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ANALYSIS OF THE SCATTERING CHARACTERISTIC OF SEVERAL WAVEGUIDE COMPONENTS USING BOUNDARY ELEMENT METHOD

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Abstract: Several kinds of waveguide components such as curved waveguide bends, arbitrary angle waveguide bends and T-junctions have been analyzed with boundary element method in this paper. A new discretization method for the boundary element method to solve the waveguide discontinuities has been given. The numerical results obtained agree well with the experimental results and numerical results in other literature. Especially, the scattering characteristics of Forked E-, H-plane T-junctions in 3mm band have been analyzed using boundary element method and the calculation results are presented.

Indexing terms: boundary element method, waveguide discontinuities, discretization method

1 Introduction

Waveguide discontinuities play an important role in the design of many microwave components. In a complicated microwave system, there may be various kinds of waveguide components such as curved waveguide bends, arbitrary angle waveguide bends and T-junctions, it is important to find a convenient and accurate method for the design of these components. Based on the field theory, many techniques [1] [2] [3] [4] [5] have been applied to analyze the scattering behavior of the waveguide components. Most of those methods have to do field expansion and field matching at the discontinuities, when the shape of the component changes, the equations obtained before is unsuitable, so the field expansion and field matching have to been applied again, which is a tedious job especially when the shape of the components have to be adjusted frequently such as in the design of forked T-junctions. There are many other excellent methods for waveguide bend structures [6] [7] [8] [9] [10], yet except the drawback mentioned above they are unsuitable for other discontinuities such as T-junctions. The finite-difference time-domain method is very effective when the shape of the component is rectangular, but in the case of curved shapes it has to use staircasing to simulate the curved shapes, which will lead to some errors. The boundary element method is a good numerical method and it is convenient to solve waveguide discontinuities with this method in the case of irregular shaped components [11] [12] [13]. In this paper a new discretization method for the boundary element method to solve the waveguide discontinuities has been given, which makes the boundary element method need less computer resources. Our method has been used to analyze curved waveguide bends, arbitrary angle waveguide bends and T-junctions, numerical results are presented and compared with the experimental results and numerical results in other literature, and good agreement has been found. Finally, S-parameters of several kinds of forked H- and E-plane junctions in 3mm-band have been given. So far as we know, there is almost no analytical results of these structures has been given.

2 Basic principle

In Fig.1, the region V is enclosed by conductor boundary Γ_c, virtual boundaries Γ_{pi} at the ports (called port boundary in the following). From Green's second identity and scalar Helmholtz equation the boundary integral equation about the region can be obtained as follows:

$$\psi_m = \int_\Gamma \psi^* g(r) d\Gamma - \int_\Gamma \psi g(r)^* d\Gamma \qquad (1)$$

where ψ is wave function, ψ can be electric field or magnetic filed which satisfies scalar Helmholtz equation, $\psi^* = \dfrac{\partial \psi}{\partial n}$, m means any point in the corresponding region, $g(r)$ is Green's function, $g^*(r) = \dfrac{\partial g}{\partial n}$, n is unit outward normal, Γ is the boundary of the region.

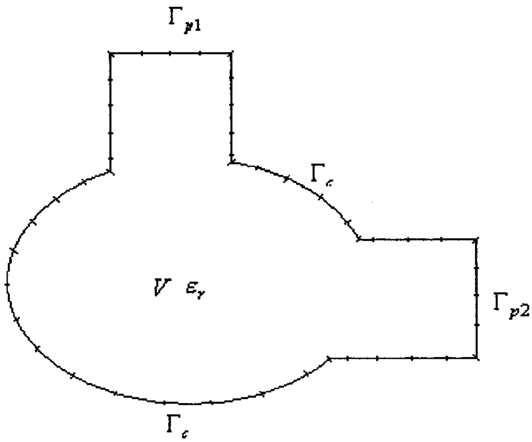


Fig.1. General discretization method

The general discretization method is shown in Fig.1, where the whole boundary including the port boundaries has been discretized. Unlike the discretization method taken in [12][13][14], we don't discretize the port boundary as shown in Fig.2.

