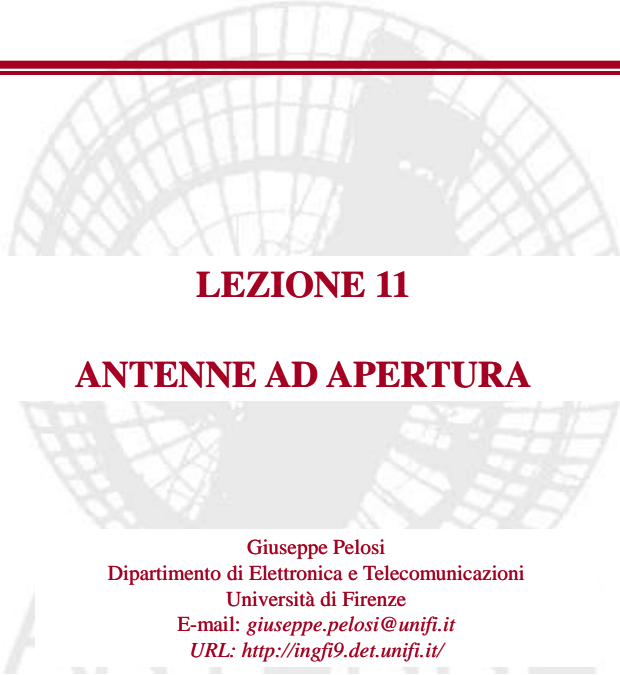


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 Prof. G. Pelosi - Laboratorio di Elettromagnetismo Numerico  
 Dipartimento di Elettronica e Telecomunicazioni – Università di Firenze

Lezione 11 – Antenne ad apertura

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## LEZIONE 11

### ANTENNE AD APERTURA

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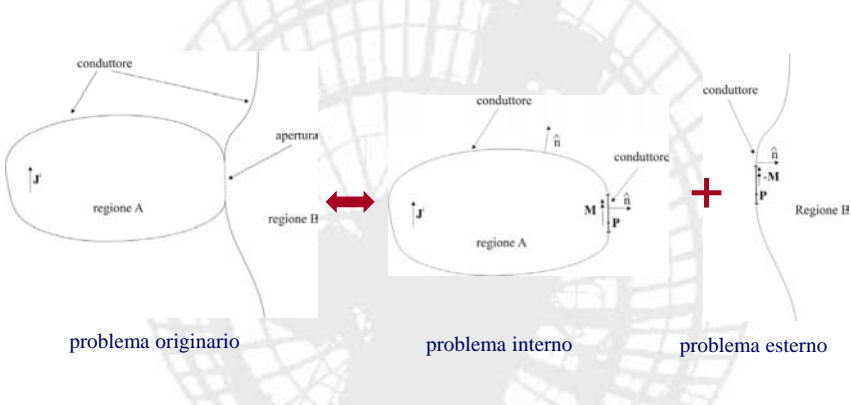


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Lezione 11 – Antenne ad apertura

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### TEOREMA DI EQUIVALENZA NELLA FORMULAZIONE DI HARRINGTON-MAUTZ



problema originario      problema interno      problema esterno

equazioni di accoppiamento

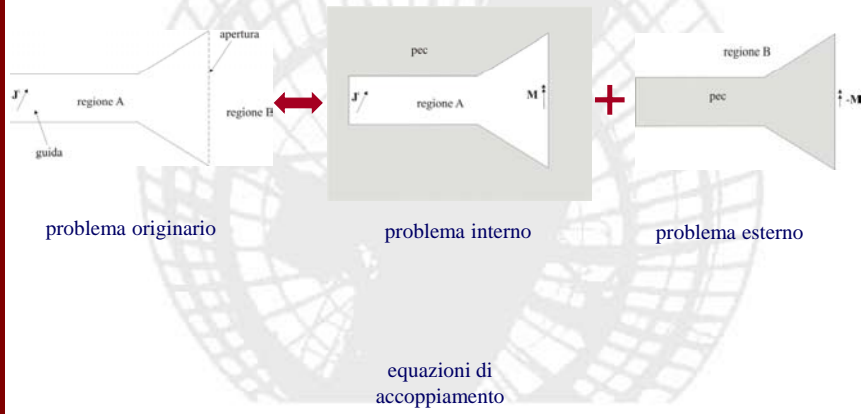
$$\mathbf{H}_t^a(\mathbf{P}) = \mathbf{H}_t^i(\mathbf{J}^i; \mathbf{P}) + \mathbf{H}_t^a(\mathbf{M}; \mathbf{P})$$

$$\mathbf{H}_t^b(\mathbf{P}) = \mathbf{H}_t^b(-\mathbf{M}; \mathbf{P}) = -\mathbf{H}_t^b(\mathbf{M}; \mathbf{P})$$

