Letters

Comments on "Double-Plane Steps in Rectangular Waveguides and their Application for Transformers, Irises, and Filters"

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In the above paper,[1] Patzelt and Arndt have presented a theory for the analysis of rectangular waveguide structures consisting of cascaded double-plane step discontinuities. The theory is based on Dr. Patzelt's dissertation [2]. The presentation of the theory contains several errors.

An attempt to derive the matrix equation [1, eq. (6)] was made because the detailed derivation was not given. The electric and magnetic fields were obtained from the eigenfunctions (3) according to [3]. The transversal fields of the incident and reflected H- and E-modes in both waveguides were matched at the step discontinuity. The resulting equations were multiplied in turn by the electric fields of every H- and E-mode and integrated over the cross section of the waveguide. Every integral, which included two different modes in the same waveguide, gave zero because of orthogonality. The resulting matrix equation was somewhat different from (6) by Patzelt and Arndt:

$$\begin{bmatrix} U & V_{hhkl_{ij}} & V_{hekl_{ij}} \\ & 0 & -V_{eekl_{ij}} \\ V_{hh_{ij}kl} \;\; 0 & & \\ V_{eh_{ij}kl} \;\; -V_{ee_{ij}kl} & -U & \end{bmatrix} \begin{bmatrix} a^{(1)}_{hkl} \\ \vdots \\ a^{(1)}_{ekl} \\ \vdots \\ a^{(2)}_{hij} \\ \vdots \\ a^{(2)}_{eij} \\ \vdots \end{bmatrix} = \begin{bmatrix} U & 0 & V_{hhkl_{ij}} & -V_{hekl_{ij}} \\ 0 & -U & 0 & V_{eekl_{ij}} \\ -V_{hh_{ij}kl} & 0 & U & 0 \\ -V_{eh_{ij}kl} & -V_{ee_{ij}kl} & 0 & -U \end{bmatrix} \begin{bmatrix} b^{(1)}_{hkl} \\ \vdots \\ b^{(1)}_{ekl} \\ \vdots \\ b^{(2)}_{hij} \\ \vdots \\ b^{(2)}_{eij} \\ \vdots \end{bmatrix} \tag{1}$$

where

$$V_{hh_{ij}kl} = \sqrt{\frac{|\gamma^{(2)}_{hij}|}{|\gamma^{(1)}_{hkl}|}} \int_{A_1} \left(e^{(1)}_{hkl}\right) \cdot \left(e^{(2)}_{hij}\right) dx\,dy$$

$$V_{hhkl_{ij}} = \frac{\gamma^{(2)}_{hij}}{\gamma^{(1)}_{hkl}} \sqrt{\frac{|\gamma^{(1)}_{hkl}|}{|\gamma^{(2)}_{hij}|}} \int_{A_1} \left(e^{(1)}_{hkl}\right) \cdot \left(e^{(2)}_{hij}\right) dx\,dy$$

$$V_{eekl_{ij}} = \sqrt{\frac{|\gamma^{(1)}_{ekl}|}{|\gamma^{(2)}_{eij}|}} \int_{A_1} \left(e^{(1)}_{ekl}\right) \cdot \left(e^{(2)}_{eij}\right) dx\,dy$$

$$V_{ee_{ij}kl} = \frac{\gamma^{(1)}_{ekl}}{\gamma^{(2)}_{eij}} \sqrt{\frac{|\gamma^{(2)}_{eij}|}{|\gamma^{(1)}_{ekl}|}} \int_{A_1} \left(e^{(1)}_{ekl}\right) \cdot \left(e^{(2)}_{eij}\right) dx\,dy$$

$$V_{hekl_{ij}} = \frac{j2\pi}{\lambda\gamma^{(1)}_{hkl}} \sqrt{\frac{|\gamma^{(1)}_{hkl}|}{|\gamma^{(2)}_{eij}|}} \int_{A_1} \left(e^{(1)}_{hkl}\right) \cdot \left(e^{(2)}_{eij}\right) dx\,dy$$

$$V_{eh_{ij}kl} = \frac{j2\pi}{\lambda\gamma^{(2)}_{eij}} \sqrt{\frac{|\gamma^{(2)}_{eij}|}{|\gamma^{(1)}_{hkl}|}} \int_{A_1} \left(e^{(1)}_{hkl}\right) \cdot \left(e^{(2)}_{eij}\right) dx\,dy \tag{2}$$

and U and 0 are, respectively, unit and zero matrices of appropriate size. Other notation, except the matrix $V_{hekl_{ij}}$, is the same as in [1].

