Electromagnetic moments determined within nuclear DFT

Herlik Wibowo

Collaborators: R. Han, B. C. Backes, G. Danneaux, J. Dobaczewski, W. Jiang, M. Kortelainen, X. Sun, A. S. Fernández, A. Nagpal, P. L. Sassarini, A. E. Stuchbery

Remote talk, School of Physics and Astronomy, Sun Yat-Sen University
September 26, 2025

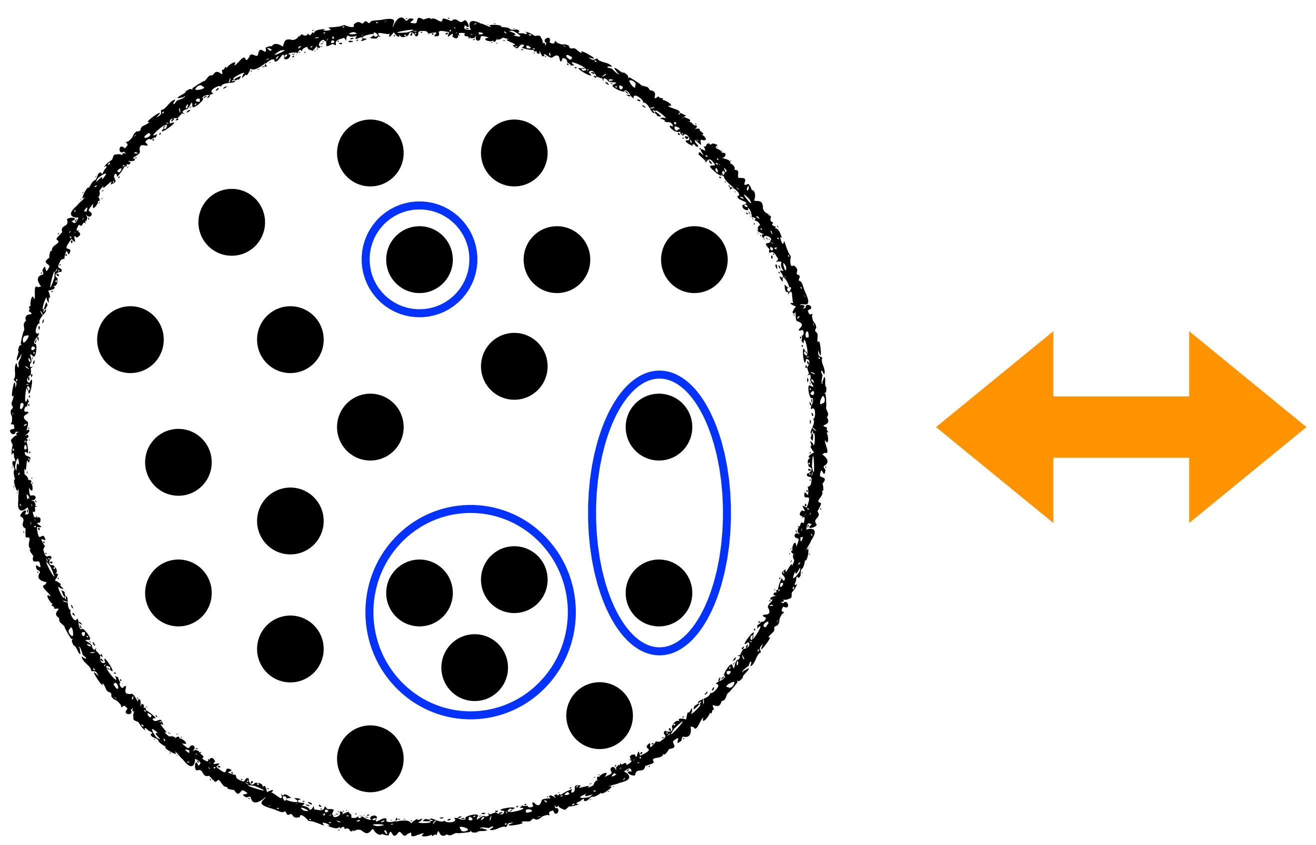
UNIVERSITY of York

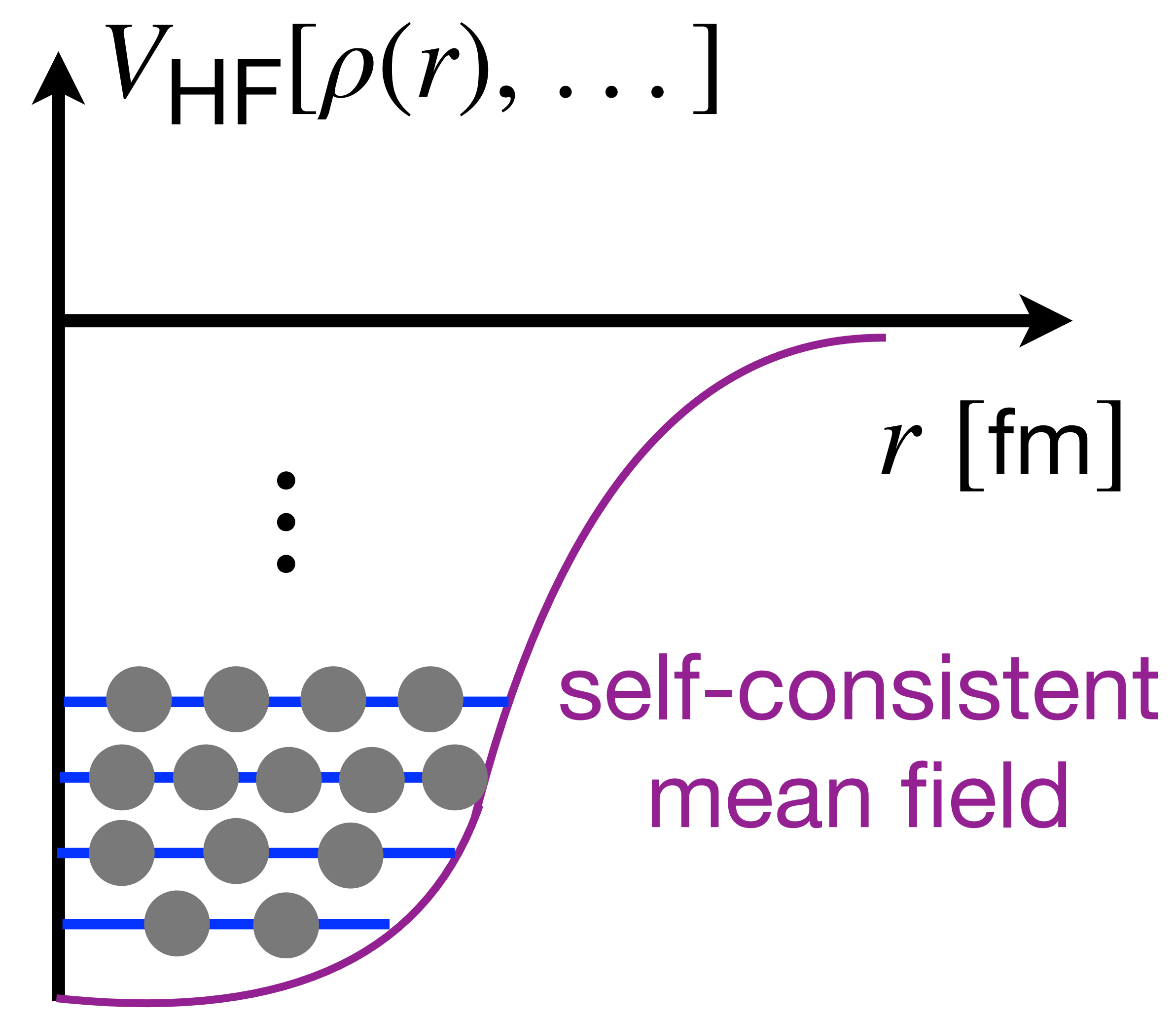
Outline

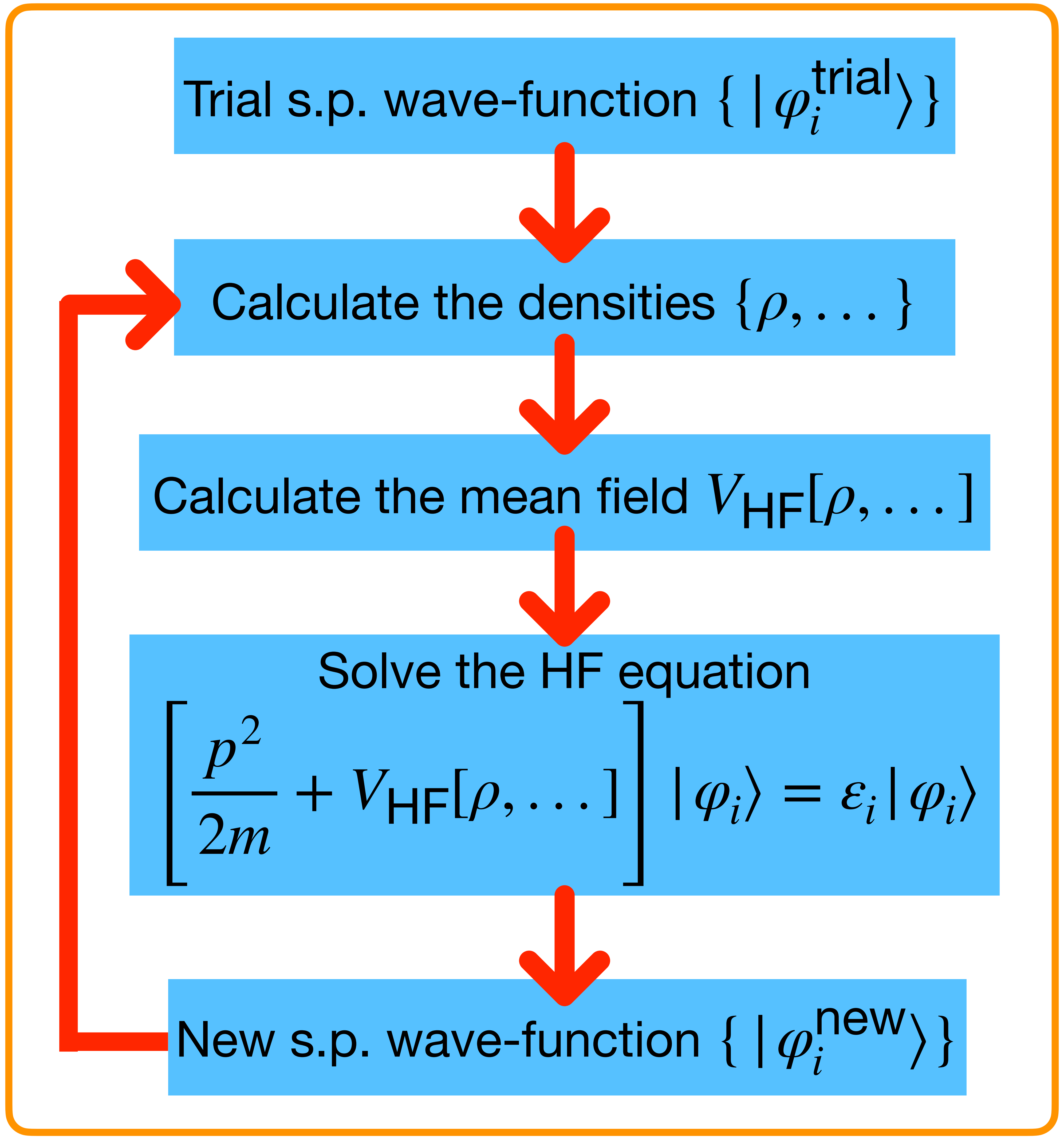
- ★ Three key concepts: self-consistency, core polarizations, symmetry restoration
- ★ Electromagnetic moments of ground and excited states calculated in heavy deformed open-shell odd nuclei
- ★ Two-body meson-exchange contributions to magnetic dipole moments
- ★ Nuclear Schiff moment of ^{227}Ac
- ★ Summary and Outlooks

1

Nuclear density functional theory







★ For the case of open-shell nuclei, the Hartree-Fock-Bogoliubov (HFB) equations are solved instead.

$$\begin{pmatrix} \hat{h}' - \lambda & \Delta \\ -\Delta^* & -\hat{h}'^* + \lambda \end{pmatrix} \begin{pmatrix} \chi_k^{\mathrm{upper}} \\ \chi_k^{\mathrm{lower}} \end{pmatrix} = \begin{pmatrix} \chi_k^{\mathrm{upper}} \\ \chi_k^{\mathrm{lower}} \end{pmatrix} E_k$$

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*
J. Dobaczewski *et al.*, CPC 180, 2361 (2009)

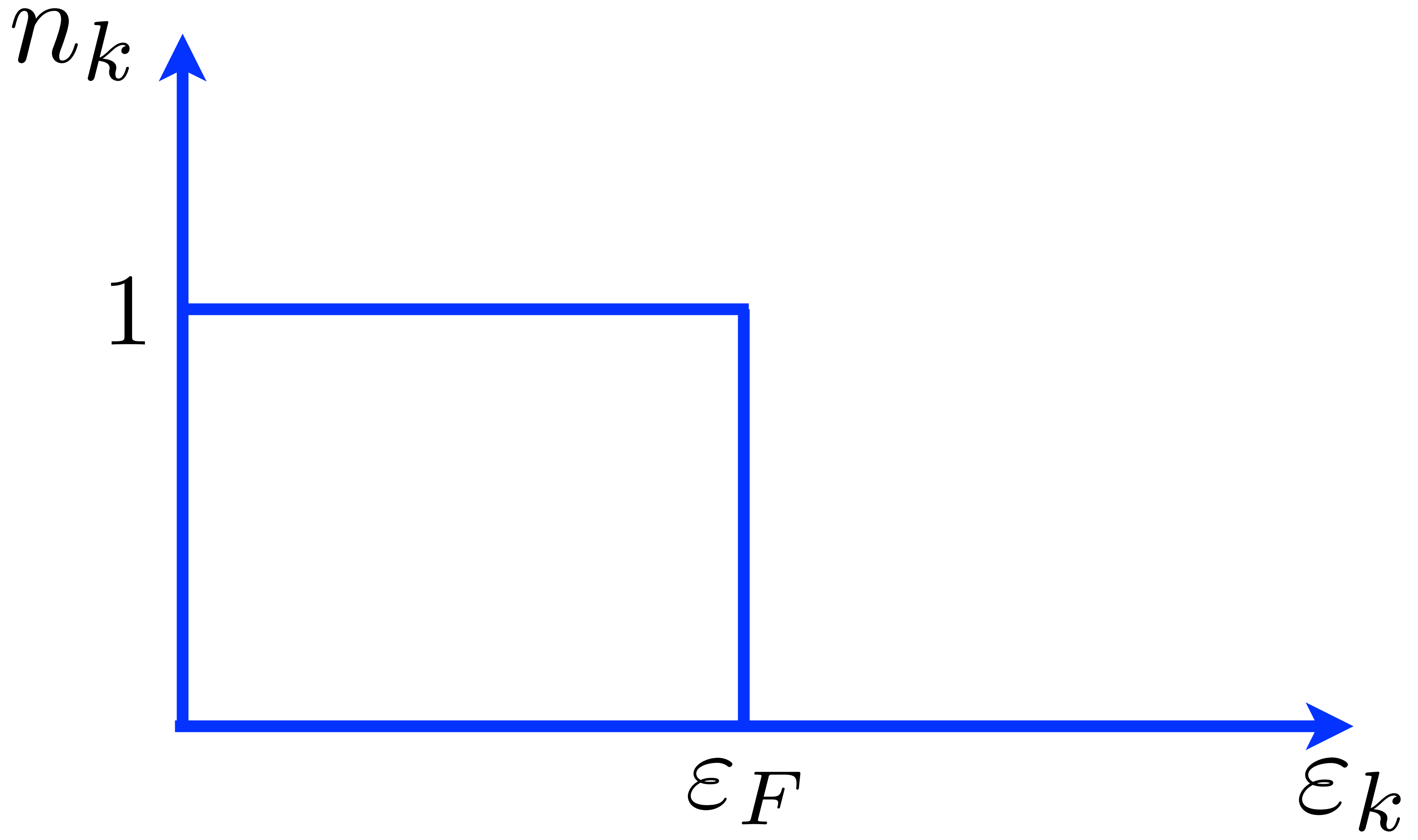
2 How do we calculate odd nuclei in nuclear DFT?

★ Without pairing:

$$A \text{ even}, \;\; p > A, \;\; h \leq A$$

$$|\Psi\rangle_{\mathrm{HF}}^{\mathrm{even}} = \hat{a}_A^\dagger \cdots \hat{a}_2^\dagger \hat{a}_1^\dagger |0\rangle$$

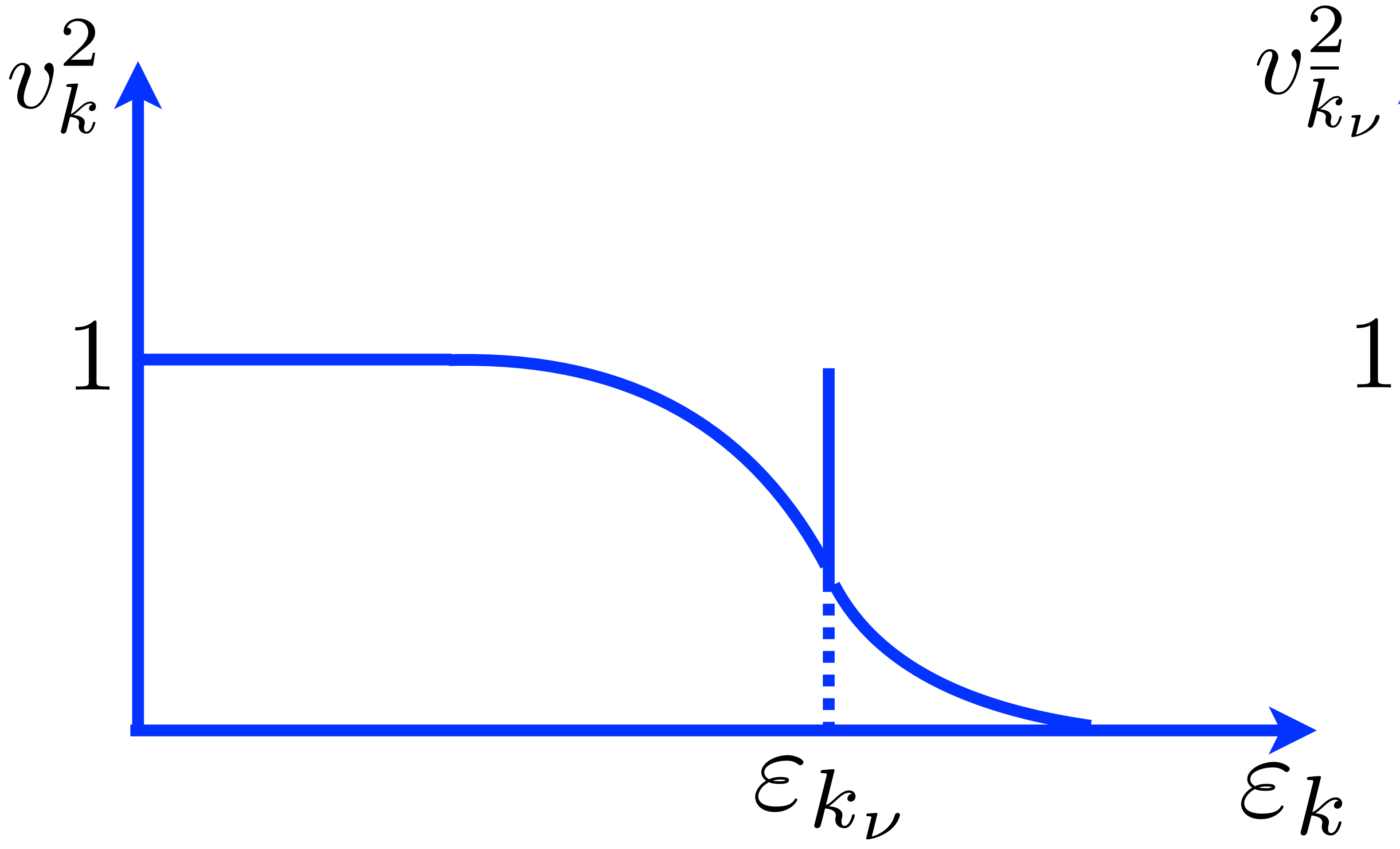
$$|\Psi\rangle_{\mathrm{HF}}^{\mathrm{odd}} = \begin{cases} \hat{a}_p^\dagger |\Psi\rangle_{\mathrm{HF}}^{\mathrm{even}} \\ \hat{a}_h |\Psi\rangle_{\mathrm{HF}}^{\mathrm{even}} \end{cases}$$

