



Radar

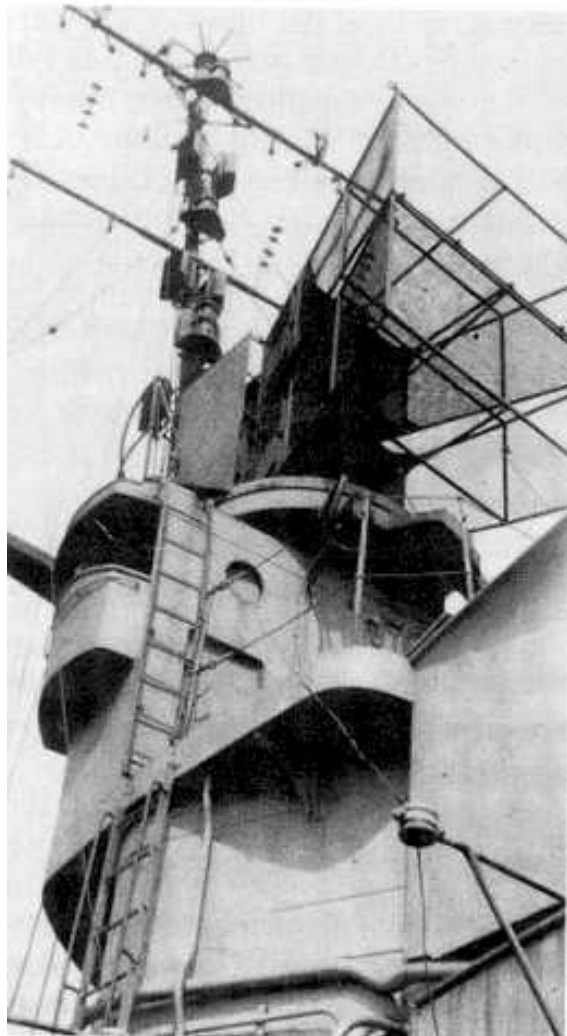
Massimiliano Pieraccini



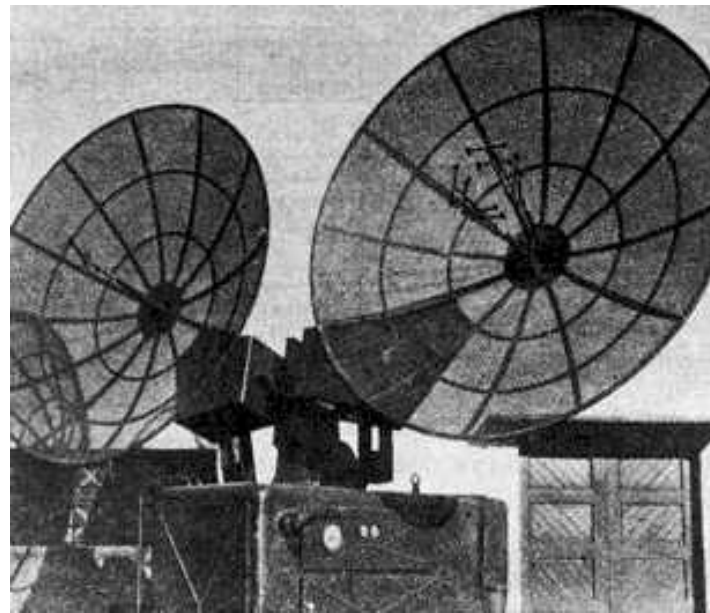
RADAR: RAdio Detection And Ranging

- 1904** Hulsmeyer brevetta un rivelatore di ostacoli a onde elettromagnetiche
- 1922** Marconi ipotizza l'impiego del radar in una conferenza negli Stati Uniti
- 1925** Breit e Tuve (USA) misurano l'altezza della ionosfera con una tecnica radar
- 1934** in Germania Kuhnold realizza il primo radar (600 MHz, 7 miglia)
- 1936** in USA il Naval Research Lab realizza un radar (200 MHz)
- 1935** prof. Tiberio propone alla Marina Italiana il Radio-Detector Telemetro
- 1941** il *Gufo* (400-750 Mhz, fino a 200 Km)

Entro l'8 settembre 1943 furono consegnati complessivamente
13 "Gufo"
4 "Folaga"

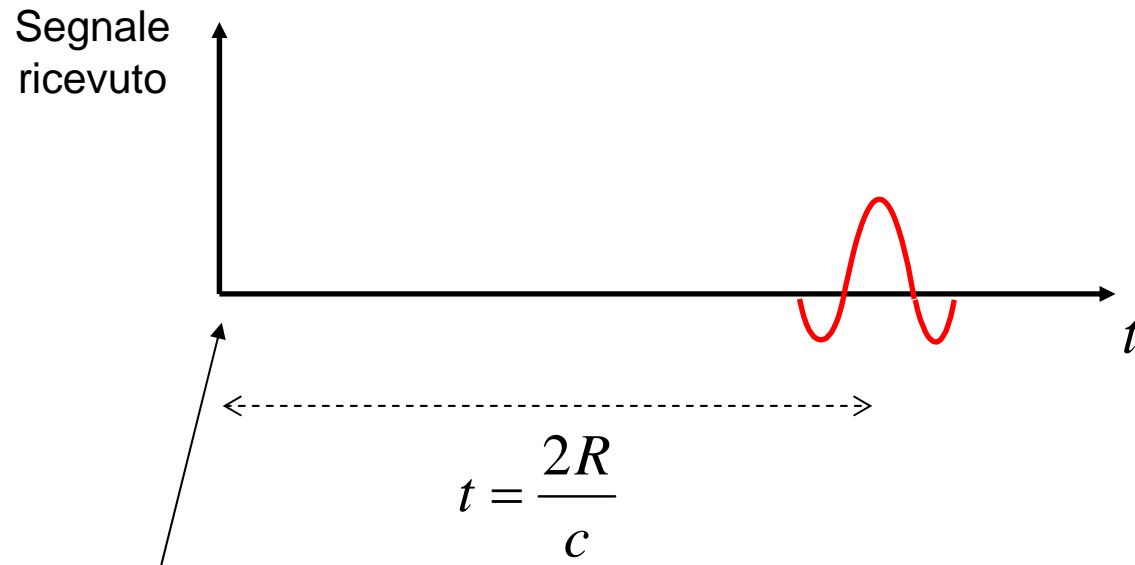
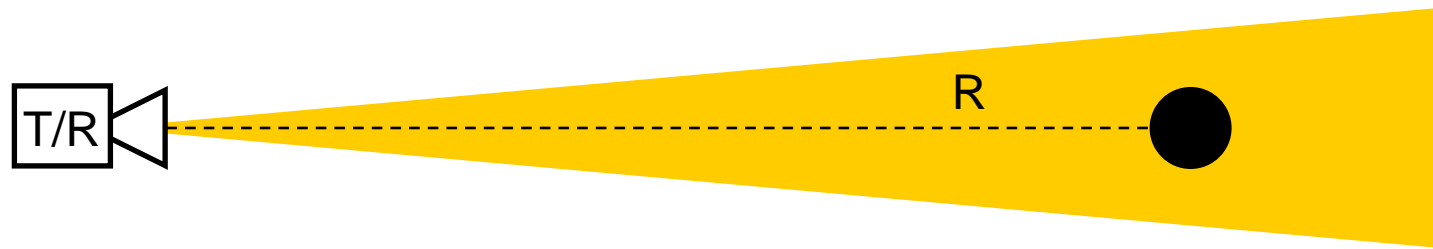


