Recurrent Neural Networks and Transformers

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Machine Learning

Outline

1. **RNNs**
   - Vanilla RNNs
   - Design Alternatives
2. **RNN Training**
   - Backprop through Time (BPTT)
   - Optimization Techniques
   - Optimization-Friendly Models & LSTM
   - Parallelism & Teacher Forcing
3. **RNNs with Attention Mechanism**
   - Attention for Image Captioning
   - Attention for Neural Machine Translation (NMT)
4. **Transformers**
   - Attention is All You Need
   - Pretrained Language Models
   - More Applications
5. **Subword Tokenization**

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  - T is call the ***horizon*** and may be different between $\boldsymbol{x}^{(n)}$ and $\boldsymbol{y}^{(n)}$ and across data points n's

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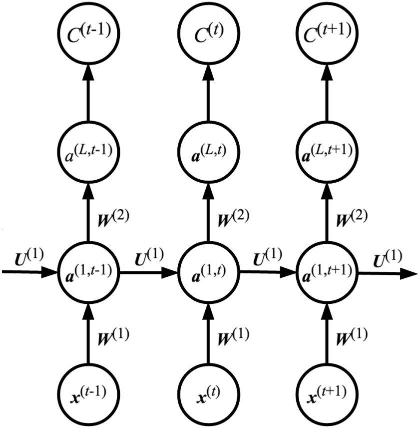

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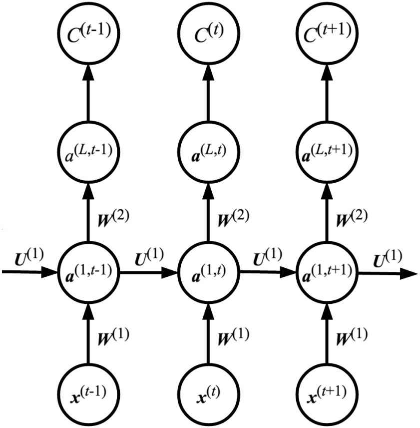

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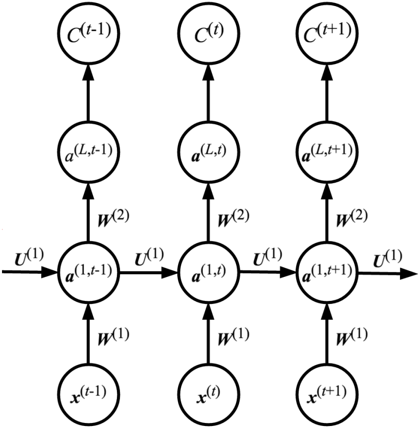

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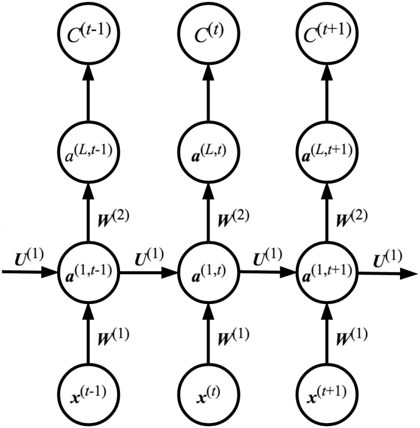

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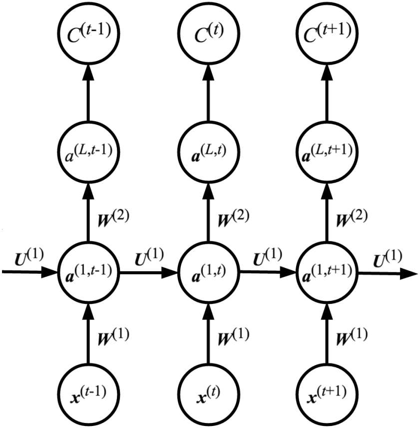
- $\mathbf{a}^{(\cdot,t)}$'s at deeper layers give more abstract summarizations



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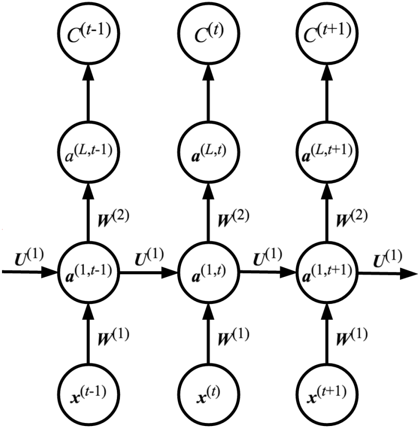

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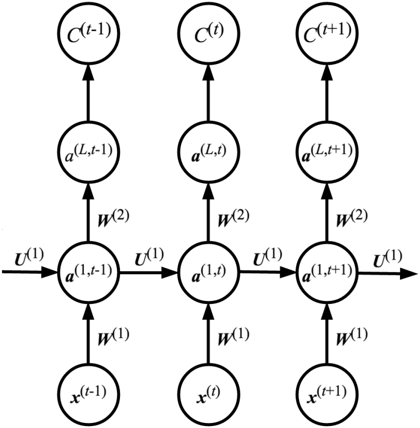

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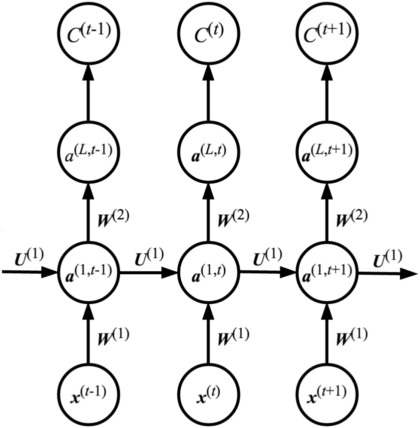
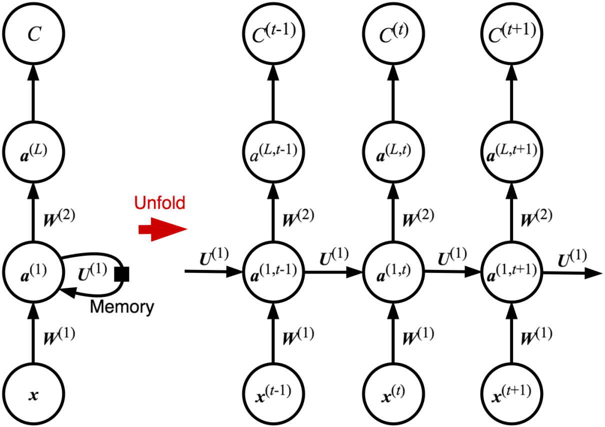
- Weights are ***shared*** across time instances
- Assumes that the "transition functions" are time invariant
- Our goal is to learn $\boldsymbol{U}^{(k)}$'s and $\boldsymbol{W}^{(k)}$'s for $k = 1, \cdots, L$



RNNs have Memory

- The computational graph of an RNN can be ***folded*** in time



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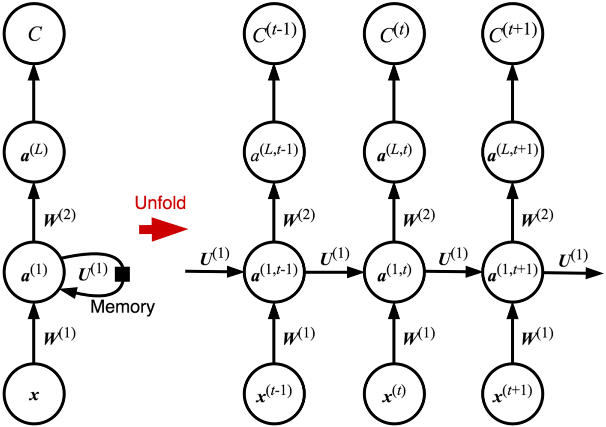
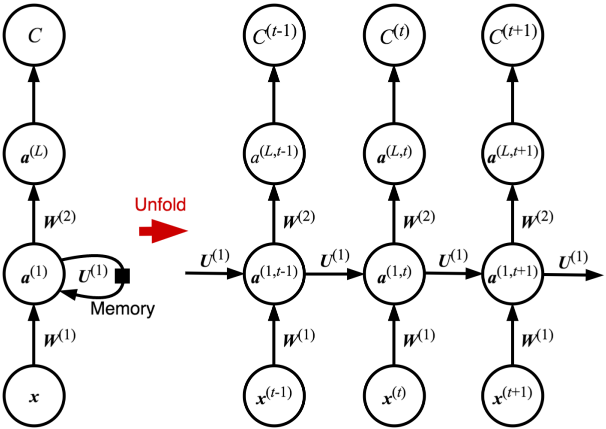
- Black squares denotes ***memory*** access



Output Layer (1/2)

- With multi-class $\mathbf{y}^{(t)}$,
  - $\mathbf{a}^{(L,t)}$ represents the probability of each class
  - $C^{(t)}$ is cross entropy
- How to obtain $\hat{\mathbf{y}}^{(t)}$ rom $\mathbf{a}^{(L,t)}$ at inference time?



Output Layer (2/2)

- ***Output sampling*** for multi-class tasks:
  - Greedy: sample $\hat{\mathbf{y}}^{(t)}$ from $\mathbf{a}^{(L,t)}$
  - Bean search: sample $\hat{\mathbf{y}}^{(t)}$ from the most probable paths of the join distribution $(\mathbf{a}^{(L,t)}, \mathbf{a}^{(L,t-1)}, \cdots \mathbf{a}^{(L,t-b)})$, where b is bean size
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- Solution?

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- Solution? ***Subword tokenization*** (to be discussed later)

RNNs vs CNNs for Sequential Data

- On processing a sequence of length T at each layer with
  - D-dimensional point input and output
  - F = the CNN filter/kernel size
  - #CNN filters = D

	#Weights	Computation	Autoregressive	Point Distance
CNN	$O(FD^2)$	$O(TFD^2)$	No	$O(\frac{T}{F})$
RNN	$O(D^2)$	$O(TD^2)$	Yes	$O(T)$

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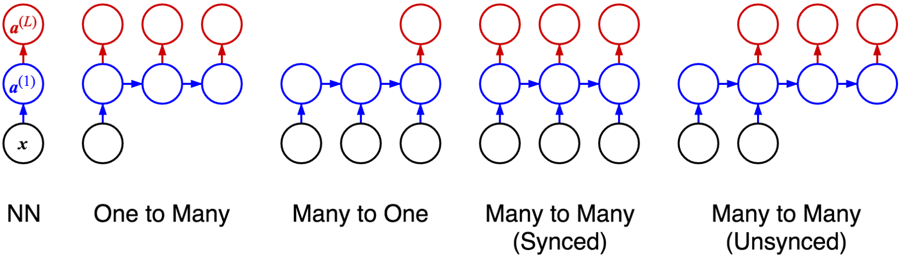
Input and Output

- $\boldsymbol{x}^{(t)}$'s and $\boldsymbol{y}^{(t)}$'s do ***not*** need to have one-to-one correspondence:



One2Many: Image Captioning

"A little girl sitting on a bed with a teddy bear."

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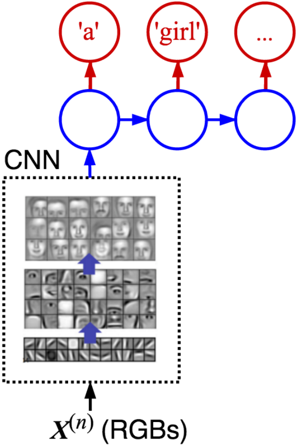
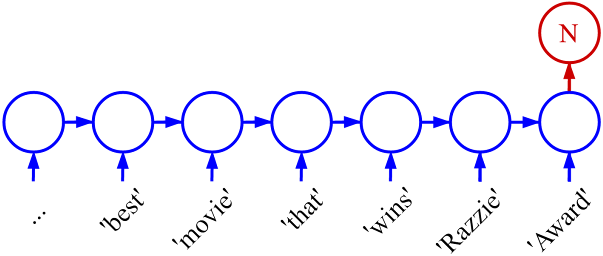


Many2One: Sentiment Analysis



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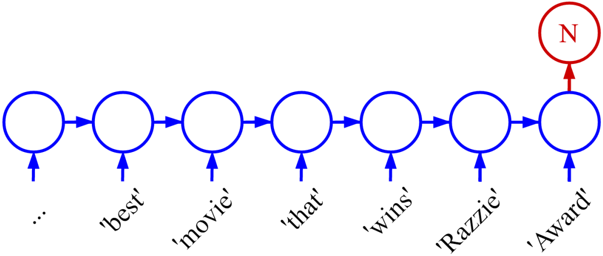
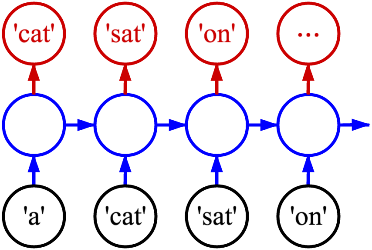


