

# Package 'CPAT'

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<b>Title</b>	Change Point Analysis Tests
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<b>Description</b>	Implements several statistical tests for structural change.
<b>Depends</b>	R (>= 3.2)
<b>Suggests</b>	coinReg (>= 0.2), foreach (>= 1.4), doRNG (>= 1.7), doParallel (>= 1.0), ggplot2 (>= 2.2), dplyr (>= 0.7), tikzDevice (>= 0.12), testthat (>= 2.0)
<b>Imports</b>	stats (>= 3.2), utils (>= 3.2), grDevices (>= 3.2), Rdpack (>= 0.9), methods (>= 3.2), Rcpp (>= 0.12), purrr (>= 0.2),
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.onAttach	
Package Attach Hook Function	
Description	
Hook triggered when package attached	
Usage	
.onAttach(lib, pkg)	
Arguments	
lib	a character string giving the library directory where the package defining the
pkg	namespace was found
Examples	
CPAT:::onAttach(libPaths()[1], "CPAT")	
Andrews.test	
Andrews' Test for End-of-Sample Structural Change	

Performs Andrews' test for end-of-sample structural change, as described in (Andrews 2003). This function works for both univariate and multivariate data depending on the nature of x and whether formula is specified. This function is thus an interface to [andrews.test](#) and [andrews.test-reg](#); see the documentation of those functions for more details.

Usage

Andrews.test(x, M, formula = NULL)

Arguments

- x  
Data to test for change in mean (either a vector or [data.frame](#))
- M  
Numeric index of the location of the first potential change point
- formula  
The regression formula, which will be passed to [lm](#)

Value

A htest-class object containing the results of the test

References

Andrews DWK (2003). "End-of-Sample Instability Tests." *Econometrica*, **71**(6), 1661–1694. ISSN 00129682, 14680262, <https://www.jstor.org/stable/1555535>.

Examples

```
Andrews.test(rnorm(1000), M = 900)
x <- rnorm(1000)
y <- 1 + 2 * x + rnorm(1000)
df <- data.frame(x, y)
Andrews.test(df, y ~ x, M = 900)
```

andrews_test	Univariate Andrews Test for End-of-Sample Structural Change
--------------	---

Description

This implements Andrews’ test for end-of-sample change, as described by Andrews (2003). This test was derived for detecting a change in univariate data. See (Andrews 2003) for a description of the test.

Usage

```
andrews_test(x, M, pval = TRUE, stat = TRUE)
```

Arguments

x	Vector of the data to test
M	Numeric index of the location of the first potential change point
pval	If TRUE, return a p-value
stat	If TRUE, return a test statistic

Value

If both pval and stat are TRUE, a list containing both; otherwise, a number for one or the other, depending on which is TRUE

References

Andrews DWK (2003). “End-of-Sample Instability Tests.” *Econometrica*, **71**(6), 1661–1694. ISSN 00129682, 14680262, <https://www.jstor.org/stable/1555535>.

Examples

```
CPAT:::andrews_test(rnorm(1000), M = 900)
```

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Arguments

n	The parameter $n$
m	The parameter $m$

Value

The number  $a_n(m)$

Examples

CPAT:::a\_n(5, 2)

Description

Data set representing the returns of an industry portfolio representing the banking industry based on company four-digit SIC codes, obtained from the data library maintained by Kenneth French. Data ranges from July 1, 1926 to October 31, 2017.

Usage

banks

Format

A data frame with 24099 rows and 1 variable:

**Banks** The return of a portfolio representing the banking industry

Row names are dates in YYYY-MM-DD format.

Source

[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Description

Extract the base name of the file without path or extension.

Usage

base\_file\_name(x)

Arguments

x	String from which to extract base name
---	--

Arguments

bool	Condition to check; if FALSE, <b>stop</b> is called
message	Message to report if <b>stop</b> is called

Examples

```
x <- 1
CPAT:::stop_with_message(x == 1, message = "x is not 1")
```

---

%s%	<i>Concatenate (With Space)</i>
-----	---------------------------------

---

Description

Concatenate and form strings (with space separation)

Usage

x %s% y

Arguments

x	One object
y	Another object

Value

A string combining x and y with a space separating them

Examples

```
`%s%` <- CPAT:::`%s%`
"Hello" %s% "world"
```

---

%s0%	<i>Concatenate (Without Space)</i>
------	------------------------------------

---

Description

Concatenate and form strings (no space separation)

Usage

x %s0% y

Arguments

x	One object
y	Another object

custom_var	Can be a vector the same length as dat consisting of variance-like numbers at each potential change point (so each entry of the vector would be the "best estimate" of the long-run variance if that location were where the change point occurred) or a function taking two parameters x and k that can be used to generate this vector, with x representing the data vector and k the position of a potential change point; if NULL, this argument is ignored
kernel	If character, the identifier of the kernel function as used in <code>coinKReg</code> (see <code>getLongRunVar</code> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <code>coinKReg</code> )
bandwidth	If character, the identifier for how to compute the bandwidth as defined in <code>coinKReg</code> (see <code>getBandwidth</code> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in <code>coinKReg</code> )
get_all_vals	If TRUE, return all values for the statistic at every tested point in the data set
fast	If TRUE, the test statistic is computed quickly but at a potential loss of numerical accuracy (by solving the normal equations); otherwise, use slower but more numerically stable solution techniques

**Details**

TODOTODO: EXTENDED DESCRIPTION

TODOTODO: THIS FUNCTION DOES NOT WORK AS MARKETED BECAUSE WE'RE STILL WORKING ON THE THEORY; use\_kernel\_var, kernel, AND bandwidth ARE IGNORED AND custom\_var SHOULD NOT BE NULL, BUT CREATE A MATRIX THE SAME DIMENSION AS THE REGRESSION MODEL.

**Value**

If both estimate and get\_all\_vals are FALSE, the value of the test statistic; otherwise, a list that contains the test statistic and the other values requested (if both are TRUE, the test statistic is in the first position and the estimated change point in the second)

<b>Examples</b>	<pre>x &lt;- rnorm(1000) y &lt;- 1 + 2 * x + rnorm(1000) df &lt;- data.frame(x, y) CPAT:::stat_zn_reg(y ~ x, data = df)</pre>
<b>Description</b>	stop_with_message
<b>Usage</b>	Check if bool is TRUE; if not, stop and report message

stop\_with\_message(bool, message = NULL)

<b>Value</b>	A string containing the base file name without extension
<b>Examples</b>	CPAT:::base_filename("~/Documents/test.txt")
<b>Description</b>	Returns the zeros of the Bessel function of the first kind, $J_\nu$ .

<b>Arguments</b>	The (one-based) index of the last zero to return
a	The (one-based) index of the first zero to return (so a = 1 represents the first positive zero)
nu	The order of the Bessel function

**Details**

This function is an interface to the function `besselJ_zeros.cpp`, a function written in C++ and serves effectively as an interface to a Boost C++ function `cyl_besselJ_zero`. Thus this function does nothing other than make the Boost function available to R.

See the references of `besselJ` for more about `bessel` functions.

**Value**

A vector containing the zeros of the Bessel function

**Examples**

```
CPAT:::besselJ_zeros(4)
CPAT:::besselJ_zeros(a = 3, b = 10, nu = 3.5)
```

bind_power_sim_objs	Power Result Data Frame Creation
<b>Description</b>	
Creates a data.frame containing power simulation results. Effectively a better, higher-level interface to <code>power_sim_Zn_to_df</code> and <code>power_sim_Vn_to_df</code> .	
<b>Usage</b>	
bind_power_sim_objs(files, crit_value, conv_func, stat_name)	
<b>Arguments</b>	
files	A character vector of file names
crit_value	The critical value against which to compare a test statistic
conv_func	The function responsible for converting a list containing simulated statistic values under different conditions to a data.frame
stat_name	The label of the statistic
<b>Value</b>	
A data.frame containing power levels	
<b>Examples</b>	
<pre>## Not run: filenames &lt;- c("powerSimulations_sdest_norm_DE.rda",                "powerSimulations_sdest_ar1_0.5_DE.rda") bind_power_sim_objs(filenames, crit_value = qdarling_erdos(.95),                     conv_func = power_sim_Vn_to_df, stat_name = "de")  ## End(Not run)</pre>	
b_n	Sequence b_n of the Darling-Erdős Law

Description

Computes  $b_n(m) = (2 \log \log(n) + (m \log \log \log n)/2 - \log(\Gamma(m/2)))^2 / (2 \log \log n)$

Usage

b\_n(n, m)

Arguments

- n The parameter  $n$
- m The parameter  $m$

Details

The definition of the statistic is

$$\max_{t_T \leq t \leq T-t_T} \hat{\sigma}_{t,T}^{-1} \left| t^{-1} \sum_{s=1}^t X_s - (T-t)^{-1} \sum_{s=t+1}^T X_s \right|$$

The parameter kn corresponds to the trimming parameter  $t_T$ .

