STK 210: Practical 2

1 PROC IML

PROC IML (Interactive Matrix Language) is a SAS procedure and has built-in operators and functions for most standard matrix operations. In general, help for procedures in SAS and for the matrix programming language IML is available through the SAS help system. This is very useful for obtaining the full list of available options and features.

1.1 To Start PROC IML

IML is called is a procedure in SAS, i.e. by issuing the command:

```
proc iml;
...statements...
quit;
```

whole programs can be submitted by pressing the  run button. Often it is more convenient in PROC IML, to submit only one / a few commands at a time. This is done by selecting the line(s) and submitting / running it. In this way it is easier to keep track of the calculations and to find syntax errors.

1.2 Defining a Matrix Manually

```
proc iml;      *invokes PROC IML;
a=2;           *scalar;
b={1 2 3 4};   *1x4 row vector;
c={2,4,6,8};   *4x1 column vector;
d={1 0.2 0.8,
   0.2 1 0.4,
   0.8 0.4 1}; *3x3 correlation matrix vector;
print 'The matrices';
print a b, c d;
print 'Expressions should be in parenthesis:', (5*b) (b'+c);
quit;          *closes PROC IML;
```

- The braces {} encloses the values into a matrix and the commas are used to separate rows.
- Comma in PRINT statement will print OUTPUT in new line.

1.3 The PRINT command

The print command can be helpful in reporting results. The syntax is

```
print 'Text' x [format = 10.4];
```

This prints the explanation `Text` followed by the matrix `x`. If `x` is an expression (e.g. `z+y`) it should be in parenthesis. The format statement indicates 4 decimal points.

1.4 Matrix Functions

The table below lists some often used matrix functions in PROC IML.

Syntax	Description	Example
`{a b, c d}`	Creates a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	`A={1 2, 5 6};`
`I(p)`	Creates a p dimensional identity matrix.	`B=I(5);`
`J(m,n,q)`	Creates a $m \times n$ matrix with a constant value q.	`C=J(2,3,0.50);`
`vecdiag(A)`	Extracts the diagonal of a square matrix $\mathbf{A}$ as a vector.	
`sum(A)`	Return the sum of the elements in the matrix $\mathbf{A}$.	
‘ or `t(A)`	Transposition	`D=A‘; OR D=t(A)`
+	Sum of two matrices.	`A=B+C;`
-	Difference between two matrices.	`A=B-C;`
*	Multiplication of two matrices.	`A=B*C;`
`inv(X)`	Returns the matrix inverse, $\mathbf{X}^{-1}$.	
`abs(A)`	Absolute function	`A=abs(B);`
`sqrt(A)`	Square root function	
#	Elementwise (not matrix-) multiplication.	`C=A#B;`
##	Elementwise power.	`C=A##B;`
/	Elementwise division.	`C=A/B;`
`A[i,j]`	Extract element a_{ij} from a matrix $\mathbf{A}$.	`b=A[i,j];`
`A[i,]`	Extracts the i^{th} row.	`c=A[i,];`
`A[,j]`	Extracts the j^{th} column.	`d=A[,j];`
`M[,+]`	Sum of all columns, for each row.	`RowSum = M[ ,+];`
`M[+,]`	Sum of all rows, for each column.	`ColSum = M[+, ];`
`M[+]`	Sum of all elements	`TotalSum = M[+];`
||	Columnwise concatenation.	`a=b||c;`
//	Rowwise concatenation.	`a=b//c;`
`nrow(A)`	Returns the number of rows of the matrix $\mathbf{A}$.	`n=nrow(A);`
`ncol(A)`	Returns the number of columns of the matrix $\mathbf{A}$.	`c=ncol(A);`

2 Exercise

1. Use PROC IML as a calculator and calculate the value of the following:

   Remember: Mathematical operations are customarily performed in a specific sequence.
   The sequence of evaluation is as follows:

   - brackets,
   - exponents,
   - multiplication and division (with equal status), and
   - addition and subtraction (with equal status).

   Note: SAS will automatically follows these rules.

   (a) $(312 + 824)(400 - 319)$ $= 92016.$

   (b) $\dfrac{319(1013 - 93)}{5}$ $= 58696.$

   (c) $\sqrt{826 + 718}$ $= 39.294$

   (d) $(212/8)^4$ $= 4.931\,6 \times 10^5$

   (e) $3 - 2 \times 5^2 \div 9 + 4$ $= 1.444\,4$

   (f) $(3 - 2 \times 5)^2 \div 9 + 4$ $= 9.444\,4$

2. A dealer in refrigerators uses matrices to represent his monthly sales. **He uses:**

   - the **rows** to indicate the **number** of the make

     AEG, Mercury and KIC

     of refrigerators sold.
   - the **columns** to indicate the **number** of the type

     Ordinary, A500, B500 and C600

     of refrigerators sold.
   - **The matrices for March $(\mathbf{M})$ and April $(\mathbf{A})$ 2003 are given by:**

$$\mathbf{M} = \begin{pmatrix} 2 & 6 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 2 & 7 & 6 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 0 & 2 & 4 & 4 \\ 2 & 3 & 3 & 2 \\ 4 & 0 & 2 & 6 \end{pmatrix}$$

- **In tabular format:**

March				
	Ordinary	A500	B500	C600
AEG	2	6	1	2
Mercury	0	1	3	5
KIC	2	7	6	0

April				
	Ordinary	A500	B500	C600
AEG	0	2	4	4
Mercury	2	3	3	2
KIC	4	0	2	6

(a) Create the matrices $\mathbf{M}$ and $\mathbf{A}$ in PROC IML and print them by making use of the PRINT statement.

