理论物理系列学术报告:第6期

中山大學 物理与天文学院
SUN YAT-SEN UNIVERSITY SCHOOL OF PHYSICS AND ASTRONOMY

Geometric Structure of Carbon-12 and Beryllium Isotopes Using Nuclear Lattice Effective Field Theory

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Beihang University

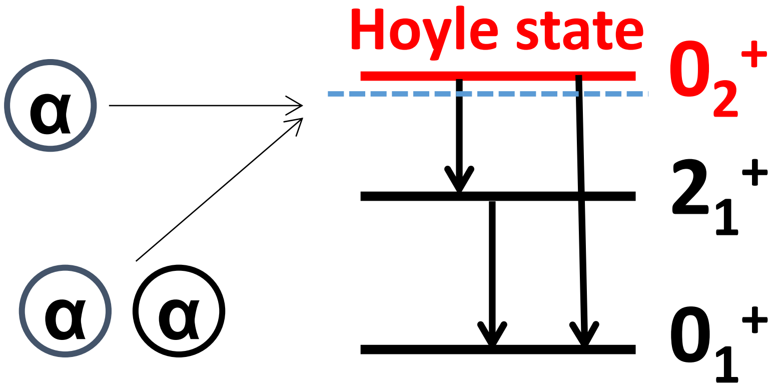
Collaborators: Serdar Elhatisari, Timo A. Lähde, Dean Lee, Bing-Nan Lu, and Ulf-G. Meißner

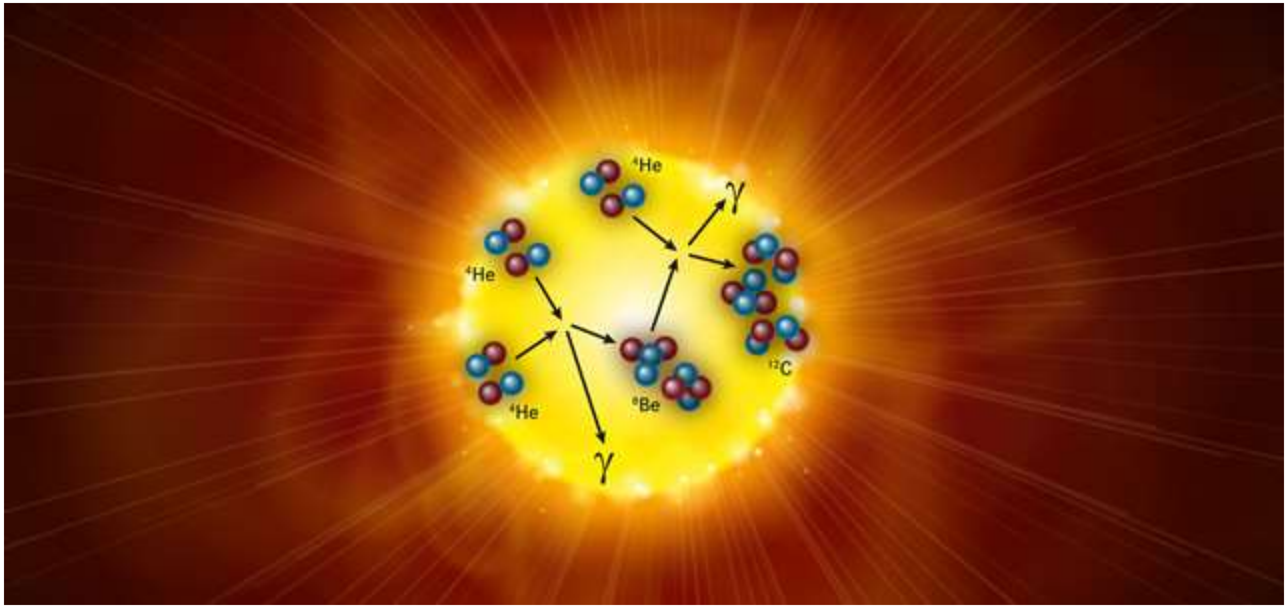
European Research Council
Established by the European Commission

Carbon-12

Life element: ^{12}C

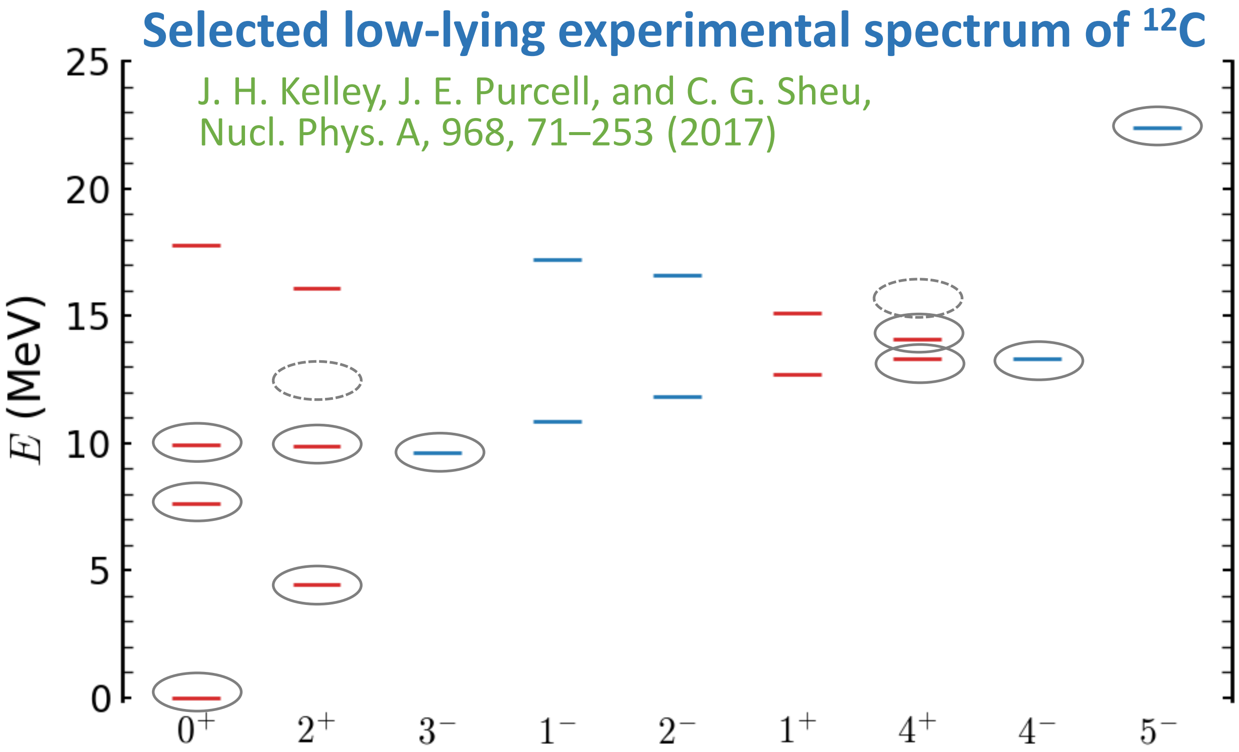
F. Hoyle, Astrophys. J. Suppl. Ser. 1, 121 (1954)

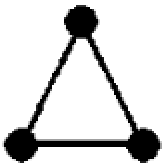




https://www.quantamagazine.org/the-physics-behind-the-elements-of-life-20121204/

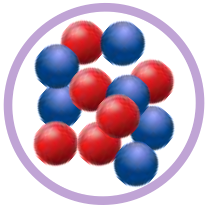
Low-Lying Levels in Carbon-12



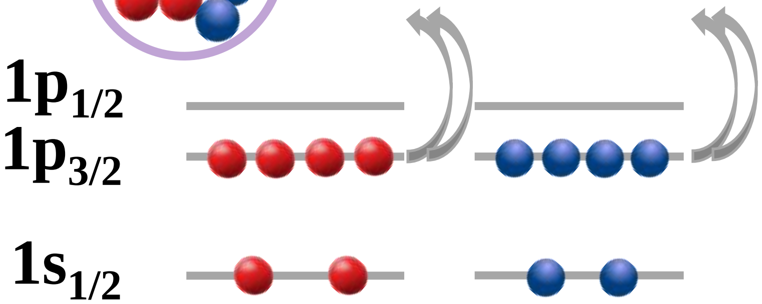


- Rotation of equilateral triangle: 0^+, 2^+, 3^-, $4^\pm$, and 5^- states (ground state band)
  R. Bijker and F. Iachello, Phys. Rev. C, 61, 067305 (2000)
- Shape of Hoyle state and Hoyle band
  M. Freer and H. O. U. Fynbo, Prog. Part. Nucl. Phys., 78, 1–23 (2014)
- Breathing modes of Hoyle state and its rotational excitations
  K. C. W. Li et al., Phys. Lett. B, 827, 136928 (2022); Z. Cheng et al. arXiv: 2406.15060
- … …

Structure and Shape

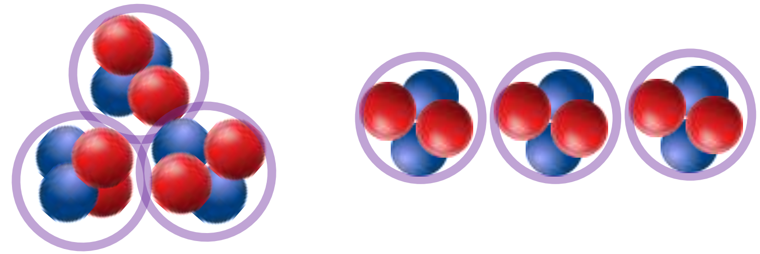


Shell model



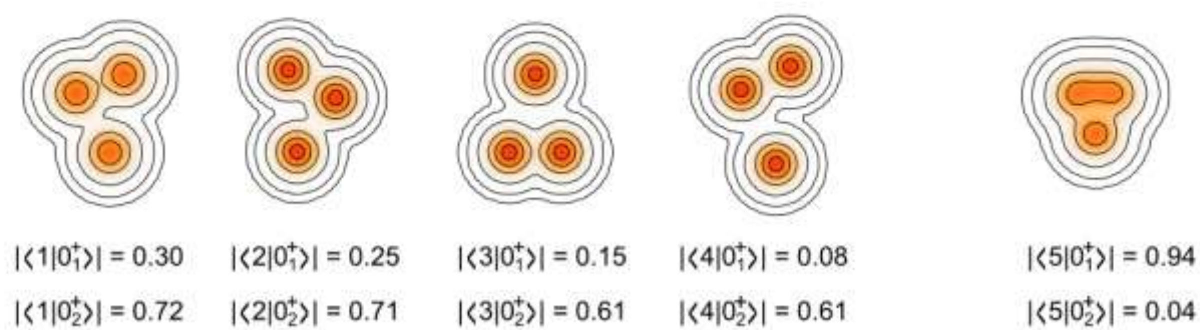
?

Cluster model



Hoyle State, debate M. Freer and H.O.U. Fynbo., Prog. Part. Nucl. Phys. 78, 1 (2014)

- Linear arrangement of 3α-particles H. Morinaga, Phys. Rev. 101, 254 (1956)
- Vibrational excitation, triangular symmetry R. Bijker and F. Iachello, PRC, 61, 067305 (2000)
- Bose Einstein Condensate of α-particles A. Tohsaki, et al., Phys. Rev. Lett. 87, 192501 (2001)
- Cluster-gas close to an equilateral triangle Y. Kanada-En'yo, Prog. Theor. Phys. 117, 655 (2007)
- Superposition of 3α arrangements M. Chernykh, et al., Phys. Rev. Lett., 98, 032501 (2007)
- … …

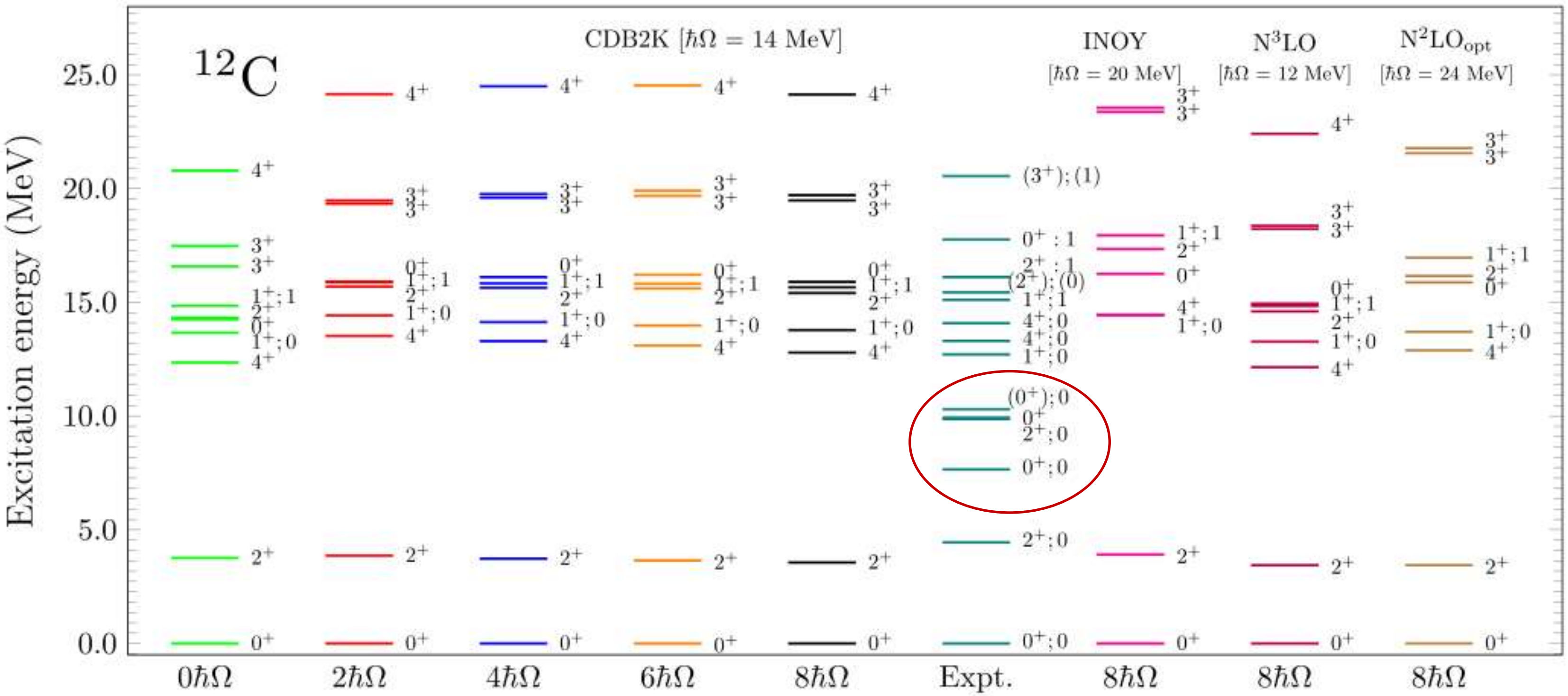


Ab initio Study

- Hoyle state is a challenge for ab initio method

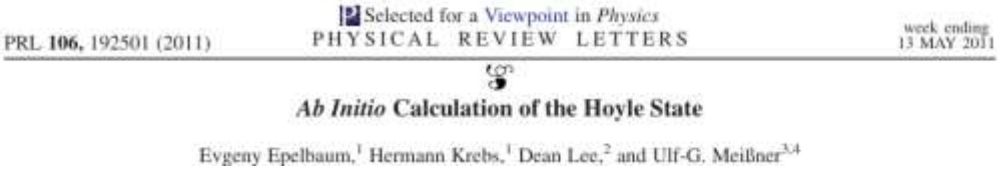
No-core shell model

P. Choudhary et al., Phys. Rev. C, 107, 014309 (2023)



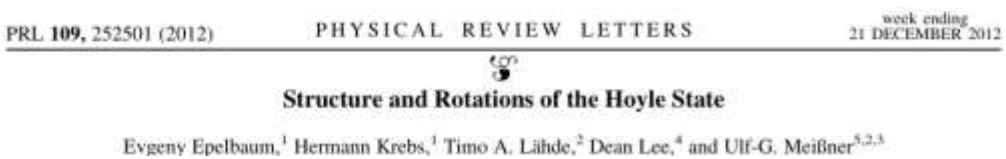
Ab initio Study

- **First successful ab initio description**: nuclear lattice effective field theory (NLEFT)


Selected for a Viewpoint in *Physics*
PRL **106,** 192501 (2011) PHYSICAL REVIEW LETTERS week ending 13 MAY 2011

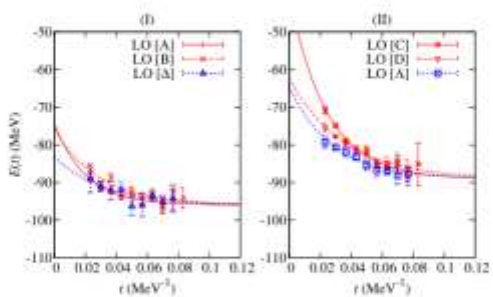
Ab Initio Calculation of the Hoyle State

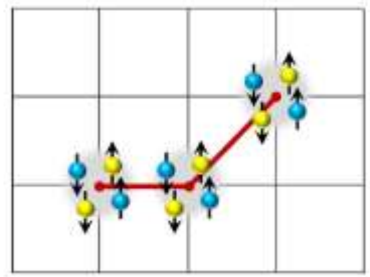
Evgeny Epelbaum,[1] Hermann Krebs,[1] Dean Lee,[2] and Ulf-G. Meißner[3,4]


PRL **109,** 252501 (2012) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2012

Structure and Rotations of the Hoyle State

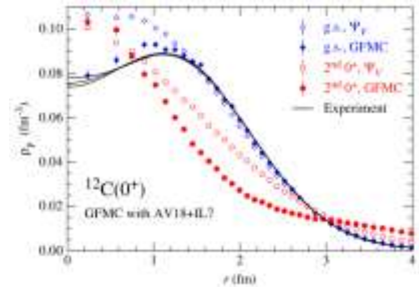
Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]





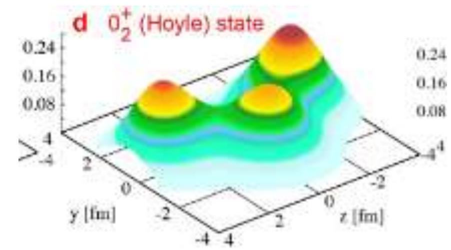
- Green's function Monte Carlo

J. Carlson, et al., Rev. Mod. Phys., 87, 1067 (2015)



- Monte Carlo Shell Model

T. Otsuka et al., Nature Commun., 13, 2234 (2022)



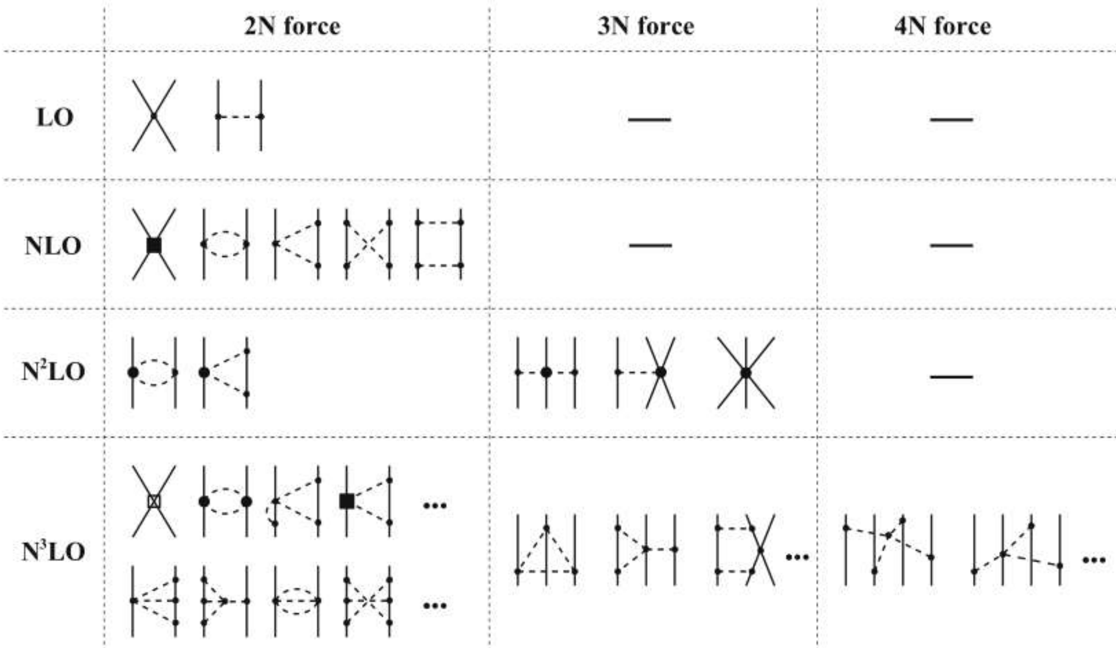
In this work

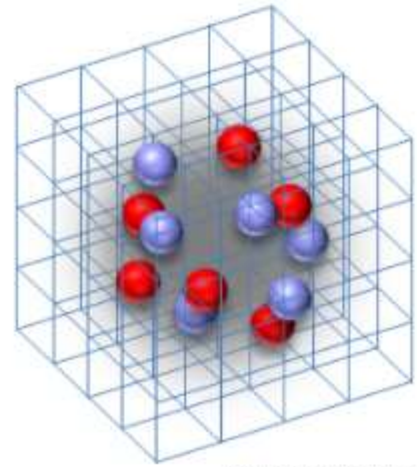
Study low-lying structure of ^{12}C in NLEFT

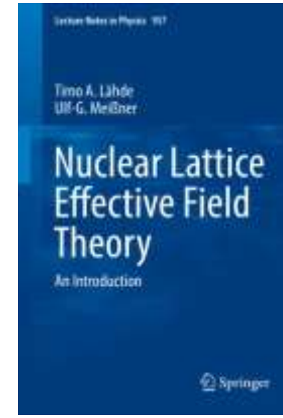
- Make use of SU(4) symmetry to alleviate sign problem
- To give a full description of all the low-lying states, both cluster and shell-model type
- To propose new method to study the geometric structure (using pinhole algorithm)
- To propose geometric description to quantify different excitation mode

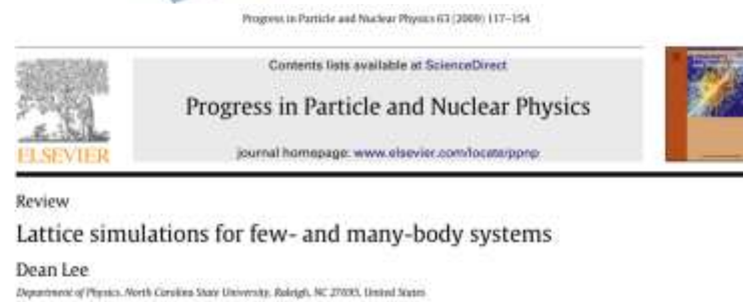
Theoretical Framework

- Nuclear lattice effective field theory (NLEFT)








Progress in Particle and Nuclear Physics 63 (2009) 117–154

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Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

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Review

Lattice simulations for few- and many-body systems

Dean Lee

Department of Physics, North Carolina State University, Raleigh, NC 27695, United States

- 16O, E. Epelbaum et al., PRL 112, 102501 (2014)
- α-α scattering, S. Elhatisari et al., Nature 528, 111 (2015)
- thermodynamics, B.-N. Lu et al., PRL 125, 192502 (2020)
- … …

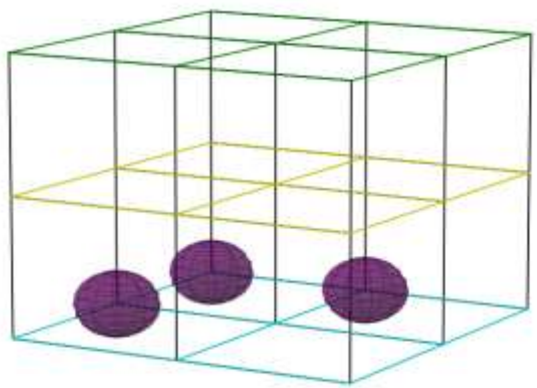
Upper left figure is in courtesy of E. Epelbaum
lattice figure from https://www.physics.ncsu.edu/ntg/leegroup/research.html

