What neutron stars tell about the properties of strong interaction

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Overview

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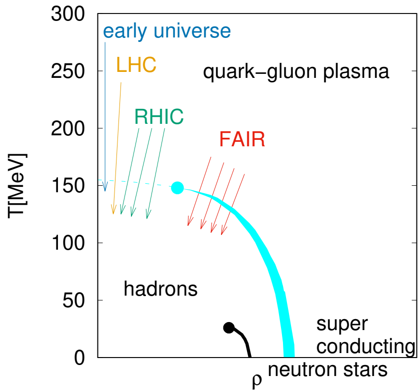
Dense strongly interacting matter



What is the phase diagram and EOS for dense strongly interacting matter?

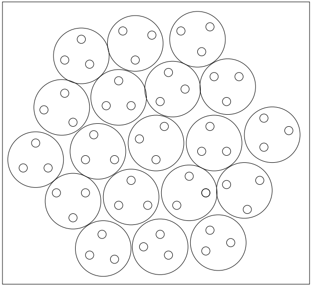
At $\mu = 0$: lattice and experiments (STAR/PHENIX and ALICE).

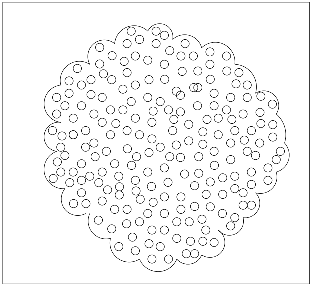
For $\mu >> 0$ no precise theory and no heavy ion experiment.

Dense matter at T=0

Are there different phases at T=0? If yes, at which densities?

heavy ion collisions: no sharp transition until 2-3 ρ_0





$V_{proton} = 2.22 fm^3$ (with r_{em}), densest packing with spheres: 74%

$\rightarrow \rho_{max} = 0.33 fm^{-3} \approx 1.8\rho_0$ by maximal packing

Model: nucleon = core + meson cloud

Reid hard core potential: $r_{hc} \approx 0.5 r_{em} \rightarrow \rho_{max} \approx 15\rho_0$ at hard core overlap

Form factors (scalar and vector em, axial vector and gluonic: μ absorption):

$r_{core} = \sqrt{< r^2 >} \approx 0.5 fm \rightarrow \rho_{max} \approx 8\rho_0$ at core overlap

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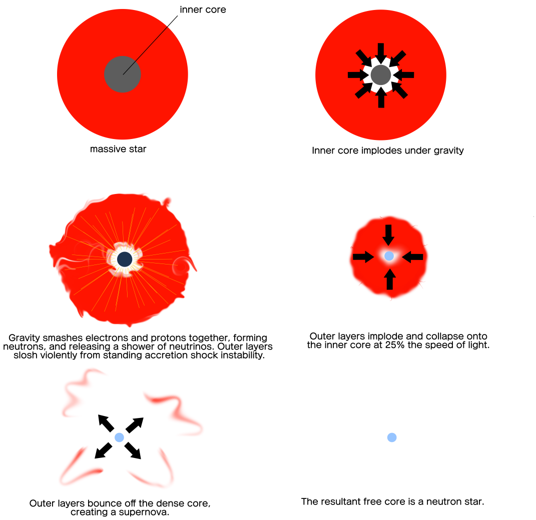
The importance of neutron stars

Neutron stars are one of the 3 final states of stars (white dwars, neutron stars, black holes) containing the densest matter in the Universe.
It is expected to be born in supernova explosion if the original star mass was between 8 $M_\odot$ and 25 $M_\odot$

What is a neutron star? What balances the gravitational attraction?
-thermal pressure: ordinary stars
-repulsion of atoms (electron shells+Pauli): white dwarfs ($\approx$ size of Earth)
-repulsion of the strong interction(+Pauli): neutron stars($\approx$ size of Budapest)
-nothing: black holes (pointlike)

very complex, general relativity, nuclear physics, solid state physics, condensed matter