- A single word (e.g., "Razzie") can negate the entire input sentence

Many2Many (Synced): Language Modeling

- ***Language modeling***: predicting the next/nearby word based on the context



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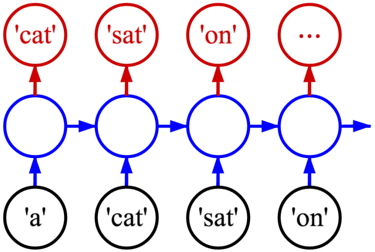
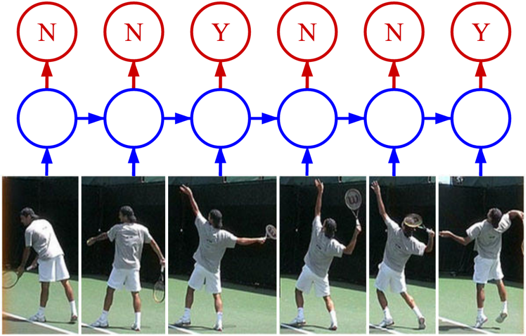


- Latent representations of RNN provide the context

Many2Many (Synced): Video Keyframe Tagging

- Video frame annotation:



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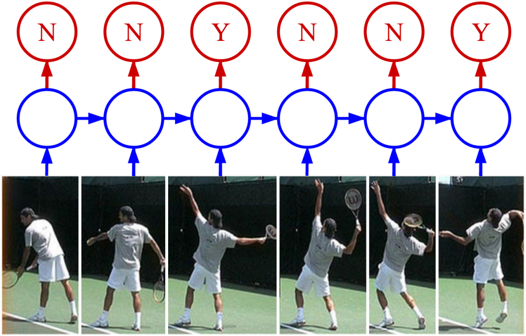
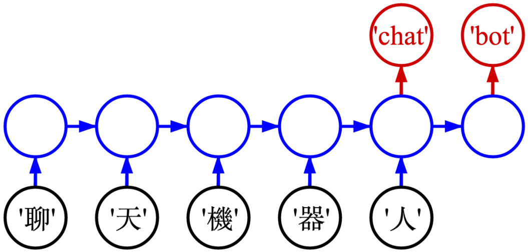


- Latent representations summarize "what's going on"

Many2Many (Unsynced): Machine Translation



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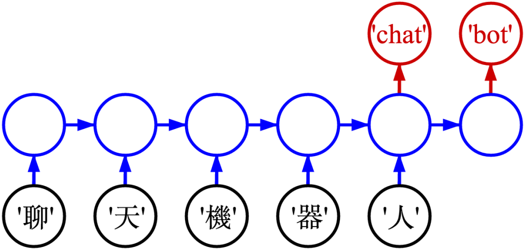

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- RNN learns the structure difference
- Also called ***sequence to sequence*** learning
  - Also used in other applications, e.g., chat bots

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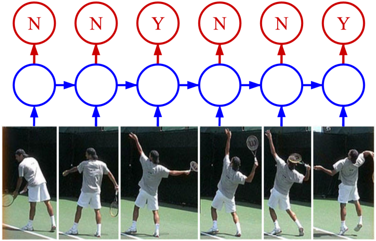

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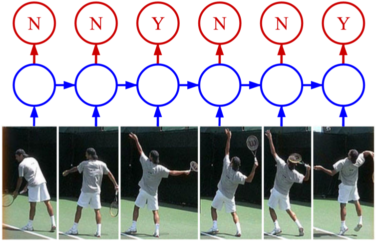

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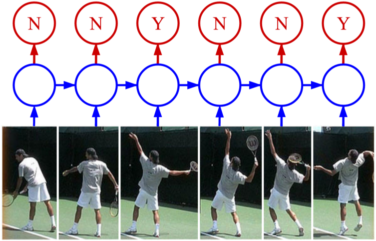
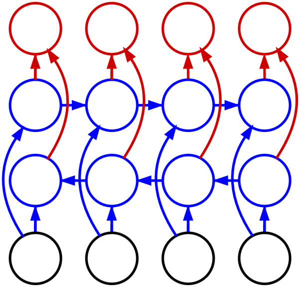
- Can we take ***future*** frames into account?
- ***Bidirectional RNNs***: output $\mathbf{a}^{(L,t)}$ depends on both $\mathbf{a}^{(k,t)}$'s and $\tilde{\mathbf{a}}^{(k,t)}$'s

$$\tilde{\mathbf{a}}^{(k,t)} = \text{act}(\tilde{\boldsymbol{U}}^{(k)}\tilde{\mathbf{a}}^{(k,t+1)} + \tilde{\boldsymbol{W}}^{(k)}\tilde{\mathbf{a}}^{(k-1,t)})$$





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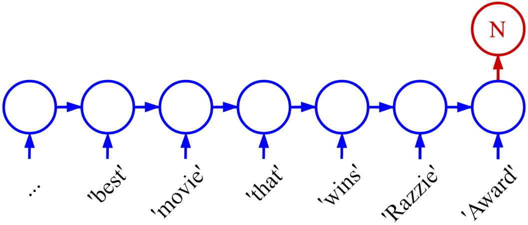

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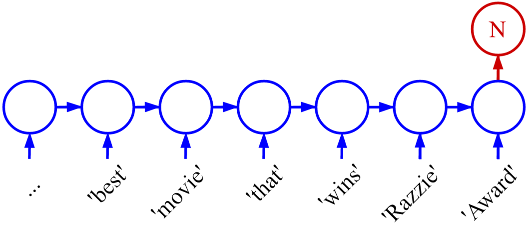
- In some applications, transitions are invariant in terms of other concepts



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- In natural language processing (NLP), we can parse the input sentence $\boldsymbol{X}^{(n)}$ into a tree
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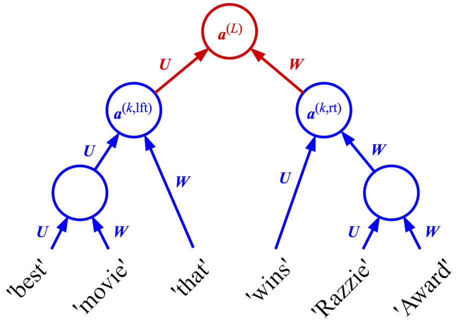

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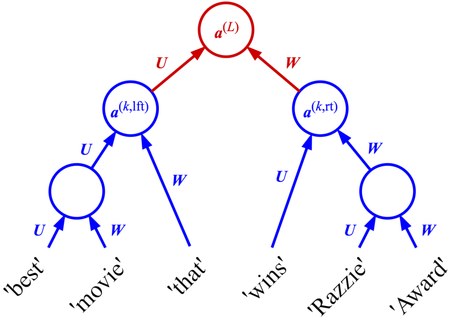
  - $\boldsymbol{U}$ and $\boldsymbol{W}$ are shared recursively in subtrees



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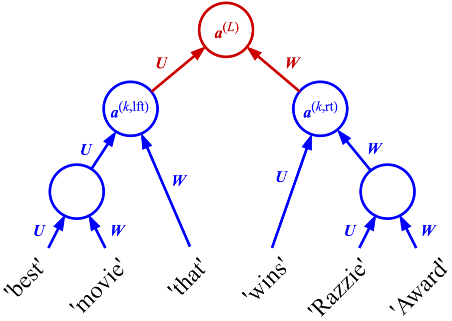
  - $\boldsymbol{U}$ and $\boldsymbol{W}$ are shared recursively in subtrees
- Given sentence length T, $\mathbf{a}^{(L)}$ and $\mathbf{a}^{(1,\cdot)}$ can be $O(\log T)$ away in the best case



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Cost Function of Vanilla RNNs

- Parameters to learn: $\Theta = \{\boldsymbol{W}^{(k)}, \boldsymbol{U}^{(k)}\}_k$ (bias terms omitted)
- Maximum likelihood:

$$
\begin{aligned}
&\arg\min_{\Theta} C(\Theta) \\
&\quad = \arg\min_{\Theta} -\log \mathrm{P}(\mathbf{X} \,|\, \Theta) \\
&\quad = \arg\min_{\Theta} -\textstyle\sum_{n,t} \log \mathrm{P}(\boldsymbol{y}^{(n,t)} \,|\, \boldsymbol{x}^{(n,t)}, \cdots, \boldsymbol{x}^{(n,1)}, \Theta) \\
&\quad = \arg\min_{\Theta} -\textstyle\sum_{n,t} C^{(n,t)}(\Theta)
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  - $\mathbf{y}^{(t)}$ depends only on $\mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(t)}$

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  - $\mathbf{y}^{(t)}$ depends only on $\mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(t)}$
- For example, in binary classification:
- Assuming $\mathrm{P}(\mathbf{y}^{(n,t)} = 1 \,|\, \boldsymbol{x}^{(n,t)}, \cdots, \boldsymbol{x}^{(n,1)}) \sim \mathrm{Bernoulli}(\rho^{(t)})$, we have

$$C^{(n,t)}(\Theta) = (\boldsymbol{a}^{(L,t)})^{y^{(n,t)}} (1 - \boldsymbol{a}^{(L,t)})^{(1-y^{(n,t)})}$$

  - $\boldsymbol{a}^{(L,t)} = \rho^{(t)}$ are based on $\boldsymbol{a}^{(\cdot,t)}$'s, which summarize $\boldsymbol{x}^{(n,t)}, \cdots, \boldsymbol{x}^{(n,1)}$

Outline

SGD-based Training

- RNN optimization problem can be solved using SGD:

$$\Theta^{(s+1)} \leftarrow \Theta^{(s)} - \eta \nabla_{\Theta} \sum_{n,t} C^{(n,t)}(\Theta^{(s)})$$

- Let $c^{(n,t)} = C^{(n,t)}(\Theta^{(s)})$, our goal is to evaluate $\frac{\partial c^{(n,t)}}{\partial U_{i,j}^{(k)}}$ and $\frac{\partial c^{(n,t)}}{\partial W_{i,j}^{(k)}}$
- Evaluation of $\frac{\partial c^{(n,t)}}{\partial W_{i,j}^{(k)}}$ is similar to that in DNNs and omitted
- We focus on:

$$\frac{\partial c^{(n,t)}}{\partial U_{i,j}^{(k)}} = \frac{\partial c^{(n,t)}}{\partial z_j^{(k,t)}} \cdot \frac{\partial z_j^{(k,t)}}{\partial U_{i,j}^{(k)}} = \delta_j^{(k,t)} \frac{\partial z_j^{(k,t)}}{\partial U_{i,j}^{(k)}}$$

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- We can get all second terms starting from the most shallow layer and ***earliest time***

Backward Pass through Time I

- The first term (error signal): $\delta_j^{(k,t)} = \frac{\partial c^{(n,t)}}{\partial z_j^{(k,t)}}$
- We have

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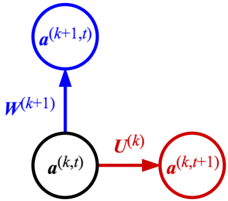

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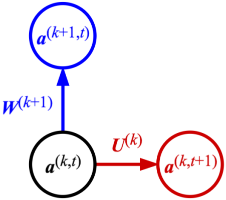

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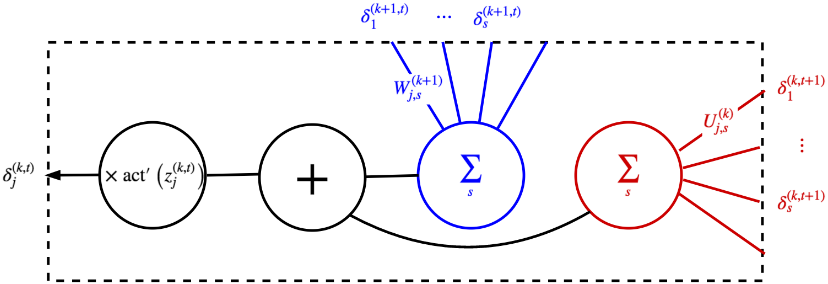
- We can evaluate all $\delta_j^{(k,t)}$'s starting from the deepest layer and ***latest time***



Backprop through Time (BPTT)

- So far, we have discussed how to compute the gradients of $U_{i,j}^{(k)}$'s (and $W_{i,j}^{(k)}$'s) for a single loss $c^{(n,t)} = C^{(n,t)}(\Theta^{(s)})$ at specific time

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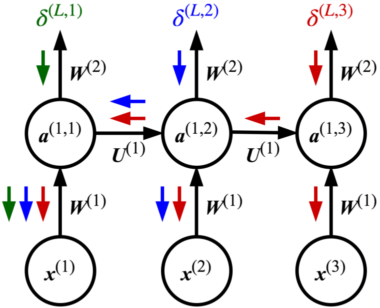
- ***BPTT***: single forward pass, ***multiple*** backward passes



Outline

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- Backward pass:

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- If $\boldsymbol{a}^{(k,i)}$ and $\boldsymbol{a}^{(k,j)}$ are far away in time, their long-term dependency causes optimization problems

Exploding/Vanishing Gradient Problem

- Ignoring activation function and depth:

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- Exploding or vanishing gradients!

