PHY180 - Pendulum Lab

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Chapter 1

1.1 General Setup

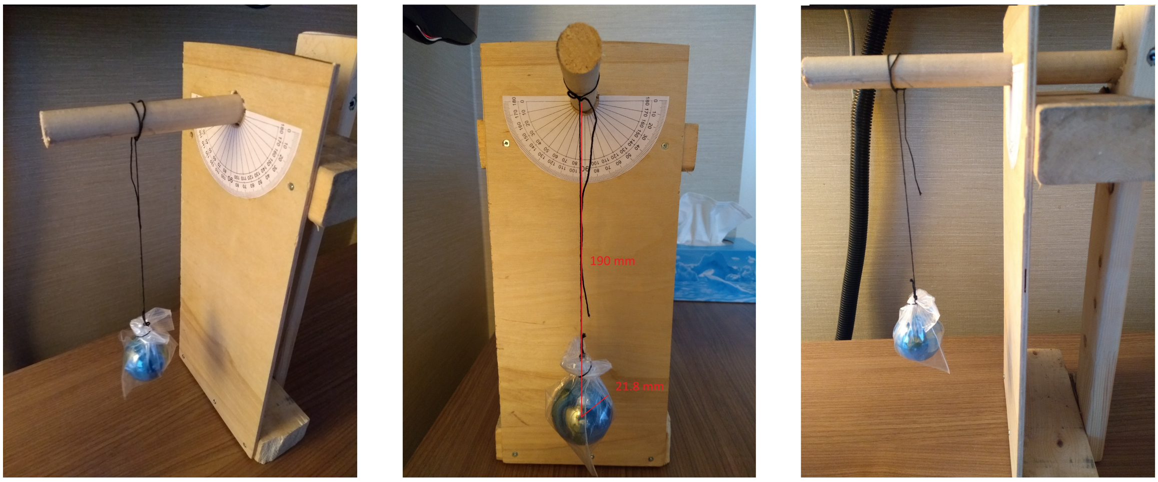

Figure 1.1: Pendulum Images

The pendulum used in this experiment (seen in Figure 1.1) consists of a few distinct parts. The base is constructed out of wooden 2x4s and plywood, with the goal of ensuring the stability of the pendulum since any potential motion of the overall structure could affect the motion of the pendulum itself, thereby skewing results. The pivot of the pendulum is a wooden dowel perpendicular to the base, allowing the pendulum's string to be attached some distance away from the base. This distance has been designed in to keep the pendulum's mass from potentially hitting the base and losing energy due to frictional forces - as this could also skew results. A piece of string tied to the pivot point provides tension between the pendulum's pivot and its mass, and is tied in such a way that it is easily removable and re-attachable - this allows for the effective string length (the distance from the pivot point to the center of mass) to be changed easily by retying the string. The bottom of the string is attached to a plastic bag containing marbles - the number of marbles can be changed to vary the tested mass.

A camera was set up parallel to the plane of the pendulum's planned oscillation with a full view of the pendulum in order to gather data about its motion. (The theoretical model only considers the pendulum's motion in one plane, so positioning the camera parallel to that plane makes it much more convenient to extract data regarding its motion in that plane.) The camera was used to record video with one frame recorded every 0.04s (24.87 FPS).

Chapter 2

2.1 Lab 1 Overview

The purpose of Lab 1 is to determine the Q factor of the experimental pendulum that will be used later in this report to evaluate the validity of the damped harmonic motion model as it applies to pendulums.

The Q factor was determined experimentally by recording a video of the pendulum swinging, and then feeding that video into the motion tracking software *Tracker* to find the relative position of the pendulum's mass and pivot in each frame of the video. This positional data was fed into a custom Python program to calculate the angle of the pendulum and the elapsed time from the beginning of the motion. Two separate methods were then used to calculate the Q value based on this data.

In the first method, the time and angle data were run through a curve fitting Python program (provided by the teaching team for this purpose) to find the parameter τ (representing the time constant of decay) for the curve. The Q factor was then calculated using the following equation where T represents period:

$$Q = \pi \frac{\tau}{T}$$

The second method is based on the idea of counting the number of oscillations for the pendulum to reach $e^{-\frac{\pi}{2}}$ (about 20%) of its original amplitude. This number is $\frac{Q}{2}$. However, counting the number of oscillations directly is not only tedious, but also inaccurate, since the theoretical amplitude of the pendulum is most likely to reach $e^{-\frac{\pi}{2}}$ in between oscillations. Therefore, this method will have an uncertainty spanning a full oscillation. An alternative involves finding the peak angle in each oscillation - this results in one data point per oscillation, representing the amplitude during that oscillation. At each of these points, the term $cos(2\pi\frac{\tau}{T}+\phi_0)$ within function $\theta(t) = ae^{-\frac{t}{\tau}}cos(2\pi\frac{\tau}{T}+\phi_0)$ should be 1, and so $\theta_{amp}(t) = ae^{-\frac{t}{\tau}}$. Fitting such a curve to the peak angle data points provides a more idealized representation of the amplitude over time; this makes it possible to more accurately solve for the time at which the amplitude is exactly $e^{-\frac{\pi}{2}}$.

2.2 Lab 1 Setup

In the experiment to determine Q factor, the length of the string was fixed at 168 ± 0.5mm and the mass was fixed at 121 ± 0.5g. The mass used was a large, spherical marble with a radius of 21.8 ± 0.08mm. The pendulum length, from pivot to center of mass, was 190 ± 0.5mm.