Gufo



Folaga

Principio di funzionamento

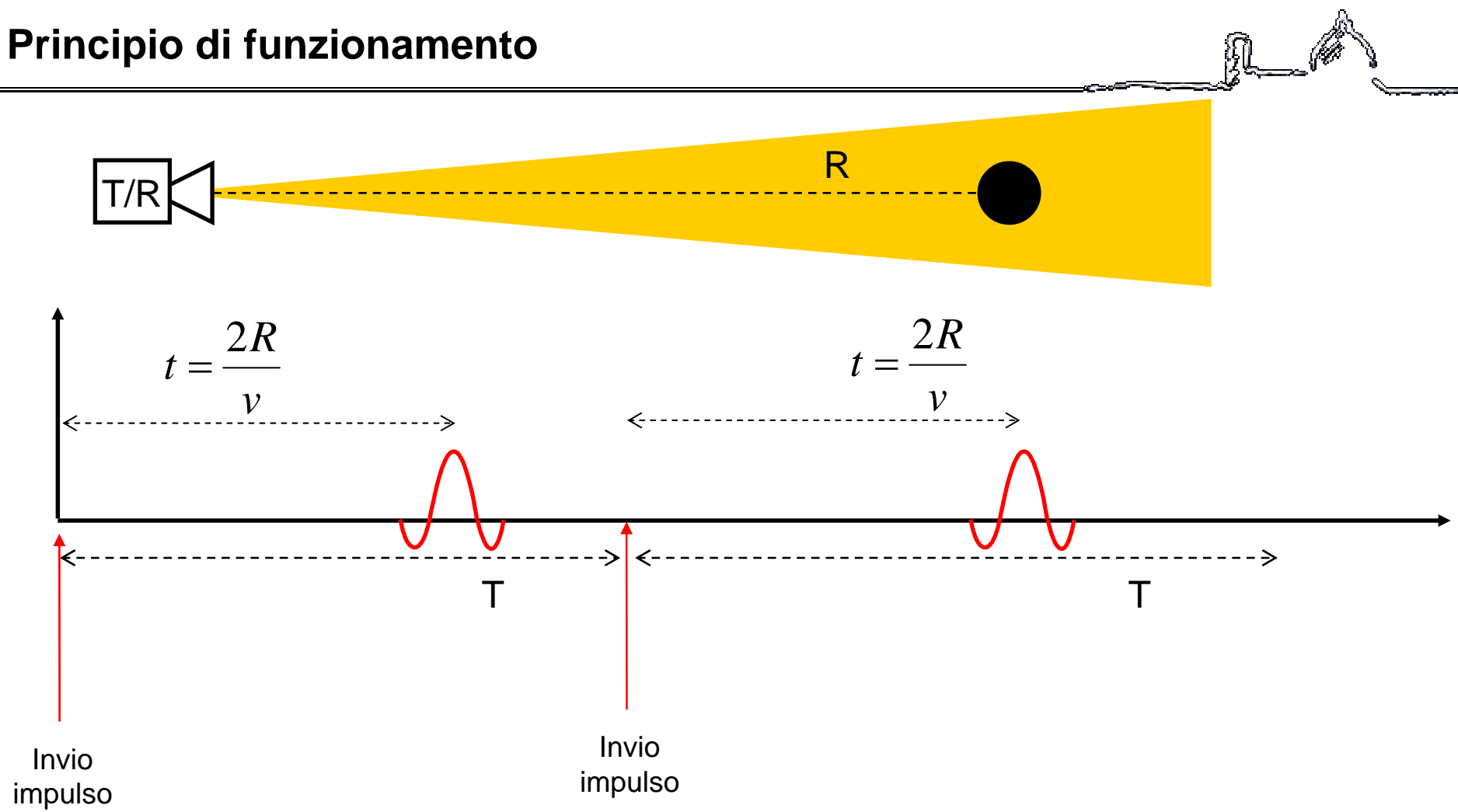


Invio dell'impulso
 $t = 0$

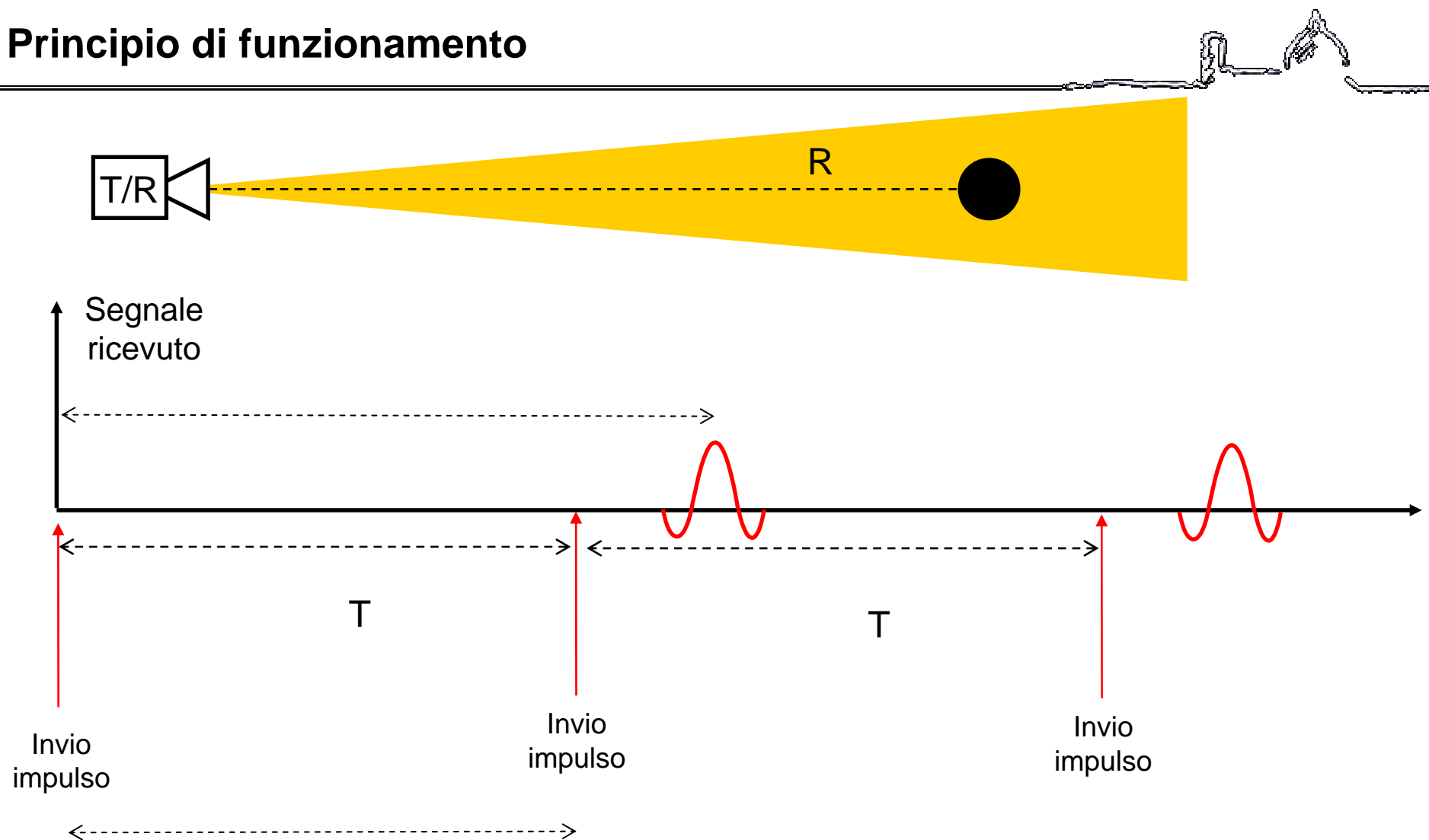
Direttività

Capacità di misurare la distanza

Principio di funzionamento

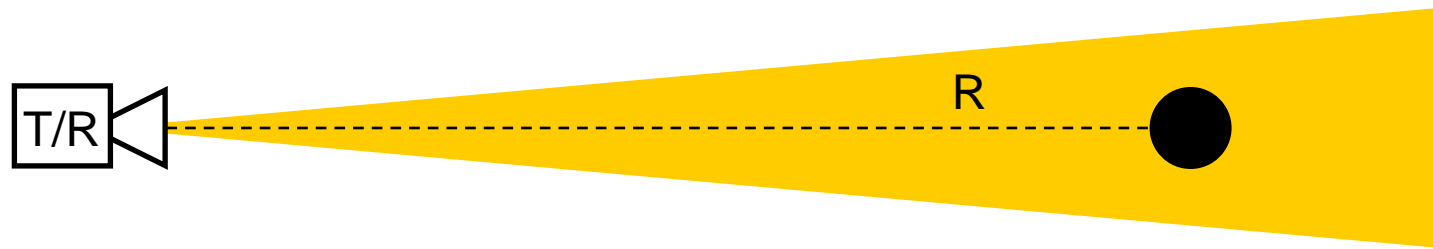


Principio di funzionamento



Range non ambiguo: $R_U = \frac{vT}{2}$

Principio di funzionamento

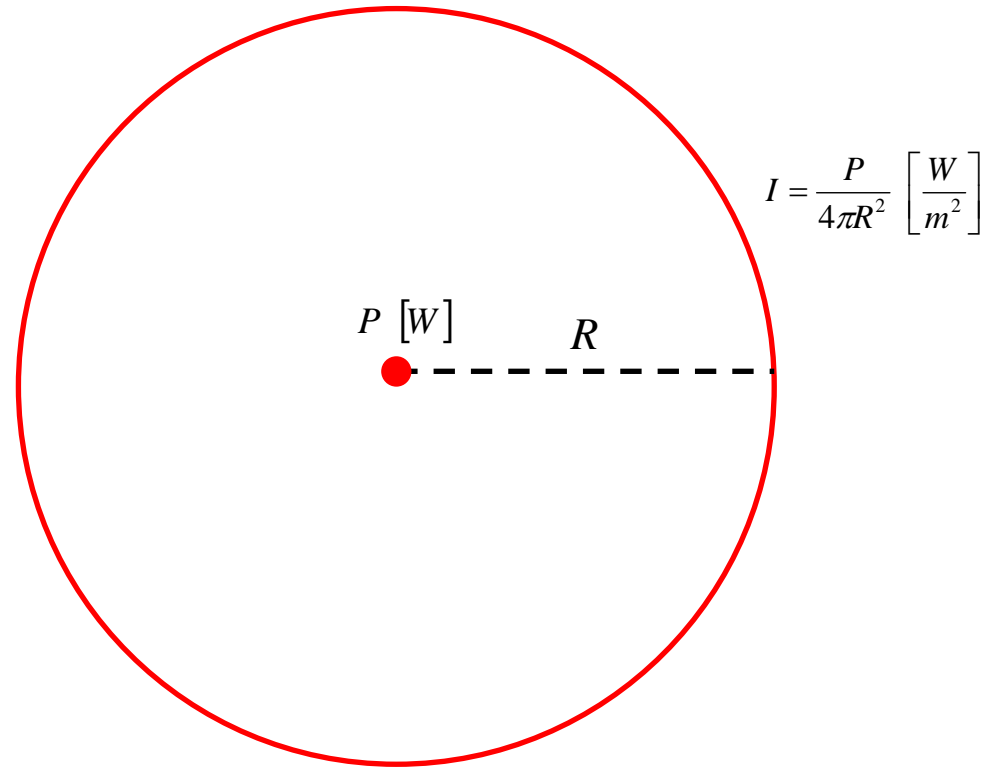


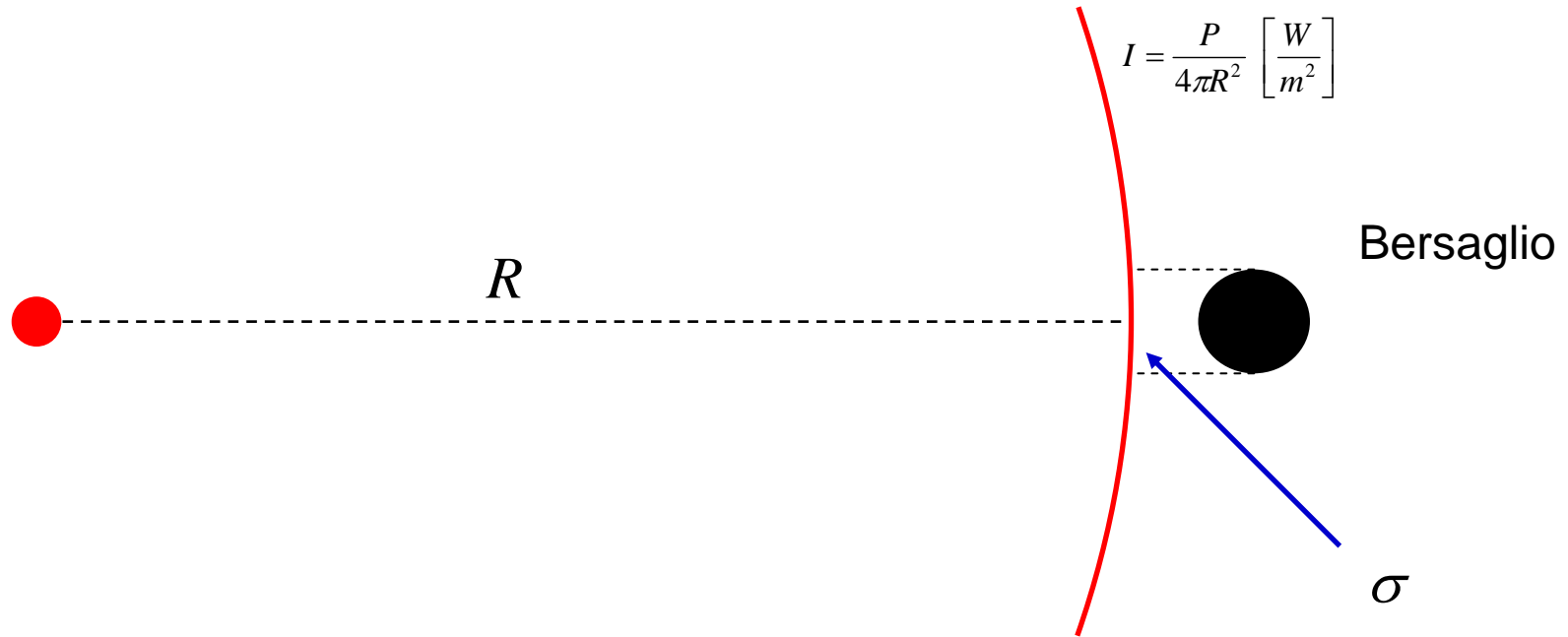
$$\tau = \frac{1}{B}$$



Risoluzione in range: $\Delta R = \frac{v\tau}{2}$

$$\Delta R = \frac{c}{2B}$$



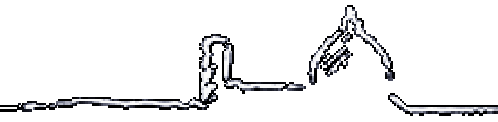


Potenza intercettata dal bersaglio $P_i = I\sigma$

Potenza intercettata dal bersaglio e diffusa $P_{scatt} = I\sigma\alpha$ $\leftarrow < 1$