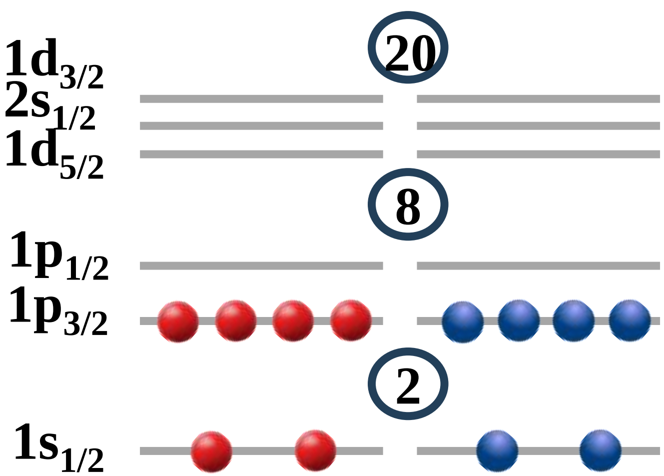
Theoretical Framework

- Starting from an initial many-body wave function:

$$|\Phi_0\rangle = \mathscr{A}\left[\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)\ldots\phi_A(\mathbf{r}_A)\right]$$

$$\phi(\mathbf{r}) = \exp\left(-(\mathbf{r}-\mathbf{r}_0)^2/2w^2\right)$$



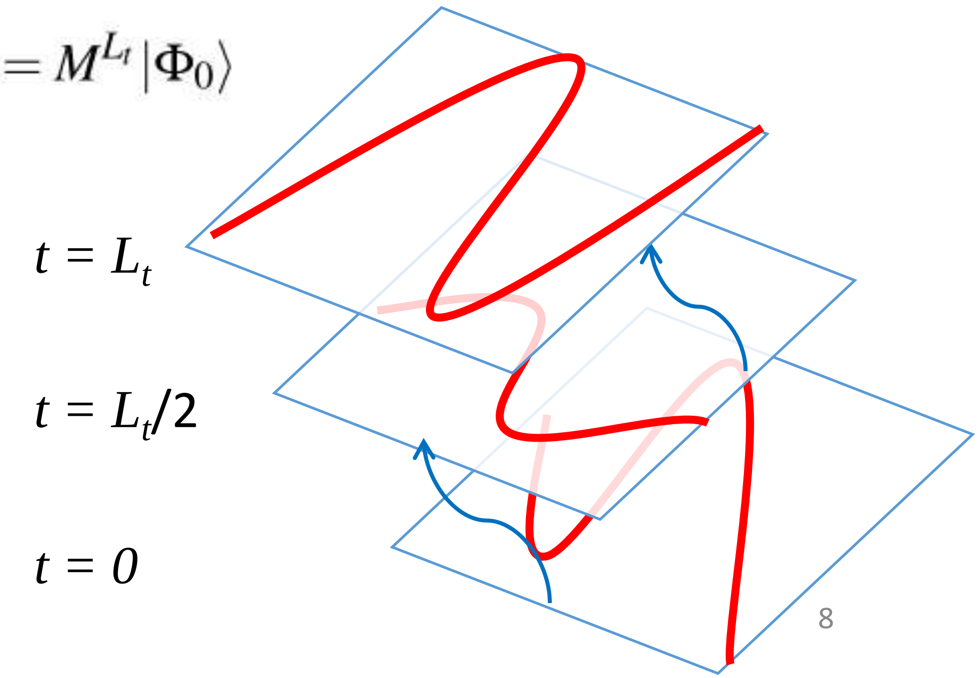


- Euclidean time projection with transfer matrix:

$$M =: \exp(-\alpha_t H) : \qquad \alpha_t = a_t/a$$

with H the many-body Hamiltonian, a_t and a the temporal and spatial lattice spacing.

$$|\Phi_{L_t}\rangle = M^{L_t}|\Phi_0\rangle$$



Theoretical Framework

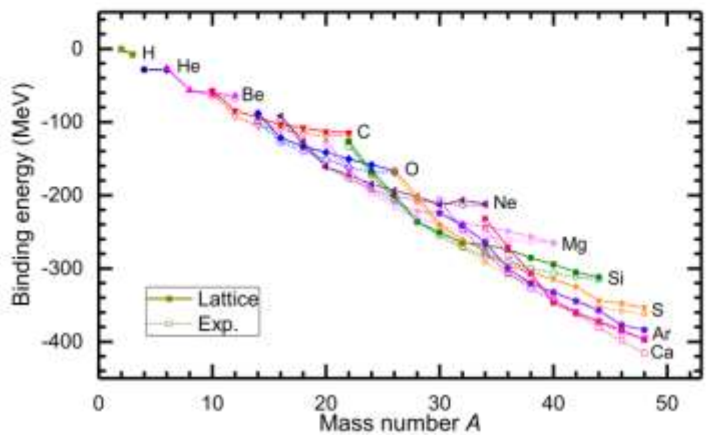
- Wigner SU(4) symmetric interaction (spin and isospin independent):

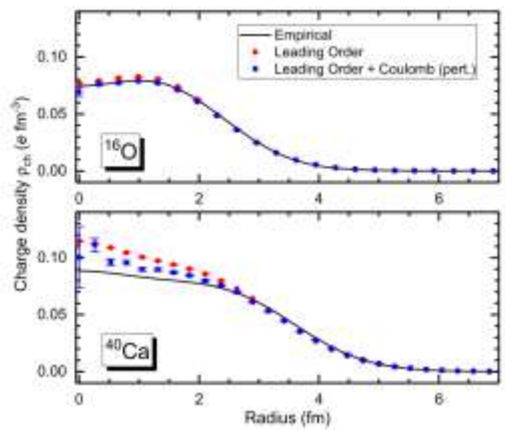
S. Elhatisari et al., PRL 119, 222505 (2017)

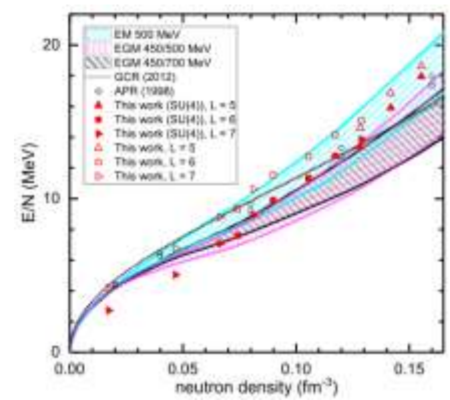
$$V = \frac{C_2}{2!}\sum_{\mathbf{n}}\tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!}\sum_{\mathbf{n}}\tilde{\rho}(\mathbf{n})^3,$$

$$\tilde{\rho}(\mathbf{n}) = \sum_{i=1}^{A}\tilde{a}_i^\dagger(\mathbf{n})\tilde{a}_i(\mathbf{n}) + s_L \sum_{|\mathbf{n}'-\mathbf{n}|=1}\sum_{i=1}^{A}\tilde{a}_i^\dagger(\mathbf{n}')\tilde{a}_i(\mathbf{n}'), \qquad \tilde{a}_i(\mathbf{n}) = a_i(\mathbf{n}) + s_{\mathrm{NL}}\sum_{|\mathbf{n}'-\mathbf{n}|=1} a_i(\mathbf{n}').$$

- Sign problem is largely suppressed J.W. Chen, D. Lee, T. Schäfer, PRL, 93, 242302 (2004)
- Ground state properties of light and medium mass nuclei, and neutron matter can be well reproduced B.-N. Luu et al., PLB 797 (2019) 134863







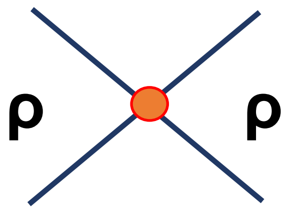
- Higher order (e.g. N3LO) can be built on with perturbation

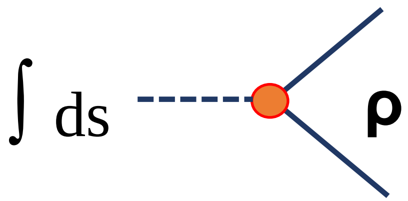
Elhatisari et al., Nature 630, 59 (2024)

Theoretical Framework

- Auxilary field with Monte-Carlo sampling

$$\exp\left(-\frac{C\alpha_t}{2}\rho^2\right) := \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds : \exp\left(-\frac{1}{2}s^2 + \sqrt{-C\alpha_t}\,s\rho\right) :$$





- Final states are a superposition of millions of configurations (Slater determinants)

$$|\Phi_{L_t}\rangle = \sum_{s_i} |\Phi_{s_i,L_t}\rangle$$

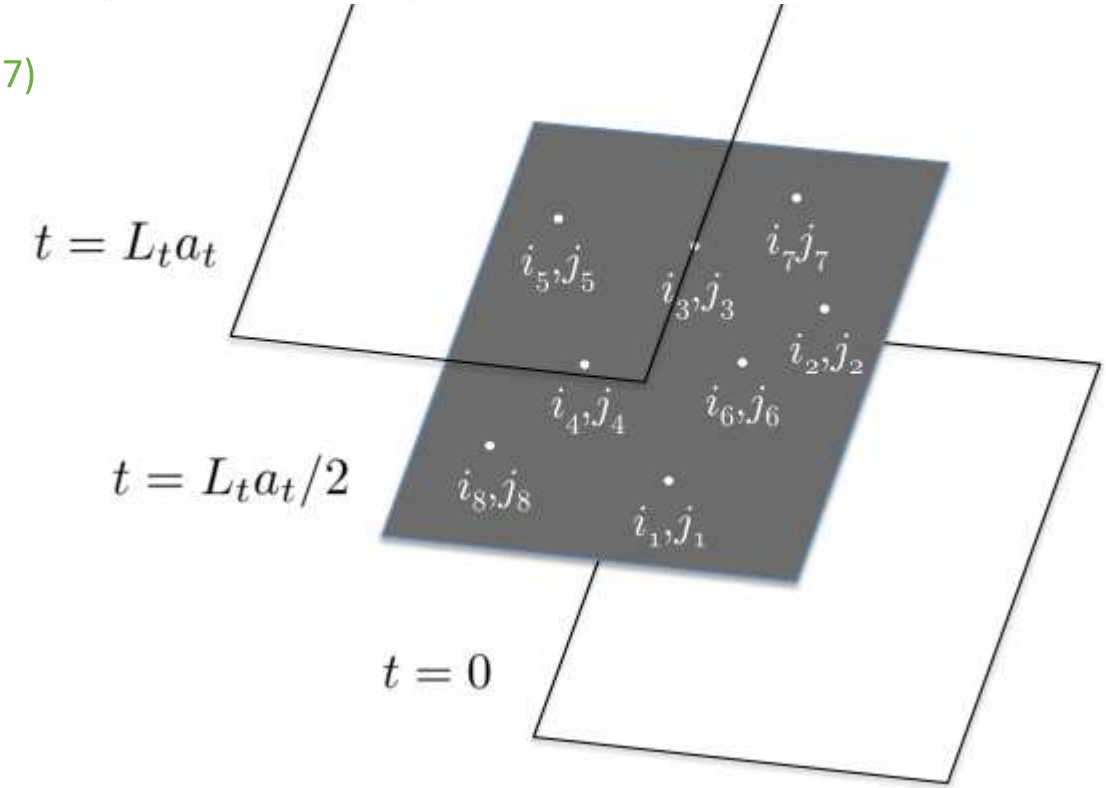
$$|\Phi_{s_i,L_t}\rangle = M_{s_i}^{L_t} |\Phi_0\rangle = \mathscr{A}\left[\phi_{s_i,1}(\mathbf{r}_1)\phi_{s_i,2}(\mathbf{r}_2)\ldots\phi_{s_i,A}(\mathbf{r}_A)\right]$$

Theoretical Framework

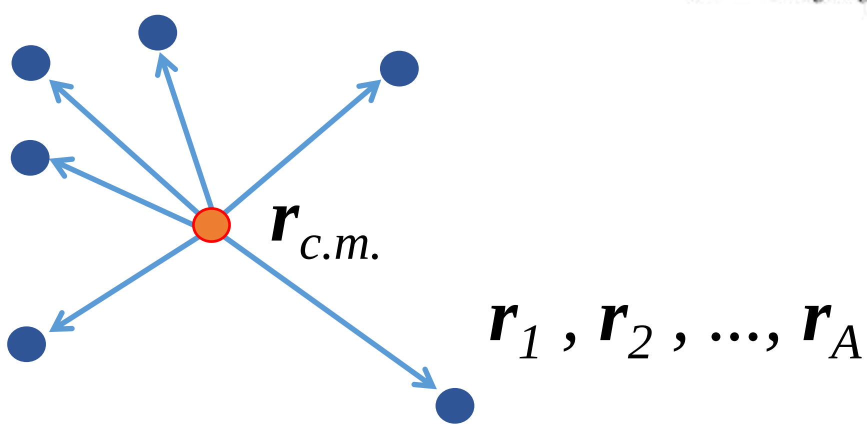
- Pinhole algorithm

S. Elhatisari et al., PRL 119, 222505 (2017)

$$\langle \Phi_0 | e^{-H\tau/2} \rho(\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_A) e^{-H\tau/2} | \Phi_0 \rangle$$



A time slice is inserted to sample the positions and spin-isospin indices in the middle time step.



Those millions of A-body positions contains all correlation information and provides a powerful tool to study the structure and geometry of nuclei

Numerical Details

- Lattice length L = 14.8 fm with spacing a = 1.64 fm ($\pi/a \sim 378$ MeV)
  Temporal lattice spacing a_t = 0.55 fm/c.

- Fitted results for SU(4) interaction

C_2 [MeV^{-2}]	C_3 [MeV^{-5}]	s_L	s_{NL}
-2.15×10^{-5}	6.17×10^{-12}	0.08	0.05

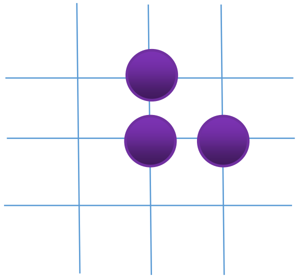
	NLEFT	Exp.
E(^{4}He) [MeV]	-28.1 (1)	-28.3
E(^{12}C) [MeV]	-91.6 (1)	-92.2
r_c(^{12}C) [fm]	2.52 (1)	2.47 (2)

And to some extent transition properties.

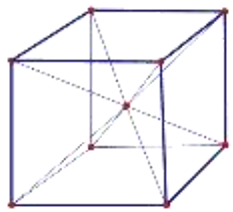
S. Shen et al., Nature Commun. (2023) 14:2777
S. Shen et al., EPJA, 57, 276 (2021)

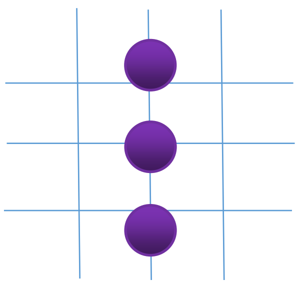
Calculation of the Hoyle State

- Hoyle state

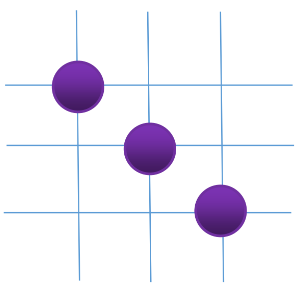


Angular momentum projection: SO(3) group reducted to cubic group O

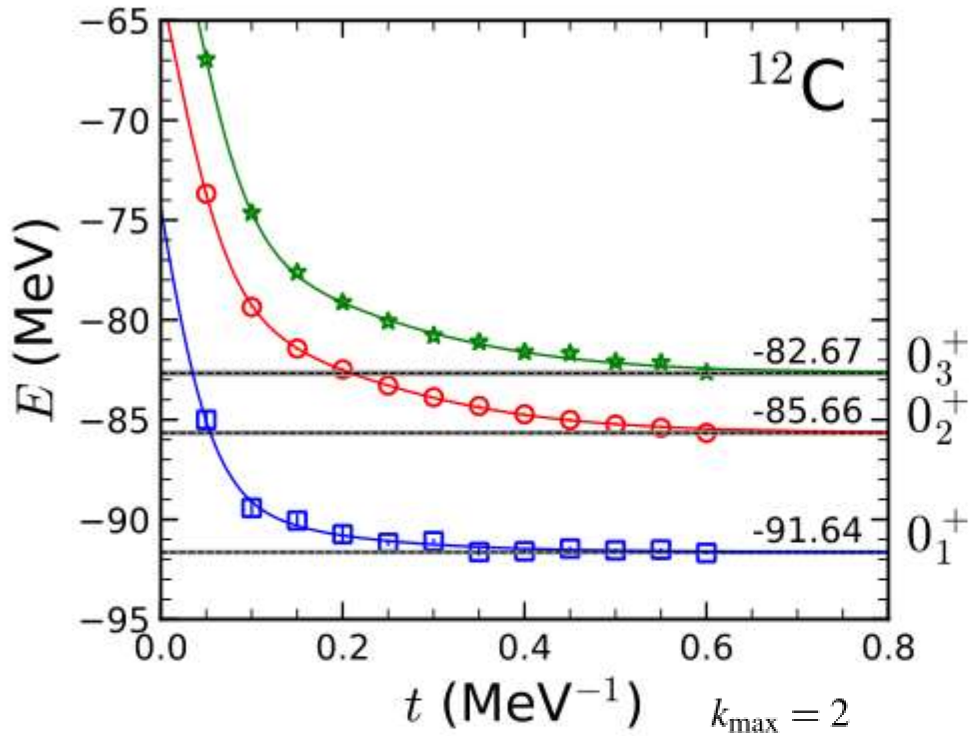




J	irrepresentation
0	A_1
1	T_1
2	$E + T_2$
3	$A_2 + T_1 + T_2$
4	$A_1 + E + T_1 + T_2$



$$\phi(\mathbf{r}) = \exp\left(-(\mathbf{r}-\mathbf{r}_0)^2/2w^2\right)$$



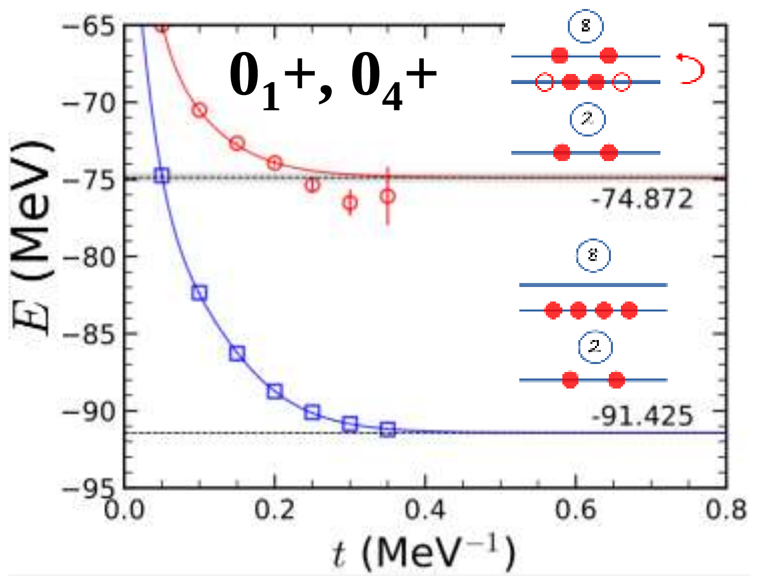
$k_{\max} = 2$

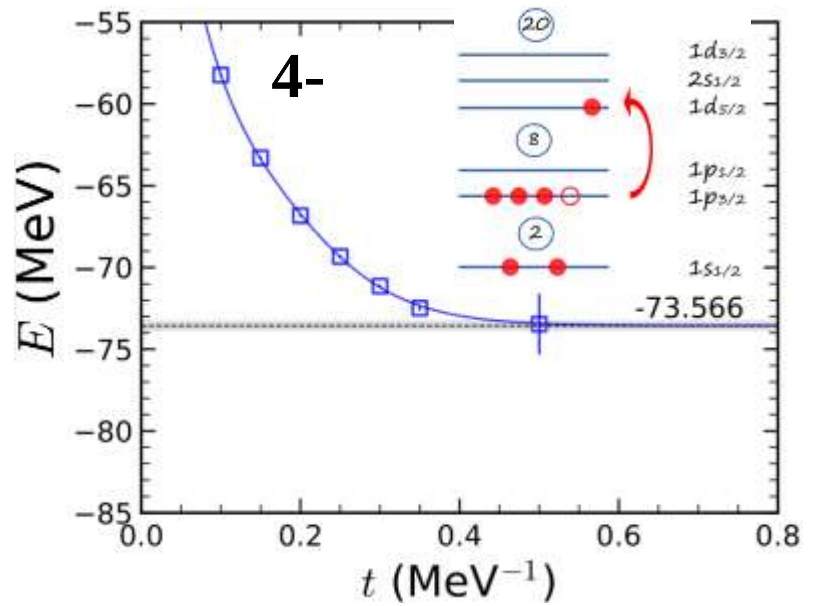
$$E_i(t) = \frac{E_i + \sum_{k=1}^{k_{\max}} (E_i + \Delta E_{i,k}) c_{i,k} e^{-\Delta E_{i,k} t}}{1 + \sum_{k=1}^{k_{\max}} c_{i,k} e^{-\Delta E_{i,k} t}}$$

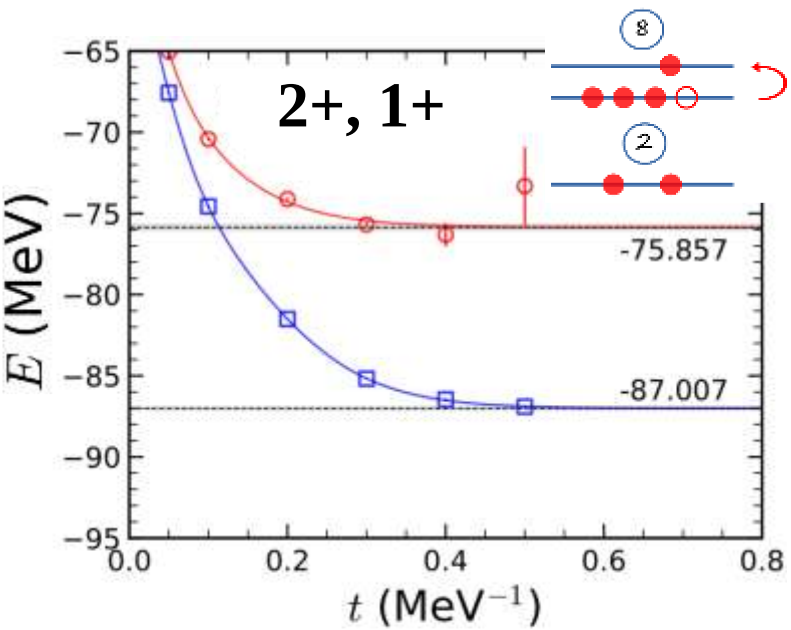
T. A. Lähde et al., JPG 42 (2015) 034012

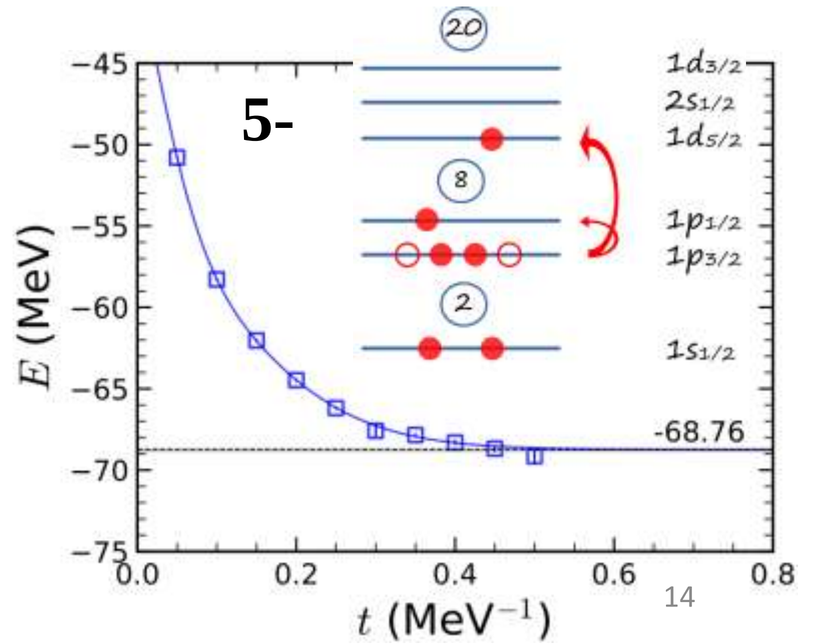
Web figures from: https://en.wikipedia.org/wiki/Sphere
https://math.ucr.edu/home/baez/icosidodecahedron/7.html

