Structural Change in a Predictive Model for GDP based on PMI; Power Demonstration

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In our example we will explore linear forecasting models for U.S. gross domestic product (GDP) based on the purchasing managers index (PMI) as computed by the Institute for Supply Management. We look at a quarterly data set starting in Q1 1950 and ending in Q1 2019. All data was obtained from Bloomberg.

Let GDP_t be the year-over-year value of GDP in quarter t and PMI_t be the value of the PMI index in quarter t. Here we investigate change points in the model:

$$\mathrm{GDP}_t = \beta_0 + \beta_1 \mathrm{PMI}_{t-1} + \beta_2 \mathrm{PMI}_{t-2} + \beta_3 \mathrm{PMI}_{t-3} + \beta_4 \mathrm{PMI}_{t-4} + \epsilon_t. \tag{1}$$

This model forecasts GDP growth using only the value of the PMI of the four previous quarters, and is the model with the lowest AIC value amongst models that use only lags of the PMI. We present estimates and other relevant statistics for the model in Table 1. While the model does not have necessarily bad characteristics, notice that according to the CUSUM test for structural change, there is a structural break in the model. According to the argmax estimator for the change point location, the change occurs around Q1 2003, near the end of the recession of the early 2000s.

Why does this change occur? One could argue that it was a result of the initial recession in the 2000s. However, there is another competing model for forecasting GDP change:

$$\mathrm{GDP}_t = \beta_0 + \beta_1 \mathrm{GDP}_{t-1} + \beta_2 \mathrm{PMI}_{t-1} + \beta_3 \mathrm{PMI}_{t-4} + \epsilon_t. \tag{2}$$

This model has the lowest AIC among models that allow for lags of GDP to be included in the model. We report parameter estimates and other relevant statistics of the model in Table 1. Notice that if one judges model quality by either AIC, R^2, or F test results, model (2) is superior to model (1). Additionally, according to the CUSUM test, there is no structural change in model (2). This suggests another possible explanation for why we see structural change in model (1); it is not specified well, and the specification breaks down around 2003.

We investigated how well the statistics considered in this paper can detect the structural change in model (1). Throughout this section, the trimming function for Rényi-type statistics will be $\log(T)$ (usually the trimming parameter was $\sqrt{T}$ in the simulation studies). Otherwise, the statistics considered here were treated the same way as they were in previous simulations.

We first wanted to establish that the relevant test statistics had good size and power properties in the neighborhood of the pre- and post-change values of the parameters of model (1). Let $\gamma_0 = (-10.706, 0.172, 0.075, 0.080, -0.059)^\top$ and $\gamma_1 = (-16.153, 0.185, 0, 0, 0)^\top$. Prior to the change, the model generating the data is γ_0, and after the change, the model generating the data is $\delta(\gamma_1 - \gamma_0)$, with $\delta \in [0, 2]$. The only sample size considered in these simulations was $T = 225$, with the change occuring at $t^* = 213$. The residuals followed a MA(3) process that changed around $t = 212$; the parameter values prior to the change were $(0.772, 0.800, 0.690)$ with the noise process being *i.i.d.* $N(0, 1.001)$ (that is, the variance is 1.001) random variables, and after the change were $(0.674, 0.537, 0.522)$ with the noise process being similar but with variance 0.42. The data matrix was randomly generated using independent $\mathrm{ARMA}(1, 3)$ processes for before and after the change, with the autoregressive coefficient being 0.204, the moving average coefficients being $(0.674, 0.537, 0.522)$, and the variance being 21.376. These conditions do not exactly match those seen in the data, but they are similar. Otherwise the simulations were done similarly as before.

	Dependent variable:	
	GDP_t	
	(1)	(2)
GDP_{t-1}		0.787^{***} (0.040)
PMI_{t-1}	0.170^{***} (0.019)	0.101^{***} (0.016)
PMI_{t-2}	0.072^{***} (0.019)	
PMI_{t-3}	0.077^{***} (0.017)	
PMI_{t-4}	-0.050^{*} (0.029)	-0.083^{***} (0.013)
Constant	-11.137^{***} (2.064)	-0.314 (1.097)
Observations	273	273
R^2	0.552	0.843
Adjusted R^2	0.545	0.841
F Statistic	42.316^{***} (df = 4; 268)	373.878^{***} (df = 3; 269)
AIC	1058.285	770.872
CUSUM Statistic	1.628^{**}	1.052
Residual Std. Error	1.660 (df = 268)	0.982 (df = 269)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 1: Regression model estimates for predictive models (1) and (2). We estimated the covariance matrix used for standard error estimates and the F test using the procedure described in [**?**]; this was also the basis for computing the CUSUM statistic, as described in [**?**]. Coefficient standard errors are reported in parentheses.