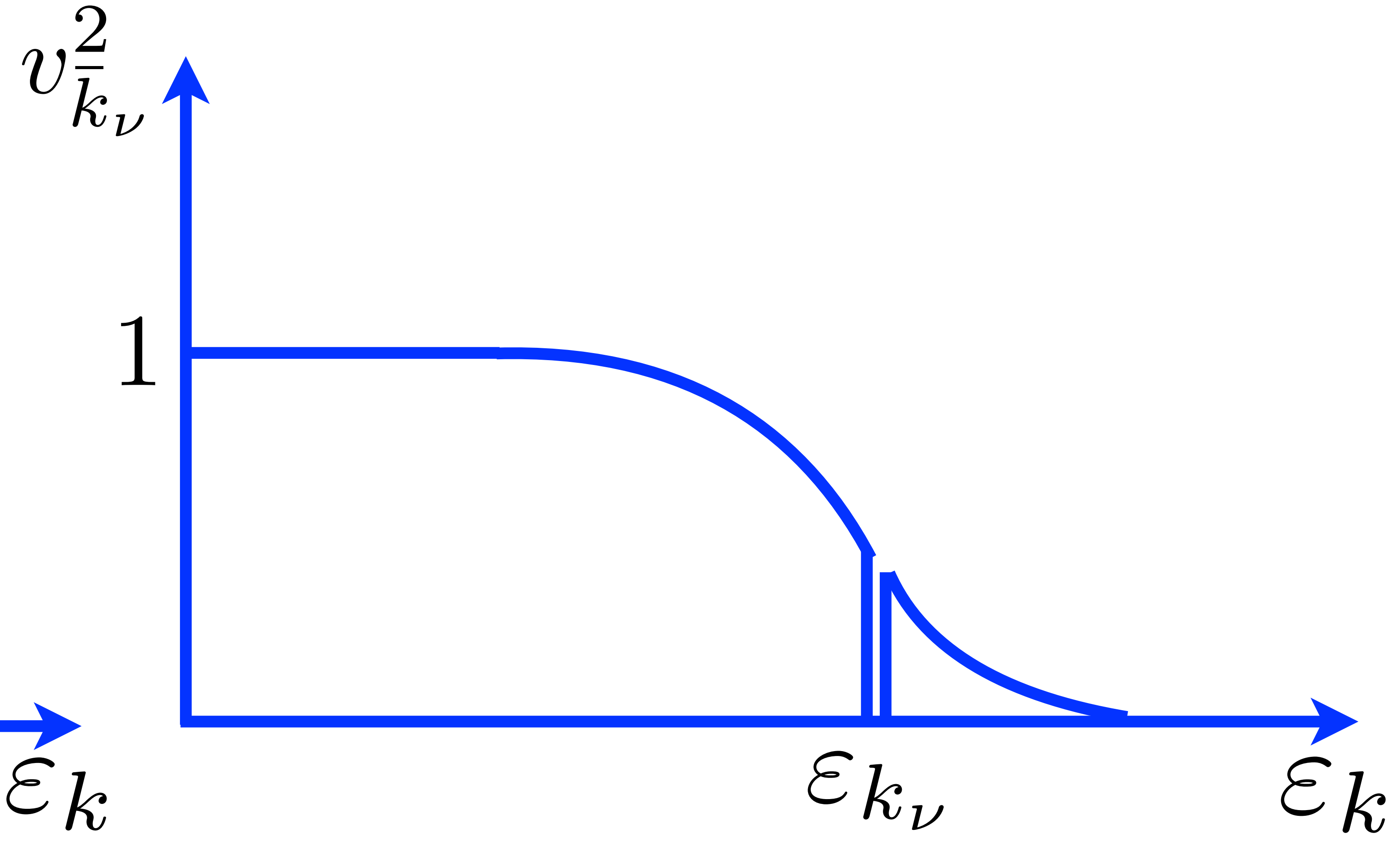


★ With pairing:

$$|\Psi\rangle_{\mathrm{HFB}}^{\mathrm{even}} = \prod_{\mu>0} \left(u_\mu + v_\mu \hat{a}_{\bar{\mu}}^\dagger \hat{a}_\mu^\dagger \right) |0\rangle$$

$$|\Psi\rangle_{\mathrm{HFB}}^{\mathrm{odd}} = \hat{\beta}_\nu^\dagger |\Psi\rangle_{\mathrm{HFB}}^{\mathrm{even}} = \hat{a}_\nu^\dagger \prod_{\nu \neq \mu > 0} \left(u_\mu + v_\mu \hat{a}_{\bar{\mu}}^\dagger \hat{a}_\mu^\dagger \right) |0\rangle$$





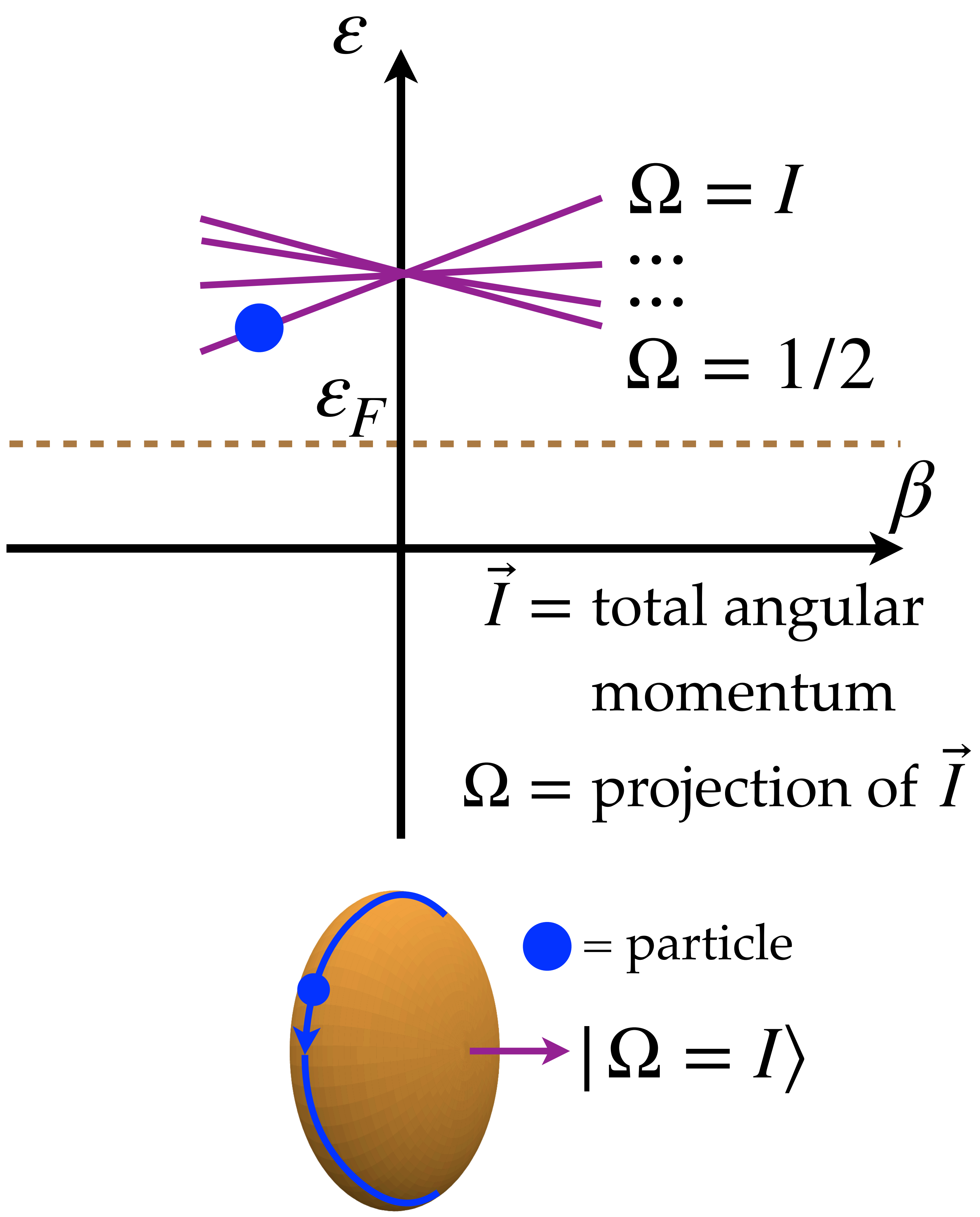
★ Tagged quasiparticle states:

$$\max_k \left\{ \langle \phi_\ell | \chi_k^{\mathrm{upper}} \rangle, \langle \phi_\ell | \chi_k^{\mathrm{lower}} \rangle \right\}$$

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*
J. Dobaczewski et al., CPC 180, 2361 (2009)
H. Wibowo, *et.al.*, *J. Phys. G: Nucl. Part. Phys.* ***52****, 065104 (2025)*

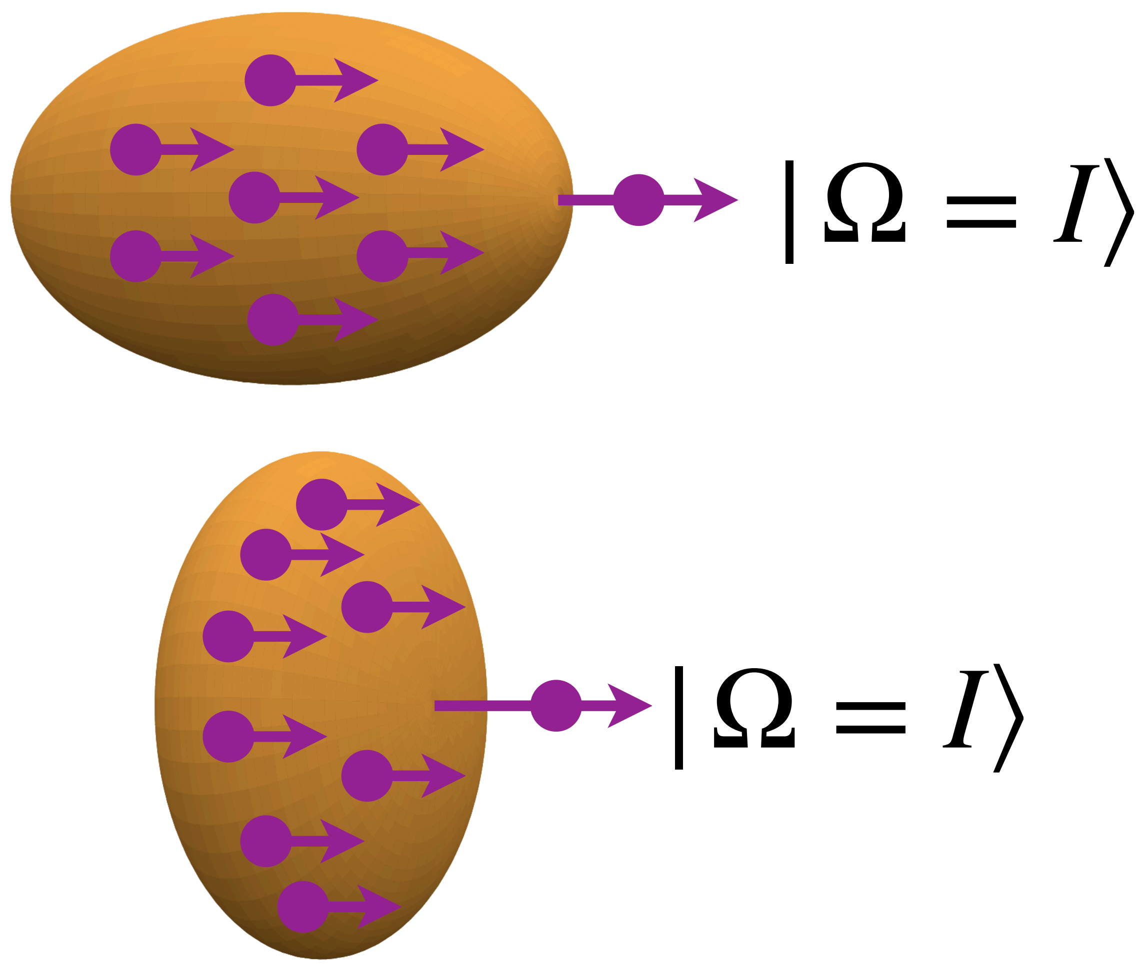
3

Shape and spin core polarizations



oblate polarization

Spin polarization



Landau parameter g_0' ($g_0' = 1.7$)

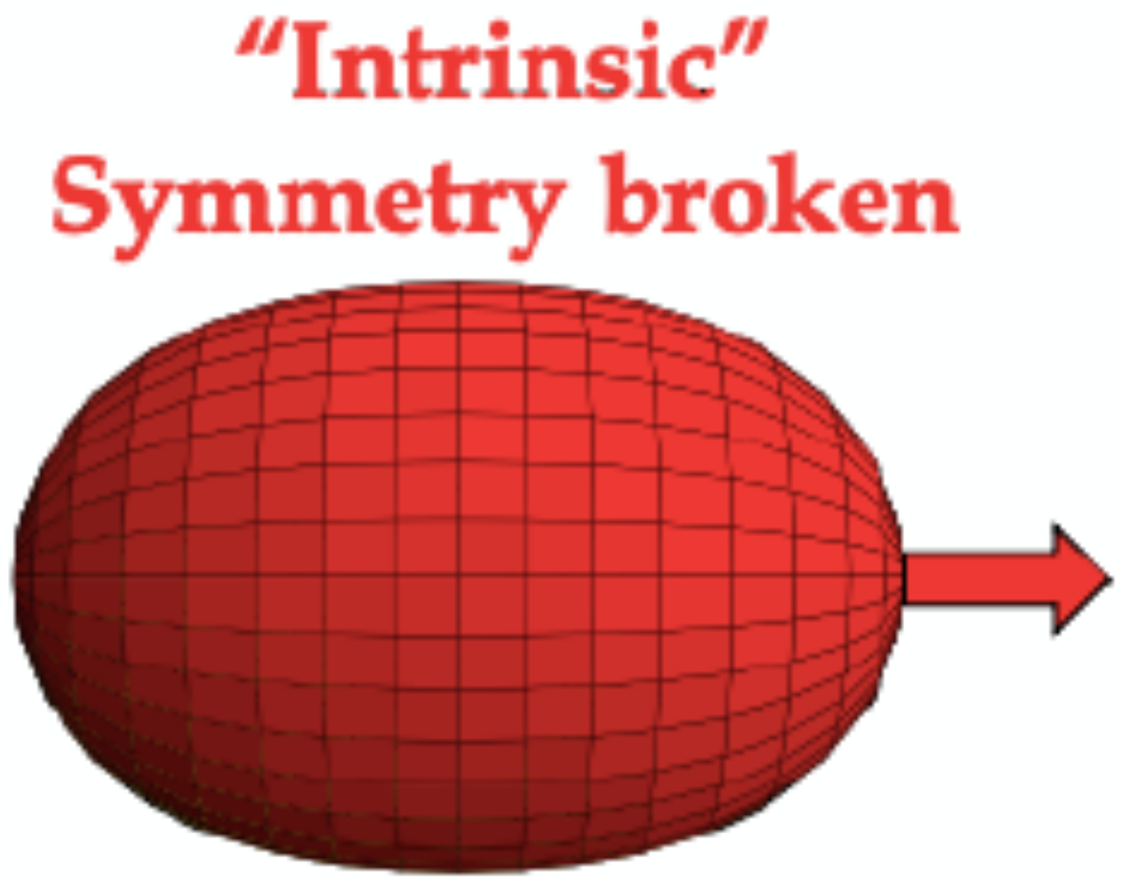
$$g_0' = N_0 \left(2C_1^s + 2C_1^T (3\pi^2\rho_0/2)^{2/3}\right)$$

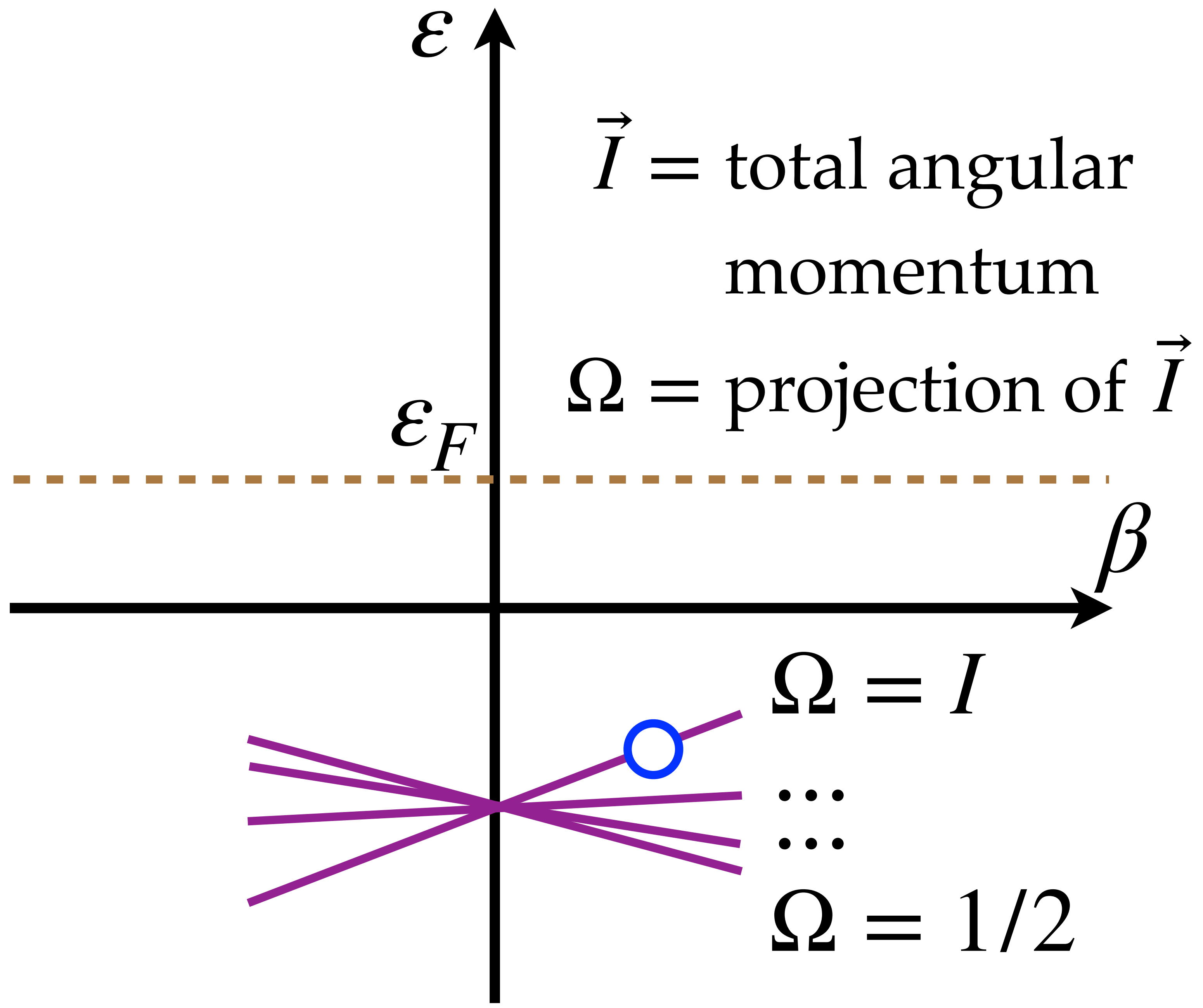
$$\frac{1}{N_0} \approx 150 \frac{m}{m^*}\ \mathrm{MeV}\cdot\mathrm{fm}^3$$

