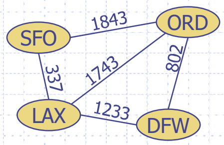
# Lecture 13: BFS Graph Traversal & Templates

Principle of Transcending



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# Wholeness Statement

Graphs have many useful applications in different areas of computer science. However, to be useful we have to be able to traverse them. One of the two primary ways that graphs are systematically explored, is using the breadth-first search algorithm. *Science of Consciousness:* The TM technique provides a simple, effortless way to systematically explore the different levels of the conscious mind until the process of thinking is transcended and unbounded silence is experienced; this is the field of wholeness of individual and cosmic intelligence.



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# List of Terms

- Graph

- Vertex, vertices
- End vertices
- Adjacent vertices
- Degree of a vertex

- Edges
- Incident edges
- Directed edge, undirected edge
- Directed graph, undirected graph, mixed graph

- Path, simple path
- Cycle, simple cycle



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# More Terms

- Subgraph
- Connectivity
  - Connected Vertices (path between them)
  - Connected Graph (all vertices are connected)
  - Connected Component (maximal connected subgraph)
- Tree (connected, no cycles)
- Forest (one or more trees)
- Spanning Tree and Spanning Forest



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# Breadth-First Search Outline and Reading

- Breadth-first search
  - Example
  - Algorithm
  - Properties
  - Analysis
  - Applications
- DFS vs. BFS
  - Comparison of applications
  - Comparison of edge labels



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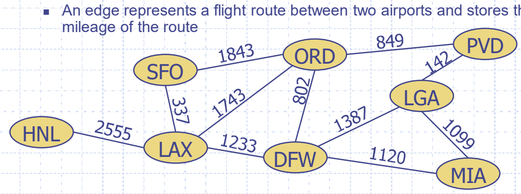
# Graph

- A graph is a pair $(V, E)$, where
  - $V$ is a set of nodes, called vertices
  - $E$ is a collection of pairs of vertices, called edges
  - Vertices and edges are positions and store elements
- Example:
  - A vertex represents an airport and stores the three-letter airport code
  - An edge represents a flight route between two airports and stores the mileage of the route



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# Properties

**Property 1**

$\Sigma_v \deg(v) = 2m$

Proof: each edge is counted twice

**Property 2**

In an undirected graph with no self-loops and no parallel edges

$m \leq n(n-1)/2$

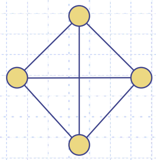
Proof: each vertex has degree at most $(n-1)$

What is the bound for a directed graph?

$m \leq n(n-1)$

**Notation**

| | |
|---|---|
| $n$ | number of vertices |
| $m$ | number of edges |
| $\deg(v)$ | degree of vertex $v$ |



**Example**

- $n = 4$
- $m = 6$
- $\deg(v) = 3$



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# Main Methods of the Undirected Graph ADT

- Vertices and edges
  - are Positions
  - store elements
- Accessor methods
  - aVertex()
  - incidentEdges(v)
  - endVertices(e)
  - opposite(v, e)
  - areAdjacent(v, w)
- Update methods
  - insertVertex(o)
  - insertEdge(v, w, o)
  - removeVertex(v)
  - removeEdge(e)
- Generic methods
  - numVertices()
  - numEdges()
  - vertices()
  - edges()
  - degree(v)



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# Graph Data Structures

Adjacency list



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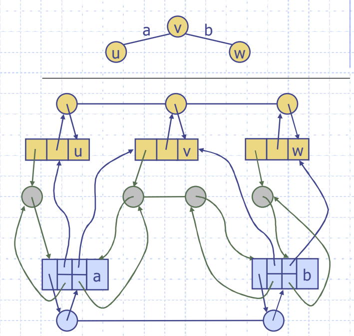
# Adjacency List Structure

- Edge list structure
- Incidence sequence for each vertex
  - sequence of references to edge objects of incident edges
- Augmented edge objects
  - references to associated positions in incidence sequences of end vertices



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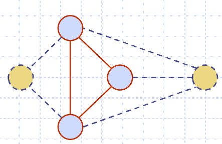
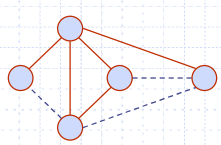
# Subgraphs

- A subgraph S of a graph G is a graph such that
  - vertices(S) $\subseteq$ vertices(G)
  - edges(S) $\subseteq$ edges(G)
- A spanning subgraph of G is a subgraph that contains all the vertices of G, i.e., vertices(S) = vertices(G)



