Standard Bucket Sort (not in text)

Standard Bucket Sort (not in text)

- Three phases
  1. Distribution into buckets
  2. Sorting the buckets
  3. Combining the buckets

1. Distribution

- Each key is examined once
  - a particular field of bits is examined or
  - some work is done to determine in which bucket it belongs
    - e.g., the key is compared to at most k preset values
- The item is then inserted into the proper bucket
- The work done in the distribution phase must be $\Theta(n)$

2. Sorting the Buckets

- Most of the work is done here
- O(m log m) operations are done for each bucket
  - m is the bucket size

3. Combining the Buckets

- The sorted sequences are concatenated
- Takes $\Theta(n)$ time

Analysis of Standard Bucket Sort

- If the keys are evenly distributed among the buckets and there are *k* buckets
  - Then the size of the buckets m = n/k
  - Thus the work (key comparisons) done would be
    c m log m for each of the k buckets
  - That is, the total work would be
    k c (n/k) log (n/k) = c n log (n/k)
- If the number of buckets k = n/20, then the size of each bucket (n/k) is equal to 20, so the number of key comparisons would be
  c n log 20
- Thus bucket sort would be linear when the input comes from a uniform distribution
- Note also that the larger the bucket size, the larger the constant (log m)

Summary of Sorting Algorithms (§4.6)

Algorithm	Time	Notes (pros and cons)
insertion-sort		
merge-sort		
quick-sort		
heap-sort		
standard bucket-sort		

Summary of Sorting Algorithms (§4.6)

Algorithm	Time	Notes (pros and cons)
insertion-sort	$O(n^2)$ or $O(n+k)$	◈ excellent for small inputs ◈ fast for 'almost' sorted inputs
merge-sort	$O(n \log n)$	◈ excels in sequential access ◈ for huge data sets
quick-sort	$O(n \log n)$ expected	◈ in-place, randomized ◈ excellent generalized sort
heap-sort	$O(n \log n)$	◈ in-place ◈ fastest for in-memory
standard bucket-sort	$O(n \log(n/k))$	◈ if keys can be distributed evenly ◈ relatively small bucket sizes

Main Point

0. In Bucket-sort, knowledge of the structure of keys allows them to be distributed into k distinct buckets that are sorted separately and recombined. The running time is O(n log(n/k)). Knowledge has organizing power. Pure knowledge has infinite organizing power for optimum efficiency in fulfilling ones desires.

Lecture 7: Dictionaries

Sequential Unfoldment of Knowledge

Wholeness Statement

The Dictionary ADT stores a searchable collection of *key-element* items that represents either an unordered or an ordered collection. Hashing solves the problem of item-lookup by providing a table whose size is not unreasonably large, yet it can store a large range of keys such that the element associated with each key can be accessed quickly ($O(1)$). SCI provides systematic techniques for accessing and experiencing total knowledge of the Universe to enhance individual life.

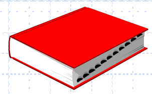
The Dictionary ADT

Two Types of Dictionaries

1. Unordered (§2.5.1)
2. Ordered (§3.1)

- Both use a key to identify a specific element
- Stores items, i.e., key-element pairs
- For the sake of generality, multiple items can have the same key

Unordered Dictionary ADT (§2.5.1)



- The dictionary ADT models a searchable collection of key-element items
- The main operations of a dictionary are searching, inserting, and deleting items
- Multiple items with the same key are allowed
- Applications:
  - address book
  - credit card authorization
  - mapping host names (e.g., cs16.net) to internet addresses (e.g., 128.148.34.101)
- Dictionary ADT methods:
  - findElement(k): if the dictionary has an item with key k, returns its element, else, returns the special element NO_SUCH_KEY
  - insertItem(k, o): inserts item (k, o) into the dictionary
  - removeElement(k): if the dictionary has an item with key k, removes it from the dictionary and returns its element, else returns the special element NO_SUCH_KEY
  - size(), isEmpty()
  - keys(), elements()

Log Files (§2.5.1)

