

Natural Language Processing with Deep Learning

CS224N/Ling284



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Lecture 9: Self-Attention and Transformers

Lecture Plan

1. From recurrence (RNN) to attention-based NLP models
2. Introducing the Transformer model
3. Great results with Transformers
4. Drawbacks and variants of Transformers

Reminders:

Assignment 4 due on Thursday!

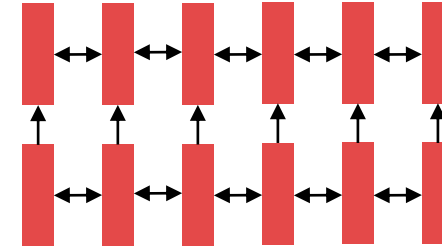
Mid-quarter feedback survey due Tuesday, Feb 16 at 11:59PM PST!

Final project proposal due Tuesday, Feb 16 at 4:30PM PST!

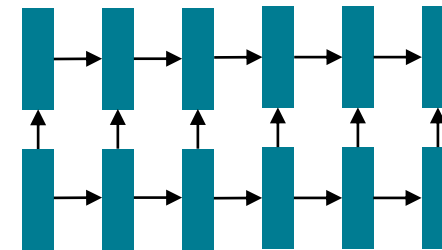
Please try to hand in the project proposal on time; we want to get you feedback quickly!

As of last week: recurrent models for (most) NLP!

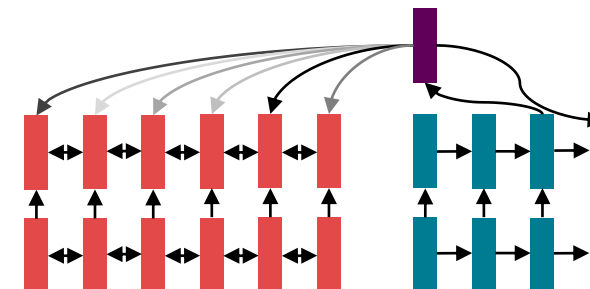
- Circa 2016, the de facto strategy in NLP is to **encode** sentences with a bidirectional LSTM: (for example, the source sentence in a translation)



- Define your output (parse, sentence, summary) as a sequence, and use an LSTM to generate it.

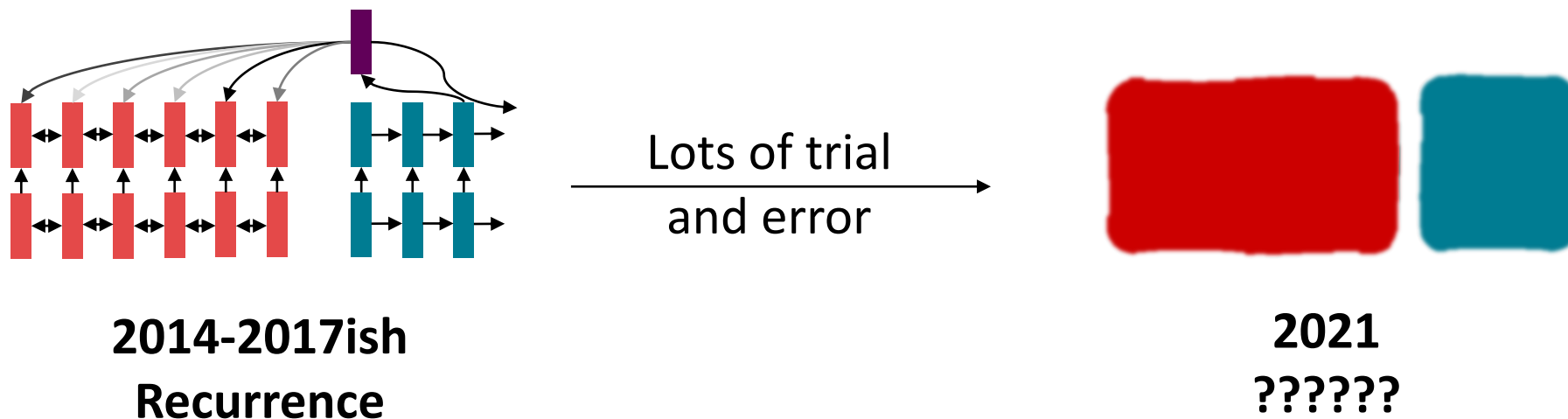


- Use attention to allow flexible access to memory



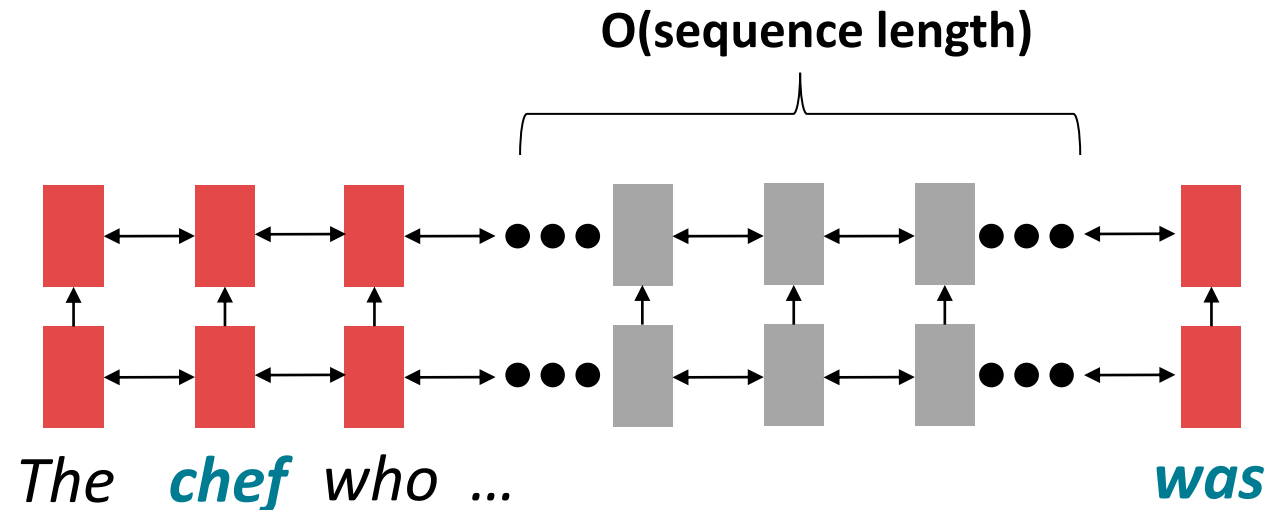
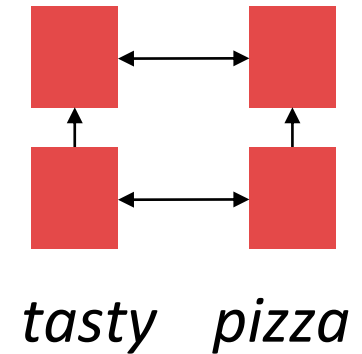
Today: Same goals, different building blocks

- Last week, we learned about sequence-to-sequence problems and encoder-decoder models.
- Today, we're **not** trying to motivate entirely new ways of looking at problems (like Machine Translation)
- Instead, we're trying to find the best **building blocks** to plug into our models and enable broad progress.



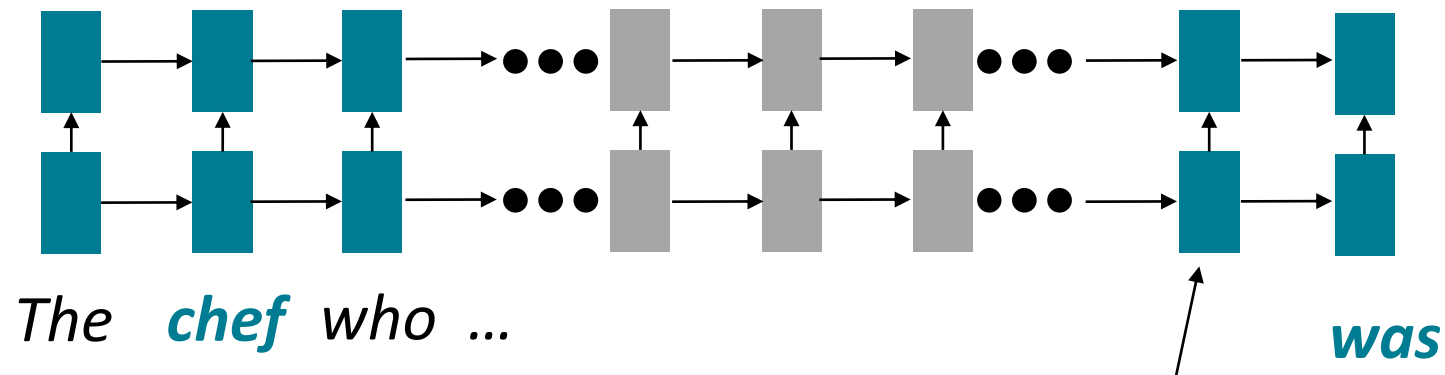
Issues with recurrent models: Linear interaction distance

- RNNs are unrolled “left-to-right”.
- This encodes linear locality: a useful heuristic!
 - Nearby words often affect each other’s meanings
- **Problem:** RNNs take $O(\text{sequence length})$ steps for distant word pairs to interact.



