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Properties of TE–TM Mode-Matching Techniques

Gian Guido Gentili

***Abstract*—A line integral formulation of TE–TM mode-matching techniques for scattering problems in waveguides is described. The procedure is convenient from a computational point of view when the modes in the waveguides must be computed numerically. Some interesting properties of TE–TM mode-matching techniques are then demonstrated.**

I. Introduction

In the analysis of scattering from waveguide discontinuities, the mode-matching technique, whenever it can be applied, is by far the most popular way to solve the problem. It has been used to solve scattering problems in several different kinds of waveguides, such as the rectangular waveguide [1]–[5], the microstrip line [9], the finline [13], and the circular waveguide (or coaxial cable, see e.g. [18]). Scattering caused by the transition between different kinds of waveguides has been dealt with too (see e.g. [17]). Although several formulations can be used to represent the fields at the discontinuity interface, the TE–TM field expansion is the most general one when homogeneous waveguides with perfectly conducting walls are considered. Such field expansion derives the tangential components of the electric and magnetic fields from the longitudinal ones: the tangential components of the fields are then matched at the discontinuity interface, yielding a infinite system of linear equations. An approximate solution of the system of equations is then found by truncating the infinite series. Some properties of such approximate solution of the system of equations have been discussed in [14] (e.g. the relative convergence problem).

This paper focuses on some general properties of TE–TM field expansions in perfectly conducting waveguides when they are matched at some arbitrarily shaped waveguide discontinuity. It can be shown that the surface integrals resulting from matching the tangential field components can almost always be expressed as line integrals along the boundary of the region over which surface integration is carried out. Such reduction in the dimensionality of the integration is convenient from a computational point of view when the modes in the waveguides must be computed numerically. To be more specific, the line integral formulation is best suited when the modes in the waveguides are computed by techniques based on some integral equation expressed on the boundary, since in that case the modal eigenfunctions are computed only on the boundary of the waveguide cross section. The application of the line integral formulation of mode matching is then directly applicable, without the need for a time-consuming computation of the eigenfunctions at internal points in the waveguide cross section. On the other hand, the reduction to line integrals has pointed out an interesting property of mode-matching techniques: some coefficients representing coupling between TE and TM modes are always null; i.e., such modes are uncoupled. Such phenomenon have been observed in [2] and [17] for two particular cases (rectangular-to-rectangular waveguide junction and rectangular-to-circular waveguide junction). It will be shown here that it is quite general and independent of the shape of the waveguide cross section.

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The author is with the Dipartimento di Electtronica, Politecnico di Milano, Piazza Leonard da Vinci, 32-20133 Milan, Italy.
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II. Formulation

Consider the scattering problem represented by the transition between two arbitrarily shaped perfectly conducting walls waveguides (Fig. 1). Let S_1 be the cross section of guide 1, σ_1 its boundary, S_2 the cross section of guide 2, and σ_2 its boundary. Let then $\Omega = S_1 \cap S_2$ and C be its boundary.

Let $\boldsymbol{E}_{\mathrm{I}n}$ and $\boldsymbol{H}_{\mathrm{I}n}$ be the transverse electric and magnetic fields of the generic mode n in region I (with S_1 the cross section of the related waveguide). Such tangential fields can be expressed as the sum of two contributions: a TE field and a TM field. They can be derived from the two longitudinal fields $E_{z\mathrm{I}n}$ and $H_{z\mathrm{I}n}$ (z being the coordinate relative to the direction of propagation, so that the z dependence of the fields is of the type $\exp(\mp j\beta z)$). By expanding the fields in the two waveguides as sums of the modal fields multiplied by unknown coefficients, one gets

$$\boldsymbol{E}_{\mathrm{I(TE)}} \cong \sum_{n}^{N_E} \left(e^{+}_{\mathrm{I(TE)}n} + e^{-}_{\mathrm{I(TE)}n}\right)\nabla\varphi_{\mathrm{I}n} \times \hat{z} \tag{1}$$

$$\boldsymbol{H}_{\mathrm{I(TE)}} \cong \sum_{n}^{N_E} \left(e^{+}_{\mathrm{I(TE)}n} - e^{-}_{\mathrm{I(TE)}n}\right)Y_{\mathrm{I(TE)}n}\nabla\varphi_{\mathrm{I}n} \tag{2}$$

$$\boldsymbol{E}_{\mathrm{I(TM)}} \cong \sum_{n}^{N_M} \left(e^{+}_{\mathrm{I(TM)}n} + e^{-}_{\mathrm{I(TM)}n}\right)\nabla\psi_{\mathrm{I}n} \tag{3}$$

$$\boldsymbol{H}_{\mathrm{I(TM)}} \cong \sum_{n}^{N_M} \left(e^{+}_{\mathrm{I(TM)}n} - e^{-}_{\mathrm{I(TM)}n}\right)Y_{\mathrm{I(TM)}n}\nabla\psi_{\mathrm{I}n} \times \hat{z}. \tag{4}$$

