Università degli Studi di Firenze

Facoltà d' Ingegneria
Dipartimento di Elettronica e Telecomunicazioni

Radar penetranti

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Radar penetranti

SPR (Surface Penetrating Radar)

GPR (Ground Penetrating Radar)

GeoRADAR

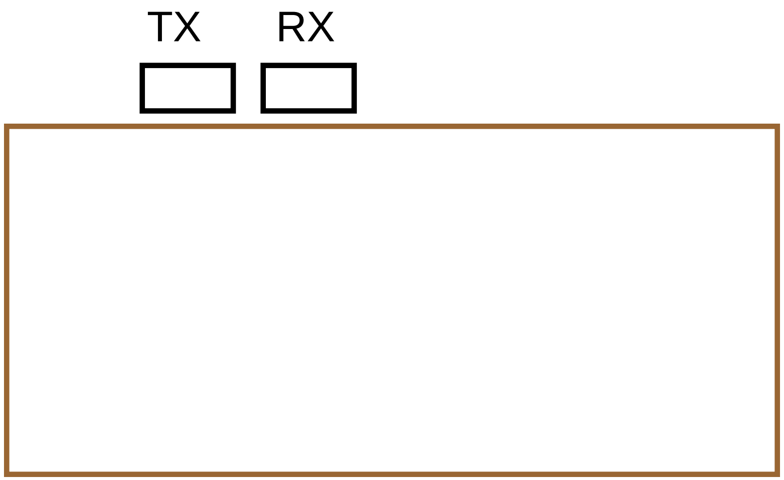
Radar penetranti

I radar penetranti impiegano onde e.m.
per sondare
materiali dielettrici fortemente attenuanti (lossy)

Frequenze impiegate: 1 MHz – 2GHz

Radar penetranti

Accessibilità solo da un lato



Configurazione monostatica

Configurazione bistatica

Equazioni di Maxwell

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \nabla \times \underline{E} = -\frac{\partial}{\partial t}\left(\nabla \times \underline{B}\right)$$

$$\nabla \times \underline{B} = \mu \underline{J} + \varepsilon_0 \mu \frac{\partial \underline{E}}{\partial t}$$

$$\nabla \times \nabla \times \underline{E} = -\frac{\partial}{\partial t}\left(\mu \underline{J} + \varepsilon_0 \mu \frac{\partial \underline{E}}{\partial t}\right)$$

$$\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \nabla \times \underline{E} + \varepsilon_0 \mu \frac{\partial^2 \underline{E}}{\partial t^2} + \mu \frac{\partial \underline{J}}{\partial t} = 0$$

Equazione d'onda

Corrente nel mezzo

Correnti

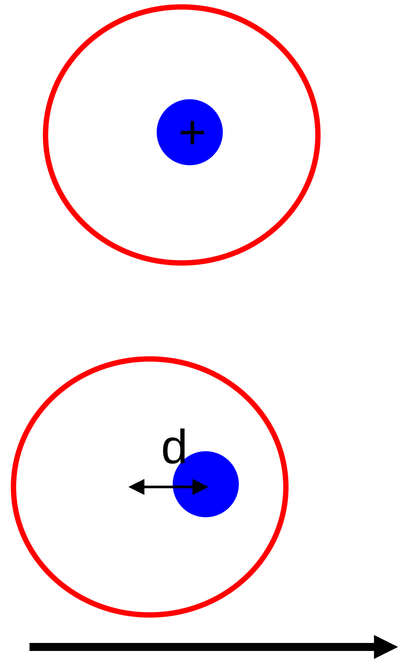
$$\boxed{J = J_{free} + J_{pol}}$$

Correnti ohmiche

$$J_{free} = \sigma' E$$

Correnti

$$J = J_{free} + J_{pol}$$



$$D = qd$$

$$\frac{\partial D}{\partial t} = q\frac{\partial d}{\partial t} = qv = J_{pol}$$

$$D = \chi' E$$

$$J_{pol} = \frac{\partial D}{\partial t} = \chi' \frac{\partial E}{\partial t}$$

Equazioni di Maxwell

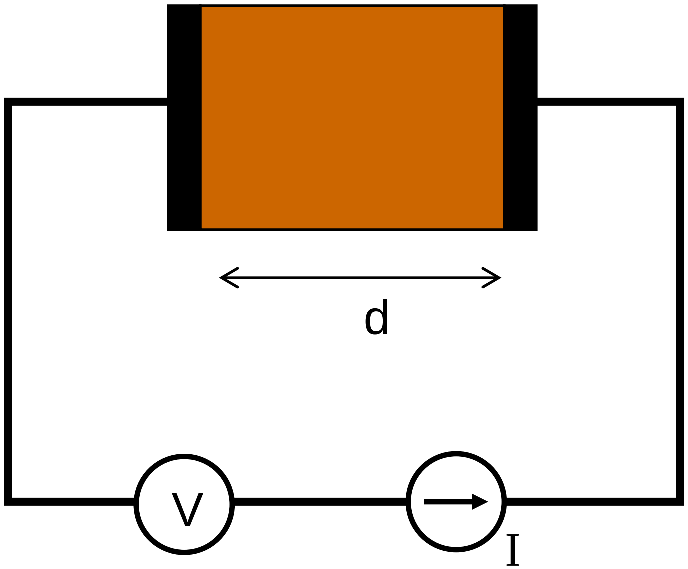
$$J = J_{free} + J_{pol} = \sigma' E + \chi' \frac{\partial E}{\partial t}$$

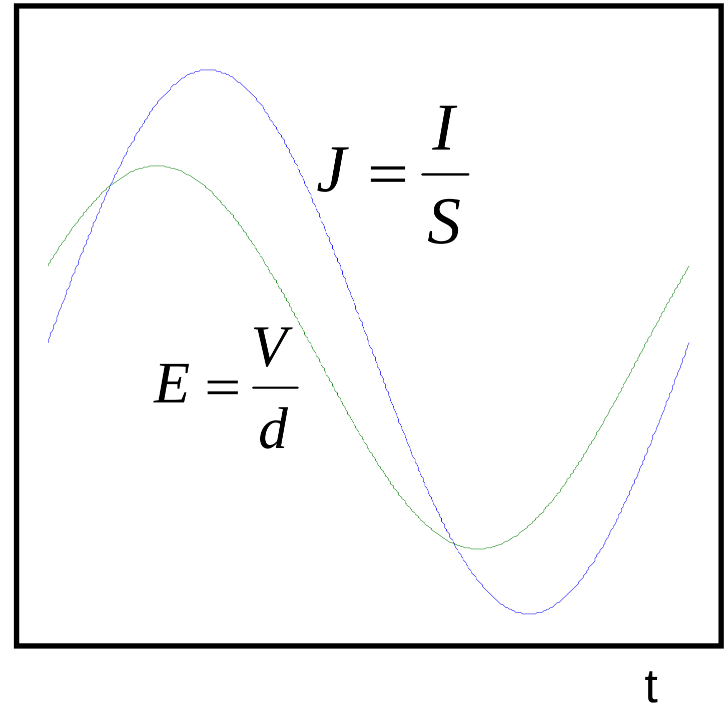
Per una sinusoide:

$$J = \sigma' E + \chi' j\omega E = \sigma E$$

complesso

La corrente in un mezzo dispersivo





$$J = \sigma E \qquad \sigma = \sigma' - j\sigma''$$

Definiamo:

$$\chi = \frac{\sigma}{j\omega\varepsilon_0} \longrightarrow J = \sigma E = j\omega\varepsilon_0 \chi E = \varepsilon_0 \chi \frac{\partial E}{\partial t}$$

$$\nabla \times \nabla \times \underline{E} + \varepsilon_0 \mu \frac{\partial^2 \underline{E}}{\partial t^2} + \mu \frac{\partial \underline{J}}{\partial t} = 0 \qquad J = \varepsilon_0 \chi \frac{\partial E}{\partial t}$$

$$\varepsilon = \varepsilon_0 (1 + \chi)$$

$$\boxed{\nabla \times \nabla \times \underline{E} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0}$$

$$\varepsilon = \varepsilon_0 (1 + \chi) = \varepsilon_0 \left(1 + \frac{\sigma}{j \omega \varepsilon_0} \right) = \varepsilon_0 + \frac{\sigma' + j \sigma''}{j \omega} = \left(\varepsilon_0 + \frac{\sigma''}{\omega} \right) - j \left(\frac{\sigma'}{\omega} \right)$$

$$\varepsilon = \varepsilon' - j \varepsilon''$$

$$\boxed{\begin{aligned} \varepsilon' &= \varepsilon_0 + \frac{\sigma''}{\omega} \\ \varepsilon'' &= \frac{\sigma'}{\omega} \end{aligned}}$$

