



## Sintetizzatori ad aggancio di fase (PLL)

**Massimiliano Pieraccini**

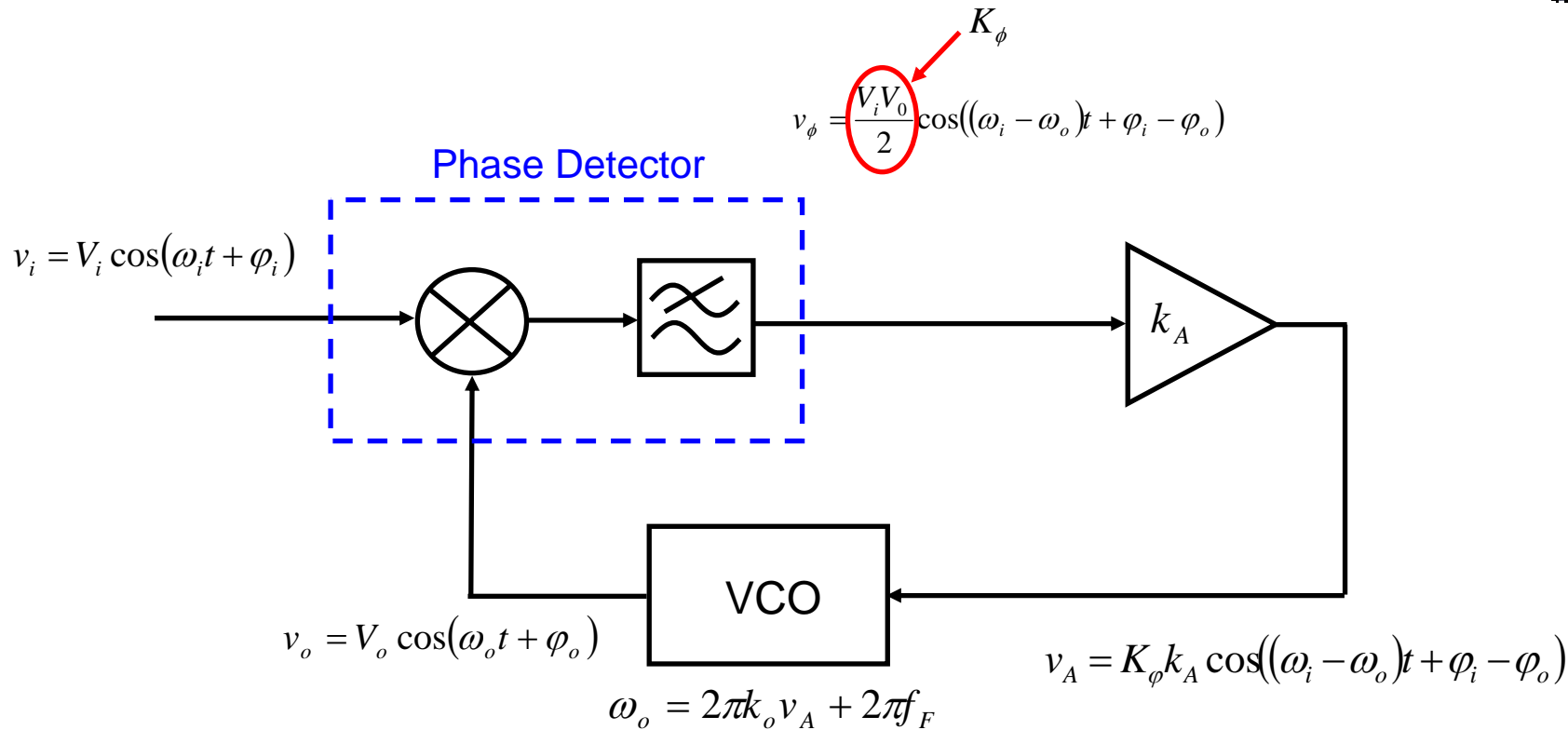


Sintetizzatore

Demodulatore FM

Sistema di aggancio della portante per demodulatori coerenti

Filtro inseguitore



$\omega_o = \omega_i$

$v_A = K_\phi k_A \cos(\varphi_i - \varphi_o)$

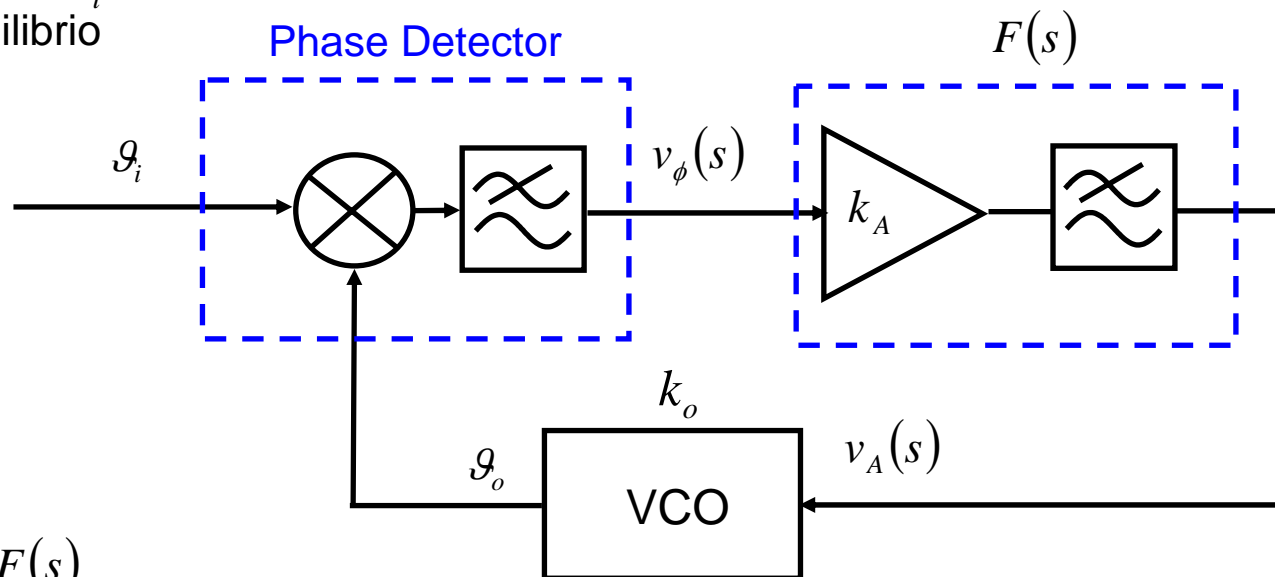
$\omega_i = 2\pi k_o v_A + 2\pi f_F$

$v_A = \frac{f_i - f_F}{k_0}$

la condizione  $\omega_o = \omega_i$   
è un punto di equilibrio  
stabile

# Analisi nel tempo

la condizione  $\omega_o = \omega_i$   
è un punto di equilibrio  
stabile



$$v_A(s) = v_\phi(s)F(s)$$

$$v_A(t) = v_A(0) + \int_0^t v_\phi(u)f(t-u)du$$

$f(t)$  antitrasformata di  $F(s)$

$$\frac{d\vartheta_o}{dt} = K_o v_A \quad v_\phi = \frac{V_i V_o}{2} \cos(\vartheta_e) \quad \vartheta_e = \vartheta_i - \vartheta_o$$

$$\frac{d\vartheta_o}{dt} = K_o v_A(0) + K_o \frac{V_i V_o}{2} \int_0^t \cos(\vartheta_e(u))f(t-u)du$$

# Analisi nel tempo

$$\frac{d\vartheta_o}{dt} = K_o v_A(0) + K_o \frac{V_i V_o}{2} \int_0^t \cos(\vartheta_e(u)) f(t-u) du$$

$$\frac{d\vartheta_e}{dt} = \frac{d\vartheta_i}{dt} - \frac{d\vartheta_o}{dt} \qquad \Delta\omega_i = \frac{d\vartheta_i}{dt}$$

$$\frac{d\vartheta_e}{dt} = \Delta\omega_i - K_v \int_0^t \cos(\vartheta_e(u)) f(t-u) du \qquad K_v = K_o \frac{V_i V_o}{2}$$

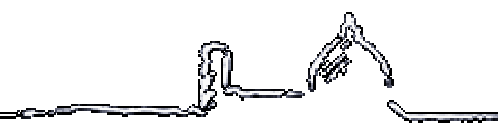
## Anello del primo ordine

$$f(t-u) = \delta(t-u)$$

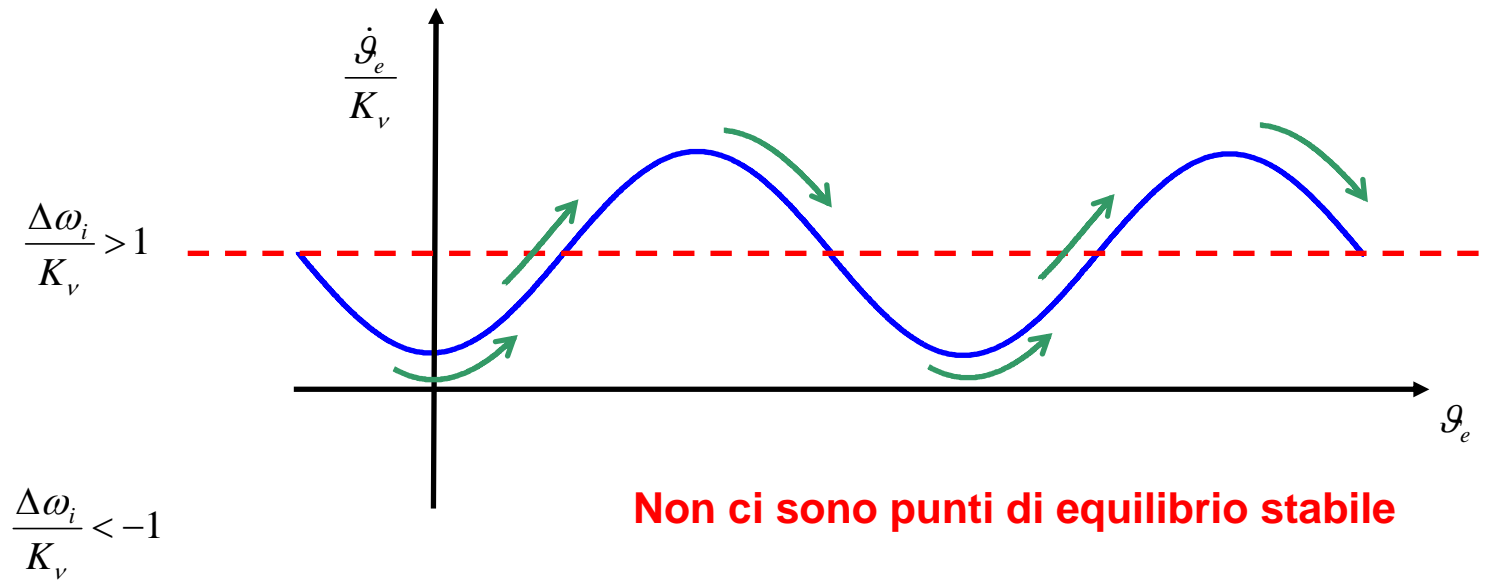
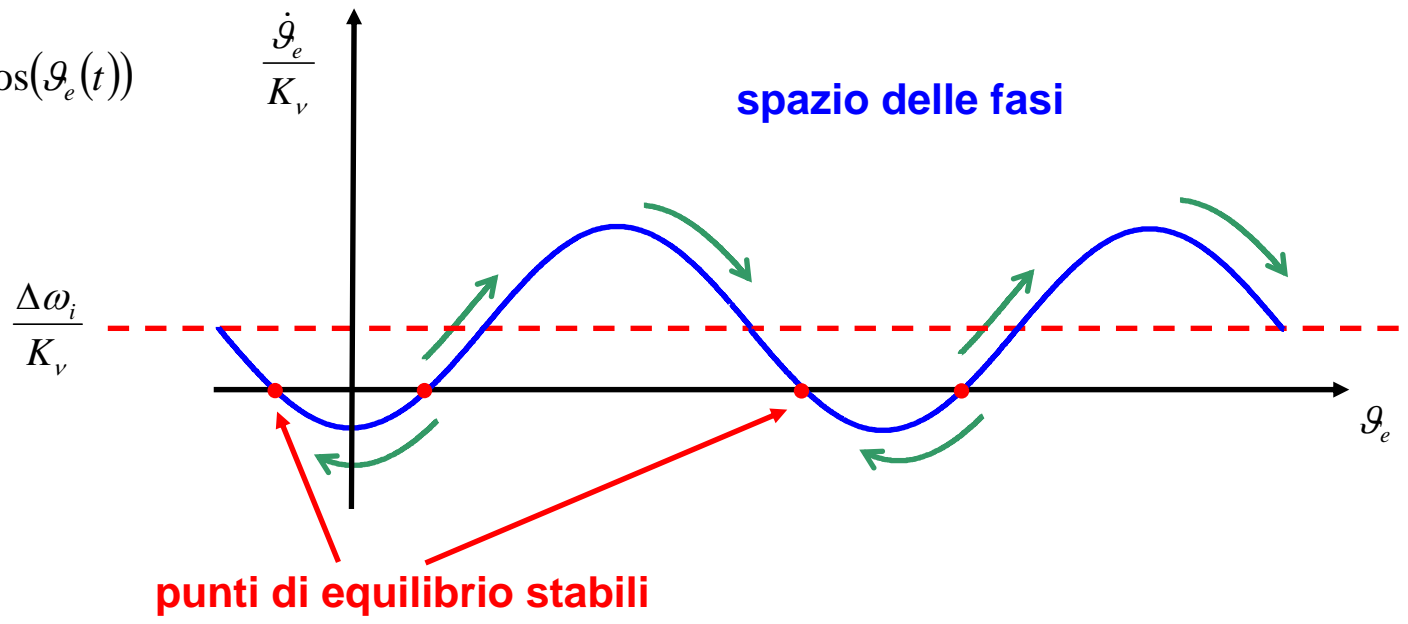
$$\frac{d\vartheta_e}{dt} = \Delta\omega_i - K_v \cos(\vartheta_e(t))$$