Here

$$Y_{\mathrm{I(TE)}n} = \beta_{\mathrm{I(TE)}n} / \omega\mu$$

and

$$Y_{\mathrm{I(TM)}n} = \omega\epsilon / \beta_{\mathrm{I(TM)}n},$$

$\mathrm{Re}[\beta_{\mathrm{I(TE)}n}]$($\mathrm{Re}[\beta_{\mathrm{I(TM)}n}]$) being the propagation constant of the nth TE(TM) mode. Also, $\hat{z}$ is the unit vector of the z axis, ∇ is the transverse gradient, ω is the angular frequency, μ is the magnetic permeability of the medium (throughout this work $\mu = \mu_0$), and ϵ is the dielectric constant. The modal series have been truncated by retaining only a limited number of modes. The unknown coefficients with suffixes "+" or "−" account for a wave traveling toward $+z$ (+) and a wave traveling toward $-z$ (−). The scalar functions $\varphi_{\mathrm{I}n}$ and $\psi_{\mathrm{I}n}$ are then the solu-

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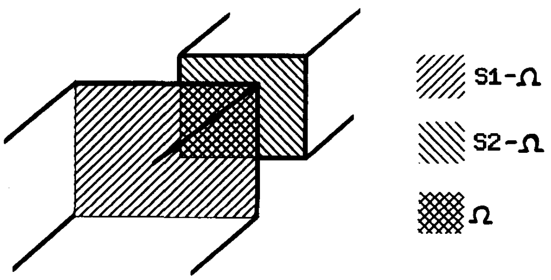

Fig. 1. Geometry of a generic scattering problem. $S1$ and $S2$ are the two waveguide cross sections. Ω is the common region.

tions of the following boundary value problems:

$$\nabla^2\varphi_{In} + k_{In}^2\varphi_{In} = 0 \quad \text{in } S_I$$
$$\nabla\varphi_{In}\cdot\hat{n} = 0 \quad \text{on } \sigma_I \tag{5}$$
$$\nabla^2\psi_{In} + \kappa_{In}^2\psi_{In} = 0 \quad \text{in } S_I$$
$$\psi_{In} = 0 \quad \text{on } \sigma_I \tag{6}$$

for which the following normalizing condition holds:

$$\int_{S_i}\varphi_{In}^2\,dS = \int_{S_i}\psi_{In}^2\,dS = 1 \tag{7}$$

and are therefore the normalized longitudinal fields. The quantity k_{In} is the eigenvalue of the nth TE mode and κ_{In} is the eigenvalue of the nth TM mode in waveguide I ($\beta_{I(TE)n}^2 = \omega^2\mu\epsilon - k_{In}^2$, $\beta_{I(TM)n}^2 = \omega^2\mu\epsilon - \kappa_{In}^2$). Continuity of the tangential components of the fields at the discontinuity region requires that

$$\boldsymbol{E}_1 = \boldsymbol{E}_2 \quad \text{in } \Omega \tag{8}$$
$$\boldsymbol{H}_1 = \boldsymbol{H}_2 \quad \text{in } \Omega. \tag{9}$$
$$\text{in } \Sigma = (S_1 \cup S_2) - \Omega \quad \begin{cases} \boldsymbol{E}_I = 0 & \text{if } \Sigma \text{ is electric} \\ \boldsymbol{H}_I = 0 & \text{if } \Sigma \text{ is magnetic.}\end{cases} \tag{10}$$