2.3 Experimental Procedure

This section focuses on the procedure of collection of raw data, and the results section will focus on how the data was processed. The raw data collected contains information about the time at which each measurement was taken, as well as the relative position (x,y) of the pendulum mass within the video at each measurement.

1. The pendulum and camera were set up as described in Sections 1.1 & 2.2
2. From the perspective of the camera, the pendulum mass was held at rest 28° counter-clockwise from its lowest point, then released
3. The camera recorded the pendulum swinging until the pendulum's amplitude became so low as to be indiscernible
4. The video was imported into *Tracker* and using the software's auto-tracking feature, the relative (x,y) position of the pendulum mass within each frame of the video was found and recorded, along with a timestamp

2.4 Results

The experiment ran for 335.6s - over 8600 frames of data were processed.

The two methods used to calculate the Q factor both require timestamp and angle data, and so it was necessary to convert the raw positional (x,y) data from each frame into angular data, centered around the pivot of the pendulum. Note that this pivot point ended up being slightly below the rigid dowel the pendulum string was attached to. A counter-clockwise rotation was assumed to be positive, with the pendulum forming an angle of 0 rad at rest. The conversion to angular data was made using a custom Python program. The program also determined which frames represented the peak of each oscillation, and stored the angle and time data for those frames separately. Additionally, the timestamp of each frame was shifted to represent elapsed time from the start of the pendulum's motion, rather than from the start of the video.

The two methods of calculating the Q factor also require knowledge of the pendulum's period, which can be calculated using the following equation:

$$T \approx 2\sqrt{L}$$

Where L represents the length of the pendulum, 190mm. Therefore, the period is 0.872 ± 0.003s.

2.4.1 Q Factor Method 1 (Q_1)

The first method of calculating the Q factor involved running the timestamp and angle data through a Python program to fit a curve of $\theta(t)$ to it.

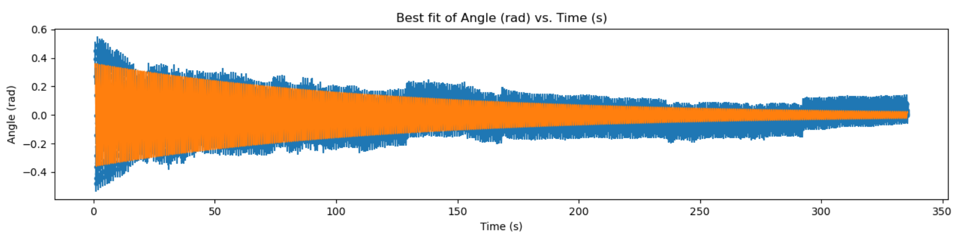

Figure 2.1: $\theta(t)$ Curve Fit

τ is a parameter in this curve. It can be used along with the period (T) to calculate the Q factor. From the program:

$$\tau = 122\text{s}$$

Using the equation:

$$Q_1 = \pi\frac{\tau}{T}$$

$$Q_1 = \pi\frac{122\text{s}}{0.872\text{s}}$$

$$Q_1 = 440$$

2.4.2 Q Factor Method 2 (Q_2)

The second method of calculating the Q factor involved running the timestamp and peak angle data through a Python program to fit a curve of the form $ae^{-\frac{t}{\tau}}$ to it.

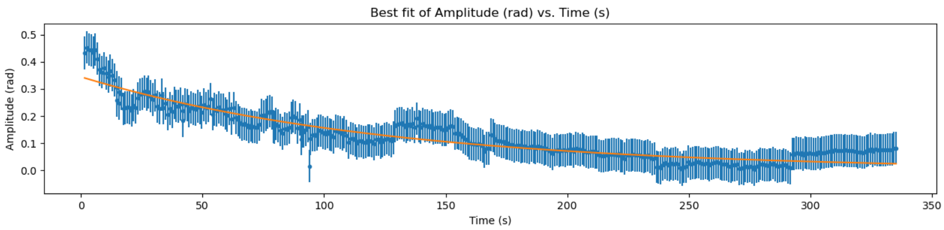

Figure 2.2: $ae^{-\frac{t}{\tau}}$ Curve Fit

This process found that $a = -0.327$ and $\tau = 132$.

$$\theta_{amp} = -0.327e^{-\frac{t}{132}}$$

The number of oscillations to reach the elapsed time where $\theta_{amp} = -0.327e^{-\frac{\pi}{2}}$ (or $e^{-\frac{\pi}{2}}$ of the peak amplitude) is $\frac{Q_2}{2}$. In this case:

$$-0.3268e^{-\frac{t}{132}} = -0.3268e^{-\frac{\pi}{2}}$$
$$-\frac{t}{132} = -\frac{\pi}{2}$$
$$t = \pi \cdot 66\text{s}$$

This can be converted to a number of oscillations to find $\frac{Q_2}{2}$. Each oscillation has a period of T, and so $\frac{Q_2}{2}$ is:

$$\frac{Q_2}{2} = \frac{t}{T} = \frac{\pi \cdot 66\text{s}}{0.872\text{s}} = 238$$
$$Q_2 = 476$$

2.5 Lab 1 Uncertainties Analysis

Potential sources of uncertainty that are specific to only Lab 1 have been chosen for analysis in this section. Uncertainties that are common to all experiments will be analyzed in Chapter 4.

2.5.1 Physical Measurement Uncertainty

The length of the pendulum string was measured using a ruler with marks to the nearest millimeter. Therefore, the measurement with error is 168 ± 0.5mm. The radius of the pendulum mass was measured by finding the circumference of the mass, which was similarly measured to 137 ± 0.5mm, or 0.4%. Therefore, the radius is 21.8 ± 0.08mm. The final length of the pendulum is the sum of the string length and the mass radius - this is 190 ± 0.5mm, with an error of 0.3%. This error propagates to the uncertainty of the period, which with error measures in at 0.872 ± 0.003s.

