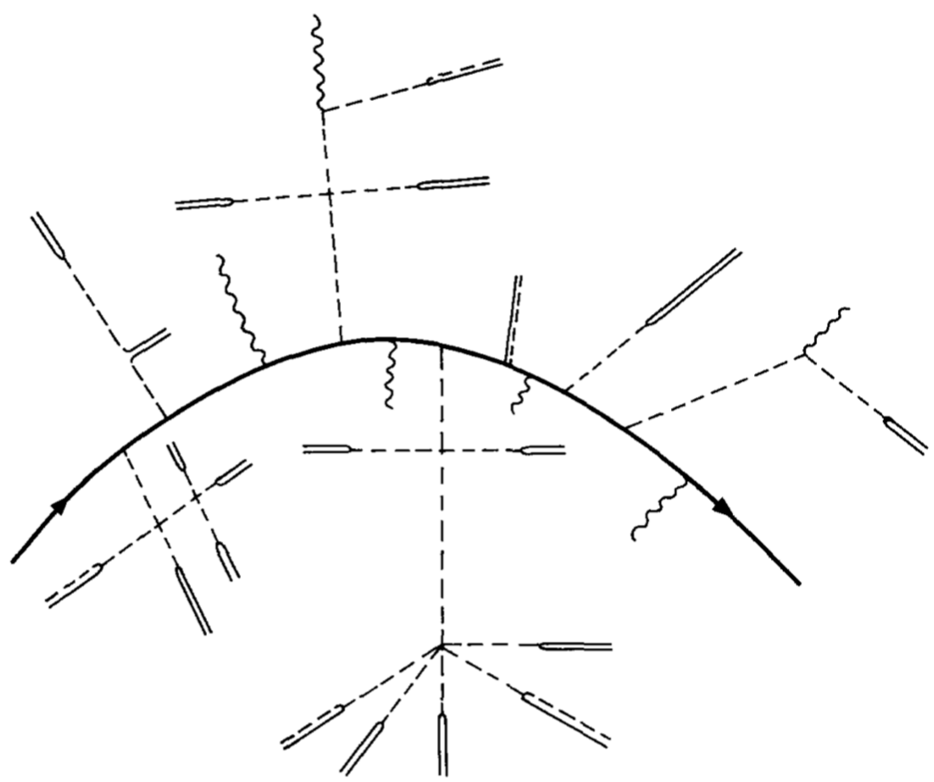
- **Baryon to baryon transition amplitude**

$$\mathscr{F}(p,p'|v,a,s,p) = \left\langle p'_{\rm out} | p_{\rm in} \right\rangle^c_{v,a,s,p} \propto \text{Residue of } \tilde{S}(p,p'|v,a,s,p)$$

- **Examples : Baryon matrix elements**

$$\langle p' | \bar{q}(x)\gamma^\mu T^a q(x) | p \rangle = \frac{\delta}{i\delta v^a_\mu(x)} \mathscr{F}|_{v=0}$$

$$\left\langle p' | \bar{q}(x)\gamma^\mu \gamma^5 T^a q(x) | p \right\rangle = \frac{\delta}{i\delta a^a_\mu(x)} \mathscr{F}|_{a=0}$$



- **Our task is to construct $D^{ab}(U; v,a,s,p)$ order by order**

$$\mathscr{L}_B = \bar{B}^a D^{ab} B^b \qquad D^{ab} = \sum_i D^{ab,(i)}$$

Part II:

Construction of Chiral Effective Lagrangians

Building Blocks

✦ Stuffs in hand

- **Meson fields:** $U \xmapsto{g} RUL^\dagger$
- **Baryon fields:** $B \xmapsto{g} KBK^\dagger$ $\quad(\Psi \xmapsto{g} K\Psi)$
- **A set of external fields:** $v_\mu\,, a_\mu\,, s\,, p$

✦ Recipe for BChPT

Building blocks transform under chiral symmetry group in the same way as baryon fields

$$A \xmapsto{g} KAK^\dagger \qquad A \in \{u_\mu, F^\pm_{\mu\nu}, \chi_\pm\} \qquad [D_\mu\,, X] = \partial_\mu X + [\Gamma_\mu\,, X]$$

- **The chiral connection**

$$\Gamma_\mu = \{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\}/2$$

- **The so-called chiral vielbein**

$$u_\mu \equiv i[u^\dagger(\partial_\mu - ir_\mu)u - u\left(\partial_\mu - il_\mu\right)u^\dagger]$$

- **Vector and axial vector sources**

$$F^\pm_{\mu\nu} \equiv u^\dagger F^R_{\mu\nu}u \pm uF^L_{\mu\nu}u^\dagger$$

- **Scalar and pseudo-scalar sources**

$$\chi_\pm \equiv u^\dagger\chi u^\dagger \pm u\chi^\dagger u$$

$$r_\mu = v_\mu + a_\mu$$

$$l_\mu = v_\mu - a_\mu$$

$$F^R_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i\left[r_\mu\,, r_\nu\right]$$

$$F^L_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i\left[l_\mu\,, l_\nu\right]$$

$$\chi = 2B\left(s + ip\right)$$

Building Blocks

✦ Lorentz transformation (also parity)

- **Bilinear**

$\bar{B}'B' = \bar{B}B$ **(Scalar)**

$\bar{B}'\gamma_5 B' = \det\Lambda\, \bar{B}\gamma_5 B$ **(Pseudo scalar)**

$\bar{B}'\gamma_\mu B' = \Lambda_\mu^{\ \nu}\bar{B}\gamma_\nu B$ **(Vector)**

$\bar{B}'\gamma_\mu\gamma_5 B' = \det\Lambda\Lambda_\mu^{\ \nu}\bar{B}\gamma_\nu\gamma_5 B$ **(Axial vector)**

- **Building blocks**

$\chi_+' = \chi_+$ **(Scalar)**

$\chi_-' = \det\Lambda\, \chi_-$ **(Pseudo Scalar)**

$D_\mu' = \Lambda_\mu^{\ \nu} D_\nu$ **(Vector)**

$u_\mu' = \det\Lambda\Lambda_\mu^{\ \nu} u_\nu$ **(Axial vector)**

$F_{\mu\nu}^{+'} = \Lambda_\mu^{\ \alpha}\Lambda_\nu^{\ \beta} F_{\alpha\beta}^+$ **(Tensor)**

$F_{\mu\nu}^{-'} = \det\Lambda\Lambda_\mu^{\ \alpha}\Lambda_\nu^{\ \beta} F_{\alpha\beta}^-$ **(Pseudo Tensor)**

Clifford algebra

$$\Gamma \in \left\{1\,,\gamma^5\,,\gamma^\mu\,,\gamma^5\gamma^\mu\,,\sigma^{\mu\nu}\right\}$$

Proof (Hints) :

0^- **particle**

$U = e^{i\Phi}$ $\quad v_\mu' = \Lambda_\mu^{\ \nu} v_\nu$

$\Phi' \stackrel{P}{=} -\Phi$ $\quad a_\mu' = \det\Lambda\Lambda_\mu^{\ \nu} a_\nu$

$U' \stackrel{P}{=} U^\dagger$ $\quad s' = s$

$u' \stackrel{P}{=} u^\dagger$ $\quad p' = \det\Lambda\, p$

$$\chi_- = u^\dagger(s+ip)u^\dagger - u(s+ip)^\dagger u$$
$$\downarrow$$
$$\chi_-' = u(s-ip)u - u^\dagger(s-ip)^\dagger u^\dagger$$
$$= -\chi_-$$

Building Blocks

✦ Charge conjugation

- **Baryon field transform as** $B^C = C\bar{B}^T$

$$
\begin{aligned}
\left(\bar{B}\right)^C &= \left(B^\dagger\gamma_0\right)^C = \left(B^C\right)^\dagger\gamma_0 \\
&= \left(C\bar{B}^T\right)^\dagger\gamma_0 = \left(\bar{B}C^T\right)^*\gamma_0 \\
&= B^T\gamma_0^*C^{T*}\gamma_0 = B^T\gamma_0C^\dagger\gamma_0 \\
&= -B^T\gamma_0C\gamma_0 \\
&= B^TC \\
&= -B^TC^{-1}
\end{aligned}
$$

- **Bilinear transform as**

$$
\begin{aligned}
\left(\bar{B}\Gamma B\right)^C &= \bar{B}^C\Gamma B^C \\
&= -B^TC^{-1}\Gamma C\bar{B}^T = -B^TC\Gamma C^{-1}\bar{B}^T \\
&= -B^T(-1)^{C_\Gamma}\Gamma^T\bar{B}^T \\
&= (-1)^{C_\Gamma}\left[-B^T\Gamma^T\bar{B}^T\right] \\
&= (-1)^{C_\Gamma}\bar{B}\Gamma B
\end{aligned}
$$

where we made use of $\left[B^T\Gamma^T\bar{B}^T\right] = -\left[\bar{B}\Gamma B\right]$

$$C\Gamma C^{-1} = (-1)^{C_\Gamma}\Gamma^T$$

Charge-conjugation matrix

$$C = i\gamma^2\gamma^0 = -C^{-1} = -C^\dagger = -C^T = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$C\gamma^\mu C^{-1} = -\gamma^{\mu T}$$

$$C\gamma^5\gamma^\mu C^{-1} = \left(\gamma^5\gamma^\mu\right)^T$$

$$C\gamma^5C^{-1} = \gamma^{5T}$$

$$C\sigma^{\mu\nu}C^{-1} = -\left(\sigma^{\mu\nu}\right)^T$$

Building Blocks

✦ Charge conjugation

- **Sources**

$$s^C = s^T \qquad p^C = p^T$$

$$v_\mu^C = -v_\mu^T \qquad a_\mu^C = a_\mu^T$$

How to prove? **QCD Lagrangian is invariant, e.g.**

$$[(\bar{q}\gamma^\mu v_\mu)q]^C = (\bar{q}\gamma^\mu v_\mu q)$$

$$\begin{aligned}\bar{q}^C\gamma^\mu v_\mu^C q^C &= -q^T C^{-1}\gamma^\mu C v_\mu^C \bar{q}^T \\ &= -q^T C\gamma^\mu C^{-1} v_\mu^C \bar{q}^T \\ &= q^T\gamma^{\mu T} v_\mu^C \bar{q}^T \\ &= -\left(\bar{q}\gamma^\mu v_\mu^{CT} q\right) = \bar{q}\gamma^\mu v_\mu q\end{aligned}$$

- **Meson fields**

$$\Phi^C = \Phi^T \quad \Longrightarrow \begin{cases} U^C = U^T \quad (U = e^{i\Phi}) \\ u^C = u^T \end{cases}$$

- **Building blocks**

$$u_\mu^C = u_\mu^T \qquad D_\mu^C = -D_\mu^T \qquad F_{\mu\nu}^{+C} = -F_{\mu\nu}^{+T} \qquad F_{\mu\nu}^{-C} = F_{\mu\nu}^{-T} \qquad \chi_\pm^C = \chi_\pm^T$$

✦ Hermiticity properties

- **Bilinear**

$$\left(\bar{B}\Gamma B\right)^\dagger = B^\dagger\Gamma^\dagger\gamma^0 B = \bar{B}\left(\gamma^0\Gamma^\dagger\gamma_0\right)B \xrightarrow{\gamma^0\Gamma\gamma^0 = (-1)^{h_\Gamma}\Gamma} \left(\bar{B}\Gamma B\right)^\dagger = (-1)^{h_\Gamma}\left(\bar{B}\Gamma B\right)$$

- **Building blocks**

$$u_\mu^\dagger = -u_\mu \qquad D_\mu^\dagger = -D_\mu \qquad F_{\mu\nu}^{\pm\dagger} = F_{\mu\nu}^\pm \qquad \chi_-^\dagger = -\chi_- \qquad \chi_+^\dagger = \chi_+$$

Building Blocks

✦ Chiral power counting

- **Building blocks**

$$U = e^{i\Phi} \quad \sim \ O(1) \qquad\qquad u = \sqrt{U} \quad \sim \ O(1)$$

$$\partial_\mu U \quad \sim O(p)$$

Explanation: **Four momentum** $p = \left(E = \sqrt{M_\phi^2 + \vec{p}^2}, \vec{p}\right)$

$M_\phi^2 \sim m_q \sim O(p^2)$ **three-momentum is small quantity**

$$\nabla_\mu U = \partial_\mu U - i\left(v_\mu + a_\mu\right)U + iU\left(v_\mu - a_\mu\right) \quad \sim O(p) \qquad \Longrightarrow v_\mu \sim a_\mu \sim O(p)$$

$$s + ip \quad \Longrightarrow \quad s \sim p \sim O(p^2)$$

$U, u \sim O(1)$ $\quad s, p \sim O(p^2)$ $\quad a_\mu, u_\mu \sim O(p)$

$u_\mu \sim O(p)$ $\quad F_{\mu\nu}^{\pm} \sim O(p^2)$ $\quad \chi_\pm \sim O(p^2)$ $\quad D_\mu \sim O(p)$

Building Blocks

✦ Chiral power counting

- **Positive-energy plane-wave solutions**

$$B(\vec{x},t) = e^{-ip\cdot x}\sqrt{E+M_B}\begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_B}\chi \end{pmatrix} \begin{array}{l} \longrightarrow \text{Large component} \\ \longrightarrow \text{Small component} \end{array} \qquad B(\vec{x},t) \sim O(p^0)$$

$$\bar{B}(\vec{x},t) = B^\dagger\gamma^0 = e^{+ip\cdot x}\sqrt{E+M_B}\left(\chi^\dagger, \frac{\vec{\sigma}\cdot\vec{p}}{E+M_B}\chi^\dagger\right)\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$= e^{+ip\cdot x}\sqrt{E+M_B}\left(\chi^\dagger, -\frac{\vec{\sigma}\cdot\vec{p}}{E+M_B}\chi^\dagger\right) \sim O(p^0)$$

$$[D_\mu, B] \sim O(1) \qquad i\gamma^\mu[D_\mu, B] - M_0 B \sim O(p)$$

- **Bilinear $\bar{B}\Gamma B$**

$$\Gamma = \gamma_5 \qquad \sim O(p^1) \qquad\qquad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Gamma = 1, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \qquad \sim O(p^0) \qquad\qquad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Construction of Chiral Effective Lagrangians

✦ Guiding rules

① Real (Hermiticity) ② Flavor neutral (Trace) ③ Scalar (Parity even)

④ Chiral transformation ⑤ Proper Lorentz transformations

⑥ Charge conjugation C ⑦ Parity P ⑧ Time reversal T

- **A general form**

$$\left\langle A_1 \bar{B} \Gamma A_2 B A_3 \right\rangle \longrightarrow \left\langle \bar{B} \Gamma A_2' B A_3' \right\rangle$$

$$A_i \in \left\{ F_{\mu\nu}^{\pm}, \chi_{\pm}, u_\mu, D_\mu \right\}$$

$$\Gamma \text{ (Clifford algebra)} \in \left\{ 1, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu} \right\}$$

Constraint from Flavor neutral, Scalar, Chiral and Proper Lorentz transformation

Lorentz indices contracted by $g_{\mu\nu}$, $\epsilon_{\mu\nu\rho\sigma}$

Chiral power counting $D = D_\Gamma + D_{A_2'} + D_{A_3'} \Longrightarrow O(p^D)$

Construction of Chiral Effective Lagrangians

✦ Guiding rules

① Real (Hermiticity) ② Flavor neutral (Trace) ③ Scalar (Parity even)

④ Chiral transformation ⑤ Proper Lorentz transformations

⑥ Charge conjugation C ⑦ Parity P ⑧ Time reversal T

- **More suitable form form by imposing charge and hermiticity transformation**

$$X = \langle \bar{B}\Gamma[A_1\,, [A_2\ \cdots\ , [A_n\,, B]_\pm \cdots]_\pm]_\pm \rangle$$

Hermiticity $$X^\dagger = (-1)^{h_1+\cdots+h_n+h_\Gamma}\langle \bar{B}\Gamma[A_n\,, [A_2\ \cdots\ , [A_1\,, B]_\pm]_\pm \cdots]_\pm \rangle$$

Charge conj. $$X^c = (-1)^{c_1+\cdots+c_n+c_\Gamma}\langle \bar{B}\Gamma[A_n\,, [A_2\ \cdots\ , [A_1\,, B]_\pm]_\pm \cdots]_\pm \rangle$$

$$\begin{cases} C\Gamma C^{-1} = (-1)^{c_\Gamma}\Gamma^T \\ \gamma_0\Gamma^\dagger\gamma_0 = (-1)^{h_\Gamma}\Gamma \end{cases} \qquad \begin{cases} A_k^C = (-1)^{c_k}A_k^T \\ A_k^\dagger = (-1)^{h_k}A_k \end{cases} \qquad X_{\text{allowed}} \Longrightarrow \begin{cases} \frac{1}{2}\left(X + X^\dagger\right) \\ \frac{1}{2}\left(X + X^c\right) \end{cases}$$

Construction of Chiral Effective Lagrangians

✦ Rewind of the differential operator D^{ab} in the generating functional

- **The effective Lagrangian**

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{GB}} + \mathscr{L}_{B} + \bar{\eta}^a B^a + \bar{B}^a \eta^a$$

$$\begin{aligned}\mathscr{L}_{\text{B}} &\sim \langle \bar{B}\Gamma[A_1[A_2\,,\cdots,[A_n\,,B]_{\pm}\cdots]_{\pm}]_{\pm}\rangle \\ &= \frac{1}{2}\bar{B}^a\langle\lambda^a\Gamma[A_1\,,[A_2\,,\cdots[A_n\,,\lambda^b]_{\pm}\cdots]_{\pm}]_{\pm}\rangle B^b \\ &= \bar{B}^a D^{ab} B^b\end{aligned}$$

$$B = \frac{1}{\sqrt{2}}B_a\lambda^a$$

Therefore, differential operator D^{ab} has the form of

$$D^{ab} = \frac{1}{2}\Gamma\langle\lambda^a[A_1\,,[A_2\,,\cdots[A_n\,,\lambda^b]_{\pm}\cdots]_{\pm}]_{\pm}\rangle$$

SU(3) Meson-Baryon Lagrangian

✦ Lowest order

- $O(p^0)$ operators $a\langle\bar{B}i\gamma^\mu[D_\mu, B]\rangle + b\langle\bar{B}B\rangle$

$$\Rightarrow \quad \langle\bar{B}i\gamma^\mu[D_\mu, B]\rangle - M_0\langle\bar{B}B\rangle \quad \sim \quad O(p^1)$$

- $O(p^1)$ operators $D\langle\bar{B}i\gamma^5\gamma^\mu\{u_\mu, B\}\rangle + F\langle\bar{B}i\gamma^5\gamma^\mu[u_\mu, B]\rangle$

- **The lowest-order chiral effective Lagrangian is**

$$\mathscr{L}^{(1)}_{\text{Baryon}} = \langle\bar{B}i\gamma^\mu[D_\mu, B]\rangle - M_0\langle\bar{B}B\rangle + D\langle\bar{B}i\gamma^5\gamma^\mu\{u_\mu, B\}\rangle + F\langle\bar{B}i\gamma^5\gamma^\mu[u_\mu, B]\rangle$$

- D and F are unknown constants, called low-energy constants (LECs)
- Determined by fitting to semileptonic decay $B \to B' + e^- + \bar{\nu}_e$

✦ Higher orders

- **Two and more traces**
- **Redundancy : minimal set (EOM, trace theorem, etc)**

	$O(p^2)$	$O(p^3)$	$O(p^4)$	
$SU(3)$:	16	84	540	[J.A. Oller, et al, JHEP 2006] [S.Z. Jiang , et al,PRD 2017]
$SU(2)$:	7	23	118	[N. Fetles, et al, Annals of physics 2000]

Electroweak interaction in BChPT

✦ Electromagnetic interaction ($\mathscr{A}_\mu$)

- $SU(2)$

$$r_\mu = l_\mu = -e\mathscr{A}_\mu \frac{\tau_3}{3} \qquad v_\mu^{(s)} = -\frac{e}{2}\mathscr{A}_\mu \qquad e > 0 \quad \text{and} \quad \frac{e^2}{4\pi} \approx \frac{1}{137}$$

- $SU(3)$

$$r_\mu = l_\mu = -e\mathscr{A}_\mu Q \qquad Q = \mathrm{diag}\{\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\}$$

✦ Weak charged interaction ($W_\mu^\pm = (W_{1\mu} \mp iW_{2\mu})/\sqrt{2}$)

$$r_\mu = 0 \qquad l_\mu = -\frac{g}{\sqrt{2}}\left(W_\mu^\pm T_+ + h.c.\right)$$

- $SU(2)$ $\quad T_+ = \begin{pmatrix} 0 & v_{ud} \\ 0 & 0 \end{pmatrix}$
- $SU(3)$ $\quad T_+ = \begin{pmatrix} 0 & v_{ud} & v_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Fermi constant $\quad G_F = \dfrac{\sqrt{2}g^2}{8M_W^2} = 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$

✦ Weak neutral interaction (Z_μ)

- $SU(2)$ $\quad r_\mu = e\tan(\theta_w) Z_\mu \dfrac{\tau_3}{2} \qquad l_\mu = -\dfrac{g}{\cos\theta_w} Z_\mu \dfrac{\tau_3}{2} + e\tan(\theta_w) Z_\mu \dfrac{\tau_3}{2} \qquad v_\mu^{(s)} = \dfrac{e\tan(\theta_w)}{2} Z_\mu$

SU(2) Meson-Baryon Lagrangian

✦ The SU(2) Lagrangian can be constructed in the same way

$O(p^1)$ $$\mathscr{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma^\mu D_\mu - m - \frac{g}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi \qquad \Psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

$O(p^2)$ $$\mathscr{L}_{\pi N}^{(2)} = \bar{\Psi}\left\{ c_1\langle\chi_+\rangle - \frac{c_2}{8m^2}[\langle u_\mu u_\nu\rangle D^{\mu\nu} + h.c.] + \frac{1}{2}c_3\langle u_\mu u^\mu\rangle + \frac{i}{4}c_4[u_\mu, u_\nu]\sigma^{\mu\nu} + c_5[\chi_+ - \frac{1}{3}\langle\chi_+\rangle] + \frac{c_6}{8m}F^+_{\mu\nu}\sigma^{\mu\nu} + \frac{c_7}{8m}\langle F^+_{\mu\nu}\rangle\sigma^{\mu\nu} \right\}\Psi$$

$O(p^3)$ $$\mathscr{L}_{\pi N}^{(3)} = \sum_{i=1}^{23} d_i O_i^{(3)}$$

- **Remarks**

  - **# of LECs grows rapidly** — **No predictive power?**
  - **Operators with dimension larger than 4** — **Non-renomalizable?**

> **"A non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a finite accuracy."—Aneesh V. Manohar**

Power counting rule is important!!!

Part III:

Phenomenological Applications at Tree Level

Meson-Baryon Lagrangian

✦ Expansion of the building blocks in terms of GB fields

- **The meson field u is defined by exponential function of matrix** $u = e^{\frac{i}{\sqrt{2}F}\Phi}$

$$u(\Phi) = 1 + \frac{i}{\sqrt{2}F}\Phi - \frac{1}{4F^2}\Phi^2 - \frac{i}{12\sqrt{2}F^3}\Phi^3 + \frac{1}{96F^4}\Phi^4 + \frac{i}{480\sqrt{2}F^5}\Phi^5 - \frac{1}{5760F^6}\Phi^6 + O(\Phi^7)$$

$$u^\dagger(\Phi) = 1 - \frac{i}{\sqrt{2}F}\Phi - \frac{1}{4F^2}\Phi^2 + \frac{i}{12\sqrt{2}F^3}\Phi^3 + \frac{1}{96F^4}\Phi^4 - \frac{i}{480\sqrt{2}F^5}\Phi^5 - \frac{1}{5760F^6}\Phi^6 + O(\Phi^7)$$

- **If** $r_\mu = l_\mu$

$$u_\mu = -\frac{\sqrt{2}}{F}\sum_{m=0}^{1} C_1^m(-\Phi)^m\left(\partial_\mu - il_\mu\right)\Phi^{1-m} + \frac{1}{6\sqrt{2}F^3}\sum_{m=0}^{3} C_3^m(-\Phi)^m\left(\partial_\mu - il_\mu\right)\Phi^{3-m}$$
$$-\frac{1}{240\sqrt{2}F^5}\sum_{m=0}^{5} C_5^m(-\Phi)^m\left(\partial_\mu - il_\mu\right)\Phi^{5-m} + O(\Phi^7)$$

- **If** $r_\mu = l_\mu = 0$

$$\Gamma_\mu = \frac{1}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right) \overset{n=2k-1}{=} \frac{1}{2}\sum_{k=1}^{\infty}\frac{-2}{(2k)(2k-1)!}\frac{i^{2k}}{2^k F^{2k}}[\underbrace{\Phi,\cdots[\Phi}_{2k-1\text{ times}},\partial_\mu\Phi]\cdots]$$
$$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k(2k-1)!2^k F^{2k}}[\underbrace{\Phi,\cdots[\Phi}_{2k-1\text{ times}},\partial_\mu\Phi]\cdots]$$

Meson-Baryon Lagrangian

✦ Expansion of the building blocks in terms of GB fields

- **If** $p=0,\ \chi=\chi^\dagger=2B_0 s$

$$\chi_+ = 4B_0 s - \frac{B_0}{F^2}\sum_{m=0}^{2} C_2^m \Phi^m s \Phi^{2-m} + \frac{B_0}{24F^4}\sum_{m=0}^{4} C_4^m \Phi^m s \Phi^{4-m} - \frac{B_0}{1440F^6}\sum_{m=0}^{6} C_6^m \Phi^m s \Phi^{6-m} + O(\Phi^8)$$

$$\chi_- = -\frac{2\sqrt{2}B_0 i}{F}\sum_{m=0}^{1} C_1^m \Phi^m s \Phi^{1-m} + \frac{B_0 i}{3\sqrt{2}F^3}\sum_{m=0}^{3} C_3^m \Phi^m s \Phi^{3-m} - \frac{B_0 i}{120\sqrt{2}F^5}\sum_{m=0}^{5} C_5^m \Phi^m s \Phi^{5-m} + O(\Phi^7)$$

- **If** $r_\mu = l_\mu = -e\mathscr{A}_\mu Q$

$$F_+^{\mu\nu} = e(\partial^\mu \mathscr{A}^\nu - \partial^\nu \mathscr{A}^\mu)\Big[-2Q + \frac{1}{2F^2}\sum_{m=0}^{2} C_2^m \Phi^m Q \Phi^{2-m} - \frac{1}{48F^2}\sum_{m=0}^{4} C_4^m \Phi^m Q \Phi^{4-m}$$
$$+\frac{1}{2880F^2}\sum_{m=0}^{6} C_6^m \Phi^m Q \Phi^{6-m} + O(\Phi^8)\Big]$$

$$F_-^{\mu\nu} = e(\partial^\mu \mathscr{A}^\nu - \partial^\nu \mathscr{A}^\mu)\Big[\frac{\sqrt{2}i}{F}\sum_{m=0}^{1} C_1^m (-\Phi)^m \partial_\mu \Phi^{1-m} - \frac{i}{6\sqrt{2}F^3}\sum_{m=0}^{3} C_3^m (-\Phi)^m \partial_\mu \Phi^{3-m}$$
$$+\frac{i}{240\sqrt{2}F^5}\sum_{m=0}^{5} C_5^m (-\Phi)^m \partial_\mu \Phi^{5-m} + O(\Phi^7)\Big]$$

Meson-Baryon Lagrangian

231126

✦ Feynman rule

- **LO Lagrangian**

$$\mathscr{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma^\mu D_\mu - m\right)\Psi - \frac{g}{2}\bar{\Psi}\gamma^\mu\gamma_5 u_\mu \Psi \qquad D_\mu = \partial_\mu + \Gamma_\mu$$

$$= \bar{\Psi}\left(i\gamma^\mu \partial_\mu - m\right)\Psi + \bar{\Psi}i\gamma^\mu\Gamma_\mu\Psi - \frac{g}{2}\bar{\Psi}\gamma^\mu\gamma_5 u_\mu \Psi$$

- **The chiral connection and vielbein** $(r_\mu = l_\mu = 0)$

$$\begin{aligned}\Gamma_\mu &= \frac{1}{2}\left\{u^\dagger\left(\partial_\mu - ir_\mu\right)u + u\left(\partial_\mu - il_\mu\right)u^\dagger\right\}\\ &= \frac{1}{2}\left\{u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right\}\\ &= -\frac{1}{4F^2}\left(\partial_\mu\Phi\Phi - \Phi\partial_\mu\Phi\right) + O(\Phi^4)\\ &= -\frac{i}{4F^2}\left(\partial_\mu\vec{\phi}\times\vec{\phi}\right)\cdot\vec{\tau} + O(\vec{\phi}^4)\end{aligned}$$

$$\begin{aligned}u_\mu &= i\left\{u^\dagger\left(\partial_\mu - ir_\mu\right)u - u\left(\partial_\mu - il_\mu\right)u^\dagger\right\}\\ &= i\left\{u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right\}\\ &= -\frac{\sqrt{2}}{F}\partial_\mu\Phi + O(\Phi^3)\\ &= -\frac{1}{F}\vec{\tau}\cdot\partial\vec{\phi} + O(\vec{\phi}^3)\end{aligned}$$

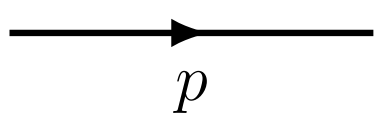
Meson-Baryon Lagrangian

✦ Feynman rule

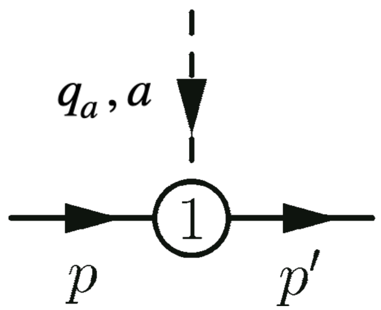
- **Rewrite Lagrangian**

$$\mathscr{L}^{(1)}_{\pi N} = \underbrace{\bar{\Psi}\left(i\gamma^\mu\partial_\mu - m\right)\Psi}_{0\phi 2N} + \frac{1}{4F^2}\underbrace{\bar{\Psi}\left(\gamma^\mu\partial_\mu\vec{\phi}\times\vec{\phi}\right)\cdot\vec{\tau}\Psi}_{2\phi 2N} - \frac{g}{2F}\underbrace{\bar{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\phi}\Psi}_{1\phi 2N} + \cdots$$

- **$O(p)$ Feynman rules**

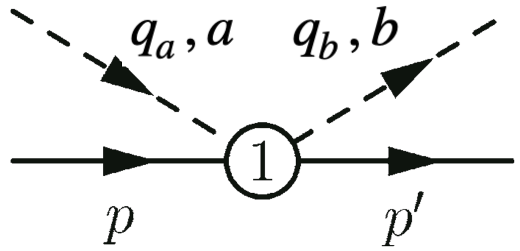


$$\frac{i}{\gamma^\mu p_\mu - m} \quad \sim \quad O(p^{-1})$$



$$-\frac{g}{2F}\gamma_\mu q_a^\mu \gamma_5 \tau_a \quad \sim \quad O(p^1)$$

- overall i
- Incoming $\partial_\mu \to -iq_\mu$



$$\frac{1}{4F^2}\epsilon_{abc}\tau_c\gamma_\mu\left(q_a^\mu + q_b^\mu\right) \quad \sim \quad O(p^1)$$

Goldberger-Treiman Relation

✦ Algebra calculation

- **Partially Conserved Axial Current (PCAC)**

$$\begin{cases} \partial_\mu V_a^\mu = i\bar{q}\left[\mathscr{M}, \frac{\tau_a}{2}\right] q & V_a^\mu = \bar{q}\gamma^\mu \frac{\tau_a}{2} q \\ \partial_\mu A_a^\mu = i\bar{q}\gamma_5 \left\{\frac{\tau_a}{2}, \mathscr{M}\right\} q & A_a^\mu = \bar{q}\gamma^\mu\gamma_5 \frac{\tau_a}{2} q \end{cases} \qquad a = 1,2,3$$

$SU(2)$ flavor symmetry is good

$$\mathscr{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} = \hat{m}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Longrightarrow \begin{cases} \partial_\mu V_a^\mu = 0 \\ \partial_\mu A^\mu = i\bar{q}\gamma_5\tau_a q = \hat{m}P(x) \end{cases}$$

$P(x)$ denotes pseudo scalar current

$$\Longrightarrow \left\langle N(p') \,|\, \partial_\mu A_a^\mu(x) \,|\, N(p) \right\rangle = \hat{m}\left\langle N(p') \,|\, P_a(x) \,|\, N(p) \right\rangle$$

Baryon to baryon transition amplitude

$$\Longrightarrow \partial_\mu \left\langle N(p') \,|\, e^{i\hat{p}\cdot x} A_a^\mu(0) e^{-i\hat{p}\cdot x} \,|\, N(p) \right\rangle = \hat{m}\left\langle N(p') \,|\, e^{i\hat{p}\cdot x} P_a(0) e^{-i\hat{p}\cdot x} \,|\, N(p) \right\rangle$$

$$\Longrightarrow \partial_\mu \left[e^{i(p'-p)\cdot x} \left\langle N(p') \,|\, A_a^\mu(0) \,|\, N(p) \right\rangle \right] = \hat{m} e^{i(p'-p)\cdot x} \left\langle N(p') \,|\, P_a(0) \,|\, N(p) \right\rangle$$

$$\overset{q^\mu = (p'-p)^\mu}{\Longrightarrow} iq_\mu \left\langle N(p') \,|\, A_a^\mu(0) \,|\, N(p) \right\rangle = \hat{m}\left\langle N(p') \,|\, P_a(0) \,|\, N(p) \right\rangle$$

Goldberger-Treiman Relation

✦ Algebra calculation

- **Form factors**

$$\left\langle N(p')\,|\,A_a^\mu(0)\,|\,N(p)\right\rangle = \bar{u}(p')\left[\gamma^\mu G_A(t) + \frac{(p'-p)^\mu}{2m_N}G_P(t)\right]\gamma_5\frac{\tau_a}{2}u(p)$$

$$\hat{m}\left\langle N(p')\,|\,P_a(0)\,|\,N(p)\right\rangle = \frac{M_\pi^2 F_\pi}{M_\pi^2 - t}G_{\pi N}(t)i\bar{u}(p')\gamma_5\tau_a u(p) \qquad (t = q^2 = (p'-p)^2)$$

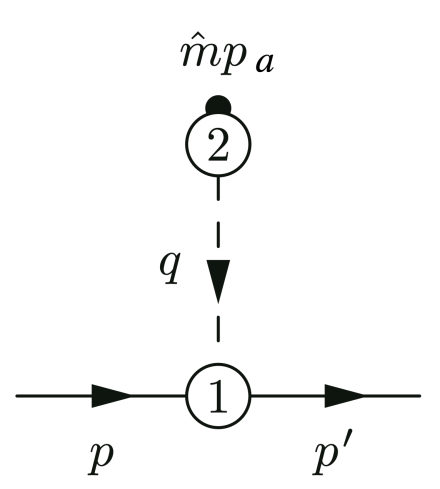
- **General relation**

$$2m_N G_A(t) + \frac{t}{2m_N}G_P(t) = 2\frac{M_\pi^2 F_\pi}{M_\pi^2 - t}G_{\pi N}(t)$$

- **GT relation** $t = 0$

$$G_A(t=0) = g_A \qquad G_{\pi N}(t=0) = g_{\pi NN} \qquad m_N g_A = F_\pi g_{\pi NN}$$

✦ ChPT calculation



$$\hat{m}2BF_\pi\frac{i}{t - M_\pi^2}\bar{u}(p')\left\{-\frac{1}{2}\frac{g_A}{F}\gamma^\mu q_\mu\gamma_5\tau_a\right\}u(p)$$

$$= M_\pi^2 F_\pi\frac{mg_A}{F}\frac{1}{M_\pi^2 - t}\bar{u}(p')\gamma_5 i\tau_a u(p)$$

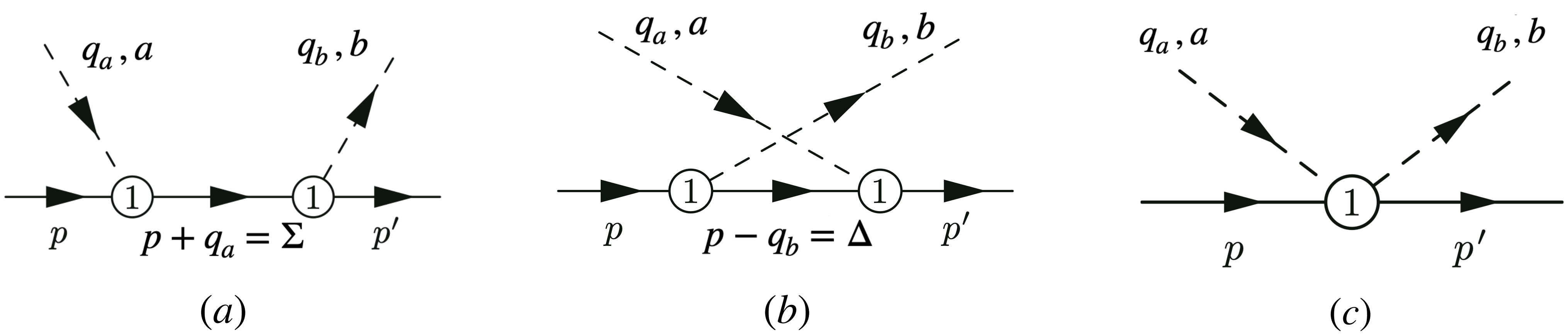
The lowest-order prediction

$$G_{\pi N}(t) = \frac{m}{F}g_A \qquad g_{\pi NN} = \frac{m_N}{F_\pi}g_A$$

Pion-Nucleon Scattering

✦ Leading-order calculation

- **Tree diagrams**



- **Tree amplitudes**

$$i\mathcal{M}_a = \bar{u}(p')\left\{\left[-\frac{g}{2F}\left(-\not{q}_b\right)\gamma_5\tau_b\right]\frac{i}{\not{p}+\not{q}_a-m}\left[-\frac{g}{2F}\left(\not{q}_a\right)\gamma_5\tau_a\right]\right\}u(p)$$

$$i\mathcal{M}_b = \bar{u}(p')\left\{\left[-\frac{g}{2F}\left(\not{q}_a\right)\gamma_5\tau_a\right]\frac{i}{\not{p}-\not{q}_b-m}\left[-\frac{g}{2F}\left(-\not{q}_b\right)\gamma_5\tau_b\right]\right\}u(p)$$

$$i\mathcal{M}_c = \bar{u}(p')\left\{\frac{1}{4F^2}\epsilon_{abc}\tau_c\left(\not{q}_a+\not{q}_b\right)\right\}u(p)$$

Pion-Nucleon Scattering

✦ Leading-order calculation

- **Mandelstam variables**

$$s=(p+q_a)^2 \qquad t=(p-p')^2 \qquad u=(p-q_b)^2$$

- **Independent scalar kinematical variables**

$$\nu=\frac{s-u}{4m_N} \qquad \nu_B=-\frac{q_a\cdot q_b}{2m_N}=\frac{t-2M_\pi^2}{4m_N}$$

- **The simplified amplitude**

$$T_{(a)}^{ab}=\frac{g_A}{4F_\pi^2}\bar{u}(p')\left\{2m_N+\frac{1}{2}\left(\not{q}_a+\not{q}_b\right)\left(-1-\frac{2m_N}{\nu-\nu_B}\right)\right\}\tau_b\tau_a u(p)$$

$$T_{(b)}^{ab}=\frac{g_A}{4F_\pi^2}\bar{u}(p')\left\{2m_N+\frac{1}{2}\left(\not{q}_a+\not{q}_b\right)\left(1-\frac{2m_N}{\nu+\nu_B}\right)\right\}\tau_a\tau_b u(p)$$

$$T_{(c)}^{ab}=\frac{1}{4F_\pi^2}\bar{u}(p')\left\{\left(\not{q}_a+\not{q}_b\right)(-i\epsilon_{abc}\tau_c)\right\}u(p)$$

Pion-Nucleon Scattering

✦ Leading-order calculation

- **Amplitude analysis**

1. **Isospin decomposition** $\left(\tau_a\tau_b = \delta_{ab} + i\epsilon_{abc}\tau_c = \frac{1}{2}\left[\tau_a\,,\tau_b\right] + \frac{1}{2}\left\{\tau_a\,,\tau_b\right\}\right)$

$$
\begin{aligned}
T^{ab} &= \frac{1}{2}\left\{\tau^b\,,\tau^a\right\}T^+ + \frac{1}{2}\left[\tau^b\,,\tau^a\right]T^- \\
&= \delta^{ab}T^+ - i\epsilon^{abc}\tau^c T^-
\end{aligned}
$$

2. **Lorentz decomposition**

$$
T^{\pm} = \bar{u}(p')\left[A^{\pm}(\nu\,,\nu_B) + \frac{1}{2}\left(\not{q}_a + \not{q}_b\right)B^{\pm}(\nu\,,\nu_B)\right]u(p)
$$

3. **Relation between** $\left\{T^+\,,T^-\right\}$ **and** $\left\{T^{1/2}\,,T^{3/2}\right\}$

$$
\begin{cases}
T^{\frac{1}{2}} = T^+ + 2T^- \\
T^{\frac{3}{2}} = T^+ - T^-
\end{cases}
$$

Ten physical processes :

e.g. $T_{\pi^+p\to\pi^+p} = T^{\frac{3}{2}}$

…

4. **A, B functions**

$$
A^+ = A^+_{(a)} + A^+_{(b)} + A^+_{(c)} = \frac{g_A^2 m_N}{F_\pi^2}
$$

$$
A^- = -\frac{g_A^2}{F_\pi^2}\frac{m_N\nu}{\nu^2 - \nu_B^2}
$$

$$
B^+ = 0
$$

$$
B^- = \frac{1-g_A^2}{2F_\pi^2} - \frac{g_A^2}{F_\pi^2}\frac{m_N\nu}{\nu^2 - \nu_B^2}
$$

Pion-Nucleon Scattering

✦ Leading-order calculation

- S-wave scattering lengths

$$a_{0+}^{\pm} \equiv \frac{1}{8\pi(m_N + M_\pi)} T^{\pm}\big|_{\rm thr} = \frac{1}{4\pi(1+\mu)}\left[A^{\pm} + M_\pi B^{\pm}\right]_{\rm thr}$$