$$\frac{\dot{\vartheta}_e}{K_v} = \frac{\Delta\omega_i}{K_v} - \cos(\vartheta_e(t))$$

# Analisi nel tempo



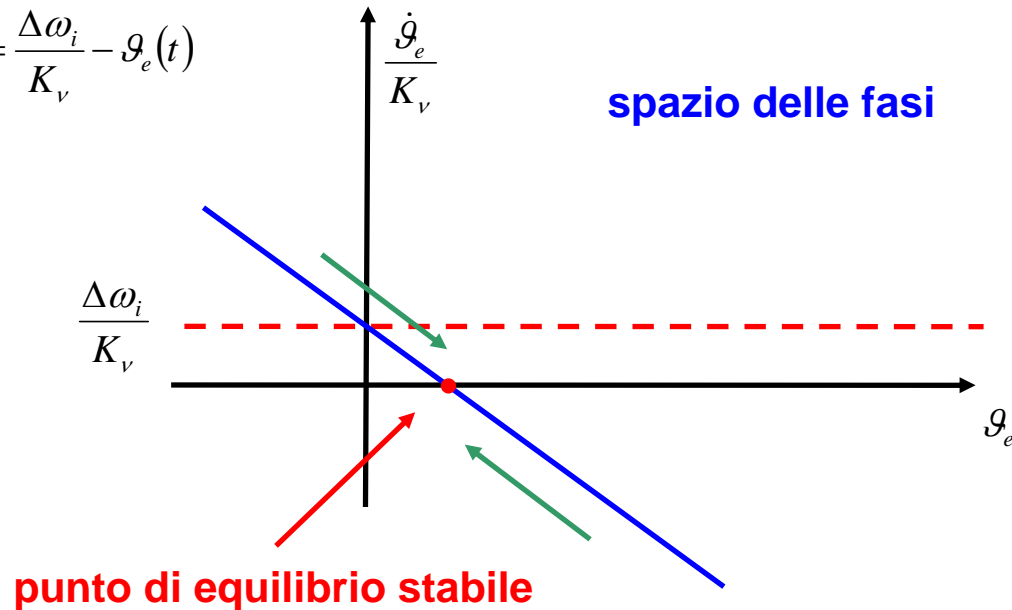
$$\frac{\dot{g}_e}{K_v} = \frac{\Delta\omega_i}{K_v} - \cos(g_e(t))$$



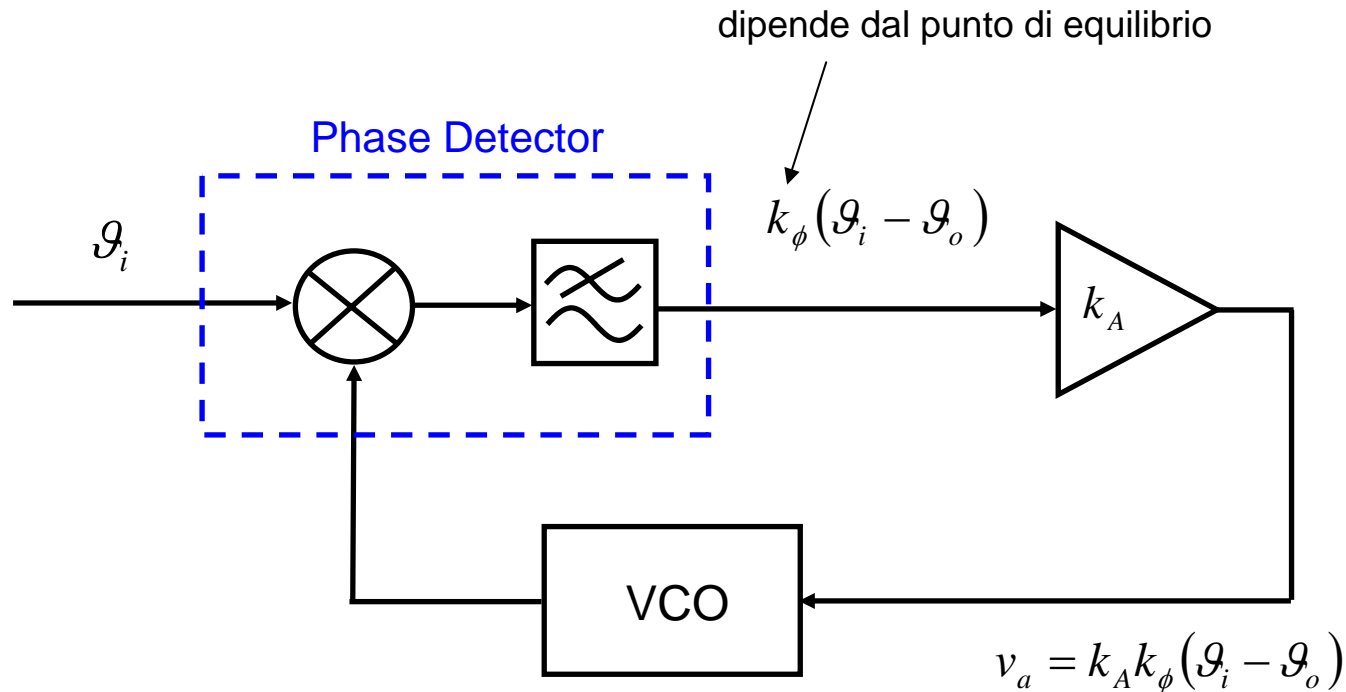
## Anello del primo ordine con rivelatore di fase lineare

$$v_\phi = \frac{V_i V_o}{2} g_e$$

$$\frac{\dot{g}_e}{K_v} = \frac{\Delta\omega_i}{K_v} - g_e(t)$$



# Analisi delle piccole variazioni intorno al punto di equilibrio stabile



$$\delta\omega_o = 2\pi k_o k_A k_\phi(\mathcal{G}_i - \mathcal{G}_o)$$

$$\delta\omega_o = 2\pi k_o v_a$$

$$\frac{d\mathcal{G}_o}{dt} = 2\pi k_o k_A k_\phi(\mathcal{G}_i - \mathcal{G}_o)$$

$$s\mathcal{G}_o = 2\pi k_o k_A k_\phi(\mathcal{G}_i - \mathcal{G}_o) \quad \boxed{k_v = 2\pi k_o k_A k_\phi}$$

$$s\mathcal{G}_o = k_v(\mathcal{G}_i - \mathcal{G}_o)$$

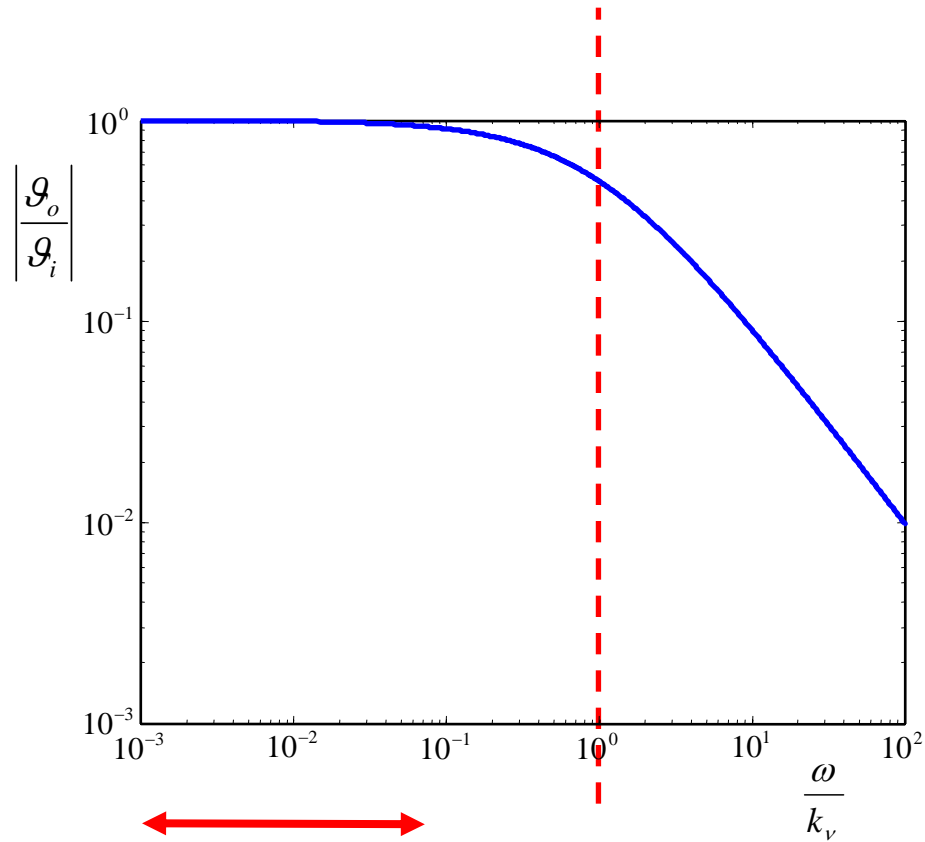
$$\frac{\mathcal{G}_o}{\mathcal{G}_i} = \frac{1}{1 + \frac{s}{k_v}}$$