Radar Cross Section $\sigma_{RCS} = \alpha\sigma$

$$P_{scatt} = I\sigma_{RCS}$$



RCS di una sfera riflettente

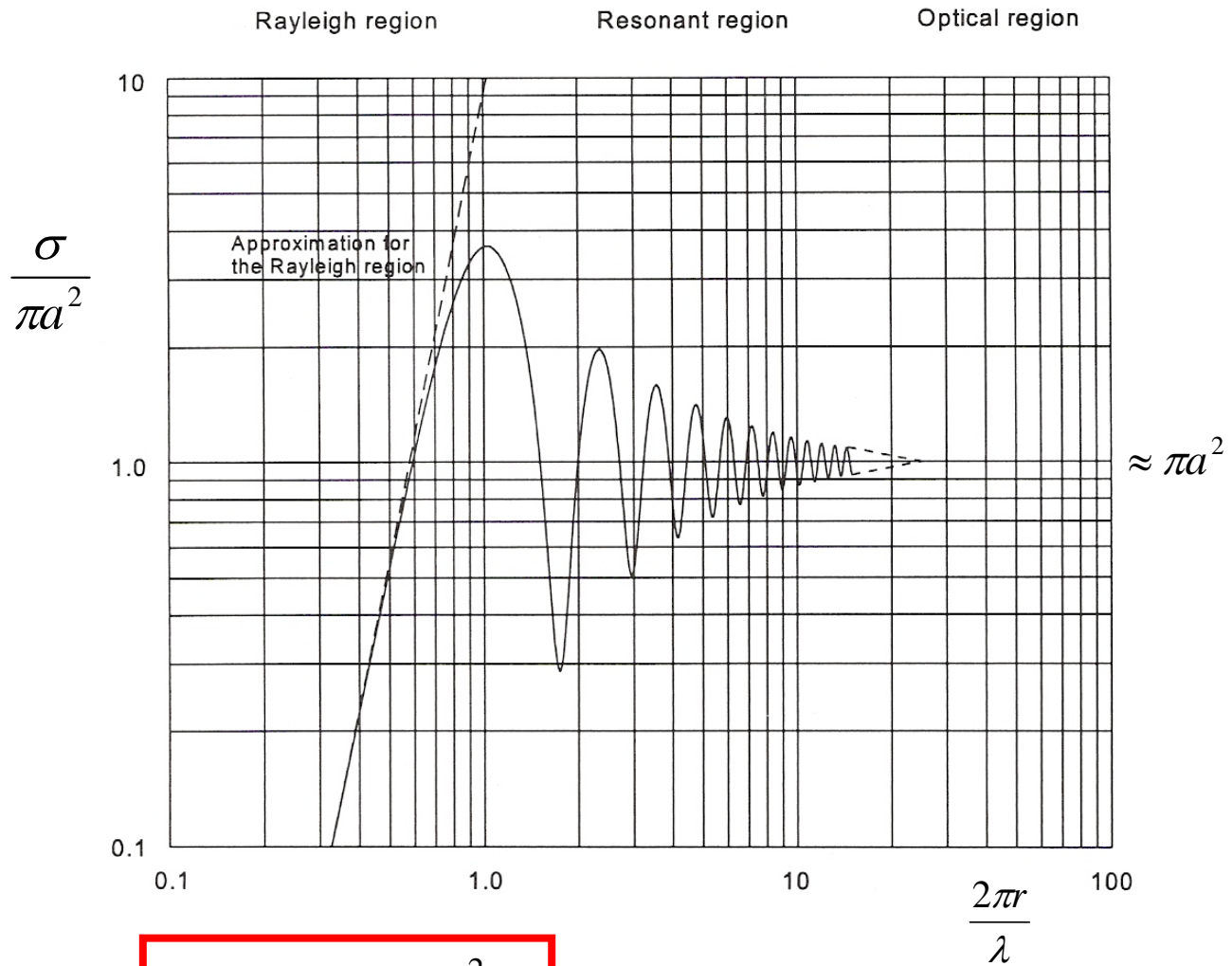
Teoria di Mie

$$\sigma = \pi \left(\left| \sum_{n=1} (-1)^n (2n+1) (a_n(r) + b_n(r)) \right| \right)^2$$

$$a_n(r) = \frac{j_n(r)}{j_n(r) - y_n(r)}$$

$$a_n(r) = \frac{-\frac{d}{dr}[rj_n(r)]}{\frac{d}{dr}[rj_n(r) - jry_n(r)]}$$

Teoria di Mie



$$\sigma \approx 9\pi \left(\frac{2\pi a}{\lambda} \right)^2 a^2$$

Tecnologia Stealth

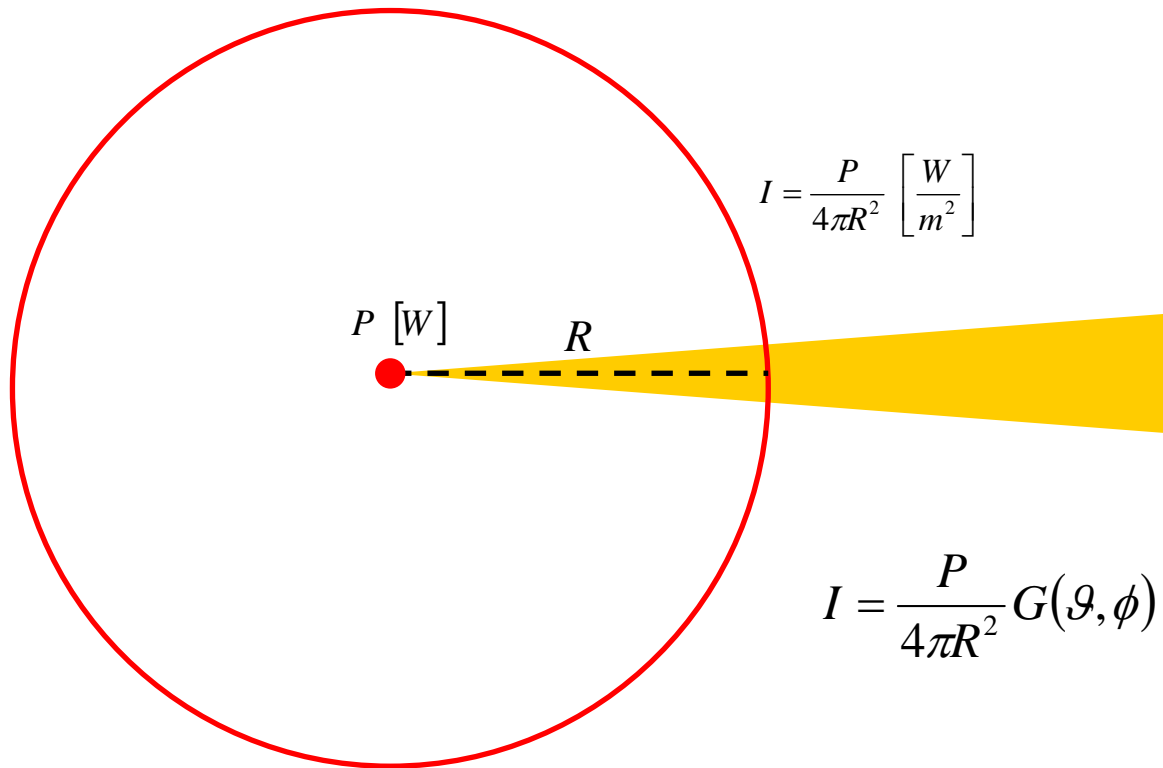
- 1) Vernici e rivestimenti assorbenti
- 2) Materiali non metallici
- 3) Superfici studiate in modo da minimizzare l'energia diffusa nella stessa direzione di incidenza
- 4) Forme studiate per produrre interferenza distruttiva alla frequenza dei radar



F-117



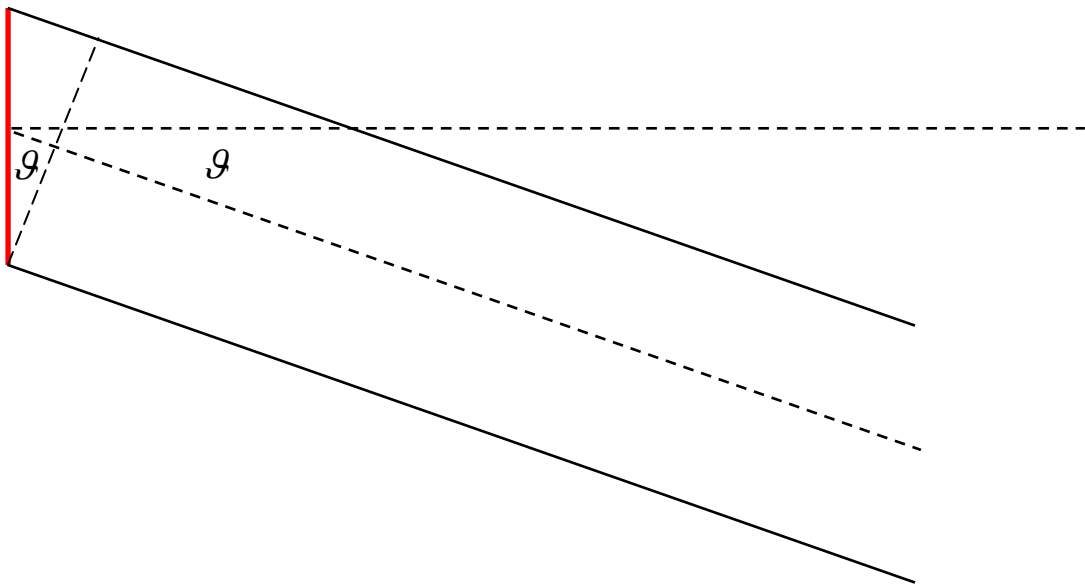
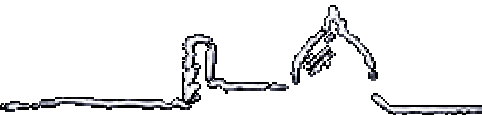
B-2

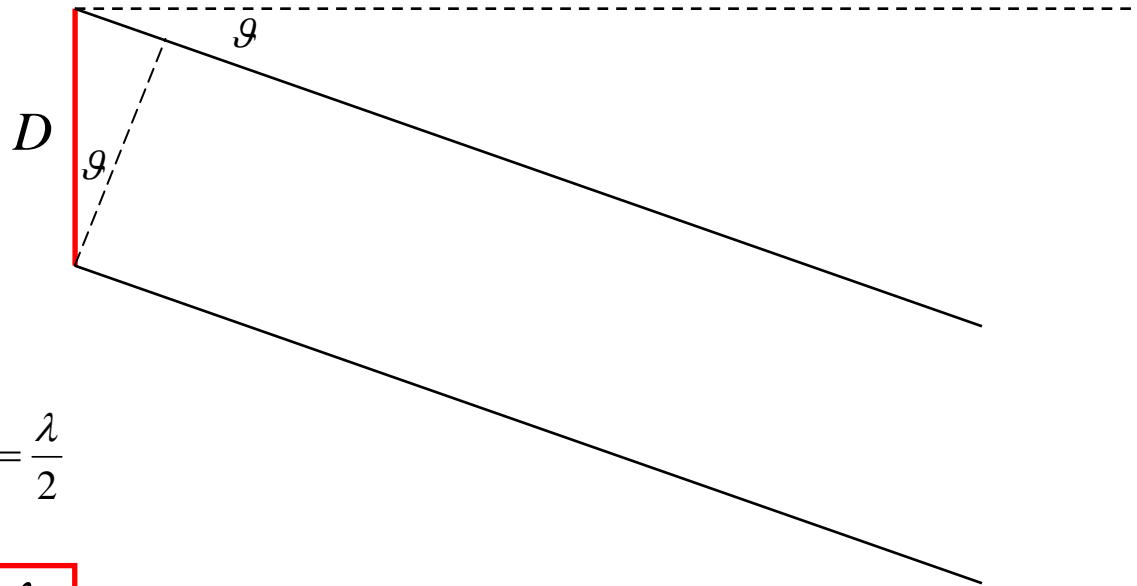


$$I = \frac{P}{4\pi R^2} \left[\frac{W}{m^2} \right]$$

$$I = \frac{P}{4\pi R^2} G(\vartheta, \phi)$$

$$\int G(\vartheta, \phi) d\Omega = 4\pi$$





$$D \sin \vartheta = \frac{\lambda}{2}$$

$$\vartheta \approx \frac{\lambda}{2D}$$

$$\Delta\Omega \approx \pi\vartheta^2$$

$$G = \frac{4\pi}{\Delta\Omega}$$

$$\Delta\Omega = \frac{4\pi}{G}$$

$$\mathcal{G} \approx \frac{\lambda}{2D} \quad \Delta\Omega \approx \pi \mathcal{G}^2 \quad G = \frac{4\pi}{\Delta\Omega}$$

$$G = \frac{4\pi}{\Delta\Omega} = \frac{16D^2}{\lambda^2}$$

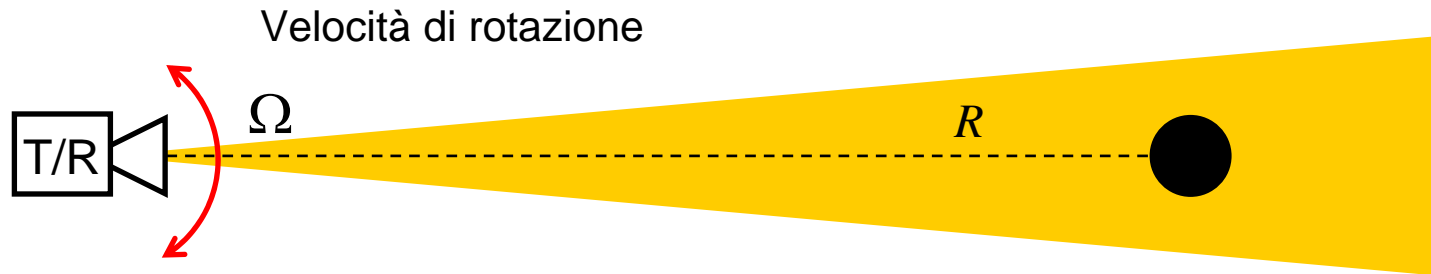
$$A = \pi \left(\frac{D}{2} \right)^2 \left(\frac{1}{2} \right)$$