- A log file (or audit trail) is a dictionary implemented by means of an unsorted sequence
  - Items are stored in the dictionary in a sequence in arbitrary order
  - Based on doubly-linked lists or a circular array
- Performance:
  - insertItem takes $O(1)$ time since we can insert the new item at the beginning or at the end of the sequence
  - findElement and removeElement take $O(n)$ time since in the worst case (the item is not found) we traverse the entire sequence to look for an item with the given key

Log File

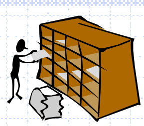
- Effective only for dictionaries of small size or
- For dictionaries on which insertions are the most common operations, while searches and removals are rarely performed

  (e.g., historical record of logins to a workstation)

- What do we do if we need to do frequent searches and removals in a large dictionary?

Hash Tables

Hash Tables and Hash Functions (§2.5.2)



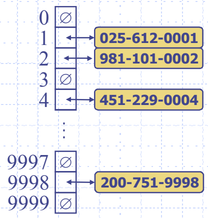
- A hash table for a given key type consists of
  - Hash function h
  - Array (called table) of size N
- A hash function h maps keys of a given type to integers in a fixed interval $[0, N-1]$
- Example:
  $h(k) = k \bmod N$
  is a hash function for integer keys
- The integer $h(k)$ is called the hash value of key k

Goals of Hash Functions

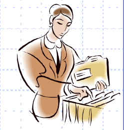
1. Store item (k, o) at index $i = h(k)$ in the table
2. Avoid collisions as much as possible
   - Collisions occur when two keys hash to the same index i
   - The average performance of hashing depends on how well the hash function distributes the set of keys (i.e., avoids collisions)

Example

- Design a hash table for a dictionary storing items (SSN, Name), where SSN (social security number) is a nine-digit positive integer
- Our hash table uses an array of size $N = 10{,}000$ and the hash function $h(x) = \text{last four digits of } x$



Hash Functions (§ 2.5.3)



- A hash function is usually specified as the composition of two functions:

  Hash code map:
  h_1: keys $\rightarrow$ integers

  Compression map:
  h_2: integers $\rightarrow [0, N-1]$

- The hash code map is applied first, and the compression map is applied next on the result, i.e.,
  $h(x) = h_2(h_1(x))$
- The goal of the hash function is to "disperse" the keys in an apparently random way

Hash Code Maps (§2.5.3)

- Memory address:
  - We reinterpret the memory address of the key object as an integer
    - (default hash code of all Java objects)
  - Good in general, except for numeric and string keys
- Integer cast:
  - We reinterpret the bits of the key as an integer
  - Suitable for keys of length less than or equal to the number of bits of the integer type (e.g., byte, short, int, and float in Java)
- Component sum:
  - We partition the bits of the key into components of fixed length (e.g., 16 or 32 bits)
  - Then we sum the components (ignoring overflows)
  - Suitable for numeric keys of fixed length greater than or equal to the number of bits of the integer type (e.g., long and double in Java)

Hash Code Maps (cont.)

- Polynomial accumulation:
  - We partition the bits of the key into a sequence of components of fixed length (e.g., 8, 16 or 32 bits)
    $a_0\, a_1 \ldots a_{n-1}$
  - We evaluate the polynomial
    $p(z) = a_0 + a_1 z + a_2 z^2 + \ldots$
    $\ldots + a_{n-1} z^{n-1}$
    at a fixed value z, ignoring overflows
  - Especially suitable for strings (e.g., the choice $z = 33$ gives at most 6 collisions on a set of 50,000 English words)
- Polynomial $p(z)$ can be evaluated in $O(n)$ time using Horner's rule:
  - The following polynomials are successively computed, each from the previous one in $O(1)$ time
    $p_0(z) = a_{n-1}$
    $p_i(z) = a_{n-i-1} + z p_{i-1}(z)$
    $(i = 1, 2, \ldots, n-1)$
- We have $p(z) = p_{n-1}(z)$

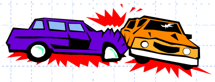
Compression Maps (§2.5.4)

