# Part II: Advances in Bayesian Optimization Sec 1: Batch Bayesian Optimization

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#### Agenda

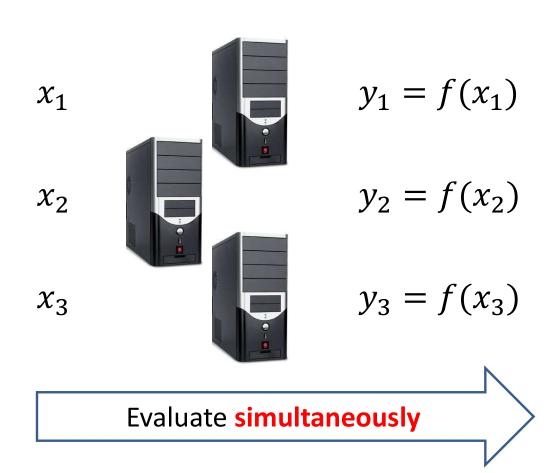
- Hyperparameter Tuning and Experimental Design as Black-Boxes
- Part I: Bayesian Optimization
- Part II: Recent Advances in Bayesian Optimization
  - Batch Bayesian Optimization
  - High dimensional Bayes Opt
  - Mixed Categorical-Continuous Bayes Opt
- Research Directions in Bayesian Optimization

#### Outline Part II.1: Batch Bayesian Optimization

- Introduction and Problem Statement
- Peak Suppression Approaches
  - Constant Liar
  - Batch Upper Confidence Bound (GP-BUCB)
  - Local Penalization
- Budgeted Batch Bayesian Optimization
- Thompson Sampling for Batch Bayes Opt
- Asynchronous Batch Bayes Opt

# Batch Bayesian Optimization

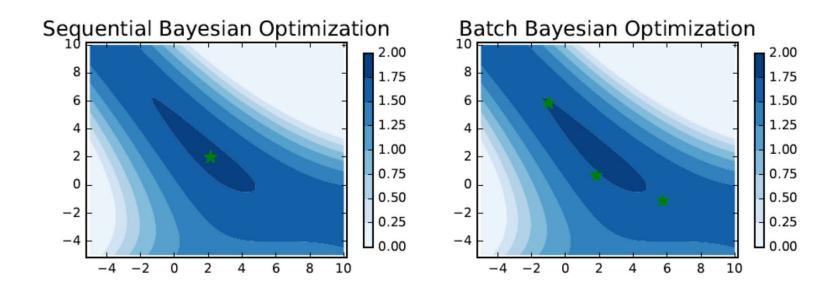
 Evaluating multiple experiments take the same time as evaluating single experiment.



#### Batch Bayesian Optimization

 When parallel experiments can be done, we estimate points for evaluations at each iteration.

$$\boldsymbol{X}_{t} = [\boldsymbol{x}_{t1}, \boldsymbol{x}_{t2}, ..., \boldsymbol{x}_{tn_{t}}] = \underset{\boldsymbol{x} \in \mathcal{X}}{\operatorname{argmax}} \alpha_{t} \left( \boldsymbol{x} \right)$$



#### Batch Bayesian Optimization

- Selects a batch of points for parallel evaluations at each iteration.
- Recent research:
  - GP-BUCB: [Desautels et al ICML'12]
  - Parallel PES: [Shah et al NIPS'15]
  - Local Penalization: [Gonzalez et al AISTATS'16]
  - Determinantal Point Process: [Kathuria et al NIPS'16]
  - Knowledge Gradient: [Wu et al NIPS'16]
  - Thompson sampling [Lobato et al ICML'17]
  - Asynchronous parallel Bayes opt [Kandasamy et al AISTATS 2018]
  - Asynchronous BO using improved local penalisation [Ahvin et al ICML19]

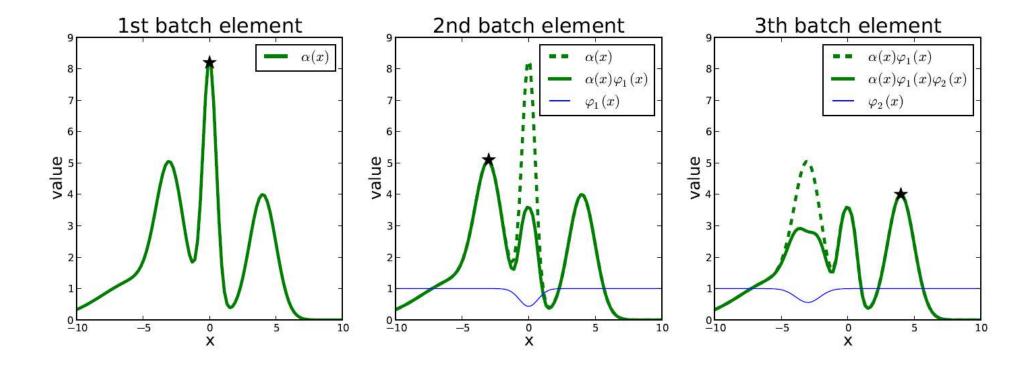
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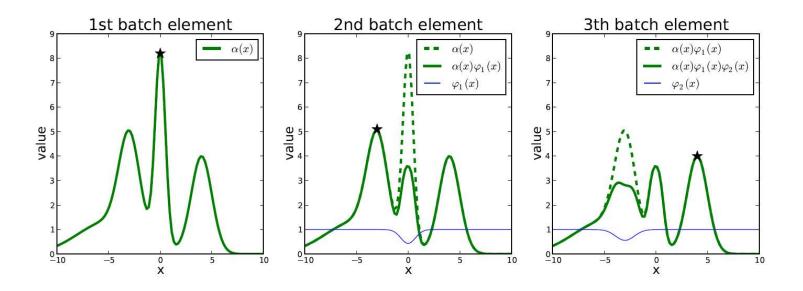
# Peak Suppression Approaches

 Intuition: Identify the peaks of the acquisition functions as the batch of points for testing.



#### Peak Suppression Approaches

- Intuition: Sequentially select a peak, then suppress this peak and move to the next one.
- There are different ways to suppress the peaks.



#### Peak Suppression Approaches

The acquisition function is a form of mean and variance.

$$\alpha_t^{GP-UCB}(x) = \mu_t(x) + \sqrt{\beta_t} \times \sigma_t(x)$$

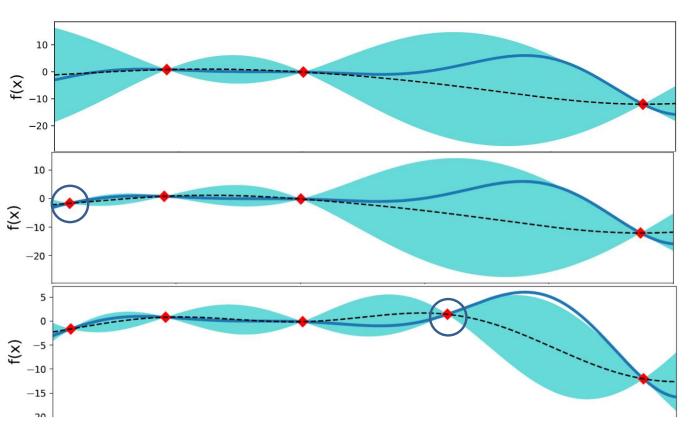
- The peaks of the acquisition function will have high mean  $\mu(x)$  and high variance  $\sigma(x)$ .
- We can suppress the peak by reducing its mean (and/or) variance.