We present the results of these simulations in Figure 1. The new statistic sees size inflation and non-monotonic power in this context. The CUSUM statistic sees more size inflation and the Hidalgo-Seo statistic is so inflated it is unreliable. However, the univariate version of the Rényi statistic appears to do well, having both appropriate size and power.

We computed each test statistic on an expanding window of data, where each window has a left end point of Q1 1950 and a right end point that varies, starting with 1999 Q4 in the first window and adding an additional quarter until eventually reaching Q1 2019. The results are presented in Figure 2.

While the univariate Rényi-type statistic appears to do well (it always rejects the null hypothesis), we note that outside of the window presented, we have yet to find a window where this statistic does *not* reject the null hypothesis. While in the simulations the statistic appears to do well, in the real data it acts like a statistic that is overpowered. A similar observation holds for the Hidalgo-Seo statistic. The CUSUM statistic behaves more conservatively than the aforementioned statistics, rejecting near the end of the period but not rejecting before. The new version of the Rényi-type statistic however rejects the null hypothesis sooner than the CUSUM statistic and seems to have more moderate behavior than the univariate version.

References

[1] D. W. K. Andrews, "Heteroskedasticity and autocorrelation consistent covariance matrix estimation," *Econometrica*, vol. 59, pp. 817–858, May 1991.

[2] L. Horváth, C. Miller, and G. Rice, "A new class of change point test statistics of Rényi type," *Journal of Business & Economic Statistics*, vol. 0, no. 0, pp. 1–19, 2019.

power_plot_Example_n225.pdf

Figure 1: Estimated power of test statistics under conditions resembling the example.

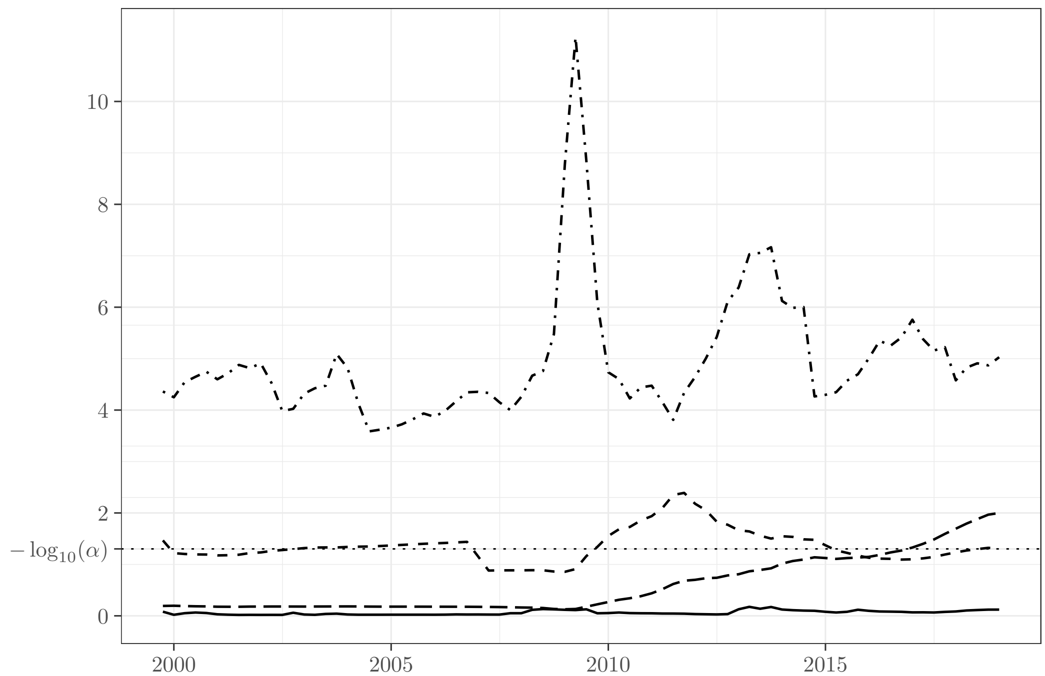

Figure 2: $-\log_{10}(p)$-values for statistics computed on an expanding window of data. Here $\alpha = 0.05$, and crossing above $-\log_{10}(\alpha)$ indicates rejection of the null hypothesis of no structural change.