Cost Surface

- The cost surface of C is either very flat or steep
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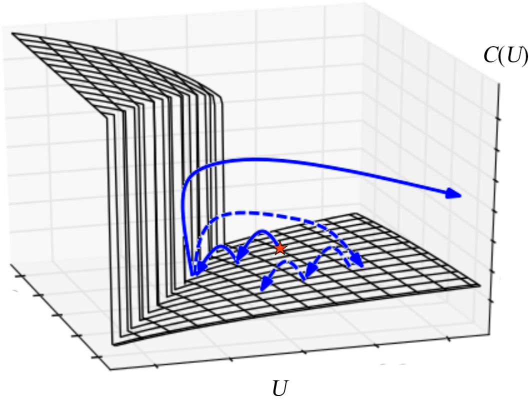

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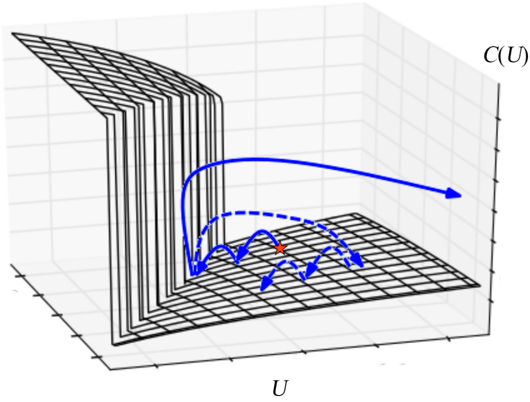
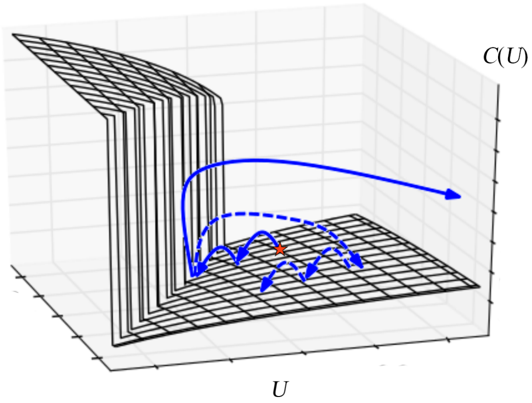
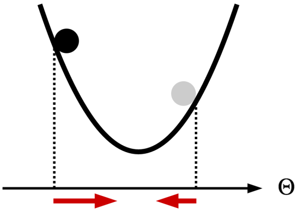


Nesterov Momentum

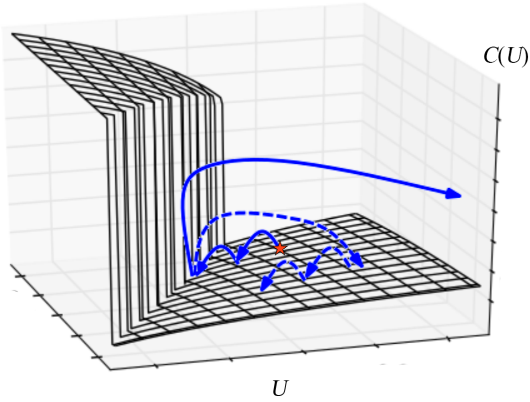
- Use Nesterov momentum to “brake” before hitting the wall





Gradient Clipping

- A simple way is to avoid the exploding gradient problem is to ***clip*** a gradient if it exceeds a predefined threshold
- Very effective in practice



RMS Prop

- Adaptive learning rate based on statistics of recent gradients:

$$\boldsymbol{r}^{(t+1)} \leftarrow \lambda \boldsymbol{r}^{(t)} + (1-\lambda)\boldsymbol{g}^{(t)} \odot \boldsymbol{g}^{(t)}$$

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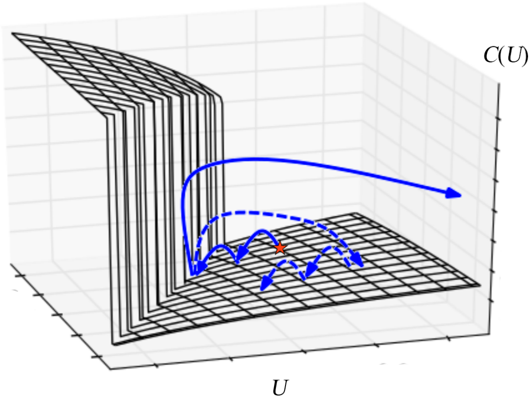

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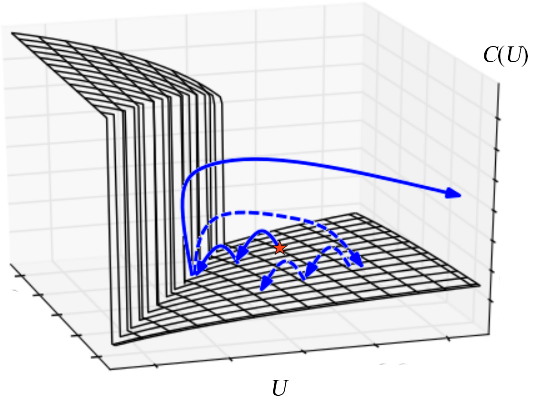
- Reduce λ



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- Bengio et al. [1] propose uRNN that learns unitary $U^{(k)}$'s explicitly

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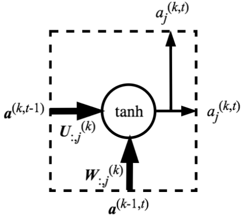

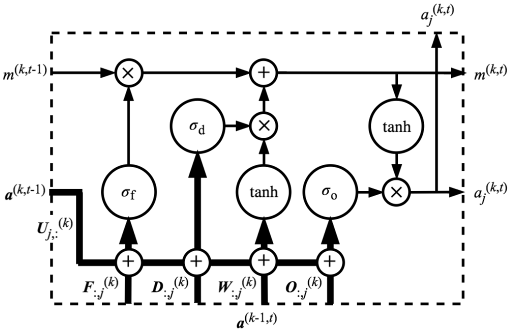

Long Short-Term Memory (LSTM)

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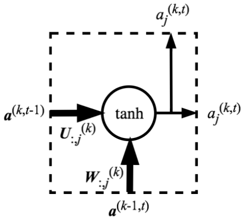

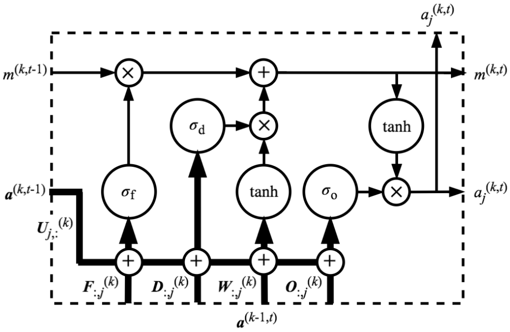

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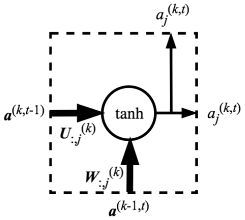

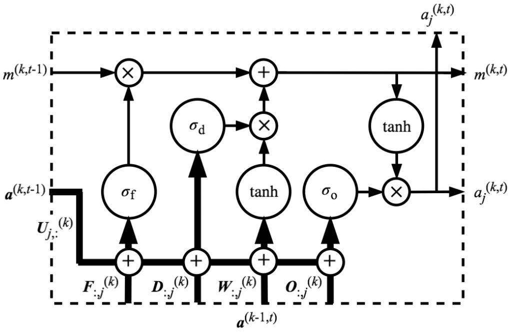

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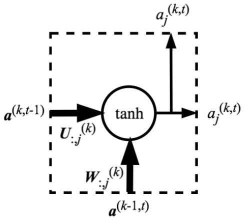

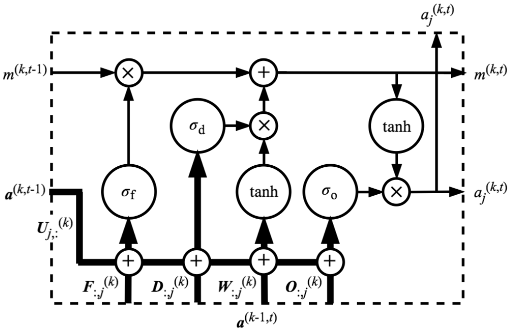

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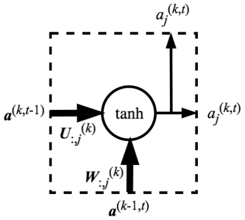
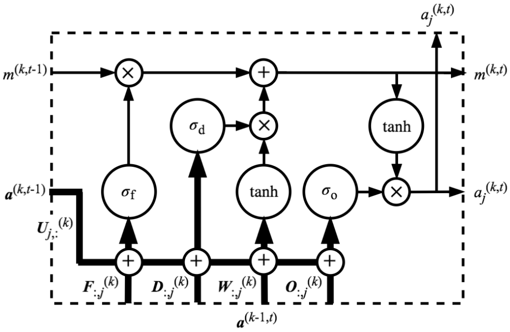
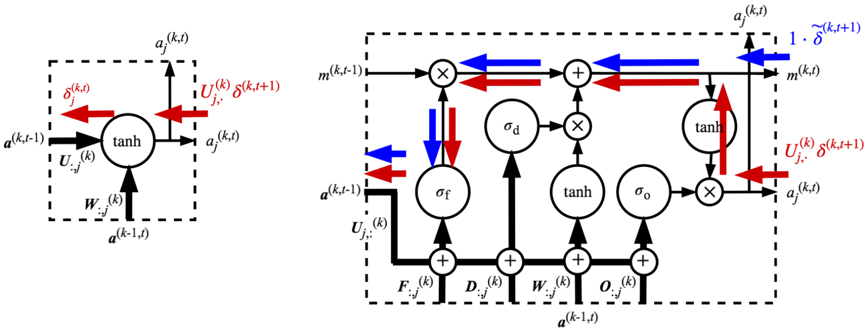




Error Signals

- Error signals now have a second path
  - If the forget gate is open, error signals won't decay (***blue arrows***)



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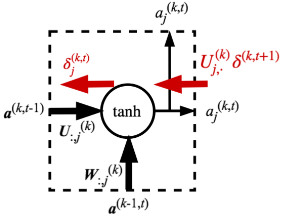

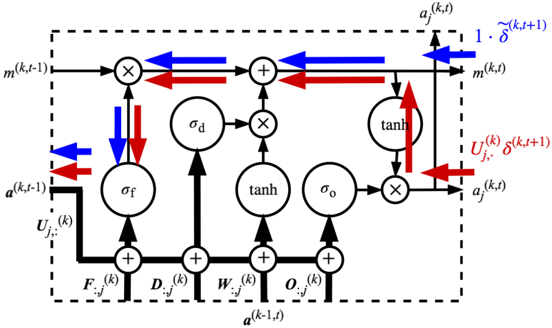

Error Signals

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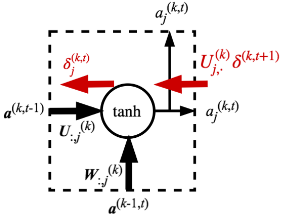
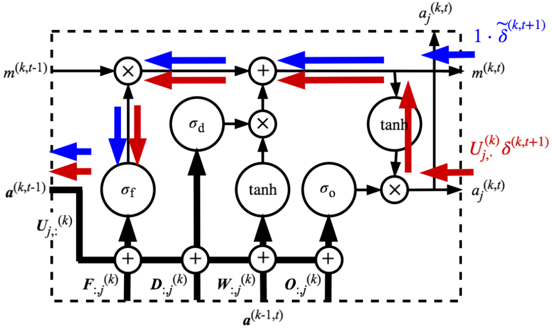
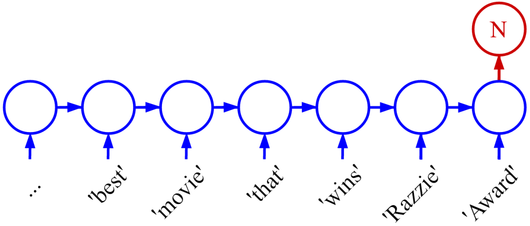
  - Avoids the vanishing gradients (but not exploding ones)
- When NN decides to close the forget gate, the vanishing gradient problem is irrelevant
- In practice, LSTM + gradient clipping works well together





Dynamic Representations for Sentiment Analysis



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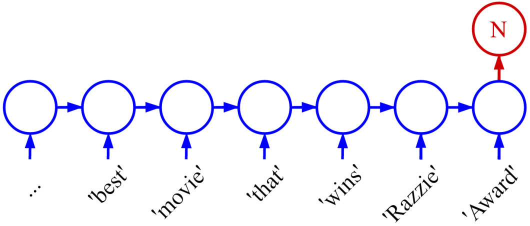

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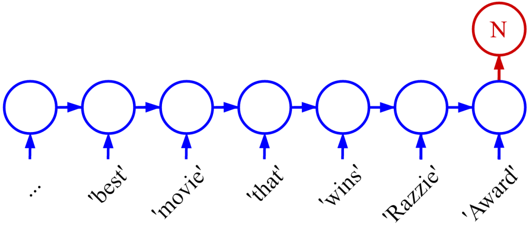

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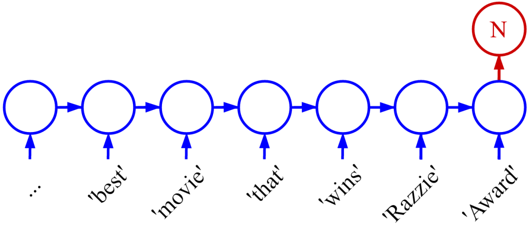