The main difference is that, if we follow Patzelt and Arndt, the upper right corner submatrix ($V^T_{h,e}$ in [1]) becomes zero, as given in [2] and [4]. This is not so. Here it is replaced by the matrix $V_{hekl_{ij}}$, which is not a zero matrix. In addition, the signs of several submatrices and diagonal elements are not the same as in [1].

According to Patzelt and Arndt, their eigenfunctions [1, eq. (3)] are normalized so that the power carried by a given wave is proportional to the square of the wave-amplitude coefficients a and b. We also require the powers of different modes to be comparable with each other according to $|a|^2$ and $|b|^2$; therefore, the eigenfunctions (3) should be multiplied by factors $\sqrt{2}/\sqrt{|\gamma^{(\nu)}_{hmn}|k_0 Z_0}$ for H-modes and $\sqrt{2}/\sqrt{|\gamma^{(\nu)}_{emn}|k_0 Y_0}$ for E-modes. k_0 is the free-space wavenumber, and $Z_0 = 1/Y_0 = \sqrt{\mu_0/\epsilon_0}$ is the intrinsic impedance of free space. Patzelt and Arndt seem to have done this implicitly, but they have not used the absolute value of the propagation constant $\gamma^{(\nu)}_{h,emn}$. This results in different matrix elements in (2) here and in [1, eqs. (A5)–(A8)]. However, for propagating modes, the expressions become equal. In (A6) and (A7), the factor $j2\pi/\lambda$ should not be under the square root. This error has been corrected in a later paper [4].

In order to obtain correct phase angles for the scattering matrix elements, the eigenfunctions have to be real also for nonpropagating modes. This is essential if correct results are to be obtained when combining the cascaded matrices. There are different ways to normalize the eigenfunctions for nonpropagating modes and make them real; here it is done by using the absolute value of $\gamma^{(\nu)}_{h,emn}$.

In [1], the propagation constant $\gamma^{(\nu)}_{h,emn}$ is defined as

$$\gamma^{(\nu)}_{h,emn} = jk\sqrt{1 - \frac{\left(k^{(\nu)}_{xmn}\right)^2 + \left(k^{(\nu)}_{ymn}\right)^2}{k^2}}\,.$$

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[1] H. Patzelt and F. Arndt, *IEEE Trans. Microwave Theory Tech.*, vol MTT-30, pp. 771–776, May 1982

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When combining the cascaded scattering matrices, this causes the nonpropagating wave amplitudes to increase exponentially, because the diagonal elements of matrix D are defined as $D_{ll} = \exp(-\gamma_l 1_{ll})$. If we assume a sign error here, we get the right amplitudes, but the phases of the propagating waves change into the wrong direction.

We avoid all these difficulties by defining $\gamma^{(\nu)}_{h,emn}$ the usual way:

$$\gamma^{(\nu)}_{h,emn} = \sqrt{\left(k^{(\nu)}_{xmn}\right)^2 + \left(k^{(\nu)}_{ymn}\right)^2 - k^2}\,.$$

The numerical results given in [1] seem to be correct, and they have partly been experimentally verified by us. It is, however, impossible to obtain these results by using the theory of Patzelt and Arndt in the form presented in [1]. Only after making the above corrections can a good agreement with experimental results be obtained.

Reply[2] by F. Arndt, U. Tucholke, T. Wriedt, and H. Patzelt[3]

We wish to disentangle the confusion that the commentator may possibly have biased in some of the readers by his hasty assertions.

We clearly describe in our paper [1, eq. (6)] that only $V_{h,e}$ and $V^T_{e,h}$ are zero matrices and that all other submatrices exist. This follows also by a correct interpretation of the common notation for a transposed matrix [3], i.e., for V^T, that, e.g., the row h and the column e have to be interchanged, too. The definition of the propagation constant given $\gamma = jk\sqrt{1-(k_c/k)^2}$ implies evidently (cf., e.g., [5]) the case $\gamma = k_c\sqrt{1-(k/k_c)^2}$ for $k < k_c$ (evanescent modes), although not explicitly mentioned in the paper. These cases are included in our program, of course.

