Bayesian Tools for Synthesis of Ecological Data

Moment Matching

Tom Hobbs

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Graduate Degree Program in Ecology
Department of Ecosystem Science and Sustainability
Colorado State University®

Objective of videos

Learn Bayesian methods for synthesizing existing data and published findings to gain new insight in ecology

Sequence

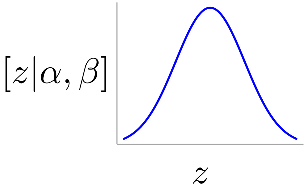
1. Video 1
   1.1 Review of components of Bayesian inference
   1.2 Why use informed priors?
   1.3 A problem using published means and standard deviations
2. Video 2: Moment matching
3. Video 3: Developing priors from multiple studies
4. Video 4: Hierarchical analysis of data sets from multiple studies

Slides for this lecture can be found at

https://github.com/nthobbs50/Bayesian_synthesis

The problem

All distributions have parameters:



α and β are parameters of the distribution of the random variable z.

The problem

The normal and the Poisson are the only distributions for which the parameters of the distribution are the mean and the variance. The parameters of all other distributions are *functions* of the mean and the variance.

$$\begin{aligned} \alpha &= f_1(\mu, \sigma^2) \\ \beta &= f_2(\mu, \sigma^2) \end{aligned}$$

What do we usually get from the literature?

Parameter	Mean	Median	SD	Quantile	
				0.025	0.975
β	1.9	1.9	0.16	1.5	2.2
f_n	0.77	0.77	0.045	0.68	0.85
f_p	0.54	0.54	0.059	0.43	0.66
f_c	0.18	0.17	0.088	0.045	0.38
m	0.47	0.47	0.041	0.39	0.55
p_1	0.95	0.96	0.044	0.84	1.00
p_2	0.89	0.88	0.023	0.84	0.93
p_3	0.93	0.93	0.039	0.84	0.99
ψ	0.031	0.027	0.021	0.004	0.082
σ_p	0.21	0.21	0.029	0.16	0.27
v	0.099	0.081	0.075	0.0068	0.29

Notes: Definitions are: β, the continuous time rate of frequency-dependent transmission (yr^{-1}); f_n, number of juveniles recruited per susceptible adult female; f_p, number of juveniles recruited per recovered adult female; f_c, number of juveniles recruited per infected and infectious adult female; m, sex ratio of juveniles surviving to yearlings; ψ, probability of recrudescence; p_1, juvenile survival probability; p_2, adult and yearling female survival probability; p_3, yearling and adult male survival probability; σ_p, process standard deviation; v, probability of vertical transmission.

Moment matching: the general theory

We seek a prior on μ using published data $\bar{x}$ and σ^2.We need numeric values for the parameters of the distribution of μ.

$$
\begin{aligned}
\alpha &= f_1(\bar{x}, \sigma^2) && (1)\\
\beta &= f_2(\bar{x}, \sigma^2) && (2)\\
\mu &\sim [\alpha, \beta] && (3)
\end{aligned}
$$

where α and $\beta$are numeric values computed from 1 and 2.

Moment matching the gamma distribution for non-negative, continuous quantities

The gamma distribution is useful for representing continuous random variables with support on $0 \to \infty$.
The mean of the gamma distribution is

$$\mu = \frac{\alpha}{\beta}$$

and the variance is

$$\sigma^2 = \frac{\alpha}{\beta^2}.$$

How do we find α and β in terms of μ and σ^2?

Moment matching the gamma distribution

$1)\mu = \frac{\alpha}{\beta}$
$2)\sigma^2 = \frac{\alpha}{\beta^2}$
Solve 1 for β, substitue for β in 2), solve for α :
3) $\alpha = \frac{\mu^2}{\sigma^2}$
Substitute rhs 3) for α in 2), solve for β :
4) $\beta = \frac{\mu}{\sigma^2}$

Example: Above ground net primary production in sagebrush steppe, a non-negative real number

Mean	SE
36.7	1.97

[1]

A gamma prior on μ:

$$\mu \sim \text{gamma}\left(\frac{36.7^2}{1.97^2}, \frac{36.7}{1.97^2}\right)$$

$$\mu \sim \text{gamma}(347.1, 9.46)$$

```
JAGS code: mu ~ dgamma(347.2, 9.46)
```

[1] Manier, D. J., and N. T. Hobbs. 2007. Large herbivores in sagebrush steppe ecosystems: livestock and wild ungulates influence structure and function. Oecologia 152:739-750.

Moment matching the beta distribution for proportions

The beta distribution gives the probability density of random variables with support on $0, ..., 1$.

$$[z|\alpha,\beta] = \frac{z^{\alpha-1}(1-z)^{\beta-1}}{B(\alpha,\beta)}$$
$$B = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\mu = \frac{\alpha}{\alpha+\beta}$$

$$\alpha = \frac{\mu^2 - \mu^3 - \mu\sigma^2}{\sigma^2}$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\beta = \frac{\mu - 2\mu^2 + \mu^3 - \sigma^2 + \mu\sigma^2}{\sigma^2}$$

You need some functions...

```
#BetaMomentMatch.R
# Function for parameters from moments
shape_from_stats <- function(mu, sigma){
 a <-(mu^2-mu^3-mu*sigma^2)/sigma^2
 b <- (mu-2*mu^2+mu^3-sigma^2+mu*sigma^2)/sigma^2
shape_ps <- c(a,b)
return(shape_ps)
}
# Functions for moments from parameters
beta.mean=function(a,b)a/(a+b)
beta.var = function(a,b)a*b/((a+b)^2*(a+b+1))
```

Example: Annual survival probability of adult female bison in Yellowstone, a 0 -1 real number

Mean	SE
.89	.023

[2]

A beta prior on ϕ:

$$\alpha = (.89^2 - .89^3 - .89 \times .023^2)/.023^2$$
$$\beta = (.89 - 2 \times .89^2 + .89^3 - .023^2 + .89 \times .023^2)/.023^2$$
$$\phi \sim \text{beta}(\alpha, \beta)$$
$$\phi \sim \text{beta}(163.8, 20.24)$$

JAGS code follows

[2] Hobbs, N. T., C. Geremia, J. Treanor, R. Wallen, P. J. White, M. B. Hooten, and J. C. Rhyan. 2015. State-space modeling to support management of brucellosis in the Yellowstone bison population. Ecological Monographs 85:3-28.

JAGS code

```
sigma <- .023
mu <- .89
a <-(mu^2-mu^3-mu*sigma^2)/sigma^2
b <- (mu-2*mu^2+mu^3-sigma^2+mu*sigma^2)/sigma^2
phi ~ dbeta(a,b)
```

Sources of functions of moments to compute parameters

- Hobbs , N. T., and M. B. Hooten. 2015. Bayesian models: a statistical primer for ecologists. Princeton University Press, Princeton, N.J.
- McCarthy, M. A. 2007. Bayesian Methods for Ecology. Cambridge University Press, Cambridge, U. K.
- Distribution cheat sheet on git hub site

Take home

Prior distributions are powerful tools for synthesizing results from ecological studies. Using them reliably requires choosing distributions with proper support. Moment matching enables use of published means and standard deviations to find parameters of distributions for which parameters and moments are not the same.