Equations (8)–(10) can be converted to a set of algebraic equations by projecting them onto some suitable functional space. The general case requires projecting (8)–(10) onto a complete space of vector functions with no simplifications (that is, no previous knowledge of the problem). By defining the scalar product of two vector fields $\boldsymbol{A}$ and $\boldsymbol{B}$ as

$$P = \int_s \boldsymbol{A}\cdot\boldsymbol{B}\,dS$$

and observing that a complete set of vector functions for the tangential $\boldsymbol{E}$ field and the tangential $\boldsymbol{H}$ field in region S satisfying the boundary conditions is given by the space of TE and TM modes themselves, a linear set of equations is obtained by first writing (8)–(10) as follows:

$$\boldsymbol{E}_{1(TE)} + \boldsymbol{E}_{1(TM)} = \boldsymbol{E}_{2(TE)} + \boldsymbol{E}_{2(TM)}. \tag{11}$$
$$\boldsymbol{H}_{1(TE)} + \boldsymbol{H}_{1(TM)} = \boldsymbol{H}_{2(TE)} + \boldsymbol{H}_{2(TM)}. \tag{12}$$
$$(\boldsymbol{H}_{I(TE)} + \boldsymbol{H}_{I(TM)}) \text{ or } (\boldsymbol{E}_{I(TE)} + \boldsymbol{E}_{I(TM)}) = 0 \quad \text{on } \Sigma. \tag{13}$$

Then by substituting (1)–(4) into (11)–(13) one obtains the basic equations to be projected onto the functional space chosen for $\boldsymbol{E}$ and $\boldsymbol{H}$.

The equations representing the generic projection are then

$$\int\nabla\psi_{In}\cdot(\boldsymbol{E}_{1(TE)} + \boldsymbol{E}_{1(TM)})\,dS = \int\nabla\psi_{In}\cdot(\boldsymbol{E}_{2(TE)} + \boldsymbol{E}_{2(TM)})\,dS \tag{14}$$

$$\int(\nabla\varphi_{In}\times\hat{z})\cdot(\boldsymbol{E}_{1(TE)} + \boldsymbol{E}_{1(TM)})\,dS = \int(\nabla\varphi_{In}\times\hat{z})\cdot(\boldsymbol{E}_{2(TE)} + \boldsymbol{E}_{2(TM)})\,dS \tag{15}$$

$$\int\nabla\varphi_{In}\cdot(\boldsymbol{H}_{1(TE)} + \boldsymbol{H}_{1(TM)})\,dS = \int\nabla\varphi_{In}\cdot(\boldsymbol{H}_{2(TE)} + \boldsymbol{H}_{2(TM)})\,dS \tag{16}$$

$$\int(\nabla\psi_{In}\times\hat{z})\cdot(\boldsymbol{H}_{1(TE)} + \boldsymbol{H}_{1(TM)})\,dS = \int(\nabla\psi_{In}\times\hat{z})\cdot(\boldsymbol{H}_{2(TE)} + \boldsymbol{H}_{2(TM)})\,dS. \tag{17}$$

It is pointed out that in effect the functional spaces used to project (14)–(17) are still undetermined. Although the set of TE and TM modes is complete, some further consideration should be given to the boundary condition. Depending on the type of scattering problem, the functional spaces in effect change. For simplicity we will consider the case $S_1 \subset S_2$ and Σ a perfectly conducting wall. In this case the functional spaces can be defined as follows: for the electric field equations a suitable functional space is the space of the tangential electric fields of TE and TM modes in region 1, while for the magnetic field equations a suitable functional space is the space of the tangential magnetic fields of TE and TM modes in region 2.

In fact, given the field expansions (1)–(4), the tangential electric field in region 1 may be regarded as a known quantity (0 on Σ and $\boldsymbol{E}_2$ on S_2); therefore it can be projected onto the modes of region 1. The magnetic field in region 2 can be regarded as a known quantity too ($\boldsymbol{H}_1$ on S_2) and it can be projected onto the modes of region 2. Therefore in (14) and (15) $I = 1$, while in (16) and (17) $I = 2$. Since $\boldsymbol{E} = 0$ on Σ, by including this condition in the projections, the integrals on the right in (14) and (15) extend over S_2. If Σ were a perfect magnetic wall, the regions of the projections would need to be interchanged.

After substitution of (1)–(4) into (14)–(17), one gets the final system of equations. By proper algebric manipulation the generalized scattering matrix can be obtained ([16]).