2.5.2 Timestamp Uncertainty

With regards to the independent variable - the elapsed time since the start of the experiment - data collection was fairly straightforward, with timestamp data captured in each frame. However, some uncertainty still exists here. Any particular event that was recorded in the video could have occurred at any time between frames, since there could have been a slight difference between when the frame was recorded and the timestamp it was recorded with. Therefore, since a frame was recorded every 0.04s, each data point has a time uncertainty of ± 0.02s. Given that the pendulum's motion was recorded for 335.6s, this corresponds to a percentage uncertainty of $\pm 0.0006\%$.

2.5.3 Curve Fit Uncertainty

Neither of the curves $\theta(t)$ and $ae^{-\frac{t}{\tau}}$ that were fit to the experimental data were perfect fits (as should be expected from experimental data). The uncertainties of the fit for each curve parameter were directly provided by the algorithm used to fit the curves. These values were:

$\theta(t)$ Parameters and Uncertainty:

$$a = -0.362 \pm 0.003 \quad [0.8\%], \quad \tau = 122 \pm 1 \quad [0.8\%]$$

$$T = 0.87404 \pm 0.00001 \quad [0.0001\%], \quad \phi = -2.126 \pm 0.007 \quad [0.3\%]$$

$ae^{-\frac{t}{\tau}}$ Parameters and Uncertainty:

$$a = -0.327 \pm 0.008 \quad [2\%], \quad \tau = 131 \pm 5 \quad [4\%]$$

A couple of these curve parameters were later used to calculate the Q factor, and so the error in these parameters directly propagated to the Q factor. The first method introduced to compute the Q factor made use of the parameter τ of the $\theta(t)$ curve. The second method made use of the parameters a and τ of the $ae^{-\frac{t}{\tau}}$ curve.

2.5.4 Overall Uncertainty Propagation

The final percentage uncertainty of each Q value is the largest percentage uncertainty it is dependent on. It turns out that angular uncertainty due to computer vision tracking error is actually the most significant uncertainty in *both* methods of computing Q factor [Section 4.1.4]. Applying this percentage error to the Q factors obtained through each method:

$$Q_1 = 440 \pm 6\% = 440 \pm 30$$
$$Q_2 = 476 \pm 6\% = 480 \pm 30$$

Note that a significant amount of precision in each measurement has been lost due to this uncertainty.

2.6 Analysis

2.6.1 Observed Rotational Behaviour

The pendulum also appears to exhibit some rotational motion. This is a discrepancy between the predicted behaviour of the pendulum and its actual behaviour, since the theoretical motion model assumes motion within one plane, while in reality, the pendulum is not fixed to any plane. The tension in the string always tries to pull the pendulum's center of mass so that it is directly underneath the pivot - so if the pendulum's initial plane of oscillation is not directly underneath the pivot point, the pendulum will experience a force slightly outside of that initial plane, and will begin to oscillate in this new direction as well. It is practically impossible to setup the pendulum exactly so that its initial plane of oscillation is directly underneath the pivot, and so some level of this rotational motion is unavoidable. Through coupling effects, this phenomenon could theoretically affect the motion of the pendulum in the observed plane, skewing results, but this effect is likely quite small, and extremely difficult to quantify. As such, the specific consequences of this rotational motion on the data are not analyzed in this report.

2.6.2 Q Factor Agreement

There is considerable overlap (within 1σ) in the range of uncertainty of each measurement, indicating agreement between the two methods of calculating Q factor. Since the uncertainty is equal in both measurements, it is sufficient to take a simple average of the two values to obtain an estimate for the Q value:

$$Q_{est} = 460 \pm 30$$

This uncertainty could be reduced in a few ways. The largest source of uncertainty is computer vision tracking error - this error could be reduced through changes to the pendulum setup. Using a background with high contrast to the pendulum mass would make it easier for *Tracker* to track its position more precisely. Indirectly, assuming that the theoretical model of the pendulum's motion is valid, reducing tracking error would also help to reduce the uncertainty of the curve fit, since the gathered data would then more closely follow the theoretical model; allowing for less error in the calculated curve parameters. The other sources of uncertainty that have been analyzed in this report are already negligible, and so no further action is required to reduce these.

2.6.3 Q Factor Consequences

The Q factor provides an idea of how quickly the pendulum's amplitude decays - a high Q factor indicates less decay in each oscillation, and more consistency between numerous consecutive oscillations. Such consistency will be useful in future experiments, since measuring and averaging data across multiple consistent oscillations often results in lower uncertainties as compared to measuring individual oscillations (note: this may not always be the case due to the nature of the type of uncertainty in question). Conversely, with a low Q factor, measuring multiple consecutive oscillations would actually introduce error, since the oscillations would be consistently biased due to the higher decay rate; skewing any averages calculated.

The Q factor measured in this experiment was extremely high: $Q = 460 \pm 30$. This implies an extraordinarily high consistency between consecutive oscillations, and indicates that it should be reasonably safe to take the average of multiple consecutive oscillations when gathering data in the future.

2.6.4 Agreement with Theoretical Model

The theoretical model predicts sinusoidal angular motion with respect to time, and an exponential decay of amplitude. The data obtained through this experiment supports these hypotheses.