Shell-Model States Used as Initial Wave



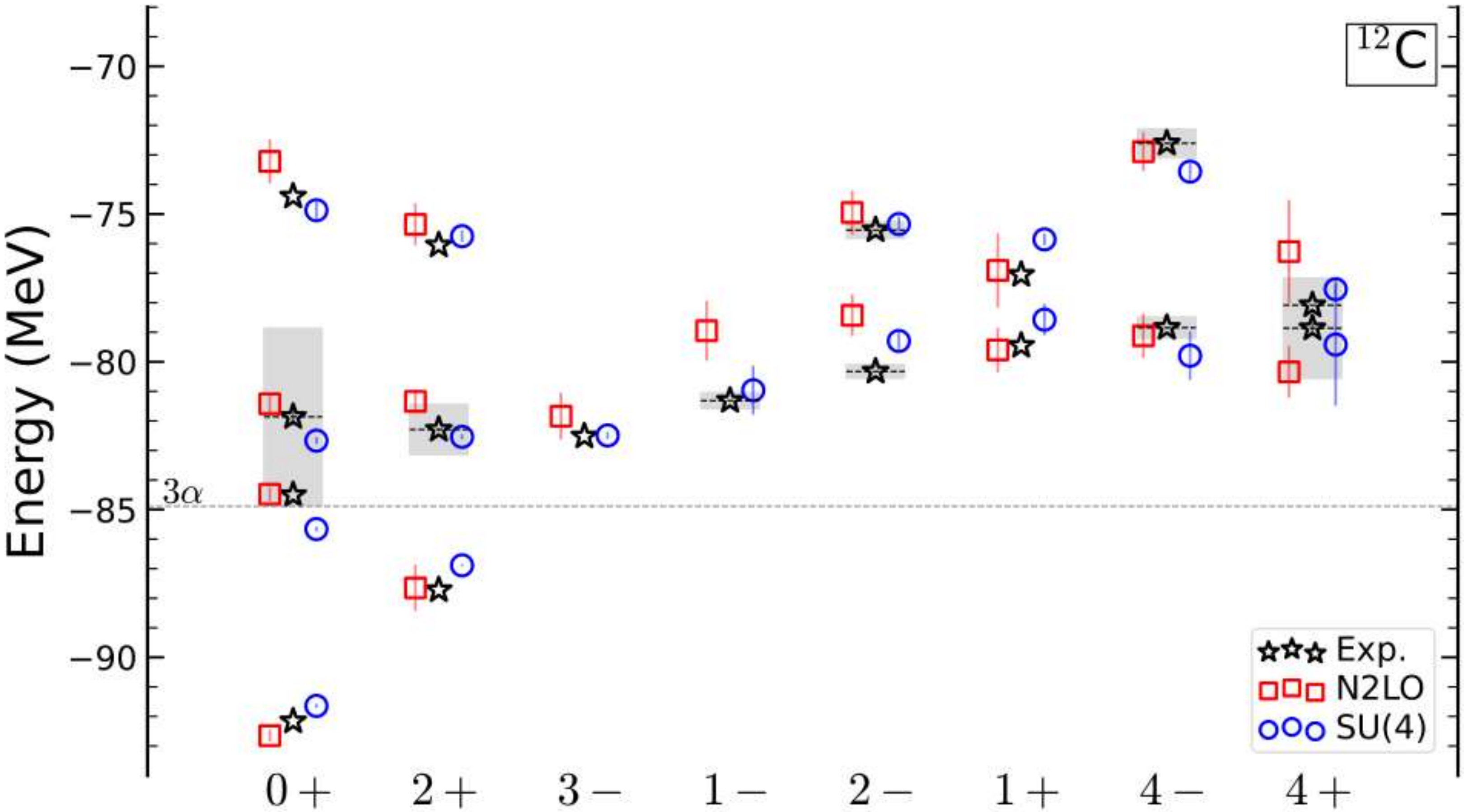






Low-lying Spectrum

- Spectrum of ^{12}C calculated by NLEFT using N2LO and SU(4) interaction in comparison with experimental data.



S. Shen et al., Nature Commun. (2023) 14:2777

Electromagnetic Properties

- Quadrupole moment and transition rates of ^{12}C calculated by NLEFT, comparing with other theoretical calculations and Experiments. Units for Q and $M(E0)$ are e fm^2 and for $B(E2)$ e^2 fm^4.

	NLEFT	FMD	α cluster	NCSM	GCM	Exp.
$Q(2_1^+)$	6.8(3)(1.2)	–	–	6.3(3)	–	8.1(2.3)
$Q(2_2^+)$	−35(1)(1)	–	–	–	–	–
$M(E0, 0_1^+ \to 0_2^+)$	4.8(3)	6.5	6.5	–	6.2	5.4(2)
$M(E0, 0_1^+ \to 0_3^+)$	0.4(3)	–	–	–	3.6	–
$M(E0, 0_2^+ \to 0_3^+)$	7.4(4)	–	–	–	47.0	–
$B(E2, 2_1^+ \to 0_1^+)$	11.4(1)(4.3)	8.7	9.2	8.7(9)	–	7.9(4)
$B(E2, 2_1^+ \to 0_2^+)$	2.4(2)(7)	3.8	0.8	–	–	2.6(4)

Future Experiments can be used as a test.

fermion molecular dynamics (FMD) M. Chernykh et al., PRL 98, 032501 (2007)

α cluster M. Chernykh et al., PRL 98, 032501 (2007)

BEC Y. Funaki et al., PRC 67, 051306 (2003); EPJA 24, 321 (2005)

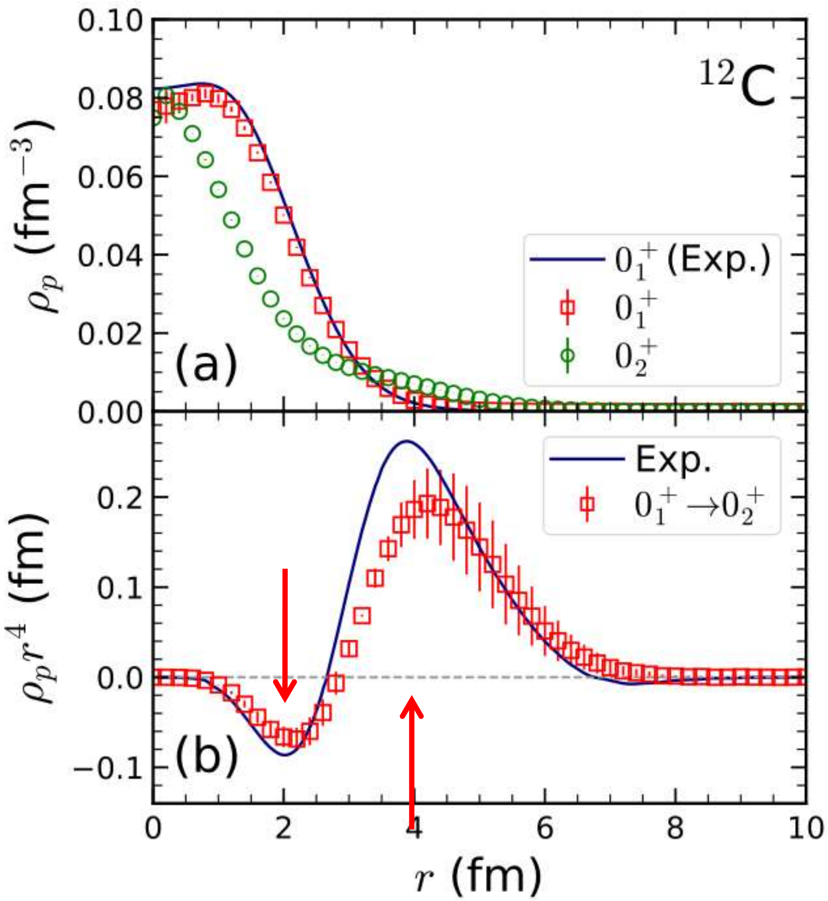
in-medium no-core shell model (NCSM) A. D'Alessio et al., PRC 102, 011302 (2020)

generator coordinate method (GCM) B. Zhou, PRC 94, 044319 (2016)

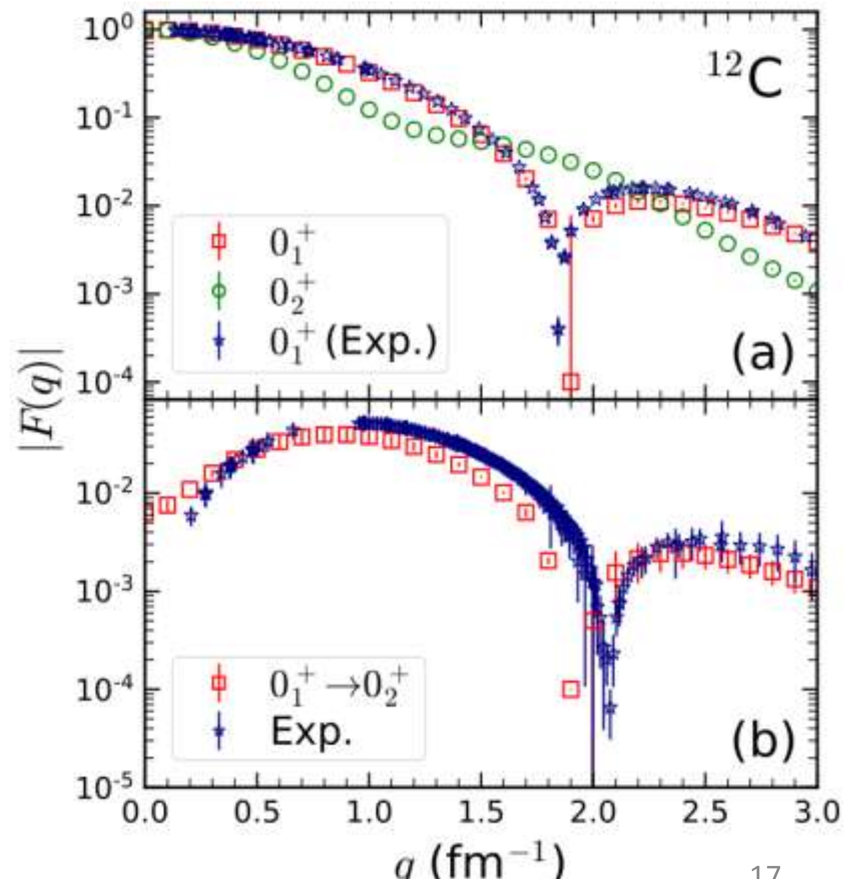
Exp. F. Ajzenberg-Selove, NPA 506, 1 (1990); J. Saiz Lomas, PhD thesis, University of York, UK (2021)

Density Profiles

- Charge density distributions (left) and form factors (right) of ground state, Hoyle state, and transitions between them.



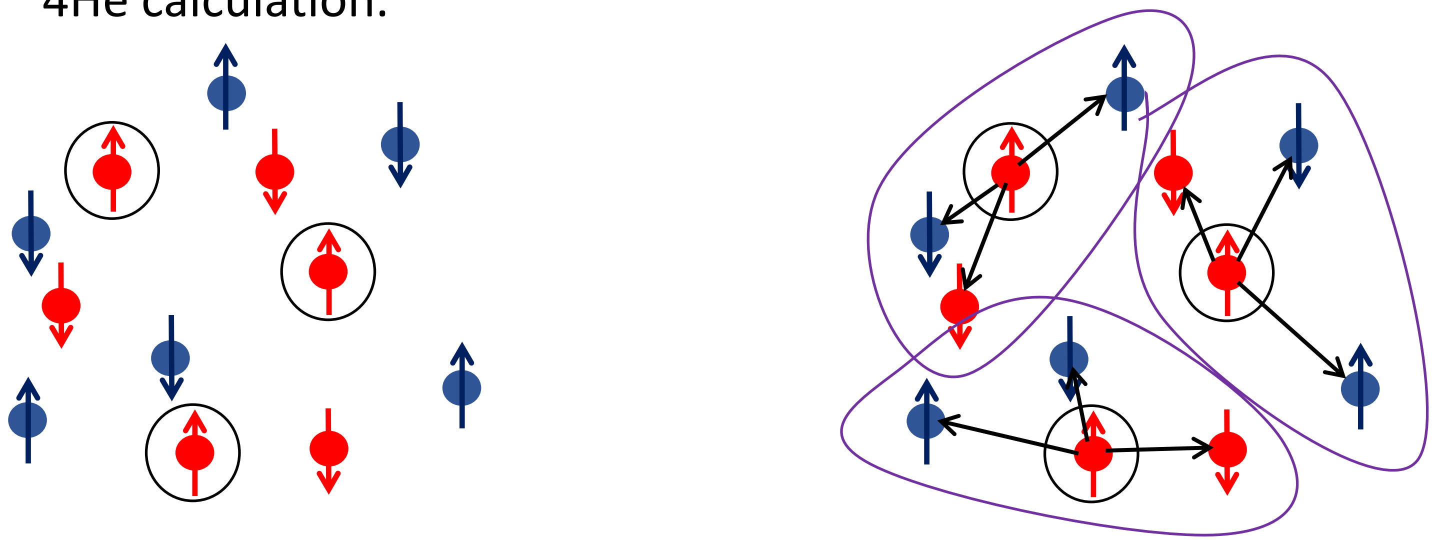
$$F(q) = \frac{4\pi}{Z} \int dr r^2 \rho_p(r) j_0(qr)$$



Exp. M. Chernykh et al., PRL 105, 022501 (2010)
I. Sick and J. S. Mccarthy, NPA 150, 631 (1970)
P. Strehl, Z. Phys. 234 (1970) 416; H. Crannell et al., NPA 758, 399 (2005)

Investigation of the Geometry

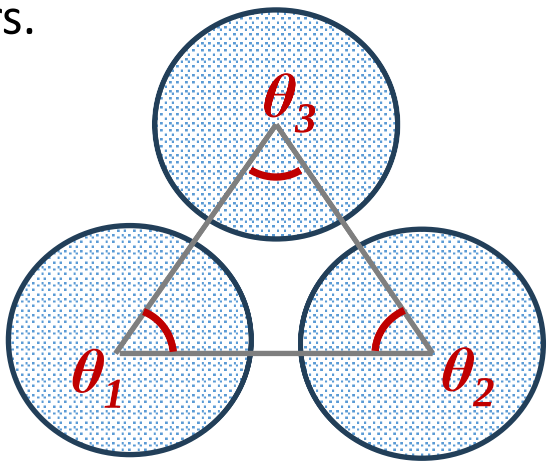
- Define (α) clusters
  1. Identify 3 spin-up protons;
  2. Find the closest possible of the other 3 types of particles (spin-down proton, spin-up neutron, spin-down neutron);
  3. Calculate the rms radius of α cluster defined this way and compare with 4He calculation.

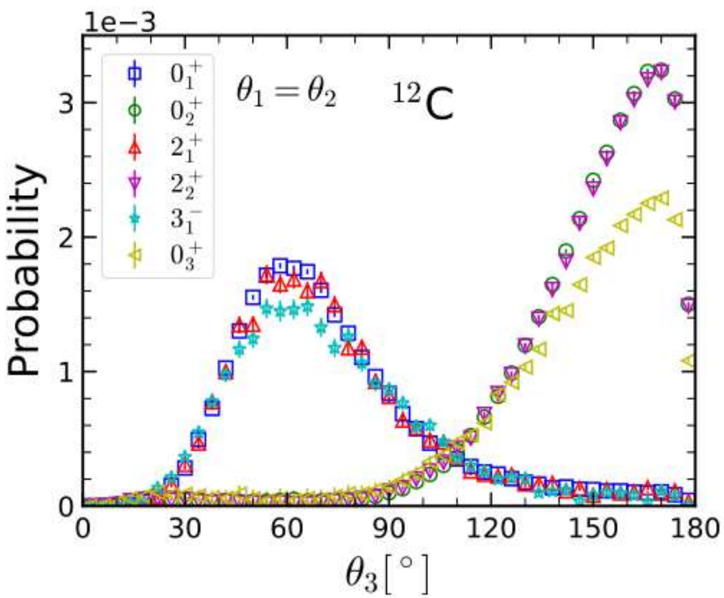


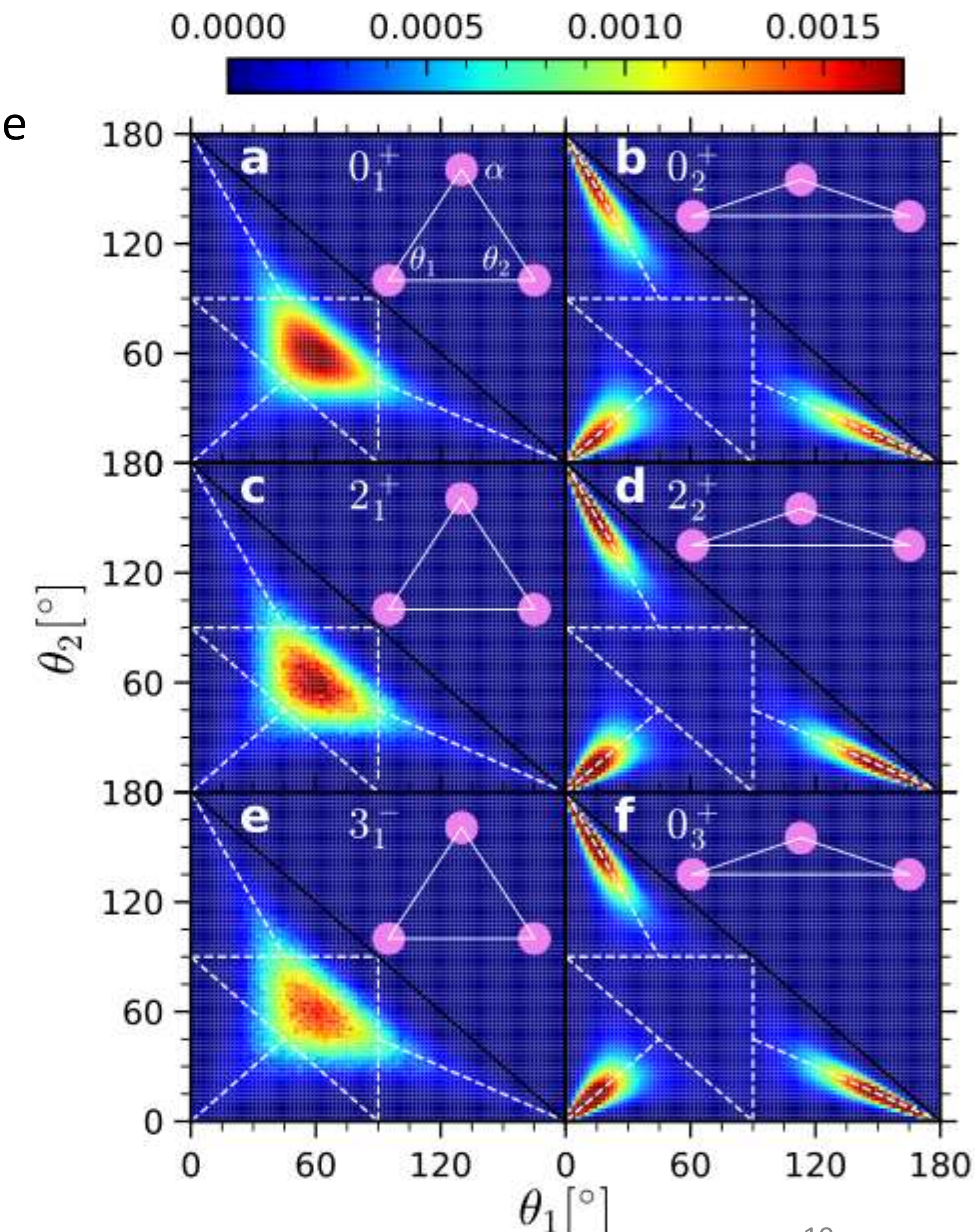
	12C, 0_1+	12C, 0_2+	4He
rms α cluster [fm]	1.65	1.71	1.63

Distribution of Angles

Probability distribution for the two inner angles of the triangle formed by the three α clusters.







Intrinsic Density Distribution

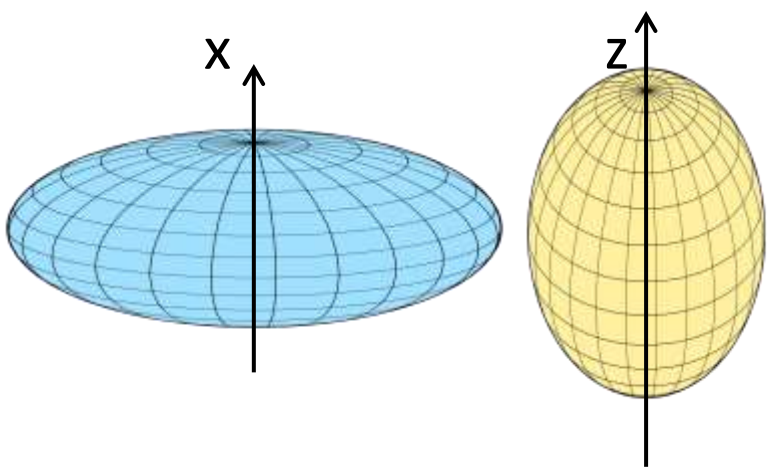
- Alignment of configurations:

For equilateral triangle type:

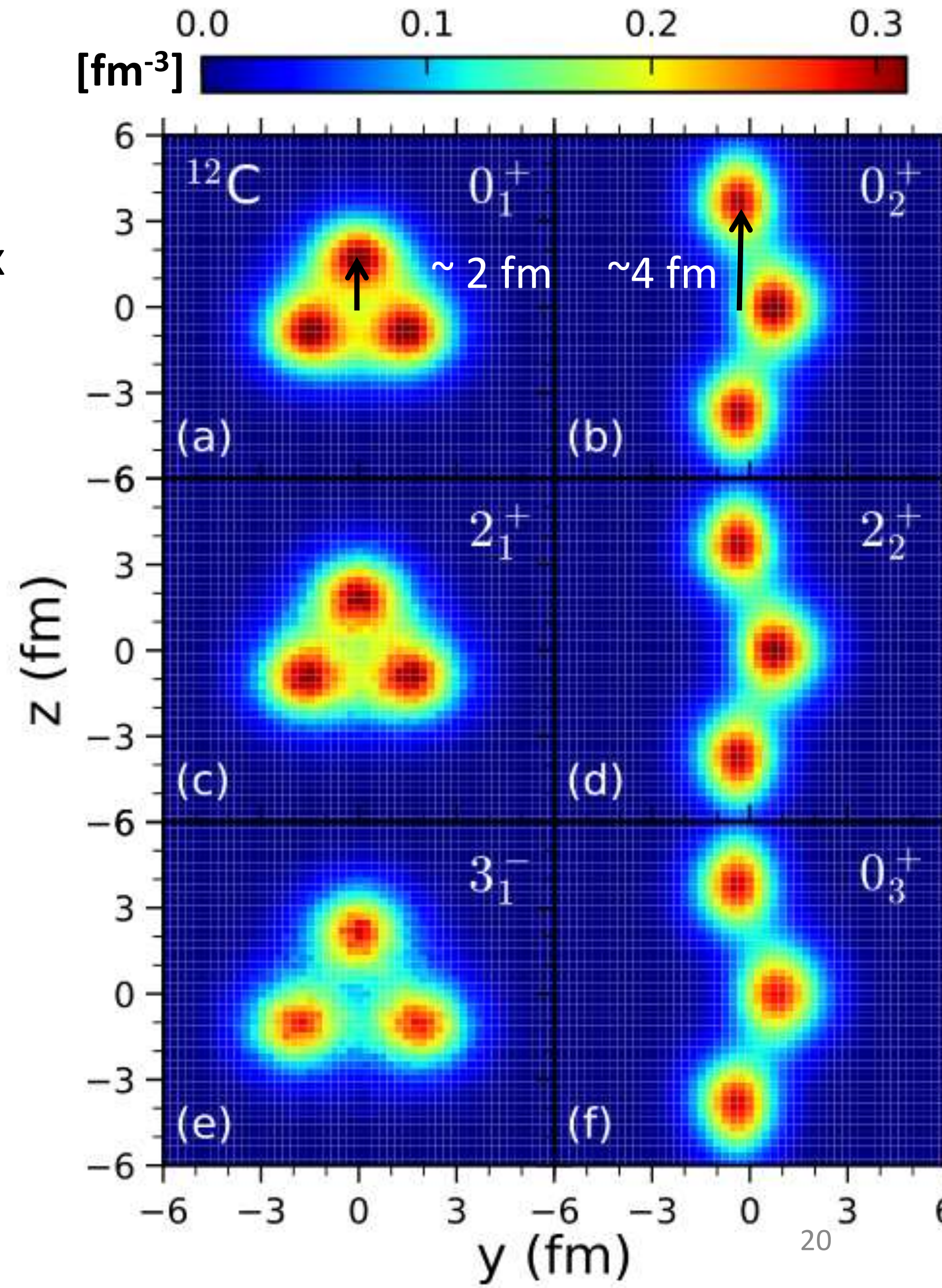
1. Align shortest principal axis to x
2. Rotate 1 α to y = 0 (positive z), and (randomly) +/- 120°.