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**TEOREMA DI EQUIVALENZA NELLA FORMULAZIONE DI HARRINGTON-MAUTZ**



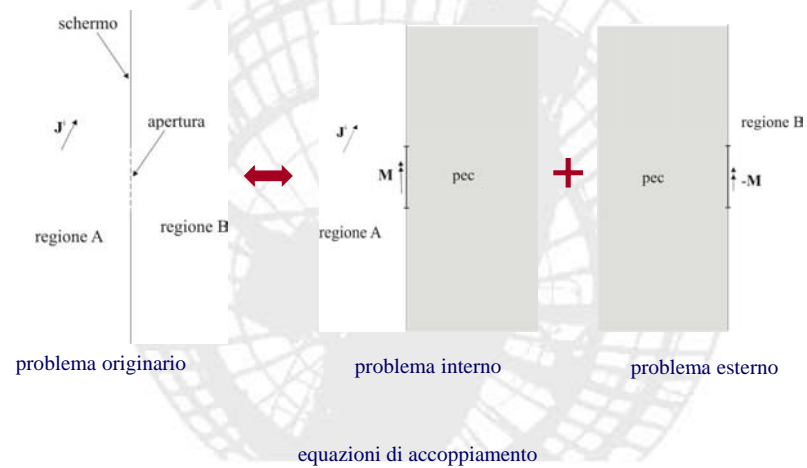
$$\mathbf{H}_t^a(\mathbf{P}) = \mathbf{H}_t^i(\mathbf{J}^i; \mathbf{P}) + \mathbf{H}_t^e(\mathbf{M}; \mathbf{P})$$

$$\mathbf{H}_t^b(\mathbf{P}) = \mathbf{H}_t^b(-\mathbf{M}; \mathbf{P}) = -\mathbf{H}_t^b(\mathbf{M}; \mathbf{P})$$

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**TEOREMA DI EQUIVALENZA NELLA FORMULAZIONE DI HARRINGTON-MAUTZ**



$$\mathbf{H}_t^a(\mathbf{P}) = \mathbf{H}_t^i(\mathbf{J}^i; \mathbf{P}) + \mathbf{H}_t^e(\mathbf{M}; \mathbf{P})$$

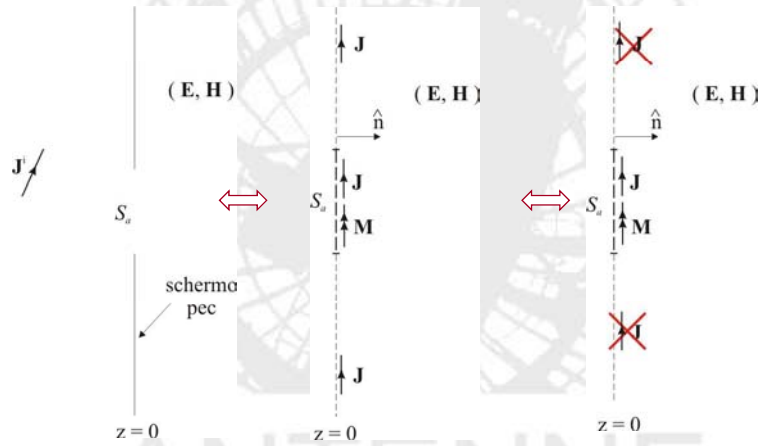
$$\mathbf{H}_t^b(\mathbf{P}) = \mathbf{H}_t^b(-\mathbf{M}; \mathbf{P}) = -\mathbf{H}_t^b(\mathbf{M}; \mathbf{P})$$

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### APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO Approssimazione di Kirchhoff

Si considera il campo sull'apertura quello che ci sarebbe in assenza dello schermo



