

Benchmarking nuclear matrix elements of $0\nu\beta\beta$ decay with high-energy nuclear collisions

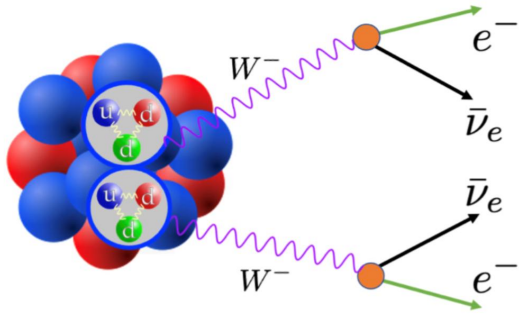
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Introduction

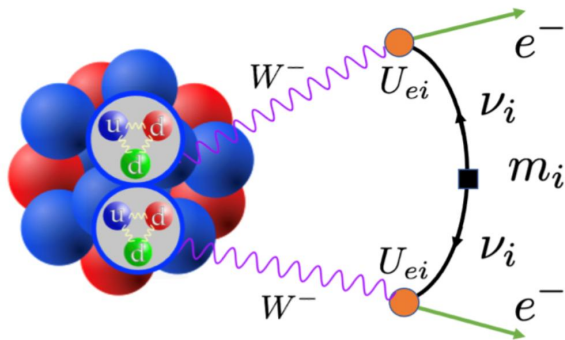
Introduction of neutrinoless double beta ($0\nu\beta\beta$) decay



■ Double beta decay

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$

Allowed by the standard model



■ Neutrinoless double beta decay

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$

Beyond the standard model

- Determine the nature of the neutrino (Dirac or Majorana)
- Demonstrate the violation of the lepton number in nature and reveal the origin of the matter-antimatter asymmetry
- Determine the effective neutrino mass $\langle m_{\beta\beta} \rangle$

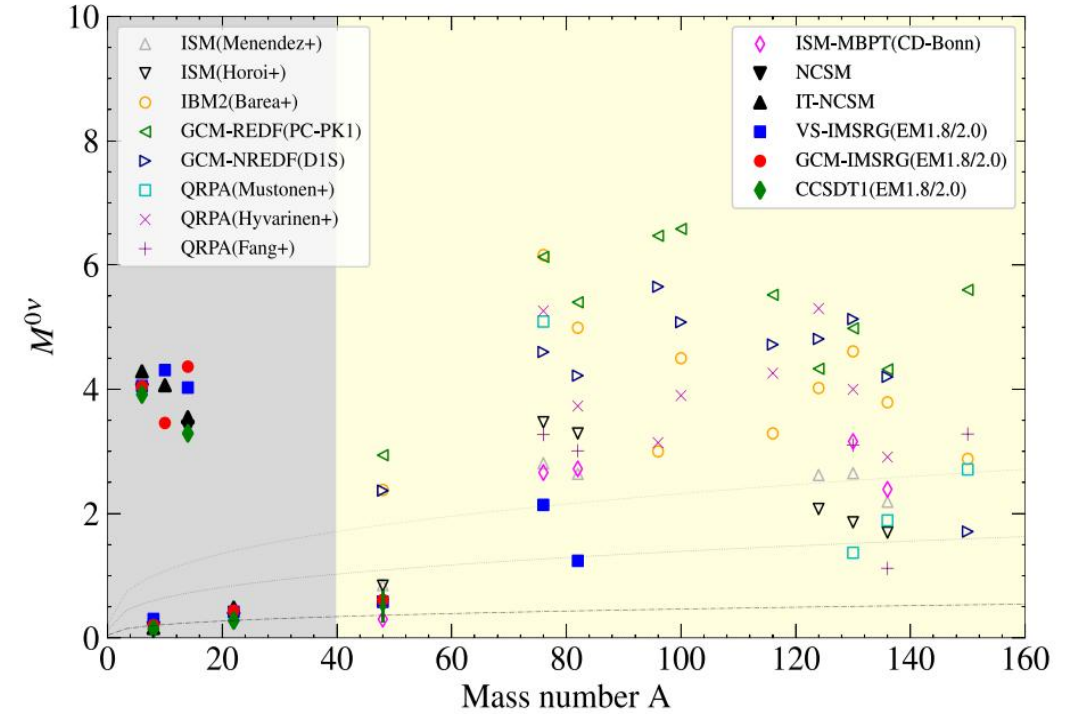
Introduction

NME of $0\nu\beta\beta$ decay

$$|\langle m_{\beta\beta} \rangle| = \left[\frac{m_e^2}{g_A^4(0) G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2} \right]^{1/2}$$