For obtuse triangle type:

1. Align longest principal axis to z;
2. Rotate central α to x = 0 (positive y).

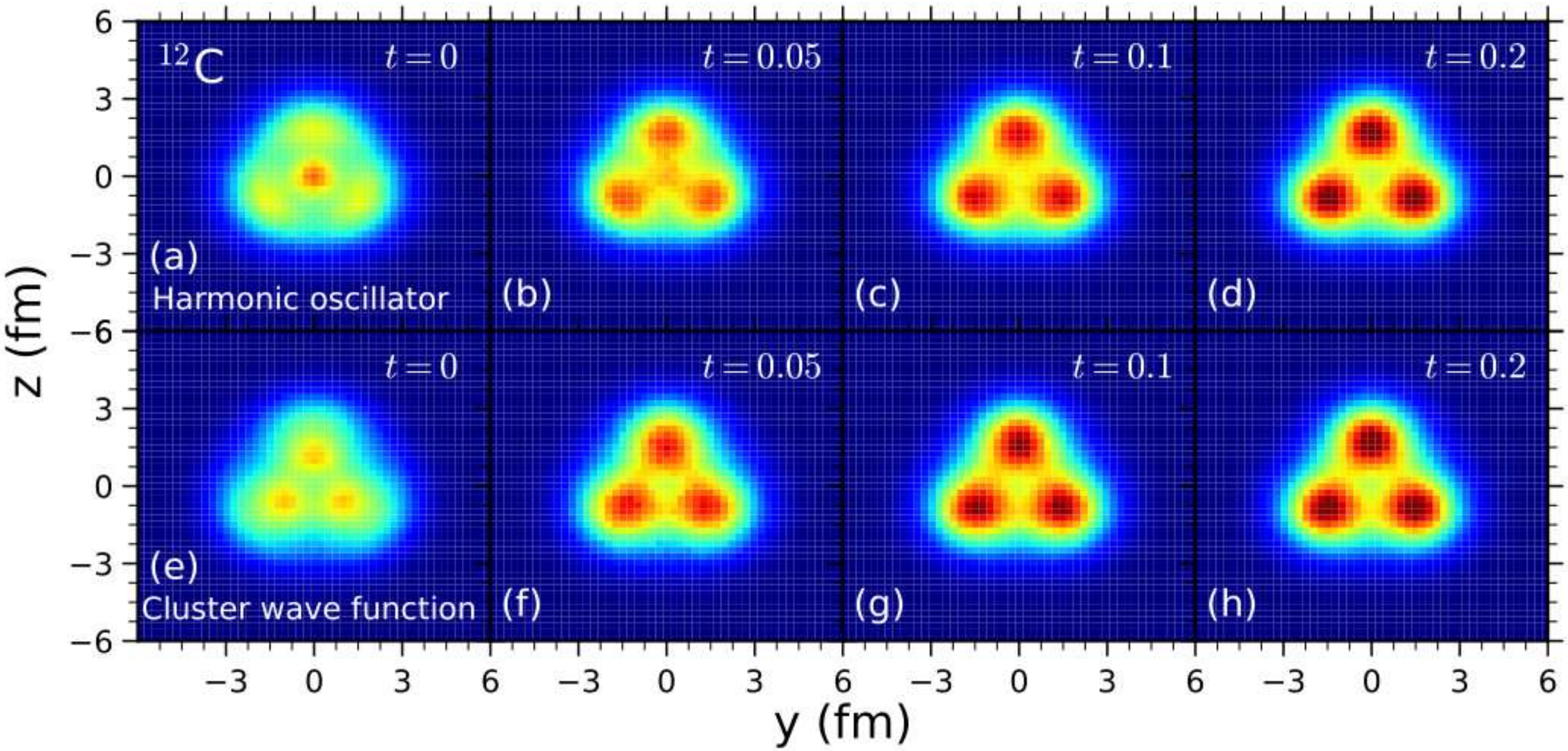


Web figure from: https://en.wikipedia.org/wiki/Spheroid



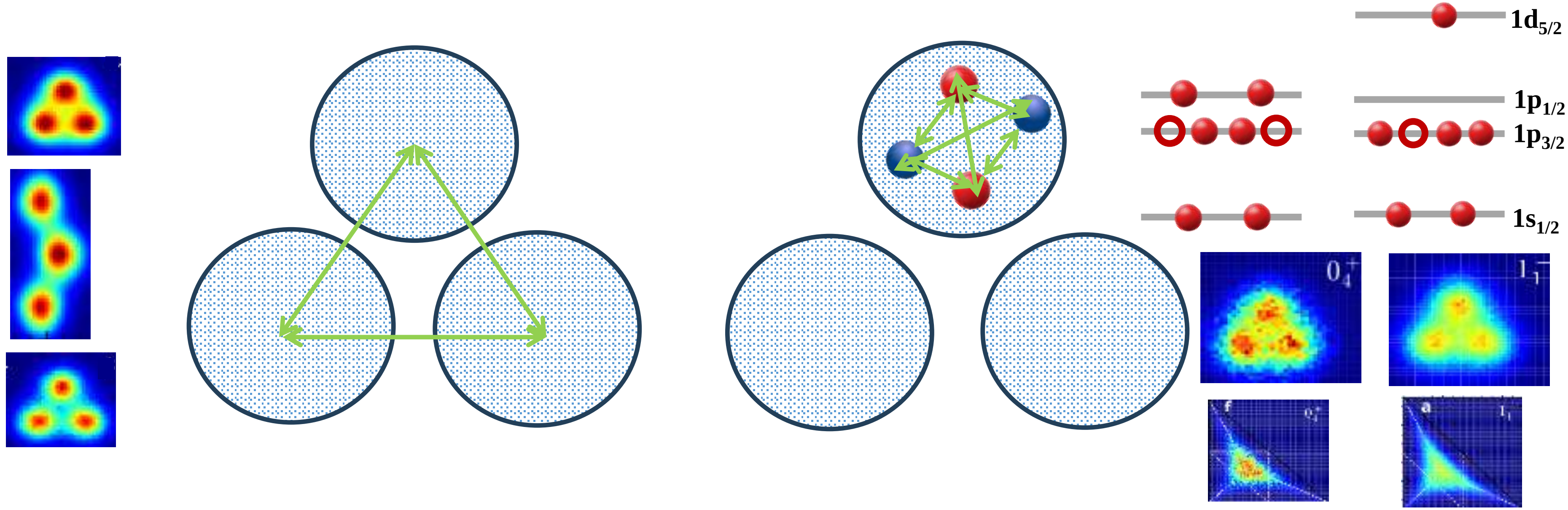
Cluster Formation

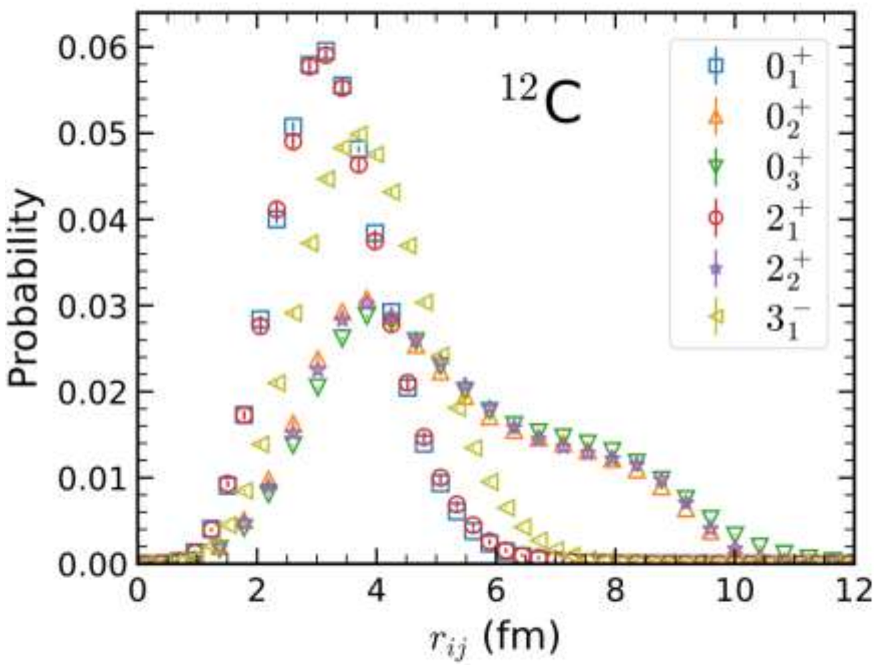
- Density distribution of ^{12}C ground state using (a-d) harmonic oscillator or (e-h) cluster wave function as initial states, with Euclidean projection time ranging from t = 0 to 0.2 MeV^{-1}.

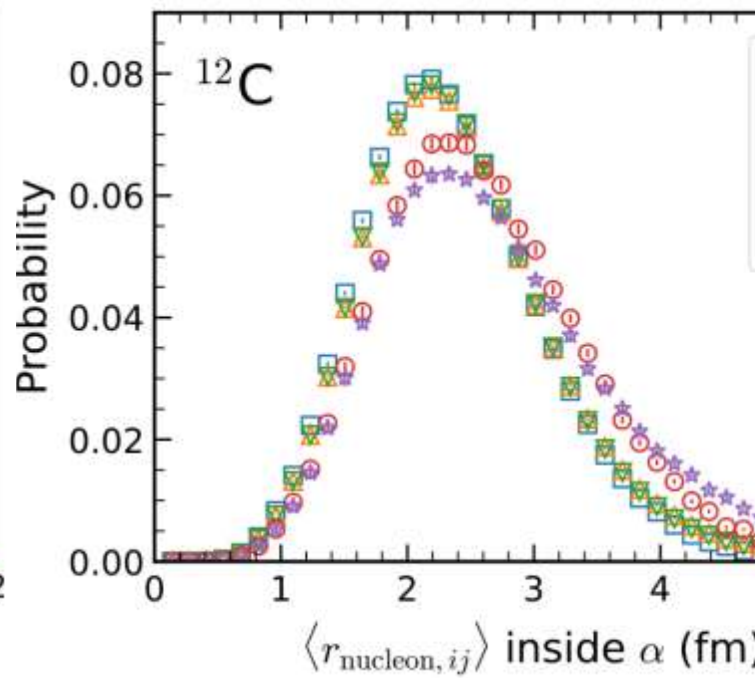


confirms the finding in Ref. E. Epelbaum et al., PRL 109, 252501 (2012)

Cluster Excitation? Single-Particle Excitation?

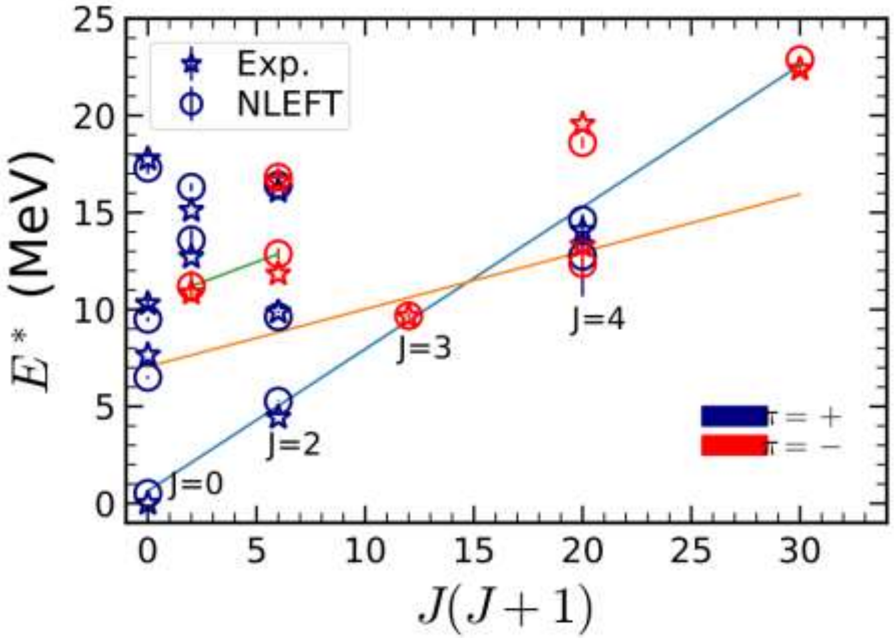


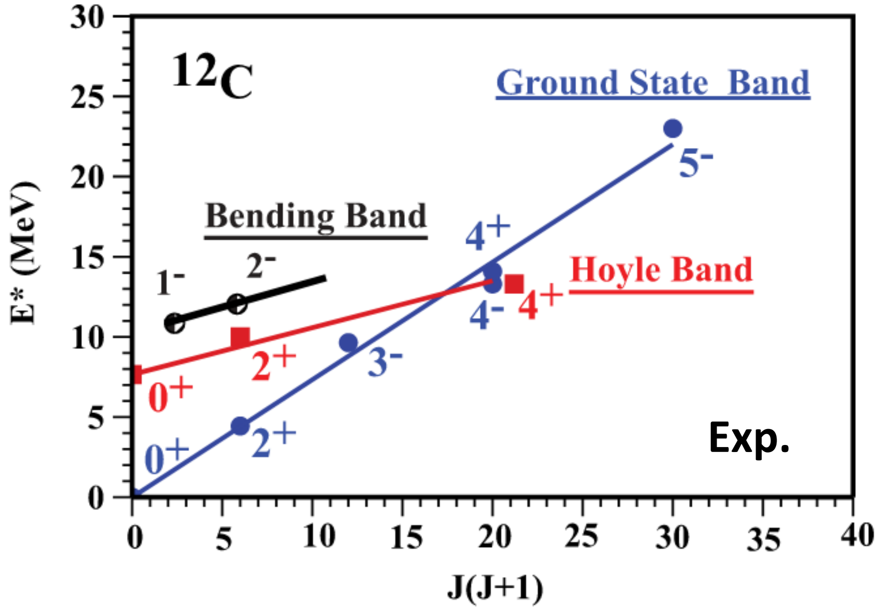


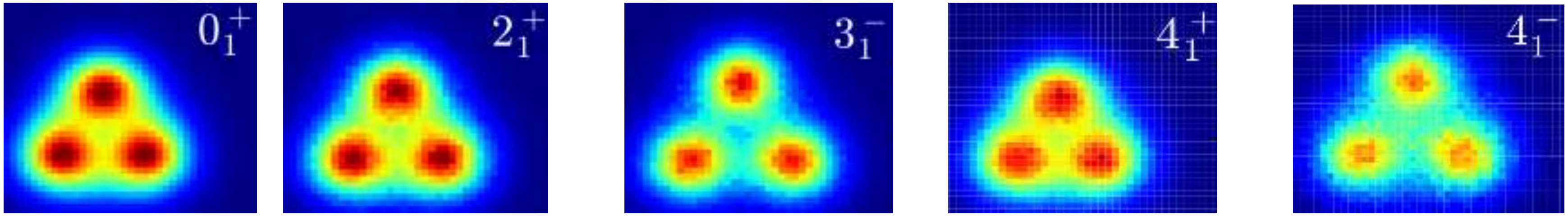


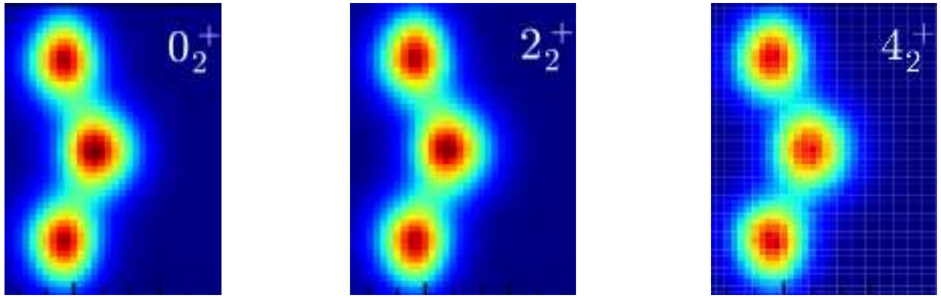
Band Structure

Marín-Lámbarri, D. J. et al., Phys. Rev. Lett. 113, 012502 (2014)
R. Bijker and F. Iachello, Phys. Rev. C, 61, 067305 (2000)



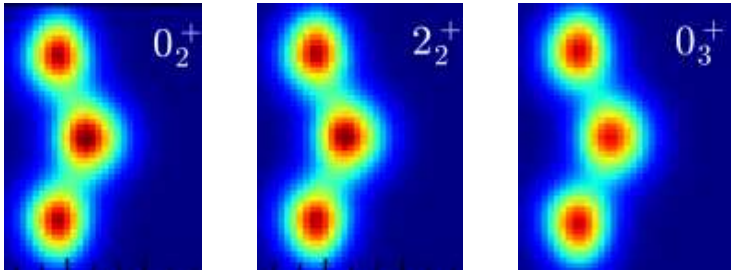






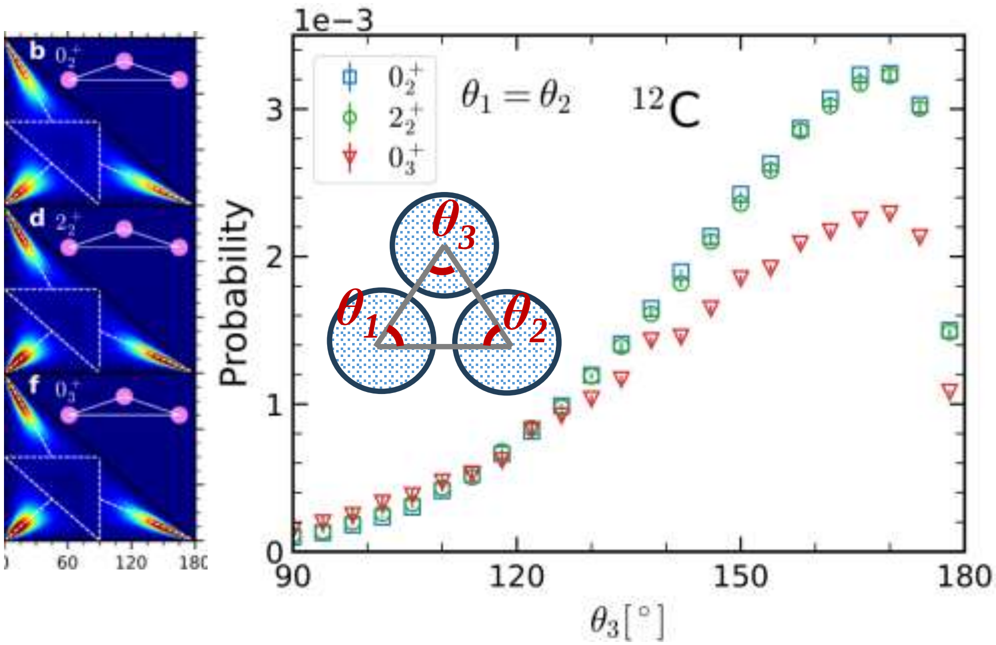
Breathing Mode of Hoyle State

- 3D density at y = 0 plane

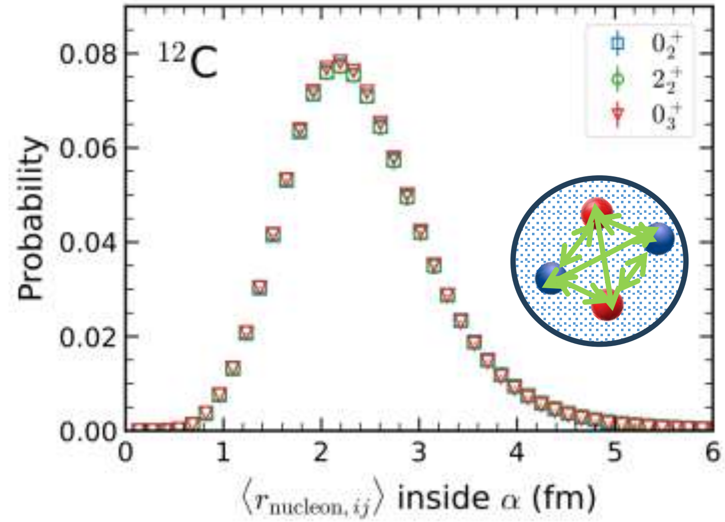


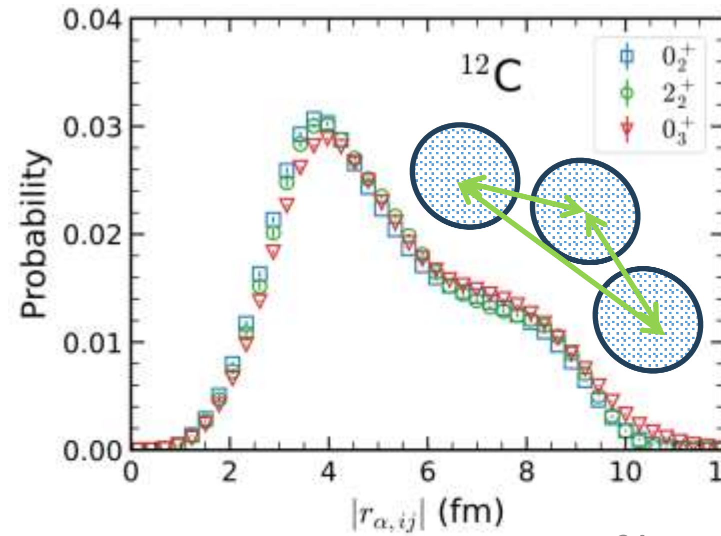
Hoyle state | Rotation | Breathing

- Angle distribution



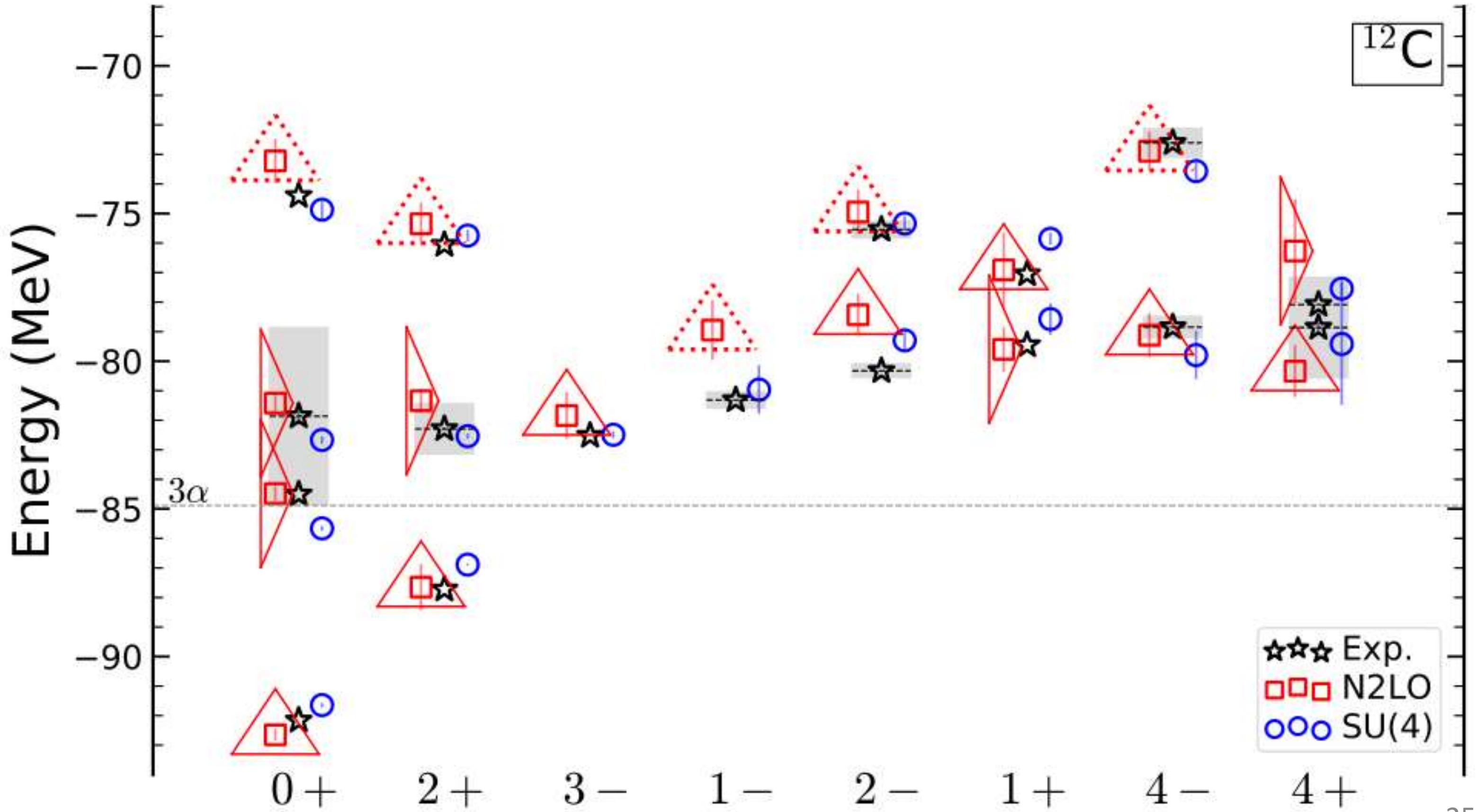
- Distance distribution





Geometry Information in the Low-Lying Spectrum

- To summarize the geometry properties of each states in the low-lying spectrum of ^{12}C calculated by NLEFT:
  - 2 types of shape: equilateral or large angle obtuse triangle.
  - α cluster is well maintained (solid triangles) or diminished (dashed ones).