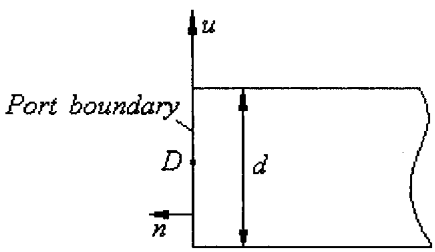


Fig.2. Port boundary

If a port boundary is sufficiently far from the discontinuities, the port boundary may be taken as one element (port element).

Assuming the value of ψ at the central point of the port element

D is ψ_{pi}, any point along the port element is known as:

$$\begin{cases} \psi(u) = \psi_{pi} \sin\left(\frac{\pi}{d} u\right) & for \quad H-plane \quad discontinities \\ \psi(u) = \psi_{pi} & for \quad E-plane \quad discontinities \end{cases} \tag{2}$$

ψ_{pi} is the only unknown value for a port element to be solved.

The difference between the port element and the other element shows in the coefficient related to the port element when it is a source element. The boundary discretized using our method is shown in Fig.3.

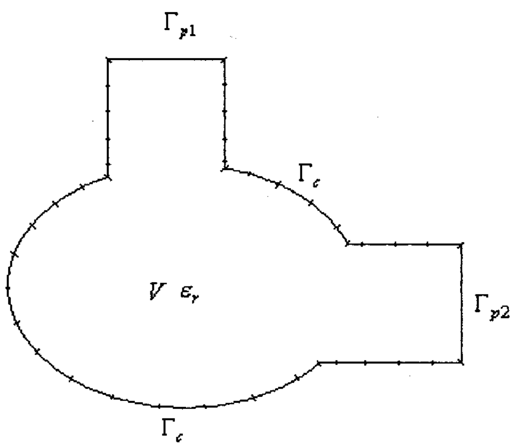


Fig.3. Improved discretization method

If a port is selected to be an exciting port, ψ at this port is the sum of the incident wave ψ^i and the reflection wave ψ^r, else

ψ contains only reflection wave ψ^r. ψ^* on the port boundary may be expressed in the form of ψ:

$$\begin{cases} \psi^{i*} = j\beta\psi^i \\ \psi^{r*} = -j\beta\psi^r \end{cases} \tag{3}$$

Here

$$\begin{cases} \beta = \sqrt{k_0^2\varepsilon_v - \left(\pi/a\right)^2} & for \quad H-plane \quad discontinuities \\ \beta = k_0\sqrt{\varepsilon_v} & for \quad E-plane \quad discontinuities \end{cases} \tag{4}$$

Assuming the exciting wave incident to port 1, the sum of the elements of all boundaries is W, according to [11][12][13][15] and considering (3) the discretization equation is obtained as follows:

$$\begin{bmatrix} [H-j\beta_1 G]^{p1} & [H-j\beta_2 G]^{p2} & [A]^c \end{bmatrix} \begin{bmatrix} \psi^{p1r} & \psi^{p2r} & \{\psi\}^c \end{bmatrix} = \begin{bmatrix} [H+j\beta_1 G]^{p1} .\psi^{p1i} \end{bmatrix} \tag{5}$$

Where $[H-j\beta_i]^{pi}$ is a $W\times 1$ matrix, p, c mean the source element related to the coefficient is in port boundary or in conductor boundary respectively. The coefficient in (5) may be written as:

$$H_j^{pi} = \int_0^a g^*(r)f(u)du, \quad i=1,2; \quad j=1,2...W \tag{6}$$

$$G_j^{pi} = \int_0^a g(r)f(u)du, \quad i=1,2; \quad j=1,2...W \tag{7}$$

Where $g(r) = \dfrac{1}{4j}H_0^{(2)}(kr)$, (8)

