Relativistic chiral nuclear forces: status and prospect

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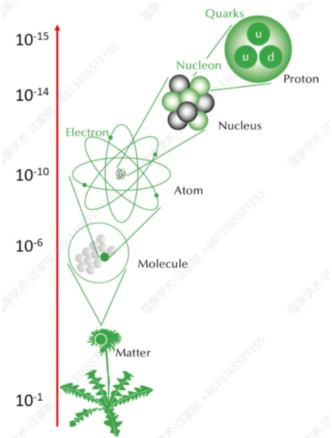
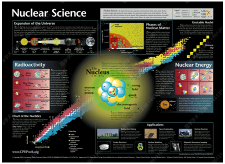
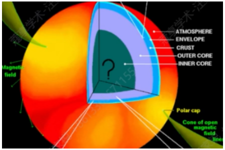
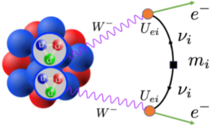
Outline

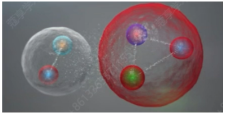
Introduction





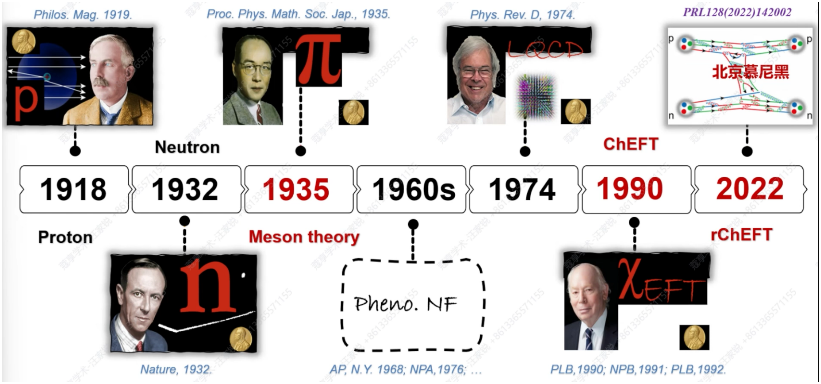






New Physics: Nuclear structures, Nuclear astrophysics, strange hadron states, beyond the Standard Model...

Brief history:



There are infinite terms in the Lagrangian that satisfy symmetry. It's necessary to find relatively important terms in the Lagrangian. This scheme is called **Chiral perturbation theory (ChPT)**.

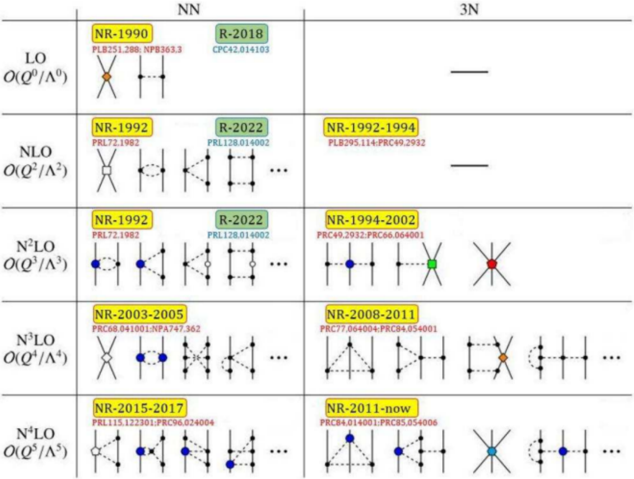
In ChPT, graphs are analyzed in terms of powers of $\left(\frac{Q}{\Lambda}\right)^{\nu}$. Determining ν is known as **power counting**.

- Nucleon propagator $\frac{1}{\not{p}-m_n} \sim \frac{1}{Q}$.
- Pion propagator $\frac{1}{q^2-m_\pi^2} \sim \frac{1}{Q^2}$.
- Loop integration $\int d^4k \sim Q^4$.
- Derivative in any interaction $\sim Q$.

Weinberg obtained for the power of an irreducible diagram involving A nucleons:

$$\nu = \begin{cases} 4-A-2C+2L+\sum_i \Delta_i & A \leq 2 \\ -2+2A-2C+2L+\sum_i \Delta_i & A > 2 \end{cases} \tag{1}$$

where C denotes the number of separately connected pieces, L denotes the number of loops, Δ_i denotes the power of the i-th vertex.


