Bayesian Tools for Synthesis of Ecological Data

Moment Matching

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Objective of videos

Learn Bayesian methods for synthesizing existing data and published findings to gain new insight in ecology

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1. Video 1
    1.1 Review of components of Bayesian inference
    1.2 Why use informed priors?
    1.3 A problem using published means and standard deviations
2. Video 2: Moment matching
3. Video 2: Developing priors from multiple studies
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The problem: How do we find parameters using tabulated means and standard deviations?

The normal and the Poisson are the only distributions for which the parameters of the distribution are the mean and the variance. The parameters of all other distributions are *functions* of the mean and the variance.

$$\begin{aligned} \alpha &= f_1(\mu, \sigma^2) \\ \beta &= f_2(\mu, \sigma^2) \end{aligned}$$

We can use these functions to "match"' the moments to the parameters.

Moment matching: the general theory

We seek a prior on μ using published data $\bar{x}$and σ^2. We need numeric values for the parameters of the distribution of μ.

$$
\begin{aligned}
\alpha &= f_1(\bar{x}, \sigma^2) && (1)\\
\beta &= f_2(\bar{x}, \sigma^2) && (2)\\
\mu &\sim [\alpha, \beta] && (3)
\end{aligned}
$$

where $\alpha$and $\beta$are numeric values computed from 1 and 2

Moment matching the gamma distribution for non-negative, continuous quantities

The mean of the gamma distribution is

$$\mu = \frac{\alpha}{\beta}$$

and the variance is

$$\sigma^2 = \frac{\alpha}{\beta^2}.$$

How do we find α and β in terms of μ and σ^2?

Answer

$1)\mu = \frac{\alpha}{\beta}$
$2)\sigma^2 = \frac{\alpha}{\beta^2}$
Solve 1 for β, substitue for β in 2), solve for α :
3) $\alpha = \frac{\mu^2}{\sigma^2}$
Substitute rhs 3) for α in 2), solve for β :
4) $\beta = \frac{\mu}{\sigma^2}$

Example: Above ground net primary production in sagebrush steppe, a non-negative real number

Mean	SE
36.7	1.97

[1]

A gamma prior on μ:

$$\mu \sim \text{gamma}\left(\frac{36.7^2}{1.97^2}, \frac{36.7}{1.97^2}\right)$$

$$\mu \sim \text{gamma}(347.1, 9.46)$$

```
JAGS code: mu ~ dgamma(347.2, 9.46)
```

[1] Manier, D. J., and N. T. Hobbs. 2007. Large herbivores in sagebrush steppe ecosystems: livestock and wild ungulates influence structure and function. Oecologia 152:739-750.

Moment matching the beta distribution for proportions

The beta distribution gives the probability density of random variables with support on $0, ..., 1$.

$$[z|\alpha,\beta] = \frac{z^{\alpha-1}(1-z)^{\beta-1}}{B(\alpha,\beta)}$$
$$B = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\mu = \frac{\alpha}{\alpha+\beta}$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\alpha = \frac{\mu^2 - \mu^3 - \mu\sigma^2}{\sigma^2}$$

$$\beta = \frac{\mu - 2\mu^2 + \mu^3 - \sigma^2 + \mu\sigma^2}{\sigma^2}$$

You need some functions...

```
#BetaMomentMatch.R
# Function for parameters from moments
shape_from_stats <- function(mu, sigma){
 a <-(mu^2-mu^3-mu*sigma^2)/sigma^2
 b <- (mu-2*mu^2+mu^3-sigma^2+mu*sigma^2)/sigma^2
shape_ps <- c(a,b)
return(shape_ps)
}
# Functions for moments from parameters
beta.mean=function(a,b)a/(a+b)
beta.var = function(a,b)a*b/((a+b)^2*(a+b+1))
```

Example: Annual survival probability of adult female bison in Yellowstone, a 0 -1 real number

Mean	SE
.89	.023

[2]

A beta prior on ϕ:

$$\alpha = (.89^2 - .89^3 - .89 \times .023^2)/.023^2$$
$$\beta = (.89 - 2 \times .89^2 + .89^3 - .023^2 + .89 \times .023^2)/.023^2$$
$$\phi \sim \text{beta}(\alpha, \beta)$$
$$\phi \sim \text{beta}(163.8, 20.24)$$

JAGS code follows

[2] Hobbs, N. T., C. Geremia, J. Treanor, R. Wallen, P. J. White, M. B. Hooten, and J. C. Rhyan. 2015. State-space modeling to support management of brucellosis in the Yellowstone bison population. Ecological Monographs 85:3-28.

JAGS code

```
sigma <- .023
mu <- .89
a <-(mu^2-mu^3-mu*sigma^2)/sigma^2
b <- (mu-2*mu^2+mu^3-sigma^2+mu*sigma^2)/sigma^2
phi ~ dbeta(a,b)
```

Sources of functions of moments to compute parameters

- Hobbs , N. T., and M. B. Hooten. 2015. Bayesian models: a statistical primer for ecologists. Princeton University Press, Princeton, N.J.
- McCarthy, M. A. 2007. Bayesian Methods for Ecology. Cambridge University Press, Cambridge, U. K.
- Distribution cheat sheet on git hub site

Take home

Prior distributions are powerful tools for synthesizing results from ecological studies. Using them reliably requires choosing distributions with proper support. Moment matching enables use of published means and standard deviations to find parameters of distributions for which parameters and moments are not the same.