(b) Create the character matrices

$$\mathbf{r} = \begin{pmatrix} \text{AEG} \\ \text{Mercury} \\ \text{KIC} \end{pmatrix} \quad \text{and} \quad \mathbf{c} = \begin{pmatrix} \text{Ordinary} & \text{A500} & \text{B500} & \text{C600} \end{pmatrix}$$

and print the matrix $\mathbf{M}$ again with ROWNAME and COLNAME by making use of

```
print M[rowname=r colname=c] A[rowname=r colname=c];
```

Note:

- Possible to use only ROWNAME or COLNAME.
- The vectors $\mathbf{r}$ and $\mathbf{c}$ can be specified as row or column vectors in PROC IML.

(c) How many KIC A500 units were sold in March? Extract the value and print.

(d) How many Mercury C600 units were sold in April? Extract the value and print.

(e) Calculate the total number of KIC units sold during March by making use of `M[3,+];`

(f) Calculate the total number of B500 units sold during April.

(g) For March and April calculate

i. the total number of refrigerators sold by making use of the `SUM()` function.

ii. the maximum number of refrigerators by making use of the `MAX()` function.

iii. the maximum number of refrigerators by making use of the `MIN()` function.

(h) Use the statements

```
X[+], X[,+] and X[+,]
```

where `X` is a matrix to calculate the

- total number
- **row totals:** AEG, Mercury and KIC
- **column totals:** Ordinary, A500, B500 and C600

of refrigerators sold for March and April.

Optional: You can use ROWNAME and COLNAME to make Output more interpretable.

i. During what month did the company sell the most refrigerators? Why?

Answer: March, during March 35 units were sold and during April 32.

ii. During what month did the company sell the most Mercury refrigerators? Why?

Answer: April, during March 9 Mercury units were sold and during April 10.

iii. During what month did the company sell the most A500 refrigerators? Why?

Answer: March, during March 14 A500 units were sold and during April only 5.

(i) We can also use matrix multiplication with a vector of ones to obtain the row and column totals of a matrix.

i. Use the `J(m,n,q)` function in SAS to print the following:

A. (1×4) column vector of ones

B. (3×1) row vector of ones

ii. Use matrix multiplication to calculate the following totals for March

A. row totals

$$\mathbf{M1}_{4\times 1} = \begin{pmatrix} 2 & 6 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 2 & 7 & 6 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ 15 \end{pmatrix}$$

B. column totals

$$\mathbf{1}_{1\times 3}\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 6 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 2 & 7 & 6 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 14 & 10 & 7 \end{pmatrix}$$

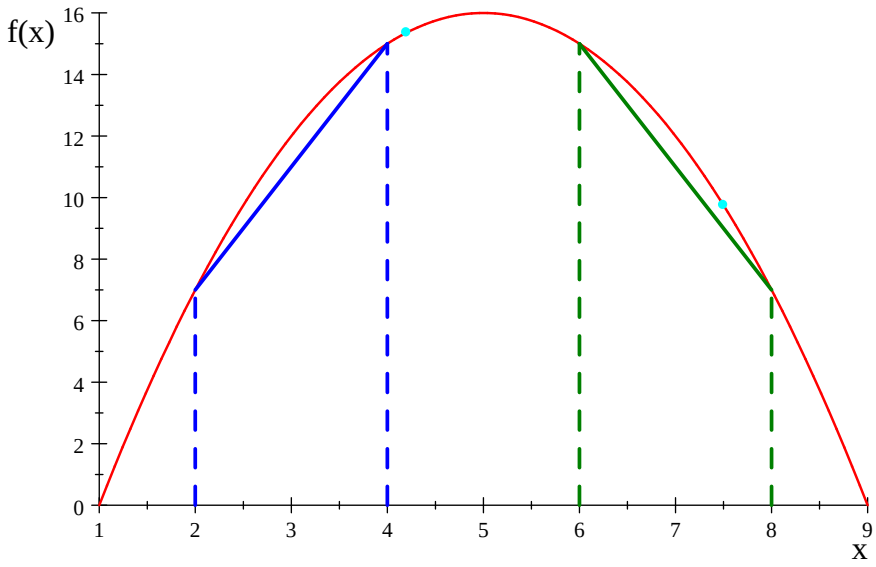
C. total

$$\mathbf{1}_{1\times 3}\mathbf{M1}_{4\times 1} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 6 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 2 & 7 & 6 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 35$$

(j) Find the matrix which represents the total of refrigerators sold in March and April for each combination of the make (AEG, Mercury, and KIC) and the type (Ordinary, A500, B500 C600) of refrigerator.

3. Assume that the relationship between the revenue y (in R100) of a producer and the production x (in thousands) of a certain item, is represented by the function

$$f(x) = -x^2 + 10x - 9$$



(a) For $x = 4.2$. Use PROC IML to calculate the values for

i. $f(x)$ $= 15.36$

ii. $f'(x)$ $= 1.6$

(b) For $x = 7.5$. Use PROC IML to calculate the values for

i. $f(x)$ $= 9.75$

ii. $f'(x)$ $= -5.0$

(c) For the interval $[2, 4]$.

i. Calculate the average rate of change i.e.

$$\frac{f(4) - f(2)}{4 - 2}$$

$= 4.0$

ii. Calculate the total area below the curve and the x axis i.e.

$$\int_2^4 f(x)\,dx = -\frac{1}{3}x^3 + 5x^2 - 9x\Big|_2^4$$

$= 23.333$

(d) For the interval $[6, 8]$.

i. Calculate the average rate of change. $= -4.0$

ii. Calculate the total area below the curve and the x-as. $= 23.333$