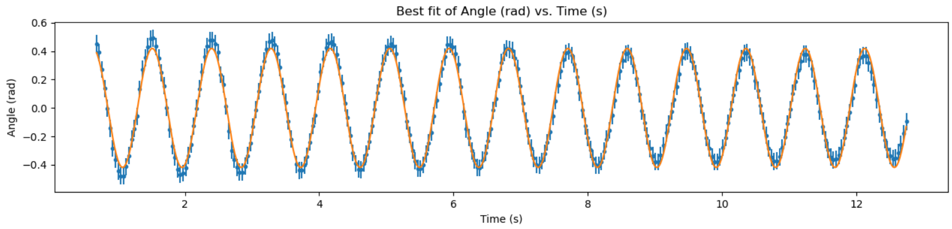

Figure 2.3: $\theta(t)$ Curve Fit [first 300 data points]

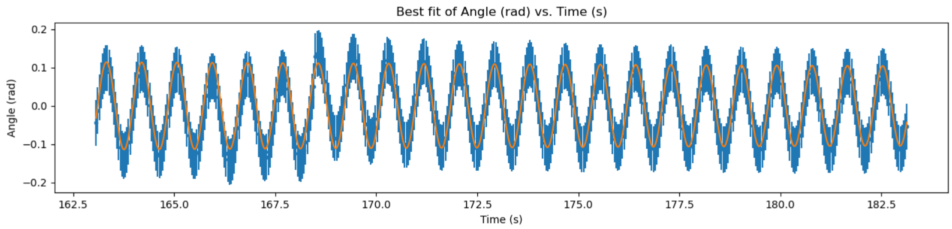

Figure 2.4: $\theta(t)$ Curve Fit [data points 4000-4500]

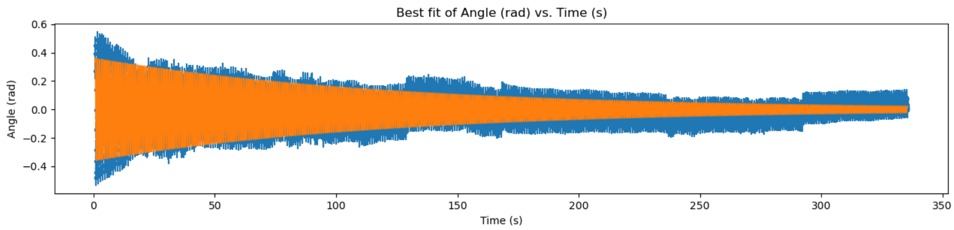

Figure 2.5: $\theta(t)$ Curve Fit [all data points]

The sinusoidal behaviour of the pendulum is clearly visible in the first two subsets of data points (Figures 2.3 and 2.4), and although it is hard to discern in the full curve fit of all data points (Figure 2.5), this behaviour is present there as well. The model $\theta(t)$ used to fit a curve to the data points was quantifiably accurate as well - its R^2 value was calculated to be 0.84, and nearly all data points appear to be within a single uncertainty interval of the curve. The residuals of the fit were also tracked, and apart from some discrete jumps that are likely attributable to computer vision tracking error, they appear fairly randomized as well, indicating a good fit.

The theoretical model also predicts an exponential decay of the pendulum's amplitude. The data also supports this hypothesis.

The exponential decay of the amplitude is clearly visible in Figure 2.7. Quantifiably, the $ae^{-\frac{t}{\tau}}$ curve had an R^2 value of 0.87, indicating a good fit. Most data points also appear to be within a single uncertainty interval of the curve. The residuals of the $ae^{-\frac{t}{\tau}}$ curve fit were also fairly randomized, indicating a good fit.

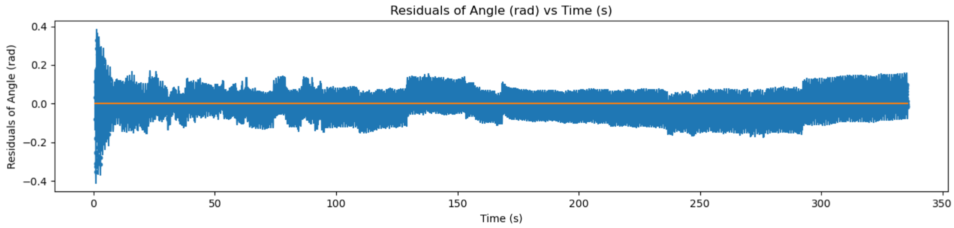

Figure 2.6: $\theta(t)$ Residuals

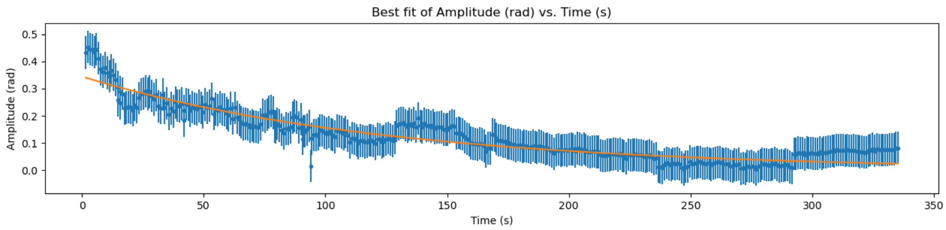

Figure 2.7: $ae^{-\frac{t}{\tau}}$ Curve Fit

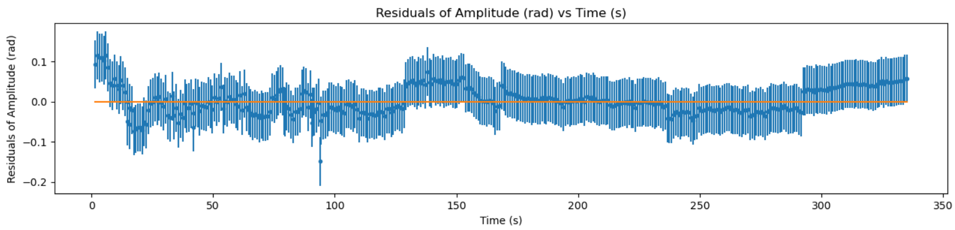

Figure 2.8: $ae^{-\frac{t}{\tau}}$ Residuals

2.7 Updated Q Factor

After the changes made to the pendulum in Lab 2 [Section 3.2], the Q factor was recalculated - using the same method in Lab 1 - to be:

$$Q = 170 \pm 10$$

Chapter 3

3.1 Lab 2 Overview

The purpose of Lab 2 is to determine the effect that varying the experimental pendulum's starting amplitude has on its period, and to compare this result with the prediction of the damped harmonic motion model to evaluate the model's validity.

As a secondary objective, the collected data also helps to verify that the experimental pendulum contains no asymmetries - this is important since any potential asymmetries would make the pendulum fundamentally different from the theoretical pendulum described by the damped harmonic motion model.

In order to test the effect of different starting amplitudes, data was collected from a single release of the pendulum. The natural decay of the pendulum's amplitude over time reduced the starting amplitude of each individual oscillation; by measuring the starting amplitude and the period of each individual oscillation, it was possible to obtain numerous data points from a single video.

This video was fed into the motion tracking software *Tracker* to find the position of the pendulum's mass in each frame. This positional data was then fed into a custom Python program to find the angular amplitude of the pendulum at the beginning of each oscillation, and using the elapsed frame count between peaks, the period of each oscillation was found as well. In an effort to reduce measurement uncertainty, the period associated with each starting amplitude was calculated by averaging the periods of a set of oscillations, all centered within a set range of the desired amplitude.

This data was processed and fit to a curve of the form:

$$T(\theta) = T_0 + B\theta_0 + C\theta_0^2$$

where $T(\theta)$ represents the pendulum's period, and θ_0 represents the motion's starting amplitude.

Curve fitting was done using a modified version of the Python curve fitting program provided by the teaching team.

3.2 Lab 2 Setup

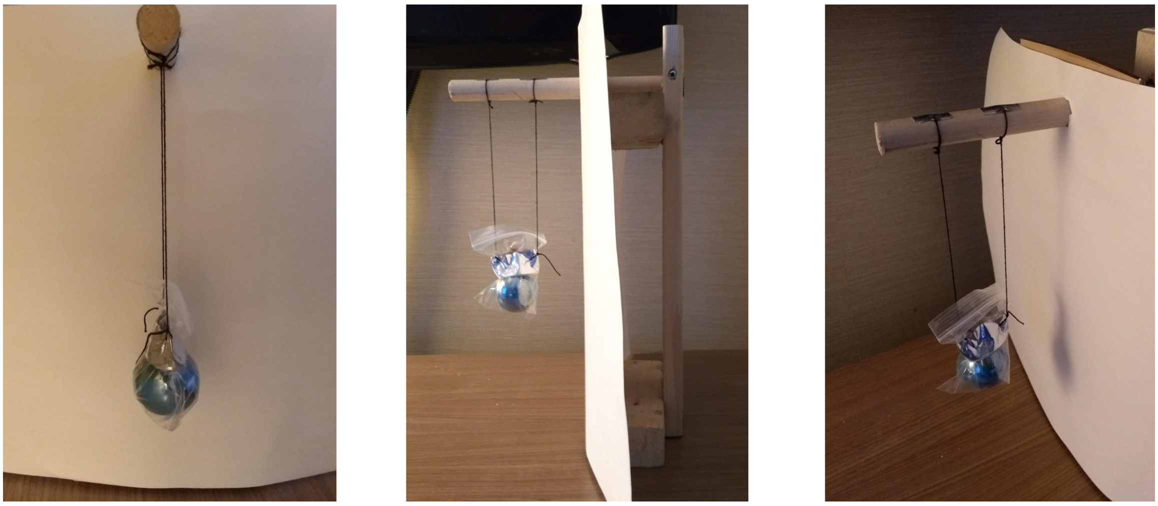

Figure 3.1: Modified Pendulum Images

In Lab 2, the pendulum was modified to reduce out of plane rotational behaviour. Although this rotational behaviour had only mild consequences in Lab 1, it has more significant consequences in Lab 2. This is because Lab 2 involves a much more extreme starting angle, which results in large out of plane oscillations that have a severe impact on the pendulum's motion within the plane being tracked. In order to remedy this, a second string was attached to the marble such that the two strings form a plane perpendicular to the plane of oscillation - this keeps the pendulum from swinging out of plane, without significantly affecting its motion within the plane. Additionally, a piece of solid white paper was attached behind the pendulum to increase contrast in recordings, making it easier for *Tracker* to track the pendulum mass. This modified setup is shown in Figure 3.1.

In this experiment, the length of the string was fixed at 154 ± 0.5mm and the mass was fixed at 121 ± 0.5g. The mass used was a large, spherical marble with a radius of 21.8 ± 0.08mm. The pendulum length, from pivot to center of mass, was 176 ± 0.5mm.

3.3 Experimental Procedure

This section focuses on the procedure of collection of raw data. The raw data collected contains information about the time at which each measurement was taken, as well as the relative position (x,y) of the pendulum mass within the video at each measurement.

