滴线附近原子核
奇异结构与跃迁研究

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S. M. Wang, W. Nazarewicz…

Outline

- Introduction
  - *ab initio* calculation in Nuclear Structure
  - Open quantum systems
  - Many-body pertubation theory (MBPT)
- Results
  - β-decay mirror symmetry breaking in dripline nuclei
  - $E2$ transitions in ^{36}Ca and ^{38}Ca
- Emulator for resonant state
- Summary

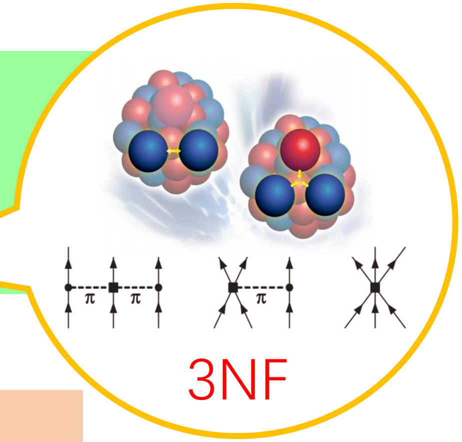
ab initio calculation in Nuclear Structure

Realistic nuclear force

Degree of freedom: nucleon (meson)

Reproduce nucleon-nucleon scattering experimental data

CD-Bonn, AV18, Chiral Effective Field Theory (χEFT) force



Renormalization of nuclear force

Decouple low- and high-momentum physics

$V_{\mathrm{low}-k}$ method, Similarity Renormalization Group (SRG)

Many-body methods

No-core shell model (NCSM), Many-body perturbation theory (MBPT), Coupled-Cluster (CC), In-Medium Similarity Renormalization Group (IMSRG), Nuclear Lattice effective field theory (NLEFT), Self-Consistent Green's Functions (SCGF), Quantum Monte Carlo (QMC), …

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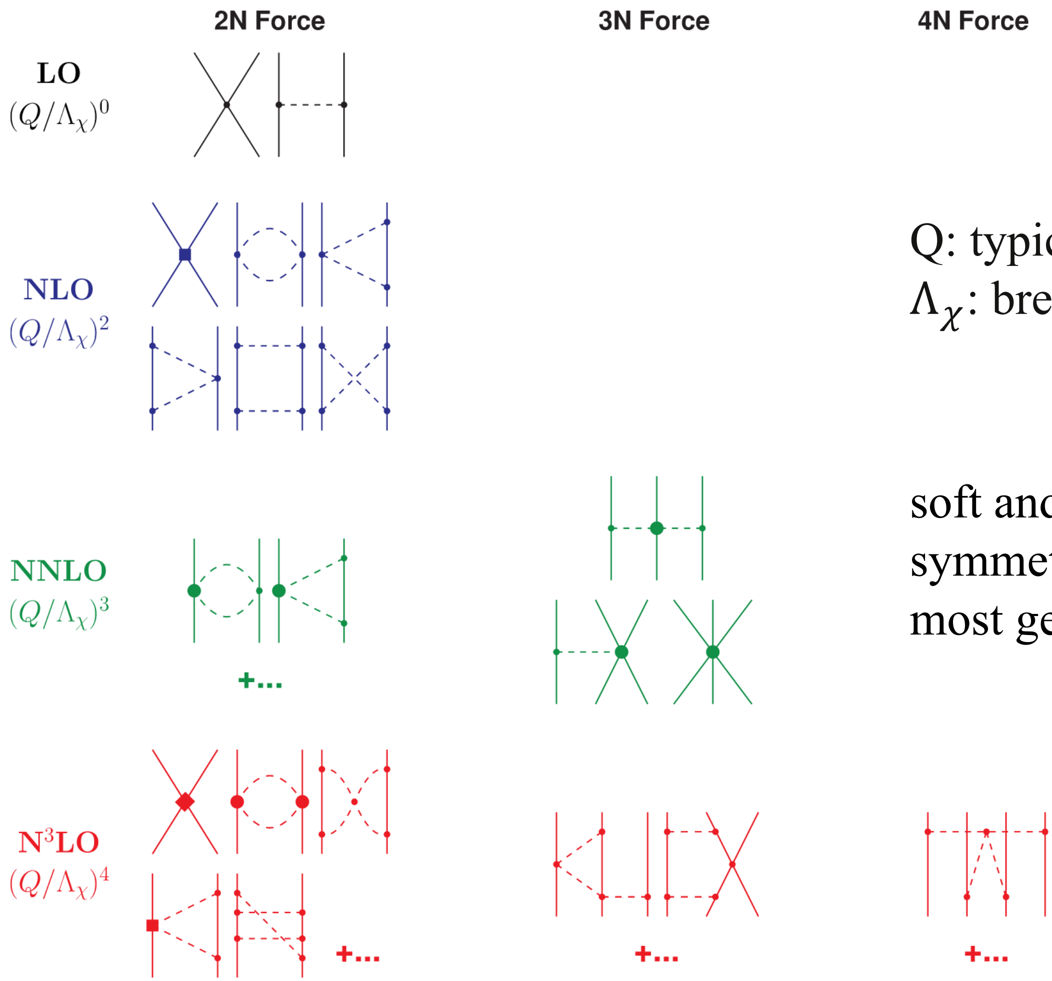
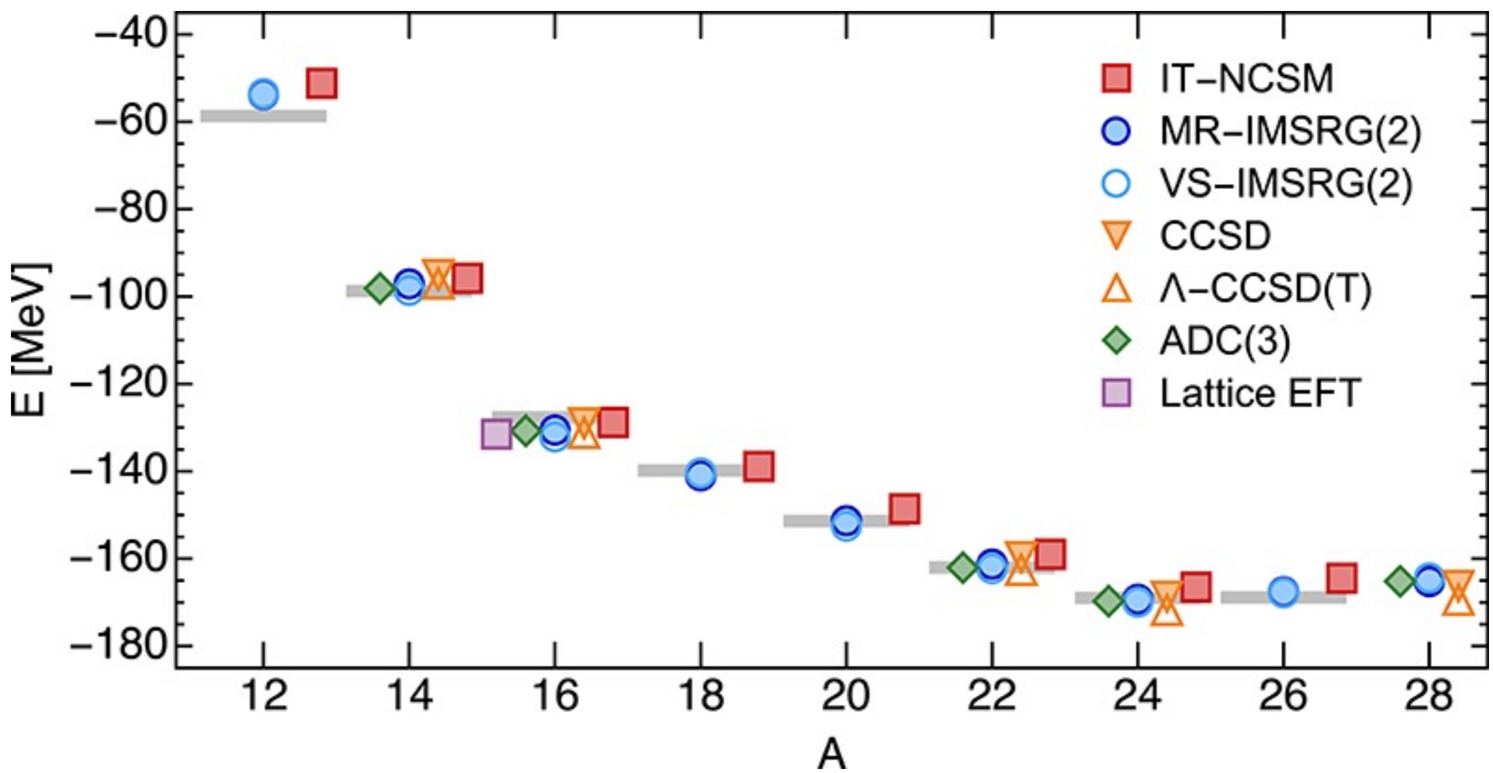