III. Transformation to Line Integrals

By inspection on the projection equations, the following integrals appear in the electric field equations:

$$\mathscr{E}_{e,TE}(m,n,I,J,S_E) = \int_{S_E}\nabla\psi_{Im}\cdot\nabla\varphi_{Jn}\times\hat{z}\,dS \tag{18}$$

$$\mathscr{E}_{e,TM}(m,n,I,J,S_E) = \int_{S_E}\nabla\psi_{Im}\cdot\nabla\psi_{Jn}\,dS \tag{19}$$

$$\mathscr{E}_{h,TE}(m,n,I,J,S_E) = \int_{S_E}\nabla\varphi_{Im}\times\hat{z}\cdot\nabla\varphi_{Jn}\times\hat{z}\,dS \tag{20}$$

$$\mathscr{E}_{h,TM}(m,n,I,J,S_E) = \int_{S_E}\nabla\varphi_{Im}\times\hat{z}\cdot\nabla\psi_{Jn}\,dS = \mathscr{E}_{e,TE}(n,m,J,I,S_E) \tag{21}$$

where $S_E = S_1$ if $I = J = 1$ and $S_E = S_1$ if $I \neq J$. In the magnetic field equation the integrals are the following:

$$\mathscr{H}_{h,\mathrm{TE}}(m,n,I,J,S_H) = \int_{S_H} \nabla\varphi_{Im}\cdot\nabla\varphi_{Jn}\, d\Omega = \mathscr{E}_{h,\mathrm{TE}}(m,n,I,J,S_H) \tag{22}$$

$$\mathscr{H}_{h,\mathrm{TM}}(m,n,I,J,S_H) = \int_{S_H} \nabla\varphi_{Im}\cdot\nabla\psi_{Jn}\times\hat{z}\, d\Omega = -\mathscr{E}_{e,\mathrm{TE}}(n,m,J,I,S_H) \tag{23}$$

$$\mathscr{H}_{e,\mathrm{TE}}(m,n,I,J,S_H) = \int_{S_H} \nabla\psi_{Im}\times\hat{z}\cdot\nabla\varphi_{Jn}\, d\Omega = -\mathscr{E}_{e,\mathrm{TE}}(m,n,I,J,S_H) \tag{24}$$

$$\mathscr{H}_{e,\mathrm{TM}}(m,n,I,J,S_H) = \int_{S_H} \nabla\psi_{Im}\times\hat{z}\cdot\nabla\psi_{Jn}\times\hat{z}\, d\Omega = \mathscr{E}_{e,\mathrm{TM}}(m,n,I,J,S_H) \tag{25}$$

where $S_H = S_2$.

Let us now identify the three fundamental types of integrations:

$$\int_S \nabla\psi_{Im}\cdot\nabla\varphi_{Jn}\times\hat{z}\, d\Omega = \Xi_{IJmn}(S) \tag{26}$$

$$\int_S \nabla\psi_{Im}\cdot\nabla\psi_{Jn}\, d\Omega = \Psi_{IJmn}(S) \tag{27}$$

$$\int_S \nabla\varphi_{Im}\times\hat{z}\cdot\nabla\varphi_{Jn}\times\hat{z}\, d\Omega = \Phi_{IJmn}(S) \tag{28}$$

where S in the generic region. Letting σ be the boundary of S, $\hat{n}$ the unit vector defined along σ pointing outward, $\hat{s}$ the unit vector identifying the direction tangent to σ (counterclockwise), according to Green's first identity the following equalities hold:

$$\int_s \nabla A\cdot\nabla B\, d\Omega = \int_\sigma A\nabla B\cdot\hat{n}\, ds - \int_s A\nabla\cdot\nabla B\, d\Omega = \int_\sigma B\nabla A\cdot\hat{n}\, ds - \int_s B\nabla\cdot\nabla A\, d\Omega \tag{29}$$

$$\int_s \nabla A\cdot\nabla B\times\hat{z}\, d\Omega = \int_\sigma A\nabla B\cdot\hat{s}\, ds = -\int_\sigma B\nabla A\cdot\hat{s}\, ds \tag{30}$$

A and B being two generic scalar fields [15]. Since

$$\nabla\cdot\nabla\varphi_{In} = -k_{In}^2\varphi_{In} \tag{31}$$

$$\nabla\cdot\nabla\psi_{In} = -\kappa_{In}^2\psi_{In}. \tag{32}$$

After some algebraic manipulations one gets

$$\Xi_{IJmn}(S) = \int_\sigma \psi_{Im}\frac{\partial\varphi_{Jn}}{\partial s}\, ds = -\int_\sigma \varphi_{Jn}\frac{\partial\psi_{Im}}{\partial s}\, ds \tag{33}$$