Subgraph



Spanning subgraph



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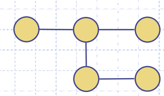
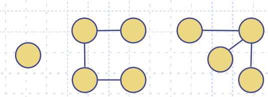
# Trees and Forests

- A (free) *tree* is an **undirected** graph T such that
  - T is **connected**
  - T has no **cycles**

  This definition is different from the definition of a rooted tree
- A *forest* is an undirected graph without cycles
- The connected components of a forest are trees



Tree



Forest



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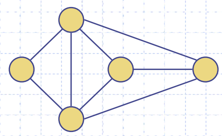
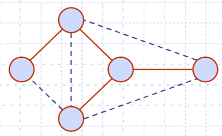
# Spanning Trees and Forests

- A spanning tree of a connected graph is a spanning subgraph that is a tree
- A spanning tree is not unique unless the graph is a tree
- Spanning trees have applications to the design of communication networks
- A spanning forest of a graph is a spanning subgraph that is a forest



Graph



Spanning tree



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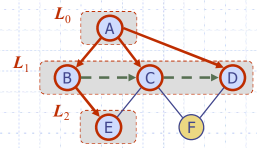
# Breadth-First Search



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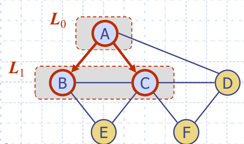
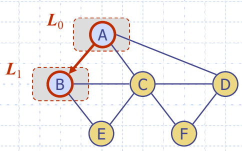
# Example

- A unexplored vertex
- A visited vertex
- unexplored edge
- discovery edge
- cross edge



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# Example

- A unexplored vertex
- A visited vertex
- unexplored edge
- discovery edge
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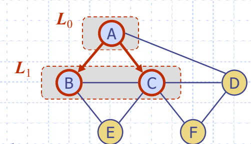


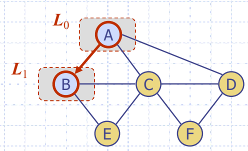


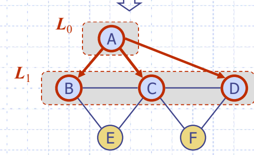




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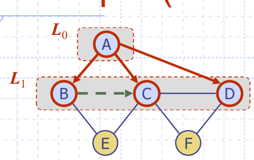
# Example (cont.)



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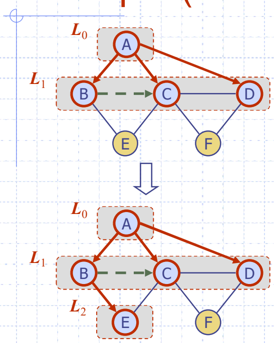
# Example (cont.)



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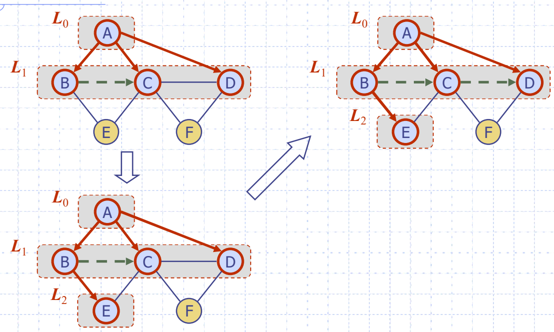
## Example (cont.)





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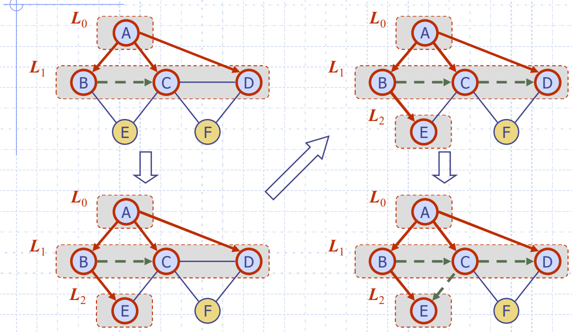
## Example (cont.)





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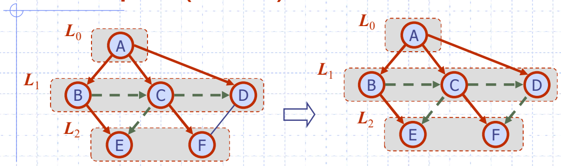
## Example (cont.)





