Short Papers

On the Line-Integral Formulation of Mode-Matching Technique

G. Figlia and G. G. Gentili

Abstract—**In this paper, a brief description of the line-integral formulation of mode matching is recalled, and the analysis is extended to the case of a TEM mode, normalization integrals, and degenerate eigenvalues.**

***Index Terms*—Mode matching, waveguide scattering problems.**

I. Introduction

In spite of the great popularity of the mode-matching (MM) technique to solve waveguide scattering problems, there are still some rather unexplored features concerning the line-integral formulation of the coupling integrals. Line integrals can be used instead of standard surface integrals leading to a more efficient implementation of MM [1]. Line integrals have been used in [2]–[5], and in [6], a generalization of [1] to the hybrid-mode case was also presented.

On the other hand, in [1], the discussion focused only on the important TE/TM case, and the TEM mode (or modes) was not considered. It is shown in this paper that inclusion of the TEM mode does not exclude the possibility to use line integrals to compute coupling coefficients between all modes. In addition, it is also shown how normalization integrals and coupling between degenerate modes can be evaluated in terms of line integrals, leading to a complete self-consistent line-integral formulation of MM.

II. Formulation

We consider two facing lossless homogeneous waveguides, waveguide "b" (the "big" waveguide) and waveguide "s" (the "small" waveguide), whose transverse cross sections are indicated by Ω_b and Ω_s, respectively [Ω_b completely encloses Ω_s, although the boundaries may have a common part (see Fig. 1)]. Let the respective boundaries be indicated by L_b and L_s.

Multiply connected boundaries allow the existence of one or more TEM modes in addition to the standard TE and TM set, and we assume that both waveguides have a multiply connected boundary. Letting z represent the propagation axis and $\mathbf{u}_z$ its normal unit vector, the transverse electric and magnetic fields $\mathbf{e}_n$, $\mathbf{h}_n$ for the generic waveguide "n" (n can be either b or s) can be obtained by two Hertz-type potentials, i.e., φ_n and ψ_n, as follows:

$$\begin{cases} \mathbf{e}_n^{\mathrm{TE}} = A_n \nabla_t \varphi_n \times \mathbf{u}_z \\ \mathbf{h}_n^{\mathrm{TE}} = A_n Y_n \mathbf{u}_z \times \mathbf{e}_n^{\mathrm{TE}} \end{cases} \quad \text{(TE)} \tag{1}$$

$$\begin{cases} \mathbf{e}_n^{\mathrm{TM}} = B_n \nabla_t \psi_n \\ \mathbf{h}_n^{\mathrm{TM}} = B_n Y_n \mathbf{u}_z \times \mathbf{e}_n^{\mathrm{TM}} \end{cases} \quad \text{(TM)} \tag{2}$$

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G. Figlia was with Satelgest Srl, 10155 Turin, Italy. He is now with MICROELIT SPA, I-20146 Milan, Italy (e-mail: gfiglia@tiscalinet.it).

G. G. Gentili is with the Centro Studi per le Telecommunicazioni Spaziali (Consiglio Nazionale delle Ricerche), Dipartimento di Elettronica e Informazione, Politecnico di Milano, I-20133 Milan, Italy (e-mail: gentili@elet.polimi.it).

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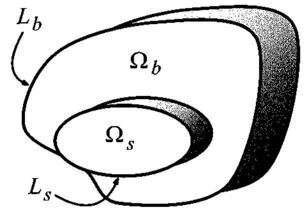

Fig. 1. Step discontinuity between arbitrarily shaped waveguides.