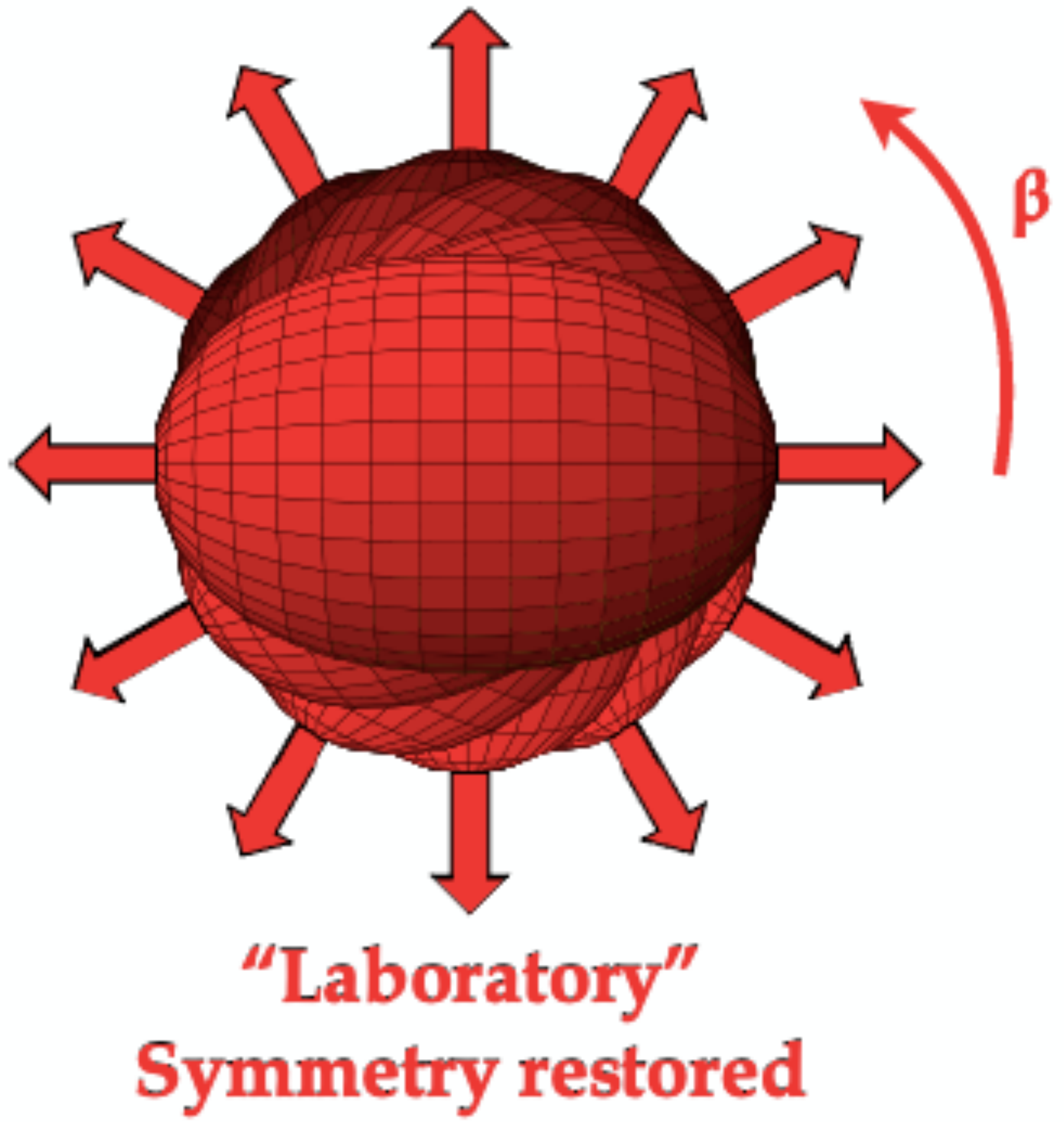
P. L. Sassarini et al., J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

A. Bohr and B. R. Mottelson, *Nuclear Structure* Vol. 1
K. L. G. Heyde, *The Nuclear Shell Model*
I. Ragnarsson and S. G. Nilsson, *Shapes and Shells in Nuclear Structure*

Symmetry restoration







Angular-momentum projected (AMP) state:

$$|IM\rangle = \mathcal{N}_I \int_0^{\pi} d\beta \; d^I_{M\Omega}(\beta)|\Omega, \beta\rangle$$

J. A. Sheikh et al., J. Phys. G: Nucl. Part. Phys. **48**, 123001 (2021)

5

Nuclear electromagnetic moments

Spectroscopic electric quadrupole Q and magnetic dipole μ moments are :

$$Q = \sqrt{\frac{16\pi}{5}} \langle II | \hat{Q}_{20} | II \rangle \quad \text{and} \quad \mu = \sqrt{\frac{4\pi}{3}} \langle II | \hat{M}_{10} | II \rangle \,.$$

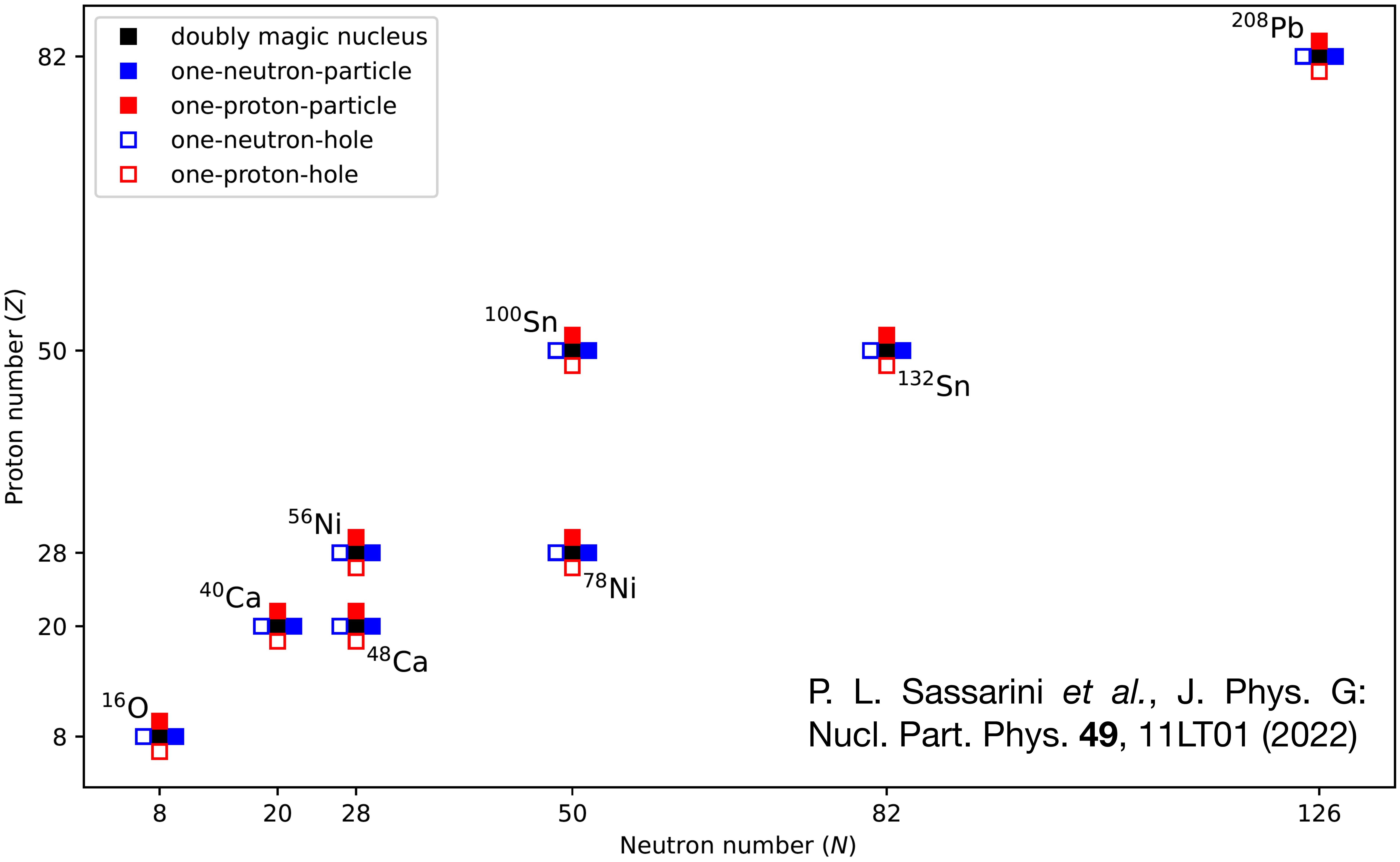
P. Ring and P. Schuck, *The Nuclear Many-Body Problem*

$$\hat{Q}_{20} = \sqrt{\frac{5}{16\pi}} e \sum_{i=1}^{A} \left(\frac{1}{2} - t_3^{(i)}\right) \{3z_i^2 - r_i^2\}; \qquad \hat{M}_{10} = \sqrt{\frac{3}{4\pi}} \mu_N \sum_{i=1}^{A} \left\{ g_s^{(i)} s_{zi} + g_\ell^{(i)} \ell_{zi} \right\};$$

$$\boxed{g_s^{(i)} = g_p(g_n) = 5.59(-3.83) \text{ and } g_\ell^{(i)} = 1(0)}$$

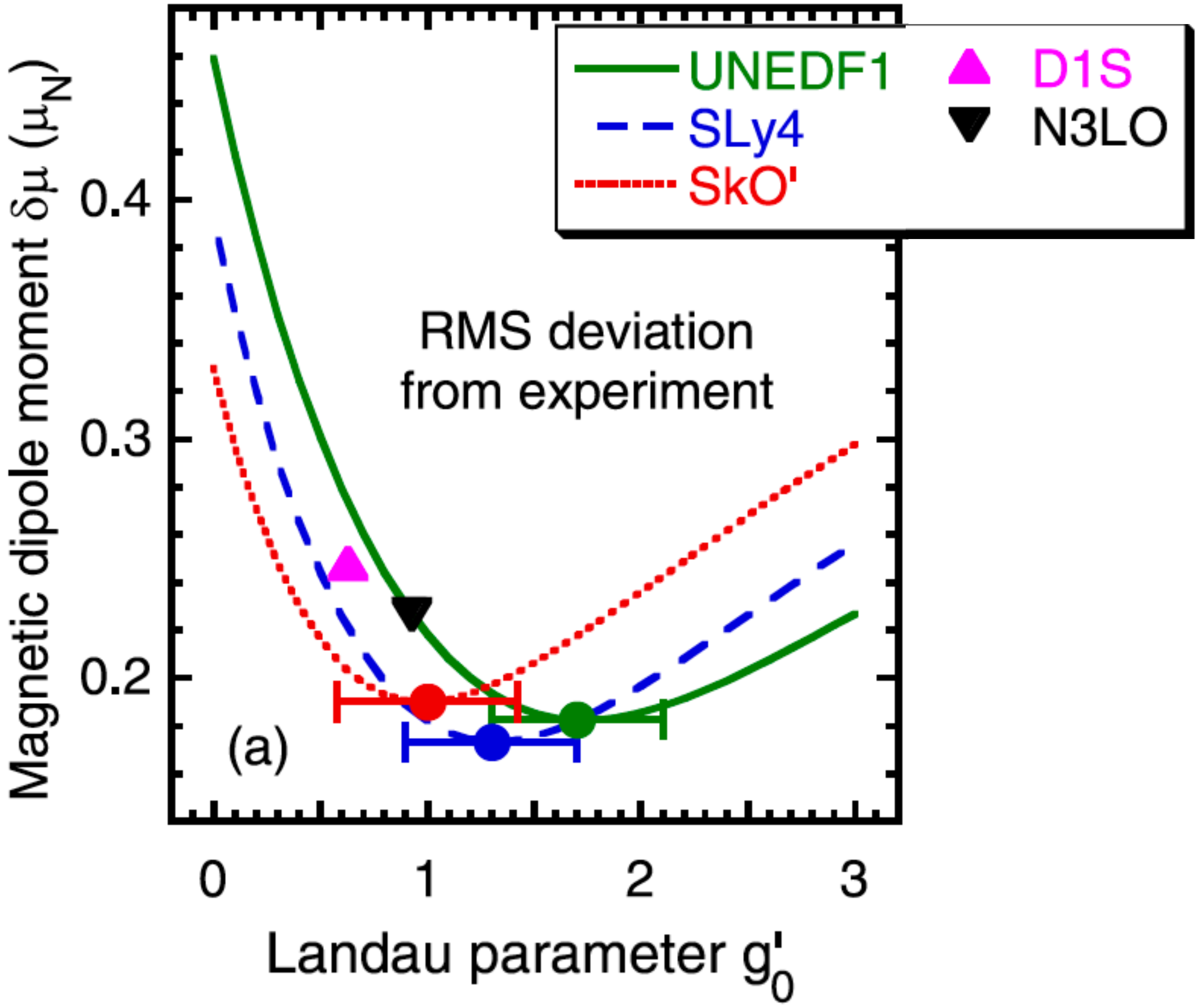
Since the self-consistent polarizations act in the full single-particle space, neither effective charges nor effective g-factors are needed!

Odd near doubly magic nuclei



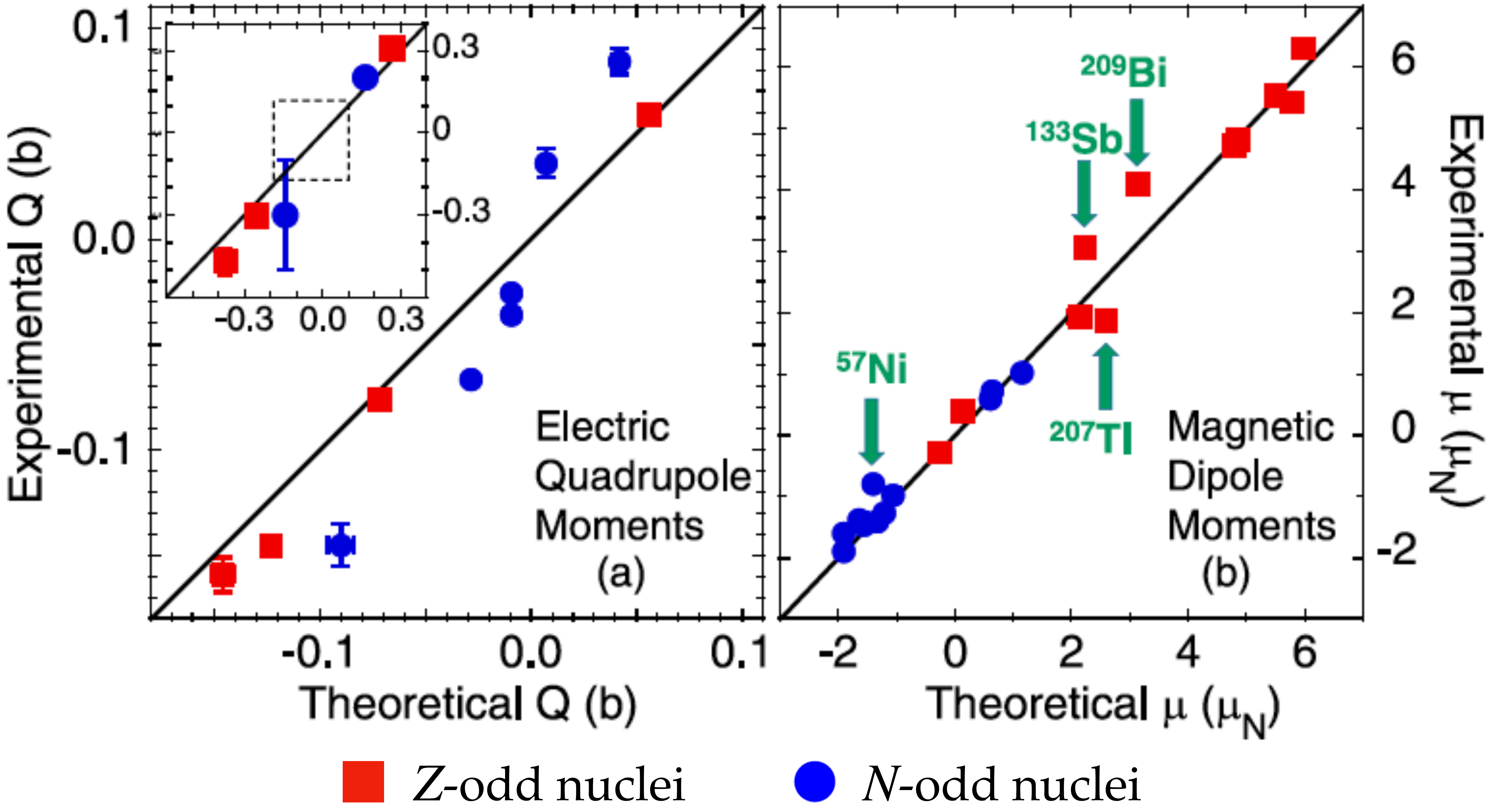
P. L. Sassarini *et al.*, J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

Theory vs. experiment



$g_0' = 1.7(4)$ (UNEDF1)

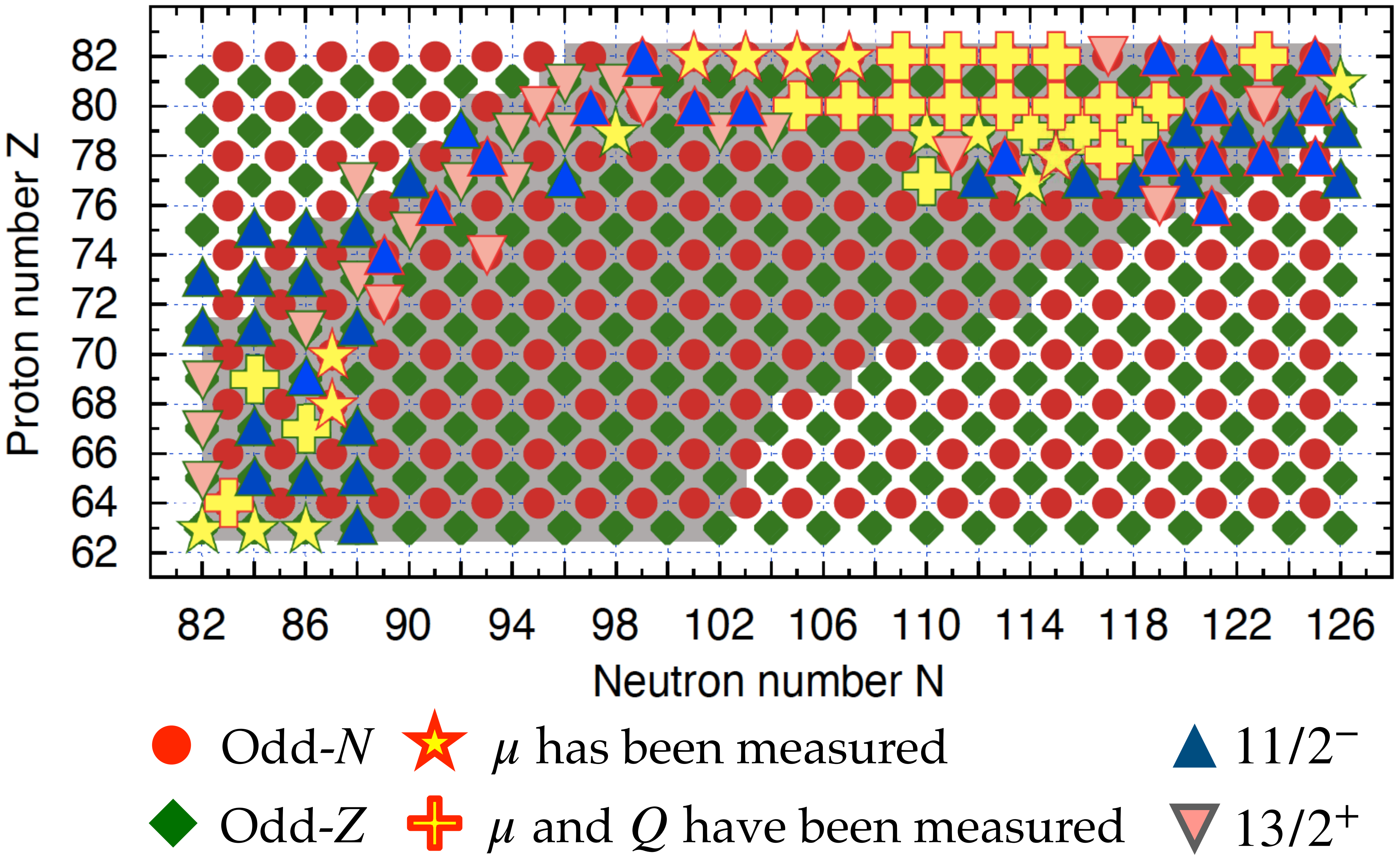
P. L. Sassarini *et al.*, J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