Equazioni di Maxwell

$$\nabla \times \nabla \times \underline{E} + \varepsilon\mu \frac{\partial^2 \underline{E}}{\partial t^2} = 0$$

$$\frac{\partial^2 E}{\partial x} + \varepsilon\mu \frac{\partial^2 E}{\partial t^2} = 0$$

$$\boxed{E(x,t) = E(x) e^{j\omega t}}$$

$$\frac{\partial^2 E(x)}{\partial x} e^{j\omega t} + \varepsilon\mu (j\omega)^2 E(x) e^{j\omega t} = 0$$

$$\frac{\partial^2 E(x)}{\partial x} - \varepsilon\mu\omega^2 E(x) = 0$$

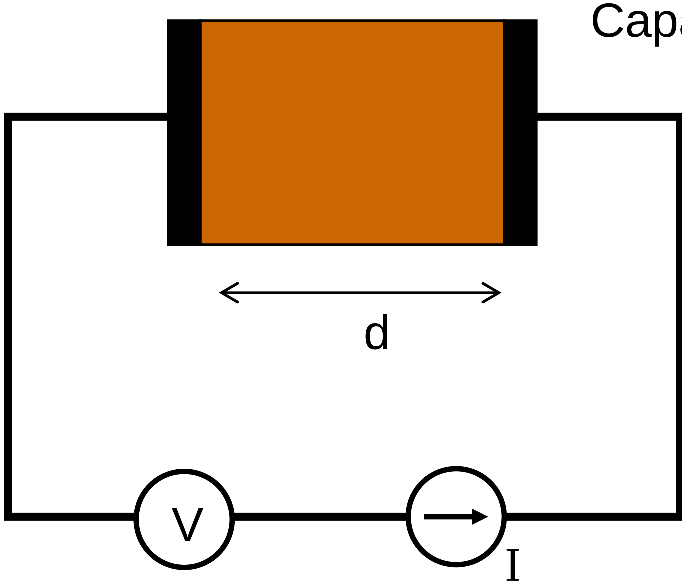
$$k = \omega\sqrt{\mu\varepsilon}$$

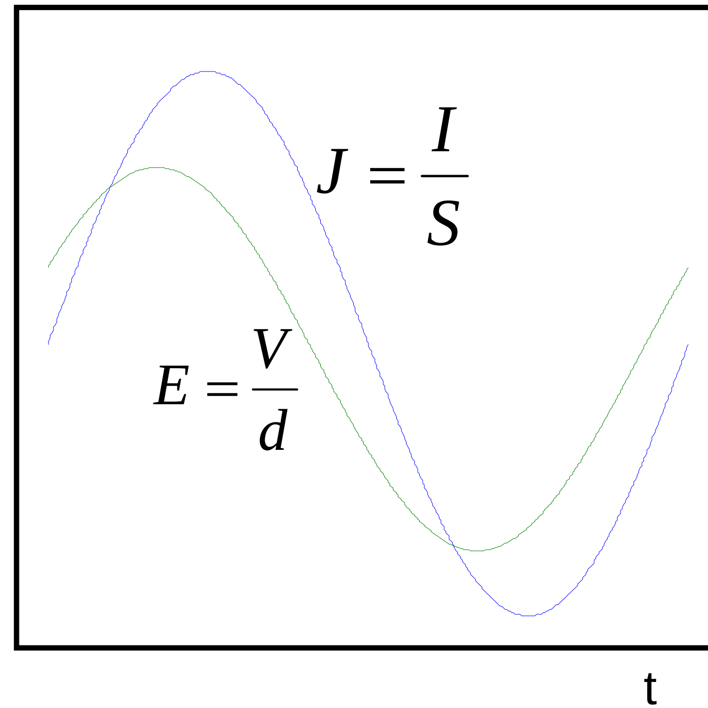
$$\frac{\partial^2 E(x)}{\partial x} - k^2 E(x) = 0 \longrightarrow E(x) = E_0 e^{\pm jkx} \longrightarrow E(x) = E_0 e^{-jkx}$$

$$\boxed{E(x,t) = E_0 e^{j(\omega t - kx)}}$$

Mezzi dispersivi

Capacità?





$$J = \sigma E$$

Conducibilità complessa $\sigma = \sigma' - j\sigma''$

Permittività complessa $\varepsilon = \varepsilon' - j\varepsilon''$

$$\varepsilon' = \varepsilon_0 + \frac{\sigma''}{\omega}$$

$$\varepsilon'' = \frac{\sigma'}{\omega}$$

Onda e.m. in un mezzo dispersivo

$$\boxed{E(x,t) = E_0 e^{j(\omega t - kx)} \qquad k = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')}}$$

$$jk = \alpha + j\beta \qquad E(x,t) = E_0 e^{j(\omega t - \beta x)} e^{-\alpha x}$$

$$\frac{\varepsilon''}{\varepsilon'} = \gamma$$

$$\beta = \omega\left[\frac{\mu\varepsilon'}{2}\left(\sqrt{1+\gamma^2}+1\right)\right]^{\frac{1}{2}}$$

$$\alpha = \omega\left[\frac{\mu\varepsilon'}{2}\left(\sqrt{1+\gamma^2}-1\right)\right]^{\frac{1}{2}}$$

Onda e.m. in un mezzo dispersivo

$$\boxed{e^{-j\beta x}}$$

$$\gamma = \frac{\varepsilon''}{\varepsilon'} << 1$$

$$\beta = \omega\left[\frac{\mu\varepsilon'}{2}\left(\sqrt{1+\gamma^2}+1\right)\right]^{\frac{1}{2}} = \omega\left[\frac{\mu\varepsilon'}{2}2\right]^{\frac{1}{2}} = \omega\sqrt{\mu\varepsilon'}$$

$$\varepsilon_r = \frac{\varepsilon'}{\varepsilon_0} \qquad c = \frac{1}{\sqrt{\mu\varepsilon_0}}$$

$$\beta = \omega\frac{\sqrt{\varepsilon_r}}{c} = \frac{\omega}{v}$$

$$\boxed{v = \frac{c}{\sqrt{\varepsilon_r}}}$$

Onda e.m. in un mezzo dispersivo

$$\boxed{e^{-\alpha x}}$$

$$\gamma = \frac{\varepsilon''}{\varepsilon'} << 1$$

$$\alpha = \omega\left[\frac{\mu\varepsilon'}{2}\left(\sqrt{1+\gamma^2}-1\right)\right]^{\frac{1}{2}} = \omega\frac{\sqrt{\mu\varepsilon'}}{\sqrt{2}}\left(1+\frac{1}{2}\gamma^2-1\right)^{\frac{1}{2}} = \frac{\omega\sqrt{\mu\varepsilon'}}{2}\gamma$$

$$\boxed{\alpha = \frac{\omega}{2v}\gamma}$$

Onda e.m. in un mezzo dispersivo

$$E(x,t) = E_0 e^{j\omega\left(t - \frac{1}{v}x\right)} e^{-\frac{\omega\gamma}{2v}x} \qquad v = \frac{c}{\sqrt{\varepsilon_r}} \qquad \gamma = \frac{\varepsilon''}{\varepsilon'}$$

La tangente di perdita

$$\omega\varepsilon_0 << \sigma''$$

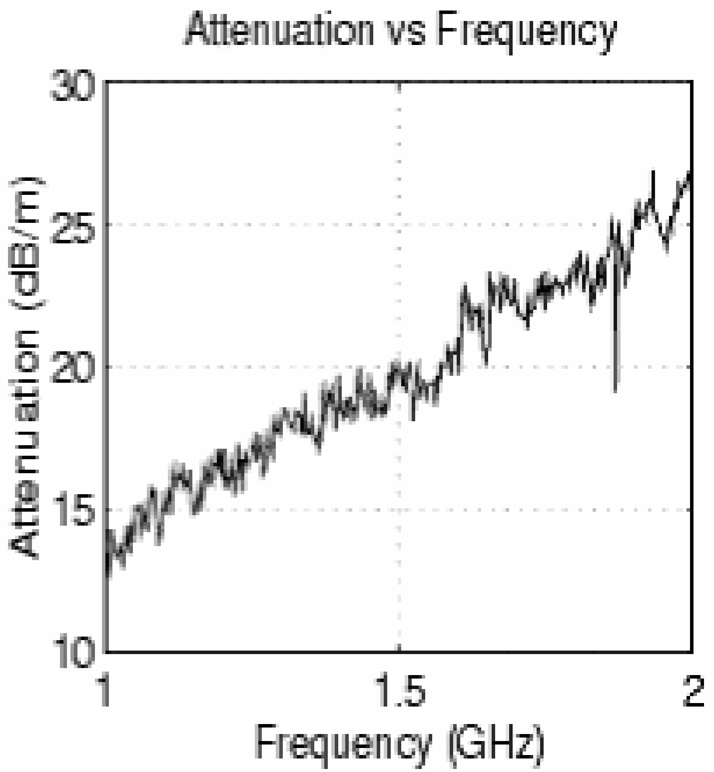
$$\gamma = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma'}{\omega\left(\varepsilon_0 + \frac{\sigma''}{\omega}\right)} \approx \frac{\sigma'}{\sigma''} = \tan\delta$$