Recent Advance in NLEFT

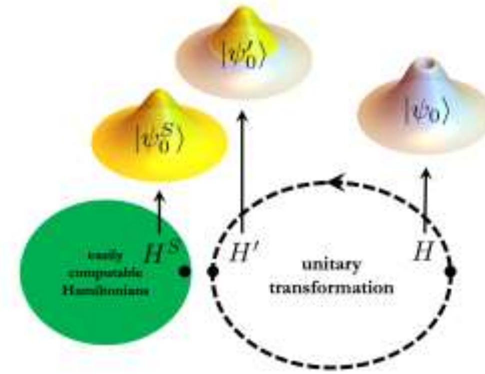
Article

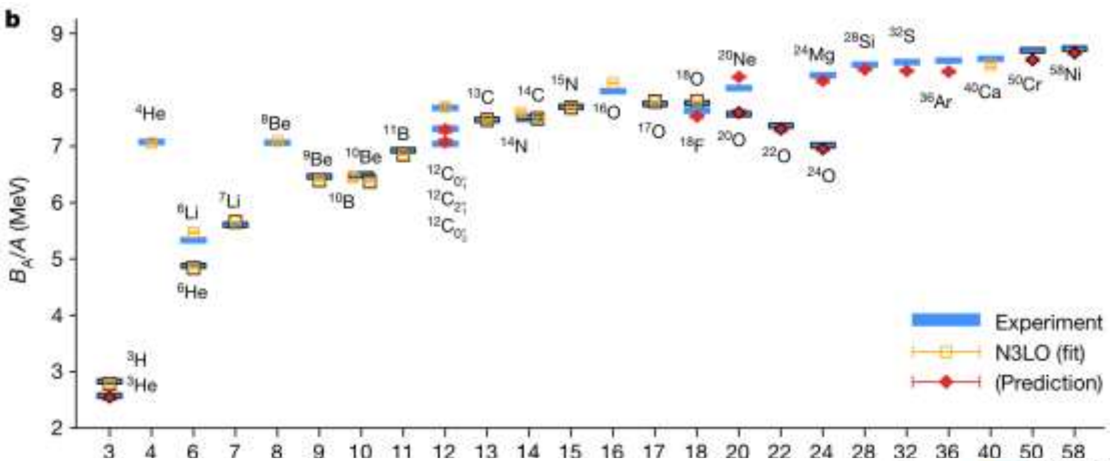
Wavefunction matching for solving quantum many-body problems

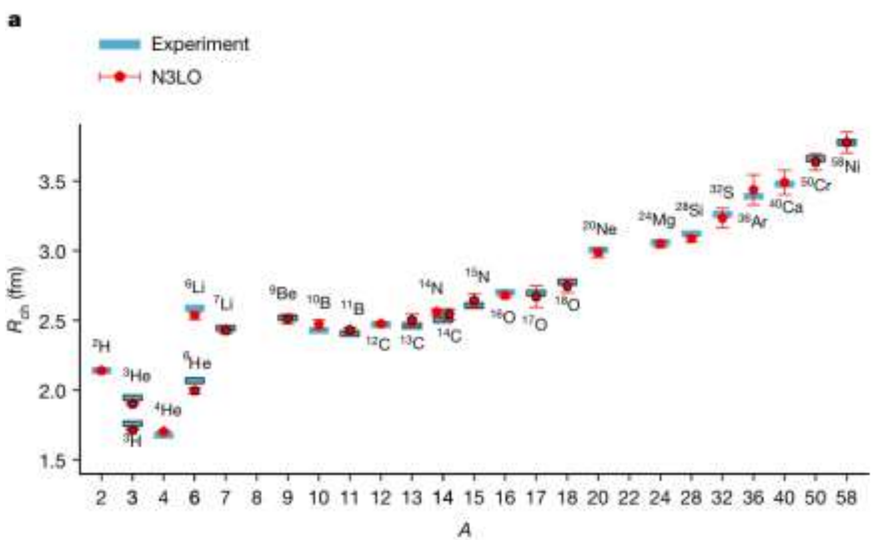
Nature 630 (2024) 59

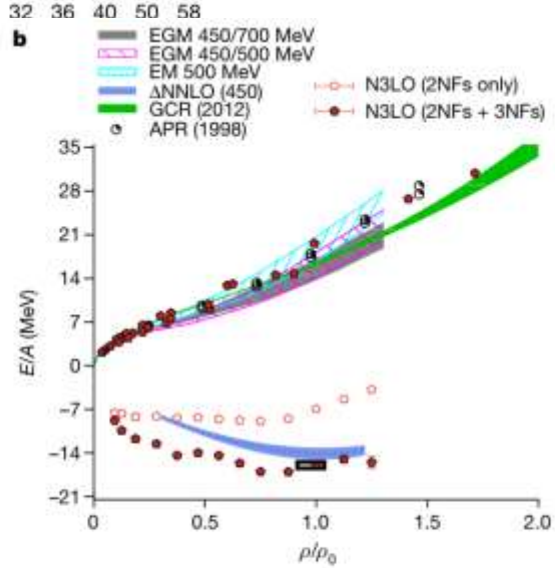
https://doi.org/10.1038/s41586-024-07422-z
Received: 23 November 2022
Accepted: 15 April 2024

Serdar Elhatisari, Lukas Bovermann, Yuan-Zhuo Ma, Evgeny Epelbaum, Dillon Frame, Fabian Hildenbrand, Myungkuk Kim, Youngman Kim, Hermann Krebs, Timo A. Lähde, Dean Lee, Ning Li, Bing-Nan Lu, Ulf-G. Meißner, Gautam Rupak, Shihang Shen, Young-Ho Song & Gianluca Stellin









Recent Advance in Nuclear Shape Imaging

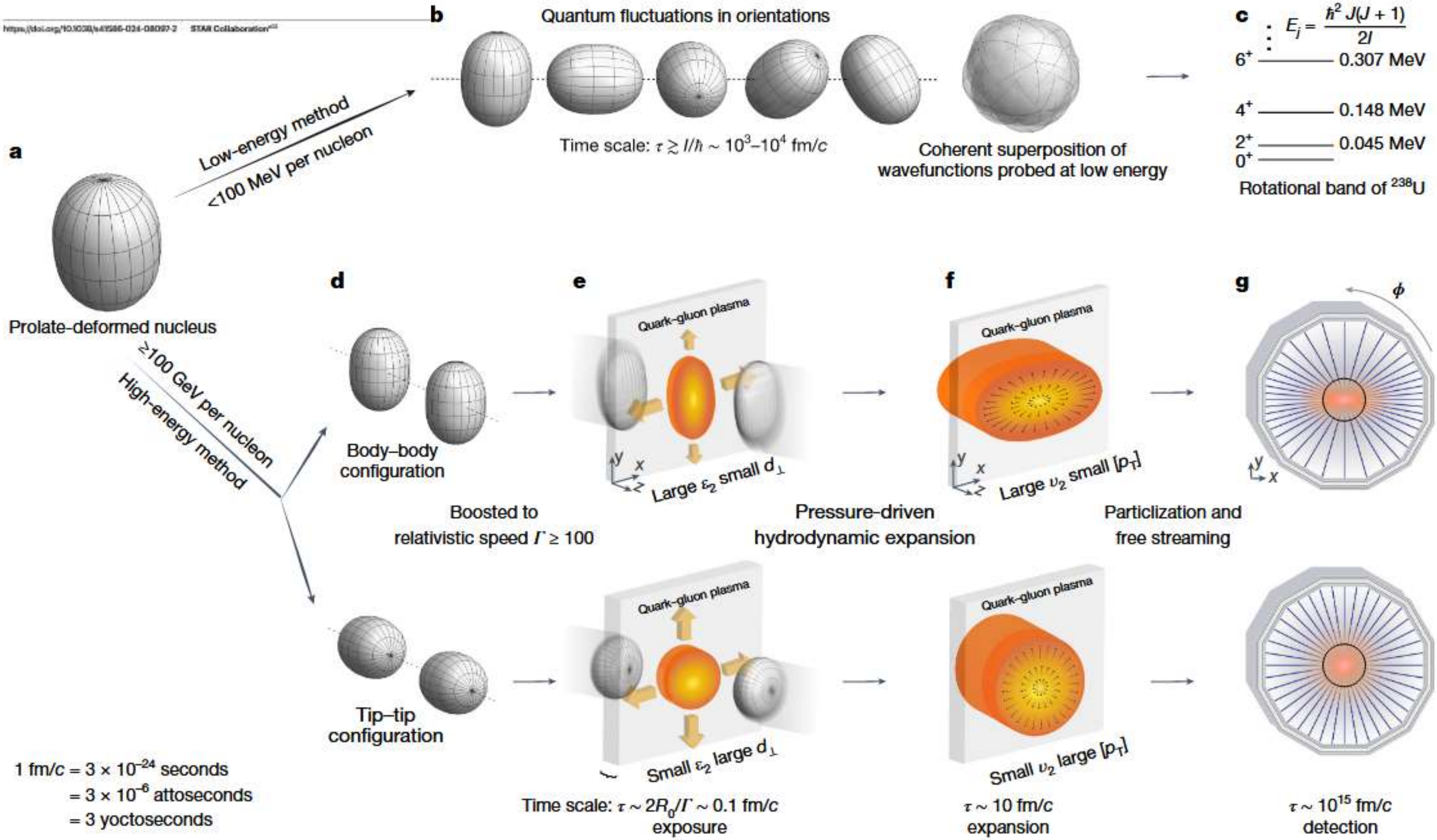
Article

Imaging shapes of atomic nuclei in high-energy nuclear collisions

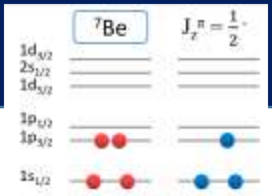
https://doi.org/10.1038/s41586-024-08097-2

STAR Collaboration

STAR, Nature 635 (2024) 67



Beryllium isotopes



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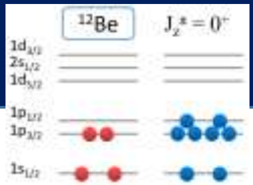
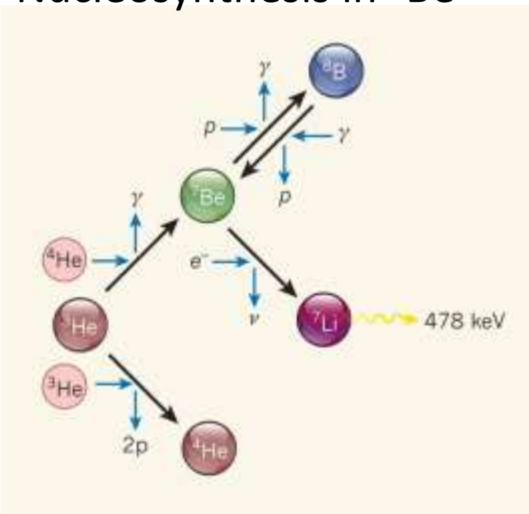
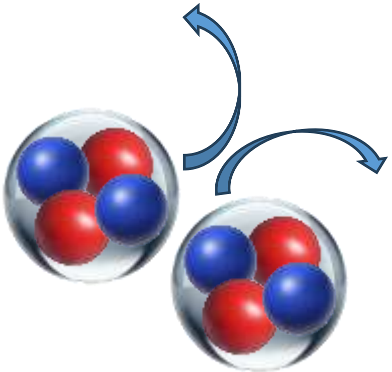


Nucleosynthesis in ^{7}Be

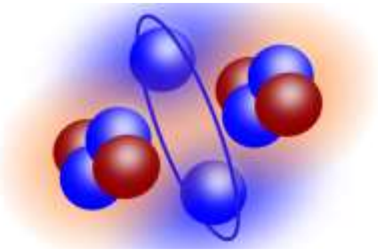


M. Hernanz., Nature **518**, 307 (2015)

alpha decay in ^{8}Be, 8.19 x 10^{-17} s

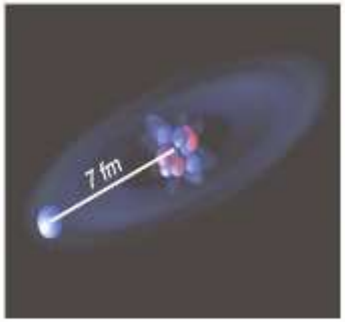


Molecule-Like Structure in ^{10}Be



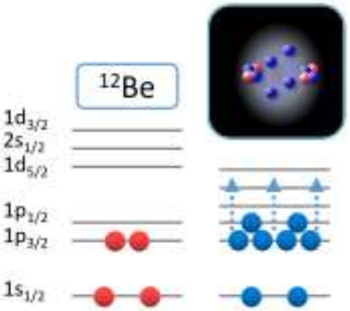
P. Li, et al., Phys. Rev. Lett. **131**, 212501 (2023)

One neutron halo in ^{11}Be



W. Nörtershäuser, et al., Phys. Rev. Lett. **102**, 062503 (2009)

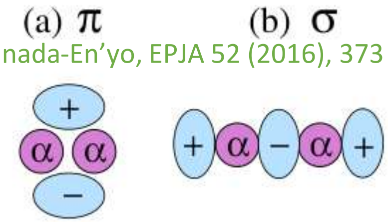
Breakdown of N = 8 shell in ^{12}Be



A. Krieger, et al., Phys. Rev. Lett. **108**, 142501 (2012)

Theoretical Studies

- Cluster models
  - Antisymmetrized Molecular Dynamics (AMD) Y. Kanada-En'yo, Phys. Rev. C 91 (2015)
  - Fermionic Molecular Dynamics (FMD) A. Krieger, et al., Phys. Rev. Lett. 108, 142501 (2012)
  - molecular-orbital W. Von Oertzen,Z. Phys. A 354 (1996) 37; N. Itagaki and S. Okabe, Phys. Rev. C 61 (2000), 044306
  - Tohsaki–Horiuchi–Schuck–Röpke (THSR) M. Lyu, et al., Eur. Phys. J. A 57 (2021). 51



M. Kimura, T. Suhara and Y. Kanada-En'yo, EPJA 52 (2016), 373

- Density functional theory

  J. Geng, P. W. Zhao, Y. F. Niu and W. H. Long, Phys. Lett. B 858 (2024), 139036
  J. P. Ebran, E. Khan, T. Niksic and D. Vretenar, Phys. Rev. C 90 (2014), 054329

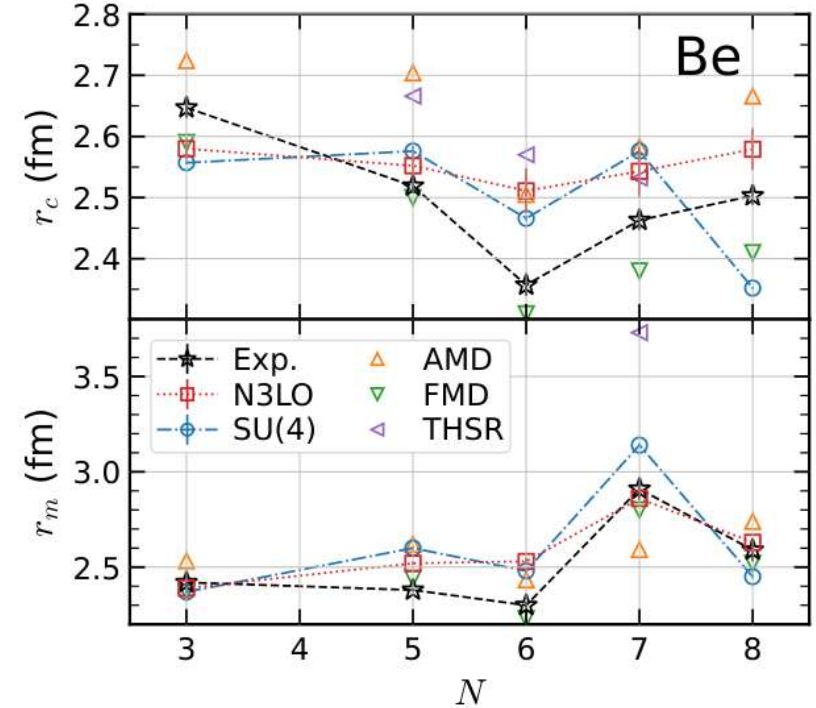
- Ab initio methods
  - no-core shell model (NCSM) A. Calci, et al., PRL 117 (2016), 242501; P. Navratil, et al., PRL 99 (2007), 042501
  - Green's function Monte Carlo E. McCutchan, et al., PRL 103 (2009), 192501; R. Wiringa, et al., PRC 62 (2000), 014001
  - Monte Carlo shell model (MCSM) T. Yoshida, et al., FBS 54 (2013), 1465; L. Liu, et al., PRC 86 (2012), 014302
  - resonating group K. Kravvaris and A. Volya, Phys. Rev. Lett. 119 (2017) 062501

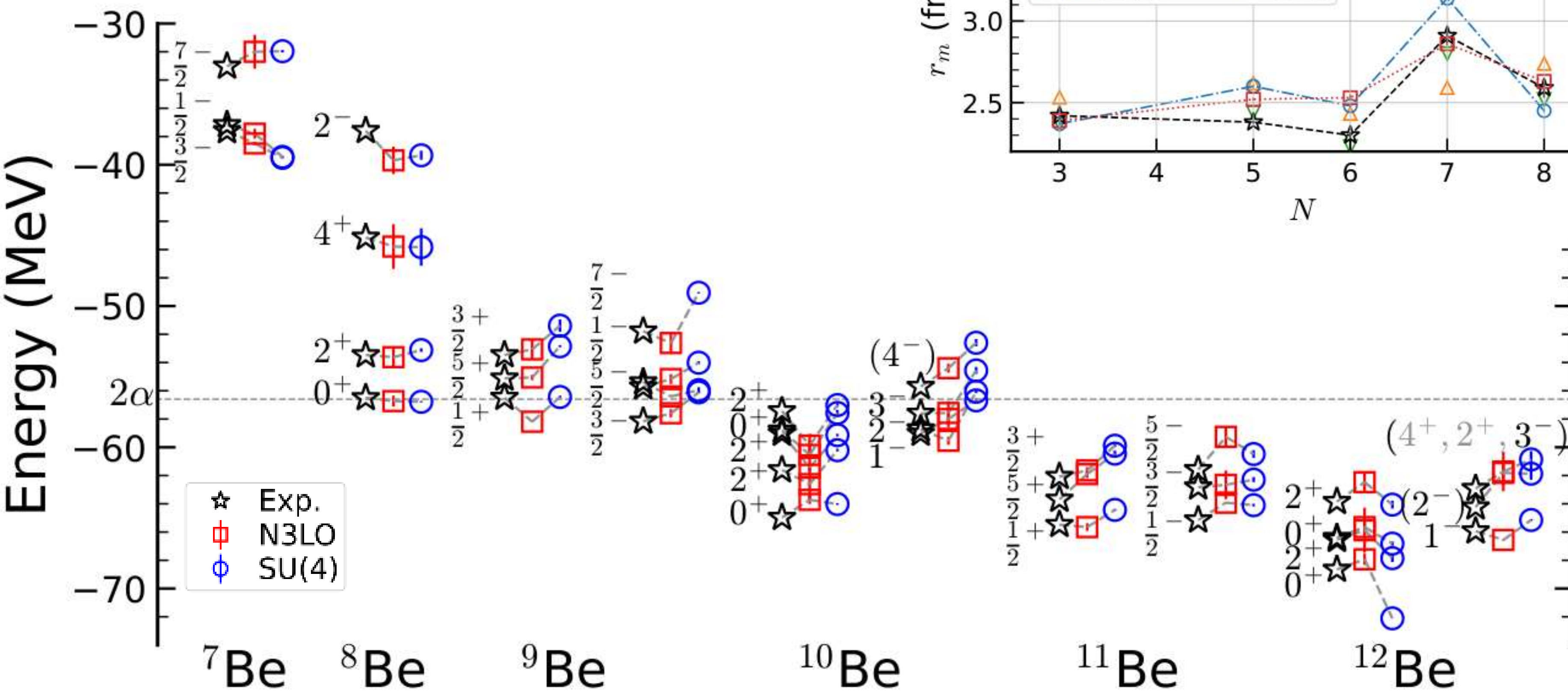
A systematic ab initio study is still lacking