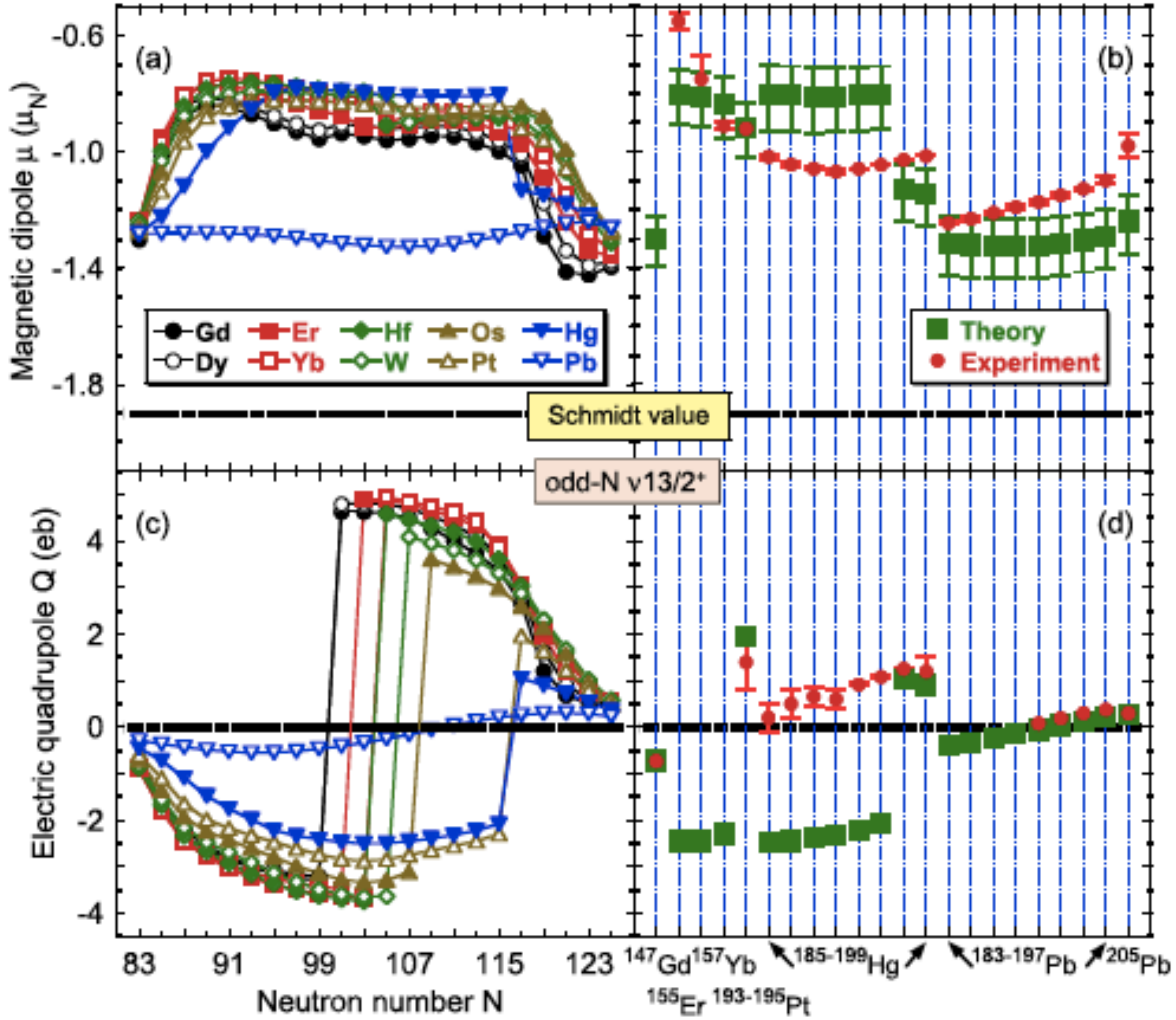


★15 experimentally measured values for Q and 23 experimentally measured values for μ.

★Theoretical error bars are always smaller than the sizes of the symbols.

Systematic nuclear-DFT calculation of the electromagnetic moments in heavy deformed open-shell Eu-Pb nuclei





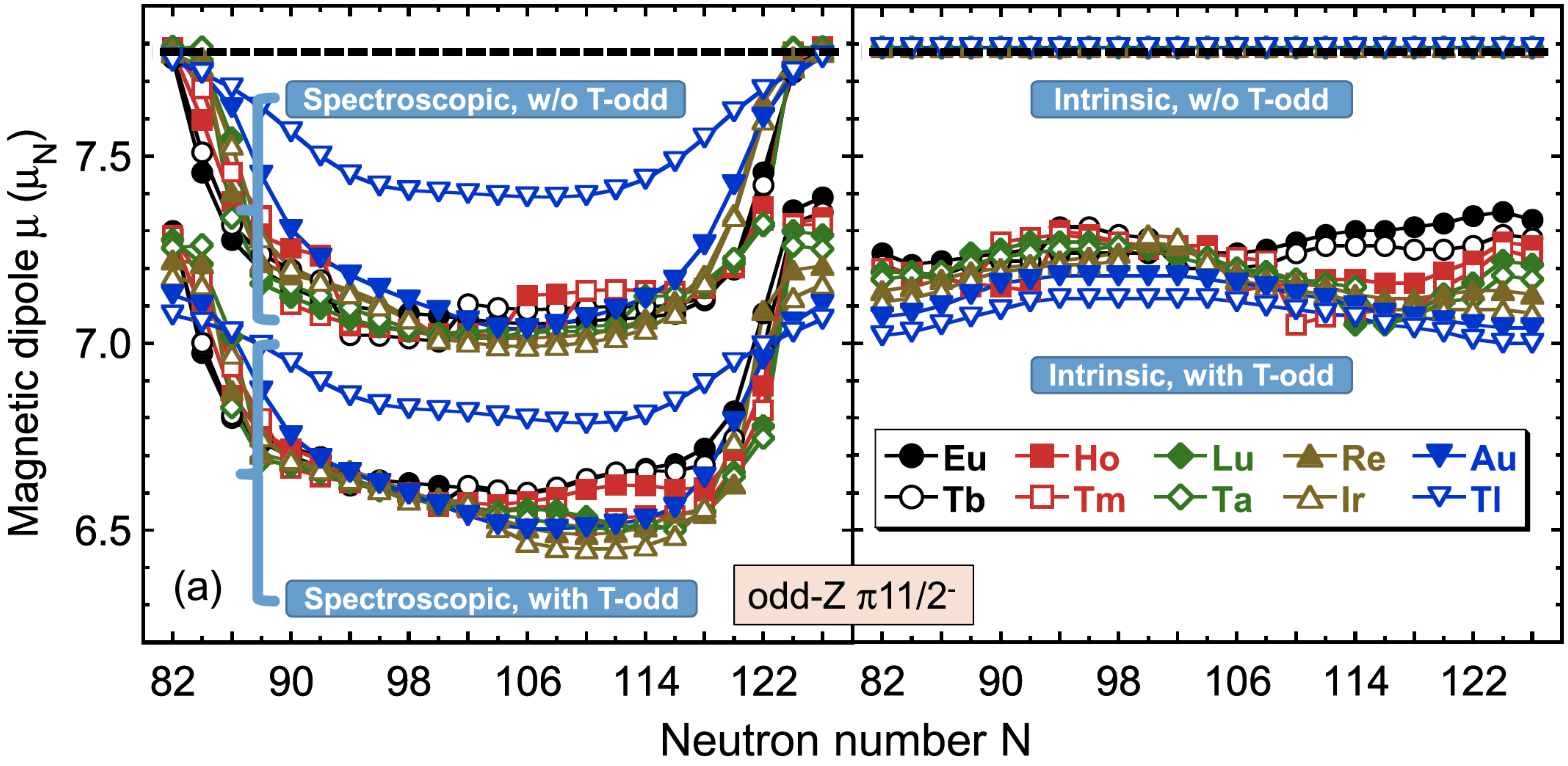
★Quasiparticle blocked states are tagged by $\pi 11/2^-$ and $\nu 13/2^+$ single-particle orbitals

★$g_0' = 1.7 \pm 0.4$

★UNEDF1 forces

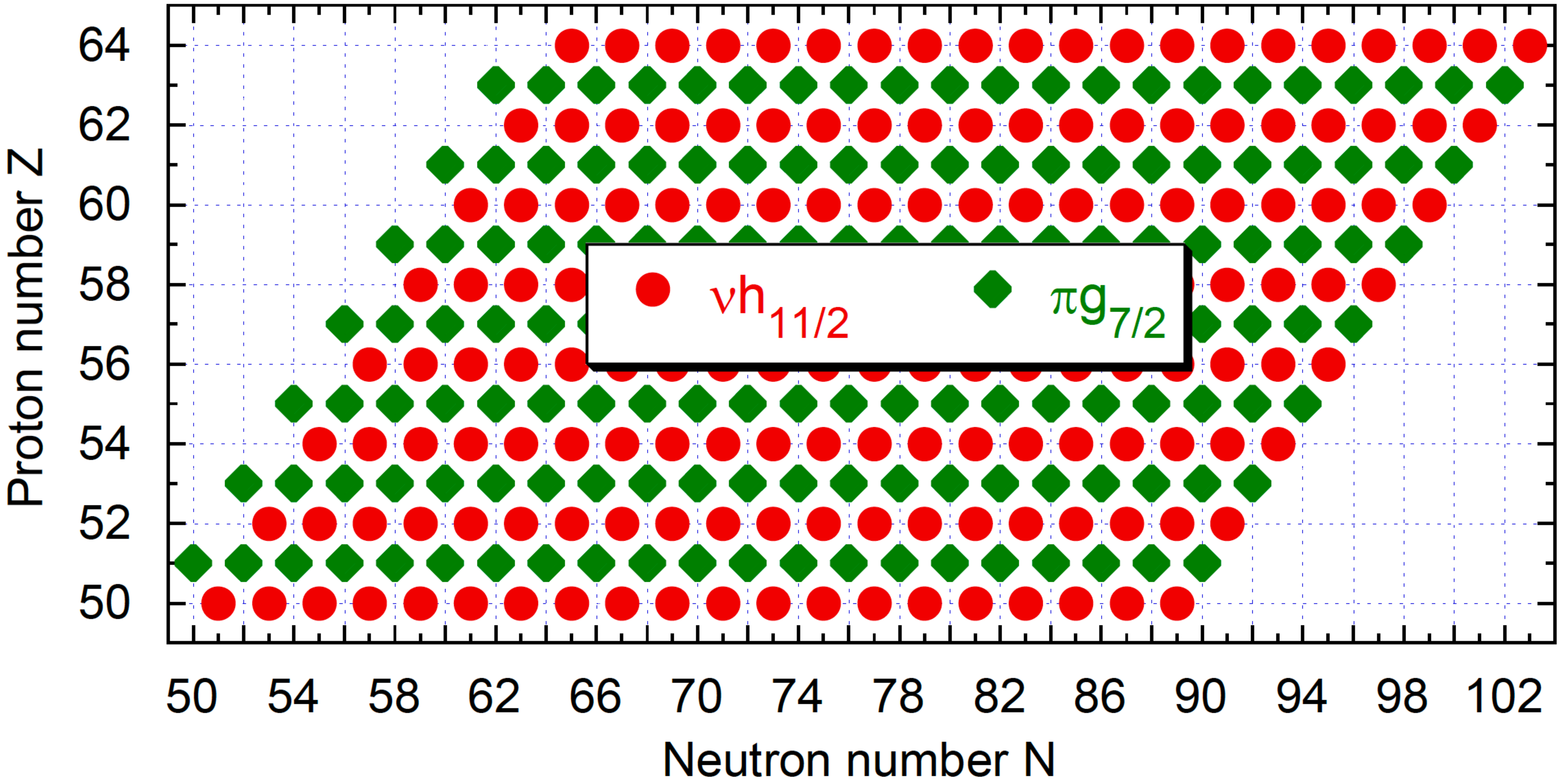
J. Bonnard, J. Dobaczewski, G. Danneaux, and M. Kortelainen, PLB **843**, 138014 (2023)

Effects of time-odd mean field



J. Bonnard, J. Dobaczewski, G. Danneaux, and M. Kortelainen, PLB 843, 138014 (2023)

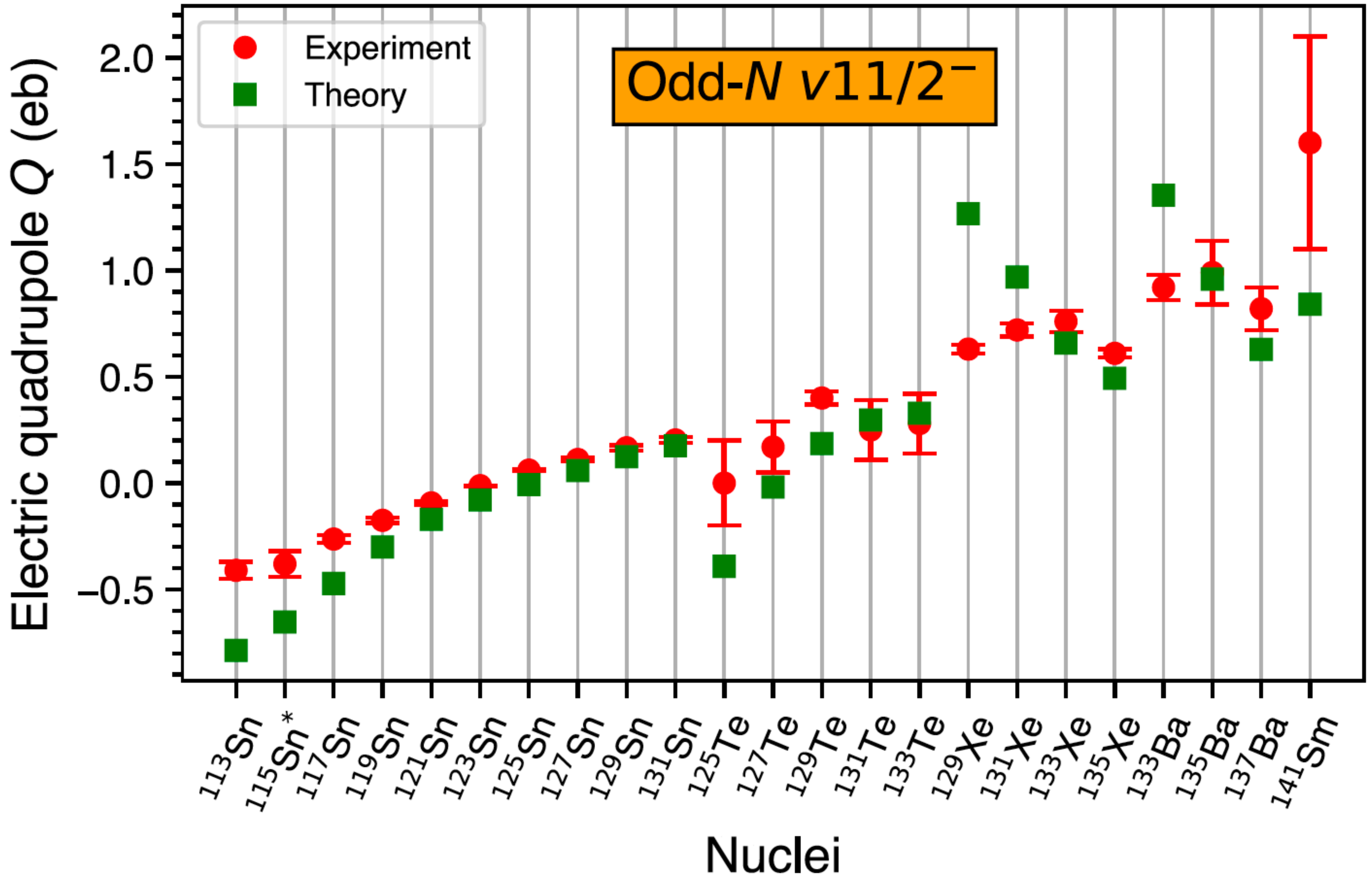
Systematic nuclear DFT calculations: Sn-Gd

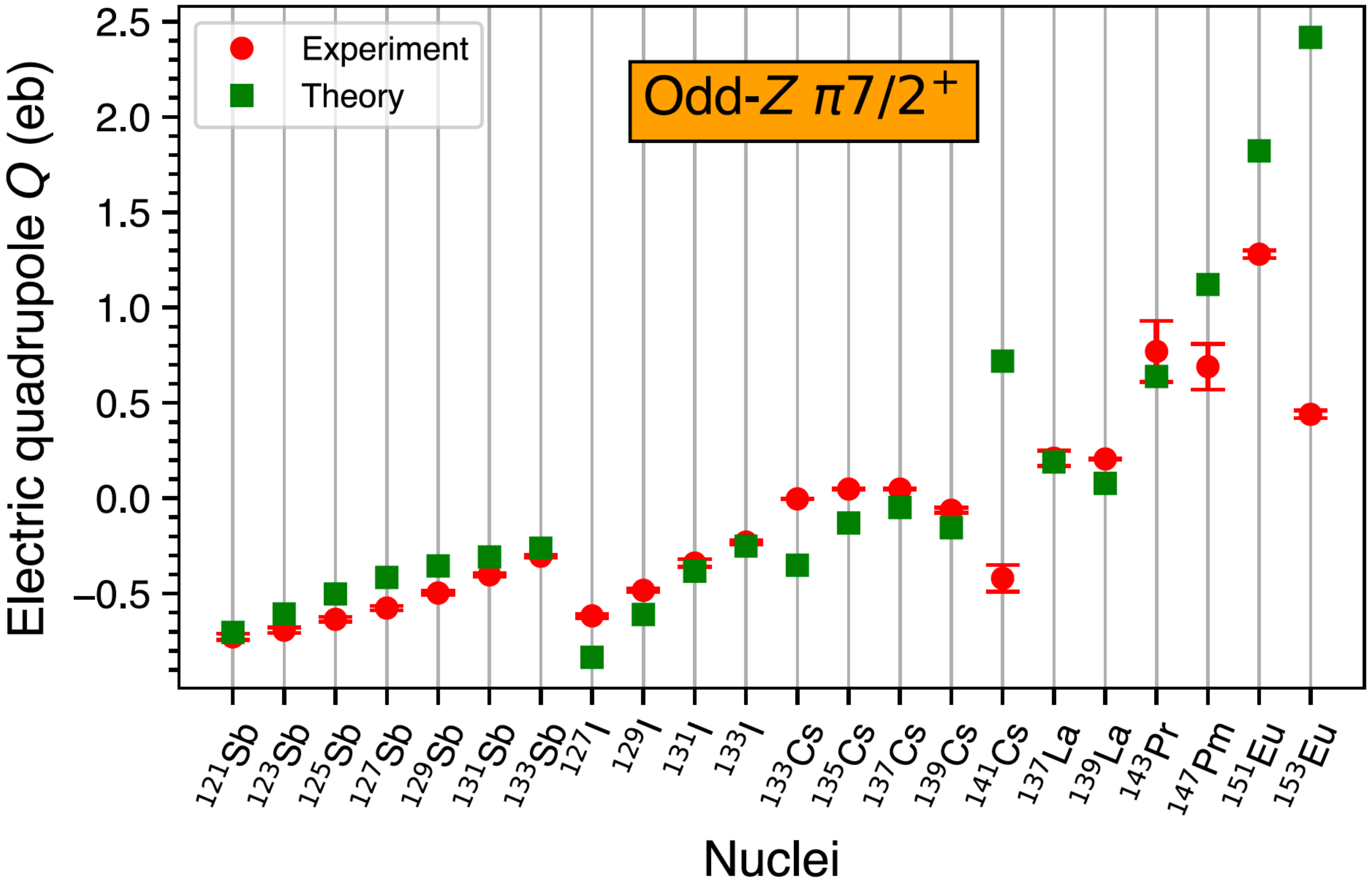


H. Wibowo, *et al.*, *J. Phys. G: Nucl. Part. Phys.* ***52****, 065104 (2025)*

$g_0' = 1.7(4)$ (UNEDF1)