1. The pendulum and camera were set up as described in Sections 1.1 and 3.2
2. From the perspective of the camera, the pendulum mass was held at rest $85°$ counter-clockwise from its lowest point, then released
3. The camera recorded the pendulum swinging until the pendulum's amplitude became so low as to be indiscernible
4. The video was imported into *Tracker* and using the software's auto-tracking feature, the relative (x,y) position of the pendulum mass within each frame of the video was found and recorded, along with a timestamp

3.4 Results

The experiment ran for 390s - over 9600 frames of data were processed.

Since the objective of this experiment is to determine the relationship between angular amplitude and period, it was necessary to first convert the positional data from *Tracker* to angular data, centered around the pivot of the pendulum. A counter-clockwise rotation was assumed to be positive, with the pendulum forming an angle of 0 rad at rest. This conversion to angular data was made using a custom Python program, which then split the data up into separate, individual oscillations, and found the period and starting amplitude of each oscillation.

A curve of the form $T(\theta) = T_0 + B\theta_0 + C\theta_0^2$ was then fit to the data (Figure 3.2).

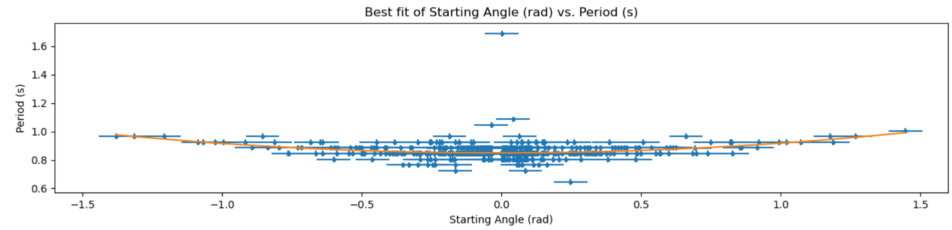

Figure 3.2: Raw $T(\theta)$ Curve Fit

The curve fit found the following parameters for the curve:

$$T_0 = 0.849, B = 9.53 \times 10^{-4}, C = 0.0674$$

Note that all values are in units such that the equation holds true when θ is measured in radians and T is measured in seconds.

When calculated, the R^2 value of this fit is 0.109 - indicating that this is fit is very loosely correlated with the data set.

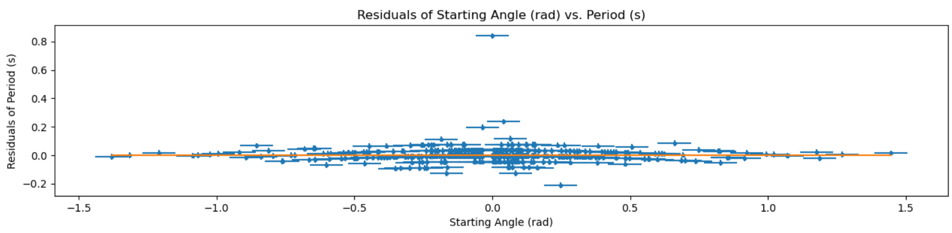

Figure 3.3: Raw $T(\theta)$ Residuals

The residuals of this curve fit are shown above in Figure 3.3 - they are reasonably randomly distributed.

In order to reduce measurement uncertainty, this curve was also fit to a modified set of data, where a smaller quantity of starting amplitude data points was considered, with each data point corresponding to a period averaged across multiple consecutive swings with similar starting amplitudes. This helped to reduce the impact of any randomized measurement uncertainty inherent in determining the period for any individual swing. This process is possible since the Q factor of the pendulum is sufficiently high ($Q = 170$) and so it can be expected that the amplitude decay between oscillations will be minimal.

30 starting amplitudes were considered - half of them were positive, and half negative:

$\pm 80°, \pm 75°, \pm 65°, \pm 60°, \pm 55°, \pm 50°, \pm 45°, \pm 40°, \pm 35°, \pm 30°, \pm 25°, \pm 20°, \pm 15°, \pm 10°, \pm 5°$

For each of these set starting amplitudes, all oscillations with starting amplitudes within 3° (approximately 0.0524rad) were considered to be similar - their periods were averaged to obtain a more accurate measure of the period at the set amplitude. This categorization was performed by another Python program.

A curve of the identical form $T(\theta) = T_0 + B\theta_0 + C\theta_0^2$ was then fit to these 30 data points (Figure 3.4).

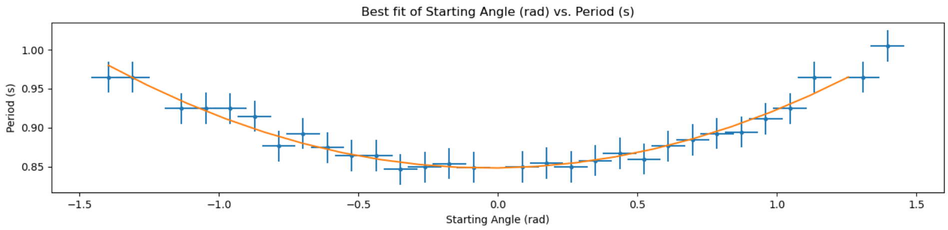

Figure 3.4: Averaged $T(\theta)$ Curve Fit

The curve fit found the following parameters for the curve:

$T_0 = 0.848, B = 4.19 \times 10^{-3}, C = 0.0705$

When calculated, the R^2 value of this fit is 0.958 - this is a much stronger fit than the previous one, which should be expected since there are fewer data points in this set.