NN
3N
LO
$O(Q^0/\Lambda^0)$
NR-1990
PLB251,288; NPB363,3
R-2018
CPC42,014103
—
NLO
$O(Q^2/\Lambda^2)$
NR-1992
PRL72,1982
R-2022
PRL128,014002
NR-1992-1994
PLB295,114;PRC49,2932
—
N^2LO
$O(Q^3/\Lambda^3)$
NR-1992
PRL72,1982
R-2022
PRL128,014002
NR-1994-2002
PRC49,2932;PRC66,064001
N^3LO
$O(Q^4/\Lambda^4)$
NR-2003-2005
PRC68,041001;NPA747,362
NR-2008-2011
PRC77,064004;PRC84,054001
N^4LO
$O(Q^5/\Lambda^5)$
NR-2015-2017
PRL115,122301;PRC96,024004
NR-2011-now
PRC84,014001;PRC85,054006

Problems:

- **Power counting:**
  When the LS equation is truncated up to a certain order of the chiral expansion, it is non-renormalizable. The ultraviolet divergence of the loop diagrams in the scattering amplitude cannot be completely absorbed by the LECs truncated to the corresponding order[1].

- **Convergence:**
  Non-relativistic expansion of the baryon propagator and Dirac spinors results in a slow convergence.

[1] E. Epelbaum et al, Eur. Phys. J. A 54, 186 (2018)

In order to overcome slow-convergence and non-renormalizable, the relativistic chiral NF was proposed.

The contact Lagrangian should be a Lorentz scalar and satisfy chiral symmetry, parity transformation, charge conjugation transformation, Hermitian conjugation transformation,and time-reversal invariance, the form of Lagrangian:

$$\mathcal{L} = \sum_i c_i O_i(\{\psi\}) \tag{2}$$

Define the chiral order of each building block:

Building blocks	$\mathbf{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	+	-	+	-	+	-	+	+
$\mathcal{C}$	+	+	-	+	-	+	-	+
h.c.	+	-	+	+	+	+	-	+
Chiral order	0	1	0	0	0	-	0	1

$\widetilde{O}_1$	$(\bar{\psi}\psi)\,(\bar{\psi}\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m_n^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar{\psi}\gamma^{\mu}\psi)\,(\bar{\psi}\gamma_{\mu}\psi)$	$\widetilde{O}_{22}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\alpha}\psi\right)\partial^2\partial_{\alpha}\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_3$	$(\bar{\psi}\gamma_5\gamma^{\mu}\psi)\,(\bar{\psi}\gamma_5\gamma_{\mu}\psi)$	$\widetilde{O}_{23}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^{\beta}\partial_{\nu}\left(\bar{\psi}\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar{\psi}\sigma^{\mu\nu}\psi)\,(\bar{\psi}\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{24}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\psi\right)\partial^4\left(\bar{\psi}\psi\right)$
$\widetilde{O}_5$	$(\bar{\psi}\gamma_5\psi)\,(\bar{\psi}\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma^{\mu}\psi\right)\partial^4\left(\bar{\psi}\gamma_{\mu}\psi\right)$
$\widetilde{O}_6$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}\psi\right)\partial^4\left(\bar{\psi}\gamma_5\gamma_{\mu}\psi\right)$
$\widetilde{O}_7$	$\frac{1}{4m_n^2}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\nu}\psi\right)\partial^4\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_8$	$\frac{1}{4m_n^2}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m_n^2}\left(\bar{\psi}\sigma^{\mu\alpha}\psi\right)\partial_{\alpha}\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{29}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\psi\right)\partial^2\left(\bar{\psi}\psi\right)$	$\widetilde{O}_{30}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma^{\mu}\psi\right)\partial^2\left(\bar{\psi}\gamma_{\mu}\psi\right)$	$\widetilde{O}_{31}$	$\frac{1}{16m_n^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\mu}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^{\alpha}\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma_5\gamma^{\mu}\psi\right)\partial^2\left(\bar{\psi}\gamma_5\gamma_{\mu}\psi\right)$	$\widetilde{O}_{32}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_{\alpha}\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\sigma^{\mu\nu}\psi\right)\partial^2\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{33}$	$\frac{1}{16m_n^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m_n^2}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m_n^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m_n^2}\left(\bar{\psi}\gamma_5\psi\right)\partial^2\left(\bar{\psi}\gamma_5\psi\right)$	$\widetilde{O}_{38}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar{\psi}\gamma_5\gamma_{\nu}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m_n^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

To describe the long-range one-pion exchange and medium-range two-pion exchanges, we need the Lagrangian for the πN interaction, which takes the form as[2][3]