$$\Psi_{IJmn}(S) = \begin{cases} \text{if } I = J & \kappa_{In}^2\delta_{mn} \\ \text{if } I \neq J & \begin{cases} \Psi'_{IJmn} & \text{if } \kappa_{In} \neq \kappa_{Jm} \\ \Psi''_{IJmn} & \text{if } \kappa_{In} = \kappa_{Jm} \end{cases} \end{cases} \tag{34}$$

where

$$\Psi'_{IJmn}(S) = \frac{1}{\kappa_{Im}^2 - \kappa_{Jn}^2}\left(\kappa_{Im}^2\int_\sigma \psi_{Im}\frac{\partial\psi_{Jn}}{\partial n}\, ds - \kappa_{Jn}^2\int_\sigma \psi_{Jn}\frac{\partial\psi_{Im}}{\partial n}\, ds\right) \tag{35}$$

$$\Psi''_{IJmn}(S) = \kappa_{Im}^2\int_s \psi_{Im}\psi_{Jn}\, dS = \kappa_{Jn}^2\int_s \psi_{Im}\psi_{Jn}\, dS. \tag{36}$$

Then

$$\Phi_{IJmn}(S) = \begin{cases} \text{if } I = J & k_{In}^2\delta_{mn} \\ \text{if } I \neq J & \begin{cases} \Phi'_{IJmn} & \text{if } k_{In} \neq k_{Jm} \\ \Phi''_{IJmn} & \text{if } k_{In} = k_{Jm} \end{cases} \end{cases} \tag{37}$$

where

$$\Phi'_{IJmn}(S) = \frac{1}{k_{Im}^2 - k_{Jn}^2}\left(k_{Im}^2\int_\sigma \varphi_{Im}\frac{\partial\varphi_{Jn}}{\partial n}\, ds - k_{Jn}^2\int_\sigma \varphi_{Jn}\frac{\partial\varphi_{Im}}{\partial n}\, ds\right) \tag{38}$$

$$\Psi''_{IJmn}(S) = k_{Im}^2\int_S \varphi_{Im}\varphi_{Jn}\, dS = k_{Jn}^2\int_S \varphi_{Im}\varphi_{Jn}\, dS. \tag{39}$$

In (33)–(39), excluding the case $\kappa_{Jn} = \kappa_{Im}$ or $k_{Jn} = k_{Im}$, only line integrals appear.

It is interesting to note that some terms are always null. This happens in the expressions for Ψ'_{IJmn} and Φ'_{IJmn}, where one of the line integrals vanishes, either the function itself or its normal derivative being null. Moreover

$$\Xi_{IJmn}(S_I) = 0 \tag{40}$$

The problem can now be cast in matrix form. Let us introduce

$$\boldsymbol{e}_{\mathrm{I,TE}} = \left[e^+_{\mathrm{I(TE)}n} + e^-_{\mathrm{I(TE)}n}\right] \tag{41}$$

$$\boldsymbol{e}_{\mathrm{I,TM}} = \left[e^+_{\mathrm{I(TM)}n} + e^-_{\mathrm{I(TM)}n}\right] \tag{42}$$

$$\boldsymbol{h}_{\mathrm{I,TE}} = \mathrm{diag}\left[Y_{\mathrm{I(TE)}n}\right]\left[e^+_{\mathrm{I(TE)}n} - e^-_{\mathrm{I(TE)}n}\right] = \boldsymbol{D}_{\mathrm{I,TE}}\left[e^+_{\mathrm{I(TE)}n} - e^-_{\mathrm{I(TE)}n}\right] \tag{43}$$

$$\boldsymbol{h}_{\mathrm{I,TM}} = \mathrm{diag}\left[Y_{\mathrm{I(TM)}n}\right]\left[e^+_{\mathrm{I(TM)}n} - e^-_{\mathrm{I(TM)}n}\right] = \boldsymbol{D}_{\mathrm{I,TM}}\left[e^+_{\mathrm{I(TM)}n} - e^-_{\mathrm{I(TM)}n}\right] \tag{44}$$

$$\boldsymbol{F}_{11}^{(1)} = \left[\Phi_{11mn}(S_I)\right] \qquad \boldsymbol{F}_{12}^{(1)} = \left[\Phi_{12mn}(S_I)\right]$$
$$\boldsymbol{F}_{21}^{(1)} = \left[\Phi_{21mn}(S_I)\right] \qquad \boldsymbol{F}_{22}^{(1)} = \left[\Phi_{22mn}(S_I)\right] \tag{45}$$

