

Information you might need.

1. Master Formula

Suppose $T(n)$ satisfied

$$T(n) = \begin{cases} d & \text{if } n = 1 \\ aT\left(\left\lceil \frac{n}{b} \right\rceil\right) + cn^k & \text{otherwise} \end{cases}$$

Where k is none negative integer and a, b, c, d are constants with $a > 0, b > 1, c > 0, d \geq 0$ then

$$T(n) = \begin{cases} \Theta(n^k) & \text{if } a < b^k \\ \Theta(n^k \log n) & \text{if } a = b^k \\ \Theta(n^{\log_b a}) & \text{if } a > b^k \end{cases}$$

2. $x = b^y \implies \text{Log}_b x = y$

3. $\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$

Q1) (14 points) Complexity Analysis

1. [4 points] What is the worst case running time in Big-Oh notation for the following functions? Show your work for partial credit.

a. `void foo(int n)`
`{`
 `int count = 0;`
 `for (int i = 0; i < n; i++)`
 `for (int j = i; j2 > 0; j--)`
 `count = count + 1;`
 `return count;`
`}`

b. `void foo(int n)`
`{`
 `m = 0;`
 `while(n >= 2)`
 `{`
 `n = n/3;`
 `m = m+1;`
 `System.out.println(m);`
 `}`
 `return m`
`}`

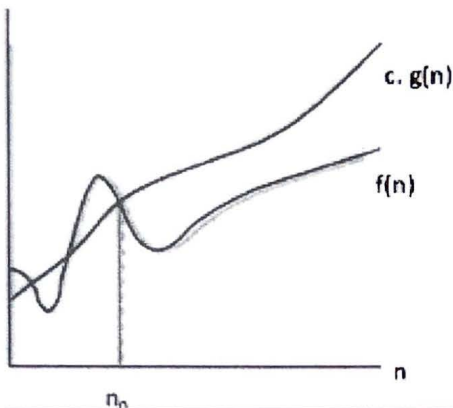
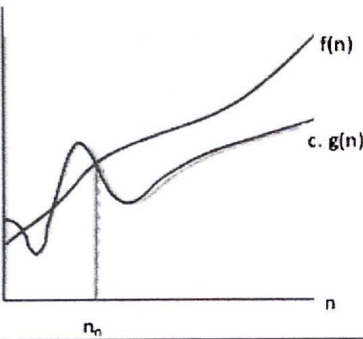
2. [4 points] Discuss whether the next statements are true for the given function $f(n) = 2n^2 + n \log n$

a) $f(n) = O(n^2 \log n)$

b) $f(n) = \Theta(n^2 \log n)$

3. [6 points] Multiple Choice Questions:

1.	What is the tightest asymptotic bound for the following $f(n)$ $f(n) = 2n^n + 2^{100}$	<ul style="list-style-type: none"> a. $O(n^n)$ b. $O(2^{100})$ c. $O(2n^n)$
2.	What is the tightest asymptotic bound for the following $f(n)$ $f(n) = n^2 \log(n^5) + 5n \log(n^5) + n^3$	<ul style="list-style-type: none"> a. $O(n^2 \log(n^5))$ b. $O(n^2 \log n)$ c. $O(n^3)$ d. $O(5n \log(n^5))$

3.	<p>For the following graph, the asymptotic relationship of f and g functions is</p> 	<p>a. $f(n) = \Theta(g(n))$ b. $f(n) = O(g(n))$ c. $f(n) = \Omega(g(n))$</p>
4.	<p>For the following graph, the asymptotic relationship of f and g functions is</p> 	<p>a. $f(n) = \Theta(g(n))$ b. $f(n) = O(g(n))$ c. $f(n) = \Omega(g(n))$</p>
5.	<p>Which of the following is not $O(n^3)$</p>	<p>A. $N^{2.98}$ B. $15^{10}n + 1000$ C. $n^4 / n^{1/2}$ D. $2^{20}n^2$</p>
6.	<p>Which of the given options provide the increasing order of asymptotic complexity of functions f1, f2, f3 and f4? $f1(n) = 2n$ $f2(n) = n!$ $f3(n) = n \log n$ $f4(n) = 7^n$</p>	<p>a. f3, f2, f4, f1 b. f3, f2, f1, f4 c. f2, f3, f1, f4 d. f1, f3, f4, f2</p>

Q2) (11 points) Analysis

1. [4 points] Give tight asymptotic bound for the following recurrences:

a. $T(n) = 4T(n/2) + 1$

b. $T(n) = T(n/2) + n^3$

2. [5 points] Given an array A with n integer elements. Write an algorithm to print the largest number and the smallest number in the array. Propose an algorithm to solve this problem in $O(n)$ worst-case time or better.
Example: $A = \{1, 3, 2, 1, 3, 5, 3\}$ the result should be the largest number is 5 and the smallest number is 1.

3. [2 points] Verify whether the following statement is true or false. The amortized cost of a sequence of n operations from S is $O(n)$, and so the amortized cost of a single operation from S is $O(n)$.

Given S consists of two operations:

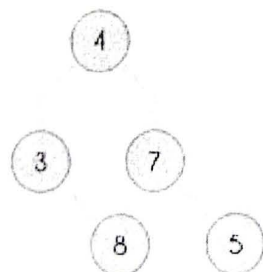
- $\text{add}(x)$ = inserts String x into next available slot
- $\text{clear}()$ = replaces all Strings in array with nulls

Q3) (14 points) Sorting algorithms:

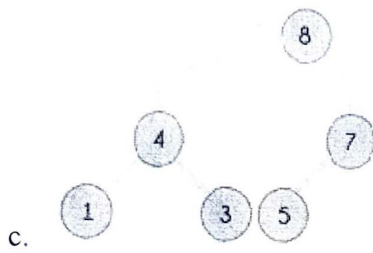
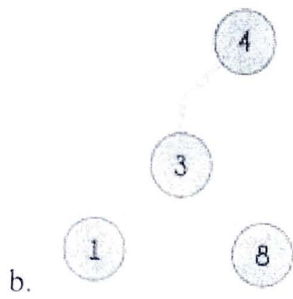
- [5 points] Answer the following questions by True or False
 - It is possible to develop a comparison based sorting algorithm that runs in $O(n)$.
 - The best time complexity of Bubble Sort is $O(n \log n)$.
 - Merge Sort makes more swap operations than Selection-Sort.
 - Suppose we have a $O(n)$ time algorithm that finds median of an unsorted array. Consider a QuickSort implementation where we first find median using the above algorithm, then use median as pivot. The worst case time complexity of this modified QuickSort is $O(n)$
 - Insertion-Sort is stable sorting algorithm.

A	B	C	D	E

- [3 points] Which of the following trees is a heap? Explain your answer for each tree.



a.



3. [2 points] Given that the running time worst case for merge-sort is better than quicksort, why quick-sort is commonly used?

4. [4 points] Use Merge Sort Algorithm to sort the following array of integers.
Show the merge-sort tree.

30 25 10 80 20 15 99 88