- **ChPT results of $O(p)$**

$$a_{0+}^{-} = \frac{M_\pi}{8\pi(1+\mu)F_\pi^2}\left(1 + \frac{g_A^2\mu^2}{4}\frac{1}{1-\frac{\mu^2}{4}}\right) = \frac{M_\pi}{8\pi(1+\mu)F_\pi^2}\left[1 + O(p^2)\right]$$

$$a_{0+}^{+} = -\frac{g_A^2 M_\pi}{16\pi(1+\mu)F_\pi^2}\frac{\mu}{1-\frac{\mu^2}{4}} = O(p^2)$$

- **Weinberg-Tomozawa relation**

Taking the linear combination $a^{\frac{1}{2}} = a_{0+}^{+} + 2a_{0+}^{-}$ **and** $a^{\frac{3}{2}} = a_{0+}^{+} - a_{0+}^{-}$ **the results satisfy**

$$a_{0+}^{I} = -\frac{M_\pi}{8\pi(1+\mu)F_\pi^2}\left[I(I+1) - \frac{3}{4} - 2\right]$$

Part IV:

Renormalization and Power Counting

Power counting of a given diagram

✦ Power counting rule

$$\frac{1}{i}\int\frac{d^4k}{(2\pi)^4} \qquad \frac{1}{k^2-m_\phi^2} \qquad \frac{1}{k\cdot\gamma-m_B}$$

$$D = 4N_L - 2I_\phi - I_B + \sum_{k=1}^{\infty}(2j)\,N_\phi^{(2j)} + \sum_{k=1}^{\infty}kN_B^{(k)}$$

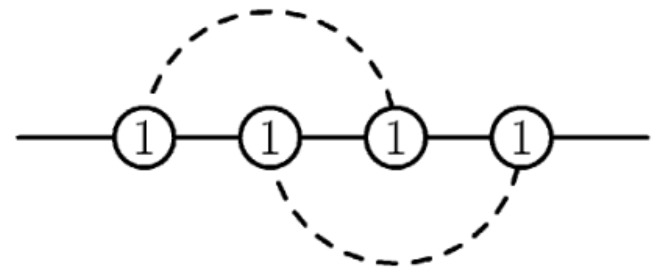
# of loops

# of internal mesons

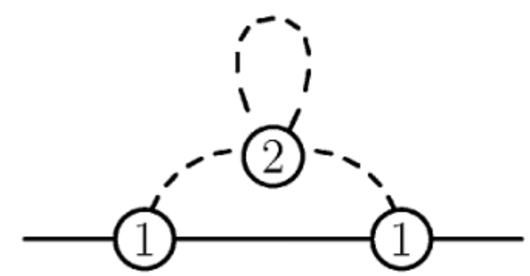
# of internal baryons

of $O(p^{2j})$ vertices

of $O(p^k)$ vertices



$$D = 4\cdot 2 - 2\cdot 2 - 2\cdot 1 + 0 + 3\cdot 1 = 5$$



$$D = 4\cdot 2 - 2\cdot 3 - 1\cdot 1 + 1\cdot 2 + 2\cdot 1 = 5$$

$$\mathscr{L}_{\rm GB} = \mathscr{L}_{\rm GB}^{(2)} + \mathscr{L}_{\rm GB}^{(4)} + \mathscr{L}_{\rm GB}^{(6)} + \cdots$$

$$\mathscr{L}_{\rm B\phi} = \mathscr{L}_{\rm B\phi}^{(1)} + \mathscr{L}_{\rm B\phi}^{(2)} + \mathscr{L}_{\rm B\phi}^{(3)} + \cdots$$

Topology relation

$$N_L = I_\phi + I_B - N_\phi - N_B + 1$$

$$D = 2N_L + I_B + \sum_{k=1}^{\infty}(2j)\,N_\phi^{(2j-2)} + \sum_{k=1}^{\infty}(k-2)N_B^{(k)}$$

Single baryon $\quad N_B = I_B + 1$

$$D = 1 + 2N_L + \sum_{j=1}^{\infty}2(j-1)\,N_\phi^{(2j)} + \sum_{k=1}^{\infty}(k-1)N_B^{(k)}$$

Power counting of a given diagram

✦ Power counting rule

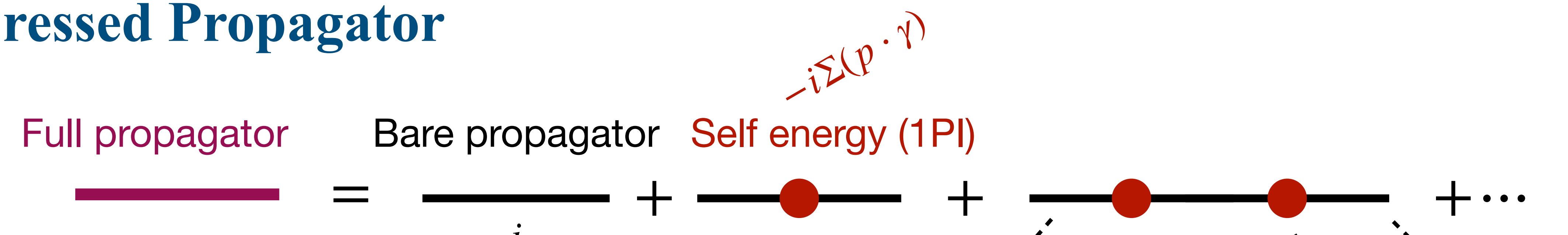
$$D = 1 + 2N_L + \sum_{j=1}^{\infty} 2(j-1)\, N_\phi^{(2j)} + \sum_{k=1}^{\infty} (k-1) N_B^{(k)}$$

- **All possible diagrams up to $O(p^3)$**

	# of loops	# of vertices				
	N_L	$N_\phi^{(2)}$	$N_\phi^{(4)}$	$N_B^{(1)}$	$N_B^{(2)}$	$N_B^{(3)}$
D=1	0 (tree)	Arbitrary	0	Arbitrary	0	0
D=2	0 (tree)	Arbitrary	0	Arbitrary	1	0
D=3	0 (tree)	Arbitrary	1	Arbitrary	0	0
		Arbitrary	0	Arbitrary	2	0
		Arbitrary	0	Arbitrary	0	1
	1 (one-loop)	Arbitrary	0	Arbitrary	0	0

The nucleon mass at one-loop order

✦ Dressed Propagator



$$i\,S(p) \qquad \frac{i}{p\cdot\gamma - m}$$

$$i\,S(p) = \frac{i}{p\cdot\gamma-m} + \frac{i}{p\cdot\gamma-m}\left[-i\Sigma(p\cdot\gamma)\right]\frac{i}{p\cdot\gamma-m} + \frac{i}{p\cdot\gamma-m}\left[-i\Sigma(p\cdot\gamma)\right]\frac{i}{p\cdot\gamma-m}\left[-i\Sigma(p\cdot\gamma)\right]\frac{i}{p\cdot\gamma-m} + \cdots$$

$$= \frac{i}{p\cdot\gamma-m}\left\{1 + \frac{\Sigma(p\cdot\gamma)}{p\cdot\gamma-m} + \left[\frac{\Sigma(p\cdot\gamma)}{p\cdot\gamma-m}\right]^2 + \cdots\right\}$$

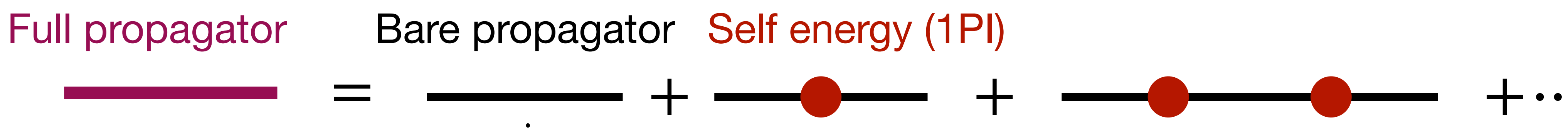
$$= \frac{i}{p\cdot\gamma-m}\,\frac{1}{1-\frac{\Sigma(p\cdot\gamma)}{p\cdot\gamma-m}}$$

$$= \frac{i}{p\cdot\gamma-m-\Sigma(p\cdot\gamma)}$$

One can also regard the SE as perturbative correction!

The nucleon mass at one-loop order

✦ Physical mass is the pole of the dressed propagator



$i\,S(p)$ $\qquad$ $\dfrac{i}{p\cdot\gamma - m}$

$$
\begin{aligned}
i\,S(p) &= \frac{i}{p\cdot\gamma - m - \Sigma(p\cdot\gamma)} \\
&= \frac{i}{p\cdot\gamma - m - \left[\Sigma(m_N) + (p\cdot\gamma - m_N)\Sigma'(p\cdot\gamma)\Big|_{p\cdot\gamma=m_N}\right]} \\
&= \frac{i}{p\cdot\gamma - \underbrace{\left[m + \Sigma(m_N)\right]}_{m_N} + (p\cdot\gamma - m_N)\Sigma'(p\cdot\gamma)\Big|_{p\cdot\gamma=m_N}} \\
&= \frac{i}{(p\cdot\gamma - m_N)\left[1 + \Sigma'(p\cdot\gamma)\,|_{p\cdot\gamma=m_N}\right]} \\
&= \frac{iZ_N}{p\cdot\gamma - m_N}
\end{aligned}
$$

- Physical nucleon mass

$$m_N = m + \Sigma(m_N)$$

- Wave function renormalization constant

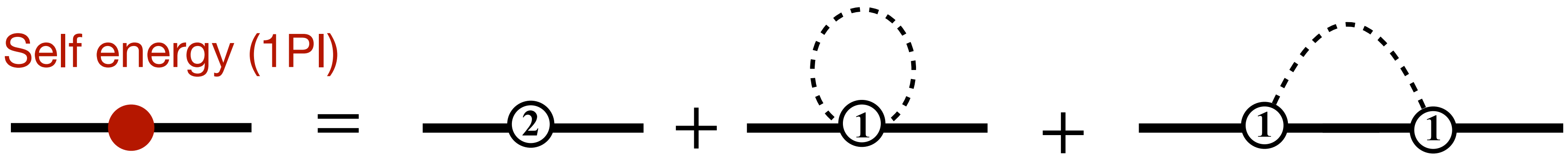
$$
\begin{aligned}
Z_N &= \left[1 + \Sigma'(p\cdot\gamma)\Big|_{p\cdot\gamma=m_N}\right]^{-1} \\
&= 1 - \Sigma'(p\cdot\gamma)\Big|_{p\cdot\gamma=m_N}
\end{aligned}
$$

The remaining task is to derive the SE in ChPT!

The nucleon mass at one-loop order

✦ Nucleon self energy in ChPT up to $O(p^3)$

Self energy (1PI)



(a) (b) (c)

Up to $O(p^3)$ $\quad$ $O(p^2)$, $D=2$ $\quad$ $O(p^3)$, $D=4-2+1=3$ $\quad$ $O(p^3)$, $D=4-2-1+2=3$

- **Diagram (a):** $$-i\Sigma_a(\hat{p}) = 4ic_1m_\pi^2$$

Antisymmetric vs symmetric

- **Diagram (b):** $$-i\Sigma_b(\hat{p}) = \int \frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{4F^2}\epsilon_{abc}\tau_c\left[\hat{k}-(-\hat{k})\right]\frac{i\delta_{ab}}{k^2-m_\pi^2} \quad = 0$$

- **Diagram (c):** $$-i\Sigma_c(\hat{p}) = \int \frac{\mathrm{d}^dk}{(2\pi)^d}\left[-\frac{g}{2F}(-\hat{k})\gamma^5\tau_b\right]\frac{i}{(\hat{p}+\hat{k})-m}\frac{i\delta_{ab}}{k^2-m_\pi^2}\left[-\frac{g}{2F}\hat{k}\gamma^5\tau_a\right]$$

Isospin $\tau_b\tau_a\delta_{ab}=3$ $\longrightarrow$ $$=\frac{3g^2}{4F^2}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{\hat{k}\gamma_5[(\hat{p}+\hat{k})+m]\hat{k}\gamma_5}{[(p+k)^2-m^2][k^2-m_\pi^2]}$$

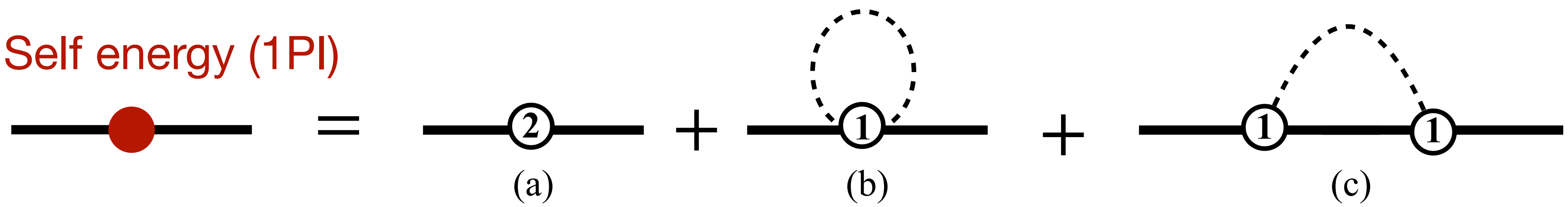
Dirac

$$\text{numerator} = \hat{k}\gamma_5[(\hat{p}+\hat{k})+m]\hat{k}\gamma_5 = -(\hat{p}+m)[(k^2-m_\pi^2)+m_\pi^2] + \{[(k+p)^2-m^2]+(m^2-p^2)\}\hat{k}$$

The nucleon mass at one-loop order

✦ Nucleon self energy in ChPT up to $O(p^3)$

Self energy (1PI)



Up to $O(p^3)$ | $O(p^2)$, $D=2$ | $O(p^3)$, $D=4-2+1=3$ | $O(p^3)$, $D=4-2-1+2=3$

- **Diagram (c):** $$-i\Sigma_c(\hat{p})=\frac{3g^2}{4F^2}\left\{-(\hat{p}+m)\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{(p+k)^2-m^2}-(\hat{p}+m)m_\pi^2\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{[(p+k)^2-m^2][k^2-m_\pi^2]}\right.$$ $$\left.+\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{\hat{k}}{k^2-m_\pi^2}+(m^2-p^2)\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{\hat{k}}{[(p+k)^2-m^2][k^2-m_\pi^2]}\right\}$$

➡ Definition of loop integrals

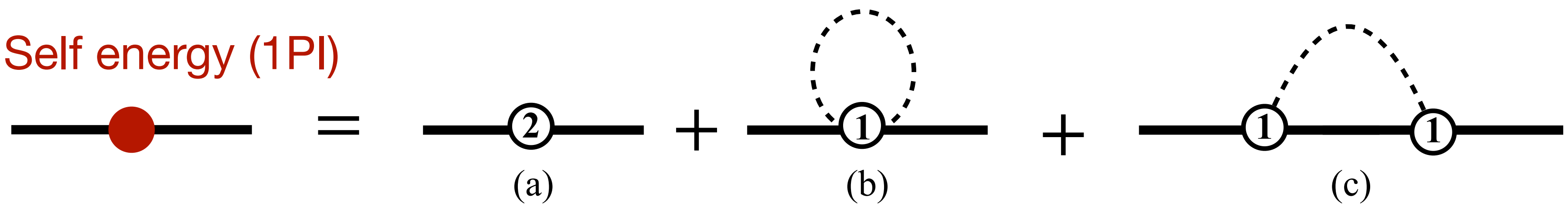
✓ Scalar integrals $$A_0(m_1^2)=\frac{1}{i}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{k^2-m_1^2}\qquad B_0(p^2,m_1^2,m_2^2)=\frac{1}{i}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{[k^2-m_1^2][(k+p)^2-m_2^2]}$$

✓ Tensor integrals $$B^\mu(p^2,m_1^2,m_2^2)=\frac{1}{i}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{k^\mu}{[k^2-m_1^2][(k+p)^2-m_2^2]}=p^\mu B_1(p^2,m_1^2,m_2^2)$$

numerator $=\hat{k}\gamma_5[(\hat{p}+\hat{k})+m]\hat{k}\gamma_5=-(\hat{p}+m)[(k^2-m_\pi^2)+m_\pi^2]+\{[(k+p)^2-m^2]+(m^2-p^2)\}\hat{k}$

The nucleon mass at one-loop order

✦ Nucleon self energy in ChPT up to $O(p^3)$



Up to $O(p^3)$ | (a) $O(p^2)$, $D=2$ | (b) $O(p^3)$, $D = 4-2+1 = 3$ | (c) $O(p^3)$, $D = 4-2-1+2 = 3$

- **Diagram (c):**

$$-i\Sigma_c(\hat{p}) = \frac{3g^2}{4F^2}\left\{ -(\hat{p}+m)\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{(p+k)^2-m^2} - (\hat{p}+m)m_\pi^2\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{[(p+k)^2-m^2][k^2-m_\pi^2]} \right.$$

$$\left. +\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{\hat{k}}{k^2-m_\pi^2} + (m^2-p^2)\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{\hat{k}}{[(p+k)^2-m^2][k^2-m_\pi^2]} \right\}$$

$$= \frac{3g^2}{4F^2}\left\{ -(\hat{p}+m)iA_0(m^2) - (\hat{p}+m)iB_0(p^2,m_\pi^2,m^2) + (m^2-p^2)i\hat{p}\,B_1(p^2,m_\pi^2,m^2) \right\}$$

✓ **Passarino-Veltman Reduction**

$$B_1(p^2,m_1^2,m_2^2) = \frac{1}{2p^2}\left\{ A_0(m_1^2) - A_0(m_2^2) + (m_2^2-m_1^2-p^2)B_0(p^2,m_1^2,m_2^2) \right\}$$

The self energy can be expressed in terms of scalar integrals eventually!

The nucleon mass at one-loop order

✦ The nucleon mass up to $O(p^3)$

$$
\begin{aligned}
m_N &= m + \Sigma(m_N) \\
&= m + \Sigma_a(m_N) + \Sigma_b(m_N) + \Sigma_c(m_N) \\
&= m - \underbrace{4c_1 m_\pi^2}_{O(p^2)} + \frac{3g^2 m_N}{2F^2}\underbrace{\left\{A_0(m_N^2) + m_\pi^2 B_0(m_N^2, m_\pi^2, m_N^2)\right\}}_{O(p^3)\text{ correction (naive power counting)}}
\end{aligned}
$$

- **Analytical expressions of scalar integrals**

✓**Dimensional regularisation (DR)** ✓$\widetilde{\mathrm{MS}} = \overline{\mathrm{MS}} - 1$ **subtraction scheme**

$$
A_0(m_1^2) = -\frac{m_N^2}{16\pi^2}\left\{R + \ln\frac{m_N^2}{\mu^2}\right\} \qquad R = \frac{2}{d-4} - [\ln(4\pi) - \gamma_E] - 1
$$

$$
B_0(p^2, m_\pi^2, m_N^2) = -\frac{1}{16\pi^2}\left\{R + \ln\frac{m_N^2}{\mu^2} - 1 + \frac{p^2 - m_N^2 - m_\pi^2}{p^2}\ln\frac{m_\pi}{m_N} + \frac{2m_N m_\pi}{p^2}F(\Omega)\right\}
$$

S. Scherer & M. R. Schindler, A primer for chiral perturbation theory

$$
F(\Omega) = \begin{cases}
\sqrt{\Omega^2 - 1}\,\ln(-\Omega - \sqrt{\Omega^2 - 1}) & \Omega \le -1 \\
\sqrt{\Omega^2 - 1}\,\arccos(-\Omega) & -1 \le \Omega \le 1 \\
\sqrt{\Omega^2 - 1}\,\ln(\Omega + \sqrt{\Omega^2 - 1}) - i\pi\sqrt{\Omega^2 - 1} & \Omega \ge 1
\end{cases}
\qquad \Omega \equiv \frac{p^2 - m_N^2 - m_\pi^2}{2m_N m_\pi}
$$

The nucleon mass at one-loop order

✦ The nucleon mass up to $O(p^3)$

$$m_N = m - 4c_1 m_\pi^2 + \frac{3g^2 m}{2F^2}\left\{ -\frac{m^2}{16\pi^2}\left[\underset{\uparrow\ \text{UV divergence}}{R} + \ln\frac{m^2}{\mu^2}\right] - \frac{m_\pi^2}{16\pi^2}\left[\underset{\uparrow\ \text{UV divergence}}{R} + \ln\frac{m^2}{\mu^2} - 1 - \frac{m_\pi^2}{m^2}\ln\frac{m_\pi}{m} + \frac{2m_\pi}{m}\sqrt{1-\frac{m_\pi^2}{4m^2}}\arccos\frac{m_\pi}{2m}\right]\right\}$$

- **UV renormalization/ cancellation**

$$m = m^r + \frac{\beta_m R}{16\pi^2 F^2} \xrightarrow{O(p^0)\ \text{UV div.}} \beta_m = \frac{3}{2}m^3 g^2$$

$$c_1 = c_1^r + \frac{\beta_{c_1} R}{16\pi^2 F^2} \xrightarrow{O(p^2)\ \text{UV div.}} \beta_{c_1} = -\frac{3}{8}m g^2$$

UV renormalized LECs are finite: m^r and c_1^r

- **Power counting breaking problem**

$$m_N = m^r - 4c_1^r m_\pi^2 - \frac{3g^2 m}{32\pi^2 F^2}\left\{ \boxed{\underbrace{m^2\ln\frac{m^2}{\mu^2}}_{O(p^0)} + \underbrace{m_\pi^2\left[\ln\frac{m^2}{\mu^2} - 1\right]}_{O(p^2)}} + \boxed{\underbrace{\frac{2m_\pi^3}{m}\sqrt{1-\frac{m_\pi^2}{4m^2}}\arccos\frac{m_\pi}{2m}}_{O(p^3)}} - \boxed{\underbrace{\frac{m_\pi^4}{m^2}\ln\frac{m_\pi}{m}}_{O(p^4)}} \right\}$$

PCB terms!!! **Naive power counting** **Higher order terms**

PCB problem & solutions

✦ The nucleon mass up to $O(p^3)$

$$m_N = m - 4c_1 m_\pi^2 + \frac{3g^2 m}{2F^2}\left\{ -\frac{m^2}{16\pi^2}\underbrace{\left[R + \ln\frac{m^2}{\mu^2}\right]}_{\text{UV divergence}} - \frac{m_\pi^2}{16\pi^2}\underbrace{\left[R + \ln\frac{m^2}{\mu^2} - 1 - \frac{m_\pi^2}{m^2}\ln\frac{m_\pi}{m} + \frac{2m_\pi}{m}\sqrt{1-\frac{m_\pi^2}{4m^2}}\arccos\frac{m_\pi}{2m}\right]}_{\text{UV divergence}}\right\}$$

- **UV renormalization/ cancellation**

$$m = m^r + \frac{\beta_m R}{16\pi^2 F^2} \xrightarrow{O(p^0)\ \text{UV div.}} \beta_m = \frac{3}{2}m^3 g^2$$

$$c_1 = c_1^r + \frac{\beta_{c_1} R}{16\pi^2 F^2} \xrightarrow{O(p^2)\ \text{UV div.}} \beta_{c_1} = -\frac{3}{8}m g^2$$

UV renormalized LECs are finite: m^r and c_1^r

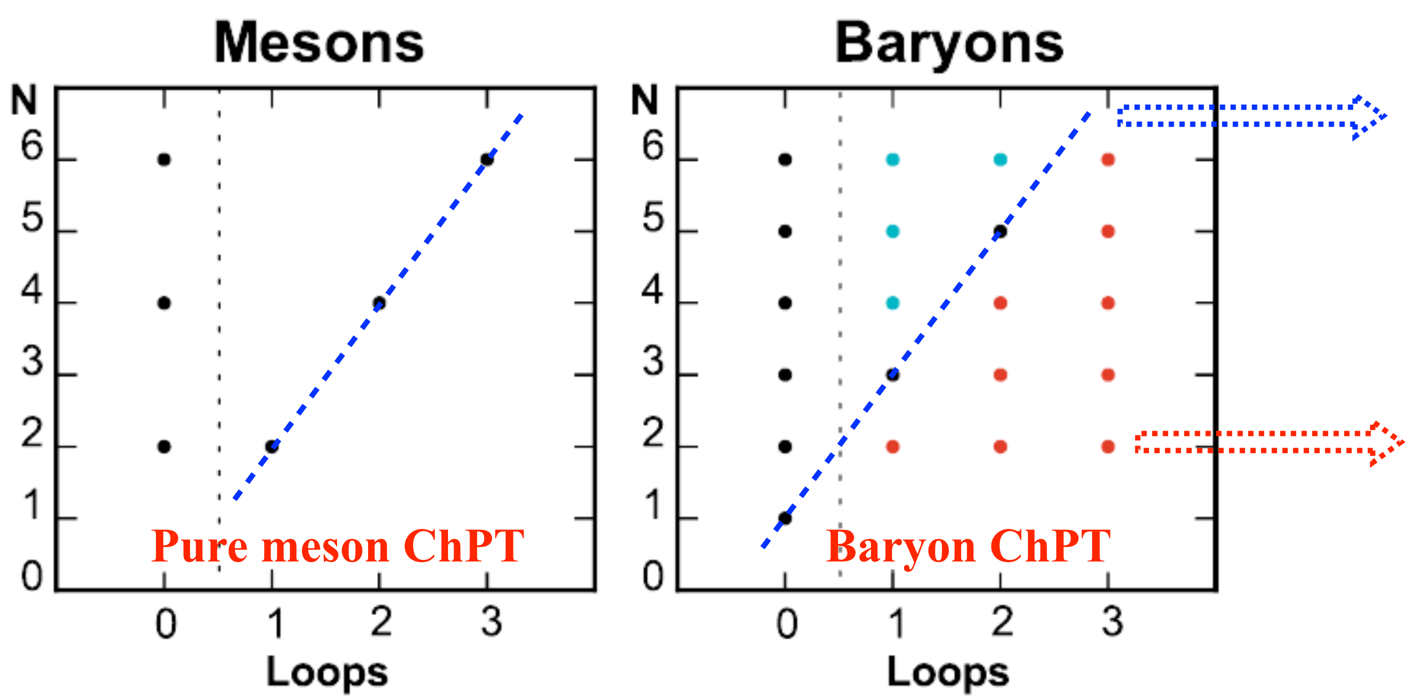
- **Power counting breaking problem**

$$m_N = m^r - 4c_1^r m_\pi^2 - \frac{3g^2 m}{32\pi^2 F^2}\left\{ \underbrace{\boxed{\underbrace{m^2\ln\frac{m^2}{\mu^2}}_{O(p^0)} + \underbrace{m_\pi^2\left[\ln\frac{m^2}{\mu^2} - 1\right]}_{O(p^2)}}}_{\text{PCB terms!!!}} + \underbrace{\boxed{\underbrace{\frac{2m_\pi^3}{m}\sqrt{1-\frac{m_\pi^2}{4m^2}}\arccos\frac{m_\pi}{2m}}_{O(p^3)}}}_{\text{Naive power counting}} - \underbrace{\boxed{\underbrace{\frac{m_\pi^4}{m^2}\ln\frac{m_\pi}{m}}_{O(p^4)}}}_{\text{Higher order terms}} \right\}$$

PCB problem & solutions

✦ PCB problem in covariant ChPT with matter fields

- **Dimensional Regularization (DR) with standard MSbar-1 subtraction (keep original symmetries)**
- **A systematic power counting rule is lost due to the non-zero mass of matter fields in the chiral limit**



Naive power counting rule !!!

red dots denote PCB terms!!!

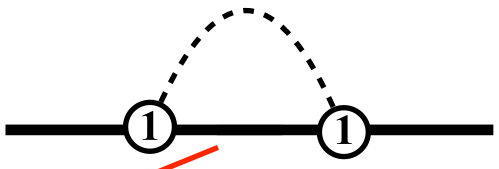
✦ Solutions

- **Heavy baryon approach (HBChPT)** [Jenkins and Manohar,PLB255'91]
- **Infrared regularisation** [T.Becher and H.Leutwyler,Eur.Phys.J.C9'99]
- **Extended on mass shell scheme** [T.Fuchs,J.Gegelia,G.Japaridze and S.Scherer,PRD68'03]

Heavy-Baryon Approach

✦ Taking the two-point scalar integral in the SE for example

$$B_0(p^2, m_\pi^2, m_N^2) = \frac{1}{i}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{[k^2-m_\pi^2][(k+p)^2-m_N^2]} \equiv \mathscr{H}$$



(c)

The origin of PCB terms

- **The baryon propagator**

$$G(\hat{k}+\hat{p}) = \frac{1}{(\hat{k}+\hat{p})-m_N}$$

k **is a small quantity and one can make an expansion**

$$= \frac{(\hat{k}+\hat{p})+m_N}{(\hat{k}+\hat{p})^2-m_N^2} = \frac{\hat{k}+\hat{p}+m_N}{2k\cdot p+p^2-m_N^2+k^2}$$

$\hat{p} \sim m_N \sim \mathcal{O}(\epsilon^0)$ $\qquad k \sim p^2 - m_N^2 \sim \mathcal{O}(\epsilon)$

$$= \frac{\hat{k}+\hat{p}+m_N}{(2k\cdot p+p^2-m_N^2)\left[1+\frac{k^2}{2k\cdot p+p^2-m_N^2}\right]} = \frac{\hat{k}+\hat{p}+m_N}{2k\cdot p+p^2-m_N^2}\left[1-\frac{k^2}{2k\cdot p+p^2-m_N^2}+\cdots\right]$$

$$= \frac{\hat{p}+m_N}{2k\cdot p+p^2-m_N^2} + \frac{1}{2k\cdot p+p^2-m_N^2}\left[\hat{k}-\frac{k^2(\hat{p}+m_N)}{2k\cdot p+p^2-m_N^2}\right]+\cdots$$

On shell

$\hat{p} = m(1,0,0,0)$
$= mv$

$$= \frac{1+\hat{v}}{2}\frac{1}{v\cdot k} + \cancel{\frac{1}{2v\cdot k}\frac{1}{m}\left[\hat{k}-\frac{1+\hat{v}}{2}\frac{k^2}{v\cdot k}\right]} + \cdots$$

$\sim k^{-1}(1/m_N)^0 \qquad \sim (k^0+k^1)(1/m_N)^1 +$

1. **A simultaneous expansion in k and $1/m_N$.**
2. **Hard scale m_N appears only in denominator.**
3. **Usually, only the leading term is kept.**

Heavy-Baryon Approach

✦ The two-point scalar integral in HB approach

DR

$$\frac{1}{i}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{[(\hat{k}+\hat{p})-m][k^2-m_\pi^2]}$$

Naive power counting ⟶ $D = 4 - 1 - 2 = 1$

Chiral expansion ↓ **1/m expansion**

Splitting of the external momentum

$$\underset{\textbf{Large}}{p^\mu = mv^\mu} + \underset{\textbf{Small}}{l^\mu}$$

HB

$$\frac{1+\hat{v}}{2}\frac{1}{i}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{[v\cdot k+v\cdot l][k^2-m_\pi^2]}$$

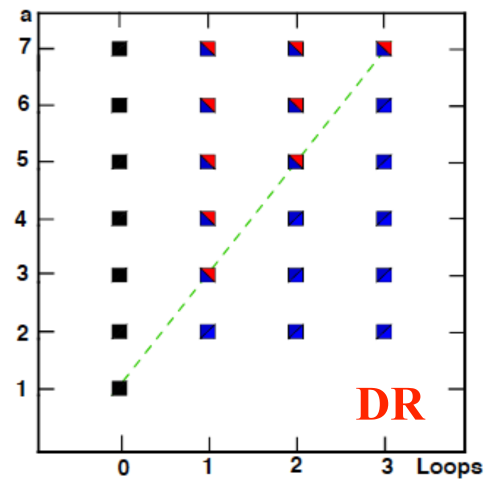
$$=\frac{1+\hat{v}}{2}\left\{4L\,v\cdot l-\frac{v\cdot l}{8\pi^2}\left[1-2\ln\frac{m_\pi}{\mu}\right]-\frac{1}{4\pi^2}\sqrt{m_\pi^2-(v\cdot l)^2}\arccos\frac{v\cdot l}{m_\pi}\right\}$$

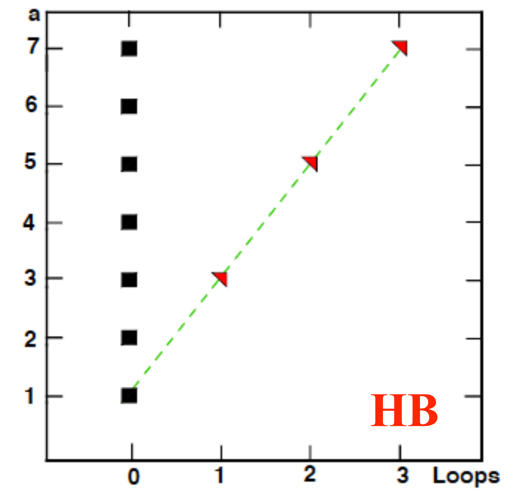
Obey naive power counting ⟶ $D = 1$

UV divergence $L = \dfrac{1}{32\pi^2}R$ [Benard, Kaiser, Meissner, Int. J. Mod. Phys.]

- **The nucleon mass in HB formalism**

$$m_N = m^r - \underbrace{4c_1 m_\pi^2}_{\text{diagram (a)}} - \underbrace{\frac{3g^2 m_\pi^3}{32\pi F^2}}_{\text{diagram (c)}}$$





Heavy-Baryon Approach

✦ Realization at the level of chiral effective Lagrangian (HBChPT)

Relativistic form **Decomposed into two velocity-dependent fields**

$$\mathscr{L}_B = \bar{B}\mathscr{D}^{ab}B \dashrightarrow \mathscr{L}_B = \bar{H}_v\mathscr{A}H_v + \bar{h}_v\mathscr{B}H_v + \bar{H}_v\mathscr{B}^\dagger h_v + \bar{h}_v\mathscr{C}h_v$$

$$B = e^{-imv\cdot x}[H_V + h_v]$$

Light component **Heavy component**

$$\hat{v}H_v = H_v$$
$$\hat{v}h_v = -h_v$$

Velocity eigenstates

- **LO Lagrangian for example**

$$\bar{B}(i\hat{v} - m)B = [\bar{H}_v + \bar{h}_v]e^{+imv\cdot x}(i\hat{\partial} - m)e^{-imv\cdot x}[H_v + h_v]$$

$$= \bar{H}_v(iv\cdot\partial)H_v + \bar{H}_v i\hat{\partial}^\perp h_v + \bar{h}_v i\hat{\partial}^\perp H_v - \bar{h}_v(iv\cdot\partial + 2m)h_v$$

$$\hat{\partial} = \hat{v}(v\cdot\partial) + \hat{\partial}^\perp$$

$\mathscr{A}$ $\mathscr{B}$ $\mathscr{C}$

✓ H_v **represents an (infinitely heavy) static source.**

✓ **The mass dependence resides entirely in new vertices which can be ordered according to their power in term of 1/m.**

Heavy-Baryon Approach

✦ Integrating out the heavy degrees of freedom

- **Path integral**

$\mathscr{L}_B$ **relativistic Lagrangian**

$$e^{i\mathcal{Z}} = N'\int \mathscr{D}U\, e^{i\int \mathrm{d}^4x\mathscr{L}_{\mathrm{GB}}} \int \mathscr{D}\bar{B}\mathscr{D}B\, e^{i\int \mathrm{d}x^4[\boxed{\bar{B}DB}+\bar{\eta}B+\bar{B}\eta]}$$

$$= N'\int \mathscr{D}U\, e^{i\int \mathrm{d}^4x\mathscr{L}_{\mathrm{GB}}} \int \mathscr{D}\bar{H}_v\mathscr{D}H_v\mathscr{D}\bar{h}_v\mathscr{D}h_v$$

$$\times\, e^{i\int \mathrm{d}x^4[\bar{H}_v\mathscr{A}H_v+\bar{h}_v\mathscr{B}H_v+\bar{H}_v\gamma_0\mathscr{B}^\dagger\gamma_0 h_v+\bar{h}_v\mathscr{C}h_v+\bar{\rho}H_v+\bar{H}_v\rho+\bar{\sigma}h_v+\bar{h}_v\sigma]}$$

$$= N''\int \mathscr{D}U\, e^{i\int \mathrm{d}^4x\mathscr{L}_{\mathrm{GB}}} \int \mathscr{D}\bar{H}_v\mathscr{D}H_v e^{i\int \mathrm{d}x^4\underline{\bar{H}_v[\mathscr{A}+(\gamma_0\mathscr{B}^\dagger\gamma_0)\mathscr{C}^{-1}\mathscr{B}]H_v}+\bar{\rho}H_v+\bar{H}_v\rho} \times \Delta_h$$

$\mathscr{L}_B$ **non-relativistic Lagrangian**

➡ **The determinant**

$$\Delta_h = \exp\{\frac{1}{2}\mathrm{tr}\ln\mathscr{C}\}$$

⟶ **a constant!**

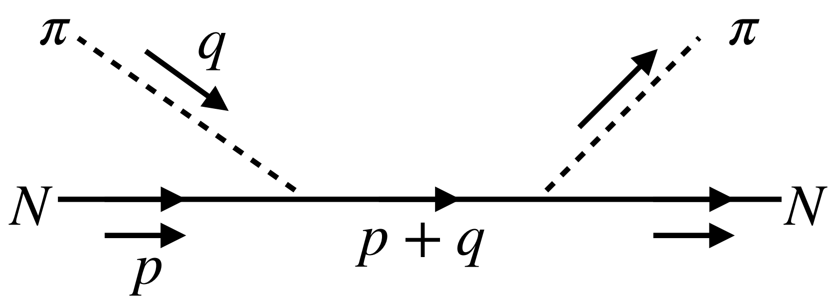
➡ $\mathscr{C}$ **contains the baryon mass,** $\mathscr{C}^{-1}$ **contains the inverse of baryon mass**

⟶ **the baryon mass only appears in the denominator!**

Heavy-Baryon Approach

✦ Shortcoming of HB formalism

- **The power counting is restored, however, at the price of losing the original analyticity!**
- **e.g. The pole in πN scattering amplitudes**



- **Relativistic formalism**

$$\textbf{Amp.} \propto \frac{1}{(p+q)^2 - m_N^2} = \frac{1}{2p\cdot q + m_\pi^2} \quad \Longrightarrow \quad \textbf{pole at } 2p\cdot q = -\, m_\pi^2$$

- **Non-relativistic formalism**

$$\textbf{Amp.} \propto \frac{1}{2p\cdot q + m_\pi^2} \xrightarrow{p^\mu = m_N v^\mu} \frac{1}{2m_N v\cdot q - m_\pi^2} = \frac{1}{2m_N}\frac{1}{v\cdot q}\left[1 - \frac{m_\pi^2}{2m_N v\cdot q} + \cdots\right]$$

pole at $2p\cdot q = 2m_N v\cdot q = 0$

Infrared Regularisation Prescription

✦ Essence of IR

- **Rewind of the two-point scalar integral**

$$\mathcal{H} = \frac{1}{i}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{[(k+p)^2 - m^2][k^2 - m_\pi^2]}$$

$$= \frac{1}{i}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\int_0^1 \mathrm{d}z \frac{1}{[(k+zp)^2 - \mathcal{M}^2]^2}$$

$$= \int_0^1 \mathrm{d}z \frac{1}{(4\pi)^{d/2}}(\mathcal{M}^2)^{\frac{d}{2}-2}\frac{\Gamma(\frac{d}{2})\Gamma(2-\frac{d}{2})}{\Gamma(\frac{d}{2})\Gamma(2)}$$

$$= \frac{1}{(4\pi)^{d/2}}\Gamma(2-\frac{d}{2})\int_0^1 \mathrm{d}z\ (\mathcal{M}^2)^{\frac{d}{2}-2}$$

$$= \frac{1}{(4\pi)^{d/2}} m^{d-4}\Gamma(2-\frac{d}{2})\int_0^1 \mathrm{d}z\ C^{\frac{d}{2}-2}$$

- **Feynman Parametrization**

$$\mathcal{M}^2 = z^2p^2 - z(p^2 - m^2) + (1-z)m_\pi^2$$

- **Master Integral of momentum integration**

$$\frac{1}{i}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{(k^2)^p}{(k^2 - \mathcal{M}^2 + i\epsilon)^q}$$

$$= (-1)^{p-q}\frac{1}{(4\pi)^{d/2}}(\mathcal{M}^2)^{p+\frac{d}{2}-q}\frac{\Gamma(p+\frac{d}{2})\Gamma(q-p-\frac{d}{2})}{\Gamma(\frac{d}{2})\Gamma(q)}$$

- **Nondimensionalization**

$$C = \frac{\mathcal{M}^2}{m^2} = z^2 - 2\alpha\Omega z(1-z) + \alpha^2(1-z)^2$$

$$\Omega = \frac{p^2 - m^2 - m_\pi^2}{2mm_\pi}$$

$O(p^0)$

$$\alpha = \frac{m_\pi}{m}$$

$O(p^1)$

Infrared Regularisation Prescription

✦ Infrared singular and regular parts

$$\mathcal{H} = \frac{1}{(4\pi)^{d/2}}\Gamma(2-\frac{d}{2})\int_0^1 \mathrm{d}z\ (\mathcal{M}^2)^{\frac{d}{2}-2}$$

$$\mathcal{M}^2 = z^2p^2 - z(p^2-m^2) + (1-z)m_\pi^2$$

$$= \frac{1}{(4\pi)^{d/2}}m^{d-4}\Gamma(2-\frac{d}{2})\int_0^1 \mathrm{d}z\ C^{\frac{d}{2}-z}$$

$$C = z^2 - 2\alpha\Omega z(1-z) + \alpha^2(1-z)^2$$

★**REMARKS:**

- **Infrared singularity**

$$\mathrm{d}z\ C^{\frac{d}{2}-2} \xrightarrow{m_\pi \to 0 \quad \alpha \to 0} \mathrm{d}z\ z^{d-4}$$

$d < 4$ **divergent**

- **Pure mesonic case (an overall rescaling of chiral expansion parameters can be done)**

$$(\mathcal{M}^2)^{\frac{d}{2}-2} \xrightarrow[p \to \epsilon p]{m \to \epsilon m \quad m_\pi \to \epsilon m_\pi} \epsilon^{d-4}(\mathcal{M}^2)^{\frac{d}{2}-2}$$

When $d < 4$, $\epsilon \to 0$, Infrared div.

