ESTIMATION PROCEDURE OF THE SPATIAL DISTRIBUTION OF RAINFALL RATE THROUGH COMBINED USE OF REFLECTIVITY RADAR AND RAIN GAUGE DATA

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Abstract

In this work we propose a procedure for the estimation of the spatial distribution of the rainfall rate in the observation time T over an area covered by a meteorological radar and a rain gauge network. T can be the whole duration of a rainfall phenomenon or a part of it. The estimation of the rainfall rate is based on the processing of a data set composed by rain gauge and horizontal reflectivity radar data gathered during a rainfall phenomenon. Besides, the proposed procedure allows the quantification of the estimation error, related to the rainfall spatial distribution, using the rain gauge network as "true" reference and the analysis of the space-time variability of the precipitation phenomenon.

Introduction

It is recognized that rainfall rate estimate based on radar measurements is affected by a high degree of uncertainty due to many sources: space time variability of drop size distribution (phenomenon to phenomenon and within the same phenomenon), the space time variation of reflectivity with height, the radar signal attenuation etc [1]. Moreover, the most common validation way to asses the reliability of the rainfall radar estimates is the comparison with rain gauge data [1].

The objective of this work is the description of a data processing method for the determination of the spatial distribution of the rainfall rate $R_T(x,y)$ over an area covered by a meteorological radar and a rain gauge network during the observation time T. The proposed method processes the instantaneous radar horizontal reflectivity data and the rain gauge data, collected during a prefixed observation time T, in order to compute the rainfall rate $R_T(x,y)$ as:

$$\log(R_T(x,y)) = A_T(x,y) + B_T(x,y)\log(Z_T(x,y)) \qquad (1)$$

where $Z_T(x,y)$ is an average function of the instantaneous horizontal reflectivity maps gathered during the observation time T and $A_T(x,y)$ and $B_T(x,y)$ are coefficients that are determined by means of ad hoc correlation functions between space-time "average" values of rain gauge data and horizontal reflectivity data.

In order to describe how the coefficients A_T and B_T are determined, the definition of a set of parameters depending on the radar and the rain gauge data set is necessary.

The rain gauge network provides a dataset of N_r x n_{rs} accumulated rainfall values, where N_r is the number of the available rain gauges and n_{rs} is the number of time samples depending on both the rain gauge accumulation time t_{rs} and the prefixed observation time T.

For each rain gauge, the reflectivity radar data is composed by N_z x n_{zs} instantaneous horizontal reflectivity values related to the air volume just above the rain gauge location and acquired at the lowest elevation angle available. N_z is the number of adjacent radar cells at the same elevation angle, again just above the rain gauge, and n_{zs} is the number of time samples depending on the radar antenna rotation period t_{zs} and the prefixed observation time T.

The instantaneous horizontal reflectivity data at the range r from the radar is related to the contribution of the air volume having a resolution of $\Delta r=c\tau/2$ in range, $\Delta\theta_E=r\theta_E$ in elevation and $\Delta\theta_A=r\theta_A\cos(\theta_E)$in azimuth, where τ is the pulse length, θ_E and θ_A the 3 dB antenna beam width in elevation and azimuth respectively. Such air volume ($\Delta r\cdot\Delta\theta_E\cdot\Delta\theta_A$) defines the radar resolution cell RC.

For a given elevation angle θ_E we have an uniform grid in polar coordinates with radial step Δr and azimuthal step $\Delta\theta_A$. We define RC_{ij} as the radar resolution cell that has a radial distance $i\Delta r$ and an azimuthal reference $j\Delta\theta_A$; consequently $Z_{ij}(t)$ is the horizontal reflectivity value at the instant t related to the RC_{ij}.

