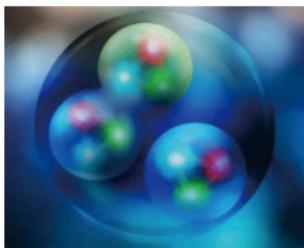


Axially deformed relativistic quasiparticle random-phase approximation based on point-coupling interactions



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Part I

Introduction

The spin-isospin excitation is one of the fundamental collective modes of a nucleus. According to the total angular momentum J and parity π , spin-isospin excitations are categorized as Fermi (0^+), Gamow-Teller (1^+), spin dipoles ($0^-, 1^-, 2^-$), and other higher order multipoles.

- ① induce spin-transitions between different nuclear states; allows proton-neutron mixing.
- ② determine the abundance patterns of r -process nucleosynthesis through β -decay rates
- ③ fundamental symmetries by determining the nuclear matrix elements of the neutrinoless double β decay
- ④ The difference between isobaric analog and Gamow-Teller (GT) resonance energies can be used to extract the neutron skin thickness , as well as from the sum rules of spin-dipole (SD) excitations

Method

- ab initio approaches
 - the study of spin-isospin modes is restricted to considering specific nuclei only.
- the nuclear shell model
 - up to the tin region of the nuclide chart, while the dimension of the Hamiltonian becomes too large
- energy density functional theory (EDF)
 - in principle, allows for the calculation of spin-isospin transitions throughout the nuclide chart, from the proton to the neutron drip line.

How

The aim of this work is to establish a new theoretical framework to study the impact of deformation on spin-isospin excitations that is able to provide fast and reliable solutions of the RPA equations. Employing the linear response formalism built for point-coupling EDFs with separable pairing interactions allows us to work in the space determined by the number of interaction channels and the coordinate mesh, which is around 10^3 , significantly lower than the two-q.p. dimension about 10^5 .

- The nuclear ground state is determined with the axially deformed relativistic Hartree-Bogoliubov (RHB) model based on relativistic point-coupling energy density functionals (EDF).
- To study the excitations in the charge-exchange channel, an axially deformed proton-neutron relativistic quasiparticle RPA (pnQRPA) is developed in the linear response approach.

Part II

Theoretical Framework

Relativistic Hartree-Bogoliubov (RHB) model

The relativistic Hartree-Bogoliubov (RHB) model describes nuclear particle-hole (ph) and particle-particle (pp) correlations .

$$\begin{pmatrix} h_D - m - \lambda & \Delta \\ -\Delta^* & -h_D^* + m + \lambda \end{pmatrix} \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix} = E_\mu \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix} \quad (1)$$

Bogoliubov transformation

$$\beta_\mu^\dagger = \sum_{l=1}^M U_{l\mu} c_l^\dagger + V_{l\mu} c_l, \quad (2)$$

$$\beta_\mu = \sum_{l=1}^M V_{l\mu}^* c_l^\dagger + U_{l\mu}^* c_l, \quad (3)$$

where M denotes the dimension of the single-particle basis. We introduce the $2M$ -dimensional set of extended q.p. states a_μ , satisfying $\{a_\mu, a_{\mu'}\} = \delta_{\mu\tilde{\mu}'}$

$$\left. \begin{array}{l} a_\mu = \beta_\mu \\ a_{\tilde{\mu}} = \beta_\mu^\dagger \end{array} \right\}, \quad \mu = 1, \dots, M; \quad \tilde{\mu} = -\mu \quad (4)$$

Extended q.p. States a_μ

In this basis, the one-body q.p. operator is represented

$$\hat{F} = \hat{F}^0 + \frac{1}{2} \sum_{\mu\mu'} \mathcal{F}_{\mu\mu'} a_\mu^\dagger a_{\mu'} \quad (5)$$

The matrix elements $\mathcal{F}_{\mu\mu'}$ with signs of the μ and μ' indices explicitly specified read

$$\mathcal{F}_{\mu\mu'} = \begin{pmatrix} \mathcal{F}_{\mu>0\mu'>0} & \mathcal{F}_{\mu>0\mu'<0} \\ \mathcal{F}_{\mu<0\mu'>0} & \mathcal{F}_{\mu<0\mu'<0} \end{pmatrix}. \quad (6)$$

Furthermore, the following relations can easily be verified:

$$\begin{aligned} \mathcal{F}_{\mu>0\mu'>0} &= F_{\mu\mu'}^{11}, \\ \mathcal{F}_{\mu>0\mu'<0} &= F_{\mu-\mu'}^{20}, \\ \mathcal{F}_{\mu<0\mu'>0} &= -F_{-\mu\mu'}^{02}, \\ \mathcal{F}_{\mu<0\mu'<0} &= -F_{-\mu-\mu'}^{11}, \end{aligned} \quad (7)$$

External Field Matrix Element

The external field matrix element is defined as

$$\langle p|F_{JK}|n\rangle = \int r dr dz d\phi [\Psi_p^\dagger F_{JK} \Psi_n]$$

We have to consider four combinations of pairs in the previous equation: $pn, p\bar{n}, \bar{p}n$, and $\bar{p}\bar{n}$, where $\bar{n}(\bar{p})$ denotes the timereversed neutron (proton) state. The Fermi transition operator is defined as

$$F_{00} = \tau_\pm$$

where τ_\pm is the isospin raising or lowering operator. The GT external field operator is defined as

$$F_{1K} = \sigma_{1K} \tau_\pm$$

where σ denotes the Pauli spin matrix. The SD operator reads

$$F_{JK} = r [\sigma_1 \otimes Y_1]_{JK}$$

where Y_{JM} is the spherical harmonic and $J = 0, 1, 2$.