#### Peak Suppression – GP-BUCB

- GP-BUCB sequentially find a batch  $X_t = [x_{t,1}, x_{t,2}, ..., x_{t,B}]$  as follows
  - Select the first element  $x_{t,1}$  in a batch like the standard BO.
  - Hallucinating the output  $y_{t,1} = \mu(x_{t,1})$  by the GP predictive mean,
  - Update the variance function by inserting  $x_{t,1}$ ,  $\sigma(.|x_{t,1})$
  - 4. Optimizing the UCB acquisition function to select the next point  $x_{t,b}$
  - Repeat the above steps (2 and 3) to find  $x_{t,b+1}$ .
- Without using the outcome y, by updating the variance function by inserting  $x_{t,1}$ , BUCB has suppressed the variance around  $x_{t,1}$ .
- The property of the variance in a Gaussian process used in BUCB:
  - the variance at the observed location is zero or close to zero (for noise setting).
  - ullet the variance depends on the location x (but not the outcome y)

Desautels et al. Parallelizing exploration-exploitation tradeoffs in Gaussian process bandit optimization. JMLR, 2014

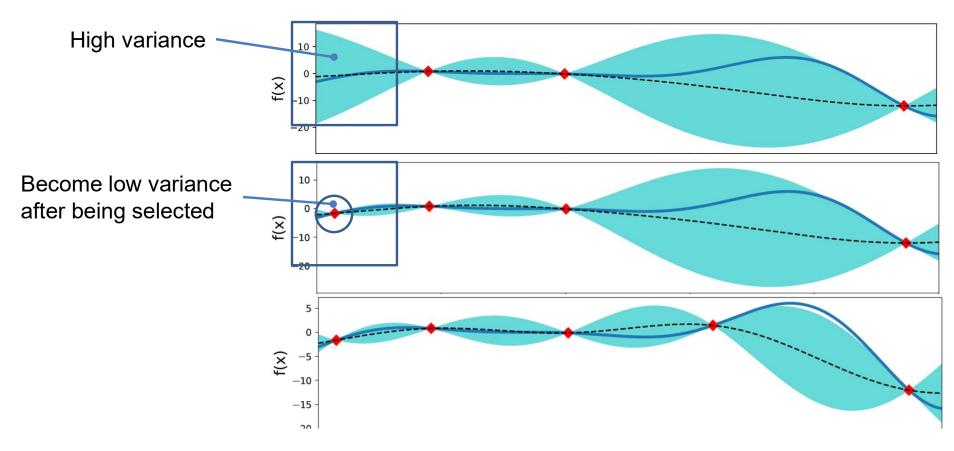
#### The Effect of Reducing Uncertainty at Selected Point



- 1. Get initial data
- 2. Fit a GP model to the data
- 3. Select  $x_{t+1}$
- 4. Collect data  $y_t = f(x_t)$ Repeat steps 2-4

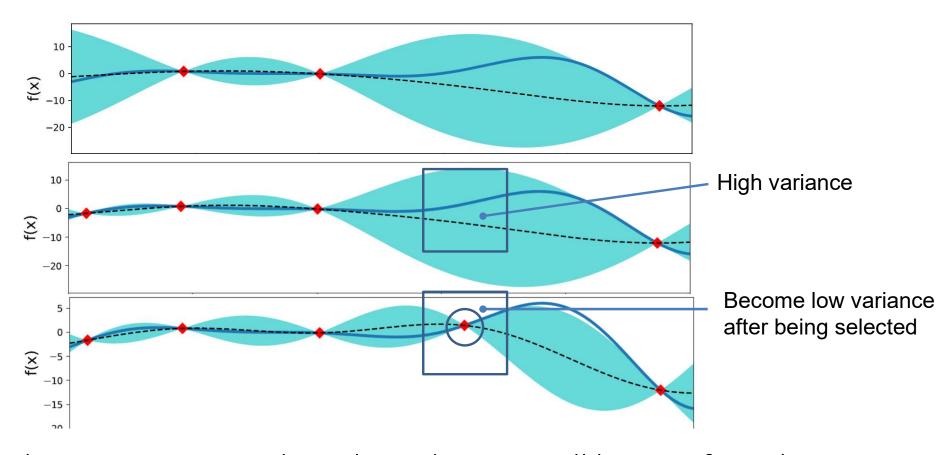
Intuitively, BO selects a point which has high mean and high variance.

#### The Effect of Reducing Uncertainty at Selected Point



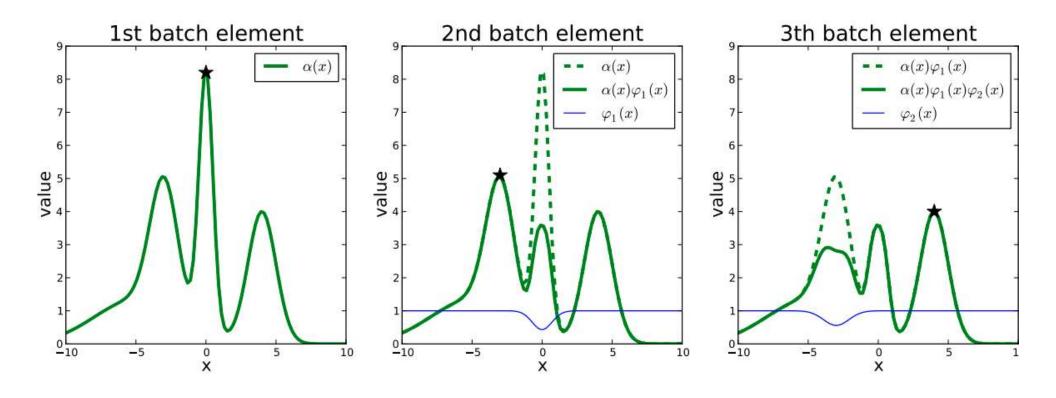
• The uncertainty at the selected points will be significantly reduced. The choice encodes the exploration.

#### The Effect of Reducing Uncertainty at Selected Point



 The uncertainty at the selected points will be significantly reduced. The choice encodes the exploration.

Illustration of peak suppression approach – Local Penalization



Gonzalez, J., et al. Batch bayesian optimization via local penalization. AISTATS 2016

ullet Since the Lipschitz constant of the original function f is unknown.

$$L_{\nabla} = \max_{\mathbf{x} \in \mathcal{X}} \|\nabla f(\mathbf{x})\|$$

We estimate Lipschitz Constant of the GP

$$\mu_{\nabla}(\mathbf{x}^*) = \partial \mathbf{K}_{n,*}(\mathbf{x}^*) \tilde{\mathbf{K}}_n^{-1} \mathbf{y}$$

where 
$$(\partial \mathbf{K}_{n,*})_{i,l} = \frac{\partial \mathbf{k}_N(\mathbf{x}^*)}{\partial x^{(i)}}$$

• Then, we choose 
$$\hat{L}_{GP-LCA} = \max_{\mathcal{X}} \|\mu_{\nabla}(\mathbf{x}^*)\|$$

Gonzalez, J., et al. Batch bayesian optimization via local penalization. AISTATS 2016

- Define the penalized function  $\varphi(x)$ 
  - indicates how much we want to exclude the nearby area around x.
  - smoothly reduce the value of the acquisition function in a neighborhood of x.

ullet Define a ball parameterized by a radius  $r_i$ 

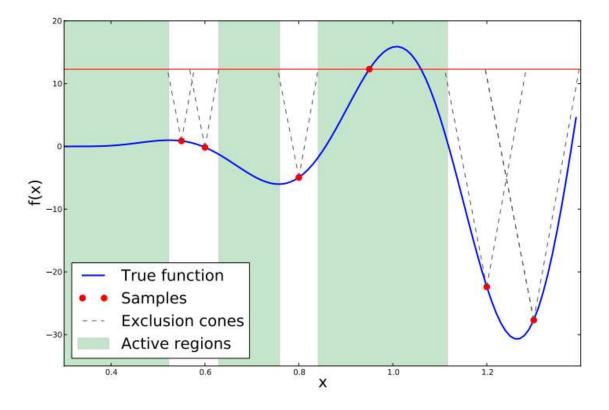
$$B_{r_j}(x_j) = \{x \in X : \left| \left| x_j - x \right| \right| \le r_j \}$$

Gonzalez, J., et al. Batch bayesian optimization via local penalization. AISTATS 2016

• Define a ball parameterized by a radius  $r_j$  where  $r_j = \frac{M - f(x_j)}{L}$ , M is the current best value  $\max_i \{y_i\}$  and L is a Lipschitz constant.

• Intuition: the radius is large for the low value region where  $f(x_i)$  is

small and vice versa.



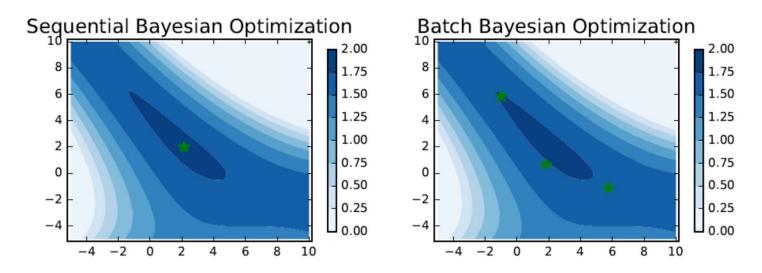
#### Drawback of Local Penalization

- ullet They use a unique value of the Lipschitz constant L to represent for the whole function.
  - Some problems may not satisfy this condition, e.g. heteroscedastic functions.
- Estimating L in high dimension is still non-trivial.