Electric quadrupole moments: theory vs. experiment



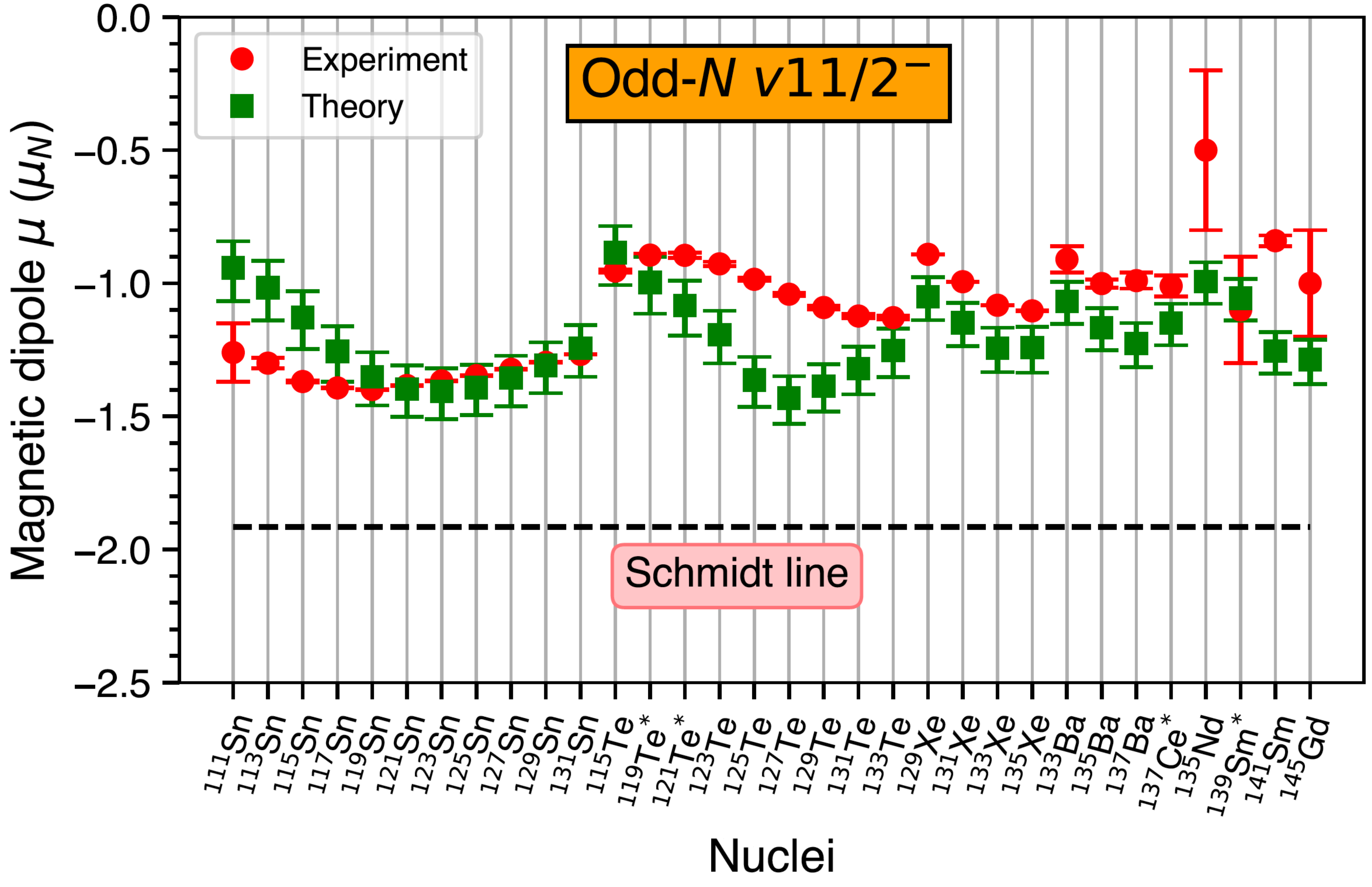


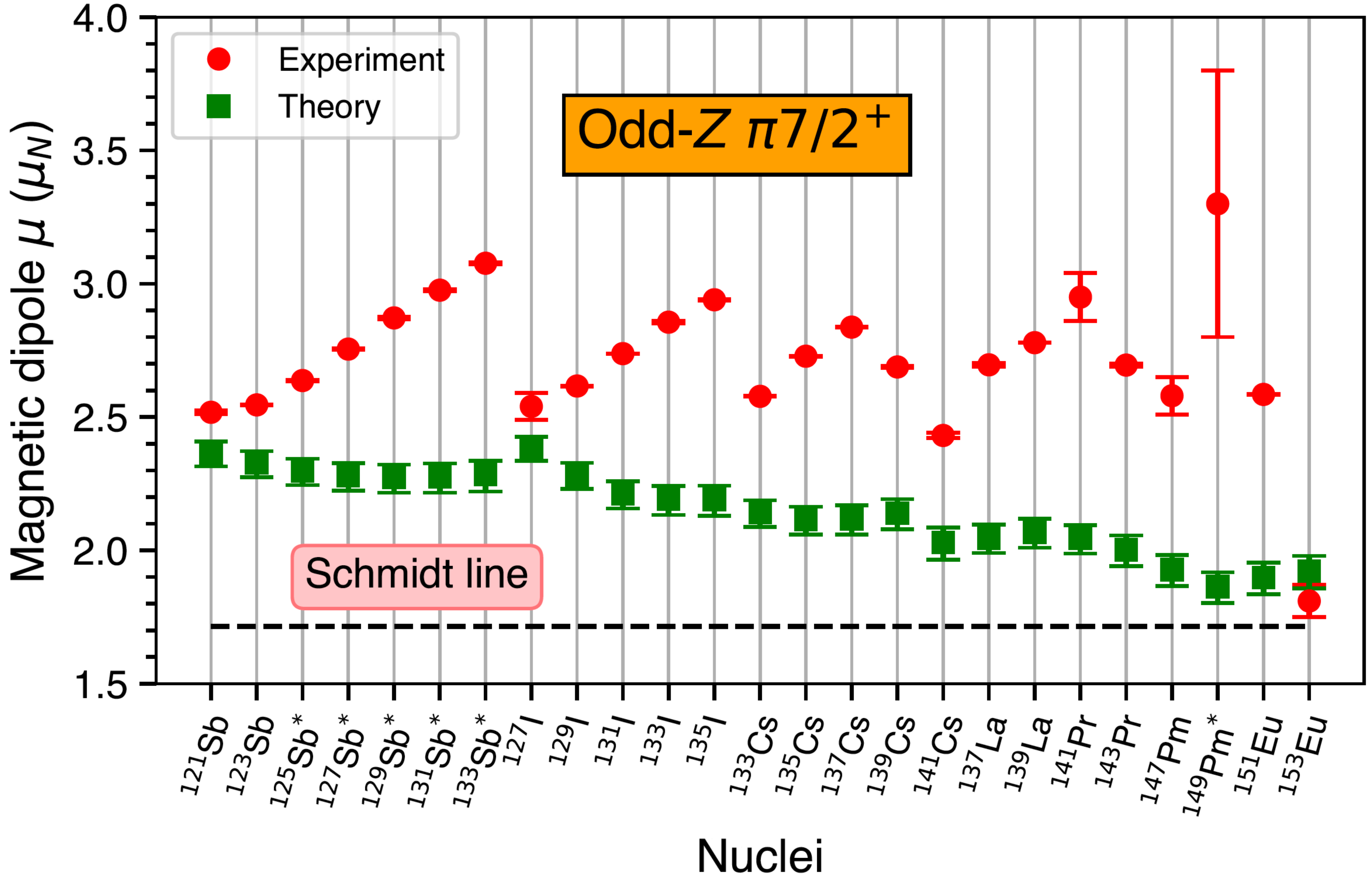
$$g_0' = 1.7(4)\ \text{(UNEDF1)}$$

H. Wibowo, *et.al.*, *J. Phys. G: Nucl. Part. Phys.* ***52****, 065104 (2025)*

N. J. Stone, INDC, report INDC(NDS)-0658
N. J. Stone, INDC, report INDC(NDS)-0794
N. J. Stone, INDC, report INDC(NDS)-0816
Yordanov D. T. *et al.*, Comm. Phys. 3, 107 (2020)
Lechner S. *et. al.*, Phys. Lett. B 847, 138278 (2023)

Magnetic dipole moments: theory vs. experiment



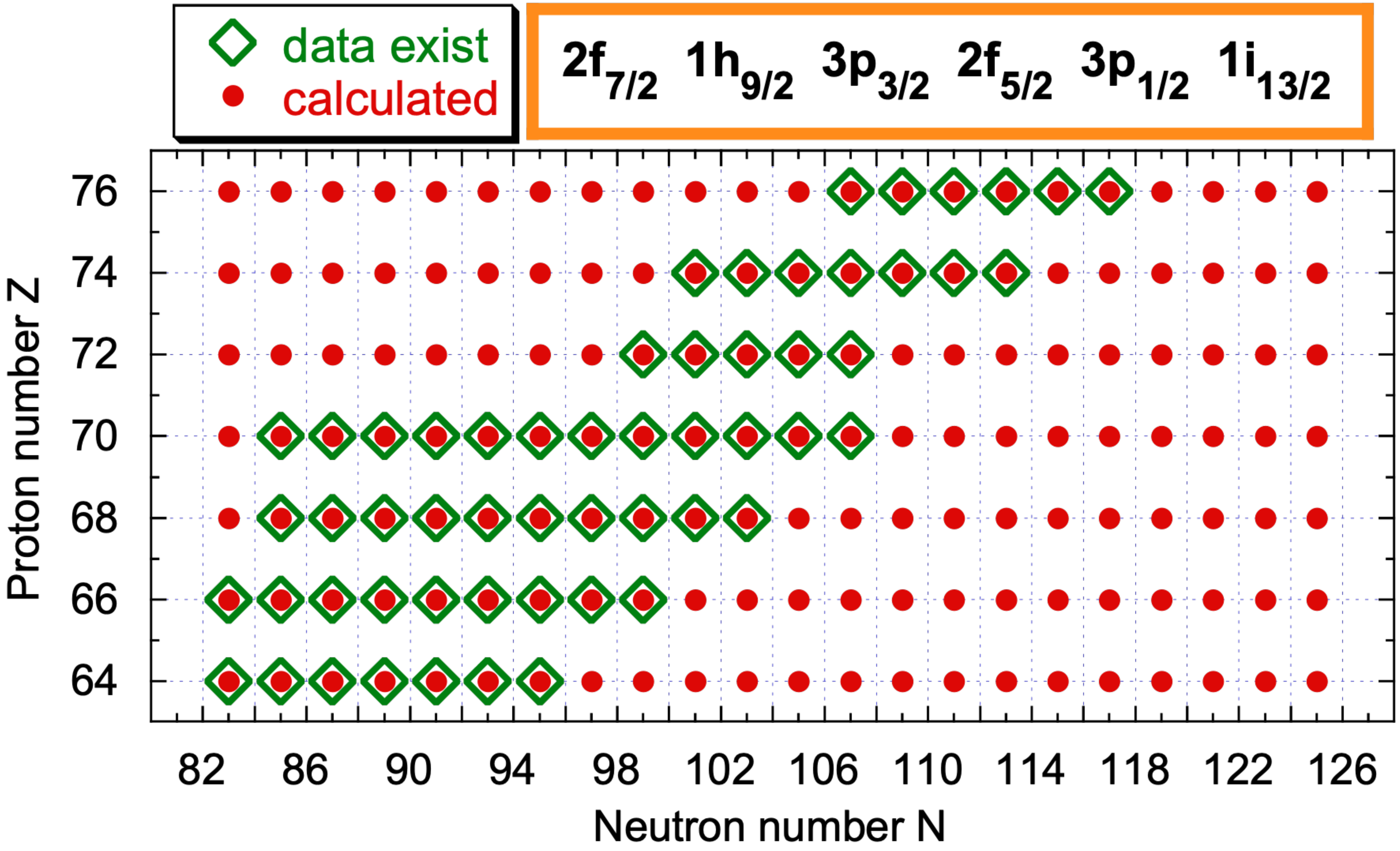


$$g_0' = 1.7(4) \text{ (UNEDF1)}$$

H. Wibowo, *et.al.*, *J. Phys. G: Nucl. Part. Phys.* ***52****, 065104 (2025)*

N. J. Stone, INDC, report INDC(NDS)-0658
N. J. Stone, INDC, report INDC(NDS)-0794
N. J. Stone, INDC, report INDC(NDS)-0816
Yordanov D. T. *et al.*, Comm. Phys. 3, 107 (2020)
Lechner S. *et. al.*, Phys. Lett. B 847, 138278 (2023)

The first systematic nuclear-DFT analysis of the electromagnetic moments in excited quasiparticle states



44 (22 prolate + 22 oblate tags)

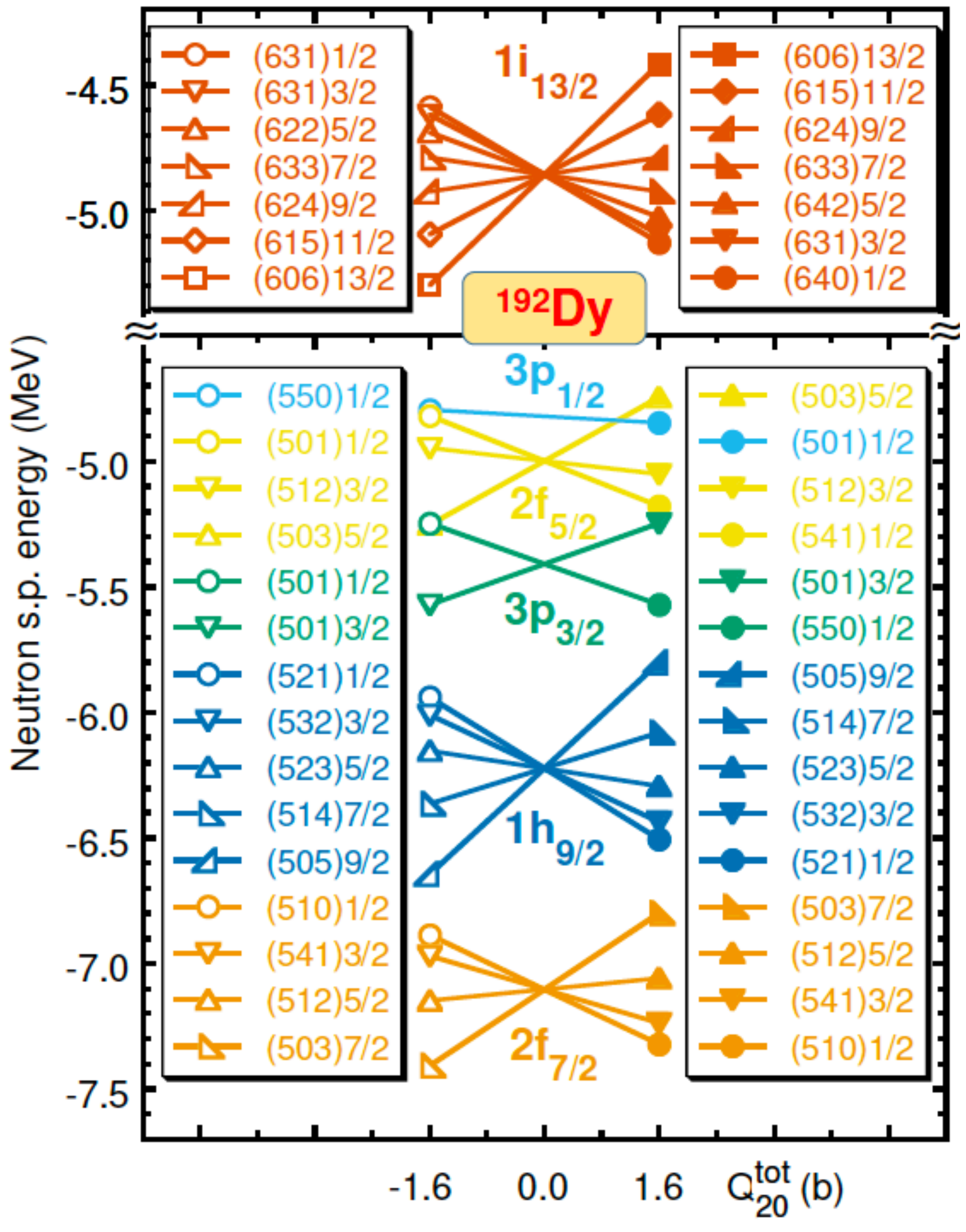
6,776 independent self-consistent calculations