External Field Matrix Element

Finally, the matrix elements have to be transformed to the q.p. space. In the q.p. basis the external field operator assumes the form

$$\begin{aligned}\hat{F}_{JK} &= \sum_{pn} \langle p | F_{JK} | n \rangle c_p^\dagger c_n \\ &= \sum_{\pi\nu} (U^\dagger F_{JK} U)_{\pi\nu} \beta_\pi^\dagger \beta_\nu + (U^\dagger F_{JK} V^*)_{\pi\nu} \beta_\pi^\dagger \beta_\nu^\dagger \\ &\quad + (V^T F_{JK} U)_{\pi\nu} \beta_\pi \beta_\nu + (V^T F_{JK} V^*)_{\pi\nu} \beta_\pi \beta_\nu^\dagger \\ &= \sum_{\pi\nu} (U^\dagger F_{JK} U)_{\pi\nu} a_{\tilde{\pi}} a_\nu + (U^\dagger F_{JK} V^*)_{\pi\nu} a_{\tilde{\pi}} a_{\tilde{\nu}} \\ &\quad + (V^T F_{JK} U)_{\pi\nu} a_\pi a_\nu + (V^T F_{JK} V^*)_{\pi\nu} a_\pi a_{\tilde{\nu}}\end{aligned}$$

where $\pi, \nu > 0$. Using the matrix structure defined in Eq. (6) and extending π, ν to negative values we can conveniently rewrite the external field operator as

$$\hat{F}_{JK} = \sum_{\pi\nu} \left(\begin{array}{cc} (U^\dagger F_{JK} U)_{\pi\nu} & (U^\dagger F_{JK} V^*)_{\pi\nu} \\ (V^T F_{JK} U)_{\pi\nu} & (V^T F_{JK} V^*)_{\pi\nu} \end{array} \right) a_\pi^\dagger a_\nu \quad (39)$$

Generalized Density Matrix \mathcal{R}

$$\mathcal{R} = \begin{pmatrix} \langle \Phi | a_{\tilde{\mu}'} a_\mu | \Phi \rangle & \langle \Phi | a_{\mu'} a_\mu | \Phi \rangle \\ \langle \Phi | a_{\tilde{\mu}'} a_{\tilde{\mu}} | \Phi \rangle & \langle \Phi | a_{\mu'} a_{\tilde{\mu}} | \Phi \rangle \end{pmatrix}, \quad \mu, \mu' > 0 \quad (8)$$

where $|\Phi\rangle$ is the vacuum state obtained by solving the RHB equations
 derive the linear response equations by considering the time-dependent generalized density
 $\mathcal{R}(t)$ in an external charge-changing field $F(t)$ which obeys the equation of motion

$$i\dot{\mathcal{R}}(t) = [\mathcal{H}(\mathcal{R}(t)) + F(t), \mathcal{R}(t)] \quad (9)$$

with $\mathcal{H} = \frac{\partial E}{\partial \mathcal{R}}$. Assuming harmonic time dependence of the external field

$$F(t) = F e^{-i\omega t} + \text{H.c.}, \quad (10)$$

with excitation energy ω , and linearizing the generalized density

$$\mathcal{R}(t) = \mathcal{R}^0 + (\delta \mathcal{R} e^{-i\omega t} + \text{H.c.}) \quad (11)$$

where $[\mathcal{H}(\mathcal{R}^0), \mathcal{R}^0] = 0$ is the static RHB equation

Bethe-Salpeter equation

Bethe-Salpeter equation for the response \mathbb{R} , up to the leading order in $\delta\mathcal{R}$, in the matrix form,

$$\mathbb{R} = \mathbb{R}^0 + \mathbb{R}^0 W \mathbb{R} \quad (12)$$

$$\delta\mathcal{R}_{\pi v} = \sum_{\pi' v'} \mathbb{R}_{\pi v \pi' v'} F_{\pi' v'} \quad (13)$$

where $\pi(v)$ labels proton (neutron) q.p. states.

$$\mathbb{R}_{\pi v \pi' v'}(\omega) = \frac{1}{\hbar} \sum_{\nu > 0} \left(\frac{\langle 0 | a_v^+ a_\pi | \nu \rangle \langle \nu | a_{\pi'}^+ a_{v'} | 0 \rangle}{\omega - \Omega_\nu + i\eta} - \frac{\langle 0 | a_\pi^+ a_{v'} | \nu \rangle \langle \nu | a_v^+ a_\pi | 0 \rangle}{\omega + \Omega_\nu + i\eta} \right)$$

Small parameter η provides finite width of the resonances. Ω_ν is the excitation energy for the excited state $|\nu\rangle$.

The effective interaction matrix \mathbb{W} is a functional derivative of the interaction Hamiltonian,

$$\mathbb{W}_{\pi v \pi' v'} = \frac{\delta \mathcal{H}_{\pi v}}{\delta \mathcal{R}_{\pi' v'}} \quad (14)$$

Unperturbed Response \mathbb{R}^0

while the unperturbed response is diagonal in the q.p. space

$$\mathbb{R}_{\pi v \pi' v'}^0 = \frac{(f_\pi - f_v) \delta_{\pi \pi'} \delta_{v v'}}{\omega - E_\pi - E_v + i\eta} \quad (15)$$

In the q.p. basis, matrices \mathcal{R}^0 and \mathcal{H}^0 are diagonal,

$$\mathcal{R}^0 = \begin{pmatrix} f_\mu & 0 \\ 0 & f_{\tilde{\mu}} \end{pmatrix}, \quad \mathcal{H}^0 = \begin{pmatrix} E_\mu & 0 \\ 0 & E_{\tilde{\mu}} \end{pmatrix}, \quad (16)$$

with the eigenvalues,

$$\begin{aligned} f_\mu &= 0, & f_{\tilde{\mu}} &= 1 \\ E_\mu &= E_k, & E_{\tilde{\mu}} &= -E_k \end{aligned} \quad (17)$$

Strength Function:

$$\begin{aligned} S_F(\omega) &= \sum_{\nu>0} |\langle \nu | F | 0 \rangle|^2 \delta(\hbar\omega - \hbar\Omega_\nu) \\ S_F(\omega) &= -\frac{1}{\pi} \text{Im} \sum_{\pi v \pi' v'} (F_{\pi v}^* \mathbb{R}_{\pi v \pi' v'} F_{\pi' v'}) \end{aligned} \quad (18)$$

Hamiltion :

$$\hat{H} = \hat{H}_0 + \sum_{cc'} v_{cc'} \hat{Q}_c^\dagger \hat{Q}_{c'} \quad (19)$$

where \hat{H}_0 is the mean-field Hamiltonian, and indices c, c' run over a set of operators \hat{Q}_c with coupling strength $v_{cc'}$. In general, index c runs over states in the discretized basis, e.g., coordinate space basis (r, z) , momentum space basis (p_r, p_z) or harmonic oscillator basis (n_r, n_z) .