Let us state again and definitely that equation (6) in our paper [1] is correct and reproducible at any time, requiring only a minimum acquaintance with the topic. Furthermore, we reproduce explicitly, for convenience, the related correct coupling integrals (with $j2\pi/\lambda$ not under the square root as has already been corrected in [4])

$$V_{h,h} = V_{hh_{IJkl}} = \sqrt{\frac{\gamma^{(2)}_{hIJ}}{\gamma^{(1)}_{hkl}}} \int_{A_1} e^{(1)}_{hkl} e^{(2)}_{hIJ}\, dA_1 \qquad (1)$$

$$V_{h,e} = V_{he_{IJkl}} = 0 \qquad (2)$$

$$V_{e,h} = V_{eh_{IJkl}} = \frac{\frac{2\pi}{j\lambda}}{\sqrt{\gamma^{(2)}_{eIJ}\gamma^{(1)}_{hkl}}} \int_{A_1} e^{(1)}_{hkl} e^{(2)}_{eIJ}\, dA_1 \qquad (3)$$

$$V_{e,e} = V_{ee_{IJkl}} = \sqrt{\frac{\gamma^{(1)}_{ekl}}{\gamma^{(2)}_{eIJ}}} \int_{A_1} e^{(1)}_{ekl} e^{(2)}_{eIJ}\, dA_1 \qquad (4)$$

[2]Manuscript received June 3, 1985.

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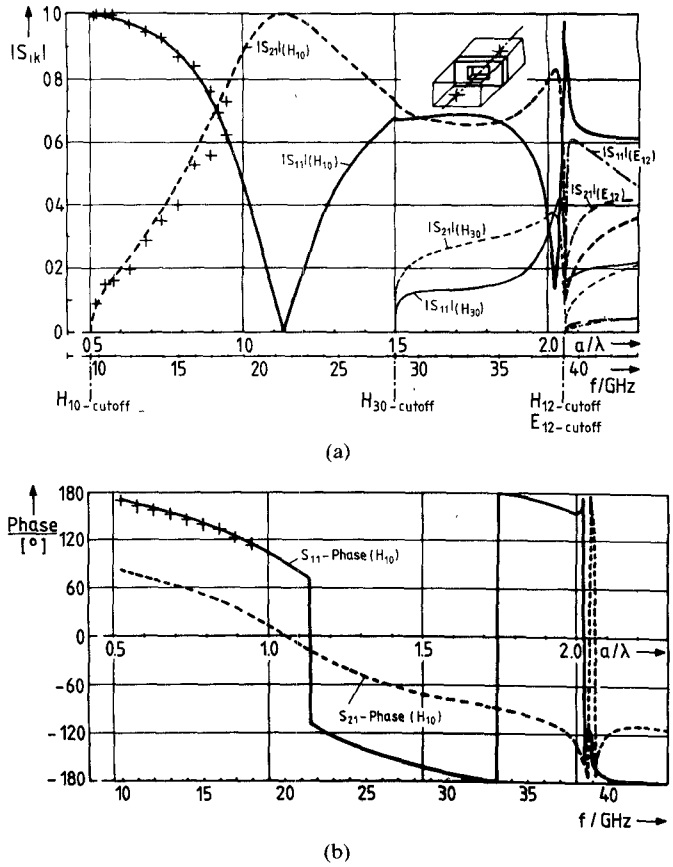

Fig. 1. Scattering coefficients of a resonant iris with finite thickness t. Waveguide dimensions: $a = 15.799$ mm, $b = a/2$. Iris dimensions: width $a' = a/2$, height $b' = b/2$, $t = 2$ mm. (a) Magnitude. (b) Phase. + Measured results.

