VI. Attenuation

Wegener[22] has computed the attenuation in a dielectric waveguide caused by the losses in the dielectric. Fig. 7 shows his theoretical curves and experimental data for a series of lucite rods. The lucite used in these experiments has a loss factor given by $\tan \delta = 0.01$.

It is frequently suggested that flexible dielectric guides can be used as connections to join sections of metallic guides. Fig. 8 shows the attenuation that results when two parallel waveguides separating a vertical distance h are connected with a flexible vinyl tube filled with Nujol. The tube is 2 feet long and has an outside diameter of 9/16 inch.

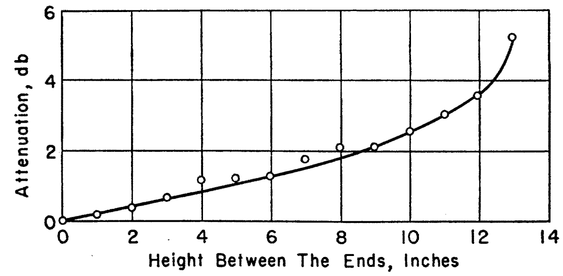

Fig. 8—The attenuation of a vinyl tube (od 9/16 inch) filled with Nujol ($n = 1.50$) as a function of the height between the ends. The data are for the HE_{11} mode.

[22] See Figs. 13, 14, and 15 of footnote reference 8

The Short-Slot Hybrid Junction*

HENRY J. RIBLET†, ASSOCIATE, IRE

Summary—**This paper describes a novel high-performance x-band hybrid junction. Its over-all dimensions are $1\frac{3}{4}'' \times \frac{1}{2}'' \times 2''$. It consists of a suitably loaded gap in the narrow common wall between two $\frac{1}{2}'' \times 1''$ waveguides. Over the frequency range 8,500- to 9,600-mc per second power equality within ± 0.25 decibels, isolation in excess of 30 decibels and a standing-wave ratio less than 1.07 may be obtained. The theory of the device is explained, and the particular advantages of this hybrid junction for a number of applications are outlined.**

Introduction

THE WAVEGUIDE hybrid junction[1] plays an important part in a number of specialized waveguide circuits. In addition to its application as a power splitter, it is useful in the construction of balanced duplexers,[2] balanced mixers,[3] and broad-band switches.[4] Although special forms of waveguide hybrids have been used on occasion, the most common are the "magic tee" and the "hybrid ring." Both of these have in common the characteristic that when power enters one of the terminals it divides between two of the others so that the outgoing voltages at equally distant terminals are either in phase or exactly out of phase. There exists, however, another large class of waveguide hybrid junctions at whose equidistant output terminals the voltages are always in quadrature. One of the earliest of these has been called a right-angle hybrid.[5] The possibility of quadrature hybrid junctions having broad-band characteristics has been pointed out by N. I. Korman, in an unpublished work, and by Riblet and Saad.[6] For many applications, however, these junctions are unduly large. It is the object of this article to describe a compact broad-band hybrid junction of the quadrature type which lends itself, for many applications, to more efficient use of space than is possible with conventional hybrids.

Although the short-slot hybrid is closely related to the family of directional couplers, it is not, strictly speaking, a member according to the definition given by Mumford.[7] Nevertheless, a structure which has the same general appearance but which is a directional coupler has been described by Surdin.[8] Moreover, the feasibility of obtaining hybrid performance from paral-

* Decimal classification: R310.9×R118. Original manuscript received by the Institute, October 30, 1950; revised manuscript received June 18, 1951. Presented, URSI, Washington, D.C., April 18, 1950.

† Microwave Development Laboratories, Inc., Waltham, Mass.

[1] W. A. Tyrrell, "Hybrid circuits for microwave," Proc. I.R.E., vol. 35, pp. 1307–1313; November, 1947.

[2] J. Reed, "Rat Race Duplexing," M.I.T. Radiation Laboratory Report 885; February, 1946.

[3] W. A. Tyrrell, *ibid.*

[4] W. D. Lewis, and L. C. Tillotson, "A non-reflecting branching filter for microwaves," *Bell Sys. Tech. Jour.*, pp. 83–84; January, 1948.

[5] C. W. Zabel, "Balanced Duplexers," Microwave Duplexers, Radiation Laboratory Series, McGraw-Hill Book Co., New York, N.Y., vol. 14, p. 367; 1948.