- Division:
  - $h_2(y) = y \bmod N$
  - The size N of the hash table is usually chosen to be a prime
    - The reason has to do with number theory and is beyond the scope of this course
- Multiply, Add and Divide (MAD):
  - $h_2(y) = (ay + b) \bmod N$
  - a and b are nonnegative integers such that
    $a \bmod N \neq 0$
  - Otherwise, every integer would map to the same value b

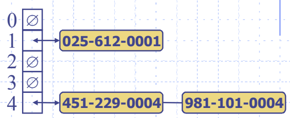
Main Point

1. The hash function solves the problem of fast table-lookup, i.e., it allows the element associated with each key to be accessed quickly (in O(1) time). A hash function is composed of a hash code function and a compression function that transforms (in constant time) each key into a specific location in the table.
   Through a process of self-referral, the unified field transforms itself into all the values of creation without making mistakes.

Collision Handling (§ 2.5.5)



- Collisions occur when different elements are mapped to the same cell
- **Chaining**: let each cell in the table point to a linked list of elements that map there

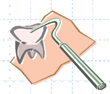


- Chaining is simple, but requires additional memory outside the table

Load Factors and Rehashing

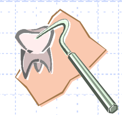
- Load factor is n/N where n is the number items in the table and N is the table size
- When the load factor goes above .75, the table is resized and the items are rehashed

Linear Probing (§2.5.5)



- Open addressing: the colliding item is placed in a different cell of the table
- **Linear probing** handles collisions by placing the colliding item in the next (circularly) available table cell
- Each table cell inspected is referred to as a "probe"
- Colliding items lump together, causing future collisions to cause a longer sequence of probes

Linear Probing (§2.5.5)



- Exercise:
  - $h(x) = x \bmod 13$
  - Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

0	1	2	3	4	5	6	7	8	9	10	11	12

Linear Probing (§2.5.5)

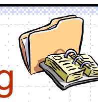
- Example:
  - $h(x) = x \bmod 13$
  - Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

0	1	2	3	4	5	6	7	8	9	10	11	12

⇩

		41			18	44	59	32	22	31	73	
0	1	2	3	4	5	6	7	8	9	10	11	12

Search with Linear Probing



- Consider a hash table A that uses linear probing
- findElement(k)
  - We start at cell $h(k)$
  - We probe consecutive locations until one of the following occurs
    - An item with key k is found, or
    - An empty cell is found, or
    - N cells have been unsuccessfully probed

```
Algorithm findElement(k)
i ← h(k)
p ← 0
repeat
    c ← A[i]
    if c = ∅ then
        return NO_SUCH_KEY
    else if c.key () = k then
        return c.element()
    else
        i ← (i + 1) mod N
        p ← p + 1
until  p = N
return NO_SUCH_KEY
```

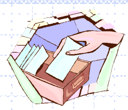
Updates with Linear Probing

- To handle insertions and deletions, we introduce a special object, called *AVAILABLE*, which replaces deleted elements
- removeElement(k)
  - We search for an item with key k
  - If such an item (k, o) is found, we replace it with the special item *AVAILABLE* and we return element o
  - Else, we return *NO_SUCH_KEY*
- insert Item(k, o)
  - We throw an exception if the table is full
  - We start at cell $h(k)$
  - We probe consecutive cells until one of the following occurs
    - A cell i is found that is either empty or stores *AVAILABLE*, or
    - N cells have been unsuccessfully probed
  - We store item (k, o) in cell i

Quadratic Probing

- Search A[(i + j^2) mod N]
  for j = 0, 1, 2, … until an empty slot is found
- Disadvantages
  - Complicates removal even more
  - Secondary clustering

Double Hashing



- Double hashing uses a secondary hash function $d(k)$ and handles collisions by placing an item in the first available cell of the series
  $(i + j*d(k)) \bmod N$
  for $j = 0, 1, \ldots, N-1$
- The secondary hash function $d(k)$ cannot have zero values
- The table size N must be a prime to allow probing of all the cells
- Common choice of compression map for the secondary hash function:
  $d_2(k) = q - (k \bmod q)$
  where
  - $q < N$
  - q is a prime
- The possible values for $d_2(k)$ are
  $1, 2, \ldots, q$

