CS435 Algorithms Mid-block exam

January 21, 2006

Name ________________________________

Answer True or False next to each question. Please write clearly.

1. ______ A logarithmic algorithm has a time complexity of O(log n).
2. ______ A quadratic algorithm has a time complexity of O(n^3).
3. ______Generally, an algorithm that runs in O(n log n) time will take longer than an algorithm that has O(log n) time complexity when $n > n_0$.
4. ______When deciding between a List and an Array data structure, if the application will be frequently accessing the elements by rank and seldom inserting elements by rank then it is better to chose a List to store the elements.
5. ______In the Queue ADT, the enqueue and dequeue operations run in O(log n) time.
6. ______In a proper binary tree, every internal node has two children.
7. ______The height of a tree T is equal to the maximum depth of an external node of T.
8. ______Post-order traversal of a tree means the node is "visited" after the node's parent is "visited".
9. ______A red-black tree that has 1000 internal nodes will have a height between 25 and 50.
10. ______Insertion-sort and selection-sort are best used only on sequences of a few hundred or fewer elements since faster sorting algorithms are available for larger sequences.
11. ______An unordered dictionary is inefficient for finding items since the entire dictionary might have to be scanned to find the key.
12. ______In Radix-sort, the key is divided into components and Bucket-sort is run on the input data using first the most-significant component, followed by Bucket-sorts using each component in order.

Multiple choice. Circle the letter of the statement with the best answer.

11. An algorithm with O(n^2) average case time complexity that takes 10 seconds to execute for an input size of 1000 elements will take how long to run when the input size is 10,000 elements.
    a) less than 50 seconds
    b) from 50 up to 500 seconds
    c) from 500 up to 5000 seconds
    d) from 5000 up to 50,000 seconds
    e) more than 50,000 seconds

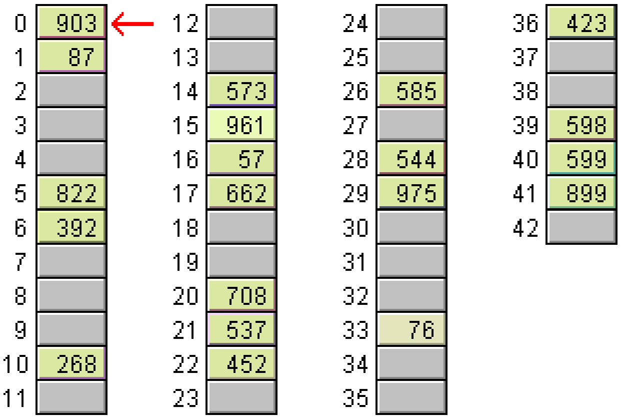
12. What is the worst case time complexity of an insertion into a red-black tree of size *n* and why?
   a) O(1) – the timing doesn't depend on the size of the dataset.
   b) O(log *n*) – the insertion has to traverse all the levels of the tree.
   c) O(*n*) – the insertion can cause a ripple effect to all the nodes.
   d) O(*n* log *n*) – the insertion depends on the height of the tree and the restructuring takes time.
   e) O(n^2) – in the worst case, it is the same as insertion-sort.

13. What is primary benefit offered by hash tables?
   a) They are very size efficient.
   b) They expand automatically with no extra operations.
   c) They handle various kinds of objects.
   d) They are very fast for insertion and retrieval.

14. The Dictionary ADT includes what methods:
   a) atRank*(r),* size(), isEmpty()
   b) insertAfter(*p*), removeElement(), first()
   c) findElement(*k*), insertItem(*k,o*), isEmpty()
   d) find(*l*), keys(), isLast(*p*)

15. Which situation is Bucket-sort the best method to use for sorting?
   a) When the input size of the data elements is less than a million.
   b) When the keys are integers in a range less than the input size.
   c) When the keys are very long and can be sub-divided evenly.
   d) When the keys are short strings less than 32 characters.