Due to the isospin selection rules, only two terms of the interaction Lagrangian density can contribute to the charge-exchange linear response equations. The first is the **isovector-vector (TV) term** with the matrix element , the second is the **isovector pseudovector (TPV) term**

$$\begin{aligned}
 V_{pnn'p'}^{\text{TV}} = & - \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \alpha_{\text{TV}} [\rho_v] \left[\bar{\Psi}_p(\mathbf{r}_1) \gamma_\mu^{(1)} \boldsymbol{\tau}^{(1)} \Psi_n(\mathbf{r}_1) \right] \\
 & \times \left[\bar{\Psi}_{n'}(\mathbf{r}_2) \gamma^\mu \boldsymbol{\tau}^{(2)} \Psi_{p'}(\mathbf{r}_2) \right] \delta(\mathbf{r}_1 - \mathbf{r}_2)
 \end{aligned} \tag{40}$$

where $\bar{\Psi} = \Psi^\dagger \gamma_0$. The coupling α_{TV} is a function of the vector density

$$\rho_v(\mathbf{r}) = \sum_k V_k^\dagger(\mathbf{r}) V_k(\mathbf{r}), \tag{41}$$

where $V_k(\mathbf{r})$ is the coordinate-space representation of the lower component of the q.p. wave function and the summation is performed by omitting the antiparticle states, within the no-sea approximation

TV Residual Interaction

The TV residual interaction term can be separated into timelike and spacelike components,

$$\begin{aligned}
 V_{pnn'p'}^{\text{TV}(t)} &= -2 \int r dr dz d\phi \alpha_{\text{TV}} [\rho_v] [\Psi_p^\dagger(\mathbf{r}) \Psi_n(\mathbf{r})] \\
 &\quad \times [\Psi_{n'}^\dagger(\mathbf{r}) \Psi_{p'}(\mathbf{r})] \\
 V_{pnn'p'}^{\text{TV}(s)} &= -2 \int r dr dz d\phi \alpha_{\text{TV}} [\rho_v] \\
 &\quad \times \sum_\mu (-)^\mu \left[\Psi_p^\dagger(\mathbf{r}) \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \Psi_n(\mathbf{r}) \right] \\
 &\quad \times \left[\Psi_{n'}^\dagger(\mathbf{r}) \begin{pmatrix} 0 & \sigma_{-\mu} \\ \sigma_{-\mu} & 0 \end{pmatrix} \Psi_{p'}(\mathbf{r}) \right]
 \end{aligned} \tag{42}$$

Interaction can be written in a separable form, where the separable channels are defined as

$$Q_{pn}^{\text{TV}(t)}(r, z) = \Psi_p^\dagger(r, z) \Psi_n(r, z), \tag{43}$$

$$Q_{pn}^{\text{TV}(s),\mu} = \Psi_p^\dagger(r, z) \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \Psi_n(r, z). \tag{44}$$

Isovector-pseudovector (TPV) Term

Therefore, we also include the isovector-pseudovector (TPV) term in the residual interaction:

$$V_{pnn'p'}^{\text{TPV}} = g_0 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \left[\bar{\Psi}_p(\mathbf{r}_1) \gamma_5^{(1)} \gamma_\mu^{(1)} \boldsymbol{\tau}^{(1)} \Psi_n(\mathbf{r}_1) \right] \\ \times \left[\bar{\Psi}_{n'}(\mathbf{r}_2) \gamma_5^{(2)} \gamma^\mu \boldsymbol{\tau}^{(2)} \Psi_{p'}(\mathbf{r}_2) \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (45)$$

we use $g_0 = 0.734(0.621)$ for the DD-PC1 (DD-PCX) EDFs

the isovector-pseudovector (TPV) residual interaction can be written as

$$V_{pnn'p'}^{\text{TPV}(t)} = 2g_0 \int r dr dz d\phi \left[\Psi_p^\dagger(\mathbf{r}) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Psi_n(\mathbf{r}) \right] \left[\Psi_{n'}^\dagger(\mathbf{r}) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Psi_{p'}(\mathbf{r}) \right] \\ V_{pnn'p'}^{\text{TPV}(s)} = 2g_0 \int r dr dz d\phi \sum_{\mu} (-)^{\mu} \left[\Psi_p^\dagger(\mathbf{r}) \begin{pmatrix} \sigma_{\mu} & 0 \\ 0 & \sigma_{\mu} \end{pmatrix} \Psi_n(\mathbf{r}) \right] \left[\Psi_{n'}^\dagger(\mathbf{r}) \begin{pmatrix} \sigma_{-\mu} & 0 \\ 0 & \sigma_{-\mu} \end{pmatrix} \Psi_{p'}(\mathbf{r}) \right] \quad (46)$$

for timelike and spacelike components, respectively. The separable channels are defined as

$$Q_{pn}^{\text{TPV}(t)}(r, z) = \Psi_p^\dagger(r, z) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Psi_n(r, z), \quad (47)$$

$$Q_{pn}^{\text{TPV}(s),\mu} = \Psi_p^\dagger(r, z) \begin{pmatrix} \sigma_\mu & 0 \\ 0 & \sigma_\mu \end{pmatrix} \Psi_n(r, z). \quad (48)$$

For both TV term and the TPV term have the $4 \times N_z^{\text{GH}} \times N_r^{\text{GL}}$ separable channels .

The separable matrix elements are transformed to the q.p. basis analogously to the external field matrix elements

$$\hat{Q}_{cc'} = \sum_{\pi v} \begin{pmatrix} (U^\dagger Q_{cc'} U)_{\pi v} & (U^\dagger Q_{cc'} V^*)_{\pi v} \\ (V^T Q_{cc'} U)_{\pi v} & (V^T Q_{cc'} V^*)_{\pi v} \end{pmatrix} a_\pi^\dagger a_v \quad (49)$$

where (c, c') label the separable interaction channels.

Particle-particle Interaction

For the particle-particle (pp) interaction we assume the separable pairing form

$$V'(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}'_1, \mathbf{r}'_2) = -fG\delta(\mathbf{R} - \mathbf{R}') P(r, z)P(r', z') \quad (50)$$

where $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ is the center-of-mass and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the relative coordinate. The overall factor f is defined as

$$f = \begin{cases} V_0^{\text{pp}}, & T = 0, S = 1 \\ 1, & T = 1, S = 0 \end{cases} \quad (51)$$

where V_0^{pp} is the isoscalar pairing strength, not constrained at the ground-state level [28]. The form factor $P(r, z)$ corresponds to the Gaussian function

$$P(r, z) = \frac{1}{(4\pi a^2)^{3/2}} e^{-\frac{z^2+r^2}{4a^2}}, \quad (52)$$

with strength G and range a parameters adjusted to reproduce the pairing gap of the Gogny pairing force .