# Analisi delle piccole variazioni intorno al punto di equilibrio stabile

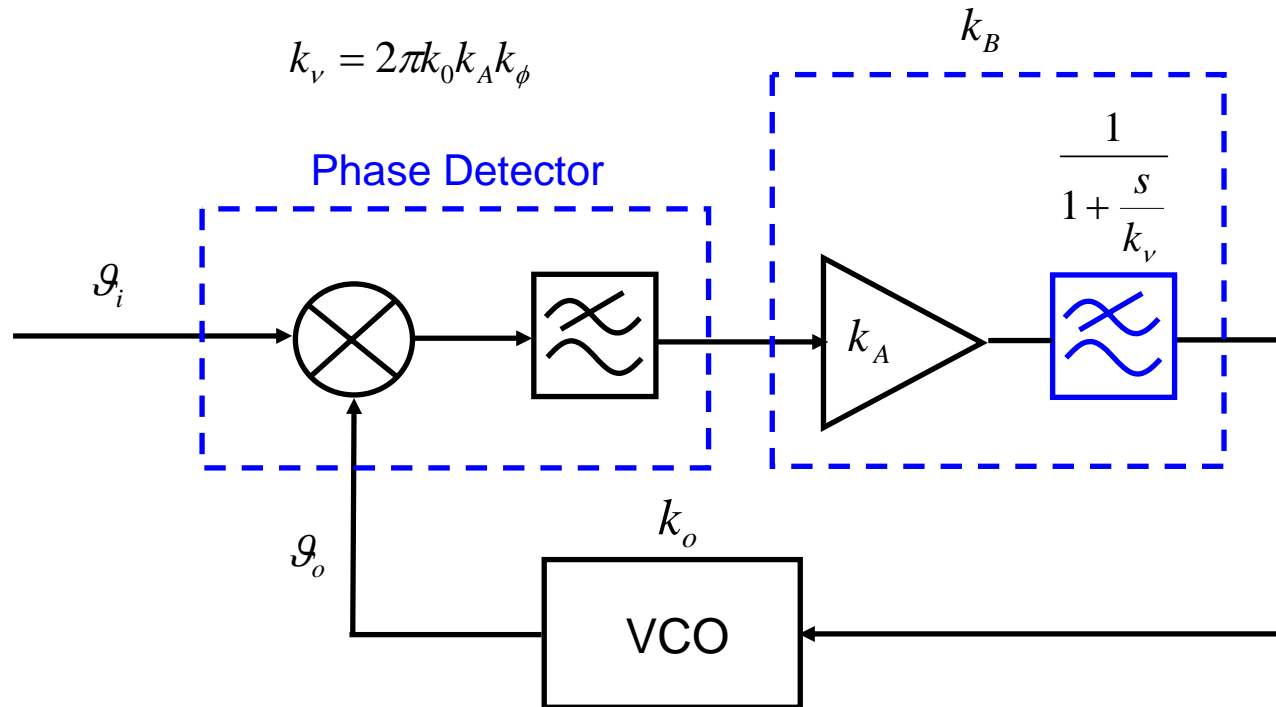


$$\frac{\mathcal{G}_o}{\mathcal{G}_i} = \frac{1}{1 + \frac{s}{k_v}}$$



**Demodulatore FM**

# Analisi delle piccole variazioni intorno al punto di equilibrio stabile



$$\frac{g_o}{g_i} = \frac{1}{1 + \frac{s}{k_q}}$$

$$k_q = 2\pi k_o k_B k_\phi$$

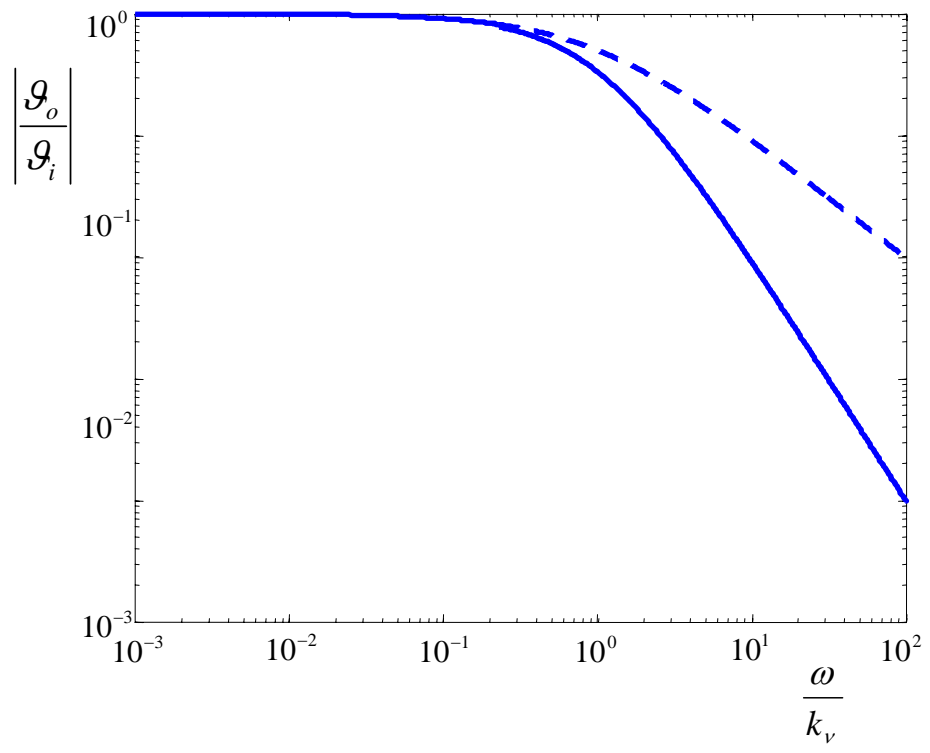
$$k_q = 2\pi k_o k_A k_\phi \frac{1}{1 + \frac{s}{k_v}} \longrightarrow k_q = k_v \frac{1}{1 + \frac{s}{k_v}}$$

$$\frac{g_o}{g_i} = \frac{1}{1 + \frac{s}{k_v} + \left(\frac{s}{k_v}\right)^2}$$

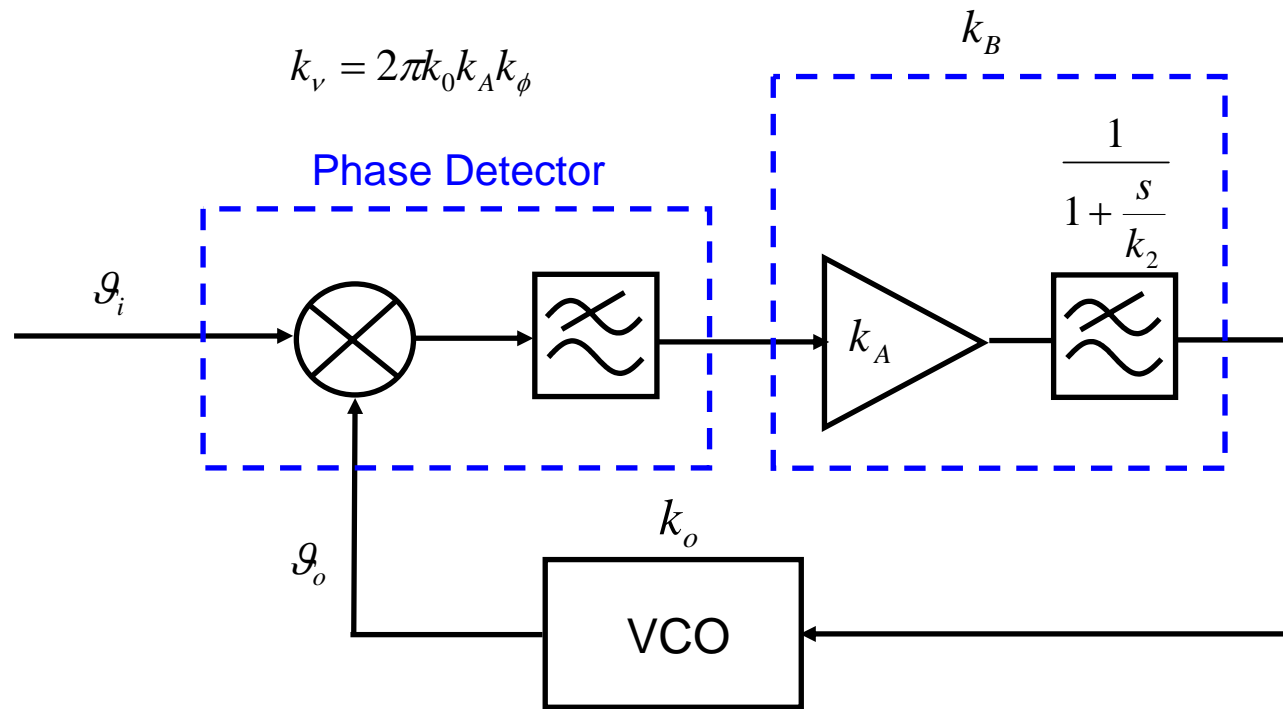
# Analisi delle piccole variazioni intorno al punto di equilibrio stabile



$$\frac{\mathcal{G}_o}{\mathcal{G}_i} = \frac{1}{1 + \frac{s}{k_v} + \left(\frac{s}{k_v}\right)^2}$$