$$g^*(r) = \frac{j}{4} k_n H_1^{(2)}(kr)\cos\alpha \quad , \tag{9}$$

$$k = \begin{cases} k_0\sqrt{\varepsilon_r} & \text{for } H-\text{plane discontinuities} \\ \sqrt{k_0^2\varepsilon_r - \left(\dfrac{\pi}{a}\right)^2} & \text{for } E-\text{plane discontinuities} \end{cases} \tag{10}$$

$$f(u) = \begin{cases} \sin\dfrac{\pi u}{a}, & \text{for } H-\text{plane discontinuities} \\ 1, & \text{for } E-\text{plane discontinuities} \end{cases} \tag{11}$$

Here α is the angle between vector r and the outward unit normal n, a is the width of the port, b is the height of the port.

The matrix $[A]^c$ takes the form of $[H]$ or $[G]$ in [12] based on the discontinuity is E-plane or H-plane type.

3 Numerical results

Program according to the above analysis can be used to calculate curved waveguide bends, arbitrary angle waveguide bends, T-junctions (including forked T-junctions). Examples of curved H-plane waveguide bend with different radius and E-plane cascaded curved waveguide bend are shown in Fig.4 and Fig.5.

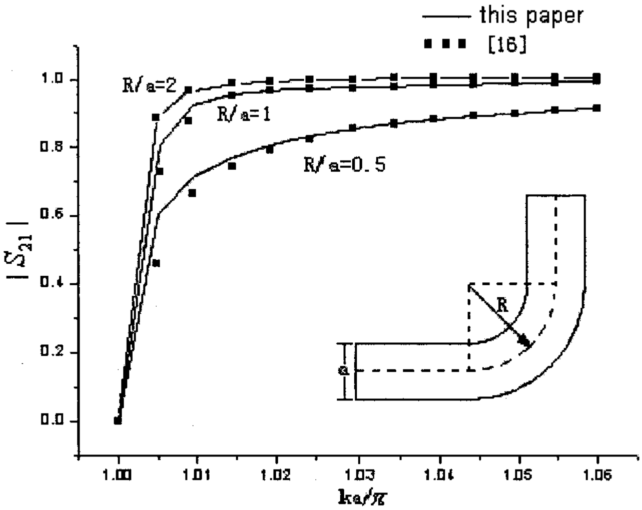
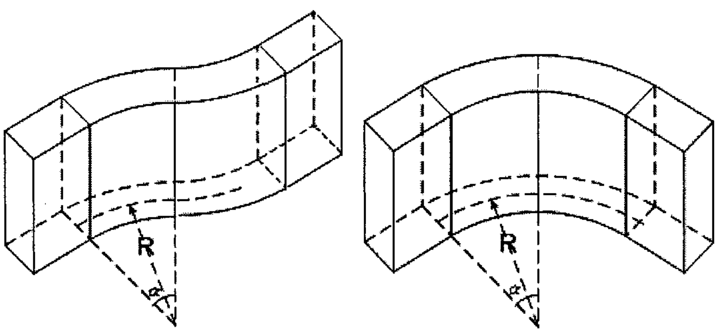
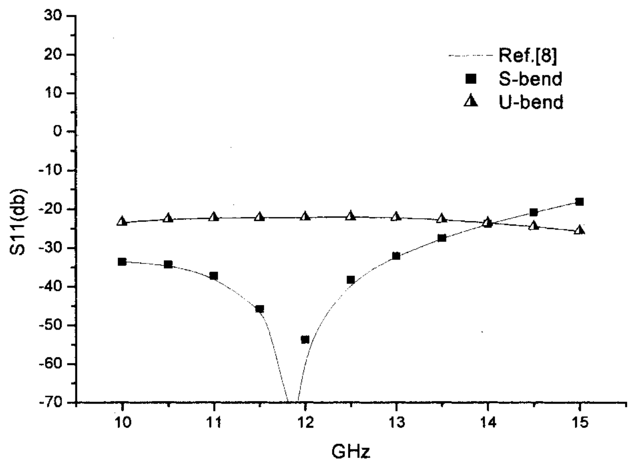


