

## 12. MAGNETISM AND SUPERCONDUCTIVITY.

The last lectures of the course are intended to give a brief overview of the basics of two subjects of importance for current and emerging technology, namely magnetism and superconductivity.

### A. Superconductivity

In this lecture we follow closely the exposition by Kittel in part of Ch. 10. The lecture will cover:

1. Basic experimental properties (resistivity=0, superconducting transition temperatures, the effect of a magnetic field; Kittel p. 259-261)
2. Superconductivity is NOT the same as perfect conduction. Rather the superconductor exhibits perfect diamagnetism, the so called Meissner effect (Kittel p. 262-265).
3. Heat capacity, energy gap (Kittel p. 264-270).
4. Theoretical overview (thermodynamics, London equation, Kittel p. 270-277).
5. Qualitative picture of the BCS theory of superconductivity (Kittel p. 277-279).
6. Some applications (including high-temperature superconductivity, Kittel p. 293).

### B. Dia- and paramagnetism.

The magnetic moment of a free atom is due to the spins of the electrons, their orbital angular momentum and the change in orbital momentum when applying a magnetic field. Magnetic properties of solids are usually expressed by the **magnetic susceptibility** (K p. 299). It can be expressed per unit volume as in Kittel, but also per unit mass (by division by the density of the material or per unit mole (multiplying by the molar mass and dividing by the density).

#### 1. Diamagnetism

A diamagnetic contribution (susceptibility<0) occurs in all materials. That is due to the contribution of the ion cores. It is dominant in the solid inert gases and is also of importance in many metals. We closely follow the exposition in Kittel p.299-301.

## 2. Paramagnetism.

Paramagnetism (susceptibility > 0) occurs when the number of electrons of an atom is odd, or when inner electron shells are partially filled. An electron has an orbital angular momentum  $\mathbf{l}$  and a spin momentum  $\mathbf{s}$ . Their sum is the total momentum  $\mathbf{j}$ . The magnetic moment of an orbiting electron is given by  $\mu = -g \mu_B \mathbf{j}$ , where  $g \geq 2$  and  $\mu_B = eh/4m$  is called the **Bohr magneton**. For a free ion or atom the corresponding moments are denoted by capital letters and the magnetic moment is given by an analogous relation, using  $\mathbf{J}$  and the appropriate g-factor for an atom (K p. 302). In a magnetic field the ground state is split into  $2J+1$  energy levels (K p.303). To determine L, S and J for atomic ground state we use **Hund's rules** (K p.306-307).

We now treat a simple classical model for the **ideal magnetic gas**. By this we mean a system of N non-interacting magnetic atoms having a magnetic moment,  $\mu$ . An applied magnetic field tends to orient the moments, but the thermal motion of the atoms makes the orientation more random. The energy of an atom in the field is  $U = -\mu \cdot \mathbf{B} = -\mu B \cos \theta$ , where  $\theta$  is the angle between the directions of the field and the atomic moment. The number of atoms within a solid angle  $d\Omega$  between  $\theta$  and  $\theta + d\theta$  is given by:

$$dN = A \exp(\mu B \cos \theta / kT) d\Omega$$

Integrating over all solid angles we can determine the constant A from  $N = (4\pi A/x) \sinh x$ , where  $x = \mu B/kT$ . The total magnetization in the field direction becomes:

$$M = \int \mu \cos \theta dN = 2\pi A \mu \int_0^\pi \exp(x \cos \theta) \sin \theta \cos \theta d\theta.$$

Evaluating the integral we obtain:

$$M = N\mu (\coth x - (1/x)).$$

In the limit that x is much smaller than unity,  $\coth x = (1/x) + (x/3) - O(x^3)$  and this simple classical model gives the same result as the quantum mechanical derivation in Kittel p. 302-305, namely:

$$M = N\mu^2 B / 3kT \quad \text{and} \quad \chi = \mu_0 N\mu^2 / 3kT = C/T,$$

since,  $\mu = g \mu_B$  in this case. This theory is best for dilute magnetic systems, where the magnetic atoms or ions are situated far from one another. The best example is of such materials are the **paramagnetic salts**, for example  $\text{MnSO}_4(\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$  or  $\text{Gd}(\text{C}_2\text{H}_5\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}$  (K p. 423). In these cases one obtains experimentally  $\chi = C/(T + \theta)$ , where  $\theta$  is small, of the order of 10 K or less, and comes from residual magnetic interactions.

The ions of the rare earth metals and the transition metals exhibit paramagnetic properties. For the rare earths, the ground states are easily obtained from Hund's rules (K p. 305-308). In the case of the transition metals the orbital angular momentum is quenched due to the effect of the so-called crystal field (K p. 307-311), and one must **only** take into account the spin momentum when calculating the ground state.

### **3. Pauli paramagnetism in a metal.**

The conduction electrons in a metal also give rise to paramagnetism, but the mechanism is quite different from those above (K p. 315-317). In an applied magnetic field, the energy of the electron orbitals with spin parallel to  $B$  will be lowered, while the orbitals with opposite spin will be raised. It will then be energetically favorable for some electrons near the Fermi level to flip their spin direction, and this effect gives rise to paramagnetism that is the dominant effect in many metals.