For a given rain gauge k (k=1···N_r) placed in (x_k ,y_k), we define the corresponding radar cell RC$^{(k)}$ as the RC$_{ij}$ that is just above the rain gauge (see fig. 1), therefore RC$_{ij}$ = RC$^{(k)}$ if the following relationship is verified:

$$\begin{gathered} i\Delta r \cdot \cos(\Delta\vartheta_E) \le \sqrt{x_k^2 + y_k^2} < (i+1)\Delta r \cdot \cos(\Delta\vartheta_E) \\ j\Delta\vartheta_E \le \text{arctg}\left(\frac{y_k}{x_k}\right) < (j+1)\Delta\vartheta_E \end{gathered} \tag{2}$$

Since $Z_{ij}(t)$ is the instantaneous horizontal reflectivity of the cell RC$_{ij}$, the $Z^{(k)}(t)$ is the reflectivity related to the radar cell RC$^{(k)}$ at the time t.
Let us define N_z=(2n_c+1)^2 , with n_c =1,2,···, as the number of adjacent radar cells. For a given rain gauge k we have i and j such that RC$^{(k)}$=RC$_{ij}$.
RC$_{lp}$ is one of the N_z adjacent radar cells of RC$^{(k)}$ if the following equation is verified:

$$\begin{gathered} |l - i| \le n_c \\ |p - j| \le n_c \end{gathered} \tag{3}$$

Posing $m=(i-l+n_c)\cdot(2n_c+1)+(j-p+n_c+1)$, m assumes values between 1 and $N_z=(2n_c+1)^2$: we define $\mathrm{RC}_m^{(k)}$ = RC$_{lp}$ as the set of adjacent radar cells of the rain gauge k. Consequently, we defined $\mathrm{Z}_m^{(k)}(t)$ the instantaneous horizontal radar reflectivity at the time t of the cell $\mathrm{RC}_m^{(k)}$ (with m=1···N_z).

Fig.1 shows the spatial disposition of the N_z=9 (n_c=1) adjacent cells related to the cell RC$^{(k)}$ and their ground projection.

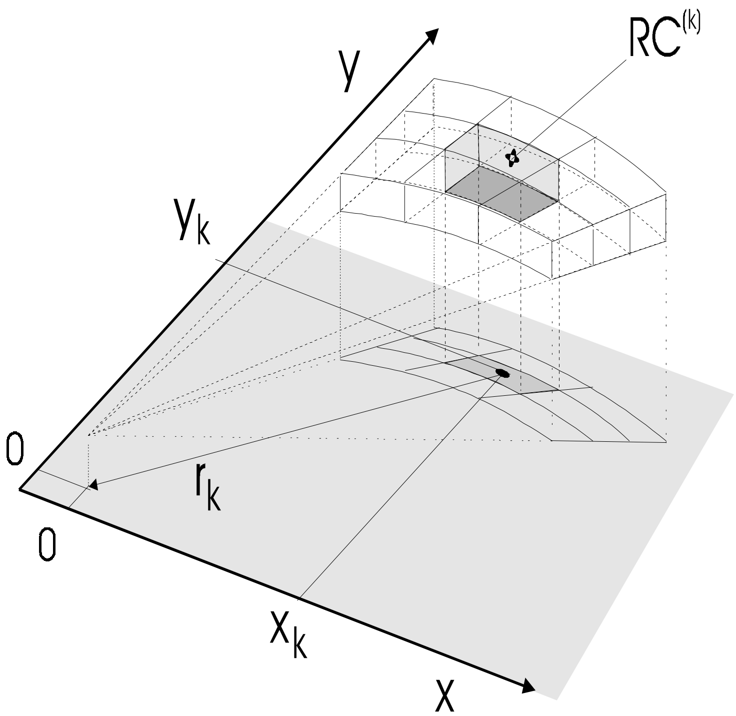

Fig. 1 – spatial disposition of the adjacent radar cells considering the location (x_k , y_k) of the k^{th} rain guage.

In synthesis, once the observation time T and the number of adjacent cells N_z have been fixed, for each rain gauge location k we have:

- $n_{rs}=T/t_{rs}$ time samples of accumulated rain fall ($\mathrm{R}_i^{(k)}$, i=1···n_{rs}) as well as they are measured by the rain gauge

- n_{zs}=T/t_{zs} time samples of instantaneous horizontal reflectivity ($\mathrm{Z}_{m,i}^{(k)}=\mathrm{Z}_m^{(k)}(j\cdot t_{zs})$ with m=1···N_z and j=1··· n_{zs}) for N_z cells as well as they are measured by the radar.