Issues with recurrent models: Linear interaction distance

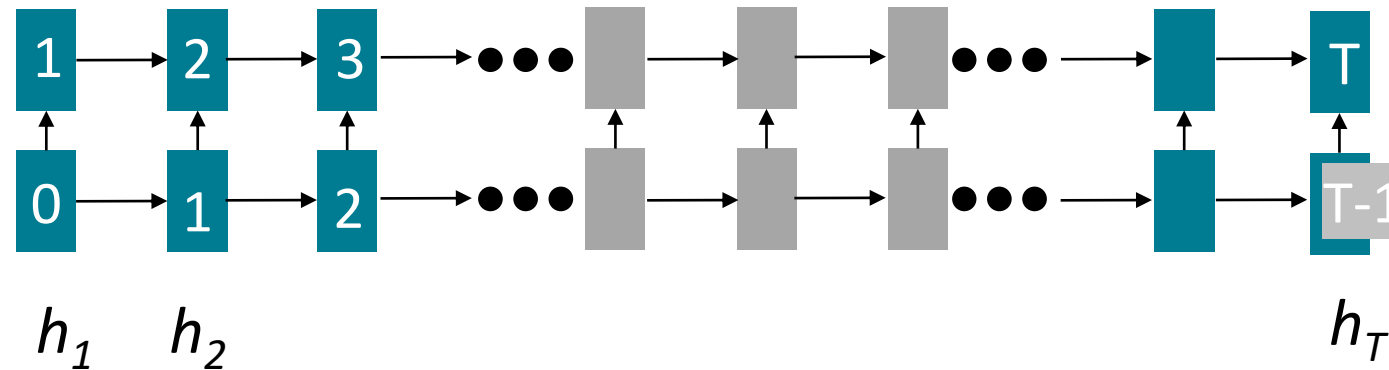
- **$O(\text{sequence length})$** steps for distant word pairs to interact means:
 - Hard to learn long-distance dependencies (because gradient problems!)
 - Linear order of words is “baked in”; we already know linear order isn’t the right way to think about sentences...



Info of *chef* has gone through $O(\text{sequence length})$ many layers!

Issues with recurrent models: Lack of parallelizability

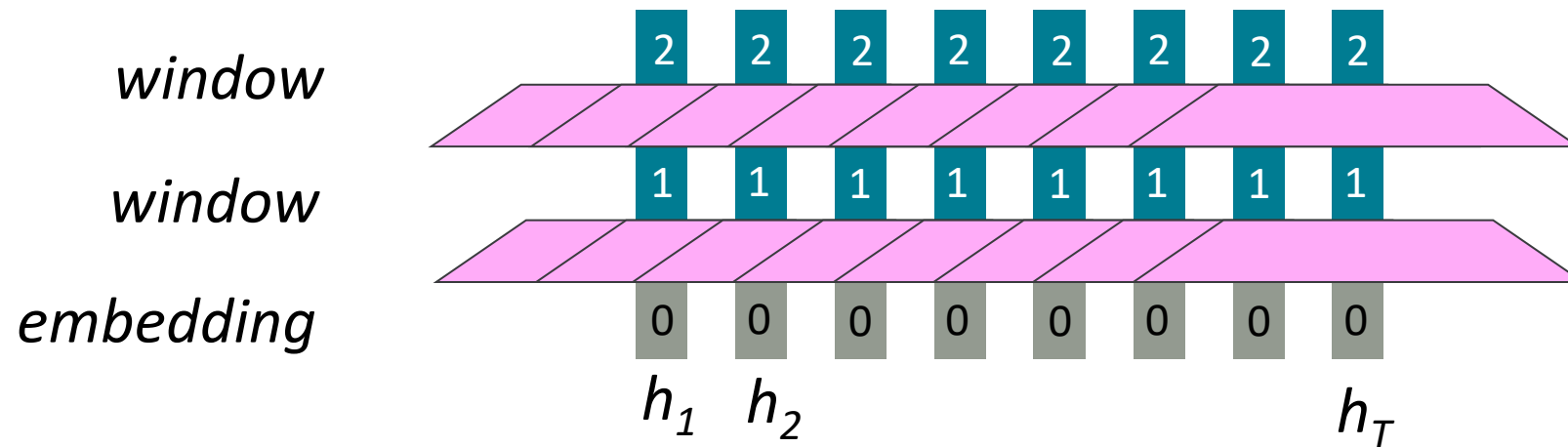
- Forward and backward passes have **$O(\text{sequence length})$** unparallelizable operations
 - GPUs can perform a bunch of independent computations at once!
 - But future RNN hidden states can't be computed in full before past RNN hidden states have been computed
 - Inhibits training on very large datasets!



Numbers indicate min # of steps before a state can be computed

If not recurrence, then what? How about word windows?

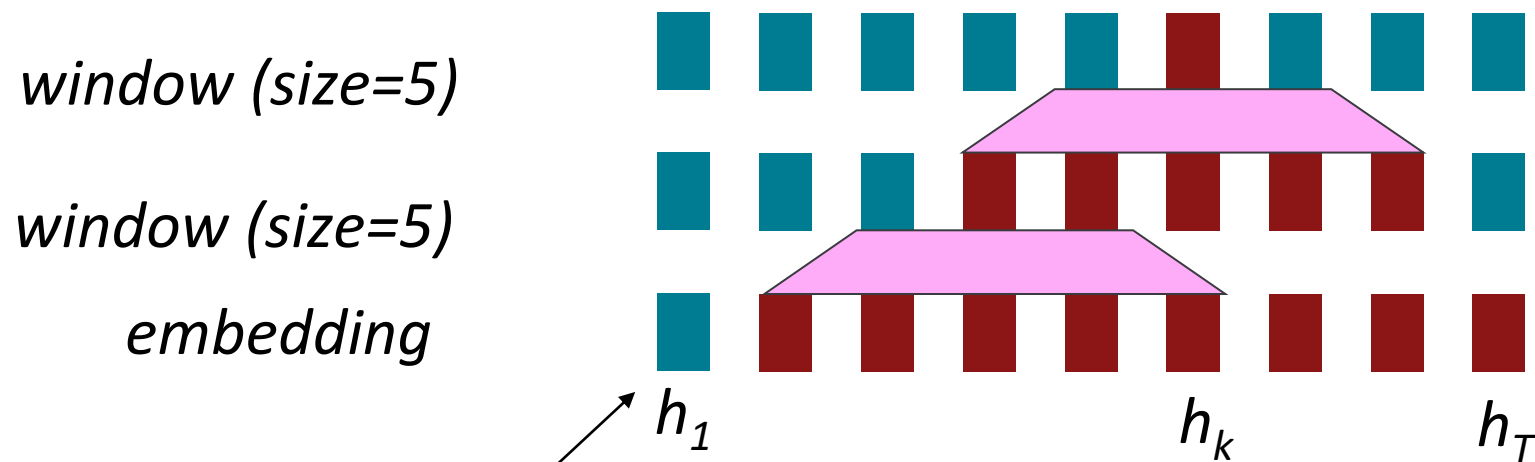
- **Word window models aggregate local contexts**
 - (Also known as 1D convolution; we'll go over this in depth later!)
 - Number of unparallelizable operations does not increase sequence length!



Numbers indicate min # of steps before a state can be computed

If not recurrence, then what? How about word windows?

- **Word window models aggregate local contexts**
- What about long-distance dependencies?
 - Stacking word window layers allows interaction between farther words
- Maximum Interaction distance = **sequence length / window size**
 - (But if your sequences are too long, you'll just ignore long-distance context)

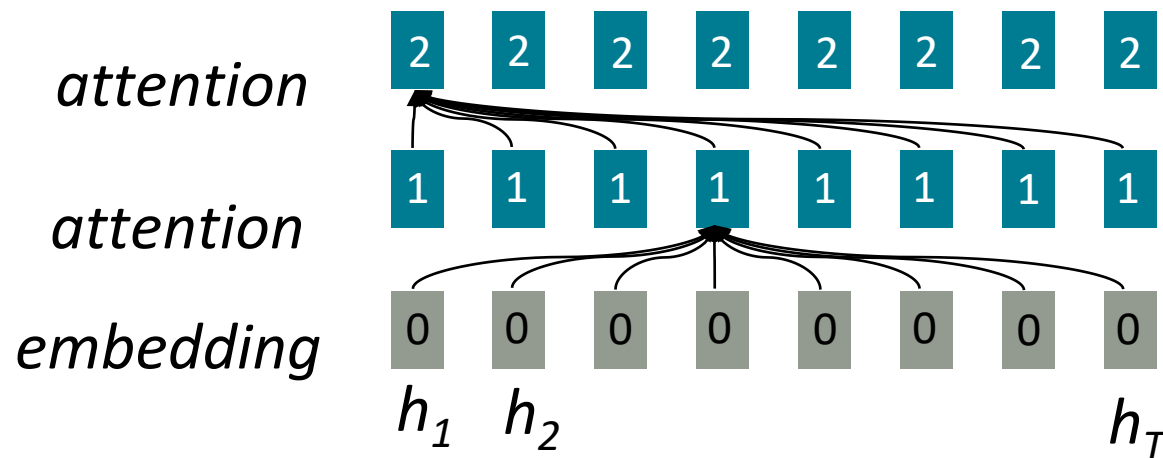


Red states indicate those "visible" to h_k

Too far from h_k to be considered

If not recurrence, then what? How about attention?