[6] H. J. Riblet and T. S. Saad, "A new type of waveguide directional coupler," Proc. I.R.E. vol. 36, p. 64; January, 1948.

[7] W. W. Mumford, "Directional couplers," Proc. I.R.E., vol. 35, pp. 160–166; February, 1947.

[8] M. J. Surdin, "Directional couplers in waveguides," *Jour. IEE* (London), pt. III, A 931 (no. 4) pp. 735–736; 1946.

lel waveguides apertured by a single large hole is suggested by Dicke.[9]

Simple General Theory[10]

To date all directional coupler-like hybrid junctions have had a plane of symmetry running their full length. Such an arrangement is shown schematically in Fig. 1. When power is incident on the main guide 2 at terminal 2_i, it proceeds along that waveguide until it encounters the coupling section. Under suitable conditions, by the time the energy reaches the end of the coupling section, it will have divided so that the energy leaving at terminal 1_o just equals that leaving at 2_o. If in addition no power leaves at terminal 1_i and none is reflected at terminal 2_i, assuming perfectly matched terminations at 1_o and 2_o, the structure is an ideal waveguide hybrid unction.

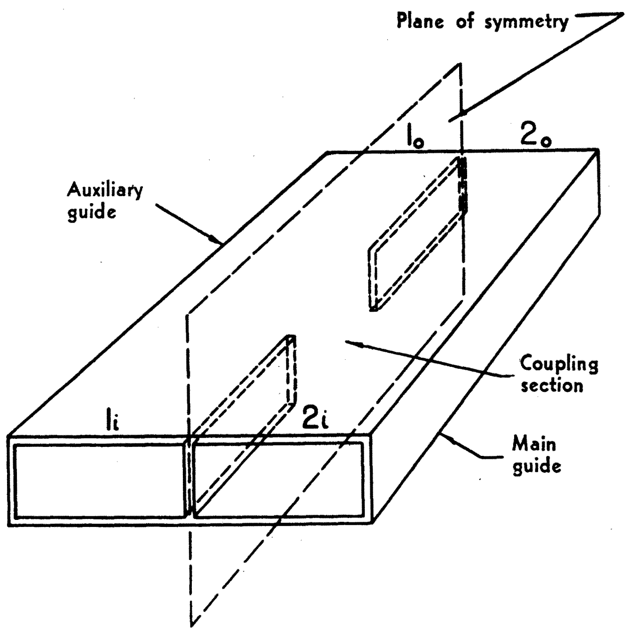

Fig. 1—Schematic hybrid junction.

It is now rather easy to derive several of the important characteristics of this type of hybrid junction and to state certain fundamental conditions which must be satisfied.

Since the reader should have no difficulty in reconstructing the arguments with the help of the Lippmann and Kyhl references, the results will be presented without details.

Conditions for complete isolation: *The reflected voltages in the even and odd modes shall both be zero.* When this condition is satisfied, the condition for power division becomes the following:

Condition for hybrid performance: *The transmitted voltages for the even and odd modes must differ from each other by 90 degrees.* This may be restated as

$$L\left(\frac{1}{\lambda_g^e} - \frac{1}{\lambda_g^o}\right) + \phi_r = 1/4, \tag{1}$$

where L is the length of the coupling section, λ_g^e and λ_g^o are the guide wave lengths for the even and odd modes, respectively, and ϕ_r is the phase shift in the even mode contributed by the reflections from the ends of the coupling section. Two useful additional facts may be inferred also from simple vector arguments concerning the voltages in the even and odd modes. They are:

(a) The output voltages at terminal 1_o leads the voltage at 2_o by 90 degrees.

(b) The output voltage at 2_o leads by 45 degrees what it would be if there were no slot at all in the waveguide.

The first conclusion is a consequence of high isolation, and is independent of the power-division characteristics of the hybrid junction. The second conclusion requires high isolation and the additional assumption that the odd mode does not see the slot.

Special Theory

Since the slot has very little effect on the odd mode, further analysis of the performance of the short-slot hybrid requires a solution of the Maxwell equations which satisfies the boundary conditions for the structure of Fig. 1 excited in the even mode. Fortunately, the solution of this problem is completely known in terms of convergent series, thanks to the work of Carlson and Heins.[11] This results from the fact that the even mode in the coupling section can be expressed in terms of plane waves so that the problem reduces to that of determining the reflection and transmission coefficients for a pair of plane waves suitably incident on an infinite array of metal plates.