Procedure for the computation of the Z-R relationship

In this section we deal with the procedure used for the computation of the functions $Z_T(x,y)$, $A_T(x,y)$ and $B_T(x,y)$ that are used in eq. 1 for the estimation of the rainfall rate. Such functions are computed by means of space time mean operations on the data set $\mathrm{R}_i^{(k)}$ and $\mathrm{Z}_{m,j}^{(k)}$.

Let us define T_w the processing time: such parameter must be lower than the observation time T and greater than twice both t_{rs} and t_{zs}. In this way T_w contains at least two values of both $\mathrm{R}_i^{(k)}$ and $\mathrm{Z}_{m,i}^{(k)}$ for each rain gauge, namely n_{rw} = T_w / t_{rs} rainfall data $\mathrm{R}_i^{(k)}$ (i=1···n_{rw}) and n_{zw} = T_w / t_{zs} horizontal reflectivity data $\mathrm{Z}_{m,j}^{(k)}$ (m=1···n_{zw}).

Let us define $\mathrm{ZS}_j^{(k)}$ as the spatial arithmetical mean of $\mathrm{Z}_{m,j}^{(k)}$ with respect to the index m:

$$\mathrm{ZS}_j^{(k)}=\frac{1}{N_z}\sum_{m=1}^{N_z}\mathrm{Z}_{m,j}^{(k)} \tag{4}$$

and $\mathrm{ZW}_p^{(k)}$ as the moving arithmetical mean of $\mathrm{ZS}_j^{(k)}$ with respect to the index j:

$$\mathrm{ZW}_p^{(k)}=\frac{1}{n_{zw}}\sum_{j=p}^{p+n_{zw}}\mathrm{ZS}_j^{(k)}\ ,\ p=1\cdots np\ , np=n_{zs}-n_{zw}+1 \tag{5}$$

Finally let us define $\mathrm{RW}_p^{(k)}$ as the moving arithmetical mean of $\mathrm{R}_i^{(k)}$ with respect to the index i:

$$\mathrm{RW}_p^{(k)}=\frac{1}{n_{rw}}\sum_{i=f(p)}^{f(p)+n_{rw}}\mathrm{R}_i^{(k)}\ ,\ f(p)=\left\lceil p\frac{t_{zs}}{t_{rs}}\right\rceil, p=1\cdots np \tag{6}$$

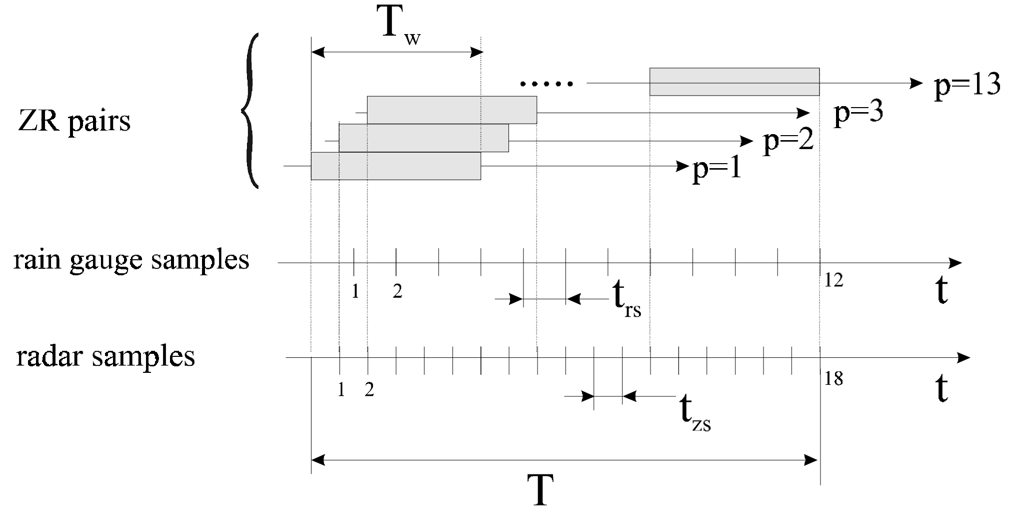

Fig. 2 – Example of the relative position of the time parameters with respect to the observation time T. In the case reported in the picture we have $3t_{zs}$=$2t_{rs}$, n_{zs}=18, n_{rs}=12 T_w=6t_{zs}, n_{rw}=4, n_{zw}=6 and np=13.