- **Attention** treats each word's representation as a **query** to access and incorporate information from **a set of values**.
 - We saw attention from the **decoder** to the **encoder**; today we'll think about attention **within a single sentence**.
- Number of unparallelizable operations does not increase sequence length.
- Maximum interaction distance: $O(1)$, since all words interact at every layer!



All words attend to all words in previous layer; most arrows here are omitted

Self-Attention

- Recall: Attention operates on **queries**, **keys**, and **values**.
 - We have some **queries** q_1, q_2, \dots, q_T . Each query is $q_i \in \mathbb{R}^d$
 - We have some **keys** k_1, k_2, \dots, k_T . Each key is $k_i \in \mathbb{R}^d$
 - We have some **values** v_1, v_2, \dots, v_T . Each value is $v_i \in \mathbb{R}^d$
- In **self-attention**, the queries, keys, and values are drawn from the same source.
 - For example, if the output of the previous layer is x_1, \dots, x_T , (one vec per word) we could let $v_i = k_i = q_i = x_i$ (that is, use the same vectors for all of them!)
- The (dot product) self-attention operation is as follows:

The number of queries can differ from the number of keys and values in practice.

$$e_{ij} = q_i^\top k_j$$

Compute **key-query** affinities

$$\alpha_{ij} = \frac{\exp(e_{ij})}{\sum_{j'} \exp(e_{ij'})}$$

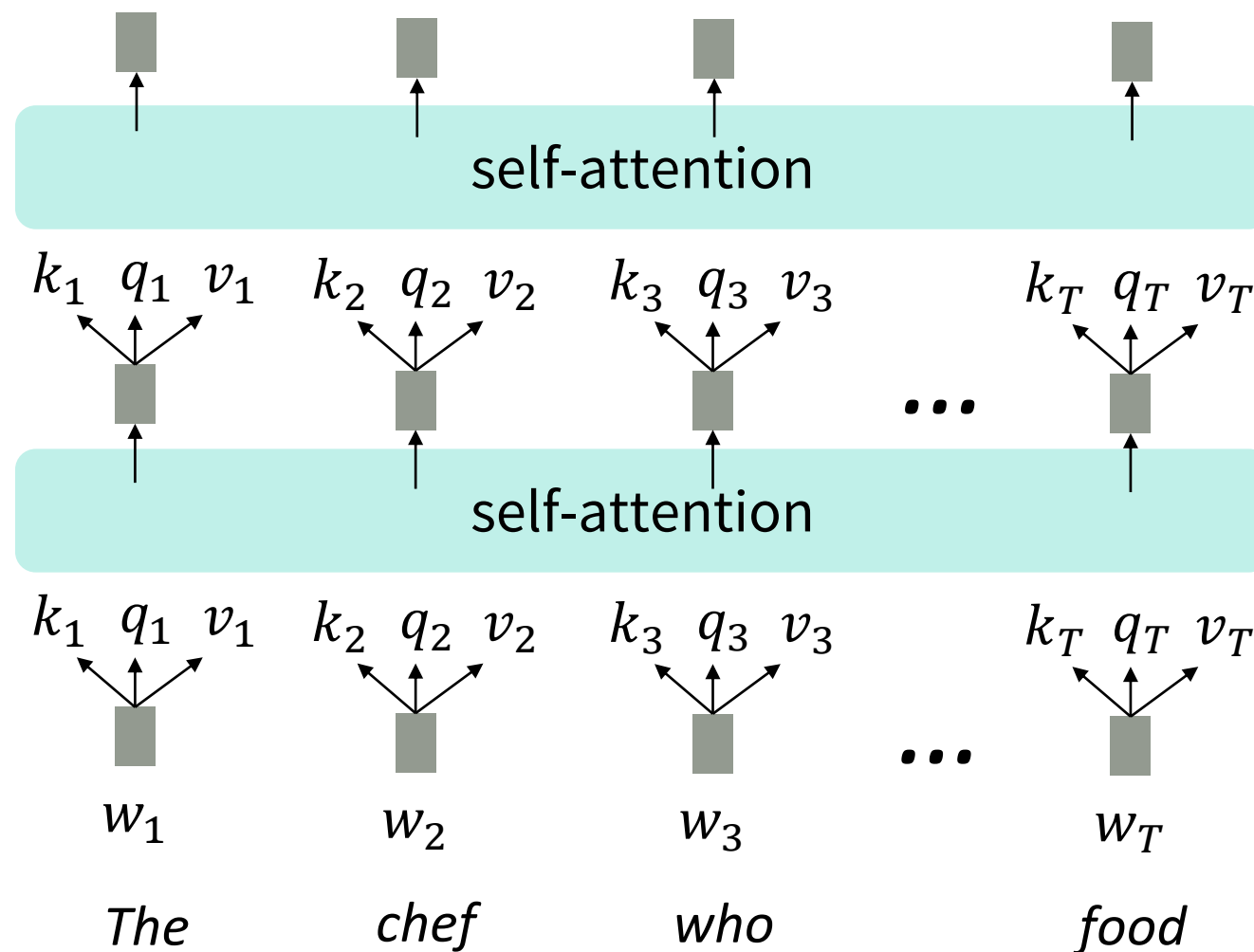
Compute attention weights from affinities (softmax)

$$\text{output}_i = \sum_j \alpha_{ij} v_j$$

Compute outputs as weighted sum of **values**

Self-attention as an NLP building block

- In the diagram at the right, we have stacked self-attention blocks, like we might stack LSTM layers.
- Can self-attention be a drop-in replacement for recurrence?
- No. It has a few issues, which we'll go through.
- First, self-attention is an operation on **sets**. It has no inherent notion of order.



Self-attention doesn't know the order of its inputs.

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!



Solutions

Fixing the first self-attention problem: **sequence order**

- Since self-attention doesn't build in order information, we need to encode the order of the sentence in our keys, queries, and values.
- Consider representing each **sequence index** as a **vector**

$p_i \in \mathbb{R}^d$, for $i \in \{1, 2, \dots, T\}$ are position vectors

- Don't worry about what the p_i are made of yet!
- Easy to incorporate this info into our self-attention block: just add the p_i to our inputs!
- Let $\tilde{v}_i, \tilde{k}_i, \tilde{q}_i$ be our old values, keys, and queries.

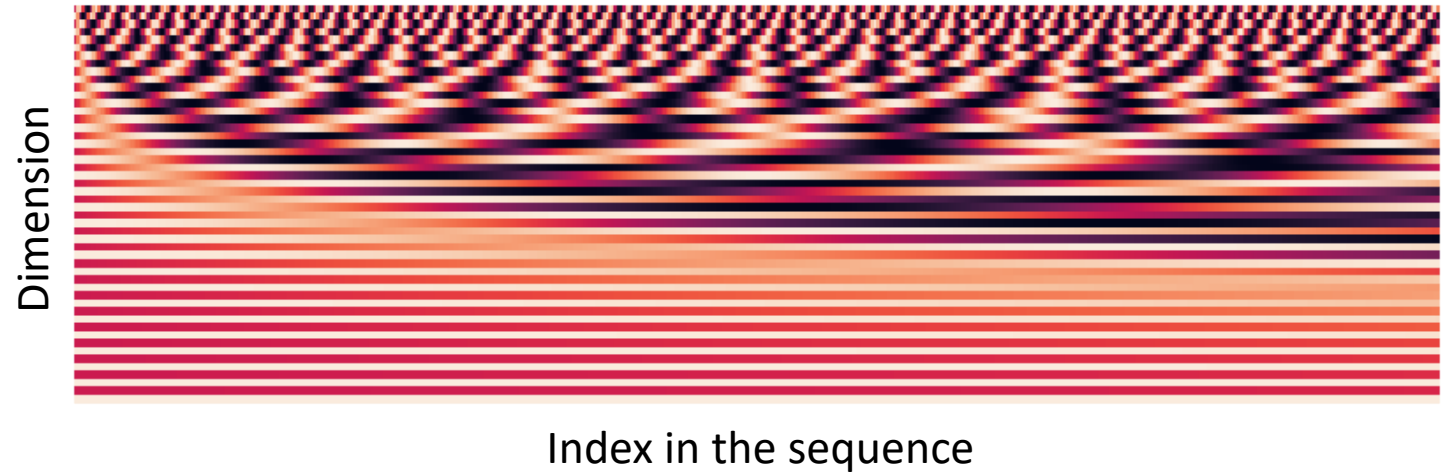
$$\begin{aligned}v_i &= \tilde{v}_i + p_i \\q_i &= \tilde{q}_i + p_i \\k_i &= \tilde{k}_i + p_i\end{aligned}$$

In deep self-attention networks, we do this at the first layer! You could concatenate them as well, but people mostly just add...

Position representation vectors through sinusoids

- **Sinusoidal position representations:** concatenate sinusoidal functions of varying periods:

$$p_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{pmatrix}$$



- Pros:
 - Periodicity indicates that maybe “absolute position” isn’t as important
 - Maybe can extrapolate to longer sequences as periods restart!
- Cons:
 - Not learnable; also the extrapolation doesn’t really work!