Q: typical momentum
Λ_χ: breakdown scale (700−1000 MeV)

soft and hard scales;
symmetries of low-energy QCD and broken;
most general Lagrangian;

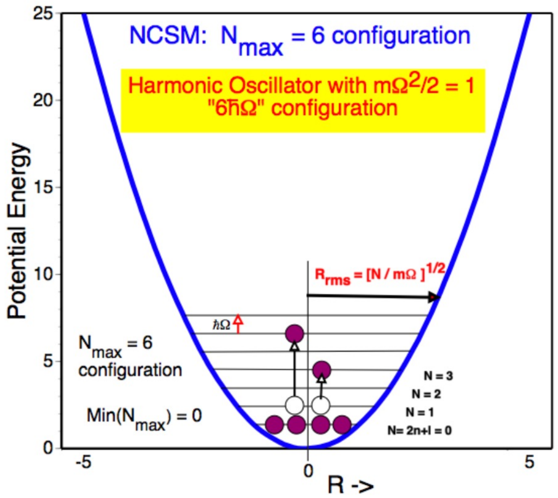
Hierarchy of nuclear forces in Chiral EFT. Two- and many-nucleon forces on an equal footing.

R. Machleidt, D.R. Entem. Physics Reports 503, 1-75 (2011)

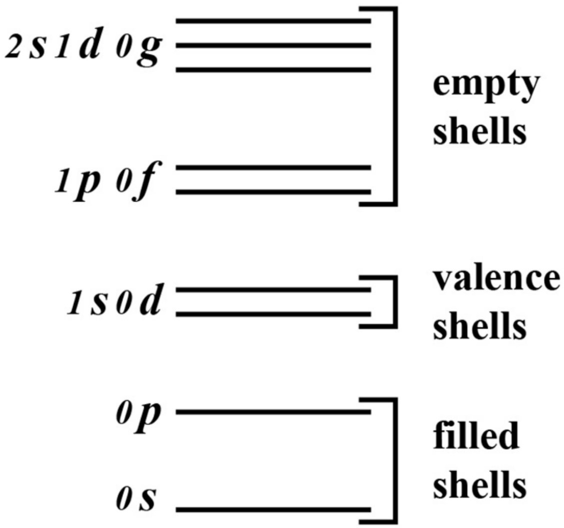


Ground-state energies of the oxygen isotopes for various many-body approaches, using the chiral NN+3N(400) interaction

Heiko Hergert. Front. Phys. 8, 379 (2020)



No core shell model



Configuration interaction

MBPT, VS-IMSRG, …

- Valence space Hamiltonian and operator
  - phenomenological approach
  - appropriate many-body theory

Open Quantum Systems

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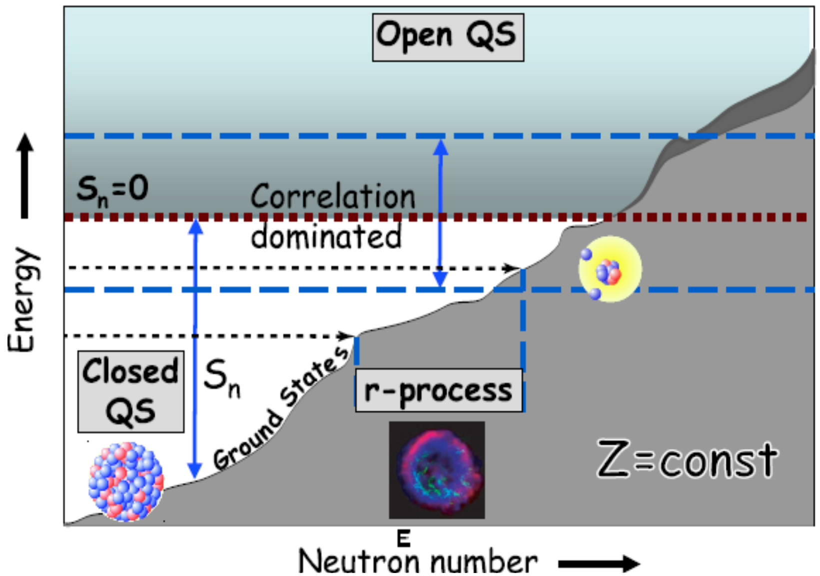
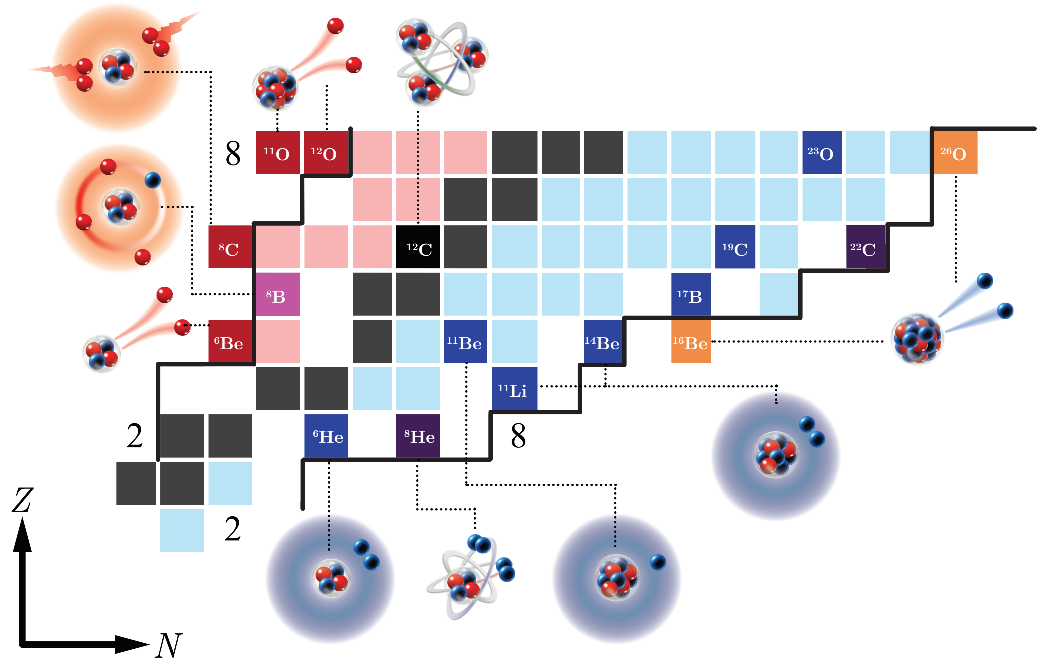
Open quantum system