The condition for complete isolation requires that the admittance of the structure in both directions as seen from the center of the coupling section shall be real. Accordingly, we determine R_g, the reflection coefficient, when two in-phase plane waves are incident on a semi-infinite set of metal plates ($\alpha = \pi/2$) along the directions $\pm\theta$. Of course θ is determined by the requirement that the electric field be zero at the outer wall of the coupling section. In the notation of Carlson and Heins

$$R_g = e^{i(\theta_1 - \theta_2)} \frac{K \cos i - k}{K \cos i + k}. \tag{2}$$

[9] R. H. Dicke, "Principles of Microwave Circuits," M.I.T. Radiation Laboratory Series, McGraw-Hill Book Co., New York, N.Y., vol. 8, pp. 447–448; 1948.

[10] This discussion constitutes a very slight extension of that given by B. A. Lippmann, "Theory of Directional Couplers," M.I.T. Radiation Laboratory Report 860, pp. 33–35; December 28, 1945, and R. L. Kyhl, "Directional Couplers," M.I.T. Radiation Laboratory Series, McGraw-Hill Book Co., New York, N.Y., vol. 11, pp. 889–890; 1947.

[11] J. F. Carlson and A. E. Heins, "The reflection of an electromagnetic plane wave by an infinite set of plates, 1*, *Quart. Appl. Math.*, vol. 4, no. 4, pp. 313–329; January, 1947.

Equation (1), which states the condition for equal power division, contains the term ϕ_r which must be evaluated. According to the reciprocity theorem, the phase shifts at the beginning and at the end of the coupling section are equal and given by the phase of T_g, the transmission coefficient for the two in-phase plane waves above. It may be shown that

$$T_g = \frac{2K}{K\cos\theta + k} e^{i(\theta_1-\theta_3)}. \tag{3}$$

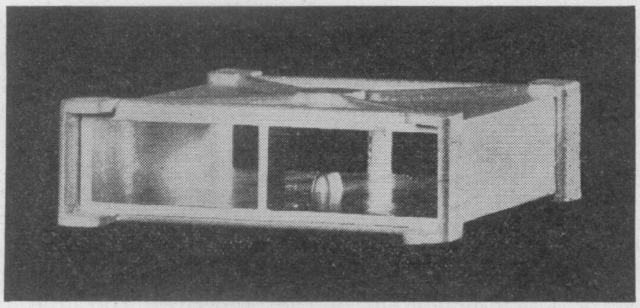

Fig. 2—Short-slot hybrid.

The actual short-slot hybrid shown in Fig. 2 differs from the schematic hybrid of Fig. 1 in two important respects: In the first place, capacitive domes have been provided in the coupling section, and secondly, the width of the coupling section has been reduced, by means of convex indentations, to below that of the two waveguides leading up to it.

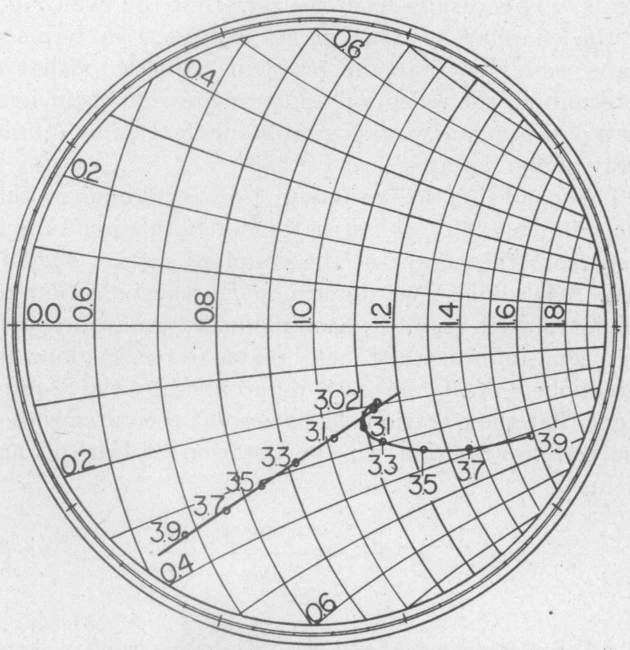

Fig. 3—Admittance of septum termination.