- **Baryonic case (a rescaling of Feynman parameter)**

$$z = \alpha u \Longrightarrow \mathcal{H} = \kappa\alpha^{d-3}\int_0^{\frac{1}{\alpha}} \mathrm{d}u D^{\frac{d}{2}-2}$$

$$D = 1 - 2\Omega u + u^2 + 2\alpha u(\Omega u - 1) + \alpha^2 u^2$$

Infrared Regularisation Prescription

✦ Essence of IR

$$\mathscr{H} = \frac{1}{(4\pi)^{d/2}}\Gamma(2-\frac{d}{2})\int_0^1 \mathrm{d}z\ (\mathscr{M}^2)^{\frac{d}{2}-2}$$

$$= \frac{1}{(4\pi)^{d/2}} m^{d-4}\Gamma(2-\frac{d}{2})\int_0^1 \mathrm{d}z\ C^{\frac{d}{2}-z}$$

$$\mathscr{M}^2 = z^2p^2 - z(p^2-m^2) + (1-z)m_\pi^2$$

$$C = z^2 - 2\alpha\Omega z(1-z) + \alpha^2(1-z)^2$$

★REMARKS:

- **Infrared singularity**

$$\mathrm{d}z\ C^{\frac{d}{2}-2} \xrightarrow{m_\pi \to 0 \quad \alpha \to 0} \mathrm{d}z\ z^{d-4}$$

$d < 4$ **divergent**

- **Pure mesonic case (an overall rescaling of chiral expansion parameters can be done)**

$$(\mathscr{M}^2)^{\frac{d}{2}-2} \xrightarrow[p \to \epsilon p]{m \to \epsilon m \quad m_\pi \to \epsilon m_\pi} \epsilon^{d-4}(\mathscr{M}^2)^{\frac{d}{2}-2}$$

When $d < 4$, $\epsilon \to 0$, Infrared div.

- **Baryonic case (a rescaling of Feynman parameter)**

$$z = \alpha u \longrightarrow \mathscr{H} = \kappa\alpha^{d-3}\boxed{\int_0^{\frac{1}{\alpha}} \mathrm{d}u D^{\frac{d}{2}-2}}$$

$$D = 1 - 2\Omega u + u^2 + 2\alpha u(\Omega u - 1) + \alpha^2 u^2$$

Divergent if $d < 3$

No divergence

Infrared Regularisation Prescription

✦ Infrared singular and regular parts

$$\mathscr{H} = \kappa\alpha^{d-3}\int_0^{\frac{1}{\alpha}} \mathrm{d}u D^{\frac{d}{2}-2} \qquad\qquad D = 1 - 2\Omega u + u^2 + 2\alpha u(\Omega u - 1) + \alpha^2 u^2$$

- **Infrared singular part**

$$\mathscr{I} = \lim_{\alpha\to 0}\mathscr{H} = \kappa\alpha^{d-3}\int_0^{\infty}\mathrm{d}u D^{\frac{d}{2}-2} = \kappa\int_0^{\infty}\mathrm{d}z C^{\frac{d}{2}-2}$$

$$= \alpha^{d-3}\times\left[\text{Talor series of } \alpha\right]$$

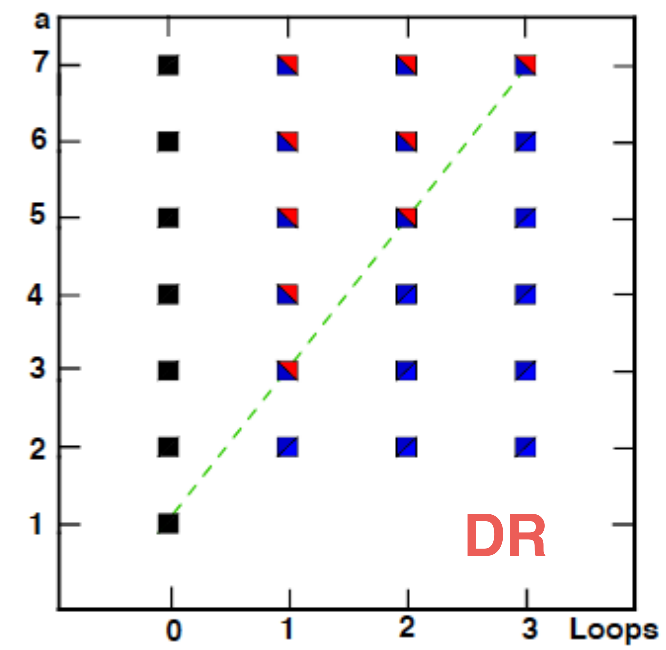
$$= c_1\alpha^{d-3} + c_2\alpha^{d-4} + c_3\alpha^{d-5} + \cdots$$ **No PCB!**

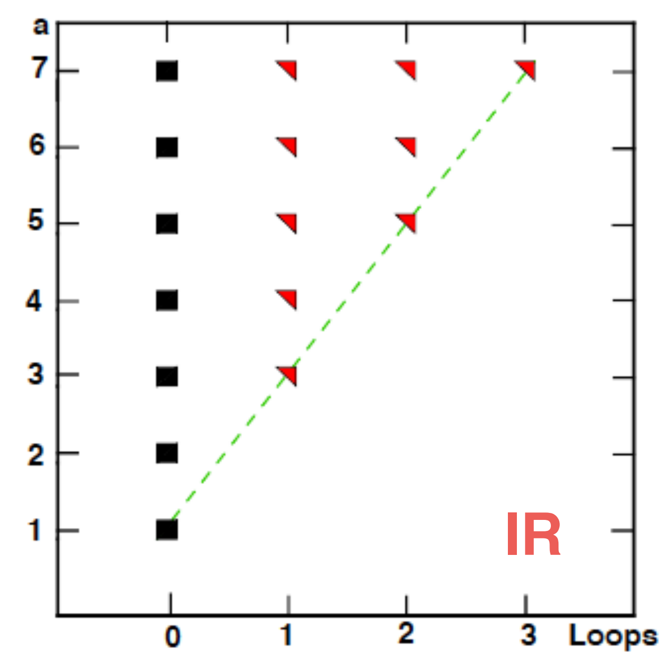
- **Infrared regular part**

$$\mathscr{R} = \mathscr{H} - \mathscr{I} = -\kappa\int_1^{\infty}\mathrm{d}z C^{\frac{d}{2}-2}$$

$$= \text{Talor series of } \alpha$$

$$= d_1\alpha^0 + d_2\alpha^1 + d_3\alpha^2 + \cdots$$ **Contains PCB!**





1. **The loop integrals can be decomposed into infrared singular and regular parts**
2. **The infrared singular part obeys PC, which regular part contains PCB.**
3. **IR prescription: drop the regular part.**

Extended on mass shell scheme

✦ Philosophy of EOMS

step 1:

Ultraviolet (UV) divergence

Dimensional regularization
MSbar-1 subtraction

finite ren. →

step 2:

PCB terms
(polynomials)

removed by redefinition of LECs
in Effective Lagrangians

Scale independent

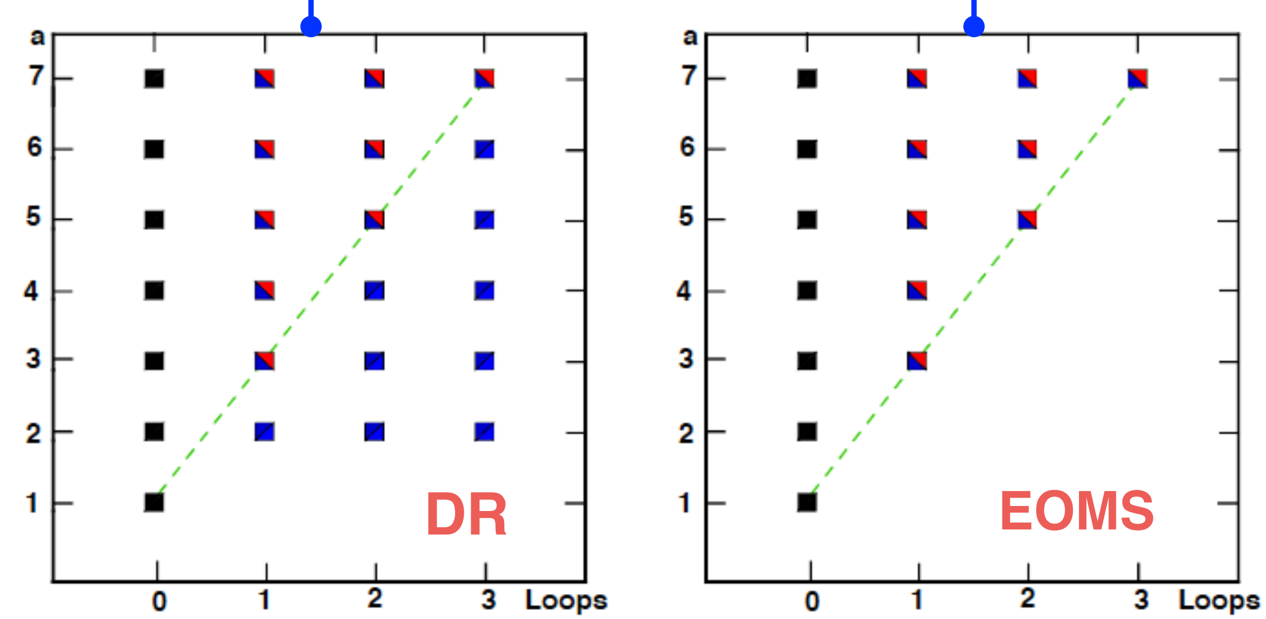
Correct power counting
(respectively faster convergence)

keep original analyticity
and all assumed symmetries

✦ Extraction of PCB terms

- **Method 1:**
  Feynman parametrization → Chiral expansion → Interchanging integration and summation

- **Method 2:**
  Chiral expansion →Interchanging momentum integration and summation



Extended on mass shell scheme

✦ A convenient way to derive the polynomial of regular part

- **Full integral**

$$\frac{\partial}{\partial p^2} = \frac{\partial p_\mu}{\partial p^2}\frac{\partial}{\partial p_\mu} = \frac{1}{2p^\mu}\frac{\partial}{\partial p_\mu} = \frac{1}{2p^2}p_\mu\frac{\partial}{\partial p_\mu}$$

$$\mathscr{H} = \frac{1}{i}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{[(k+p)^2-m^2][k^2-m_\pi^2]}$$

$$= \frac{1}{i}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\sum_{\ell,j=0}^{\infty}\left\{\frac{(p^2-m^2)^\ell(m_\pi^2)^j}{\ell!\,j!}\left[\left(\frac{1}{2p^2}p_\mu\frac{\partial}{\partial p_\mu}\right)^\ell\left(\frac{\partial}{\partial m_\pi^2}\right)^j\frac{1}{[(k+p)^2-m^2][k^2-m_\pi^2]}\right]_{p^2=m^2,m_\pi^2=0}\right.$$

Interchange the order of integration and summation

- **Regular part**

$$\mathscr{R} = \sum_{\ell,j=0}^{\infty}\frac{(p^2-m^2)^\ell(m_\pi^2)^j}{\ell!\,j!}\frac{1}{i}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\left[\left(\frac{1}{2p^2}p_\mu\frac{\partial}{\partial p_\mu}\right)^\ell\left(\frac{\partial}{\partial m_\pi^2}\right)^j\frac{1}{[(k+p)^2-m^2][k^2-m_\pi^2]}\right]_{p^2=m^2,m_\pi^2=0}$$

$$= \frac{1}{i}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{(k+2k\cdot p)k^2}\Bigg|_{p^2=m^2} + \mathscr{O}(p^1)$$

→ The origin of EOMS

$$= -\frac{1}{16\pi^2}\left[\ln\frac{m^2}{\mu^2}-1\right] + \mathscr{O}(p^1)$$

→ PCB term

Extended on mass shell scheme

✦ The nucleon mass in EOMS scheme

$$m_N = m^r - 4c_1^r m_\pi^2 - \frac{3g^2 m}{32\pi^2 F^2}\left\{ \underbrace{m^2 \ln\frac{m^2}{\mu^2}}_{O(p^0)} + \underbrace{m_\pi^2\left[\ln\frac{m^2}{\mu^2} - 1\right]}_{O(p^2)} + \underbrace{\frac{2m_\pi^3}{m}\sqrt{1-\frac{m_\pi^2}{4m^2}}\arccos\frac{m_\pi}{2m}}_{O(p^3)} - \underbrace{\frac{m_\pi^4}{m^2}\ln\frac{m_\pi}{m}}_{O(p^4)} \right\}$$

PCB terms!!! Naive power counting Higher order terms

- **Finite renormalization/ cancellation**

$$\begin{cases} m^r = \tilde{m} + \dfrac{\tilde{\beta}_m m}{16\pi^2 F^2} \\ c_1^r = \tilde{c}_1 + \dfrac{\tilde{\beta}_{c_1} m}{16\pi^2 F^2} \end{cases} \xrightarrow{\text{cancel PCB}} \begin{cases} \tilde{\beta}_m = \dfrac{3m^2 g^2}{2}\ln\dfrac{m^2}{\mu^2} \\ \tilde{\beta}_{c_1} = \dfrac{3}{8}g^2\left(1 - \ln\dfrac{m^2}{\mu^2}\right) \end{cases}$$

EOMS renormalized LECs:
$\widetilde{m}$ **and** $\widetilde{c}_1$

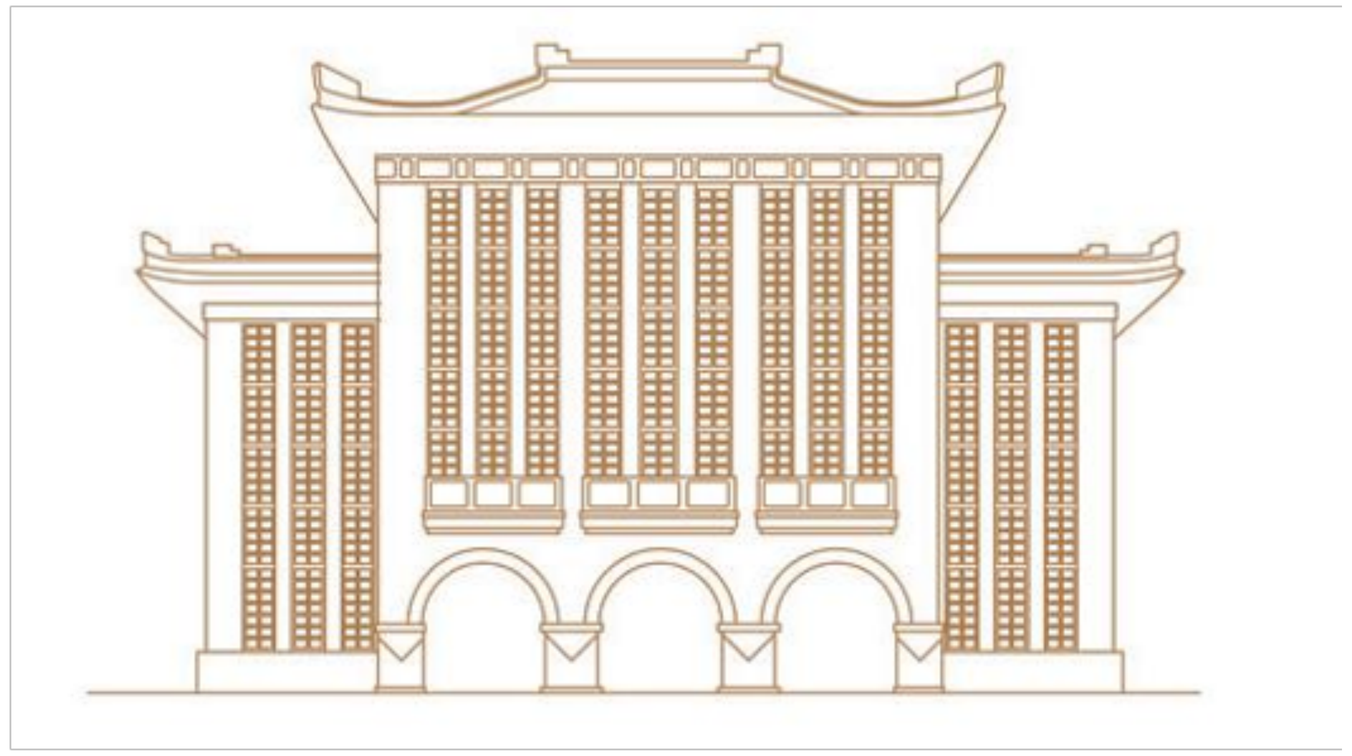
- **The final results**

$$m_N = \tilde{m} - 4\tilde{c}_1 m_\pi^2 - \frac{3g^2 m}{32\pi^2 F^2}\left\{ \frac{2m_\pi^3}{m}\sqrt{1-\frac{m_\pi^2}{4m^2}}\,\mathrm{arcos}\frac{m_\pi}{2m} - \frac{m_\pi^4}{m}\ln\frac{m_\pi}{m} \right\}$$

➡ **Correct power counting** ➡ **Original analyticity** ➡ **Better convergence**

\- **Amplitude analysis** - **Chiral extrapolation**

Thanks!



手征有效场论·系列报告3

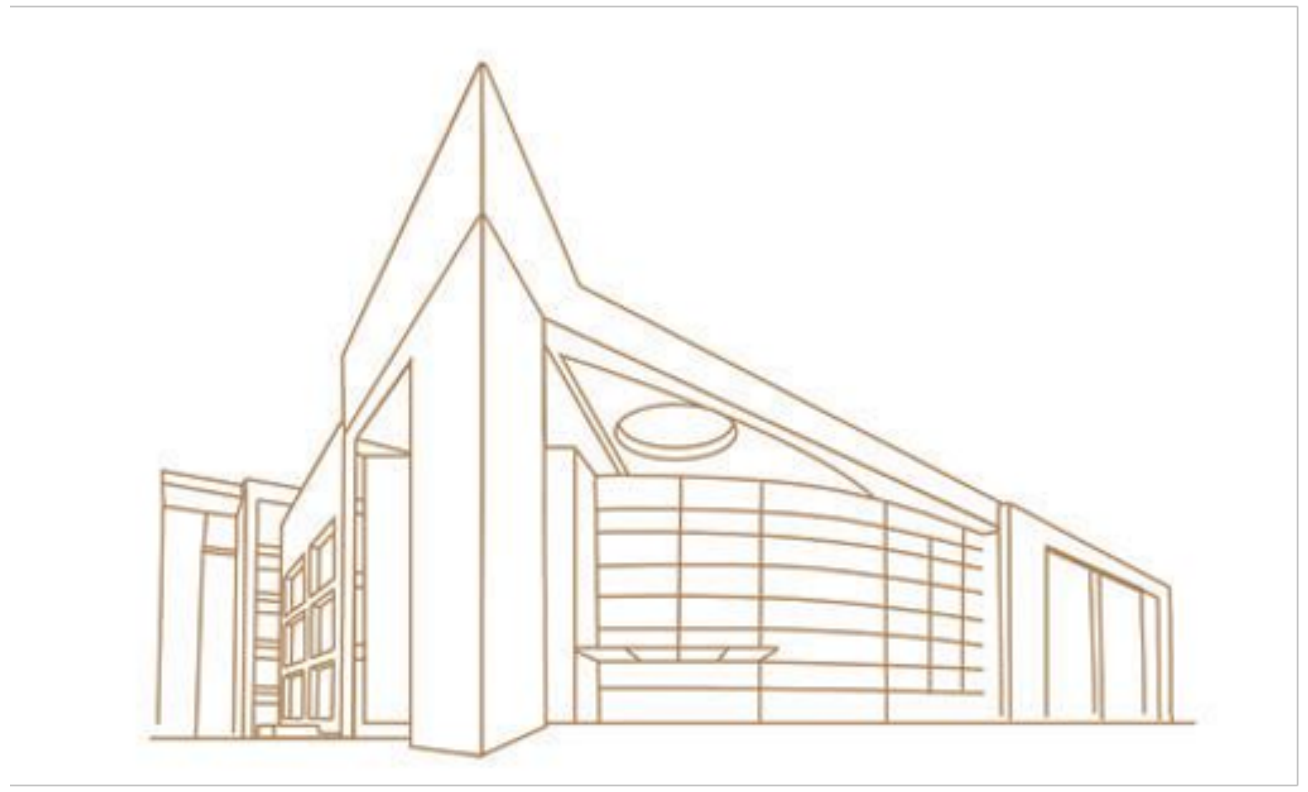
Chiral effective theories and their applications to low energy strong interactions

De-Liang Yao

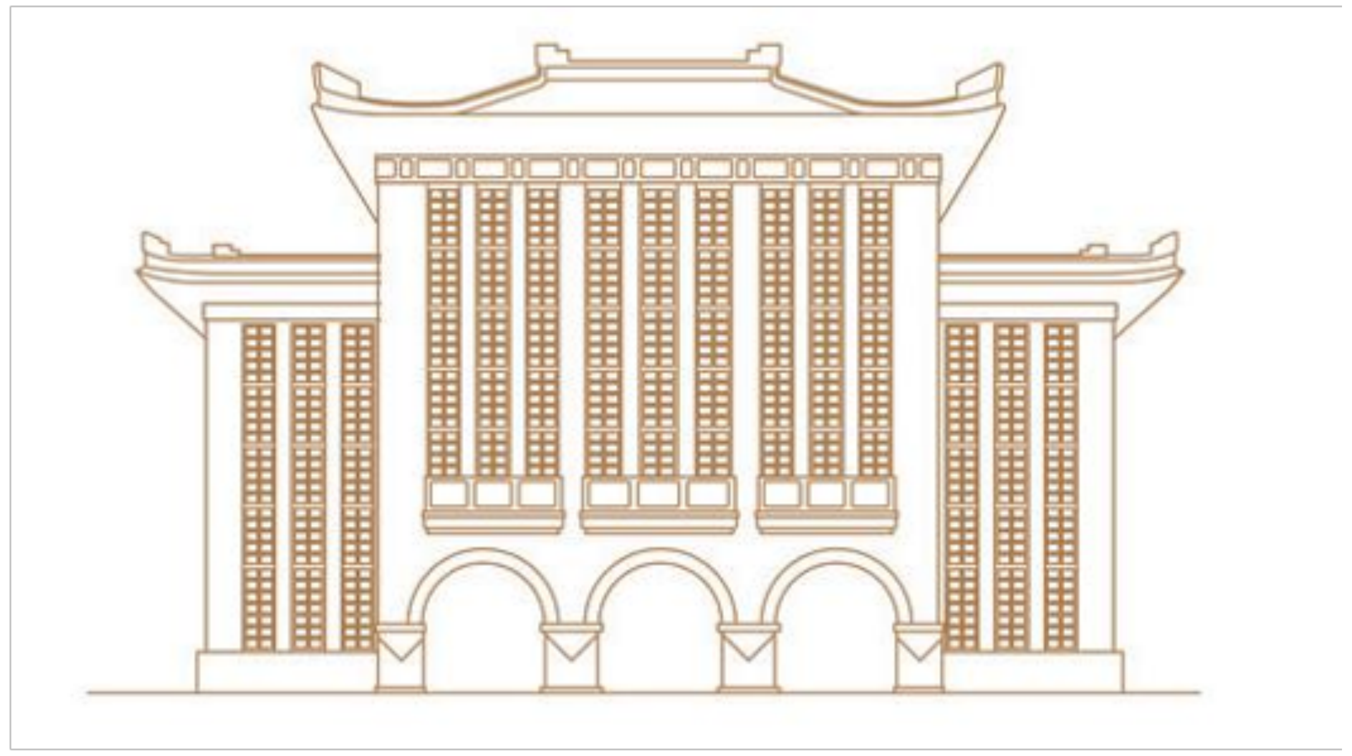
姚德良

Hunan University

Nov 26-27, 2023, Zhuhai



OUTLINE



1. Introduction
2. Progress of BChPT with EOMS scheme
3. Prospect of BChPT
4. A two-loop calculation of nucleon mass
5. Summary and outlook

ChPT: EFT of QCD at low energies

✦ Facets of QCD — Asymptotic Freedom & Color Confinement

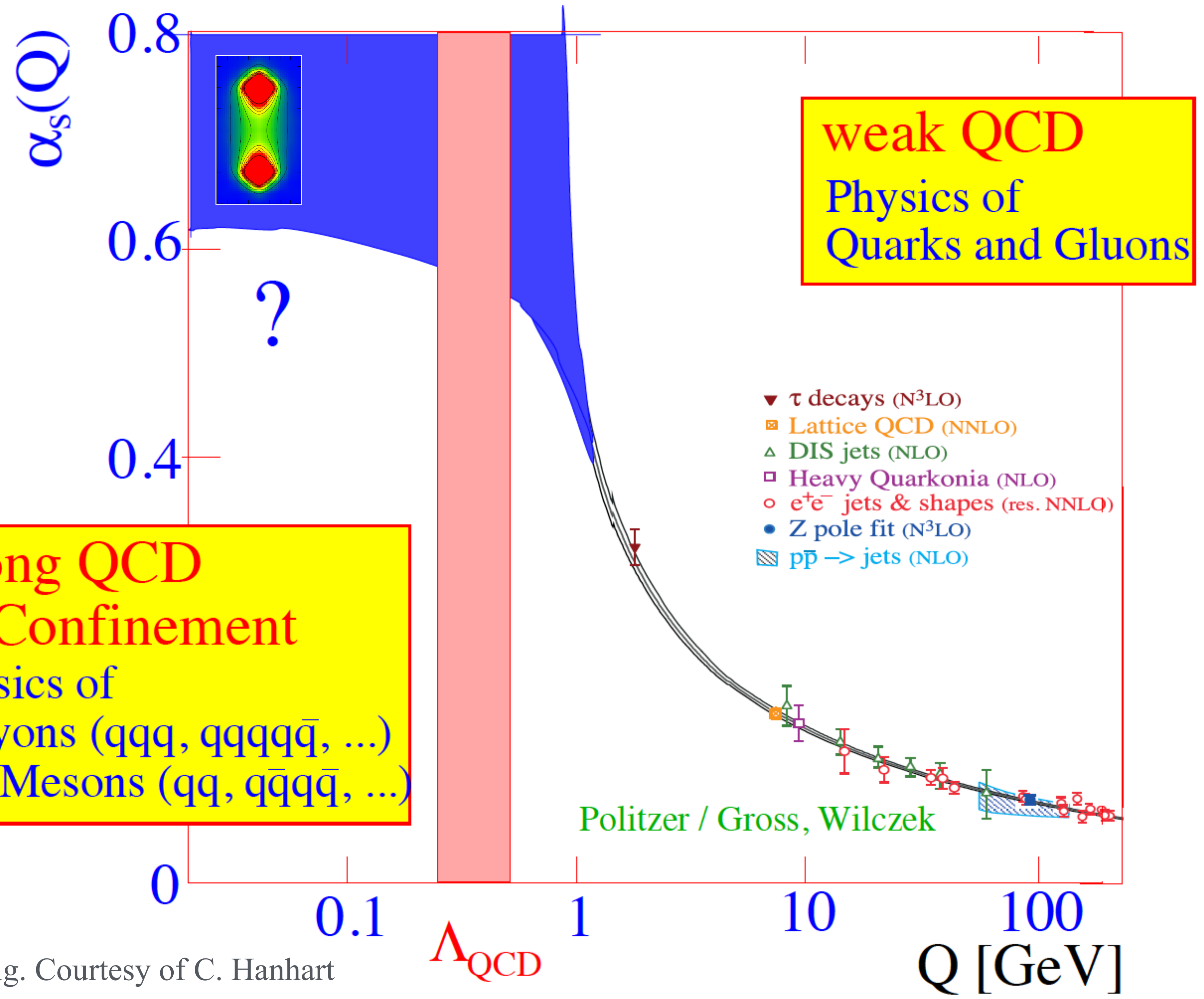
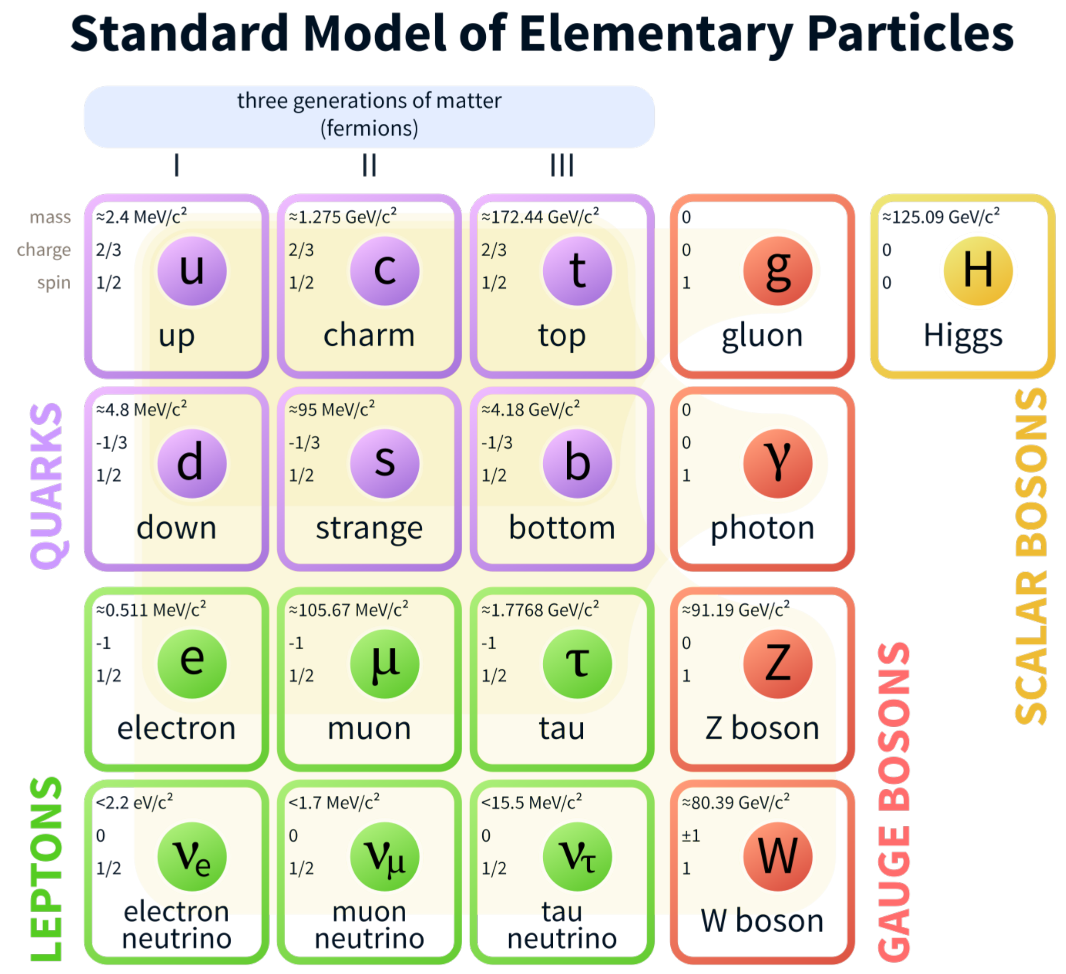
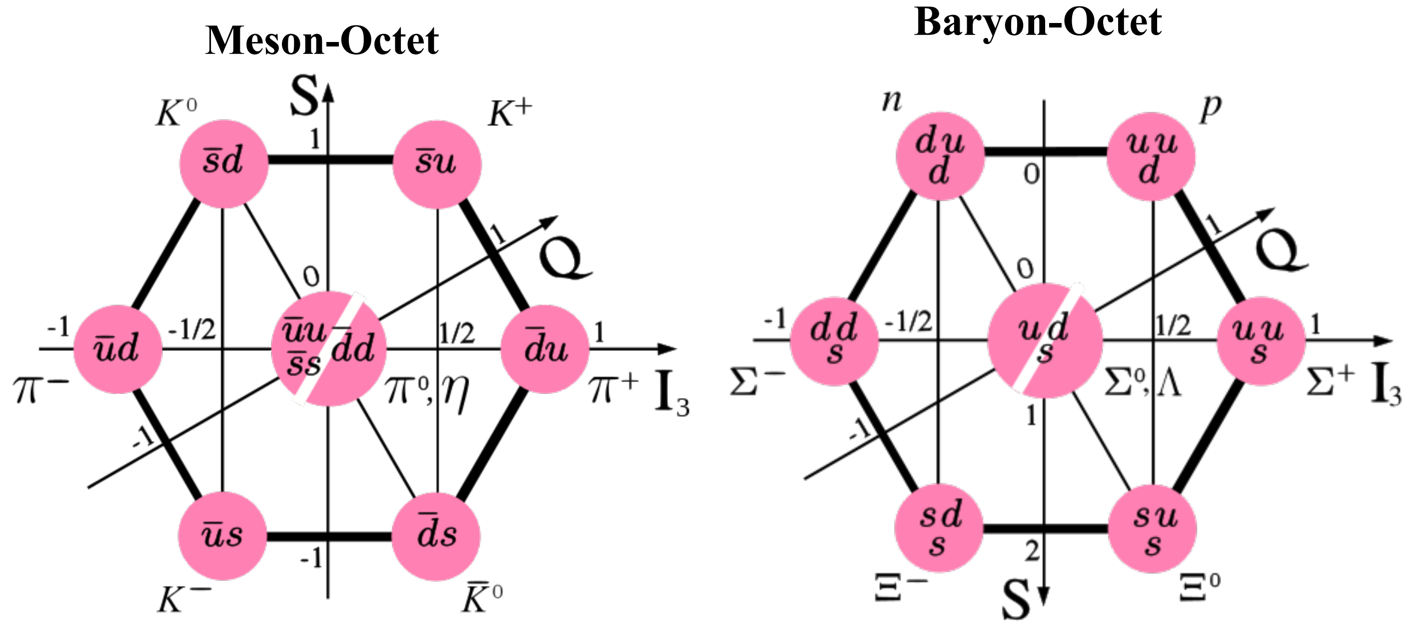


Fig. Courtesy of C. Hanhart

Quarks



Hadrons



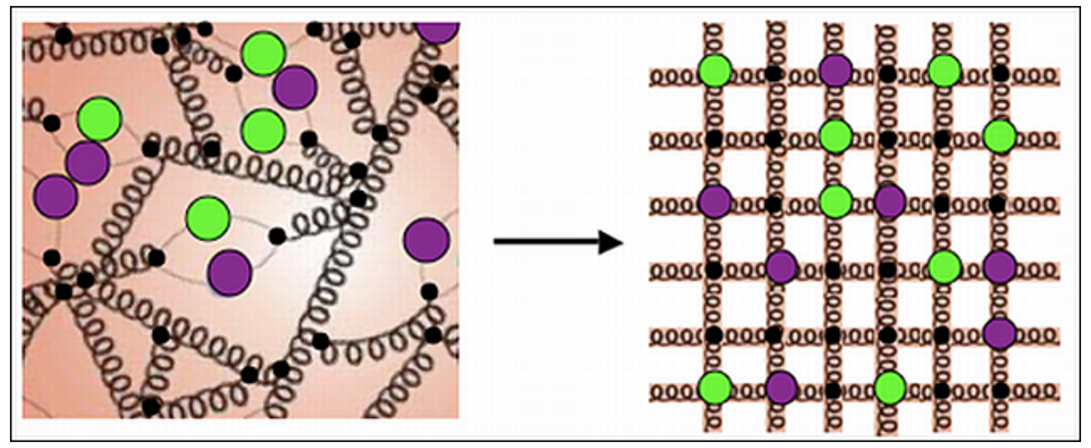
✦ Low-energy region — Quarks are glued together by gluons to form hadrons

Phenomenological Models

⇩

- **Linear Sigma Model**
- **Nambu-Jona-Lasinio Model**
- **Jülich Model**
- **…**

Lattice QCD



Effective Field Theories

⇩ Chiral Symmetry

- **Chiral Perturbation theory (ChPT)**

➡ hadron fields as degrees of freedom

➡ Expansion in masses and external momenta

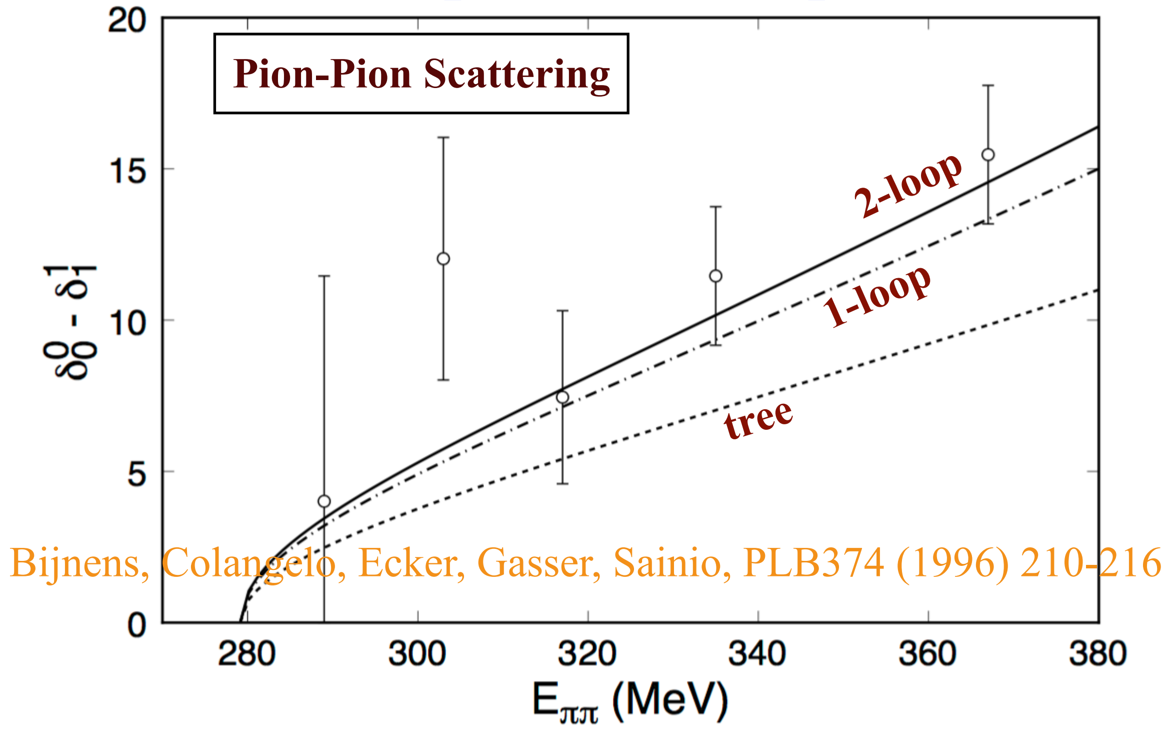
BChPT: Power Counting Breaking (PCB) problem

✦ **Pure Goldstone Bosons: ChPT has gained great achievements**

➡High-order calculations become standard

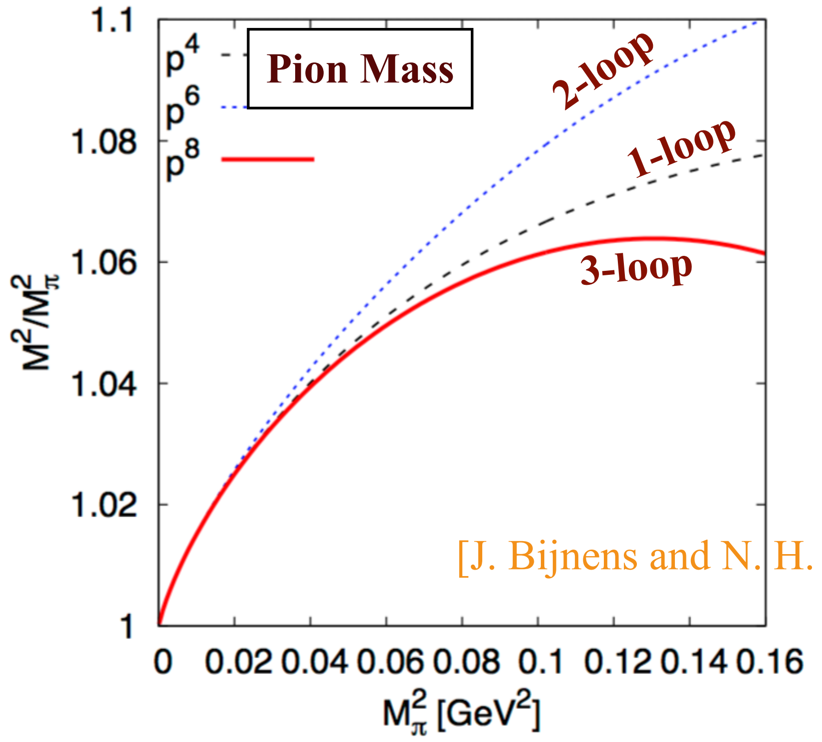
➡Fast convergence

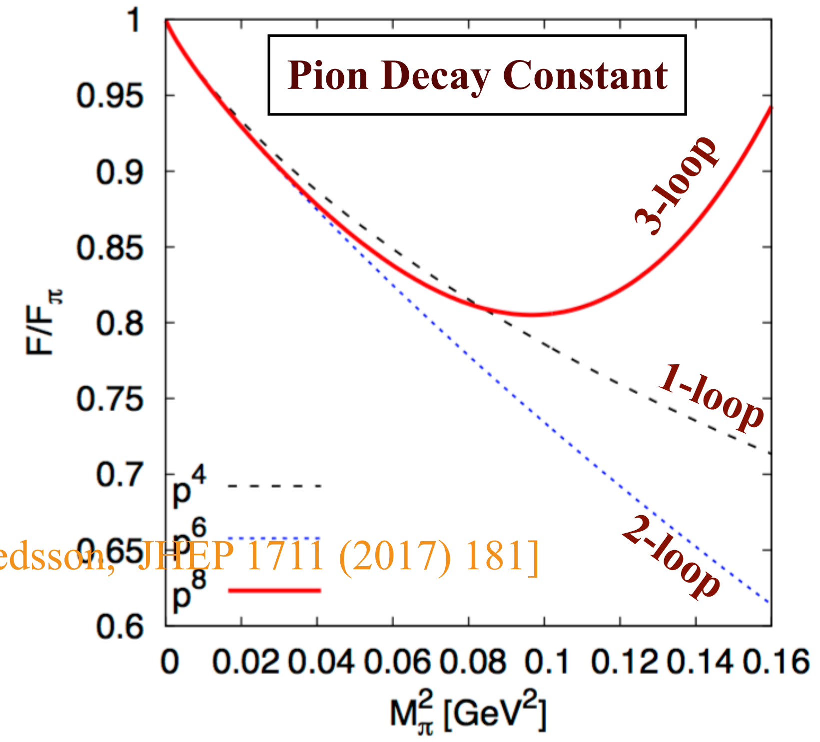
up to Two-Loop



Bijnens, Colangelo, Ecker, Gasser, Sainio, PLB374 (1996) 210-216

up to Three-Loop





[J. Bijnens and N. H. Truedsson, JHEP 1711 (2017) 181]

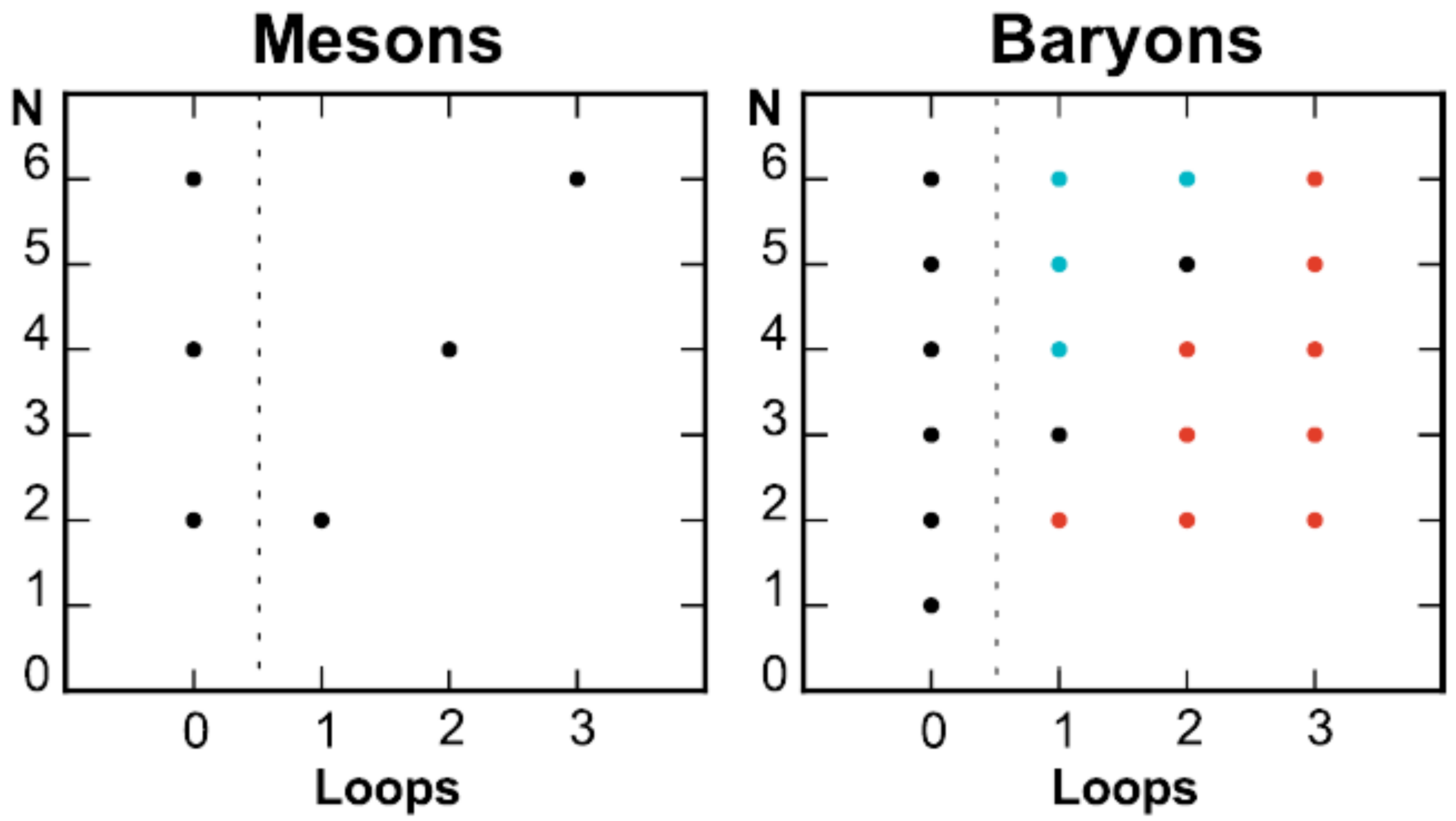
✦ **Covariant ChPT including matter fields** (Baryons, D/B mesons)

- **Dimensional Regularization (DR) with standard MSbar-1 subtraction**
- **A systematic power counting rule is lost due to the non-zero mass of matter fields in the chiral limit**

Feynman Diagram

$$\mathcal{O}(p^N)$$

How important?