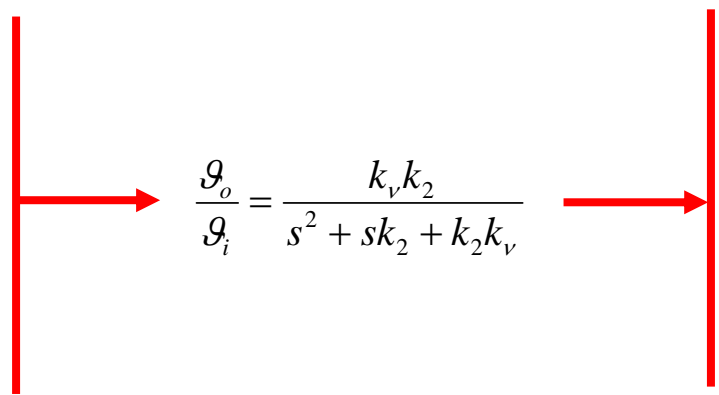


# Compensazione dell'anello



$$\frac{\mathcal{G}_o}{\mathcal{G}_i} = \frac{1}{1 + \frac{s}{k_q}}$$

$$k_q = 2\pi k_0 k_B k_\phi \rightarrow k_q = k_v \frac{1}{1 + \frac{s}{k_2}}$$



$$\frac{\mathcal{G}_o}{\mathcal{G}_i} = \frac{k_v k_2}{s^2 + s k_2 + k_2 k_v}$$

$$A(s) = \frac{A_0 \omega_0^2}{s^2 + 2z \omega_0 s + \omega_0^2}$$

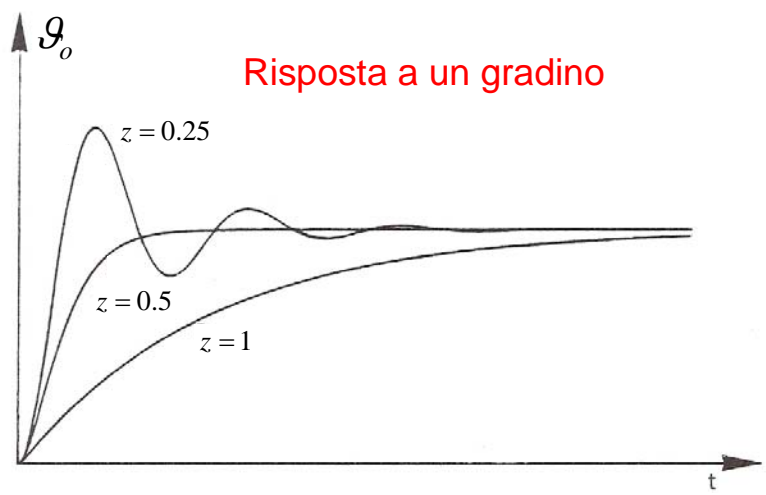
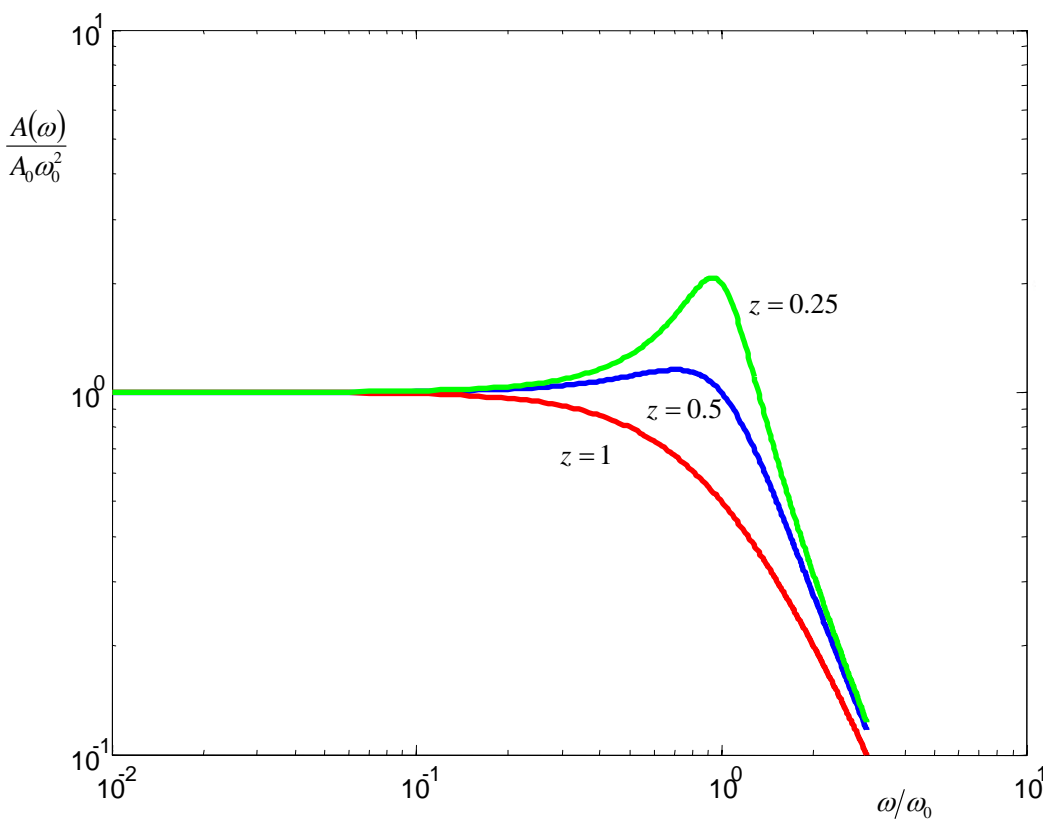
$$\omega_0 = \sqrt{k_v k_2}$$

$$z = \sqrt{\frac{k_2}{k_v}}$$

# Compensazione dell'anello



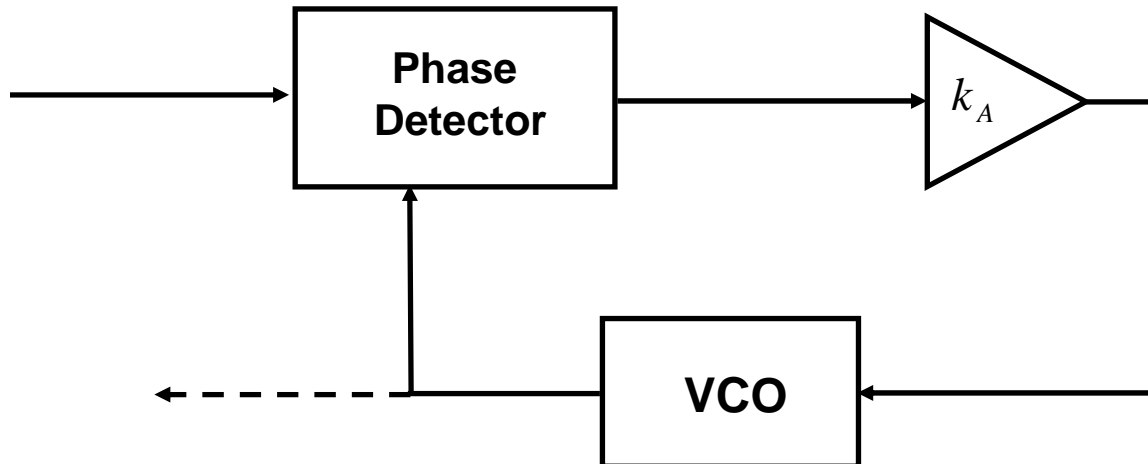
$$A(s) = \frac{A_0 \omega_0^2}{s^2 + 2z \omega_0 s + \omega_0^2} \quad \omega_0 = \sqrt{k_v k_2} \quad z = \sqrt{\frac{k_2}{k_v}}$$



## Demodulatore FM

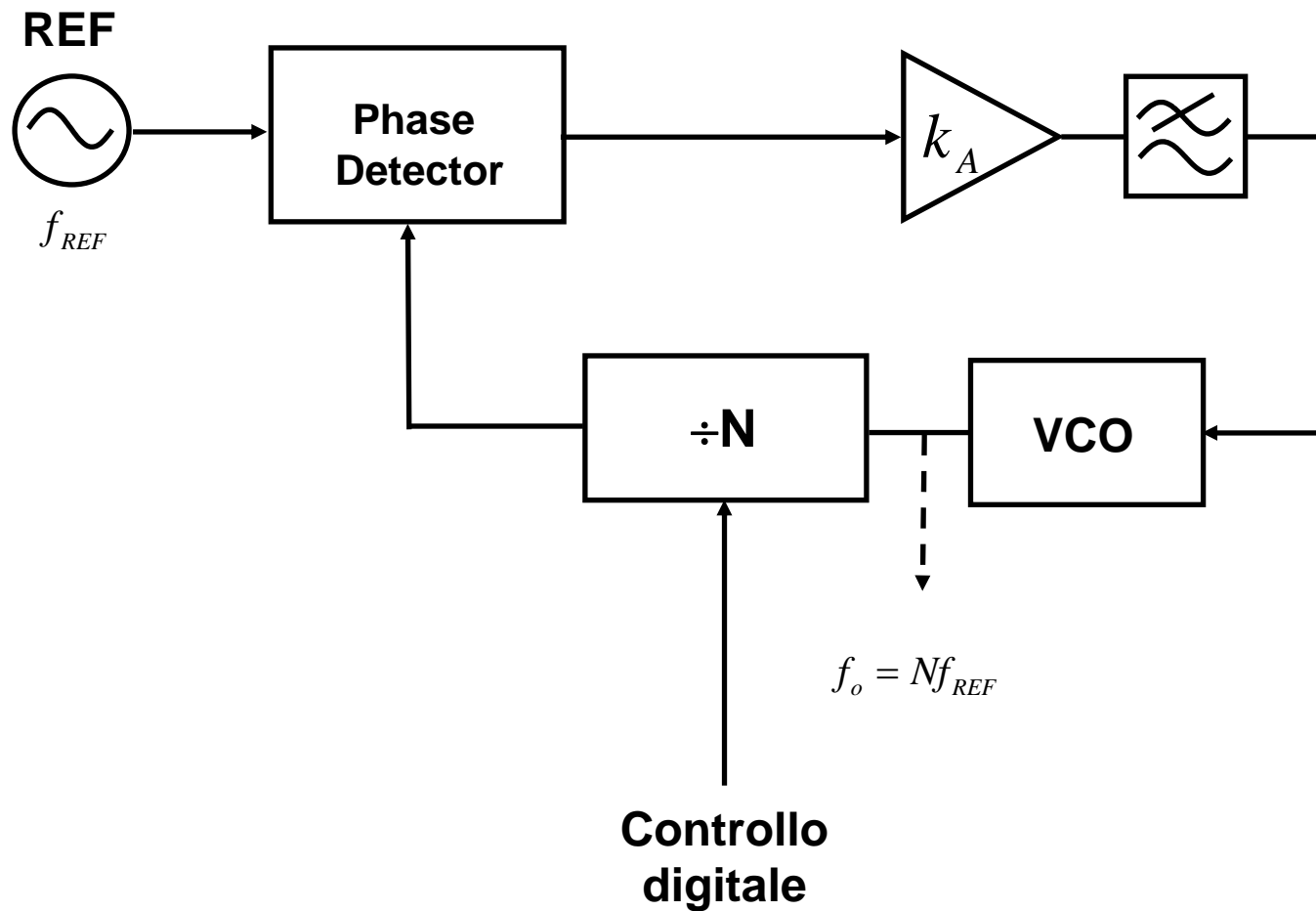
$$v_A = \frac{f_i - f_F}{k_0} \longrightarrow \boxed{\delta v_A = \frac{\delta f_i}{k_0}} \quad f_m < \frac{k_v}{2\pi}$$

