$$\boxed{\rho = \frac{\rho_0}{J}}$$

$\Leftrightarrow$ $M := \rho J$

$$\dot{M} = 0 \quad \Leftrightarrow \quad M\Big|_{\Gamma_u} = \text{const.}$$

$\parallel$

$$(\rho J)^{\cdot} = (\rho_t + \nabla\cdot(\rho u))\underset{\neq 0}{J} \quad \Leftrightarrow \quad \rho_t + \nabla\cdot(\rho u) = \cancel{0}$$

$\underline{\text{How about } \dot{M} \neq 0 \text{ ?}}$ e.g. Mass-action ?

$$\rho_t + \nabla\cdot(\rho\vec{u}) = \underline{\sum \vec{r}}$$

Kinematics : $\rho = \rho_0 + \sum \vec{R}$

$$M = \underline{m(t)} = \int_\Omega \rho(x,t)\,dx \overset{\text{C.V.T.}}{=} \int_{\Omega_X} \rho(\underline{x(X,t),t})\left|\frac{\partial x}{\partial X}\right| dX$$

if $\parallel$ $\qquad x = x(X,t)$ $\qquad \parallel$ $\rho(x,t),\ J(x,t)$

$$m\Big|_{t=0} = \int_{\Omega_{\underline{X}}} \underbrace{\rho(\underline{X}, 0)}_{=\rho_0(\underline{X}) = \rho_0} d\underline{X}$$

$$\boxed{\rho = \frac{\rho_0}{J}}$$

$$\boxed{\rho = \rho_0 + \sum \vec{R}}$$

$$m(t) = \int_{\Omega} \rho(x,t)\,dx = \int_{\Omega_{\underline{X}}} \rho_0(\underline{X}) + \sum \vec{R}(t, \vec{R})\,d\underline{X}$$

$$\downarrow \frac{d}{dt} \qquad\qquad \downarrow \frac{d}{dt}$$

$$\int_{\Omega_{\underline{X}}} (\rho_t + \nabla\cdot(\rho u)) J\,d\underline{X} \qquad \int_{\Omega_{\underline{X}}} \sum \vec{r}\,d\underline{X}$$

$$= \int_{\Omega} \rho_t + \nabla\cdot(\rho u)\,dx \qquad = \int_{\Omega} \sum \vec{r}\,dx$$

$$\rho_t + \nabla\cdot(\rho u) = \sum \vec{r}, \qquad \vec{r} = \vec{r}(\vec{\rho})$$

e.g.: $\vec{\rho} = \begin{bmatrix} B \\ R \\ E \end{bmatrix}$ $\quad \boxed{\vec{u}_B \equiv \vec{u}_R \equiv \vec{u}_E = 0. \qquad (\text{ODE})}$

? $\vec{r} = \vec{r}_{NN}(\rho)$ ~~of~~ $\vec{u} = \vec{u}_{NN}(\rho)$

$$\dot{B} = \left(rB(1 - B/K) \right) = rB - \frac{r}{K} B^2$$

For $n = 0, 1, 2, \ldots$

$$B \underset{k_-}{\overset{k_+}{\rightleftharpoons}} 2B$$

$$\dot{B}^{(+)} = \left(k_+ B - k_- B^2 \right)$$

$$u^{n+1} = u^n_{NN}(\rho^n)$$

$$r_1^{n+1} = r^n_{1,NN}(\vec{\rho}^{\,n}) \; x \; \checkmark$$

$$R_1^{n+1} = R_1^n + \Delta t \left(\cancel{\textstyle\sum_i} \; r_1^n \right) \checkmark$$

$$J^{n+1} = J^n + \Delta t (\nabla \cdot u) J^n - u \cdot \nabla J$$

$$\rho^{n+1} = \frac{\rho_0 + \cancel{B} R^{n+1}}{J^{n+1}}$$

$$\rho_t = \Delta\rho + k^+\rho - k^-\rho^2$$

$$r_{NN} = r(\rho)$$

$$x_t = \frac{x^{n+1}-x^n}{\Delta t} = u$$

$$R_t = \frac{R^{n+1}-R^n}{\Delta t} = r_{NN}$$

$$R(t=0) = 0.$$

ν_{PINN}

Wei:

$$\rho_t = \Delta\rho + \rho - \rho^2 \qquad \rho|_{t=0} = \rho_0$$