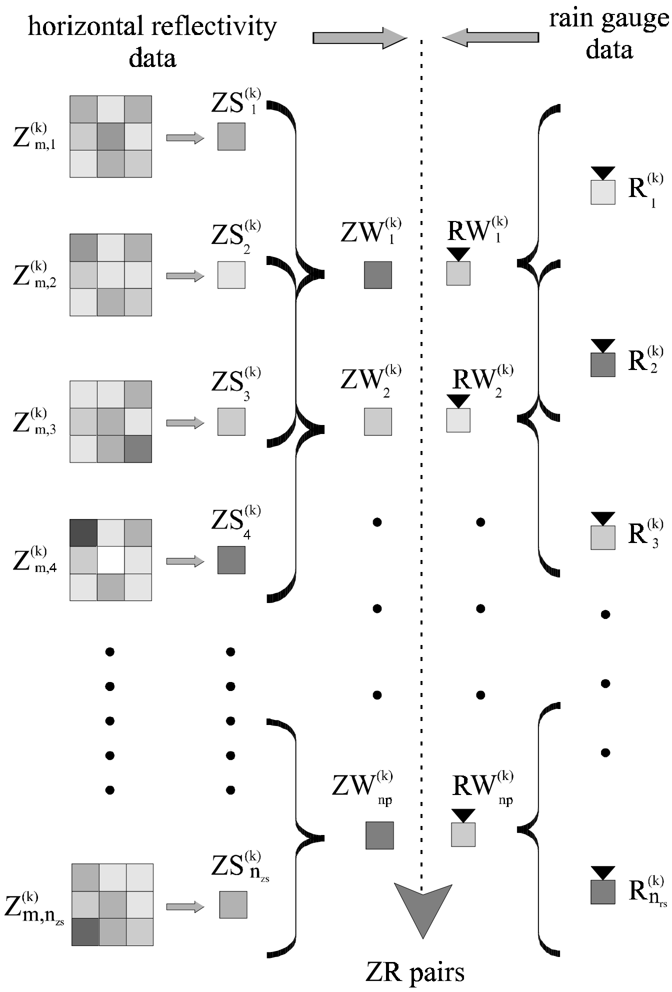

Fig. 3 – The processing chain to be applied on the data set $R_i^{(k)}$ (right side) and $Z_{m,j}^{(k)}$ (left side) in order to compute the ZR pairs ($RW_p^{(k)}$, $ZW_p^{(k)}$) accordingly to the procedure described in the text. The right arrows on the left side (radar data) represent the mean operation of eq. 4 (space average value of the radar data), the right braces on the left side (radar data) represent the average operation of eq. 5 (time average value of the radar data), the left braces on the right side (rain gauge data) represent the mean operation of eq. 6(time mean value of the rain gauge data). The top-down arrow represents the observation time *T* and the vertical dimension of the braces represents the processing time T_w.

In this way, for each rain gauge *k*, we have *np* pairs of values Z-R in the observation time *T* that are related to the moving time window T_w.

Fig, 2 shows a time graph with some time parameters that are defined and used in this work, while Fig. 3 shows the processing chain to be applied on the on the data set $R_i^{(k)}$ and $Z_{m,j}^{(k)}$ in order to compute the values $RW_p^{(k)}$ and $ZW_p^{(k)}$ accordingly to the procedure described above.

Fig 4 and 5 show some examples of the ZR pairs computation using true data acquired in Tuscany during 1999 in autumn considering two different rain gauge locations. Notice that we have different results with different rain gauges and with different T_w. Such differences highlight the rainfall phenomenon was not stationary and not homogeneous.