- LSTM units learn dynamic representations
- Closing forget gate for "Razzie"
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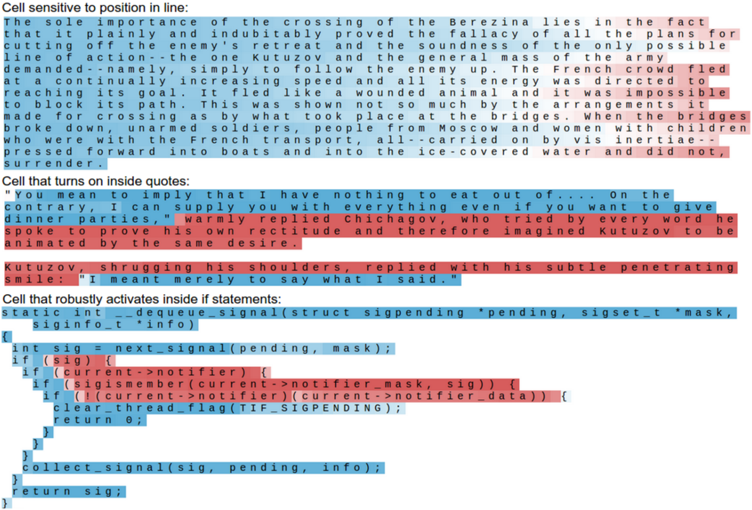
- Closing input gate for "movie"
  - Not to learn, to keep the same summarization
- Closing output gate for "that"
  - To let the next neuron decide the activation/gate values by its own

Dynamic Representations for Language Models

Dynamic Representations for Language Models

- Neuron activations for language modeling [6] Interactive Tool



Learning Process of LSTMs

- Output at epoch 100:

  "*... tyntd-iafhatawiaoihrdemot lytdws e ,tfti, astai f ogoh...*"

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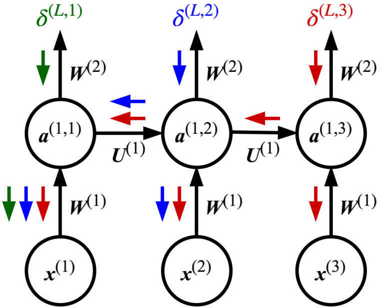
- At 2000 (topics and longer-term dependencies):

  "*... "Why do what that day," replied Natasha, ...*"

Outline

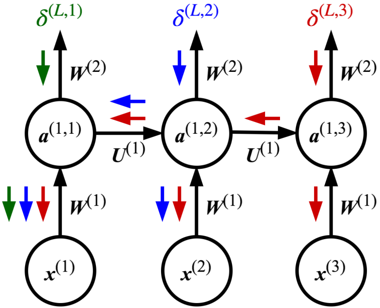
Parallelism

- A forward/backward pass through time in BPTT cannot be parallelized



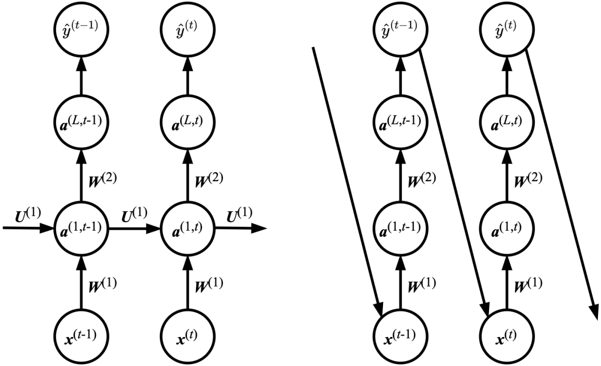
Parallelism

- A forward/backward pass through time in BPTT cannot be parallelized
- The ***hidden-to-hidden*** recurrent connections in a vanilla RNN create dependency between
  - $\boldsymbol{a}^{(k,t)}$'s in forward pass
  - $\delta^{(k,t)}$'s in backward pass



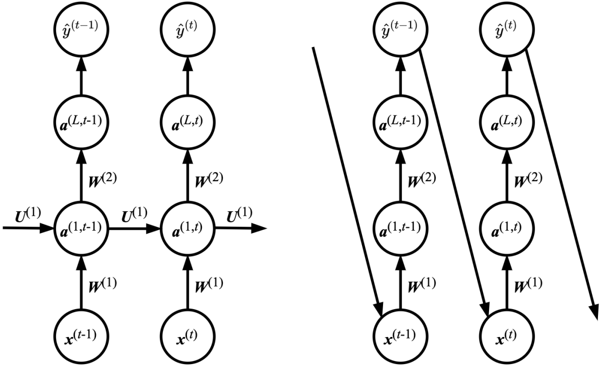
Output Recurrence and Teacher Forcing

- *Teacher forcing*: replace hidden-to-hidden recurrence with output-to-hidden or output-to-input recurrence



Output Recurrence and Teacher Forcing

- ***Teacher forcing***: replace hidden-to-hidden recurrence with output-to-hidden or output-to-input recurrence
- At training time, use ***correct labels*** $\boldsymbol{y}^{(\cdot)}$**'s** to train the model
  - So, the forward/backward pass through time can be parallelized
- At test time, switch back to using model output $\hat{\boldsymbol{y}}^{(\cdot)}$'s



Exposure Bias

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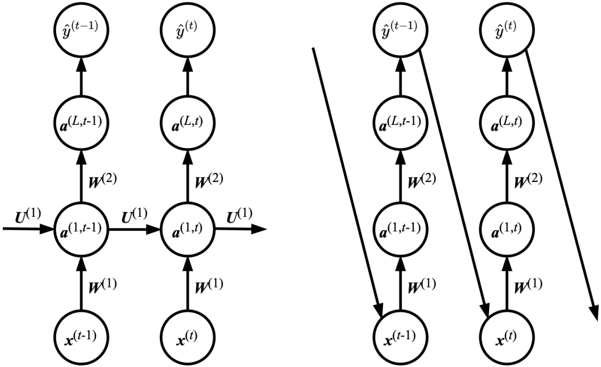
- Solution? Scheduled sampling
- At training time,
  1. Use $\boldsymbol{y}^{(\cdot)}$'s initially
  2. Gradually mix in $\hat{\boldsymbol{y}}^{(\cdot)}$'s later

Reduced Expressiveness

- The vanilla RNNs are universal in the sense that they can simulate Turing machines [18]



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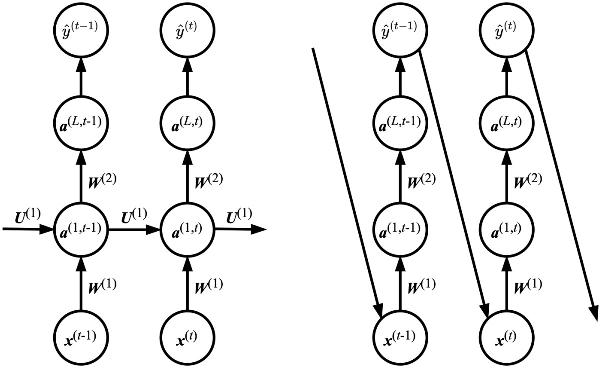

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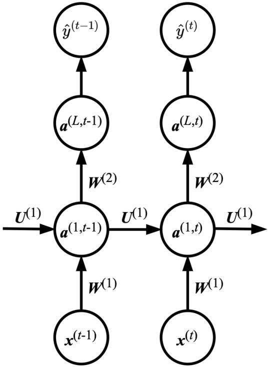
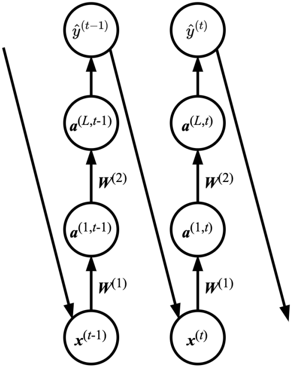
- The output $\boldsymbol{a}^{(L,\cdot)}$'s are explicitly trained to match training targets
    - Cannot capture all required information in the past to predict the future





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- In some RNNs, a hidden representation $\mathbf{a}^{(\cdot,t)}$ needs to support:
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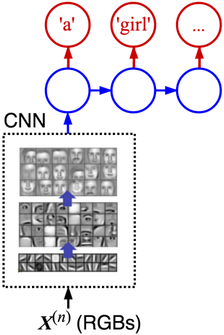
"A little girl sitting on a bed with a teddy bear."



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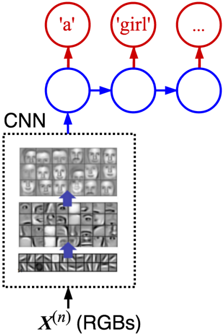

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- Can we ease the job of $\mathbf{a}^{(\cdot,t)}$?

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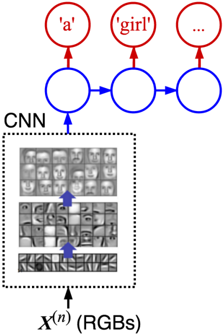

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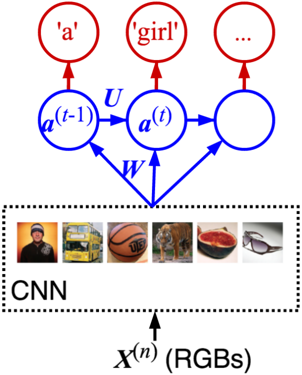

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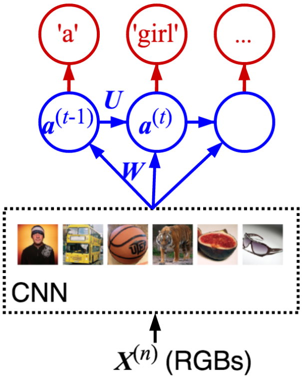

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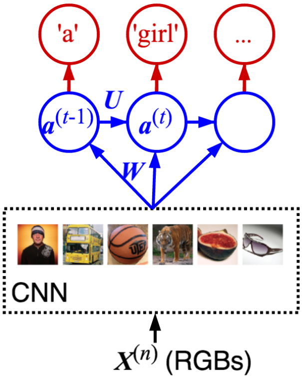

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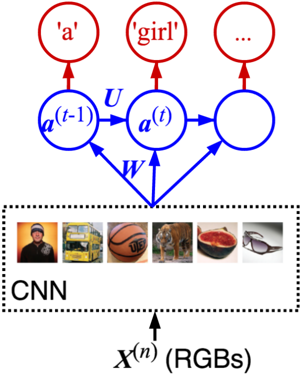
- Why not model the attention explicitly?
  - So we can see where $\mathbf{a}^{(\cdot,t)}$ is "looking at"



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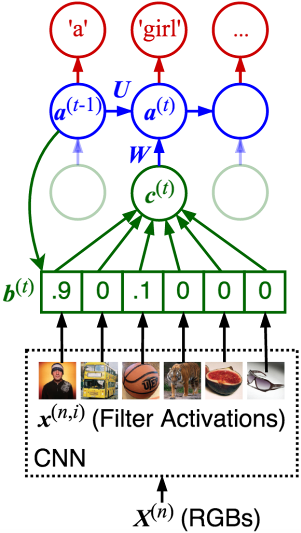
  - E.g., with CNN, $\mathbf{x}^{(i)}$ could be the activation values of a filter



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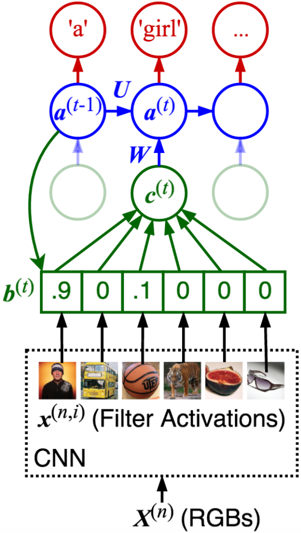

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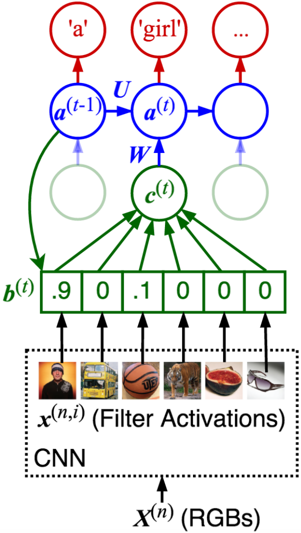

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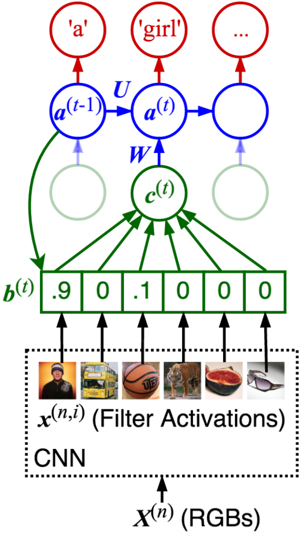

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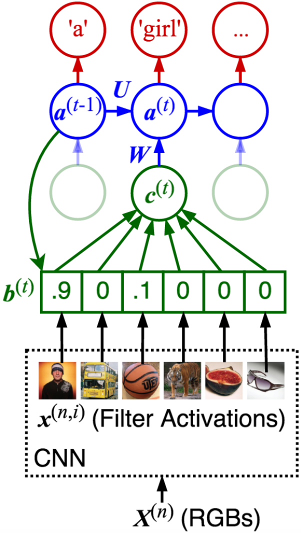
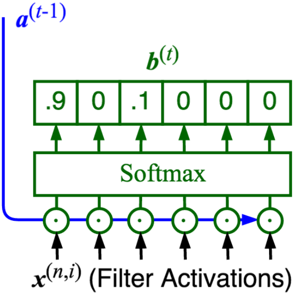
- Reduces size of $\boldsymbol{W}$ from $O(|\mathbf{X}| \cdot |\mathbf{a}^{(\cdot,t)}|)$ to $O(|\mathbf{x}^{(i)}| \cdot |\mathbf{a}^{(\cdot,t)}|)$
- How to obtain $\mathbf{b}^{(t)}$?