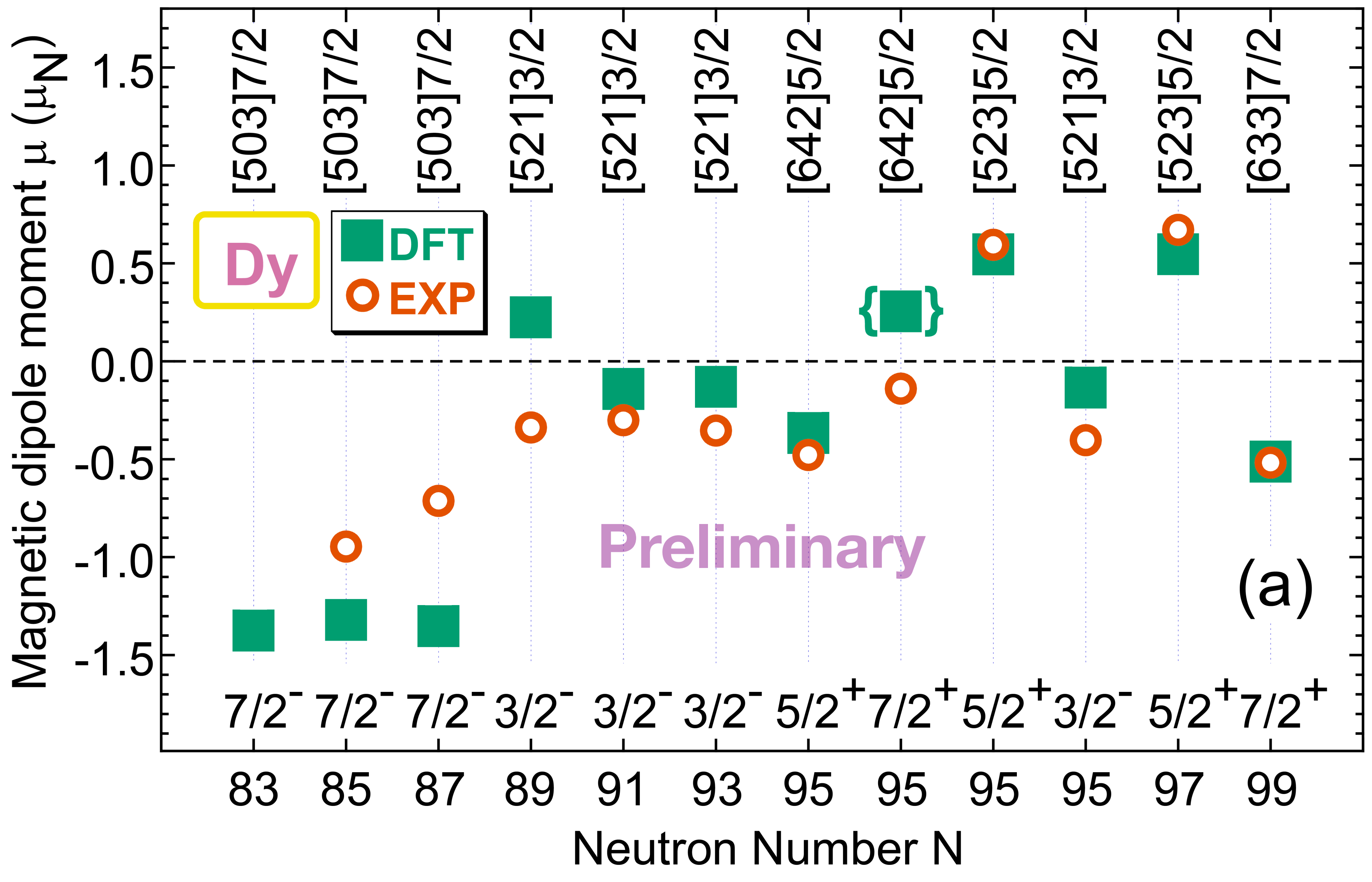
7 elements

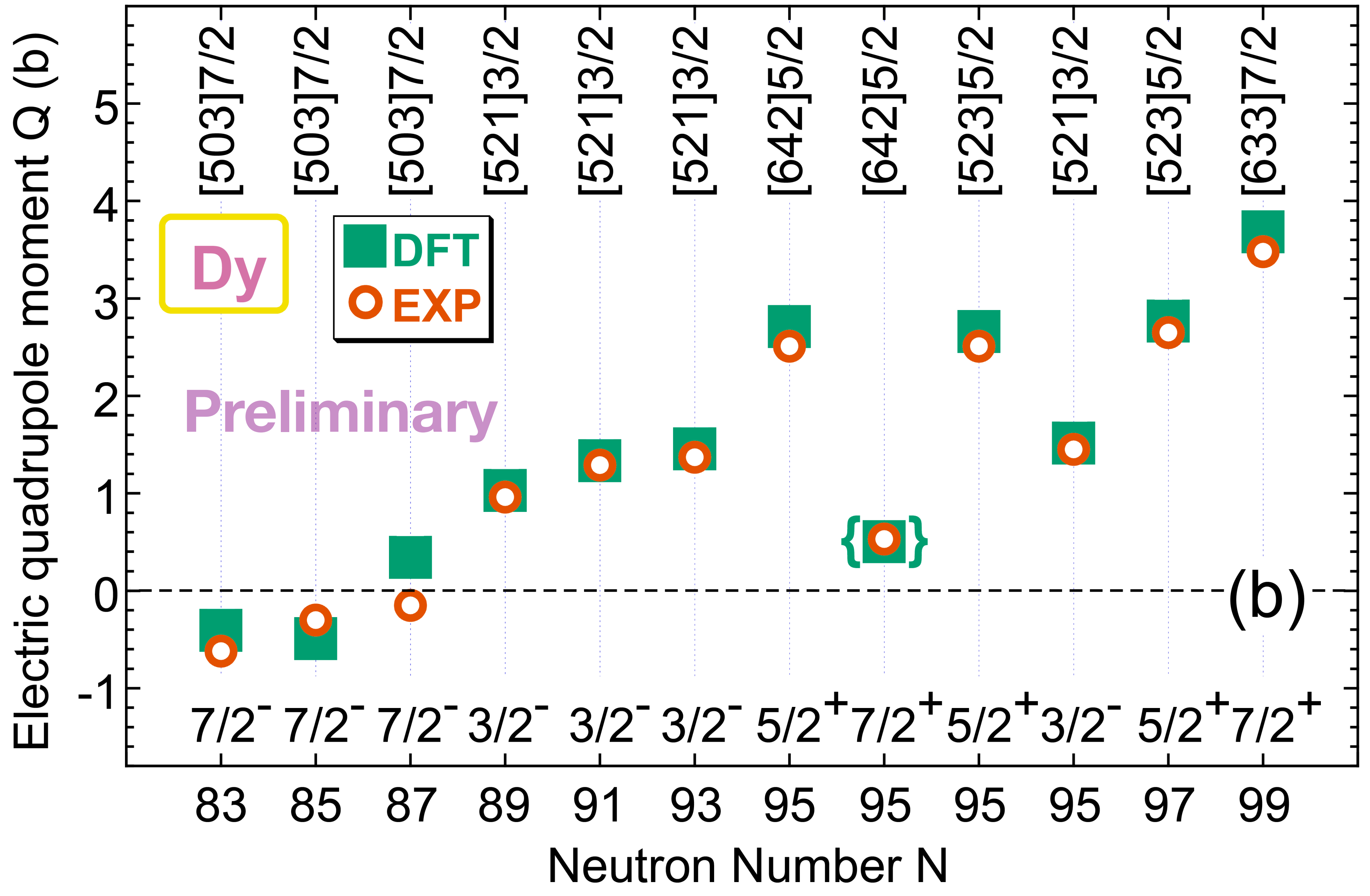
22 isotopes

J. Dobaczewski, A. E. Stuchbery *et al.*, **Electromagnetic moments of ground and excited states calculated in heavy odd-N open-shell nuclei**, *to be published (2025)*

14 Electromagnetic moments of odd dysprosium isotopes







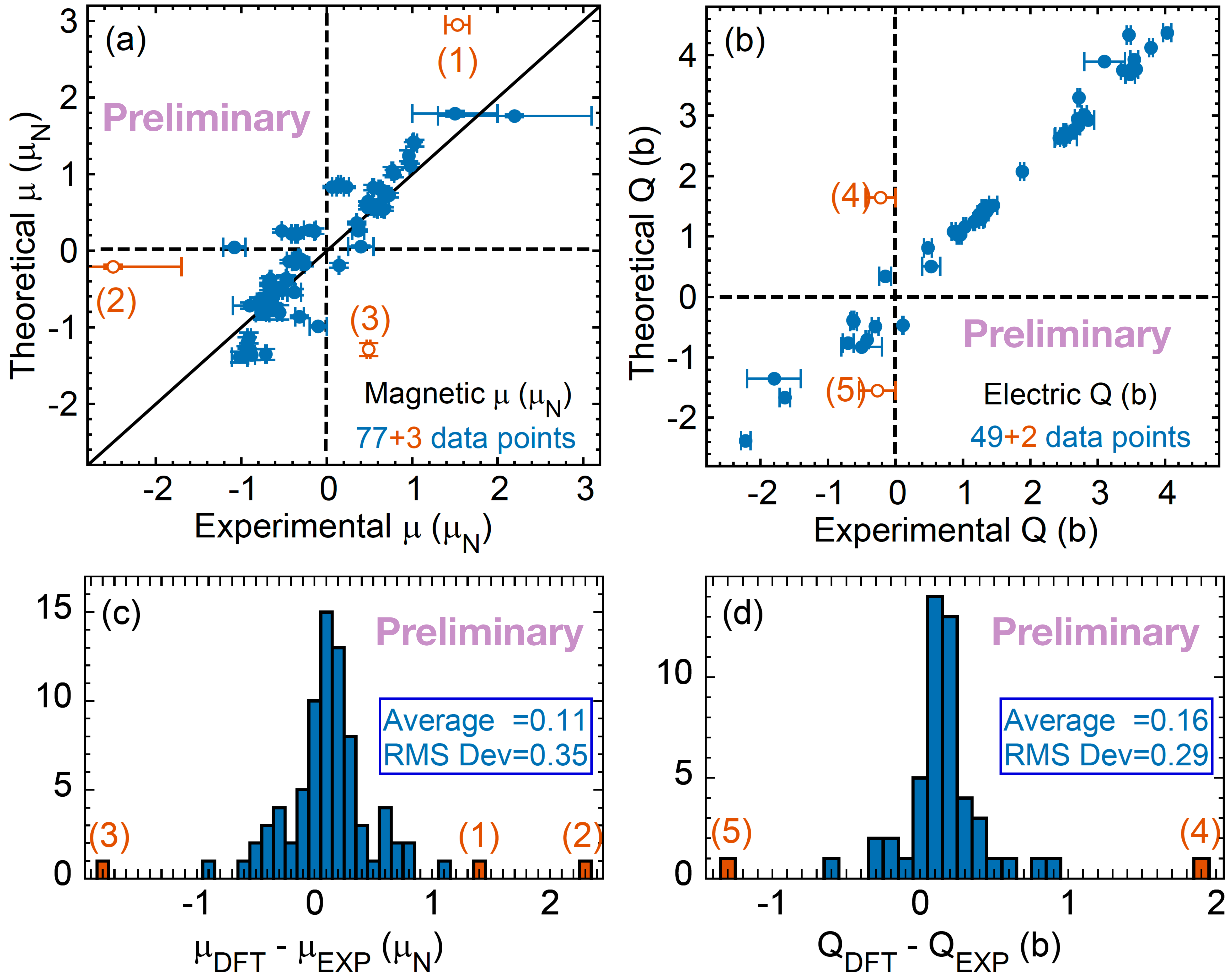
{■} Member of rotational band

J. Dobaczewski, A. E. Stuchbery *et al.*, *to be published (2025)*

15

Summary of electromagnetic moments: Gd-Os

Odd-*N* Gd-Os nuclei



J. Dobaczewski, A. E. Stuchbery *et al., to be published (2025)*

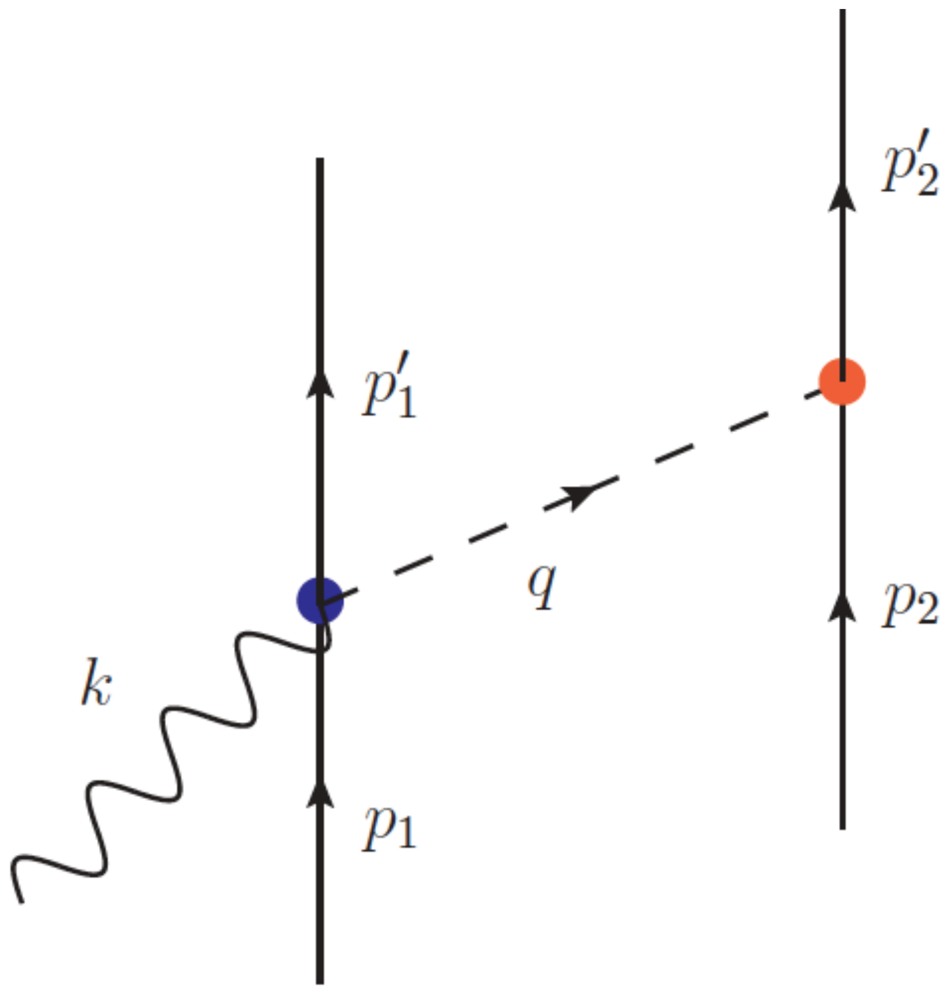
Meson-exchange contributions

Magnetic moment operator: $$\hat{\boldsymbol{\mu}} = \sum_{k}^{A} \hat{\boldsymbol{\mu}}_{1\mathrm{b},k} + \sum_{k<\ell}^{A} \hat{\boldsymbol{\mu}}_{2\mathrm{b},k\ell} + \cdots$$

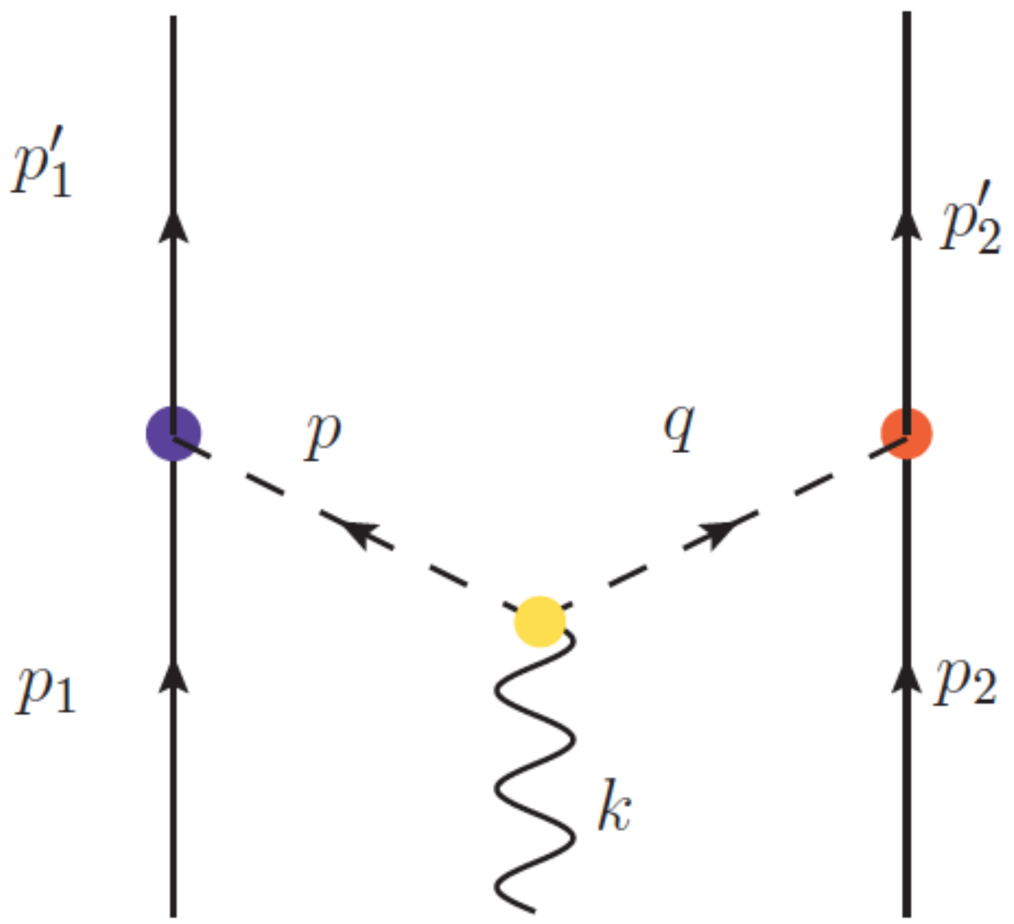
One-body: $$\hat{\boldsymbol{\mu}}_{1\mathrm{b},k} = g_{\ell}^{(k)} \hat{\boldsymbol{\ell}}_k + g_s^{(k)} \hat{\mathbf{s}}_k$$

Two-body meson-exchange: $$\hat{\boldsymbol{\mu}}_{2\mathrm{b}}^{\mathrm{NLO}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} \int d^3\mathbf{x}\, \mathbf{x} \times \mathbf{j}_{2\mathrm{b}}^{\mathrm{NLO}}(\mathbf{x}, \mathbf{r}_1, \mathbf{r}_2)$$

Next-to-leading order (NLO):



Seagull graph

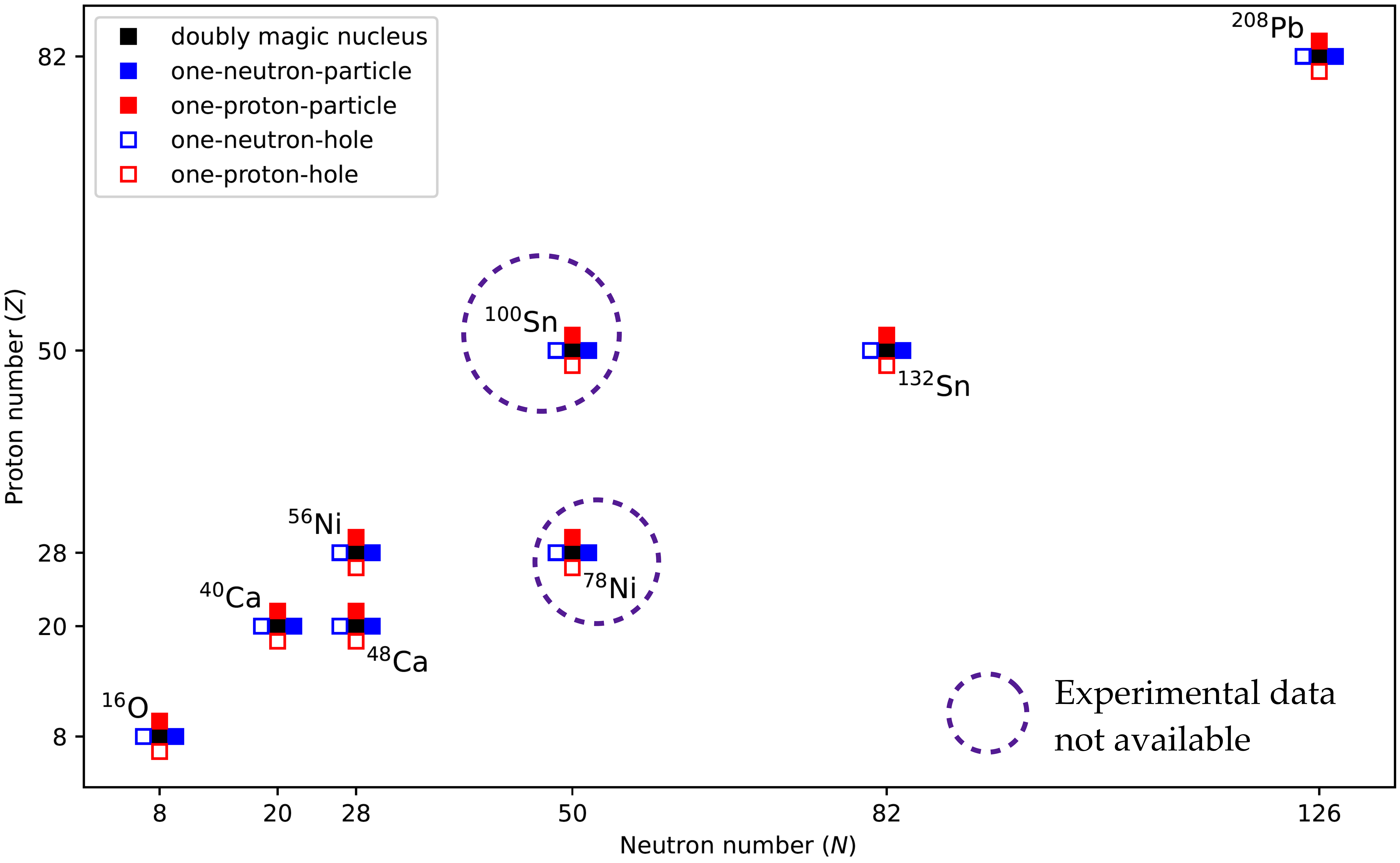


Pion-in-flight graph

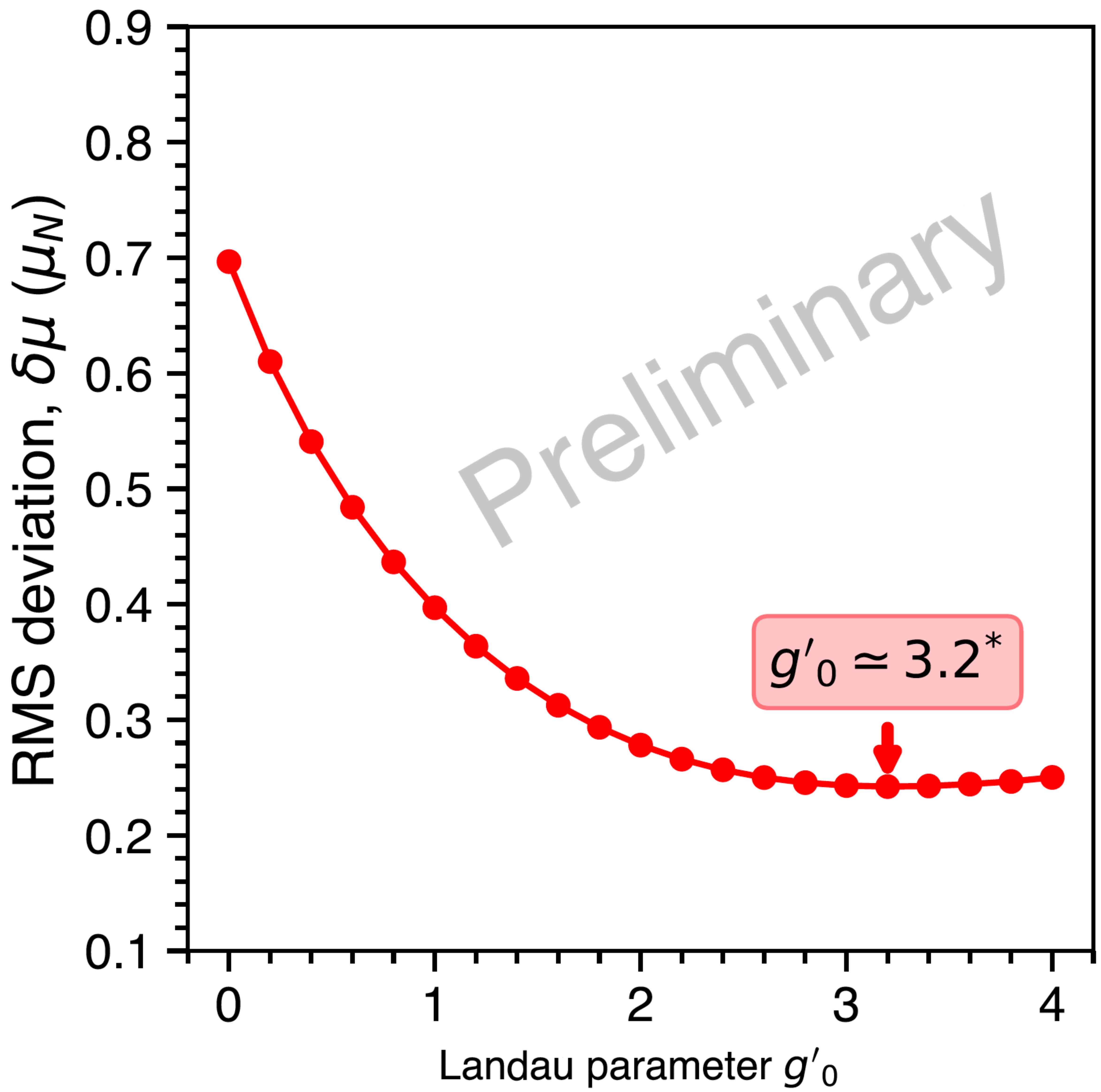
R. Seutin, *et al.*, PRC **108**, 054005 (2023)