Example of Double Hashing

- Consider a hash table storing integer keys that handles collision with double hashing
  - $N = 13$
  - $h(k) = k \bmod 13$
  - $d(k) = 7 - (k \bmod 7)$
- Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

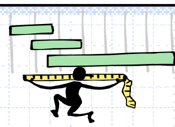
k	$h(k)$	$d(k)$	Probes		
18	5	3	5		
41	2	1	2		
22	9	6	9		
44	5	5	5	10	
59	7	4	7		
32	6	3	6		
31	5	4	5	9	0
73	8	4	8		

0	1	2	3	4	5	6	7	8	9	10	11	12

⇩

31		41			18	32	59	73	22	44		
0	1	2	3	4	5	6	7	8	9	10	11	12

Performance of Hashing



- In the worst case, searches, insertions and removals on a hash table take $O(n)$ time
- The worst case occurs when all the keys inserted into the dictionary collide
- The load factor $\alpha = n/N$ affects the performance of a hash table
- Assuming that the hash values are like random numbers, it can be shown that the expected number of probes for an insertion with open addressing is
  $1 / (1 - \alpha)$
- The expected running time of all the dictionary ADT operations in a hash table is $O(1)$
- In practice, hashing is very fast provided the load factor is not close to 100%
- Applications of hash tables:
  - small databases
  - compilers
  - browser caches

Universal Hashing

- If allowed to pick the keys to be hashed, then a malicious adversary can choose n keys that all hash to the same slot
  - Any fixed hash function is vulnerable to this sort of worst-case behavior
- The only effective way to improve the situation
  - Choose the hash function randomly in a way that is independent of the keys to be stored
- This approach is called universal hashing
  - The hash function is chosen randomly at beginning of execution
- Yields good performance no matter what keys are chosen by an adversary

Universal Hashing (§ 2.5.6)

- A family of hash functions is **universal** if, for any $0 \le j,k \le M-1$, $Pr(h(j)=h(k)) \le 1/N$.
- Keys are in the range [0, M-1]
- A hash function maps to the range [0, N-1]
- Theorem: The set of all functions, h, as defined here, is universal.
- Choose p as a prime between M and 2M.
- Randomly select $0<a<p$ and $0 \le b<p$, and define $h(k)=(ak+b \bmod p) \bmod N$

Main Point

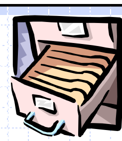
2. A hash table is an example of a highly efficient implementation of an unordered Dictionary ADT (its operations have complexity *O*(1)). Access to Pure Consciousness is simple, effortless, and spontaneous through the introduction of the proper techniques.

Ordered Dictionaries

Ordered Dictionaries

- Keys are assumed to come from a total order.
- New operations:
  - closestKeyBefore(k)
    Return the key of the item with largest key less than or equal to *k*
  - closestElemBefore(k)
    Return the element of the item with largest key less than or equal to *k*
  - closestKeyAfter(k)
    Return the key of the item with smallest key greater than or equal to *k*
  - closestElemAfter(k)
    Return the element of the item with smallest key greater than or equal to *k*

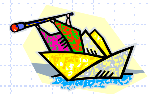
Lookup Tables (§3.1.1)



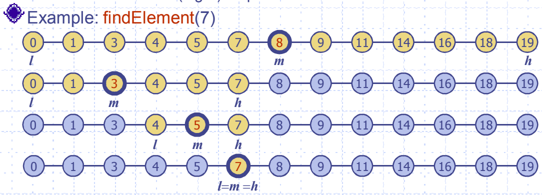
Lookup Table (§3.1.1)

- A dictionary implemented by means of a sorted sequence
  - store the items of the dictionary in an array-based sequence, sorted by key
  - use an external comparator for the keys

Binary Search (§3.1.1)