Fig.4. Transmission coefficient of $WR62$ waveguide H-plane curved bend



(a) (b)



(c)

Fig.5. Cascaded 45^0 E-plane bends with R=8mm in *WR*75 waveguide: (a) S-shaped E-plane bend; (b) U-shaped E-plane bend; (c) Reflection coefficient

In [17], equivalent T-shaped network for arbitrary angle H-plane waveguide bend (shown in Fig.6) and their impedance parameters have been given out, so the S-parameters of the bend can be obtained from these impedance parameters. We have also calculated H-plane waveguide bends of several angles, and compared the S-parameters with that derived from [17], the results are shown in table 1, table 2 and table3.

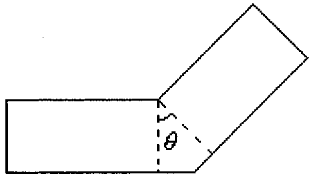


Fig.6. Side view of arbitrary angle H-plane waveguide bend

Table1: Comparison of calculated results and that from literature for $\lambda = 32mm$

θ	$\lvert S_{11} \rvert$		$\lvert S_{21} \rvert$	
	From literature	Calculated	From literature	Calculated
30°	0. 0399	0. 0378	0. 9992	0. 9990
60°	0. 1558	0. 1521	0. 9878	0. 9905
90°	0. 3824	0. 3810	0. 9240	0. 9223

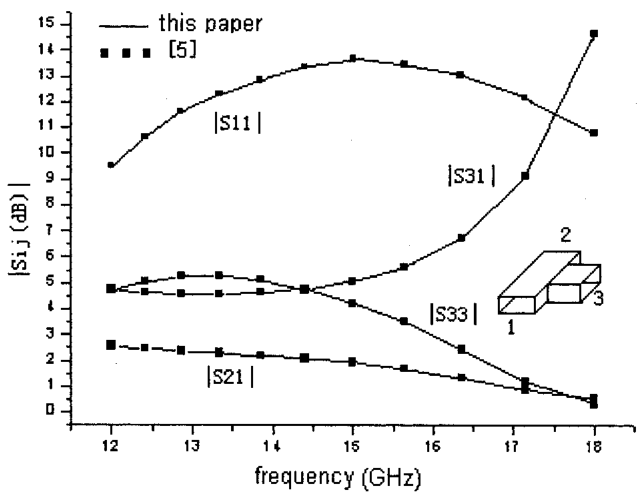
Table2: Comparison of calculated results and that from literature for $\lambda = 34mm$

θ	$\lvert S_{11} \rvert$		$\lvert S_{21} \rvert$	
	From literature	Calculated	From literature	Calculated
30°	0. 0418	0. 0436	0. 9991	0. 9991
50°	0. 1090	0. 1151	0. 9940	0. 9937
60°	0. 1613	0. 1640	0. 9869	0. 9898
90°	0. 3726	0. 3721	0. 9280	0. 9277

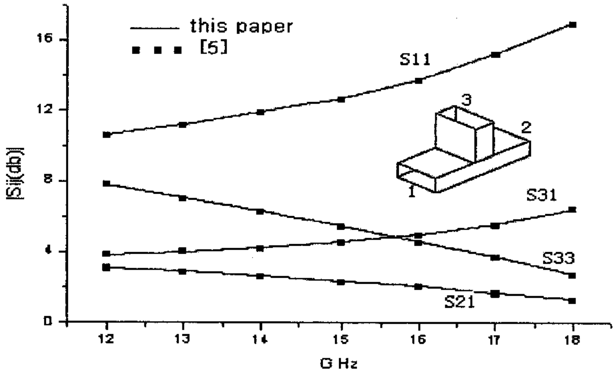
Table3: Comparison of calculated results and that from literature for $\lambda = 30mm$

θ	$\lvert S_{11}\rvert$		$\lvert S_{21}\rvert$	
	From literature	Calculated	From literature	Calculated
30°	0.0347	0.0331	0.9994	0.9988
50°	0.0975	0.1017	0.9952	0.9940
60°	0.1497	0.1522	0.9887	0.9904
75°	0.2610	0.2581	0.9653	0.9656
90°	0.4198	0.4332	0.9076	.9022

Fig.7 is the example of waveguide Ku band (12-18 GHz, $WR62$ housing: $15.799mm \times 7.899mm$).