Anche nel migliore dei casi (adattamento di impedenza perfetto) il 50% della potenza incidente su un'antenna è riflessa

$$\frac{A}{G} = \frac{\lambda^2}{10.2}$$

$$\frac{A}{G} = \frac{\lambda^2}{4\pi}$$

$$A = \frac{\lambda^2}{4\pi} G$$



$$\Delta\Omega = \frac{4\pi}{G}$$

$$T_{\max} = \frac{2R_{\max}}{c}$$

$$\pi\vartheta^2 = \frac{4\pi}{G}$$

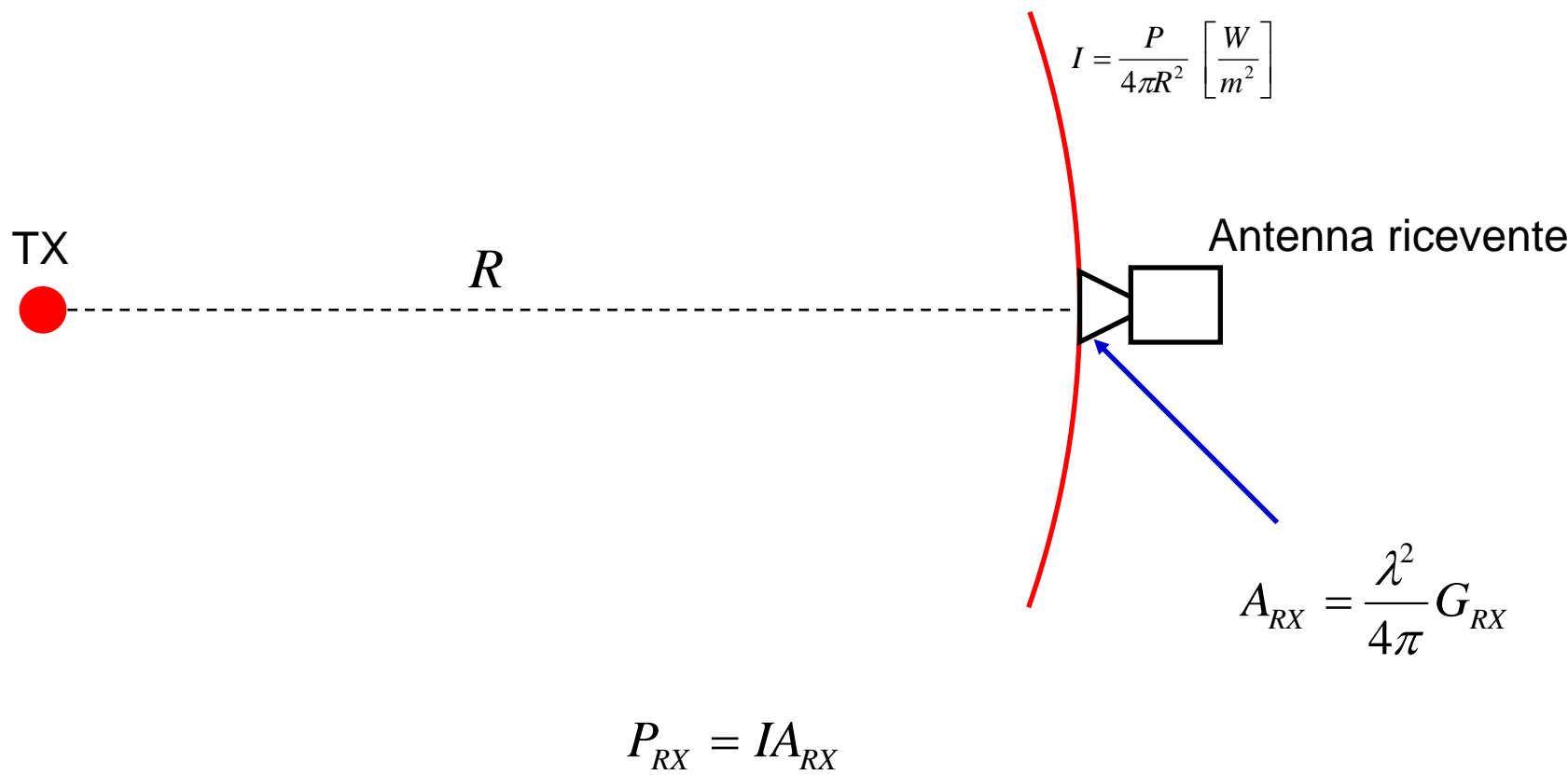
$$\Omega T < \vartheta$$

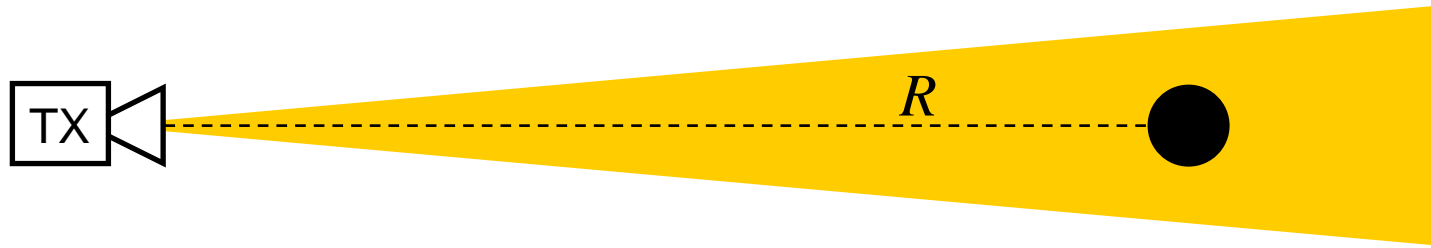
$$\vartheta = \frac{2}{\sqrt{G}}$$

$$\Omega < \frac{c}{R_{\max} \sqrt{G}}$$

Esempio: $R_{\max}=100 \text{ Km}$, $G=40\text{dB} \rightarrow 5 \text{ giri/sec}$

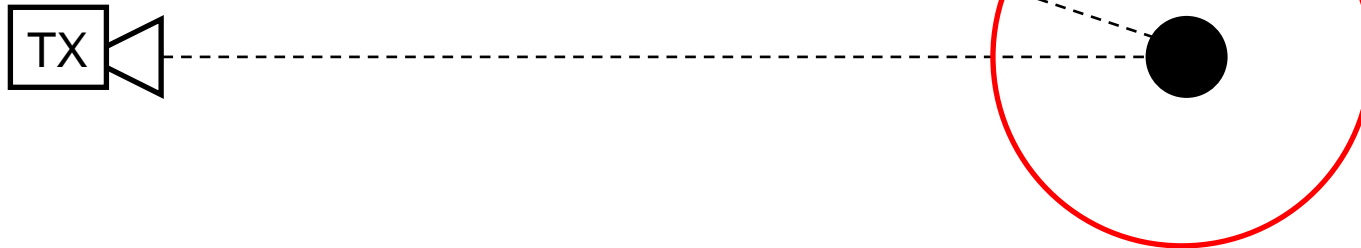
Area efficace di un'antenna



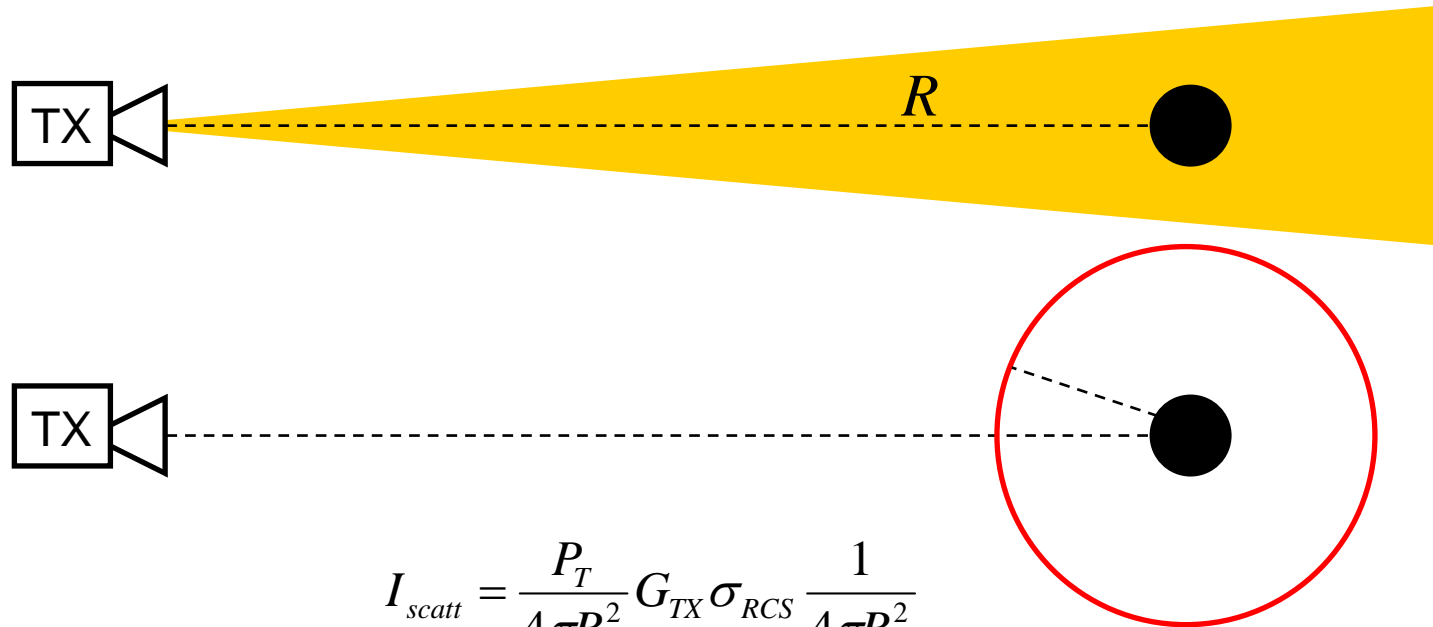


$$P_{scatt} = I \sigma_{scatt} = \frac{P_T}{4\pi R^2} G_{TX} \sigma_{RCS}$$

$$I_{scatt} = \frac{P_{scatt}}{4\pi R^2} = \frac{P_T}{4\pi R^2} G_{TX} \sigma_{RCS} \frac{1}{4\pi R^2}$$



Equazione radar



$$I_{scatt} = \frac{P_T}{4\pi R^2} G_{TX} \sigma_{RCS} \frac{1}{4\pi R^2}$$

$$A_R = \frac{\lambda^2}{4\pi} G_{RX}$$

$$P_{RX} = I_{scatt} A_R$$

$$P_{RX} = \frac{P_T}{4\pi R^2} G_{TX} \sigma_{RCS} \frac{1}{4\pi R^2} \frac{\lambda^2}{4\pi} G_{RX}$$

$$P_{RX} = \frac{P_T}{4\pi R^2} G_{TX} \sigma_{RCS} \frac{1}{4\pi R^2} \frac{\lambda^2}{4\pi} G_{RX}$$