- Binary search performs operation findElement(k) on a dictionary implemented by means of an array-based sequence, sorted by key
  - similar to the high-low game
  - at each step, the number of candidate items is halved
  - terminates after O(log n) steps
- Example: findElement(7)



Binary Search Algorithm (recursive)

Algorithm BinarySearch(*S, k, low, high*):
  Input: *An ordered vector S storing n items, accessed by keys()*
  Output: *An element of S with key k and rank between low & high.*

```
if low > high then
    return NO_SUCH_KEY
else
    mid ← (low + high)/2
    if k = key(mid) then
        return elem(mid)
    else if k < key(mid) then
        return BinarySearch(S, k, low, mid-1)
    else
        return BinarySearch(S, k, mid + 1, high)
```


Running Time of Binary Search

- Running time proportional to number of recursive calls performed.
- Recurrence equation:

$$T(n) \le \begin{cases} b & \text{if } n < 2 \\ T(n/2) + b & \text{else,} \end{cases}$$

Exercise: solve this recurrence equation (use the guess-and-test method).

Running Time of Binary Search

- Recurrence equation:

$$T(n) \le \begin{cases} b & \text{if } n < 2 \\ T(n/2) + b & \text{else,} \end{cases}$$

Guess: T(n) is O(log n)

Assume $T(n/2) \le c \log n/2$

$$\begin{aligned} T(n) &\le c \log n/2 + b \\ &= c (\log n - \log 2) + b \\ &= c (\log n - 1) + b \\ &= c \log n - c + b \\ &\le c \log n \quad \text{for } b \le c \end{aligned}$$

Binary Search Algorithm (iterative)

Algorithm BinarySearch(*S, k*):
  Input: *An ordered vector S storing n items, accessed by keys()*
  Output: *An element of S with key k.*

```
low ← 0
high ← S.size() - 1
while low ≤ high do
    mid ← (low + high)/2
    if k = key(mid) then
        return elem(mid)
    else  if k < key(mid) then
        high ← mid - 1
    else
        low ← mid + 1
return NO_SUCH_KEY
```


Lookup Table (§3.1.1)

- A dictionary implemented by means of a sorted sequence
  - store the items of the dictionary in an array-based sequence, sorted by key
  - use an external comparator for the keys
- Performance:
  - findElement
  - insertItem
  - removeElement

Lookup Table (§3.1.1)

- A dictionary implemented by means of a sorted sequence
  - store the items of the dictionary in an array-based sequence, sorted by key
  - use an external comparator for the keys
- Performance:
  - findElement takes $O(\log n)$ time, using binary search
  - insertItem takes $O(n)$ time since in the worst case we have to shift $n/2$ items to make room for the new item
  - removeElement take $O(n)$ time since in the worst case we have to shift $n/2$ items to compact the items after the removal

Lookup Table (§3.1.1)

- Effective only
  - for dictionaries of small size or
  - for dictionaries on which
    - searches are the most common operation, and
    - insertions and removals are rarely performed
    - (e.g., credit card authorizations)
- What do we do if this is not the case?

Main Point

3. A lookup table is an example of an ordered Dictionary ADT allowing elements to be efficiently accessed in order by key. When implemented as an ordered sequence, searching for a key is relatively efficient, O(log n), but insertion and deletion are not, O(n).
   The unified field of natural law always operates with maximum efficiency.

Connecting the Parts of Knowledge with the Wholeness of Knowledge

1. A hash table is a very efficient way of implementing an unordered Dictionary ADT; the running time of search, insertion, and deletion is expected O(1) time.
2. To achieve efficient behavior of the hash table operations takes a careful choice of table size, load factor, hash function, and handling of collisions.

3. **Transcendental Consciousness** is the silent field of perfect efficiency and frictionless flow for coordinating all activity in the universe.
4. **Impulses within Transcendental Consciousness**: The dynamic natural laws within this unbounded field create and maintain the order and balance in creation, all spontaneously without effort.
5. **Wholeness moving within itself:** In Unity Consciousness, the diversity of creation is experienced as waves of intelligence, perfectly efficient fluctuations of one's own self-referral consciousness.