Quiz

Algorithms, Corazza

For this quiz, you may use the document MathReview.pdf.

1. Compute the following:
   (A) $\{1,2\} \cup \{3,4\}$
   (B) $\{1,2\} \cup \{2,3\}$
   (C) $\{1,2\} \cap \{2,3\}$
   (D) $[0,1] \cup [1,2]$, where $[a,b]$ denotes the set of all real numbers between — and including — a and b.
   (E) $(0,2) \cup (1,3)$, where (a,b) denotes the set of all real numbers between — but *not* including — a and b
   (F) $[0,2] \cap [1,3]$
   (G) $\mathbf{E} \cup \mathbf{O}$, where $\mathbf{E}$ is the set of even natural numbers and $\mathbf{O}$ is the set of odd natural numbers.
   (H) $\mathbf{E} \cap \mathbf{O}$, where $\mathbf{E}$ is the set of even natural numbers and $\mathbf{O}$ is the set of odd natural numbers.

2. Show that for any $b > 0$, $\log_b(\frac{1}{c}) = -\log_b(c)$ and $\log_b(c) = \frac{1}{\log_c(b)}$.
3. Show that for any $x, b, c > 0$, $\frac{\log_b(x)}{\log_c(x)} = \log_b(c)$.
4. Define functions f, g, h from $\{0,1,2\}$ to $\{0,1,2\}$ as follows:

$$f(n) = n$$
$$g(n) = (1+n) \bmod 3$$
$$h(n) = (2+n) \bmod 3$$

   Let $S = \{f, g, h\}$. Define F on S by

$$F(u)(n) = 2u(n) \bmod 3$$

   for any $u \in S$. Is it true that $F(u) \in S$ for every $u \in S$? Explain.
5. Prove by induction that for all $n > 3, 2^n > 3n$. What value will you use in the base case?

6. Prove by induction that the function $f(n) = n \log n$ is increasing.

7. The Division Algorithm guarantees that, given the numbers 31 and 7, there are unique numbers q and r such that $31 = 7q + r$ with $0 \le r < 7$. Find q and r in this case.

8. The Division Algorithm guarantees that, given the numbers -31 and 7, there are unique numbers q and r such that $-31 = 7q + r$ with $0 \le r < 7$. Find q and r in this case.

9. Suppose a, b, c, d are nonzero integers. Prove the following:
   a. If $a|b$ and $b|c$ then $a|c$.
   b. If $a|b$ and $a|c$ then $a|(b+c)$ and $a|(b-c)$

10. Suppose a, b, d, x, y are integers satisfying $1 = ax + by$. Show that $\gcd(a,b) = 1$.

11. Suppose p, q are distinct primes. Prove that there are integers x, y such that $1 = px + qy$.
12. Find integers x, y so that $1 = 3x + 5y$. (See the previous exercise.)
13. Write out the first 10 Fibonacci numbers.