## Aggancio della portante per demodulazione coerente (filtro)

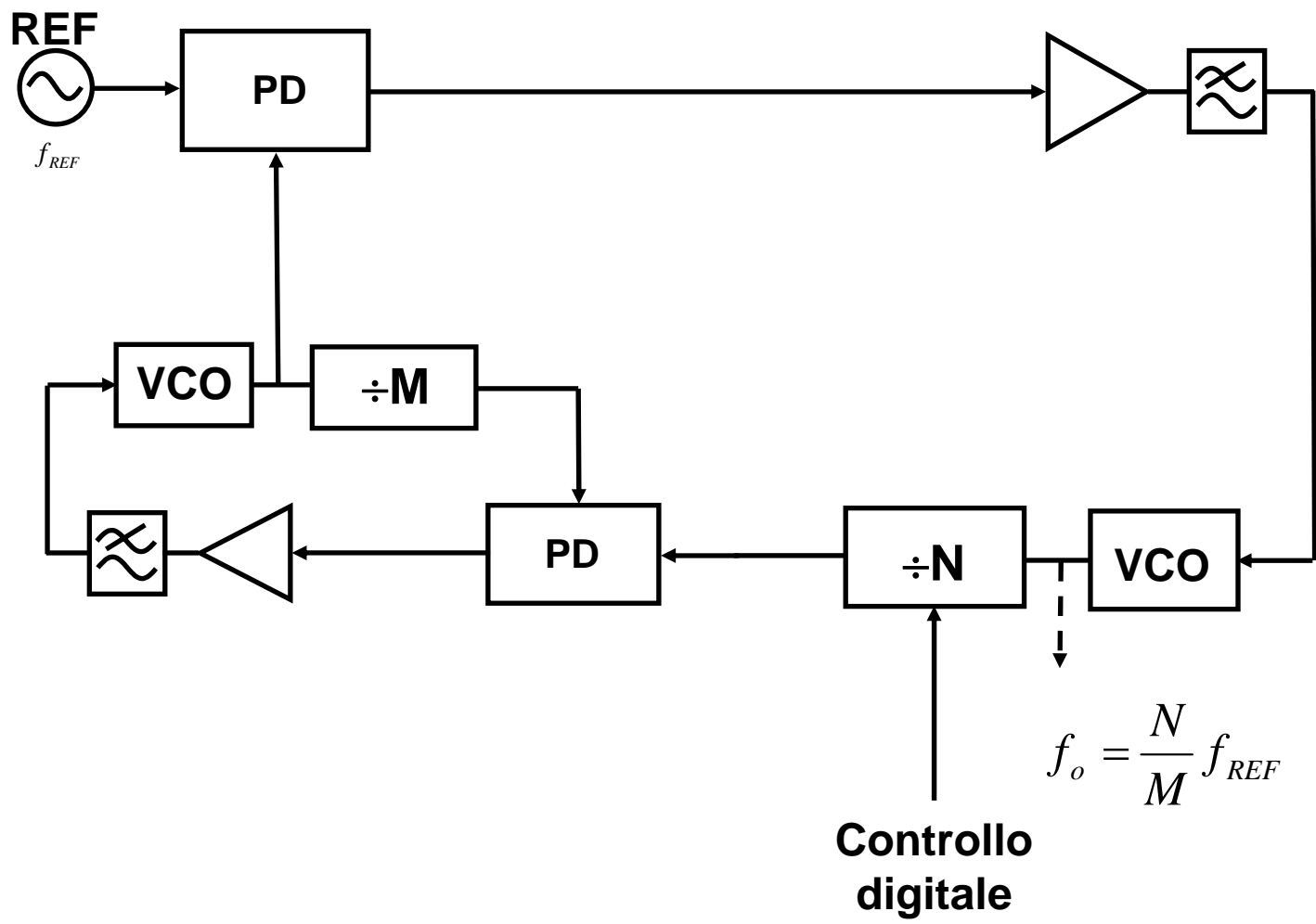




## Sintetizzatore

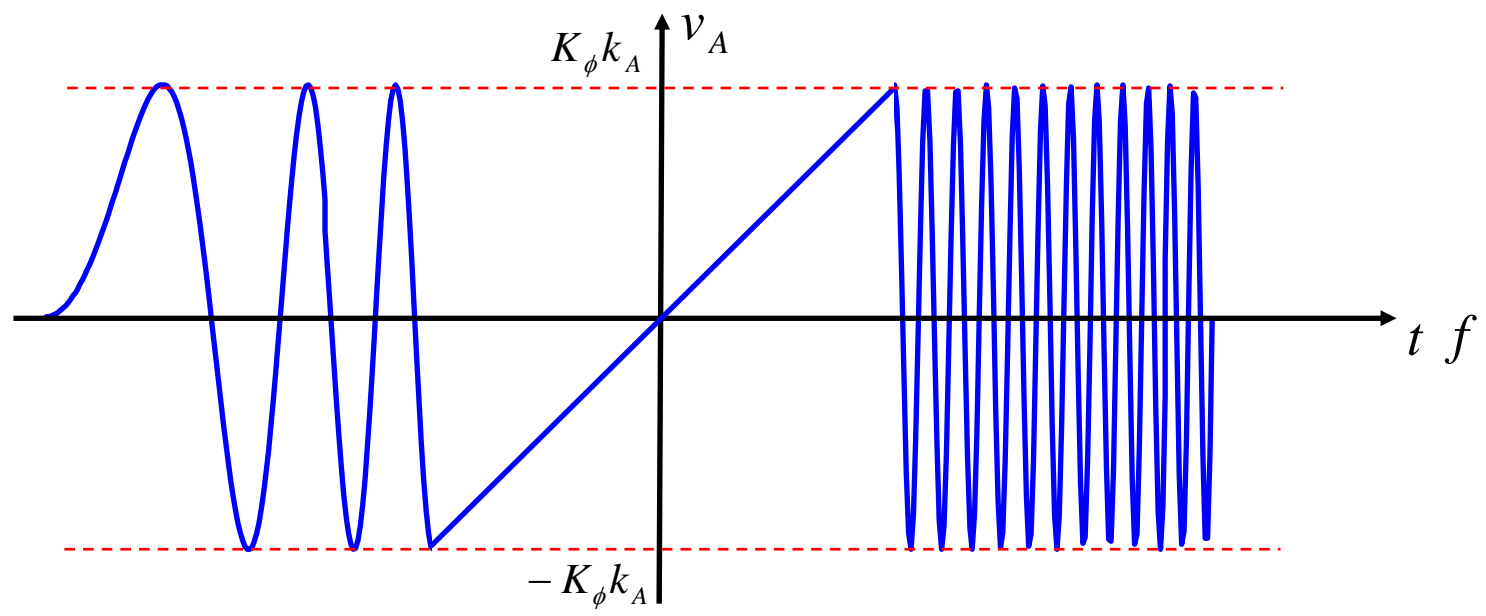
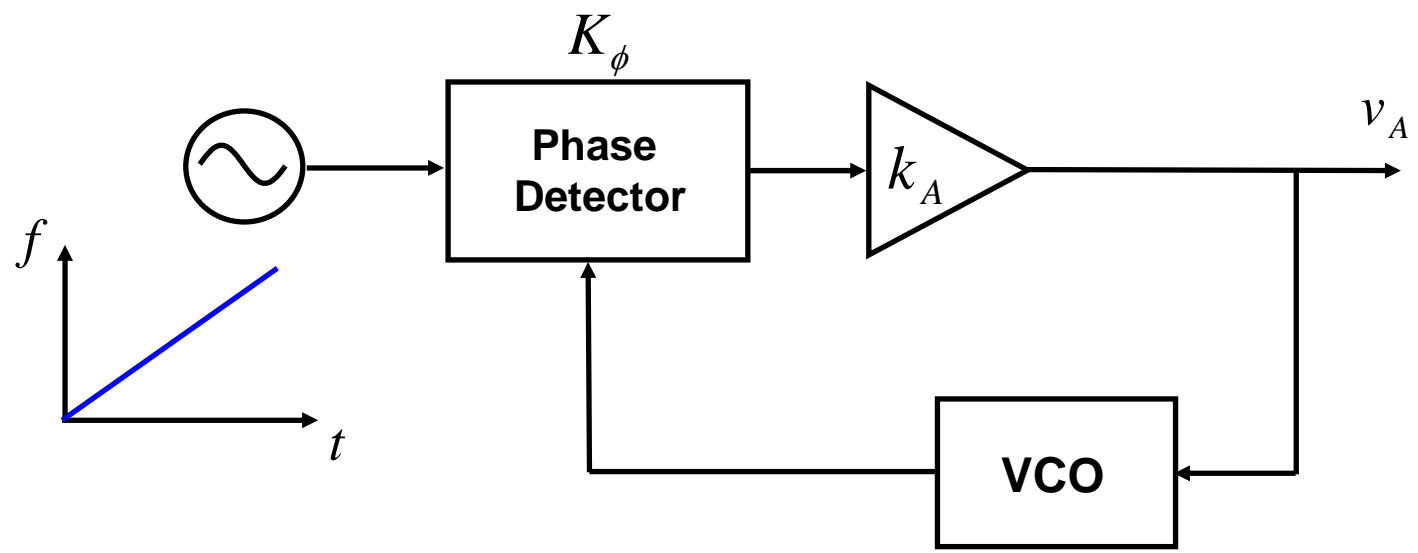