Energy spectrum and radii

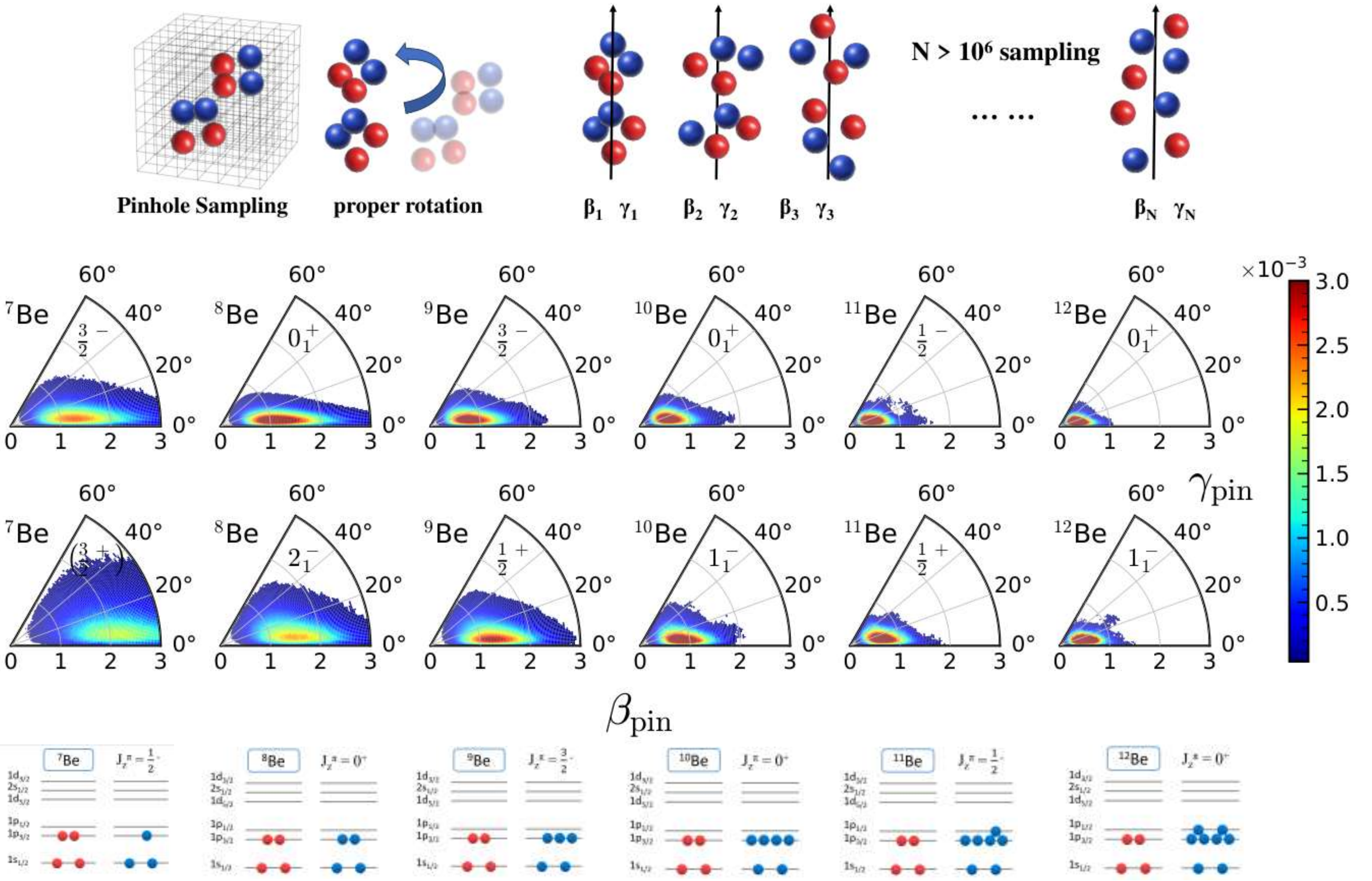
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, arXiv:2411.14935

	SU(4)	N^{3}LO	Exp.
^{11}Be , $\frac{1}{2}^+$	−64.6(1)	−65.6(5)	−65.5
^{11}Be , $\frac{1}{2}^-$	−64.1(1)	−63.5(13)	−65.2



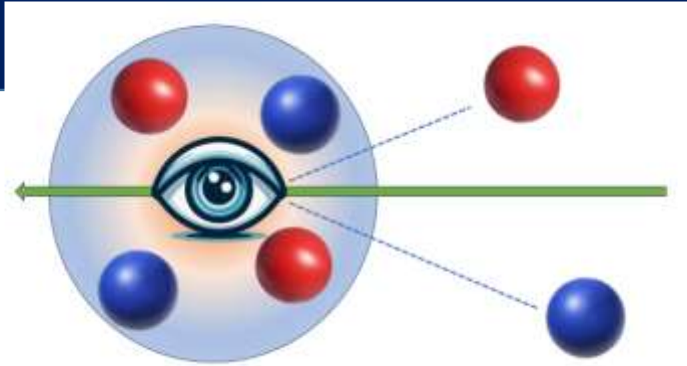


Nuclear Shape Imaging from ab initio Study



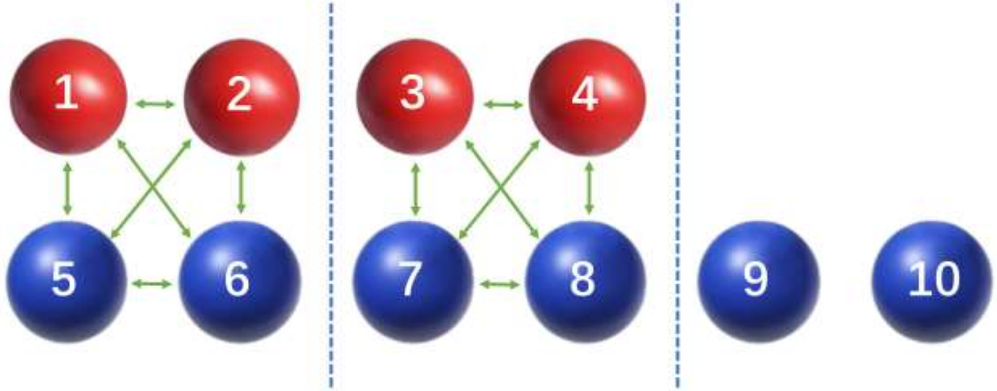
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, arXiv:2411.14935

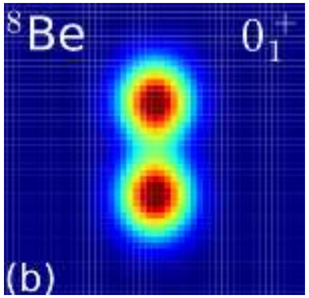
α-cluster-view intrinsic density



- Group 2 α cluster and align them to z-axis

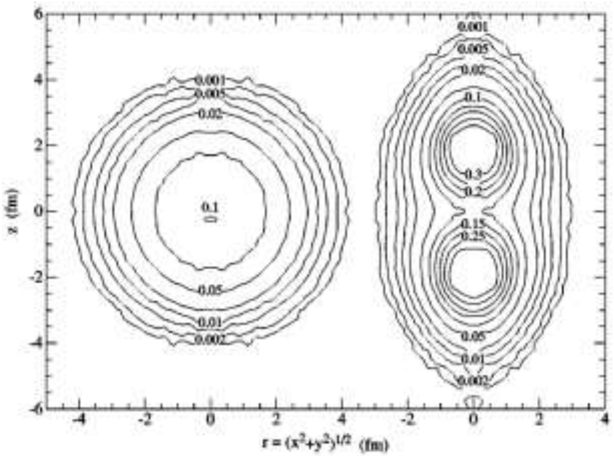
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, arXiv:2411.14935





- Comparing with principal-axis alignment view:

$$\mathcal{M}=\sum_{i=1}^{A}\begin{pmatrix} x_i^2 & x_i y_i & x_i z_i \\ y_i x_i & y_i^2 & y_i z_i \\ z_i x_i & z_i y_i & z_i^2 \end{pmatrix}$$

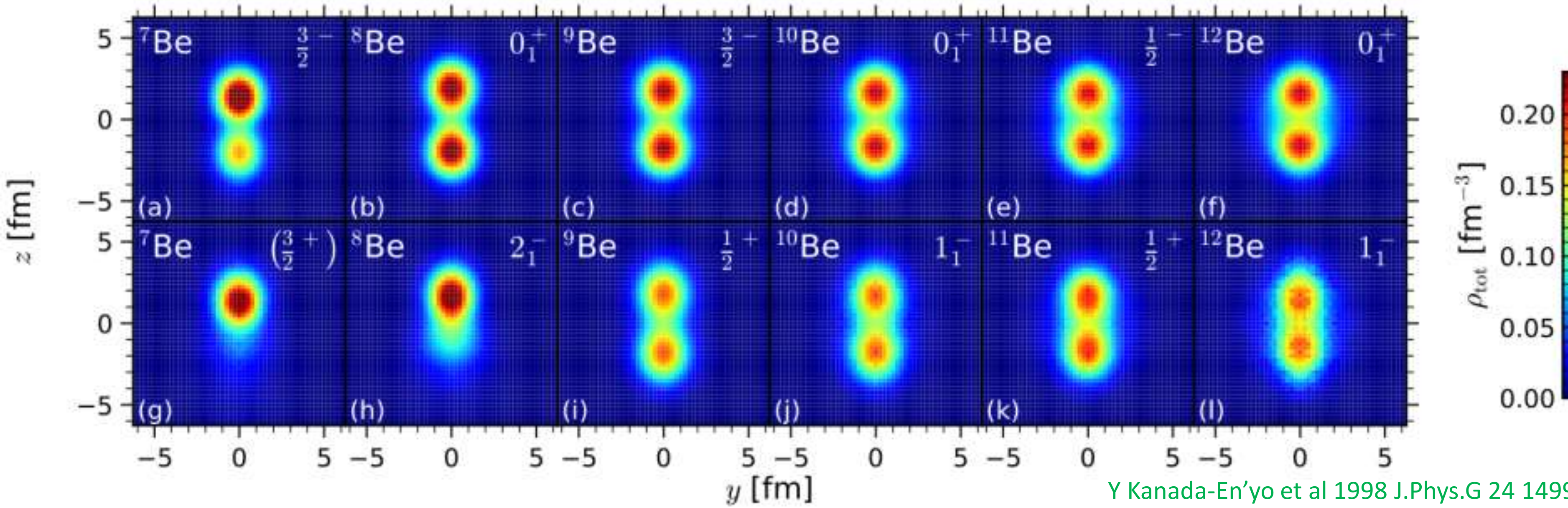


R. B. Wiringa, S. C. Pieper, J. Carlson, V. R. Pandharipande, PRC 62, 014001 (2000)

α-cluster-view intrinsic density

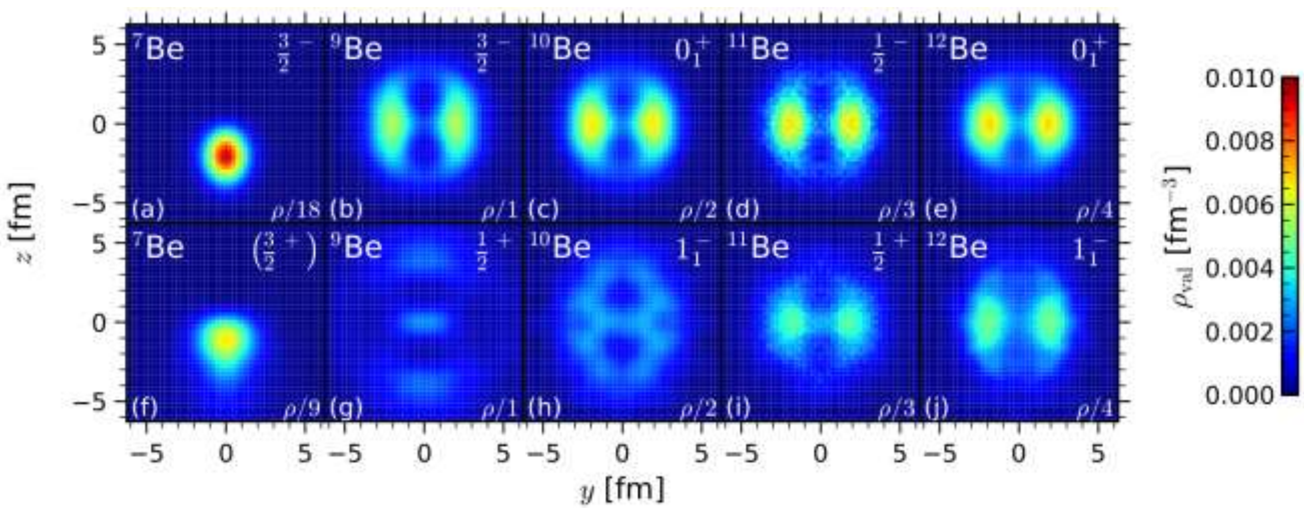
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, arXiv:2411.14935

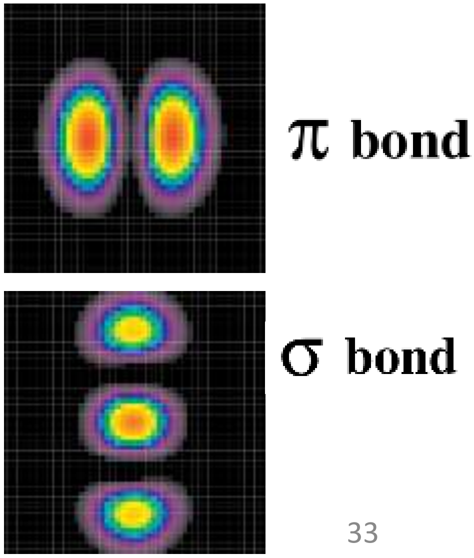
- Total density



Y Kanada-En'yo et al 1998 J.Phys.G 24 1499

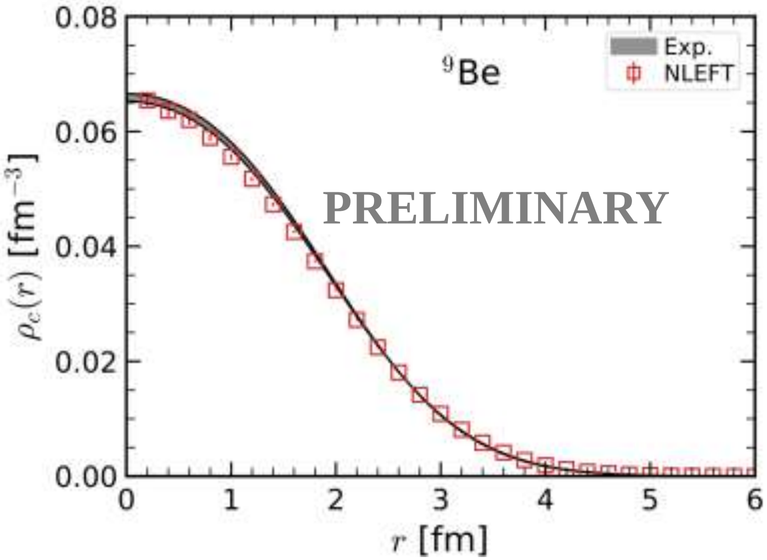
- Valence particle density

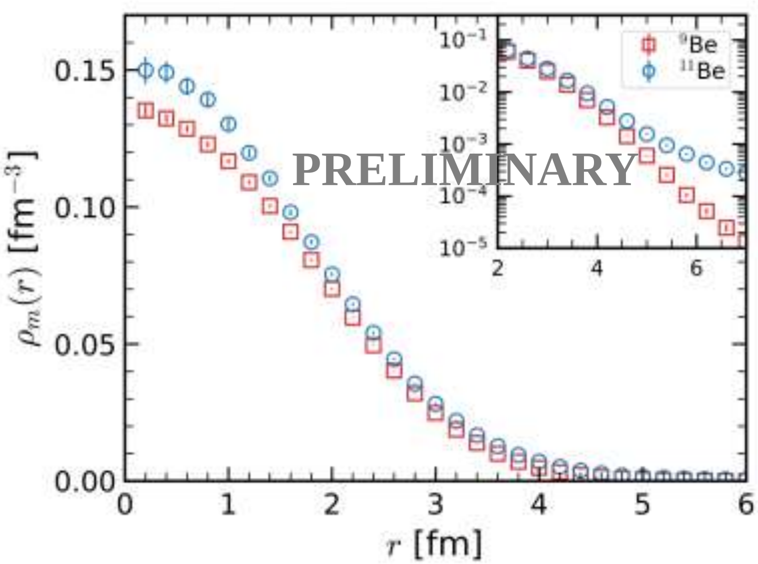




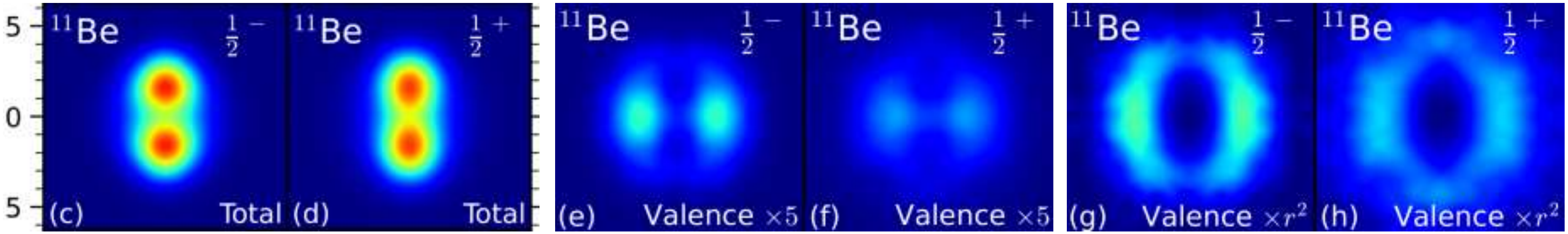
Density distribution and Halo structure

- Charge density of ^{9}Be (left) and matter density of ^{9}Be and ^{11}Be (right)





- Intrinsic density in ^{11}Be



E(Exp.): -65.2 MeV, -65.5 MeV

Summary

- Nuclear Lattice Effective Field Theory (NLEFT) provided an unified ab initio framework to study various nuclear structure phenomena.
- Low-lying spectrum of ^{12}C and p-shell Beryllium isotopes have been studied by NLEFT, the agreement with experiment is impressive, not only energies, but also electromagnetic transitions and density profiles.
- A tomographic scan of the three-dimensional geometry of the nuclear states has been introduced. The Hoyle state and its rotational excitations, as already stated in E. Epelbaum et al., PRL 109, 252501 (2012), are found to be an obtuse triangle with large angle.
- Geometric structure has been discussed, characteristics of cluster structure and single-particle structure are shown.
- Prominent two-center cluster structures, the emergence of one-neutron halo, and complex nuclear molecular dynamics in Beryllium isotopes are revealed.

APPENDIX

4He: Puzzle for Nuclear Forces?

PHYSICAL REVIEW LETTERS **130**, 152502 (2023)

Editors' Suggestion   Featured in Physics

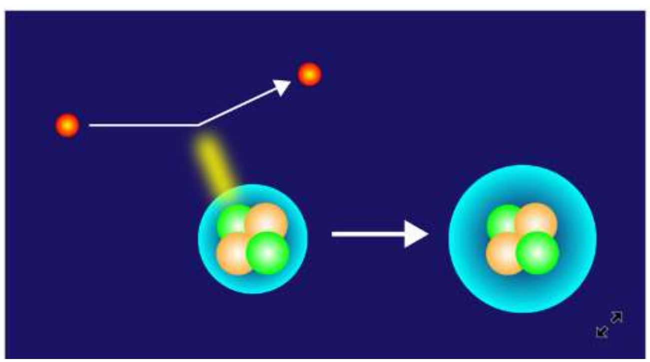
Measurement of the α-Particle Monopole Transition Form Factor Challenges Theory: A Low-Energy Puzzle for Nuclear Forces?

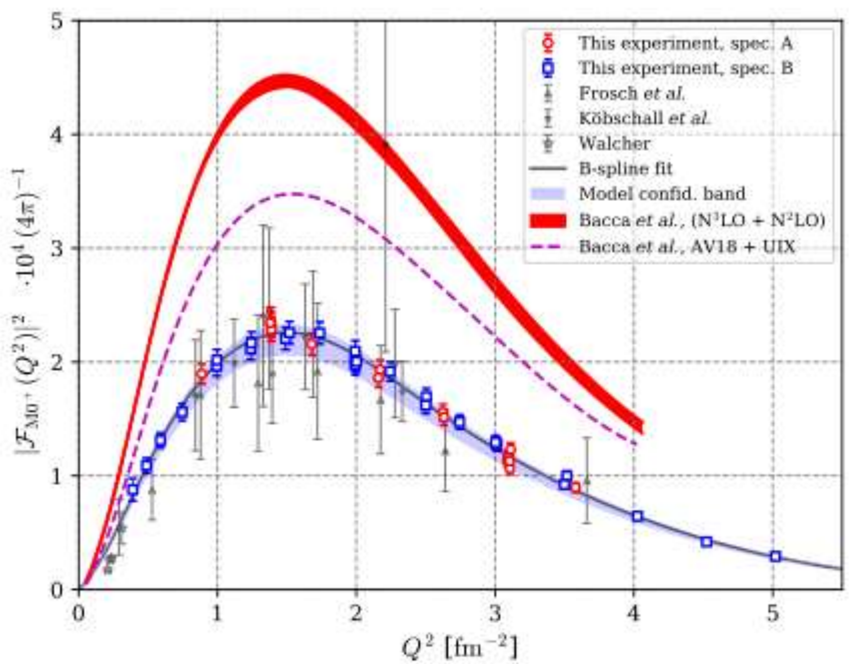
Probing the Helium Nucleus beyond the Ground State

Evgeny Epelbaum
Faculty of Physics and Astronomy, Ruhr University Bochum, Bochum, Germany

April 10, 2023 • *Physics* 16, 58

A new electron-scattering experiment challenges our understanding of the first excited state of the helium nucleus.





Energies and Threshold

- SU(4) interaction from B.-N. Luu et al., PLB 797 (2019) 134863

L [fm]	$E(0_1^+)$ [MeV]	$E(0_2^+)$ [MeV]	ΔE [MeV]
13.2	$-28.32(3)$	$-8.37(14)$	0.28(14)
14.5	$-28.30(3)$	$-8.02(14)$	0.42(14)
15.7	$-28.30(3)$	$-7.96(9)$	0.40(9)

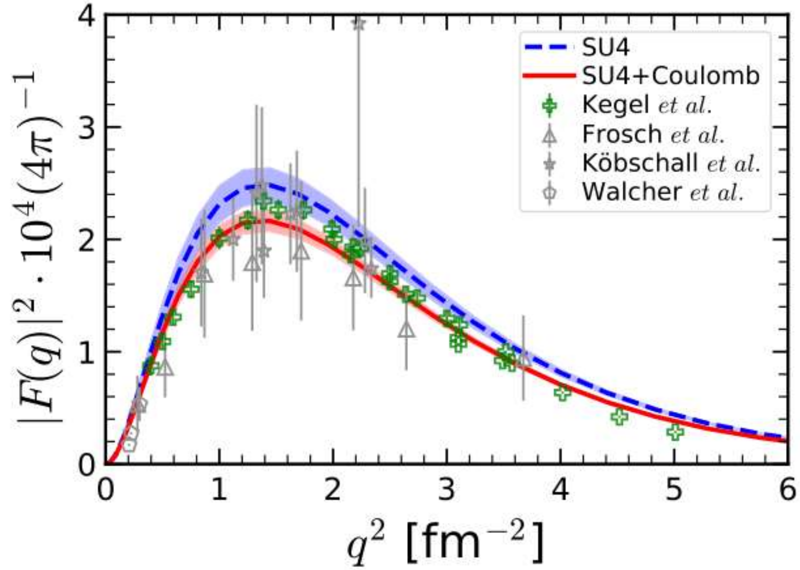
U.-G. Meißner, SS, S. Elhatisari, D. Lee, PRL 132, 062501 (2024)

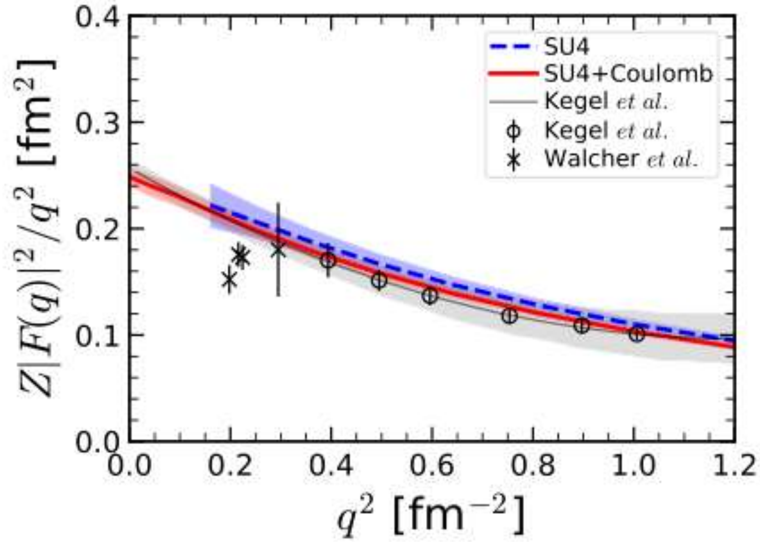
	NLEFT	Exp.
4He (0_1+)	-28.30(3)	-28.30
4He (0_2+)	-7.96(9)	-8.09
3H	-8.36	-8.48
3He	-7.65	-7.72
ΔE = 4He (0_2+) - 3H	**0.40(9)**	**0.39**

N. Michel, W. Nazarewicz, and M. Płoszajczak, PRL 131, 242502 (2023)

The explicit reproduction of particle-emission thresholds is crucial for the theoretical understanding of the 0_2+ state.