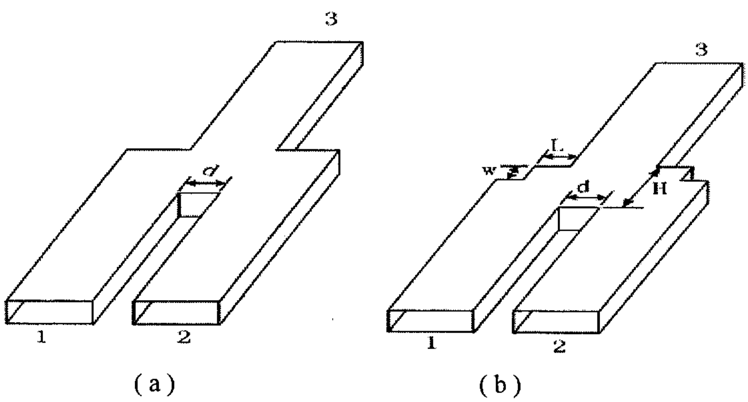
(a)

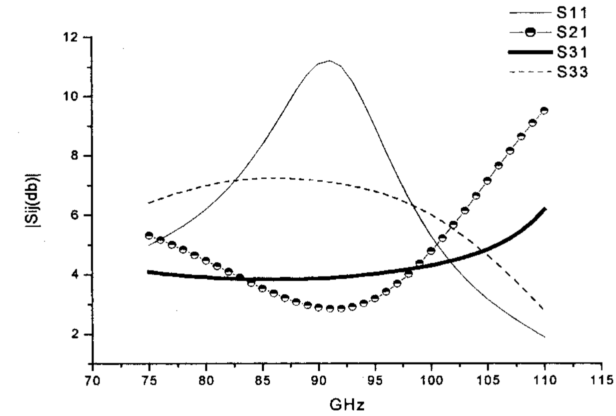


(b)

Fig.7. S-parameter of $WR62$ waveguide T-junctions: (a) H-plane T-junction; (b) E-plane T-junction

Forked T-junctions are very useful components in waveguide circuits, but there are little literatures for the analysis of those structures. Two kinds of forked H-plane T-junctions and two kinds of forked E-plane T-junctions have analyzed in this paper, and the results are shown in Fig.8 and Fig.9.





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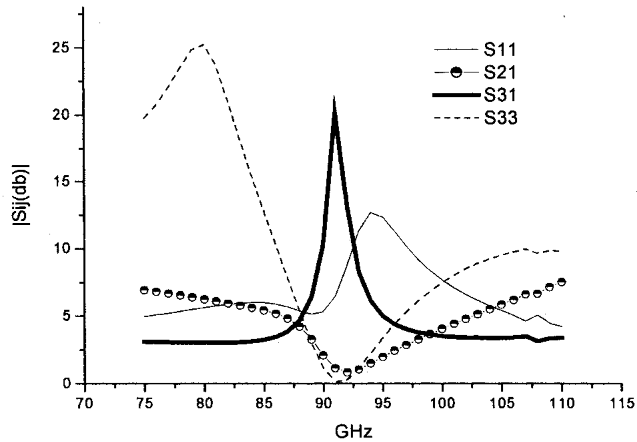
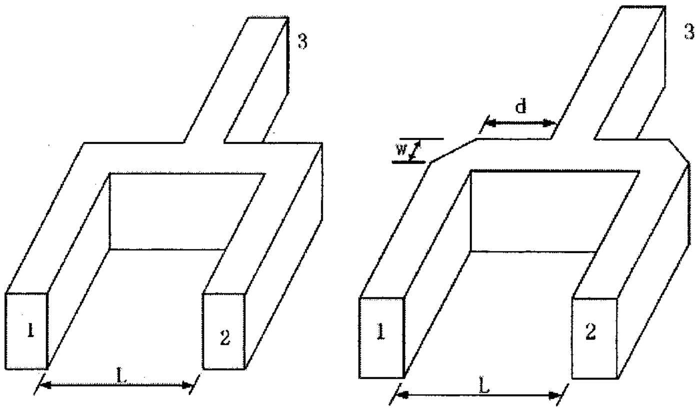