N. Michel *et al.*, JPG: NPP 36, 013101 (2009)



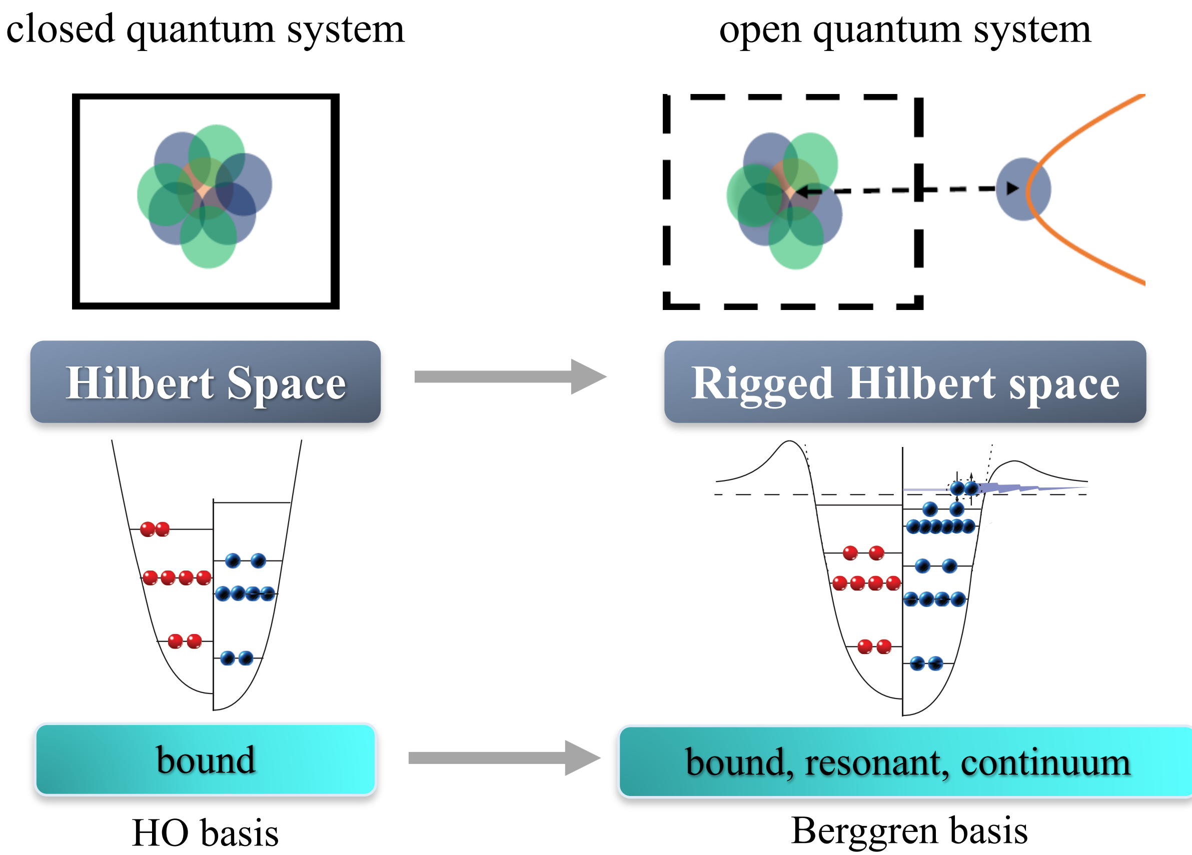
From S. M. Wang's talk

Resonance & continuum near threshold

- The asymptotic behavior of wavefunction
- The decay channel open & clustering

- Halo & clustering
- Exotic decay
- Deformation
- New magic number
- …

Berggren basis & Gamow Shell Model



Gamow Shell Model

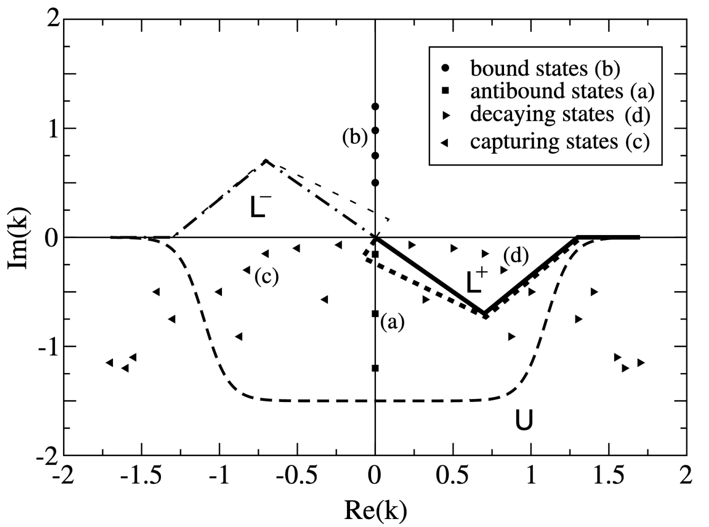
N. Michel *et al*, Phys. Rev. Lett. 89, 042502 (2002)

Gamow Coupled-Channel

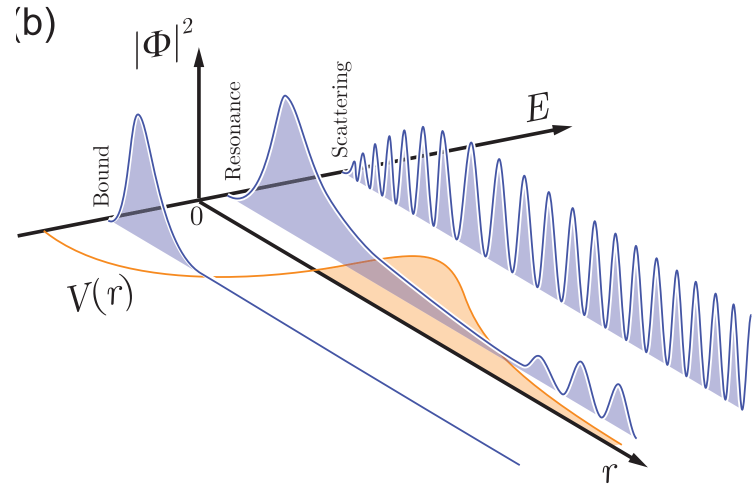
S. Wang *et al*, Phys. Rev. C. 96, 044307 (2017)

Gel'fand I M *et al*, Generalized Functions vol 4 (1961)
T. Berggren, Nucl. Phys. A 109, 265 (1968)

Berggren basis



N. Michel *et al.*, JPG: NPP 36, 013101 (2009)



Hamiltonian Non-Hermitian

Berggren basis completeness relation:

$$\sum_n u_n(E_n, r)u_n(E_n, r') + \int_{L^+} dE u(E, r)u(E, r') = \delta(r - r')$$

T. Berggren, Nucl. Phys. A 109, 265 (1968)

Gamow Hartree-Fock method: with three nucleon force

$$H_{int} = \sum_{i=1}^{A}(1 - \frac{1}{A})\frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A}(V_{NN,ij} - \frac{\boldsymbol{p}_i \cdot \boldsymbol{p}_i}{mA}) + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

1. One body Hartree-Fock potential
$$\hat{h}_{pq} = (t_{pq} + \sum_i V^{\text{NN}}_{piqi} + \frac{1}{2}\sum_{ij} V^{3N}_{pij,qij}) : \hat{a}_p^\dagger \hat{a}_q :$$
2. Analytical extension to complex-k plane
$$\langle k| h |k'\rangle = \frac{\hbar^2 k^2}{2\mu}\delta(k - k') + \sum\nolimits_{\alpha\beta} \langle\alpha| U_{\text{HF}} |\beta\rangle \langle k \,|\, \alpha\rangle \langle\beta \,|\, k'\rangle$$

S. Zhang *et al.*, Phys. Rev. C 108, 064316 (2023)

Many-Body Pertubation Theory

$$\begin{bmatrix} PHP & PHQ \\ QHP & QHQ \end{bmatrix} \xrightarrow[\text{Decoupling Equation}]{\text{Similarity Transformation}} \begin{bmatrix} P\mathcal{H}P & P\mathcal{H}Q \\ 0 & Q\mathcal{H}Q \end{bmatrix}$$

$Q\mathcal{H}P = 0$

Projection Operator: $P = \sum_{i\in P} |\Phi_i\rangle\langle\Phi_i|$

Similarity Transformation: $\mathcal{H} = X^{-1}HX,\ X = e^{\omega} \approx 1 + \omega,\ \omega = Q\omega P, P\omega P = Q\omega Q = P\omega Q = 0$

Decoupling Equation: $QHP + QHQ\omega - \omega PHP - \omega PHQ\omega = 0$

Decoupled Hamiltonian: $H_{\mathrm{eff}}(\omega) = P\mathcal{H}P = PHP + PHQ\omega$

$\hat{Q}$-box: $\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$

$H_{\mathrm{eff}}(\omega) = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)H_{\mathrm{eff}}(\omega)^k$

L. Coraggio and N. Itaco, Front. Phys. 8, 345 (2020)

With EKK method, an H_{eff} the within non-degenerate model spaces can be derived.

