A COMBINED DOMAIN DECOMPOSITION – MODEL ORDER REDUCTION TECHNIQUE FOR FAST FINITE ELEMENT PARAMETRIC SWEEPS

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I – Introduction

Although the increasing computer power allows for numerical solution of electromagnetically large problems via accurate full-wave techniques like the finite element (FE) method, computing times can still be unaffordable when such analyses are cast into parametric studies, optimization schemes or tolerance analysis problems.

Among the many possible ways to obtain faster analyses, model order reduction techniques (MOR) [1] and domain decomposition methods (DD) [2] represent quite complementary approaches. The goal of MOR is the efficient characterization of a system over a whole parameter space by reducing the dimension of the discretized model. While MOR was originally applicable to single-parameter systems only, it has recently been extended to the multi-parameter case [3], [4]. MOR approaches can be classified as either single-point [3], [5], [6] or multi-point methods [4], [7]. Multi-point schemes are more robust but come along with the drawback of requiring a full FE solution at each sampling point. DD, on the other hand, is a technique for reducing a large problem, which would not be numerically solvable as a whole, to a set of smaller, coupled problems which can be solved one-by-one in a divide-and-conquer scheme [8], [9].

In this paper, the two techniques are combined in the following way: first, the model is partitioned so that all parameter-dependent features are located in a single subdomain [10]. Then, the DD technique is employed to reduce the FE system to this subdomain only, and, finally, a multi-point MOR method is applied to produce a low-dimensional model for this region, which can be evaluated efficiently at any point in parameter space.

As a test case, passive waveguide devices are considered, and, for the sake of simplicity, only devices uniform either along the E-plane or the H-plane, i.e. devices that can be solved by 2D FEM, are considered. The next Section briefly presents the methodology, while Section III gives some numerical results, and Section IV draws conclusions.

II – Methodology

This work employs a 2D E-field FE formulation for H-plane devices [11]. Figure 1 sketches a generic 2-port waveguide device in the H-plane, subdivided into an arbitrary number of domains Ω_i which are separated by boundaries Γ_{ij}. Specifically, Γ_{0j} denotes the outer boundary of domain Ω_i, where Dirichlet or Neumann boundary conditions are

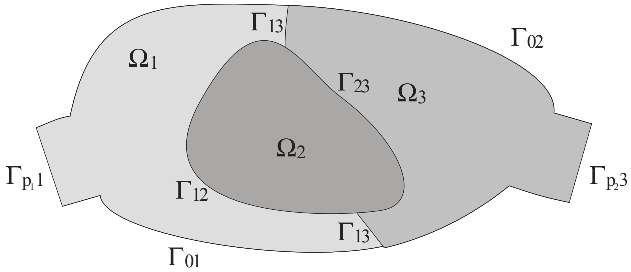

***Figure 1** – Geometry of the problem: a generic 2 port waveguide device subdivided onto three subdomains.*

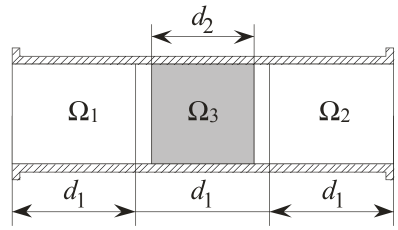

***Figure 2** – Test case: a WR90 (a=22.86mm, b=10.16mm) waveguide section with a dielectric slab in its center.*

to be imposed, and $\Gamma_{p_k j}$ represents the portion of the boundary of $\mathbf{\Omega}_j$ pertaining to port p_k of the device, onto which a suitable modal expansion is to be imposed. It can be shown that the FEM solution of this problem in a DD framework leads to a linear system of equations of the form

$$\begin{bmatrix} \mathbf{M}_1 & 0 & \cdots & 0 & \mathbf{E}_1 \\ 0 & \mathbf{M}_2 & \cdots & 0 & \mathbf{E}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{M}_N & \mathbf{E}_N \\ \mathbf{E}_1^T & \mathbf{E}_2^T & \cdots & \mathbf{E}_N^T & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \mathbf{c} \end{bmatrix}, \tag{1}$$

in which the matrices $\mathbf{M}_i$ correspond to the internal unknowns $\mathbf{x}_i$ of subdomain Ω_i, matrix $\mathbf{C}$ is associated with unknowns $\mathbf{y}$ on boundaries, and the matrices $\mathbf{E}_i$ represent interactions between internal and boundary unknowns. The solution of (1) in a DD framework resorts to the Schur complement $\mathbf{S}$

$$\mathbf{S} = \sum_{i=1}^{N} \mathbf{C}_i - \mathbf{E}_i^T \mathbf{M}_i^{-1} \mathbf{E}_i \ , \tag{2}$$