$$\begin{cases} \mathbf{e}_n^{\mathrm{TEM}} = B_{0n} \nabla_t \psi_{0n} \\ \mathbf{h}_n^{\mathrm{TEM}} = B_{0n} Y_{0n} \mathbf{u}_z \times \mathbf{e}_n^{\mathrm{TEM}} \end{cases} \quad \text{(TEM)} \tag{3}$$

where ∇_t represents the "transverse" gradient operator, Y_n (Y_{0n}) is the modal admittance, A_n, B_n, and B_{0n} are arbitrary normalization constants, and, for clarity, we have used a different symbol ψ_{0n} to indicate the TEM hertzian potential. The potentials satisfy the following equations:

$$\begin{cases} (\nabla_t^2 + k_n^2)\varphi_n = 0, & \text{in } \Omega_n \\ \partial\varphi_n/\partial\nu = 0, & \text{on } L_n \end{cases} \tag{4}$$

$$\begin{cases} (\nabla_t^2 + \kappa_n^2)\psi_n = 0, & \text{in } \Omega_n \\ \psi_n = 0, & \text{on } L_n \end{cases} \tag{5}$$

$$\begin{cases} \nabla_t^2 \psi_{0n} = 0, & \text{in } \Omega_n \\ \psi_{0n} = \text{const}, & \text{on } L_n \end{cases} \tag{6}$$

where ν is the outward normal direction. The constant value of ψ_{0n} can be different for any simply connected part of the boundary L_n.

A. Normalization

In order to apply MM, one must first define a normalization for the electric or magnetic field. We choose an explicit normalization for the electric field and assume that the transverse electric mode functions are real (this is possible since we only deal with the case of lossless homogeneous waveguides). Under this assumption, to simplify the notation we define

$$\langle \mathbf{a}, \mathbf{b} \rangle = \iint_{\Omega} \mathbf{a} \cdot \mathbf{b} \, d\Omega.$$

By applying Green's first identity

$$\iint_{\Omega} \nabla_t f \cdot \nabla_t g d\Omega = -\iint_{\Omega} g \nabla_t^2 f \, d\Omega + \oint_L g \frac{\partial f}{\partial \nu} \, dl \tag{7}$$

in a region Ω with boundary L, one can easily obtain that

$$\langle \mathbf{e}_n^{\mathrm{TE}}, \mathbf{e}_n^{\mathrm{TE}} \rangle = A_n^2 k_n^2 \iint_{\Omega_n} \varphi_n^2 \, d\Omega \tag{8}$$

$$\langle \mathbf{e}_n^{\mathrm{TM}}, \mathbf{e}_n^{\mathrm{TM}} \rangle = B_n^2 \kappa_n^2 \iint_{\Omega_n} \psi_n^2 \, d\Omega \tag{9}$$

$$\langle \mathbf{e}_n^{\mathrm{TEM}}, \mathbf{e}_n^{\mathrm{TEM}} \rangle = B_{0n}^2 \oint_{L_n} \psi_{0n} \frac{\partial \psi_{0n}}{\partial \nu} \, dl. \tag{10}$$

We impose the following normalization condition:

$$\langle \mathbf{e}_n, \mathbf{e}_n \rangle = 1. \tag{11}$$

Equation (11) can be expressed as a line integral. For any function F satisfying

$$(\nabla_t^2 + k^2)F = 0 \tag{12}$$

in a region Ω with boundary L, by applying L'Hopital rule, one can obtain the following identity (see the Appendix):

$$\iint\limits_{\Omega} F^2 \, d\Omega = -\frac{1}{2k} \oint_L \left(F \frac{\partial^2 F}{\partial k \, \partial \nu} - \frac{\partial F}{\partial k} \frac{\partial F}{\partial \nu} \right) dl. \tag{13}$$

Applying (11) and (13) with the appropriate boundary conditions for the potentials leads to

$$A_n = \pm \left| -\frac{k_n}{2} \oint_{L_n} \left(\varphi_n \frac{\partial^2 \varphi_n}{\partial k_n \, \partial \nu} \right) dl \right|^{-1/2} \tag{14}$$

$$B_n = \pm \left| \frac{\kappa_n}{2} \oint_{L_n} \left(\frac{\partial \psi_n}{\partial \kappa_n} \frac{\partial \psi_n}{\partial \nu} \right) dl \right|^{-1/2} \tag{15}$$

$$B_{0n} = \pm \left| \oint_{L_n} \psi_{0n} \frac{\partial \psi_{0n}}{\partial \nu} \, dl \right|^{-1/2} \tag{16}$$

completely specifying the normalization constants in terms of line integrals.