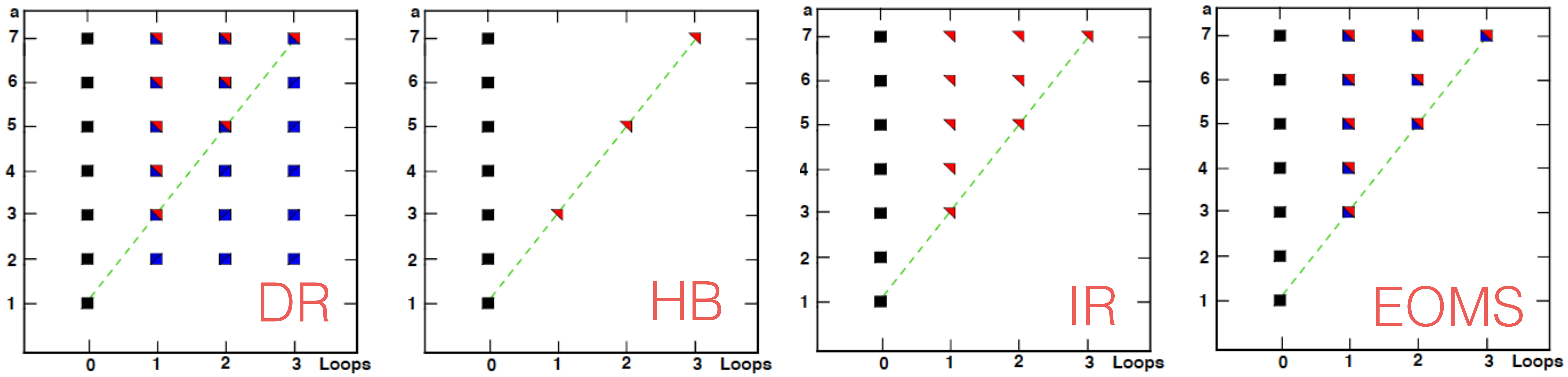


Chiral order: $N = 4L - 2N_M - N_B + \Sigma_k k V_k$

red dots denote possible PCB terms

➡Is there a solution?

BChPT: HB approach, IR prescription & EOMS scheme



✦ Heavy Baryon ChPT (HBChPT):

[Jenkins and Manohar,PLB255'91]

A simultaneous expansion in external momenta and 1/mB.

— Non-covariant and slowly convergent in the threshold region.

[N.Fettes,Ulf-G.Meissner and S.Steininger, NPA'98], [M.Mojˇziˇs,Eur.Phys.J.C2'98]

— Even divergent in the sub-threshold region (e.g. scalar form factor).

[V.Bernard,N.Kaiser and Ulf-G.Meissner,Int.J.Mod.Phys.E4'95], [T.Becher and H.Leutwyler,Eur.Phys.J.C9'99]

✦ Infrared Regularization (IR):

[T.Becher and H.Leutwyler,Eur.Phys.J.C9'99]

The full integral is separated into Infared singular and ~~Regular parts.~~

—Scale-dependence: amplitude and observables. [T.Becher and H.Leutwyler,JHEP0106'01]

—Unphysical cuts(u=0) [J.M.Alarcon,J.Martin Camalich,J.A.Oller and L.Alvarez-Ruso,PRC83'11]

—Bad predictions: e.g., huge Goldberger-Treiman relation violation (20-30%).

[J.M.Alarcon,J.Martin Camalich,J.A.Oller and L.Alvarez-Ruso,PRC83'11]

✦ Extended-on-mass-shell (EOMS): [T.Fuchs,J.Gegelia,G.Japaridze and S.Scherer,PRD68'03]

Is it an approach to solve the above problems?

EOMS scheme

✦ **EOMS is a two-step renormalization scheme**

step 1: **Ultraviolet (UV) divergence** — Dimensional regularization, MSbar-1 subtraction

↓ Scale independent

finite ren. →

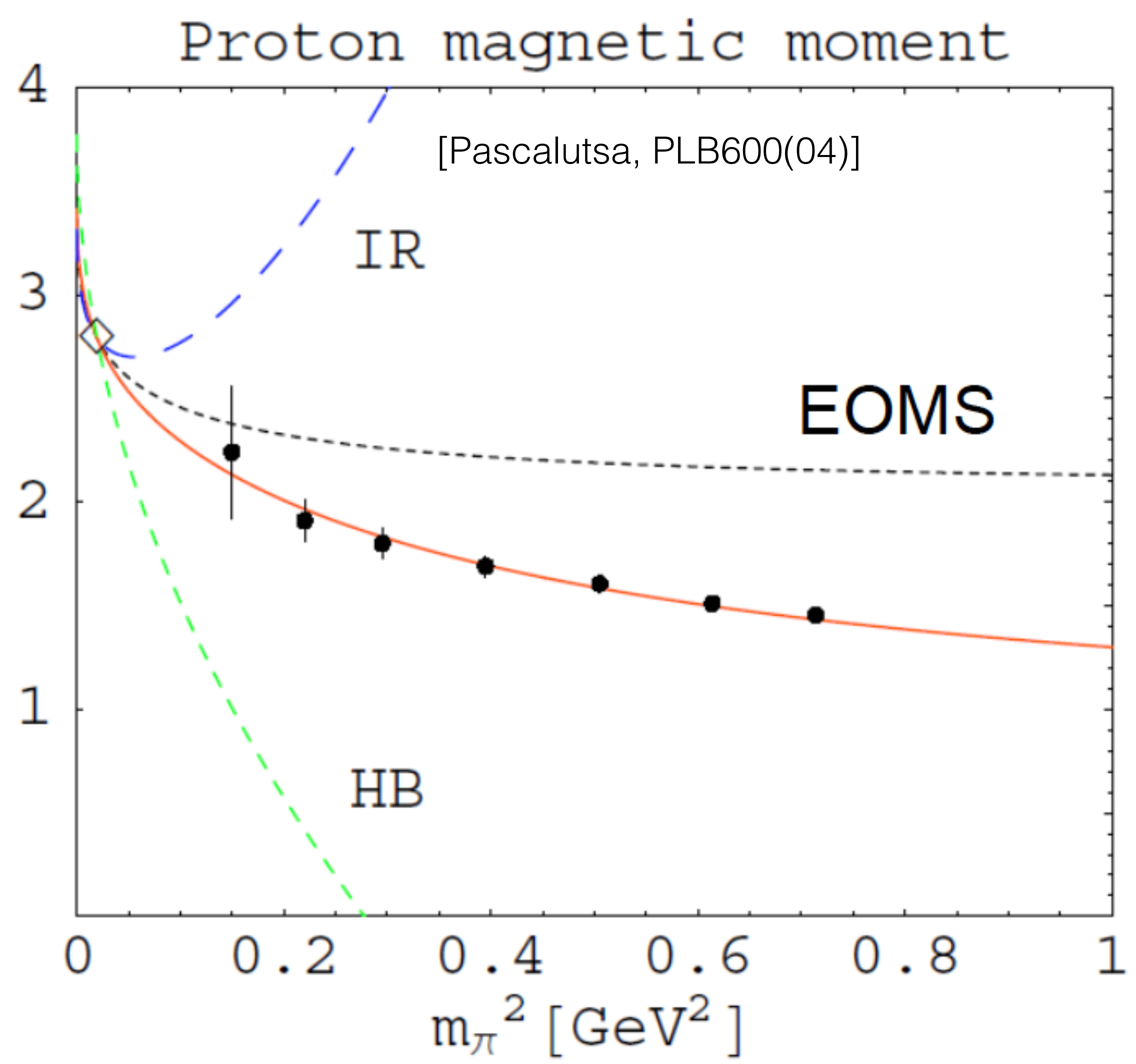
step 2: **PCB terms** (polynomials) — removed by redefinition of LECs in Effective Lagrangians

↓ Correct power counting (respectively faster convergence)

↓ keep original analyticity and all assumed symmetries

✦ **…related to other theories:**

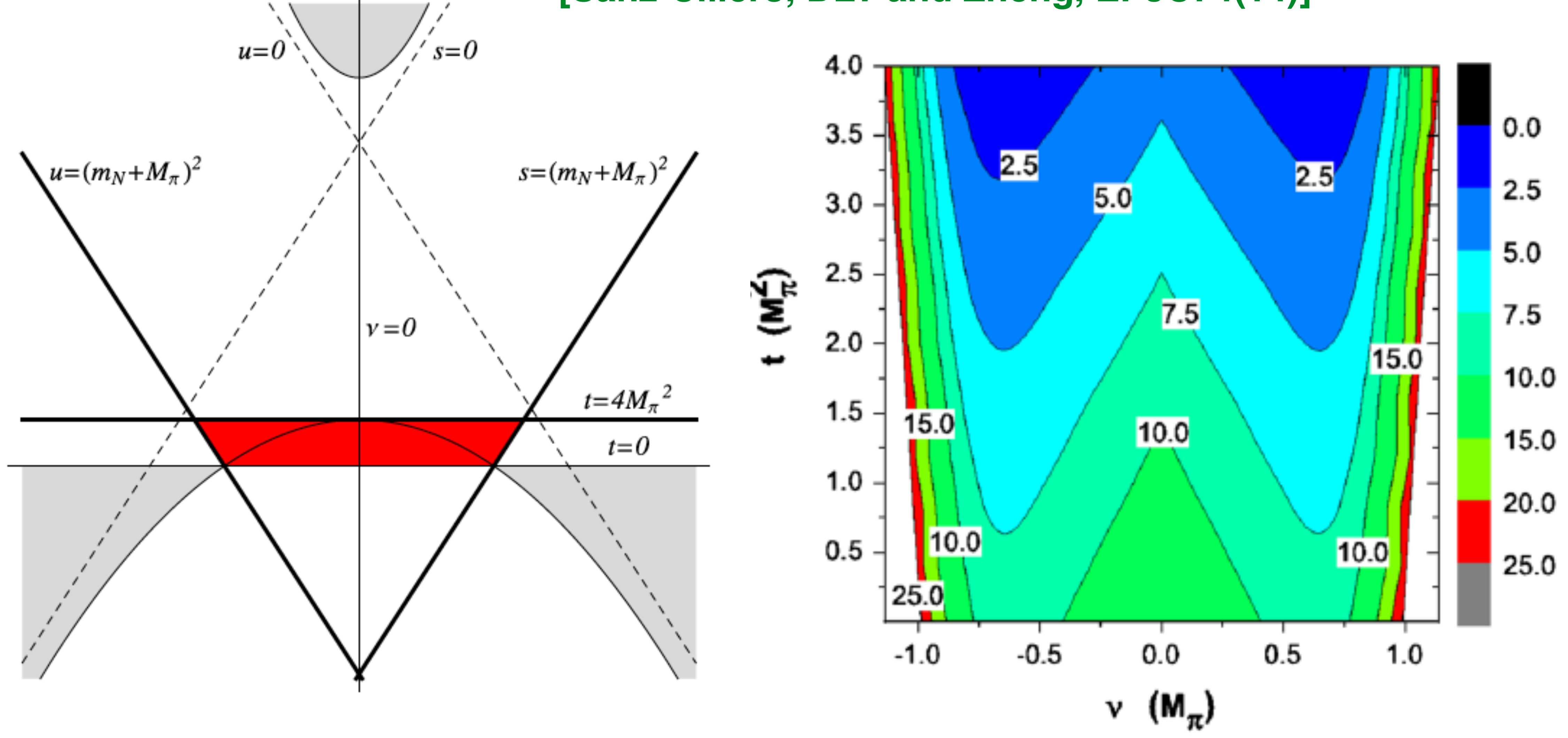
Lattice QCD



Chiral extrapolation

Dispersion Theory

[Sanz-Cillero, DLY and Zheng, EPJC74(14)]



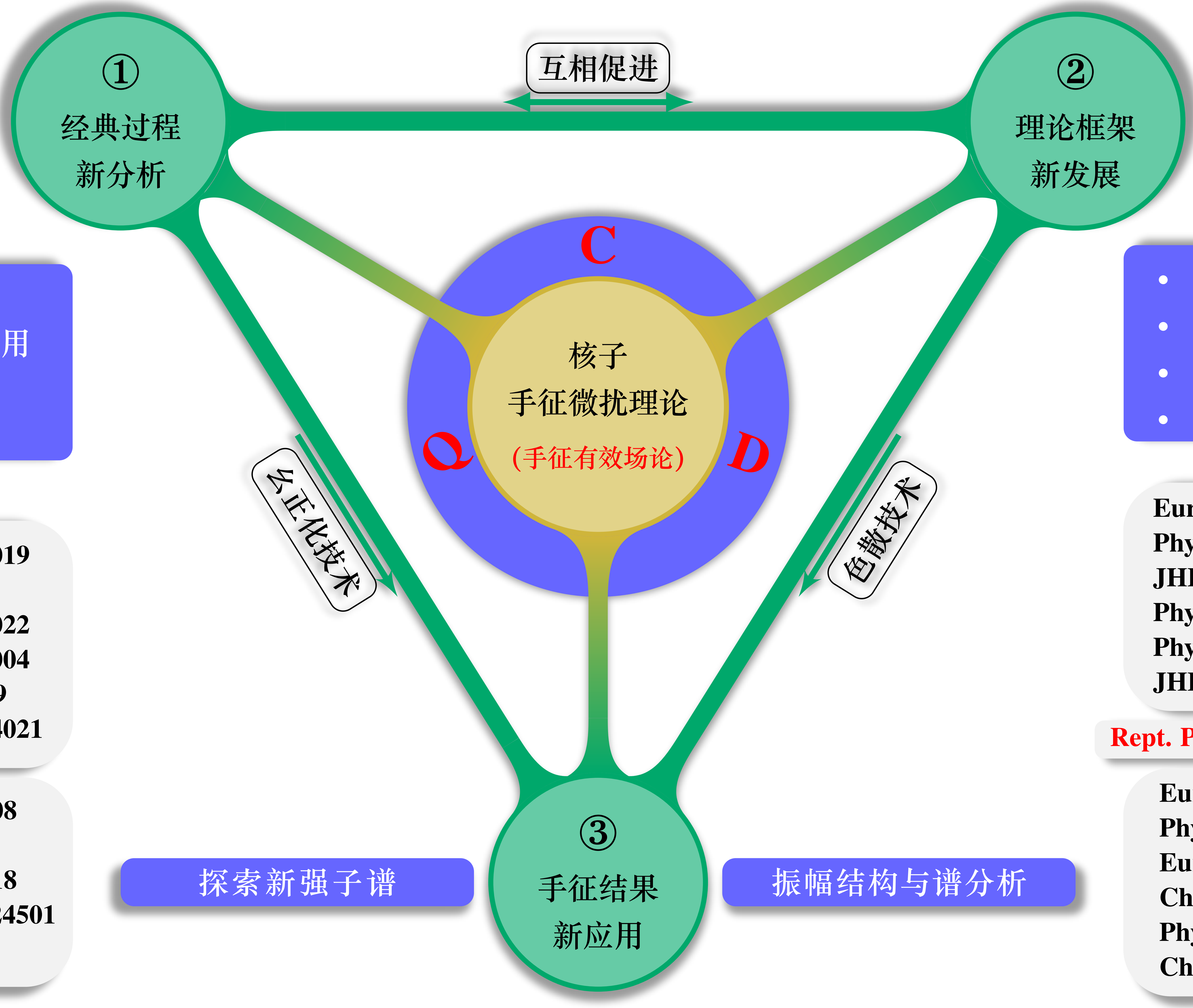
Dispersion Relation -> Positivity Bounds

Progress of BChPT with EOMS scheme

- 介子核子散射
- 核子电磁相互作用
- 中微子核子散射
- 介子光生过程

Phys.Rev.D87(2013)054019
JHEP1605(2016)038
Phys.Rev.D96(2017)116022
Phys.Rev.D98(2018)076004
Phys.Lett.B794(2019)109
Phys.Rev.D100(2019)094021

Phys.Rev.D92(2015)094008
Eur.Phys.J.C77(2017)728
Phys.Rev.D98(2018)094018
Front.Phys.(BJ)14(2019)24501
Eur.Phys.J.C80(2020)433



- 中高能区域推广
- 加入不稳定粒子
- 其它重轻系统
- 有限体积修正效应

Eur.Phys.J.C75(2015)499
Phys.Lett.B760(2016)736
JHEP11(2015)058
Phys.Rev.D97(2018)034012
Phys.Rev.D103(2021)034006
JHEP12(2022)029

Rept. Prog. Phys. 84, 076201 (2021)

Eur.Phys.J.C74(2014)2763
Phys.Rev.D94(2016)094037
Eur.Phys. J.C78(2018)310
Chin.Phys.C43(2019)064110
Phys.Rev.D101 (2020)034014
Chin.Phys.C45(2021)014104

Selected Progress 1: πN scattering and the crazy $N^*(890)$ resonance

✦ Peking university parametrization

$$S(s) = \underbrace{\left[\prod_b S_b(s) \cdot \prod_v S_v(s) \cdot \prod_R S_R(s)\right]}_{\text{poles}} \cdot e^{2i\rho(s)f(s)}$$

Left-hand cut & faraway right-hand cut

$$f(s) = \frac{s}{\pi}\int_{s_c}^{s_L} \frac{-\ln|S(s')|ds'}{2\rho(s')s'(s'-s)} + \frac{s}{\pi}\int_0^{\theta_c} \frac{\ln[S_{in}(\theta)/S_{out}(\theta)]}{2i\rho(s')(s'-s)}\Big|_{s'=(M_N^2-m_\pi^2)e^{i\theta}} d\theta$$

$$+\frac{s}{\pi}\int_{(M_N^2-m_\pi^2)^2/M_N^2}^{M_N^2+2m_\pi^2} \frac{\mathrm{Arg}[S(s')]ds'}{2is'\rho(s')(s'-s)} + \frac{s}{\pi}\int_{(2m_\pi+M_N)^2}^{\Lambda_R^2} \frac{\ln[1/\eta(s')]ds'}{2\rho(s')s'(s'-s)}$$

- **Bound / virtual states**

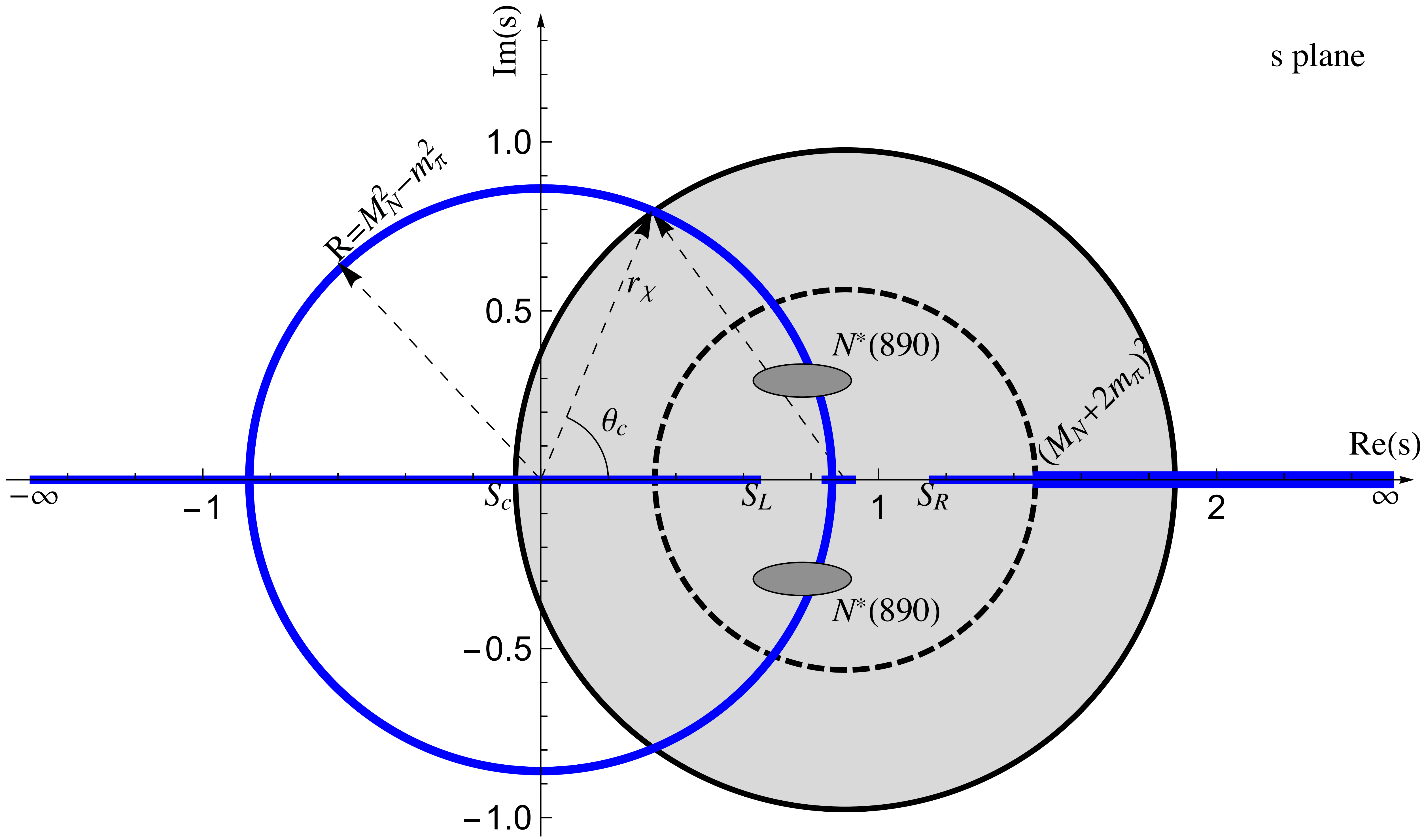
$$S_{b/v}(s) = \frac{1 \pm i\rho(s)a(s_0)}{1 \mp i\rho(s)a(s_0)} \qquad a(s_0) = \pm\frac{2\sqrt{s_R}}{s_R - s_L}\sqrt{\frac{s_0 - s_L}{s_R - s_0}}$$

- **Resonance**

$$S_R(s) = \frac{M^2(z_0) - s + i\rho(s)sG[z_0]}{M^2(z_0) - s - i\rho(s)sG[z_0]}$$

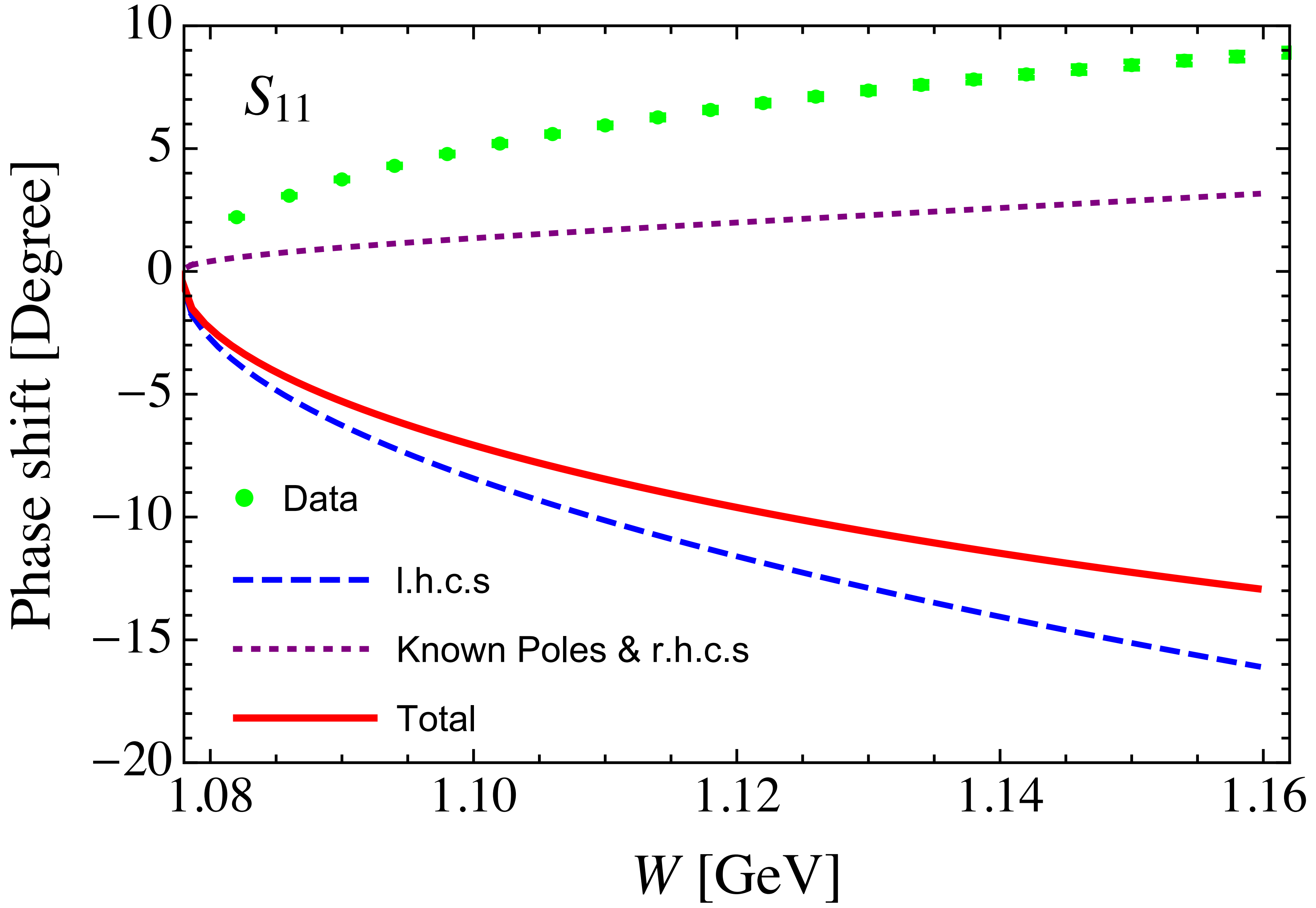
$$M^2(z_0) = \mathrm{Re}[z_0] + \frac{\mathrm{Im}[z_0]\mathrm{Im}[z_0\rho(z_0)]}{\mathrm{Re}[z_0\rho(z_0)]}$$

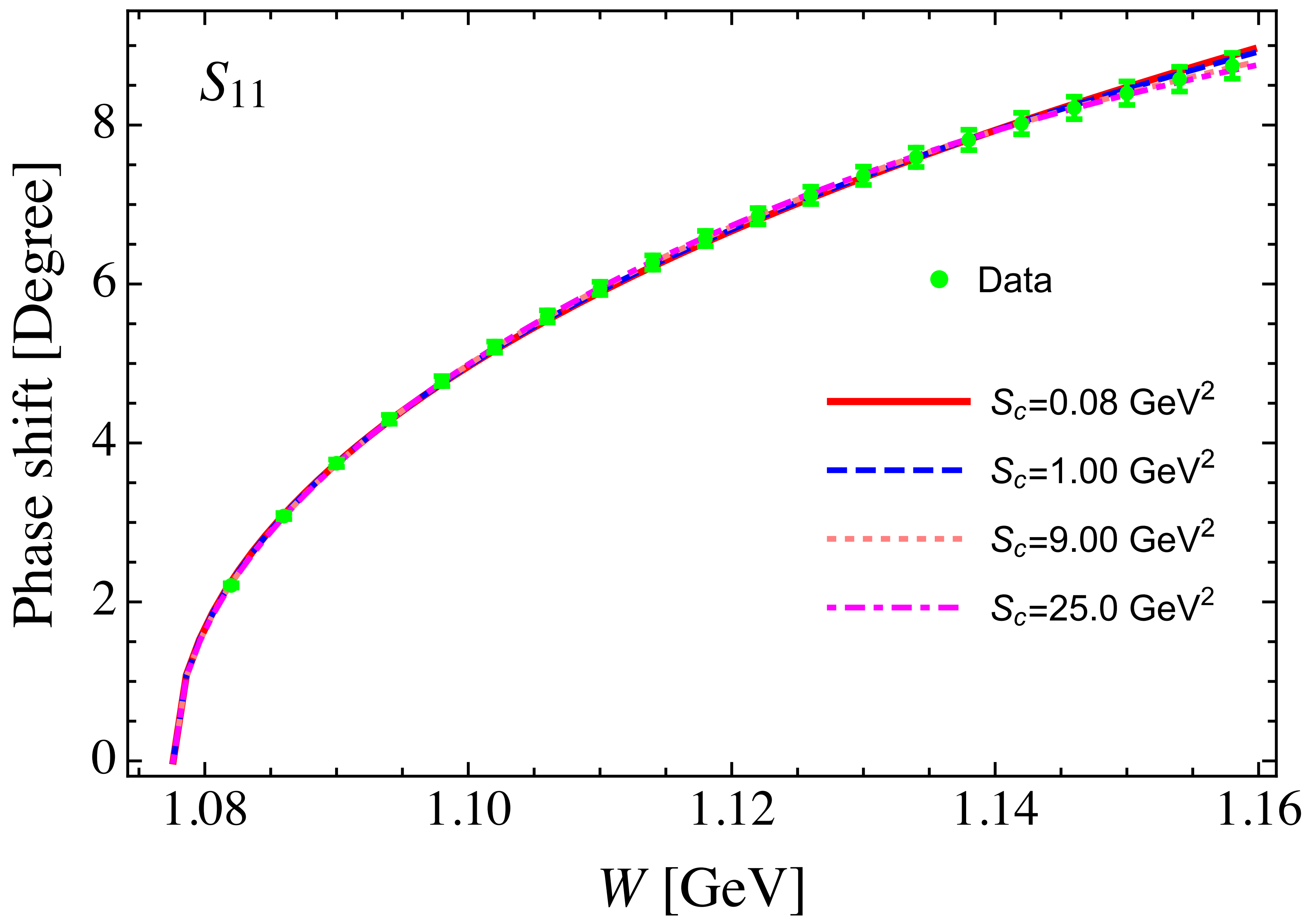
$$G[z_0] = \frac{\mathrm{Im}[z_0]}{\mathrm{Re}[z_0\rho(z_0)]}$$



Selected Progress 1: πN scattering and the crazy $N^*(890)$ resonance

✦ Phase shifts





A crazy resonance is indispensable to compensate the discrepancy!

- **Bound state → negative phase shift**
- **Virtual state → positive phase shift**
- **Resonance→ positive phase shift**
- **Left-hand cut → (empirically) negative phase shift**

$$N^*(890) : \sqrt{s} = (895 \pm 81) - i\,(164 \pm 23)\ \text{MeV}$$

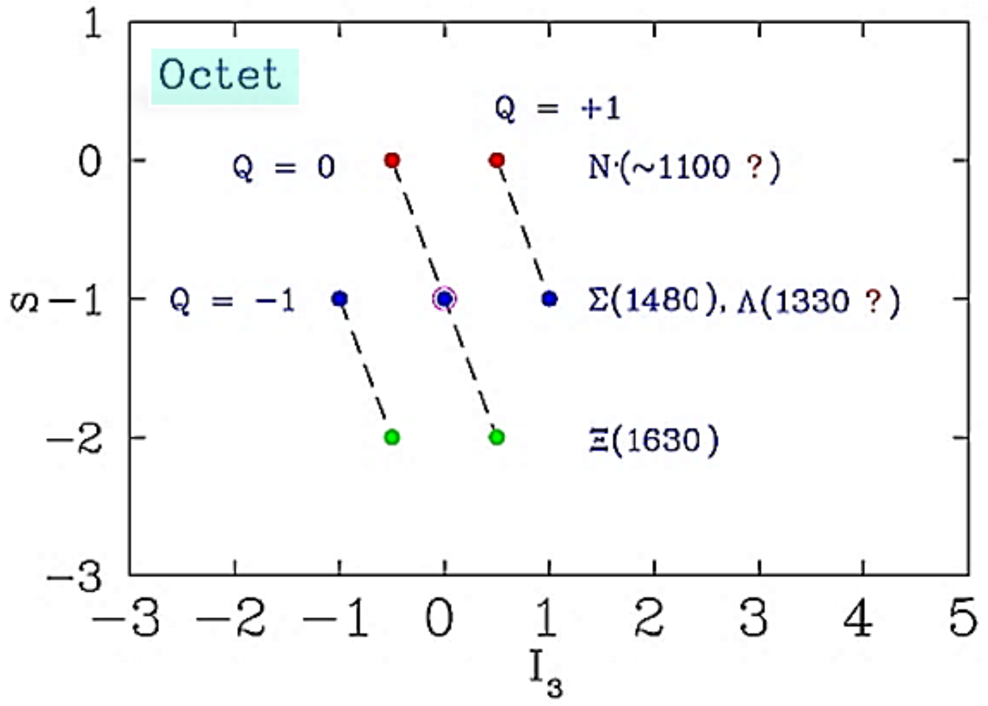
Selected Progress 1: πN scattering and the crazy $N^*(890)$ resonance

✦ **The N' state searched for half a century ?**

Courtesy of Igor Strakovsky]

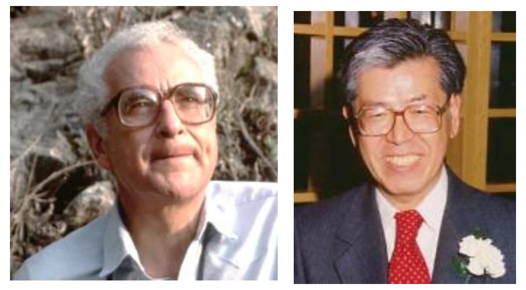
Unitarity Partners (?)

Ya. Azimov, R. Arndt, IS, R. Workman, Phys Rev C 68, 045204 (2003)



$$M = a_0 + a_1 Y + a_2 \left[I\,(I+1) - \frac{1}{4} Y^2 \right]$$

- Mixing is able to shift some masses for Gell-Mann-Okubo mass formula.



State	Mass (MeV)	Width (MeV)	Decay Modes	Hadron Production Xsections
N′	~1100 ?	<0.05	Nγ ?	$< 10^{-4}$ of "normal"
Λ	1330 ?		Λγ	$\sim 10\mu b$
Σ	1480	30-80 ?	$\Lambda\pi$, $\Sigma\pi$, $N\bar{K}$	$\sim 1\mu b$
Ξ	1630	20-50 ?	Ξπ	$\sim 1\mu b$

On base of positive observations.

- PhotoProd Xsection has additional ~α/π factor.
- ElectroProd has ~$(\alpha/\pi)^2$.