## Sintetizzatore multianello

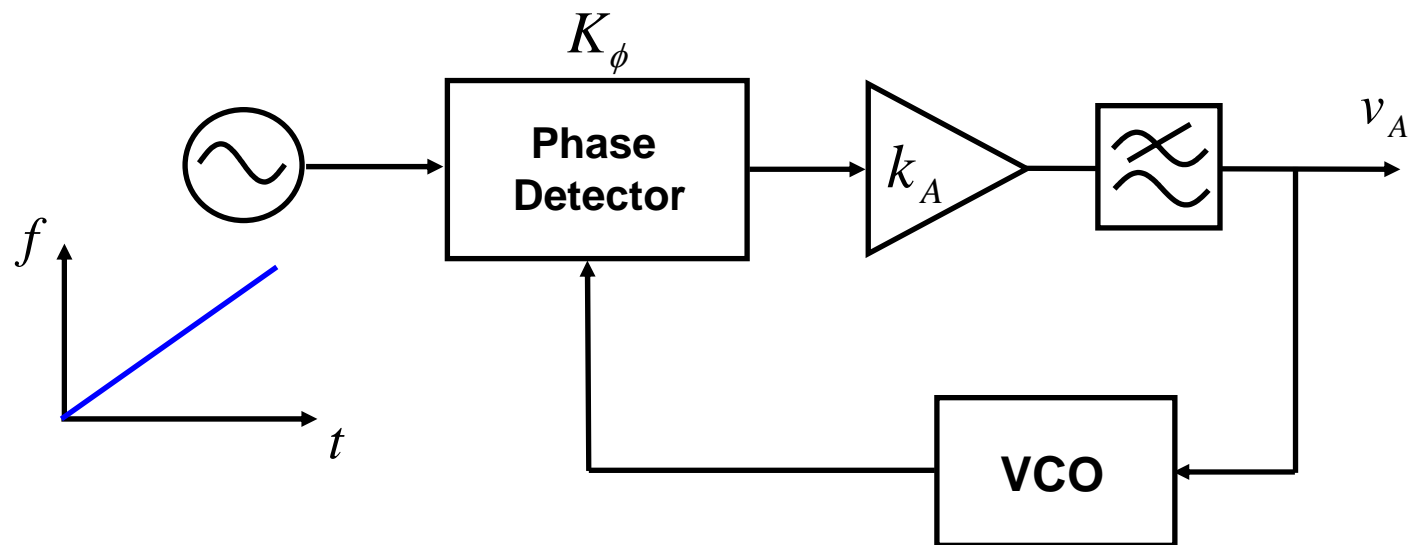




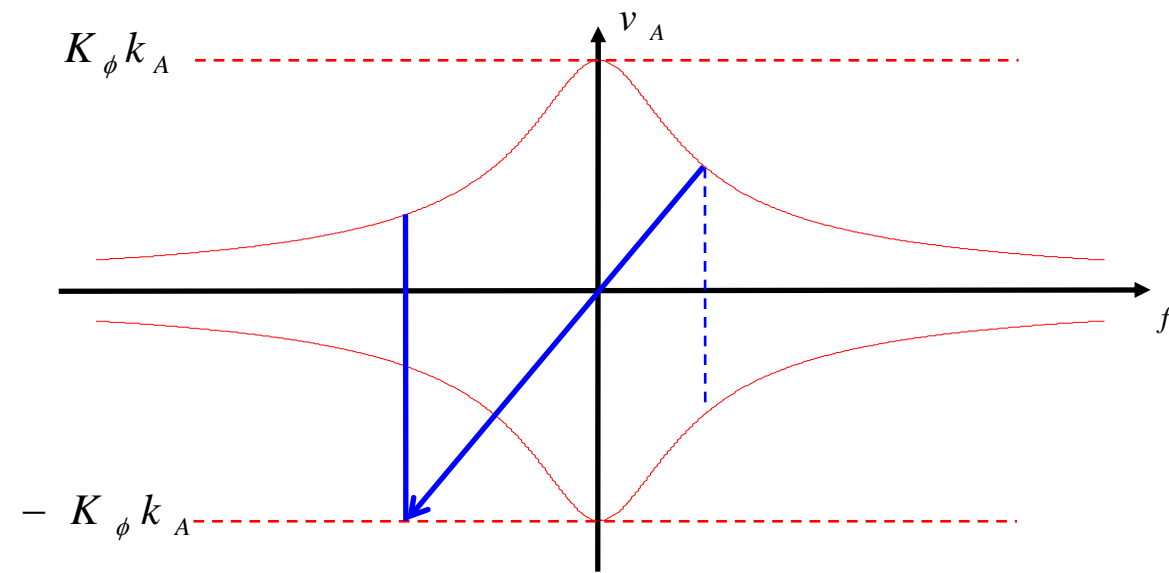
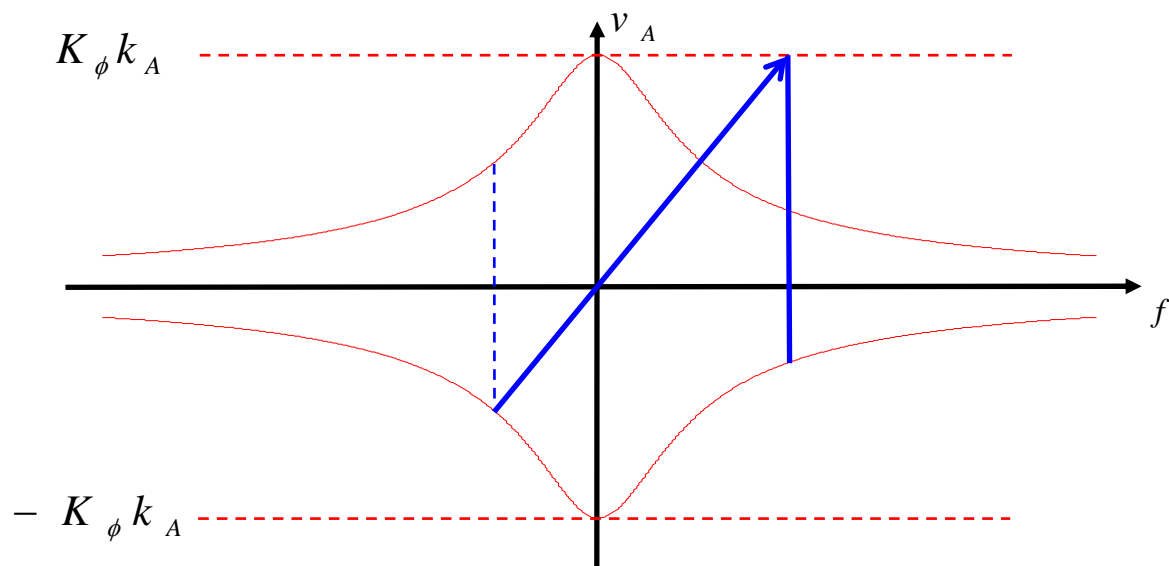
# Caratteristica a farfalla



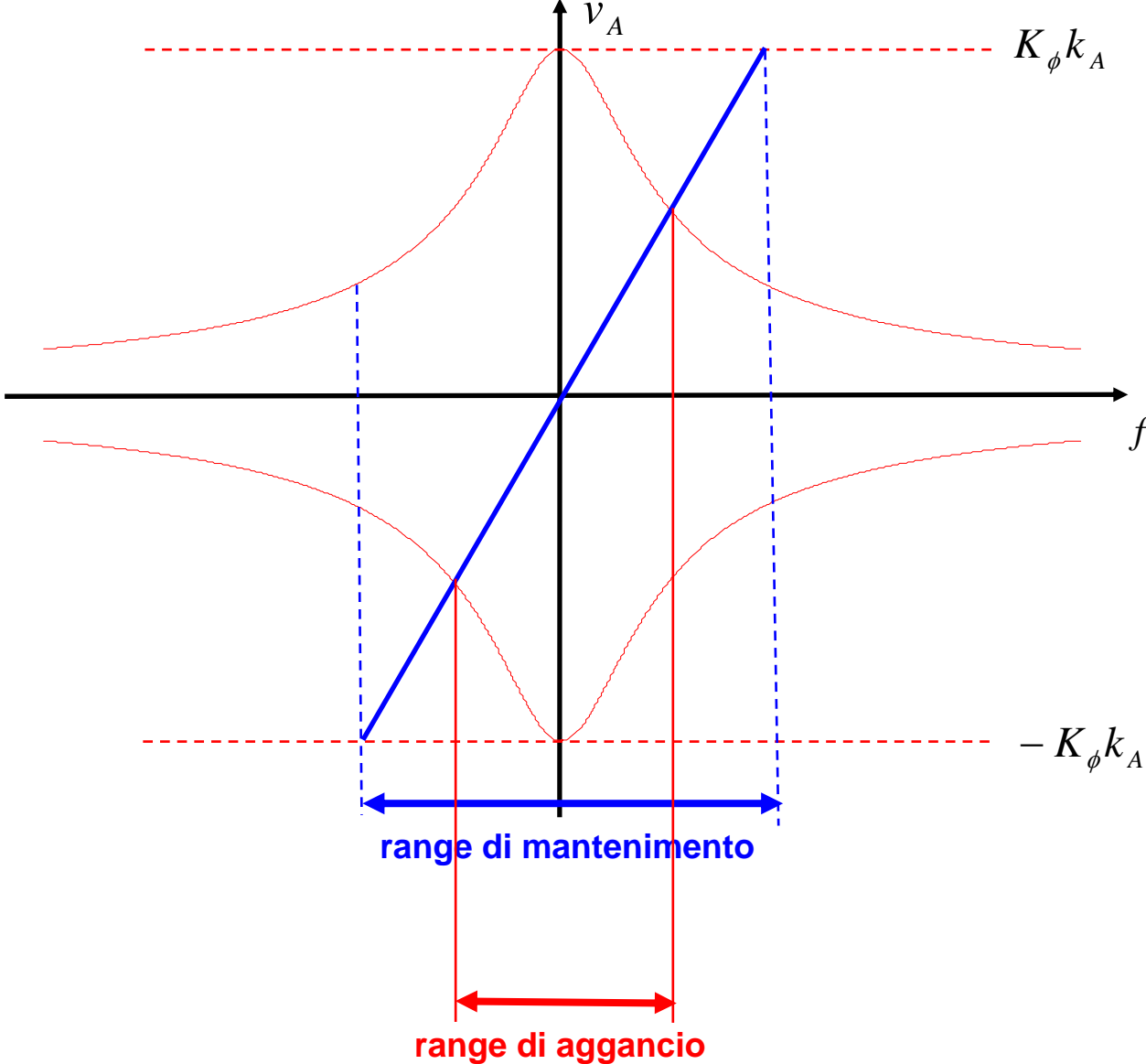
# Caratteristica a farfalla



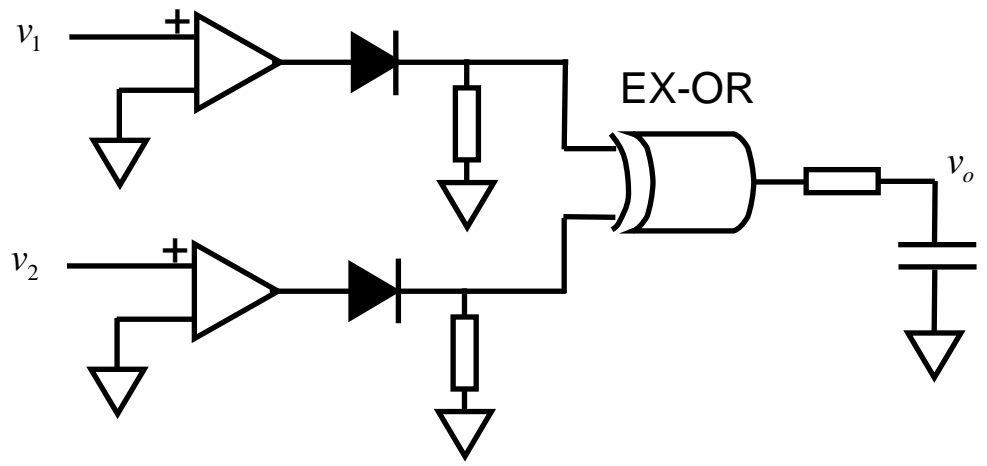
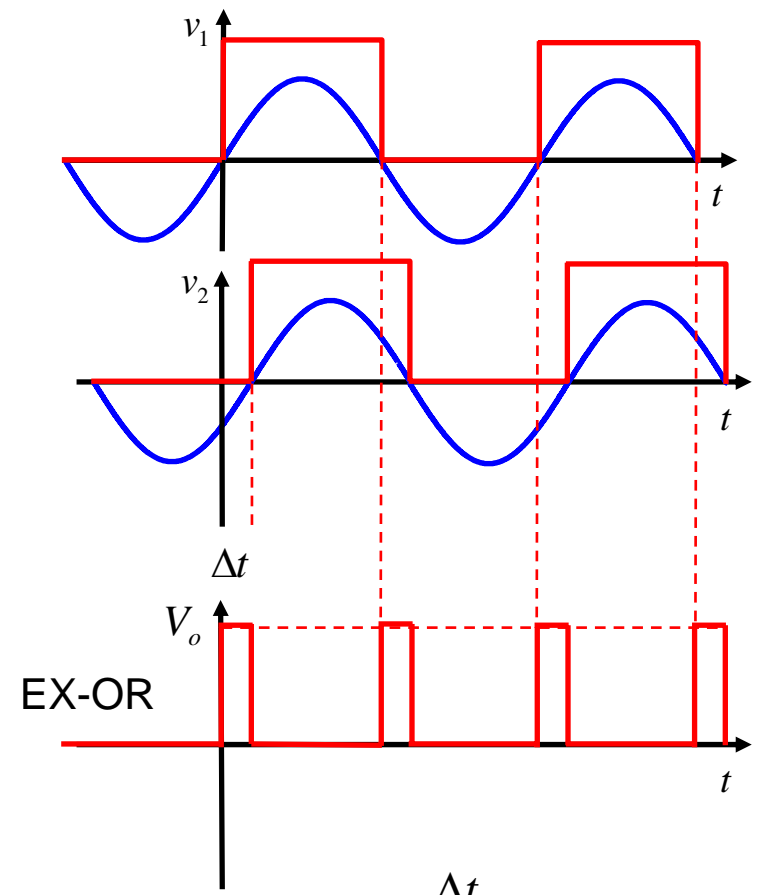
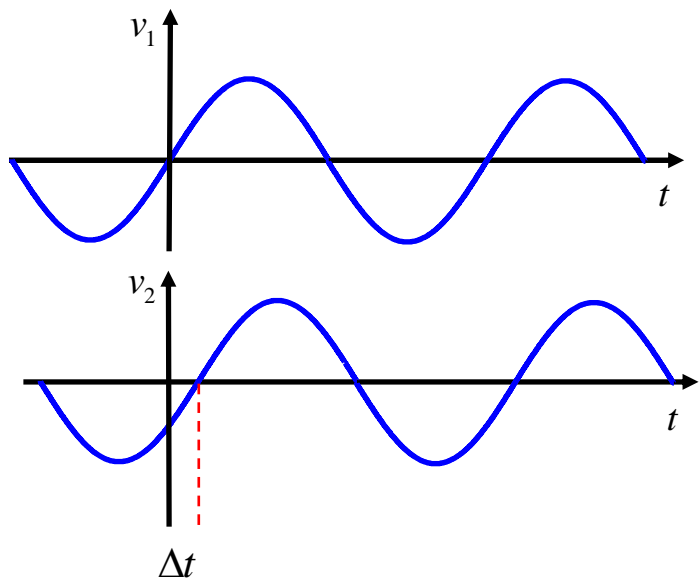
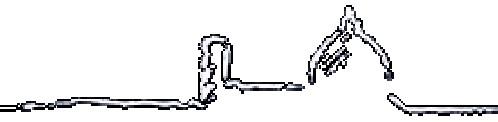
# Caratteristica a farfalla