Value

If both estimate and get\_all\_vals are FALSE, the value of the test statistic; otherwise, a list that contains the test statistic and the other values requested (if both are TRUE, the test statistic is in the first position and the estimated change point in the second)

Examples

```
CPAT:::stat_Zn(rnorm(1000))
CPAT:::stat_Zn(rnorm(1000), kn = function(n) {floor(log(n))})
CPAT:::stat_Zn(rnorm(1000), use_kernel_var = TRUE, bandwidth = "nw",
               kernel = "bo")
```

stat_Zn_reg	Compute the Rényi-Type Statistic for Stability in Linear Regression Models
<b>Description</b>	
This function computes the Rényi-type statistic for detecting structural change in linear regression models.	
<b>Usage</b>	
stat_Zn_reg(formula, data, kn = function(n) { floor(sqrt(n)) }, estimate = FALSE, use_kernel_var = FALSE, custom_var = NULL, kernel = "ba", bandwidth = "and", get_all_vals = FALSE, fast = FALSE)	

Arguments

- formula The regression formula, which will be passed to `lm`
- data data.frame containing the data
- kn A function corresponding to the trimming parameter  $t_T$ ; by default, the square root function
- estimate Set to TRUE to return the estimated location of the change point
- use\_kernel\_var Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using  $\hat{\sigma}_{T,t}^2 = T^{-1} \left( \sum_{s=1}^t (X_s - \bar{X}_t)^2 + \sum_{s=t+1}^T (X_s - \bar{X}_{T-t})^2 \right)$ , where  $\bar{X}_t = t^{-1} \sum_{s=1}^t X_s$  and  $\bar{X}_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$ ; if custom\_var is not NULL, this argument is ignored





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cpt_consistent_var	<i>Variance Estimation Consistent Under Change</i>
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---

### Description

Estimate the variance (using the sum of squared errors) with an estimator that is consistent when the mean changes at a known point.

### Usage

```
cpt_consistent_var(x, k)
```

### Arguments

x                      A numeric vector for the data set  
k                      The potential change point at which the data set is split

### Details

This is the estimator

$$\hat{\sigma}_{T,t}^2 = T^{-1} \left( \sum_{s=1}^t (X_s - \bar{X}_t)^2 + \sum_{s=t+1}^T (X_s - \tilde{X}_{T-t})^2 \right)$$

where  $\bar{X}_t = t^{-1} \sum_{s=1}^t X_s$  and  $\tilde{X}_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$ . In this implementation,  $T$  is computed automatically as `length(x)` and  $k$  corresponds to  $t$ , a potential change point.

### Value

The estimated change-consistent variance

### Examples

```
CPAT::cpt_consistent_var(c(rnorm(500, mean = 0), rnorm(500, mean = 1)), k = 500)
```

---

CUSUM.test	<i>CUSUM Test</i>
------------	-------------------

---

### Description

Performs the CUSUM test for change in mean, as described in (Rice et al. ).

### Usage

```
CUSUM.test(x, formula = NULL, use_kernel_var = FALSE,  
stat_plot = FALSE, kernel = "ba", bandwidth = "and")
```

### Arguments

dat                      The data vector  
kn                      A function corresponding to the trimming parameter  $t_T$  in the trimmed CUSUM variant; by default, is a function returning 1 (for no trimming)  
tau                      The weighting parameter  $\tau$  for the weighted CUSUM statistic; by default, is 0 (for no weighting)  
estimate                Set to TRUE to return the estimated location of the change point  
use\_kernel\_var        Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using  $\hat{\sigma}_{T,t}^2 = T^{-1} \left( \sum_{s=1}^t (X_s - \bar{X}_t)^2 + \sum_{s=t+1}^T (X_s - \tilde{X}_{T-t})^2 \right)$ , where  $\bar{X}_t = t^{-1} \sum_{s=1}^t X_s$  and  $\tilde{X}_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$   
custom\_var            Can be a vector the same length as `dat` consisting of variance-like numbers at each potential change point (so each entry of the vector would be the "best estimate" of the long-run variance if that location were where the change point occurred) or a function taking two parameters `x` and `k` that can be used to generate this vector, with `x` representing the data vector and `k` the position of a potential change point; if NULL, this argument is ignored  
kernel                If character, the identifier of the kernel function as used in **cointReg** (see [getLongRunVar](#)); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in **cointReg**)  
bandwidth            If character, the identifier for how to compute the bandwidth as defined in **cointReg** (see [getBandwidth](#)); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in **cointReg**)  
get\_all\_vals        If TRUE, return all values for the statistic at every tested point in the data set

### Details

The definition of the statistic is

$$T^{-1/2} \max_{1 \leq t \leq T} \hat{\sigma}_{t,T}^{-1} \left| \sum_{s=1}^t X_s - \frac{t}{T} \sum_{s=1}^T X_s \right|$$

A more general version is

$$T^{-1/2} \max_{t_T \leq t \leq T-t_T} \hat{\sigma}_{t,T}^{-1} \left( \frac{t}{T} \left( \frac{T-t}{T} \right) \right)^{\tau} \left| \sum_{s=1}^t X_s - \frac{t}{T} \sum_{s=1}^T X_s \right|$$

The parameter `kn` corresponds to the trimming parameter  $t_T$  and the parameter `tau` corresponds to  $\tau$ .

See (Rice et al. ) for more details.

### Value

If both `estimate` and `get_all_vals` are FALSE, the value of the test statistic; otherwise, a list that contains the test statistic and the other values requested (if both are TRUE, the test statistic is in the first position and the estimated change point in the second)

$$\mathcal{LM}(s) = \left( \frac{\Delta(\hat{g})s(n-s)}{n} \right) \left( \sum_s^{\iota=1} x_{\iota} u_{\iota} \right) \left( \sum_s^{\iota=1} x_{\iota} u_{\iota} \right)^T$$

and  $\hat{\Delta}(\beta)$  is the long-run variance estimator

$$\hat{\Delta}(\beta) = \frac{1}{m} \sum_{j=1}^m I \left( \frac{n}{2\pi j}; \beta \right)$$

where  $I(\cdot; \beta)$  is the periodogram estimated from the residuals when the regression model coefficients are given by  $\beta$ . This is the test statistic suggested by the procedure introduced in (Hidalgo and Seo 2013).

The parameter  $m$  described above can be controlled via the function parameter  $m$ , which can be either numeric or a function that returns numeric values.

Value

If both estimate and get\_all\_vals are FALSE, the value of the test statistic; otherwise, a list that contains the test statistic and the other values requested (if both are TRUE, the test statistic is in the first position and the estimated change point in the second)

References

Hidalgo J, Seo MH (2013). "Testing for structural stability in the whole sample." *Journal of Econometrics*, **175**(2), 84 - 93. ISSN 0304-4076, doi: 10.1016/j.jeconom.2013.02.008, <http://www.sciencedirect.com/science/article/pii/S0304407613000626>.

Examples

```
x <- rnorm(100)
y <- 1 + 2 * x + rnorm(100)
df <- data.frame("x" = x, "y" = y)
CPAT:::stat_hs_reg(y ~ x, data = df)
```

Description

This function computes the CUSUM statistic (and can compute weighted/trimmed variants, depending on the values of kn and tau).

Usage

```
stat_Vn(dat, kn = function(n) { 1 }, tau = 0, estimate = FALSE,
use_kernel_var = FALSE, custom_var = NULL, kernel = "ba",
bandwidth = "and", get_all_vals = FALSE)

CUSUM.test(df, formula = y ~ x, use_kernel_var = TRUE)

CUSUM.test(rnorm(1000), use_kernel_var = TRUE, kernel = "nw",
bandwidth = "nw")
x <- rnorm(1000)
y <- 1 + 2 * x + rnorm(1000)
df <- data.frame(x, y)
CUSUM.test(df, formula = y ~ x, use_kernel_var = TRUE)
```

Examples

Ploberger W, Krämer W (1992). "The CUSUM test with OLS residuals." *Econometrica*, **60**(2), 271–285.  
Rice G, Miller C, Horváth L (???). "A new class of change point test of Rényi type." in-press.

References

A htest-class object containing the results of the test

Value

This is effectively an interface to `stat_Vn`; see its documentation for more details.  
When  $x$  is a (numeric) vector, the CUSUM test is performed directly on the data. When  $x$  is a `data.frame` and formula is not NULL, then a regression model is estimated first with `lm` and the test is performed on the residuals of the regression model (see (Ploberger and Krämer 1992)).  
p-values are computed using `pkoImogorov`, which represents the limiting distribution of the statistic under the null hypothesis.

Details

**stat\_plot** Whether to create a plot of the values of the statistic at all potential change points  
If character, the identifier of the kernel function as used in `coinKReg` (see `getLongRunVar`);  
if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in `coinKReg`)  
**bandwidth** If character, the identifier for how to compute the bandwidth as defined in `coinKReg` (see `getBandwidth`); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in `coinKReg`)  
**use\_kernel\_var** Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using  $\hat{\sigma}_{T,t}^2 = T^{-1} \left( \sum_{s=1}^t (X_s - \bar{X}_t)_2 + \sum_{s=t+1}^T (X_s - \bar{X}_{T-t})_2 \right)$ , where  $\bar{X}_t = T^{-1} \sum_{s=1}^t X_s$  and  $\bar{X}_{T-t} = T^{-1} \sum_{s=t+1}^T X_s$   
**formula** Formula used for defining the regression model, if applicable  
**x** Data to test for change in mean (either `numeric` or a `data.frame`)

Arguments

dBst	<i>Density Function of the First Hitting Time of a Bessel Process</i>
------	---

### Description

Density function of the distribution of the first time a Bessel process with parameter  $\nu > 1$  hits  $b > 0$ .