B. MM

Since MM is a well-known technique [7], only a brief summary is sufficient for the purpose of this paper. Application of MM requires the following:

- fields expansion in both waveguides by a sum of modes with unknown coefficients;
- application of the continuity conditions to the electric and magnetic fields;
- projection of the continuity equations on to a suitable set of waveguide modes [usually surface $\Omega_b - \Omega_s$ is a metallic one and, in this case, the electric-field continuity equations are projected using the set of modes of the "big" waveguide and the magnetic-field continuity equations are projected using the "small" waveguide mode set (alternative projection methods can be found in [7])].

For simplicity, we redefine the potentials so that (11) applies and we can get rid of constants A_n, B_n, and B_{0n}. The generic coupling coefficient (projection) takes on the generic form $\langle \mathbf{e}_b, \mathbf{e}_s \rangle$, where we use subscript b for the fields (and potentials) relative to the "big" waveguide and subscript s for those of the "small" waveguide. The reduction to a line-integral form is dependent on the type of the two modes. From [1] and using (1) and (2), one gets

$$\langle \mathbf{e}_b^{\mathrm{TE}}, \mathbf{e}_s^{\mathrm{TE}} \rangle = \frac{k_s^2}{k_s^2 - k_b^2} \oint_{L_s} \varphi_s \frac{\partial \varphi_b}{\partial \nu} \, dl \tag{17}$$

$$\langle \mathbf{e}_b^{\mathrm{TM}}, \mathbf{e}_s^{\mathrm{TM}} \rangle = \frac{\kappa_b^2}{\kappa_b^2 - \kappa_s^2} \oint_{L_s} \psi_b \frac{\partial \psi_s}{\partial \nu} \, dl \tag{18}$$

$$\langle \mathbf{e}_b^{\mathrm{TM}}, \mathbf{e}_s^{\mathrm{TE}} \rangle = \oint_{L_s} \psi_b \frac{\partial \varphi_s}{\partial l} \, dl = -\oint_{L_s} \varphi_s \frac{\partial \psi_b}{\partial l} \, dl \tag{19}$$

$$\langle \mathbf{e}_b^{\mathrm{TE}}, \mathbf{e}_s^{\mathrm{TM}} \rangle = 0. \tag{20}$$

When a TEM mode is considered, some additional work is needed. Simple algebraic manipulations yield

$$\langle \mathbf{e}_b^{\mathrm{TEM}}, \mathbf{e}_s^{\mathrm{TEM}} \rangle = \oint_{L_s} \psi_{0b} \frac{\partial \psi_{0s}}{\partial \nu} \, dl = \oint_{L_s} \psi_{0s} \frac{\partial \psi_{0b}}{\partial \nu} \, dl \tag{21}$$

$$\langle \mathbf{e}_b^{\mathrm{TM}}, \mathbf{e}_s^{\mathrm{TEM}} \rangle = \oint_{L_s} \psi_b \frac{\partial \psi_{0s}}{\partial \nu} \, dl \tag{22}$$

$$\langle \mathbf{e}_b^{\mathrm{TE}}, \mathbf{e}_s^{\mathrm{TEM}} \rangle = 0 \tag{23}$$

$$\langle \mathbf{e}_b^{\mathrm{TEM}}, \mathbf{e}_s^{\mathrm{TE}} \rangle = \oint_{L_s} \psi_{0b} \frac{\partial \varphi_s}{\partial l} \, dl = -\oint_{L_s} \varphi_s \frac{\partial \psi_{0b}}{\partial l} \, dl \tag{24}$$

$$\langle \mathbf{e}_b^{\mathrm{TEM}}, \mathbf{e}_s^{\mathrm{TM}} \rangle = 0 \tag{25}$$

(the proofs follow in the Appendix).