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}\left(i\not{D} - m + \frac{g_A}{2}\not{u}\gamma_5\right)\psi \tag{3}$$

$$\begin{aligned}\mathcal{L}_{\pi N}^{(2)} =& c_1\left\langle\chi_+\right\rangle\bar{\psi}\psi - \frac{c_2}{4m^2}\left\langle u^\mu u^\nu\right\rangle\left(\bar{\psi}D_\mu D_\nu\psi + h.c.\right)\\ &+ \frac{c_3}{2}\left\langle u^2\right\rangle\bar{\psi}\psi - \frac{c_4}{2}\bar{\psi}\gamma^\mu\gamma^\nu\left[u_\mu, u_\nu\right]\psi\end{aligned} \tag{4}$$

where D_μ is the covariant derivative defined as $D_\mu + \partial_\mu + \Gamma_\mu$, with $\Gamma_\mu = \frac{1}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right)$, $u = \exp\left(\frac{i\Phi}{2f}\right)$, Φ is the pion field. The axial current $u_\mu = i\left(u^\dagger\partial_\mu - u\partial_\mu u^\dagger\right)$, and the chiral symmetry breaking term $\chi_+ = u^\dagger\chi u + u\chi u^\dagger$, with $\chi = \mathrm{diag}\left(m_\pi^2, m_\pi^2\right)$.

[2] Y.-H. Chen et al, Phys. Rev. D 87, 054019 (2013)

[3] More details: Y. Xiao et al, Phys. Rev. C 102, 054001 (2020), C.-X. Wang et al, Phys. Rev. C 105, 014003 (2022)

NN scattering in the relativistic framework

In order to describe NN interaction, we solve relativistic Blankenbecler-Sugar (BbS) equation[4]:

$$T\left(\mathbf{p}',\mathbf{p},s\right)=V\left(\mathbf{p}',\mathbf{p},s\right)+\int\frac{d^3\mathbf{k}}{(2\pi)^3}V\left(\mathbf{p}',\mathbf{k},s\right)\frac{m^2}{E_k}\frac{1}{\mathbf{q}_{\rm cm}^2-\mathbf{k}^2-i\epsilon}T\left(\mathbf{k},\mathbf{p},s\right)\quad(5)$$

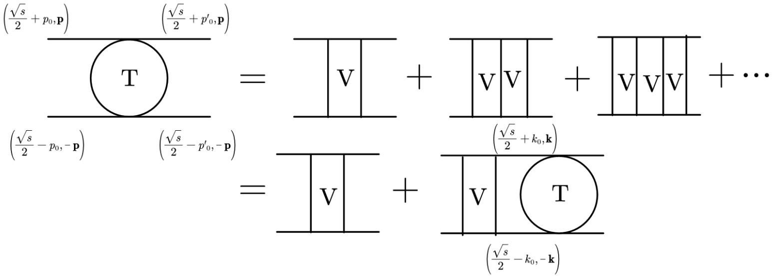


Figure: Schematic diagram for the kinematics of NN scattering.

[4] Detailed derivations refer to Note_NNScat by C.C. Wang.

To remove the ultraviolet divergence, the potential is regularized as follows,

$$V_{l'l}^{sj}\left(p', p|\sqrt{s}\right) = f_R\left(p\right) V\left(p', p|\sqrt{s}\right) f_R\left(p'\right) \tag{6}$$

the regulator is taken to be

$$f_R\left(p\right) = f_R^{\text{sharp}}\left(p\right) = \theta\left(\Lambda^2 - p^2\right) \tag{7}$$

The S-matrix for each partial wave can be expressed as

$$S_{l'l}^{sj}\left(p_{\text{cm}}\right) = \delta_{l'l}^{sj} + 2\pi i \rho T_{l'l}^{sj}\left(p_{\text{cm}}\right), \quad \rho = -\frac{|p_{\text{cm}}| m^2}{16\pi^2 E_{\text{cm}}} \tag{8}$$

The phase shifts ($s = 0, 1$) are then calculated as

$$S_{jj}^{0j} = \exp\left(2i\delta_j^{0j}\right), \; S_{jj}^{1j} = \exp\left(2i\delta_j^{1j}\right) \tag{9}$$