### Usage

```
dBst(x, b, nu = -1/2, summands = NULL)
```

### Arguments

x	Points at which to evaluate the density function
b	Point in space Bessel process hits
nu	The parameter $\nu > -1$ of the Bessel process
summands	Number of summands to use in summation; default is to pick the number of summands with <a href="#">dBst_summand_solver</a> (it could be slow, so for performance it may be best to pick a fixed number)

### Details

Let  $\tau_b^{(\nu)}$  be the first time a Bessel process with parameter  $\nu$  hits  $b > 0$ . Let  $J_\nu(x)$  be the Bessel function (of the first kind) with order  $\nu$ , and let  $j_{\nu,k}$  be the  $k$ th zero of  $J_\nu(x)$ . Let  $\Gamma(x)$  be the gamma function. Then the density function of  $\tau_b^{(\nu)}$  is

$$\frac{1}{2^\nu b^2 \Gamma(\nu + 1)} \sum_{k=1}^{\infty} \frac{j_{\nu,k}^{\nu+1}}{J_{\nu+1}(j_{\nu,k})} e^{-\frac{j_{\nu,k}^2}{2b^2} t}$$

This was found by differentiating the CDF computed by [pBst](#).

### Value

The value of the density function at x

### Examples

```
CPAT:::dBst(0.1, 1)
```

### Value

If both estimate and get\_all\_vals are FALSE, the value of the test statistic; otherwise, a list that contains the test statistic and the other values requested (if both are TRUE, the test statistic is in the first position and the estimated change point in the second)

### References

Hidalgo J, Seo MH (2013). “Testing for structural stability in the whole sample.” *Journal of Econometrics*, **175**(2), 84 - 93. ISSN 0304-4076, doi: [10.1016/j.jeconom.2013.02.008](https://doi.org/10.1016/j.jeconom.2013.02.008), <http://www.sciencedirect.com/science/article/pii/S0304407613000626>.

### Examples

```
CPAT:::stat_hs(rnorm(1000))
CPAT:::stat_hs(rnorm(1000), corr = FALSE)
```

stat_hs_reg	<i>Regression Model Hidalgo-Seo Statistic</i>
-------------	---

### Description

Compute the Hidalgo-Seo statistic intended for detecting change in linear models (estimated via least squares regression).

### Usage

```
stat_hs_reg(formula, data, m = sqrt, estimate = FALSE,
  get_all_vals = FALSE)
```

### Arguments

formula	A <a href="#">formula</a> that describes the regression model
data	A <a href="#">data.frame</a> -like object containing the data set; should be able to be passed to the data argument of <a href="#">lm</a>
m	If numeric, the number of terms of the periodogram to sum; if a function, how to compute the number of terms to sum (will be passed the number of rows of data)
estimate	Set to TRUE to return the estimated location of the change point
get_all_vals	If TRUE, return all values for the statistic at every tested point in the data set

### Details

For a data set  $(y_t, x_t)$  with  $n$  observations,  $y_t \in \mathbf{R}$ , and  $x_t \in \mathbf{R}^d$ , the test statistic is

$$\max_{d < s \leq n-d} (\mathcal{LM}(s) - B_n)/A_n$$

where  $a_n = \sqrt{2 \log \log n}$ ;  $b_n = a_n^2 + d \log \log \log n / 2 - \log \Gamma(d/2)$ ;  $A_n = b_n/a_n^2$ ;  $B_n = b_n^2/a_n^2$ ;  $\hat{\beta}$  is the least-squares estimate of the linear regression model coefficients;  $\hat{u}_t = y_t - \hat{\beta}^T x_t$  are the residuals of the model;

Usage	
stat_hs(dat, estimate = FALSE, corr = TRUE, m = sqrt, get_all_vals = FALSE, custom_var = NULL, use_kernel_var = FALSE, kernel = "ba", bandwidth = "and")	
Arguments	
dat	The data vector
estimate	Set to TRUE to return the estimated location of the change point
corr	If TRUE, the long-run variance will be computed under the assumption of correlated residuals; ignored if custom_var is not NULL or use_kernel_var is TRUE
m	Either numeric or a function that returns numeric; corresponds to $m$ used in computing the estimate of the long-run variance
get_all_vals	If TRUE, return all values for the statistic at every tested point in the data set
custom_var	Can be a vector the same length as dat consisting of variance-like numbers at each potential change point (so each entry of the vector would be the "best estimate" of the long-run variance if that location were where the change point occurred) or a function taking two parameters $x$ and $k$ that can be used to generate this vector, with $x$ representing the data vector and $k$ the position of a potential change point; if NULL, this argument is ignored
use_kernel_var	Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using $\hat{\sigma}_{T,t}^2 = T^{-1} \left( X_s - \bar{X}_t \right)^2 + \sum_{T^{s=t+1}}^T X_s - \bar{X}_{T-t} \Bigg)$ where $\bar{X}_t = t^{-1} \sum_{s=1}^t X_s$ and $\bar{X}_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$ ; if custom_var is not NULL, this argument is ignored
kernel	If character, the identifier of the kernel function as used in <b>coinKreg</b> (see <b>getLongRunVar</b> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <b>coinKreg</b> )
bandwidth	If character, the identifier for how to compute the bandwidth as defined in <b>coinKreg</b> (see <b>getBandwidth</b> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in <b>coinKreg</b> )
Details	
kernel	The number of summands needed is determined by using a loop that runs over the summands until it encounters a summand that is not greater than the specified level of numerical accuracy. The index of that last summand is then returned.
Value	Integer for number of summands
Examples	
dbst_summand_solver(1, 1)	
Description	
Find the number of summands needed to achieve numerical accuracy of the sum involved in <a href="#">dbst</a> .	
dbst_summand_solver	Find Number of Summands Needed for Numerical Accuracy of dbst

**Arguments**

x	Data to test for change in mean (either a <a href="#">numeric</a> vector or a <a href="#">data.frame</a> )
formula	Formula used for defining the regression model, if applicable
a	The function that will be composed with $l(x) = (2 \log x)^{1/2}$
b	The function that will be composed with $u(x) = 2 \log x + \frac{1}{2} \log \log x - \frac{1}{2} \log \pi$
use_kernel_var	Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using $\hat{\sigma}_{T,t}^2 = T^{-1} \left( \sum_{s=1}^t (X_s - \bar{X}_t)^2 + \sum_{s=t+1}^T (X_s - \bar{X}_{T-t})^2 \right)$ , where $\bar{X}_t = t^{-1} \sum_{s=1}^t X_s$ and $\bar{X}_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$
stat_plot	Whether to create a plot of the values of the statistic at all potential change points
kernel	If character, the identifier of the kernel function as used in <b>cointReg</b> (see <a href="#">getLongRunVar</a> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <b>cointReg</b> )
bandwidth	If character, the identifier for how to compute the bandwidth as defined in <b>cointReg</b> (see <a href="#">getBandwidth</a> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in <b>cointReg</b> )

**Details**

This is effectively an interface to [stat\\_de](#); see its documentation for more details.

When x is a (numeric) vector, the CUSUM test is performed directly on the data. When x is a [data.frame](#) and formula is not NULL, then a regression model is estimated first with [lm](#) and the test is performed on the residuals of the regression model.

p-values are computed using [pdarling\\_erdos](#), which represents the limiting distribution of the test statistic under the null hypothesis when a and b are chosen appropriately. (Change those parameters at your own risk!)

**Value**

A htest-class object containing the results of the test

**References**

Rice G, Miller C, Horváth L (????). "A new class of change point test of Rényi type." in-press.

**Examples**

```
DE.test(rnorm(1000))
DE.test(rnorm(1000), use_kernel_var = TRUE, kernel = "bo", bandwidth = "nw")
x <- rnorm(1000)
y <- 1 + 2 * x + rnorm(1000)
df <- data.frame(x, y)
DE.test(df, formula = y ~ x, use_kernel_var = TRUE)
```

custom_var	Can be a vector the same length as dat consisting of variance-like numbers at each potential change point (so each entry of the vector would be the "best estimate" of the long-run variance if that location were where the change point occurred) or a function taking two parameters x and k that can be used to generate this vector, with x representing the data vector and k the position of a potential change point; if NULL, this argument is ignored
kernel	If character, the identifier of the kernel function as used in <b>cointReg</b> (see <a href="#">getLongRunVar</a> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <b>cointReg</b> )
bandwidth	If character, the identifier for how to compute the bandwidth as defined in <b>cointReg</b> (see <a href="#">getBandwidth</a> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in <b>cointReg</b> )
get_all_vals	If TRUE, return all values for the statistic at every tested point in the data set

**Details**

If  $\bar{A}_T(\tau, t_T)$  is the weighted and trimmed CUSUM statistic with weighting parameter  $\tau$  and trimming parameter  $t_T$  (see [stat\\_Vn](#)), then the Darling-Erdős statistic is

$$l(a_T) \bar{A}_T(1/2, 1) - u(b_T)$$

with  $l(x) = \sqrt{2 \log x}$  and  $u(x) = 2 \log x + \frac{1}{2} \log \log x - \frac{1}{2} \log \pi$  ( $\log x$  is the natural logarithm of  $x$ ). The parameter a corresponds to  $a_T$  and b to  $b_T$ ; these are both  $\log$  by default.

See (Rice et al. ) to learn more.

**Value**

If both estimate and get\_all\_vals are FALSE, the value of the test statistic; otherwise, a list that contains the test statistic and the other values requested (if both are TRUE, the test statistic is in the first position and the estimated change point in the second)

**References**

Rice G, Miller C, Horváth L (????). "A new class of change point test of Rényi type." in-press.