- $m_e \approx 0.511\text{MeV}$ is electron mass.
- $g_A(0)$ is factorized out from the nuclear matrix element (NME) $M^{0\nu}$ (Its free-space value is around 1.27).
- $G_{0\nu} \approx 10^{-14}\text{yr}^{-1}$ is the phase-space factor.
- $T_{1/2}^{0\nu}$ is the half-life of the decaying nucleus, which can be measured experimentally.
- $M^{0\nu}$ is calculated theoretically using a formula:

$$M^{0\nu} = \langle \Psi_F | O^{0\nu} | \Psi_I \rangle$$



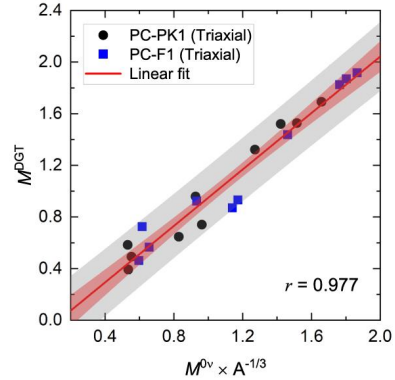
Yao J M, Meng J, Niu Y F, et al. Prog. Part. Nucl. Phys. 126, 103965 (2022)

The NME given by different models varies by 2~3 times !

Introduction

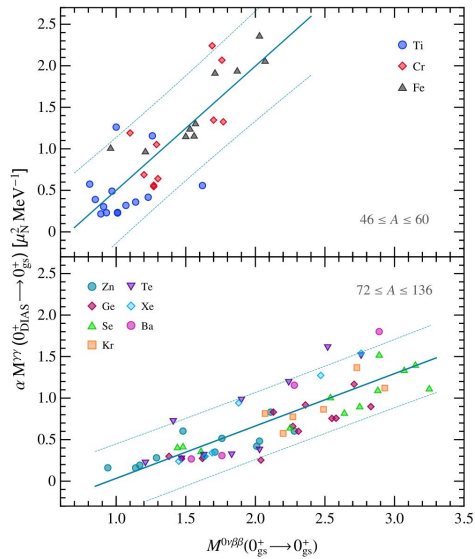
Correlations between the NMEs and observables

double Gamow-Teller transitions



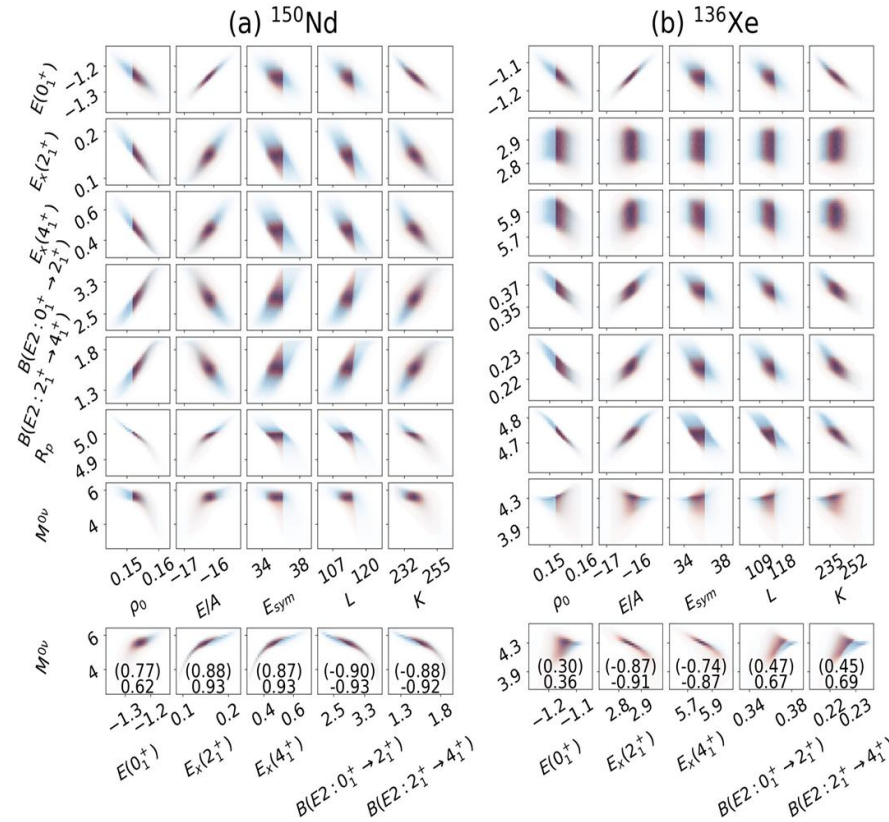
Y.K.Wang, P.W.Zhao et al, PLB 855, 138796(2024)