It is convenient to calculate the matrix element in the axially deformed harmonic oscillator (h.o.) basis

$$\langle 12 | V | 1'2' \rangle = \langle 12 | V' (1 - P^r P^\sigma P^\tau) | 1'2' \rangle \quad (53)$$

where each state is denoted with the h.o. quantum numbers $|1\rangle \equiv |n_{z_1} n_{r_1} \Lambda_1 m_{s_1} m_{t_1}\rangle$, where n_z and n_r are quantum numbers in z and r directions, respectively. Λ is the projection of the orbital angular momentum on the z axis, m_s is the spin projection, and m_t denotes the isospin projection. In the coordinate space the h.o. eigenfunction has the form

$$\langle \mathbf{r} | n_{z_1} n_{r_1} \Lambda_1 m_{s_1} m_{t_1} \rangle = \phi_{n_{z_1}}(z, b_z) \phi_{n_{r_1}}^{\Lambda_1}(r, b_r) \frac{e^{i\phi\Lambda_1}}{\sqrt{2\pi}} \chi_{1/2m_{s_1}} \xi_{1/2m_{t_1}} \quad (54)$$

where $\chi_{1/2m_{s_1}}$ denotes the spin and $\xi_{1/2m_{t_1}}$ the isospin wave functions. b_r and b_z are the oscillator lengths .

The projector operators exchange the position, spin, and isospin of two nucleons:

$$\begin{aligned} P^r |\mathbf{r}_1 \mathbf{r}_2\rangle &= |\mathbf{r}_2 \mathbf{r}_1\rangle, & P^\sigma |SM_S\rangle &= (-)^{S-1} |SM_S\rangle, \\ P^\tau |TM_T\rangle &= (-)^{T-1} |TM_T\rangle \end{aligned} \quad (55)$$

where S and T denote the total spin and isospin of two states, with projections M_S and M_T , respectively. The wave function coupled to total spin S and isospin T reads

$$\begin{aligned} |12\rangle = & \phi_{n_{z_1}}(z_1, b_z) \phi_{n_{r_1}}^{\Lambda_1}(r_1, b_r) \phi_{n_{z_2}}(z_2, b_z) \phi_{n_{r_2}}^{\Lambda_2}(r_2, b_r) \\ & \times \frac{1}{2\pi} e^{i\phi_1 \Lambda_1} e^{i\phi_2 \Lambda_2} \sum_{SM_S} C_{1/2m_{s_1} 1/2m_{s_2}}^{SM_S} |SM_S\rangle \\ & \times \sum_{TM_T} C_{1/2m_{t_1} 1/2m_{t_2}}^{TM_T} |TM_T\rangle. \end{aligned} \quad (56)$$

Transform the Wave Function

In order to calculate the matrix elements, we have to transform the h.o. wave functions from the laboratory to the center-of-mass frame.

- the product of z -component wave functions can be written as

$$\phi_{n_{z1}}(z_1) \phi_{n_{z2}}(z_2) = \sum_{N_z n_z} M_{N_z n_z}^{n_{z2} n_{z2}} \phi_{N_z}(Z, \tilde{b}_Z) \phi_{n_z}(z, \tilde{b}_z) (-)^{n_z} \quad (57)$$

where $\tilde{b}_Z = \sqrt{2}b_z$ and $\tilde{b}_z = b_z/\sqrt{2}$. $M_{N_z}^{n_{z2} n_{z2}}$ is the onedimensional Talmi-Moshinsky coefficient

The TalmiMoshinsky coefficients imply the following selection rule that connects quantum numbers in the intrinsic and laboratory frame [50]:

$$n_{z1} + n_{z2} = N_z + n_z \quad (59)$$

Transform the Wave Function

In order to calculate the matrix elements, we have to transform the h.o. wave functions from the laboratory to the center-of-mass frame.

- transformation to the radial wave functions

$$\phi_{n_{r_1}}^{\Lambda_1}(\mathbf{r}_1) \phi_{n_{r_2}}^{\Lambda_2}(\mathbf{r}_2) = \sum_{N_r \Lambda} \sum_{n_r \lambda} M_{N_r \Lambda n_r \lambda}^{n_{r_1} \Lambda_1 n_{r_2} \Lambda_2} \times \phi_{N_r}^{\Lambda}(\mathbf{R}, \tilde{b}_R) \phi_{n_r}^{\lambda}(\mathbf{r}, \tilde{b}_r) (-)^{\lambda} \quad (58)$$

where $\tilde{b}_R = \sqrt{2}b_r$ and $\tilde{b}_r = b_r/\sqrt{2}$. $M_{N_r \Lambda n_r \lambda}^{n_{r_1} \Lambda_1 n_{r_2} \Lambda_2}$ is the two dimensional Talmi-Moshinsky coefficient

The TalmiMoshinsky coefficients imply the following selection rule that connects quantum numbers in the intrinsic and laboratory frame [50]:

$$n_r + N_r = n_{r_1} + n_{r_2} + \frac{|\Lambda_1| + |\Lambda_2| + |\Lambda_1 + \Lambda_2|}{2} \quad (60)$$

$$\Lambda_1 + \Lambda_2 = \Lambda + \lambda \quad (61)$$

Particle-Particle Matrix Element

The total matrix element in the coupled basis reads

$$\begin{aligned} \langle 1\bar{1}\bar{2}|V|1'\bar{2}'\rangle &= -G\delta_{\lambda 0}\delta_{\lambda' 0}\delta_{\Lambda\Lambda'}\frac{1}{b_z b_r^2}\Sigma(S, T) \\ &\times \sum_{N_z N_r} W_{1\bar{2}}^{N_z} W_{1\bar{2}}^{N_r} W_{1'\bar{2}'}^{N_z} W_{1'\bar{2}'}^{N_z} \end{aligned} \quad (62)$$

where we have defined the separable terms analogously to Ref. [23]:

$$\begin{aligned} W_{1\bar{2}}^{N_z} &= \frac{1}{\sqrt{b_z}} M_{n_{z_1} n_{z_2}}^{N_z N_z} \delta_{n_z, \text{even}} \frac{(-)^{n_z/2}}{(2\pi)^{1/4}} \frac{\sqrt{n_z!}}{2^{n_z/2} (n_z/2)!} \\ &\times \left(\frac{b_z^2}{a^2 + b_z^2} \right)^{1/2} \left(\frac{b_z^2 - a^2}{b_z^2 + a^2} \right)^{n_z/2}, \end{aligned} \quad (63)$$

$$W_{1\bar{2}}^{N_r} = \frac{1}{b_r} M_{n_{r_1} \Lambda_1 n_{r_2} \Lambda_2}^{N_r \Lambda n_r 0} \frac{1}{(2\pi)^{1/2}} \frac{b_r^2}{b_r^2 + a^2} \left(\frac{b_r^2 - a^2}{b_r^2 + a^2} \right)^{n_r}. \quad (64)$$

notice that n_z can only assume even values

Spin-isospin part $\Sigma(S, T)$

Spin-isospin part $\Sigma(S, T)$ has the form

$$\begin{aligned} \Sigma(S, T) = & \sum_{SM_S} \sum_{TM_T} \frac{1}{2} [1 - (-)^{S+T}] (-)^{1/2-m_{s_2}} (-)^{1/2-m'_{s_2}} \\ & \times C_{1/2m_{s_1} 1/2-m_{s_2}}^{SM_{1/2m'_{s_1}}} {}^{1/2-m'_{s_2}} \\ & \times C_{1/2m'_{t_1}}^{SM_T} {}^{1/2m'_{t_2}} C_{1/2m_{t_1} 1/2m_{t_2}}^{TM_T} \end{aligned} \quad (65)$$

from which it follows that $S + T$ assumes only odd values. Two cases can be distinguished corresponding to either isovector ($T = 1, S = 0$) or isoscalar ($T = 0, S = 1$) pairing interaction.

$$W_{N_r, N_z}^{T=1, S=0} = \frac{1}{\sqrt{2}} W_{12}^{N_z} W_{1\bar{2}}^{N_r} (-)^{1/2-m_{s_2}} C_{1/2m_{s_1} 1/2-m_{s_2}}^{00} \quad (66)$$

$$W_{N_r, N_z, M_S}^{T=0, S=1} = -\frac{1}{\sqrt{2}} W_{12}^{N_z} W_{12}^{N_r} (-)^{1/2-m_{s_2}} C_{1/2m_{s_1} 1/2-m_{s_2}}^{1M_S} \quad (67)$$

Total pp Residual Interaction Matrix Element

$$\begin{aligned}
 V_{pn\bar{p}'n'}^{\text{ppp}} &= \langle p\bar{n}|V|p'\bar{n}'\rangle c_p^\dagger c_{\bar{n}}^\dagger c_{\bar{n}'} c_{p'} \\
 &= \sum_{N_z N_r M_S} (W_{pn}^{N_z N_r M_S})^* c_p^\dagger c_{\bar{n}}^\dagger W_{p'n'}^{N_z N_r M_S} c_{\bar{n}'} c_{p'} \\
 &= \sum_{N_z N_r M_S} (\hat{Q}_{pn}^{N_z N_r M_S})^\dagger \hat{Q}_{p'n'}^{N_z N_r M_S}
 \end{aligned} \tag{68}$$

defined a separable term as $(\hat{Q}_{pn}^{N_z N_r M_S})^\dagger = (W_{pn}^{N_z N_r M_S})^* c_p^\dagger c_{\bar{n}}^\dagger$.

transform the pp separable matrix elements from the singleparticle to the q.p. basis. Here one has to take into account the transformation properties of time-reversed states [63] to obtain the correct expression,

$$\hat{W}_{cc'} = \sum_{\pi\nu} \begin{pmatrix} - (U^T W_{cc'} V^*)_{\pi\nu} & (U^\dagger W_{cc'} U)_{\pi\nu} \\ - (V^T W_{cc'} V^*)_{\pi\nu} & (V^\dagger W_{cc'} U^*)_{\pi\nu} \end{pmatrix} a_\pi^\dagger a_\nu \tag{69}$$

where (c, c') denote the separable pp residual interaction channels.

Strength Function

$$S_F(\omega) = -\frac{1}{\pi} \operatorname{Im} \sum_{\pi v \pi' v'} (F_{\pi v}^* \mathbb{R}_{\pi v \pi' v'} F_{\pi' v'}) = -\frac{1}{\pi} \operatorname{Im} R_{FF}$$

obtain the reduced response R_{FF} as

$$R_{FF} = R_{FF}^0 + \sum_{cc'} R_{Fc}^0 v_{cc'} R_{c'F} \quad (25)$$

with the following definitions:

$$R_{FF} = \sum_{\pi v \pi' v'} (F_{\pi v}^* \mathbb{R}_{\pi v \pi' v'} F_{\pi' v'}) \quad (26)$$

$$R_{FF}^0 = \sum_{\pi v \pi' v'} (F_{\pi v}^* \mathbb{R}_{\pi v \pi' v'}^0 F_{\pi' v'}) \quad (27)$$

$$R_{Fc}^0 = \sum_{\pi v \pi' v'} (F_{\pi v}^* \mathbb{R}_{\pi v \pi' v'}^0 Q_{\pi' v'}^c) \quad (28)$$

Part III

Numerical Test

Residual Interaction

- use doubly magic neutron-rich ^{28}O isotope to test for the implementation of the pp channel in the residual interaction
- Comparison between the spherical and axially deformed pnQRPA results for ^{28}O for the IAS $^-$ (a) and GT $^-$ (b) strength function, with $N_{\text{osc}} = 8$ oscillator shells.
- $\eta = 0.25\text{MeV}$
- $N_z^{\text{GL}} = N_r^{\text{GL}} \sim N_{\text{osc}}$ mesh points