Equazione radar

$$P_R = P_T G_T G_R \frac{\lambda^2}{(4\pi)^3 R^4} \sigma_{RCS}$$

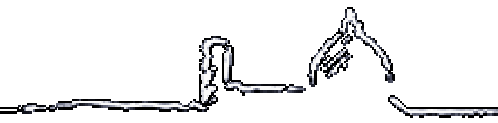
$$P_R = P_T G_T G_R \frac{\lambda^2}{(4\pi)^3 R^4} \sigma_{RCS}$$

$$\frac{P_R}{P_T} = G_T G_R \frac{1}{(4\pi)^3 \left(\frac{R}{\lambda}\right)^4} \frac{\sigma_{RCS}}{\lambda^2}$$

$$20\log_{10}\left(\frac{P_R}{P_T}\right) = 20\log_{10} G_T + 20\log_{10} G_R - 20\log_{10}\left((4\pi)^3 \left(\frac{R}{\lambda}\right)^4\right) + 20\log_{10}\left(\frac{\sigma_{RCS}}{\lambda^2}\right)$$

Attenuazione
geometrica

Equazione radar



$$20\log_{10}\left(\frac{P_R}{P_T}\right) = 20\log_{10} G_T + 20\log_{10} G_R - 20\log_{10}\left((4\pi)^3\left(\frac{R}{\lambda}\right)^4\right) + 20\log_{10}\left(\frac{\sigma_{RCS}}{\lambda^2}\right)$$

Esempio:

Attenuazione
geometrica

$$20\log_{10} G_{TX} = 20\text{dB}$$

$$20\log_{10} G_{RX} = 20\text{dB}$$

$$3\text{GHz} \longrightarrow \lambda = 0.1\text{m}$$

$$R = 10000\text{m}$$

$$\longrightarrow A = 466\text{dB}$$

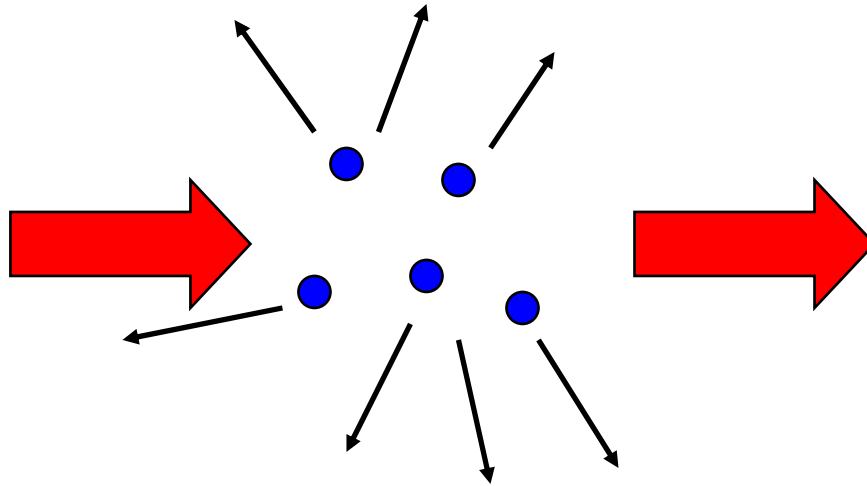
$$3\text{GHz} \longrightarrow \lambda = 0.1\text{m}$$

$$\sigma_{RCS} = 100\text{m}^2$$

$$\longrightarrow RCS = 60\text{dB}$$

$$40 - 466 + 60 = -366\text{dB}$$

$$\text{Dinamica} = 366\text{dB}$$



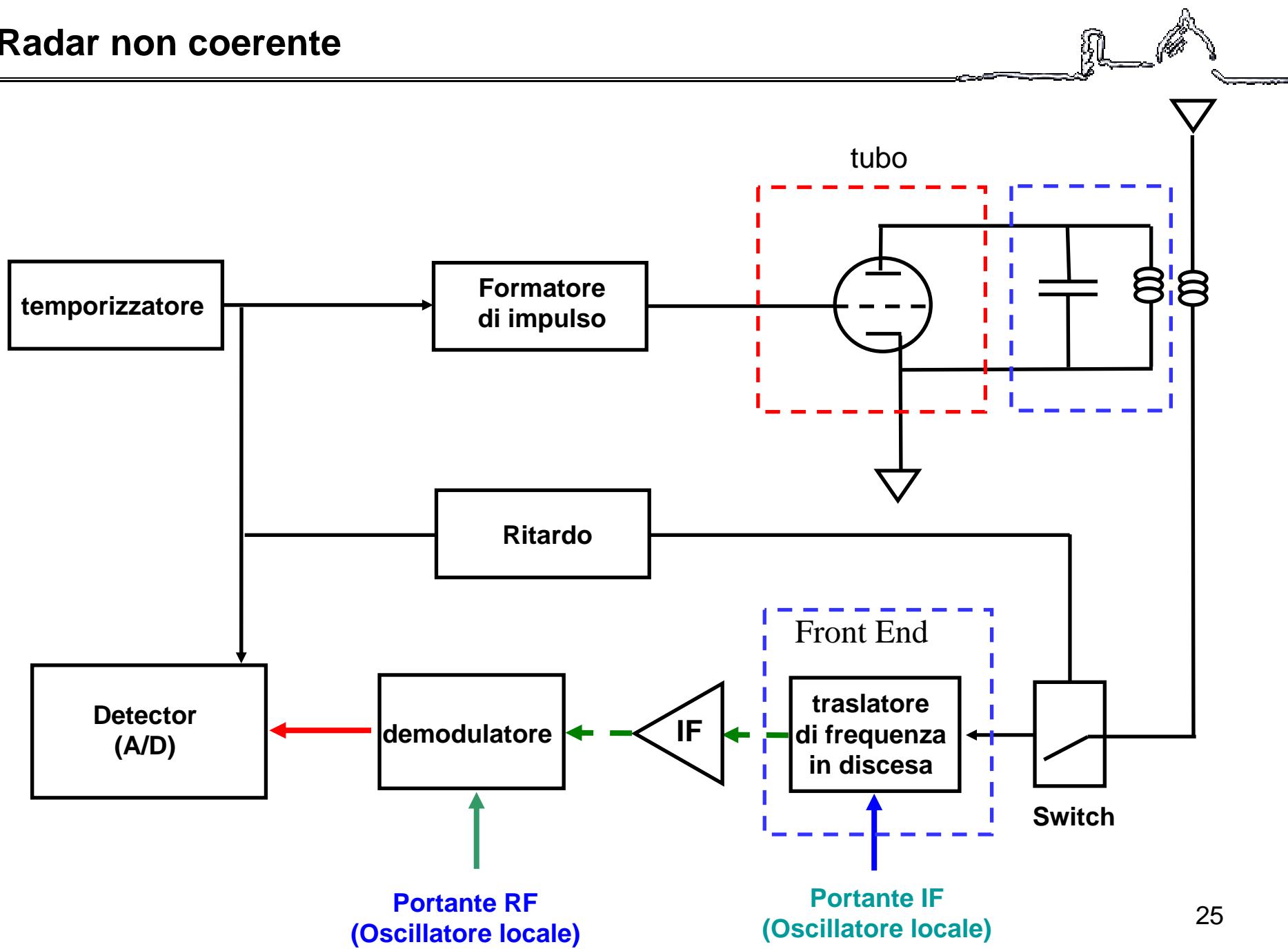
$$\sigma \approx 9\pi \left(\frac{2\pi a}{\lambda} \right)^2 a^2$$

$$a = 1\text{mm} \longrightarrow \pi a^2 = 3.14 \times 10^{-6}$$

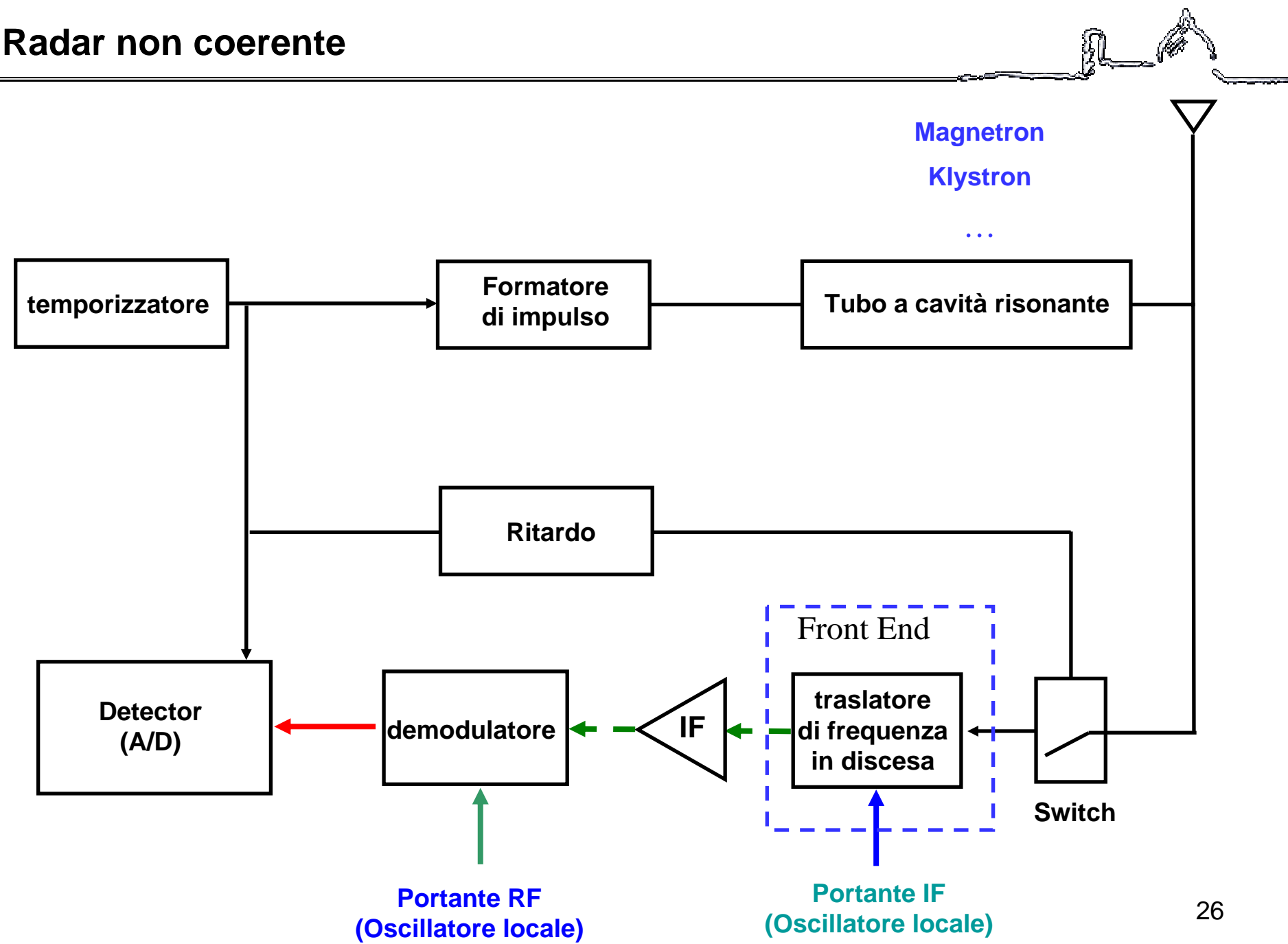
$$a = 1\text{mm} \longrightarrow \sigma = 1.12 \times 10^{-7}$$