double-gamma transitions



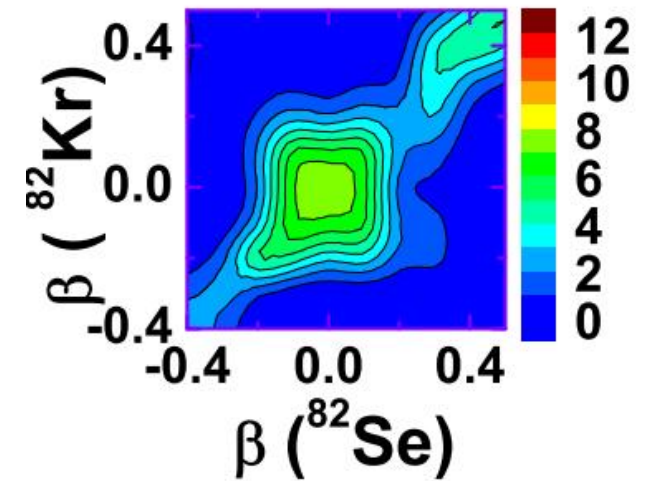
B. Romeo, D. Stramaccioni, et al, PLB 860, 139186 (2025)

low-lying states



X.Zhang et al, arXiv:2408.00691(2024)

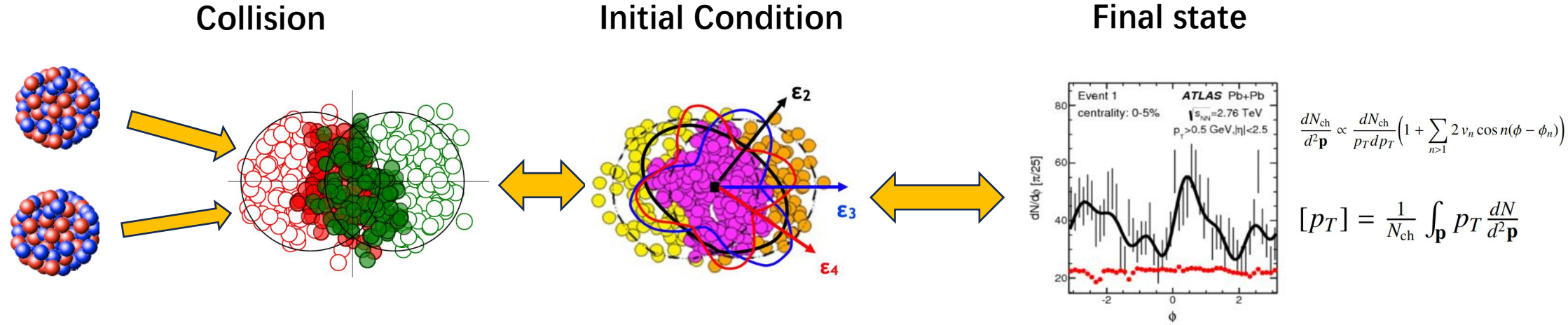
deformation



J.M.Yao etc, PRC 91, 024316(2015)