K. Takayanagi et al. Nucl. Phys. A 852, 61 (2005)
K. Takayanagi et al. Nucl. Phys. A 864, 91 (2011)

MBPT effective operator

Model space and full space operator:

$$\langle\tilde{\psi}^r_\lambda|\Theta|\psi^r_\mu\rangle = \langle\tilde{\psi}^m_\lambda|\Theta_{\text{eff}}|\psi^m_\mu\rangle$$

Same projection get wavefunction :

$$|\psi^r_\lambda\rangle = X|\psi^m_\lambda\rangle = (1+\omega)|\psi^m_\lambda\rangle = (P+\omega)|\psi^m_\lambda\rangle$$

and

$$\langle\tilde{\psi}^r_\lambda| = \langle\psi^m_\lambda|(P+\omega^\dagger\omega)^{-1}(P+\omega^\dagger)$$

Effective operator：

$$\Theta_{\text{eff}} = (P+\omega^\dagger\omega)^{-1}\hat{\Theta}$$

$\hat{\Theta}$ defined as：

$$\hat{\Theta} = (P+\omega^\dagger)\Theta(P+\omega) = P\Theta P + P\Theta Q\omega + \omega^\dagger Q\Theta P + \omega^\dagger Q\Theta Q\omega$$

Define $\hat{\Theta}$-box:

$$\hat{\Theta}(\epsilon) = P\Theta Q\frac{1}{\epsilon - QHQ}QH_1P$$

$$\hat{\Theta}(\epsilon_1,\epsilon_2) = PH_1Q\frac{1}{\epsilon - QHQ}Q\Theta Q\frac{1}{\epsilon - QHQ}QH_1P$$

Define:

$$\chi_0 = P\Theta P + (\hat{\Theta}_0 + h.c.) + \hat{\Theta}_{00}$$

$$\chi_1 = (\hat{\Theta}_1\hat{Q} + h.c.) + (\hat{\Theta}_{01}\hat{Q} + h.c.)$$

$$\chi_2 = (\hat{\Theta}_1\hat{Q}_1\hat{Q} + h.c.) + (\hat{\Theta}_2\hat{Q}\hat{Q} + h.c.) + (\hat{Q}_{02}\hat{Q}\hat{Q} + h.c.) + \hat{Q}\hat{\Theta}_{11}\hat{Q}$$

With ω:

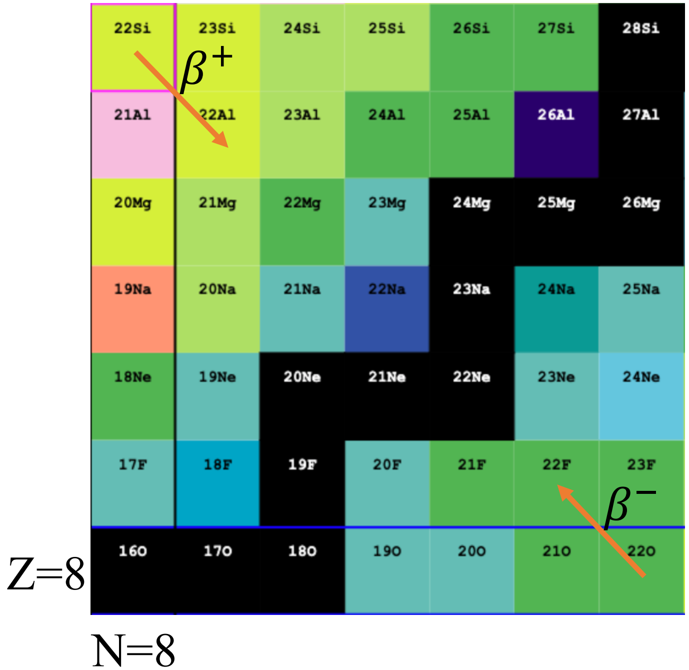
$$\omega^\dagger\omega = -\hat{Q}_1 + (\hat{Q}_2\hat{Q} + h.c.) + (\hat{Q}_3\hat{Q}\hat{Q} + h.c.) + \cdots$$

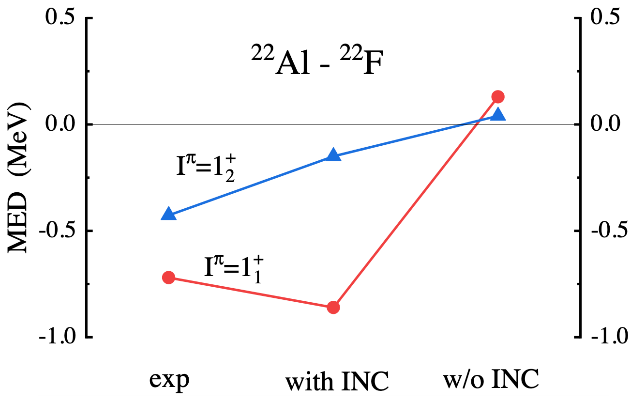
$$\boxed{\Theta_{\text{eff}} = (P + \hat{Q}_1 + \hat{Q}_1\hat{Q}_1 + \hat{Q}_2\hat{Q} + \hat{Q}\hat{Q}_2 + \cdots) \times (\chi_0 + \chi_1 + \chi_2 + \cdots)}$$

K. Suzuki, R. Okamoto. Prog. Theor. Phys. 93,905 (1995)
L. Coraggio and N. Itaco, Front. Phys. 8, 345 (2020)

We extend the model space effective operator to Berggren basis.

Mirror symmetry breaking near dripline





mirror energy differences:
$$\mathrm{MED} = \mathrm{E}_x(I,T,T_z=-T) - \mathrm{E}_x(I,T,T_z=T)$$

	^{22}Si → ^{22}Al		^{22}O → ^{22}F	
	Experiment	Calculations	Experiment	Calculations
I_i^π	$\lvert M_{\mathrm{GT}}^+\rvert^2$	$\lvert M_{\mathrm{GT}}^+\rvert^2$	$\lvert M_{\mathrm{GT}}^-\rvert^2$	$\lvert M_{\mathrm{GT}}^-\rvert^2$
1_1^+	0.0310 (58)	0.0587 [0.1138]	0.096 (20)	0.1831 [0.1059]
1_2^+	0.563 (61)	0.7449 [0.7193]	0.60 (12)	0.7196 [0.7948]

J. Lee, *et al*. Phys. Rev. Lett. 125,192503 (2020)

- Two MSB:
  - larger MED of two 1^+ states.
  - Gamow-Teller transition
- USDA with isospin-nonconserving (INC) forces related to the $s_{1/2}$ orbit, mimic the effect of continuum.

How continuum effect?

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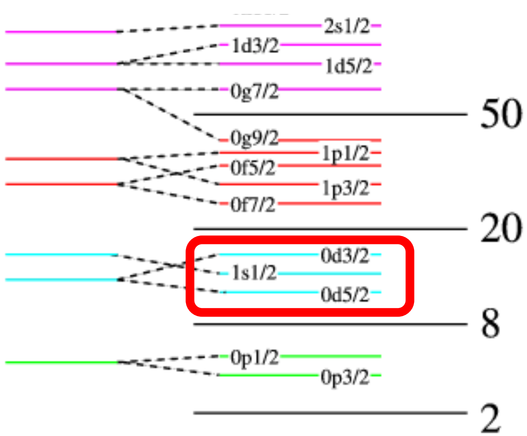
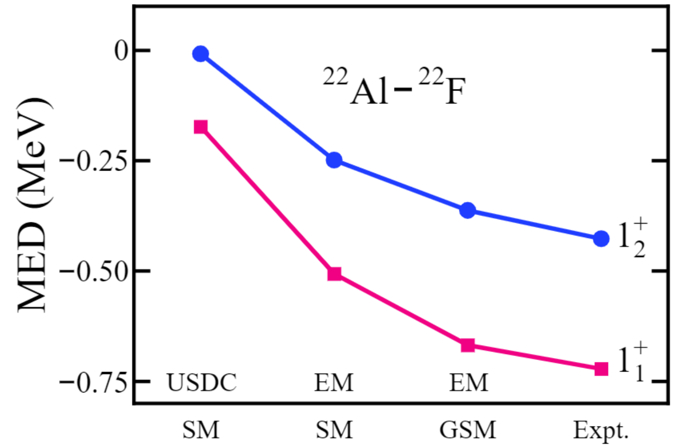
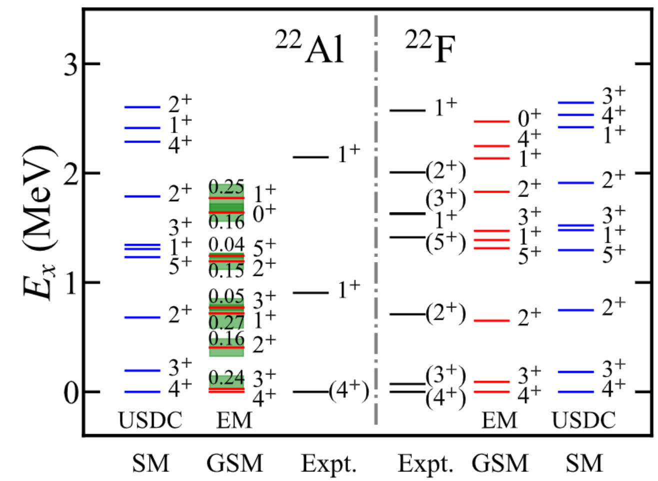