**Examples**

```
CPAT:::stat_de(rnorm(1000))
CPAT:::stat_de(rnorm(1000), use_kernel_var = TRUE, bandwidth = "nw", kernel = "bo")
```

stat\_hs

Compute the Univariate Hidalgo-Seo Statistic

**Description**

This function computes the Hidalgo-Seo statistic for a change in mean model.

dist_conv_plot_tikz	Create Tikz Plot Demonstrating Rényi-Type Statistic's Convergence in Distribution
---------------------	---

**Description**

Create a Tikz file containing a plot demonstrating that the Rényi-type statistic converges in distribution. Optionally, create a PDF as well.

**Usage**

```
dist_conv_plot_tikz(obj, dist, trim, size, title = "", width = 4, height = 3, filename = NULL, makePDF = TRUE, verbose = TRUE)
```

**Arguments**

obj The list containing the simulations

dist The identifier of the data-generating process that generated the datasets on which

the Rényi-type statistic was computed

trim The identifier of the trimming parameter of the Rényi-type statistic

size The sample size of the simulated data sets

title The title of the plot

width The width of the plot

height The height of the plot

filename The name of the output file (without extensions: .tex and maybe .pdf files will be created); if NULL, the name will automatically be determined (of the form

makePDF Automatically compile the resulting .tex file

verbose Print updates about progress (via link[base]{cat})

**Examples**

```
## Not run:
ZnSimulations <- list(
  "norm" = list(
    "log" = list(
      "n500" = c(3.206, 1.32, 0.776, 1.262, 0.676)
    )
  )
)
dist_conv_plot_tikz(ZnSimulations, "norm", "log", 500)
## End(Not run)
```

**Details**

This differs from sim\_Zn() in that the long-run variance is estimated with this function, while sim\_Zn() assumes the long-run variance is known. Estimation can be done in a variety of ways. If use\_kernel\_var is set to TRUE, long-run variance estimation using kernel-based techniques will be employed; otherwise, a technique resembling standard variance estimation will be employed. Any technique employed, though, will account for the potential break points, as described in Rice et al. (). See the documentation for [stat\\_Zn](#) for more details.

The parameters kernel and bandwidth control parameters for long-run variance estimation using kernel methods. These parameters will be passed directly to [stat\\_Zn](#).

**Value**

A vector of simulated realizations of the Rényi-type statistic

**References**

Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, **59**(3), 817-858.

Rice G, Miller C, Horvath L (???). "A new class of change point test of Rényi type." in-press.

**Examples**

```
CAPT:::sim_Zn_stat(100)
# kn = function(n) { floor(log(n)) },
# gen_func = CAPT:::rchangepoint,
# args = list(changepoint = 250, mean2 = 1) )
```

**Description**

This function computes the Darling-Erdős statistic.

**Usage**

```
stat_de(dat, a = log, b = log, estimate = FALSE,
use_kernel_var = FALSE, custom_var = NULL, kernel = "ba",
bandwidth = "and", get_all_vals = FALSE)
```

**Arguments**

dat The data vector

a The function that will be composed with  $l(x) = (2 \log x)^{1/2}$

b The function that will be composed with  $u(x) = 2 \log x + \frac{1}{2} \log \log x - \frac{1}{2} \log \pi$

estimate Set to TRUE to return the estimated location of the change point

use\_kernel\_var Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using  $\phi_{T,t}^2$

$\left( X_s - X_{T-t} \right)_2 + \sum_{s=t+1}^T \left( X_s - X_{T-t} \right)_2$

where  $X_t = t^{-1} \sum_{s=1}^t X_s$  and  $X_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$

---

dZn	<i>Rényi-Type Statistic Limiting Distribution Density Function</i>
-----	--

---

### Description

Function for computing the value of the density function of the limiting distribution of the Rényi-type statistic.

### Usage

```
dZn(x, d = 1, summands = NULL)
```

### Arguments

x	Point at which to evaluate the density function (note that this parameter is not vectorized)
d	Dimension parameter
summands	Number of summands to use in summation (the default should be machine accurate)

### Details

The density function was found by differentiating the CDF, as described by [pZn](#).

### Value

Value of the density function at  $x$

### Examples

```
CPAT:::dZn(1)
```

---

ff	<i>Fama-French Five Factors</i>
----	---------------------------------

---

### Description

Data set containing the five factors described by Fama and French (2015), from the data library maintained by Kenneth French. Data ranges from July 1, 1963 to October 31, 2017.

### Usage

```
ff
```

### Value

A vector of simulated realizations of the Rényi-type statistic

### Examples

```
CPAT:::sim_Zn(100, kn = function(n) {floor(log(n))})
CPAT:::sim_Zn(100, kn = function(n) {floor(log(n))},
  gen_func = CPAT:::rchangeoint, args = list(changepoint = 250,
                                             mean2 = 1))
```

---

sim_Zn_stat	<i>Rényi-Type Statistic Simulation</i>
-------------	--

---

### Description

Simulates multiple realizations of the Rényi-type statistic.

### Usage

```
sim_Zn_stat(size, kn = function(n) { floor(sqrt(n)) },
  use_kernel_var = FALSE, kernel = "ba", bandwidth = "and",
  n = 500, gen_func = rnorm, args = NULL, parallel = FALSE)
```

### Arguments

size	Number of realizations to simulate
kn	A function returning a positive integer that is used in the definition of the Rényi-type statistic effectively setting the bounds over which the maximum is taken
use_kernel_var	Set to TRUE to use kernel-based long-run variance estimation (FALSE means this is not employed)
kernel	If character, the identifier of the kernel function as used in the <b>cointReg</b> (see documentation for <code>cointReg::getLongRunVar</code> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <b>cointReg</b> ); this parameter has no effect if <code>use_kernel_var</code> is FALSE
bandwidth	If character, the identifier of how to compute the bandwidth as defined in the <b>cointReg</b> package (see documentation for <code>cointReg::getLongRunVar</code> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth to use (the default behavior is to use the Andrews (1991) method, as used in <b>cointReg</b> ); this parameter has no effect if <code>use_kernel_var</code> is FALSE
n	The sample size for each realization
gen_func	The function generating the random sample from which the statistic is computed
args	A list of arguments to be passed to <code>gen_func</code>
parallel	Whether to use the <b>foreach</b> and <b>doParallel</b> packages to parallelize simulation (which needs to be initialized in the global namespace before use)



<code>getLongRunWeights</code>							
<b>Details</b>	This differs from <code>sim_vn()</code> in that the long-run variance is estimated with this function, while <code>sim_vn()</code> assumes the long-run variance is known. Estimation can be done in a variety of ways. If <code>use_kernel_var</code> is set to <code>TRUE</code> , long-run variance estimation using kernel-based techniques will be employed; otherwise, a technique resembling standard variance estimation will be employed. Any technique employed, though, will account for the potential break points, as described in Rice et al. ( ). See the documentation for <code>stat_vn</code> for more details.						
<b>Value</b>	A vector of simulated realizations of the CUSUM statistic						
<b>References</b>	Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." <i>Econometrica</i> , <b>59</b> (3), 817-858. Rice G, Miller C, Horvath L (???) <i> </i> ,"A new class of change point test of Rényi type." in-press.						
<b>Examples</b>	<pre>CPAT:::sim_vn_stat(100)   use_kernel_var = TRUE, gen_func = CPAT:::change_point,   args = list(change_point = 250, mean2 = 1))</pre>						
<b>sim_Zn</b>	<i>Rényi-Type Statistic Simulation (Assuming Variance)</i>						
<b>Description</b>	Compute some weights for long-run variance. This code comes directly from the source code of <code>coinReg</code> ; see <code>getLongRunWeights</code> .						
<b>Usage</b>	<code>getLongRunWeights(n, bandwidth, kernel = "ba")</code>						
<b>Arguments</b>	<table><tr><td><code>n</code></td><td>Length of weights' vector</td></tr><tr><td><code>bandwidth</code></td><td>A number for the bandwidth</td></tr><tr><td><code>kernel</code></td><td>The kernel function; see <code>getLongRunVar</code> for possible values</td></tr></table>	<code>n</code>	Length of weights' vector	<code>bandwidth</code>	A number for the bandwidth	<code>kernel</code>	The kernel function; see <code>getLongRunVar</code> for possible values
<code>n</code>	Length of weights' vector						
<code>bandwidth</code>	A number for the bandwidth						
<code>kernel</code>	The kernel function; see <code>getLongRunVar</code> for possible values						
<b>Value</b>	List with components <code>w</code> containing the vector of weights and upper, the index of the largest non-zero entry in <code>w</code>						
<b>Examples</b>	<pre>CPAT:::getLongRunWeights(10, 1)</pre>						