# Caratteristica a farfalla



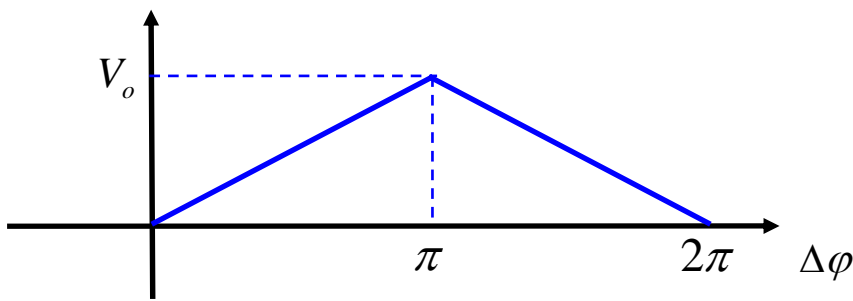
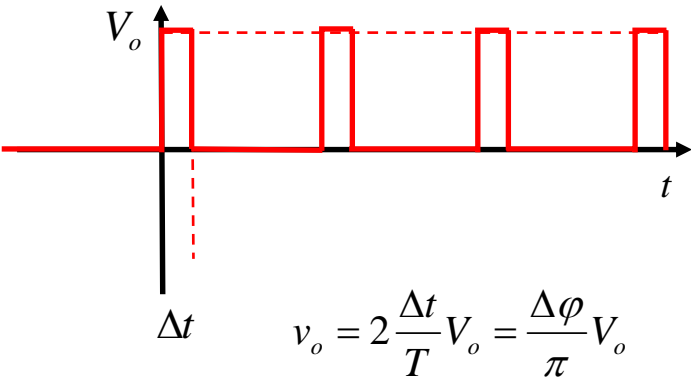
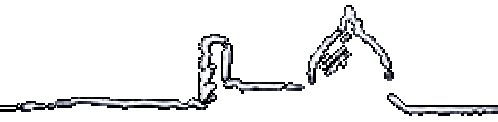
# Rivelatori di fase lineari



$$v_o = 2 \frac{\Delta t}{T} V_o$$

$$v_o = 2 \frac{\Delta \phi}{2\pi} V_o = \frac{\Delta \phi}{\pi} V_o$$

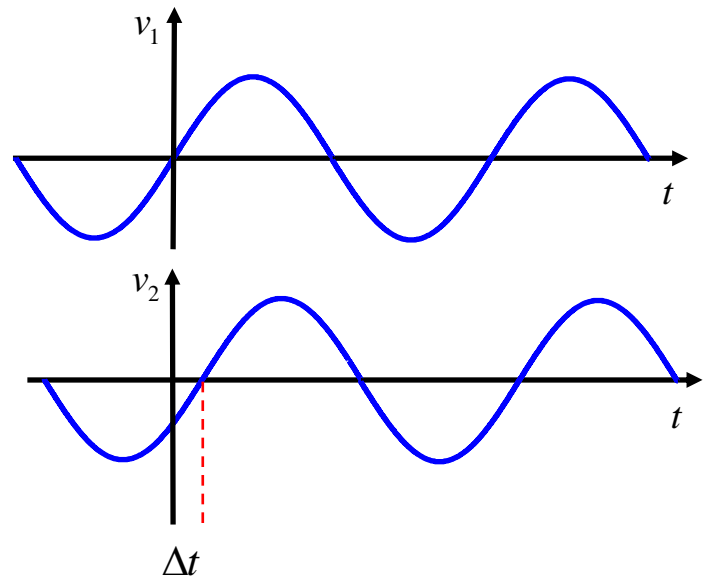
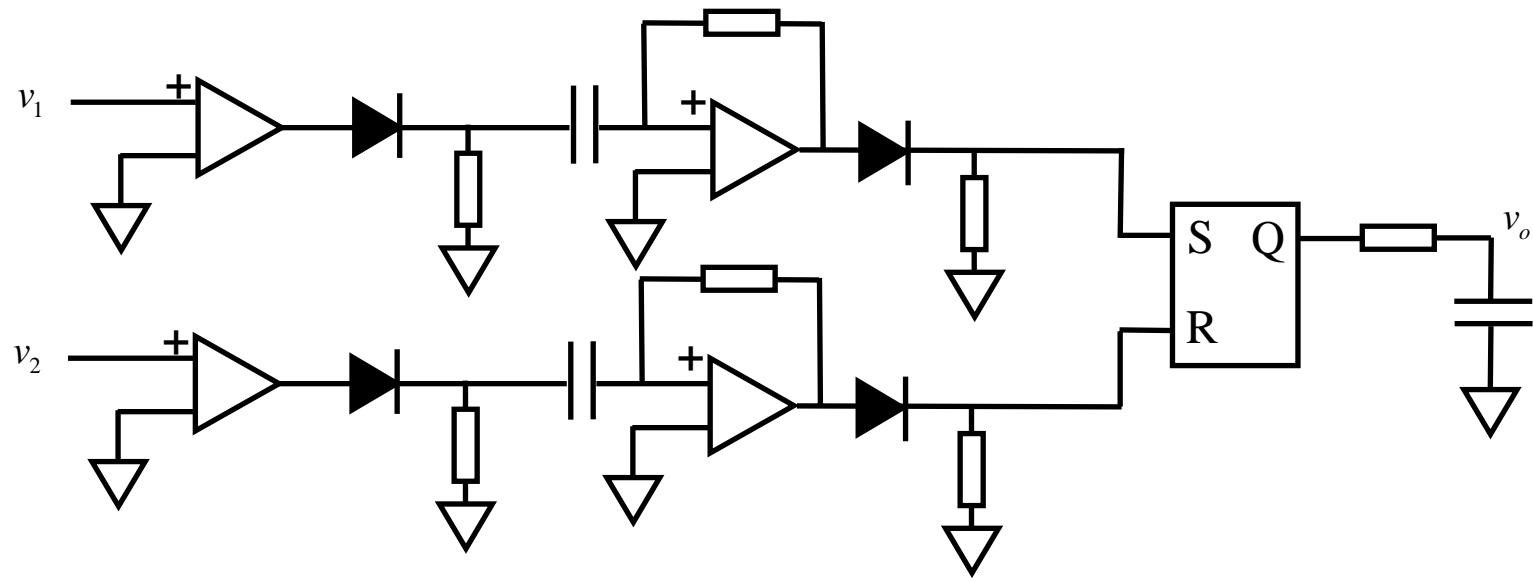
# Rivelatori di fase lineari



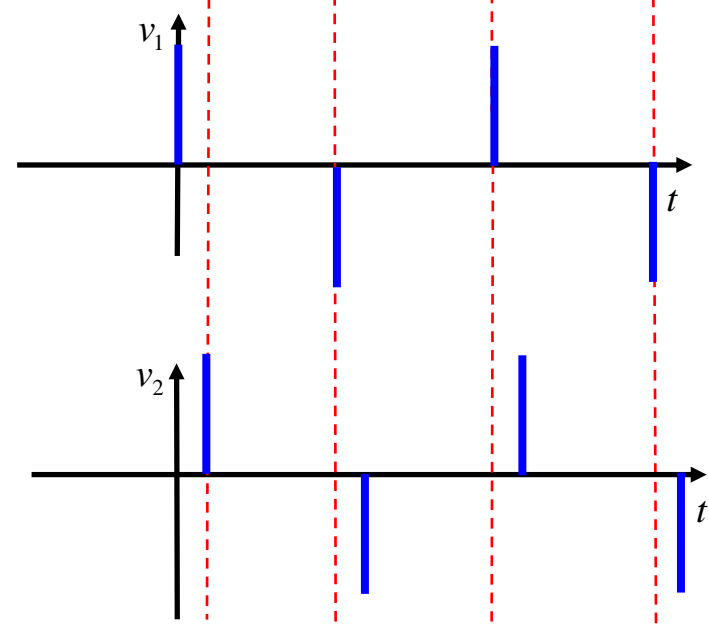
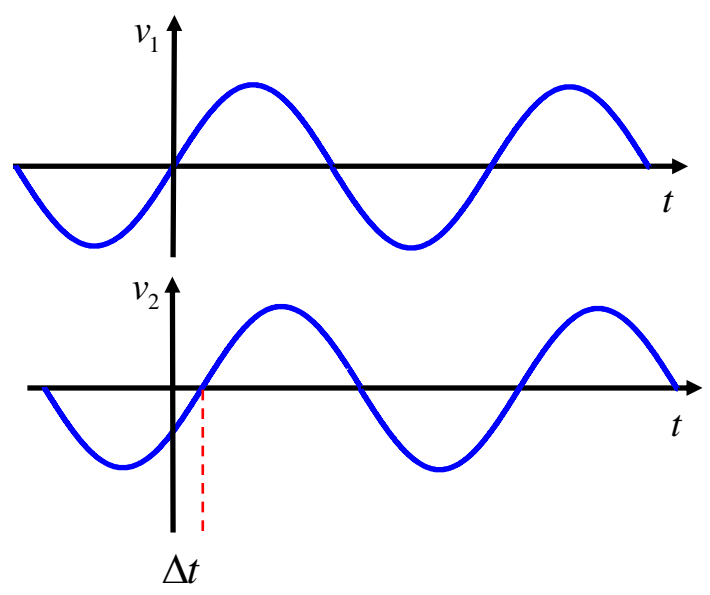
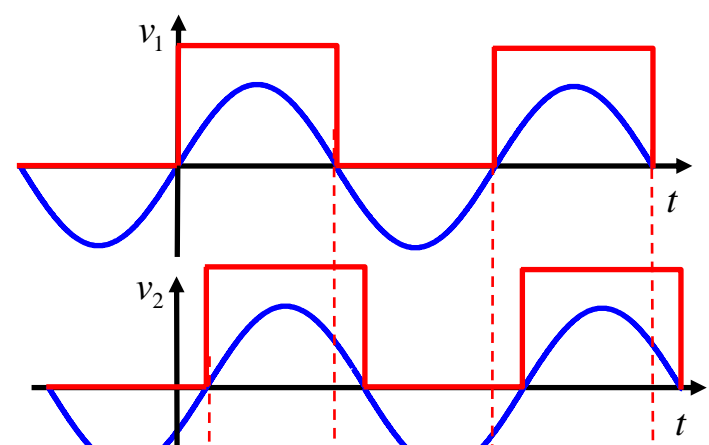
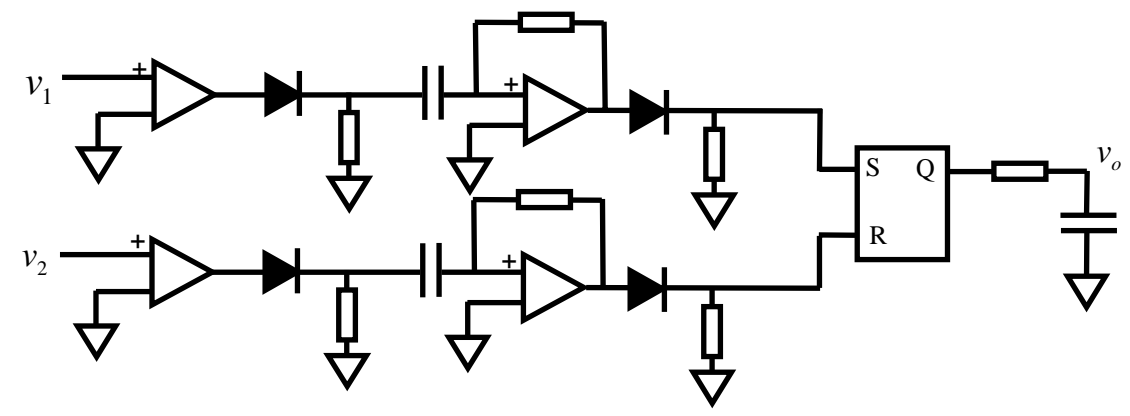
1) range:  $\pi$