Costante
[100MHz-2GHz]

$$\alpha = \frac{\omega\gamma}{2v} = \frac{\omega}{2v}\tan\delta$$

$$\alpha = \frac{\pi f}{v}\tan\delta$$

$$\tan\delta = \frac{1}{Q}$$



$$\boxed{\alpha = \frac{\pi f}{Qv}}$$

Velocità di propagazione e attenuazione

Velocità di propagazione

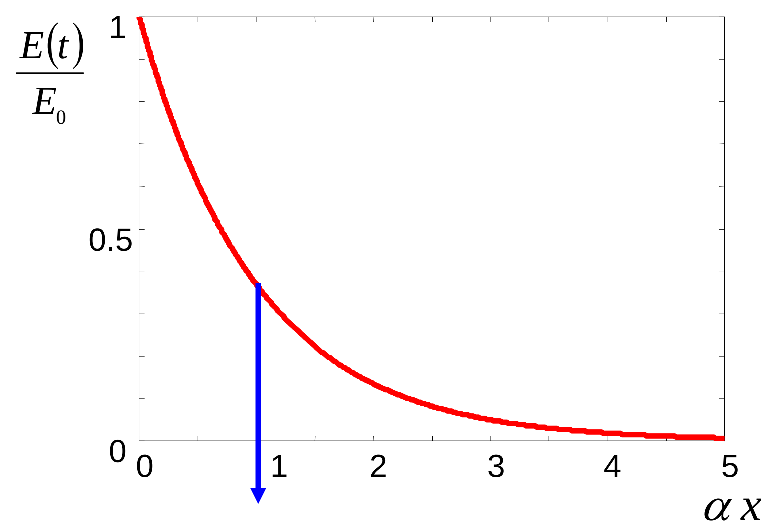
$$v = \frac{c}{\sqrt{\varepsilon_r}} \qquad \varepsilon_r = \frac{\varepsilon'}{\varepsilon_0} \qquad \varepsilon_r > 1 \longrightarrow$$

Correnti di polarizzazione

Attenuazione

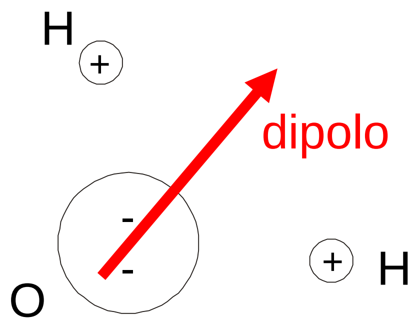
$$\alpha = \frac{\mu v \sigma'}{2} \qquad \sigma' \longrightarrow$$

Correnti di conduzione



$$x = \frac{1}{\alpha}$$

Acqua



Molecola di acqua

Elevata suscettività → $\varepsilon_r \cong 80$ Correnti di polarizzazione

Molecola polare → Dissolve ioni → σ Correnti di conduzione

Materiali

	ε_r	σ'
		mS/m
acqua dolce	80	0.5
acqua di mare	80	3000
sabbia asciutta	3	0.01
sabbia bagnata (satura)	20-30	0.1-1
calcare	4-8	0.5-2
argilla	5-40	2-1000
granito	4-6	0.01-1
ghiaccio	3-4	0.01

$$\varepsilon_r = \frac{\varepsilon'}{\varepsilon_0}$$

$$\varepsilon' = \varepsilon_0 + \frac{\sigma''}{\omega}$$

$$\varepsilon'' = \frac{\sigma'}{\omega}$$

Dispersione

$$v = \frac{c}{\sqrt{\varepsilon_r(\omega)}}$$

Impulso

Δt = durata

ω_0 = frequenza portante

Banda

$$B \approx \frac{1}{\Delta t}$$

ω_0 = frequenza portante

Componenti spettrali a frequenze diverse si propagano con velocità diversa

L'impulso si allarga

Dispersione

$$\delta t = \frac{L}{v_2} - \frac{L}{v_1} \quad = \frac{L(v_1 - v_2)}{v_1 v_2} \approx -\frac{L\Delta v}{v^2}$$

$$v = \frac{c}{\sqrt{\varepsilon_r}}$$

$$\Delta v = \frac{\partial v}{\partial \varepsilon_r}\Delta\varepsilon_r = c\left(-\frac{1}{2}\right)\varepsilon_r^{-\frac{3}{2}}\Delta\varepsilon = -\frac{c}{2}\frac{1}{\varepsilon_r\sqrt{\varepsilon_r}}\Delta\varepsilon_r = -\frac{v}{2\varepsilon_r}\Delta\varepsilon_r$$

$$\delta t = \frac{L}{2v\varepsilon_r}\Delta\varepsilon_r$$

$$\boxed{\delta t = \frac{L}{2v\varepsilon}\frac{\partial \varepsilon}{\partial f}B}$$

1) Discontinuità localizzate

Teoria di Mie

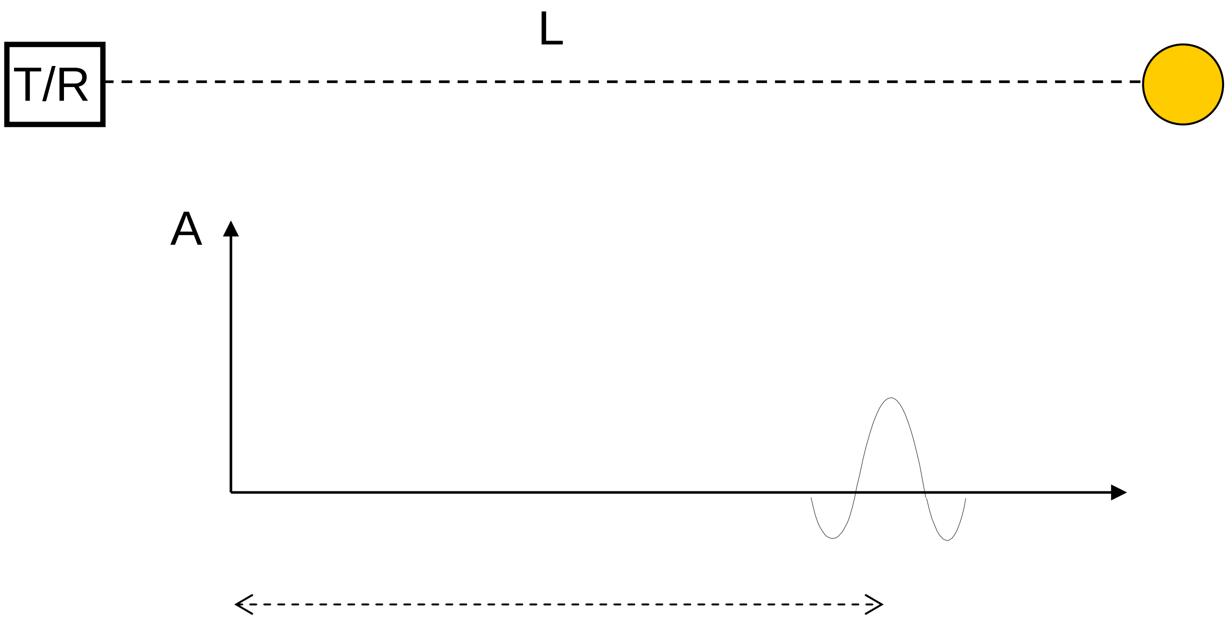
$$\boxed{P_{scatt} = I\sigma_{RCS}}$$

2) Superfici di discontinuità

Incidenza perpendicolare

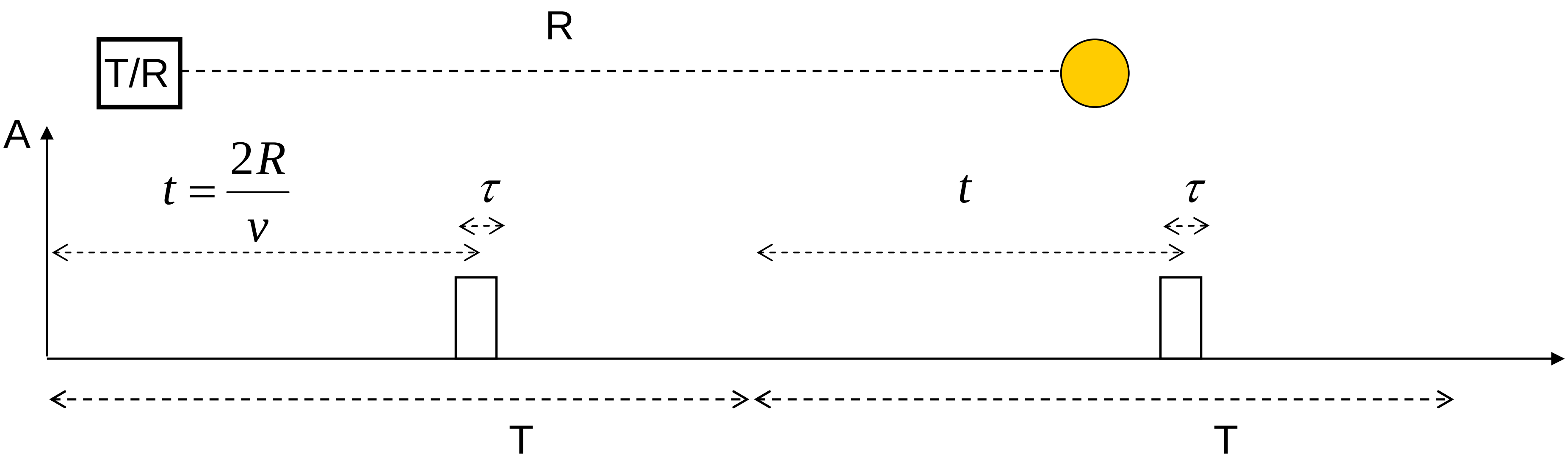
$$R = \frac{\sqrt{(\varepsilon_r)_2} - \sqrt{(\varepsilon_r)_1}}{\sqrt{(\varepsilon_r)_1} + \sqrt{(\varepsilon_r)_2}}$$