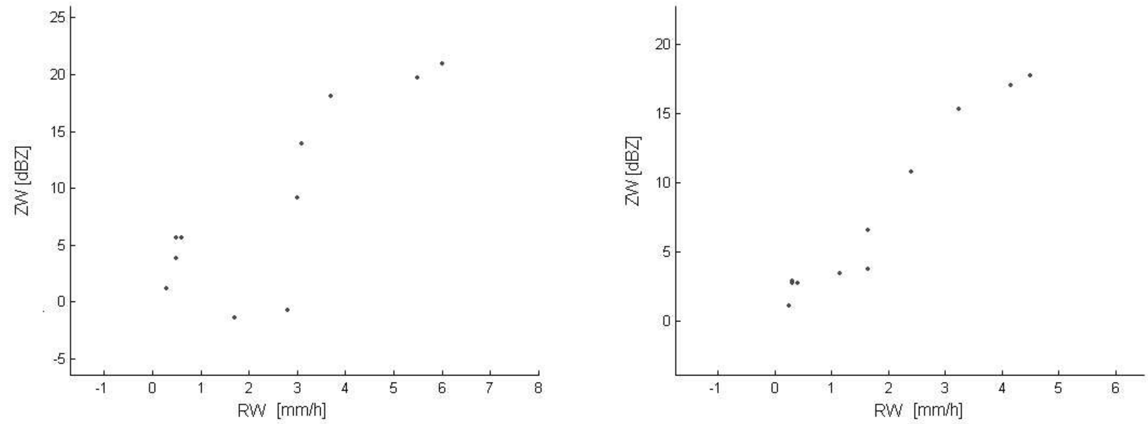

Fig. 4 – ZR pairs computed on rain gauge and radar data acquired during a rainfall phenomenon in Tuscany (Italy) on 21 October 1999 from 11:40:00 to 22:20:00 local time; radar-rain gauge distance: 42.675 km; data processing parameters: t_{zs}=10 min, t_{rs}=15 min, N_z=9, T=640 min.
(left) T_w=120 min, *(right)* T_w=240 min.

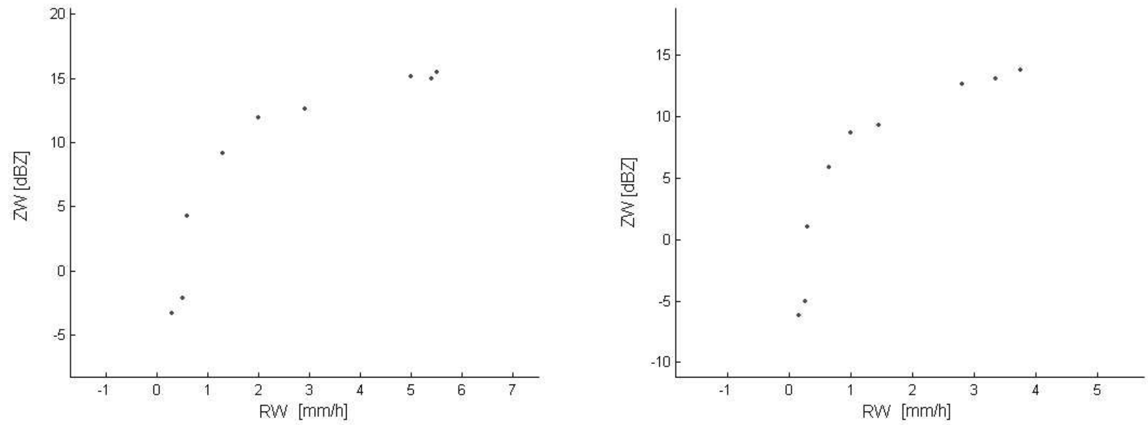

Fig. 5 – As in fig 4 with a different rain gauge: radar-rain gauge distance:16.764 km.
(left) T_w=120 min, *(right)* T_w=240 min.

If a linear regression method is applied to the *np* pairs ($\mathrm{RW}_p^{(k)}$, $\mathrm{ZW}_p^{(k)}$) we obtain two coefficients of the regression line, $a_0^{(k)}$ and $b_0^{(k)}$ for each rain gauge location. Such coefficients are used to assign the values to the functions A_T and B_T in the rain gauge locations, therefore:

$$\begin{aligned} A_T(x_k, y_k) &= a_0^{(k)} \\ B_T(x_k, y_k) &= b_0^{(k)} \end{aligned} \qquad (7)$$