Si stimano  $\mathbf{J}$  ed  $\mathbf{M}$  sull'apertura

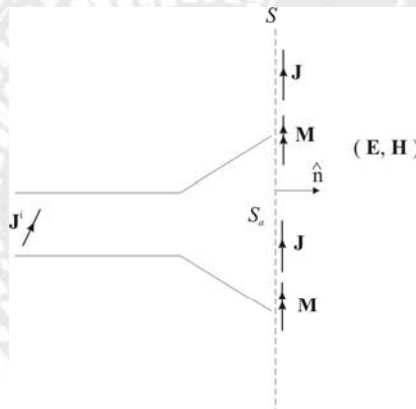
$$\mathbf{M} = \mathbf{E}^i \times \hat{\mathbf{n}}$$

$$\mathbf{J} = \hat{\mathbf{n}} \times \mathbf{H}^i$$

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### ANTENNA A TROMBA



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## APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO

### Approssimazione di Kirchhoff

Si supponga di aver determinato in qualche modo il campo elettrico,  $\mathbf{E}_a$ , e magnetico,  $\mathbf{H}_a$ , sulla superficie  $S$  di equivalenza

Si supponga che i campi siano diversi da zero solo nella porzione di superficie  $S_a$

$$\begin{cases} \mathbf{J}_s = \hat{n} \times \mathbf{H}_a \\ \mathbf{M}_s = -\hat{n} \times \mathbf{E}_a \end{cases} \quad \text{su } S_a$$

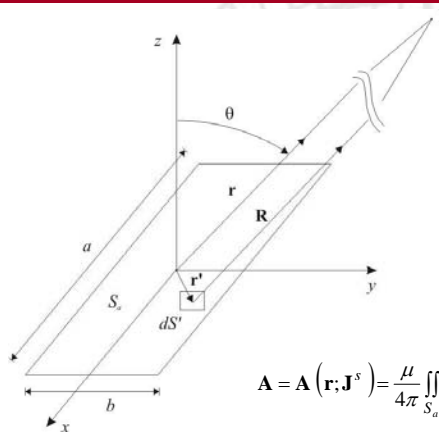
$$\begin{cases} \mathbf{J}_s = 0 \\ \mathbf{M}_s = 0 \end{cases} \quad \text{altrove}$$

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## APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO

### Approssimazione di Kirchhoff



$$\mathbf{r}' = x' \hat{x} + y' \hat{y}$$

$$R = |\mathbf{r} - \mathbf{r}'|$$

$$\mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \epsilon \mu} - \frac{1}{\epsilon} \nabla \times \mathbf{F}$$

$$\begin{cases} \mathbf{J}_s = \hat{n} \times \mathbf{H}_a \\ \mathbf{M}_s = -\hat{n} \times \mathbf{E}_a \end{cases}$$

$$\mathbf{A} = \mathbf{A}(\mathbf{r}; \mathbf{J}^s) = \frac{\mu}{4\pi} \iint_{S_a} \mathbf{J}^s(\mathbf{r}') \frac{e^{-jkR}}{R} dS' = \frac{\mu}{4\pi} \hat{n} \times \iint_{S_a} \mathbf{H}_a(\mathbf{r}') \frac{e^{-jkR}}{R} dS'$$

$$\mathbf{F} = \mathbf{F}(\mathbf{r}; \mathbf{M}^s) = -\frac{\epsilon}{4\pi} \hat{n} \times \iint_{S_a} \mathbf{E}_a(\mathbf{r}') \frac{e^{-jkR}}{R} dS'$$

al denominatore  $\frac{1}{R} \approx \frac{1}{r}$

all'esponente  $R \approx r - \mathbf{r}' \cdot \hat{r}$

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## APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO

### Approssimazione di Kirchhoff

Potenziali vettori in campo lontano

$$\mathbf{A} = \mu \frac{e^{-jkr}}{4\pi r} \hat{n} \times \iint_{S_a} \mathbf{H}_a e^{jk\hat{r} \cdot \mathbf{r}'} dS'$$

$$\mathbf{Q} = \iint_{S_a} \mathbf{H}_a e^{jk\hat{r} \cdot \mathbf{r}'} dS' = Q_x \hat{x} + Q_y \hat{y}$$

$$\mathbf{F} = -\varepsilon \frac{e^{-jkr}}{4\pi r} \hat{n} \times \iint_{S_a} \mathbf{E}_a e^{jk\hat{r} \cdot \mathbf{r}'} dS'$$

$$\mathbf{P} = \iint_{S_a} \mathbf{E}_a e^{jk\hat{r} \cdot \mathbf{r}'} dS' = P_x \hat{x} + P_y \hat{y}$$

$$\hat{r} \cdot \mathbf{r}' = (\hat{x} \sin \vartheta \cos \phi + \hat{y} \sin \vartheta \sin \phi + \hat{z} \cos \vartheta) \cdot (\hat{x} x' + \hat{y} y') = x' \sin \vartheta \cos \phi + y' \sin \vartheta \sin \phi$$



## APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO

### Approssimazione di Kirchhoff

$$P_x = \iint_{S_a} E_{ax}(x', y') e^{jk(x' \sin \vartheta \cos \phi + y' \sin \vartheta \sin \phi)} dx' dy'$$

$$P_y = \iint_{S_a} E_{ay}(x', y') e^{jk(x' \sin \vartheta \cos \phi + y' \sin \vartheta \sin \phi)} dx' dy'$$

$$Q_x = \iint_{S_a} H_{ax}(x', y') e^{jk(x' \sin \vartheta \cos \phi + y' \sin \vartheta \sin \phi)} dx' dy'$$

$$Q_y = \iint_{S_a} H_{ay}(x', y') e^{jk(x' \sin \vartheta \cos \phi + y' \sin \vartheta \sin \phi)} dx' dy'$$

$$\hat{n} = \hat{z}$$

$$\mathbf{A} = \mu \frac{e^{-jkr}}{4\pi r} (-Q_y \hat{x} + Q_x \hat{y})$$

$$\mathbf{F} = -\varepsilon \frac{e^{-jkr}}{4\pi r} (-P_y \hat{x} + P_x \hat{y})$$

**APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO**  
**Approssimazione di Kirchhoff**

Esprimendo i versori in coordinate sferiche e lasciando soltanto le componenti trasverse alla direzione radiale

$$\hat{x} = \hat{r} \sin \vartheta \cos \phi + \hat{\vartheta} \cos \vartheta \cos \phi - \hat{\phi} \sin \phi$$

$$\hat{y} = \hat{r} \sin \vartheta \sin \phi + \hat{\vartheta} \cos \vartheta \sin \phi + \hat{\phi} \cos \phi$$

$$\mathbf{A} = \mu \frac{e^{-jkr}}{4\pi r} \left[ \hat{\vartheta} \cos \vartheta (Q_x \sin \phi - Q_y \cos \phi) + \hat{\phi} (Q_x \cos \phi + Q_y \sin \phi) \right]$$

$$\mathbf{F} = -\varepsilon \frac{e^{-jkr}}{4\pi r} \left[ \hat{\vartheta} \cos \vartheta (P_x \sin \phi - P_y \cos \phi) + \hat{\phi} (P_x \cos \phi + P_y \sin \phi) \right]$$

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**APERTURA RETTANGOLARE IN UNO SCHERMO METALLICO**  
**Approssimazione di Kirchhoff**

$$\mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \varepsilon \mu} - \frac{1}{\varepsilon} \nabla \times \mathbf{F}$$

in campo lontano:  $\mathbf{E} \approx -j\omega (A_\theta \hat{\vartheta} + A_\phi \hat{\phi}) + j\omega \zeta \hat{r} \times \mathbf{F}$

$$E_\vartheta = jk \frac{e^{-jkr}}{4\pi r} \left[ P_x \cos \phi + P_y \sin \phi + \zeta \cos \vartheta (Q_y \cos \phi - Q_x \sin \phi) \right]$$

$$E_\phi = jk \frac{e^{-jkr}}{4\pi r} \left[ \cos \vartheta (P_y \cos \phi - P_x \sin \phi) - \zeta (Q_y \sin \phi + Q_x \cos \phi) \right]$$

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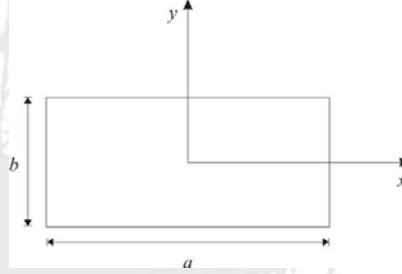
## APERTURA RETTANGOLARE UNIFORME