In early experiments, it was found that a slot length of 1.25 inches and a coupling section width corresponding to a = 0.892 inch gave satisfactory hybrid performance over the frequency range from 8,500 to 9,600 mc per second. As a theoretical check, the admittance of the septated termination seen from the coupling section was calculated and has been plotted on a circle diagram as shown in Fig. 3. These points fall closely along a straight line over the frequency range from 7,700 to 9,900 mc per second. When these admittances are plotted relative to the center of the coupling section, the other curve of the figure is obtained. It is clear that a centrally located capacity may be provided which will give high isolation over relatively broad bands of frequencies.

Fig. 4 shows the relative phase shift due to the length of the aperture and also that due to the ends of the aperture. The compensating tendencies of these phase shifts explains the broad-band power-division characteristics obtained. It will be noticed that the sum of these phase shifts differs from the required 90 degrees. Of course, the central capacity and multiple reflections (ignored in this discussion) supply the remaining phase shift.

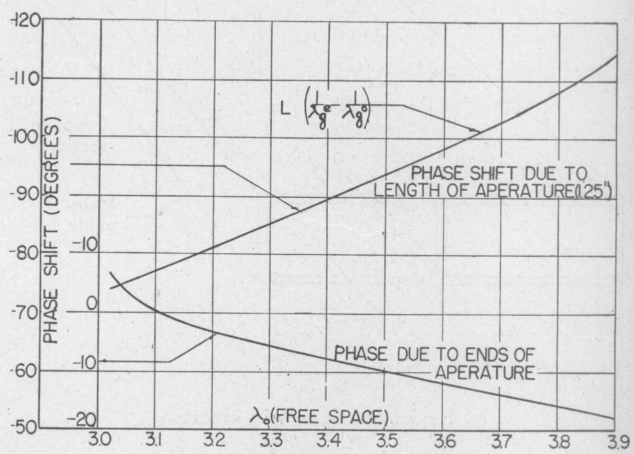

Fig. 4—Relative phase shift.

Present Performance

The present design of the short-slot hybrid junction was arrived at by a process of iterated selections from a small number of alternatives. As far as is known, it is not the optimum design and no effort has been made to obtain precise agreement between theory and practice. The theory is quite useful, however, in predicting the general effect of small changes in dimensions. Fig. 5 shows the best performance which has been obtained with one of the short-slot hybrids.

Applications

A review of all the applications of the hybrid junction would cover much of the waveguide art. It will be suf-

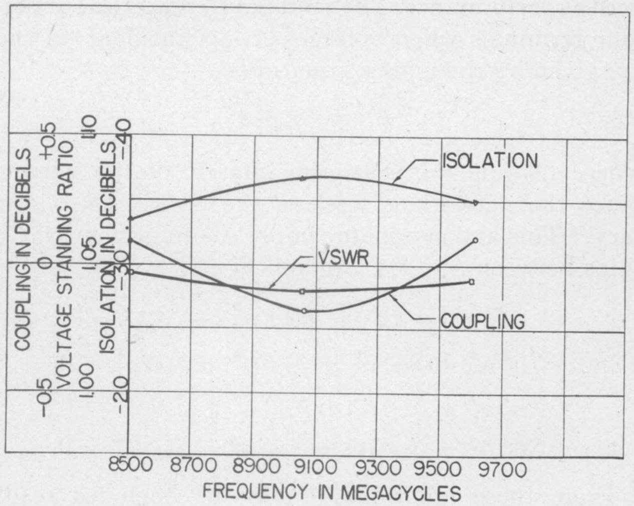

Fig. 5—Best performance.

ficient for our purposes to limit this discussion to two of the most important—namely the balanced mixer and the balanced duplexer. Fig. 6 shows a balanced mixer which uses this type of hybrid junction. Careful examination of the phase relationships shows that the 90-degree phase-shift characteristic of the hybrid junction in no way affects the ability of the mixer to discriminate against local oscillator noise. Pound[12] has gone through this argument in detail and indicated certain advantages for this arrangement in so far as image frequency power is concerned.

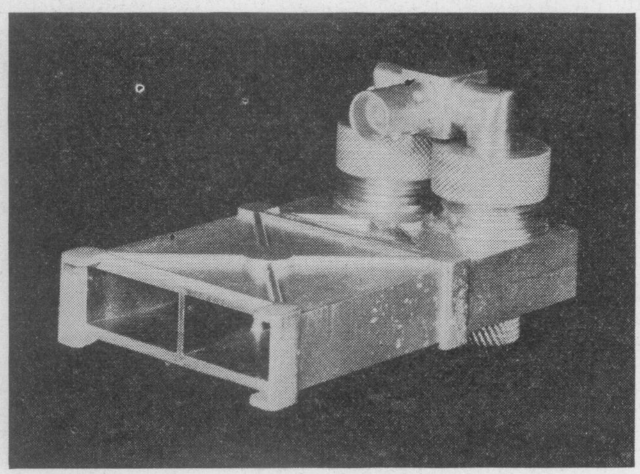

Fig. 6—Balanced mixer.