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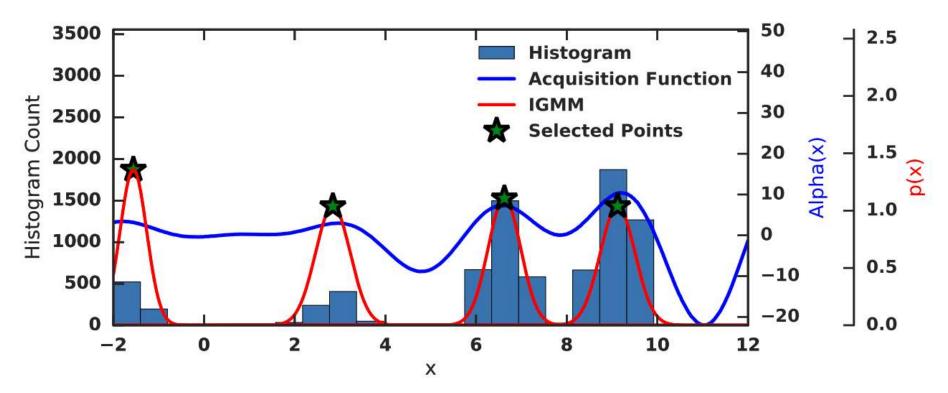
#### Batch Size in Batch Bayesian Optimization



- Existing approaches for batch BO require a fixed batch size of points:
  - Over-specify: wasting time and resources to evaluate redundant points.
  - Under-specify: missing important points that affect the performance.
- We aim to save the number of evaluations, but preserve the performance, by controlling the batch size in a principled way.

#### Acquisition Function as Multi-modal Functions

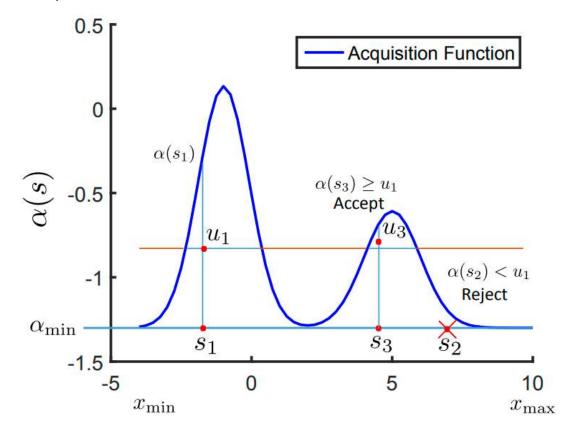
- The acq function is multi-modal with unknown number of peaks.
- Identifying the peaks (star) is equivalent to finding the mean locations in the infinite mixture of Gaussian (IGMM) (Red curve).



Vu Nguyen et al. Budgeted batch Bayesian optimization. In ICDM 2016,

# Budgeted Batch Bayesian Optimization

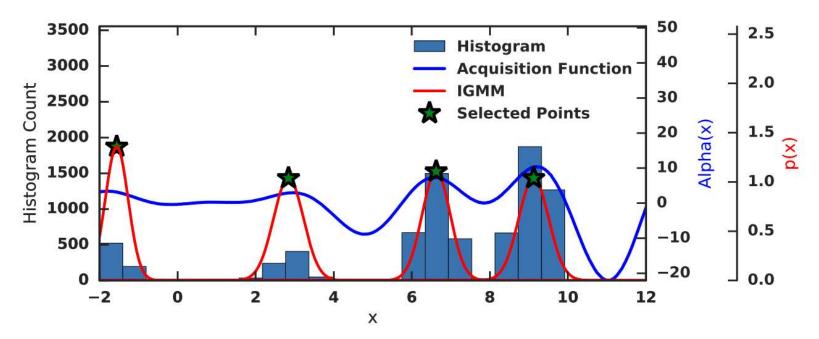
 We propose the Generalized Batch Slice Sampling to draw samples under the acquisition function to learn the IGMM.



Vu Nguyen et al. Budgeted batch Bayesian optimization. In ICDM 2016,

#### Budgeted Batch Bayesian Optimization

- Using Generalized Batch Slice Sampling to draw samples under the acquisition function (see the Histogram).
- 2. Fit the samples to the Infinite Gaussian Mixture Model.
- IGMM can detect the unknown number of peaks.



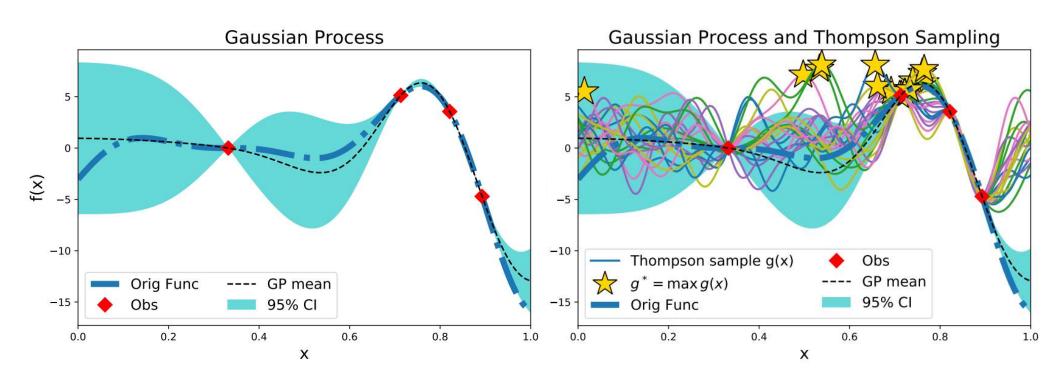
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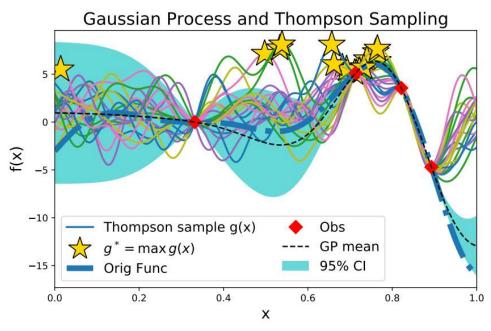
#### Thompson Sampling to Sample the Optimum Locations

- Thompson Sampling draws samples g() from GP.
- Each yellow stars  $x^*$  is the maximizer of the sampled function g()
- ullet We consider  $x^*$  as the perceived optimal samples



# Thompson Sampling for Large Scale Batch BO

- For large batch size (e.g.,  $B \ge 100$ ), the penalization-based batch approaches may not be scalable.
- We can use different Thompson samples from a GP to draw multiple suggestions.



# Thompson Sampling for Distributed BO

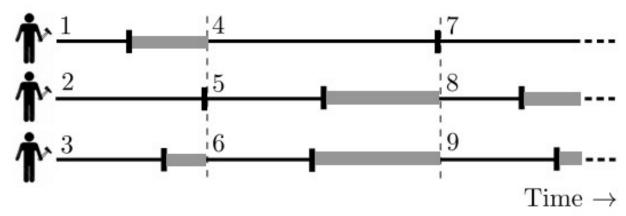
Thompson Sampling for Parallel and Distributed BO

Node updating model comunicating information between nodes optimizing acquisition function evaluating objective function Parallel EI Parallel and Distributed TS idle node

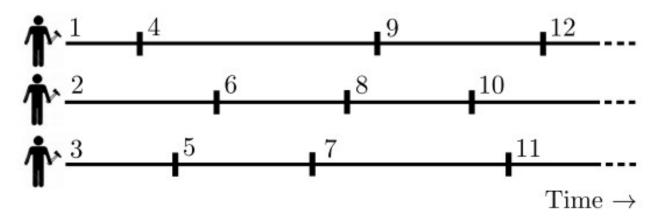
Hernández-Lobato, J. M., et al. "Parallel and Distributed Thompson Sampling for Large-scale Accelerated Exploration of Chemical Space." ICML 2017

# TS for Asynchronous Optimisation

Synchronous optimization



Asynchronous optimization

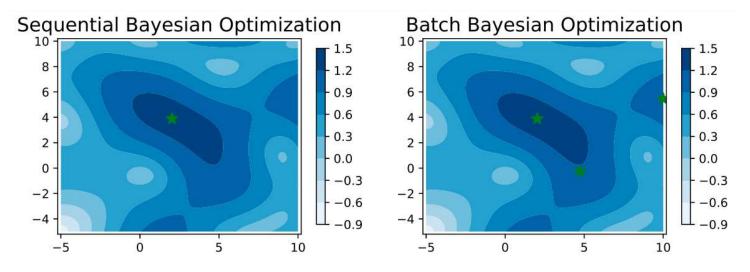


Kandasamy, K., et al,. Parallelised Bayesian Optimisation via Thompson Sampling. AISTATS, 2018.