Campo elettrico sull'apertura  
polarizzato lungo y e uniforme in  
ampiezza e fase

$$\mathbf{E}_a = E_0 \hat{y}$$

$$|x| \leq \frac{a}{2}$$

$$|y| \leq \frac{b}{2}$$



$$\begin{cases} P_y = E_0 dy' \int_{-a/2}^{+a/2} e^{jkx' \sin \vartheta \cos \phi} dx' \int_{-b/2}^{+b/2} e^{jky' \sin \vartheta \sin \phi} dy' = E_0 ab \frac{\sin u}{u} \frac{\sin v}{v} \\ P_x = 0 \end{cases}$$

$$u = \frac{ka}{2} \sin \vartheta \cos \phi$$

$$v = \frac{kb}{2} \sin \vartheta \sin \phi$$

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## APERTURA RETTANGOLARE UNIFORME

Si applica il teorema di equivalenza nella versione con sole correnti magnetiche

$$\mathbf{J}_s = 0$$

$$\mathbf{M}_s = -2 \hat{n} \times \mathbf{E}_a$$



$$E_\vartheta = jk \frac{e^{-jkr}}{2\pi r} P_y \sin \phi$$

$$E_\phi = jk \frac{e^{-jkr}}{2\pi r} \cos \vartheta P_y \cos \phi$$

$$E_\vartheta = jk \frac{e^{-jkr}}{2\pi r} E_0 ab \sin \phi \frac{\sin u}{u} \frac{\sin v}{v}$$

$$E_\phi = jk \frac{e^{-jkr}}{2\pi r} E_0 ab \cos \vartheta \cos \phi \frac{\sin u}{u} \frac{\sin v}{v}$$

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## APERTURA RETTANGOLARE UNIFORME

## Piano E

$$\phi = \frac{\pi}{2}$$

diagramma normalizzato

$$E_{\theta} = jk \frac{e^{-jkr}}{2\pi r} E_0 a b \frac{\sin[(kb/2)\sin\theta]}{(kb/2)\sin\theta}$$

$$E_r = E_{\phi} = 0$$

$$F_E(\theta) = \frac{\sin[(kb/2)\sin\theta]}{(kb/2)\sin\theta}$$

Nulli diagramma di radiazione

$$\frac{kb}{2} \sin\theta = n\pi, \quad n = 1, 2, 3, \dots$$

$$\theta_n = \sin^{-1} \frac{n\lambda}{b}, \quad n = 1, 2, 3, \dots$$

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## APERTURA RETTANGOLARE UNIFORME

## Piano H

$$\phi = 0$$

diagramma normalizzato

$$E_{\phi} = jk \frac{e^{-jkr}}{2\pi r} E_0 a b \cos\theta \frac{\sin[(ka/2)\sin\theta]}{(ka/2)\sin\theta}$$

$$E_{\theta} = 0$$

$$F_H(\theta) = \cos\theta \frac{\sin[(ka/2)\sin\theta]}{(ka/2)\sin\theta}$$

L'andamento del campo è identico a quello sul piano H a meno del fattore  $\cos\theta$ .Quando  $a$  è grande rispetto alla lunghezza d'onda questo fattore è lentamente variabile rispetto al "sinc"In prima approssimazione le relazioni valide per il piano E risultano ancora valide a patto di scambiare  $a$  con  $b$ 

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### APERTURA RETTANGOLARE UNIFORME

#### Solido di radiazione

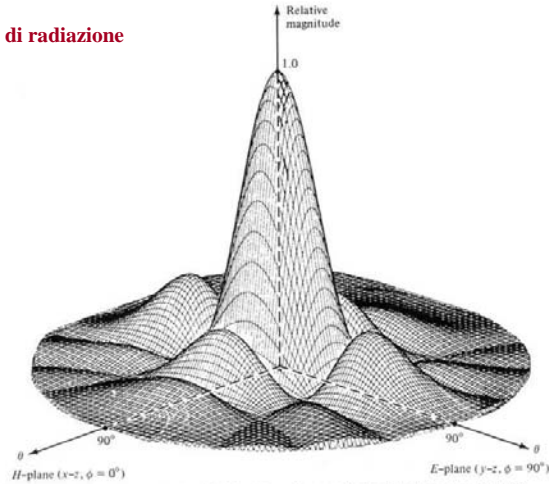


Figure 11.8 Three-dimensional field pattern of a constant field rectangular aperture mounted on an infinite ground plane ( $a = 3\lambda$ ,  $b = 2\lambda$ ).