The principal advantage of the short-slot hybrid for mixer applications results from the close spacing between crystals which is made possible. Not only does it lend itself to more compact IF amplifier construction using the conventional balanced input transformer design, but it becomes a very simple matter to operate the crystals with a common dc bias. Fig. 6 shows such a mixer.[13] This circuit requires that one of the crystals be inverted with respect to the other. The possible advantages of this circuit over the conventional arrangement using a balanced input transformer appear to be a simpler mechanical arrangement, a perceptible tendency for the rf impedances of the crystal to balance more closely with a common dc bias, and a better over-all noise figure due possibly to the elimination of the closely coupled coils and their inherent losses and unbalances. Unfortunately, the mixer measurements were made with a small number of crystals, so the conclusions regarding noise figure and impedance balance must be considered tentative.

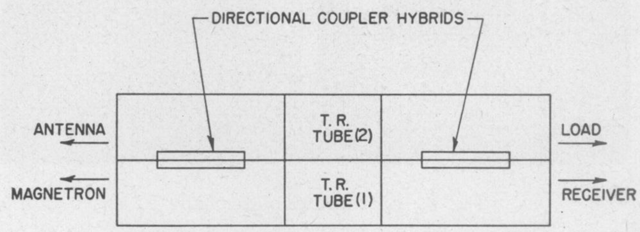

Fig. 7—Schematic balanced duplexer.

Fig. 7 gives a schematic drawing of a balanced duplexer using this type of hybrid. The inherent compactness of this arrangement as compared with the "model city" duplexer described by John Reed is apparent. The 90-degree phase-shift characteristic of the hybrid makes it possible to construct a perfectly symmetrical circuit without a frequency-critical quarter-wave-line length difference. This comes about as follows: Energy from the magnetron splits at the hybrid. The voltage crossing over leads the voltage going straight through by 90 degrees. At the TR tubes, which fire and thus act as short circuits, the energy is reflected without relative phase shift. Voltages in the magnetron branch arising from reflection at TR tube (2) experience an additional 90-degree phase advance, and thus destructively interfere with those which are reflected from TR tube (1). On the other hand, they reinforce in the antenna branch. On reception, the signal level is insufficient to fire the TR tubes, and the energy passes through the second hybrid when it recombines in the receiver branch, with little signal lost in the load.

[12] R. V. Pound, "Microwave Mixers," M.I.T. Radiation Laboratory Series, McGraw-Hill Book Co., New York, N. Y., vol. 16, pp. 277–279; 1948.

[13] The dc series features of this mixer were investigated by the author while employed at the Submarine Signal Co., Boston, Mass. The work was done under a subcontract supported by the Electronics Div., Glenn L. Martin Co., Baltimore, Md.

Fig. 8 is a photograph of a portion of a balanced duplexer which has been designed by the Reeves Instrument Company using these hybrids. One of the short-slot hybrid junctions is shown in the figure together with the two TR tubes and an ingenious mechanism for clamping the tubes in place.

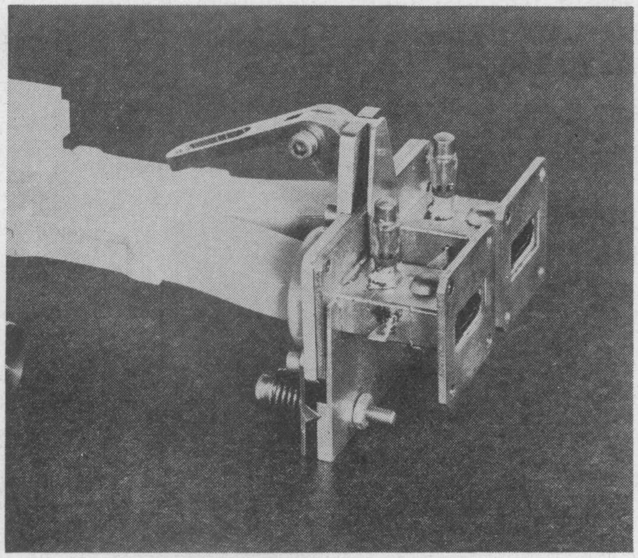

Fig. 8—Balanced duplexer half.