<code>sim_Zn</code>													
<b>Details</b>	This differs from <code>sim_vn()</code> in that the long-run variance is estimated with this function, while <code>sim_vn()</code> assumes the long-run variance is known. Estimation can be done in a variety of ways. If <code>use_kernel_var</code> is set to <code>TRUE</code> , long-run variance estimation using kernel-based techniques will be employed; otherwise, a technique resembling standard variance estimation will be employed. Any technique employed, though, will account for the potential break points, as described in Rice et al. ( ). See the documentation for <code>stat_vn</code> for more details.												
<b>Value</b>	A vector of simulated realizations of the CUSUM statistic												
<b>References</b>	Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." <i>Econometrica</i> , <b>59</b> (3), 817-858. Rice G, Miller C, Horvath L (???) <i> </i> ,"A new class of change point test of Rényi type." in-press.												
<b>Examples</b>	<pre>CPAT:::sim_vn_stat(100)   use_kernel_var = TRUE, gen_func = CPAT:::change_point,   args = list(change_point = 250, mean2 = 1))</pre>												
<b>sim_Zn</b>	<i>Rényi-Type Statistic Simulation (Assuming Variance)</i>												
<b>Description</b>	Simulates multiple realizations of the Rényi-type statistic when the long-run variance of the data is known.												
<b>Usage</b>	<code>sim_Zn(size, kn, n = 500, gen_func = rnorm, args = NULL, sd = 1)</code>												
<b>Arguments</b>	<table><tr><td><code>size</code></td><td>Number of realizations to simulate</td></tr><tr><td><code>kn</code></td><td>A function returning a positive integer that is used in the definition of the Rényi-type statistic effectively setting the bounds over which the maximum is taken</td></tr><tr><td><code>n</code></td><td>The sample size for each realization</td></tr><tr><td><code>gen_func</code></td><td>The function generating the random sample from which the statistic is computed</td></tr><tr><td><code>args</code></td><td>A list of arguments to be passed to <code>gen_func</code></td></tr><tr><td><code>sd</code></td><td>The square root of the second moment of the data</td></tr></table>	<code>size</code>	Number of realizations to simulate	<code>kn</code>	A function returning a positive integer that is used in the definition of the Rényi-type statistic effectively setting the bounds over which the maximum is taken	<code>n</code>	The sample size for each realization	<code>gen_func</code>	The function generating the random sample from which the statistic is computed	<code>args</code>	A list of arguments to be passed to <code>gen_func</code>	<code>sd</code>	The square root of the second moment of the data
<code>size</code>	Number of realizations to simulate												
<code>kn</code>	A function returning a positive integer that is used in the definition of the Rényi-type statistic effectively setting the bounds over which the maximum is taken												
<code>n</code>	The sample size for each realization												
<code>gen_func</code>	The function generating the random sample from which the statistic is computed												
<code>args</code>	A list of arguments to be passed to <code>gen_func</code>												
<code>sd</code>	The square root of the second moment of the data												

---

get_expanding_window_pvals
<i>Expanding Window p-Values</i>

---

**Description**

Gets p-values for the CUSUM, Darling-Erdős, Hidalgo-Seo, Andrews, and Rényi-type tests when applied to an expanding window of data.

**Usage**

```
get_expanding_window_pvals(dat, m = Inf)
```

**Arguments**

**dat** The dataset for which to test for change in mean  
**m** The location of the first potential change point for Andrews' test

**Value**

A matrix containing p-values for an expanding sample size, with each row corresponding to one observation larger; columns are labeled for each statistic

**Examples**

```
if (require("foreach") & require("doParallel")) {
  CPAT:::get_expanding_window_pvals(rnorm(1000), m = 900)
}
```

---

get_expanding_window_pvals_reg
<i>Expanding Window p-Values for Regression Models</i>

---

**Description**

Gets p-values for the CUSUM, Darling-Erdős, Hidalgo-Seo, Andrews, and Rényi-type tests when applied to an expanding window of data for a regression model.

**Usage**

```
get_expanding_window_pvals_reg(formula, data, min_n = 3, m = Inf,
  verbose = FALSE)
```

**Arguments**

**formula** The regression model formula, which will be passed to `lm`  
**data** A `data.frame`, the dataset for which to test for structural change  
**min\_n** An integer; the minimum sample size  
**m** The location of the first potential change point for Andrews' test  
**verbose** If TRUE, send messages to output

**Examples**

```
CPAT:::sim_Vn(100)
CPAT:::sim_Vn(100, gen_func = CPAT:::rchangepoint,
  args = list(changepoint = 250, mean2 = 1))
```

---

sim_Vn_stat
<i>CUSUM Statistic Simulation</i>

---

**Description**

Simulates multiple realizations of the CUSUM statistic.

**Usage**

```
sim_Vn_stat(size, kn = function(n) { 1 }, tau = 0,
  use_kernel_var = FALSE, kernel = "ba", bandwidth = "and",
  n = 500, gen_func = rnorm, args = NULL, parallel = FALSE)
```

**Arguments**

**size** Number of realizations to simulate  
**kn** A function returning a positive integer that is used in the definition of the trimmed CUSUM statistic effectively setting the bounds over which the maximum is taken  
**tau** The weighting parameter for the weighted CUSUM statistic (defaults to zero for no weighting)  
**use\_kernel\_var** Set to TRUE to use kernel-based long-run variance estimation (FALSE means this is not employed)  
**kernel** If character, the identifier of the kernel function as used in the **cointReg** (see documentation for `cointReg::getLongRunVar`); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in **cointReg**); this parameter has no effect if `use_kernel_var` is FALSE  
**bandwidth** If character, the identifier of how to compute the bandwidth as defined in the **cointReg** package (see documentation for `cointReg::getLongRunVar`); if function, a function to use for computing the bandwidth; if numeric, the bandwidth to use (the default behavior is to use the method described in (Andrews 1991), as used in **cointReg**); this parameter has no effect if `use_kernel_var` is FALSE  
**n** The sample size for each realization  
**gen\_func** The function generating the random sample from which the statistic is computed  
**args** A list of arguments to be passed to `gen_func`  
**parallel** Whether to use the **foreach** and **doParallel** packages to parallelize simulation (which needs to be initialized in the global namespace before use)

Details	Value
A matrix containing p-values for an expanding sample size, with each row corresponding to one observation larger; columns are labeled for each statistic	
<b>Examples</b>	
<pre>x &lt;- rnorm(1000) y &lt;- 1 + 2 * x + rnorm(1000) df &lt;- data.frame(x, y) if (require("foreach") &amp; require("doparallel")) {   CPT:::get_expanding_window_vals_reg(y ~ x, data = df, min_n = 4, m = 900) }</pre>	
<b>Description</b>	
<i>Long-Run Variance Estimation With Possible Change Points</i>	

Computes the estimates of the long-run variance in a change point context, as described in (Rice et al. ). By default it uses kernel and bandwidth selection as used in the package **coinReg**, though changing the parameters `kernel` and `bandwidth` can change this behavior. If **coinReg** is not installed, the Bartlett internal (defined internally) will be used and the bandwidth will be the square root of the sample size.

**Usage**

```
get_Lrv_vec(dat, kernel = "ba", bandwidth = "and")
```

Arguments	Value
<code>dat</code>	The data vector
<code>kernel</code>	If character, the identifier of the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <b>coinReg</b> )
<code>bandwidth</code>	If character, the identifier for how to compute the bandwidth as defined in <b>coinReg</b> (see <b>getBandwidth</b> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in <b>coinReg</b> )

A vector of estimates of the long-run variance

References	Examples
Rice G, Miller C, Horvath L (???). "A new class of change point test of Rényi type." in-press.	<pre>x &lt;- rnorm(1000) CPT:::get_Lrv_vec(x) CPT:::get_Lrv_vec(x, kernel = "pa", bandwidth = "nw")</pre>

Details	Value
If <code>corr</code> is TRUE, then the residuals of the data-generating process are assumed to be correlated and the test accounts for this in long-run variance estimation; see the documentation for <b>stats</b> for more details. Otherwise, the sample variance is the estimate for the long-run variance, as described in Hidalgo and Seo (2013).	
A vector of simulated realizations of the Hidalgo-Seo statistic	
References	
Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." <i>Econometrica</i> , <b>59</b> (3), 817-858.	
Hidalgo J, Seo MH (2013). "Testing for structural stability in the whole sample." <i>Journal of Econometrics</i> , <b>175</b> (2), 84 - 93. ISSN 0304-4076, doi: 10.1016/j.jeconom.2013.02.008, <a href="http://www.sciencedirect.com/science/article/pii/S0304407613000626">http://www.sciencedirect.com/science/article/pii/S0304407613000626</a> .	

**Description**

Simulates multiple realizations of the CUSUM statistic when the long-run variance of the data is known.

**Usage**

```
sim_Vn(size, n = 500, gen_func = rnorm, sd = 1, args = NULL)
```

Arguments	Value
<code>size</code>	Number of realizations to simulate
<code>n</code>	The sample size for each realization
<code>gen_func</code>	The function generating the random sample from which the statistic is computed
<code>sd</code>	The square root of the second moment of the data
<code>args</code>	A list of arguments to be passed to <code>gen_func</code>

**Value**

A vector of simulated realizations of the CUSUM statistic

HR.test	Rényi-Type Test
---------	-----------------

### Description

Performs the (univariate) Rényi-type test for change in mean, as described in (Rice et al. ). This is effectively an interface to `stat.Zn`; see its documentation for more details. p-values are computed using `pZn`, which represents the limiting distribution of the test statistic under the null hypothesis, which represents the limiting distribution of the test statistic under the null hypothesis when `kn` represents a sequence  $t_T$  satisfying  $t_T \rightarrow \infty$  and  $t_T/T \rightarrow 0$  as  $T \rightarrow \infty$ . (`log` and `sqr` should be good choices.)