$$\boldsymbol{P}_{11}^{(1)} = \left[\Psi_{11mn}(S_I)\right] \qquad \boldsymbol{P}_{12}^{(1)} = \left[\Psi_{12mn}(S_I)\right]$$
$$\boldsymbol{P}_{21}^{(1)} = \left[\Psi_{21mn}(S_I)\right] \qquad \boldsymbol{P}_{22}^{(1)} = \left[\Psi_{22mn}(S_I)\right] \tag{46}$$

$$\boldsymbol{X}_{11}^{(1)} = \left[\Xi_{11mn}(S_I)\right] \qquad \boldsymbol{X}_{12}^{(1)} = \left[\Xi_{12mn}(S_I)\right]$$
$$\boldsymbol{X}_{21}^{(1)} = \left[\Xi_{21mn}(S_I)\right] \qquad \boldsymbol{X}_{22}^{(1)} = \left[\Xi_{22mn}(S_I)\right]. \tag{47}$$

Equations (13)–(20) can be written as follows:

$$\begin{bmatrix} \boldsymbol{F}_{11}^{(1)} & \boldsymbol{X}_{11}^{(1)\prime} \\ \boldsymbol{X}_{11}^{(1)} & \boldsymbol{P}_{11}^{(1)} \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_{1,\mathrm{TE}} \\ \boldsymbol{e}_{1,\mathrm{TM}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{12}^{(2)} & \boldsymbol{X}_{21}^{(2)\prime} \\ \boldsymbol{X}_{12}^{(2)} & \boldsymbol{P}_{12}^{(2)} \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_{2,\mathrm{TE}} \\ \boldsymbol{e}_{2,\mathrm{TM}} \end{bmatrix} \tag{48}$$

$$\begin{bmatrix} \boldsymbol{F}_{12}^{(2)\prime} & \boldsymbol{X}_{12}^{(2)\prime} \\ \boldsymbol{X}_{21}^{(2)} & \boldsymbol{P}_{12}^{(2)\prime} \end{bmatrix}\begin{bmatrix} \boldsymbol{h}_{1,\mathrm{TE}} \\ \boldsymbol{h}_{1,\mathrm{TM}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{22}^{(2)} & \boldsymbol{X}_{22}^{(2)\prime} \\ \boldsymbol{X}_{22}^{(2)} & \boldsymbol{P}_{22}^{(2)} \end{bmatrix}\begin{bmatrix} \boldsymbol{h}_{2,\mathrm{TE}} \\ \boldsymbol{h}_{2,\mathrm{TM}} \end{bmatrix} \tag{49}$$

where the prime denotes the transpose matrix. Equations (48) and (49) represent respectively the E-field and the H-field equations imposing the continuity of the fields at the interface. The upper matrix equations are the projections onto the space of TE modes (matrices $F_{11}^{(1)}, X_{11}^{(1)'}, F_{12}^{(2)}, X_{21}^{(2)'}, F_{12}^{(2)'}, -X_{12}^{(2)'}, F_{22}^{(2)}, -X_{22}^{(2)'}$); the lower matrix equations are the projections onto the space of TM modes (matrices $X_{11}^{(1)}, P_{11}^{(1)'}, X_{12}^{(2)}, P_{12}^{(2)}, -X_{21}^{(2)}, P_{12}^{(2)'}, -X_{22}^{(2)}, P_{22}^{(2)}$). According to the previous discussion the following equalities hold:

$$F_{11}^{(1)} = \operatorname{diag}\left[k_{1n}^2\right] f_1 \qquad F_{22}^{(2)} = \operatorname{diag}\left[k_{2n}^2\right] = f_2 \tag{50}$$

$$P_{11}^{(1)} = \operatorname{diag}\left[\kappa_{1n}^2\right] = p_1 \qquad P_{22}^{(2)} = \operatorname{diag}\left[\kappa_{2n}^2\right] = p_2$$

$$X_{I1}^{(1)} = X_{I2}^{(1)} = 0. \tag{51}$$

The matrix equations now become

$$\begin{bmatrix} f_1 & 0 \\ 0 & p_1 \end{bmatrix} \begin{bmatrix} e_{1,\mathrm{TE}} \\ e_{1,\mathrm{TM}} \end{bmatrix} = \begin{bmatrix} F_{12}^{(2)} & 0 \\ X_{12}^{(2)} & P_{12}^{(2)} \end{bmatrix} \begin{bmatrix} e_{2,\mathrm{TE}} \\ e_{2,\mathrm{TM}} \end{bmatrix} \tag{52}$$

$$\begin{bmatrix} F_{12}^{(2)} & X_{12}^{(2)'} \\ 0 & P_{12}^{(2)'} \end{bmatrix} \begin{bmatrix} h_{1,\mathrm{TE}} \\ h_{1,\mathrm{TM}} \end{bmatrix} = \begin{bmatrix} f_2 & 0 \\ 0 & p_2 \end{bmatrix} \begin{bmatrix} h_{2,\mathrm{TE}} \\ h_{2,\mathrm{TM}} \end{bmatrix}. \tag{53}$$