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### APERTURA RETTANGOLARE UNIFORME

#### Diagrammi di radiazione sui piani principali

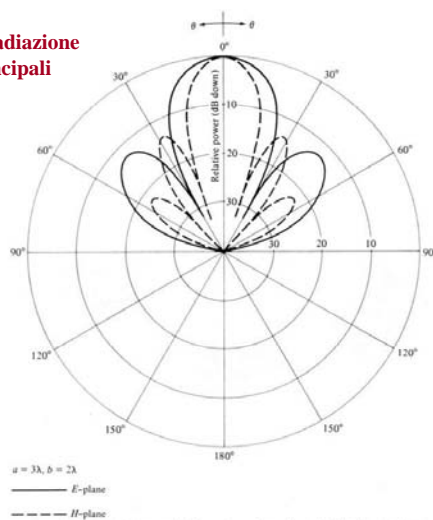


Figure 11.10 E- and H-plane amplitude patterns for uniform distribution aperture mounted on an infinite ground plane ( $a = 3\lambda$ ,  $b = 2\lambda$ ).

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**DIRETTIVITA'**

per qualunque antenna

$$D(\vartheta, \phi) = \lim_{r \rightarrow \infty} \frac{4\pi r^2 \frac{1}{2} \frac{|E|^2}{\zeta}}{P_r} \quad D_{\max} = \lim_{r \rightarrow \infty} \frac{4\pi r^2 \frac{1}{2} \frac{|E|_{\max}^2}{\zeta}}{P_r}$$

$$P_r = \frac{1}{2\zeta} \int_0^{\pi} \int_0^{2\pi} \left( |E_\vartheta|^2 + |E_\phi|^2 \right) r^2 \sin \vartheta \, d\vartheta \, d\phi$$

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**DIRETTIVITA'**

Se sull'apertura si suppone di avere un'onda piana (casi pratici) la potenza totale radiata può essere valutata come potenza che fluisce attraverso l'apertura

$$P_r = \frac{1}{2\zeta} \iint_{S_a} |E_a|^2 \, dS'$$

L'approssimazione risulta valida se i campi sull'apertura variano lentamente rispetto alla lunghezza d'onda la distribuzione di fase sull'apertura è quasi uniforme

$$P_r = \frac{1}{2\zeta} \int_{-a/2}^{+a/2} \int_{-b/2}^{+b/2} E_0^2 \, dx' \, dy' = \frac{1}{2\zeta} E_0^2 \, a \, b$$

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## DIRETTIVITA'

Per una apertura rettangolare illuminata uniformemente si ha:

massimo valore del campo elettrico ( $\theta = 0$ )

$$E_{\theta \max} = jk \frac{e^{-jkr}}{2\pi r} E_0 ab \quad D_{\max} = \lim_{r \rightarrow \infty} \frac{4\pi r^2 \frac{1}{2} \frac{|E|_{\max}^2}{\zeta}}{P_r}$$

$$|E|_{\max}^2 = \frac{k^2 E_0^2 a^2 b^2}{4\pi^2 r^2}$$



area fisica dell'apertura

$$D_{\max} = \frac{4\pi \frac{1}{2\zeta} \frac{k^2 E_0^2 a^2 b^2}{4\pi^2}}{\frac{1}{2\zeta} E_0^2 ab} = \frac{4\pi}{\lambda^2} S_a$$

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## DIRETTIVITA'

Se la distribuzione del campo sull'apertura non è uniforme si ha:

$$D_{\max} = \frac{4\pi}{\lambda^2} A_e$$

$A_e$  area efficace dell'antenna

$A_e < S_a$

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Lezione 11 – Antenne ad apertura