### Usage

```
HR.test(x, formula = NULL, kn = log, use_kernel_var = FALSE,
       stat_plot = FALSE, kernel = "ba", bandwidth = "and")
```

### Arguments

x	Data to test for change in mean
formula	The regression formula, which will be passed to <code>lm</code>
kn	A function corresponding to the trimming parameter $t_T$ ; by default, the square root function
use_kernel_var	Set to TRUE to use kernel methods for long-run variance estimation (typically used when the data is believed to be correlated); if FALSE, then the long-run variance is estimated using $\hat{\sigma}_{T,t}^2 = T^{-1} \left( \sum_{s=1}^t (X_s - \bar{X}_t)^2 + \sum_{s=t+1}^T (X_s - \bar{X}_{T-t})^2 \right)$ , where $\bar{X}_t = t^{-1} \sum_{s=1}^t X_s$ and $\bar{X}_{T-t} = (T-t)^{-1} \sum_{s=t+1}^T X_s$ ; if <code>custom_var</code> is not NULL, this argument is ignored
stat_plot	Whether to create a plot of the values of the statistic at all potential change points
kernel	If character, the identifier of the kernel function as used in <code>cointReg</code> (see <code>getLongRunVar</code> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <code>cointReg</code> )
bandwidth	If character, the identifier for how to compute the bandwidth as defined in <code>cointReg</code> (see <code>getBandwidth</code> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth value to use (the default is to use Andrews' method, as used in <code>cointReg</code> )

### Value

A `htest`-class object containing the results of the test

### References

Rice G, Miller C, Horváth L (????). "A new class of change point test of Rényi type." in-press.

### References

- Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, **59**(3), 817-858.
- Rice G, Miller C, Horváth L (????). "A new class of change point test of Rényi type." in-press.

### Examples

```
CPAT:::sim_de_stat(100)
CPAT:::sim_de_stat(100, use_kernel_var = TRUE,
                  gen_func = CPAT:::rchangepoint,
                  args = list(changepoint = 250, mean2 = 1))
```

sim_hs_stat	Hidalgo-Seo Statistic Simulation
-------------	----------------------------------

### Description

Simulates multiple realizations of the Hidalgo-Seo statistic.

### Usage

```
sim_hs_stat(size, corr = TRUE, gen_func = rnorm, args = NULL,
            n = 500, parallel = FALSE, use_kernel_var = FALSE, kernel = "ba",
            bandwidth = "and")
```

### Arguments

size	Number of realizations to simulate
corr	Whether long-run variance should be computed under the assumption of correlated residuals
gen_func	The function generating the random sample from which the statistic is computed
args	A list of arguments to be passed to <code>gen_func</code>
n	The sample size for each realization
parallel	Whether to use the <b>foreach</b> and <b>doParallel</b> packages to parallelize simulation (which needs to be initialized in the global namespace before use)
use_kernel_var	Set to TRUE to use kernel-based long-run variance estimation (FALSE means this is not employed); <i>TODO: NOT CURRENTLY IMPLEMENTED</i>
kernel	If character, the identifier of the kernel function as used in the <code>cointReg</code> (see documentation for <code>cointReg::getLongRunVar</code> ); if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in <code>cointReg</code> ); this parameter has no effect if <code>use_kernel_var</code> is FALSE; <i>TODO: NOT CURRENTLY IMPLEMENTED</i>
bandwidth	If character, the identifier of how to compute the bandwidth as defined in the <code>cointReg</code> package (see documentation for <code>cointReg::getLongRunVar</code> ); if function, a function to use for computing the bandwidth; if numeric, the bandwidth to use (the default behavior is to use the Andrews (1991) method, as used in <code>cointReg</code> ); this parameter has no effect if <code>use_kernel_var</code> is FALSE; <i>TODO: NOT CURRENTLY IMPLEMENTED</i>

Description	
<i>sim_de_stat</i>	<i>Darling-Erds Statistic Simulation</i>

Simulates multiple realizations of the Darling-Erds statistic.

Usage

`sim_de_stat(size, a = log, b = log, use_kernel_var = FALSE, kernel = "ba", bandwidth = "and", n = 500, gen_func = rnorm, args = NULL, parallel = FALSE)`

Arguments

**size** Number of realizations to simulate  
**a** The function that will be composed with  $l(x) = (2 \log(x))^{1/2}$   
**b** The function that will be composed with  $u(x) = 2 \log(x) + \frac{1}{2} \log(\log(x)) - \frac{1}{2} \log(\pi)$   
**use\_kernel\_var** Set to TRUE to use kernel-based long-run variance estimation (FALSE means this is not employed)

**kernel** If character, the identifier of the kernel function as used in the **coIntReg** (see documentation for `coIntReg::getLongRunVar`): if function, the kernel function to be used for long-run variance estimation (default is the Bartlett kernel in **coIntReg**); this parameter has no effect if `use_kernel_var` is FALSE

**bandwidth** If character, the identifier of how to compute the bandwidth as defined in the **coIntReg** package (see documentation for `coIntReg::getLongRunVar`); if function, a function to use for computing the bandwidth; if numeric, the bandwidth to use (the default behavior is to use the Andrews (1991) method, as used in **coIntReg**); this parameter has no effect if `use_kernel_var` is FALSE

**n** The sample size for each realization  
**gen\_func** The function generating the random sample from which the statistic is computed  
**args** A list of arguments to be passed to `gen_func`  
**parallel** Whether to use the **foreach** and **doparallel** packages to parallelize simulation (which needs to be initialized in the global namespace before use)

Details

If `use_kernel_var` is set to TRUE, long-run variance estimation using kernel-based techniques will be employed; otherwise, a technique resembling standard variance estimation will be employed. Any technique employed, though, will account for the potential break points, as described in Rice et al. (). See the documentation for **stat\_de** for more details.  
The parameters `kernel` and `bandwidth` control parameters for long-run variance estimation using kernel methods. These parameters will be passed directly to **stat\_de**.

A vector of simulated realizations of the Darling-Erds statistic

Value

A `htest`-class object containing the results of the test

Value

Otherwise, the function tests for structural change in a linear regression model (estimated via least squares), and serves as an interface to **stat\_hs\_reg**; see its documentation for more details. In this mode the parameter `corr` is effectively ignored.  
p-values are computed using **phidalgo\_seo**, which represents the limiting distribution of the test statistic when the null hypothesis is true.

This function can perform both univariate and regression versions of the test described by Hidalgo and Seo.  
If `formula` is NULL and `x` is **numeric**, this function performs the (univariate) Hidalgo-Seo test for change in mean, as described in (Rice et al. ). This is effectively an interface to **stat\_hs**; see its documentation for more details.

Details

**x** Data to test for change in mean (either a vector or **data.frame**)  
**formula** The **formula** defining the regression model, when applicable  
**m** Either numeric or a function that returns numeric; corresponds to *m* used in computing the estimate of the long-run variance  
**corr** If TRUE, the long-run variance will be computed under the assumption of correlated residuals; ignored if `custom_var` is not NULL or `use_kernel_var` is TRUE  
**stat\_plot** Whether to create a plot of the values of the statistic at all potential change points

Arguments

`HS.test(x, formula = NULL, m = sqrt, corr = TRUE, stat_plot = FALSE)`

Usage

Performs the Hidalgo-Seo test for structural change, as proposed by Hidalgo and Seo (2013).

Description

<i>HS.test</i>	<i>Hidalgo-Seo Test</i>
----------------	-------------------------

Examples

```
HR.test(rnorm(1000))
HR.test(rnorm(1000), use_kernel_var = TRUE, kernel = "bo", bandwidth = "nw")
x <- rnorm(1000)
y <- 1 + 2 * x + rnorm(1000)
df <- data.frame(x, y)
HR.test(df, formula = y ~ x, kn = sqrt, use_kernel_var = FALSE)
```

References

Hidalgo J, Seo MH (2013). “Testing for structural stability in the whole sample.” *Journal of Econometrics*, **175**(2), 84 - 93. ISSN 0304-4076, doi: [10.1016/j.jeconom.2013.02.008](https://doi.org/10.1016/j.jeconom.2013.02.008), <http://www.sciencedirect.com/science/article/pii/S0304407613000626>.

Rice G, Miller C, Horváth L (????). “A new class of change point test of Rényi type.” in-press.

Examples

```
HS.test(rnorm(1000))
HS.test(rnorm(1000), corr = FALSE)
x <- rnorm(1000)
y <- 1 + 2 * x + rnorm(1000)
df <- data.frame(x, y)
HS.test(df, formula = y ~ x)
```

is.formula	Check For Formulas
------------	--------------------

Description

Checks if an object is a formula.

Usage

```
is.formula(x)
```

Arguments

x                      Object to check

Value

TRUE if x is a [formula](#), FALSE otherwise

Examples

```
CPAT:::is.formula(y ~ x)
CPAT:::is.formula(2)
```

rchangepoint	Simulate Univariate Data With a Single Change Point
--------------	---

Description

This function simulates univariate data with a structural change.

Usage

```
rchangepoint(n, changepoint = NULL, mean1 = 0, mean2 = 0,
  dist = rnorm, meanparam = "mean", ...)
```

Arguments

- n                      An integer for the data set’s sample size
- changepoint           An integer for where the change point occurs
- mean1                  The mean prior to the change point
- mean2                  The mean after the change point
- dist                   The function with which random data will be generated
- meanparam             A string for the parameter in dist representing the mean
- ...                    Other arguments to be passed to dist

Details

This function generates artificial change point data, where up to the specified change point the data has one mean, and after the point it has a different mean. By default, the function simulates standard Normal data with no change. If changepoint is NULL, then by default the change point will be at about the middle of the data.