Risposta radar



$$t = \frac{2L}{v}$$

Radar ad impulsi



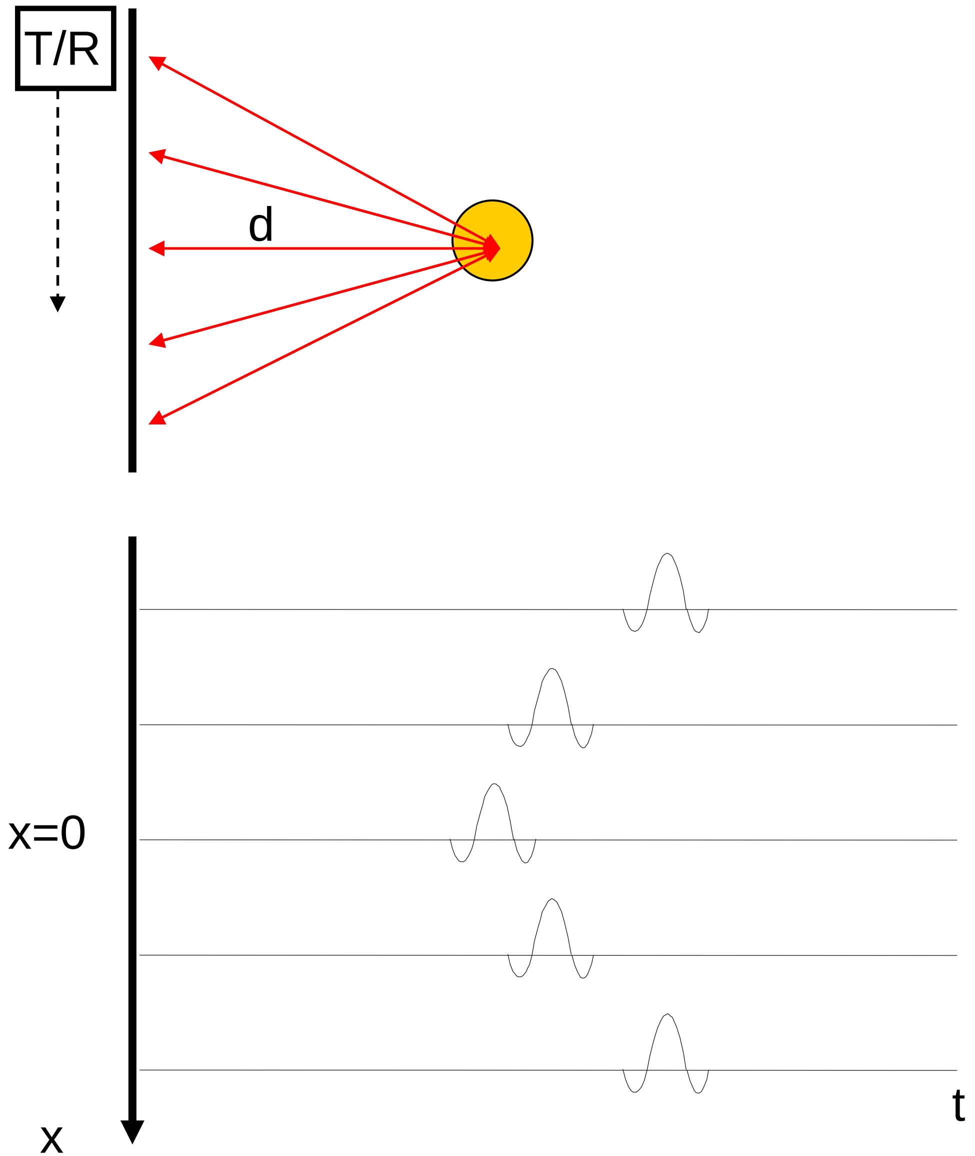
Distanza target: $R = \frac{vt}{2}$

Risoluzione in range: $\Delta R = \frac{v\tau}{2}$

Range non ambiguo: $R_U = \frac{vT}{2}$

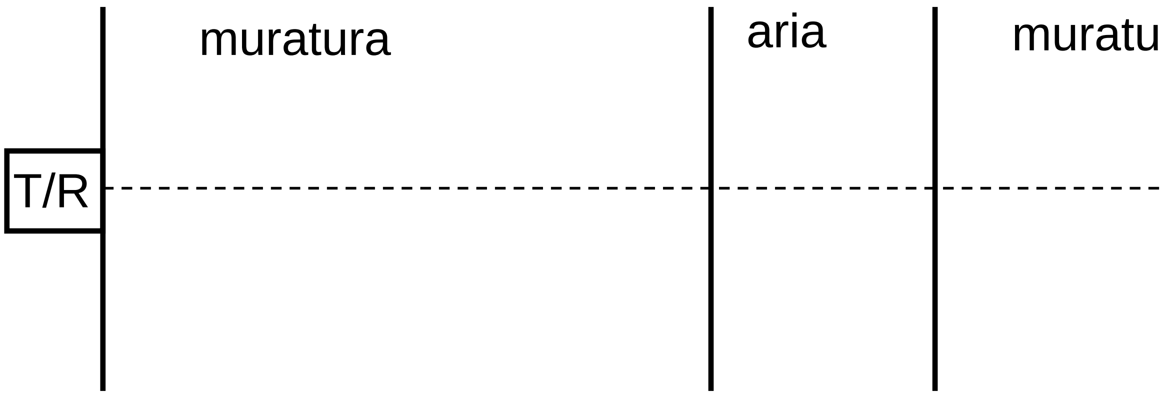
$$\boxed{B = \frac{1}{\tau}}$$

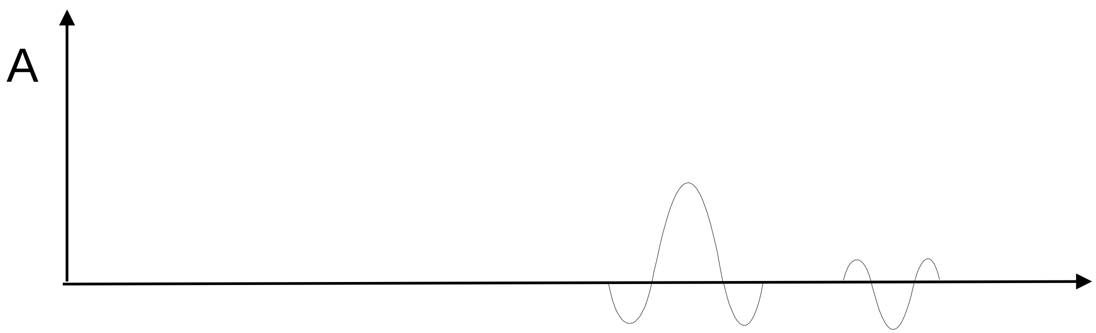
Risposta radar



$$t = \frac{2\sqrt{x^2 + d^2}}{v}$$

Polarità





Incidenza perpendicolare

$$R = \frac{\sqrt{(\varepsilon_r)_2} - \sqrt{(\varepsilon_r)_1}}{\sqrt{(\varepsilon_r)_1} + \sqrt{(\varepsilon_r)_2}}$$

Risoluzione in range

$$\Delta t = \frac{1}{B}$$

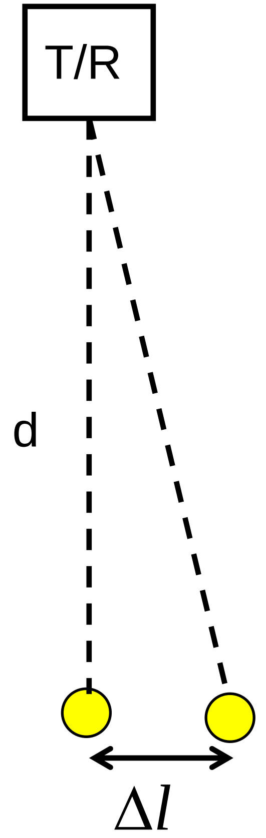
$$\Delta R = \frac{v}{2}\Delta t$$

Bassa velocità di propagazione

Migliore risoluzione

$$\boxed{\Delta R = \frac{v}{2B}}$$

Risoluzione laterale (cross-range)



$$t_1 = \frac{2d}{v}$$

$$t_2 = \frac{2\sqrt{d^2 + (\Delta l)^2}}{v}$$

$$\Delta t = t_2 - t_1 = \frac{2}{v}\left(\sqrt{d^2 + (\Delta l)^2} - d\right)$$

$$\approx \frac{2}{v}\left[d\left(1 + \frac{1}{2}\left(\frac{\Delta l}{d}\right)^2\right) - d\right] = \frac{(\Delta l)^2}{vd}$$

$$\Delta t = \frac{1}{B} = \frac{(\Delta l)^2}{vd}$$

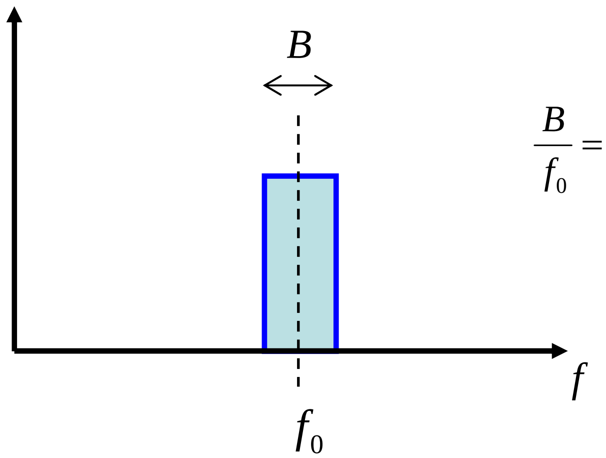
$$\boxed{\Delta l = \sqrt{\frac{vd}{B}}}$$