Position representation vectors learned from scratch

- **Learned absolute position representations:** Let all p_i be learnable parameters!
Learn a matrix $p \in \mathbb{R}^{d \times T}$, and let each p_i be a column of that matrix!
- Pros:
 - Flexibility: each position gets to be learned to fit the data
- Cons:
 - Definitely can't extrapolate to indices outside $1, \dots, T$.
- Most systems use this!
- Sometimes people try more flexible representations of position:
 - Relative linear position attention [\[Shaw et al., 2018\]](#)
 - Dependency syntax-based position [\[Wang et al., 2019\]](#)

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning! It's all just weighted averages



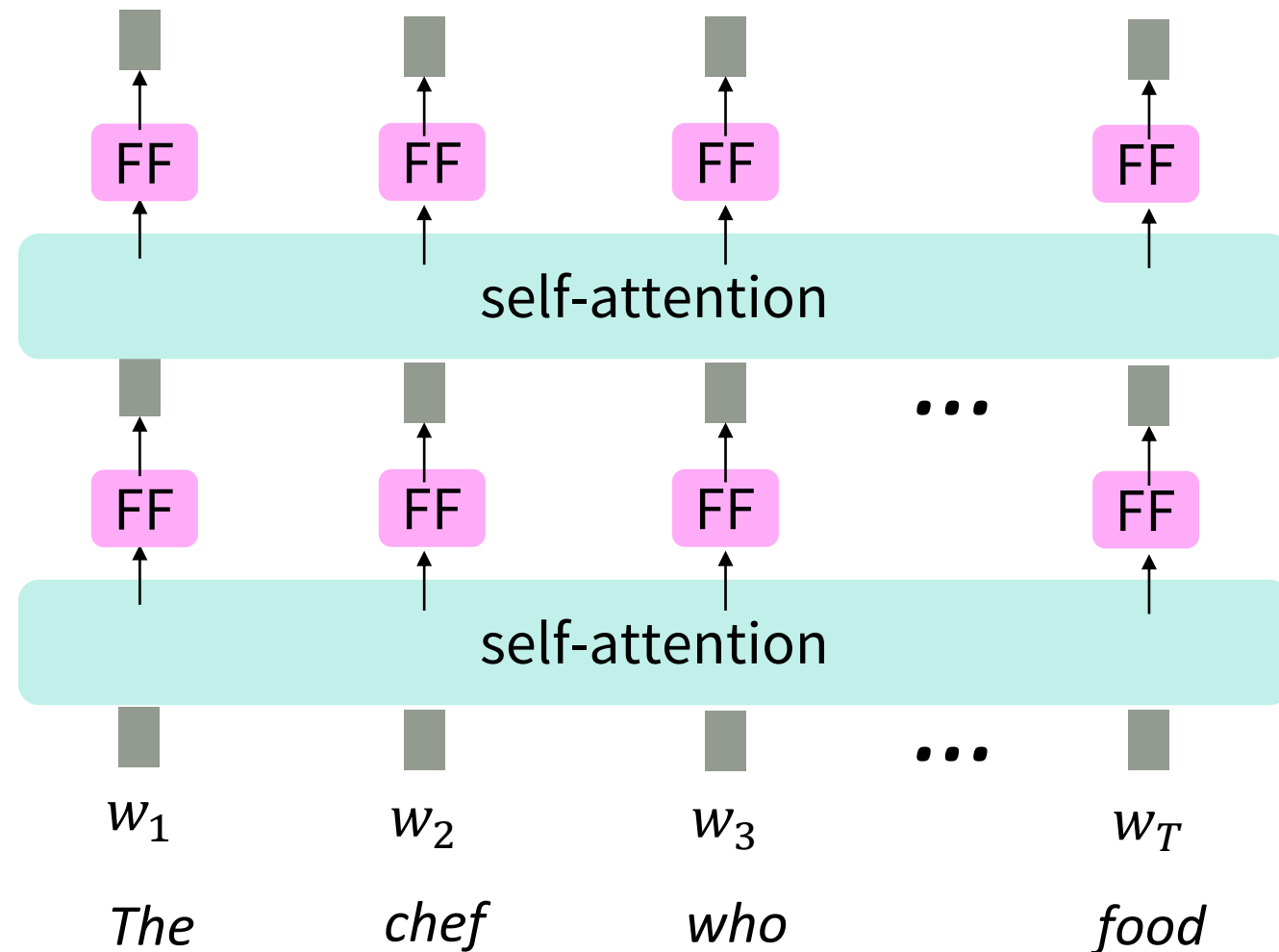
Solutions

- Add position representations to the inputs

Adding nonlinearities in self-attention

- Note that there are no elementwise nonlinearities in self-attention; stacking more self-attention layers just re-averages **value** vectors
- Easy fix: add a **feed-forward network** to post-process each output vector.

$$\begin{aligned} m_i &= MLP(\text{output}_i) \\ &= W_2 * \text{ReLU}(W_1 \times \text{output}_i + b_1) + b_2 \end{aligned}$$



Intuition: the FF network processes the result of attention

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
 - Like in machine translation
 - Or language modeling



Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.

Masking the future in self-attention

- To use self-attention in **decoders**, we need to ensure we can't peek at the future.
- At every timestep, we could change the set of **keys and queries** to include only past words. (Inefficient!)
- To enable parallelization, we **mask out attention** to future words by setting attention scores to $-\infty$.

$$e_{ij} = \begin{cases} q_i^\top k_j, j < i \\ -\infty, j \geq i \end{cases}$$

For encoding these words

[START]

We can look at these
(not greyed out) words

[START] The chef who

[The matrix of e_{ij} values]

Masking the future in self-attention

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$$e_{ij} = \begin{cases} q_i^\top k_j, j < i \\ -\infty, j \geq i \end{cases}$$

For encoding these words

We can look at these (not greyed out) words

	[START]	The	chef	who
[START]	$-\infty$	$-\infty$	$-\infty$	$-\infty$
The		$-\infty$	$-\infty$	$-\infty$
chef			$-\infty$	$-\infty$
who				$-\infty$

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
 - Like in machine translation
 - Or language modeling



Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.
- Mask out the future by artificially setting attention weights to 0!

Necessities for a self-attention building block:

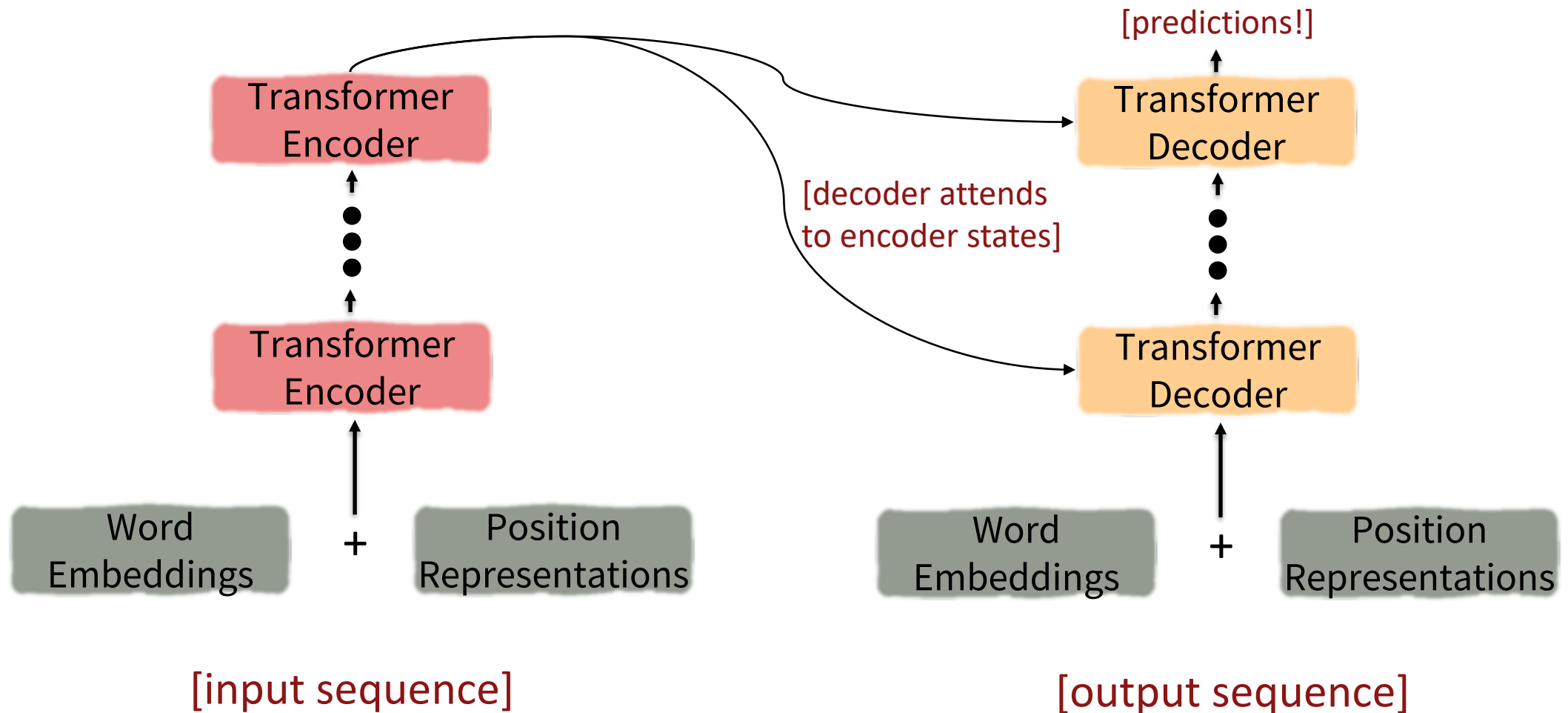
- **Self-attention:**
 - the basis of the method.
- **Position representations:**
 - Specify the sequence order, since self-attention is an unordered function of its inputs.
- **Nonlinearities:**
 - At the output of the self-attention block
 - Frequently implemented as a simple feed-forward network.
- **Masking:**
 - In order to parallelize operations while not looking at the future.
 - Keeps information about the future from “leaking” to the past.
- That’s it! But this is not the **Transformer** model we’ve been hearing about.