sd-shell model space

$d_{3/2}$, $s_{1/2}$ wave with continuum





- Calculation:
  - USDC + SM (no continuum)
  - EM + SM (no continuum)
  - EM + GSM (with continuum)
- Continuum improve MSB and spectrum
- Proton $s_{1/2}$ wave occupation increased

ZX, *et al.*, Phys. Rev. C. 108, L031301(2023)

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$|M_{GT}|$ values

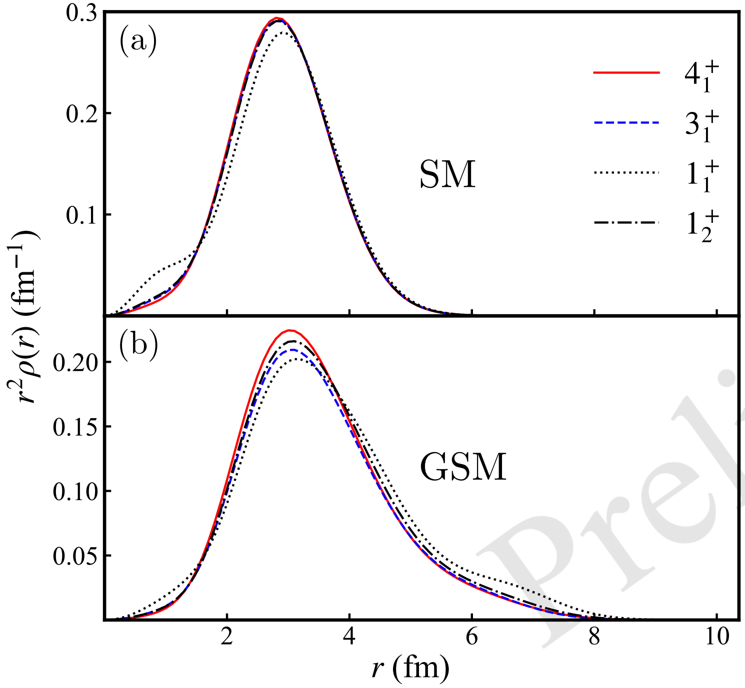
		SM		GSM	Ref. [54]	
		USDC	EM	EM	Expt.	Cal.
$^{22}Si \rightarrow {}^{22}Al$	1_1^+	0.236	0.343	0.257	0.176(16)	0.242
	1_2^+	0.721	1.042	1.012	0.750(41)	0.863
$^{22}O \rightarrow {}^{22}F$	1_1^+	0.198	0.569	0.497	0.310(32)	0.428
	1_2^+	0.719	1.092	1.068	0.775(77)	0.848

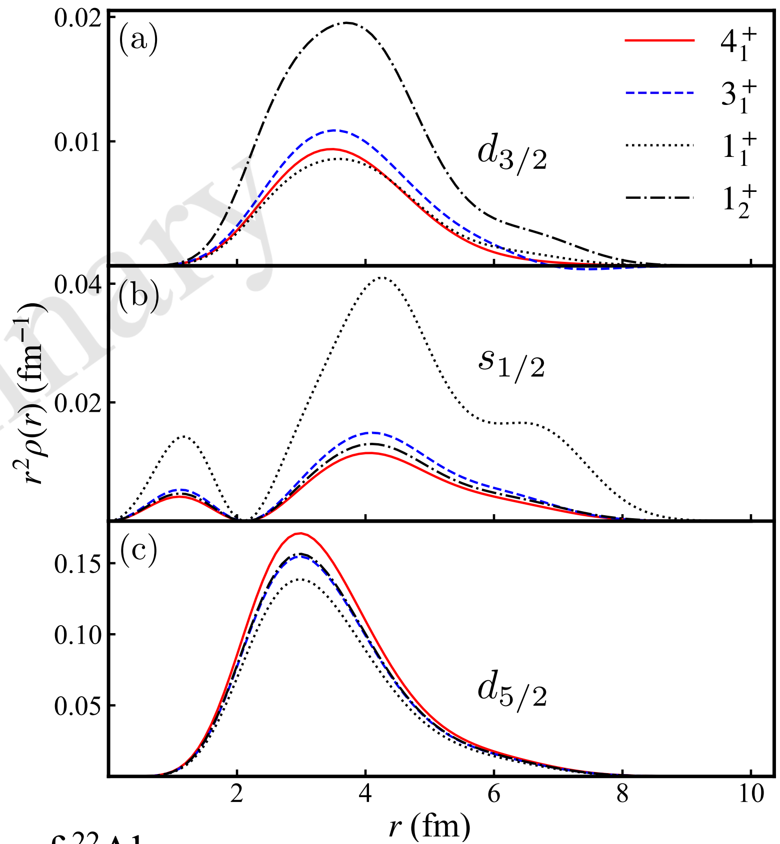
- $|M_{GT}|$ of ^{22}Si & ^{22}O mirror pair
  - The USDA with INC (Cal.) consider the effect from continuum
  - USDC cannot give the MSB in GT transition
  - The proton $s_{1/2}$ occupation increased with continuum, which reduce the $E(1^+)$ and $|M_{GT}|$

The realistic force & continuum are vital for MSB near dripline

ZX, *et al.*, Phys. Rev. C. 108, L031301(2023)

