Preliminary Power Plots

Curtis Miller

March 14, 2019

The plots below are power simulations comparing the ability of different test statistics to detect structural change in linear regression models. Here, $\beta = (1, 2)^T$, so the regression model is $y_t = 1 + 2x_t + \epsilon_t$, where x_t was generated from a $N(1, 1)$ distribution and $\{\epsilon_t\}_{t\in\mathbb{N}}$ is a mean-zero stationary process.

The test statistics displayed include the Rényi-type statistic with trimming parameter $t_T = T^{1/2}$ computed with the regression model coefficients directly (———) the Rényi-type statistic with trimming parameter $t_T = T^{1/2}$ computed on the residuals (– – – –), the CUSUM statistic (— — –), and the Hidalgo-Seo statistic (-·-·-·-·). The Rényi-type statistic and CUSUM statistics are the univariate statistics computed on the estimated residuals of the regression model. The Hidalgo-Seo statistic was tailored to the regression context as described in their paper.

For a sample size T, the change point occurs near $T^{2/3}$ if there is a change at all. If it does, the new model the data follows is $y_t = (1 + \frac{\delta}{\sqrt{2}}) + (2 + \frac{\delta}{\sqrt{2}})x_t + \epsilon_t$, where $\delta \in [0, 2]$. One may then say that, if β^* is the new model after the change, $\|\beta - \beta^*\| = \delta$. Thus the case $\delta = 0$ represents no change in the model. ϵ_t is as before.

In this document, ϵ_t follows one of three models: independent and identically distributed $N(0, 1)$ random variables; an ARMA(2, 2) process with $\phi_1 = 0.4$, $\phi_2 = -0.03$, $\theta_1 = 0.5$, $\theta_2 = -0.06$, and the variance of the noise parameter is chosen so that the long-run variance is one; and a GARCH(1, 1) process with $\omega = 0.2$, $\alpha = 0.3$, and $\beta = 0.5$ (again the long-run variance is one).

For every combination of sample size, test statistic, δ, and data-generating process for ϵ_t, 20,000 independent test statistics were simulated, and their asymptotically-appropriate p-values were computed. (The Rényi-type statistic using the regression model coefficients, though, was not computed for when the residuals followed an ARMA(2, 2) process, since this would require long-run variance estimation procedures not yet implemented.) H_0 is rejected for p-values less than $\alpha = 0.05$; the value of α is signified with a dotted line (··········). Values of δ increment by 0.1, and the sample sizes considered are $T \in \{50, 250, 1000\}$.

The best-performing statistics use only the residuals of the estimated models; in many situations, the Rényi-type statistic that uses the regression model coefficients directly has the worst performance (except in smaller sample sizes compared to the Hidalgo-Seo statistic). While all statistics seem to have appropriate power curves, the Rényi-type statistic often has the worst power, except for the version of the statistic that uses the residuals of the regression model only; that statistic seems to do well in the limit.

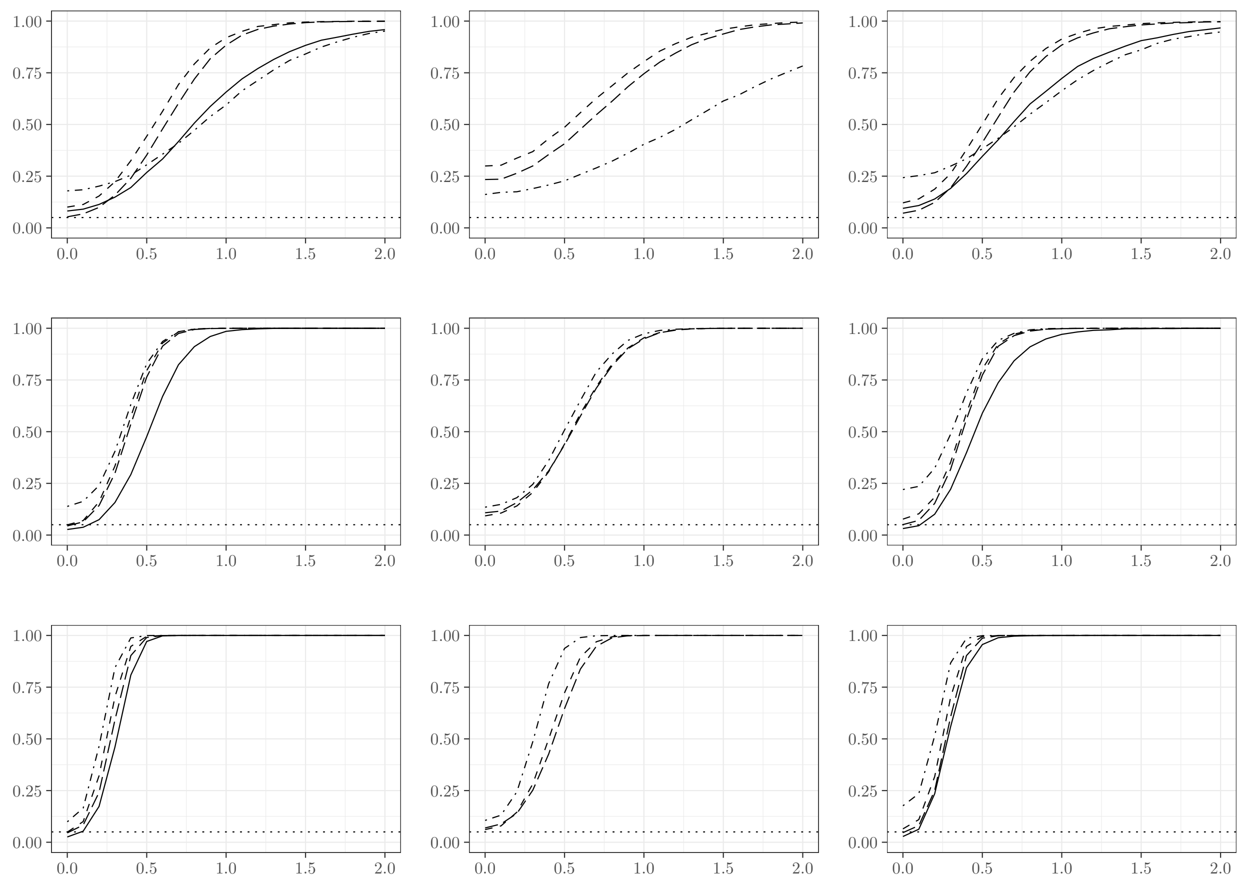

Figure 1: Power plots; rows correspond to the sample sizes 50, 250, and 1000 respectively, the first column is for *iid* Normal residuals, the second column ARMA(2, 2) residuals, and the third column GARCH(1, 1) residuals. The x-axis tracks values of δ and the y-axis tracks observed power.