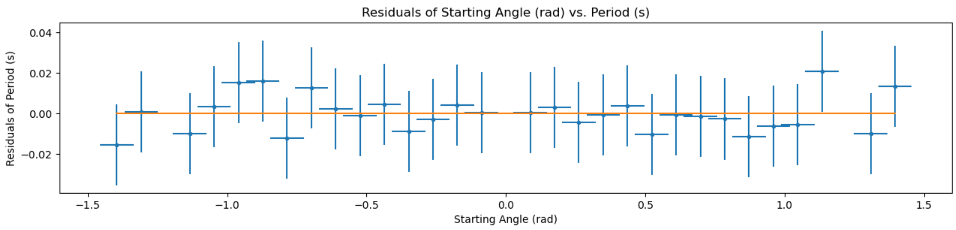

Figure 3.5: Averaged $T(\theta)$ Residuals

The residuals of this curve fit are shown above in Figure 3.5 - they are reasonably randomly distributed.

In the rest of this report, the second (averaged) data set's curve parameters will be used, and the first (full sized) data set's results will be ignored. This is because there is less measurement uncertainty in the averaged data set.

3.5 Lab 2 Uncertainties Analysis

Potential sources of uncertainty that are specific to only Lab 2 have been chosen for analysis in this section. Uncertainties that are common to all experiments will be analyzed in Chapter 4.

3.5.1 Physical Measurement Uncertainty

The length of the pendulum string was measured using a ruler with marks to the nearest millimeter. Therefore, the measurement with error is 154 ± 0.5mm. The radius of the pendulum mass was measured by finding the circumference of the mass, which was similarly measured to 137 ± 0.5mm, or 0.4%. Therefore, the radius is 21.8 ± 0.08mm. The final length of the pendulum is the sum of the string length and the mass radius - this is 176 ± 0.5mm, with an error of 0.3%.

3.5.2 Timestamp Uncertainty

Any particular event that was recorded in the video could have occurred at any time between frames, since there could have been a slight difference between when the frame was recorded and the timestamp it was recorded with. Therefore, since a frame was recorded every 0.04s, each oscillation has a time uncertainty of ± 0.02s. However, in the averaged data set, each data point has an uncertainty spread among multiple oscillations - this should lower uncertainty. Unfortunately, there are some starting amplitude data points (near the beginning of the motion, where the amplitude decay rate is high) that are based off of only one oscillation - and so it is not possible to say that the general uncertainty in the data set is any lower than ± 0.02s. Given that the pendulum's motion was recorded for 390s, this corresponds to a percentage uncertainty of $\pm 0.0005\%$.

3.5.3 Curve Fit Uncertainty

The uncertainties of the fit for each curve parameter of the $T(\theta)$ curve were directly provided by the algorithm used to fit the curve. For the averaged data set curve fit, these values were:

$T(\theta)$ Parameters and Uncertainty:

$$T_0 = 0.848 \pm 0.003 \quad [0.4\%], \quad B = 0.004 \pm 0.002 \quad [50\%], \quad C = 0.071 \pm 0.003 \quad [4.3\%]$$

3.5.4 Overall Uncertainty

The final percentage uncertainty of each term in the $T(\theta)$ expression is the largest percentage uncertainty it is dependent on. Angular uncertainty due to computer vision tracking error is the most significant uncertainty for the terms T_0 and C [Section 4.1.4]. Meanwhile, uncertainty in the curve fit is the most significant uncertainty for the term B. Applying these uncertainties:

$$T_0 = 0.848 \pm 6\% = 0.85 \pm 0.05$$
$$B = 0.004 \pm 50\% = 0.004 \pm 0.002$$
$$C = 0.071 \pm 6\% = 0.071 \pm 0.004$$

Note that the B term is within 2 uncertainty intervals, or 2σ of 0 - and so it is consistent with 0.

3.6 Analysis

3.6.1 Out of Plane Energy Dissipation

The experimental pendulum was physically constrained (using multiple strings) to a single plane of motion in order to more accurately test the predictions of the theoretical model, which assumes motion in a single plane. However, in reality, since it is practically impossible to set up the pendulum so that its motion is fully contained within the plane, a component of the pendulum's momentum always acts slightly outside of the plane - the energy involved in this out of plane motion is dissipated by the physical constraints. This energy loss is not consistent with the assumptions of the theoretical model, and so it is possible that it could skew results - however, it is extremely difficult to quantify this energy loss, and it is likely that its effects are quite small compared to the other uncertainties involved in this experiment. Therefore, the consequences of this energy loss are not analyzed in this report.

3.6.2 Curve Fit Consequences

Important conclusions can be drawn directly from the parameters of the curve fit.

The second term in the curve fit ($B\theta$) implies symmetry within the experimental pendulum. This is the only term in the expression that is not symmetrical about the y axis, and so it is the only term that could cause an asymmetry in the model. However, since this term is consistent with 0 (within 2 curve fit uncertainty intervals, or 2σ), it has no effect in describing the pendulum's motion, and so it is possible to say that there are no experimentally detectable asymmetries within the pendulum's motion. This is important since the theoretical model is meant to describe a perfectly symmetrical pendulum - a deviation from this symmetry in the experimental pendulum would make it impossible to test the model accurately.

The third term in the curve fit ($C\theta^2$) implies that the period of the pendulum is higher for more extreme starting amplitudes. This is because this term is more dominant than the others for higher angles, since it is raised to the power of 2, and since it is significantly positive. The observation that the period of the pendulum increases with larger starting amplitudes is inconsistent with the hypothesis that period and starting amplitude are independent, and so that hypothesis must be rejected.

3.6.3 Small Amplitude Approximations

Considering only small starting amplitudes from the data set, it is not possible to reject the hypothesis that period and starting amplitude are independent. This is due to uncertainty in the measured values being more significant than the dependence caused by the $B\theta$ and $C\theta^2$ terms in the model.