Transition form factor





$$\frac{Z|\mathcal{F}_{M0^+}(q^2)|}{q^2} = \frac{\langle r^2 \rangle_{\mathrm{tr}}}{6}\left[1 - \frac{q^2}{20}\mathcal{R}^2_{\mathrm{tr}} + \mathcal{O}(q^4)\right]$$

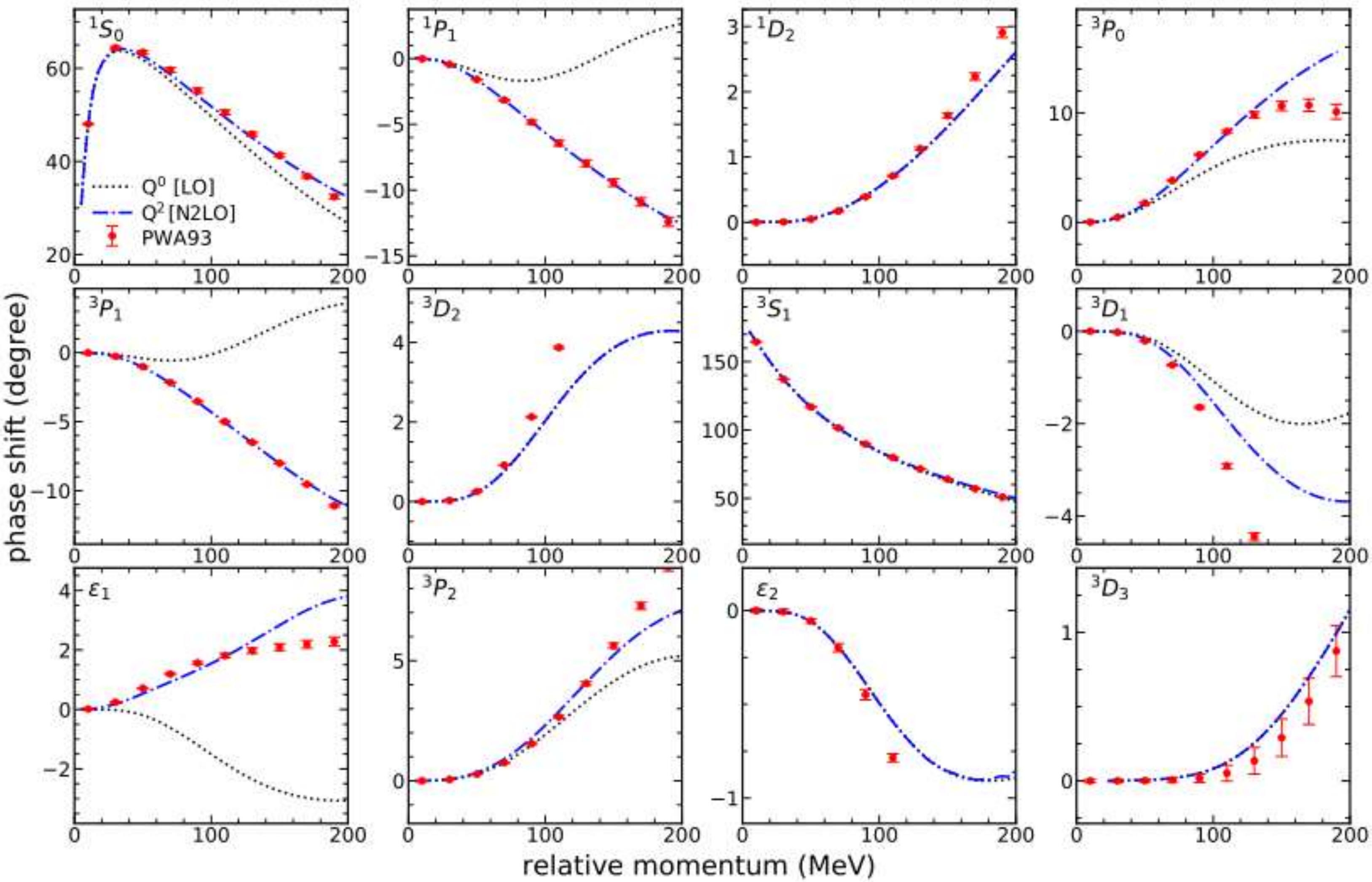
	<r^2>	R
Exp.	1.53 (5)	4.56 (15)
NLEFT	1.49 (1)	4.00 (4)
Bacca et al.	1.83 (1)	3.97 (5)

		SU(4)	N^3LO	Exp.
^{7}Be	$E2, \frac{3}{2}^- \to \frac{1}{2}^-$	16.0(2)	15.2(5)	26(6)(3) [15]
^{9}Be	$Q(\frac{3}{2}^-)$	7.3(1)	7.4(1.0)	5.29(4) [25]
	$E1, \frac{1}{2}^+ \to \frac{3}{2}^-$	0.131(3)	0.060(15)	0.136(2) [26]
	$E1, \frac{5}{2}^+ \to \frac{3}{2}^-$	0.045(14)	0.049(5)	0.010(8) [9]
	$E2, \frac{5}{2}^- \to \frac{3}{2}^-$	35.7(1.8)	27.8(1.9)	27.1(2.0) [9]
	$E2, \frac{7}{2}^- \to \frac{3}{2}^-$	11.6(2.5)	5.3(8)	9.5(4.1) [9]
^{10}Be	$E1, 3_1^- \to 2_1^+$	0.026(2)	0.004(3)	0.009(1) [9]
	$E2, 2_1^+ \to 0_1^+$	10.6(4)	8.5(9)	9.2(3) [31]
^{11}Be	$E1, \frac{1}{2}^- \to \frac{1}{2}^+$	0.023(3)	0.038(3)	0.102(2) [33]
^{12}Be	$E1, 0_1^+ \to 1_1^-$	0.049(2)	0.056(26)	0.051(13) [39]
	$E2, 2_1^+ \to 0_1^+$	7.8(1.1)	9.0(3.1)	14.2(1.0)(2.0) [40]

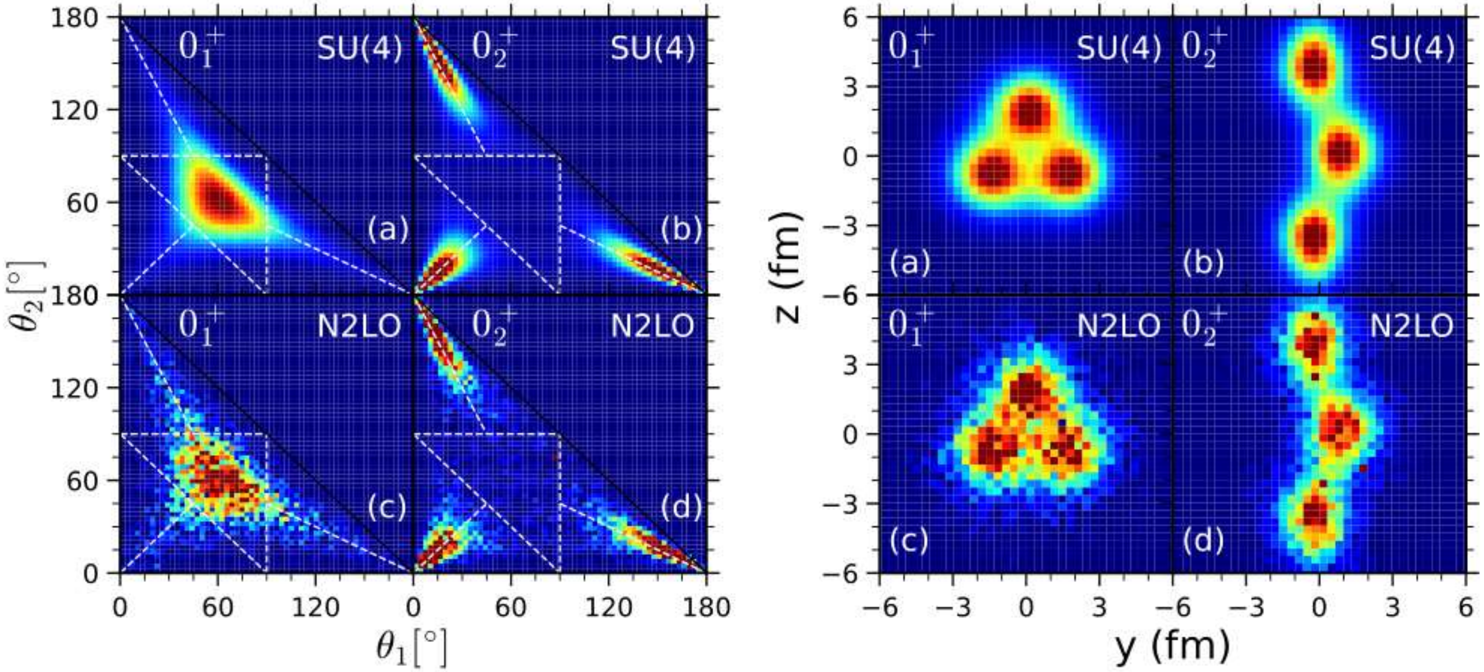
	C_0	$C_{\mathrm{GIR},0}$	$C_{\mathrm{GIR},1}$	$C_{\mathrm{GIR},2}$	s_{NL}	s_{L}
SU(4)	−0.17395	−0.07001	0.01417	−0.00125	0.1	0.06

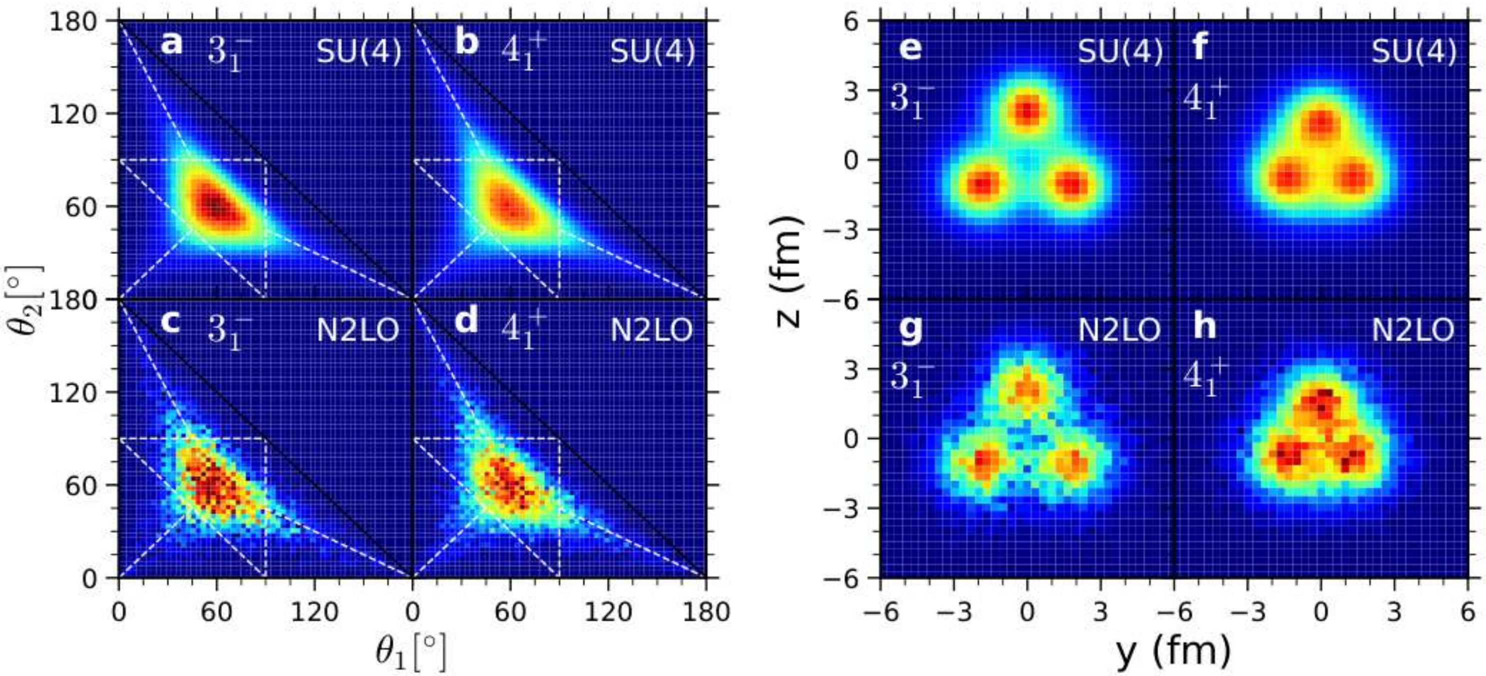
	C_0	$C_{\mathrm{GIR},0}$	$C_{\mathrm{GIR},1}$	$C_{\mathrm{GIR},2}$
Q^0, ${}^1S_0$	0.44365	0.07410	−0.00980	−0.00128
Q^0, ${}^3S_1$	−0.25149	−0.04505	0.01092	−0.00170
Q^2, ${}^1S_0$	0.55249	0.02521	0.01665	−0.01042
Q^2, ${}^3S_1$	−0.01090	1.15209	−0.64469	0.22634
Q^2, ${}^1S_0^{0,1}$	0.03241	−0.03062	0.00780	−0.00135
Q^2, ${}^3S_1^{0,1}$	0.02738	0.18172	−0.09556	0.03264
Q^2, ${}^3S_1-{}^3D_1$	−0.49342	0.09280	0.03828	−0.02687
Q^2, ${}^1P_1$	0.96569	0.95481	−0.21826	0.02956
Q^2, ${}^3P_0$	−0.19448	−0.07901	0.01729	−0.00206
Q^2, ${}^3P_1$	0.92671	0.91735	−0.20962	0.02836
Q^2, ${}^3P_2$	−0.04801	−0.03012	0.00730	−0.00114

	c_D	c_E	s_{L}
3N	−0.77527	0.67901	0.2

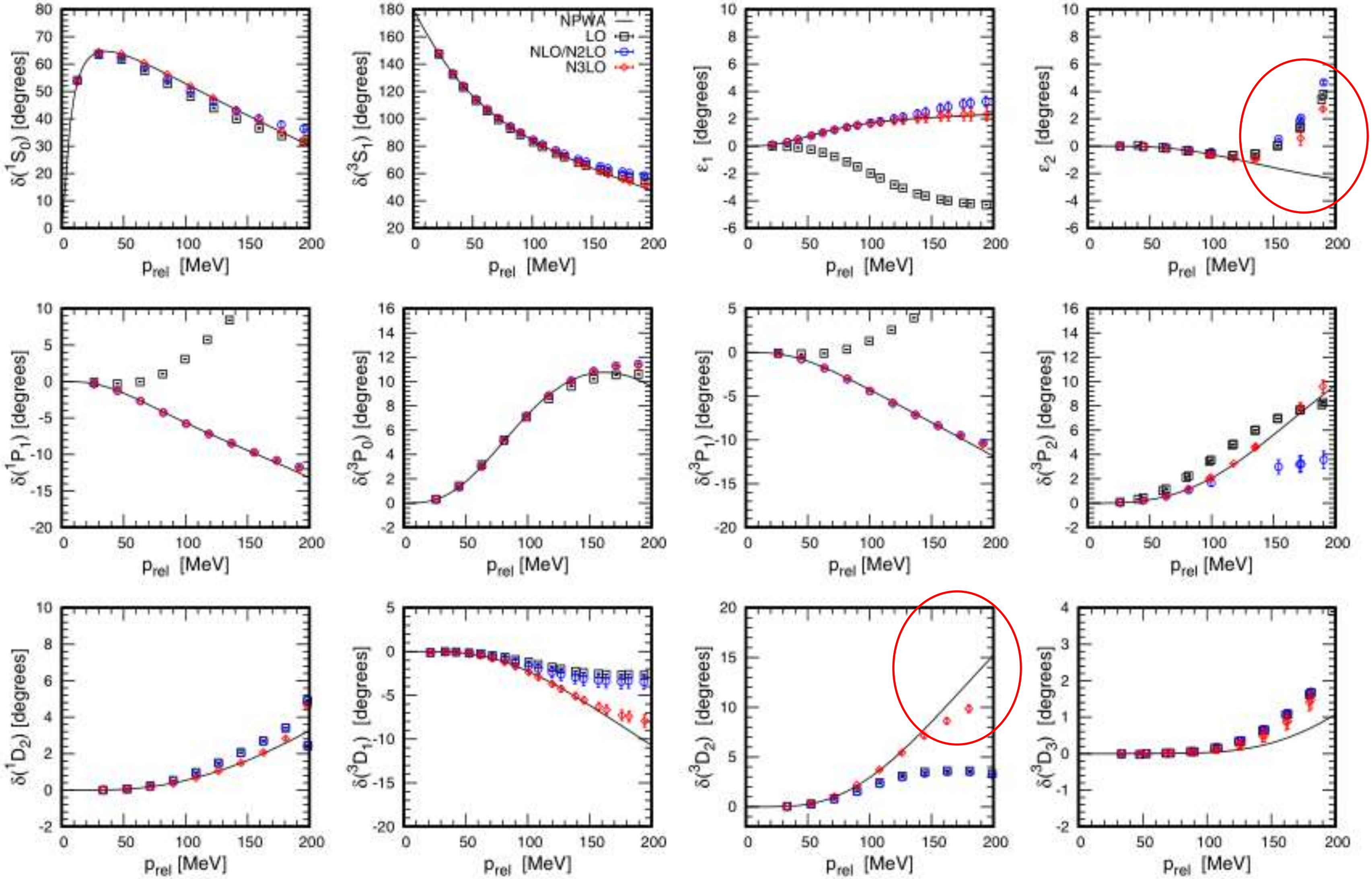


$^1S_0$
$^1P_1$
$^1D_2$
$^3P_0$
$^3P_1$
$^3D_2$
$^3S_1$
$^3D_1$
ε_1
$^3P_2$
ε_2
$^3D_3$
Q^0 [LO]
Q^2 [N2LO]
PWA93
phase shift (degree)
relative momentum (MeV)

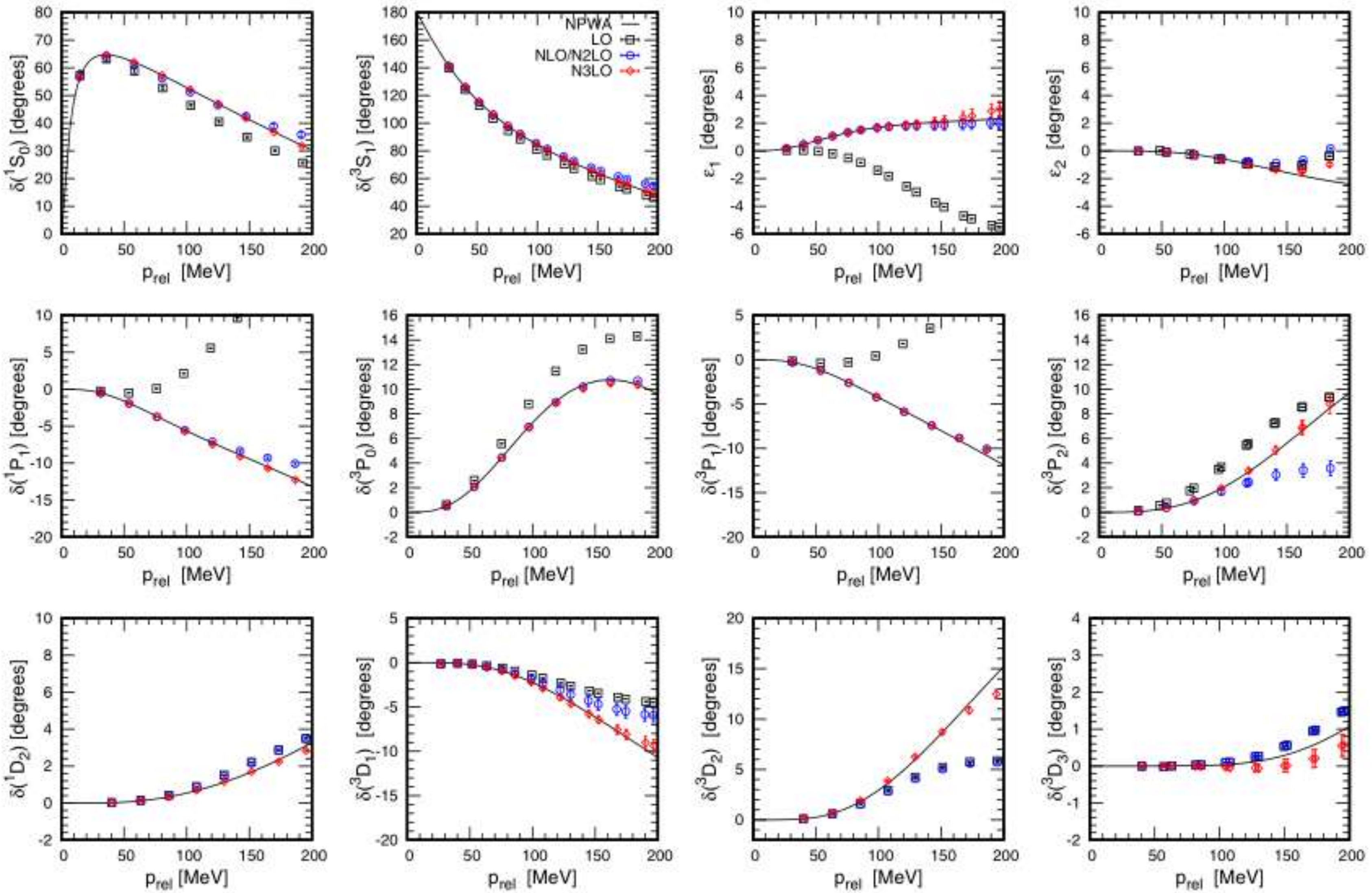

0_1^+ SU(4)
0_2^+ SU(4)
0_1^+ N2LO
0_2^+ N2LO
(a)
(b)
(c)
(d)
$\theta_1[°]$
$\theta_2[°]$
y (fm)
z (fm)


a 3_1^- SU(4)
b 4_1^+ SU(4)
c 3_1^- N2LO
d 4_1^+ N2LO
e SU(4) 3_1^-
f SU(4) 4_1^+
g N2LO 3_1^-
h N2LO 4_1^+
$\theta_1[°]$
$\theta_2[°]$
y (fm)
z (fm)