# **Short Summary**

• In the standard setting, Bayesian optimization suggests one experiment to test at each iteration.

 When parallel facilities are available, batch Bayesian optimization is desired to suggest multiple experiments and thus boost the optimization more efficient.



#### MiniBayesOpt for Batch Bayesian Optimization

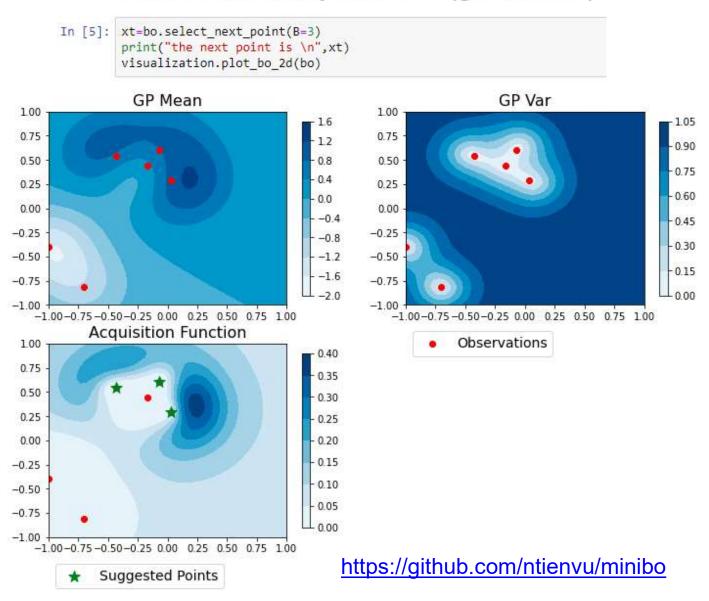
- Code: <u>vu-nguyen.org/BOTutorial ACML20</u>
- Github repository: MiniBayesOpt
- git+https://github.com/ntienvu/minibo



https://github.com/ntienvu/minibo

# Using MiniBO for batch Bayes Opt

#### Select a batch of points B=3 (green stars)



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#### Challenges for High-dimensional Bayes Opt

Standard BO often works on less than 10 dimensions.

 High dimension causes problem for optimization – the search space grows exponentially with the dimension.

#### High-dimensional Bayesian Optimization

Optimize high-dimensional acquisition functions?

- High-dimensional (d>10) acquisition functions feature only a few peaks and a large terrain of almost flat surface [Rana et al ICML 2017]
- Global optimizers (DIRECT) fail to return an optimum within limited time and resource;
- Gradient-dependent Local optimizers get stuck due to non-significant gradients in the flat surface of acquisition functions;

# High dimensional Bayes Opt

Random Embedding: Wang et al IJCAI 2013

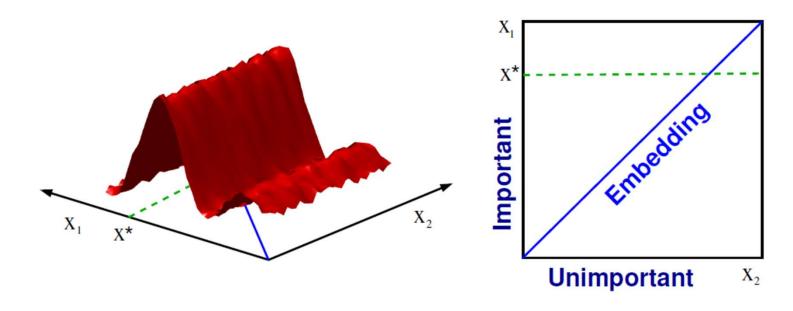
Additive GP: Kandasamy et al ICML 2015

Drop-out: Li et al. IJCAI 2017

TuRBO: Eriksson et al NeurIPS 2019

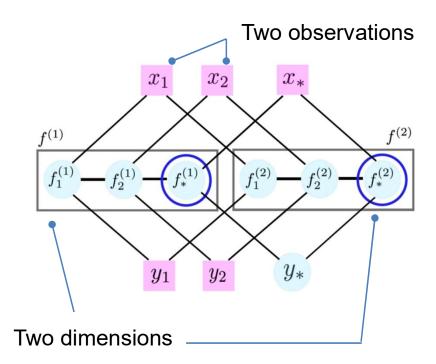
## High-dimensional Bayes Opt

- Random embeddings: [Wang et al IJCAI 2013]
  - project high-dimensional space to low-dimensional space
- This 2D function only has d=1 effective dimension.
- It is more efficient to search for the optimum along the 1-dimensional random embedding than in the original 2-dimensional space.



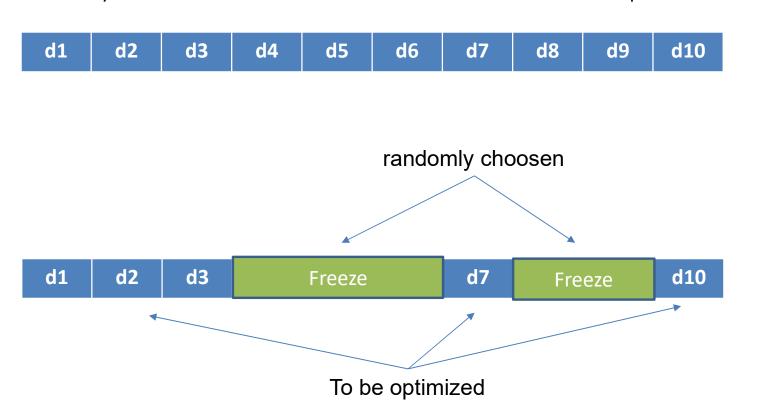
### High-dimensional Bayes Opt

- Additive kernel: [Kandasamy et al ICML 2015]
  - decompose high dimensions into disjointed groups
  - infer the GP posterior of the individual GP (for each disjointed group).
  - Define the additive acquisition function.



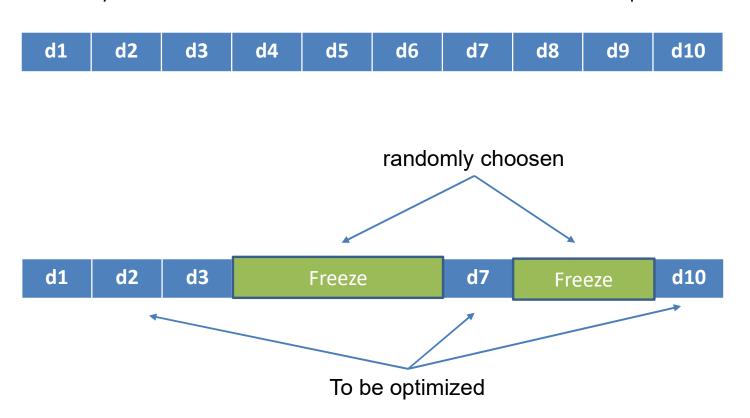
## High-dimensional Bayes Opt

- Drop-out [Li et al. IJCAI 2017]
  - Randomly select a smaller number of dimension for optimization.