The largest uncertainty in the period is 0.02s [Section 3.5.2]. When the terms that imply dependence ($B\theta + C\theta^2$) are ignored, the period (for any starting amplitude) is given by $T_0 = 0.848$. The data points from the experiment can be plotted against this curve, along with the $T(\theta)$ curve and the bounds of its uncertainty:

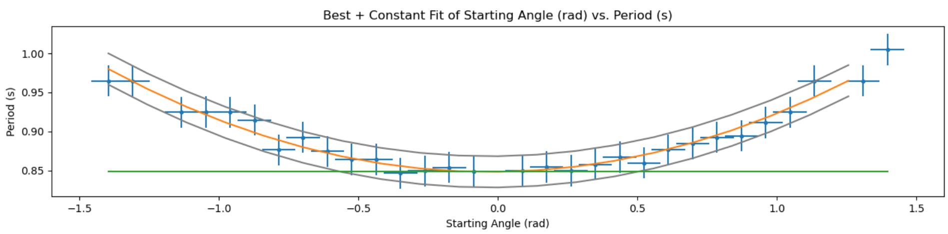

Figure 3.6: $T(\theta)$ Independence Region

As can be seen in Figure 3.6, there is a region where the hypothesis of period and starting amplitude being independent (this behaviour denoted by the green line) cannot be rejected, since measurement uncertainty can explain away any apparent dependence between the two variables. Solving for this region:

$$0.02 > |T(\theta) - T_0|$$

$$0.02 > |0.848 + 4.19 \cdot 10^{-3}\theta + 0.0705\theta^2 - 0.848|$$

$$|\theta| < 0.563\text{rad}$$

Considering an overall percentage error of 6% [Section 4.1.4], this is:

$$|\theta| < 0.563 \pm 0.03\text{rad}$$

In this case, it is safer to choose the lower bound for this value since that will more likely bound the true range of the region:

$$|\theta| < 0.56\text{rad}$$

Therefore, with this experimental setup, any dependence between period and starting amplitude can effectively be ignored for starting amplitudes between -0.56rad and 0.56rad, or $-32°$ to $32°$.

Chapter 4

4.1 General Uncertainties Analysis

Although there are numerous potential sources of uncertainty in this experiment, a few of the most significant have been chosen for analysis in this section. Note that the uncertainties analyzed in this section apply generally to all labs in this report.

4.1.1 Camera Plane Positioning Uncertainty

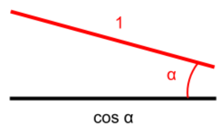

Figure 4.1: Impact of error in camera plane positioning

Ideally, the plane of the recorded video should have been completely parallel to the plane of the pendulum's motion. This was likely not the case in reality due to human error in positioning the camera and pendulum; however, the uncertainty attributable to this is negligible. This is because the accuracy of a measurement from a plane angled by α is proportional to $cos\alpha$ (as pictured in Figure 4), and for any small angle deviations of α, this is approximately equal to 1.

4.1.2 Angular Uncertainty

There are a few potential sources of error in the dependent variable (pendulum angle), since there were a number of steps taken to convert the data in each frame to a final angle, with each step introducing some error.

4.1.3 Angular Uncertainty - Center of Mass

One potential source of angle error is due to the center of mass of the pendulum being in a slightly different location than expected. In this experiment, it was assumed that the center of mass was at the center of the marble. This is probably a fairly accurate assumption, since the mass of the string is negligible, and because the marble is spherical, its center of mass should be at its center - however, it is worthwhile to note that this potential source of error exists.

4.1.4 Angular Uncertainty - Computer Vision

Another source of error is the computer vision tracking of the pendulum, since *Tracker* could not always pinpoint the center of the pendulum's mass accurately. However, since it was tracking the pendulum based on its colour and shape, it is safe to say that it always identified the center of mass point well within the marble's figure. It is a reasonable assumption that the center of mass was never identified more than half the marble's radius away from the true center of mass.

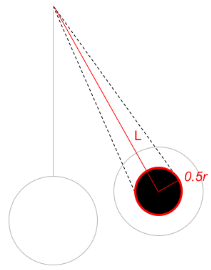

Figure 4.2: Angular uncertainty due to computer vision error

As illustrated in Figure 5, this general area of uncertainty corresponds to a final angular uncertainty. Given the length L from the pivot to the center of mass, and the radius of the area of uncertainty (assumed to be half the radius of the marble) $0.5r$, the angular uncertainty is given by the expression $\pm \tan^{-1} \frac{0.5r}{L}$. Using the length of the pendulum $L = 190$mm and the radius of the marble 21.8mm, the angular uncertainty from this error source is ± 0.06rad. Expressed as a percentage of the total range of angles recorded, 0.97 ± 0.06rad, this corresponds to a 6% uncertainty.

4.1.5 Angular Uncertainty - Angular Data Conversion

A potential source of uncertainty that was rectified was the conversion from positional data within the video to angular data. This conversion could have been problematic, since the camera was slightly tilted within a plane parallel to the plane of the pendulum's motion - making the pendulum at rest appear very slightly tilted from the camera's perspective. Although the effect of this would have likely been quite small, the Python program that converted positional data to angular data took this into account; consistently finding the angle relative to the pendulum's true rest position. Therefore, the effect of any error in converting positional data to angular data should be entirely negligible.

Chapter 5

5.1 Appendix

Code and raw/processed data can be found here:

`https://github.com/freezedcheese/Uni-Projects/tree/main/PHY180`