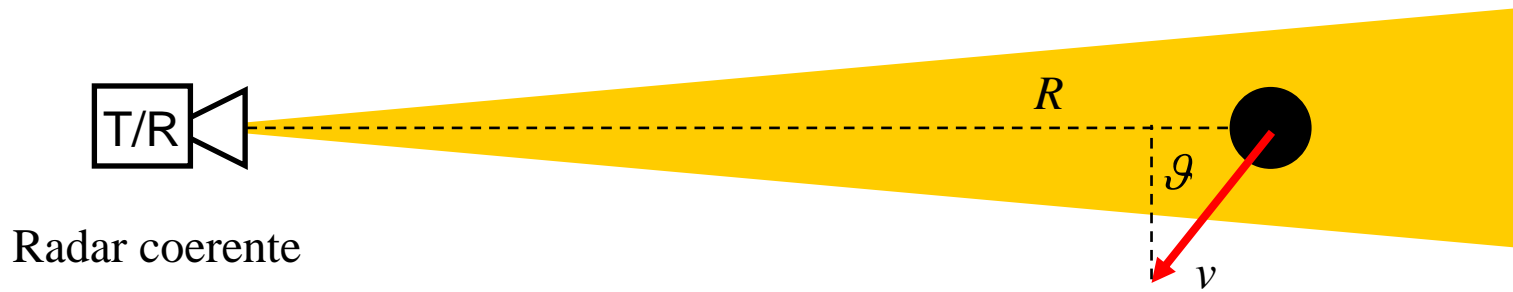
$$\lambda = 0.1\text{m}$$

Radar non coerente



Radar non coerente



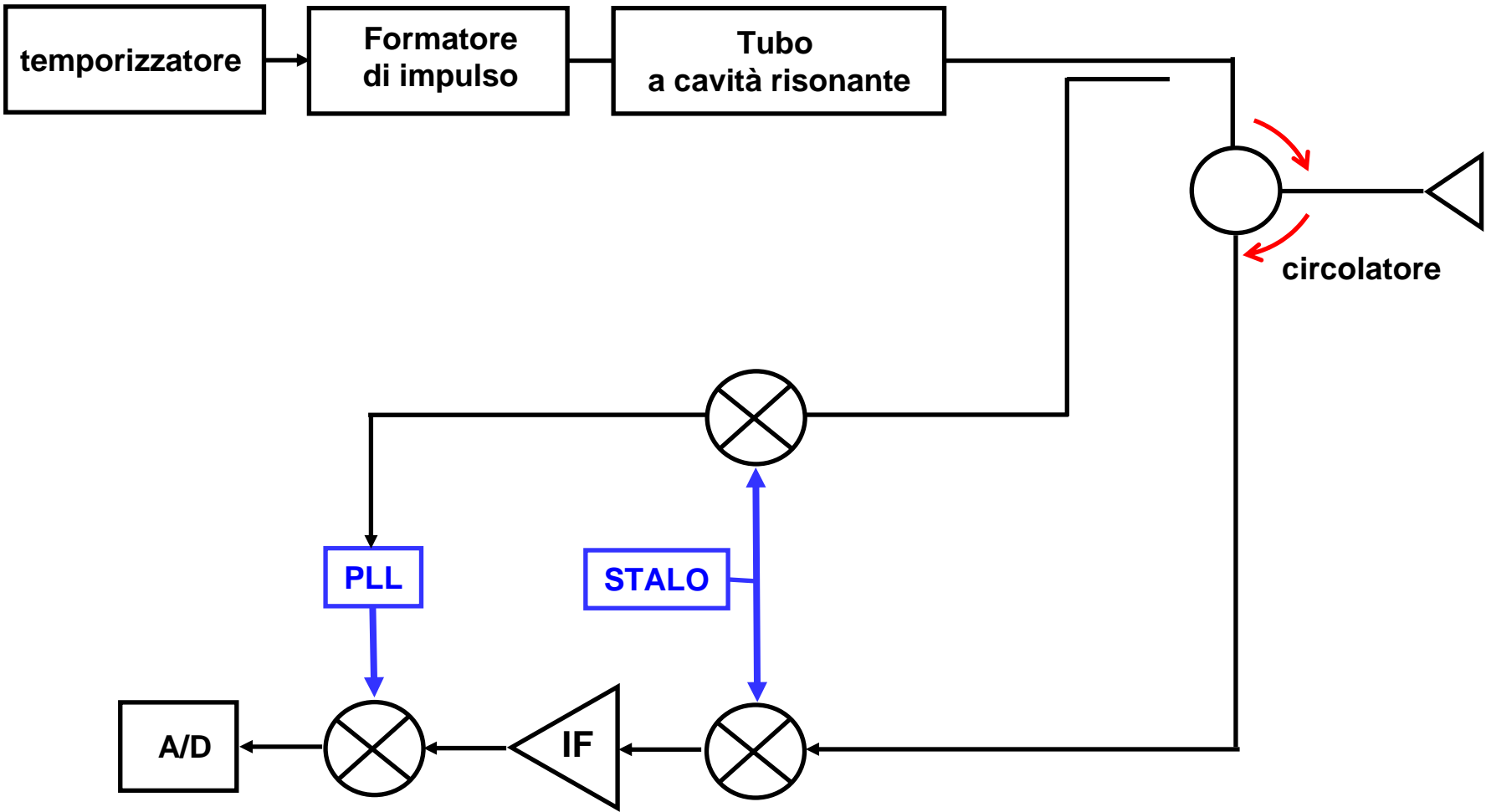


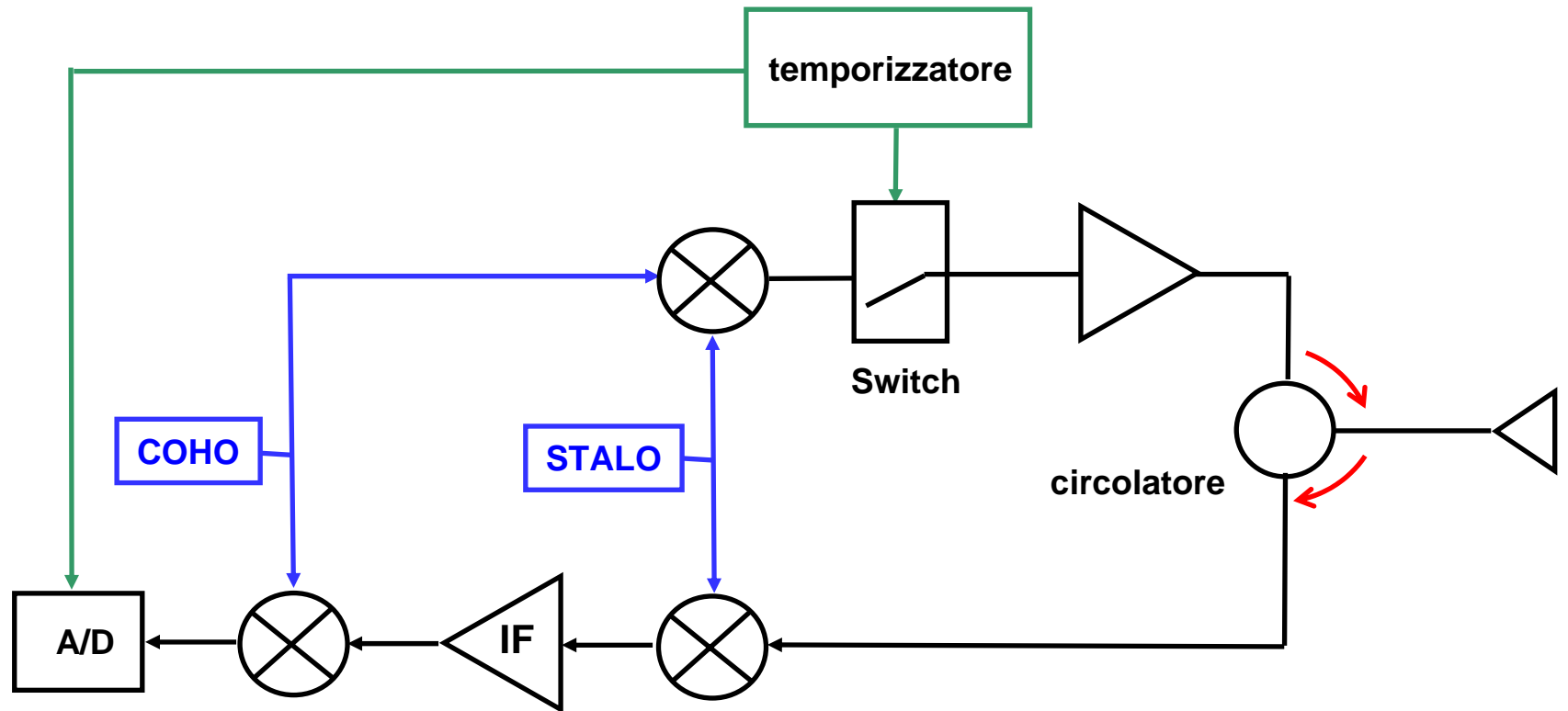
Radar coerente

$$\varphi(t) = 2\pi \frac{2v \cos(\vartheta) t}{\lambda}$$

$$\omega_{\text{doppler}} = 2\pi \frac{2v \cos(\vartheta)}{\lambda}$$

Radar coerente





MTI: Moving Target Indicator

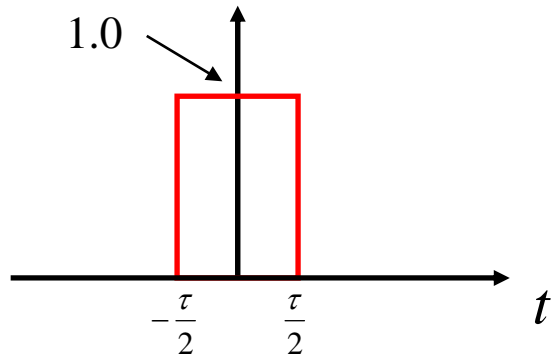
Sottrazione dell'impulso



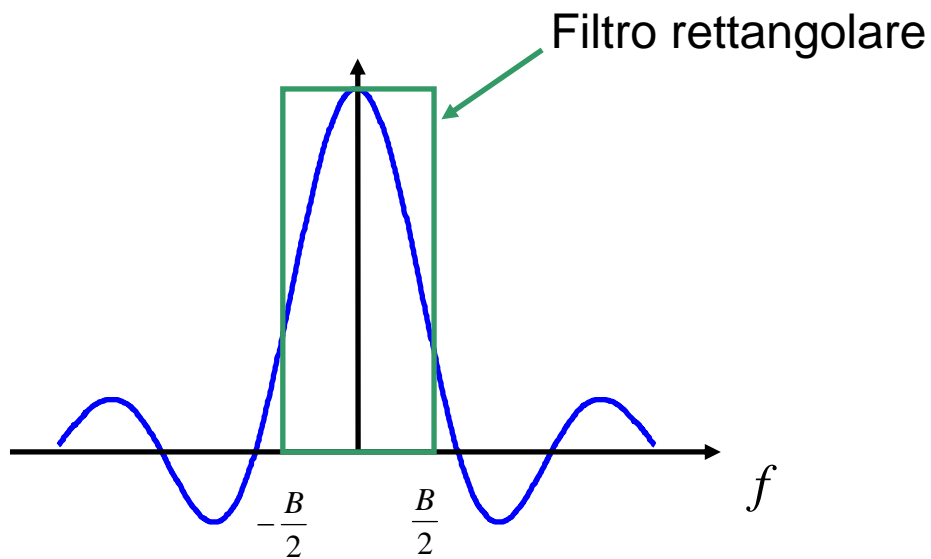
Velocità cieca e ambiguità

Stagger

Filtro adattato



$$S(f) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j2\pi ft} dt = \dots = \frac{\sin(\pi f \tau)}{\pi f \tau}$$