Introduction

Interfacing nuclear structure with *high-energy nuclear collisions*

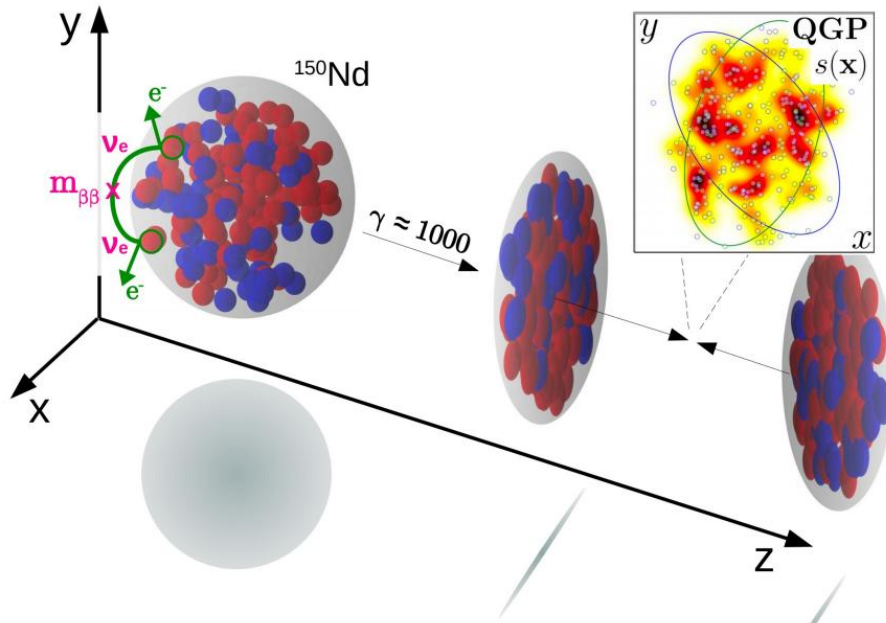


- In the early stages of the collision, a quark-gluon plasma (QGP) is generated under extreme temperature and high energy density conditions.
- After expansion and freeze-out, the QGP eventually forms observable particles that are detected by the detectors in the final state.
- The shape of the atomic nucleus can influence the geometric properties of the QGP that are experimentally accessible via collective flow measurements of final particles.

$$v_n = \kappa_n \epsilon_n$$

Introduction

Motivation



- NMEs are sensitive to the deformation parameters of the candidate isotopes.
- Nuclear deformation information can be probed through high-energy nuclear collisions.

Benchmarking NMEs of $0\nu\beta\beta$ decay with high-energy nuclear collisions !

Theoretical Framework

MR-CDFT

- The relativistic energy density functional (EDF) are composed of the kinetic energy, the electromagnetic energy, as well as the nucleon nucleon (NN) interaction energy

$$E[\tau, \rho, \nabla \rho; \mathbf{C}] = \int d^3r [\tau(\mathbf{r}) + \mathcal{E}^{\text{em}}(\mathbf{r}) + \sum_{\ell=1}^9 c_{\ell} \mathcal{E}_{\ell}^{\text{NN}}(\mathbf{r}; \rho, \nabla \rho)]$$

where the parameter of NN interaction energy denoted by $\mathbf{C} = \{\alpha_S, \beta_S, \gamma_S, \delta_S, \alpha_V, \gamma_V, \delta_V, \alpha_{TV}, \delta_{TV}\}$.

- The single-particle states are determined by minimizing the EDF

$$[\gamma_{\mu}(i\partial^{\mu} - V^{\mu}) - (M + \Sigma_S)]\psi_k = 0.$$

- The mean-field wave functions $|\Phi(\mathbf{q})\rangle$ are determined through a deformation-constrained relativistic mean-field (RMF) model coupled to Bardeen-Cooper-Schrieffer (BCS) theory

$$|\Phi(\mathbf{q})\rangle = \prod_{k>0} (u_k + v_k c_k^{\dagger} c_{\bar{k}}^{\dagger}) |0\rangle$$

- The total number density $\rho_V(\mathbf{r})$ is given by the sum of the densities of each nucleon:

$$\rho_V(\mathbf{r}) = \sum_k v_k^2 \psi_k^{\dagger}(\mathbf{r}) \psi_k(\mathbf{r})$$

Theoretical Framework

MR-CDFT

PNAMP

- Apply angular-momentum(J) and particle-number(N,Z) projections to mean-field wave functions $|\Phi(\mathbf{q})\rangle$

$$|JMNZ, \mathbf{q}\rangle = \hat{P}_{M0}^J \hat{P}^N \hat{P}^Z |\Phi(\mathbf{q})\rangle$$

where q denote a set of collective variables, such as deformation parameter.