Risoluzione e penetrazione

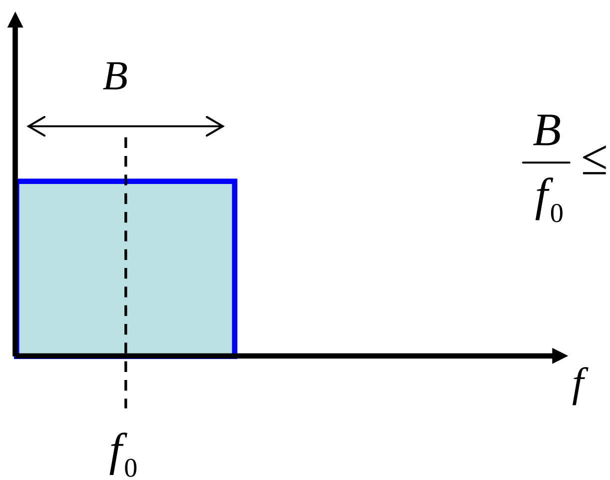
Risoluzione ⇨ **Larga banda**

Penetrazione ⇨ **Bassa frequenza**

Banda frazionaria



$$\frac{B}{f_0} = \text{ Banda frazionaria}$$



$$\frac{B}{f_0} \leq 2$$

Equazione radar

Intensità (W/m^2)
incidente sull'antenna

$$P_R = \frac{P_T}{4\pi R^2} G_T e^{-2\alpha R} \sigma_{RCS} \frac{1}{4\pi R^2} e^{-2\alpha R} A_R$$

$$A_R = \frac{\lambda^2}{4\pi} G_R$$

Intensità (W/m^2)
incidente sul bersaglio

Equazione radar

$$\boxed{P_R = P_T G_T G_R \frac{\lambda^2 \sigma_{RCS} e^{-4\alpha R}}{(4\pi)^3 R^4}}$$

Profondità di penetrazione

$$P_{R\min} = P_{T\max} G_T G_R \frac{\lambda^2 \sigma_{RCS} e^{-4\alpha R_{\max}}}{(4\pi)^3 R_{\max}^4}$$

ambiente + target

strumentale

$$\frac{P_T}{P_R} G_T G_R = \frac{(4\pi)^3 R_{\max}^4}{\lambda^2 \sigma_{RCS}} e^{4\alpha R_{\max}}$$

$$(SP) G_{TX} G_{RX} = L(R_{\max})$$

All'aumentare della frequenza, aumenta linearmente α quindi deve diminuire R_{max} mantenere L costante

Esempio:
SP=80dB
G_{TX}=G_{RX}=10dB

→ L=100dB

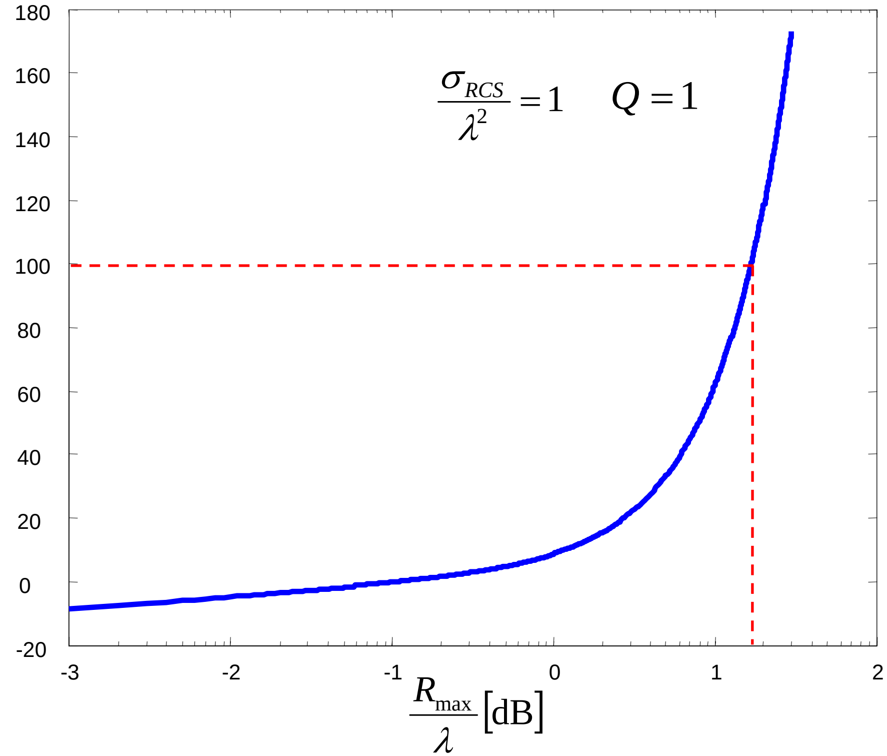
Profondità di penetrazione

$$(SP)G_{TX}G_{RX} = L(R_{\max})$$

$$\alpha = \frac{\pi f}{Qv}$$

$$L(R_{\max}) = \frac{(4\pi)^3 R_{\max}^4}{\lambda^2 \sigma_{RCS}} e^{4\alpha R} = \frac{(4\pi)^3 \left(\frac{R_{\max}}{\lambda}\right)^4}{\left(\frac{\sigma_{RCS}}{\lambda^2}\right)} e^{4\frac{\pi}{Q}\left(\frac{R_{\max}}{\lambda}\right)}$$

$$SPG_{TX}G_{RX} = L \quad [\text{dB}]$$



Risoluzione in range

$$\boxed{\Delta R = \frac{v}{2B}}$$

$$\eta = \frac{\pi R}{Qv}$$

$$B = \frac{\int_0^\infty A(f)\,df}{A_{\max}} \qquad A(f) = e^{-\frac{\pi f}{Qv}R} = e^{-\eta f}$$

$$B = \frac{\int_{f_c-\frac{B_0}{2}}^{f_c+\frac{B_0}{2}} e^{-\eta f}\,df}{e^{-\eta f_{\min}}} = e^{\eta f_{\min}} \frac{-1}{\eta}\left[e^{-\eta\left(f_c+\frac{B_0}{2}\right)} - e^{-\eta\left(f_c-\frac{B_0}{2}\right)} \right] = e^{\eta f_{\min}} \frac{1}{\delta} e^{-\eta f_c}\left[e^{\eta\frac{B_0}{2}} - e^{-\eta\frac{B_0}{2}} \right]$$

$$B = e^{\eta f_{\min}} \frac{1}{\eta} e^{-\eta f_c}\, 2\sinh\left(\frac{\eta B_0}{2}\right) = \frac{2}{\eta} e^{-\eta(f_c - f_{\min})} \sinh\left(\frac{\eta B_0}{2}\right) = \frac{2}{\eta} e^{-\frac{\eta B_0}{2}} \sinh\left(\frac{\eta B_0}{2}\right)$$

$$B = \frac{2}{\eta} e^{-\frac{\eta B_0}{2}} \sinh\left(\frac{\eta B_0}{2}\right) \approx \frac{2}{\eta}\left(1 - \frac{\eta B_0}{2}\right)\left(\frac{\eta B_0}{2}\right) = B_0 - \frac{\eta^2 B_0^2}{2}$$