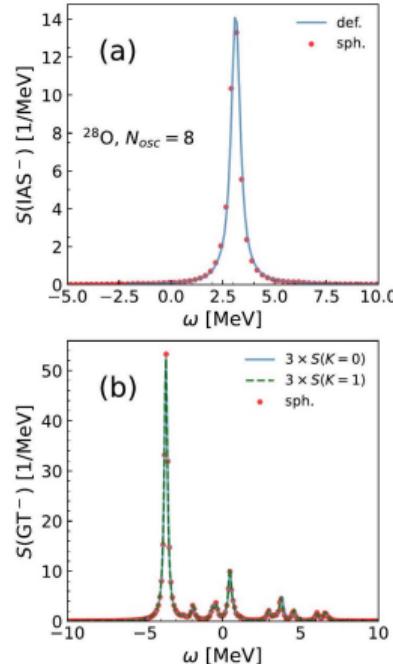


Figure: 1.Comparison between the Spherical and the deformed pnQRPA result

Ikeda Sum Rule

For instance, in the case of GT response, the total strength satisfies the Ikeda sum rule

$$\Sigma(\text{GT}^-) - \Sigma(\text{GT}^+) = 3(N - Z) \quad (71)$$

where

$$\Sigma(\text{GT}^\pm) = \frac{1}{2\pi i} \oint_{\mathcal{C}} R_{FF}(\omega) d\omega \quad (72)$$

the integration contour \mathcal{C} can be written as a sum of two contours; $\mathcal{C} = \mathcal{C}_{\alpha h} + \mathcal{C}_{q.p.}$, where $\mathcal{C}_{\alpha h}$ encircles the negative energy poles and $\mathcal{C}_{q.p.}$ includes the positive energy poles.

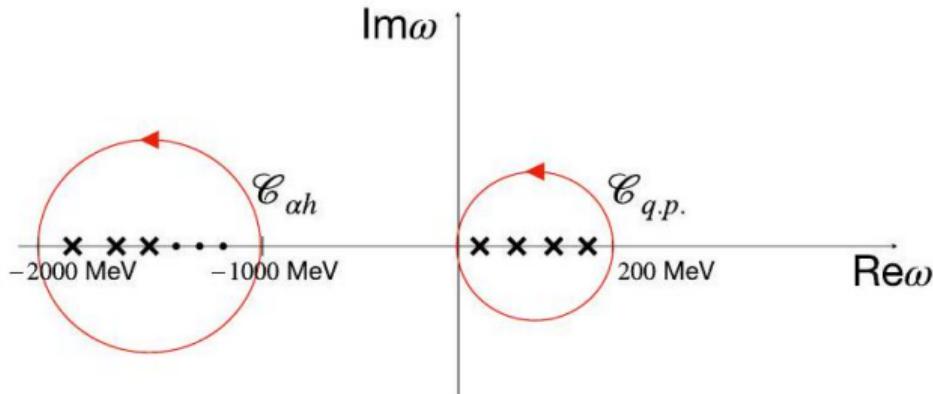
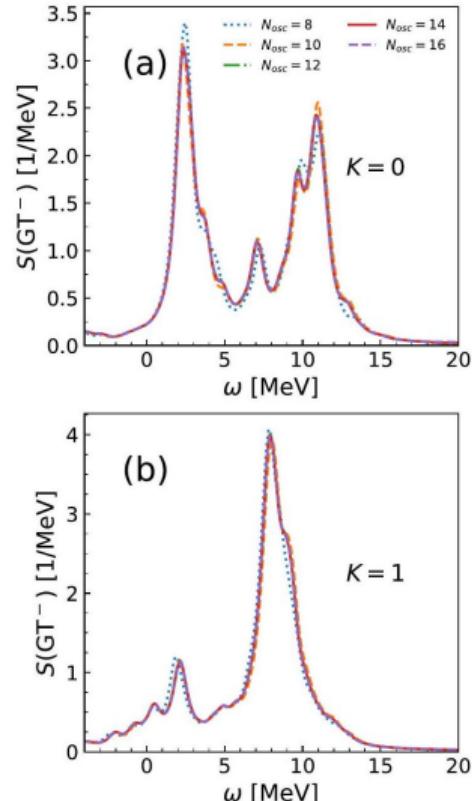


Figure: 2.Coutour Integral

Nucleus	$\Sigma(\text{GT}^-) - \Sigma(\text{GT}^+)$	% sum rule (no antipar.)
^{28}O	35.99999	99.99997(95.69733)
^{48}Ca	24.00000	100.00000(94.09129)

Basis Size

calculated the GT^- response in the neutron-rich ^{70}Fe isotope for several numbers of h.o. shells ranging from $N_{\text{osc}} = 8$ to $N_{\text{osc}} = 16$. The RHB ground state in each calculation is constrained to $\beta_2 = 0.3$. Results are shown in Fig. 3 for both $K = 0$ (a) and $K = 1$ (b) projections. From the figure, we conclude that reasonable convergence for the GT^- strength is achieved already with $N_{\text{osc}} = 12$.



Part IV

Application

Isobaric Analog Resonance

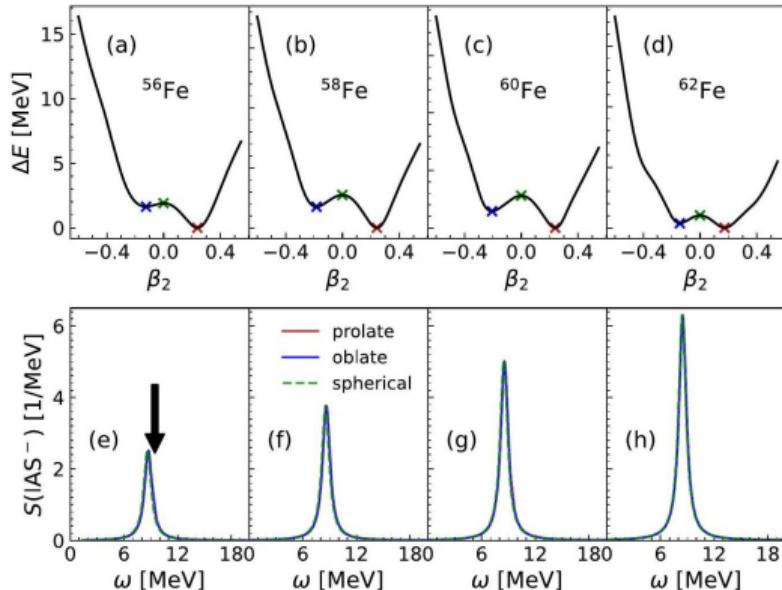


Figure: 4.Fermi strength function for the $^{56},^{58},^{60},^{62}\text{Fe}$

The deformation has almost no influence on the Fermi strength function.