Outline

1. From recurrence (RNN) to attention-based NLP models
2. Introducing the Transformer model
3. Great results with Transformers
4. Drawbacks and variants of Transformers

The Transformer Encoder-Decoder [\[Vaswani et al., 2017\]](#)

First, let's look at the Transformer Encoder and Decoder Blocks at a high level



The Transformer Encoder-Decoder [\[Vaswani et al., 2017\]](#)

Next, let's look at the Transformer Encoder and Decoder Blocks

What's left in a Transformer Encoder Block that we haven't covered?

1. **Key-query-value attention:** How do we get the k, q, v vectors from a single word embedding?
2. **Multi-headed attention:** Attend to multiple places in a single layer!
3. **Tricks to help with training!**
 1. Residual connections
 2. Layer normalization
 3. Scaling the dot product
 4. These tricks **don't improve** what the model is able to do; they help improve the training process. Both of these types of modeling improvements are very important!

The Transformer Encoder: Key-Query-Value Attention

- We saw that self-attention is when keys, queries, and values come from the same source. The Transformer does this in a particular way:
 - Let x_1, \dots, x_T be input vectors to the Transformer encoder; $x_i \in \mathbb{R}^d$
- Then keys, queries, values are:
 - $k_i = Kx_i$, where $K \in \mathbb{R}^{d \times d}$ is the key matrix.
 - $q_i = Qx_i$, where $Q \in \mathbb{R}^{d \times d}$ is the query matrix.
 - $v_i = Vx_i$, where $V \in \mathbb{R}^{d \times d}$ is the value matrix.
- These matrices allow *different aspects* of the x vectors to be used/emphasized in each of the three roles.

The Transformer Encoder: Key-Query-Value Attention

- Let's look at how key-query-value attention is computed, in matrices.
 - Let $X = [x_1; \dots; x_T] \in \mathbb{R}^{T \times d}$ be the concatenation of input vectors.
 - First, note that $XK \in \mathbb{R}^{T \times d}$, $XQ \in \mathbb{R}^{T \times d}$, $XV \in \mathbb{R}^{T \times d}$.
 - The output is defined as $\text{output} = \text{softmax}(XQ(XK)^T) \times XV$.

First, take the query-key dot products in one matrix multiplication: $XQ(XK)^T$

The diagram illustrates the first step of the attention mechanism. It shows a vertical pink box labeled XQ multiplied by a horizontal pink box labeled $K^T X^T$. The result is a larger pink box labeled $XQK^T X^T$, which is then followed by $\in \mathbb{R}^{T \times T}$. To the right of this equation, the text "All pairs of attention scores!" is written in blue.

$$XQ \cdot K^T X^T = XQK^T X^T \in \mathbb{R}^{T \times T}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

The diagram illustrates the second step of the attention mechanism. It shows the expression $\text{softmax} \left(XQK^T X^T \right)$ in a large pink box, followed by a vertical pink box labeled XV , and then an equals sign followed by an empty vertical pink box. To the right of this box is the text "output $\in \mathbb{R}^{T \times d}$ ". A curved arrow points from the $XQK^T X^T$ box in the equation above to the $XQK^T X^T$ box in this equation.

$$\text{softmax} \left(XQK^T X^T \right) \cdot XV = \text{output} \in \mathbb{R}^{T \times d}$$

The Transformer Encoder: **Multi-headed attention**

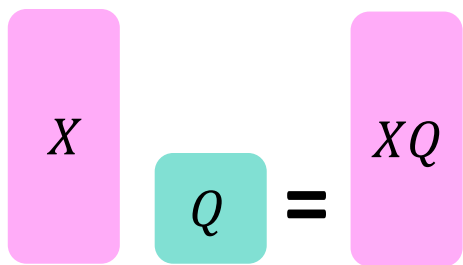
- What if we want to look in multiple places in the sentence at once?
 - For word i , self-attention “looks” where $x_i^\top Q^\top K x_j$ is high, but maybe we want to focus on different j for different reasons?
- We’ll define **multiple attention “heads”** through multiple Q,K,V matrices
- Let, $Q_\ell, K_\ell, V_\ell \in \mathbb{R}^{d \times \frac{d}{h}}$, where h is the number of attention heads, and ℓ ranges from 1 to h .
- Each attention head performs attention independently:
 - $\text{output}_\ell = \text{softmax}(X Q_\ell K_\ell^\top X^\top) * X V_\ell$, where $\text{output}_\ell \in \mathbb{R}^{d/h}$
- Then the outputs of all the heads are combined!
 - $\text{output} = Y[\text{output}_1; \dots; \text{output}_h]$, where $Y \in \mathbb{R}^{d \times d}$
- Each head gets to “look” at different things, and construct value vectors differently.

The Transformer Encoder: **Multi-headed attention**

- What if we want to look in multiple places in the sentence at once?
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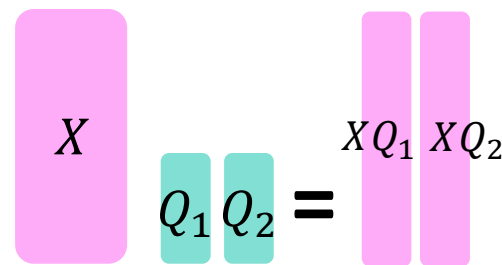
Single-head attention

(just the query matrix)



Multi-head attention

(just two heads here)



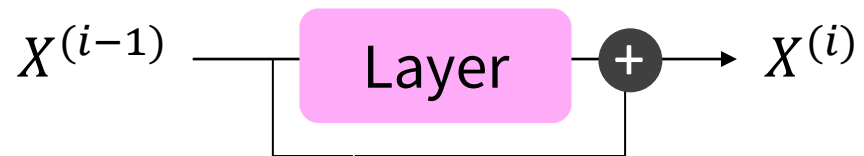
Same amount of computation as single-head self-attention!

The Transformer Encoder: **Residual connections** [[He et al., 2016](#)]

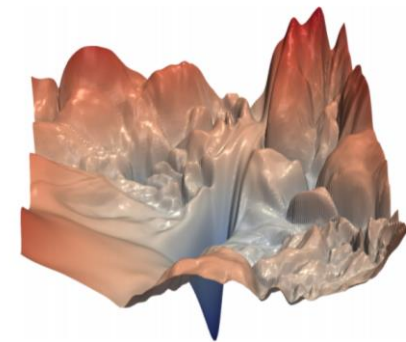
- **Residual connections** are a trick to help models train better.
 - Instead of $X^{(i)} = \text{Layer}(X^{(i-1)})$ (where i represents the layer)



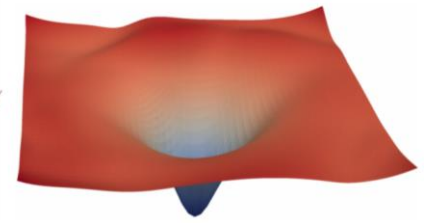
- We let $X^{(i)} = X^{(i-1)} + \text{Layer}(X^{(i-1)})$ (so we only have to learn “the residual” from the previous layer)



- Residual connections are thought to make the loss landscape considerably smoother (thus easier training!)



[no residuals]



[residuals]

[Loss landscape visualization,
[Li et al., 2018](#), on a ResNet]

The Transformer Encoder: **Layer normalization** [[Ba et al., 2016](#)]

- **Layer normalization** is a trick to help models train faster.
- Idea: cut down on uninformative variation in hidden vector values by normalizing to unit mean and standard deviation **within each layer**.
 - LayerNorm's success may be due to its normalizing gradients [[Xu et al., 2019](#)]
- Let $x \in \mathbb{R}^d$ be an individual (word) vector in the model.
- Let $\mu = \sum_{j=1}^d x_j$; this is the mean; $\mu \in \mathbb{R}$.
- Let $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^d (x_j - \mu)^2}$; this is the standard deviation; $\sigma \in \mathbb{R}$.
- Let $\gamma \in \mathbb{R}^d$ and $\beta \in \mathbb{R}^d$ be learned “gain” and “bias” parameters. (Can omit!)
- Then layer normalization computes:

Normalize by scalar
mean and variance

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon}$$

The Transformer Encoder: **Layer normalization** [[Ba et al., 2016](#)]

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- Let $\mu = \sum_{j=1}^d x_j$; this is the mean; $\mu \in \mathbb{R}$.
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- Let $\gamma \in \mathbb{R}^d$ and $\beta \in \mathbb{R}^d$ be learned “gain” and “bias” parameters. (Can omit!)
- Then layer normalization computes:

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

Normalize by scalar mean and variance

Modulate by learned elementwise gain and bias

The Transformer Encoder: **Scaled Dot Product** [Vaswani et al., 2017]

- **“Scaled Dot Product”** attention is a final variation to aid in Transformer training.
- When dimensionality d becomes large, dot products between vectors tend to become large.
 - Because of this, inputs to the softmax function can be large, making the gradients small.