from which the solution for the boundary unknowns can be obtained as $\mathbf{Sy} = \mathbf{c}$. Provided that all structures with parameter-dependent material properties ε_r and μ_r are placed in domain N, the first N-1 components of the Schur complement can be computed and stored once for all. The decomposition $\mathbf{M}_N = \mathbf{M}_N^0 + \varepsilon_r \mathbf{M}_N^\varepsilon + \mu_r^{-1} \mathbf{M}_N^\mu$ leads to a parameterized linear system of equations of the form

$$\left(\mathbf{A}_0 + \varepsilon_r \mathbf{A}_1 + \mu_r^{-1} \mathbf{A}_2 \right) \mathbf{z} = \mathbf{b} \ , \tag{3}$$

where $\dim \mathbf{A}_i = \dim \mathbf{C} + \dim \mathbf{M}_N = n$, $i \in \{0,1,2\}$. The ROM is constructed by applying to (3) a Galerkin projection with a suitable orthonormal matrix $\mathbf{V} \in \mathbb{C}^{n \times m}$, $m \ll n$. Here, $\mathbf{V}$ is chosen so that $\operatorname{colsp} \mathbf{V} = \operatorname{span}\{\mathbf{z}_1, \ldots, \mathbf{z}_m\}$, where $\mathbf{z}_k$ is the solution of (3) at an interpolation point $p_k = \left(\varepsilon_r^k, \mu_r^k \right)$ in parameter space. Since (3) stems from a single subdomain, it is of much lower dimension than the original system (1). Hence solution vectors and also the final ROM based on (3) can be constructed more efficiently.

III - Numerical Results

The geometry of the test case is depicted in Fig. 2. It comprises a WR90 waveguide segment, longitudinally subdivided into 3 subdomains, each of length $d_1 = 9mm$. Within the last, inner, domain there is a slab of diameter $d_2 = 7mm$, whose relative permittivity and permeability vary in the range $\varepsilon_r = (1\ldots9) - j0.01$ and $\mu_r = 1\ldots9$, respectively. The problem is indeed so simple that it has an analytical solution, against which numerical results are validated. The problem is discretized using 57600 first order triangular elements and by expanding the field on each port into 1 mode, yielding a total of 29081 degrees of freedom. By applying the DD technique to the outer domains once, a reduced FE system for the core region (3) of dimension 10348 is obtained. The latter is solved at 9×9 equidistant points in (ε_r, μ_r) space to construct the MOR projection matrix $\mathbf{V}$.

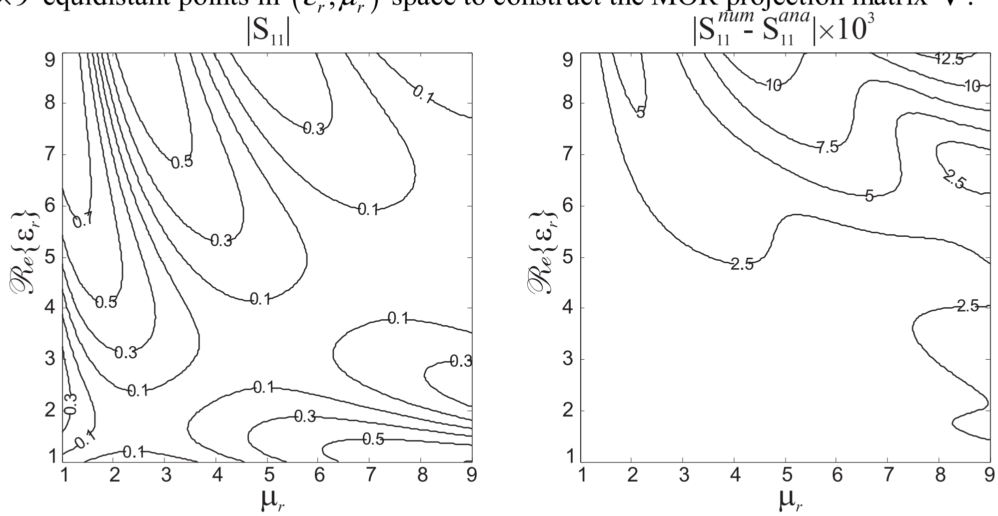

***Figure 3** – Numerical results for the test case at 11 GHz over a material parameter sweep: numerically evaluated reflection coefficient amplitude (left); amplitude of the difference between the numerical and the analytical solutions (right).*

Fig. 3 reports the contour lines relative to the numerically computed reflection coefficient and to the error with respect to the analytical solution. It is worth mentioning that the contour plots were generated over a grid of 151×151 values, thus requiring 22801 computations. Fig. 4 reports, for fixed values of ε_r and as a function of μ_r, the absolute error between the reflection coefficients computed by FEM+DD and FEM+DD+MOR. The values ε_r have been chosen so as to avoid passing through a MOR interpolation point, where the error is minimal. Fig. 4 demonstrates the very high accuracy of the MOR technique, and that enough interpolation points were taken. Finally, Tab. I reports a comparison between the time needed for the computation with the proposed technique, with a DD approach alone, and by a straightforward FEM implementation. It can be seen that the evaluation of the ROM is by three orders of magnitude faster than the two other methods. By incorporating the DD approach, the time for ROM generation is halved. This speed-up increases with the dimension of (1) and the number of interpolation points.