- The calculations require strength functions for two projections $K = 0$ and $K = 1$ because of the degeneracy of $K = +1$ and $K = -1$ modes. The total strength is calculated as

$$S(GT^\pm, \omega) = S(K=0, \omega) + 2 \times S(K=1, \omega) \quad (73)$$

- oblate minimum at $\beta_2 = -0.19$, prolate local minimum $\beta_2 = 0.13$
- the fragmentation in the deformed pnQRPA strength stems from the degeneracy breaking of the Nilsson q.p. orbitals.

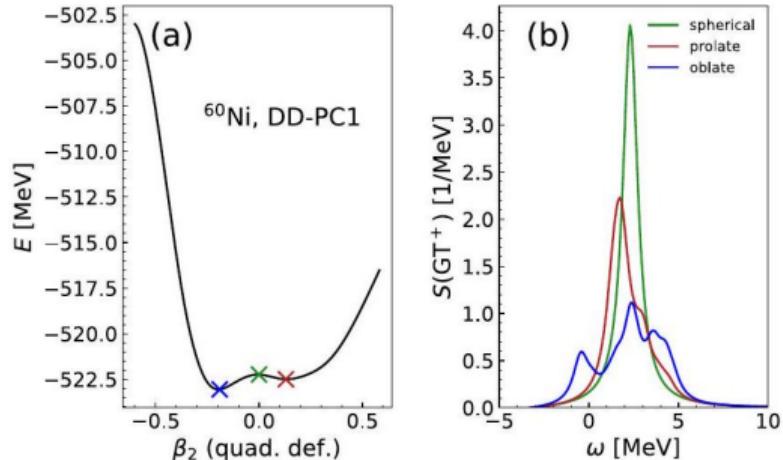


Figure: 5. The PEC for ^{60}Ni calculated with the axially deformed RHB with the DD-PC1 interaction. (b) The GT⁺ strength as a function of the excitation energy ω .

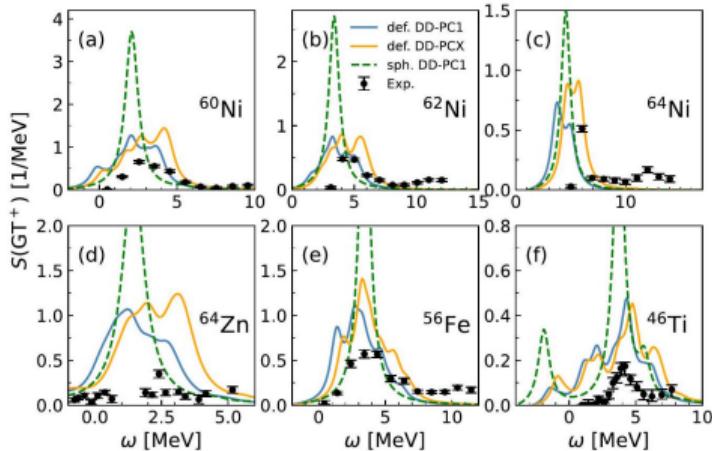


Figure: 6.Comparison the GT⁺ strength function in $^{60,62,64}\text{Ni}$, ^{64}Zn , ^{56}Fe , and ^{46}Ti with available experimental data

The difference between spherical and deformed calculations can be attributed to increased density of states and splittings between the Nilsson orbitals for deformed configurations

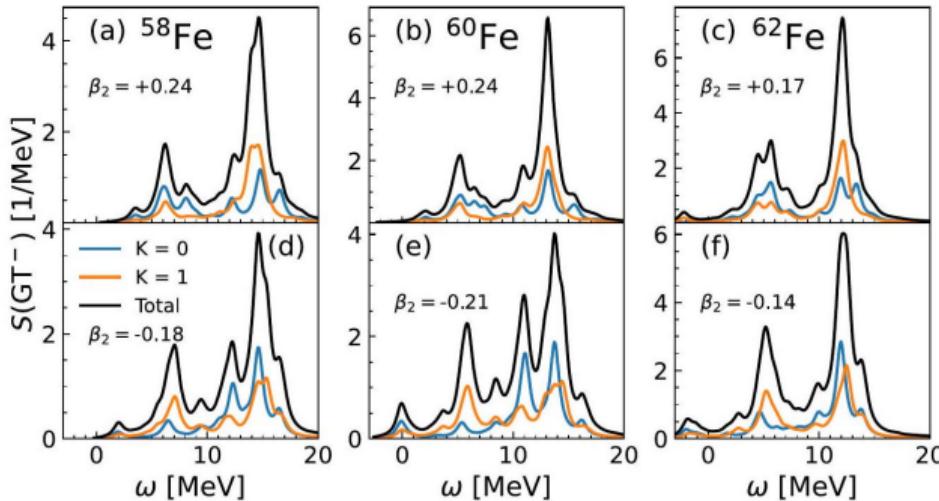


Figure: 7. The GT^- strength function for selected even-even isotopes of iron

For prolate shapes ($\beta_2 > 0$), the dominant contribution to the low-lying strength comes from the $K = 0$ component, while the strength in resonance region is dominated by the $K = 1$ component

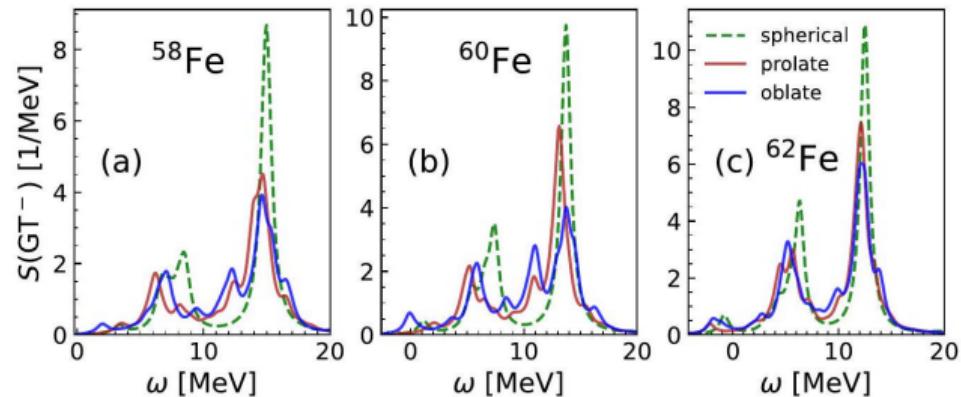


Figure: 8. The total GT⁻ strength function in ^{58}Fe , ^{60}Fe , and ^{62}Fe

Spin-dipole Resonance

transitions coupled to total spin $\Delta S = 1$ and orbital angular momentum $\Delta L = 1$. This results in coupling to three possible values of angular momentum, $J^\pi = 0^-, 1^-, \text{ and } 2^-$.