Risoluzione in range

$$\boxed{\Delta R = \frac{v}{2B}}$$

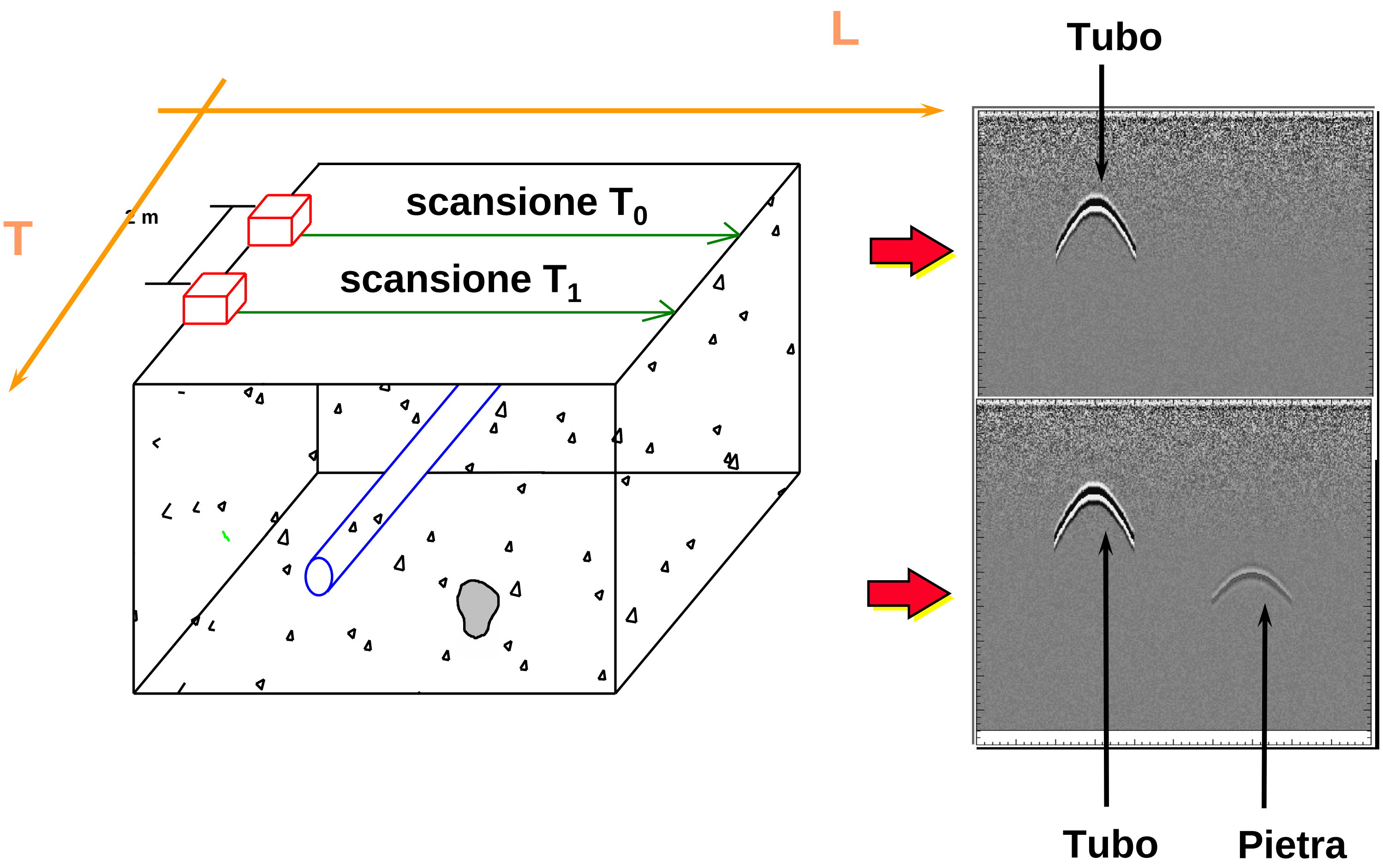
$$B = B_0 - \frac{\eta^2 B_0^2}{2} \qquad \eta = \frac{\pi R}{Qv}$$

$$\Delta R = \frac{v}{2B_0} = \frac{v}{2\left(B_0 - \frac{\eta B_0^2}{2}\right)} = \frac{v}{2B_0}\frac{1}{1-\frac{\eta B}{2}} = \frac{v}{2B_0}\left(1+\frac{\eta B_0}{2}\right)$$

$$\Delta R = \Delta R_0\left(1+\left(\frac{\pi R}{Qv}\right)\frac{B}{2}\right) = \Delta R_0 + \frac{v}{2B_0}\frac{\pi R}{Qv}\frac{B_0}{2}$$