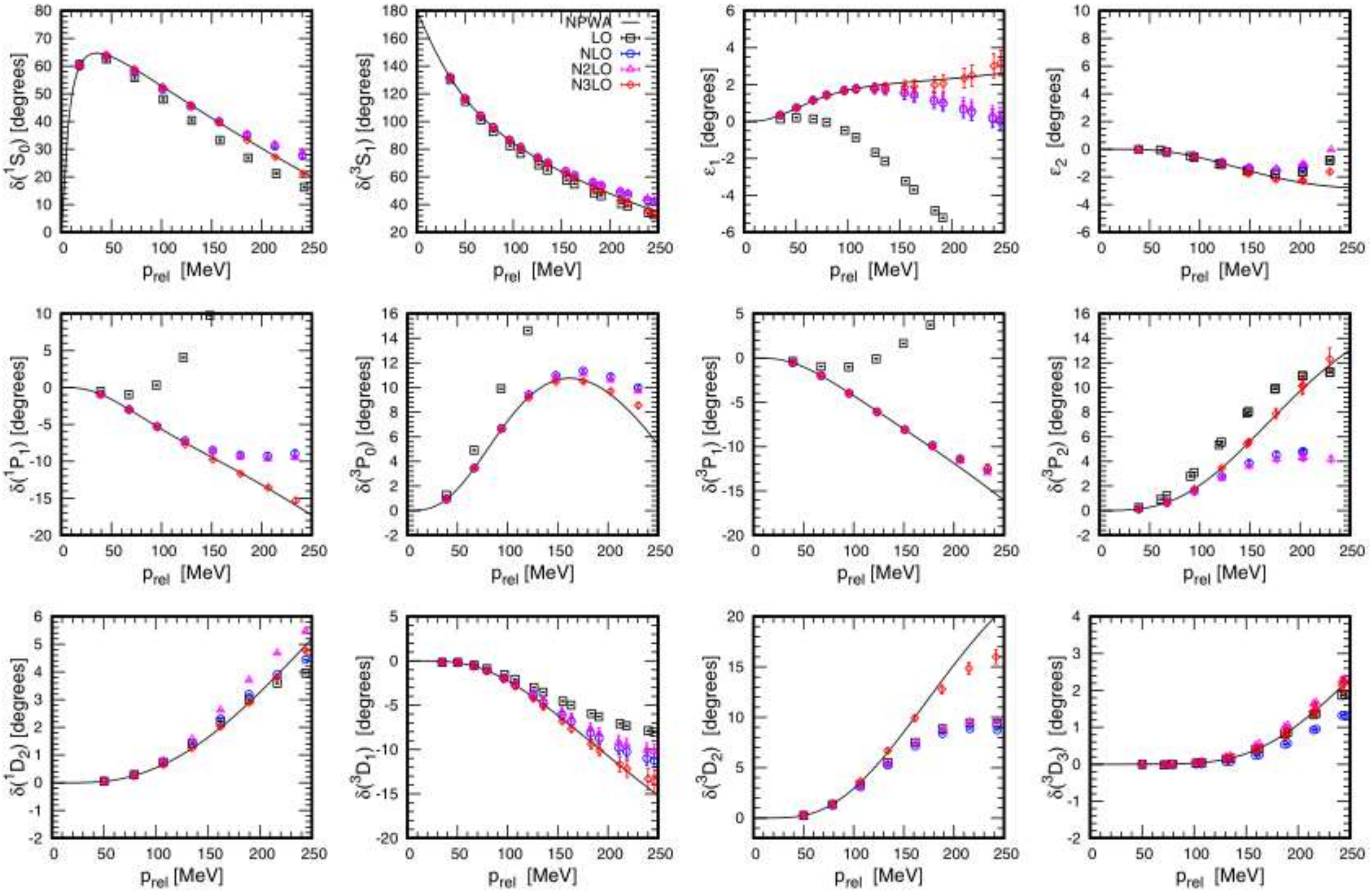
a = 1.97 fm π/a ~ 314 MeV



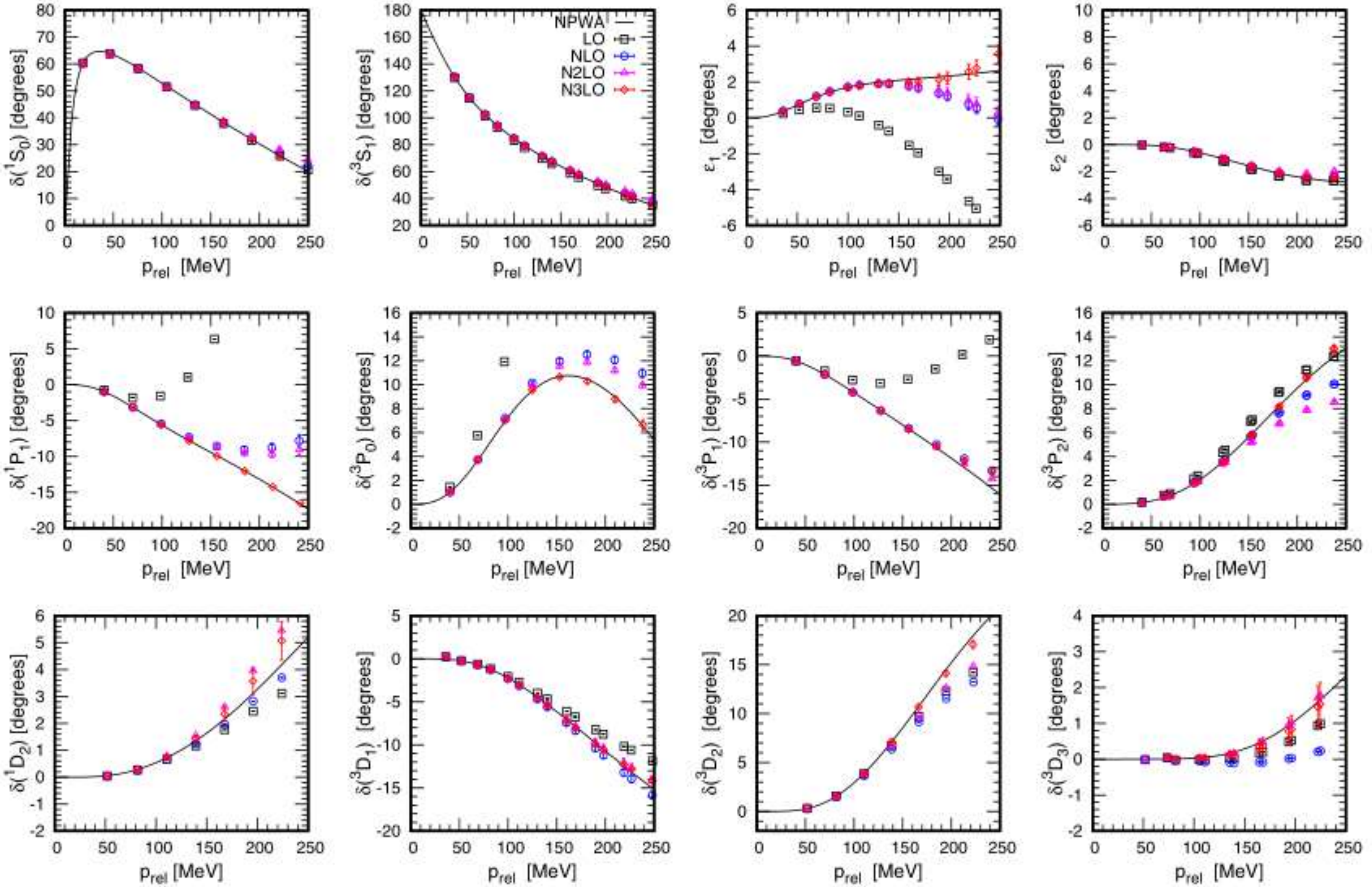
a = 1.64 fm π/a ~ 378 MeV



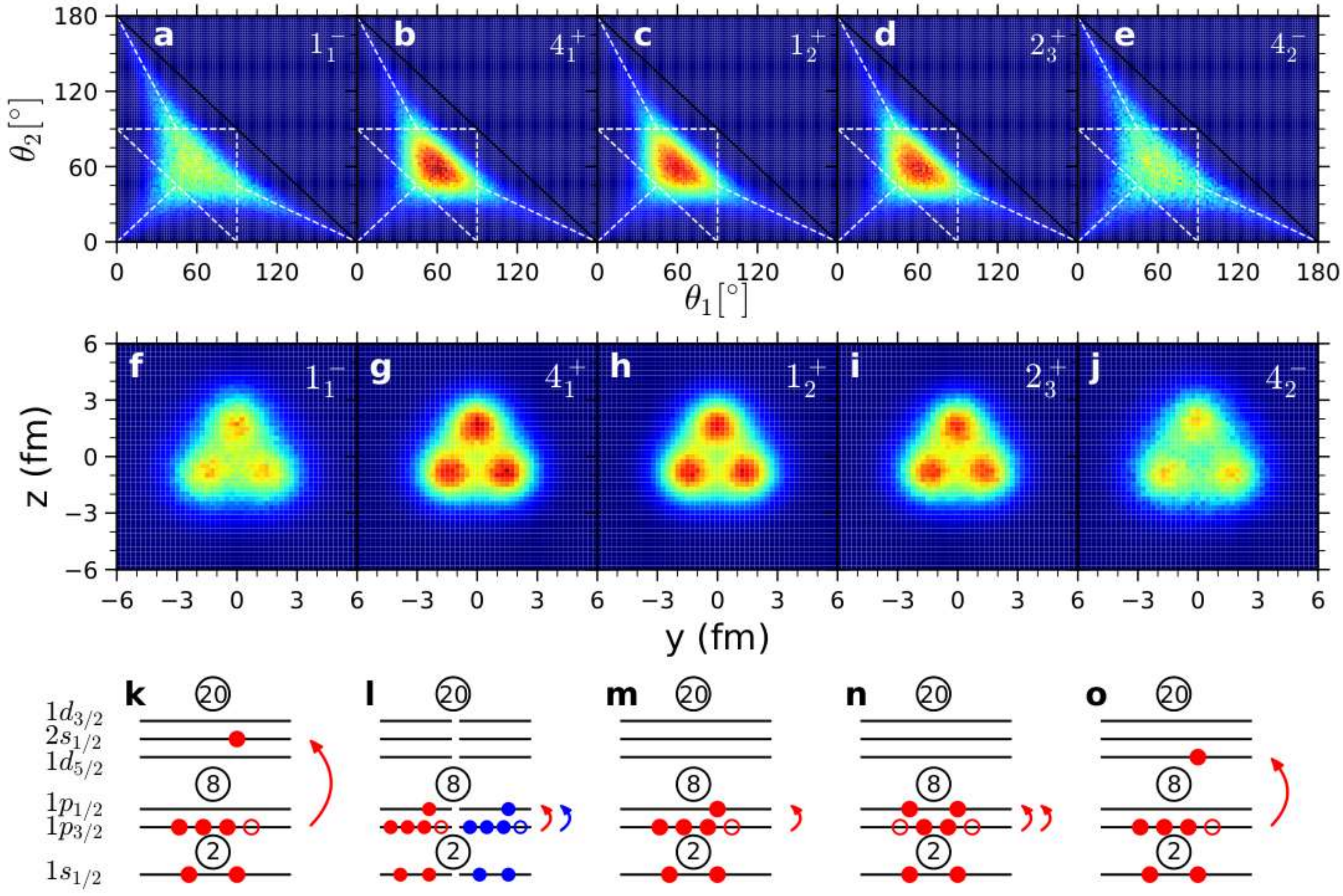
a = 1.32 fm π/a ~ 469 MeV



$a = 0.99$ fm $\pi/a \sim 626$ MeV

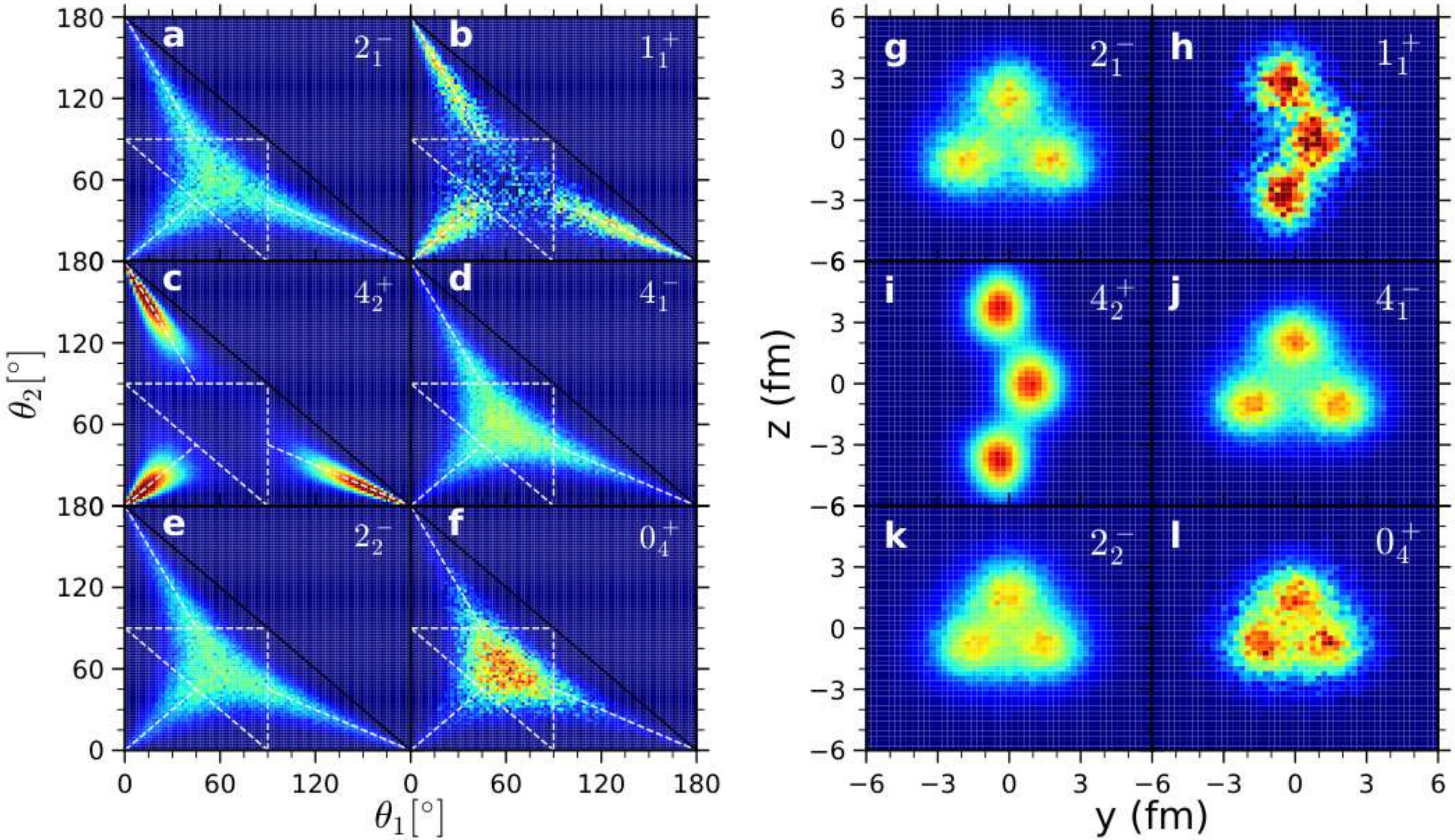


Shell-Model States as Initial Wave

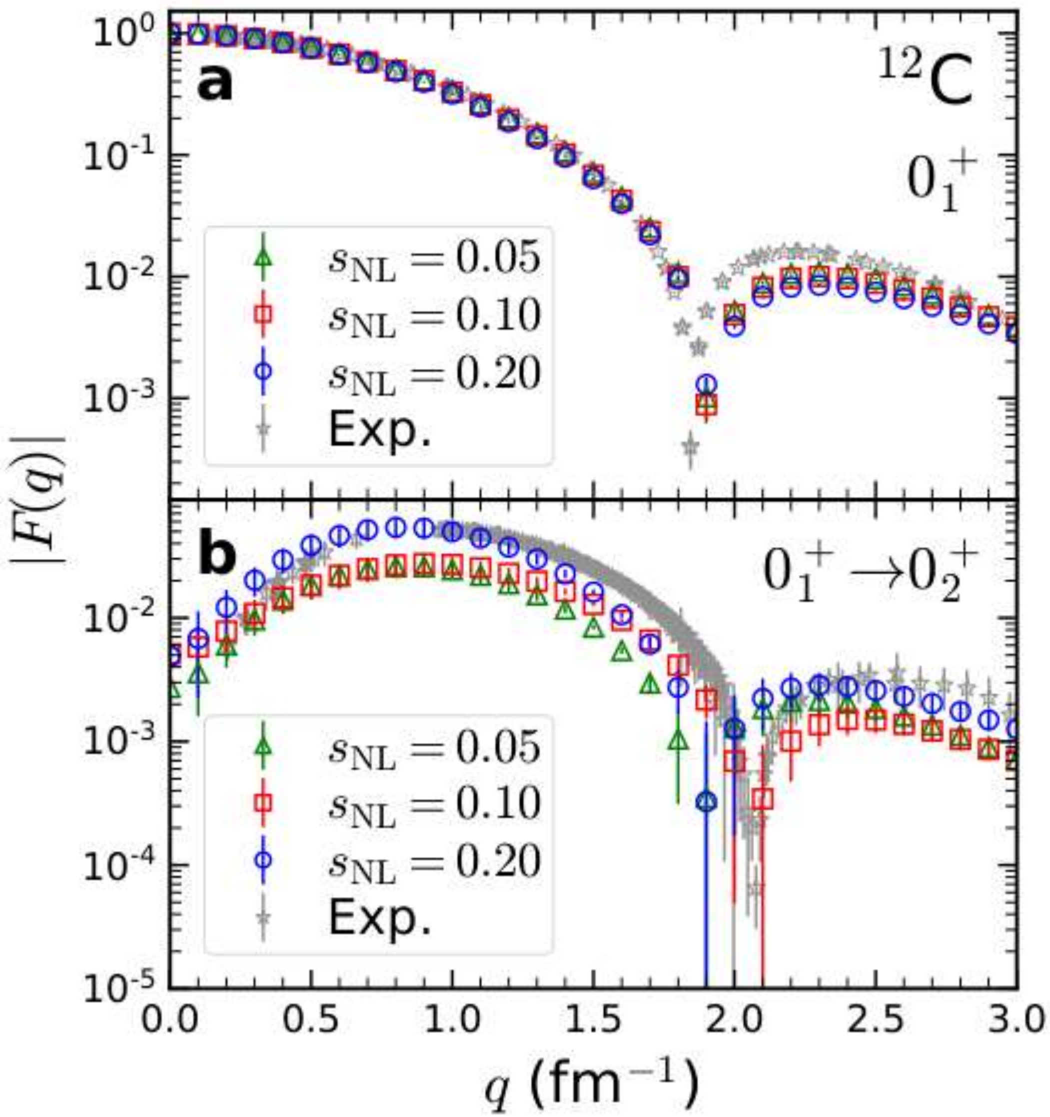


- α cluster structure is less clear due to single-particle excitation, especially when excited to the next shell.

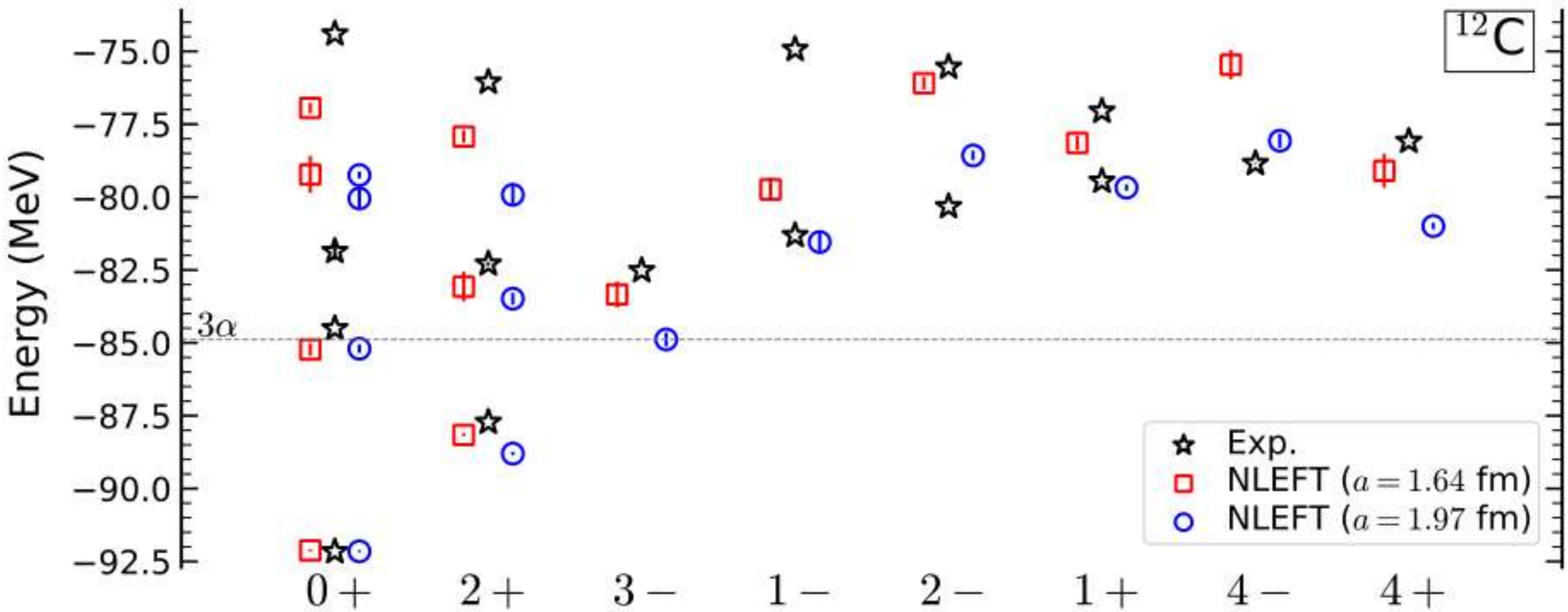
Shell-Model States as Initial Wave



	V1	V2	V3
s_{NL}	0.05	0.1	0.2
s_{L}	0.08	0.071	0.06
C_2 [MeV^{-2}]	-2.15×10^{-5}	-1.11×10^{-5}	-3.47×10^{-6}
C_3 [MeV^{-5}]	6.17×10^{-12}	-5.92×10^{-13}	-1.46×10^{-12}
$E_{^4\mathrm{He}}$ [MeV]	$-28.1(1)$	$-28.3(1)$	$-27.3(1)$
$E_{^{12}\mathrm{C}}$ [MeV]	$-91.6(1)$	$-91.8(1)$	$-90.7(2)$
$r_{c,\,^{12}\mathrm{C}}$ [fm]	2.52(1)	2.55(1)	2.58(1)
E_{Hoyle} [MeV]	$-84.2(1)$	$-84.8(5)$	$-83.2(11)$
$E_{^3\mathrm{H}}$ [MeV]	$-10.1(1)$	$-8.1(1)$	$-5.1(1)$
$r_{c,\,^4\mathrm{He}}$ [fm]	1.63(1)	1.63(1)	1.64(1)
$Q(2_1^+)$ [$e\,\mathrm{fm}^2$]	6.9(3)	7.2(4)	6.1(8)
$M(E0, 0_1^+ \to 0_2^+)$ [$e\,\mathrm{fm}^2$]	4.3(3)	2.9(3)	5.9(7)
$B(E2, 2_1^+ \to 0_1^+)$ [$e^2\mathrm{fm}^4$]	10.3(2)	10.7(3)	12.0(5)
$B(E2, 2_1^+ \to 0_2^+)$ [$e^2\mathrm{fm}^4$]	1.8(1)	3.6(2)	4.1(5)



a
^{12}C
0_1^+
$s_{\mathrm{NL}} = 0.05$
$s_{\mathrm{NL}} = 0.10$
$s_{\mathrm{NL}} = 0.20$
Exp.
b
$0_1^+ \to 0_2^+$
$s_{\mathrm{NL}} = 0.05$
$s_{\mathrm{NL}} = 0.10$
$s_{\mathrm{NL}} = 0.20$
Exp.
$|F(q)|$
q (fm^{-1})



S. Shen, T. A. Lähde, D. Lee, U.-G. Meißner, EPJA 57, 276 (2021)