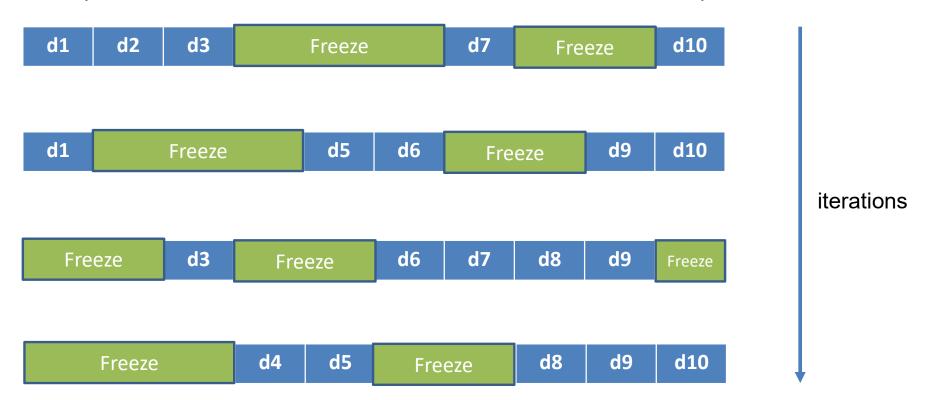
### High-dimensional Bayes Opt via Drop-out

- Drop-out [Li et al. IJCAI 2017]
  - Randomly select a smaller number of dimension for optimization.



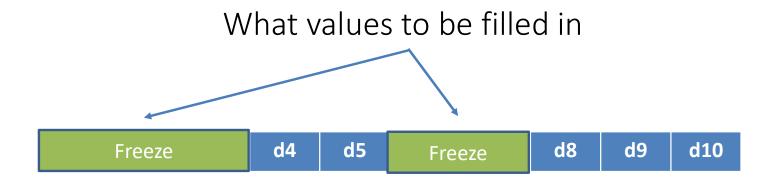
### High-dimensional Bayes Opt via Drop-out

Randomly select a smaller number of dimension for optimization.



• The number of effective dimensions to be optimized is small.

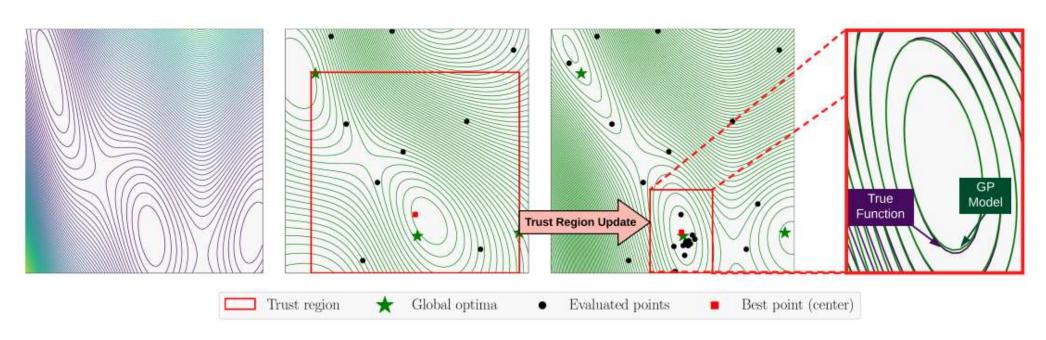
#### Fill-in strategies



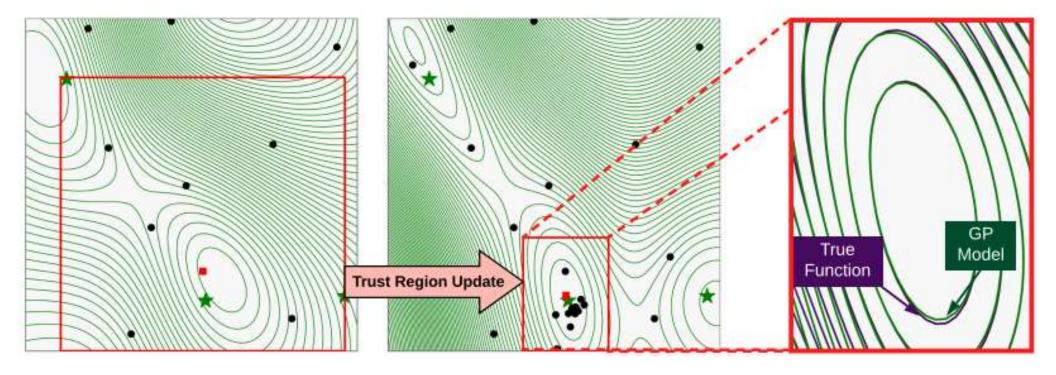
- Dropout-Random: use a random value in the domain
- Dropout-Copy: copy the value of the variables from the best function value so far.
- ullet Dropout-Mix: Random with a probability p and Copy with a probability 1-p

### Trust Region Bayesian Optimization

- High-level idea:
  - Build the trust regions
  - Perform local optimization in each trust region
  - Repeat

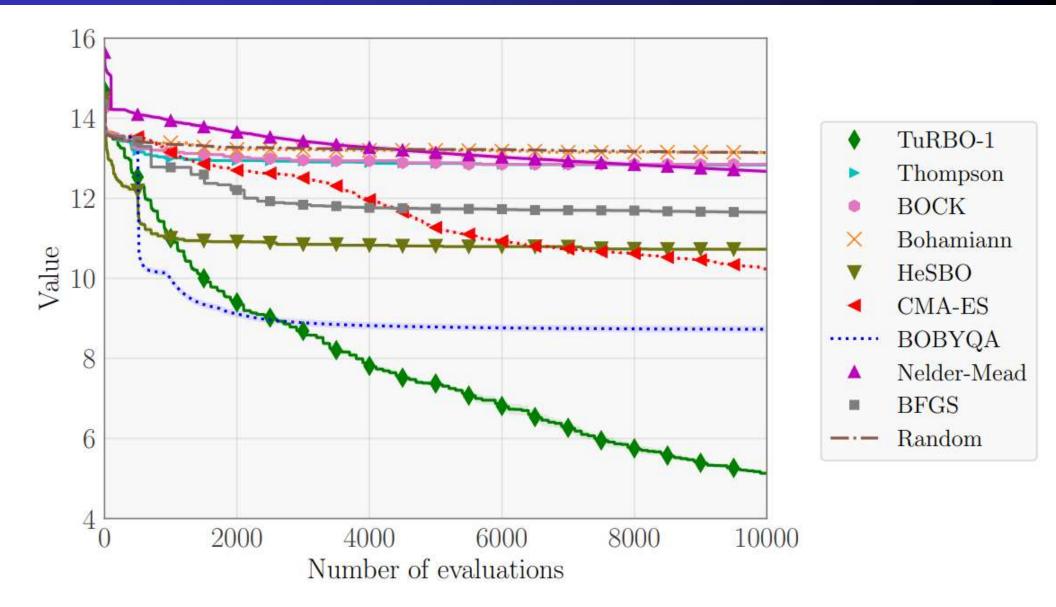


# TuRBO



• Eriksson, David, et al. "Scalable global optimization via local bayesian optimization." *NeurIPS*. 2019.

# TurBO in optimizing 200 dimension



### **Short Summary**

 Bayesian optimization can work effectively up to 10 dimensions.

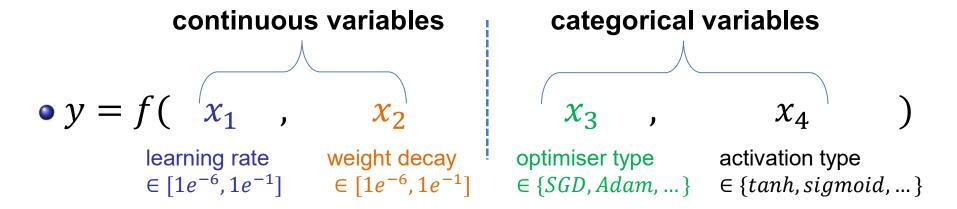
 In real-world scenarios, we may tackle the problems with large number of dimensions.

Bayesian optimization research in high dimension is essential.