Equations (52) and (53) are quite general: they represent the matrix equations resulting from the application of the mode-matching technique to a quite arbitrary structure (with the assumptions previously stated).

It can be observed that the TM fields in waveguide 2 (the smaller one) and the TE fields in waveguide 1 never couple, independently of the boundary shape.

The elements of the matrices are then the following:

$$F_{12}^{(2)} = [f_{mn}],$$

$$f_{mn} = \begin{cases} k_{1m}^2 \displaystyle\int_{S_2} \varphi_{1m}\varphi_{2n}\, dS & \text{if } k_{1m} = k_{2n} \\ \dfrac{1}{k_{2n}^2 - k_{1m}^2} k_{2n}^2 \displaystyle\int_{\sigma_2} \varphi_{2n} \frac{\partial \varphi_{1m}}{\partial n}\, ds & \text{otherwise} \end{cases} \tag{54}$$

$$P_{12}^{(2)} = [p_{mn}],$$

$$p_{mn} = \begin{cases} \kappa_{1m}^2 \displaystyle\int_{S_2} \psi_{1m}\psi_{2n}\, dS & \text{if } \kappa_{1m} = \kappa_{2n} \\ \dfrac{1}{\kappa_{1m}^2 - \kappa_{2n}^2} \kappa_{1m}^2 \displaystyle\int_{\sigma_2} \psi_{1m} \frac{\partial \psi_{2n}}{\partial n}\, ds & \text{otherwise} \end{cases} \tag{55}$$

$$X_{12}^{(2)} = [x_{mn}], \qquad x_{mn} = \int_{\sigma_2} \psi_{1m} \frac{\partial \varphi_{2n}}{\partial s}\, ds. \tag{56}$$

As a further comment, it is pointed out that although the integrals are extended over σ_2, if a part of σ_2 belongs to σ_1 too, all the line integrals of (54)–(56) relative to that part vanish, since throughout σ_1 $\psi_{1m} = 0$ and $\partial\varphi_{1m}/\partial n = 0$. In that case, letting l be the part of σ_2 not belonging to σ_1 too, all the line integrals in (54)–(56) extend over l.

It can then be observed that the coupling of tangential fields of TE modes in the different waveguides depends on the coupling of the longitudinal components when the modes have the same eigenvalue (see eq. (39)) and the same happens for TM modes (eq. (36)).

When S_1 does not enclose S_2 completely, a different formulation should be employed. Although several cases have been analyzed in the literature and several formulations can be used, by considering the junction of the two waveguides as a cascade connection of two junctions: $(S_1, S_1 \cap S_2) + (S_1 \cap S_2, S_2)$, the projections can be done as described in Section II (this procedure may not be the most convenient one, since it requires a knowledge of the modes in region $S_1 \cap S_2$). For the two single junctions the formulas presented in this paper still hold, but this does not imply that TE modes in a waveguide do not couple with TM modes in the other. Coupling may be present as a result of the cascade connection of the two junctions.

IV. Conclusions

Some properties of TE–TM mode-matching formulations for the analysis of scattering in waveguides can be easily demonstrated by reducing the surface integrals representing the projections of the equations of continuity of the tangential fields to line integrals. In particular it has been observed that whenever the cross section of a waveguide completely encloses the other, TE modes in the smaller waveguide never couple with TM modes in the larger waveguide, independently of the shape of both waveguides.

The reduction to line integrals is then convenient from a computational point of view, especially when a numerical method must be employed to compute the waveguide modes. In particular the results of numerical methods based on the formulation of an integral equation expressed on the boundary of the waveguide are directly applicable in (54)–(56).