16. The following hash table stores integer keys in the range of [0, 999] and uses the hash function and compression map, *h(k)* = *k%* 43. Collisions are handled using the quadratic probing strategy.



0	903	12		24		36	423
1	87	13		25		37	
2		14	573	26	585	38	
3		15	961	27		39	598
4		16	57	28	544	40	599
5	822	17	662	29	975	41	899
6	392	18		30		42	
7		19		31			
8		20	708	32			
9		21	537	33	76		
10	268	22	452	34			
11		23		35			

   a) Into which slot will the integer key 58 be inserted? _________
   b) Into which slot will the integer key 43 be inserted? _________
   c) What is the load factor for the hash table as shown. __________
   d) How is the delete operation handled in a hash table using quadratic probing strategy?
   ____

17. What is the heap-order property for a min-heap?

a) The key stored at a node is greater than or equal to the key stored at the parent.
b) The external nodes do not store keys or key-element pairs.
c) All the internal nodes on a level are “to the left” of the external nodes on the same level.
d) The last internal node of the tree stores the minimum key.

18. What is the primary advantage for implementing the Priority-Queue ADT using the heap data structure?

a) Heaps are more space-efficient than arrays or lists.
b) The heap provides random access to any key stored in the heap.
c) Inserting items in the heap always puts them on the end.
d) Removing the minimum item in the heap is faster than an array.

19. A total order relation for keys is necessary to satisfy the ....

a) object hierarchy.
b) comparison rule.
c) substitution property.
d) bin assignment.

20. The worst-case time complexity of Quick-sort is no better than Selection-sort or Insertion-sort. Why is Quick-sort so widely used in applications?

a) The in-place Quick-sort is more memory efficient than the other sorts.
b) Recursive algorithms are very appealing to programmers.
c) Divide-and-conquer is a better approach for comparison-based sorting.
d) In the average case, Quick-sort is O(*n* log *n*).

$$T(n) = \begin{cases} b & \text{if } n < 2 \\ 2T(n/2) + cn & \text{if } n \geq 2 \end{cases}$$

21. Consider the following recurrence equation. Which statement about this equation is **false**?

a) The equation describes the running time of a recursive algorithm.
b) The equation can be reformulated to a closed-form expression.
c) The closed-form version is O(*n* log *n*).
d) The equation describes the running time of bottom-up heap construction.

22. In a Red-Black tree, the restructuring and recoloring operations...

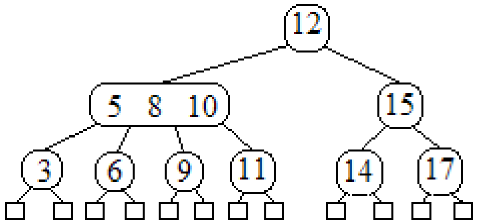
a) ... keep the balance between red and black nodes so they are always the same.
b) ...cause updating to take twice as much time as searching.
c) ...are performed when searching, inserting and deleting key-element pairs.
d) ...are designed to maintain the depth so ordered dictionary searches are fast.

23. Let *T* be a (2, 4) tree shown below, which stores items with integer keys.

a) Insert an item with key 7 into *T* .

b) Remove an item with key 17 from *T* .

Note that the above operations should be performed independently; that is, on separate copies of *T* ..No credit will be given for part (b) if you perform the removal on the tree resulting from the insertion of part (a) (or the other way around).



24. A 10-element complete binary tree can be represented by a vector with these values [5, 9, 6, 15, 12, 7, 20, 16, 25, 14, 13, 11]. Draw the complete binary tree. Is this binary tree a min-heap? Why or why not?

25. Given a set *S* of positive integers of size *n* and an integer *M*, describe an algorithm to find if any pair of two numbers in the set add to *M*. A simple algorithm that tests every integer in *S* with every other integer runs in O(n^2) time. Find a significantly faster algorithm and describe it using either pseudo-code or sentences and drawings.

Optional bonus question

Describe either the Merge-sort or Quick-sort algorithms with pseudo-code, written description and drawing. More points for better, more concise description.