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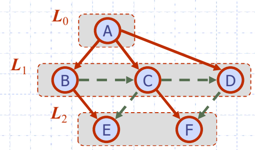
## BFS Levels

When actually implemented, the levels are normally merged into a single list/queue



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## BFS Algorithm

- The BFS algorithm using a single list/sequence/Queue

```
Algorithm BFS(G)
    Input graph G
    Output labeling of the edges
        and partition of the
        vertices of G
  for all u ∈ G.vertices() do
    setLabel(u, UNEXPLORED)
  for all e ∈ G.edges() do
    setLabel(e, UNEXPLORED)
  for all v ∈ G.vertices() do
    if getLabel(v) = UNEXPLORED
      BFScomponent(G, v)
```

```
Algorithm BFScomponent(G, s)
  Q ← new empty Queue
  Q.enqueue(s)
  setLabel(s, VISITED)
  while Q.size() > 0 do
    v ← Q.dequeue ()
    for all e ∈ G.incidentEdges(v) do
      if getLabel(e) = UNEXPLORED then
        w ← G.opposite(v,e)
        if getLabel(w) = UNEXPLORED
        then
          setLabel(e, DISCOVERY)
          setLabel(w, VISITED)
          Q.enqueue(w)
        else
          setLabel(e, CROSS)
```



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## Properties

Notation

$G_s$: connected component of $s$

Property 1

***BFScomponent***$(G, s)$ visits all the vertices and edges of $G_s$

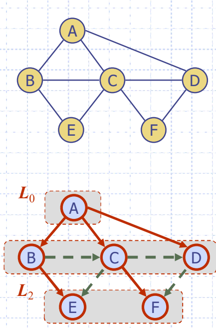
Property 2

The discovery edges labeled by ***BFScomponent***$(G, s)$ form a spanning tree $T_s$ of $G_s$

Property 3

For each vertex $v$ in $L_i$

- The path of $T_s$ from $s$ to $v$ has $i$ edges
- Every path from $s$ to $v$ in $G_s$ has at least $i$ edges





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## Analysis

- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into the queue $Q$
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$





## Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph
- A BFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G





## Breadth-First Search

- BFS on a graph with $n$ vertices and $m$ edges takes $O(n + m)$ time
- BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one





## Applications

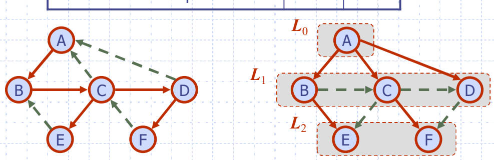
- Using the template method pattern, we can specialize the BFS traversal of a graph $G$ to solve the following problems in $O(n + m)$ time
  - Compute the connected components of $G$
  - Compute a spanning forest of $G$
  - Find a simple cycle in $G$, or report that $G$ is a forest
  - Given two vertices of $G$, find a path in $G$ between them with the minimum number of edges, or report that no such path exists





## DFS vs. BFS

| Applications | DFS | BFS |
|---|---|---|
| Spanning forest, connected components, paths, cycles | √ | √ |
| Shortest paths | | √ |
| Biconnected components | √ | |




DFS BFS

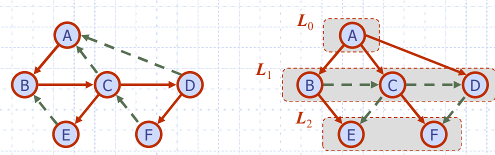




## DFS vs. BFS (cont.)

Back edge $(v,w)$
- $w$ is an ancestor of $v$ in the tree of discovery edges

**Cross edge** $(v,w)$
- $w$ is in the same level as $v$ or in the next level in the tree of discovery edges




DFS BFS

## Main Point

1. During breadth-first search of a graph, the search repeatedly takes one step in all directions until all vertices and edges are visited. This is a bit like searching for fulfillment in waking state, i.e., floating on the surface of the mind or through one's daily activity.
   *Science of Consciousness:* In contrast, Transcendental Meditation takes the mind immediately and effortlessly to the deepest levels where true fulfillment can be gained.