- Instead of the self-attention function we’ve seen:

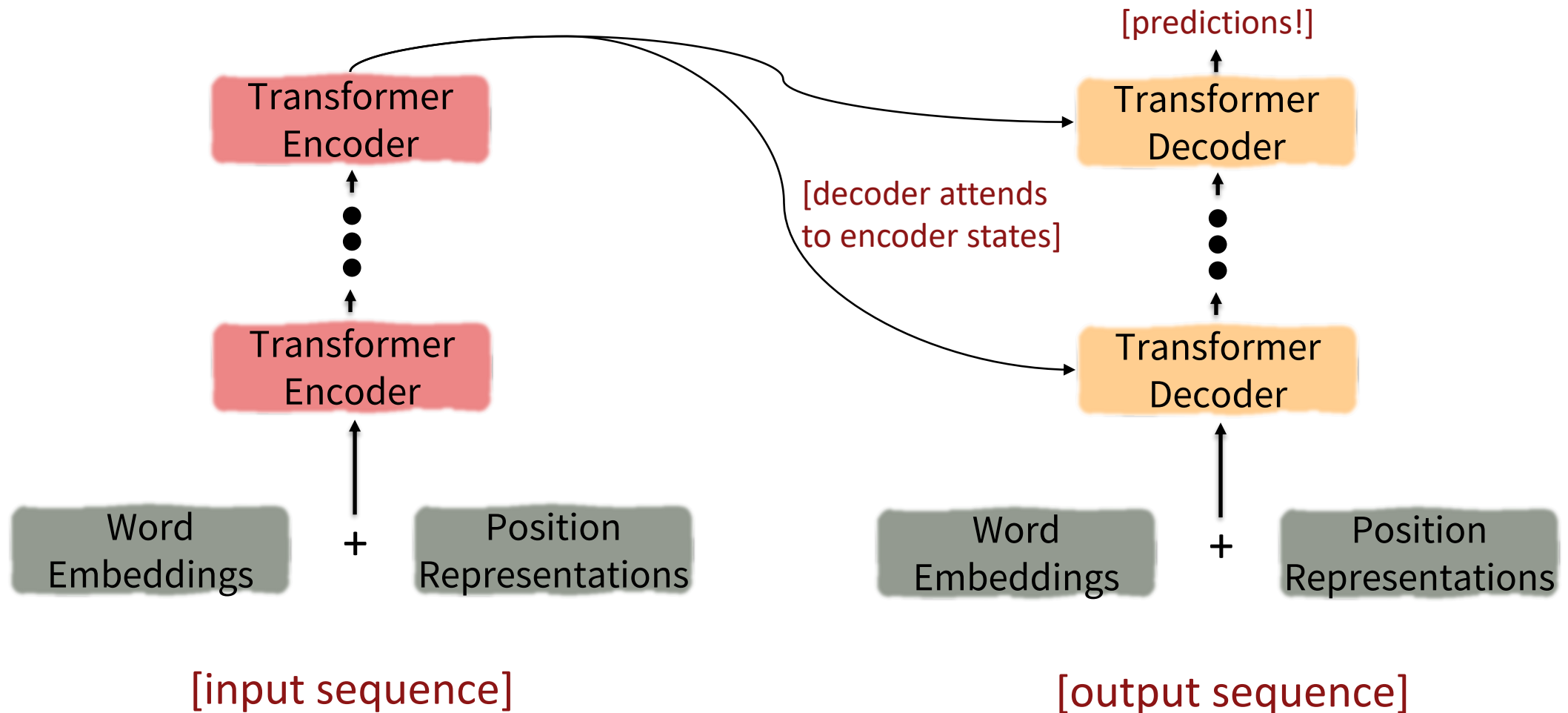
$$\text{output}_\ell = \text{softmax}(XQ_\ell K_\ell^\top X^\top) * XV_\ell$$

- We divide the attention scores by $\sqrt{d/h}$, to stop the scores from becoming large just as a function of d/h (The dimensionality divided by the number of heads.)

$$\text{output}_\ell = \text{softmax}\left(\frac{XQ_\ell K_\ell^\top X^\top}{\sqrt{d/h}}\right) * XV_\ell$$

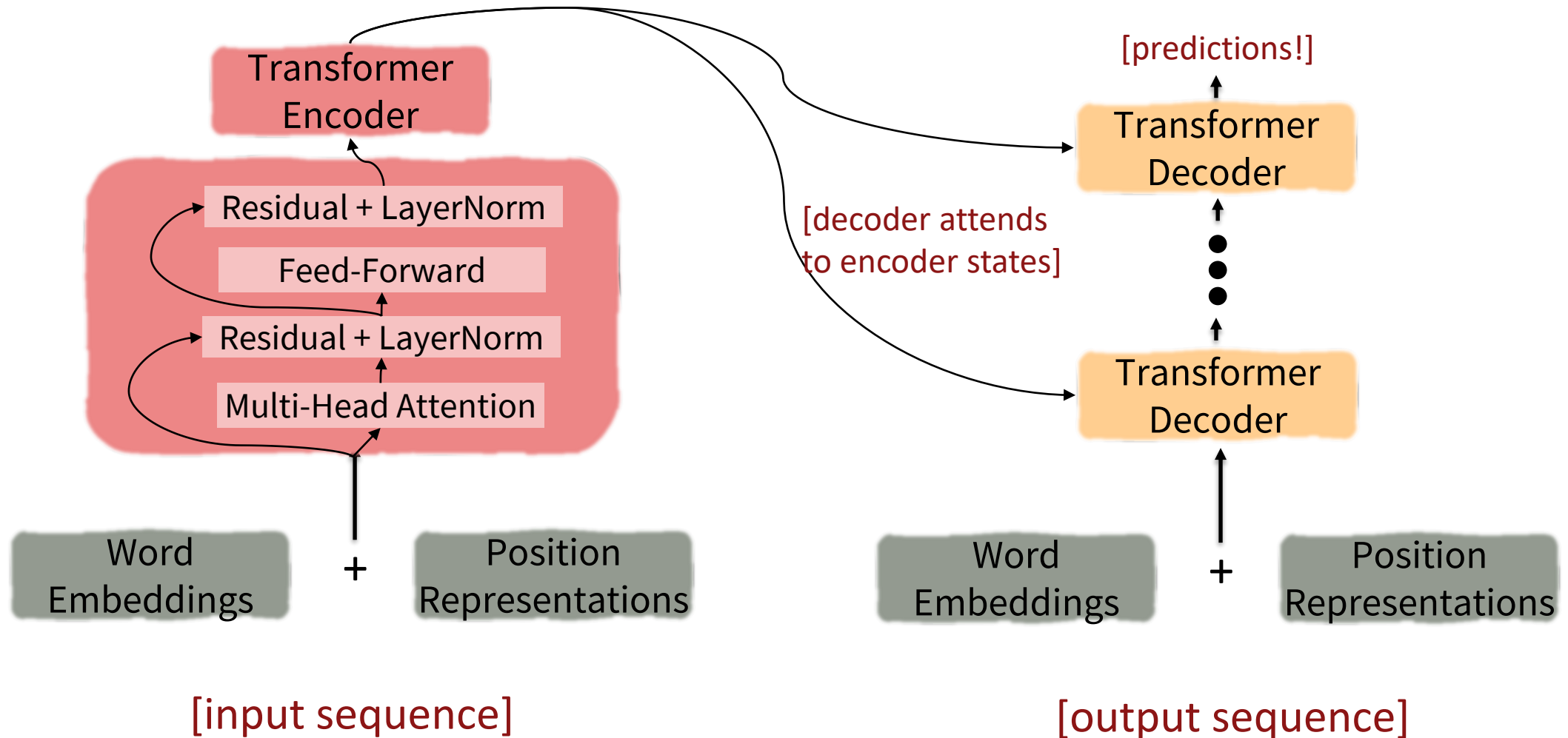
The Transformer Encoder-Decoder [\[Vaswani et al., 2017\]](#)

Looking back at the whole model, zooming in on an Encoder block:



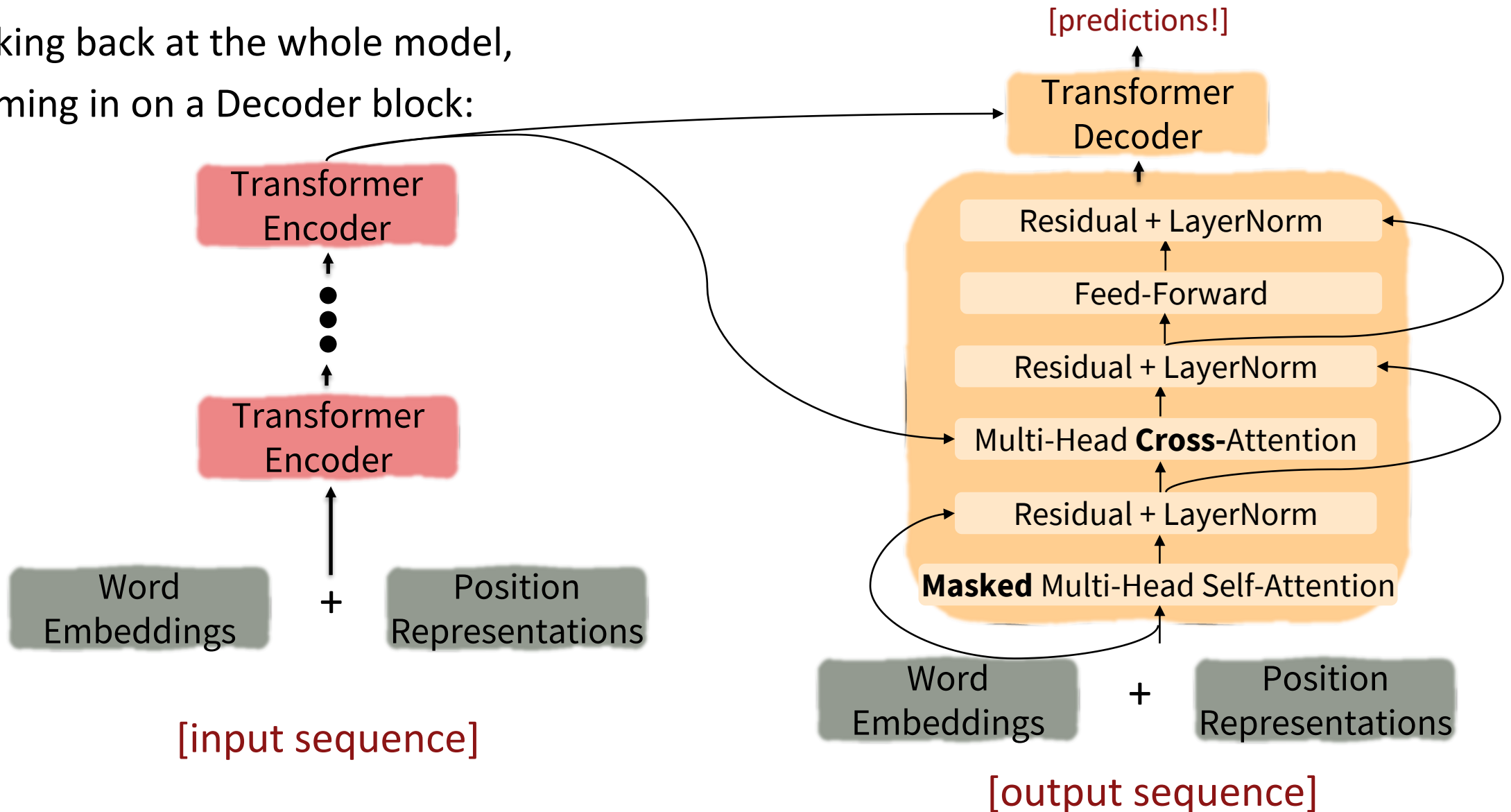
The Transformer Encoder-Decoder [Vaswani et al., 2017]

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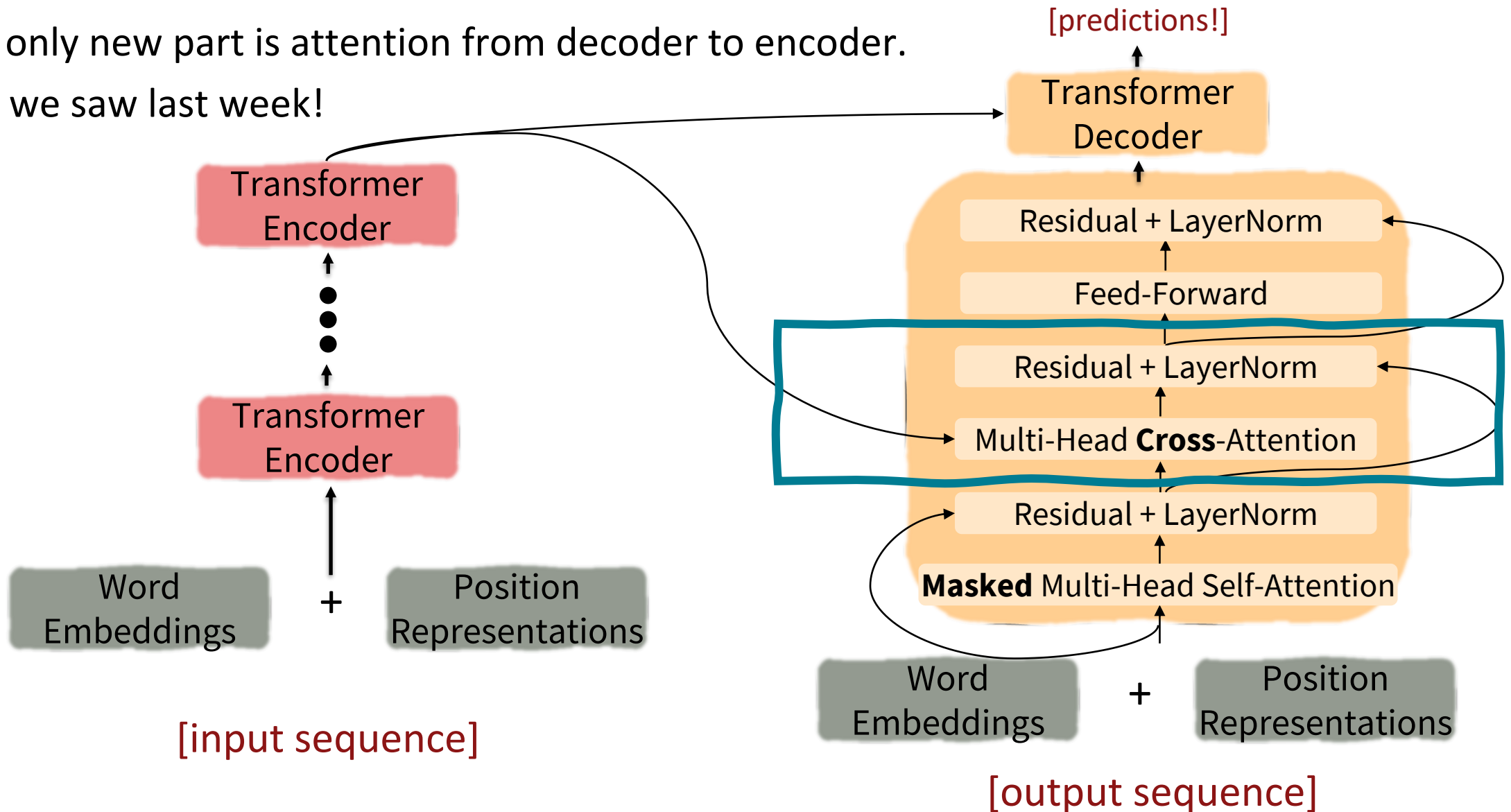
The Transformer Encoder-Decoder [Vaswani et al., 2017]

Looking back at the whole model,
zooming in on a Decoder block:



The Transformer Encoder-Decoder [Vaswani et al., 2017]

The only new part is attention from decoder to encoder.
Like we saw last week!



The Transformer Decoder: Cross-attention (details)

- We saw that self-attention is when keys, queries, and values come from the same source.
- In the decoder, we have attention that looks more like what we saw last week.
- Let h_1, \dots, h_T be **output** vectors **from** the Transformer **encoder**; $x_i \in \mathbb{R}^d$
- Let z_1, \dots, z_T be input vectors from the Transformer **decoder**, $z_i \in \mathbb{R}^d$
- Then keys and values are drawn from the **encoder** (like a memory):
 - $k_i = Kh_i, v_i = Vh_i$.
- And the queries are drawn from the **decoder**, $q_i = Qz_i$.

The Transformer Encoder: Cross-attention (details)

- Let's look at how cross-attention is computed, in matrices.
 - Let $H = [h_1; \dots; h_T] \in \mathbb{R}^{T \times d}$ be the concatenation of encoder vectors.
 - Let $Z = [z_1; \dots; z_T] \in \mathbb{R}^{T \times d}$ be the concatenation of decoder vectors.
 - The output is defined as $\text{output} = \text{softmax}(ZQ(HK)^\top) \times HV$.

First, take the query-key dot products in one matrix multiplication: $ZQ(HK)^\top$

$$ZQ \quad K^\top H^\top = ZQK^\top H^\top \in \mathbb{R}^{T \times T}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

$$\text{softmax} \left(ZQK^\top H^\top \right) HV = \text{output} \in \mathbb{R}^{T \times d}$$

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Great Results with Transformers

First, Machine Translation from the original Transformers paper!

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	41.29	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$

Great Results with Transformers

Next, document generation!

Model	Test perplexity	ROUGE-L
<i>seq2seq-attention, $L = 500$</i>	5.04952	12.7
<i>Transformer-ED, $L = 500$</i>	2.46645	34.2
<i>Transformer-D, $L = 4000$</i>	2.22216	33.6
<i>Transformer-DMCA, no MoE-layer, $L = 11000$</i>	2.05159	36.2
<i>Transformer-DMCA, MoE-128, $L = 11000$</i>	1.92871	37.9
<i>Transformer-DMCA, MoE-256, $L = 7500$</i>	1.90325	38.8

The old standard



Transformers all the way down.