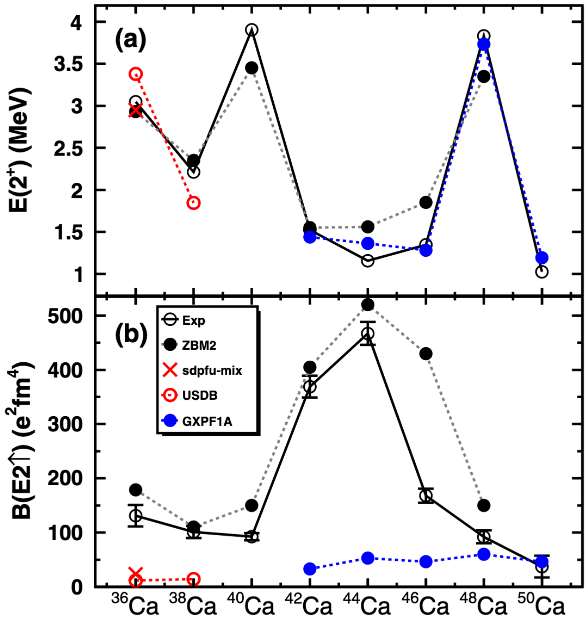
Is ^{22}Al a halo nuclei

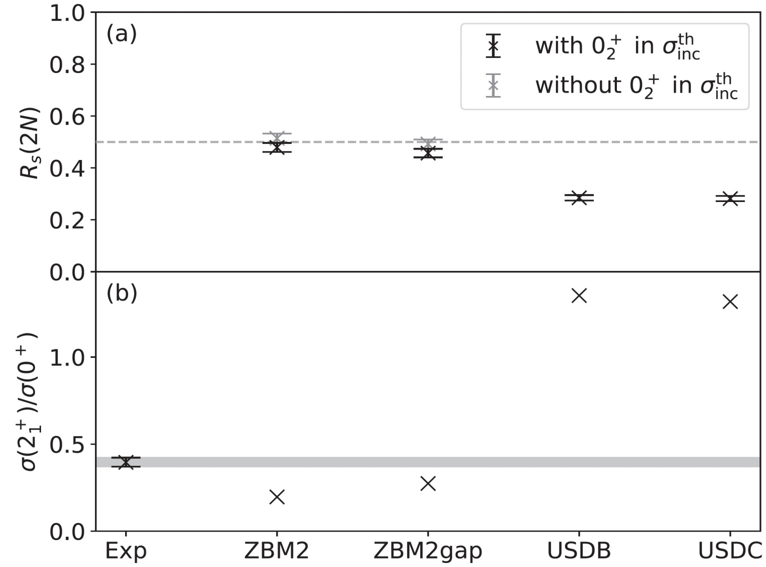


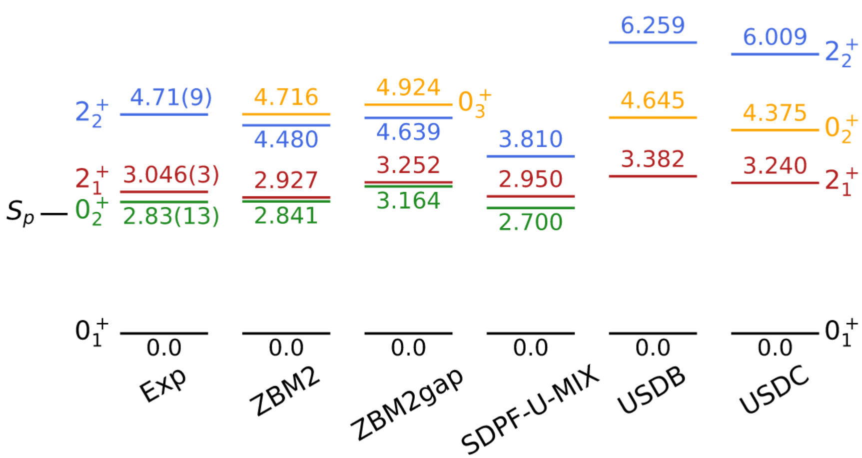


- Valence nucleon density and partial wave density of ^{22}Al
- The density extend when continuum effect included
- Halo structure found in first 1^+ state
- There are no halo structure in 4^+ and 3^+ states, which lead 22Al not a halo nuclei

$E2$ transition in ^{36}Ca and ^{38}Ca







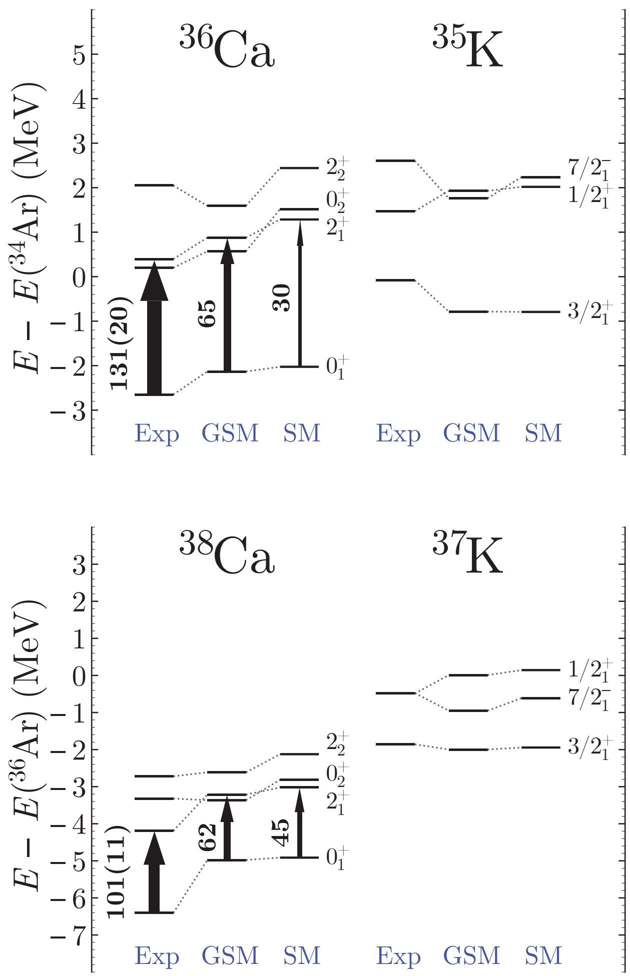
- Abnormal $E(2^+)$ and $B(E2)$ trend
- Two-neutron knockout of ^{38}Ca
- Only describe by proton cross shell excitation
- 2^+ state in ^{36}Ca is above threshold

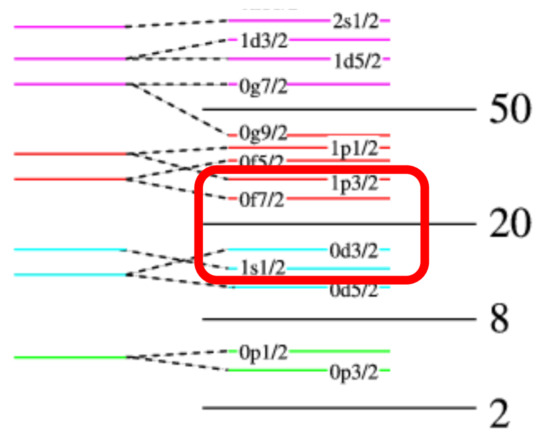
How continuum effect?

N. Dronchi, *et al.*, Phys. Rev. C. 107, 034306 (2023)
T. Beck *et al.*, Phys. Rev. C. 108, L061301 (2023)
E. Caurier *et al.*, Phys. Lett. B 522, 240 (2001)

$E2$ transition in ^{36}Ca and ^{38}Ca

Spectrum and $B(E2)$ values





ZBM2 valence space

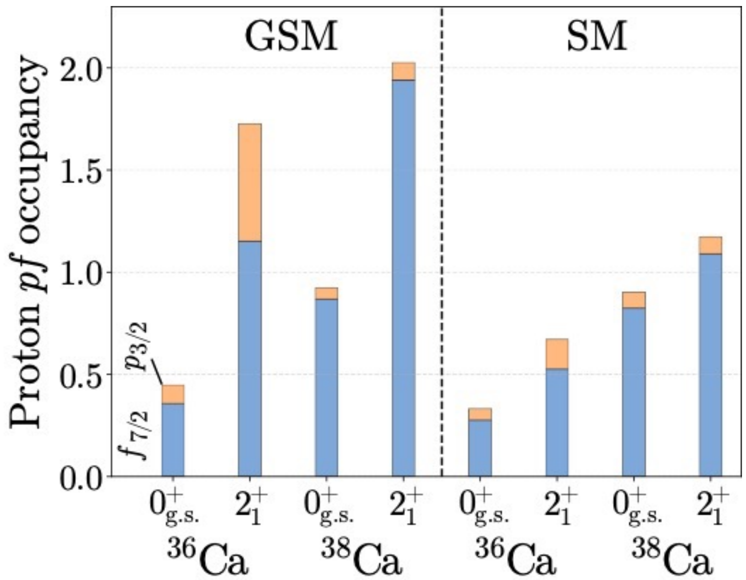
$f_{7/2}$, $p_{3/2}$ wave with continuum

A. P. Zuker, *et al*. Phys. Rev. C 92, 024320 (2015)

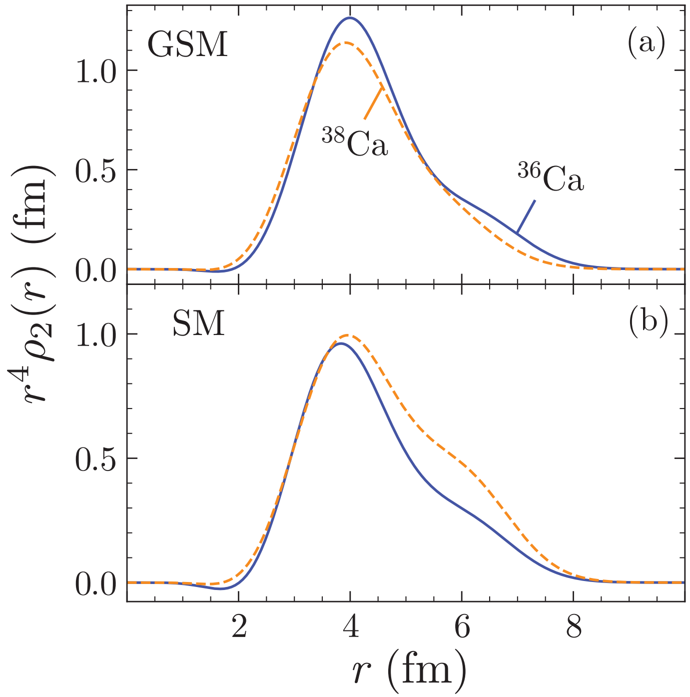
- The 2^+ states relative to threshold can properly reproduced
- $B(E2)$ & $E(2^+)$ trend can be reproduced with continuum
- The B($E2$) values are smell