It is stressed that (21)–(25) hold independently of the shape of boundary L_s.

C. Degenerate Modes

The case of degenerate modes (modes having the same eigenvalues) can be treated as a limiting case. We are interested in coefficients representing TE–TE coupling and TM–TM coupling. In the case of degenerate modes, one finds

$$\langle \mathbf{e}_b^{\mathrm{TE}}, \mathbf{e}_s^{\mathrm{TE}} \rangle = -\frac{k}{2} \oint_{L_s} \varphi_s \frac{\partial^2 \varphi_b}{\partial k \, \partial \nu} \, dl \tag{26}$$

where $k = k_b = k_s$

$$\langle \mathbf{e}_b^{\mathrm{TM}}, \mathbf{e}_s^{\mathrm{TM}} \rangle = \frac{\kappa}{2} \oint_{L_s} \frac{\partial \psi_b}{\partial \kappa} \frac{\partial \psi_s}{\partial \nu} \, dl \tag{27}$$

where $\kappa = \kappa_b = \kappa_s$.

III. Conclusions

We have extended the line-integral formulation of MM to include the case of TEM modes of normalization constants and of degenerate TE–TE and TM–TM modes. The resulting formulation of MM completely avoids surface integrals, with benefits in terms of computation time and physical understanding.

Appendix

Consider two functions f and g satisfying the following equations:

$$(\nabla_t^2 + \tau_f^2) f = 0 \tag{28}$$

$$(\nabla_t^2 + \tau_g^2) g = 0 \tag{29}$$

in a region Ω with boundary L. By multiplying the first equation by g, the second by f, and by subtracting them, one gets

$$(\tau_g^2 - \tau_f^2) f g + f \nabla_t^2 g - g \nabla_t^2 f = 0. \tag{30}$$

By integrating in Ω

$$\iint\limits_{\Omega} f g \, d\Omega = -\frac{1}{\tau_g^2 - \tau_f^2} \iint\limits_{\Omega} (f \nabla_t^2 g - g \nabla_t^2 f) \, d\Omega. \tag{31}$$

The right-hand-side member can be transformed into

$$\iint\limits_{\Omega} (f \nabla_t^2 g - g \nabla_t^2 f) \, d\Omega = \oint_L \left(f \frac{\partial g}{\partial \nu} - g \frac{\partial f}{\partial \nu} \right) dl \tag{32}$$

to get

$$\iint\limits_{\Omega} f g \, d\Omega = \frac{1}{\tau_f^2 - \tau_g^2} \oint_L \left(f \frac{\partial g}{\partial \nu} - g \frac{\partial f}{\partial \nu} \right) dl. \tag{33}$$

We now compute the limit

$$\Lambda = \oint_L \lim_{\tau_g \to \tau_f} \lim \frac{\dfrac{d}{d\tau_g}\left(f\dfrac{\partial g}{\partial \nu} - g\dfrac{\partial f}{\partial \nu}\right)}{\dfrac{d}{d\tau_g}(\tau_f^2 - \tau_g^2)}\, dl \tag{34}$$

to get

$$\Lambda = -\frac{1}{2\tau} \oint_L \left(f\frac{\partial^2 g}{\partial \tau \partial \nu} - \frac{\partial g}{\partial \tau}\frac{\partial f}{\partial \nu}\right) dl. \tag{35}$$

Equation (35) can be used to get (11), (26), and (27).

