Preliminary Power Plots

Curtis Miller

March 4, 2019

The plots below are power simulations comparing the ability of different test statistics to detect structural change in linear regression models. Here, $\beta = (1, 2)^T$, so the regression model is $y_t = 1 + 2x_t + \epsilon_t$, where x_t was generated from a $N(1, 1)$ distribution and $\{\epsilon_t\}_{t \in \mathbb{N}}$ is a mean-zero stationary process.

The test statistics displayed include the Rényi-type statistic with trimming parameter $t_T = T^{1/2}$ (- - - -), the CUSUM statistic (— — –), and the Hidalgo-Seo statistic (-·-·-·-·-). The Rényi-type statistic and CUSUM statistics are the univariate statistics computed on the estimated residuals of the regression model. The Hidalgo-Seo statistic was tailored to the regression context as described in their paper.

For a sample size T, the change point occurs near $T^{2/3}$ if there is a change at all. If it does, the new model the data follows is $y_t = (1 + \frac{\delta}{\sqrt{2}}) + (2 + \frac{\delta}{\sqrt{2}})x_t + \epsilon_t$, where $\delta \in [0, 2]$. One may then say that, if β^* is the new model after the change, $\|\beta - \beta^*\| = \delta$. Thus the case $\delta = 0$ represents no change in the model. ϵ_t is as before.

In this document, ϵ_t follows one of three models: independent and identically distributed $N(0, 1)$ random variables; an ARMA(2, 2) process with $\phi_1 = 0.4$, $\phi_2 = -0.03$, $\theta_1 = 0.5$, $\theta_2 = -0.06$, and the variance of the noise parameter is chosen so that the long-run variance is one; and a GARCH(1, 1) process with $\omega = 0.2$, $\alpha = 0.3$, and $\beta = 0.5$ (again the long-run variance is one).

For every combination of sample size, test statistic, δ, and data-generating process for ϵ_t, 20,000 independent test statistics were simulated, and their asymptotically-appropriate p-values were computed. H_0 is rejected for p-values less than $\alpha = 0.05$; the value of α is signified with a dotted line (··········). Values of δ increment by 0.1, and the sample sizes considered are $T \in \{50, 250, 1000\}$.

From the plots one sees that the Hidalgo-Seo statistic often seems to do better than the other statistics. However, it suffers significant size inflation especially at smaller sample sizes, and even for larger sample sizes it experiences the most size inflation of the three statistics considered. Additionally, it is the only statistic shown that is tailored to regression models specifically. For small sample sizes, the statistic has bad power properties.

The Rényi-type statistic and the CUSUM statistic often have similar power properties. The latter seems to suffer more from size inflation but generally has better power, particularly in larger samples with the size inflation has largely vanished.

All statistics struggle the most when the residuals follow an ARMA(2, 2) process. The Hidalgo-Seo statistic struggles particularly in the GARCH(1, 1) context even when the other statistics seem to perform reasonably well.

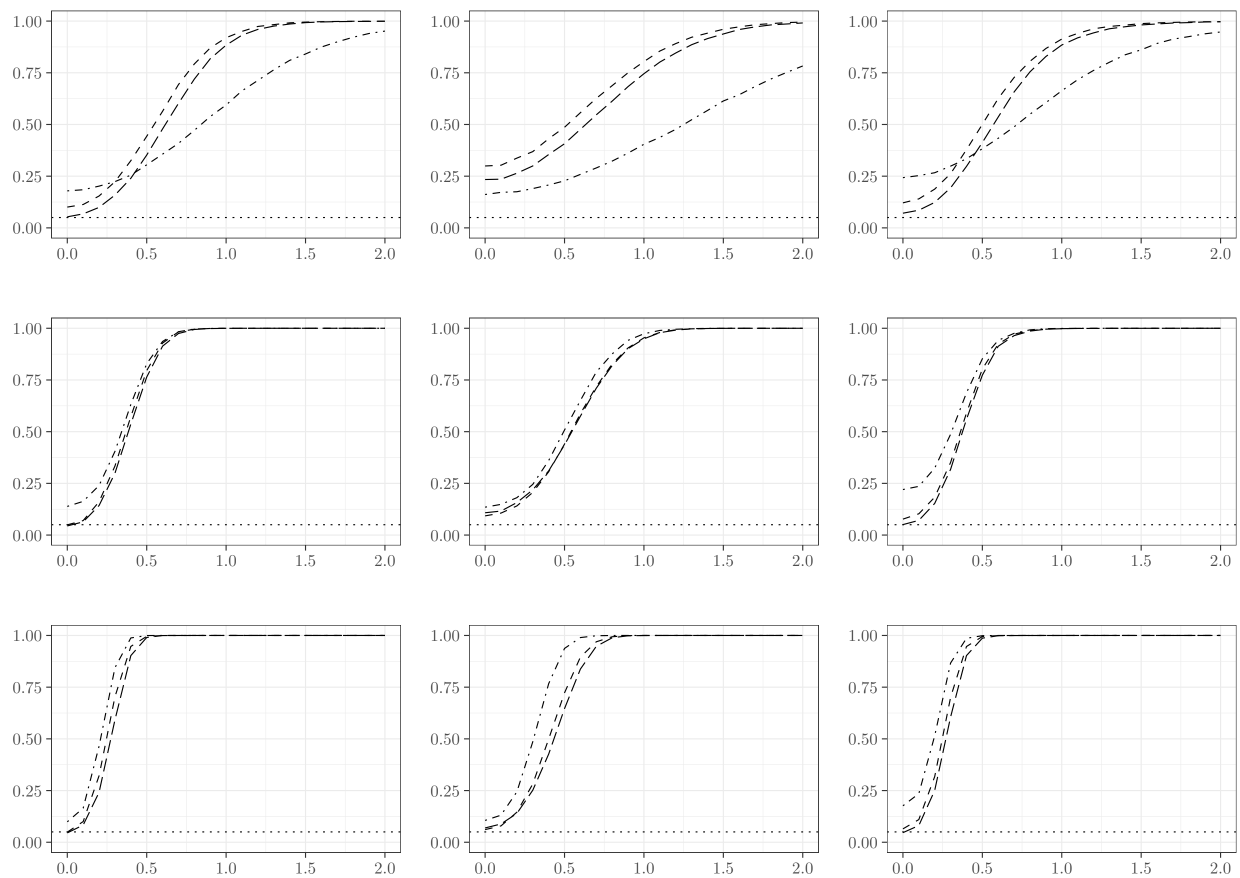

Figure 1: Power plots; rows correspond to the sample sizes 50, 250, and 1000 respectively, the first column is for *iid* Normal residuals, the second column ARMA(2, 2) residuals, and the third column GARCH(1, 1) residuals. The x-axis tracks values of δ and the y-axis tracks observed power.