Computing Attention Vector

1. Use $\mathbf{a}^{(L-1,t-1)}$ as a "query" to get a match score for each input part by using, e.g., a simple NN [2, 20]:

$$z_i = \mathrm{act}(\boldsymbol{p}^\top \mathbf{a}^{(L-1,t-1)} + \boldsymbol{q}^\top \mathbf{x}^{(i)} + r)$$



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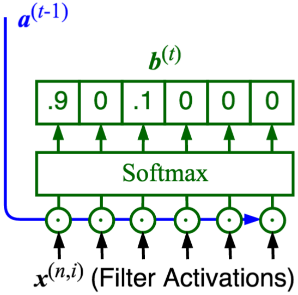
$$\mathrm{b}_i = \mathrm{softmax}(\mathrm{z})_i = \frac{\exp(\mathrm{z}_i)}{\sum_j \exp(\mathrm{z}_j)}$$



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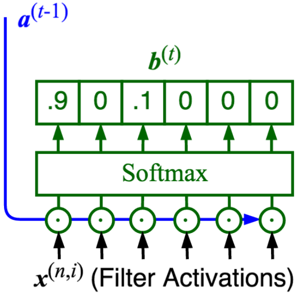

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   - Jointly trained with the main RNN
   - $\boldsymbol{p}$, $\boldsymbol{q}$, and r are shared by different i's and t's (weight tying)

2. Normalize and concentrate on few larger scores by:

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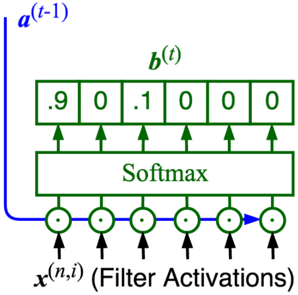
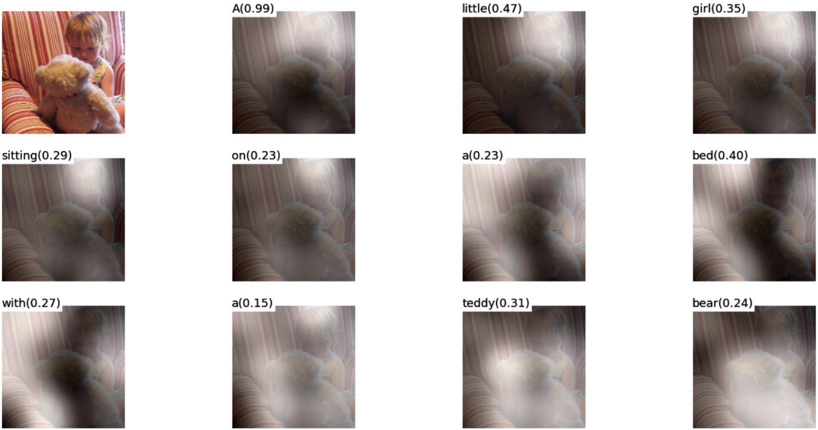
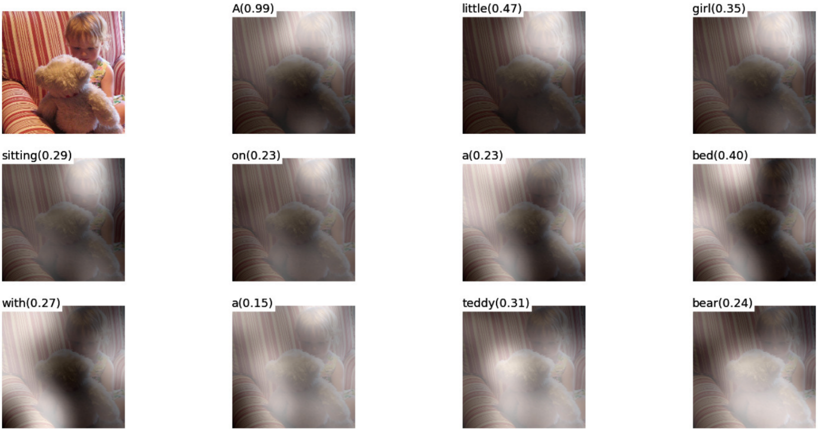


Visualizing Attention



- How to draw a mask?

Visualizing Attention

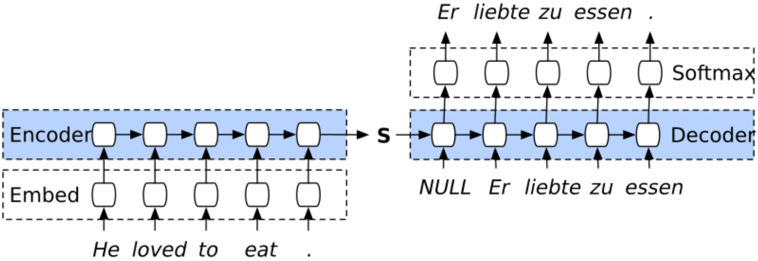


- How to draw a mask? Threat $\mathbf{c}^{(t)} = \sum_i \mathrm{b}_i^{(t)} \mathbf{x}^{(i)}$ as image and enlarge it

Outline

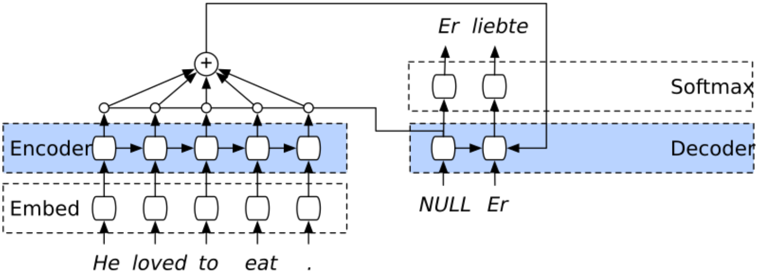
GoogleNMTv1: Encoder-Decoder

- LSTM-based RNN
- Hidden state S encodes an entire input sequence
- Then supplied to the decoder



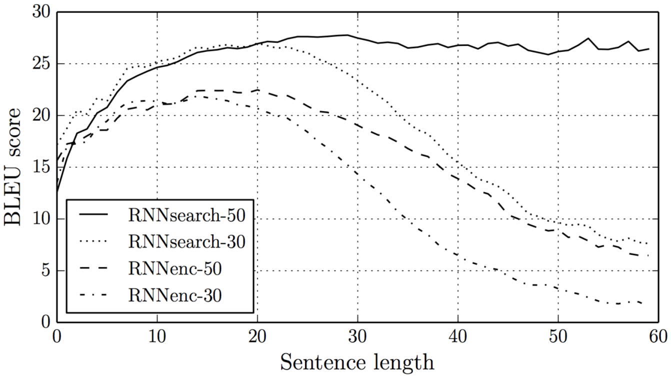
GoogleNMTv2: Attention-based Encoder-Decoder

- No feed-forward connection between the encoder and decoder
- Instead, uses previous output as query to get attention and next output
  - Allows for retrieving different parts of input sentence depending on decoding context



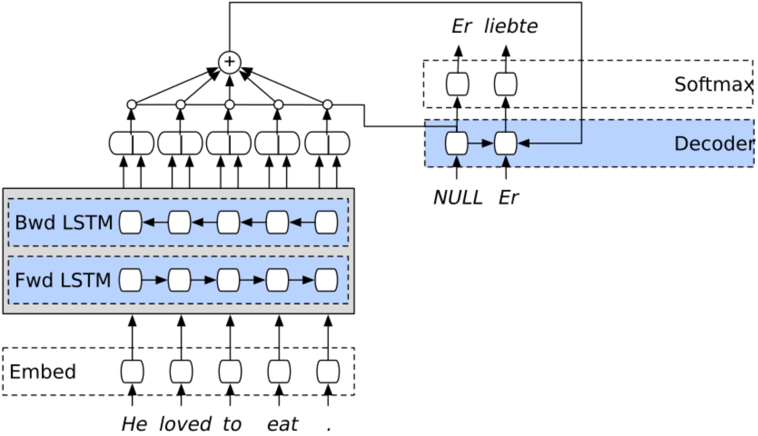
Long Sequences

- Attention-based model generates long sequences better



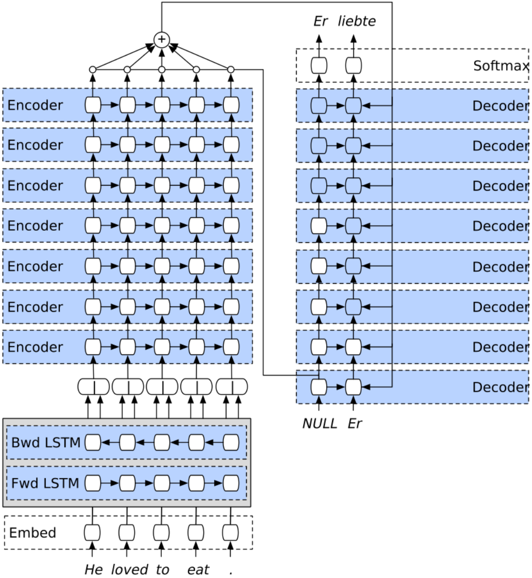
GoogleNMTv3: Bidirectional Encoder Layer

- Takes into account future words when summarizing input sequence
- Better determines the meaning/context



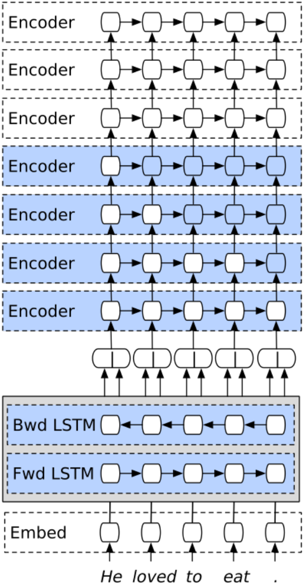
GoogleNMTv4: Going Deep

- Encoder:
  - ***1*** bi-directional layer
  - 7 uni-directional layers
- Decoder:
  - 8 uni-directional layers
  - ***Lowest*** decoder layer for querying attention

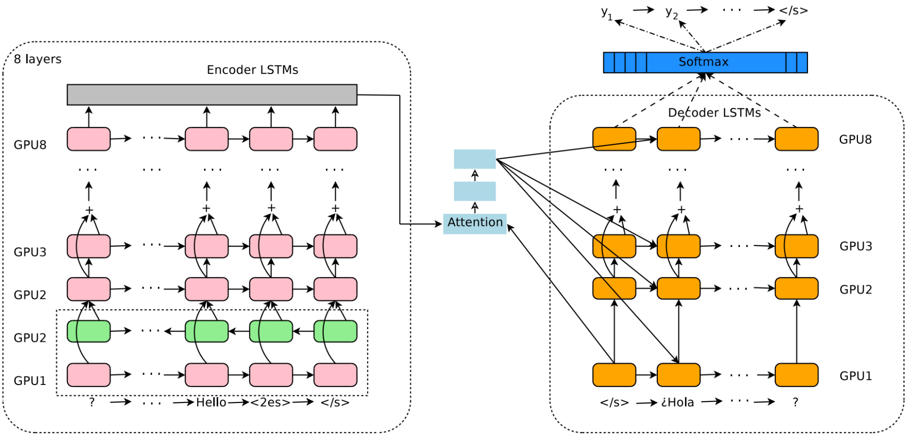


GoogleNMTv5: Parallelization

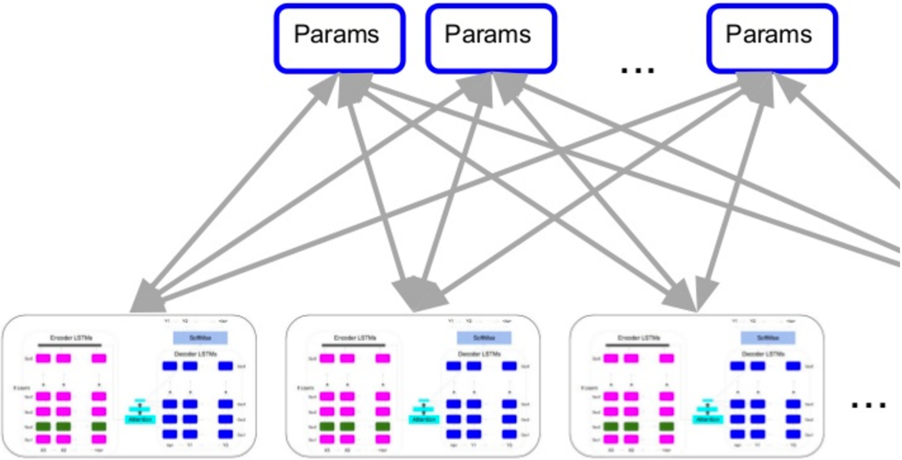
- Each layer trained by a GPU
- Forward pass:
  - Encoder: 7 uni-directional encoder layers trained in a pipeline
  - Decoder: pipeline starts as soon as encoder layers are ready
- Backward pass:
  - Teacher forcing