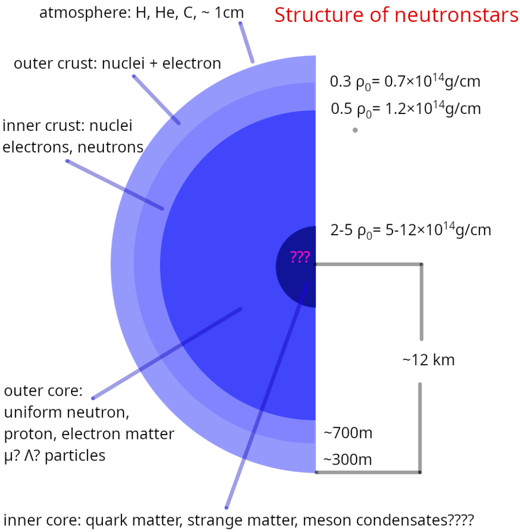
Supernova explosion



Discovery of neutron stars

- 1934: Baade and Zwicky predicted the existence of heavy stars composed mostly by neutrons, Landau even in 1931 speculated about such stars
- 1939: Tolman and independently Oppenheimer, Volkov derived the equation for spherical stars in hydrodynamical equilibrium (TOV-equation)
- Calculated the star sturcture by using assumptions about its matter
  Wheeler: free gas
  Cameron, 1959: nucleon-nucleon interaction
  in the 60ies: superfluidity, quark matter, ..., cooling of neutron stars
- 1968: Crab and Vela pulsars in the remnants of supernova explosions
- 1974: Hulse and Taylor binary pulsars PSR J1913+16, (determining the mass M$\approx 1.4 M_\odot$)
- 2010 observation of M $>$2 $M_\odot$ neutron stars
- 2017 GW170817 binary collision

Neutron Stars a challenge and a possibility

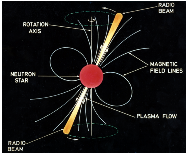


Neutron stars are contain cold, dense matter (T≈0, $\rho > 3\rho_0$) not available in terrestial experiments (Laboratory for strong interaction)

What is the structure of neutron stars (what are the constituents), hybrid stars? Superfluids?

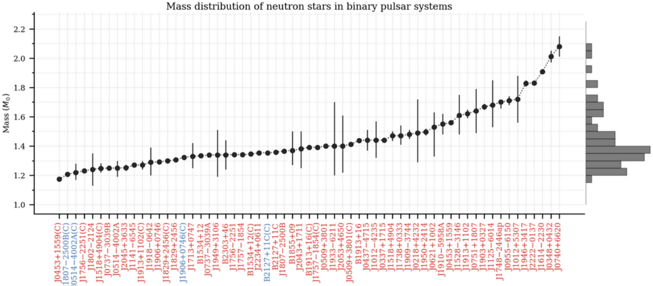
YN, YNN interactions are important, three-body repulsion for Λ, Σ (Weise)

Pulsar mass distribution



lighthouse effect, very precise frequency (1-700 Herz)

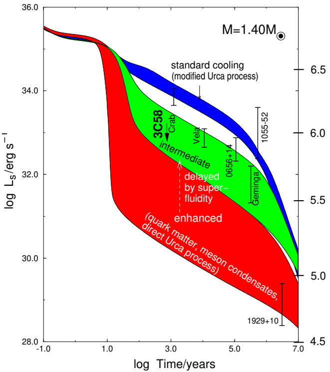
Pulsar mass measurements and tests of general relativity



Measurables of Neutron Stars

- mass: in binary systems the orbital motion, Shapiro-decay, $1M_\odot \geq M \geq 2.3M_\odot$ though limits are not very well known
- radius: from temperature and radiation, and NICER X measurement 10-15km
- rotation frequency: 0.1 - 700 H (Kepler-frequency 1.4kH)
- surface temperature $10^{11}K \sim 10MeV \rightarrow 10^6K$, by neutrino radiation (URCA process)
- magnetic field: $10^8 - 10^{12}$ G, in magnetars $10^{15} - 10^{18}$ G
- glitches: sudden increase of the frequency (star quakes, vortices in superfluid matter)
- tidal defomability $\Lambda < 800$

Cooling of Neutron Stars



observed explosion, temperature: X ray radiation

Main process: direct Urca $n \to pe\overline{\nu_e}$ $ep \to n\overline{\nu_e}$

Low proton/neutron ratio: direct Urca does not work (triangle low $k_F^n < k_F^e + k_F^p$)

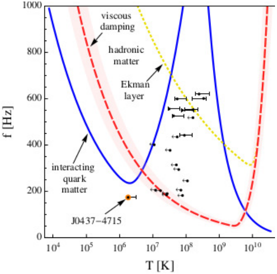
Indirect Urca: direct Urca + 1 nucleon on both sides

It is not enough for the cooling (Superfluidity?)