$$\boxed{\Delta R = \Delta R_0 + \frac{\pi}{4}\frac{1}{Q}R}$$

Modalità operativa



Radar penetranti

Antenne

Elettronica

Elaborazione

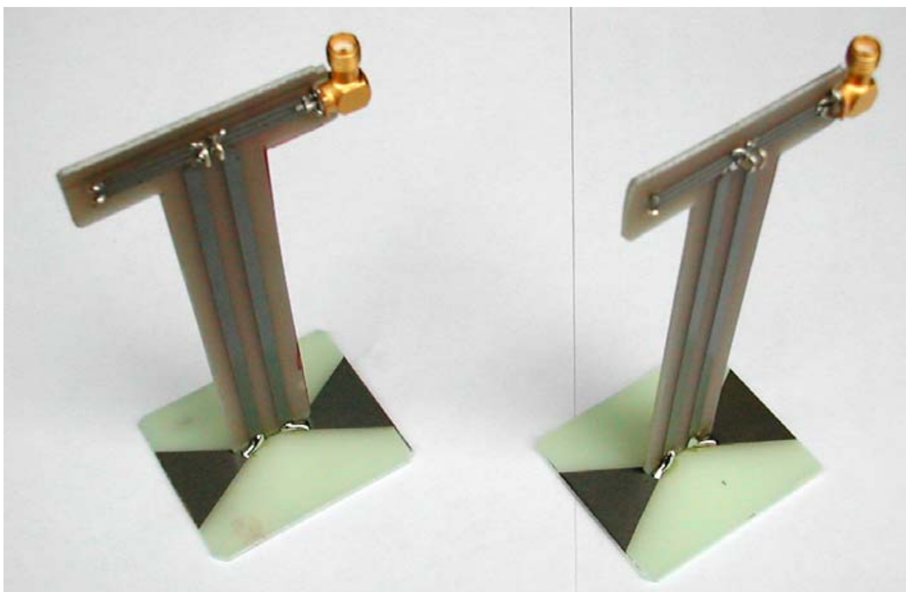
- on line
- post-processing

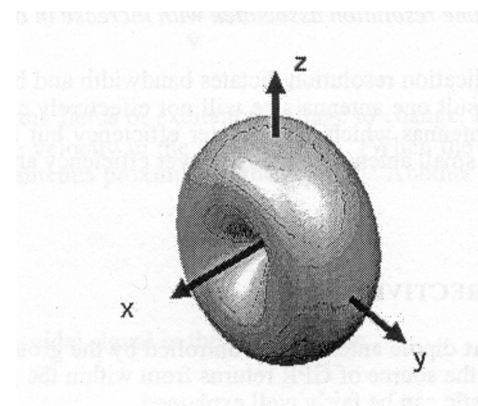
Antenne

Antenne

Larga Banda $\Leftrightarrow$ Frequenza bassa

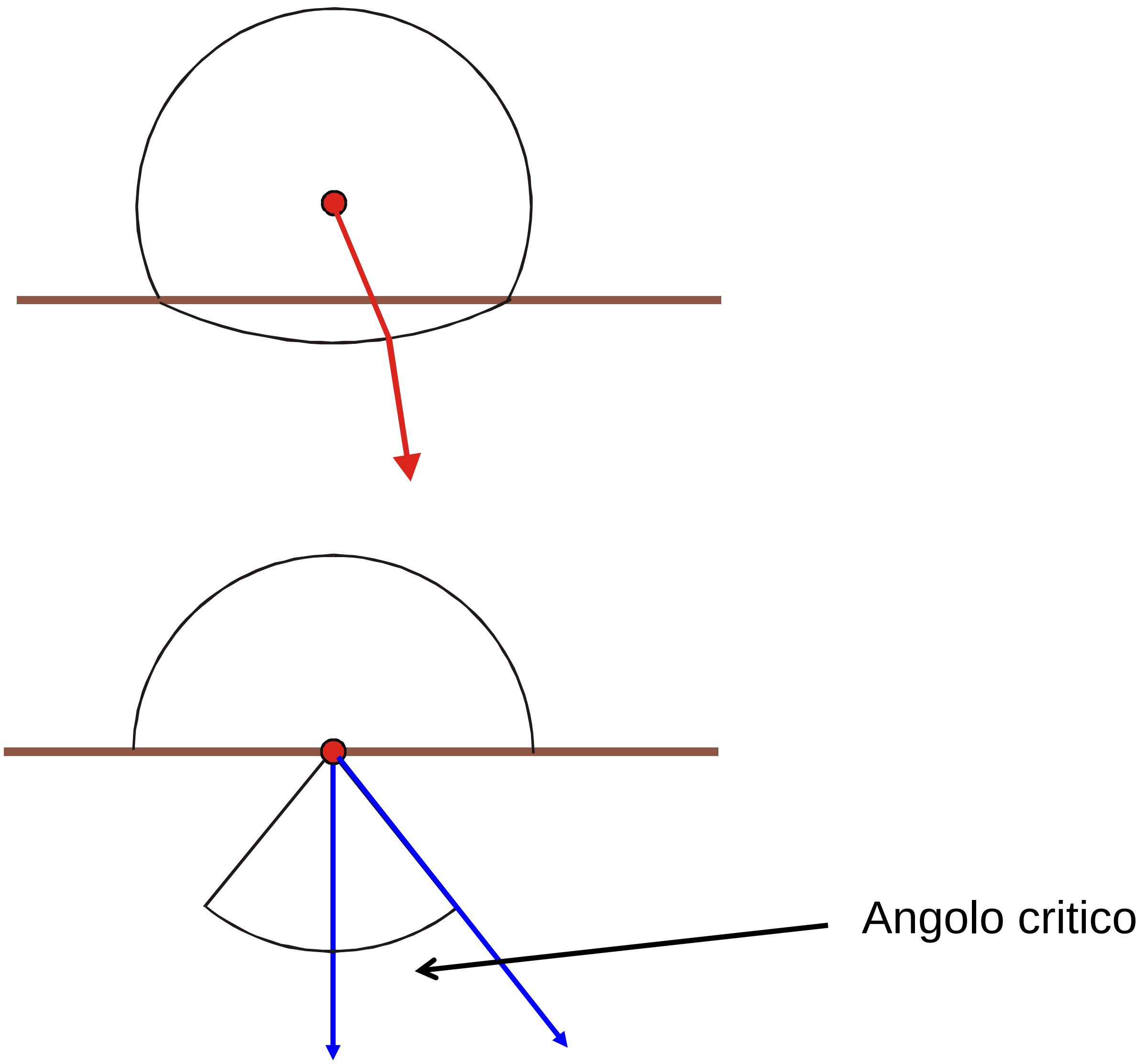
Bow-tie



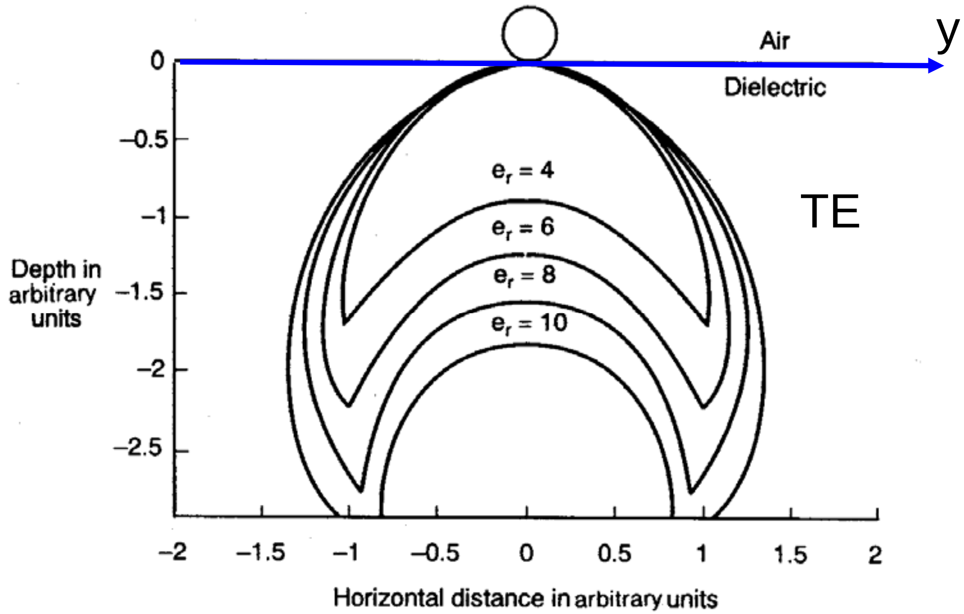


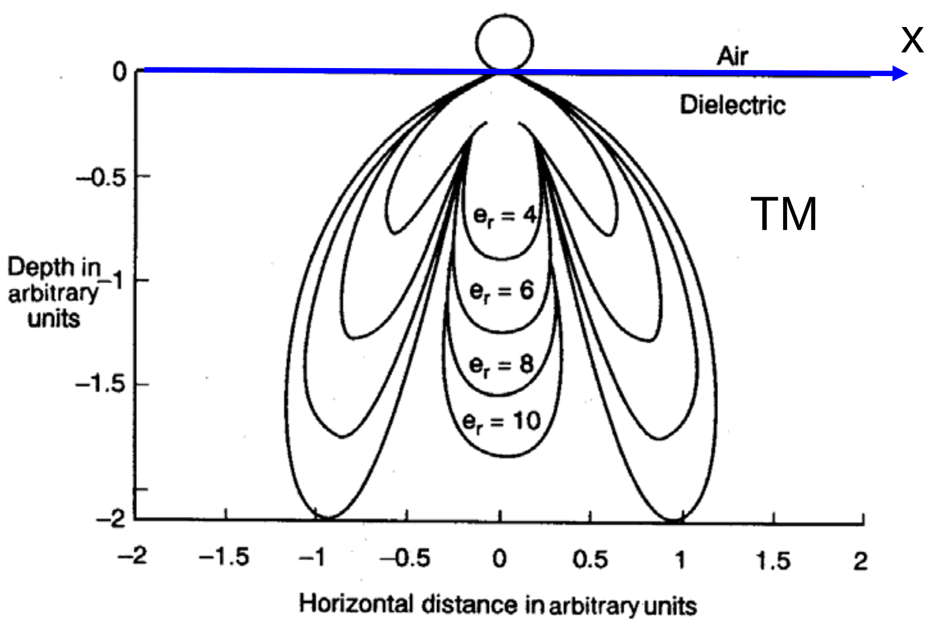
Schermatura

Antenne



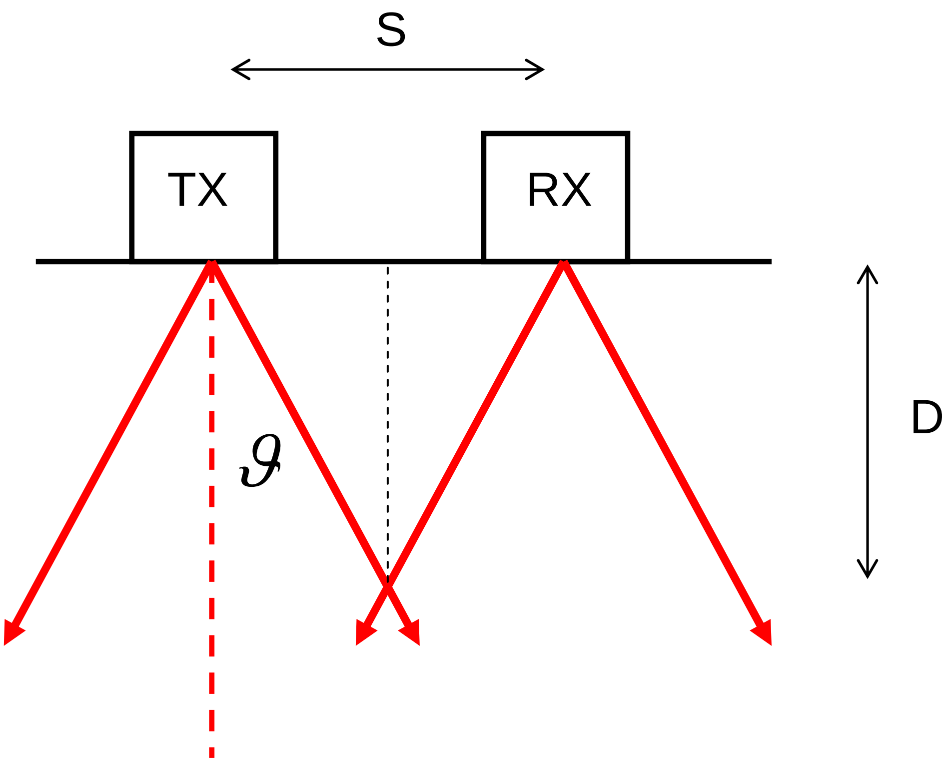
Antenne





(x asse del dipolo)