## Template Method Pattern

Depth-first search is to graphs what the Euler tour is to binary trees





## Recall Our Earlier Example of the Template Method Pattern in Java

```
public abstract class EulerTour {
    protected void visitExternal(BinaryTree tree, Position p, Object[ ] r) { }
    protected void visitPreOrder(BinaryTree tree, Position p, Object[ ] r) { }
    protected void visitInOrder(BinaryTree tree, Position p, Object[ ] r) { }
    protected void visitPostOrder(BinaryTree tree, Position p, Object[ ] r) { }
    protected Object eulerTour(BinaryTree tree, Position p) {
        Object[ ] result = new Object[3];
        if tree.isExternal(p) { visitExternal(tree, p, result); }
        else {
            visitPreOrder(tree, p, result);
            result[1] = eulerTour(tree, tree.leftChild(p));
            visitInOrder(tree, p, result);
            result[2] = eulerTour(tree, tree.rightChild(p));
            visitPostOrder(tree, p, result);
        }
        return result[0];
    }
}
```





## Template Method Pattern

- Generic algorithm that can be specialized by redefining certain steps
- Implemented by means of an abstract Java class
- Visit methods that can be redefined/overridden by subclasses
- Template method eulerTour
  - Recursively called on the left and right children
  - A result array that keeps track of the output of the recursive calls to eulerTour
  - result[0] keeps track of the <u>final</u> output of the eulerTour method
  - result[1] keeps track of the output of the recursive call of eulerTour on the left child
  - result[2] keeps track of the output of the recursive call of eulerTour on the right child





## Specializations of EulerTour

```
public class Sum extends EulerTour {
// Sums the integers in a Binary Tree of Integers
    public Integer sum(BinaryTree tree) {
        return eulerTour(tree, tree.root());
    }
    protected void visitExternal(BinaryTree t, Position p, Object[ ] res) {
        result[0] = new Integer(0);
    }
    protected void visitPostOrder(BinaryTree t, Position p, Object[ ] result) {

        result[0] = (Integer) result[1] + (Integer) result[2] + p.element()

    }
    …
}
```



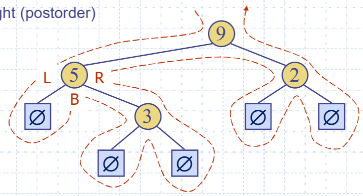


## Specializations of EulerTour

```
public class Sum extends EulerTour {
// Sums the integers in a Binary Tree of Integers (another way)
    public Integer sum(BinaryTree tree) {
        return eulerTour(tree, tree.root());
    }
    protected void visitExternal(BinaryTree t, Position p, Object[ ] result) {
        result[0] = new Integer(0);
    }
    protected void visitPreOrder(BinaryTree t, Position p, Object[ ] result) {
        result[0] = p.element()
    }
    protected void visitInOrder(BinaryTree t, Position p, Object[ ] result) {
        result[0] = (Integer) result[1] + (Integer) result[0]
    }
    protected void visitPostOrder(BinaryTree t, Position p, Object[ ] result) {
        result[0] = (Integer) result[2] + (Integer) result[0]
    }
}
```

## Euler Tour Traversal

- Generic traversal of a binary tree
- Includes as special cases the preorder, postorder, and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



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## Exercise on Binary Trees

- Generic methods:
  - integer size()
  - boolean isEmpty()
  - objectIterator elements()
  - positionIterator positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)
- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update methods:
  - swapElements(p, q)
  - object replaceElement(p, o)
- Additional BinaryTree methods:
  - position leftChild(p)
  - position rightChild(p)
  - position sibling(p)

Exercise:
- Write a method to calculate the height of a binary tree

Algorithm height(T)

Hint: you also need a helper function with argument Position p

Algorithm heightHelper(T, p)



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## Example

- Using the template, design a Java method height(T) to calculate the height of a given binary tree T.



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## Example

```
class Height extends EulerTour { // too much Java

   Object height(T) {
      return eulerTour(T, T.root());
   }
}
```

- We want to abstract away as many details as we can when designing without omitting too many details;
- This is why we use pseudo code



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## Euler Tour Template (pseudo-code)

**Algorithm** *eulerTour*(T, *v*)
  result ← new Array(3)   // 3 element array
  **if** T.*isExternal* (*v*) **then**
    *visitExternal*(T, v, result)
  **else**
    *visitPreOrder*(T, v, result)
    result[1] ← *eulerTour*(T, T.*leftChild*(*v*))
    *visitInOrder*(T, v, result)
    result[2] ← *eulerTour*(T, T.*rightChild*(*v*))
    *visitPostOrder*(T, v, result)

  **return** result[0]



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## Exercise

- Using the template, design a pseudo code algorithm height(T) to calculate the height of a given tree T.