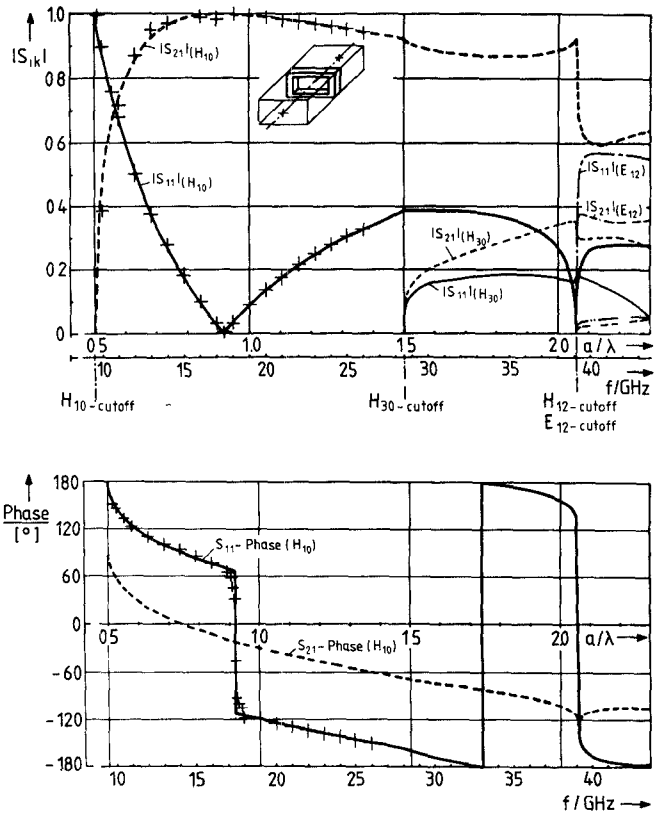

Fig. 2. Scattering coefficients of a resonant iris with finite thickness t. Waveguide dimensions: same as Fig. 1. Iris dimensions: $a' = a/\sqrt{2}$, $b' = b/\sqrt{2}$, $t = 2$ mm (a) Magnitude. (b) Phase + Measured results.

$$V^T_{h,h} = V_{hhkl_{ij}} = \sqrt{\frac{\gamma^{(2)}_{hkl}}{\gamma^{(1)}_{hij}}} \int_{A_1} e^{(1)}_{hij} e^{(2)}_{hkl}\, dA_1 \tag{5}$$

$$V^T_{h,e} = V_{ehkl_{ij}} = \frac{j\frac{2\pi}{\lambda}}{\sqrt{\gamma^{(2)}_{ekl}\gamma^{(1)}_{hij}}} \int_{A_1} e^{(1)}_{hij} e^{(2)}_{ekl}\, dA_1 \tag{6}$$

$$V^T_{e,h} = V_{hekl_{ij}} = 0 \tag{7}$$

$$V^T_{e,e} = V_{eekl_{ij}} = \sqrt{\frac{\gamma^{(1)}_{eij}}{\gamma^{(2)}_{hkl}}} \int_{A_1} e^{(1)}_{eij} e^{(2)}_{ekl}\, dA_1. \tag{8}$$

As a simple example to check the phases, where, e.g., three scattering matrices are combined (double-plane step, waveguide section of length t, double-plane step), Figs. 1 and 2 present the scattering coefficients of the resonant irises with finite thickness t already shown in our paper but now including the phases. The measured results are found to be in excellent agreement with the values theoretically predicted by our program using the theory presented in our paper [1].

Apart from the above information on the facts, let us add the following comment: In principle, we agree that criticism can be a fruitful force to advance scientific knowledge. But, in this instance, the criticism has obviously been based on a failure of sound research and an unfamiliarity with the related literature. Maybe the information given above will finally help the commentator to reproduce the results that we have extracted three [1] or seven [6] years ago and have utilized successfully since then.

References

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[2] H. Patzelt, "Berechnung von Querschnittssprüngen im Rechteckhohlleiter, inhomogenen Rechteckhohlleitertransformatoren, Blenden endlicher Dicke im Rechteckhohlleiter und längsgekoppelten Rechteckhohlleiterfiltern mit der Methode der Orthogonal-Reihen-Entwicklung," Dr.-Ing. dissertation, Univ. of Bremen, Bremen, Federal Republic of Germany, Oct. 1979.

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[5] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961, pp. 148–152, 171–177.