ZX, *et al*., Phys. Rev. C 112, L011302 (2025)

$E2$ transition in ^{36}Ca and ^{38}Ca



$E2$ transition density



- Continuum effect:
  - Proton *sd*→*pf* shell gap narrowed by continuum
  - Extended character of continuum wave functions
- The *p*-wave component are vital for transition and wavefunction

$$B(E2; 0^+ \to 2^+) = 5e^2 \left[\int_0^\infty \rho_2(r) r^4 dr \right]^2$$

ZX, *et al.*, Phys. Rev. C 112, L011302 (2025)

Paramatric parameter method (PMM)

Noval machine learning algorithms

a) Maintains quantum mechanical interpretability
b) Train energy and observable simultaneously
c) Efficiency & Small Scale
d) Universal Approximators
e) Physics-Informed Integration

Good for both Scientific Computing and General Machine Learning

Underlying equations

$$\begin{cases} H(c) = H_0 + cH_1 \\ H(c)\,|\psi(c)\rangle = E(c)\,|\psi(c)\rangle \\ \langle\psi(c)|O|\psi(c)\rangle = f(c) \end{cases}$$

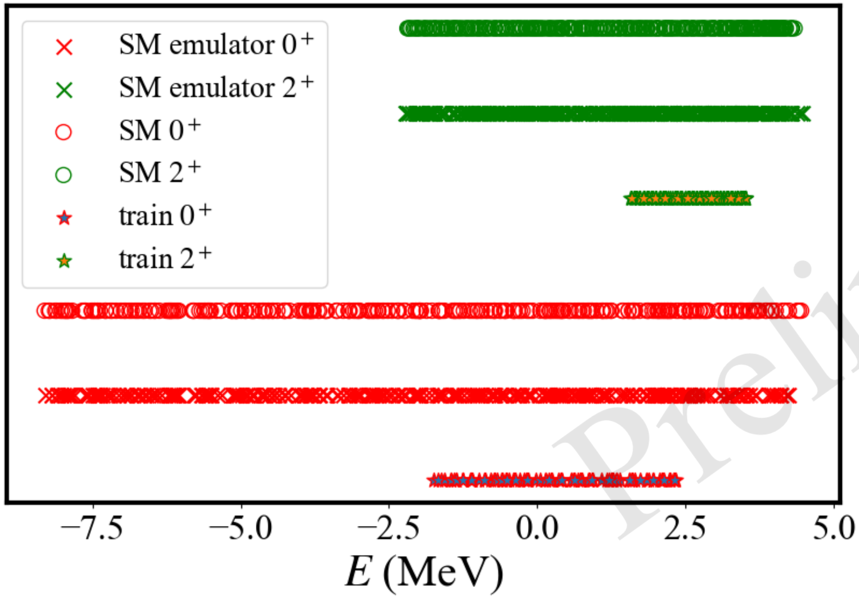
PMM

$$\begin{cases} M(c) = \underline{M_0} + c\underline{M_1} \\ M(c)\,|\tilde{\psi}(c)\rangle = \tilde{E}(c)\,|\tilde{\psi}(c)\rangle \\ \langle\tilde{\psi}(c)|\underline{X}|\tilde{\psi}(c)\rangle = \tilde{f}(c) \end{cases}$$

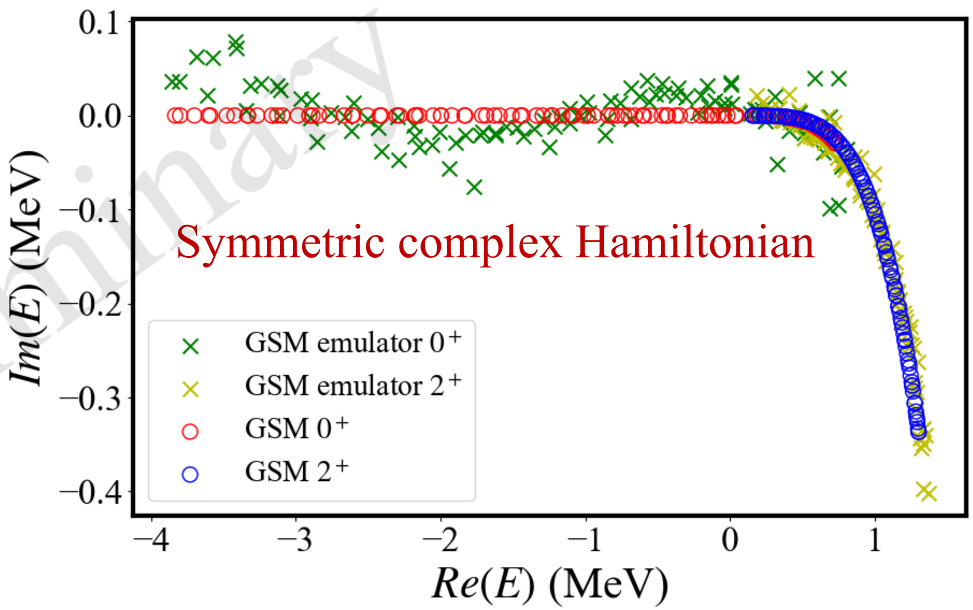
Patrick Cook, *et al.*, Nature Communications 16, 5929 (2023)

PMM emulator

PMM for closed systems



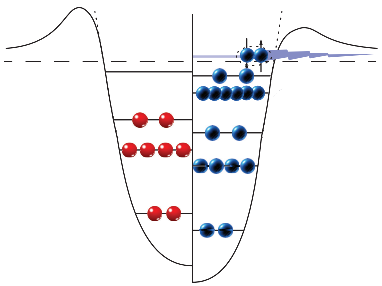
PMM for open systems

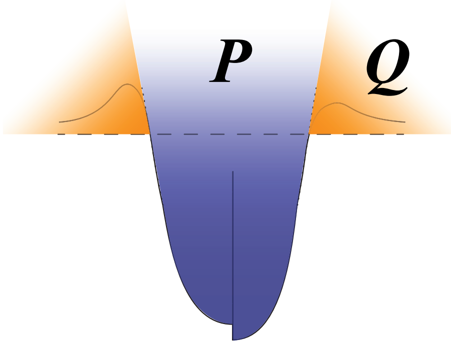


- PMM works very well for closed quantum systems (CQSs)
- Extend to rigged Hilbert space for open quantum systems (OQSs)
- Fails to reproduce the eigen values directly

Emulator for asymptotic observables

Open quantum system





Closed quantum physics

$$\begin{pmatrix} H_{\mathcal{PP}} & H_{\mathcal{PQ}} \\ H_{\mathcal{QP}} & H_{\mathcal{QQ}} \end{pmatrix} \begin{pmatrix} \mathcal{P}\Psi \\ \mathcal{Q}\Psi \end{pmatrix} = E \begin{pmatrix} \mathcal{P}\Psi \\ \mathcal{Q}\Psi \end{pmatrix}$$

Continuum physics

Continuum coupling

Prepare training data (E, O) for close and open systems

Train CQSs

$$M_{\mathcal{PP}}(c) = M^a_{\mathcal{PP}} + c\, M^b_{\mathcal{PP}}$$

$$M_{\mathcal{PP}}(c)|\Psi_{\mathcal{P}}\rangle = E_{\mathcal{P}}(c)|\Psi_{\mathcal{P}}\rangle$$