For $s = 1$ coupled channel we adopt the Stapp parametrization,

$$S = \begin{pmatrix} S^{1j}_{--} & S^{1j}_{-+} \\ S^{1j}_{+-} & S^{1j}_{++} \end{pmatrix} = \begin{pmatrix} \exp\left(i\delta^{1j}_{-}\right) & \\ & \exp\left(i\delta^{1j}_{+}\right) \end{pmatrix} \begin{pmatrix} \cos(2\epsilon) & i\sin(2\epsilon) \\ i\sin(2\epsilon) & \cos(2\epsilon) \end{pmatrix} \begin{pmatrix} \exp\left(i\delta^{1j}_{-}\right) & \\ & \exp\left(i\delta^{1j}_{+}\right) \end{pmatrix}$$
$$= \begin{pmatrix} \cos(2\epsilon)\exp\left(2i\delta^{1j}_{-}\right) & i\sin(2\epsilon)\exp\left(i\delta^{1j}_{-} + i\delta^{1j}_{+}\right) \\ i\sin(2\epsilon)\exp\left(i\delta^{1j}_{-} + i\delta^{1j}_{+}\right) & \cos(2\epsilon)\exp\left(2i\delta^{1j}_{+}\right) \end{pmatrix} \tag{10}$$

The subscripts "+" denotes $j+1$, while "-" denotes $j-1$, there exits a mixing angle ϵ. The mixing angles and phase shifts can be obtained by

$$\tan 2\epsilon = \frac{-iS^{1j}_{+-}}{\sqrt{S^{1j}_{--}S^{1j}_{++}}}, \quad \tan 2\delta^{1j}_{\pm} = \frac{\mathrm{Im}S^{1j}_{\pm\pm}}{\mathrm{Re}S^{1j}_{\pm\pm}} \tag{11}$$

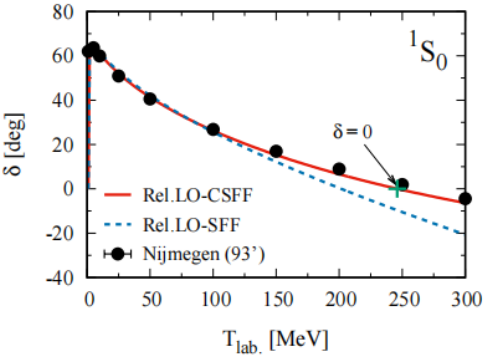


Figure: The red solid and blue dashed lines represent the leading-order relativistic chiral nuclear force results with the separable covariant and non-covariant regulators, respectively. The crosses in the figure mark the positions where the phase shift is zero. The black solid dots represent the results of the Nijmegen partial wave analysis.

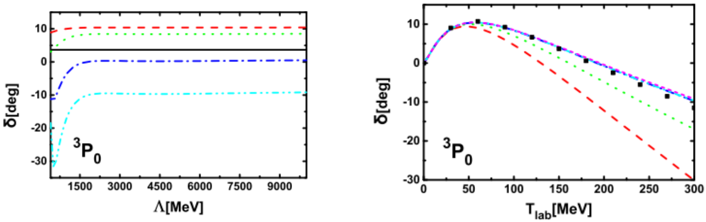


Figure: **Left panel**: the $^3P_0$ partial wave phase shift as a function of the momentum cutoff Λ for Tlab equals 10 MeV (black solid line), 50 MeV (red dashed line), 100 MeV (green dotted line), 190 MeV (blue dot-dashed line), and 300 MeV (cyan double-dot-dashed line), respectively. **Right panel**: the $^3P_0$ partial wave phase shift for momentum cutoffs of 600 MeV (red dashed line), 1000 MeV (green dotted line), 2000 MeV (blue dot-dashed line), 5000 MeV (cyan double-dot-dashed line), and 10000 MeV (purple short-dashed line), respectively

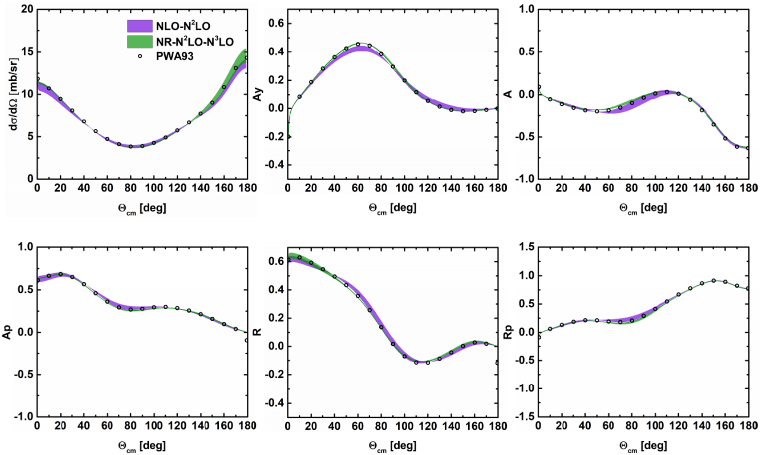


Figure: Descriptions of $d\sigma/d\Omega, A_y, A, A_p, R, R_p$ for $T_{\text{lab}} = 100$ MeV. The purple bands are the results of the relativistic chiral nuclear force at NLO and NNLO, the green bands represent the results of the Weinberg chiral nuclear force at NNLO and N3LO, and the circles represent the PWA93 results.