12/11/2019 EHS-2019, York, UK, December 2019 Igor Strakovsky 4

Where Did N' from

Volume 32B, number 6 PHYSICS LETTERS 17 August 1970

ON THE POSSIBLE EXISTENCE OF A NEW NUCLEON STATE

Ya. I. AZIMOV

A.F. Ioffe Physical-Technical Institute, Leningrad, USSR

PHYSICAL REVIEW C 68, 045204 (2003)

Light baryon resonances: Restrictions and perspectives

Ya. I. Azimov*

Petersburg Nuclear Physics Institute, Gatchina St. Petersburg 188300, Russia

R. A. Arndt,† I. I. Strakovsky,‡ and R. L. Workman§

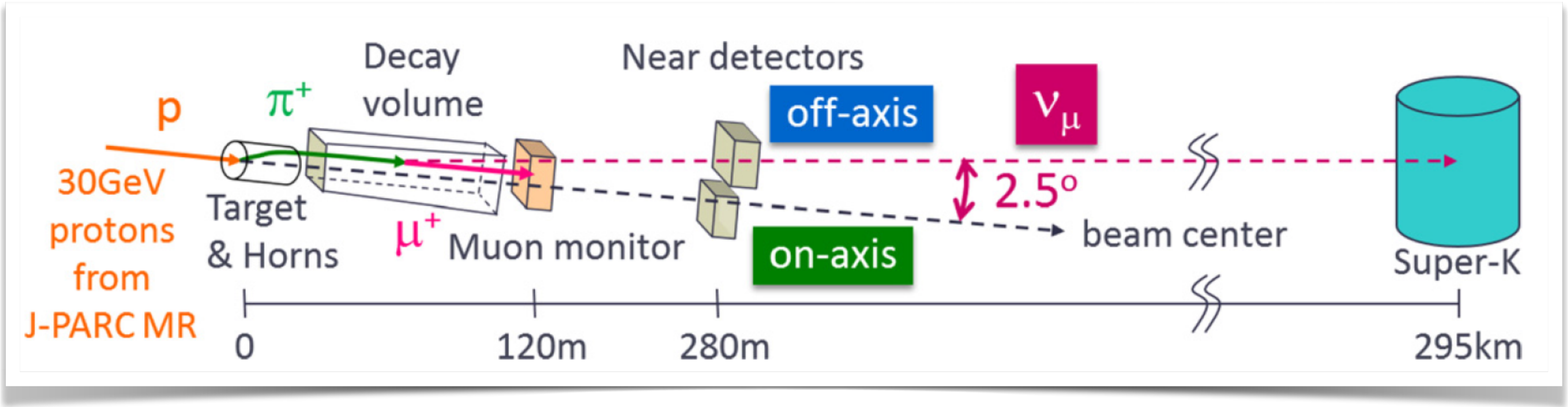
Center for Nuclear Studies, Department of Physics, The George Washington University, Washington, D.C. 20052, USA

Selected Progress 2: Applications in Neutrino Physics

✦ Oscillation experiments (e.g T2K)

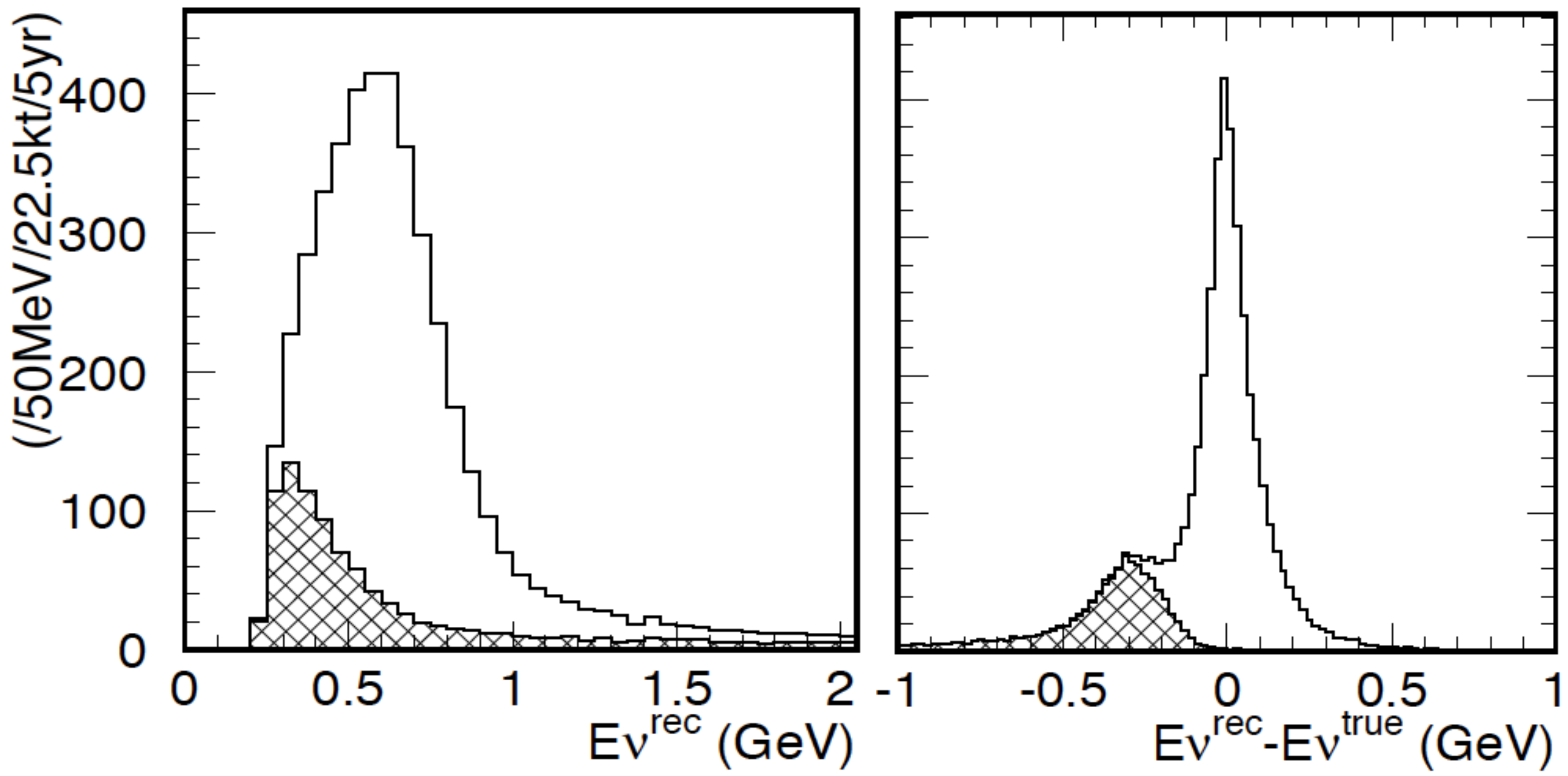
survival probability of ν_μ

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{\mu\tau} \cdot \sin^2 \frac{\Delta m_{23} L}{E_\nu}$$



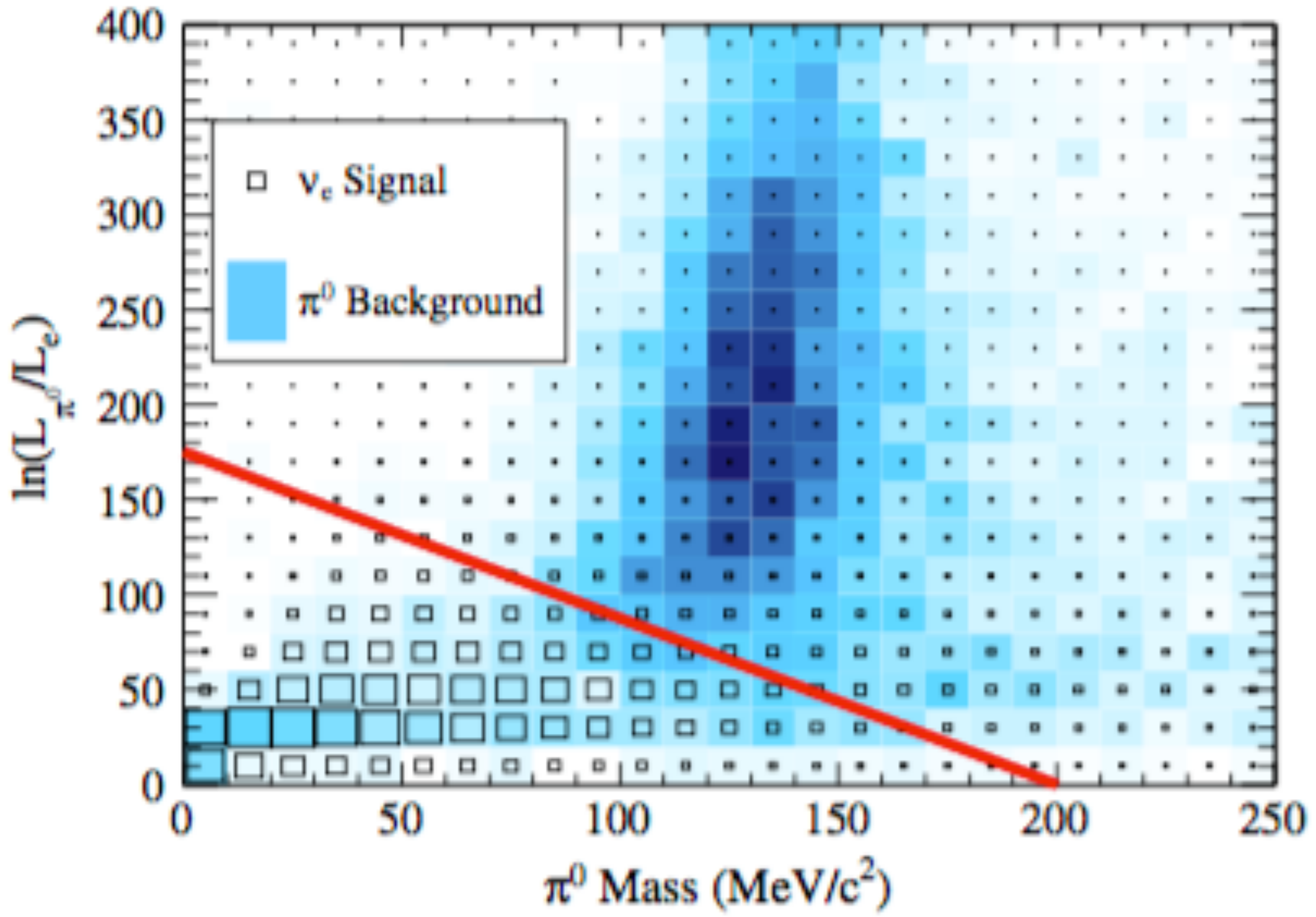
✦ CC1π:

- Source of CCQE-like events
  - ➡Misidentification of pion
- to be subtracted for a good Eν reconstruction



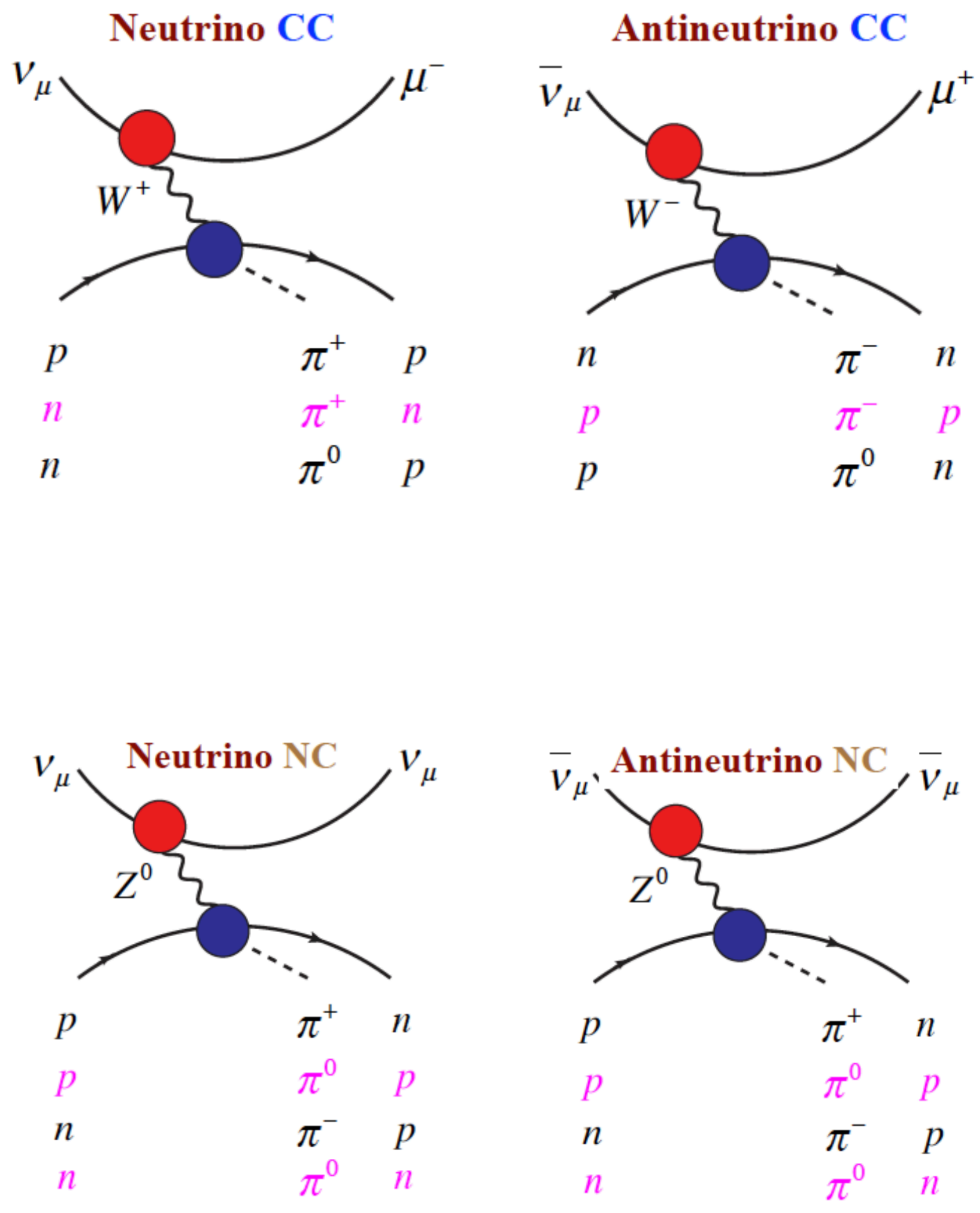
✦ NC1π:

- e-like background to $\nu_\mu \to \nu_e$ searches
- Improved at T2K with a π^0 rejection cut

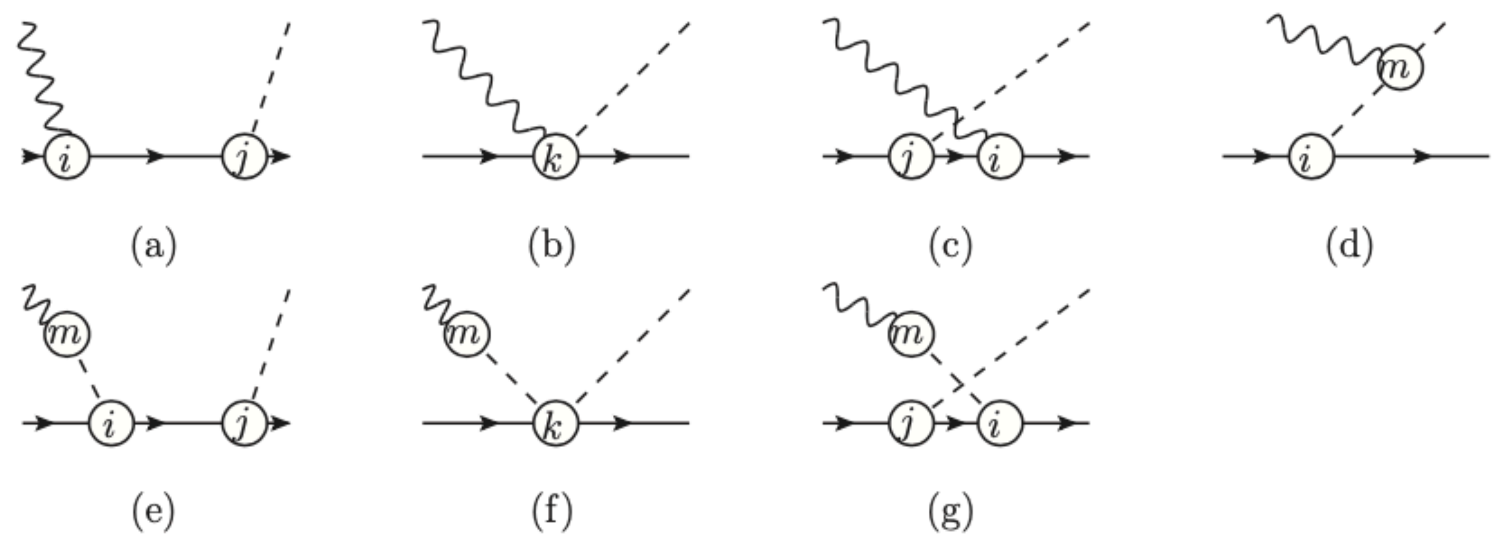


Selected Progress 2: Applications in Neutrino Physics

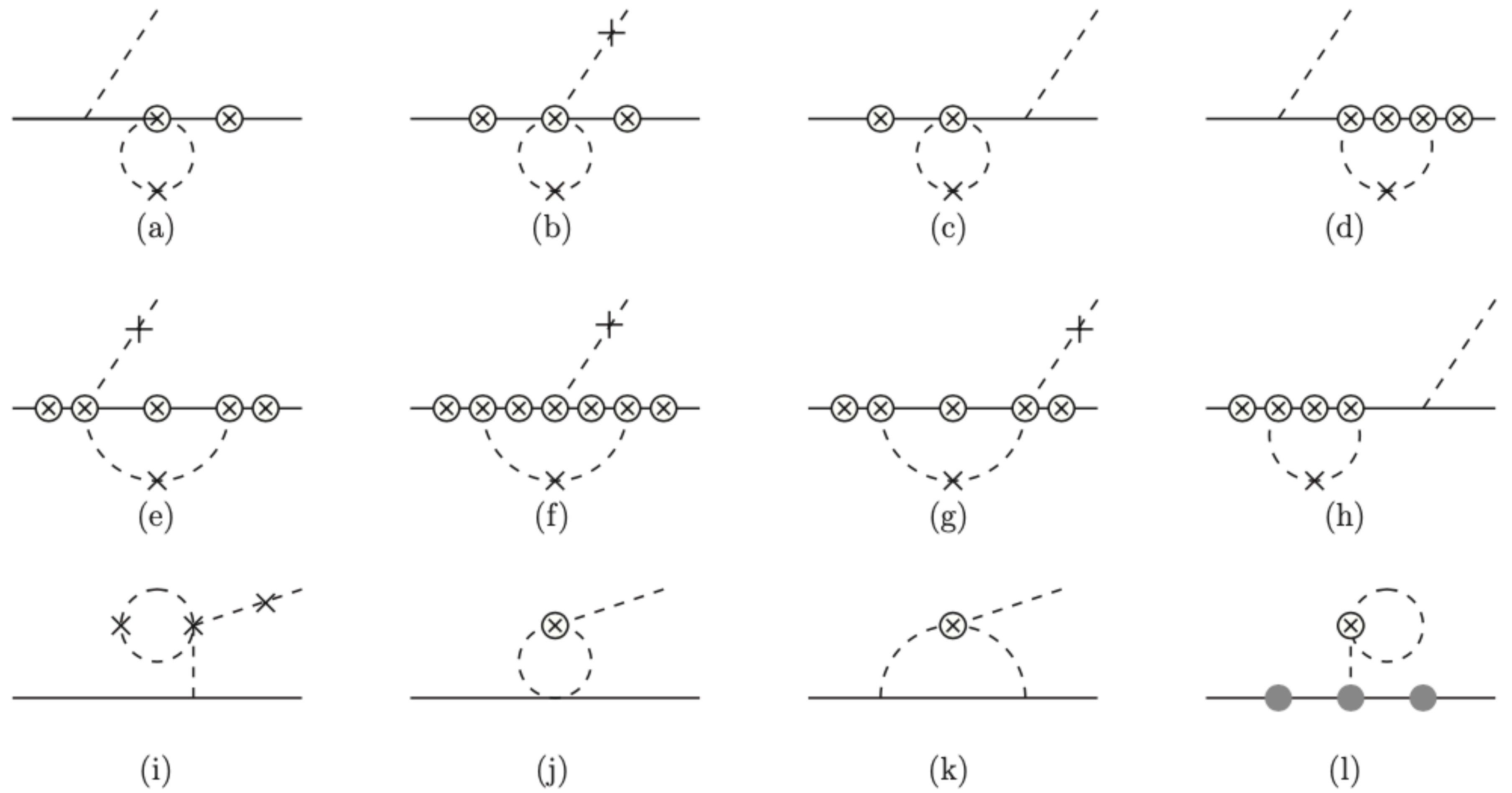
✦ First results from relativistic BChPT

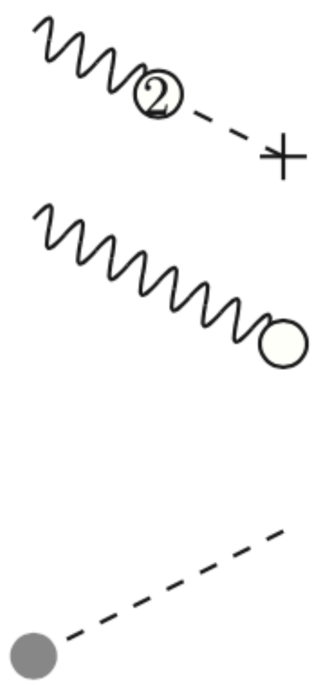


Tree diagrams up through $O(p^3)$:



All possible loop diagrams at $O(p^3)$:



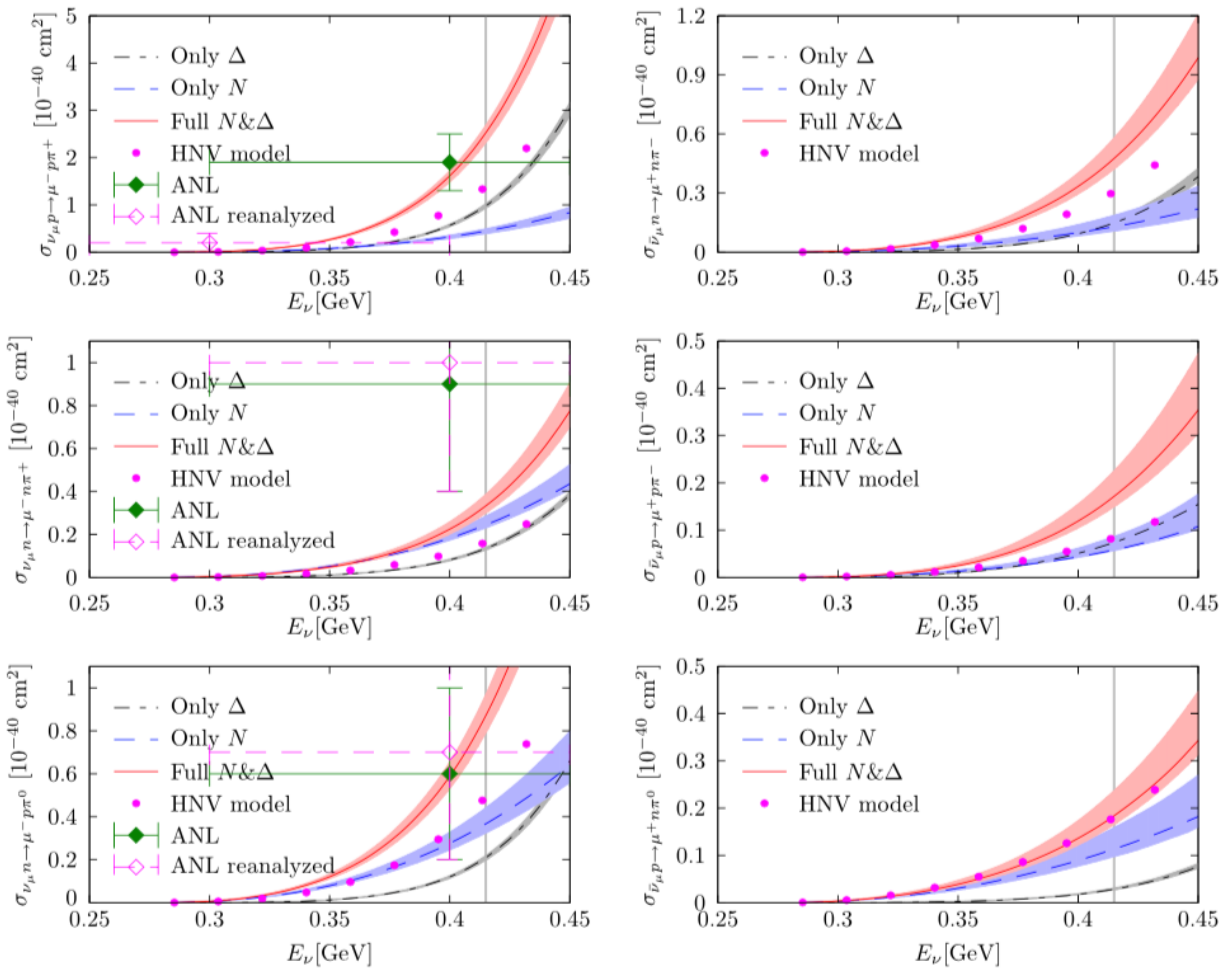


89 diagrams (*xAmpCalc*) & wave function renormalization & EOMS

Selected Progress 2: Applications in Neutrino Physics

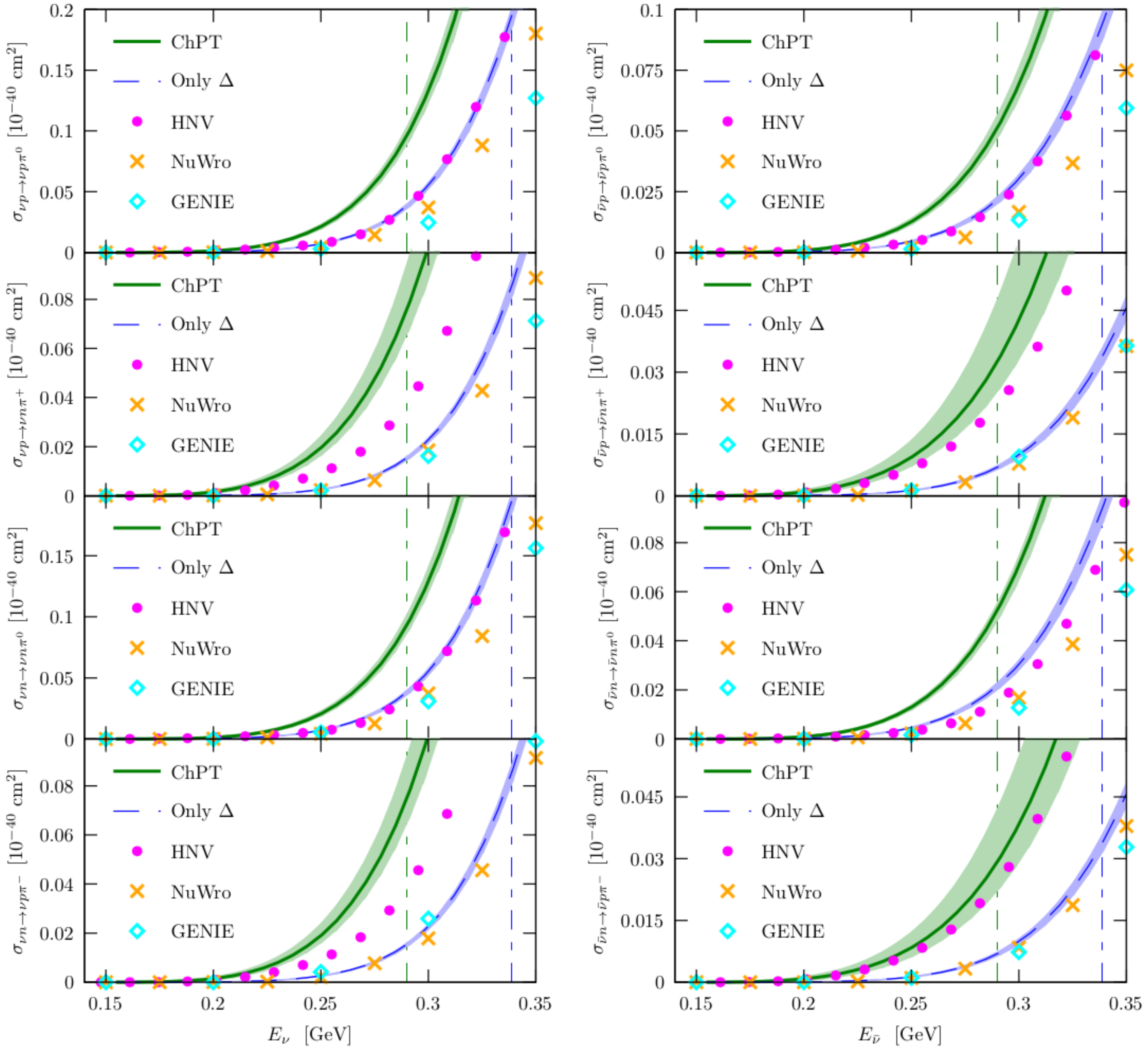
✦ First results from relativistic BChPT

ν-CC1π　　　　anti-ν-CC1π



[DLY, Alvarez-Ruso, Hiller-Blin and Vicente-Vacas, PRD98(2018)076004]

ν-NC1π　　　　anti-ν-NC1π



[DLY, Alvarez-Ruso and Vicente-Vacas, PRD794(2019)109]

✦ More outcomes from BChPT:

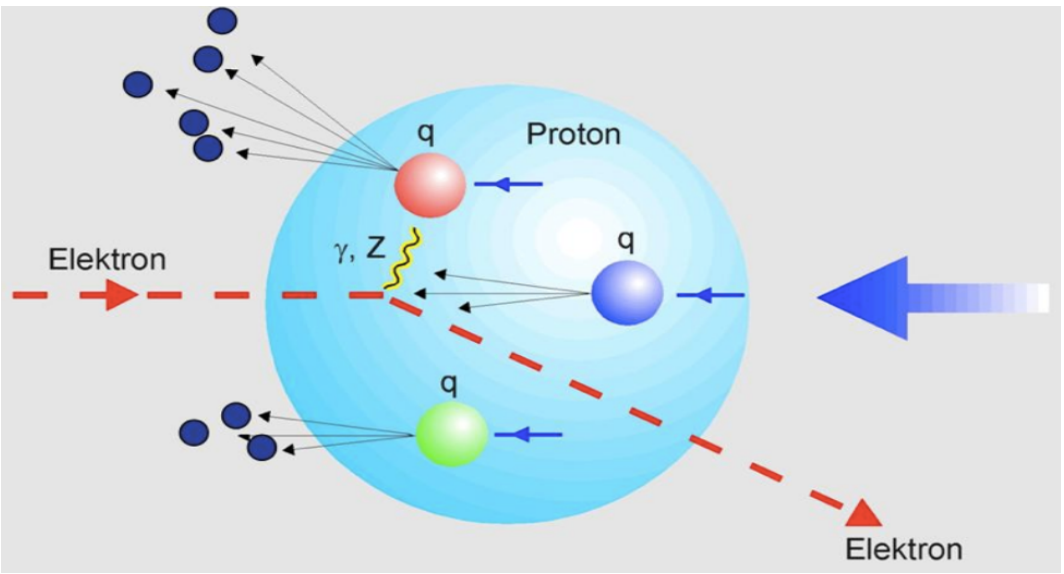
➡Extended to intermediate energies.

➡Multi pion productions & Coherent pion productions (nuclear effects)

➡Production of pseudo-scalar with strangeness

➡The same game can be played for neutinoless beta decays?

➡Strangeness of the nucleon



Selected Progress 3: Extensions in Heavy Flavour Physics

✦ Chiral potentials for charmed meson interactions from ChPT & Unitarization

✓ **NLO:** [Kolomeitsev & Lutz PLB582 (2004) 39] [Guo, et al, PLB641 (2006) 278] [Liu, et al, PRD87, 014508 (2013)] [Altenbuchinger, et al, PRD89, 014026 (2014)] ……

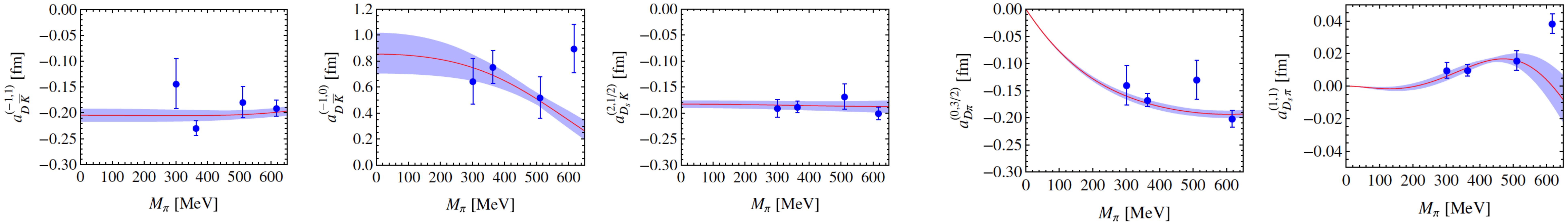
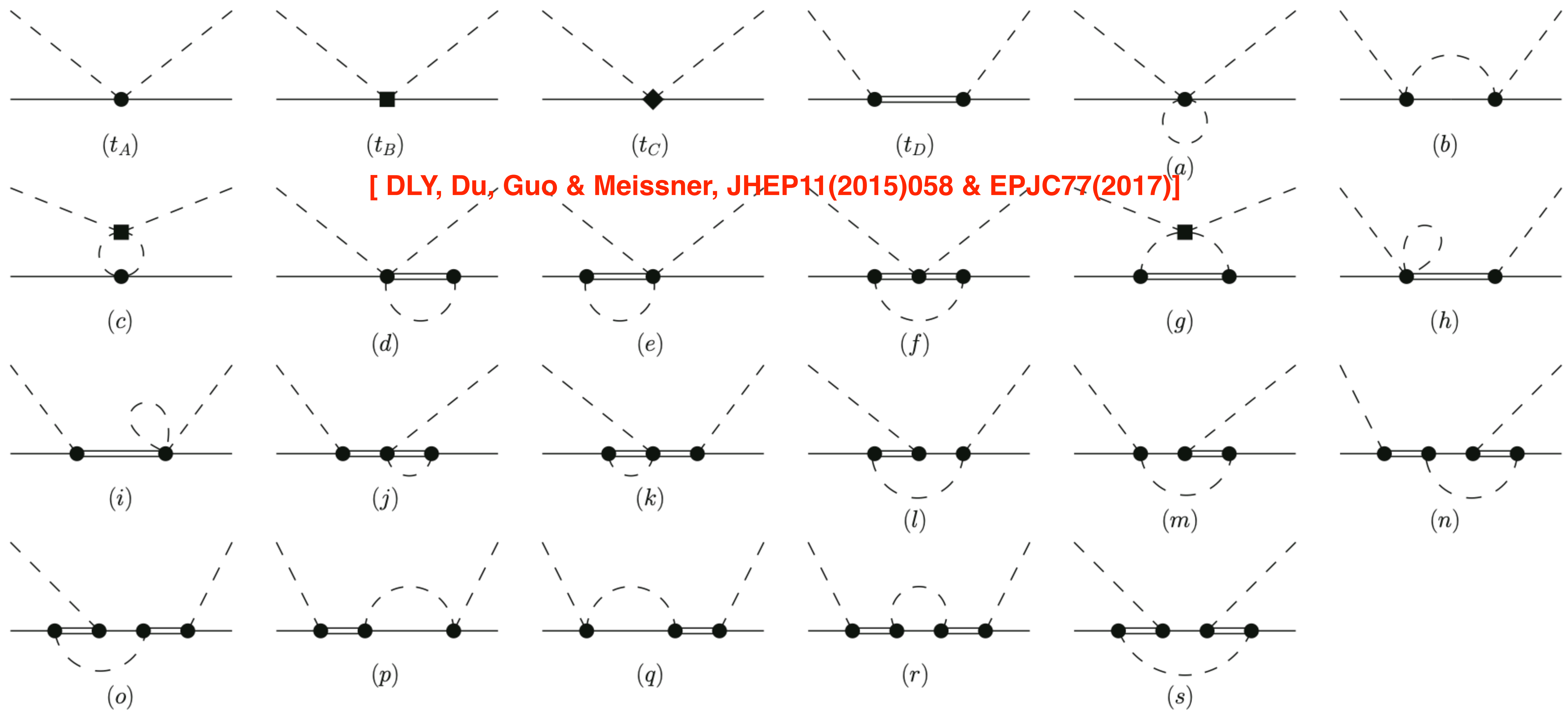
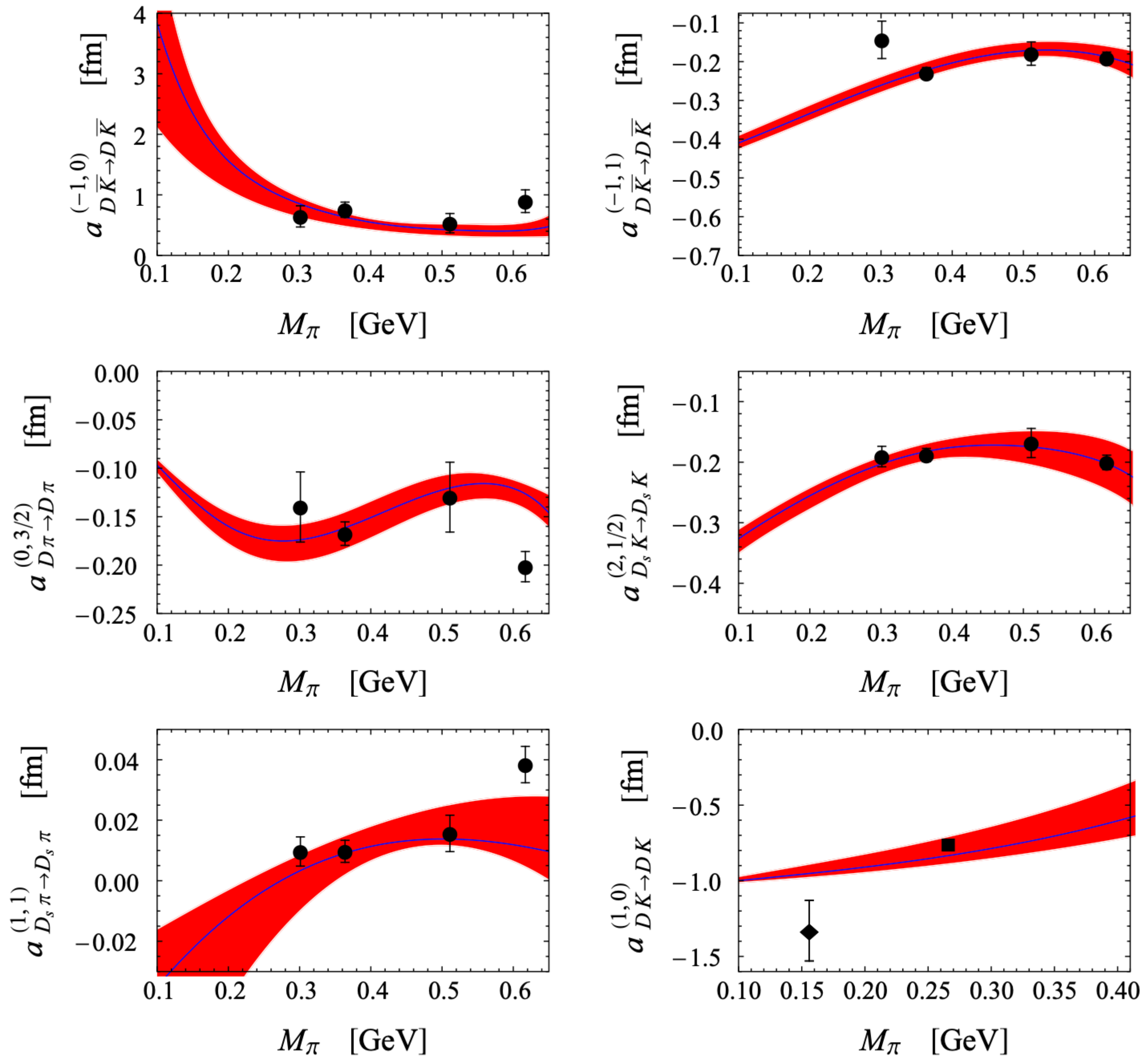


Fig from [Liu, et al, PRD87, 014508 (2013)]

✓ **NNLO:** [Geng, et al, PRD82, 054022 (2010)], …



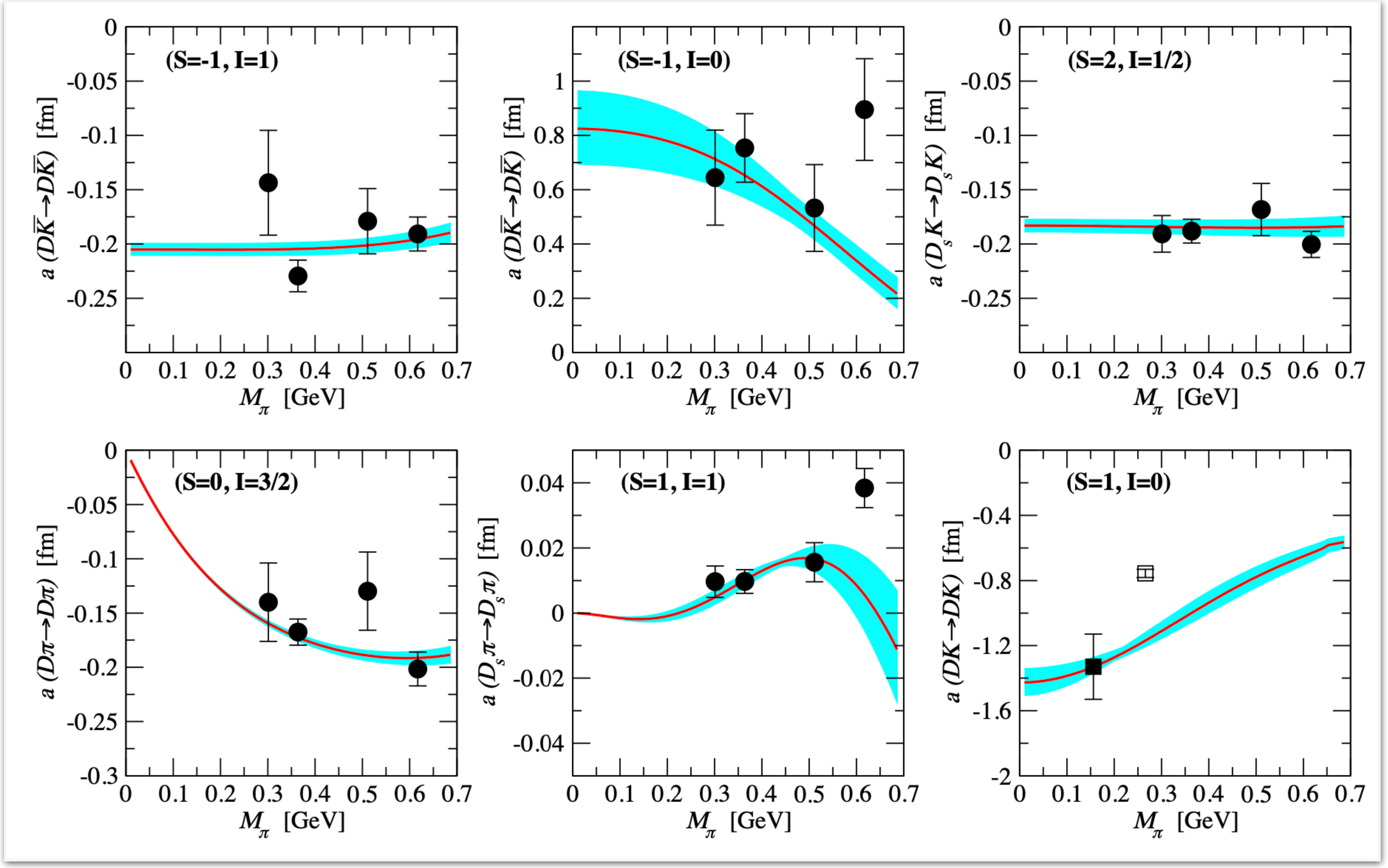
[DLY, Du, Guo & Meissner, JHEP11(2015)058 & EPJC77(2017)]



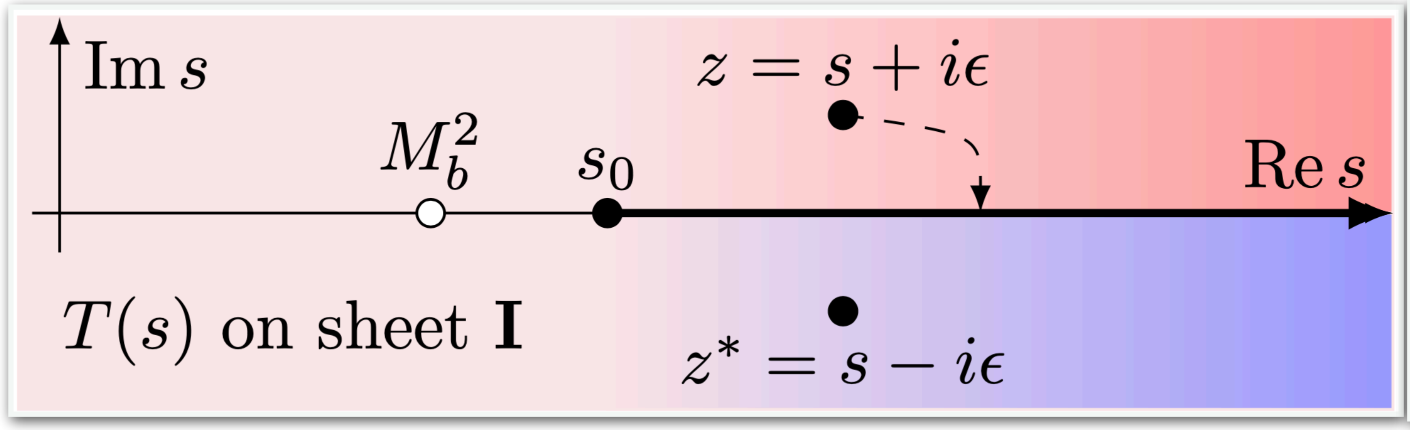
Selected Progress 3: Extensions in Heavy Flavour Physics