Conclusions

We have presented a combined DD-MOR method, which extends existing approaches by incorporating the DD in the generation process of the ROM. The DD method allows the efficient use of a multipoint MOR procedure, since only the subdomain containing parameter-dependent features has to be solved at the ROM interpolation points. In future work, we will extend our approach to frequency and geometry parameters, and demonstrate its efficiency in optimization scenarios.

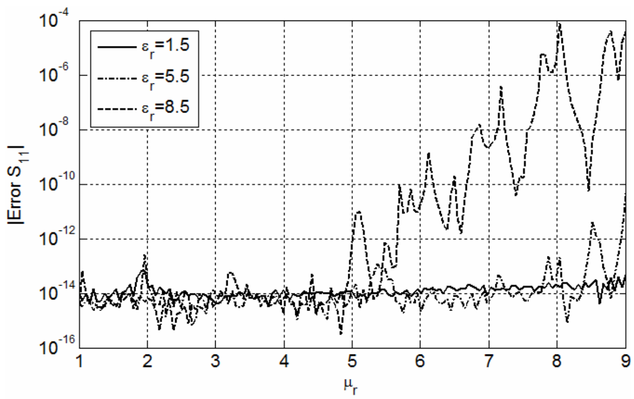

***Figure 4** – Absolute error between the FEM+DD and the FEM+DD+MOR computed reflection coefficients.*

TABLE I – CPU TIMES IN S

EVALUATION

	PER SAMPLE	TOTAL
FEM	0.923	21046
FEM+DD	0.536	12217
MOR	0.00672	153

ROM GENERATION

	TOTAL
FEM	94.41
FEM+DD	49.45

References

[1] A. C. Antoulas, *Approximation of Large-Scale Dynamical Systems*, SIAM, Philadelphia (PA), 2005.

[2] Y. Saad, *Iterative methods for sparse linear systems*, SIAM, Philadelphia (PA), 2ed., 2003.

[3] D. S. Weile, E. Michielssen; E. Grimme, K. Gallivan, "A method for generating rational interpolant reduced order models of two-parameter linear systems", *Applied Mathematics Letters* 12 (1999), Juli, pp. 93–102.

[4] C. Prud'homme, D. V. Rovas, K. Veroy, L. Machiels, Y. Maday, A. Patera, G. Turinici, "Reliable real-time solution of parametrized partial differential equations: Reduced-basis output bound methods", *Journal of Fluids Engineering* 124 (2002), pp. 70–80.

[5] O. Farle, V. Hill, P. Nickel, R. Dyczij-Edlinger, "Multivariate Finite Element Model Order Reduction for Permittivity or Permeability Estimation", *IEEE Trans. Magn.*, Vol. 42, No. 4, Apr. 2006, pp. 623-626.

[6] O. Farle, V. Hill, P. Ingelström, R. Dyczij-Edlinger, "Multi-parameter polynomial order reduction of linear finite element models", *Mathematical and Computer Modelling of Dynamical Systems*. – accepted for publication.

[7] D. S. Weile, E. Michielssen, "Analysis of frequency selective surfaces using two-parameter generalized rational Krylov model-order reduction", *IEEE Trans. Antennas Propag*. 49 (2001), November, S. 1539–1549.

[8] B. Stupfel, M. Mognot, "A domain decomposition method for the vector wave equation", *IEEE Trans. Antennas Propag*., Vol. 48, No. 5, May 2000, pp. 653 - 60.

[9] M. N. Vouvakis, Z. J. Cendes, J.-F. Lee, "A FEM Domain Decomposition Method for Photonic and Electromagnetic Band Gap Structures", *IEEE Trans. Antennas Propag.*, Vol. 54, No. 2, Feb. 2006, pp. 721-733.

[10] G. Guarnieri, G. Pelosi, L. Rossi, S. Selleri, "A new FEM/BEM domain decomposition approach for solving high diversity electromagnetic problems," 8th International Workshop on Finite Elements for Microwave Engineering, 25-26 May 2006, Stellenbosh (South Africa), p. 60.

[11] G. Pelosi, R. Coccioli, S. Selleri, *Quick Finite Elements for Electromagnetic Waves*, Artech House, London, 1998.