Heat conductivity and capacity are important

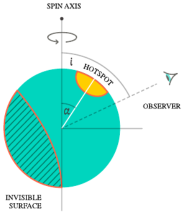
Oscillations of Rotating Neutron Stars, GW creation

polar modes: f(undamental), g(ravitational), p(ressure, aucustic) waves
axial modes (rotating stars): R(ossby, i(nertial) modes
f-waves: NS radial acustic modes, eigenfrequencies are independent of the EOS
r-waves: Coriolis force
If the oscillation is in the same direction with the rotation in the inertial frame (prograde), but opposite in the star's eigenframe (retrograde):
Chandrashekar-Friedmann-Schutz instability (GW increase the amplitude).
GW frequency in the range of LIGO-Virgo
Damping by viscosity, can explain the observed $\nu_{max} \sim 700H << \nu_{Kepler}$



Neutron Star observations

- Discovering heavy neutron stars $M > 2M_{\odot}$ Demorest, et al., Nature. 467, 1081-1083 (2010). largest mass observed: $2.14 + 0.20 - 0.18M_{\odot}$ (2019)
  (Shapiro-delay: pulsar+another star, at almost full covering the second member of the binary delays the radiation of the pulsar)
- Advanced gravitation wave detectors: Advanced Ligo and Virgo (soon Kagra):single neutron stars, multichannel astronomy
  neutron star collision: GW170817 (130 million lightyears)
- Modern telescopes: NICER X-ray telescope: precise (<5%) mass and radius measurements (2020) "for nearby" neutron stars,
  only 2 stars yet, but more to come
  radiation bent by strong gravity: hotspots observation: M, R



Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \tag{1}$$

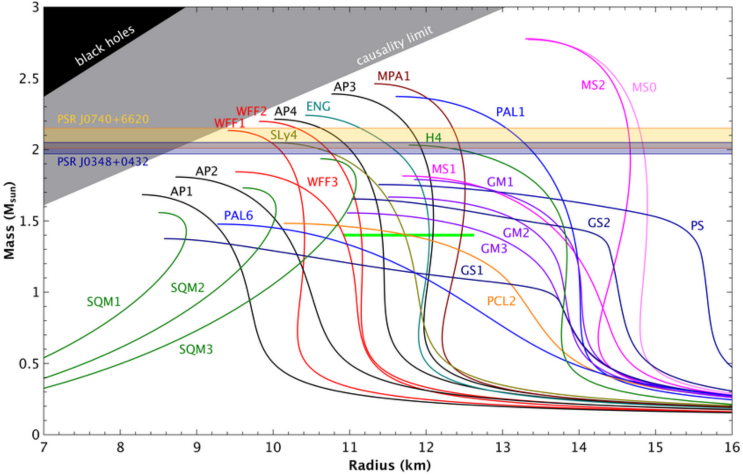
with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

MR curves


black holes
causality limit
PSR J0740+6620
PSR J0348+0432
MPA1
AP3
MS2
MS0
WFF2
AP4
ENG
PAL1
WFF1
SLy4
H4
MS1
AP2
WFF3
GM1
AP1
PAL6
GS2
PS
GM2
GM3
GS1
SQM1
SQM2
PCL2
SQM3
Mass (M_{sun})
Radius (km)
3
2.5
2
1.5
1
0.5
0
7
8
9
10
11
12
13
14
15
16

Observables for dense strongly interacting matter

1. Nuclear physics
   - $\rho = 0$
     nucleon-nucleon scattering, YN and YNN data from femtoscopy (ALICE), BB interaction and potential (lattice: HALQCD)
     femtoscopy data and HALQCD calculations are consistent
   - $\rho \approx \rho_0$
     masses of nuclei, isobaric analog states, hypernuclei, giant dipole and pigmy resonances, nuclear dipole polarizabilities, neutron skin thickness $\rightarrow$ normal nuclear density: ρ_0, binding energy, compressibility, symmetry energy (1st order in asymmetry expanded in density the 0th and 1st term)
2. Perturbative QCD: $\rho \approx 40\rho_0$
   N^3LO calculation, hard thermal loops: μ =2.6 GeV, p = 3.8 GeV/fm^3.
   T. Gorda, A. Kurkela, et al., Phys. Rev. Lett. 127 (2021) 162003, arXiv:2103.05658.
3. Heavy ion collisions: $\rho : 1 - 8\rho_0$
   not very conclusive; there are many competing effects, like momentum dependent interaction, nonequilibrium, nonzero temperature
4. Neutron stars: $\rho : 1 - 8\rho_0$
   M, R, Λ. Quite strong constraints even with not yet very precise data