$Z_T(x_k,y_k)$ is the arithmetical mean of $\mathrm{ZS}_j^{(k)}$ with respect to the index j:

$$Z_T(x_k, y_k) = \frac{1}{n_{zs}} \sum_{j=1}^{n_{zs}} ZS_j^{(k)} \qquad (8)$$

The values of A_T and B_T in all the other points (x,y) are computed by bilinear interpolation considering the ensembles $A_T(x_k,y_k)$ and $B_T(x_k,y_k)$, respectively, while $Z_T(x,y)$ is computed as $Z_T(x_k,y_k)$: each point (x,y) is related to a radar cell RC_{ij} and then to a set of adjacent reflectivity cells N_z, therefore we have n_{zs} x N_z values of horizontal reflectivity data $Z_{m,j}(x,y)$ for each (x,y) point. So we have:

$$Z_T(x, y) = \frac{1}{n_{zs}} \frac{1}{N_z} \sum_{j=1}^{n_{zs}} \sum_{m=1}^{N_z} Z_{m,j}(x, y) \qquad (9)$$

If $\mathrm{Z}_{m,j}^{(k)}$ is replaced by $Z_{m,j}(x,y)$ in eq 4, eq. 8 gives the same result of eq. 9. Once A_T, B_T and Z_T are computed the rainfall rate R_T can be estimated by eq. 1.

Performances of the proposed method

The proposed procedure allows to estimate the rainfall rate at time interval T once defined the extension of the air volume to be correlated to the rain gauges, through the number of adjacent radar cells Nz, and the processing time T_w to obtain a number of pairs Z-R through which the coefficients $a_0^{(k)}$ and $b_0^{(k)}$ are computed in each rain gauge location.

The method does not use a priori parameters and the relationship between reflectivity and rainfall rate is variable in time and space. Such approach allows to account for the dynamics of the precipitation phenomenon. In fact, the use of rain gauge data and of radar reflectivity data simultaneously allows to modify in real time the relationship between the rainfall rate and the radar observation.

As described above, the coefficients $A_T(x_k,y_k)$ and $B_T(x_k,y_k)$, are determined by linear regression methods applied to the np pairs $(\mathrm{RW}_p^{(k)}, \mathrm{ZW}_p^{(k)})$. Each pair can be given a score in terms of reliability and uncertainty in order to quantify the level of the correlation between radar reflectivity and rain gauge rainfall rate. We assume that the less stationary and homogeneous the precipitation phenomenon, the lower the correlation between radar reflectivity and rain gauge rainfall estimates. Consequently, we assume that if the precipitation phenomenon is stationary and homogeneous during the processing time T_W it is highly probable than the values of $\mathrm{RW}_p^{(k)}$ and $\mathrm{ZW}_p^{(k)}$ are strongly correlated.

We quantify the level of reliability and uncertainty of the pair $(\mathrm{RW}_p^{(k)}, \mathrm{ZW}_p^{(k)})$ on the base of the level of the space time variability of the ensemble $\mathrm{Z}_{m,j}^{(k)}$ and $\mathrm{R}_i^{(k)}$. For this purpose we define 3 parameters of reliability: $\mathrm{RRt}_p^{(k)}$, $\mathrm{RZt}_p^{(k)}$ and $\mathrm{RZs}_p^{(k)}$.

$\mathrm{RRt}_p^{(k)}$ and $\mathrm{RZt}_p^{(k)}$ are the root mean square values of $\mathrm{R}_i^{(k)}$ and $\mathrm{ZS}_j^{(k)}$ with respect to the index i and j, respectively, and $\mathrm{RZs}_p^{(k)}$ is an “adapted” space root mean square value of $\mathrm{Z}_{m,j}^{(k)}$ with respect to the indexes m and j.

The condition of stationary phenomenon during the processing time in (x_k,y_k) is $\mathrm{RRt}_p^{(k)} = 0$ and $\mathrm{RZt}_p^{(k)} = 0$ while the condition of homogeneity is $\mathrm{RZs}_p^{(k)} = 0$.

Depending on the regression method used for the computation of $a_0^{(k)}$ and $b_0^{(k)}$, different global uncertainty indexes on both the $\mathrm{RW}_p^{(k)}$ and $\mathrm{ZW}_p^{(k)}$ dataset can be defined as function of the 3 parameters of reliability.