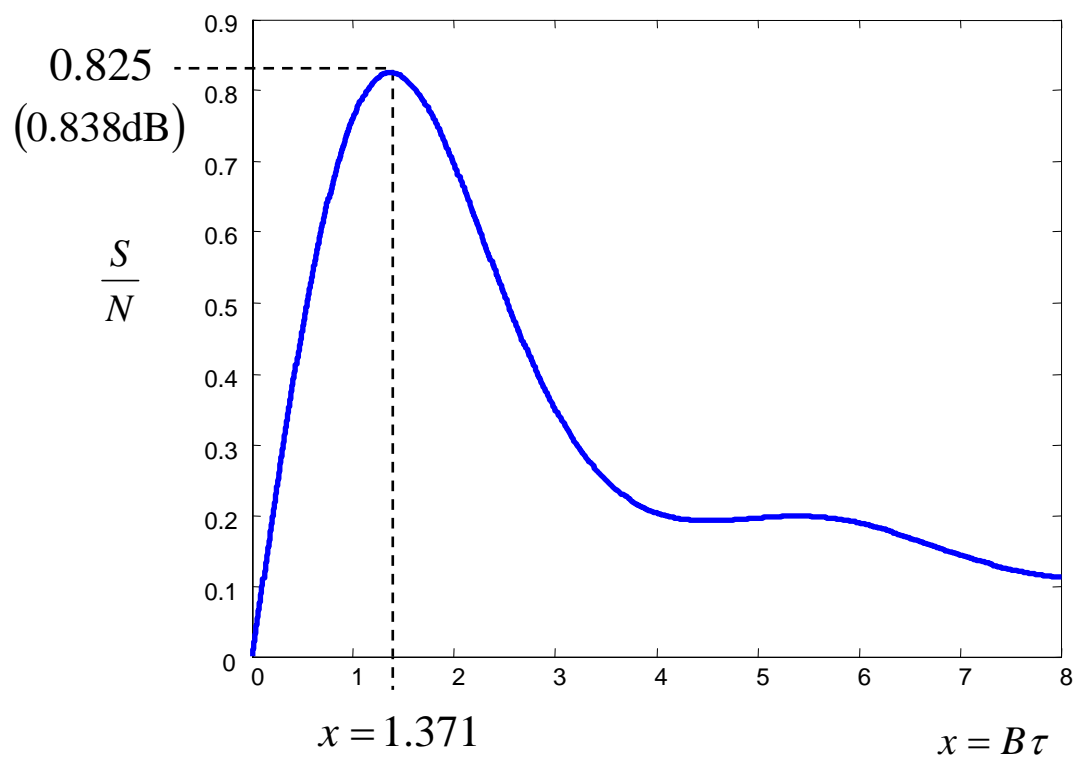


$$\frac{S}{N} = \frac{\left[\int_{-\frac{B}{2}}^{\frac{B}{2}} \frac{\sin(\pi f \tau)}{\pi f \tau} df \right]^2}{\int_{-\frac{B}{2}}^{\frac{B}{2}} n^2(f) df}$$

Filtro adattato



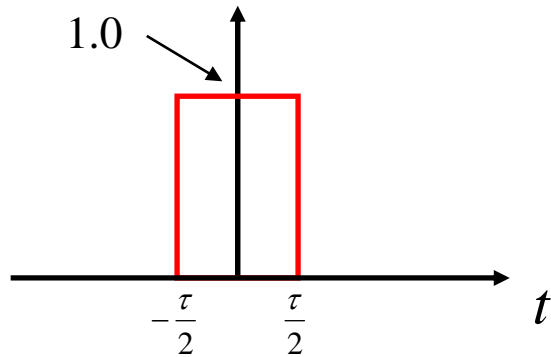
$$\frac{S}{N} = \frac{\left[\int_{-\frac{B}{2}}^{\frac{B}{2}} \frac{\sin(\pi f \tau)}{\pi f \tau} df \right]^2}{\int_{-\frac{B}{2}}^{\frac{B}{2}} n^2(f) df} = \frac{\left[\int_{-\frac{B}{2}}^{\frac{B}{2}} \frac{\sin(\pi f \tau)}{\pi f \tau} df \right]^2}{kTB} = \frac{\left[\int_{-\frac{B\tau}{2}}^{\frac{B\tau}{2}} \frac{\sin(\pi f \tau)}{\pi f \tau} df \tau \right]^2}{kTB\tau} = \frac{1}{KT} \frac{\left[\int_{-\frac{x}{2}}^{\frac{x}{2}} \frac{\sin(\pi u)}{\pi u} du \right]^2}{x}$$



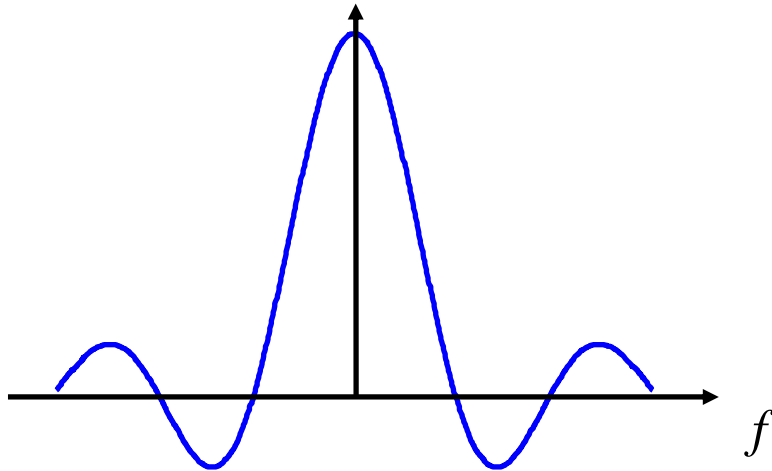
$B\tau = 1.371$

$B = \frac{1.371}{\tau}$

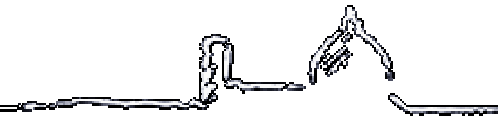
Filtro adattato



$$S(f) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j2\pi ft} dt = \dots = \frac{\sin(\pi f \tau)}{\pi f \tau}$$



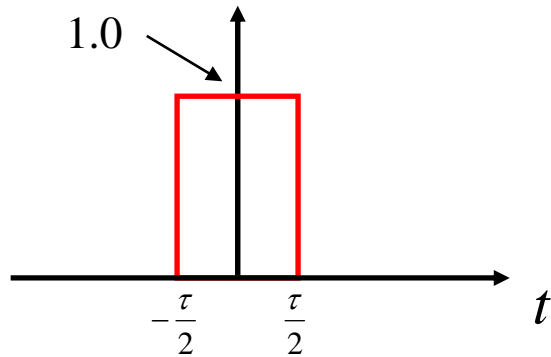
$$\frac{S}{N} = \frac{\left[\int_{-\infty}^{\infty} \frac{\sin(\pi f \tau)}{\pi f \tau} \frac{\sin(\pi f \tau)}{\pi f \tau} df \right]^2}{\int_{-\infty}^{\infty} \left[n(f) \frac{\sin(\pi f \tau)}{\pi f \tau} \right]^2 df}$$



$$\frac{S}{N} = \frac{\left[\int_{-\infty}^{\infty} \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right)^2 df \right]^2}{\int_{-\infty}^{\infty} \left[n(f) \frac{\sin(\pi f \tau)}{\pi f \tau} \right]^2 df} = \frac{\left[\int_{-\infty}^{\infty} \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right)^2 df \right]^2}{kT \int_{-\infty}^{\infty} \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right)^2 df} = \frac{1}{kT} \int_{-\infty}^{\infty} \left(\frac{\sin(\pi u)}{\pi u} \right)^2 du = \frac{1}{kT}$$

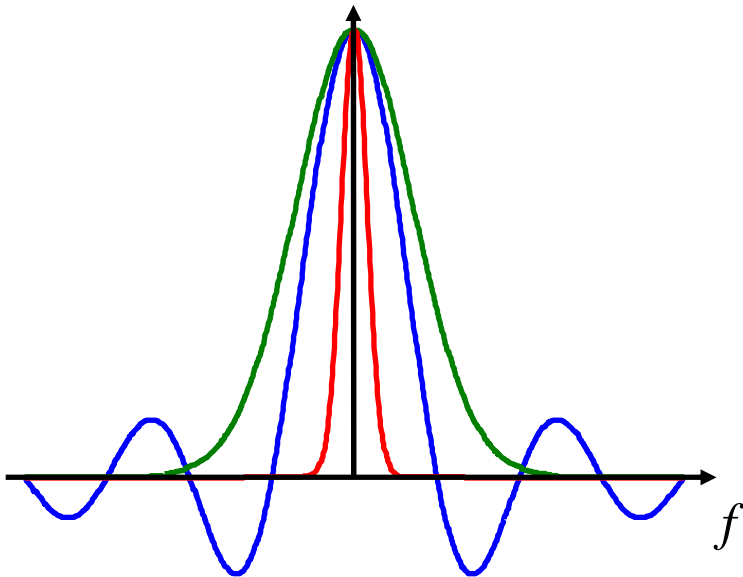
In generale S/N è massimo se uso come filtro lo spettro del segnale trasmesso

Filtro adattato



$$S(f) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j2\pi ft} dt = \dots = \frac{\sin(\pi f \tau)}{\pi f \tau}$$

$$F(f) = e^{-\frac{1}{2}\left(\frac{f}{B}\right)^2} \quad \text{Filtro gaussiano}$$

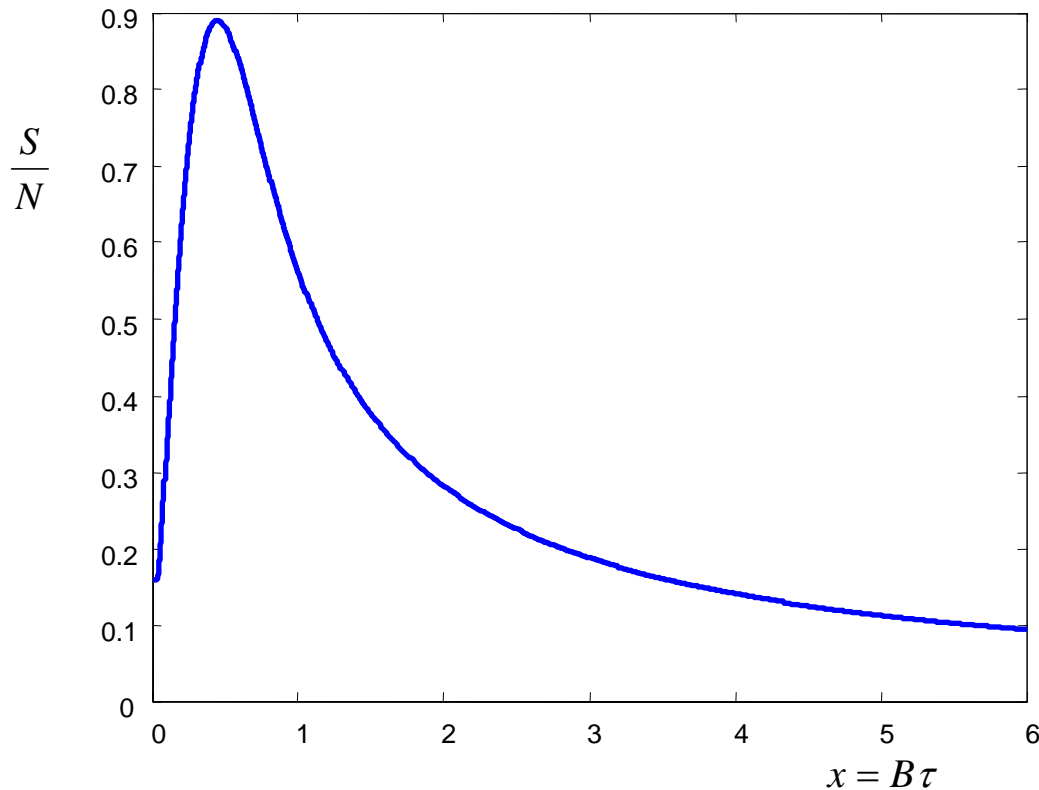


$$\frac{S}{N} = \frac{\left[\int_{-\infty}^{\infty} \frac{\sin(\pi f \tau)}{\pi f \tau} e^{-\frac{1}{2}\left(\frac{f}{B}\right)^2} df \right]}{\int_{-\infty}^{\infty} n^2(f) e^{-\frac{1}{2}\left(\frac{f}{B}\right)^2} df}$$