EOS

1. $\rho \leq 2-4\rho_0$ ordinary nuclear potentials, CEFT, ...
2. $2-4\rho_0 \leq \rho \leq 6-8\rho_0$ quark matter model
3. $6-8\rho_0 \leq \rho$ extrapolation to the pQCD point

hadronic matter - soft: SFHo
(Steiner, A. W., Hempel, M., Fischer, T. Astrophys. J. 774 (2013) 17) and Hempel, M., Schaffner-Bielich, J. Nucl. Phys. A837 (2010) 210)
relativistic mean-field model (nucleons, σ, ω, ρ with quartic copulings), with K=245 MeV, L=47.1 MeV, m^*/m_n=0.76.

hadronic matter - stiff: DD2
(S. Typel, et al., Phys. Rev. C81 (2010) 015803
relativistic mean-field + light clusters, K = 243 MeV, L=58 MeV m^*/m_n=0.63

Quark matter: Quark-meson model - chiral $U(3)xU(3) \rightarrow SU(2)xU(1)$ model
degrees of freedom: 4 meson nonets, constituent quarks, Polyakov loops
condensates: 2 scalar (N,S), Polyakov loops (T>0), vector mesons ($\mu > 0$)
P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Concatenation

It seems that a strong first order phase transition is ruled out by astrophysical constraints: J.-E. Christian and J. Schaffner-Bielich, Phys. Rev. D 103, 063042 (2021),
The allowed $p(\varepsilon)$ functions are in a rather narrow band, there can be no big jump

Hadron-quark crossover with polynomial interpolation ($\rho = \rho_B$):

$$\begin{aligned} \varepsilon(\rho_B) &= \varepsilon_{hadronic}(\rho_B) \qquad \rho_B < \rho_{BL}, \\ \varepsilon(\rho_B) &= \sum_{k=0}^{5} C_k \rho_B^k \qquad \rho_{BL} \leq \rho_B \leq \rho_{BU} \\ \varepsilon(\rho_B) &= \varepsilon_{qm}(\rho_B) \qquad \rho_{BU} < \rho_B. \end{aligned}$$

C_k is determined by the requirement that the energy density, ε and its first two derivatives with respect to ρ_B, pressure and sound velocity is continuous at the boundaries.
2 parameters: $\Gamma = 0.5 * (\rho_{BU} - \rho_{BL})$ and $\overline{\rho_B} = 0.5 * (\rho_{BL} + \rho_{BU})$

Modelling the strongly interacting matter: Quark-meson model

Plan is to have an interaction with the right global symmetry pattern describing the hadronic (with binding) and the quark phase as well.

1. Starting point is an SU(3) linear sigma model with (pseudo)scalar and (axial)vector nonets. We obtained a very good description for the meson masses and decay widths.

   D. Parganlija, P. Kovács, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

2. We added te isospin breaking

   P. Kovács, Gy. Wolf, N. Weickgenannt and D.H. Rischke, Phys. Rev. D (2024)

3. We added the baryon octet and decuplet, baryon masses are from spontanous breaking of chiral symmetry

   P. Kovács, Á. Lukács, J. Váróczy, Gy. Wolf, M. Zétényi, Phys. Rev. D89 (2014) 054004

4. Nonzero temperature, chemical potential: We added Polyakov-loops, quarks: Quark-meson model, very good agreement with lattice at $\mu = 0$.

   P. Kovács, Zs. Szép and Gy. Wolf, Phys. Rev. D93 (2016) 114014

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980$ or $1450), K_0^\star(800$ or $1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Lagrangian (2/1)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Tot}} = & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
& + \bar{\Psi}i\partial\!\!\!/\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi + g_V\bar{\Psi}\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi \\
& + \text{Polyakov loops}
\end{aligned}
$$

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

Lagrangian (2/2)

where

$$D^\mu \Phi = \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) - i e A_e^\mu [T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: \quad -g_1\phi_N a^\mu_{1N}\partial_\mu\pi_N, \\
\pi - a^\mu_1 &: \quad -g_1\phi_N(a_1^{\mu+}\partial_\mu\pi^- + a_1^{\mu0}\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\pi_S - a^\mu_{1S} &: \quad -\sqrt{2}g_1\phi_S a^\mu_{1S}\partial_\mu\pi_S, \\
K_S - K^\star_\mu &: \quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star0}_\mu\partial^\mu K^0_S + K^{\star-}_\mu\partial^\mu K^+_S) + \mathrm{h.c.}, \\
K - K^\mu_1 &: \quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K_1^{\mu0}\partial_\mu\bar{K}^0 + K_1^{\mu+}\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Diagonalization$\rightarrow$ Wave function renormalization

Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i}\right]^2,$$

where $(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

Inclusion of the vector meson- quark interaction

$$\mathcal{L}_{\mathrm{Vq}} = -g_V \sqrt{6} \bar{\Psi} \gamma_\mu V_0^\mu \Psi$$

$$V_0^\mu = \frac{1}{\sqrt{6}} \mathrm{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2} v_8) \tag{2}$$

vector fields: like Walecka model, nonzero expectation values are built up at nonzero chemical potential. For simplicity

$$< v_0^\mu >= v_0 \delta^{0\mu}, \quad < v_8^\mu >= 0$$

Modification of the grand canonical potential:

$$\Omega(T = 0, \mu_q, g_v) = \Omega(T = 0, \tilde{\mu}_q, gv = 0) - \frac{1}{2} m_v^2 v_0^2,$$

with $\tilde{\mu}_Q = \mu_q - g_v v_0$

Features of our approach

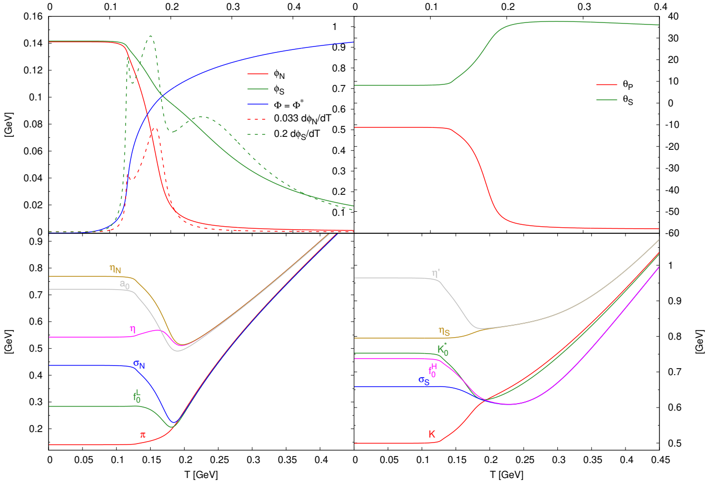
Effective field theory: the same symmetry pattern as in QCD

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

With low mass scalars, $m_{f_0^L} = 300$ MeV

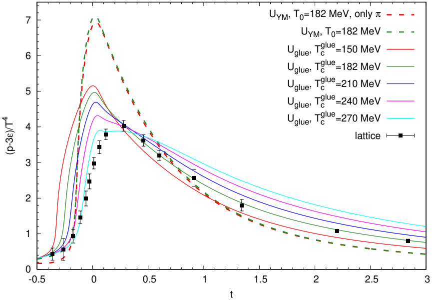


chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

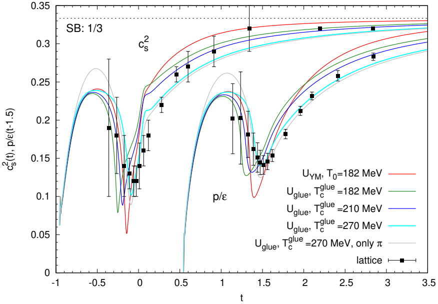
$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate

Observables

interaction measure

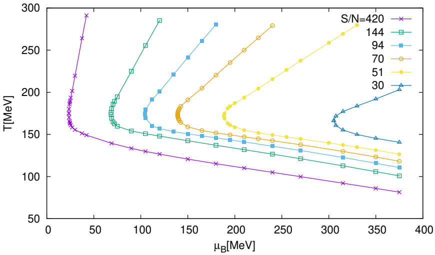


speed of sound



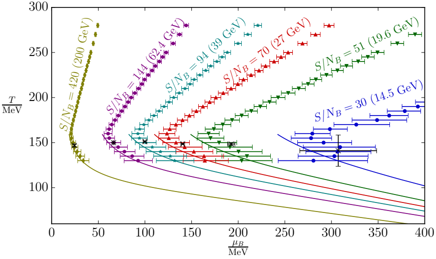
Isentropic trajectories in the $T - \mu_{\mathrm{B}}$ plane

our model, where $\mu_B^{\mathrm{CEP}} > 850\mathrm{MeV}$



lattice (analytic continuation)

Günther *et al.*, arXiv:1607.02493



same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV

$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of μ_B

Bayesian inference

Unsetted parameters: $m_\sigma, g_v, \quad \overline{\rho_B} \equiv 0.5(\rho_{BL} + \rho_{BU}), \quad \Gamma \equiv 0.5(\rho_{BU} - \rho_{BL})$