GCM

- The collective wave functions of nuclear low-lying states within the generator coordinate method (GCM)

$$|\Psi_{I/F}(J^+)\rangle = \sum_{\mathbf{q}} f_{\nu}^{JNZ}(\mathbf{q}) |JMNZ, \mathbf{q}\rangle$$

where weight function $f_{\nu}^{JNZ}(\mathbf{q})$ is determined by solving the Hill-Wheeler-Griffin equation

$$\sum_{\mathbf{q}'} \left[\mathcal{H}(\mathbf{q}, \mathbf{q}') - E_{\nu}^{JNZ} \mathcal{N}(\mathbf{q}, \mathbf{q}') \right] f_{\nu}^{JNZ}(\mathbf{q}') = 0.$$

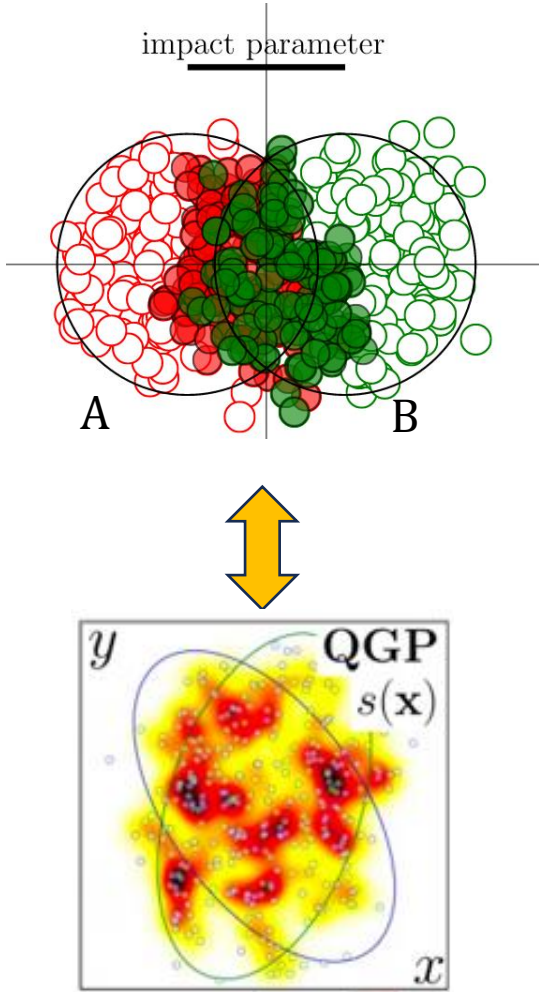
- Calculation of NME

$$M^{0\nu} = \langle \Psi_F(0_1^+) | \hat{O}^{0\nu} | \Psi_I(0_1^+) \rangle$$

where $\hat{O}^{0\nu}$ is the transition operator based on the standard mechanism of exchange of light Majorana neutrinos.

Theoretical Framework

Collision simulations and observables



- The coordinates of A nucleons are sampled for each of the colliding ions using $\rho_V(\mathbf{r})$ as a particle density.
- A given nucleon in nucleus A is flagged as a *participant* if it lies within a radius $\sqrt{\sigma_{NN}/\pi}$ from another nucleon belonging to nucleus B, or vice versa ($\sigma_{NN} = 7\text{fm}^2$ at top LHC energy).

- The thickness functions of the nuclei $t_{A/B}$ are given by a superposition of nucleonic profiles:

$$t_{A/B}(\mathbf{x}) = \sum_{i=1}^{N_{\text{part},A/B}} \frac{\gamma_i}{2\pi w^2} \exp\left(-\frac{(\mathbf{x} - \mathbf{x}_i)^2}{2w^2}\right).$$