Filtro adattato

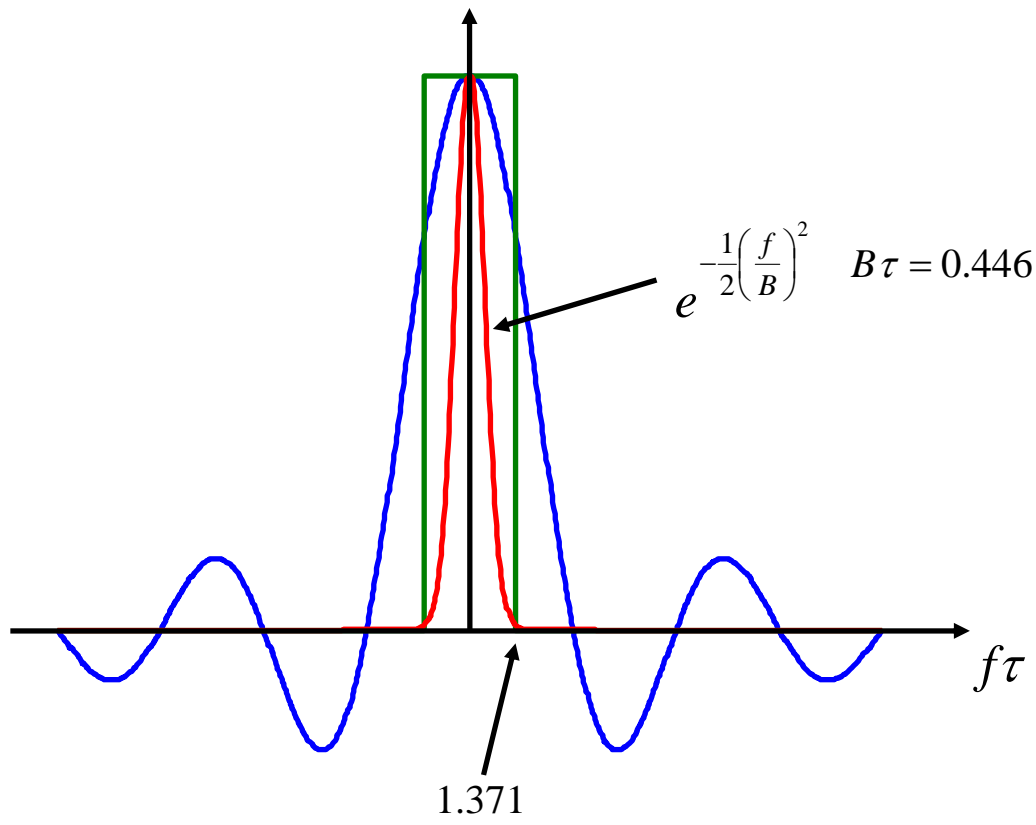


$$\frac{S}{N} = \frac{\left[\int_{-\infty}^{\infty} \frac{\sin(\pi f \tau)}{\pi f \tau} e^{-\frac{1}{2} \left(\frac{f}{B} \right)^2} df \right]^2}{\int_{-\infty}^{\infty} \left[n(f) e^{-\frac{1}{2} \left(\frac{f}{B} \right)^2} \right]^2 df} = \frac{\left[\int_{-\infty}^{\infty} \frac{\sin(\pi f \tau)}{\pi f \tau} e^{-\frac{1}{2} \left(\frac{f}{B} \right)^2} df \right]^2}{kT \int_{-\frac{B}{2}}^{\frac{B}{2}} \left[e^{-\frac{1}{2} \left(\frac{f}{B} \right)^2} \right]^2 df} = \frac{\left[\int_{-\infty}^{\infty} \frac{\sin(\pi u)}{\pi u} e^{-\frac{1}{2} \left(\frac{u}{B\tau} \right)^2} du \right]^2}{kT \int_{-\infty}^{\infty} \left[e^{-\frac{1}{2} \left(\frac{u}{B\tau} \right)^2} \right]^2 du}$$



$$\left(\frac{S}{N} \right)_{\max} = 0.890 \quad (-0.506\text{dB})$$

$$B\tau = 0.446$$





Segnali coerenti

$s(t)$

Segnale inviato

$\tilde{s}(\omega)$

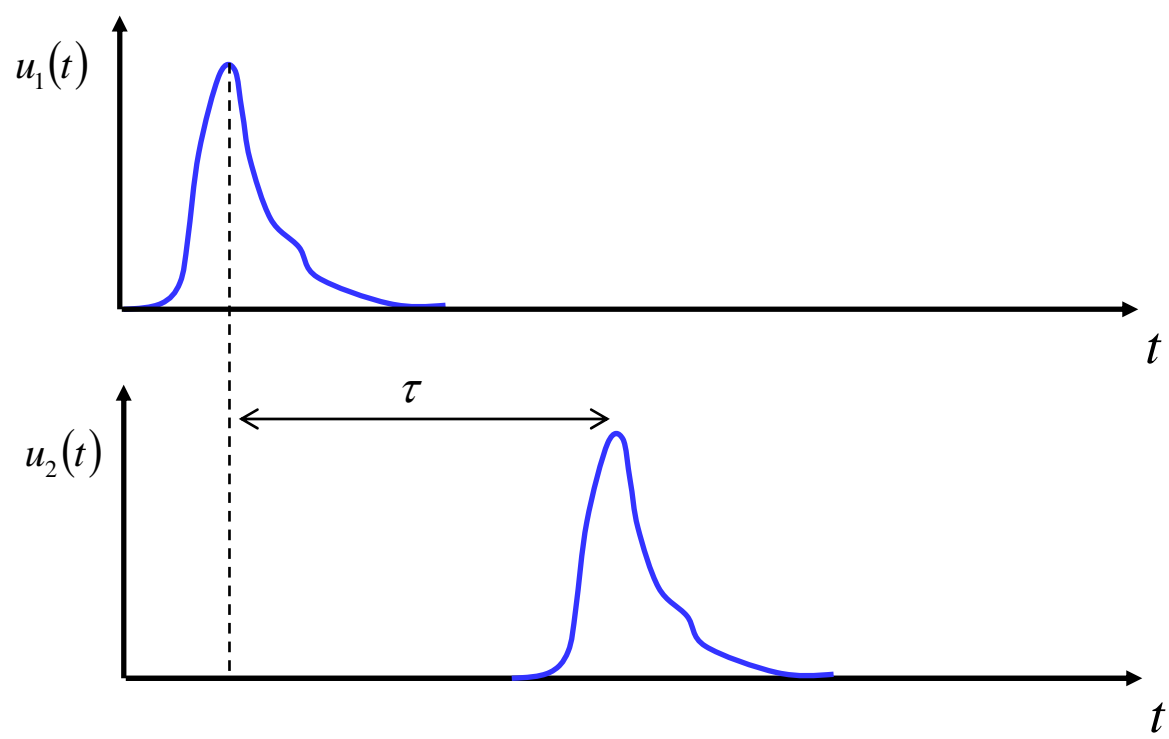
Trasformata di Fourier del segnale

$$\frac{S}{N} = \frac{\left[\int_{-\infty}^{\infty} \tilde{s}(f) a(f) df \right]^2}{kT \int_{-\infty}^{\infty} |a(f)|^2 df} = \frac{1}{kT}$$

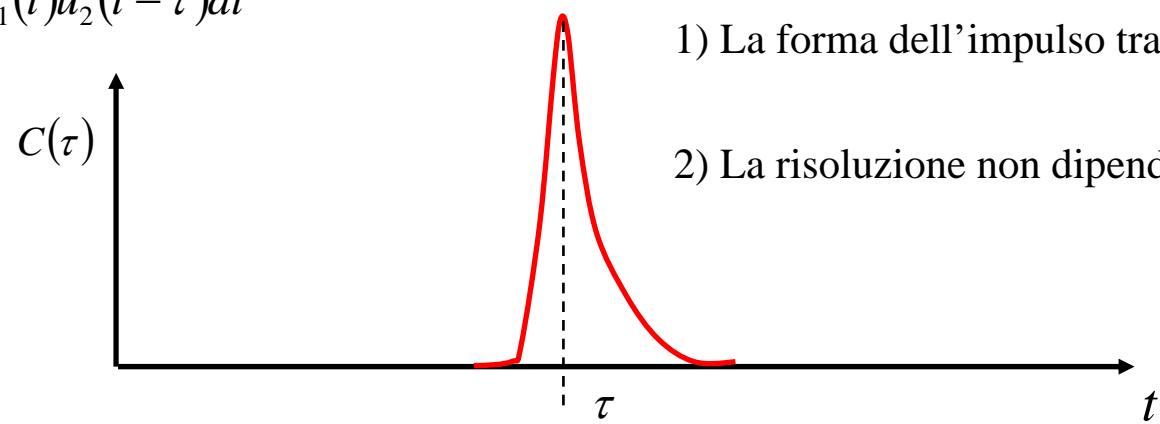
$$a(f) = \tilde{s}^*(f)$$

Filtro adattato

La correlazione



$$C(\tau) = \int u_1(t) u_2(t - \tau) dt$$



- 1) La forma dell'impulso trasmesso agisce da filtro
- 2) La risoluzione non dipende dalla durata dell'impulso

$$C(\tau) = \int_0^T u_{TX}(t) u_{RX}(t - \tau) dt$$

$$\tilde{C}(f) = |\tilde{u}(f)|^2$$

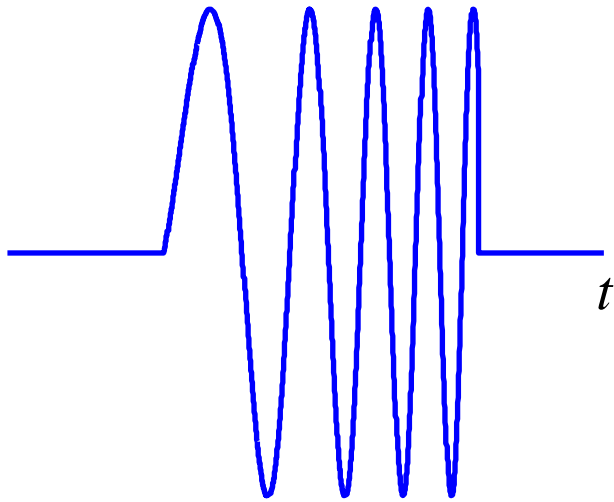
$$\Delta\tau = \frac{1}{B}$$

Separa la durata dell'impulso dalla risoluzione:
la risoluzione non dipende dalla durata dell'impulso, ma dal contenuto
spettrale (banda) della modulazione dell'impulso

L'ampiezza del segnale compresso $C(\tau)$ dipende non dall'ampiezza assoluta
del segnale trasmesso, ma dal suo integrale: quindi a parità di ampiezza del
segnale trasmesso, l'ampiezza del segnale compresso aumenta all'aumentare
della durata dell'impulso

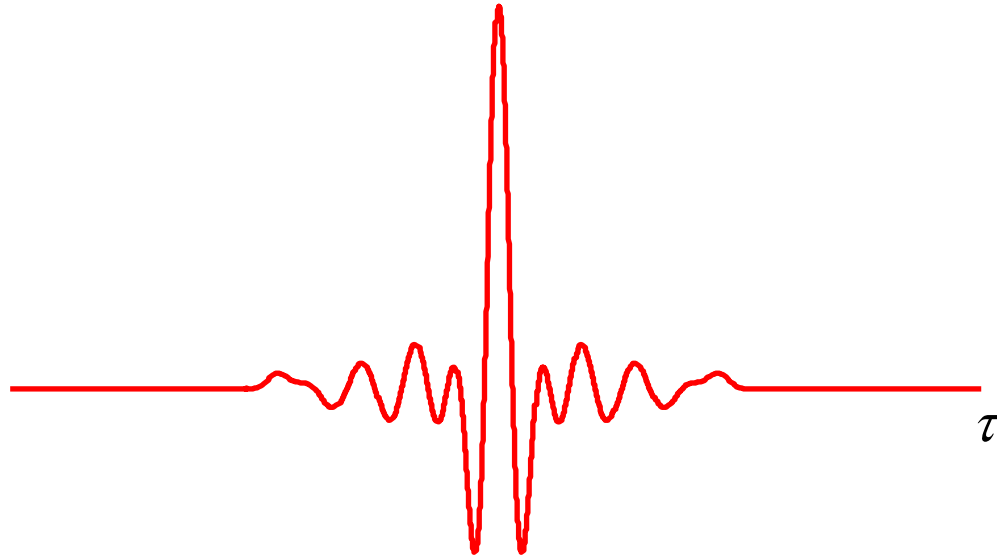
Nota: la compressione di impulso realizza un filtro adattato

Compressione di impulso



$$s(t) = \sin(2\pi(f_0 + kt)t) = \sin(2\pi(f_0t + kt^2))$$

$$C(\tau) = \int_0^T s(t)s(t-\tau)dt$$



$$C(\tau) = \int_0^T u_{TX}(t) u_{RX}^*(t - \tau) dt$$