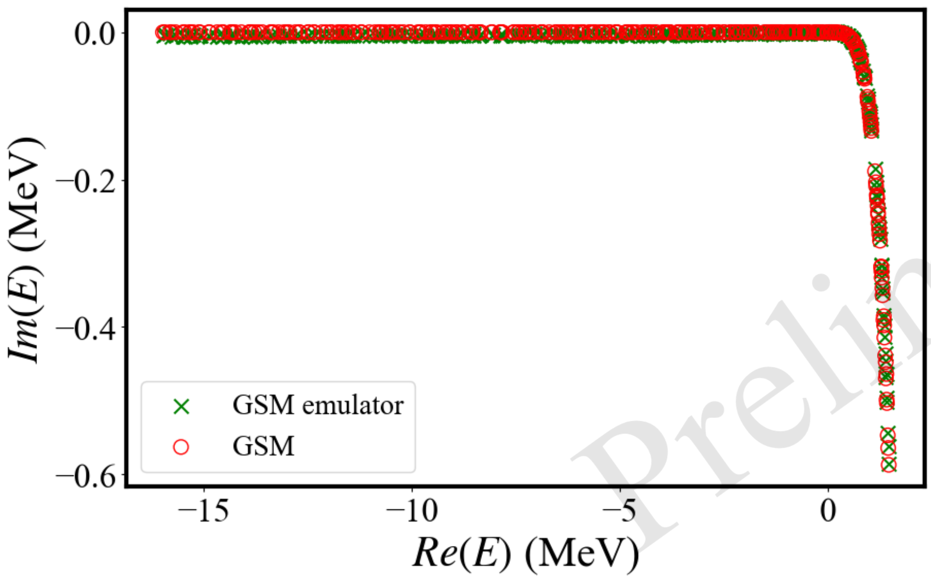
$$M = \begin{pmatrix} M_{\mathcal{PP}} & M_{\mathcal{PQ}} \\ M_{\mathcal{QP}} & M_{\mathcal{QQ}} \end{pmatrix}$$

Train OQSs similarly

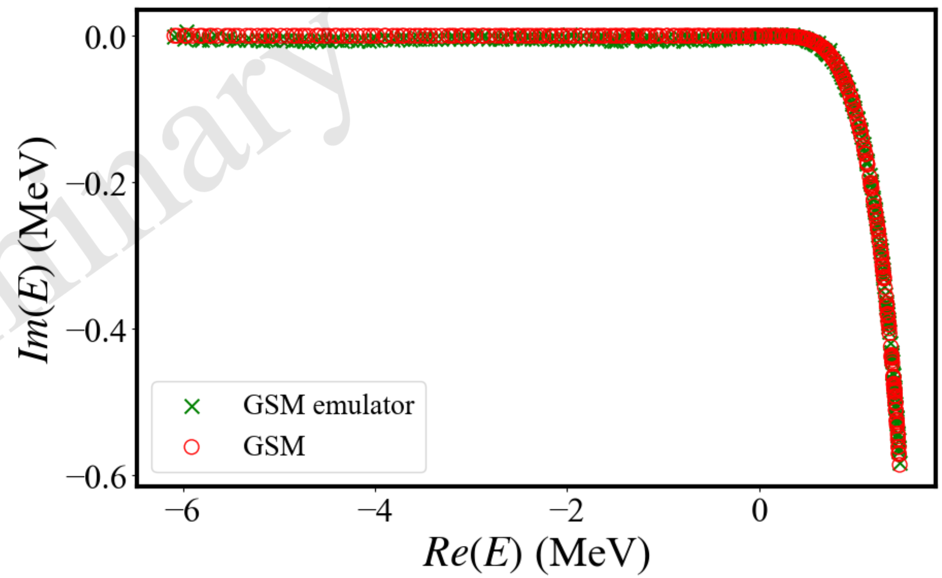
Find E through $\langle \Psi | \Psi_{\mathcal{P}} \rangle$

Emulator for resonant state

^{6}**He Ground state**



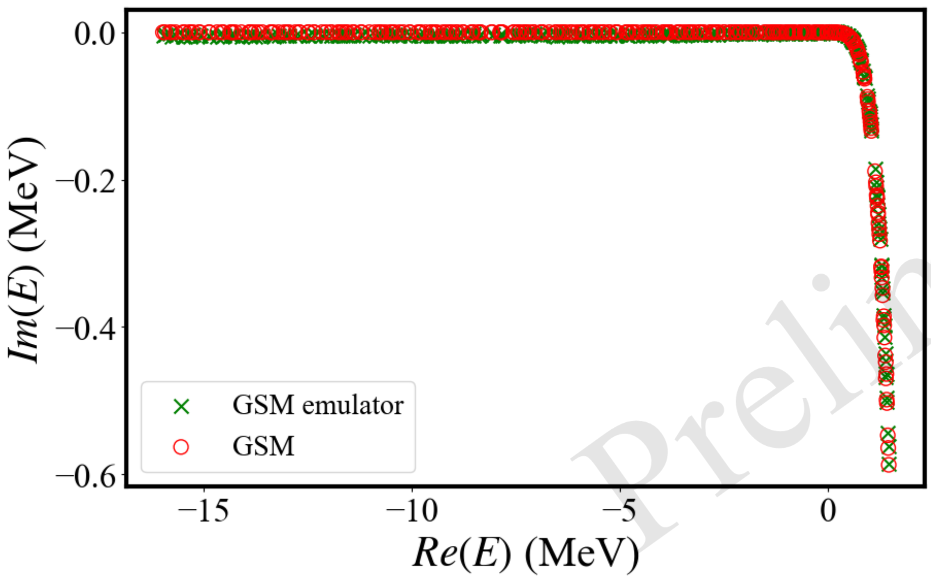
^{6}**He 2^{+} state**



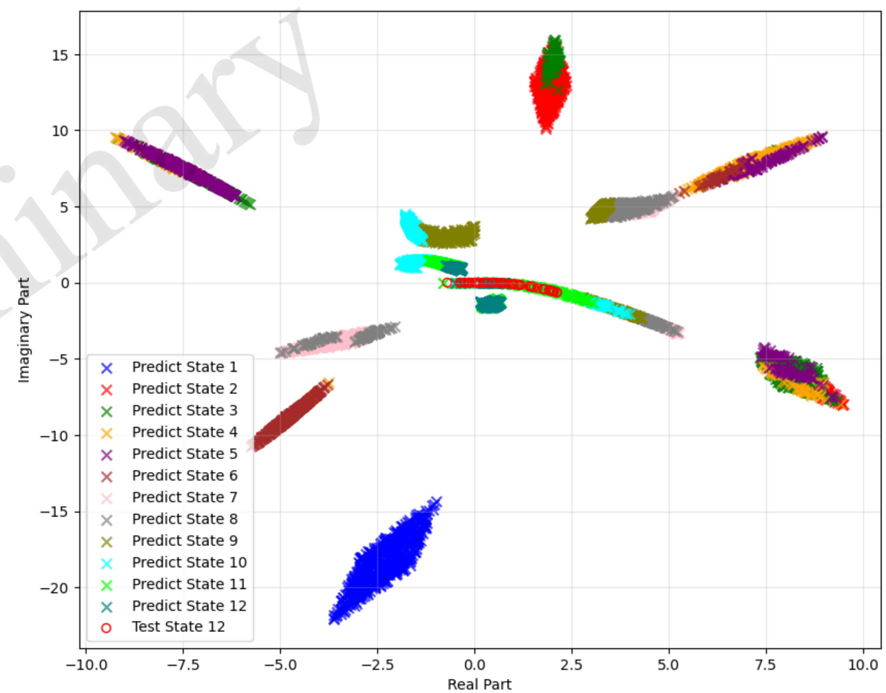
- Based on PQ method, construct GSM emulator
- Use SM emulator to obtain eigenstates, guide the selection of corresponding states
- GSM emulator reproduce the high-fidelity results very well

Emulator for resonant state

^{6}He Ground state



Eigenvalue distribution



- Based on PQ method, construct GSM emulator
- Use SM emulator to obtain eigenstates, guide the selection of corresponding states
- GSM emulator reproduce the high-fidelity results very well

Summary

- The framework of *ab initio* complex valence space effective operator.
- The mirror symmetry breaking and abnormal $E2$ transition near dripline nuclei are studied by this framework.
- The continuum effect are vital for the observables of nuclei near the dripline.
- The emulator of asymptotic observables with PMM

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Thank you for your attention!

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