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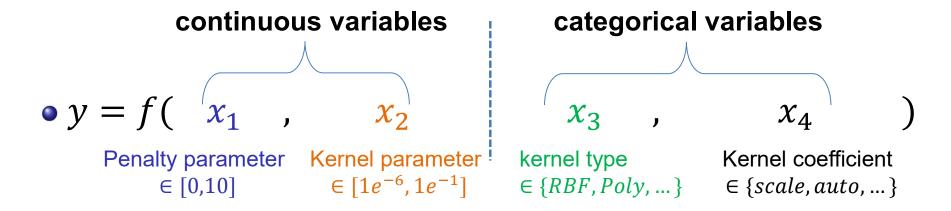
Tuning hyperparameters for deep neural network



Multiple categorical - each categorical has multiple options



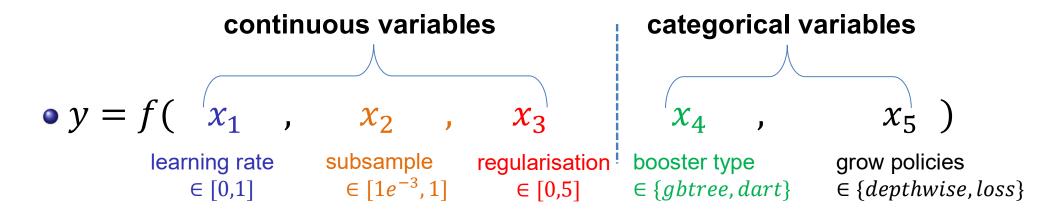
Tuning hyperparameters for support vector machine



Multiple categorical - each categorical has multiple options



Tuning hyperparameters for XGBoost



Multiple categorical - each categorical has multiple options



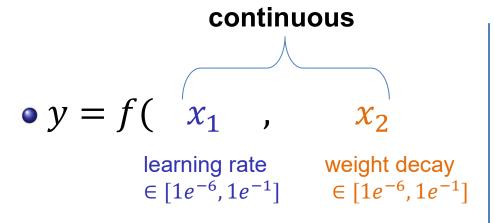
One-hot encoding:

• Red: [1,0,0] Green: [0,1,0] Blue: [0,01]

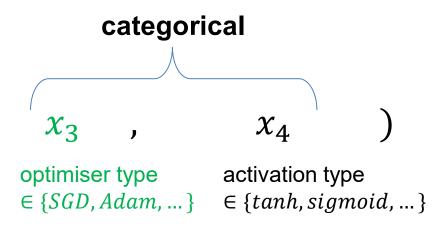
- Drawbacks:
  - Make the search space large.

if C=4 categories, each has V=5 choices=> 20 extra dimensions.

- Non-continuous and non-differentiable space
- Challenging in optimizing mixed-type: categorical continuous



Bayes Opt



Multi-armed Bandit

#### Agenda

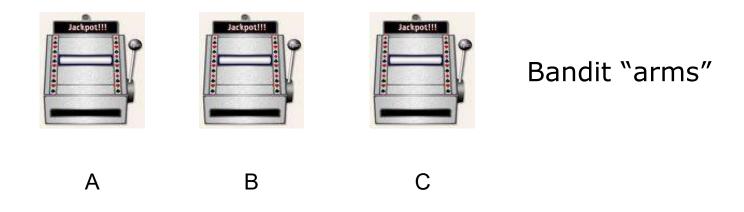
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    - Categorical-(non-)specific continuous optimization
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#### Multi-Armed Bandit Setting

A fixed set of arms, each of which returns a reward.

Observe the reward before the next pull.

The goal is to pull and receive as high reward as possible.



#### General Algorithm for Multi-Armed Bandit

- Input is a fixed set of action [C]
- ullet Initialize the model heta
- For t=1.....T
  - ullet Compute the probability  $p_c$  of selecting arm c from heta
  - Select an arm  $c \in [C]$  using the probability  $p_c$
  - Pull an arm c
  - Observe the reward  $g_t(c)$  at the arm c
  - ullet Update the model heta using the reward

There involves randomness here

#### General Algorithm for Multi-Armed Bandit

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  - ullet Update the model heta using the reward

These steps are the key of MAB

### UCB1 algorithm

Input is a fixed set of action [C]

**Exploration-Exploitation** 

- ullet Initialize the model heta
- For t=1.....T

utility score  $\int_{C} = \overline{g_C} + \sqrt{\frac{2 \log T}{g_C}}$ uncertainty

- Compute the probability  $p_c = \overline{g_c} + \sqrt{\frac{2 \log T}{n_c}}$
- Select an arm  $c \in [C]$  by maximizing  $p_c$
- Pull an arm c
- Observe the reward  $g_t(c)$  at the arm c
- ullet Update the model heta

#### EXP3 Algorithm

 EXP3 algorithm doesnot assume the underlying distribution over the process generating the reward.

#### **Algorithm 1** Exp3 Algorithm for Categorical Selection.

Input:  $\gamma \in [0,1]$ , C #categorical choice, T #max iteration

- 1: Init  $\omega_c = 1, \forall c = 1...C$
- 2: for t = 1 to T do
- 3: Compute the probability  $p_t^c = (1 \gamma) \frac{\omega_c}{\sum_{c=1}^C \omega_c} + \frac{\gamma}{C}, \forall c = 1...C$  This step is different
- 4: Choose a categorical variable  $h_t \in [1, ... \tilde{C}]$  at random according to distribution  $p_t$ .
- 5: Observe the reward  $g_t(c) = f(h_t = c)$
- 6: Normalize  $\hat{g}_t(c) = \frac{g_t(c)}{p_c^c}$
- 7: Update the weight  $\omega_c = \omega_c \times \exp(\gamma \hat{g}_t(c)/C)$

This step is different

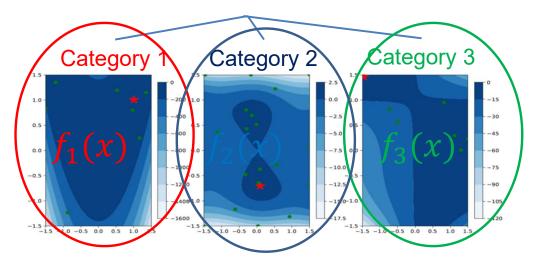
8: end for

Output:  $\mathcal{D}_T$ 

Auer P et al, "The non-stochastic multi-armed bandit problem" 2002

#### Two settings in mixed variables optimization

1. Categorical-specific



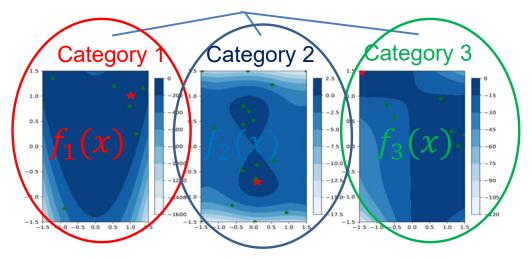
Continuous variable is **specific** to categorical variable

Each categorical variable forms an independent function

$$f_c^* = \max_{x \in X} f_c(x)$$

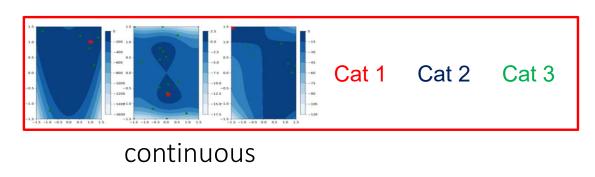
#### Two settings in mixed variables optimization

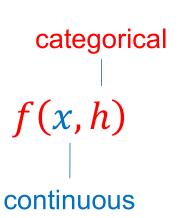
1. Categorical-specific



Continuous variable is specific to categorical variable

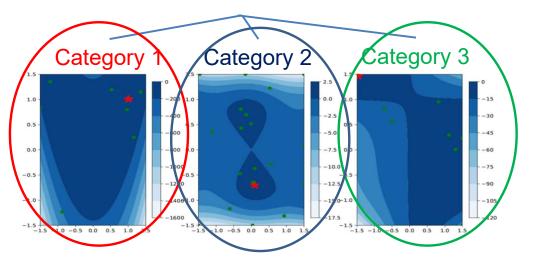
2. Continuous is not specific to categorical