T. Miyagi, *et al.*, PRL **132**, 232503 (2024)

Odd near doubly magic nuclei (two-body currents)



Optimum Landau parameter g_0' (two-body currents)



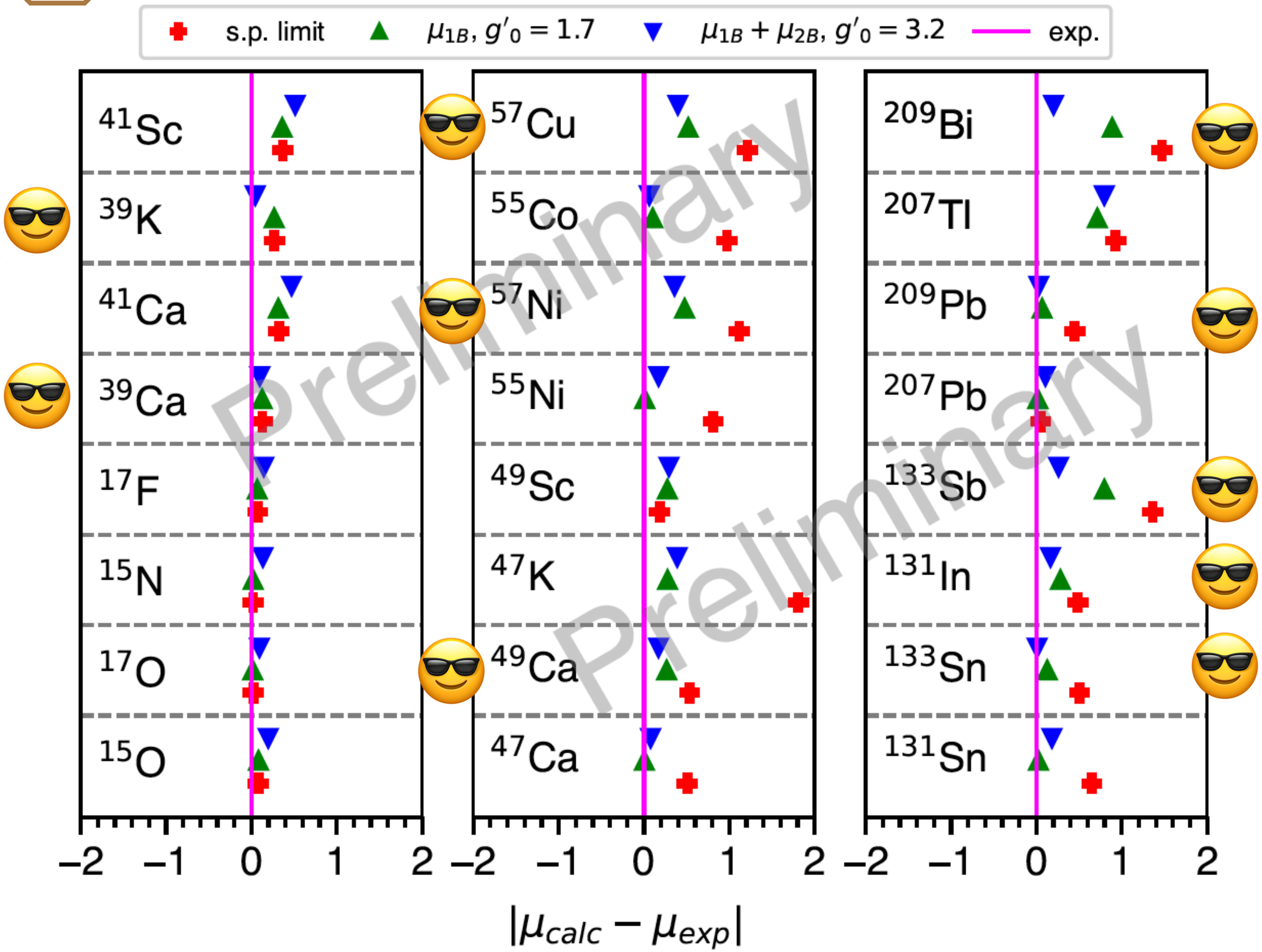
$$\delta\mu(g_0') = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{\text{calc}}(i, g_0') - \mu_{\text{exp}}(i, g_0')\right]^2}$$

i = odd nucleus
N = number of odd nuclei

(*) 23 odd near doubly magic nuclei.

H. Wibowo, *et al.*,
to be published

Magnetic dipole moments: theory vs. experiment



- In 10/14 cases, the two-body-current corrections **improve**/**deteriorate** agreement with experimental data.
- The complete statistical analysis will follow.
- Results in deformed nuclei will follow.

H. Wibowo, *et.al.*, to be published

Nuclear Laboratory Schiff Moment

$$S_{\text{lab}} \approx \sum_{i\neq 0} \frac{\langle \Psi_0 | \hat{S}_0 | \Psi_i \rangle \langle \Psi_i | \hat{V}_{\text{PT}} | \Psi_0 \rangle}{E_0 - E_i} + \text{c.c.}$$

$$\hat{S}_0 = \frac{e}{10}\sqrt{\frac{4\pi}{3}} \sum_p \left(r_p^3 - \frac{5}{3}\overline{r_{\text{ch}}^2}\, r_p \right) Y_0^1(\Omega_p) \quad \text{(Schiff operator)}$$

$$\hat{V}_{\text{PT}} = \text{P,T-violating } NN \text{ interaction}$$

Ac
$Z = 89$
$N = 138$

$3/2^{+}$ $|\widetilde{\Psi}_0\rangle$

$\Delta E = 27.369$ keV

$3/2^{-}$ $|\Psi_0\rangle$

$$S_{\text{lab}} \approx -2\text{Re}\left\{ \frac{\langle \Psi_0 | \hat{S}_0 | \widetilde{\Psi}_0 \rangle \langle \widetilde{\Psi}_0 | \hat{V}_{\text{PT}} | \Psi_0 \rangle}{\Delta E} \right\}$$

$$\hat{Q}_0^3 = e \sum_p r_p^3 Y_0^3(\Omega_p) \quad \text{(Octupole operator)}$$

C. Maples, Nuclear Data Sheets **22**, 275, 277-323 (1977)

J. Dobaczewski *et al.*, PRL **121**, 232501 (2018) and the supplemental material

P, T-violating interaction

$$\hat{V}_{\mathrm{PT}}(\boldsymbol{r}_1-\boldsymbol{r}_2)=-\frac{gm_\pi^2}{8\pi m_N}\left\{(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)\left[\bar{g}_0\vec{\tau}_1\cdot\vec{\tau}_2-\frac{\bar{g}_1}{2}(\tau_{1z}+\tau_{2z})+\bar{g}_2(3\tau_{1z}\tau_{2z}-\vec{\tau}_1\cdot\vec{\tau}_2)\right]\right.$$
$$\left.-\frac{\bar{g}_1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)(\tau_{1z}-\tau_{2z})\right\}\frac{\exp(-m_\pi|\boldsymbol{r}_1-\boldsymbol{r}_2|)}{m_\pi|\boldsymbol{r}_1-\boldsymbol{r}_2|^2}\left[1+\frac{1}{m_\pi|\boldsymbol{r}_1-\boldsymbol{r}_2|}\right]$$
$$+\frac{1}{2m_N^3}[\bar{c}_1+\bar{c}_2\vec{\tau}_1\cdot\vec{\tau}_2](\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{\nabla}\delta^3(\boldsymbol{r}_1-\boldsymbol{r}_2),$$

C. M. Maekawa, E. Mereghetti, J. de Vries, and U. van Kolck, Nucl. Phys. **A872**, 117 (2011)
W. C. Haxton and E. M. Henley, Phys. Rev. Lett. **51**, 1937 (1983)
P. Herczeg, Hyperfine Interact. **43**, 75 (1988)

$$\langle\hat{V}_{\mathrm{PT}}\rangle_{\mathrm{int}}=v_0 g\bar{g}_0+v_1 g\bar{g}_1+v_2 g\bar{g}_2+w_1\bar{c}_1+w_2\bar{c}_2$$

Nuclear Laboratory Schiff Moment

Rigid deformation approximation:

$$\langle \Psi_0 | \hat{S}_0 | \widetilde{\Psi}_0 \rangle_{\text{rigid}} = \frac{J}{J+1} S_{\text{int}}$$

$$\langle \widetilde{\Psi}_0 | \hat{V}_{\text{PT}} | \Psi_0 \rangle_{\text{rigid}} = \langle \hat{V}_{\text{PT}} \rangle_{\text{int}}$$

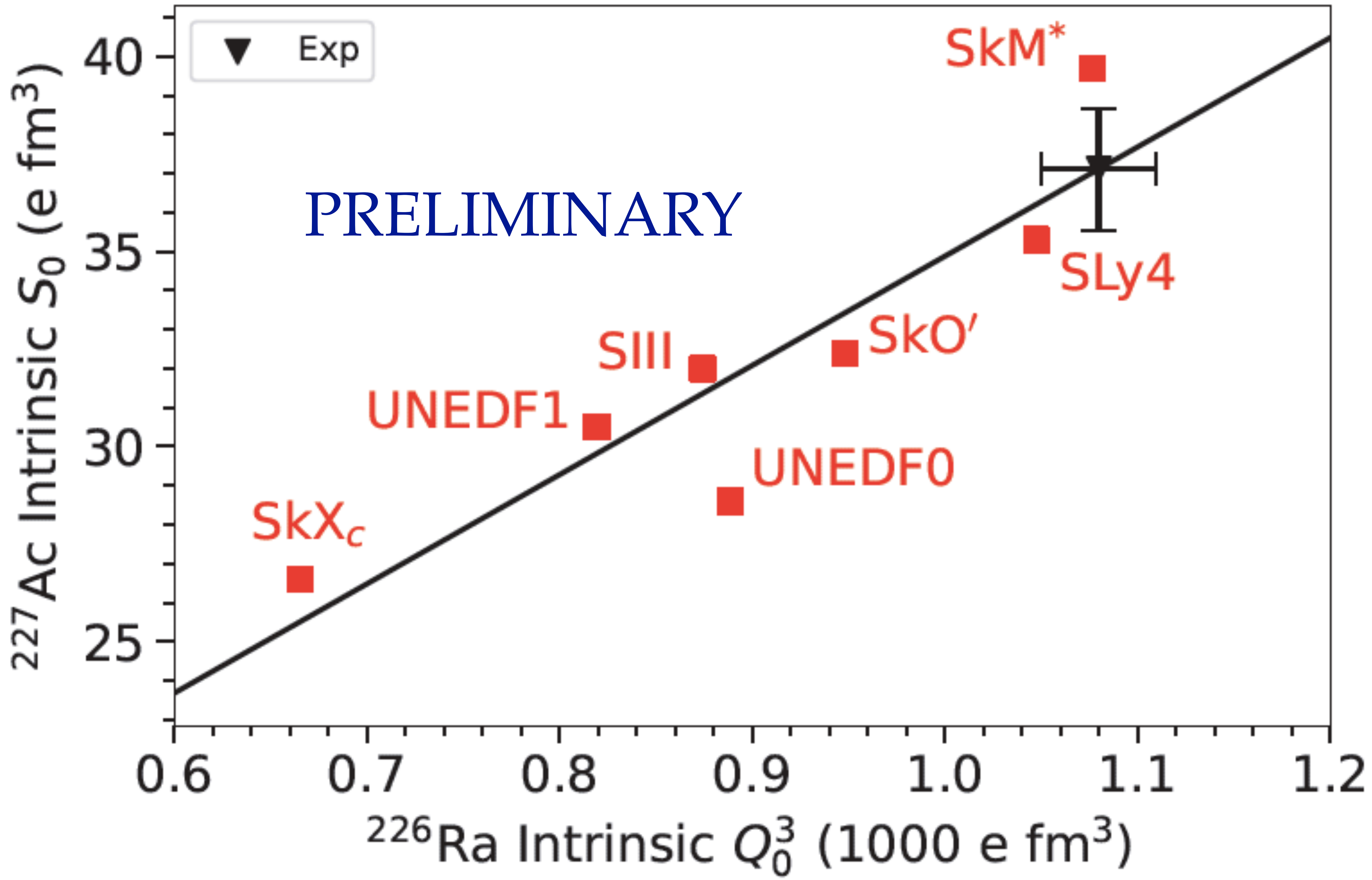
S_{int} and $\langle \hat{V}_{\text{PT}} \rangle_{\text{int}}$ are calculated within nuclear DFT.

$$\langle \hat{V}_{\text{PT}} \rangle_{\text{int}} = v_0 g \bar{g}_0 + v_1 g \bar{g}_1 + v_2 g \bar{g}_2 + w_1 \bar{c}_1 + w_2 \bar{c}_2$$

$$S_{\text{lab}} = a_0 g \bar{g}_0 + a_1 g \bar{g}_1 + a_2 g \bar{g}_2 + b_1 \bar{c}_1 + b_2 \bar{c}_2$$

$$a_i = -\frac{2J}{J+1} \frac{S_{\text{int}} v_i}{\Delta E} \quad \text{and} \quad b_i = -\frac{2J}{J+1} \frac{S_{\text{int}} w_i}{\Delta E}$$

Correlation between Octupole and Schiff Moments



M. Athanasakis-Kaklamanakis, *et al.*,
ArXiv:2507.05224

Estimated Intrinsic Schiff Moment

Intrinsic Schiff moment: $S_0(\mathrm{est}) = a + bQ_0^3(\mathrm{exp})$

Measured Octupole moment: $Q_0^3(^{226}\mathrm{Ra}) = 1080(30)\ \mathrm{e\ fm}^3$

Total uncertainty: $\Delta S_0(\mathrm{est}) = \sqrt{[\Delta S_0(\mathrm{the})]^2 + [\Delta S_0(\mathrm{exp})]^2}$

Experimental uncertainty: $\Delta S_0(\mathrm{exp}) = b\Delta Q_0^3(\mathrm{exp})$

Theoretical uncertainty: $[\Delta S_0(\mathrm{the})]^2 = \mathcal{C}_{aa} + 2\mathcal{C}_{ab}\bar{Q}_0^3(\mathrm{exp}) + \mathcal{C}_{bb}[\bar{Q}_0^3(\mathrm{exp})]^2$

$$S_0(^{227}\mathrm{Ac}) = 37.1(1.6)\ \mathrm{e\ fm}^3$$

M. Athanasakis-Kaklamanakis, *et al.*, ArXiv:2507.05224

H. Wollersheim et al., Nucl. Phys. A **556**, 261 (1993)

J. Dobaczewski *et al.*, PRL **121**, 232501 (2018) and the supplemental material

Summary and Outlook

Summary

- ★ We have demonstrated systematic nuclear-DFT calculations of electromagnetic moments in wide range of heavy-deformed open-shell odd nuclei.
- ★ The inclusion of the meson-exchange currents improves the predictions of magnetic dipole moments in ^{39}K, ^{39}Ca, ^{57}Cu, ^{57}Ni, ^{49}Ca, ^{209}Bi, ^{209}Pb, ^{133}Sb, ^{131}In, and ^{133}Sn.
- ★ We have demonstrated the nuclear-DFT calculation of laboratory Schiff moment in ^{227}Ac nucleus.

Outlook

- ★ Systematic nuclear DFT calculations of magnetic dipole moments across the nuclear chart with the inclusion of meson-exchange currents.
- ★ Calculations of anapole moments in heavy open-shell odd nuclei within nuclear-DFT framework.