If the values $(\mathrm{RW}_p^{(k)}, \mathrm{ZW}_p^{(k)})$ are assumed without uncertainty for all $p=1\cdots np$, they are equally reliable and no uncertainty indexes are needed, therefore a standard linear regression method is sufficient.

If the values $\mathrm{RW}_p^{(k)}$ and $\mathrm{ZW}_p^{(k)}$ are assumed without and with uncertainty, respectively, for all $p=1\cdots np$, the uncertainty index $\sigma_{Zp}^{(k)} = f_z\left(\mathrm{RZt}_p^{(k)}, \mathrm{RZs}_p^{(k)}\right)$ is defined in order to account for the conditions of homogeneity and stationariety on the $\mathrm{ZW}_p^{(k)}$ dataset. Therefore, at least a standard least square method should be used.

If both the values $\mathrm{RW}_p^{(k)}$ and $\mathrm{ZW}_p^{(k)}$ are assumed with uncertainty for all $p=1\cdots np$, $\sigma_{Zp}^{(k)} = f_z\left(\mathrm{RZt}_p^{(k)}, \mathrm{RZs}_p^{(k)}\right)$ and $\sigma_{Rp}^{(k)} = f_R\left(\mathrm{RRt}_p^{(k)}\right)$ are defined in order to account for the conditions of homogeneity and stationary phenomenon on the $\mathrm{ZW}_p^{(k)}$ dataset and the stationary phenomenon on the $\mathrm{RW}_p^{(k)}$ dataset. In this case an effective variance method [8] should be used.

Fig. 6 shows an example of Z-R pairs with their uncertainty parameters $\sigma_{Zp}^{(k)}$ and $\sigma_{Rp}^{(k)}$ where f_z and f_r are assumed simply as follow:

$$f_z\left(\mathrm{RZt}_p^{(k)}, \mathrm{RZs}_p^{(k)}\right) = \frac{\mathrm{RZt}_p^{(k)} + \mathrm{RZs}_p^{(k)}}{2} \tag{10}$$

$$f_R\left(\mathrm{RRt}_p^{(k)}\right) = \mathrm{RRt}_p^{(k)} \tag{11}$$

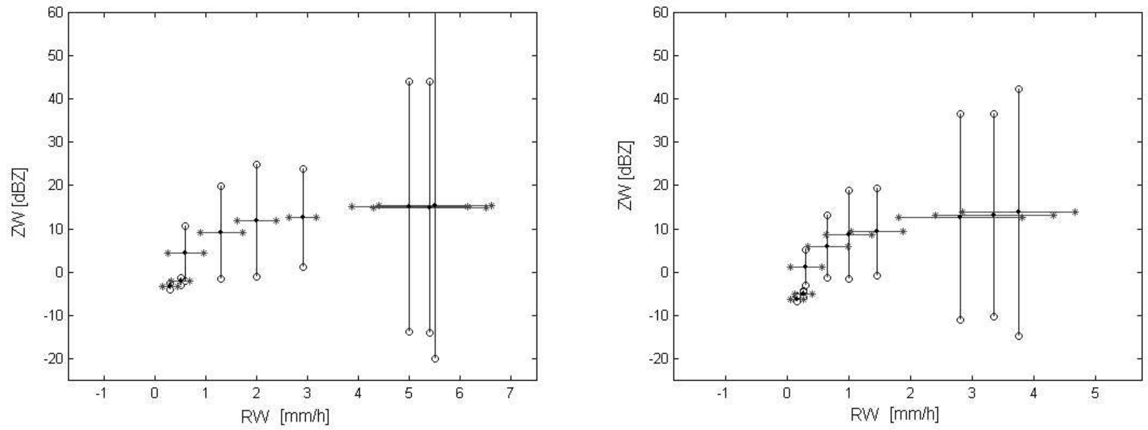

Fig. 6 – ZR pairs of Fig 5 with their uncertainty parameters $\sigma_{Zp}^{(k)}$ (vertical bars) and $\sigma_{Rp}^{(k)}$ (horizontal bars) computed as in eq 10 and 11.