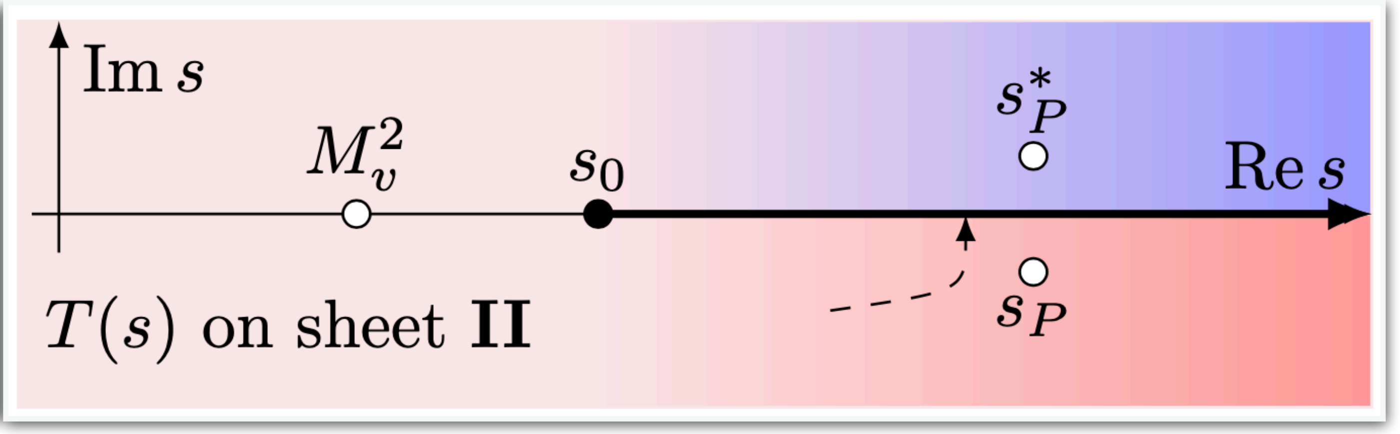
✦ New insights into the 0^+ charm scalar mesons

- **Lattice data are well described by U(3) UChPT amplitudes. LEC determined**

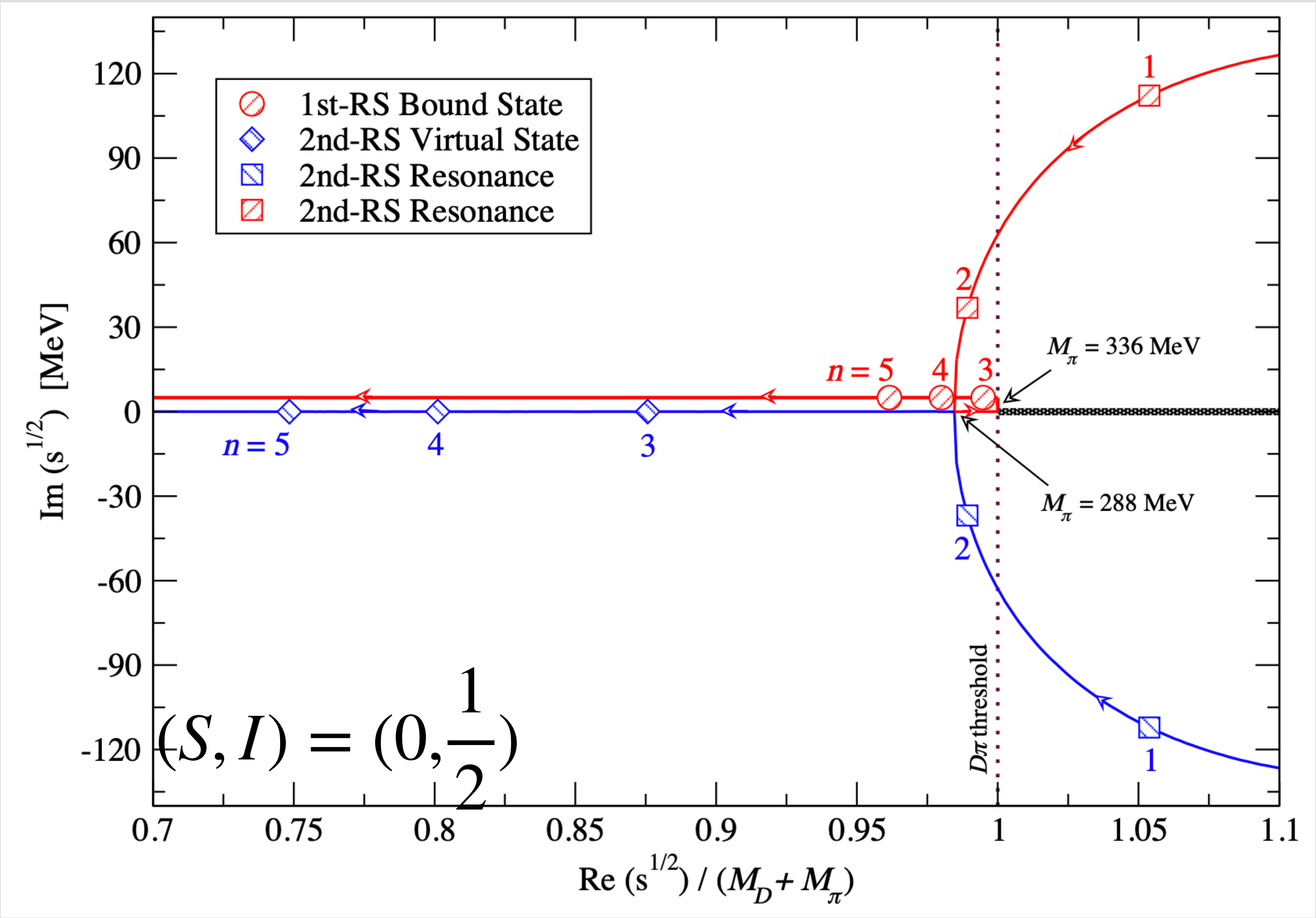


- **Bound state, virtual state and resonance**

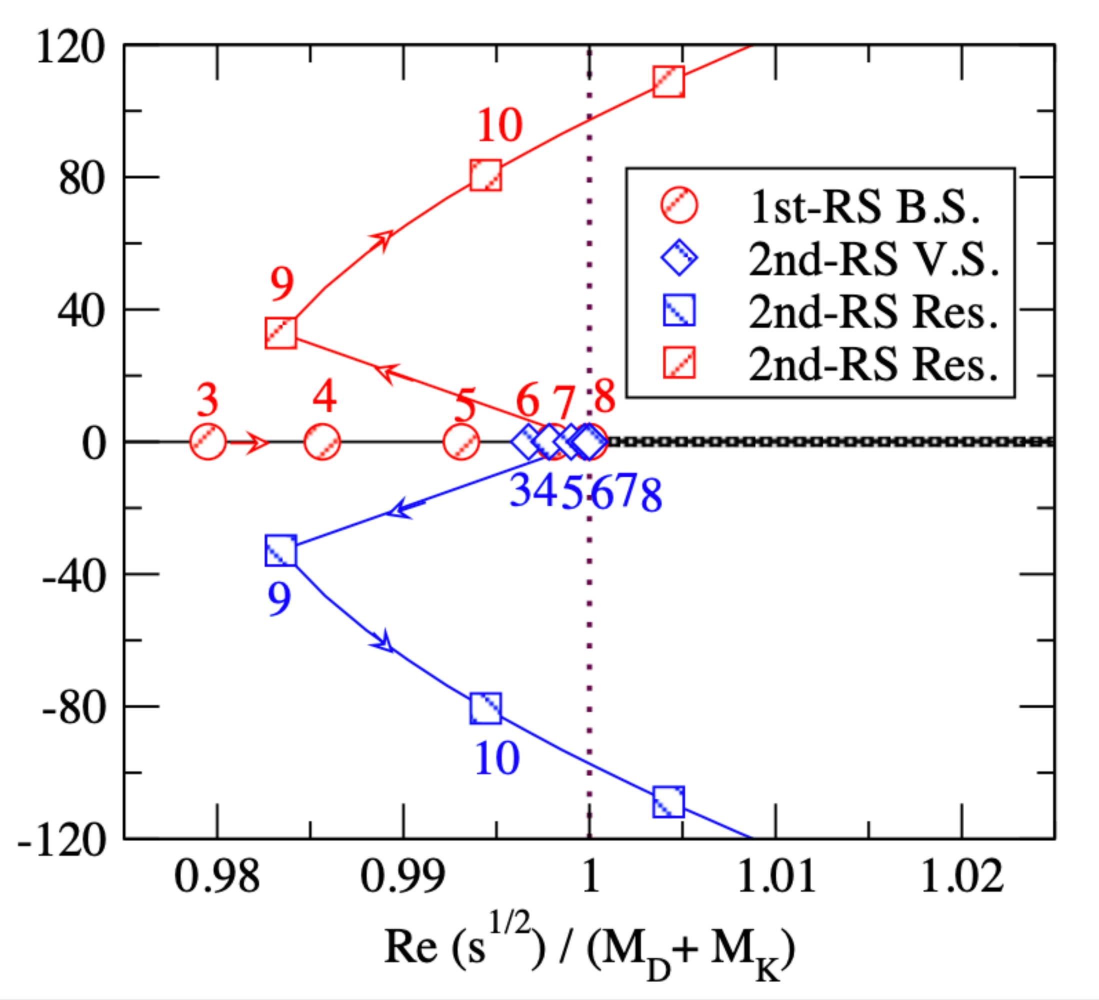




M_π trajectory of the resonance $D^*(2100)$



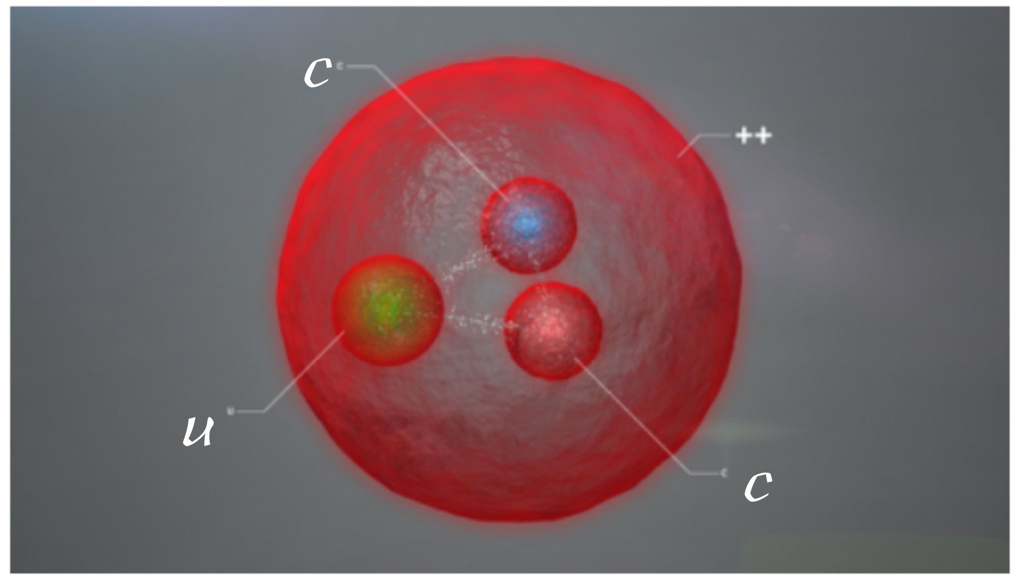
N_C trajectory of the state $D_s^*(2317)$



do not behave like a standard quark-antiquark meson at large N_c

Selected Progress 3: Extensions in Heavy Flavour Physics

✦ **Time to study doubly charmed baryons (DCB)**



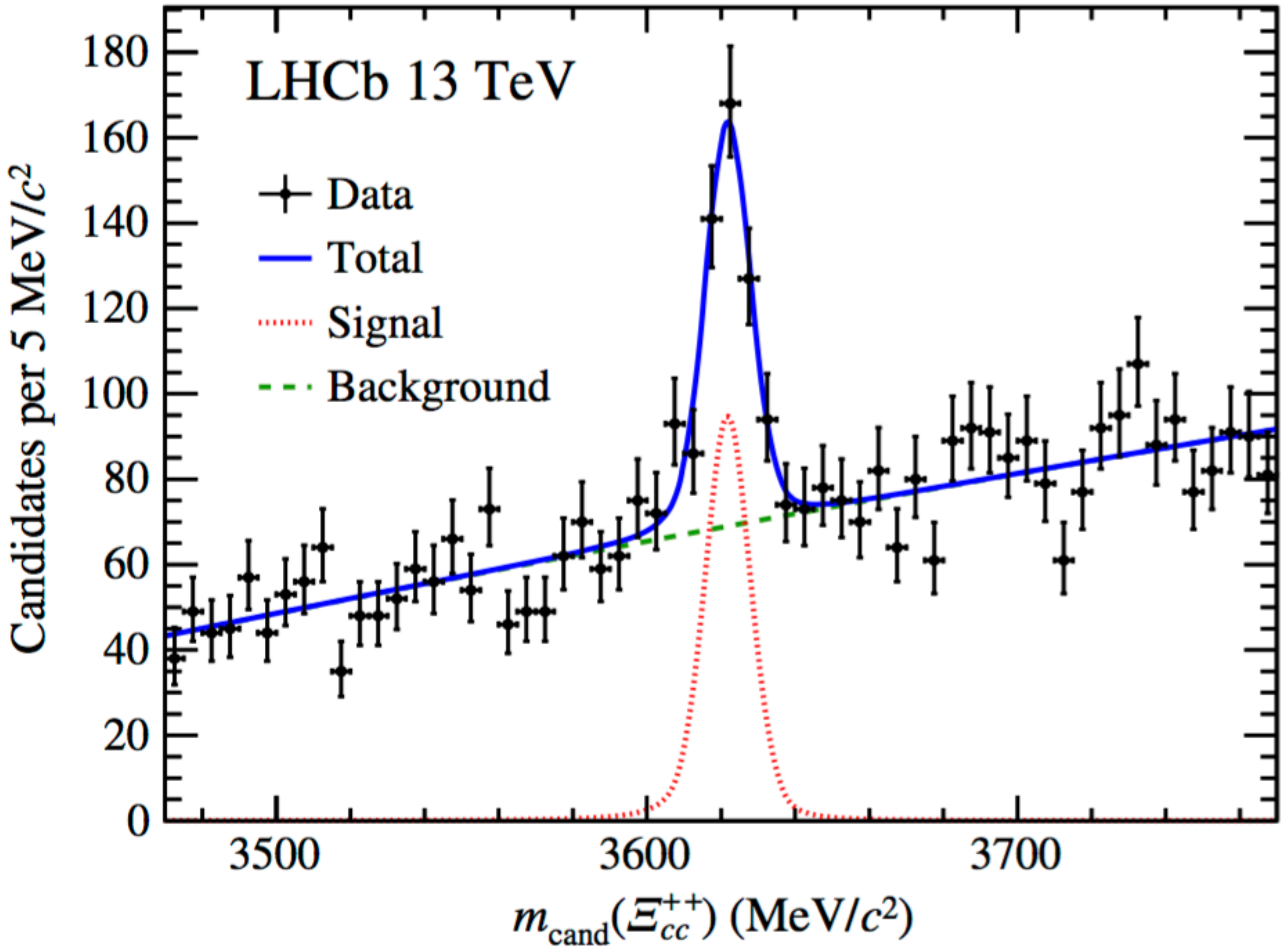
LHCb announces a charming new particle

The LHCb experiment has reported the observation of a baryon containing two charm quarks and one up quark

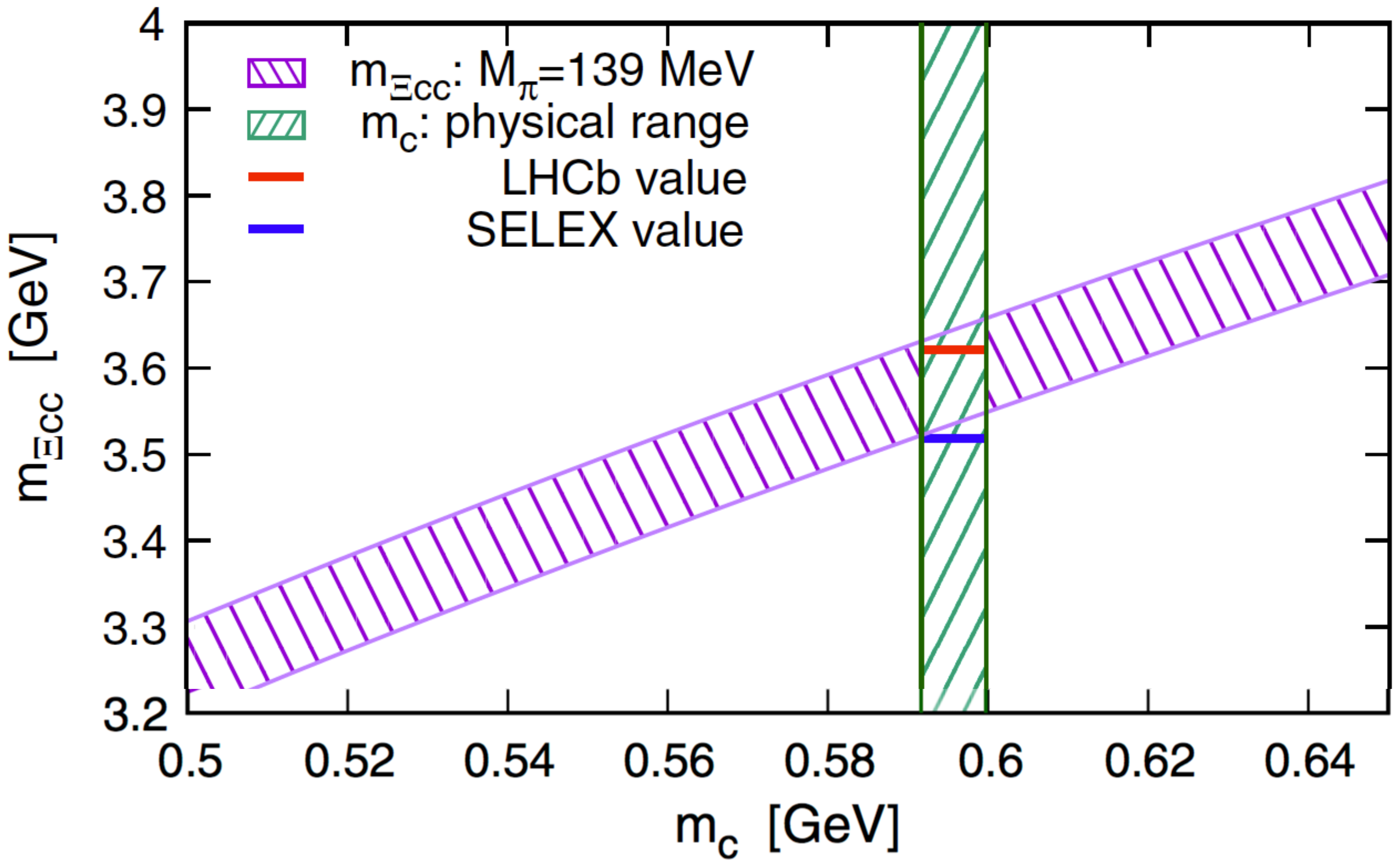
News | Experiments | 06 July, 2017

LHCb-2017: $m(\Xi_{cc}^{++}) = (3621.4 \pm 0.78)$ MeV

SELEX-2002: $m(\Xi_{cc}^{+}) = (3519 \pm 2)$ MeV



[LHCb, PRL119(2017)112001]



[DLY, PRD97(2018)034012]

✦ **Towards a new paradigm for negative-parity doubly charmed baryons?**

➡**Interactions between Goldstone Bosons and DCB**

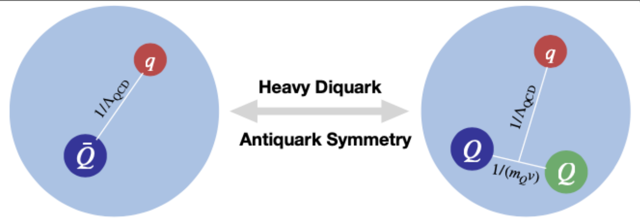
Selected Progress 3: Extensions in Heavy Flavour Physics

✦ Interactions between Goldstone Bosons and DCB

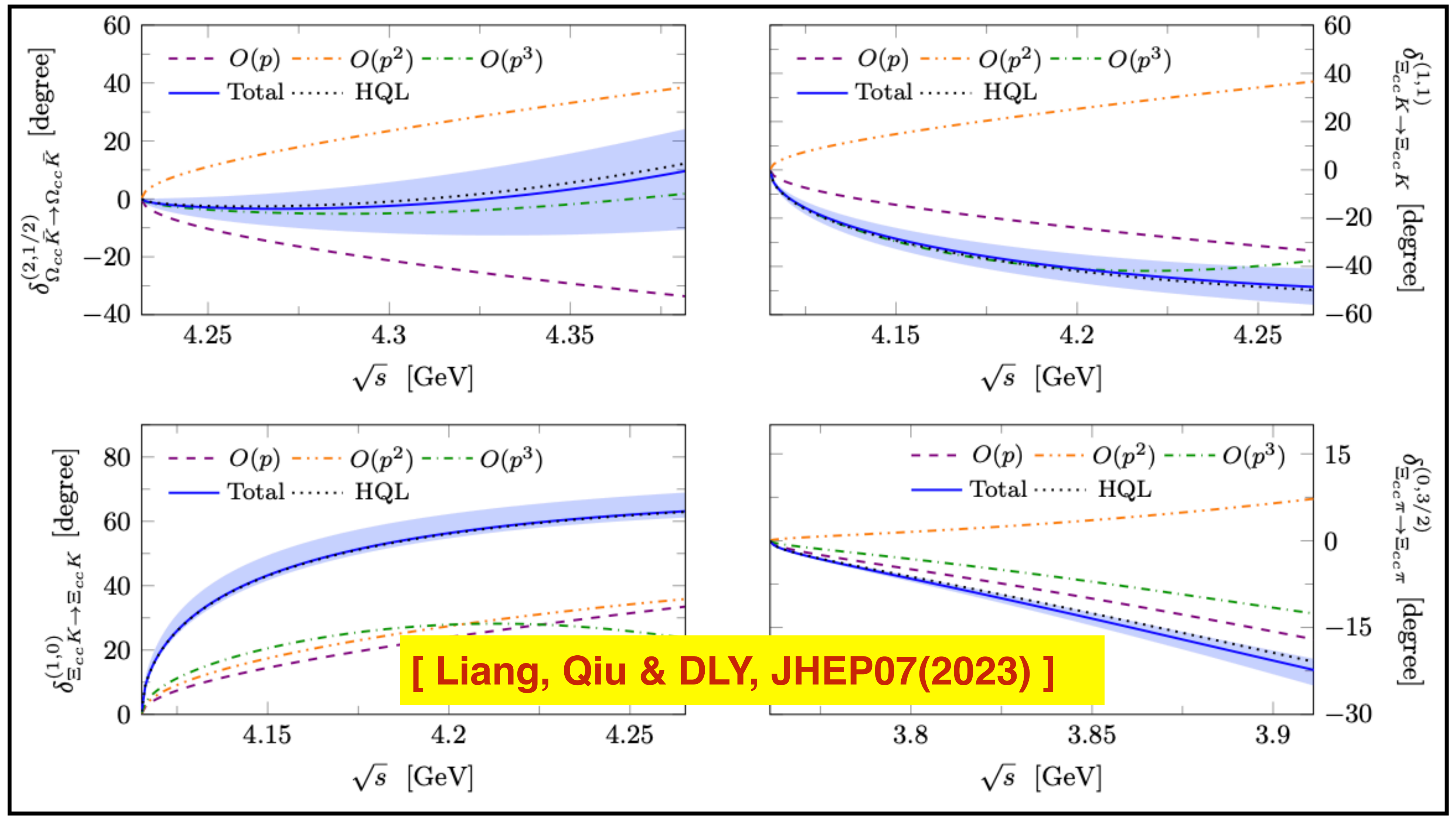
$$\mathcal{L}_{M\psi} = \bar{\psi}\Big\{\Big[(i\not{D} - m) + \frac{g_A}{2}\not{u}\gamma_5\Big] + \sum_{i=1}^{8} b_i\mathcal{O}_i^{(2)} + \sum_{j=1}^{32} c_j\mathcal{O}_j^{(3)} + \sum_{k=1}^{218} d_k\mathcal{O}_k^{(4)} + \cdots\Big\}\psi$$

		Relativistic	Non-relativistic
$O(p^2)$	1	$\langle\chi_+\rangle$	$\langle\chi_+\rangle$
	2	$\widetilde{\chi}_+$	$\widetilde{\chi}_+$
	3	u^2	u^2
	4	$\langle u^2\rangle$	$\langle u^2\rangle$
	5	$\{u^\mu, u^\nu\}D_{\mu\nu} + h.c.$	$-8m^2(v\cdot u)^2$
	6	$\langle u^\mu u^\nu\rangle D_{\mu\nu} + h.c.$	$-4m^2\langle(v\cdot u)^2\rangle$
	7	$i[u^\mu, u^\nu]\sigma_{\mu\nu}$	$2[S_\mu, S_\nu][u^\mu, u^\nu]$
	8	[illegible]	$S_\nu]f_+^{\mu\nu}$

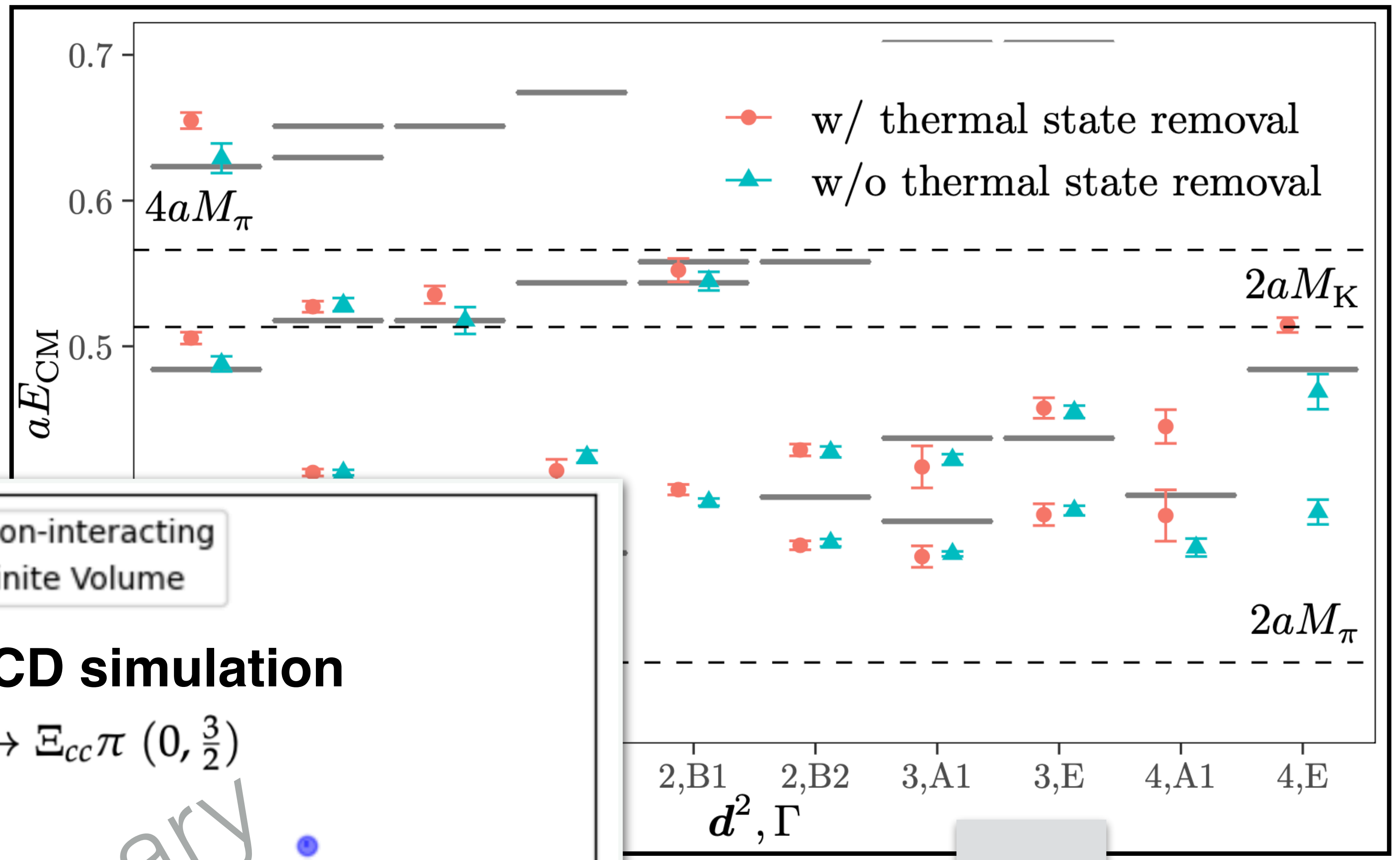
[Qiu & DLY, PRD103(2021)]

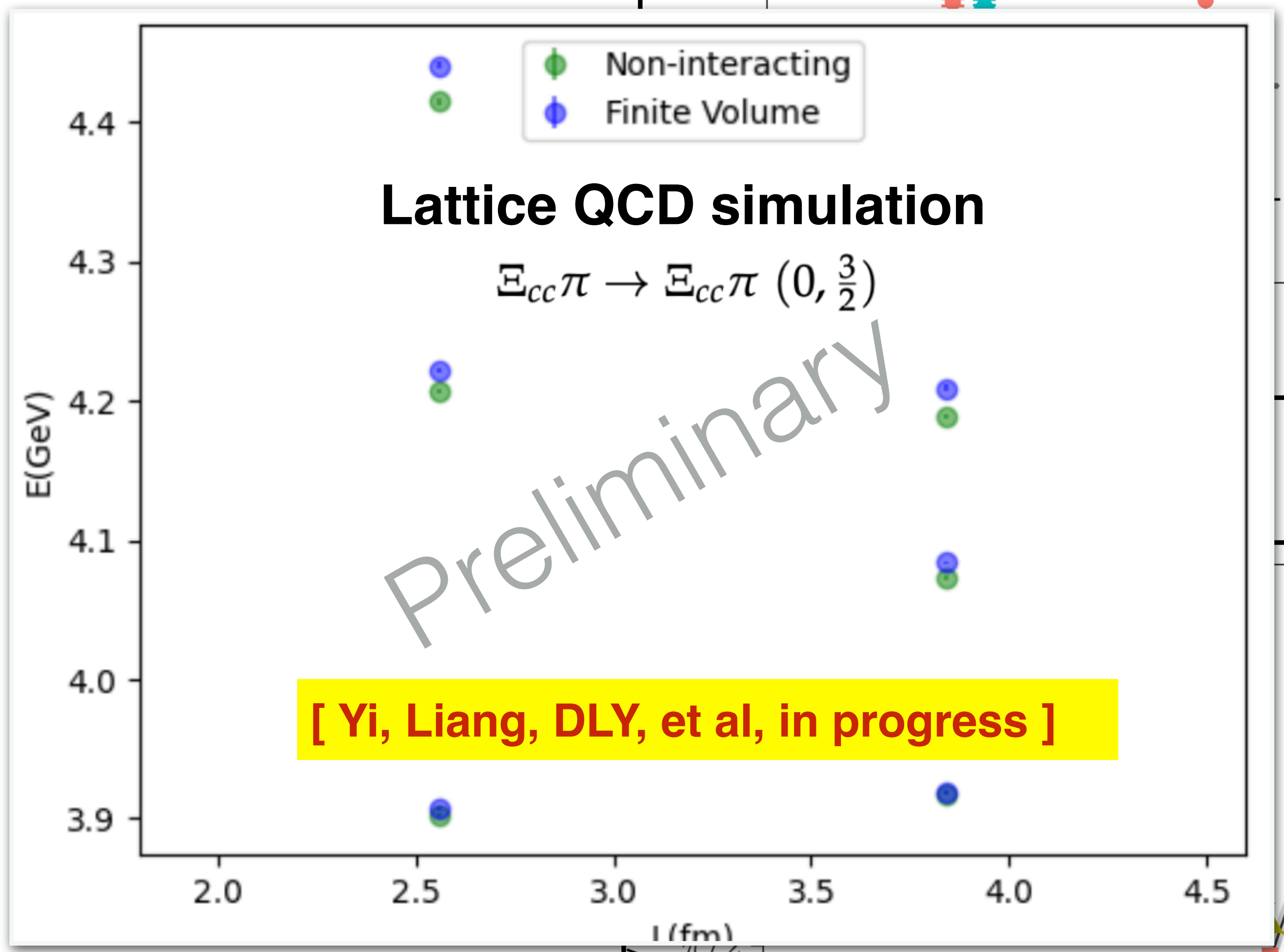


Heavy Diquark-Antiquark Symmetry

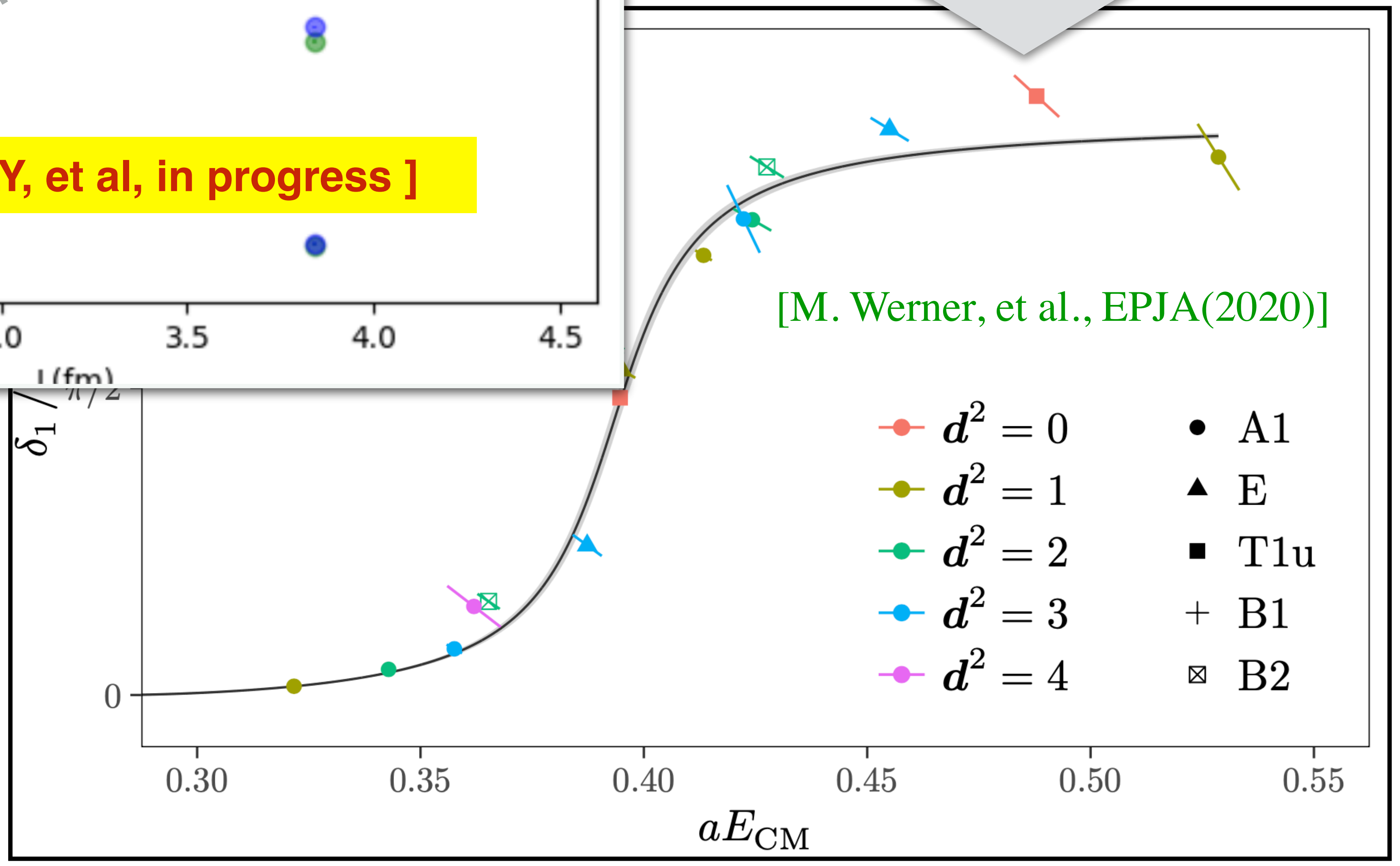


[Liang, Qiu & DLY, JHEP07(2023)]





[Yi, Liang, DLY, et al, in progress]

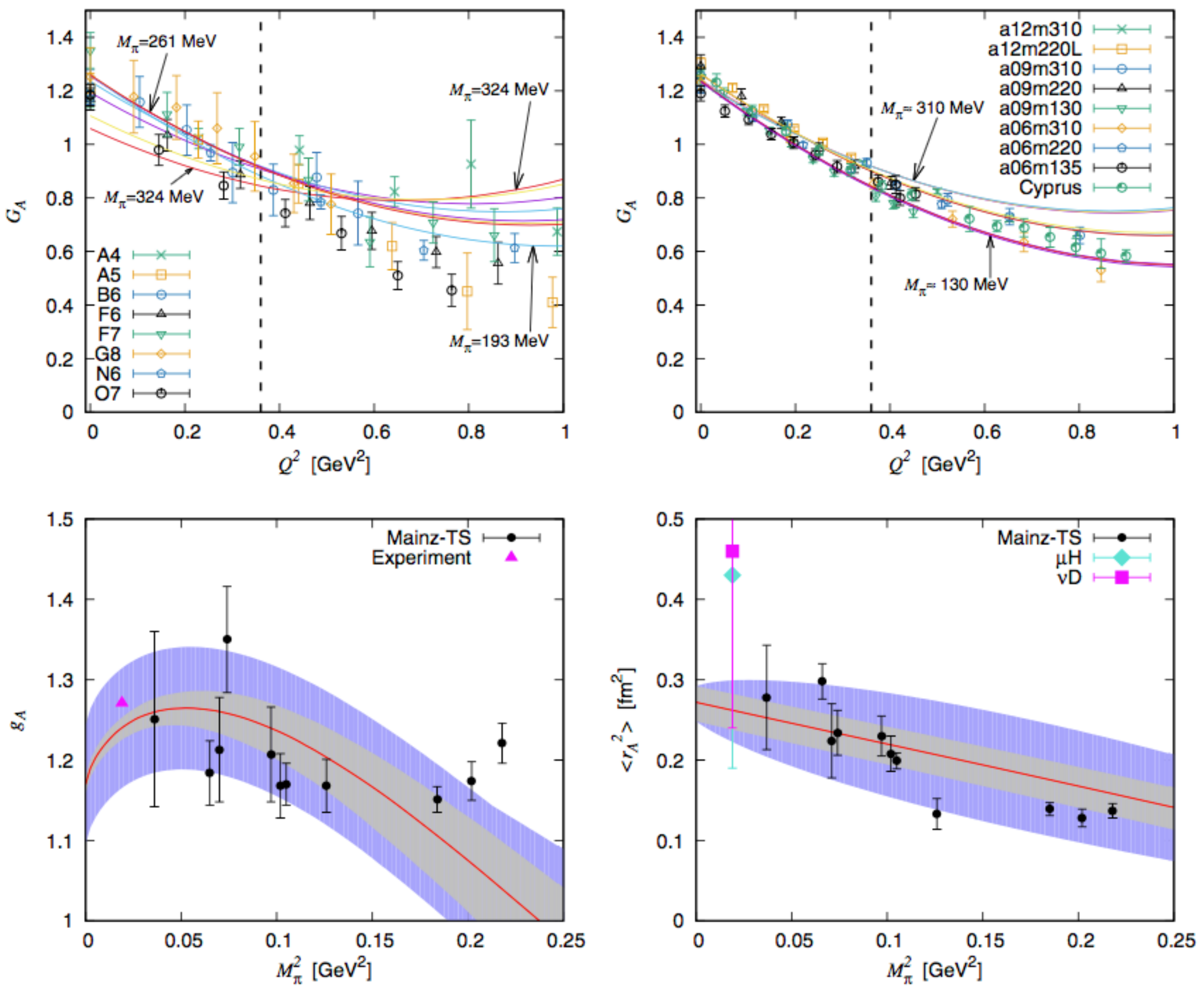


[M. Werner, et al., EPJA(2020)]