Back up slides

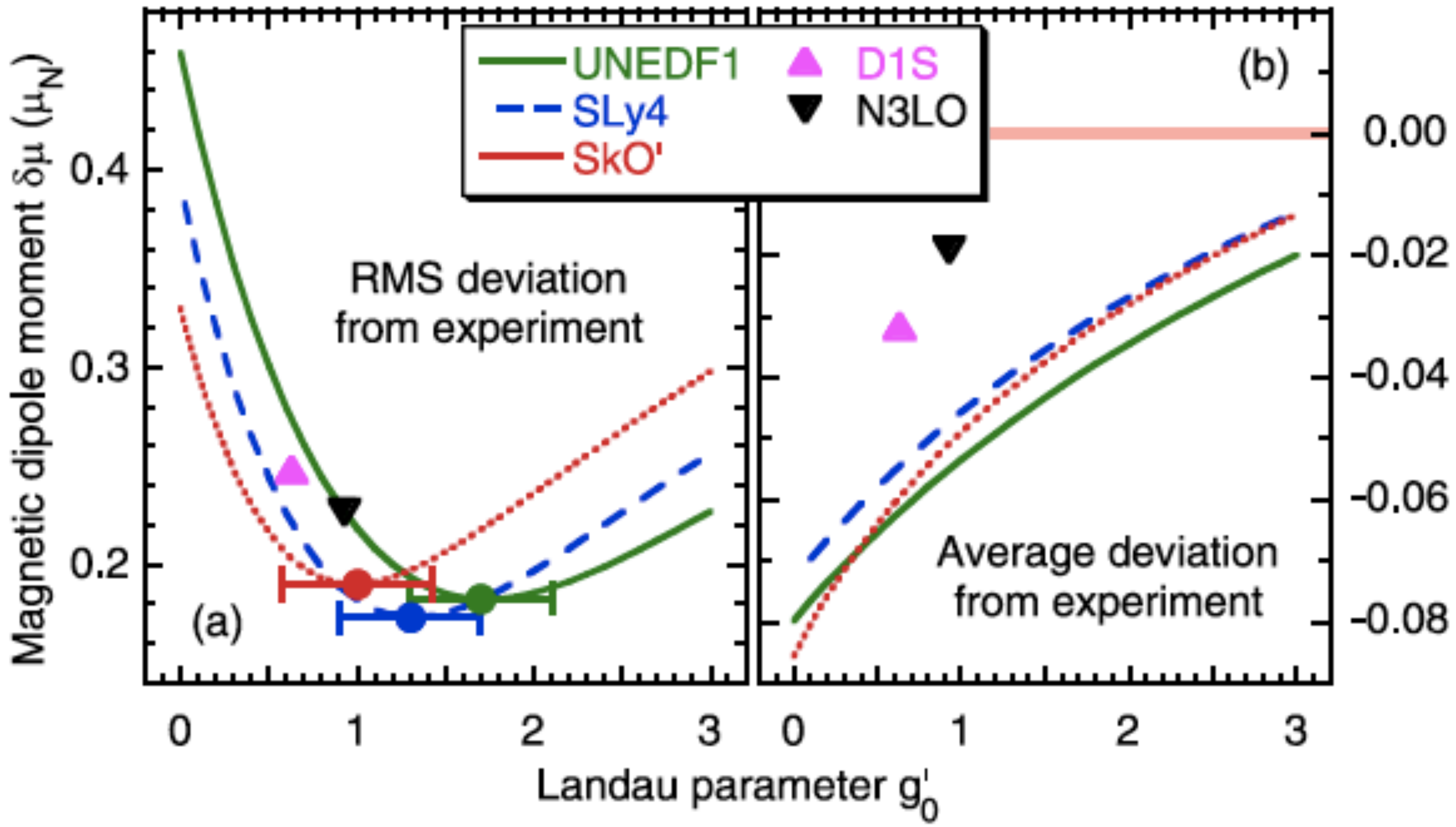
Landau Parameters

$$g_0 = N_0 \left\{ 2C_0^s[\rho_{\rm sat}] + 2C_0^T \beta \rho_{\rm sat}^{2/3} \right\}$$

$$g_0' = N_0 \left\{ 2C_1^s[\rho_{\rm sat}] + 2C_1^T \beta \rho_{\rm sat}^{2/3} \right\}$$

$$\beta = (3\pi^2/2)^{2/3}$$

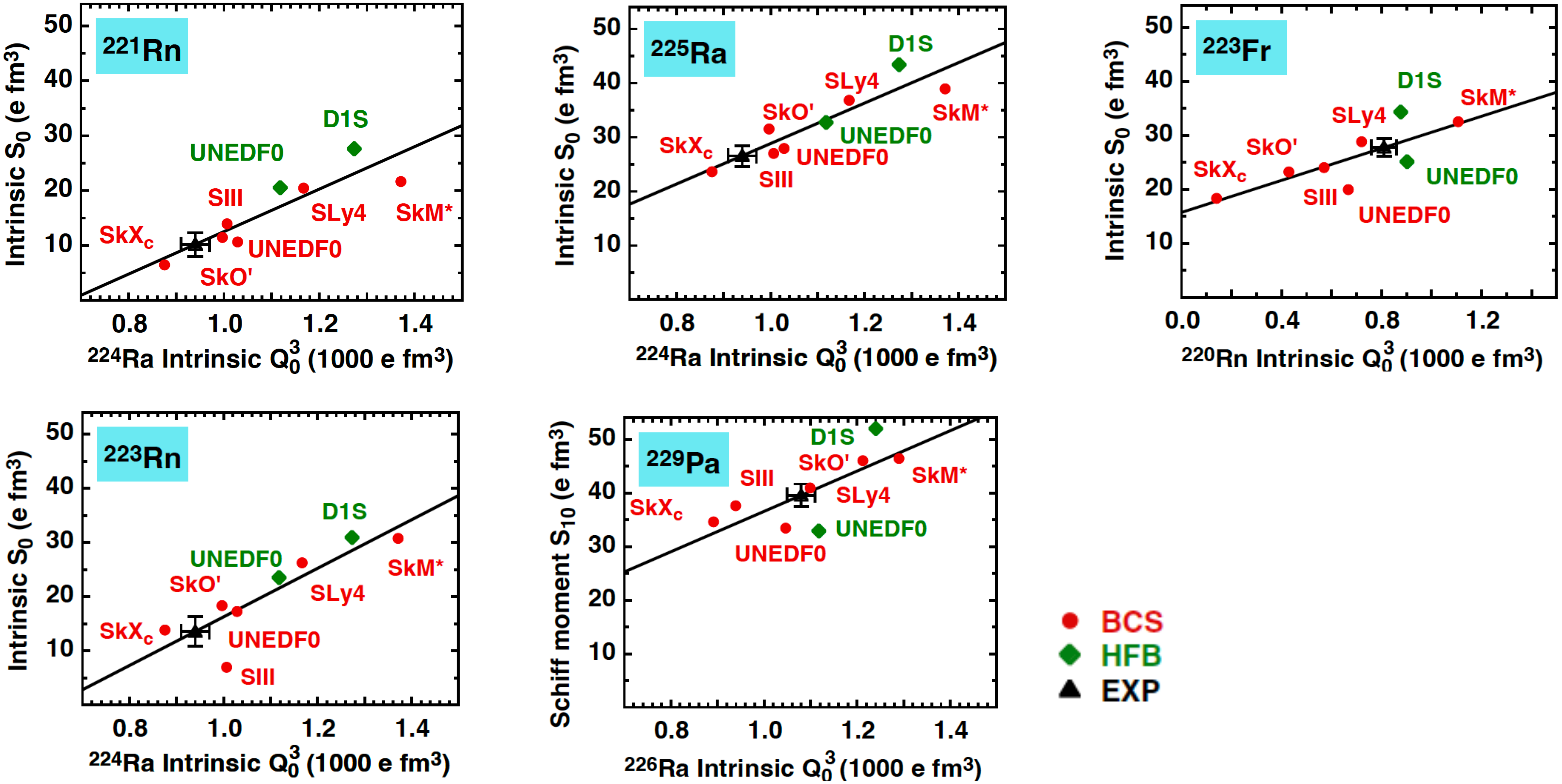
$$N_0 = \pi^{-2} \left(\frac{\hbar^2}{2m} \right)^{-1} \left(\frac{m^*}{m} \right) \left(\frac{3\pi^2 \rho_{\rm sat}}{2} \right)^{1/3}$$



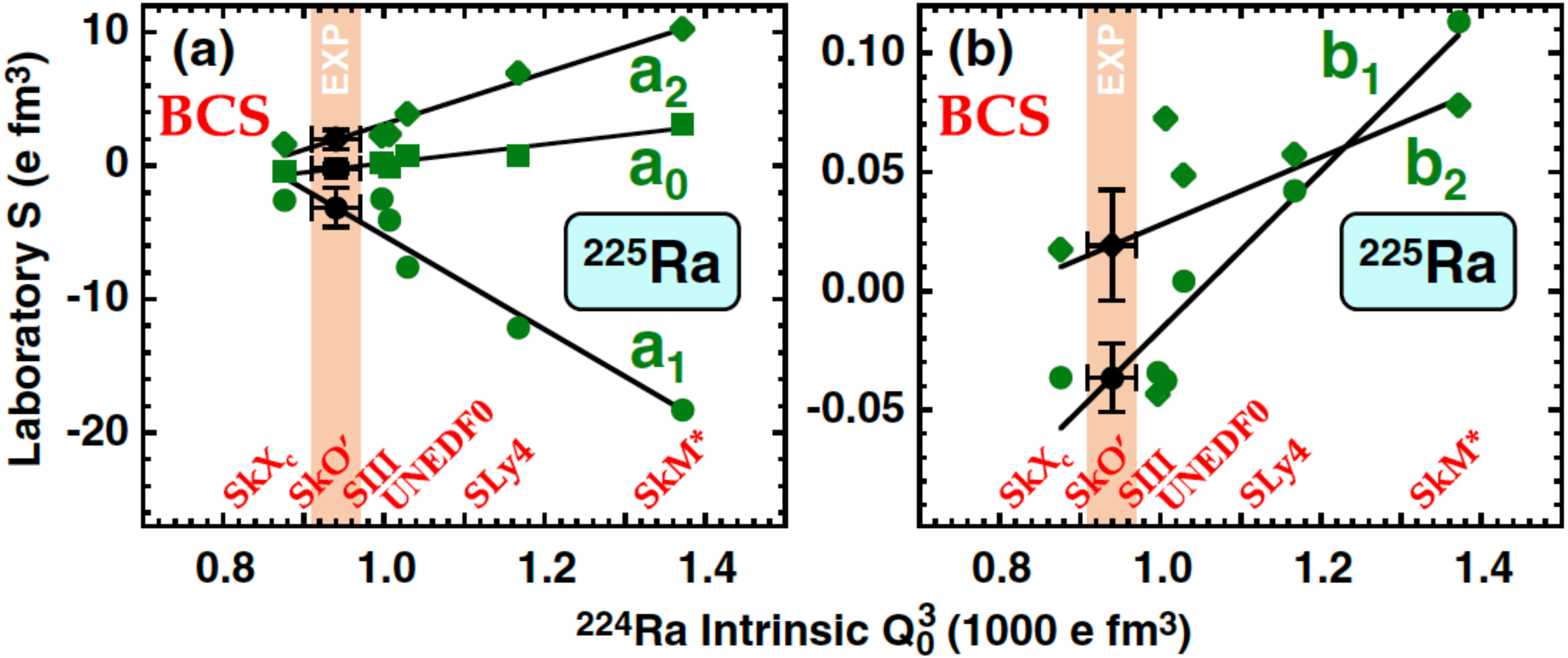
- ★ one-hole or one-particle of neighboring magic nuclei: ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{78}Ni, ^{100}Sn, ^{132}Sn, ^{208}Pb
- ★ $g_0 = 0.4$
- ★ $g_0' = 1.0$, 1.3, and 1.7 for SkO′, SLy4, and UNEDF1

P. L. Sassarini et al., J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

J. Dobaczewski et al., PRL **121**, 232501 (2018) and the supplemental material



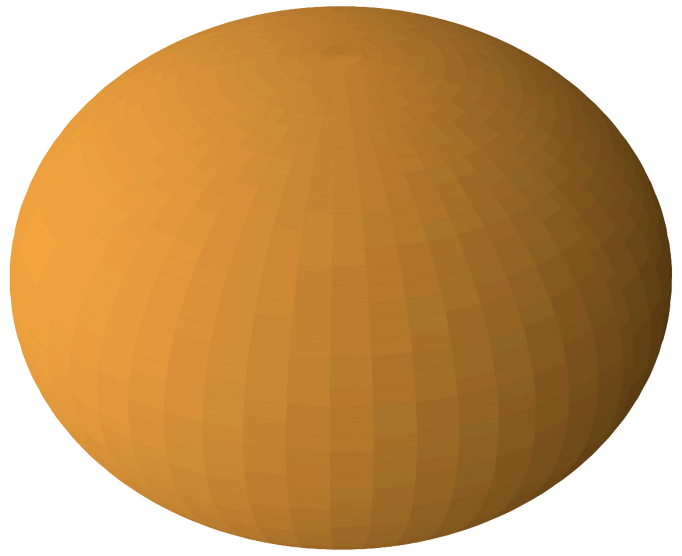
J. Dobaczewski et al., PRL **121**, 232501 (2018) and the supplemental material



Numerical scheme

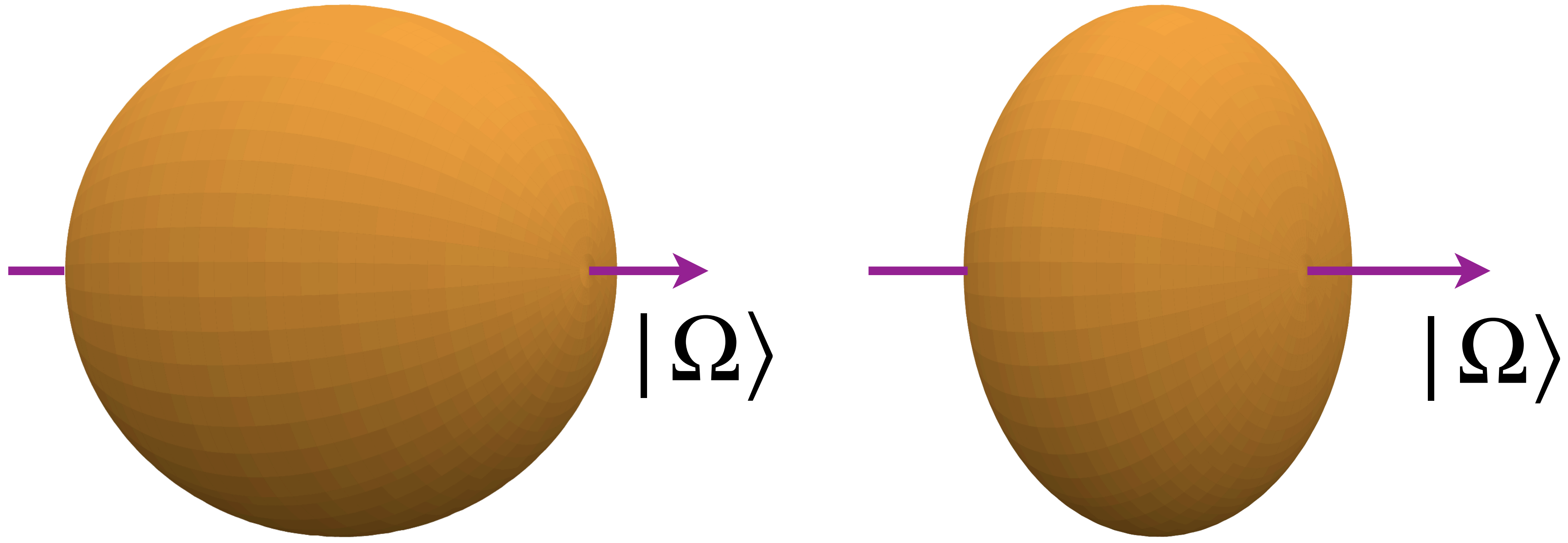
H. Wibowo, *et al.*, J. Phys. G: Nucl. Part. Phys. **52**, 065104 (2025)

1



- ✓ symmetries
- ✗ pairing correlations

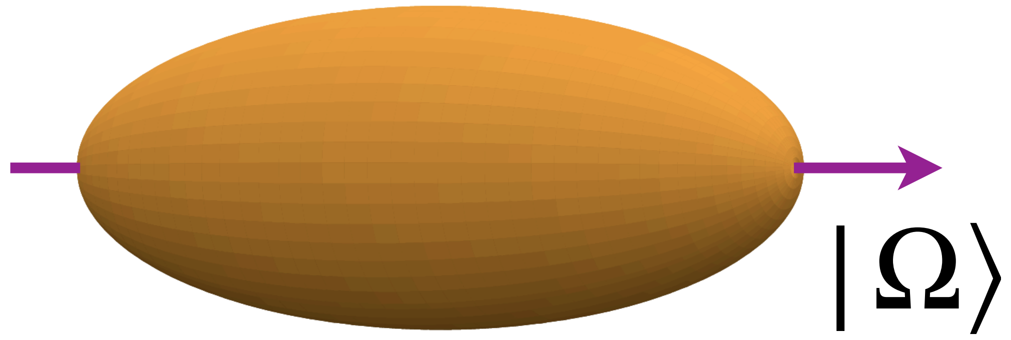
2 ✓ parity ✓ axial symmetry

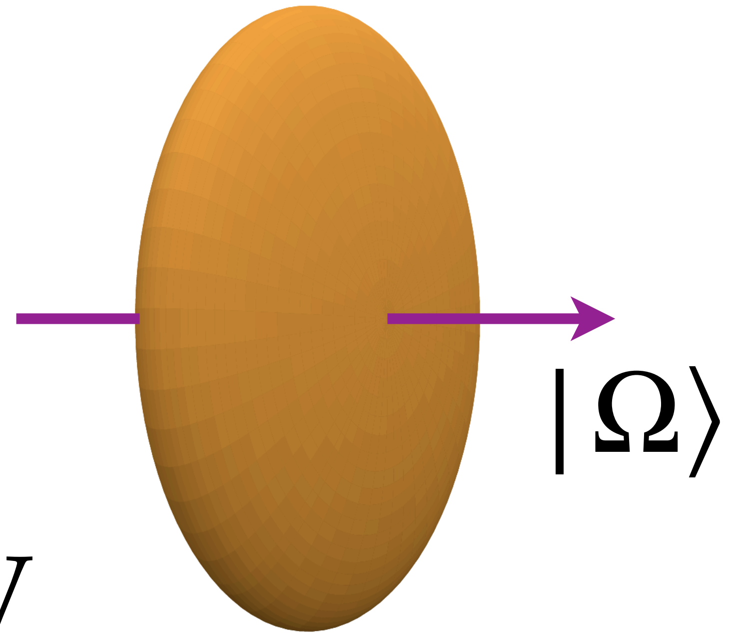


$\bar{Q}_{20} = +0.858$ b $\qquad \bar{Q}_{20} = -0.858$ b

tagged configurations : $\pi[404]7/2^{+}$ and $\nu[505]11/2^{-}$

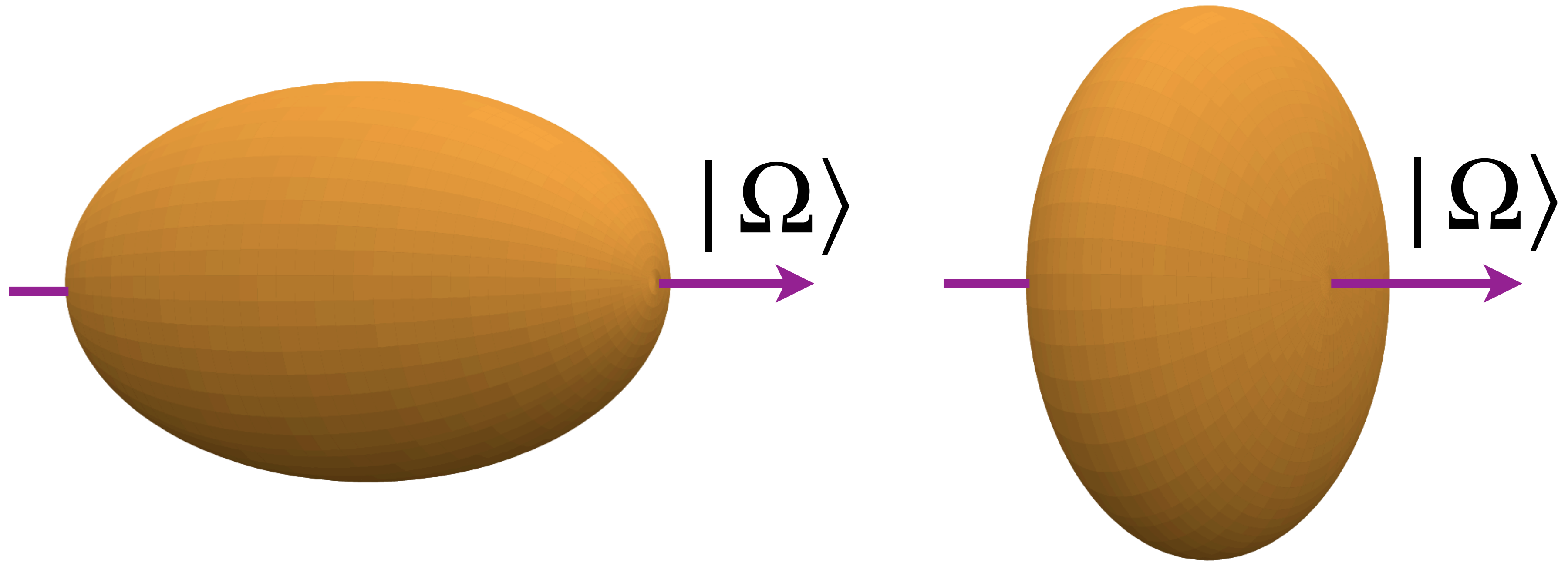
3 $\bar{Q}_{20} = +10$ b $\qquad \bar{Q}_{20} = -10$ b





$\Delta_n = \Delta_p = 1$ MeV

4



converged prolate and oblate shapes

5 $|\Omega\rangle \xrightarrow{\text{AMP}} |II\rangle \rightarrow Q, \mu$