Great Results with Transformers

Before too long, most Transformers results also included **pretraining**, a method we'll go over on Thursday.

Transformers' parallelizability allows for efficient pretraining, and have made them the de-facto standard.

On this popular aggregate benchmark, for example:



All top models are Transformer (and pretraining)-based.

Rank Name		Model	URL	Score
1	DeBERTa Team - Microsoft	DeBERTa / TuringNLRv4	↗	90.8
2	HFL iFLYTEK	MacALBERT + DKM		90.7
+ 3	Alibaba DAMO NLP	StructBERT + TAPT	↗	90.6
+ 4	PING-AN Omni-Sinitic	ALBERT + DAAF + NAS		90.6
5	ERNIE Team - Baidu	ERNIE	↗	90.4
6	T5 Team - Google	T5	↗	90.3

More results Thursday when we discuss pretraining.

[[Liu et al., 2018](#)]

Outline

1. From recurrence (RNN) to attention-based NLP models
2. Introducing the Transformer model
3. Great results with Transformers
4. Drawbacks and variants of Transformers

What would we like to fix about the Transformer?

- **Quadratic compute in self-attention (today):**
 - Computing all pairs of interactions means our computation grows **quadratically** with the sequence length!
 - For recurrent models, it only grew linearly!
- **Position representations:**
 - Are simple absolute indices the best we can do to represent position?
 - Relative linear position attention [\[Shaw et al., 2018\]](#)
 - Dependency syntax-based position [\[Wang et al., 2019\]](#)

Quadratic computation as a function of sequence length

- One of the benefits of self-attention over recurrence was that it's highly parallelizable.
- However, its total number of operations grows as $O(T^2 d)$, where T is the sequence length, and d is the dimensionality.

$$\begin{matrix} \boxed{XQ} \\ \boxed{K^T X^T} \end{matrix} = \boxed{XQK^T X^T} \in \mathbb{R}^{T \times T}$$

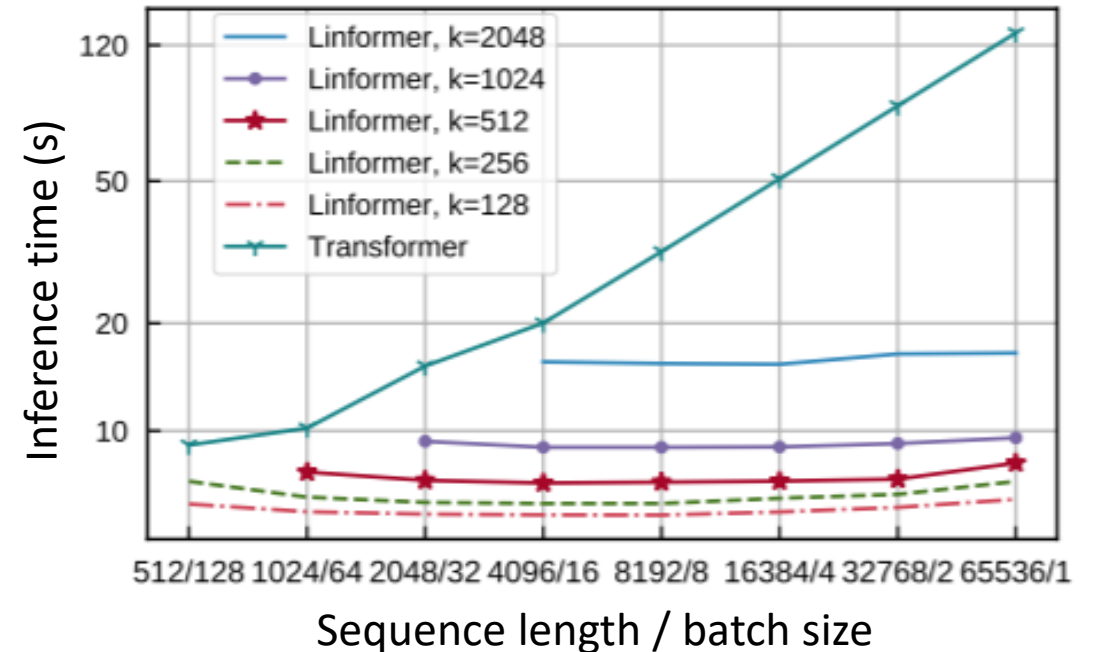
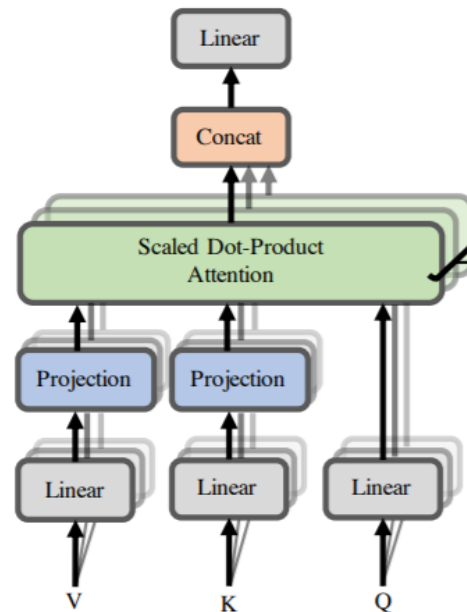
Need to compute all pairs of interactions!
 $O(T^2 d)$

- Think of d as around **1,000**.
 - So, for a single (shortish) sentence, $T \leq 30$; $T^2 \leq \mathbf{900}$.
 - In practice, we set a bound like $T = 512$.
 - **But what if we'd like $T \geq 10,000$?** For example, to work on long documents?

Recent work on improving on quadratic self-attention cost

- Considerable recent work has gone into the question, *Can we build models like Transformers without paying the $O(T^2)$ all-pairs self-attention cost?*
- For example, **Linformer** [\[Wang et al., 2020\]](#)

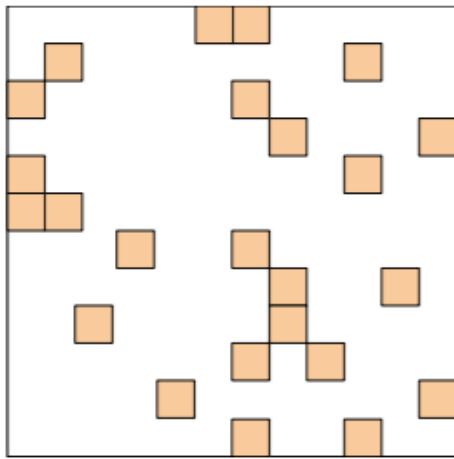
Key idea: map the sequence length dimension to a lower-dimensional space for values, keys



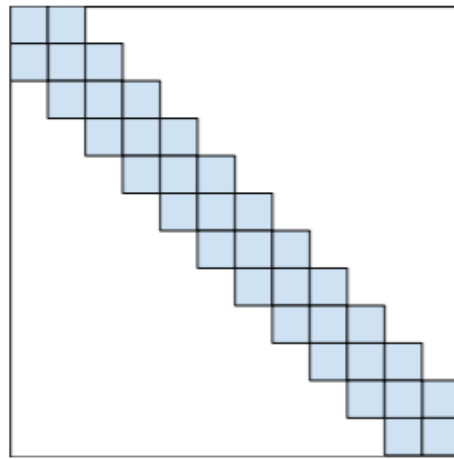
Recent work on improving on quadratic self-attention cost

- Considerable recent work has gone into the question, *Can we build models like Transformers without paying the $O(T^2)$ all-pairs self-attention cost?*
- For example, **BigBird** [\[Zaheer et al., 2021\]](#)

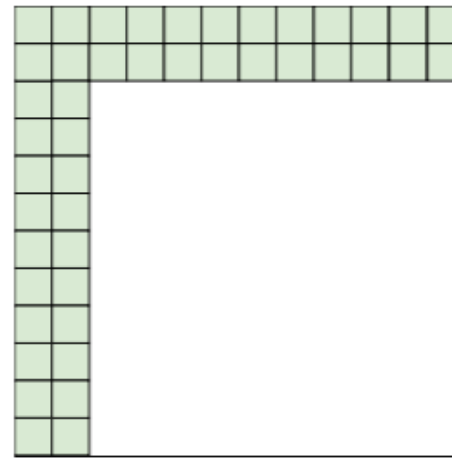
Key idea: replace all-pairs interactions with a family of other interactions, **like local windows, looking at everything**, and **random interactions**.



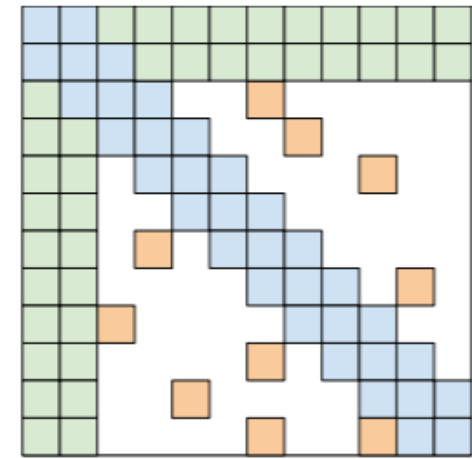
(a) Random attention



(b) Window attention



(c) Global Attention



(d) BIGBIRD

Parting remarks

- Pretraining on Thursday!
- Good luck on assignment 4!
- Remember to work on your project proposal!