290 MeV $\leq m_\sigma \leq$ 700 MeV
$0 \leq g_v \leq 10$
$2\rho_0 \leq \overline{\rho_B} \leq 5\rho_0$
$\rho_0 \leq \Gamma \leq 4\rho_0$ with the constraint: $\rho_{BL} = \overline{\rho_B} - \Gamma > \rho_0$
We created $\sim$ 18000 EOSs to be used in the Bayesian analysis
Bayes theorem:
θ is a parameter set, $p(\theta)$ is the prior probability for θ, $p(data|\theta)$ is the probability that for given θ, the data is measured. Then

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

$p(data)$ is a normalization constant. We assume $p(\theta)$ is uniform in the allowed hypersurface. For independent observations:

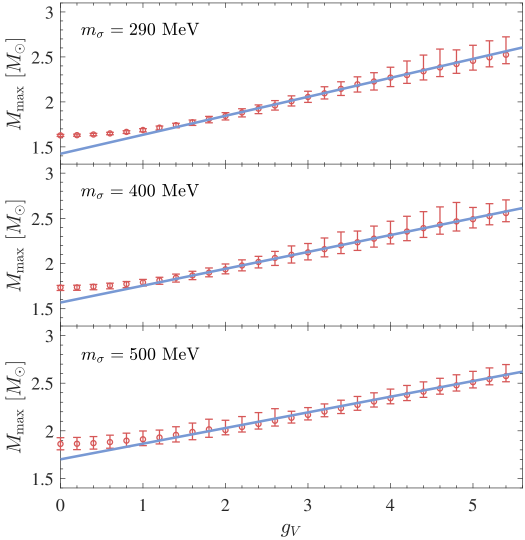
$$p(data|\theta) = p(M_{max}|\theta)p(NICER|\theta)p(\overline{\Lambda}|\theta)$$

Phys. Rev. D105 (2022) 103014, Phys. Rev. D108 (2023) 043002.

Data

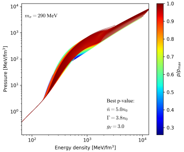
- $2M_\odot$: PSR J0348+0432 with a mass $2.01 \pm 0.04 M_\odot$, PSR J1614-2230 with a mass $1.908 \pm 0.016 M_\odot$
- perturbative QCD EOS should converge to it keeping $c_s < 1$, $\mu_{QCD} = 2.6GeV, n_{QCD} = 6.471/fm^3, p_{QCD} = 3823MeV/fm^3$
- NICER: (M,R) values for PSR J0030+0451, PSR J0740+6620 (Miller)
- tidal deformability GW170817: : $70 < \Lambda(1.4M_\odot) < 720$ Abbot (2019)
- Hess J1731-347 neutron star: mass= $0.77 \pm 0.19 M_\odot$, $R = 10.4 \pm 0.8 km$
- massgap neutron star: $2.59 \pm 0.09 M_\odot$

g_v dependence of the M_{max}

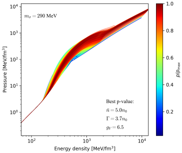


The errorbars are obtained by varying $\overline{\rho}$ and Γ.

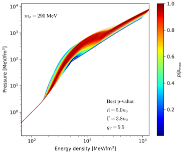
Bayesian analysis: EOS



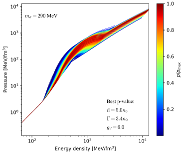
prior (M_{max} + p_{QCD})



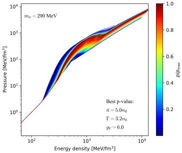
prior + NICER



prior + Nicer + GW

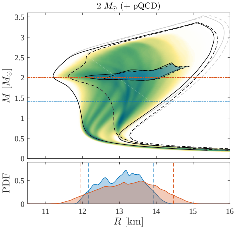


prior + Nicer + GW + HESS

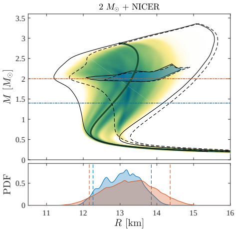


prior + Nicer + GW + HESS + GAP

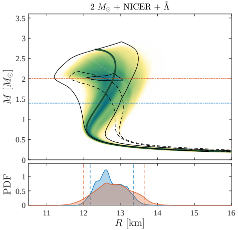
Bayesian analysis



prior (M_{max} + p_{QCD})