Selected Progress 4: Combinations with Lattice Techniques

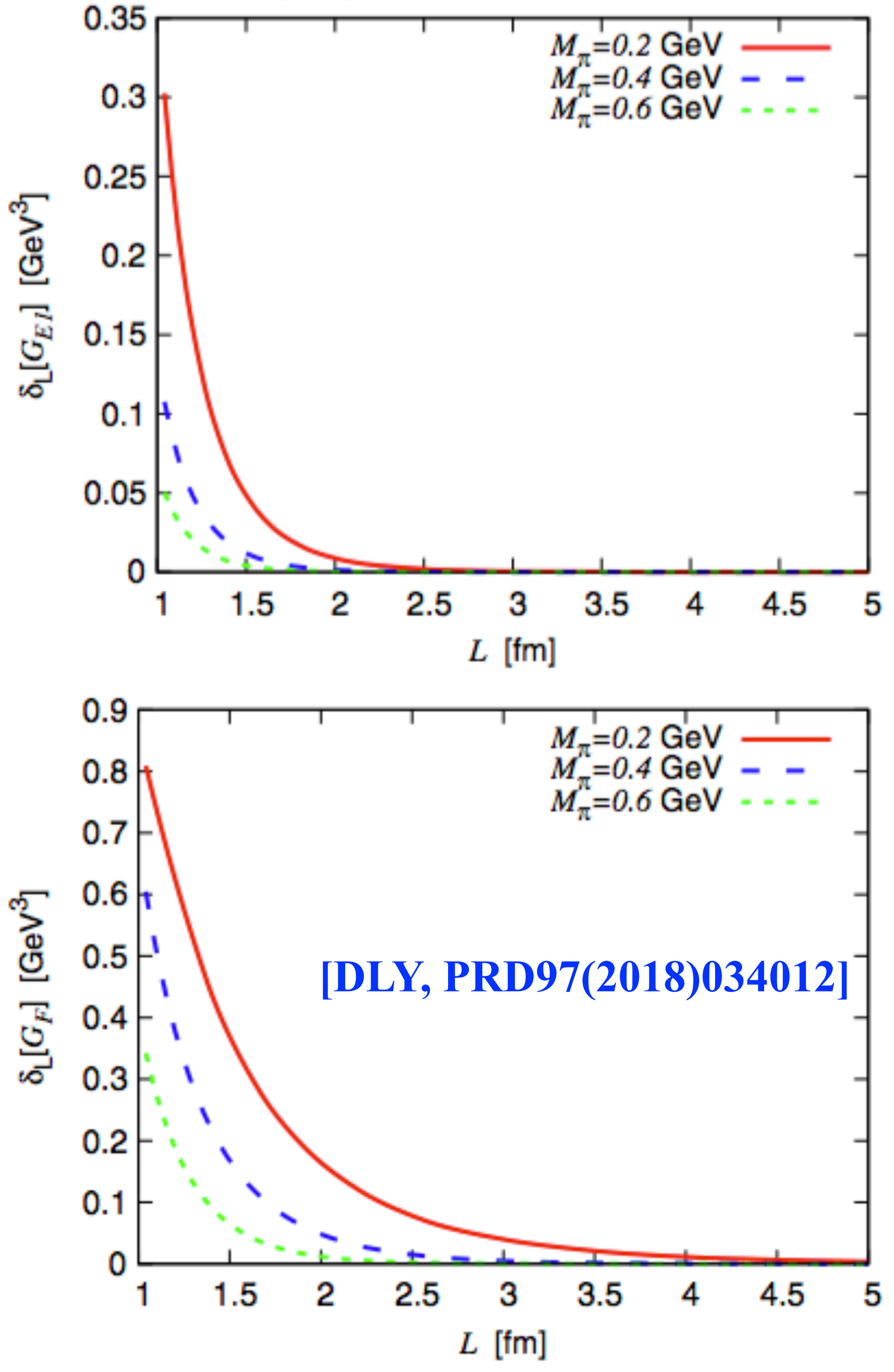
✦ Chiral Extrapolation & Finite volume correction…

[DLY, Alvarez-Ruso and Vicente-Vacas PRD96(2017)116022]



Axial coupling and radius of the nucleon

$$\delta_L[\mathcal{Q}] = \mathcal{Q}(L) - \mathcal{Q}(\infty)$$



Lattice-size (L) dependence

Finite volume correction has to be computed case by case

A unified formulation of one-loop tensor integrals for finite volume effects

[Liang, DLY, JHEP12(2022)]

$$\mathcal{A}_a(L) = \frac{3g_A^2 m_N}{4F_\pi^2}\left\{\widetilde{A}_0(m_N^2; L) + M_\pi^2 \widetilde{B}_0(m_N^2, m_N^2, M_\pi^2; L)\right\},$$

$$\mathcal{B}_a(L) = \frac{1}{m_N}\mathcal{A}_a(L), \quad m_N^{\rm FVC}(L) = \left[\mathcal{A}(L) + m_N\mathcal{B}(L)\right]$$

Systematical computation of FVC in ChPT

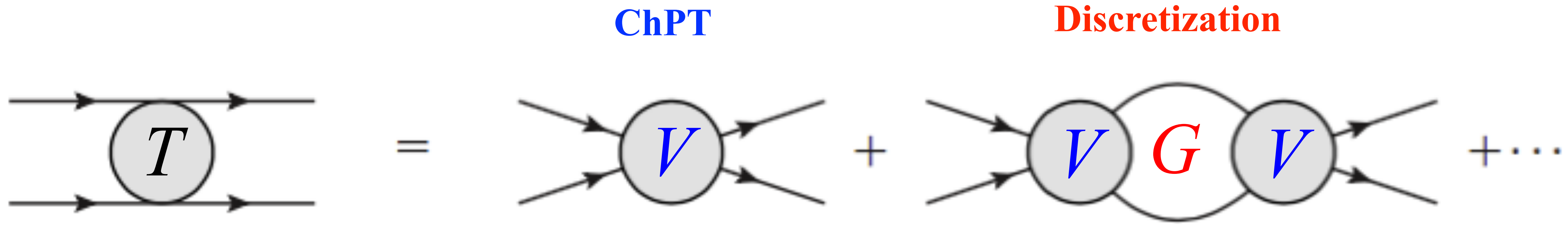
Automatization

Selected Progress 4: Combinations with Lattice Techniques

✦ **Unitarized ChPT in a finite volume**

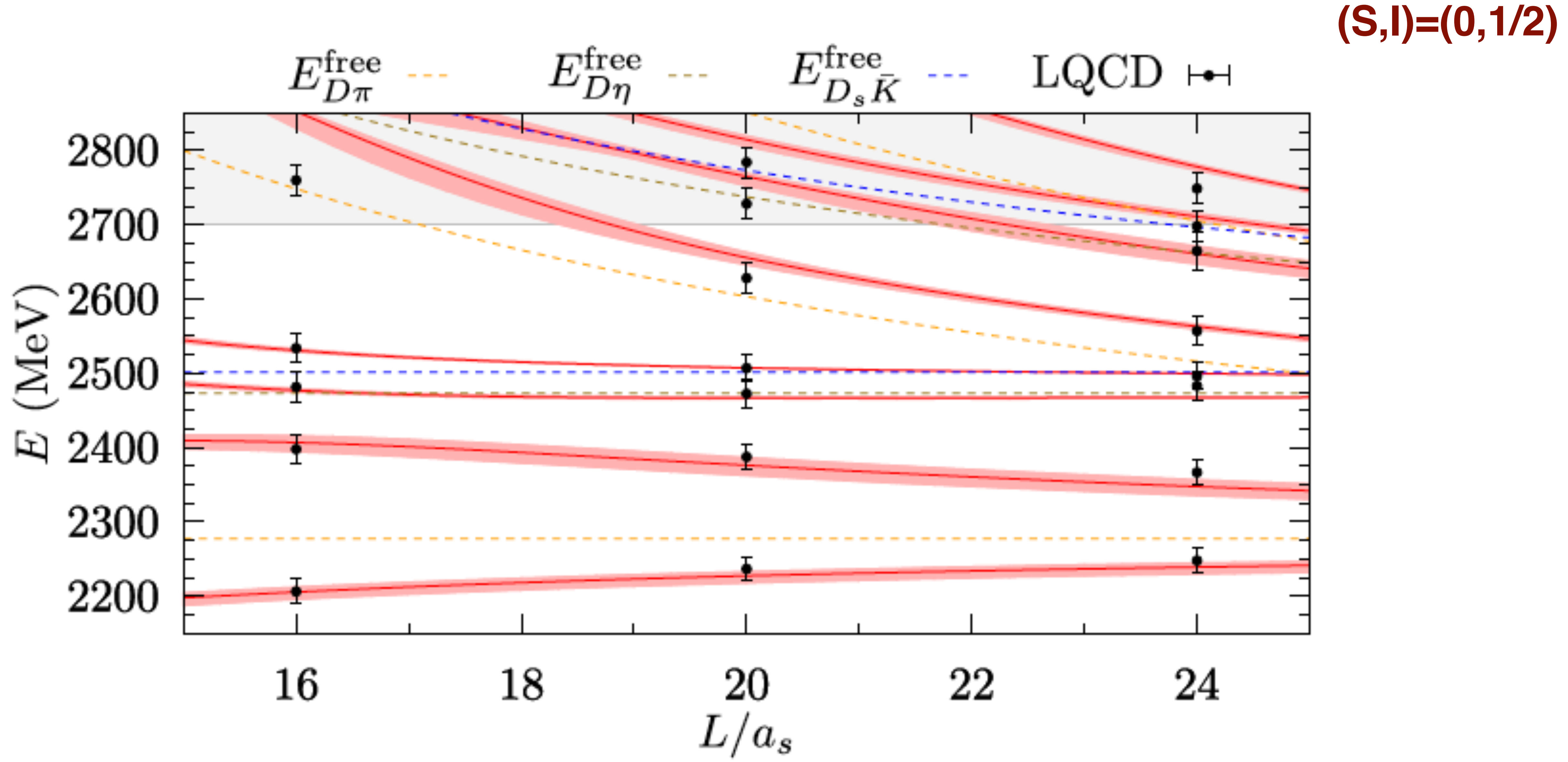
$$T = V \bullet (1 - V \bullet G)^{-1}$$

[Doring, Meissner, Oset and Rusetsky, EPJA(2011)139]

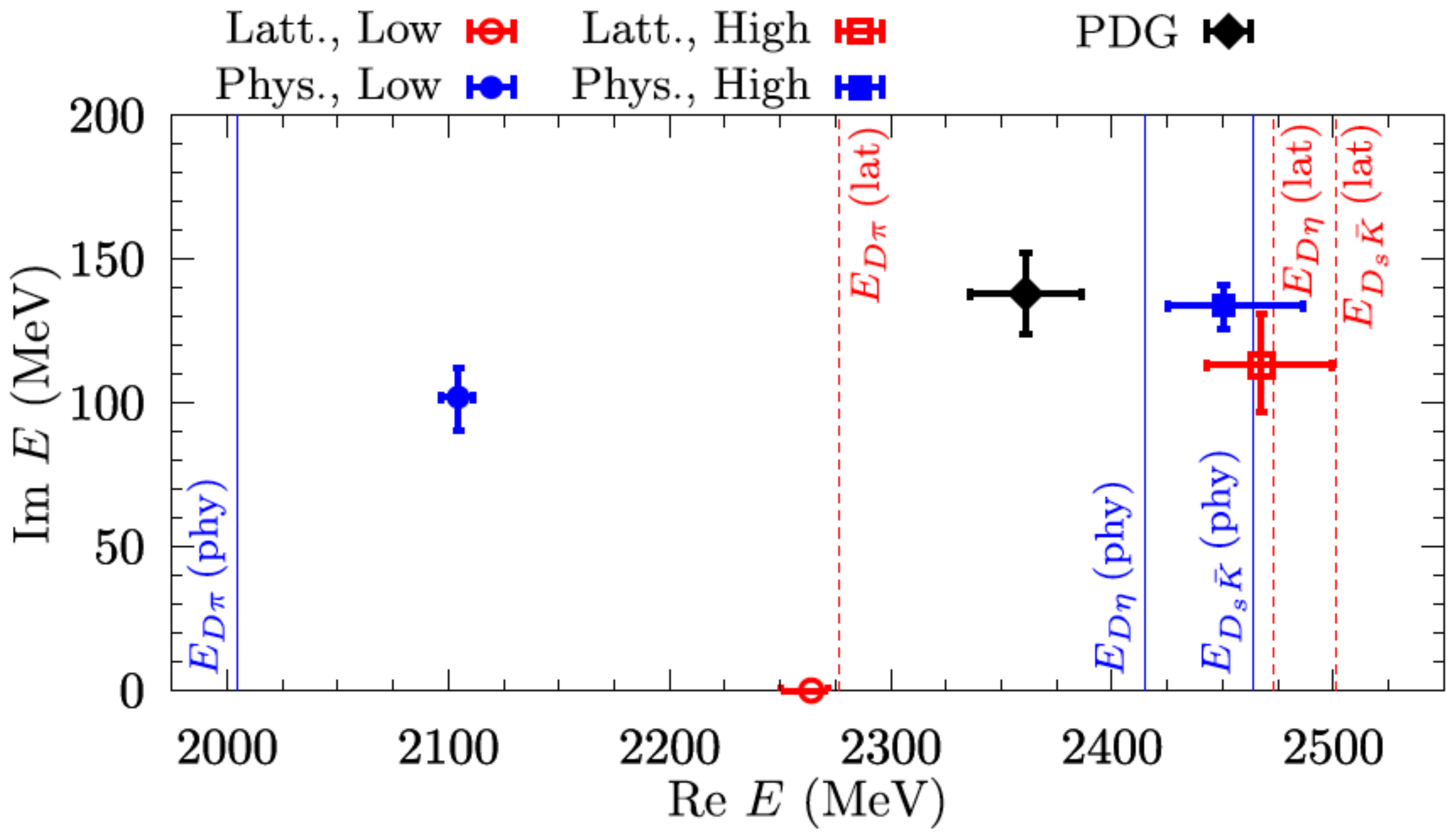


✦ **Applications of NLO potentials by Liu, et al**

Energy Level



Two-pole structure of D*(2400)



Selected Progress 4: Combinations with Lattice Techniques

✦ Nuclear Lattice Chiral Effective Field Theory (NLEFT)

Lattice QCD

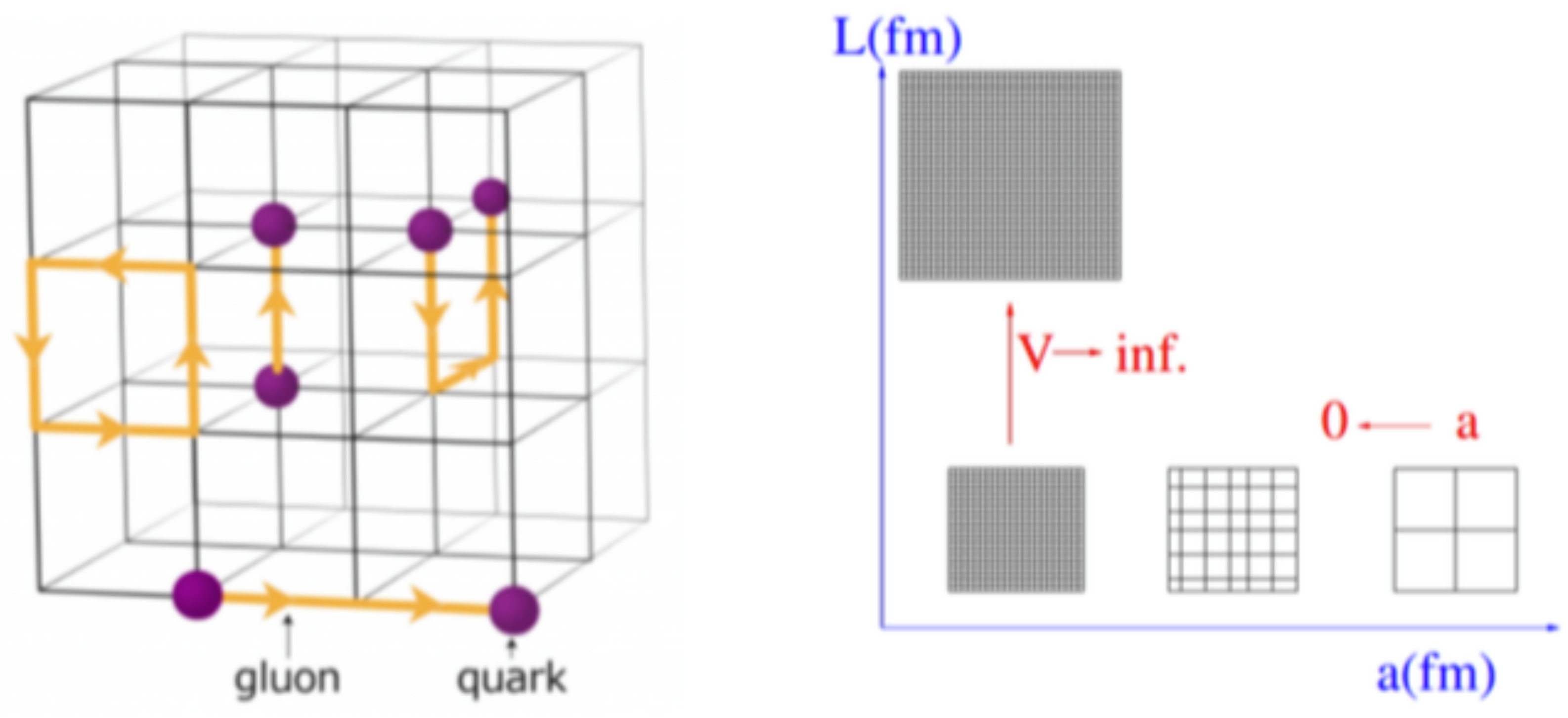
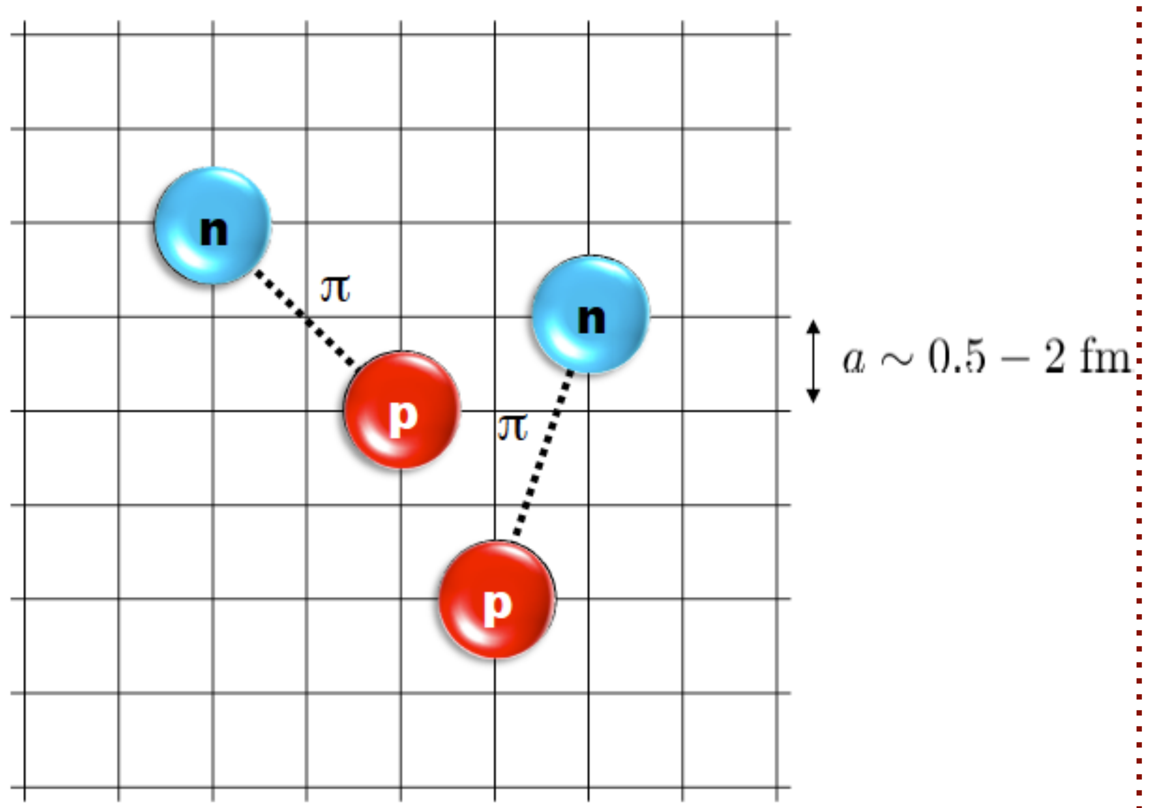
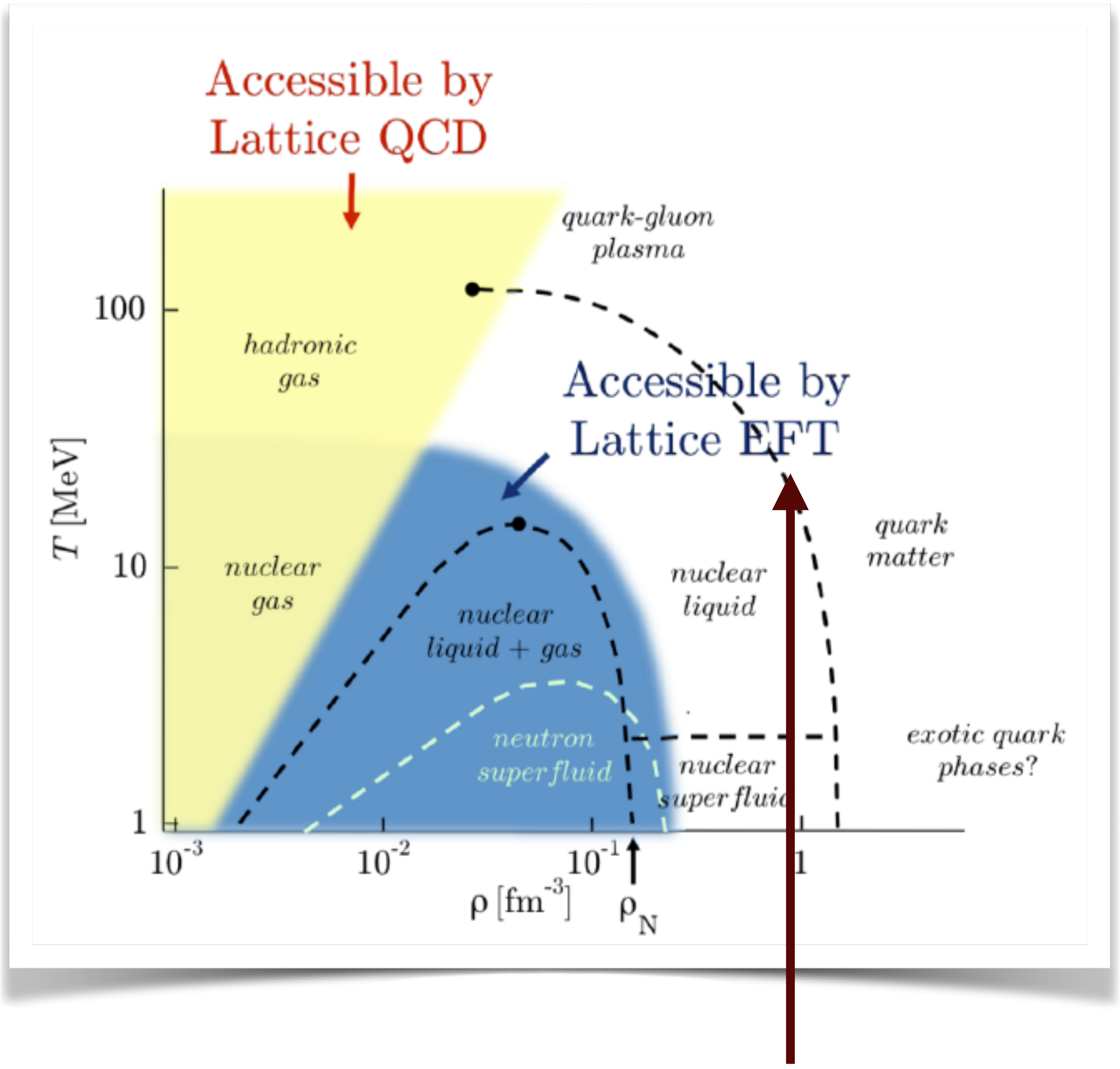


Figure: Taken from S.Ryan MESON2016

Nuclear Lattice Simulation



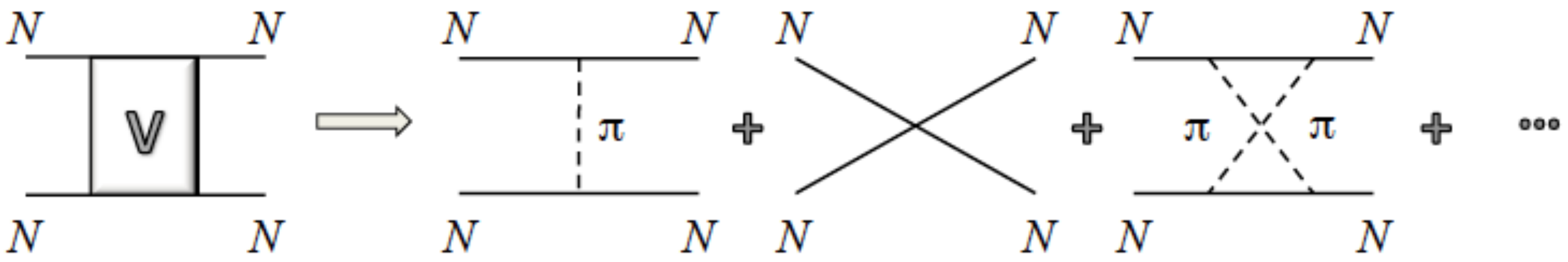
Review: D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)
TALENT summer school lectures: qmc2016.wordpress.ncsu.edu



BChPT is a widely-used EFT to be used more widely

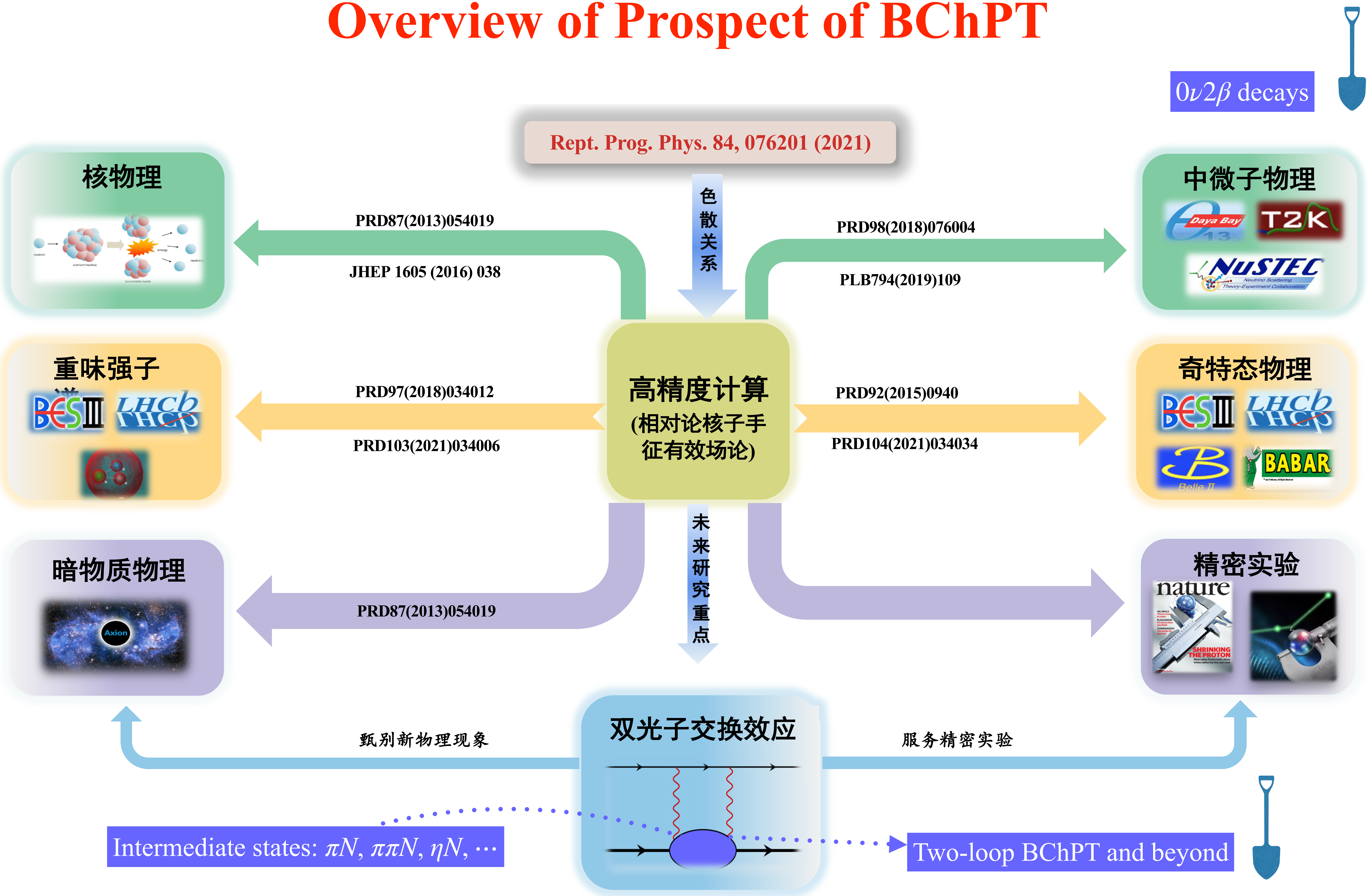
✦ Inputs: chiral effective Lagrangians

Effective Potential:

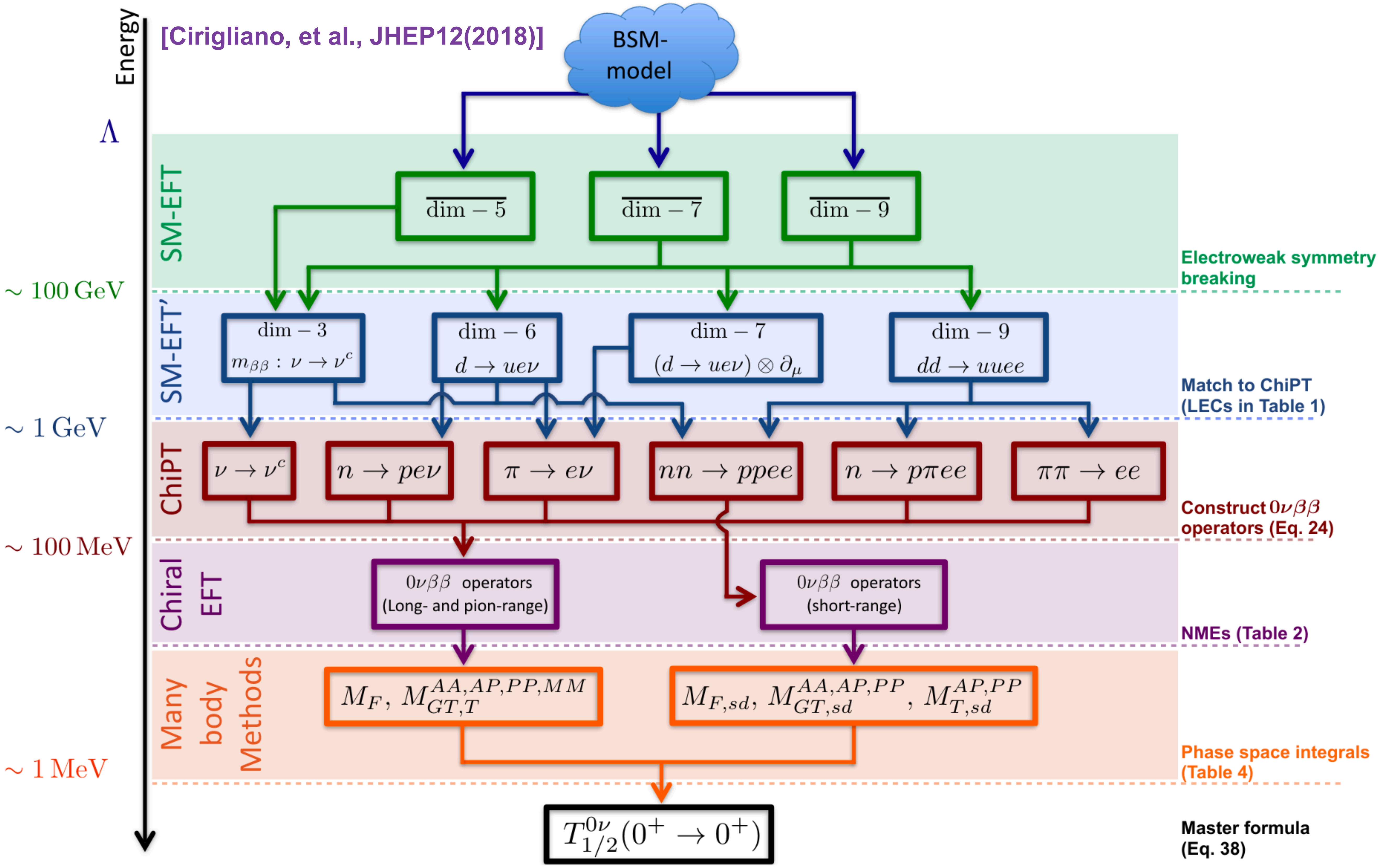


➡ πN scatterings are sub-processes ➡ one-nucleon sector to two- or multi-nucleon sectors ➡ study also strangeness?…

Overview of Prospect of BChPT



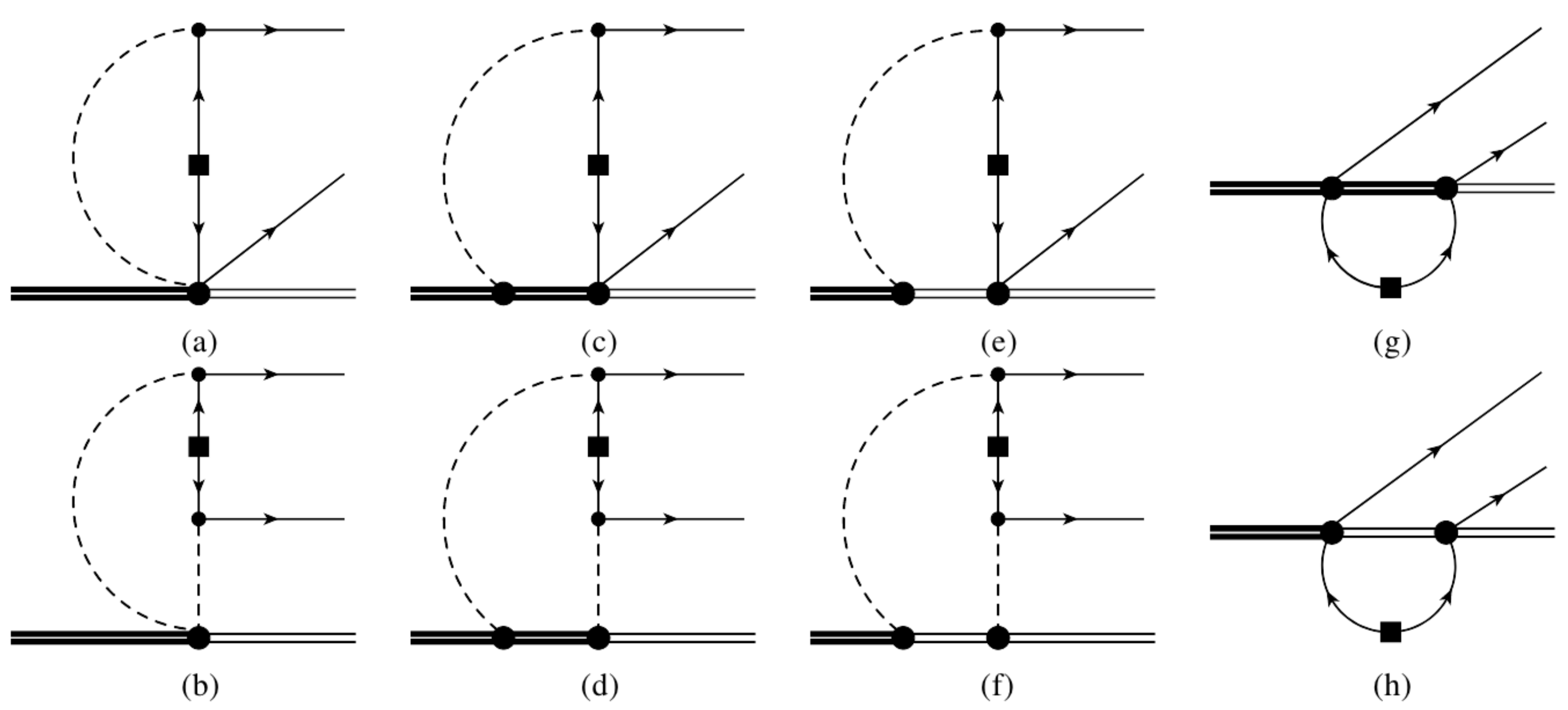
Prospect I: neutrinoless double beta decay



- $\Delta^- \to p e^- e^-$
- $\Sigma^- \to p e^- e^- \ldots$

$\Delta L = 2$ operator

$$\mathscr{L}_{\nu\nu} = -\frac{1}{2} m_{\beta\beta} \left(\nu_L^T C \nu_L + \bar{\nu}_L C^\dagger \bar{\nu}_L^T\right)$$



work in progress

- **Pion-range coupling via resonance saturation + lattice QCD.**
- **Hyperon factory @ BESIII & STCF** $J/\psi \to \Sigma\bar{\Sigma}$

- $K^\pm \to \pi^\mp l_\alpha^\pm l_\beta^\pm$ [Liao, Ma and Wang, JHEP03(2020)]
- $\pi^-\pi^- \to e^- e^-$ [Cirigliano, et al, PRC97(2018)]

Light neutrino, heavy neutrino,….

$\nu_e = U_{ei}\nu_i + R_{ej}N_j \qquad i = 1, 2, 3$

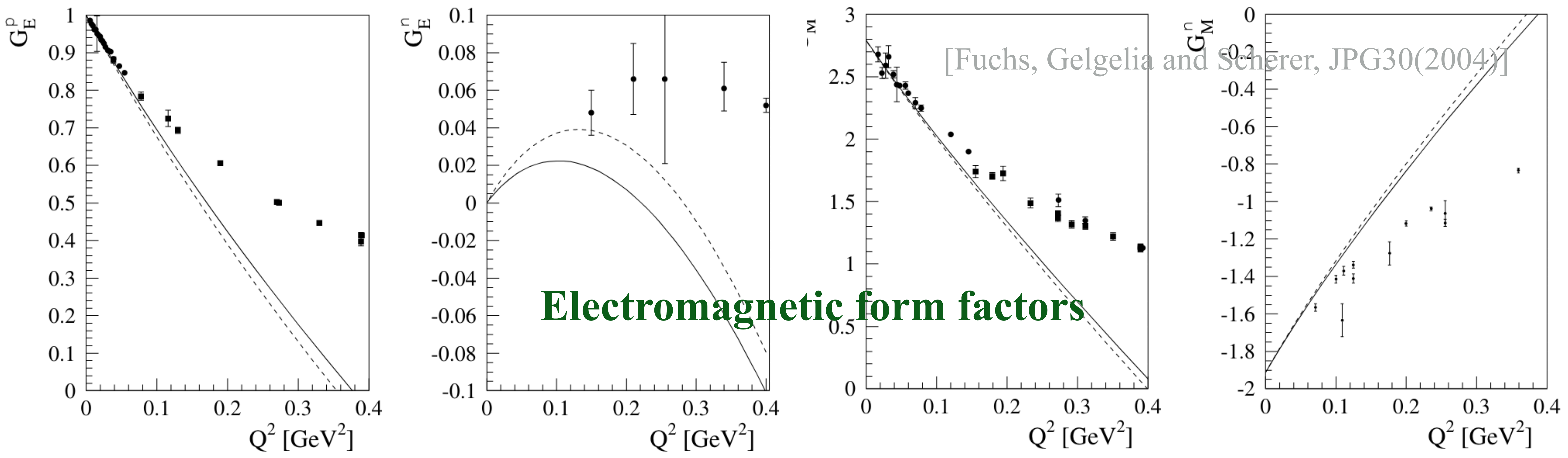
- ν_i is massive active neutrino, N sterile neutrino
- U is PMNS matrix, R mixing matrix of N

Prospect II: Two-loop BChPT

✦ Meson sector

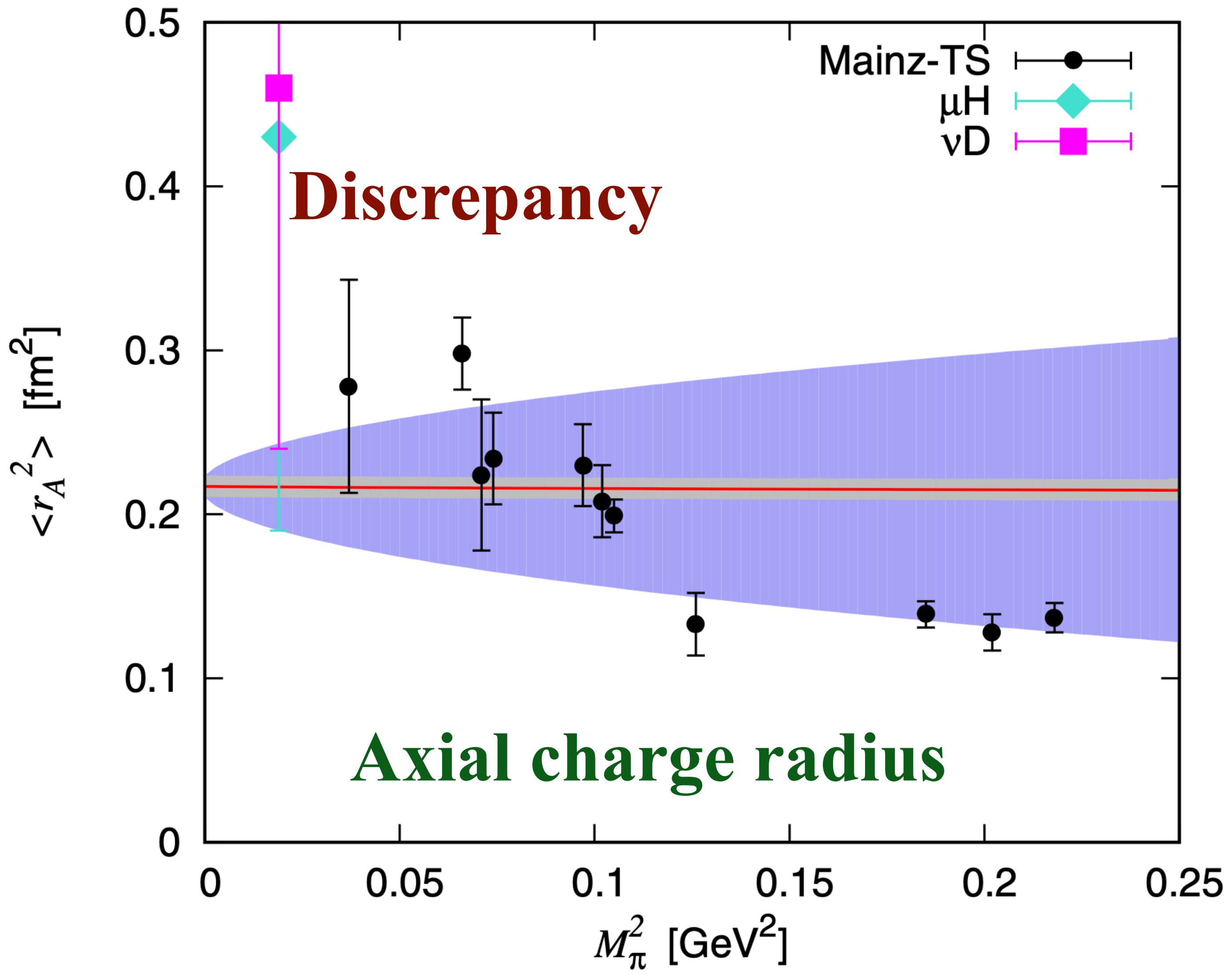
- $\pi\pi$ scattering, πK scattering, pion mass, pion decay constant, $K_{\ell 4}$, Electromagnetic form factor, …

[Bijnens, Colangelo, Ecker, Gasser and Sainio, PLB374(1996)]
[Bijnens, Colangelo, Ecker, Gasser and Sainio, NPB508(1997)]
[Bijnens, Talavera, JHEP03(2002)]
[Kaiser, Schweizer, JHEP06(2006)]
[Kaiser, JHEP09(2007)]
[Bijnens,Truedsson, JHEP11(2017)]
…



✦ Baryon sector

- Nucleon mass up to $O(p^5)$ in HBChPT [McGovern & Birse, PLB446(1999)]
- Nucleon axial-vector coupling beyond one-loop (renormalization group techniques) [Bernard & Meissner, PLB639(2006)]
- Nucleon mass to $O(p^6)$ in IR-BChPT [Schindeler, Djukanovic, Gegelia & Scherer, PLB649(2007), NPA803(2008)]



The full one-loop chiral results can only describe data at very low energies

Two-loop contributions should play a important role!

$$G_A(t) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle t + \mathcal{O}(t^2)\right]$$

LO contribution is of one-loop order!