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Clarification to "Direct Method of Obtaining Capacitance From Finite-Element Matrices"

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***Abstract*—In the letter by Daly and Helps [1], they suggest that energy can be minimized on an element-by-element basis. The functional, however, must be treated globally because energy minimization has meaning only over the whole system. This discussion follows Daly and Helps' reasoning, but on a global basis, and draws different conclusions based on our work with finite elements.**

The electric potential can be found in any three-dimensional structure having appropriate boundary conditions by minimizing the variational energy expression [2]

$$W = \frac{1}{2}\int_S \mathbf{E}\cdot\mathbf{D}\, dS. \tag{1}$$

In the finite-element method, the energy in one element can be written [3]

$$W^{(e)} = \frac{1}{2}\int_s \epsilon|\nabla\phi|^2\, ds = \frac{1}{2}\underline{v}^T\underline{\underline{S}}^{(e)}\underline{v}. \tag{2}$$

The electric potential ϕ can be expressed as

$$\phi = \sum_{i=1}^{n} v_i \alpha_i(x, y, z)$$

where v_i are the unknown nodal potentials and α_i are shape functions. $\underline{v}$ is a vector (column matrix) of unknown nodal potential values and $\underline{\underline{S}}^{(e)}$ is a square matrix having values

$$S^{(e)}_{ij} = \int_s \nabla\alpha_i \cdot \nabla\alpha_j\, ds.$$

The total energy can be summed from the elemental contributions

$$W = \sum_{\substack{\text{all}\\ \text{elements}}} W^{(e)} = \frac{1}{2}\underline{V}^T\underline{\underline{S}}\underline{V}.$$

In partitioned form, this can be written as

$$W = \frac{1}{2}\underline{V}^T\underline{\underline{S}}\underline{V} = \frac{1}{2}\left[\underline{V}_p^T\underline{V}_f^T\right]\begin{bmatrix}\underline{\underline{S}}_{pp} & \underline{\underline{S}}_{pf}\\ \underline{\underline{S}}_{fp} & \underline{\underline{S}}_{ff}\end{bmatrix}\begin{bmatrix}\underline{V}_p\\ \underline{V}_f\end{bmatrix} \tag{3}$$

where $\underline{V}_p$ is a column vector of prescribed nodal potentials, and $\underline{V}_r$ is a column vector of unprescribed nodal potentials. Since $\underline{\underline{S}}$ is symmetric, the following relations hold:

$$\underline{\underline{S}}_{pp} = \underline{\underline{S}}^T_{pp},\ \underline{\underline{S}}_{ff} = \underline{\underline{S}}^T_{ff} \text{ and } \underline{\underline{S}}_{pf} = \underline{\underline{S}}^T_{fp}. \tag{4}$$

Minimization of the energy leads to [3]

$$\left[\underline{\underline{S}}_{fp}\underline{\underline{S}}_{ff}\right]\begin{bmatrix}\underline{V}_p\\ \underline{V}_f\end{bmatrix} = 0 \tag{5a}$$

or

$$\underline{\underline{S}}_{ff}\underline{V}_f = -\underline{\underline{S}}_{fp}\underline{V}_p \triangleq \underline{b}. \tag{5b}$$

Substituting (5a) into (3) yields

$$W = \frac{1}{2}\underline{V}^T\underline{\underline{S}}\underline{V} = \frac{1}{2}\underline{V}_p^T\left[\underline{\underline{S}}_{pp}\underline{\underline{S}}_{pf}\right]\begin{bmatrix}\underline{V}_p\\ \underline{V}_f\end{bmatrix} \tag{6}$$

W, being only a number, is equal to its own transpose; therefore

$$W = \left(\frac{1}{2}\underline{V}^T\underline{\underline{S}}\underline{V}\right)^T = \left(\frac{1}{2}\underline{V}_p^T\left[\underline{\underline{S}}_{pp}\underline{\underline{S}}_{pf}\right]\begin{bmatrix}\underline{V}_p\\ \underline{V}_f\end{bmatrix}\right)^T$$

$$W = \frac{1}{2}\left[\underline{V}_p^T\underline{V}_f^T\right]\left[\underline{\underline{S}}^T_{pp}\underline{\underline{S}}^T_{pf}\right]\underline{V}_p.$$

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