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## Specialization (Subclass) of EulerTour

- We show how to specialize class EulerTour to calculate the height of a binary tree
- Create a subclass Height of EulerTour

```
// class Height extends EulerTour

Algorithm height(T) // always need top level method
      return eulerTour(T, T.root()) // need to call template method

Algorithm visitExternal(T, p, result) // override hook method to insert actions
      result[0] = 0

Algorithm visitPostOrder(T, p, result) // override hook method to insert actions
      result[0] = 1 + MAX(result[1], result[2])
```





## Template Version of DFS
## Exercise: write isConnected(G)

```
Algorithm DFS(G)
   Input graph G
   Output the edges of G are
         labeled as discovery edges
         and back edges

    initResult( G )
    for all  u ∈ G.vertices()
       setLabel(u, UNEXPLORED)
       preInitVertex(u)
    for all  e ∈ G.edges()
       setLabel(e, UNEXPLORED)
       preInitEdge(e)
    for all  v ∈ G.vertices()
       if  getLabel(v) = UNEXPLORED
          preComponentVisit(G, v)
          DFScomponent(G, v)
          postComponentVisit(G, v)

    return result( G )
```

```
Algorithm DFScomponent(G, v)
   setLabel(v, VISITED)
   startVertexVisit(G, v)
   for all  e ∈ G.incidentEdges(v)
      preEdgeVisit(G, v, e)
      if  getLabel(e) = UNEXPLORED
         w ← opposite(v,e)
         edgeVisit(G, v, e, w)
         if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            preDiscoveryVisit(G, v, e, w)
            DFScomponent(G, w)
            postDiscoveryVisit(G, v, e, w)
         else
            setLabel(e, BACK)
            backEdgeVisit(G, v, e, w)
   finishVertexVisit(G, v)
```





## Path Finding
## Override hook operations

```
Algorithm DFScomponent(G, v)
   setLabel(v, VISITED)
   startVertexVisit(G, v)
   for all  e ∈ G.incidentEdges(v)
      preEdgeVisit(G, v, e)
      if  getLabel(e) = UNEXPLORED
         w ← opposite(v,e)
         edgeVisit(G, v, e, w)
         if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            preDiscoveryVisit(G, v, e, w)
            DFScomponent(G, w)
            postDiscoveryVisit(G, v, e, w)
         else
            setLabel(e, BACK)
            backEdgeVisit(G, v, e, w)
   finishVertexVisit(G, v)
```

```
Algorithm pathDFS(G, v, z, S)
   setLabel(v, VISITED)
   S.push(v)
   if  v = z  then
      path ← S.elements()
   for all  e ∈ G.incidentEdges(v)  do
      if  getLabel(e) = UNEXPLORED  then
         w ← opposite(v, e)
         if getLabel(w) = UNEXPLORED then
            setLabel(e, DISCOVERY)
            S.push(e)
            pathDFS(G, w, z, S)
            S.pop()        { e must be popped }
         else
            setLabel(e, BACK)

   S.pop()              { v must be popped }
```





## Overriding hook methods in a subclass FindSimplePath

```
Algorithm findSimplePath(G, u, v)   // always need top level method that calls DFS
   S ← new empty stack   {S is a subclass field}
   z ← v                 {z is a subclass field & is the target vertex}
   path ← ∅              {path is a subclass field & is the path from u to v}
   for all  u ∈ G.vertices()
      setLabel(u, UNEXPLORED)
   for all  e ∈ G.edges()
      setLabel(e, UNEXPLORED)
   DFScomponent(G, u)
   return(path)

Algorithm startVertexVisit(G, v)
      S.push(v)
      if  v=z  then        {z is a subclass field & is the target}
         path ← S.elements()  {path is a subclass field & is the result}

Algorithm preDiscoveryVisit(G, v , e, w)
      S.push(e)

Algorithm postDiscoveryVisit(G, v, e, w)
      S.pop()         {pop e off the stack}

Algorithm finishVertexVisit(G, v)
      S.pop()         {pop v off the stack}
```