Model Parallelism (1 Machine, 8 GPUs)

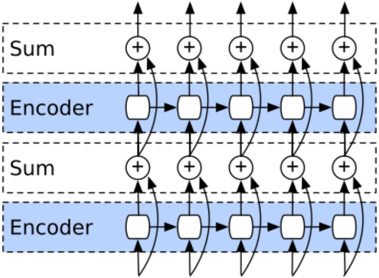


Data Parallelism (Multiple Param Servers)



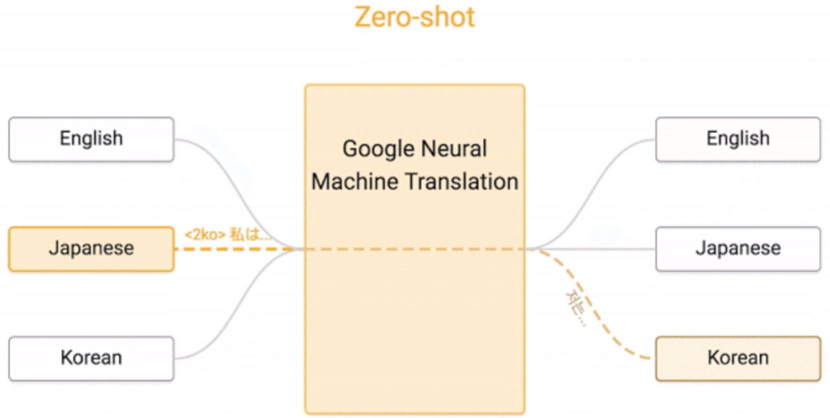
GoogleNMTv6: Residuals

- Upper layer learns the ***delta*** function to the lower one
- Easier to train a deep NN



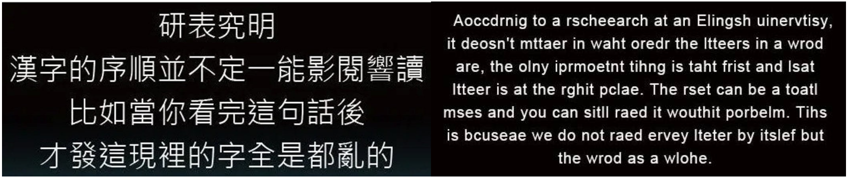
GoogleNMTv7: Multilingual & Zero-Shot Translation

- Training: input x augmented with task identifier (language pair)
  - E.g., (eng, jp), (kr, en), (jp, en)
- Inference: ***unknown*** language-pair identifier
  - E.g., (kr, jp)

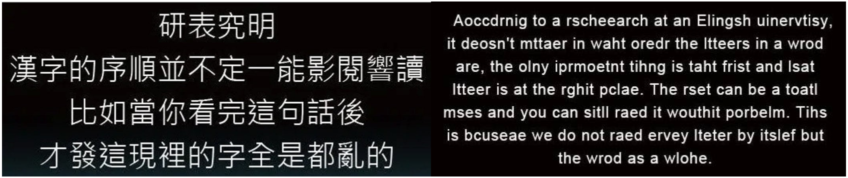


Outline

How Humans Process Text?



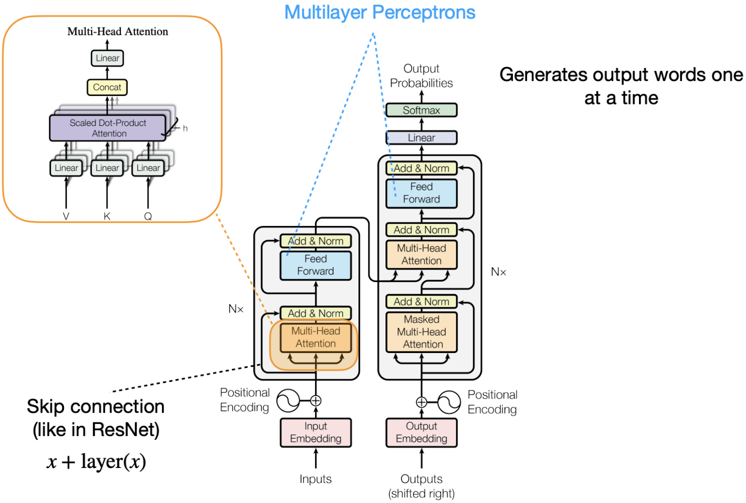
How Humans Process Text?



- ***Not entirely sequential*** as in RNNs
- ***Not entirely based on local patterns*** as in CNNs
- Any other “better” architectures?

Outline

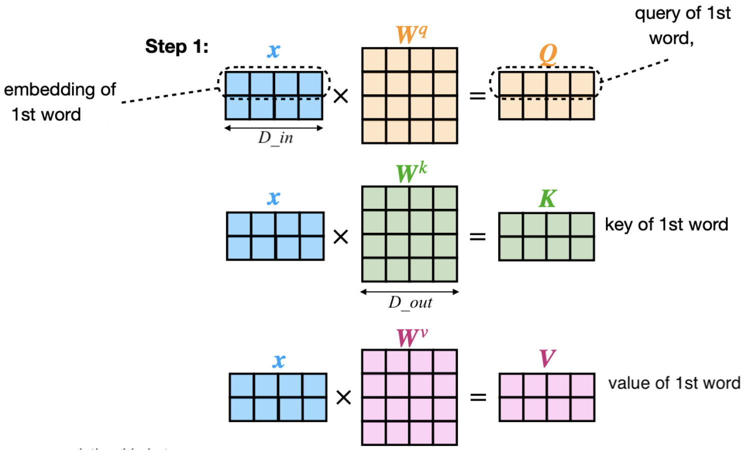
Attention is All You Need [19]



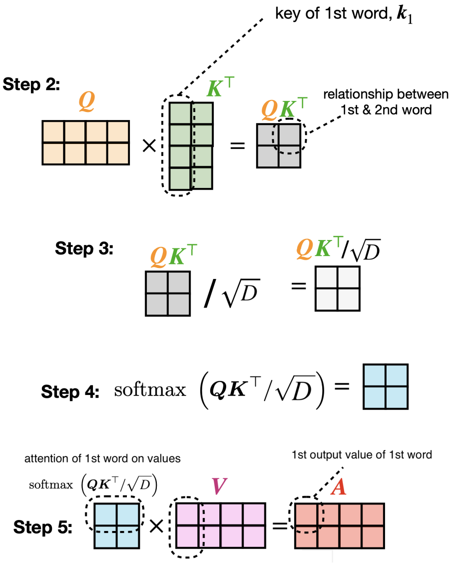
- RNN layers are replaced with ***self-*** and ***cross-attention*** blocks

Self-Attention

- Weights to learn at each layer: $\boldsymbol{W}_{\text{query}}, \boldsymbol{W}_{\text{key}}, \boldsymbol{W}_{\text{value}} \in \mathbb{R}^{D \times D}$
- Batch, non-autoregressive processing of sequences



Self-Attention



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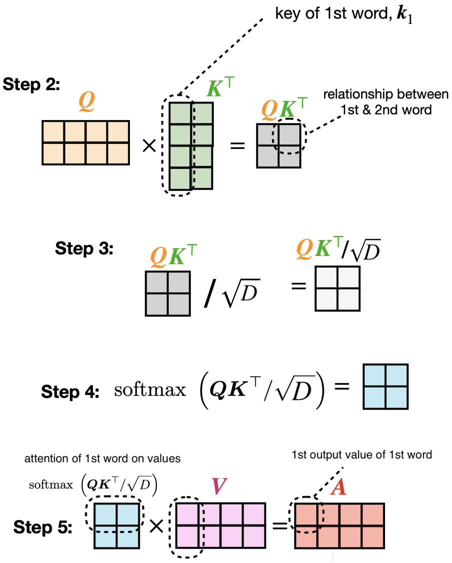
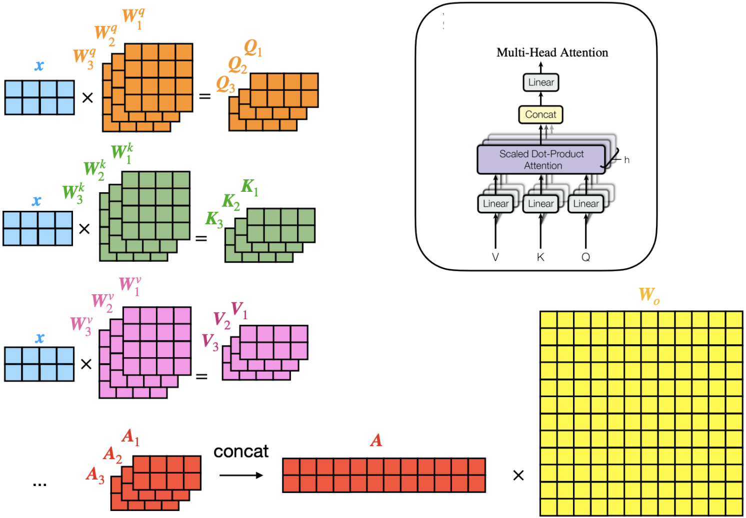


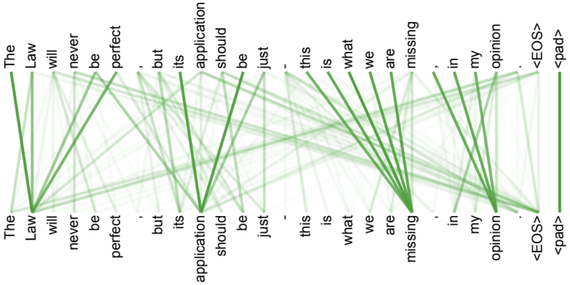
- Input: $\boldsymbol{A}^{(l-1)} \in \mathbb{R}^{T \times D}$
- Output: $\boldsymbol{A}^{(l)} \in \mathbb{R}^{T \times D}$
  - ***Context*** augmented
  - E.g, "**train** a model" vs. "get on a **train**"

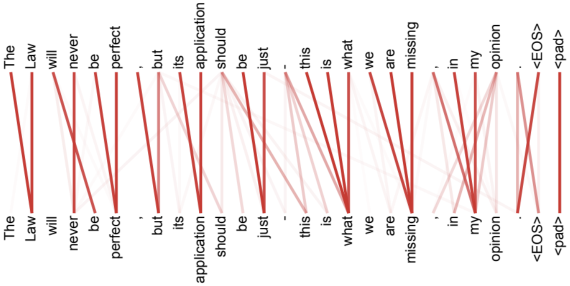
Multi-head Self-Attention



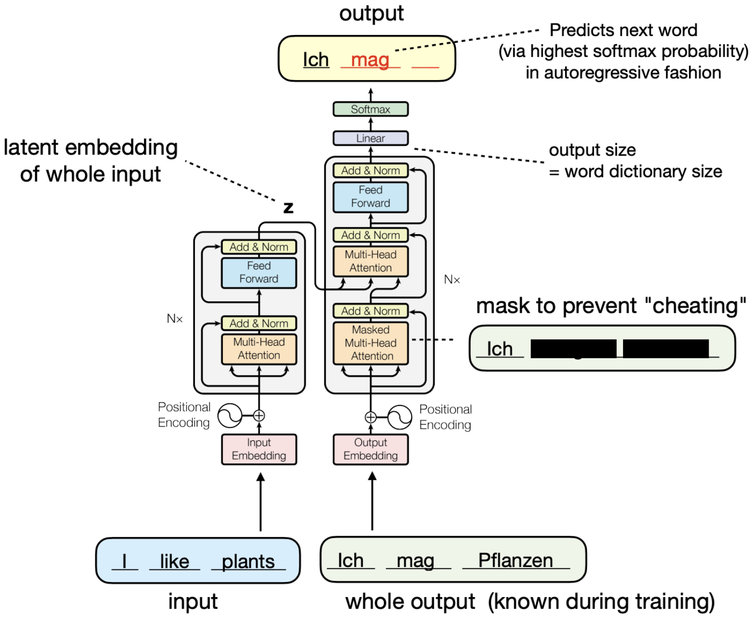
- Given H heads, we have $\boldsymbol{W}_{\text{query}}^{(h)}, \boldsymbol{W}_{\text{key}}^{(h)}, \boldsymbol{W}_{\text{value}}^{(h)} \in \mathbb{R}^{D \times \frac{D}{H}}$ and $\boldsymbol{A} \in \mathbb{R}^{T \times D}$