Near-threshold $\bar{N}N$ enhancements in $\left\{\begin{array}{c}\text{charmomium decays}\\ B \text{ meson decays}\\ e^+e^- \to \bar{p}p\end{array}\right\}$ provided an opportunity to elucidate the existence of speculated $\bar{N}N$ molecules.

The $\bar{N}N$ interaction contains real and imaginary parts, which reads

$$V_{\bar{N}N} = V^R + V^I \tag{12}$$

V^R is similar to NN potential. V^I describes $\bar{N}N$ pair annihilates into mesons or other particles.

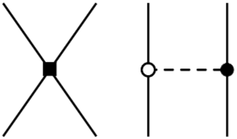


Figure: LO of $\bar{N}N$ interaction. The solid lines denote $\bar{N}/N$, and the dashed line represents the pion. The box denotes the vertex from $\mathcal{L}^{(0)}_{\bar{N}N}$ while the circle/dot shows vertex from$\mathcal{L}^{(1)}_{\pi\bar{N}}/\mathcal{L}^{(1)}_{\pi N}$.

The annihilation process leads to **complex phase shifts** for the $\bar{N}N$ interaction.

For uncoupled channels,

$$\mathrm{Re}\,(\delta_L) = \frac{1}{2}\arctan\frac{\mathrm{Im}\,(S_L)}{\mathrm{Re}\,(S_L)}, \quad \mathrm{Im}\,(\delta_L) = -\frac{1}{2}\log|S_L| \tag{13}$$

For couple channels,

$$\mathrm{Re}\,(\delta_{L\pm1}) = \frac{1}{2}\arctan\frac{\mathrm{Im}\,(\eta_{L\pm1})}{\mathrm{Re}\,(\eta_{L\pm1})}, \quad \mathrm{Im}\,(\delta_{L\pm1}) = -\frac{1}{2}\log|\eta_{L\pm1}| \tag{14}$$

where $\eta_L = \frac{S_{LL}}{\cos 2\epsilon_J}$, $\epsilon_J = \frac{1}{2}\arctan\left(\frac{i}{2}\frac{S_{L-1,L-1}+S_{L+1,L+1}}{\sqrt{S_{L-1,L-1}S_{L+1,L+1}}}\right)$.

Phase shifts of $\bar{N}N$ scattering

The partial waves are labeled in the spectral notation $^{(2I+1)(2S+1)}L_J$.

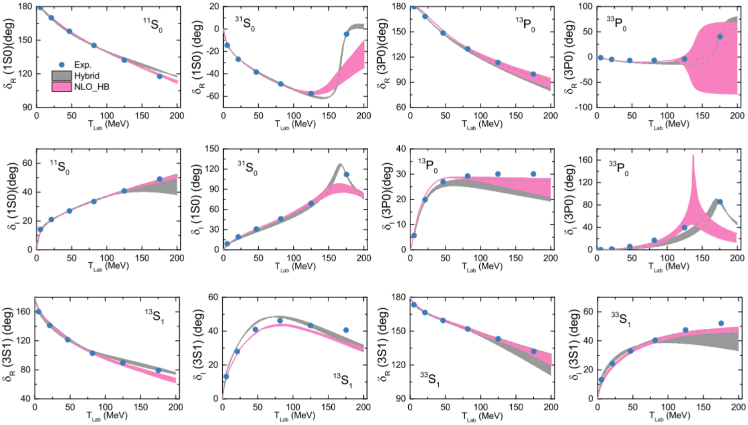


Figure: Real and imaginary parts of the phase shifts for the $^1S_0$ and $^3P_0$ partial waves. The gray bands show our results with the cutoff in the range $\Lambda = 450$–600 MeV. The pink bands show the NLO HB chiral EFT results. The blue dots refer to the PWA results

Summary and prospects

- Relativistic NN interaction has been constructed up to NNLO that can describe the Nijmegen partial-wave phase shifts. the NLO and NNLO results are similar in describing the scattering phase shifts for $T_{\rm lab} \leq 200$ MeV, and show the good convergence of the covariant chiral expansion.
- Relativistic NN interaction can be extended to $\bar{N}N$ interaction, the studies employing $\bar{N}N$ interactions have found bound states in some channels, although the predicted binding energies are rather different. Therefore, more studies are needed to confirm the nature of this state.
- The relativistic framework has been extended to study hyperon-nucleon/hyperon interactions. Such studies are currently extended to higher orders and infinite nuclear matter.