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High-Performance HEMT Amplifiers with a Simple Low-Loss Matching Network

R. Peter, M. V. Schneider, and Y. S. Wu

***Abstract*—We report on the design and performance of a K-band HEMT amplifier whose passive circuit consists of low-loss suspended stripline elements. The single-stage amplifiers were built at 4 GHz and 22 GHz by using readily available commercial HEMT devices. In the desired frequency range from 21 to 23 GHz for the high-frequency design, the best spot noise temperatures were 150 K and 65 K at 21.5 GHz for room and liquid nitrogen temperatures, respectively.**

I. Introduction

Recent advances in the performance of HEMT's make it possible to use the devices in low-noise applications, such as front ends for high-data-rate communications links or remote sensing circuits, instead of complex and expensive maser systems [1]–[4]. The main objective of the work described in this paper was to design and construct an amplifier at 22.235 GHz for a water vapor radiometer. Another objective was to use inexpensive and readily available chips embedded in a low-loss suspended microstrip structure (air line) instead of the more widely used coaxial air line design [5], [6].

Encouraged by the good results of a cryogenic IF amplifier at 4 GHz consisting of an air line structure, we used a similar design at 22 GHz. The absence of dielectric losses and a stripline to waveguide transition are the main features of our low-loss tuned circuit design. Since these circuits do not have a substrate, problems with thermal contraction of different materials will not occur when the amplifier is cooled. The air line design offers higher flexibility in making changes and reduces turnaround times for achieving optimized noise performance. Furthermore, no expensive substrates or precision photolithographic facilities are necessary.

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R. Peter is with the Institute of Applied Physics, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland.
M. V. Schneider and Y. S. Wu are with AT&T Bell Laboratories, Holmdel, NJ 07744.

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II. Amplifier Design

A schematic drawing showing the top view of the single-stage K-band amplifier is displayed in Fig. 1. The microstrip quarter-wave transformer consists of a milled gold-plated brass strip (0.15 mm thick) which is supported by two 0.25-mm-thick quartz slabs. The end of the low-impedance section is tapered in order to reduce the capacitive interaction between the air line and the grounded ridge in which the HEMT chip (Fujitsu FHR02X) is embedded (Fig. 2). In order to achieve a low parasitic inductance we used short gold wires with a diameter of 18 μm to connect the chip to the circuit by thermocompression bonding.

The input and output matching networks do include the air line–waveguide transitions, which are realized by a probe (50 Ω line section) extending 2.6 mm into the waveguide [7], [8]. The transitions have been tested with a 50 Ω trough line which is 1.1 mm wide and 0.25 mm above the ground plane of the housing. We measured a total insertion loss of the trough line of 0.5 dB (i.e., approximately 0.25 dB per transition) over a bandwidth of 2 GHz. Since this simple transition has a relatively narrow bandwidth, optimum tuning can be obtained at other center frequencies in the K band by adjusting the broad-band backshort.

The low-impedance sections of the transformers optimized for room temperature operation have a width of 2 mm and a length of 4.5 mm at the input and a length of 5.75 mm at the output, respectively. For the bias circuits a 90° radial stub on a 0.25-mm-thick alumina substrate was used with a $\lambda/4$ high-impedance bond wire. In order to improve the stability of the amplifier we added a 50 Ω series resistor and a shunt capacitor of 10 pF to the bias network and filled the cavity of the bias network with absorber material. In the following section we consider a simple calculation of the different noise contributions and the associated loss of the input matching circuit.

The noise performance of an amplifier is determined by the ohmic loss of the matching network to the input of the transistor and by the noise characteristics of the transistor. The noise temperature of an HEMT amplifier with a lossy input matching network can be expressed as the sum of two terms:

$$T_A = (L-1)T_L + LT_{\mathrm{HEMT}} \tag{1}$$

where L is the ohmic loss factor, T_L is the physical temperature of the network, and T_{HEMT} is the noise temperature of the transistor referred to the input of the matching network. T_{HEMT} is characterised by four noise parameters. T_{MIN}, the minimum noise temperature; $Z_{\mathrm{OPT}} = R_{\mathrm{OPT}} + X_{\mathrm{OPT}}$, the optimum source impedance; and g_N, the noise conductance. Over a narrow bandwidth, T_{HEMT} of a well noise matched transistor is approximately equal to T_{MIN}.

The total loss, L, of the matching circuit and transition at 22 GHz can be calculated with $T_{\mathrm{HEMT}} = 105$ K (from the manufacturer's data sheet for the FHR02X) and the measured amplifier noise temperature $T_A = 170$ K at room temperature ($T_L = 300$ K) using (1).

Table I shows the estimated losses of the input circuit (including the transitions) for the 22 GHz amplifier and the 4 GHz amplifier (similar construction).

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