APERTURE RETTANGOLARI

Table 11.1 EQUIVALENTS, FIELDS, BEAMWIDTHS, SIDE LOBE LEVELS, AND DIRECTIVITIES OF RECTANGULAR APERTURES

	UNIFORM DISTRIBUTION APERTURE ON GROUND PLANE	UNIFORM DISTRIBUTION APERTURE IN FREE-SPACE	TE <sub>10</sub> -MODE DISTRIBUTION APERTURE ON GROUND PLANE
Aperture distribution of tangential components (analytical)	$\mathbf{E}_a = \hat{a}_x E_0 \left\{ \begin{array}{l} -a/2 \leq x' \leq a/2 \\ -b/2 \leq y' \leq b/2 \end{array} \right.$	$\mathbf{E}_a = \hat{a}_x E_0 \left\{ \begin{array}{l} -a/2 \leq x' \leq a/2 \\ -b/2 \leq y' \leq b/2 \end{array} \right.$ $\mathbf{H}_a = -\hat{a}_y \frac{E_0}{\eta}$	$\mathbf{E}_a = \hat{a}_x E_0 \cos\left(\frac{\pi}{a} x'\right) \left\{ \begin{array}{l} -a/2 \leq x' \leq a/2 \\ -b/2 \leq y' \leq b/2 \end{array} \right.$
Aperture distribution of tangential components (graphical)			
Equivalent	$\mathbf{M}_s = \left\{ \begin{array}{l} -2\hat{n} \times \mathbf{E}_a \quad -a/2 \leq x' \leq a/2 \\ 0 \quad \text{elsewhere} \end{array} \right.$ $\mathbf{J}_s = 0 \quad \text{everywhere}$	$\mathbf{M}_s = -\hat{n} \times \mathbf{E}_a \left\{ \begin{array}{l} -a/2 \leq x' \leq a/2 \\ -b/2 \leq y' \leq b/2 \end{array} \right.$ $\mathbf{J}_s = \hat{n} \times \mathbf{H}_a \left\{ \begin{array}{l} -a/2 \leq x' \leq a/2 \\ -b/2 \leq y' \leq b/2 \end{array} \right.$ $\mathbf{M}_s \approx \mathbf{J}_s = 0 \quad \text{elsewhere}$	$\mathbf{M}_s = \left\{ \begin{array}{l} -2\hat{n} \times \mathbf{E}_a \quad -a/2 \leq x' \leq a/2 \\ 0 \quad \text{elsewhere} \end{array} \right.$ $\mathbf{J}_s = 0 \quad \text{everywhere}$
Far-zone fields	$E_r = H_r = 0$ $E_\theta = C \sin \phi \frac{\sin X}{X} \frac{\sin Y}{Y}$ $E_\phi = C \cos \theta \cos \phi \frac{\sin X}{X} \frac{\sin Y}{Y}$ $H_\theta = -E_\phi / \eta$ $H_\phi = E_\theta / \eta$ $C = j \frac{abk E_0 e^{-jkr}}{2\pi r}$	$E_r = H_r = 0$ $E_\theta = \frac{C}{2} \sin \phi (1 + \cos \theta) \frac{\sin X}{X} \frac{\sin Y}{Y}$ $E_\phi = \frac{C}{2} \cos \phi (1 + \cos \theta) \frac{\sin X}{X} \frac{\sin Y}{Y}$ $H_\theta = -E_\phi / \eta$ $H_\phi = E_\theta / \eta$	$E_r = H_r = 0$ $E_\theta = -\frac{\pi}{2} C \sin \phi \frac{\cos X}{(X)^2 - (\frac{\pi}{2})^2} \frac{\sin Y}{Y}$ $E_\phi = -\frac{\pi}{2} C \cos \theta \cos \phi \frac{\cos X}{(X)^2 - (\frac{\pi}{2})^2} \frac{\sin Y}{Y}$ $H_\theta = -E_\phi / \eta$ $H_\phi = E_\theta / \eta$

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Lezione 11 – Antenne ad apertura

APERTURE RETTANGOLARI

Half-power beamwidth (degrees)	E-plane $b \gg \lambda$	$\frac{50.6}{b/\lambda}$	$\frac{50.6}{b/\lambda}$	$\frac{50.6}{b/\lambda}$
	H-plane $a \gg \lambda$	$\frac{50.6}{a/\lambda}$	$\frac{50.6}{a/\lambda}$	$\frac{68.8}{a/\lambda}$
First null beamwidth (degrees)	E-plane $b \gg \lambda$	$\frac{114.6}{b/\lambda}$	$\frac{114.6}{b/\lambda}$	$\frac{114.6}{b/\lambda}$
	H-plane $a \gg \lambda$	$\frac{114.6}{a/\lambda}$	$\frac{114.6}{a/\lambda}$	$\frac{171.9}{a/\lambda}$
First side lobe max. (to main max.) (dB)	E-plane	-13.26	-13.26	-13.26
	H-plane $a \gg \lambda$	-13.26 $a \gg \lambda$	-13.26 $a \gg \lambda$	-23 $a \gg \lambda$
Directivity $D_0$ (dimensionless)		$\frac{4\pi}{\lambda^2} (\text{area}) = 4\pi \left( \frac{ab}{\lambda^2} \right)$	$\frac{4\pi}{\lambda^2} (\text{area}) = 4\pi \left( \frac{ab}{\lambda^2} \right)$	$\frac{8}{\pi^2} \left[ 4\pi \left( \frac{ab}{\lambda^2} \right) \right] = 0.81 \left[ 4\pi \left( \frac{ab}{\lambda^2} \right) \right]$