## Template Version of DFS (v2)

```
Algorithm DFS(G)
   Input graph G
   Output the edges of G are
         labeled as discovery edges
         and back edges

    initResult( G )
    for all  u ∈ G.vertices()
       setLabel(u, UNEXPLORED)
       preInitVertex(u)
    for all  e ∈ G.edges()
       setLabel(e, UNEXPLORED)
       preInitEdge(e)
    for all  v ∈ G.vertices()
       if  isNextComponent(G, v)
          preComponentVisit(G, v)
          DFScomponent(G, v)
          postComponentVisit(G, v)

    return result( G )

Algorithm isNextComponent(G, v)
   return getLabel(v) = UNEXPLORED
```

```
Algorithm DFScomponent(G, v)
   setLabel(v, VISITED)
   beginVertexVisit(G, v)
   for all  e ∈ G.incidentEdges(v)
      preEdgeVisit(G, v, e, w)
      if  getLabel(e) = UNEXPLORED
         w ← opposite(v,e)
         edgeVisit(G, v, e, w)
         if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            preDiscoveryVisit(G, v, e, w)
            DFScomponent(G, w)
            postDiscoveryVisit(G, v, e, w)
         else
            setLabel(e, BACK)
            backEdgeVisit(G, v, e, w)
   finishVertexVisit(G, v)
```





## Overriding hook methods in a subclass FindSimplePath (v2)

```
Algorithm findSimplePath(G, u, v)  // always need top level method that calls DFS
   start ← u             {start is a subclass field & is the starting vertex}
   dest ← v              {dest is a subclass field & is the destination vertex}
   S ← new empty stack   {S is a subclass field}
   path ← ∅              {path is a subclass field & is the path from u to v}
   return DFS(G)

Algorithm result(G)
   return(path)

Algorithm isNextComponent(G, v)
      return v=start           {start the component traversal at vertex start}

Algorithm beginVertexVisit(G, v)
      S.push(v)
      if  v=dest  then      {dest is a subclass field & is the destination vertex}
         path ← S.elements()  {path is a subclass field & is the result}

Algorithm preDiscoveryVisit(G, v , e, w)
      S.push(e)

Algorithm postDiscoveryVisit(G, v, e, w)
      S.pop()         {pop e off the stack}

Algorithm finishVertexVisit(G, v)
      S.pop()         {pop v off the stack}
```

## Overriding hook methods in a subclass FindSimplePath (v3)

```
Algorithm findSimplePath(G, u, v)   // always need top level method that calls DFS
    start ← u                      {start is a subclass field & is the starting vertex}
    dest ← v                       {dest is a subclass field & is the destination vertex}
    return DFS(G)

Algorithm initResult( G )  // simplify top level by moving as much as possible to template methods
    S ← new empty stack        {S is a subclass field}
    path ← ∅                   {path is a subclass field & is the path from u to v}

Algorithm result(G)
    return(path)

Algorithm isNextComponent(G, v)
    return v=start             {start the component traversal at vertex start}

Algorithm beginVertexVisit(G, v)
    S.push(v)
    if  v=dest  then           {dest is a subclass field & is the destination vertex}
        path ← S.elements()  {path is a subclass field & is the result}

Algorithm preDiscoveryVisit(G, v , e, w)
    S.push(e)

Algorithm postDiscoveryVisit(G, v, e, w)
    S.pop()          {pop e off the stack}

Algorithm finishVertexVisit(G, v)
    S.pop()          {pop v off the stack}
```

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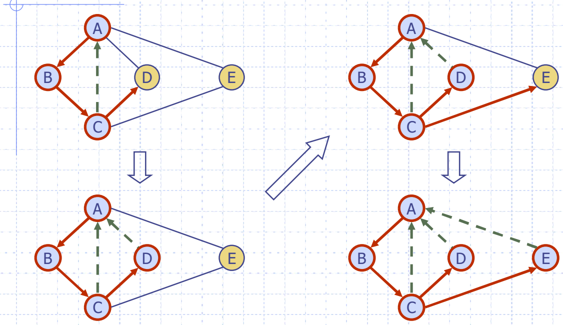
## DFS Example (cont.)




Depth-First Search 50

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## Template Version of DFS (v2)

```
Algorithm DFS(G)
  Input graph G
  Output the edges of G are
         labeled as discovery edges
         and back edges

  initResult( G )
  for all  u ∈ G.vertices()
    setLabel(u, UNEXPLORED)
    preInitVertex(u)
  for all  e ∈ G.edges()
    setLabel(e, UNEXPLORED)
    preInitEdge(e)
  for all  v ∈ G.vertices()
    if  isNextComponent(G, v)
        preComponentVisit(G, v)
        DFScomponent(G, v)
        postComponentVisit(G, v)

  return result( G )

Algorithm isNextComponent(G, v)
  return getLabel(v) = UNEXPLORED
```

```
Algorithm DFScomponent(G, v)
  setLabel(v, VISITED)
  beginVertexVisit(G, v)
  for all  e ∈ G.incidentEdges(v)
    preEdgeVisit(G, v, e, w)
    if  getLabel(e) = UNEXPLORED
      w ← opposite(v,e)
      edgeVisit(G, v, e, w)
      if getLabel(w) = UNEXPLORED
        setLabel(e, DISCOVERY)
        preDiscoveryVisit(G, v, e, w)
        DFScomponent(G, w)
        postDiscoveryVisit(G, v, e, w)
      else
        setLabel(e, BACK)
        backEdgeVisit(G, v, e, w)
  finishVertexVisit(G, v)
```

