Rigorous Mode Matching Analysis of Mitered E-Plane Bends in Rectangular Waveguide

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***Abstract*—A rigorous full-wave analysis of 90° E-plane mitered bends is obtained in a simple and accurate way by using the segmentation technique. The bend is divided into various regions of rectangular and triangular shapes, characterized by the generalised admittance matrix representation. The method is used to investigate the properties of 90° E-plane bends in rectangular waveguide. It is shown that it is possible to design the mitering so as to locate the matching frequency as desired within the operational frequency band. Design charts are given providing the optimal mitering as well as the relative bandwidth of the reflection coefficient.**

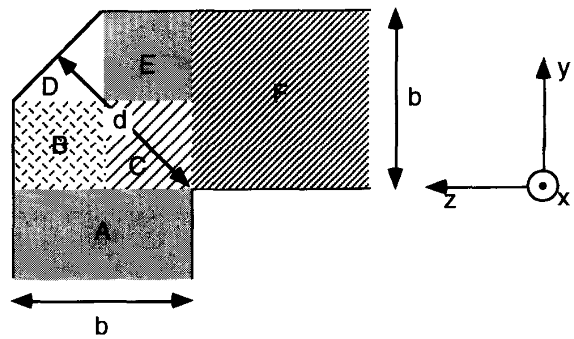

Fig. 1. Side view of an E-plane bend in rectangular waveguide. The bend is divided into various regions of space (A, B, C, D, E, F), each described by its generalized admittance matrix.

I. Introduction

BENDS ARE essential components in virtually all waveguide microwave systems, such as radar seekers, satellite beam-forming networks, etc. [1]–[3]. In order to reduce return loss, it is a common practice to chamfer the external waveguide wall. The design is generally based on approximate formulas, such as those provided in [4], and eventually requires tedious and expensive experimental adjusting. This approach becomes particularly unpractical at millimeter-wave frequencies.

It is therefore important to develop full-wave CAD tools for the analysis and design of bends. The mitered bend (Fig. 1), however, is not easily suited for standard mode-matching analysis. On the other hand, purely numerical algorithms such as finite difference or finite element method are numerically expensive. Recently, in order to retain the efficiency of the mode-matching method while extending its range of applicability, two hybrid approaches have been proposed [5]–[7]. Both of them make use of modal field description in regions A, F (see Fig. 1), while employing finite difference method [5], [6] or boundary element method [7] for the analysis of the central part corresponding to regions B, C, D, and E in Fig. 1.

It can be observed, however, that by segmenting the bend region as shown in Fig. 1, a rigorous analysis of E-plane mitered bends can be performed in a simple manner. One of these regions (region D in Fig. 1) has a triangular (right-angled isosceles) shape. The generalized admittance matrix of this region can easily be calculated using symmetry considerations as illustrated in the next section.

In this manner, a rigorous mode-matching technique can be implemented, leading to an accurate and efficient computer model of the mitered bend.

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II. Method of Analysis

The analysis of waveguide discontinuities by the generalized admittance matrix has been described in [2], [3], [8]. This approach consists of dividing the space region containing the discontinuities into several simple regions (cells) where the modes, thus the relative Green's function, are known. In the admittance formulation we choose as unknowns the tangential components of the electric field at the interface between different regions. The magnetic field in the various cells is then readily obtained as a modal expansion. The continuity of the tangential components of the magnetic field at the interfaces between the different cells provides a set of functional equations that, after Galerkin discretization, yields the desired solution. After the discretization process, each region is described by a generalized admittance matrix. The number of basis functions used to represent the tangential components of the electric and magnetic field corresponds to the number of electrical ports present in the network.

A suitable segmentation of the mitered E-plane bend is shown in Fig. 1. In the general case, five rectangular cells (A, B, C, E, F) and one triangular cell (region D) are present. In the case of no mitering ($d/b = 1.41$) the triangular region D and regions B, E are absent, while for $d/b = 0.707$, only the triangular region is present in addition to the waveguide regions A and F.

The field of the E-plane bend is conveniently described in terms of a LSE^x_{1n} modal set, where the x-dependence, common to all regions is of the type $\sin(\frac{\pi}{a}x)$, a being the waveguide broad wall. The y-dependence of LSE^x_{1n} modes in region F is

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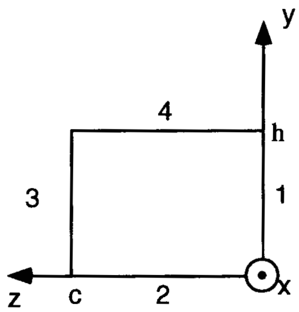

Fig. 2. Geometry of the rectangular resonator.

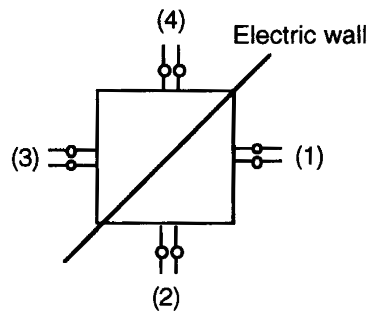

Fig. 3. The admittance matrix for the triangular region is obtained by considering the admittance matrix of a square region and by using symmetry.

given by

$$\Psi_n(y) = \sqrt{\frac{\varepsilon_n}{b}} \cos\left(\frac{n\pi}{b}y\right) \quad \begin{Bmatrix} \varepsilon_n = 1 & n = 0 \\ \varepsilon_n = 2 & n \neq 0 \end{Bmatrix} \tag{1}$$

and similarly for the z-coordinate in region A. In (1), b is the waveguide narrow wall.