- The entropy of the initial state $s(\mathbf{x})$ following the T_RENTo Ansatz

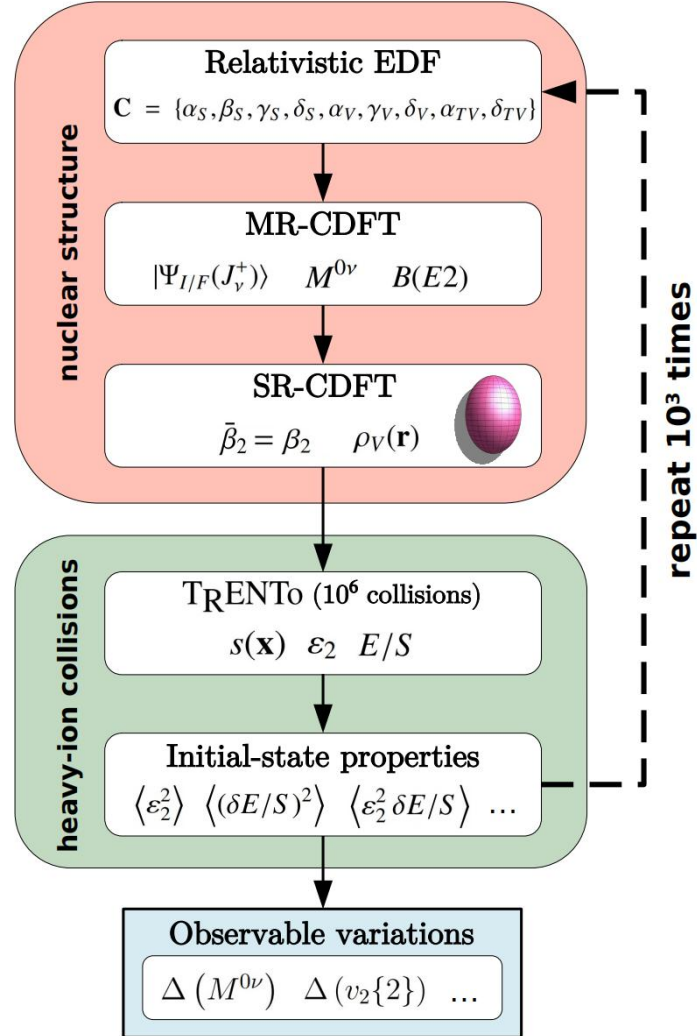
$$s(\mathbf{x}) \propto \left(\frac{t_A(\mathbf{x})^p + t_B(\mathbf{x})^p}{2} \right)^{1/p} \xrightarrow{p=0} \sqrt{t_A(\mathbf{x})t_B(\mathbf{x})}$$

- The spatial anisotropy of QGP ε_n and the energy over the entropy of the system are determined as

$$\varepsilon_n = |\mathcal{E}_n| \quad \mathcal{E}_n = -\frac{\int_{\mathbf{x}} |\mathbf{x}|^n e^{in\phi} s(\tau_0, \mathbf{x})}{\int_{\mathbf{x}} |\mathbf{x}|^n s(\tau_0, \mathbf{x})} \quad E/S = \frac{\int_{\mathbf{x}} e(\mathbf{x})}{\int_{\mathbf{x}} s(\mathbf{x})} \quad e(\mathbf{x}) \propto s(\mathbf{x})^{4/3}$$

Theoretical Framework

Flow chart of framework



- Generate a set \mathcal{C} of parameters of the nuclear EDF

- Evaluate the $M^{0\nu}$ and $BE(2)$ transition strength

- Constraint quadrupole deformation parameter $\bar{\beta}_2 = \beta_2$ by SR-CDFT and calculate the density $\rho_V(\mathbf{r})$, where β_2 determined as

$$\beta_2 = \frac{4\pi}{3ZR_0^2} \sqrt{B(E2; 0_1^+ \rightarrow 2_1^+)}.$$

- Simulate 10^6 times collisions and evaluate the averages

$$\langle \varepsilon_2^2 \rangle, \langle (\delta E/S)^2 \rangle, \langle \varepsilon_2^2 \delta E/S \rangle, 2\langle \varepsilon_2^2 \rangle^2 - \langle \varepsilon_2^4 \rangle$$