A. TEM–TEM coupling

Equation (21) follows trivially from Green's first identity

$$\iint_{\Omega_s} \nabla_t \psi_{0b} \cdot \nabla_t \psi_{0s}\, d\Omega = -\iint_{\Omega_s} \psi_{0b} \nabla_t^2 \psi_{0s}\, d\Omega + \oint_{L_s} \psi_{0b} \frac{\partial \psi_{0s}}{\partial \nu}\, dl$$

since $\nabla_t^2 \psi_{0s} = 0$.

B. TM–TEM coupling

Equation (22) follows similarly from Green's first identity.

C. TE–TEM coupling

Using the general result [1]

$$\iint_{\Omega} \nabla_t f \cdot \nabla_t g \times \mathbf{u}_z\, d\Omega = \oint_L f \frac{\partial g}{\partial l}\, dl$$

we can write

$$\langle \mathbf{e}_b^{\mathrm{TE}}, \mathbf{e}_s^{\mathrm{TEM}} \rangle = -\oint_{L_s} \varphi_b \frac{\partial \psi_{0s}}{\partial l}\, dl = 0$$

since ψ_{0s} is constant on L_s.

D. TEM–TM coupling

The integral is

$$\iint_{\Omega_s} \nabla_t \psi_{0b} \cdot \nabla_t \psi_s\, d\Omega = -\iint_{\Omega_s} \psi_s \nabla_t^2 \psi_{0b}\, d\Omega + \oint_{L_s} \psi_s \frac{\partial \psi_{0b}}{\partial \nu}\, dl \tag{36}$$

and since $\nabla_t^2 \psi_{0b} = 0$ in Ω_s and $\psi_s = 0$ on L_s, the result is zero.

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Improved Automatic Parameter Extraction of InP-HBT Small-Signal Equivalent Circuits

B. Willén, Marcel Rohner, I. Schnyder, and H. Jäckel

***Abstract*—An improved automatic extraction technique for determination of the element values of an InP heterojunction-bipolar-transistor small-signal T-model is presented. Numerical optimization is shown to yield reproducible and physically relevant results when using a suitable figure-of-merit. The outcome of such an extraction is displayed for a range of operation points and the resulting bias dependencies of the element values is shown to be in good agreement with theoretical effects. The technique is further used to validate the quality of the extraction itself by showing a significant sensitivity to a deliberate error in the value of each element.**

***Index Terms*—HBT, InP, numerical parameter extraction.**

I. INTRODUCTION

Computer-aided design of integrated circuits relies on equivalent transistor models that are able to describe the terminal characteristics of individual devices properly. Element values of such models for InP-based heterojunction bipolar transistors (HBTs) are regularly extracted by fitting simulated S-parameters to measured numerically. Reproducibility, as well as physical relevance of the extracted small-signal model elements, are important prerequisites for process monitoring and device optimization, but it is well known that this extraction represents an overdetermined optimization problem. The frequency range covered by state-of-the-art measurement equipments does not extend to all relevant circuit poles, and lumped-circuit models become questionable at high frequencies. Several methods have been proposed to overcome this nondeterministic behavior as follows.

- Some element values, i.e., the ratio of the internal to external collector area and the resistance of the metal contacts, are taken from the geometrical layout or test structures to reduce the number of elements needed to be extracted. These values should ideally also be derived from the measurements to be able to reveal fabrication deficiencies.
- Most notable are analytical extraction methods where some element values are found by extrapolation rather than numerical optimization, e.g., from $Z_{11} - Z_{12}$, $Z_{22} - Z_{21}$, $Z_{12} - Z_{21}$, and Z_{12} [1]–[3].

By fixing some element values *a priori*, the dimension of the parameter space is decreased so that subsequent tuning becomes better defined.

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B. Willén is with the Department of Electronics, Royal Institute of Technology, S-164 40 Kista, Sweden.

M. Rohner, I. Schnyder, and H. Jäckel are with the Electronics Laboratory, Swiss Federal Institute of Technology Zürich, CH-8092 Zürich, Switzerland (e-mail: rohner@ife.ee.ethz.ch).

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