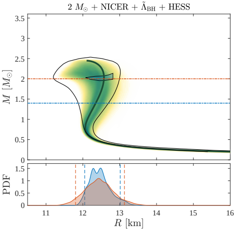


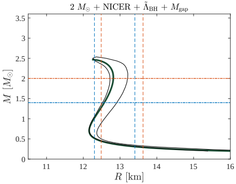
prior + NICER



prior + NICER + GW

Bayesian analysis

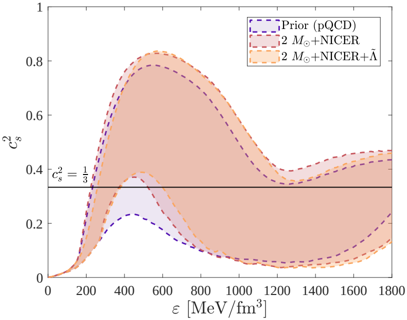


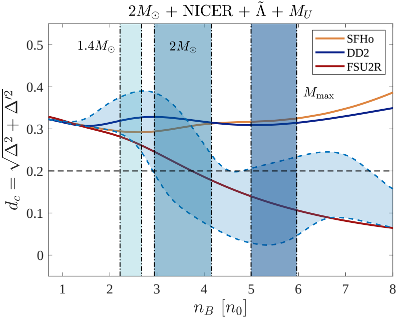


prior + NICER + GW + HESS

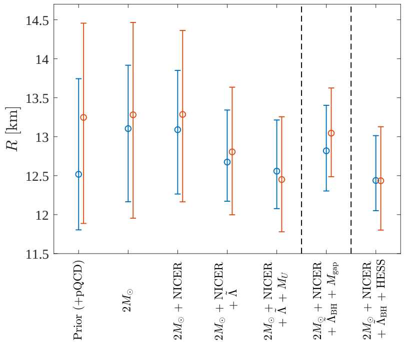
prior + NICER + GW + Mgap

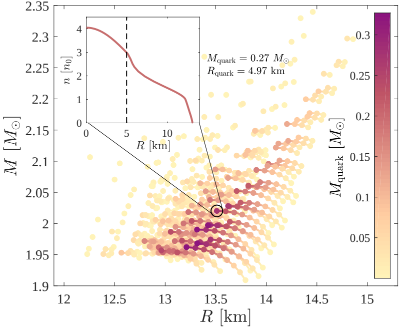
Sound velocity and Bayesian analysis





Bayesian analysis





Summary and Conclusions

- Our model can reproduce the lattice calculations at $\mu = 0$
- With our model we can fulfill the present astronomical constraints
- The central density do not go above $6\rho_0$.
- The radius of the neutron stars are 12.8 ± 0.8 km.
- strangeness should be included into the hadronic model
- hadronic and quark phase ought to be handled with the same model to drop ad-hoc parameters

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:
Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$
Scalars: $a_0(980$ or $1450), K_0^\star(800$ or $1430)$,
(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: \quad -g_1\phi_N a^\mu_{1N}\partial_\mu\pi_N, \\
\pi - a^\mu_1 &: \quad -g_1\phi_N(a^{\mu+}_1\partial_\mu\pi^- + a^{\mu 0}_1\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\pi_S - a^\mu_{1S} &: \quad -\sqrt{2}g_1\phi_S a^\mu_{1S}\partial_\mu\pi_S, \\
K_S - K^\star_\mu &: \quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star 0}_\mu\partial^\mu K^0_S + K^{\star -}_\mu\partial^\mu K^+_S) + \mathrm{h.c.}, \\
K - K^\mu_1 &: \quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K^{\mu 0}_1\partial_\mu\bar{K}^0 + K^{\mu +}_1\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Diagonalization→ Wave function renormalization

Thermodynamical Observables

We include mesonic thermal contribution to p for (π, K, f_0^l)

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

Inclusion of the vector meson- quark interaction

$$\mathcal{L}_{\mathrm{Vq}} = -g_V\sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu \Psi$$
$$V_0^\mu = \frac{1}{\sqrt{6}}\mathrm{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8) \tag{3}$$

vector fields: like Walecka model, nonzero expectation values are built up at nonzero chemical potential. For simplicity

$$< v_0^\mu >= v_0\delta^{0\mu}, \quad < v_8^\mu >= 0$$

Modification of the grand canonical potential:

$$\Omega(T=0,\mu_q,g_v) = \Omega(T=0,\tilde{\mu}_q,gv=0) - \frac{1}{2}m_v^2 v_0^2,$$

with $\quad \tilde{\mu}_Q = \mu_q - g_v v_0$

Lagrangian (2/1)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Tot}} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
&\quad - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] \\
&\quad + c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&\quad + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
&\quad + \bar\Psi i\partial\!\!\!/\Psi - g_F\bar\Psi\left(\Phi_S + i\gamma_5\Phi_{PS}\right)\Psi + g_V\bar\Psi\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi \\
&\quad + \text{Polyakov loops}
\end{aligned}
$$