✦ Long-standing challenges

1. Computation of two-loop integrals

Auxiliary mass flow (AMFlow) [Liu and Ma, Comput.Phys.Commun. 283 (2023)]

2. Power counting breaking issue

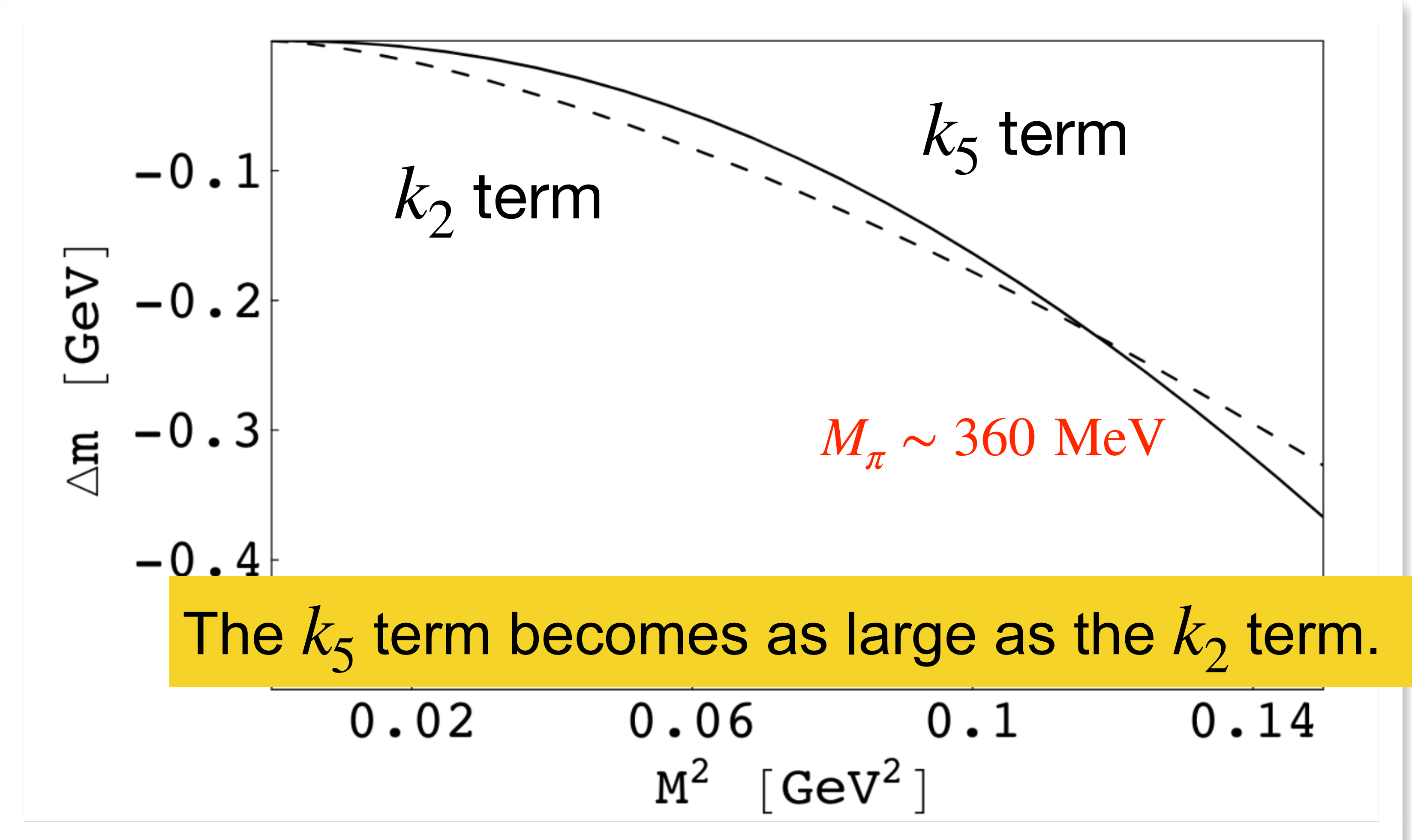
Applicability of EOMS at two-loop order?

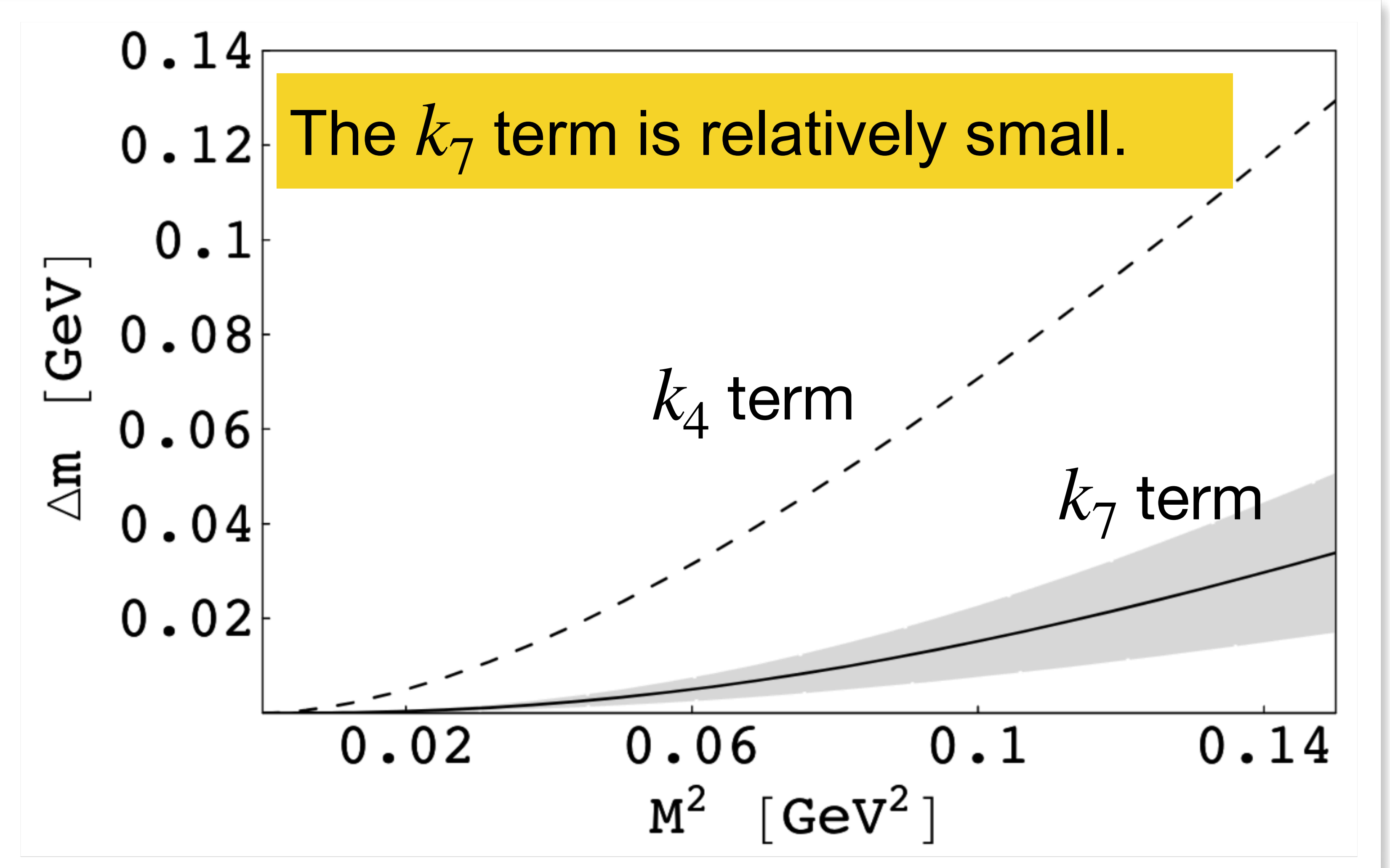
Nucleon mass at two-loop order

✦ Previous works

- **Heavy baryon formalism up to $O(p^5)$** [McGovern & Birse, PLB446(1999)]
- **Infrared Regularisation prescription up to $O(p^6)$**
  [Schindeler, Djukanovic, Gegelia & Scherer, PLB649(2007), NPA803(2008)]

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6$$





✦ Necessity of a leading two-loop study with EOMS scheme

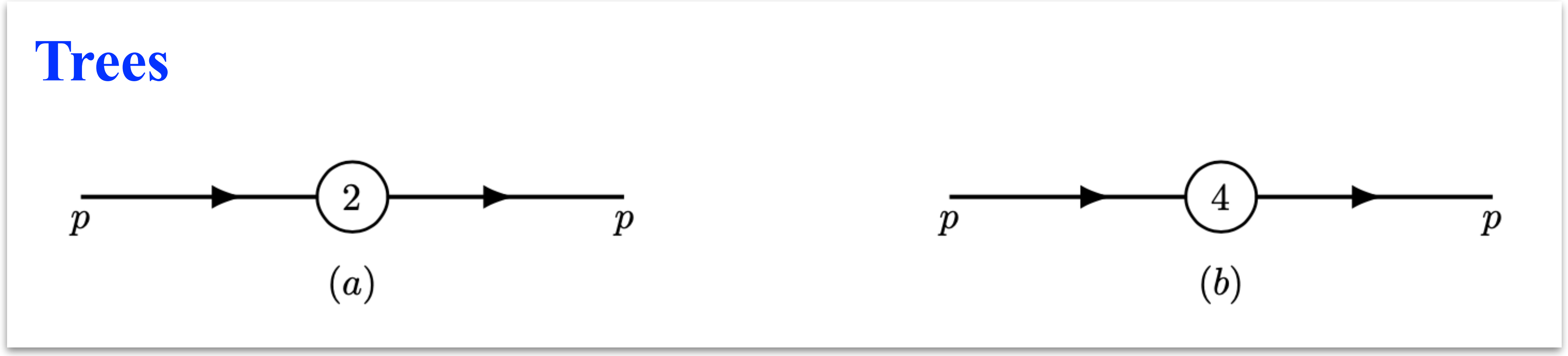
- **Leading two-loop contribution [$O(p^5)$] is sizeable, while the next one [$O(p^6)$] is slight.**
- **A reliable chiral extrapolation relies on correct analytic property.**

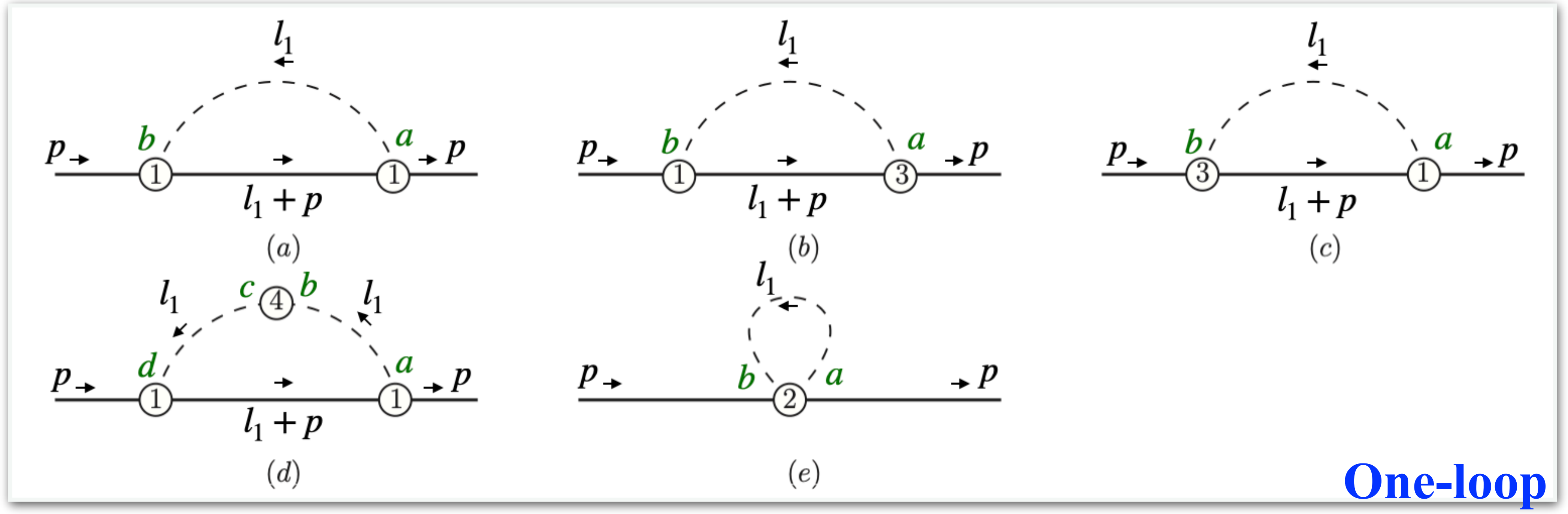
EOMS at $O(p^5)$!

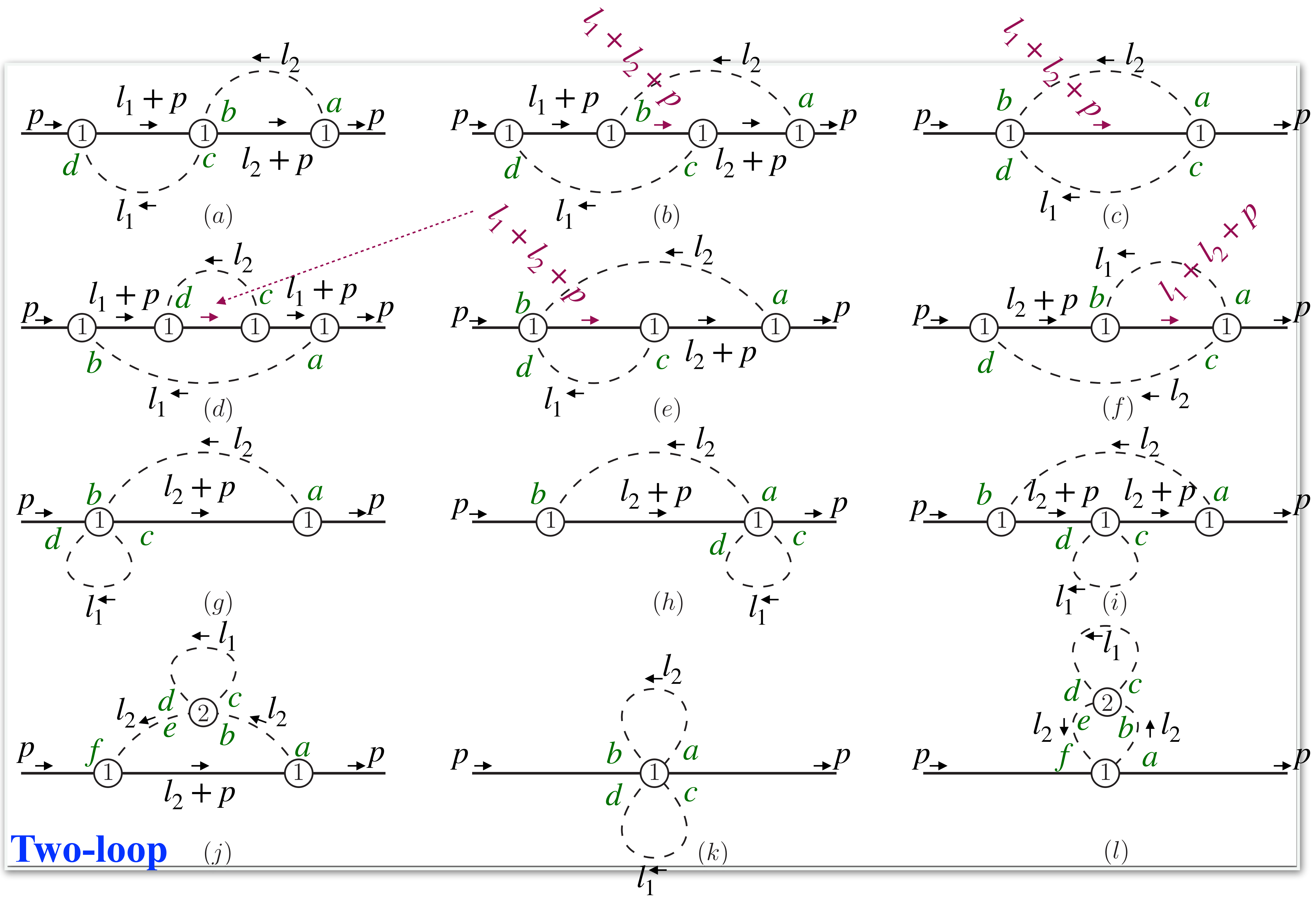
Nucleon mass at two-loop order

✦ Chiral effective Lagrangians and Feynman Diagrams

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$







✦ Calculation of self-energies

xAmplCalc — A mathematica package

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5} = \iint \frac{d^d\ell_1}{(i\pi^{d/2})}\frac{d^d\ell_2}{(i\pi^{d/2})}\frac{1}{\mathscr{D}_1^{\nu_1}\mathscr{D}_2^{\nu_2}\mathscr{D}_3^{\nu_3}\mathscr{D}_4^{\nu_4}\mathscr{D}_5^{\nu_5}}$$

ν_i's are integers

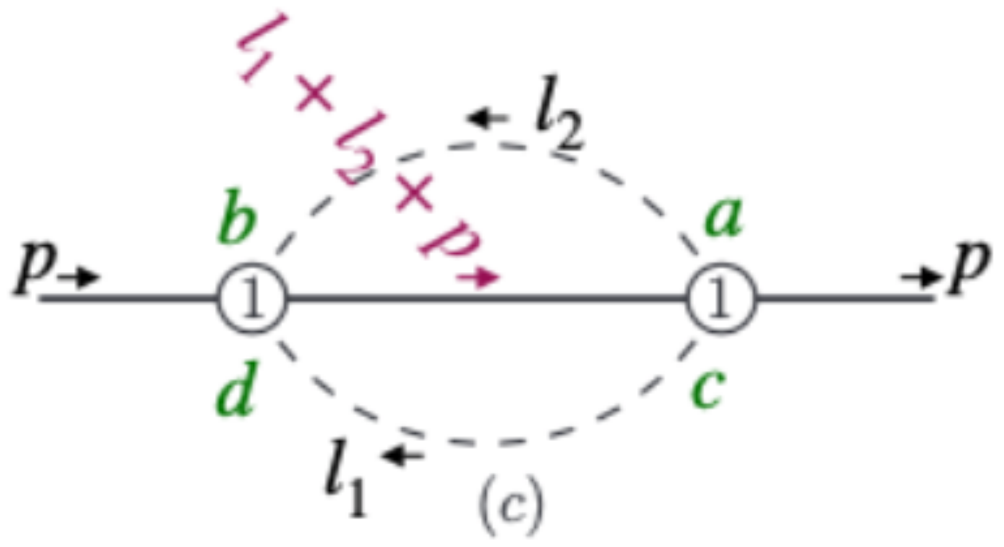
Number of independent scalar products

$$N = L \times E + \frac{1}{2}L(L+1)$$

$$\mathscr{D}_1 = \ell_1^2 - M^2\,,\mathscr{D}_2 = \ell_2^2 - M^2\,,\mathscr{D}_3 = (p+\ell_1+\ell_2)^2 - m^2\,,\mathscr{D}_4 = (p+\ell_1)^2 - m^2\,,\mathscr{D}_5 = (p+\ell_2)^2 - m^2\,.$$

IBP procedure and DSR relations

✦ Reduction to master integrals



$$\Sigma_c = -\frac{1}{32F_0^4}\int \frac{d^d\ell_1}{(i\pi^{d/2})}\frac{d^d\ell_2}{(i\pi^{d/2})}\left[\frac{(\ell\!\!/_2-\ell\!\!/_1)((p\!\!/+\ell\!\!/_1+\ell\!\!/_2)+m)(\ell\!\!/_2-\ell\!\!/_1)}{[\ell_1^2-M^2][\ell_2^2-M^2][(p+\ell_1+\ell_2)^2-m^2]}\right]$$

$$\Sigma_c = -\frac{1}{32F_0^4}\int \frac{d^d\ell_1}{(i\pi^{d/2})}\frac{d^d\ell_2}{(i\pi^{d/2})}\frac{1}{\mathcal{D}_1\mathcal{D}_2\mathcal{D}_3}\Big[m[\mathcal{D}_1+\mathcal{D}_2-\mathcal{D}_3+\mathcal{D}_4+\mathcal{D}_5+(m^2+2M^2-s)]$$
$$+[\mathcal{D}_3-\mathcal{D}_4-\mathcal{D}_5-\mathcal{D}_1-\mathcal{D}_2-(m^2+2M^2-s)]p\!\!/-[2\mathcal{D}_2-\mathcal{D}_3+2\mathcal{D}_5+(m^2+2M^2-s)]\ell\!\!/_1$$
$$-[2\mathcal{D}_1-\mathcal{D}_3+2\mathcal{D}_4+(m^2+2M^2-s)]\ell\!\!/_2\Big] \quad = A + p\!\!/B$$

$$m_N^{(2c)} = -\frac{3}{16m_NF^4}\Big|2M_\pi^2\left(2I_{11100}M_\pi^2+2I_{01100}+2I_{10100}-I_{11000}\right)+4I_{00100}-I_{01000}-I_{10000}+I_{110(-1)0}+I_{1100(-1)}-4I_{111(-1)(-1)}\Big| \,,$$

Scalar integrals in d dim.

Integration by parts (IBP) — Laporta algorithm

Master integrals

ISP-basis

Dot-basis ($\nu_i \geq 0$) → suited for implementing EOMS scheme!

Tensor integrals in d dim.

Scalar integrals in d+2 dim.

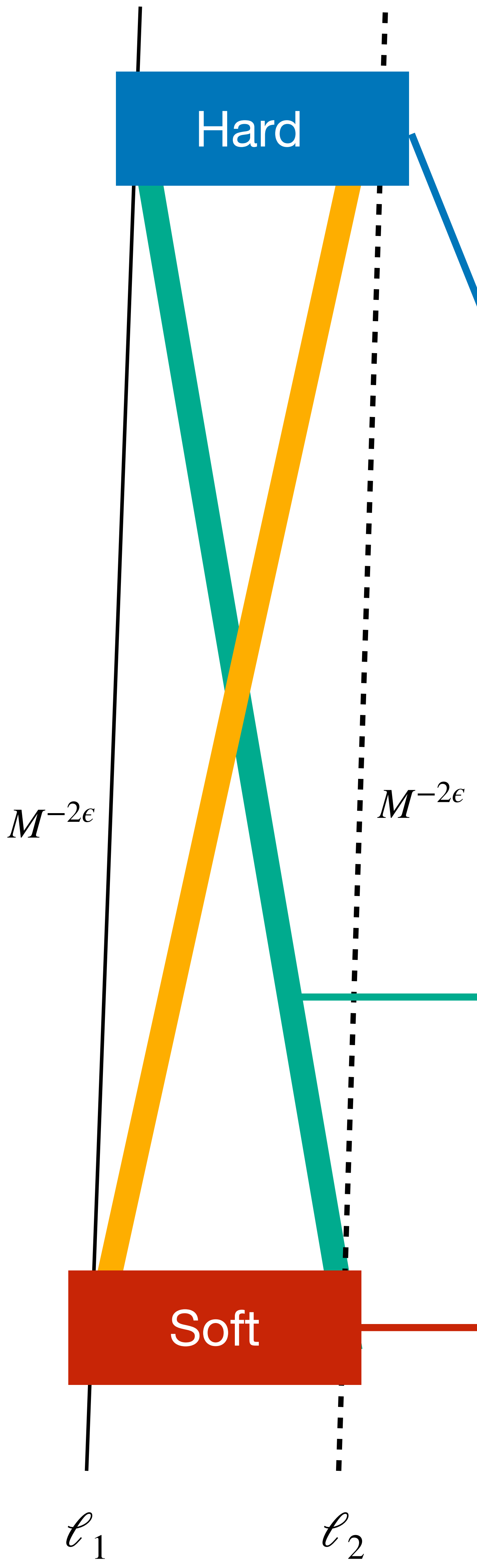
Dimensional Shift Relations (DSR)

Dimensional counting method

Strategy of region:

[Beneke and Smirnov, NPB522(1998)]

[Gegelia, Japaridze and Turashvili, Theor. Math. Phys. 101(1994)]



$$I_{11111} = \iint \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{1}{[\ell_1^2 - M_\pi^2][\ell_2^2 - M_\pi^2][(\ell_1+\ell_2+p)^2 - m_N^2][(\ell_1+p)^2 - m_N^2][(\ell_2+p)^2 - m_N^2]}$$

$$I_{11111} = I_{11111}^{(0,0)} + I_{11111}^{(0,1)} + I_{11111}^{(1,0)} + I_{11111}^{(1,1)} \qquad \ell_1 \to \tilde{\ell}_1\left(\frac{M_\pi}{m_N}\right)^\alpha \qquad \ell_2 \to \tilde{\ell}_2\left(\frac{M_\pi}{m_N}\right)^\beta$$

Infrared regular part

$$I_{11111}^{(0,0)} = \sum_{i,j} M_\pi^{2i+2j}\iint \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{1}{[\ell_1^2+i0^+]^{1+i}[\ell_2^2+i0^+]^{1+j}\mathscr{D}_3\mathscr{D}_4\mathscr{D}_5}$$

Polynomial of M_π^2 → **PCB issue occurs!**

$M_\pi^{-2\epsilon}$ × **Polynomial of M_π^2**

$$I_{11111}^{(0,1)} = \sum_{i,j,l}(-1)^{j+l}M_\pi^{d-3+2i+j+l}m_N^{3-d-j-l}\iint \frac{\mathrm{d}^d\tilde{\ell}_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{(\tilde{\ell}_1^2)^j(\frac{M_\pi}{m_N}\tilde{\ell}_1^2 + 2p\cdot\tilde{\ell}_1 + 2\tilde{\ell}_1\cdot\ell_2)^l}{[\tilde{\ell}_1^2 - m_N^2 + i0^+][\ell_2^2+i0^+]^{1+i}[2p\cdot\tilde{\ell}_1+i0^+]^{1+i}[\ell_2^2+2p\cdot\ell_2+i0^+]^{2+l}}$$

$$I_{11111}^{(1,1)} = \sum_{i,j,l}(-1)^{i+j+l}\left(\frac{M_\pi}{m_N}\right)^{2d-7+i+j+l}\iint \frac{\mathrm{d}^d\tilde{\ell}_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{(\tilde{\ell}_1^2)^i(\tilde{\ell}_2^2)^j(\tilde{\ell}_1^2 + 2\tilde{\ell}_1\cdot\tilde{\ell}_2 + \tilde{\ell}_2^2)^l}{[\tilde{\ell}_1^2 - m_N^2 + i0^+][(\tilde{\ell}_2^2 - m_N^2 + i0^+][2p\cdot\tilde{\ell}_1+i0^+]^{1+i}[2p\cdot\tilde{\ell}_2+i0^+]^{1+j}[2p\cdot\ell_1+2p\cdot\ell_2+i0^+]^{2+l}}$$

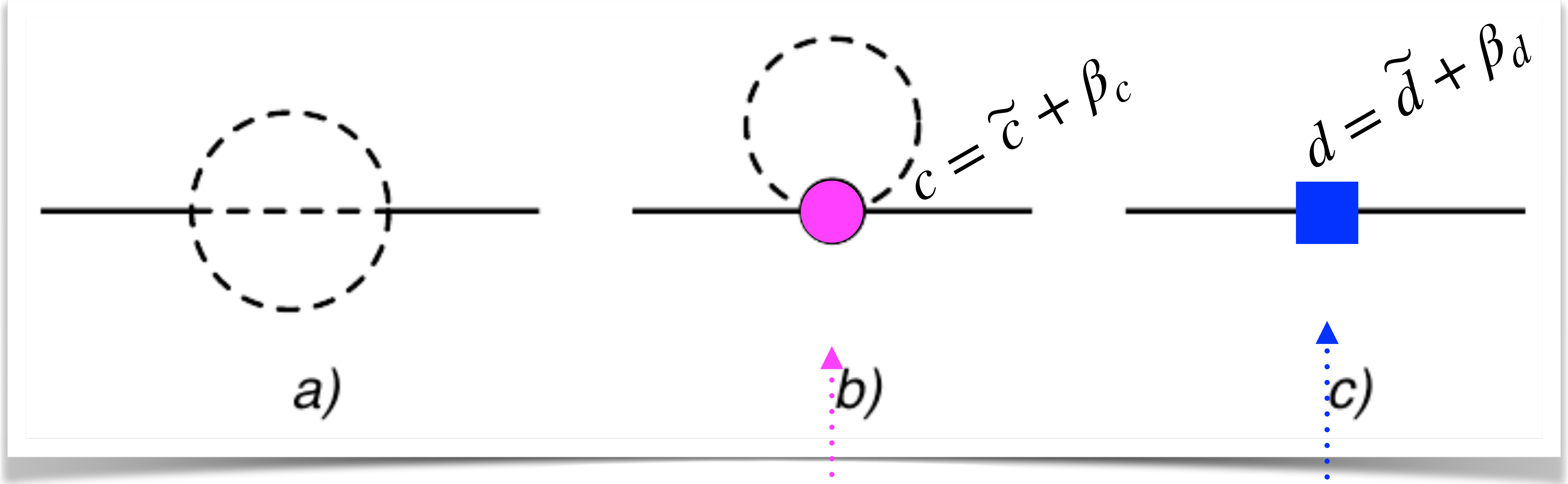
Infrared singular part

No PCB problem!

Applicability of (complex) EOMS scheme at two-loop order

✦ **A EFT toy model for the sake of easy illustration:**

$$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \pi \partial^\mu \pi - M^2\pi^2) + \frac{1}{2}(\partial_\mu \Psi \partial^\mu \Psi - m^2\Psi^2) - \frac{g}{3!}\pi^3\Psi + \mathscr{L}_c$$



Unrenormalized two-loop contribution

$$\Sigma_a = -\frac{g^2}{6(2\pi)^{2n}} \iint \frac{\mathrm{d}^n k_1 \mathrm{d}^n k_2}{[k_1^2 - M^2 + i0^+][k_2^2 - M^2 + i0^+][(k_1+k_2+p)^2 - M^2 + i0^+]}$$

$$= M^{2n-4} H(p^2, M^2; n) + M^{n-2} G(p^2, M^2; n) + F(p^2, M^2; n)$$

No PCB

Taylor series of M^2 multiplied by M^{n-2}

Taylor series of M^2

Analytical demonstration

PCB term by G in diagram (a)

$$M^{n-2}\Delta G = \frac{g^2 M^{n-2}(-m^2 - i0^+)^{\frac{n}{2}-5}}{96(4\pi)^n}\Gamma\left(1-\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)\Gamma\left(\frac{n}{2}-1\right)^2 \left\{\frac{48\,m^6}{\Gamma(n-2)}\right.$$
$$+ 6m^2\left[\frac{(m^2-p^2)^2(n-4)(n-6)}{\Gamma(n-2)} - \frac{8m^2M^2(n+4)}{n\Gamma(n-3)}\right] - \frac{24m^4(m^2-p^2)(n-4)}{\Gamma(n-2)}$$
$$\left. - (n-6)(m^2-p^2)\left(\frac{(m^2-p^2)^2(n-4)(n-8)}{\Gamma(n-2)} - \frac{24m^2M^2(n+4)}{n\Gamma(n-3)}\right)\right\}. \quad (13)$$

Cancel with each other

Counter term by diagram (b)

$$-i\Sigma_{CT} = -\frac{ig^2\lambda(m,n)}{48m^6(n-4)n}\Big\{m^6(n-10)(n-8)(n-6)n$$
$$- 3m^4(n-8)n\left[M^2(n-2)(n+4) + (n-10)(n-4)p^2\right]$$
$$+ 3m^2(n-6)\Big[M^4(n-2)n(n+4)$$
$$+ 2M^2(n+4)((n-8)n+20)p^2 + (n-10)(n-4)np^4\Big]$$
$$- (n-8)(n-6)(n-4)(M^2+p^2)(M^4n + 2M^2(n+6)p^2 + np^4)\Big\} I_\pi$$

Applicability also holds true for nucleon mass!

Nucleon-mass Physics

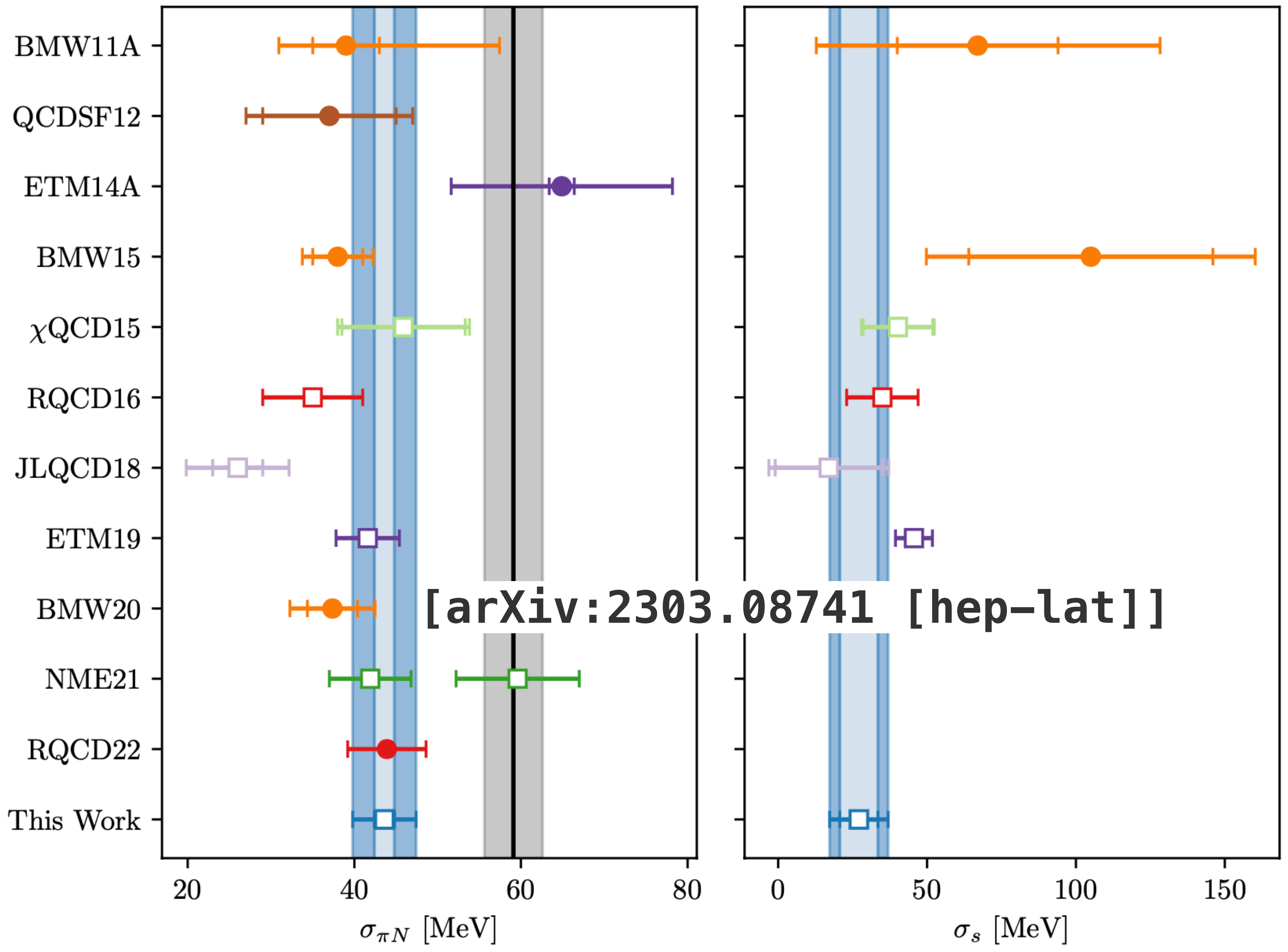
✦ Pion-nucleon sigma term

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle = M_\pi^2 \frac{\partial m_N}{\partial M_\pi^2}$$

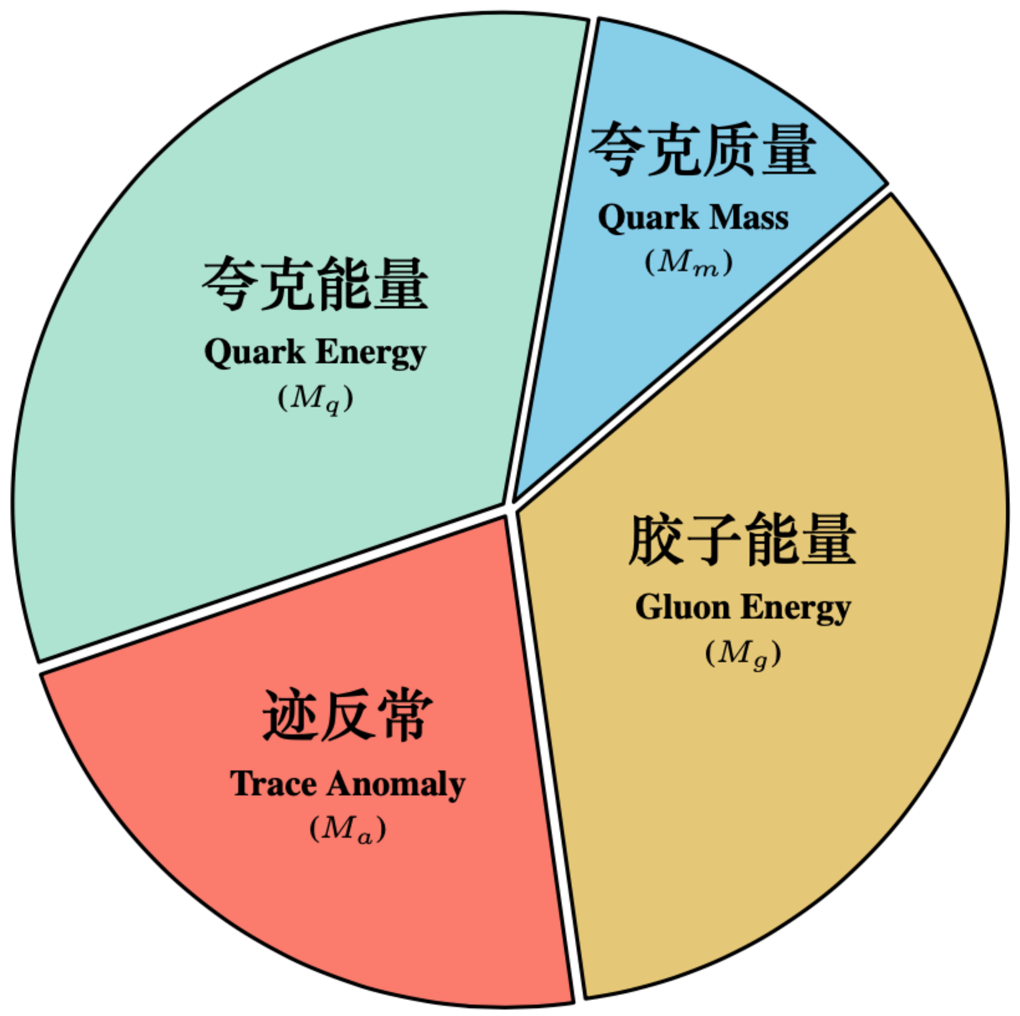
Feynman—Hellmann Theorem

$$m_N = \langle N(p)|\theta^\mu_\mu|N(p)\rangle \qquad \textbf{Energy—Momentum Tensor}$$
$$= \langle N(p)|\underbrace{\frac{\beta_{\rm QCD}}{2g}G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{field energy}} + \underbrace{m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s}_{\text{Higgs}}|N(p)\rangle$$

➡ **Tension between lattice QCD and phenomenological results**



➡ **The proton mass decomposition**



[X. Ji, PRL74(1995) & PRD52(1995)]

- $M_q = \frac{3}{4}\left(a - \frac{b}{1+\gamma_m}\right)M$
- $M_a = \frac{1}{4}(1-b)M$
- $M_m = \frac{4+\gamma_m}{4(1+\gamma_m)}bM$
- $M_g = \frac{3}{4}(1-a)M$

➡ **Inputs to the study of WIMP dark matter**

- **scalar coupling of the nucleon**

A two-loop study of nucleon mass may shed new light on the related physics!

Summary and Outlook

✦ Successfulness of covariant BChPT at one-loop order

✓ EOMS BChPT has been successfully applied to classic processes

- pion-nucleon scattering
- pion photo-/electro-production off the nucleon
- neutral and charged current weak pion production off the nucleon

✓ Extended versions for heavy hadrons and for unstable particles

- $\Delta(1232)$ resonance, Roper resonance, ….
- Charmed mesons → Exotic heavy meson spectrum
- Doubly charmed baryons → Aiming at predicting new negative-parity 1/2 DC baryons

✓ Interplay with Lattice techniques :

- Chiral extrapolation
- Finite volume correction → Systematical computation
- Unitarized ChPT in finite volume → Energy level
- Nuclear Lattice Chiral EFT

✦ Perspective of BChPT

✓ **Era of two-loop accuracy:** nucleon mass, axial radius, electromagnetic form factors, pion-nucleon scattering, ….

✓ **New physics:** neutrinoless double beta decays, dark matter, …

Thank you for your attention!