Value

A vector of the simulated data

Examples

```
CPAT:::rchangepoint(500)
CPAT:::rchangepoint(500, changepoint = 10, mean2 = 2, sd = 2)
CPAT:::rchangepoint(500, changepoint = 250, dist = rexp, meanparam = "rate",
  mean1 = 1, mean2 = 2)
```

<code>lrv_plot_tikz</code>	<code>Long-Run Variance Estimation Simulations Plot</code>
<b>Description</b>	
Create a Tikz plot of the estimated distribution of LRV estimators	
<b>Usage</b>	
<pre>lrv_plot_tikz(data, n, ker_name, true_lrv, phi = NULL, xrange = NULL, width = 4.5, height = 3.5, filename = NULL, verbose = FALSE, makePDF = TRUE)</pre>	
<b>Arguments</b>	
<code>data</code>	A <code>data.frame</code> containing the data to plot
<code>n</code>	The sample size of simulated data sets for which to plot an estimated distribution
<code>ker_name</code>	The name of the kernel function used in the LRV estimator
<code>true_lrv</code>	The value of the true long-run variance
<code>phi</code>	The autocorrelation parameter of the simulated data sets to plot; if <code>NULL</code> , the data is assumed to have been generated with a <code>GARCH(1,1)</code> process
<code>xrange</code>	The limits of the horizontal axis of the plot
<code>width</code>	The width of the plot
<code>height</code>	The height of the plot
<code>filename</code>	The name of the file to save output (without stems; files with this string appended with <code>.tex</code> and maybe <code>.pdf</code> will be created); if <code>NULL</code> , a file name will automatically be chosen (of the form <code>lrv_est_plot_ker_name_phi</code> )
<code>verbose</code>	Print updates about progress (via <code>link[base]{cat}</code> )
<code>makePDF</code>	Automatically compile the resulting <code>.tex</code> file
<b>Examples</b>	
<pre>## Not run: plotlist &lt;- data.frame(val = c(0.649, 0.965, 0.905), n = c(50, 50, 50), phi = c(0, 0, 0)) lrv_plot_tikz(plotlist, 50, ker_name = "bartlett", true_lrv = 1)  ## End(Not run)</pre>	

<code>qzn</code>	<code>Rényi-Type Statistic Quantile Function</code>
<b>Description</b>	
Quantile function for the limiting distribution of the Rényi-type statistic.	
<b>Usage</b>	
<pre>qzn(d, d = 1, summands = 500, interval = c(0, 100), tol = .Machine\$double.eps, ...)</pre>	
<b>Arguments</b>	
<code>p</code>	Value of the CDF at the quantile
<code>d</code>	Dimension parameter
<code>summands</code>	Number of summands for infinite sum
<code>interval, tol, ...</code>	Arguments to be passed to <code>uniroot</code>
<b>Details</b>	
This function uses <code>uniroot</code> for finding this quantity, and many of the the accepted parameters are arguments for that function; see its documentation for more details.	
<b>Value</b>	
The quantile associated with <code>p</code>	
<b>Examples</b>	
<pre>CPAT:::qkolmogorov(0.5)</pre>	
<b>Examples</b>	
The quantile associated with <code>p</code>	
<b>Value</b>	
This function uses <code>uniroot</code> for finding this quantity, and many of the the accepted parameters are arguments for that function; see its documentation for more details.	
<b>Details</b>	
This function uses <code>uniroot</code> for finding this quantity, and many of the the accepted parameters are arguments for that function; see its documentation for more details.	

pBst	<i>CDF of First Hitting Time of Bessel Process</i>
------	--

Description

CDF of the distribution of the first time a Bessel process with parameter  $\nu > -1$  hits  $b > 0$ .

Usage

```
pBst(q, b, nu = -1/2, summands = NULL)
```

Arguments

q	Quantile input to CDF
b	Point in space Bessel process hits
nu	The parameter $\nu > -1$ of the Bessel process
summands	Number of summands to use in summation; default is to pick the number of summands with <code>pBst_summand_solver</code> (it could be slow, so for performance it may be best to pick a fixed number)

Details

Let  $\tau_b^{(\nu)}$  be the first time a Bessel process with parameter  $\nu$  hits  $b > 0$ . Let  $J_\nu(x)$  be the Bessel function (of the first kind) with order  $\nu$ , and let  $j_{\nu,k}$  be the  $k$ th zero of  $J_\nu(x)$ . Let  $\Gamma(x)$  be the gamma function. Then the CDF of  $\tau_b^{(\nu)}$  is

$$1 - \frac{1}{2^{\nu-1}\Gamma(\nu+1)} \sum_{k=1}^{\infty} \frac{j_{\nu,k}^{\nu-1}}{J_{\nu+1}(j_{\nu,k})} e^{-\frac{j_{\nu,k}^2}{2b^2}t}$$

(This was obtained in (Kent 1980), but the formula above was given in (Hamana and Matsumoto 2013).)

Value

If  $T$  is the random variable as described,  $P(T \leq q)$

References

Hamana Y, Matsumoto H (2013). “The probability distributions of the first hitting times of Bessel processes.” *Transactions of the American Mathematical Society*, **365**(10), 5237–5257.

Kent JT (1980). “Eigenvalue expansions for diffusion hitting times.” *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, **52**(3), 309–319. ISSN 1432-2064, doi: [10.1007/BF00538895](https://doi.org/10.1007/BF00538895), <https://doi.org/10.1007/BF00538895>.

Examples

```
CPAT:::pBst(1, 1)
```

Arguments

p	The probability associated with the desired quantile
---	--

Value

The quantile associated with p

Examples

```
CPAT:::qdarling_erdos(0.5)
```

qhidalgo_seo	<i>Hidalgo-Seo Statistic Limiting Distribution Quantile Function</i>
--------------	--

Description

Quantile function for the limiting distribution of the Hidalgo-Seo statistic

Usage

```
qhidalgo_seo(p)
```

Arguments

p	The probability associated with the desired quantile
---	--

Value

A The quantile associated with p

Examples

```
CPAT:::qhidalgo_seo(0.5)
```

qkolmogorov	<i>Kolmogorov Distribution Quantile Function</i>
-------------	--

Description

Quantile function for the Kolmogorov distribution.

Usage

```
qkolmogorov(p, summands = 500, interval = c(0, 100),  
tol = .Machine$double.eps, ...)
```

Arguments

p	Value of the CDF at the quantile
summands	Number of summands for infinite sum
interval, tol, ...	Arguments to be passed to <code>uniroot</code>



qbst	<i>Bessel Process First Hitting Time Quantile Function</i>		
<b>Description</b>			
Quantile function of the distribution of the first time a Bessel process with parameter $\nu > -1$ hits $b > 0$ .			
<b>Usage</b>			
qbst(p, b, nu = -1/2, summands = NULL, interval = c(0, 100), tol = .Machine\$double.eps, ...)			
<b>Arguments</b>			
p	The probability associated with the desired quantile		
b	Point in space Bessel process hits		
nu	The parameter $\nu > -1$ of the Bessel process		
summands	Number of summands to use in summation; default is to pick the number of summands with <a href="#">pbst_summand_solver</a> (it could be slow, so for performance it may be best to pick a fixed number)		
interval, tol, ...	Arguments to be passed to <a href="#">unifroot</a>		
<b>Details</b>			
This function uses <a href="#">unifroot</a> for finding this quantity, and many of the the accepted parameters are arguments for that function; see its documentation for more details.			
<b>Value</b>			
The quantile associated with p			
<b>Examples</b>			
CPAT:::qbst(0.5, b = 1)			
qdarling_erdos	<i>Darling-Erdős Statistic Limiting Distribution Quantile Function</i>		
<b>Description</b>			
Quantile function for the limiting distribution of the Darling-Erdős statistic.			
<b>Usage</b>			
qdarling_erdos(p)			

pbst_summand_solver		<i>Find Number of Summands Needed for Numerical Accuracy of pbst</i>	
<b>Description</b>			
Find the number of summands needed to achieve numerical accuracy of the sum involved in <a href="#">pbst</a> .			
<b>Usage</b>			
pbst_summand_solver(q, b, nu = -1/2, error = .Machine\$double.eps)			
<b>Arguments</b>			
q	Quantile input to CDF		
b	Point in space Bessel process hits		
nu	The parameter $\nu > -1$ of the Bessel process		
error	The desired numerical error of the sum		
<b>Details</b>			
The number of summands needed is determined by using a loop that runs over the summands until it encounters a summand that is not greater than the specified level of numerical accuracy. The index of that last summand is then returned.			
<b>Value</b>			
Integer for number of summands			
<b>Examples</b>			
pbst_summand_solver(1, 1)			
pdarling_erdos		<i>Darling-Erdős Statistic CDF</i>	
<b>Description</b>			
CDF for the limiting distribution of the Darling-Erdős statistic.			
<b>Usage</b>			
pdarling_erdos(q)			
<b>Arguments</b>			
q	Quantile input to CDF		
<b>Value</b>			

Examples

```
CPAT:::pdarling_erdos(0.1)
```

---

phidalgo_seo	<i>Hidalgo-Seo Statistic CDF</i>
--------------	----------------------------------

---

Description

CDF of the limiting distribution of the Hidalgo-Seo statistic

Usage

```
phidalgo_seo(q)
```

Arguments

q	Quantile input to CDF
---	-----------------------

Value

If  $Z$  is the random variable following the limiting distribution, the quantity  $P(Z \leq q)$

Examples

```
CPAT:::phidalgo_seo(0.1)
```

---

pkolmogorov	<i>Kolmogorov CDF</i>
-------------	-----------------------

---

Description

CDF of the Kolmogorov distribution.