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## APERTURE CIRCOLARI

**Table 11.2** EQUIVALENTS, FIELDS, BEAMWIDTHS, SIDE LOBE LEVELS, AND DIRECTIVITIES OF CIRCULAR APERTURES

	UNIFORM DISTRIBUTION APERTURE ON GROUND PLANE	TE <sub>11</sub> -MODE DISTRIBUTION APERTURE ON GROUND PLANE
Aperture distribution of tangential components (analytical)	$\mathbf{E}_a = \hat{a}_\rho E_0 \quad \rho' \leq a$	$\left. \begin{aligned} \mathbf{E}_a &= \hat{a}_\rho E_\rho + \hat{a}_\phi E_\phi \\ E_\rho &= E_0 J_1(\chi_{11}' \rho' / a) \sin \phi' / \rho' \\ E_\phi &= E_0 J_1'(\chi_{11}' \rho' / a) \cos \phi' \end{aligned} \right\} \begin{aligned} \rho' &\leq a \\ \chi_{11}' &= 1.841 \\ \rho' &= \frac{\partial}{\partial \rho'} \end{aligned}$
Aperture distribution of tangential components (graphical)		
Equivalent	$\mathbf{M}_s = \begin{cases} -2\hat{n} \times \mathbf{E}_a & \rho' \leq a \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = 0 \quad \text{everywhere}$	$\mathbf{M}_s = \begin{cases} -2\hat{n} \times \mathbf{E}_a & \rho' \leq a \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = 0 \quad \text{everywhere}$
Far-zone fields	$E_r = H_r = 0$ $Z = ka \sin \theta$ $C_1 = j \frac{ka^2 E_0 e^{-jkr}}{r}$ $C_2 = j \frac{ka E_0 J_1(\chi_{11}') e^{-jkr}}{r}$ $\chi_{11}' = 1.841$ $E_\theta = jC_1 \sin \phi \frac{J_1(Z)}{Z}$ $E_\phi = jC_1 \cos \theta \cos \phi \frac{J_1(Z)}{Z}$ $H_\theta = -E_\phi / \eta$ $H_\phi = E_\theta / \eta$	$E_r = H_r = 0$ $Z = ka \sin \theta$ $E_\theta = C_2 \sin \phi \frac{J_1'(Z)}{Z}$ $E_\phi = C_2 \cos \theta \cos \phi \frac{J_1'(Z)}{1 - (Z/\chi_{11}')^2}$ $H_\theta = -E_\phi / \eta$ $H_\phi = E_\theta / \eta$ $J_1'(Z) = J_0'(Z) - J_1(Z)/Z$

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## APERTURE CIRCOLARI

Half-power beamwidth (degrees)	E-plane $a \gg \lambda$	$\frac{29.2}{a/\lambda}$	$\frac{29.2}{a/\lambda}$
	H-plane $a \gg \lambda$	$\frac{29.2}{a/\lambda}$	$\frac{37.0}{a/\lambda}$
First null beamwidth (degrees)	E-plane $a \gg \lambda$	$\frac{69.9}{a/\lambda}$	$\frac{69.9}{a/\lambda}$
	H-plane $a \gg \lambda$	$\frac{69.9}{a/\lambda}$	$\frac{98.0}{a/\lambda}$
First side lobe max. (to main max.) (dB)	E-plane	-17.6	-17.6
	H-plane	-17.6	-26.2
Directivity $D_0$ (dimensionless)		$\frac{4\pi}{\lambda^2} (\text{area}) = \frac{4\pi}{\lambda^2} (\pi a^2) = \left( \frac{2\pi a}{\lambda} \right)^2$	$0.836 \left( \frac{2\pi a}{\lambda} \right)^2 = 10.5 \pi \left( \frac{a}{\lambda} \right)^2$

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