#### Two settings in mixed variables optimization

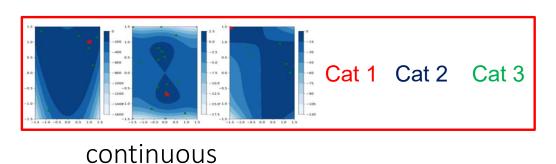
1. Categorical-specific



$$f_1(x)$$
  $f_2(x)$   $f_3(x)$ 

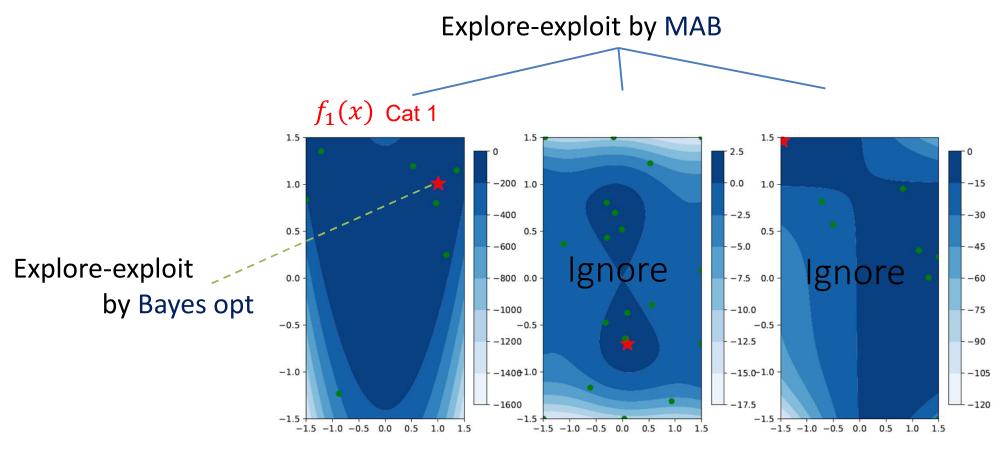
Three independent functions

2. Continuous is not specific to categorical



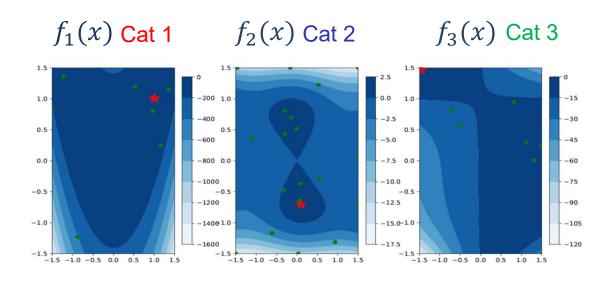
$$f(x,h)$$
  
Single function

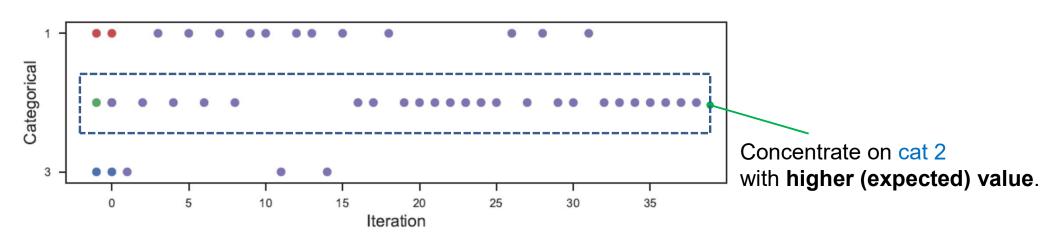
#### 1. Categorical-specific



- Sequentially pick a category c using multi-armed bandit (MAB)
- ullet Then optimize the continuous variables given  $oldsymbol{c}$  using Bayes opt

### Visualization of the Algorithm





#### Agenda

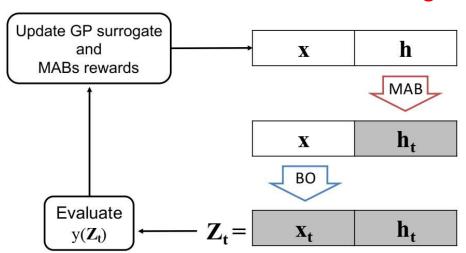
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    - Multi-armed bandits
    - Categorical-specific continuous optimization
    - Categorical-(non-)specific continuous optimization
- Research Directions in Bayesian Optimization

#### 2. Continuous is not specific to categorical variable

- MAB picks multiple categories: h={SGD optimizer, tanh activation}
- $\bullet$  Then optimize the continuous x given h in a single function

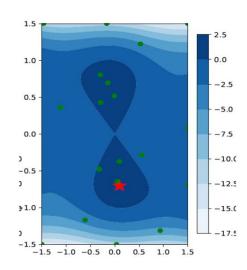
$$f(\mathbf{x}, \mathbf{h})$$

#### continuous categorical



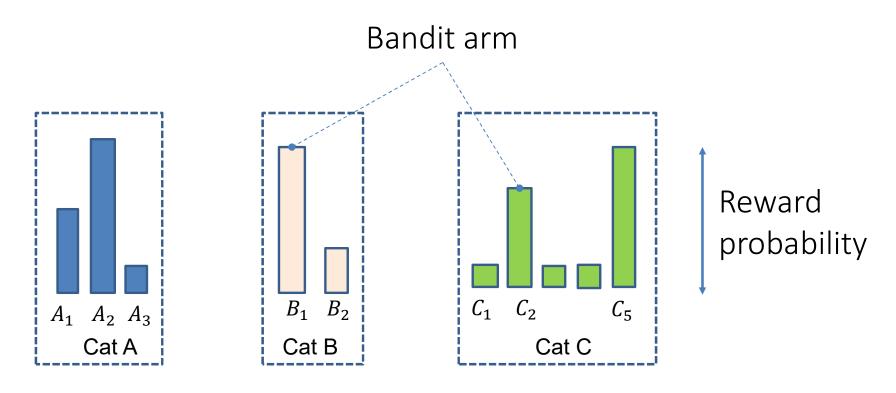
## Given h, we optimize the continuous x

- ullet Given the categorical  $oldsymbol{h}$ , optimize continuous variable  $oldsymbol{x}$
- Consider the joint kernel for learning the surrogate model
  - Additive k(x, x') + k(h, h')
  - Multiplicative  $k(x, x') \times k(h, h')$
  - $k(z,z') = (1-\lambda)k(x,x') \times k(h,h') + \lambda[k(x,x') + k(h,h')]$
  - $\bullet$   $\lambda$  can be estimated from the data.
- Optimizing in the continuous space



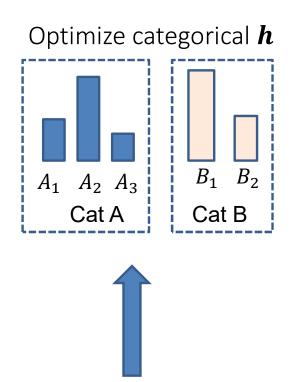
# Given the feedback f([x, h]), we optimize h

- Select h by EXP3 algorithm
- There is no assumption on the distribution of the reward



Multiple categories

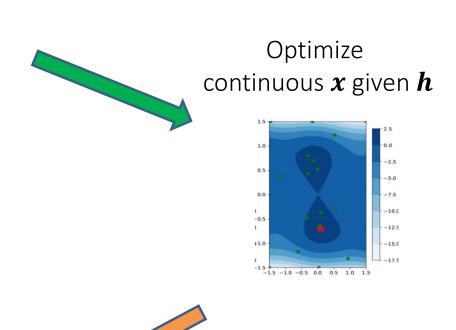
# Given the feedback f([x, h]), we optimize h

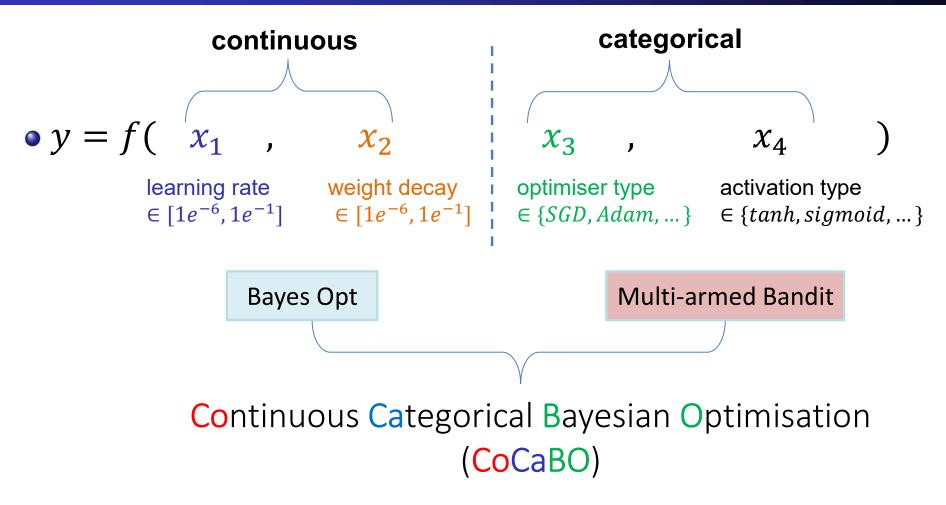




- Bandit feedback
- GP feedback







Bayes opt over multiple continuous and categorical inputs. Ru Binxin et al 2020.