Same Sequence, Different Context Augmentations



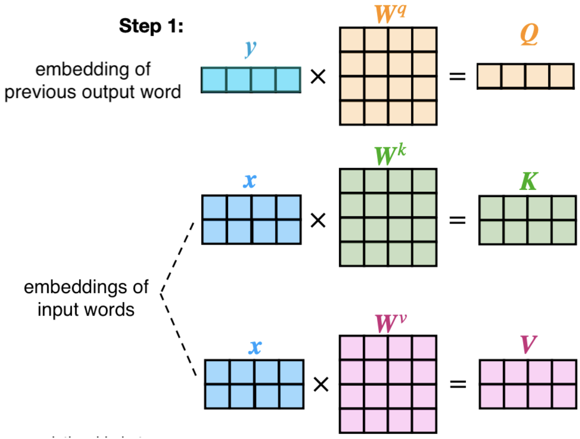


Masked Self-Attention for Decoding Output

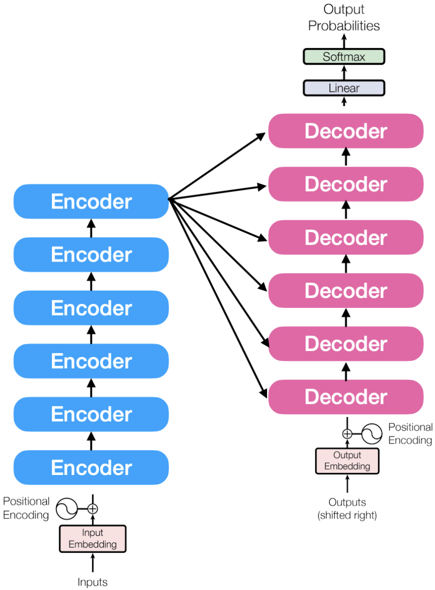


Cross-Attention

- Users previous output as query (autoregressive)
- Input as context

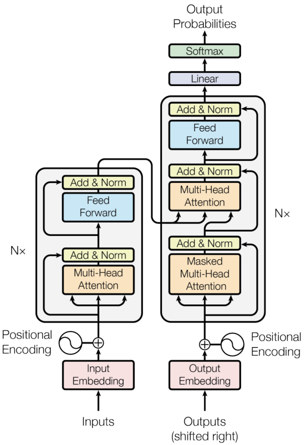


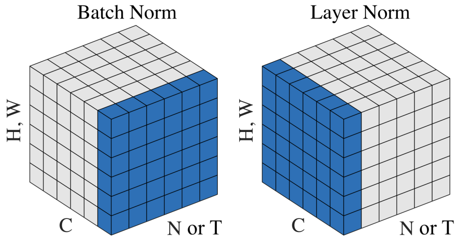
Going Deep



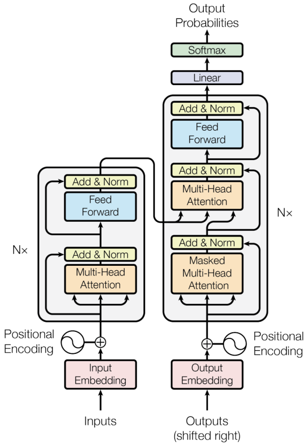
Output
Probabilities
Softmax
Linear
Decoder
Decoder
Decoder
Decoder
Decoder
Decoder
Encoder
Encoder
Encoder
Encoder
Encoder
Encoder
Positional
Encoding
Output
Embedding
Outputs
(shifted right)
Positional
Encoding
Input
Embedding
Inputs

Layer Normalization





Positional Encoding



- "John loves Mary" $\neq$ "Mary loves John"
- So far, augmented context does ***not*** change when the input words change order

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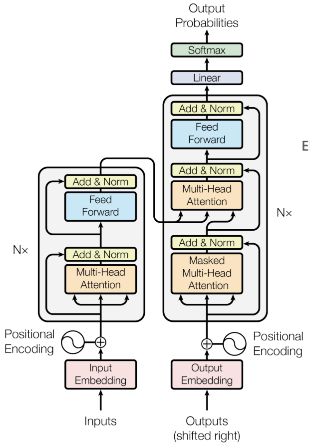

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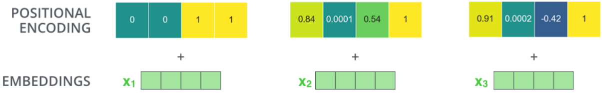
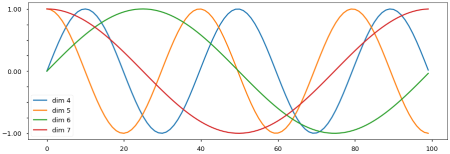


- Each word position corresponds to multiple $\sin(\cdot)$ / $\cos(\cdot)$ values of different periods



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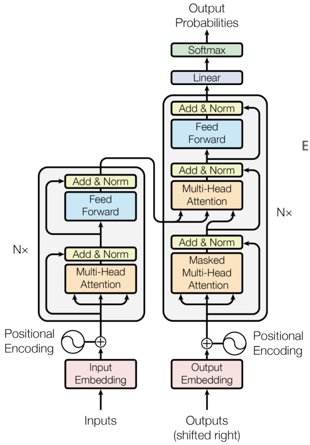

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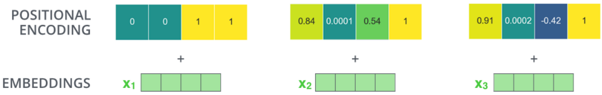
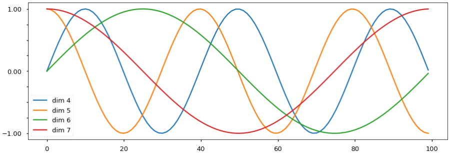


- Each word position corresponds to multiple $\sin(\cdot)$ / $\cos(\cdot)$ values of different periods
  - Why not linear values (e.g, 1, 2, 3, ...)?



RNNs vs. CNNs vs. Transformers

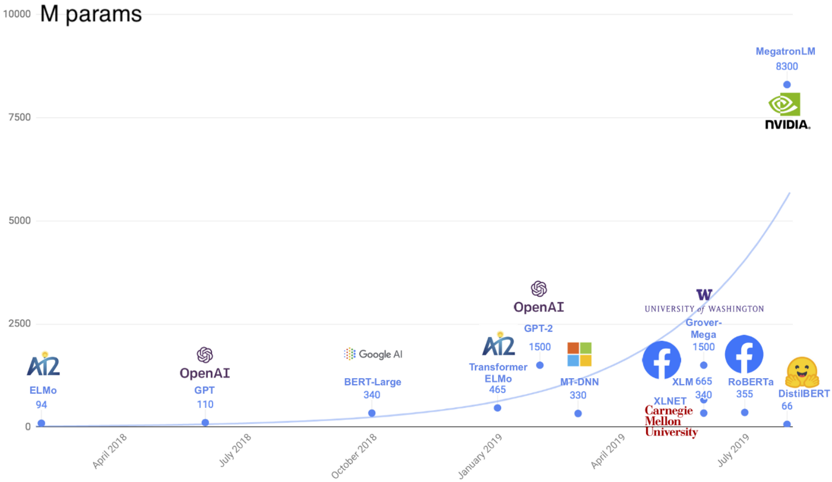
- On processing a sequence of length T at each layer with
  - D-dimensional point input and output
  - F = the CNN filter/kernel size
  - #CNN filters = D
  - #attention heads = H
  - Query, key, and value size = $\frac{D}{H}$

	#Weights	Computation	Auto-reg.	Point Dist.
CNN	$O(FD^2)$	$O(TFD^2)$	No	$O(\frac{T}{F})$
RNN	$O(D^2)$	$O(TD^2)$	Yes	$O(T)$
Self-attention	$O(D^2)$	$O(TD^2 + T^2D)$	No	$O(1)$

Outline

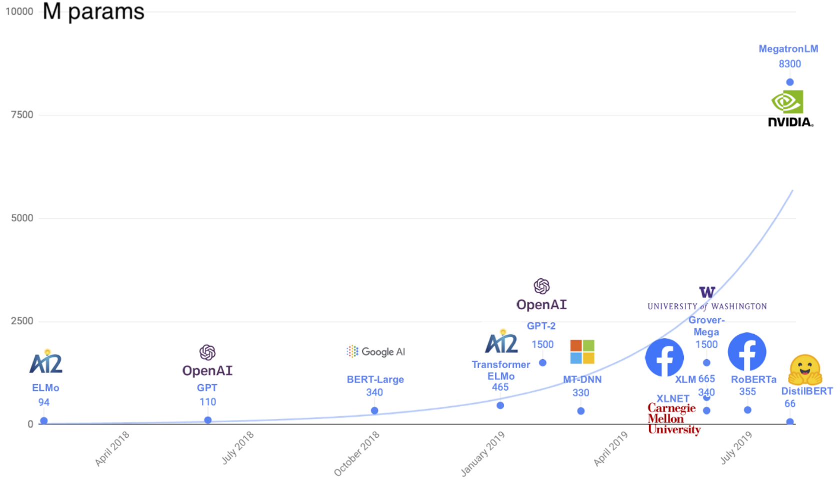
Recent Growth of Language Transformers

- All based on ***self-supervised pre-training***



Recent Growth of Language Transformers

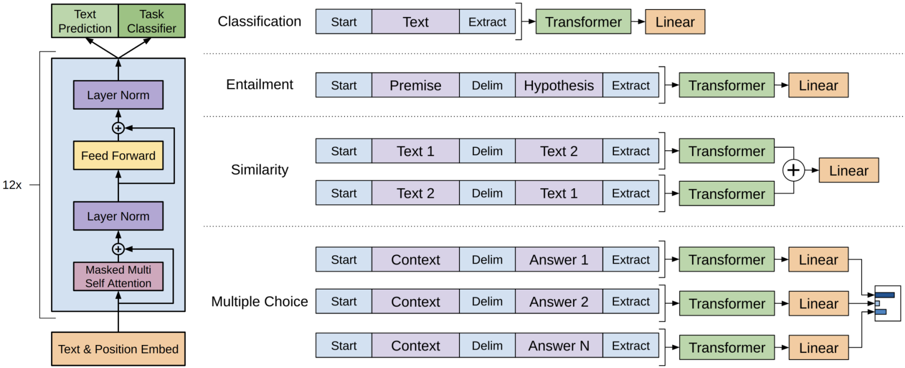
- All based on ***self-supervised pre-training***
- Costs [17]: \$2.5k–\$50k (110M params), \$10k–\$200k (340M), \$80k–\$1.6m (1.5B)



GPTv1 [12]

- 2-step training process:
  1. Generative pre-training on unlabeled data
     - to predict next word in a sentence (language model)
  2. Discriminative fine-tuning on labeled data
- Architecture based on the decoder of original transformer ("Attention is All You Need")
  - Autoregressive

Architecture & Fine-tuning Tasks



- Fine-tuning tasks: whether to language model (LM) loss or not?

Ablation Study

Table 5: Analysis of various model ablations on different tasks. Avg. score is a unweighted average of all the results. (*mc*= Mathews correlation, *acc*=Accuracy, *pc*=Pearson correlation)

Method	Avg. Score	CoLA (mc)	SST2 (acc)	MRPC (F1)	STSB (pc)	QQP (F1)	MNLI (acc)	QNLI (acc)	RTE (acc)
Transformer w/ aux LM (full)	74.7	45.4	91.3	82.3	82.0	**70.3**	**81.8**	**88.1**	**56.0**
Transformer w/o pre-training	59.9	18.9	84.0	79.4	30.9	65.5	75.7	71.2	53.8
Transformer w/o aux LM	**75.0**	**47.9**	**92.0**	**84.9**	**83.2**	69.8	81.1	86.9	54.4
LSTM w/ aux LM	69.1	30.3	90.5	83.2	71.8	68.1	73.7	81.1	54.6

BERT [4]

- Architecture based on the encoder of original transformer ("Attention is All You Need")
  - Non-autoregressive
- Pre-training tasks:
  - Masked language model ("Cloze" task)
    - "A quick brown [MASK] jumps over the lazy dog" → "fox" 11%, "ant" 5%, ...
  - Next sentence prediction
    - "[MASK] go to store [SEP] to buy a [MASK] of milk"→ True 93%, False 7%

Pre-training & Fine-tuning Tasks

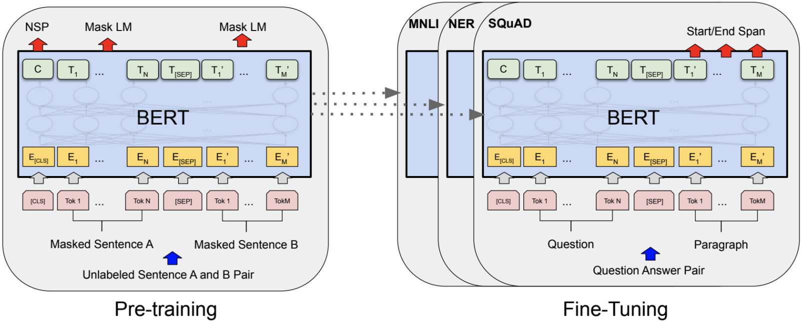


Figure 1: Overall pre-training and fine-tuning procedures for BERT. Apart from output layers, the same architectures are used in both pre-training and fine-tuning. The same pre-trained model parameters are used to initialize models for different down-stream tasks. During fine-tuning, all parameters are fine-tuned. `[CLS]` is a special symbol added in front of every input example, and `[SEP]` is a special separator token (e.g. separating questions/answers).

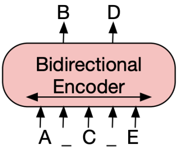
BART [11]

- BERT: bidirectional, non-autoregressive (autoencoder like)
  - Good for embedding tasks
- GPT: unidirectional, autoregressive
  - Good for generative tasks

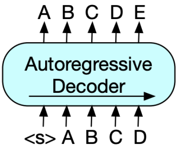
BART [11]

- BERT: bidirectional, non-autoregressive (autoencoder like)
  - Good for embedding tasks
- GPT: unidirectional, autoregressive
  - Good for generative tasks
- BART: why not combine both?