- Repeat for 10^3 sets of \mathcal{C} and calculate the variation of observables

$$\Delta(v_2\{2\}) \equiv \frac{1}{2} \Delta(\langle v_2^2 \rangle), \quad \text{elliptic flow}$$

$$\Delta(O) = \frac{O - \langle O \rangle_{\mathcal{C}}}{|\langle O \rangle_{\mathcal{C}}|}$$

$$\Delta(\delta[p_T]) \equiv \frac{1}{2} \Delta(\langle (\delta[p_T])^2 \rangle), \quad \text{fluctuation of transverse momentum}$$

$$v_2 \propto \varepsilon_2$$

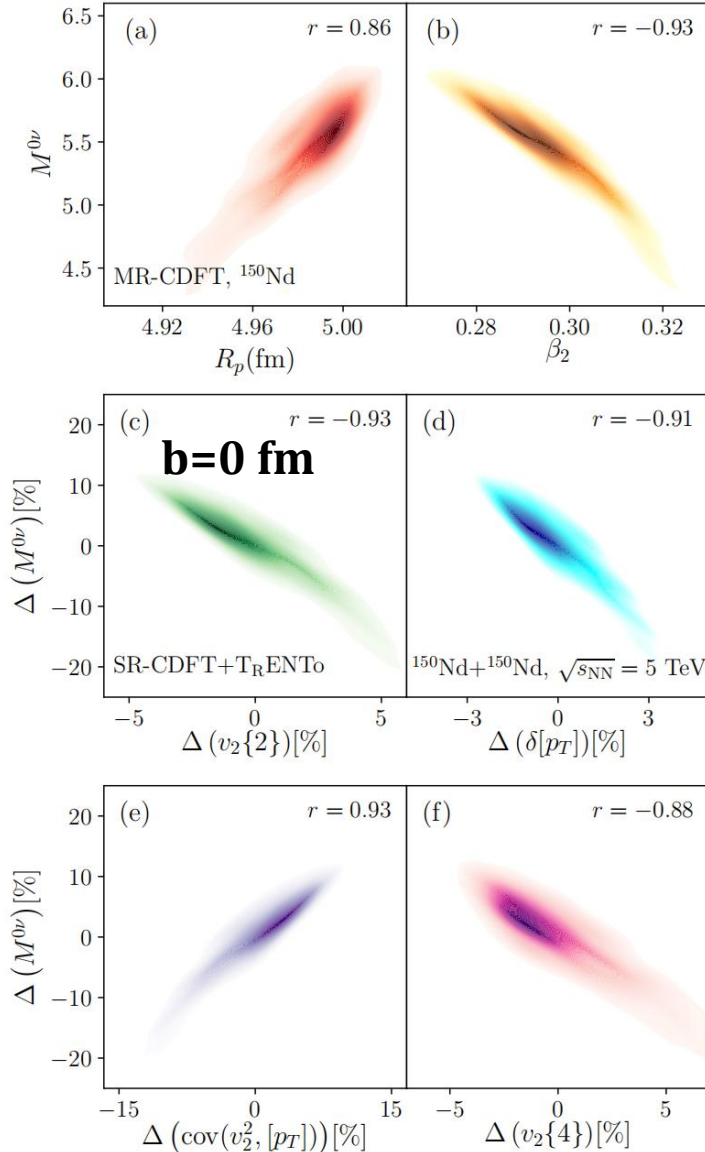
$$[p_T] \propto E/S$$

$$\Delta(\text{cov}(v_2^2, [p_T])) \equiv \frac{1}{3} \Delta(\langle v_2^2 \delta[p_T] \rangle), \quad \text{covariance between } v_2^2 \text{ and } \delta[p_t]$$

$$\Delta(v_2\{4\}) \equiv \frac{1}{4} \Delta(2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle) \quad \text{fourth-order cumulant of the elliptic flow vector}$$

Results

Results and discussion (^{150}Nd)



- NME of is correlated with R_p and β_2 . The relative systematic uncertainty on R_p is less than 0.5% , while β_2 shows variation up to 10%.

- For ultra-central $^{150}\text{Nd} + ^{150}\text{Nd}$ collisions, v_2 , $[p_T]$ fluctuation and $v_2\{4\}$ vary by serval percent. The covariance of v_2^2 and $[p_T]$ presents the strongest variation, changing by about 10%.

Summary

- All of these variations are strongly correlated with the value of the NME.
- By combining Bayesian analysis of low- and high-energy heavy-ion data would will lead to an improved determination of β_2 and thus on the NME.

Perspectives

- Include the **octupole** and the **triaxial** deformation of the candidate isotopes
- Investigate whether combining data sets from $^{150}\text{Nd} + ^{150}\text{Nd}$ and $^{150}\text{Sm} + ^{150}\text{Sm}$ collisions (or other pairs of candidates) gives access to *relative observables* that may present an even tighter correlation with the NME.

Thanks!
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