#### Alternative solutions

• Instead of using MAB, we can utilize decision tree for the categorical variable: Jenatton, R., Archambeau, C., González, J., & Seeger, M. Bayesian optimization with treestructured dependencies. ICML, 2017.

#### • TPE:

Bergstra, James S., Rémi Bardenet, Yoshua Bengio, and Balázs Kégl. "Algorithms for hyper-parameter optimization." NeurIPS. 2011.

#### SMAC:

Hutter, Frank, Holger H. Hoos, and Kevin Leyton-Brown. "Sequential model-based optimization for general algorithm configuration." In *International conference on learning and intelligent optimization*, pp. 507-523. Springer, Berlin, Heidelberg, 2011.

### **Short Summary**

 Bayesian optimization can work effectively up to 10 dimensions.

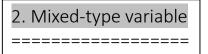
 In real-world scenarios, we may tackle the problems with large number of dimensions.

Bayesian optimization research in high dimension is essential.

#### Agenda

- Hyperparameter Tuning and Experimental Design as Black-Boxes
- Part I: Bayesian Optimization
- Part II: Recent Advances in Bayesian Optimization
  - Batch Bayesian Optimization
  - High dimensional Bayes Opt
  - Mixed Categorical-Continuous Bayes Opt
- Research Directions in Bayesian Optimization

# Research Summary in Bayesian Optimization



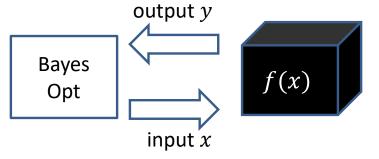
3. Exploiting knowledge from the function

4. Multi-fidelity BO

1. Neural architecture search

Surrogate function

Acquisition function



5. Safe BO Constraints BO

6. High dimensional

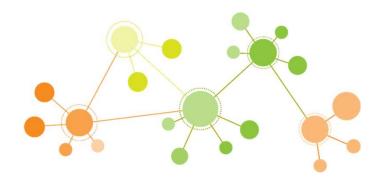
10. Gaussian process Student-t process Bayesian neural net

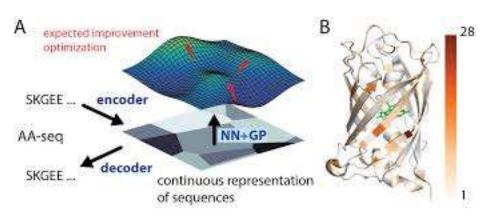
9. Information theoretic Multi-step look ahead



7. Theoretical Analysis

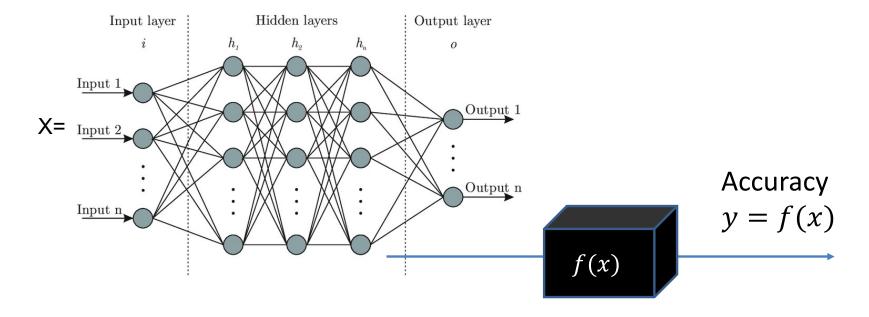
- Optimization in structured domains.
  - For example, how can we efficiently optimize over graphs, discrete sequences, trees, computer programs, etc.?





Eissman, Stephan, et al. "Bayesian optimization and attribute adjustment." UAI. 2018.

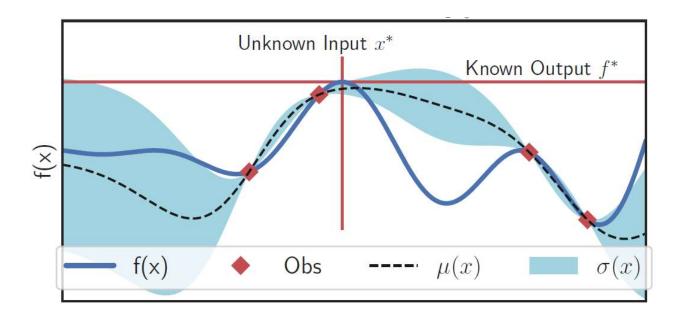
#### Neural Architecture Search



Finding the best architecture for the highest accuracy.

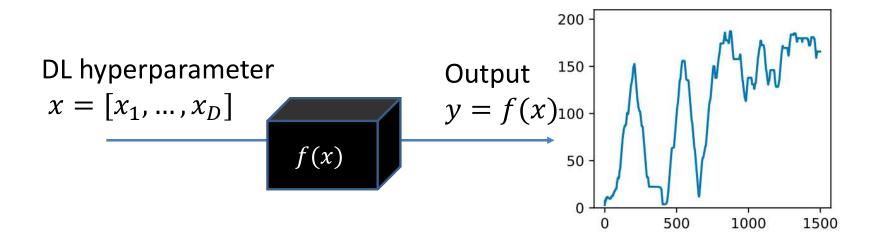
Kandasamy, K., at el NeurlPS 2018.

- Incorporating domain specific knowledge.
  - How can we easily encode and transfer available knowledge into BO methods in an easy and fast manner?
  - Knowing the optimum value of the function?



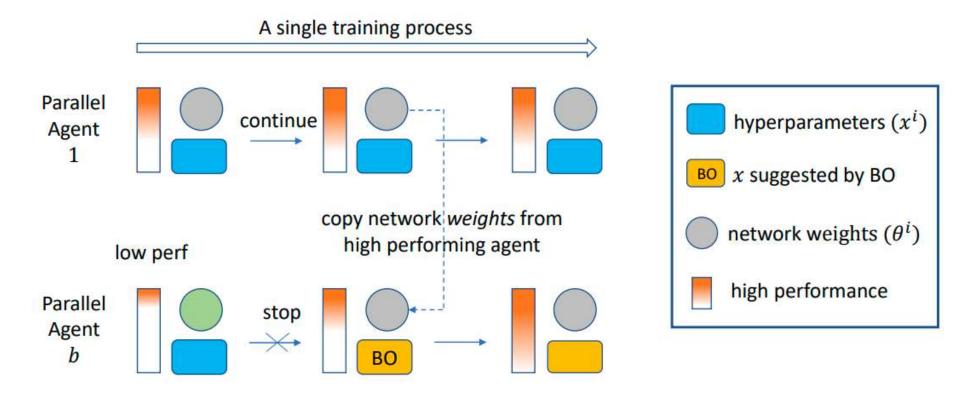
**Vu Nguyen** and Micheal Osborne. Knowing the what but not the where in Bayesian optimization. ICML 2020.

- Optimization with structured output response.
  - In training deep learning or deep reinforcement learning, the output is not only a single accuracy, but the whole training curve over epochs.



Vu Nguyen et al. "Bayesian optimization for iterative learning." NeurIPS 2020

One-shot Bayesian optimization



Parker-Holder, Jack, et al NeurlPS 2020.

- High dimension
  - Although previous researches have addressed high dimensional BO in different ways, the problem is still open.
- Mixed-type: categorical, continuous, discrete, binary variables
  - Categorical: Red, Green, Blue
  - Continuous: [0,1]
  - Discrete: 1,2,3,4,5
  - Binary: Yes, No

## Take Home Messages

- Bayes opt is efficient for optimizing the black-box function.
- Bayes opt sequentially makes suggestion to evaluate the blackbox.
- Optimizing the black-box function with
  - Parallel optimization
  - High dimensional optimization
  - Mixed categorical-continuous optimization.

#### Final Remark

• Existing works in Bayesian optimization are rich, this tutorial is by no means to handle every aspect of the field.

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# Question and Answer