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## Overriding hook methods in a subclass FindSimplePath (v3)

```
Algorithm findSimplePath(G, u, v)  // always need top level method that calls DFS
    start ← u                  {start is a subclass field & is the starting vertex}
    dest ← v                   {dest is a subclass field & is the destination vertex}
    return DFS(G)

Algorithm isNextComponent(G, v)
    return v=start             {start the component traversal at vertex start}

Algorithm preDiscoveryVisit(G, v, e, w)
    setParent(w, e)

Algorithm result(G)
  if  getLabel(dest) = UNEXPLORED   then        // dest is a subclass field
      return ∅
  else
      S ← buildPath(G, dest)      // S is a local variable, buildPath is defined in Lesson 12
      return S.elements()         // return an iterator over the path
```

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## Exercise: Cycle Finding Override hook operations

```
Algorithm DFScomponent(G, v)
  setLabel(v, VISITED)
  startVertexVisit(v)
  for all  e ∈ G.incidentEdges(v)
    preEdgeVisit(G, v, e, w)
    if  getLabel(e) = UNEXPLORED
      w ← opposite(v,e)
      edgeVisit(G, v, e, w)
      if getLabel(w) = UNEXPLORED
        setLabel(e, DISCOVERY)
        preDiscoveryVisit(G, v, e, w)
        DFScomponent(G, w)
        postDiscoveryVisit(G, v, e, w)
      else
        setLabel(e, BACK)
        backEdgeVisit(G, v, e, w)
  finishVertexVisit(G, v)
```

```
Algorithm cycleDFS(G, v)
  setLabel(v, VISITED)
  if  cycle ≠ null  then return
  S.push(v)
  for all  e ∈ G.incidentEdges(v)
    if  getLabel(e) = UNEXPLORED
      w ← opposite(v,e)
      if getLabel(w) = UNEXPLORED
        setLabel(e, DISCOVERY)
        S.push(e)
        cycleDFS(G, w)
        S.pop()
      else
        setLabel(e, BACK)
        S.push(w)
        S.push(e)
        cycle ← new empty sequence
        o ← w
        do
          cycle.insertLast(o)
          o ← S.pop()
        while o ≠ w
        return
  S.pop()
```

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## Overriding template methods in subclass FindCycles Version 1

```
Algorithm startVertexVisit(G, v)
   if ! cycleFound   then      S.push(v)

Algorithm finishVertexVisit(G, v)
   if ! cycleFound   then      S.pop()

Algorithm preDiscoveryVisit(G, v, e, w)
   if ! cycleFound   then      S.push(e)

Algorithm postDiscoveryVisit(G, v, e, w)
   if ! cycleFound   then      S.pop()

Algorithm backEdgeVisit (G, v, e, w)
   if ! cycleFound   then
        S.push(e)
        cycle ← new empty sequence
        o ← w
        do
           cycle.insertLast(o)
           o ← S.pop()
        while o ≠ w
        cycleFound ← true   {cycleFound is a subclass field, initially false}
```

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## Exercise: Cycle Finding
## Override hook operations

```
Algorithm DFScomponent(G, v)
   setLabel(v, VISITED)
   startVertexVisit(v)
   for all e ∈ G.incidentEdges(v)
      preEdgeVisit(G, v, e, w)
      if getLabel(e) = UNEXPLORED
         w ← opposite(v,e)
         edgeVisit(G, v, e, w)
         if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            preDiscoveryVisit(G, v, e, w)
            DFScomponent(G, w)
            postDiscoveryVisit(G, v, e, w)
         else
            setLabel(e, BACK)
            backEdgeVisit(G, v, e, w)
   finishVertexVisit(G, v)
```

```
Algorithm cycleDFS(G, v)
   setLabel(v, VISITED)
   for all e ∈ G.incidentEdges(v)
      if getLabel(e) = UNEXPLORED
         w ← opposite(v,e)
         if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            setParent(w, e)
            cycleDFS(G, w)
         else
            setLabel(e, BACK)
            cycle ← buildCycle(G, v, e, w)
            cycles.insertLast(cycle)

// buildCycle is defined on the next slide
```