Usage

```
pkolmogorov(q, summands = ceiling(q * sqrt(72) + 3/2))
```

Arguments

q	Quantile input to CDF
summands	Number of summands for infinite sum (the default should have machine accuracy)

Value

If  $Z$  is the random variable following the Kolmogorov distribution, the quantity  $P(Z \leq q)$

Examples

```
CPAT:::pkolmogorov(0.1)
```

Examples

```
saveobj <- list("norm" = list(
  "log" = list(
    "n50" = list(
      "c4rt" = list(
        "d_0" = c(1.551, 1.276, 1.348, 1.982, 1.423)
      )
    )
  )
)
CPAT:::power_sim_Zn_to_df(saveobj, CPAT:::qZn(.95))
```

---

pZn	<i>Rényi-Type Statistic CDF</i>
-----	---------------------------------

---

Description

CDF for the limiting distribution of the Rényi-type statistic.

Usage

```
pZn(q, d = 1, summands = NULL)
```

Arguments

q	Quantile input to CDF
d	Dimension parameter
summands	Number of summands for infinite sum; if NULL, automatically determined using <a href="#">pBst_summand_solver</a> (which isn't necessarily fast, so consider picking a fixed number if speed is important)

Details

If  $G_{\nu,b}(x)$  is the CDF of the first time a Bessel process with parameter  $\nu$  hits  $b > 0$  (as described by [pBst](#)) then the CDF of the Rényi-type statistic when the null hypothesis is true is  $F(x) = (1 - G_{d/2-1,x}(1))^2$ , where  $d$  is the dimensionality parameter of the statistic. (This comes from combining the limiting distribution of the statistic described in (Rice et al. ) with the expression for the CDF of the hitting time of the Bessel process described in (Hamana and Matsumoto 2013).)

Value

If  $Z$  is the random variable following the limiting distribution, the quantity  $P(Z \leq q)$

References

Hamana Y, Matsumoto H (2013). “The probability distributions of the first hitting times of Bessel processes.” *Transactions of the American Mathematical Society*, **365**(10), 5237–5257.  
Rice G, Miller C, Horváth L (????). “A new class of change point test of Rényi type.” in-press.

Examples

```
CPAT:::pZn(0.1)
```

power_plot_tikz	<i>Power Curve Plot (By Statistic)</i>
-----------------	--

Description

Create a Tikz plot of the power curves of a statistic, with each sample size having its own curve.

Usage

```
power_plot_tikz(data, d, t, c, s, title = "", legend_pos = "none",  
width = 4.5, height = 3.5, filename = NULL, verbose = FALSE,  
makePDF = TRUE)
```

Arguments

**data** A data.frame containing the data to plot  
**d** Label for data-generating process used to simulate the data on which the statis-  
tics were computed  
**t** Label for the trimming parameter of the Kenyi-type statistic  
**c** Label for the process that computes the location of change points  
**s** The statistic for which to plot a power curve  
**title** The title of the plot  
**legend\_pos** A string to be passed to link[ggplot2]{theme} (the legend.position argu-  
ment) identifying where to place the legend  
**width** The width of the plot  
**height** The height of the plot  
**filename** The name of the file to save output (without stems; files with this string ap-  
pend with .tex and maybe .pdf will be created); if NULL, the name will be  
automatically determined  
**verbose** Print updates about progress (via link[base]{cat})  
**makePDF** Automatically compile the resulting .tex file

Examples

```
## Not run:  
pdatt <- data.frame(power = c(0.8926, 0.8714, 0.8296, 0.7936),  
stat = c("de", "de", "de", "de"),  
dist = c("norm", "norm", "norm", "norm"),  
kn = c("10g", "10g", "10g", "10g"),  
n = c(50, 50, 50, 50),  
cpt = c("c4rt", "c4rt", "c4rt", "c4rt"),  
delta = c(-2.0, -1.9, -1.8, -1.7))  
power_plot_tikz(pdatt, "norm", "10g", "c4rt", "de")  
## End(Not run)
```

Usage

```
power_sim_vn_to_df(obj, crit)
```

Arguments

**obj** A list containing simulated statistic values  
**crit** The critical value determining whether a statistic should lead to the rejection of  
the null hypothesis

Value

A data.frame summarizing the results of the data stored in obj

Examples

```
saveobj <- list("norm" = list(  
"n50" = list(  
"c4rt" = list(  
"d_0" = c(1.551, 1.276, 1.348, 1.982, 1.423)  
))))  
CPAT:::power_sim_vn_to_df(saveobj, CPAT:::qkolmogorov(.95))
```

power_sim_Zn_to_df	<i>Convert Kenyi-Type Statistic Power Simulation Save List to Data Frame</i>
--------------------	--

Description

This function will convert the power simulation data generated in a list in our simulation scripts to a data.frame. Given such a list and a critical value to determine whether the null hypothesis should be rejected, the function will return a data.frame with columns power, stat, dist, kn, n, cpt, and delta, which correspond to: the empirical power of the statistic; the identifier of the statistic; the generating distribution of the statistic was computed on; the kn parameter; the identifier of how change points were computed; and the size of the change.

Usage

```
power_sim_Zn_to_df(obj, crit)
```

Arguments

**obj** A list containing simulated statistic values  
**crit** The critical value determining whether a statistic should lead to the rejection of  
the null hypothesis

Value

A data.frame summarizing the results of the data stored in obj

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power_plot_tikz_by_n	<i>Power Curve Plot</i>
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### Description

Create a Tikz plot of the power curves of simulated statistics.

### Usage

```
power_plot_tikz_by_n(data, d, t, c, N, statlines, title = "",
  legend_pos = "none", width = 4.5, height = 3.5, filename = NULL,
  verbose = FALSE, makePDF = TRUE)
```

### Arguments

data	A data.frame containing the data to plot
d	Label for data-generating process used to simulate the data on which the statistics were computed
t	Label for the trimming parameter of the Rényi-type statistic
c	Label for the process that computes the location of change points
N	The sample size of the simulated data sets on which the statistics were computed
statlines	A character vector where the names of the entries are the labels of the statistics in the stat column of data and the entries define the line types used by the values entry of <a href="#">scale_linetype_manual</a>
title	The title of the plot
legend_pos	A string to be passed to <code>link[ggplot2]{theme}</code> (the <code>legend.position</code> argument) identifying where to place the legend
width	The width of the plot
height	The height of the plot
filename	The name of the file to save output (without stems; files with this string appended with .tex and maybe .pdf will be created); if NULL, the name will be automatically determined
verbose	Print updates about progress (via <code>link[base]{cat}</code> )
makePDF	Automatically compile the resulting .tex file

### Examples

```
## Not run:
pdat <- data.frame(power = c(0.8926, 0.8714, 0.8296, 0.7936),
  stat = c("de", "de", "de", "de"),
  dist = c("norm", "norm", "norm", "norm"),
  kn = c("log", "log", "log", "log"),
  n = c(50, 50, 50, 50),
  cpt = c("c4rt", "c4rt", "c4rt", "c4rt"),
  delta = c(-2.0, -1.9, -1.8, -1.7))
power_plot_tikz_by_n(pdat, "norm", "log", "c4rt", 50, c("de" = "solid"))

## End(Not run)
```

---

power_sim_stat_df_creator	<i>Create Power Simulation Results Data Frame</i>
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### Description

Creates a data.frame that contains power simulation results from files containing power simulations. This function should automate the use of [power\\_sim\\_Zn\\_to\\_df](#) and [power\\_sim\\_Vn\\_to\\_df](#) for collecting power simulation data. It takes two CSV files, one passed (as a character string) to file\_meta and the other to stat\_meta, describing how the files (named and described in file\_meta) should be handled.

### Usage

```
power_sim_stat_df_creator(file_meta, stat_meta, prefix = "",
  alpha = 0.05)
```

### Arguments

file_meta	The location of a CSV file that contains file names and the statistics that those files correspond to
stat_meta	The location of a CSV file that contains statistic (stat) labels (used in file_meta), the name of the variable for the statistic, and the name of the function that converts a file (mentioned in file_meta) to a data.frame of power data
prefix	Character string representing a prefix for file names mentioned in file_meta; could be used for adding path information to those names, in case the files are not in the working directory and there is no desire to edit file_meta's data
alpha	Numeric for level of significance used in power calculations

### Value

A data frame containing the power simulation data

### Examples

```
## Not run:
power_sim_stat_df_creator("FileStatMeta.csv", "StatMeta.csv")

## End(Not run)
```

---

power_sim_Vn_to_df	<i>Convert CUSUM-Type Statistic Power Simulation Save List to Data Frame</i>
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### Description

This function will convert the power simulation data generated in a list in our simulation scripts to a data.frame. Given such a list and a critical value to determine whether the null hypothesis should be rejected, the function will return a data.frame with columns power, stat, dist, n, cpt, and delta, which correspond to: the empirical power of the statistic; the identifier of the statistic; the generating distribution of the statistic was computed on; the identifier of how change points were computed; and the size of the change.