Summarizing:

1. if the phenomenon is stationary and homogeneous then $\mathrm{RRt}_p^{(k)} = \mathrm{RZt}_p^{(k)} = \mathrm{RZs}_p^{(k)} = 0$ and a standard linear regression method is used.
2. if the phenomenon is stationary then $\mathrm{RRt}_p^{(k)} = 0$ and a standard least square method is used
3. in all other cases an effective variance method is used

The goodness of the final estimation of rainfall rate $R_T(x,y)$ by means of eq.1 can be evaluated through the comparison between $R_T(x_k, y_k)$ and $\mathrm{R}_T^{(k)}$ that is defined as:

$$\mathrm{R}_T^{(k)} = \frac{1}{n_{rs}} \sum_{i=1}^{n_{rs}} \mathrm{R}_i^{(k)} \tag{12}$$

Since $\mathrm{R}_T^{(k)}$ is the “true” rainfall rate as well as is measured by the rain gauge k during the observation time T and $R_T(x_k, y_k)$ is an estimate of $\mathrm{R}_T^{(k)}$, the comparison between them allows to quantify the estimation error in the rain gauge location k. Therefore we define the relative error $\varepsilon_T^{(k)}$ as:

$$\varepsilon_T^{(k)} = \frac{R_T(x_k, y_k) - R_T^{(k)}}{R_T^{(k)}} \tag{13}$$

At this point we associate the error $\varepsilon_T^{(k)}$ to the global uncertainty index $\sigma_{ZT}^{(k)}$ defined as arithmetical mean of the uncertainty indexes $\sigma_{Zp}^{(k)}$ that depend on the parameters of reliability, $\mathrm{RZt}_p^{(k)}$ and $\mathrm{RZs}_p^{(k)}$:

$$\sigma_{ZT}^{(k)} = \frac{1}{np}\sum_{p=1}^{np}\sigma_{Zp}^{(k)} = \frac{1}{np}\sum_{p=1}^{np} f_z\left(\mathrm{RZt}_p^{(k)}, \mathrm{RZs}_p^{(k)}\right) \tag{14}$$

Therefore we have a set of pairs ($\varepsilon_T^{(k)}$, $\sigma_{ZT}^{(k)}$) through which we make a relationship between the error on the rainfall estimate and the data radar. With such association we assume that the error on the rain fall estimate using eq. 1 is only related to parameters of reliability computed on the horizontal reflectivity data.

Such assumption allows to compute a global uncertainty index $\sigma_{ZT}(x, y)$ in all the points (*x*,*y*) independently of the presence of rain gauges. $\sigma_{ZT}(x, y)$ can be computed in the same manner as made in eq. 12:

$$\sigma_{ZT}(x, y) = \frac{1}{n_{zs}}\sum_{j=1}^{n_{zs}} f_z\left(\mathrm{RZt}(x, y), \mathrm{RZs}(x, y)\right) \tag{15}$$

Therefore, we can provide an estimate of the error $\hat{\varepsilon}_T(x, y)$ in all the points (*x*,*y*) on the base of:

1. the association between the errors made using eq. 1 in the computation of the rain fall rate in all the rain gauge locations and the global uncertainty index $\sigma_{ZT}^{(k)}$
2. the global uncertainty index $\sigma_{ZT}(x, y)$

an expression for $\hat{\varepsilon}_T(x, y)$ is:

$$\hat{\varepsilon}_T(x, y) = f_{RZT}\left(\sigma_{ZT}^{(k)}, \varepsilon_{ZT}^{(k)}, \sigma_{ZT}(x, y)\right) \tag{16}$$

Conclusions

The main result of this work is the description of a method to estimate the area rainfall rate by means of radar and rain gauge data. Moreover, a procedure to quantify the goodness of the final estimate is proposed. Such procedure is based on a set of two parameters $\varepsilon_T^{(k)}$ and $\sigma_{ZT}^{(k)}$ that we assumed depending on the functions f_Z and f_{RZT}. Here, such functions are not given in explicit form (except for f_Z in eq. 10 as uncertainty example), since they must account for the best correlation between the reliability parameters on the radar data and the errors on the rainfall estimate. Therefore the next step of this work will be the implementation of a software for the automatic analysis of real radar and rain gauge data in order to extract the right parameters for the relationships between the reliability parameters and the errors on the rainfall rate estimate.

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