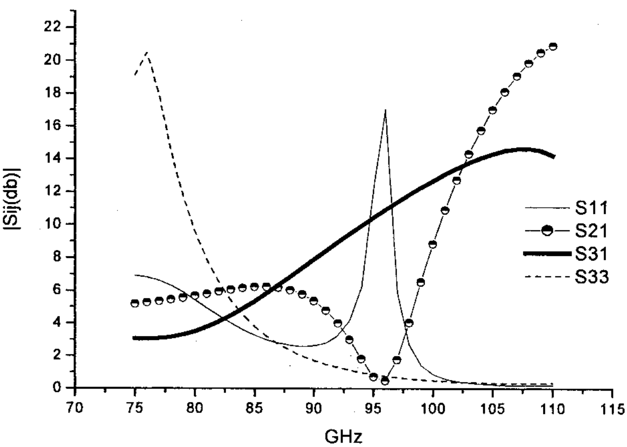
(d)

Fig.8. Forked H-plane T-junctions in $2.54\times1.27mm$ waveguide: (a) Forked H-plane T-junction with $d=1.27mm$; (b) Forked H-plane

T-junction with $d = 1.27mm$, $H = 2.54mm$, $L = 0.799mm$, $W = 1.143mm$; (c) s-parameters of Forked H-plane T-junction in (a); (d) s-parameters of Forked H-plane T-junction in (b);



(a) (b)



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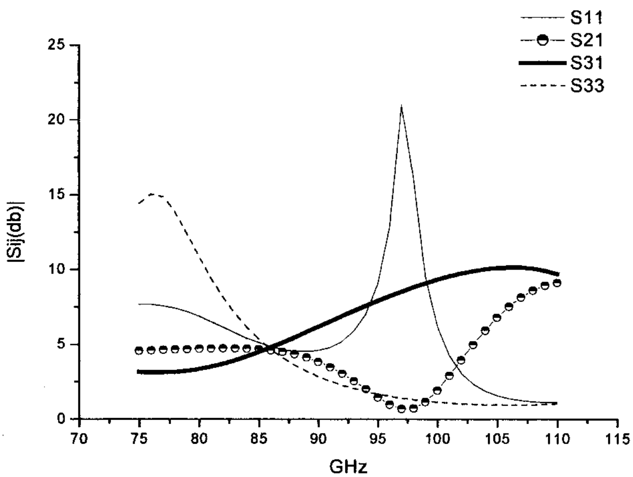

(d)

Fig.9. Forked E-plane T-junctions in $2.54 \times 1.27mm$ waveguide: (a) Forked E-plane T-junction with $L = 3.81mm$; (b) Forked E-plane T-junction with $L = 3.81mm$, $d = 2.04mm$, $W = 0.5mm$; (c) s-parameters of Forked E-plane T-junction in (a); (d) s-parameters of Forked E-plane T-junction in (b);

4 Conclusion

The boundary element method is a good method to solve waveguide discontinuities. It can be used to solve a variety of problems including curved waveguide bends, arbitrary angle waveguide bends and T-junctions. It is easy to deal with irregular boundary value problem with this method, and there is no need to do field expansion and field matching as in [1] [2] [3] [4] [5]. In the case of E-plane discontinuities, the problem may be treated as a 2-D one, but using other method such as FD-TD this kind of problem must be treated as a 3-D one.

5 References

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