It is clear that the TR tubes may be replaced by shutters to obtain a broad-band mechanical switch or by band-rejection elements in which case one has the elements of a rejection filter. In addition, if the TR tubes are replaced by a pair of ganged movable pistons, a broad-band phase shifter or line stretcher results.[14]

ACKNOWLEDGMENTS

I am indebted to Miss Eileen Quigley for the calculation on which the graphs of Figs. 3 and 4 are based. The numerical values were obtained by straightforward summation of the series involved. Dr. Bela Lingyel of the Naval Research Laboratory provided me with unpublished calculations based on very careful summation of these series. Fortunately, the two calculations agree (wherever they overlap) within the accuracy with which the graphs are plotted.

The photograph in Fig. 8 was provided by Mr. John Guarrera of the Reeves Instrument Company.

APPENDIX

It is useful to examine qualitatively some of the consequences when the hybrid junctions do not have perfect performance. The voltages B_i reflected out of the four terminals when voltages A_j are incident on them are given by the matrix equation

$$(B_i) = (S_{ij})(A_j),$$

where (S_{ij}) is the scattering matrix of the network. Since the network is lossless, the matrix (S_{ij}) is unitary.[15] This and symmetry imply the following relationships between S_{11}, S_{12}, S_{13}, and S_{14}:

$$\begin{aligned} |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 &= 1 \\ S_{11}S_{12}^* + S_{12}S_{11}^* + S_{13}S_{14}^* + S_{14}S_{13}^* &= 0 \\ S_{11}S_{13}^* + S_{12}S_{14}^* + S_{13}S_{11}^* + S_{14}S_{12}^* &= 0 \\ S_{11}S_{14}^* + S_{12}S_{13}^* + S_{13}S_{12}^* + S_{14}S_{11}^* &= 0. \end{aligned} \tag{a}$$

From these equations, it may be concluded readily that

$$S_{14}\cdot S_{13}^* - S_{13}S_{14}^* = -2S_{13}S_{14}^* - S_{11}S_{12}^* - S_{12}S_{11}^*, \tag{b}$$

and

$$\begin{aligned} |S_{11}|^2\{S_{14}S_{13}^* - S_{13}S_{14}^*\} &= |S_{12}|^2\{S_{14}S_{13}^* - S_{13}S_{14}^*\} \\ &\quad + \{|S_{14}|^2 - |S_{13}|^2\}\{S_{11}\cdot S_{12}^* - S_{12}S_{11}^*\}. \end{aligned} \tag{c}$$

Since S_{12} is small compared with S_{13} and S_{14}, the coefficient of $|S_{11}|^2$ in (c) cannot vanish. Thus the vanishing of S_{12} implies that $S_{11}=0$. In words, complete isolation implies perfect match. Moreover, if $|S_{14}| = |S_{13}|$ we conclude immediately that $|S_{11}| = |S_{12}|$. If $|S_{14}|$ and $|S_{13}|$ differ by 0.25 db, which is the maximum inequality of output for these hybrids, the S_{11} will differ from S_{12} by an error of at most 2.2 per cent. Thus, for all practical purposes, we may immediately infer the maximum SWR from the measurement of isolation. In fact, the isolation measurement can be made with more accuracy than the SWR measurement.

If we write $S_{1j} = A_j e^{i\phi_j}$, we may conclude from the second equation of (a) that

$$A_1A_2 \cos(\phi_1 - \phi_2) = -A_3A_4 \cos(\phi_3 - \phi_4). \tag{d}$$

Assuming, in the most unfavorable case, that $\phi_3 = \phi_4$, we have

$$|\cos(\phi_3 - \phi_4)| \le \frac{A_1A_2}{A_3A_4}.$$

Now the voltage isolation is given by

$$\frac{A_1}{\sqrt{2}A_3} \approx \frac{A_2}{\sqrt{2}A_4}.$$

Hence the isolation immediately determines the maximum amount by which the output phases may differ from 90 degrees. For 20 db of isolation the maximum phase error is of the order of 1 degree; for 30 db of isolation the possible error has dropped to 0.1 degree.

[14] This very interesting application was suggested to the author by Mr. Werner Koppl of the Glenn L. Martin Co.

[15] R. H. Dicke, ibid., pp. 148–149.