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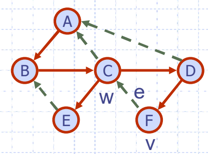
## buildCycle Helper Method




DFS

```
Algorithm buildCycle(G, v, e, w)
   cycle ← new empty Sequence
   cycle.insertLast(w)
   cycle.insertLast(e)
   x ← v
   while x ≠ w do
      cycle.insertLast(x)
      e2 ← getParent(x)
      cycle.insertLast(e2)
      x ← G.opposite(e2, x)

   cycle.insertLast(w)
```



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- What additional method(s) do we need to create or need to override?
- We need the findCycle(G) method that calls DFS(G)
  - // always need top level method that calls DFS or BFS
- Initialize cycleFound boolean variable in method initResult
- Otherwise nothing can be executed, e.g., the hook methods are not executed



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## Template Version of DFS

```
Algorithm DFS(G)
   Input graph G
   Output the edges of G are
          labeled as discovery edges
          and back edges

   initResult( G )
   for all u ∈ G.vertices()
      setLabel(u, UNEXPLORED)
      postInitVertex(u)
   for all e ∈ G.edges()
      setLabel(e, UNEXPLORED)
      postInitEdge(e)
   for all v ∈ G.vertices()
      if getLabel(v) = UNEXPLORED
         preComponentVisit(G, v)
         DFScomponent(G, v)
         postComponentVisit(G, v)

   return result( G )
```

```
Algorithm DFScomponent(G, v)
   setLabel(v, VISITED)
   startVertexVisit(G, v)
   for all e ∈ G.incidentEdges(v)
      preEdgeVisit(G, v, e, w)
      if getLabel(e) = UNEXPLORED
         w ← opposite(v,e)
         edgeVisit(G, v, e)
         if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            preDiscoveryVisit(G, v, e, w)
            DFScomponent(G, w)
            postDiscoveryVisit(G, v, e, w)
         else
            setLabel(e, BACK)
            backEdgeVisit(G, v, e, w)
   finishVertexVisit(G, v)
```



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## Overriding template methods in subclass FindCycles Version 2

```
Algorithm findCycle(G)  // here is the top-level method that calls DFS
   return DFS(G)

Algorithm initResult(G)
   cycle ← null
   cycleFound ← false
Algorithm result(G)
   return cycle
Algorithm preDiscoveryVisit(G, v, e, w)
   setParent(w, e)
Algorithm backEdgeVisit (G, v, e, w)
   if ! cycleFound then
      cycle ← buildCycle(G, v, e, w)
      cycleFound ← true  // cycleFound is a subclass field, initially false
```



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## FindCycles Version 3
## return as many cycles as we can

```
Algorithm findCycle(G)  // here is the top-level method that calls DFS
   return DFS(G)

Algorithm initResult(G)
   cycles ← new empty Sequence // collect all cycles in this Sequence
Algorithm result(G)
   return cycles
Algorithm preDiscoveryVisit(G, v, e, w)
   setParent(w, e)
Algorithm backEdgeVisit (G, v, e, w)
   cycle ← buildCycle(G, v, e, w)
   cycles.insertLast(cycle) // collect all cycles, initially empty
```



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## Main Point

2. The Template Method Pattern implements the changing and non-changing parts of an algorithm in the superclass; it then allows subclasses to override certain (changeable) steps of an algorithm without modifying the basic structure of the original algorithm.

   *Science of Consciousness:* The changing and non-changing aspects of creation are unified in the field pure intelligence that we experience every day during our TM program.



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## Connecting the Parts of Knowledge with the Wholeness of Knowledge

1. Almost any algorithm for solving a problem on a graph or digraph requires examining or processing each vertex or edge.
2. Depth-first and breadth-first search are two particularly useful and efficient search strategies requiring linear time if implemented using adjacency lists.



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3. **Transcendental Consciousness** is the goal of all searches, the field of complete fulfillment.
4. **Impulses within Transcendental Consciousness**: The dynamic natural laws within this unbounded field govern all activities and evolution of the universe.
5. **Wholeness moving within itself:** In Unity Consciousness, one experiences that the self-referral activity of the unified field gives rise to the whole breadth and depth of the universe.



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