2) duty cycle: 50%

# Rivelatori di fase lineari

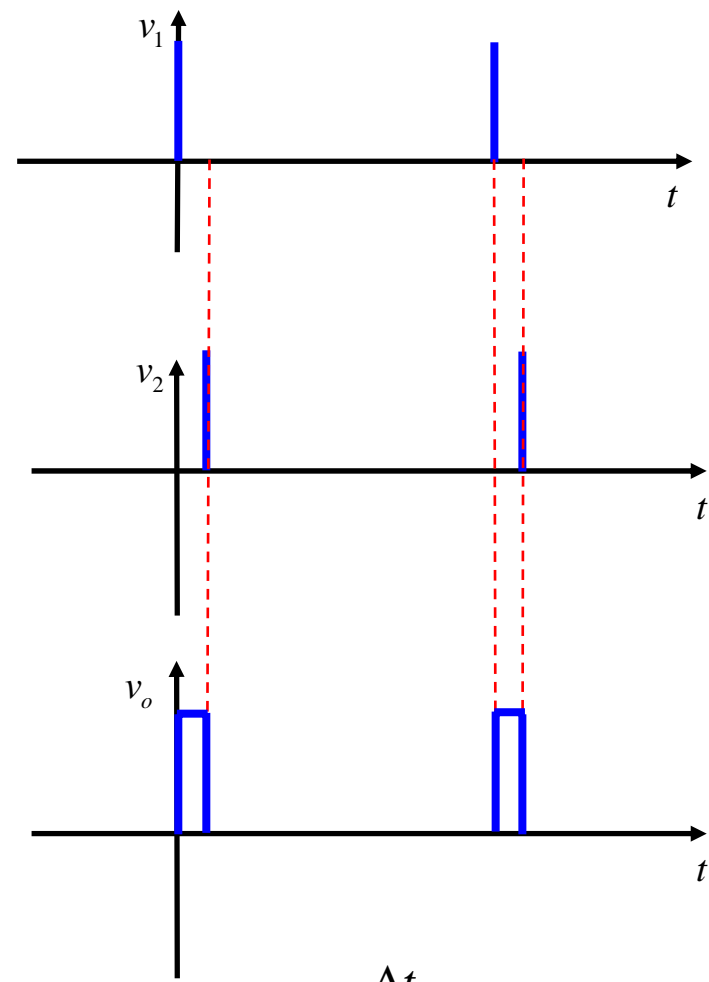
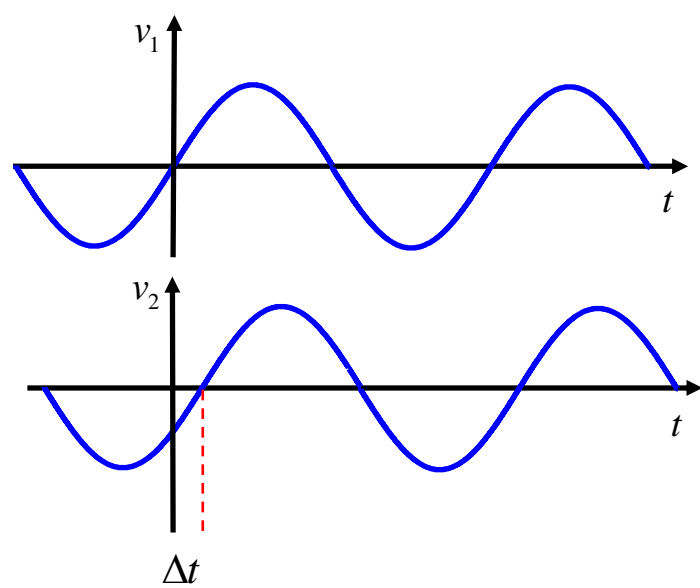
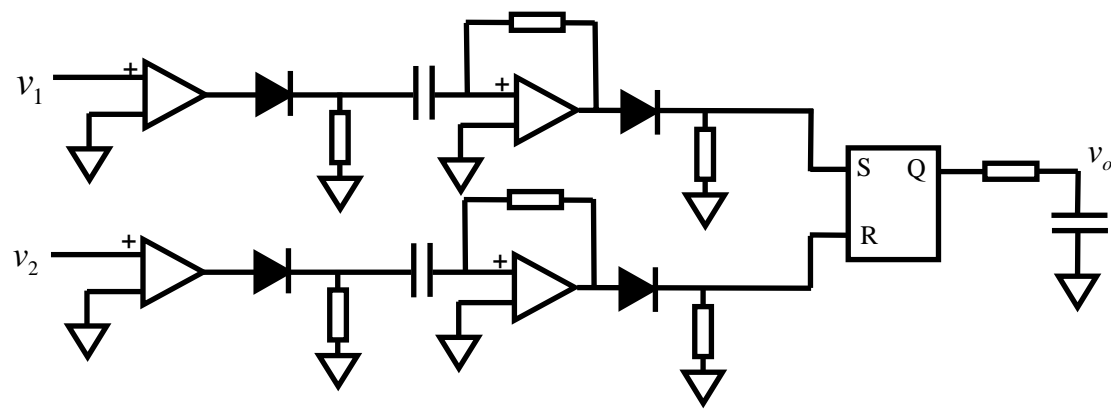


# Rivelatori di fase lineari





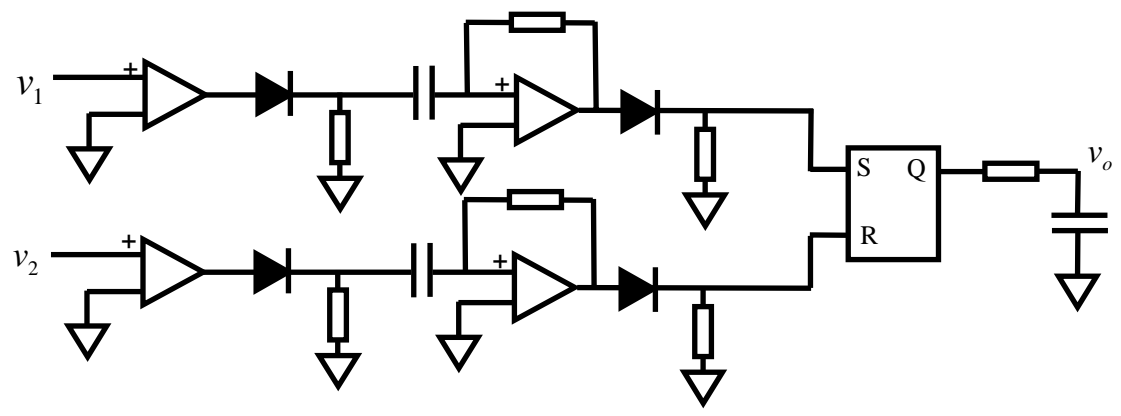
# Rivelatori di fase lineari



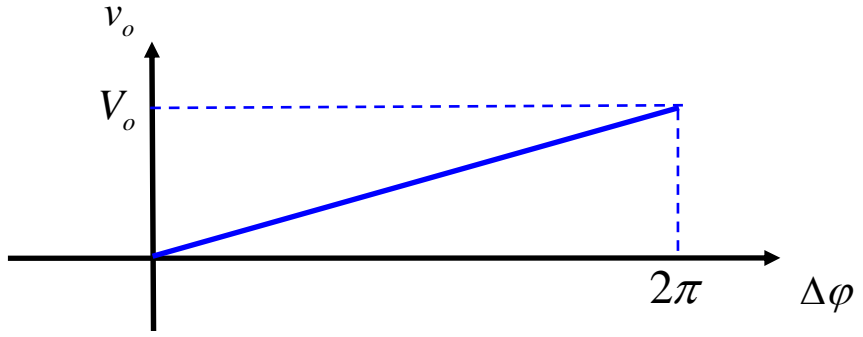
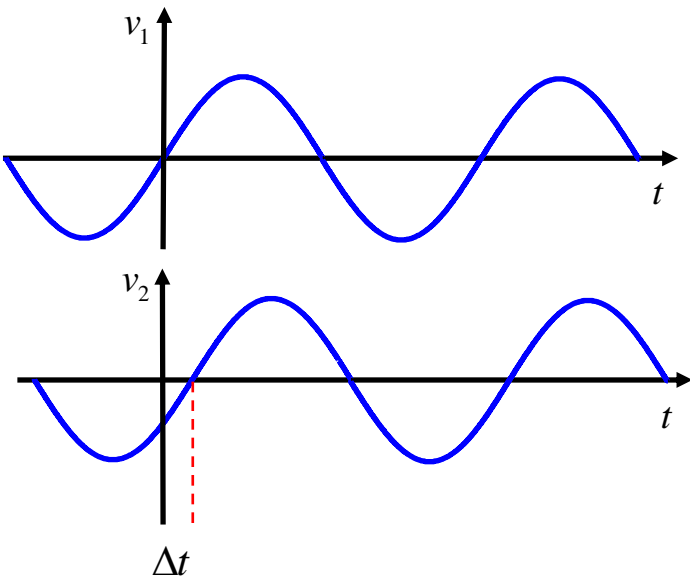
$$v_o = \frac{\Delta t}{T} V_o$$

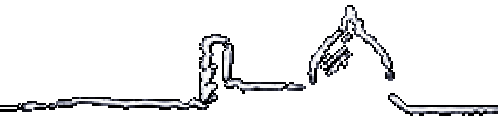
$$v_o = \frac{\Delta \varphi}{2\pi} V_o$$

# Rivelatori di fase lineari

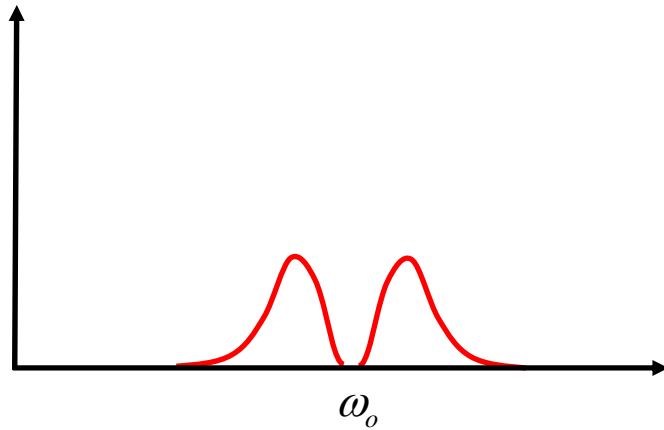


$$v_o = \frac{\Delta t}{T} V_o \quad v_o = \frac{\Delta \phi}{2\pi} V_o$$





(SC DSB) Suppressed Carrier Double Side Band



AM SC DSB: 
$$v_1 = V_1 \cos(\omega_1 t) \cos(\omega_m t)$$

# Anello di Costas