$$S(\text{SD}^-, \omega) = S(K=0, \omega) + 2 \times S(K=1, \omega) \\ + 2 \times S(K=2, \omega) \quad (74)$$

for each multipole and different nuclear shapes, defined as $E_{\text{cent.}} = m_1/m_0$, where $m_k = \int d\omega \omega^k S(\text{SD}^-, \omega)$ is k th moment of the strength distribution.

$$E_{\text{cent. .}}(2^-) < E_{\text{cent. .}}(1^-) < E_{\text{cent. .}}(0^-)$$

	^{58}Fe			^{60}Fe			^{62}Fe		
	oblate	spherical	prolate	oblate	spherical	prolate	oblate	spherical	prolate
0^-	28.333	28.445	28.368	26.407	26.856	26.538	25.098	25.398	25.133
1^-	27.074	26.855	26.972	25.203	25.414	25.194	24.135	24.089	24.112
2^-	22.039	22.374	21.970	20.268	20.759	20.269	19.117	19.351	19.111

Spin-dipole Resonance

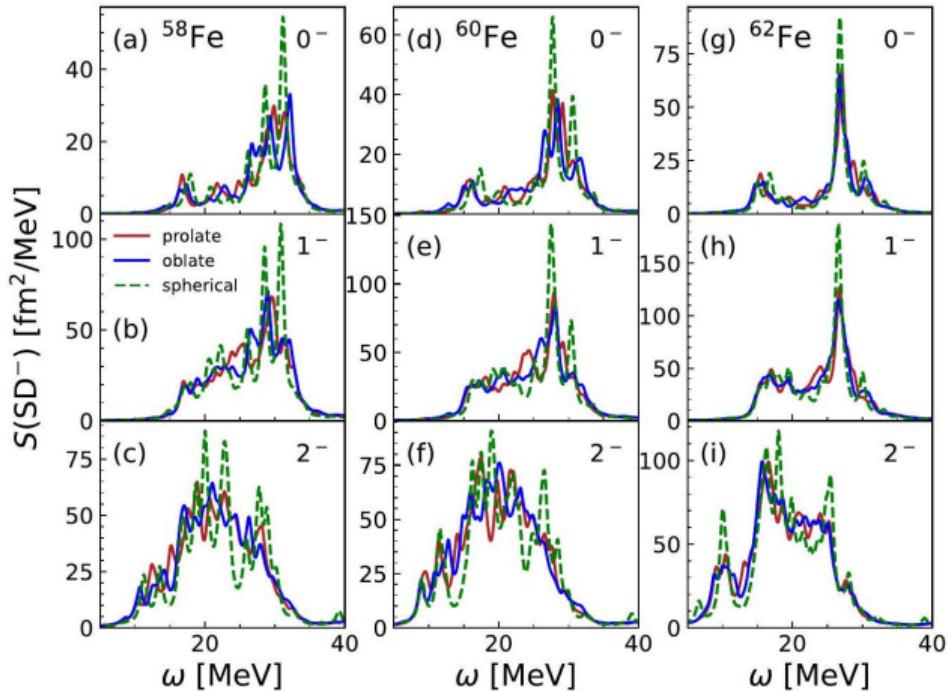


Figure: 9.the total SD^- strength function in $^{58,60,62}\text{Fe}$ isotopes for $J^\pi = 0^-, 1^-,$ and 2^- modes

Spin-dipole Resonance

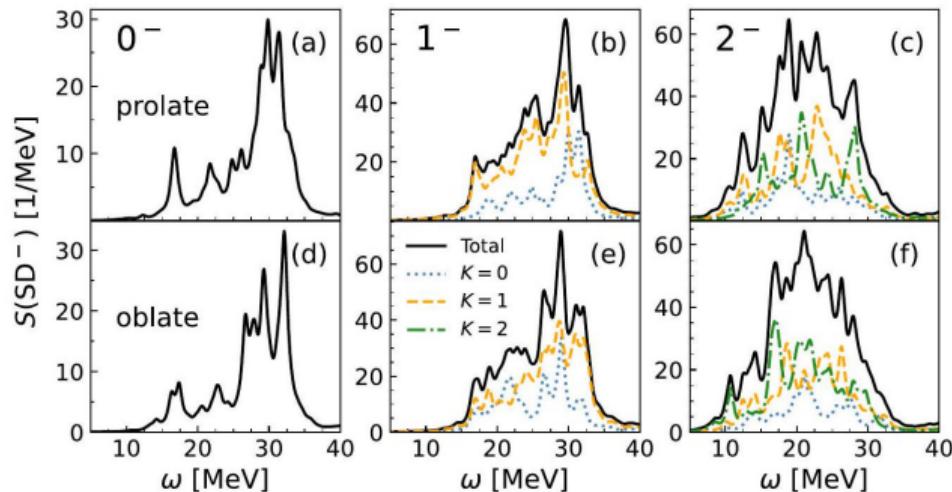


Figure: 10. The SD^- strength function for ^{58}Fe decomposed to different K modes

The splitting between the $K = 0$ and $K = 1$ modes depend on the nuclear shape.

We have developed an axially deformed pnRQRPA based on covariant energy density functional theory. The solver utilizes point-coupling EDFs with separable pairing interaction for which the residual interaction Hamiltonian can be written as a sum of products of separable terms.

- the deformation effects do not contribute to the Fermi transitions because the quadrupole deformation leads to second-order correction to the Coulomb energy which determines the position of the IAS centroid.
- For the GT strength, we show that deformation effects lead to splitting between the $K = 0$ and $K = 1$ modes, resulting in pronounced fragmentation of the strength function. the direction of the $K = 0$ and $K = 1$ splitting in deformed nuclei is proportional to the strength of quadrupole deformation $|\beta_2|$, while its direction depends on the sign of β_2 .
- The SD transitions show a complicated pattern. The fragmentation of strength for deformed shapes is still pronounced, especially for 0^- and 1^- modes. Due to the less restrictive selection rule for 2^- transitions, already the spherical strength shows a complicated structure without clear resonance peaks, which is again more fragmented for deformed nuclei.



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Thanks for everyone!