P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Lagrangian (2/2)

where

$$D^{\mu}\Phi = \partial^{\mu}\Phi - ig_1(L^{\mu}\Phi - \Phi R^{\mu}) - ieA_e^{\mu}[T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^{\mu} = \sum_{i=0}^{8}(\rho_i^{\mu} - b_i^{\mu})T_i, \quad L^{\mu} = \sum_{i=0}^{8}(\rho_i^{\mu} + b_i^{\mu})T_i$$

$$L^{\mu\nu} = \partial^{\mu}L^{\nu} - ieA_e^{\mu}[T_3, L^{\nu}] - \{\partial^{\nu}L^{\mu} - ieA_e^{\nu}[T_3, L^{\mu}]\}$$

$$R^{\mu\nu} = \partial^{\mu}R^{\nu} - ieA_e^{\mu}[T_3, R^{\nu}] - \{\partial^{\nu}R^{\mu} - ieA_e^{\nu}[T_3, R^{\mu}]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i} \right]^2 ,$$

where $(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi},$
  $\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x},\tau) \right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

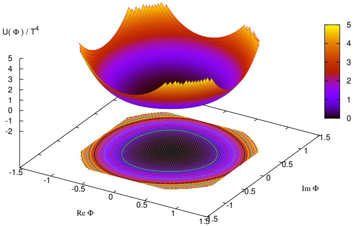
Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space
Effects of the gauge fields:

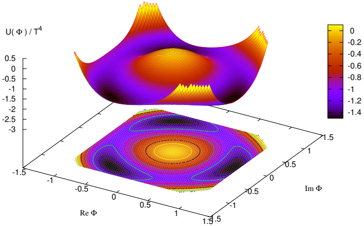
- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential
  $\longrightarrow$ modified quark distribution function.
- Polyakov potential: $\mathcal{U}\left(\Phi, \bar{\Phi}\right)$ models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum

"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected





from H. Hansen et al., PRD75, 065004 (2007)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1+3\left(\bar\Phi+\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1+3\left(\Phi+\bar\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}$$

$\Phi, \bar\Phi \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\Phi, \bar\Phi \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

T/μ_B dependence of the condensates

Ω: grand canonical potential

$$\frac{\partial \Omega}{\partial \Phi} = \left.\frac{\partial \Omega}{\partial \bar{\Phi}}\right|_{\varphi_N=\phi_N, \varphi_S=\phi_S} = 0$$

$$\frac{\partial \Omega}{\partial \phi_N} = \left.\frac{\partial \Omega}{\partial \phi_S}\right|_{\Phi,\bar{\Phi}} = 0, \qquad \text{(after the SSB)}$$

$$\frac{\partial \Omega}{\partial v_0} = 0 \qquad \text{(only contribute at} \quad \mu > 0)$$

five order parameters:

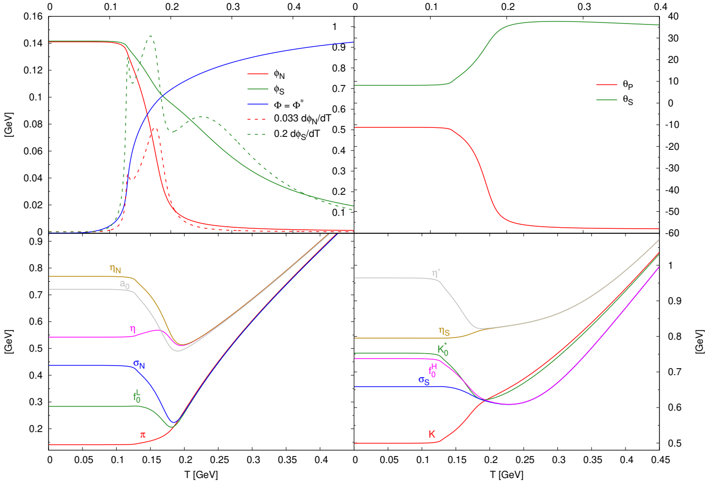
$(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

ϕ_N, ϕ_S: expectation values for the scalar field

$\Phi, \bar{\Phi}$: expectation values for the Polyakov loops

v_0 expectation value for the vector field

With low mass scalars, $m_{f_0^L} = 300$ MeV



chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate

Thank you for your attention!