We now describe the field solutions in the rectangular cells (B, E, C) and in the triangular cell (D).

Consider a rectangular region of sides $c \times h$. With reference to the notation of Fig. 2, the admittance matrix element y_{mn}^{il} relating the magnetic field amplitude of mode m at the side i of the rectangle ($i = 1, 2 \cdots 4$) to the E-field amplitude of mode n at side l ($l = 1, 2 \cdots 4$) is given by the following expressions:

$$y_{mn}^{11} = j\frac{(\frac{\pi}{a})^2 - k^2}{\beta_z \omega \mu} \operatorname{cot}g(\beta_z c)\delta_{mn} \tag{2a}$$

$$y_{mn}^{12} = j\omega\varepsilon\sqrt{\frac{\varepsilon_m \varepsilon_n}{ac}} \frac{k^2 - (\frac{\pi}{a})^2}{k^2(k^2 - \overline{k}^2)} \tag{2b}$$

$$y_{mn}^{13} = -j\frac{(\frac{\pi}{a})^2 - k^2}{\beta_z \omega \mu} \csc(\beta_z c)\delta_{mn} \tag{2c}$$

$$y_{mn}^{14} = (-1)^m y_{mn}^{12} \tag{2d}$$

where k is the free space wave number, ω is the radial frequency, and

$$\overline{k}^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{n\pi}{h}\right)^2 \tag{3a}$$

$$\beta_z = \sqrt{k^2 - \overline{k}^2} \tag{3b}$$

δ_{mn} is the Kronecker delta.

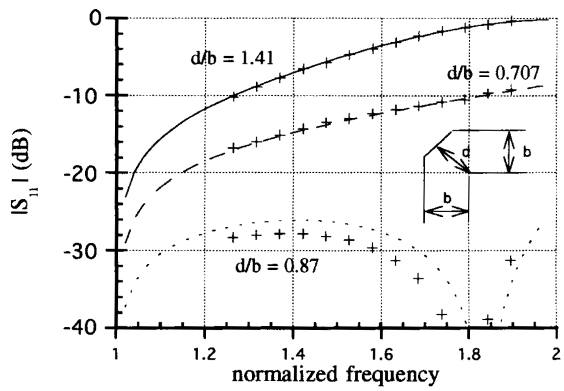

Fig. 4. Return loss of a 90° mitered E-plane bend in rectangular waveguide. The frequency is normalized with respect to the cut-off frequency. Crosses represent results from [7].

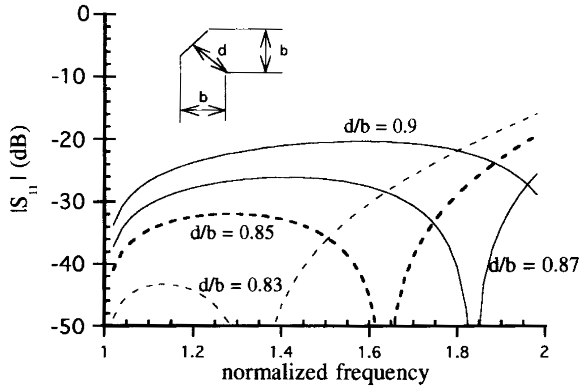

Fig. 5. Return loss of a mitered E-plane bend in rectangular waveguide. The frequency is normalized with respect to the cut-off frequency. The dept of the mitering is described by the parameter d/b.

Let us now consider a triangular region obtained by cutting a square along its diagonal with an electric wall (Fig. 3). From simple symmetry considerations, the following relation between the elements of the triangle ('t') and the square ('s') is obtained

$$\begin{aligned} y_t^{11} &= y_t^{22} = y_s^{11} - y_s^{14} \\ y_t^{12} &= y_t^{21} = y_s^{12} - y_s^{13} \end{aligned} \tag{4}$$

where the modal indexes mn have been dropped for simplicity of notation.

Once the generalized admittance matrices of all cells of Fig. 1 have been derived, the scattering parameters of the bend are computed by solving the relative network problem.

III. Results

The method described has first been checked against published results. Fig. 4 shows the comparison of the present results with those in [7]. It is seen that in both limit cases of no mitering ($d/b = 1.41$) and diagonal mitering ($d/b = 0.707$), the return loss is quite high, while a matching better than -26 dB in the whole band is obtained for the optimum value of $d/b = 0.87$.

A more detailed analysis of the behavior of the bend for mitering values d/b in the range $d/b = 0.8$– 0.9 has led to the results shown in Fig. 5. By varying the mitering ratio, the reflection zero frequency can be shifted upward and backward. Therefore, depending on the frequency band of interest, different "optimum" mitering ratios are found.

All simulations of Figs. 4 and 5 have been obtained with a laptop computer 486DX2 50 MHz. The calculation of one frequency point requires less than 1 second.

IV. Conclusion

Based on the segmentation technique, a rigorous and very efficient analysis of of E-plane mitered bends in rectangular waveguide has been presented. The applicability of the mode-matching technique has been extended by introducing right-angled isosceles triangular cells. It has been pointed out that the optimum mitering depends on the frequency band for which matching is sought.

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