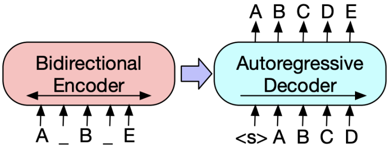
Architecture



(a) BERT: Random tokens are replaced with masks, and the document is encoded bidirectionally. Missing tokens are predicted independently, so BERT cannot easily be used for generation.

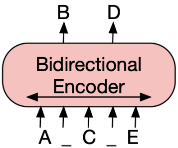


(b) GPT: Tokens are predicted auto-regressively, meaning GPT can be used for generation. However words can only condition on leftward context, so it cannot learn bidirectional interactions.



(c) BART: Inputs to the encoder need not be aligned with decoder outputs, allowing arbitary noise transformations. Here, a document has been corrupted by replacing spans of text with mask symbols. The corrupted document (left) is encoded with a bidirectional model, and then the likelihood of the original document (right) is calculated with an autoregressive decoder. For fine-tuning, an uncorrupted document is input to both the encoder and decoder, and we use representations from the final hidden state of the decoder.

Architecture



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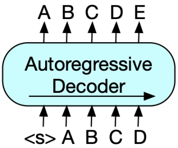
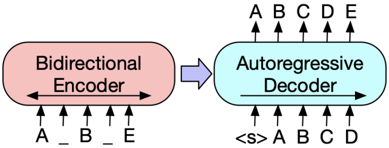


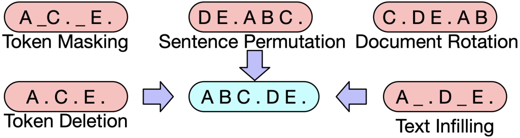
(b) GPT: Tokens are predicted auto-regressively, meaning GPT can be used for generation. However words can only condition on leftward context, so it cannot learn bidirectional interactions.



(c) BART: Inputs to the encoder need not be aligned with decoder outputs, allowing arbitary noise transformations. Here, a document has been corrupted by replacing spans of text with mask symbols. The corrupted document (left) is encoded with a bidirectional model, and then the likelihood of the original document (right) is calculated with an autoregressive decoder. For fine-tuning, an uncorrupted document is input to both the encoder and decoder, and we use representations from the final hidden state of the decoder.

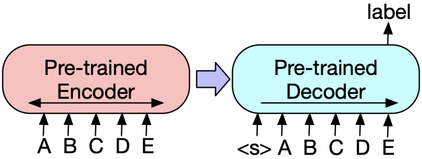
- Isn't it "Attention is All You Need?"

Denoising S2S Pre-training

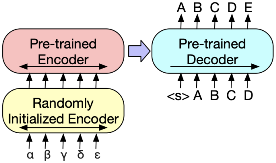


- Similar to denoising autoencoder

Fine-tuning Tasks



(a) To use BART for classification problems, the same input is fed into the encoder and decoder, and the representation from the final output is used.



(b) For machine translation, we learn a small additional encoder that replaces the word embeddings in BART. The new encoder can use a disjoint vocabulary.

GPTv2 [13] & GPTv3 [3]

- Still autoregressive
- But focus on ***transfer*** and ***few-shot learning*** scenarios
    - No need for fine-tuning ▸ Demo 1 and ▸ Demo 2
- How?

GPTv2 [13] & GPTv3 [3]

- Still autoregressive
- But focus on ***transfer*** and ***few-shot learning*** scenarios
  - No need for fine-tuning ▸ Demo 1 and ▸ Demo 2
- How? ***Multitask Language Model***
- Pre-training: to maximize $\Pr(\text{output}|\text{input}, \text{task})$ in multiple LM tasks
  - Translation: $\Pr(\text{french text}|\text{en text}, \text{translation})$
  - Question answering: $\Pr(\text{answer}|\text{question}, \text{qa})$
  - Reading comprehension: $\Pr(\text{answer}|\text{document}, \text{question}, \text{reading})$

Implicit Task Learning

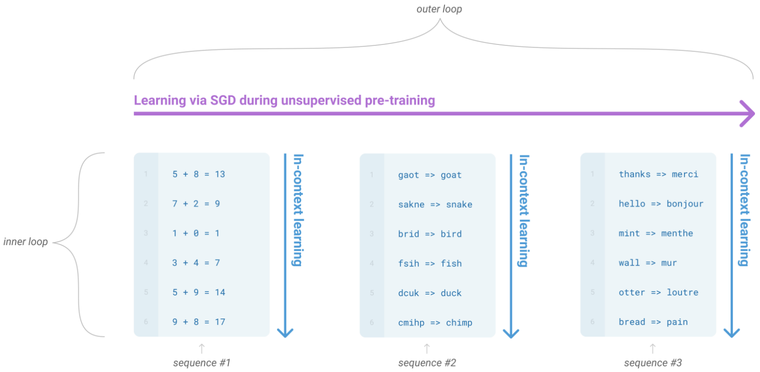
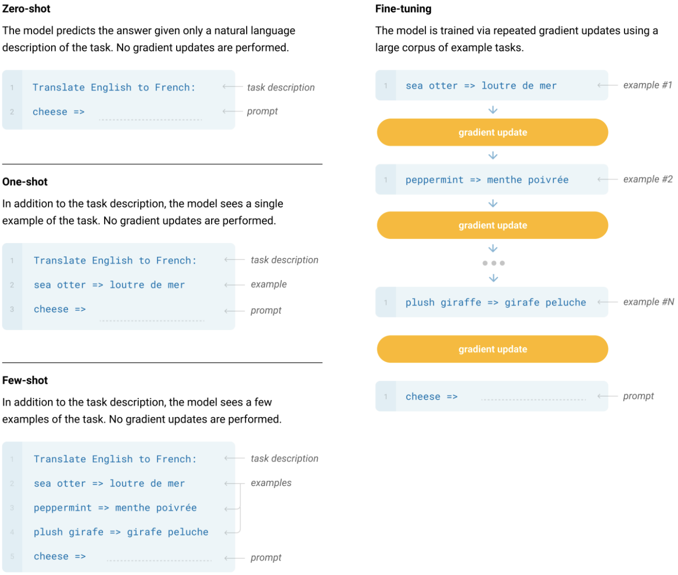


Figure 1.1: Language model meta-learning. During unsupervised pre-training, a language model develops a broad set of skills and pattern recognition abilities. It then uses these abilities at inference time to rapidly adapt to or recognize the desired task. We use the term "in-context learning" to describe the inner loop of this process, which occurs within the forward-pass upon each sequence. The sequences in this diagram are not intended to be representative of the data a model would see during pre-training, but are intended to show that there are sometimes repeated sub-tasks embedded within a single sequence.

Zero/Few Shot Learning vs. Fine-tuning



Performance vs. Model Size

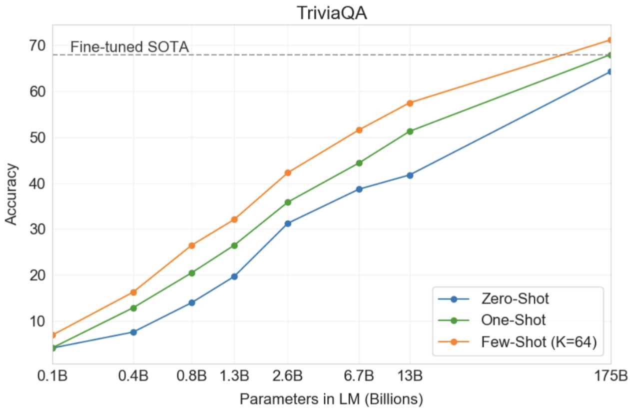
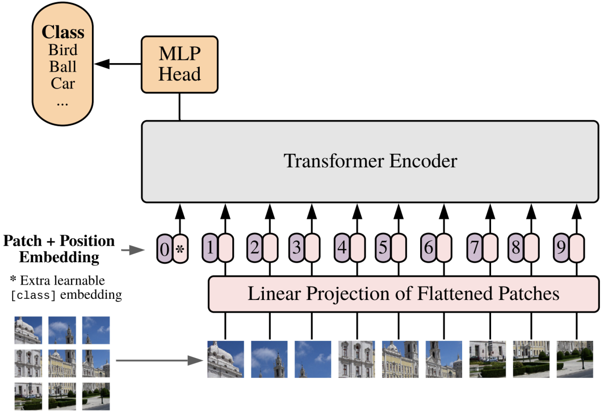


Figure 3.3: On TriviaQA GPT3's performance grows smoothly with model size, suggesting that language models continue to absorb knowledge as their capacity increases. One-shot and few-shot performance make significant gains over zero-shot behavior, matching and exceeding the performance of the SOTA fine-tuned open-domain model, RAG [LPP+20]

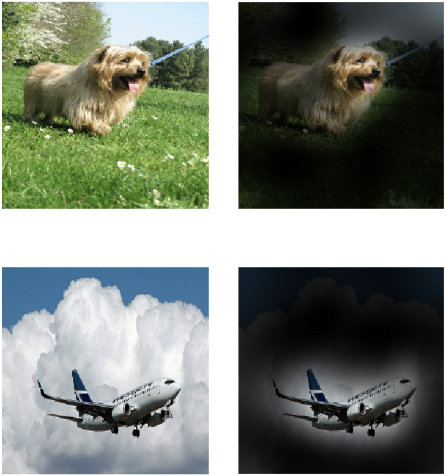
Outline

Vision Transformers (ViT) [5]

- Splits an image into 16×16 patches (words)
- Like BERT, uses [CLS] input word to get class predictions

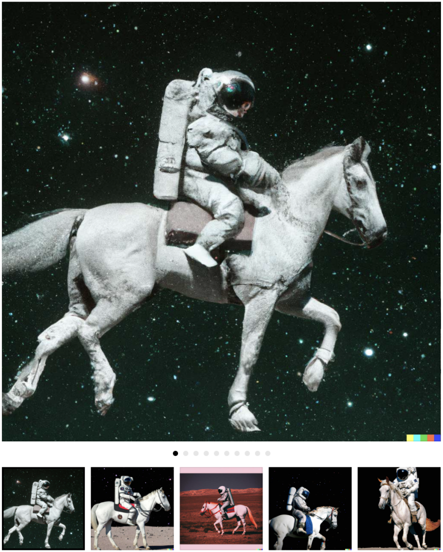


Attention



DALL-E [14]

- "An astronaut riding a horse in a photorealistic style" ▸ Live Demo



Outline

Sequence Tokenization

- Word-level
  - High input/output dimension (D)
  - Out-of-vocabulary (OOV) problem
  - "old" $\neq$ "older" $\neq$ "oldest"
- Char-level
  - Long sequence (T)

Sequence Tokenization

- Word-level
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  - Out-of-vocabulary (OOV) problem
  - "old" $\neq$ "older" $\neq$ "oldest"
- Char-level
  - Long sequence (T)
- Subword-level?
  - How to deal with OOV problem?
  - How to support different languages?

Byte Pair Encoding (BPE) [16]

- Given a training set of words
  1. Uses Unicode bytes as base symbols $\{\boldsymbol{s}^{(1)}, \boldsymbol{s}^{(2)}, \cdots\}$
  2. Merge two symbol $\boldsymbol{s}^{(i)}$ and $\boldsymbol{s}^{(j)}$ having highest $\Pr(\boldsymbol{s}^{(i)})$ and $\Pr(\boldsymbol{s}^{(j)})$
  3. Repeat step 2 until target #symbols is met

	Vocabulary	Encoded Sentence
Initialization	['a', 'c', 'b', 'e', 'i', '</w>', 'k', 'm', 'o', 'n', 'p', 's', 'r', 'u', 't', 'v', 'x']	v i e t n a m </w> t a k e s </w> m e a s u r e s </w> t o </w> b o o s t </w> r i c e </w> e x p o r t s </w>
After 1 merge operation	['a', 'c', 'b', 'e', 'p', '</w>', 'k', 'm', 'o', 'n', 'i', 's', 'r', 'u', 't', 'v', 'x', 's</w>']	v i e t n a m </w> t a k e s</w> m e a s u r e s</w> t o </w> b o o s t </w>r i c e </w> e x p o r t s</w>
After 10 merge operations	['</w>', 'vi', 'as', 'es</w>', 's</w>', 'nam', 'to', 'ri', 't</w>', 'ort', 'a', 'c', 'b', 'e', 'k', 'm', 'o', 'p', 's', 'r', 'u', 't', 'x']	vi e t nam </w> t a k es</w> m e as u r es</w> to </w> b o o s t</w> ri c e </w> e x p ort s</w>
After 34 merge operations	['takes</w>', 'measures</w>', 'exports</w>', 'boost</w>', 'rice</w>', 'vietnam</w>', 'to</w>']	vietnam</w> takes</w> measures</w> to</w> boost</w> rice</w> exports</w>

Other Variants

- BPE is used by GPT
- WordPiece [15]:
    - Merge $\boldsymbol{s}^{(i)}$ and $\boldsymbol{s}^{(j)}$ having highest $\frac{\Pr(\boldsymbol{s}^{(i)},\boldsymbol{s}^{(j)})}{\Pr(\boldsymbol{s}^{(i)})\Pr(\boldsymbol{s}^{(j)})}$
    - Used by BERT
- Unigram Language Model [8]
    - Top-down, probabilistic
- SentencePiece [9]
    - Treat space "_" as symbols to support difference languages
    - Merge algorithm: BPE or Unigram Language Model
    - Used by ALBERT, XLNet, Marian, T5

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