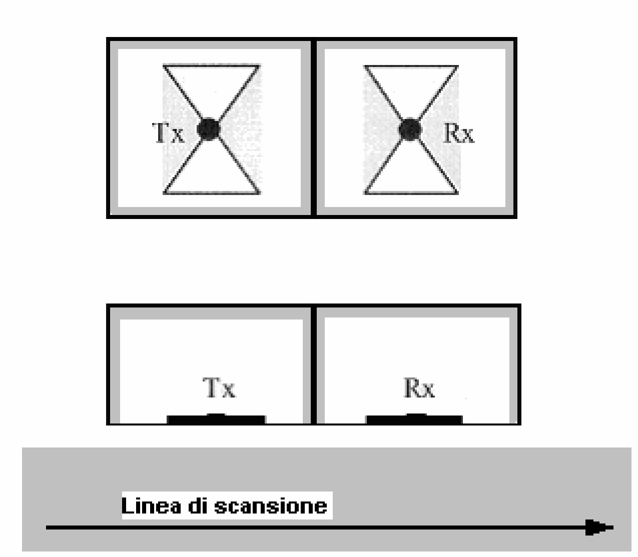
Antenne bow-tie

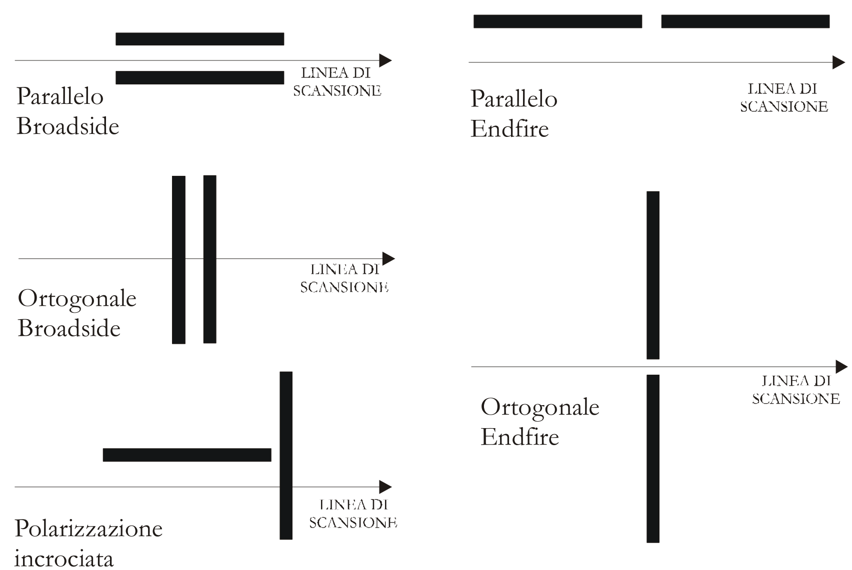


$$\sin(\vartheta) = \frac{1}{\sqrt{\varepsilon_r}}$$

$$\boxed{D = \frac{S}{2\sqrt{\varepsilon_r - 1}}}$$

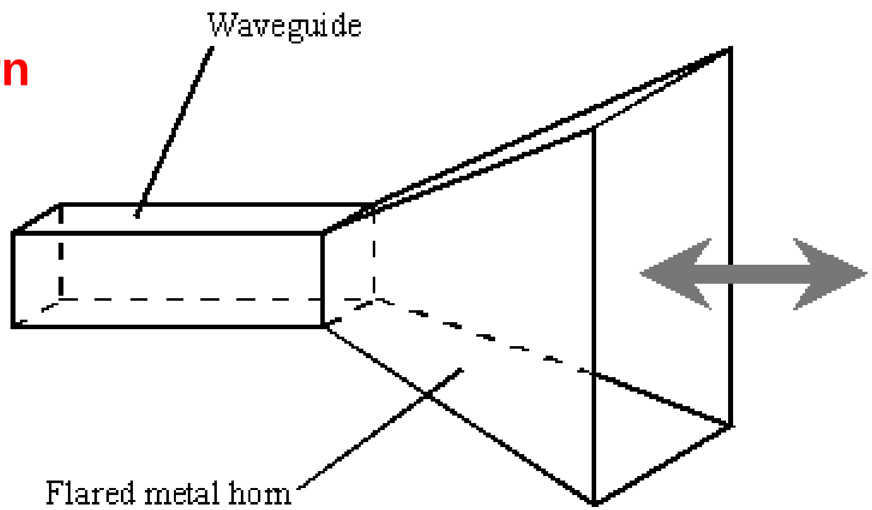
Antenne

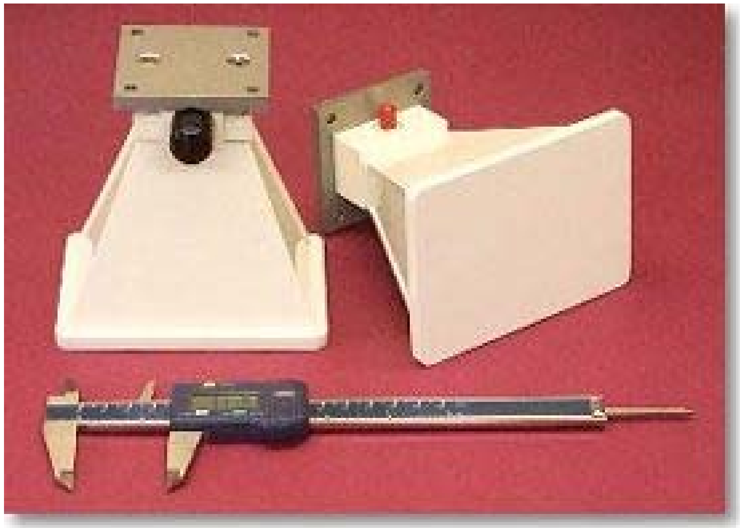

Tx
Rx
Tx
Rx
Linea di scansione

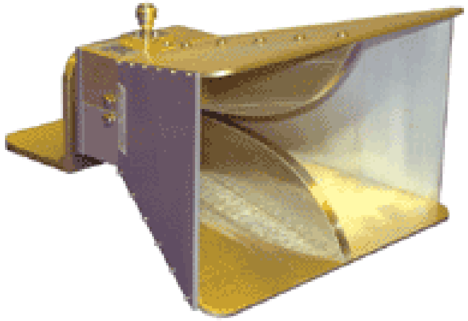

Parallelo Broadside
LINEA DI SCANSIONE
Parallelo Endfire
LINEA DI SCANSIONE
Ortogonale Broadside
LINEA DI SCANSIONE
Ortogonale Endfire
LINEA DI SCANSIONE
Polarizzazione incrociata
LINEA DI SCANSIONE

Antenne

Horn

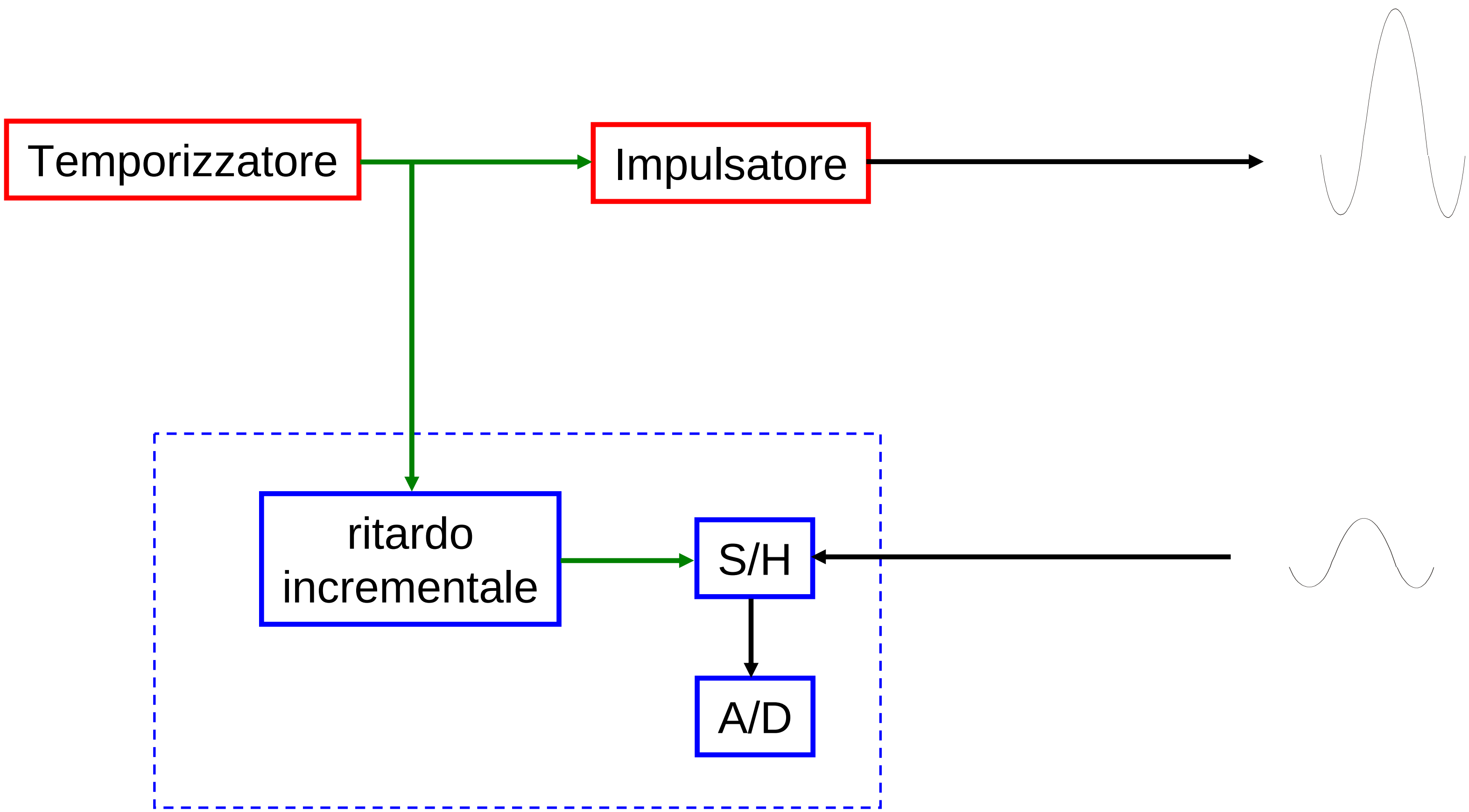




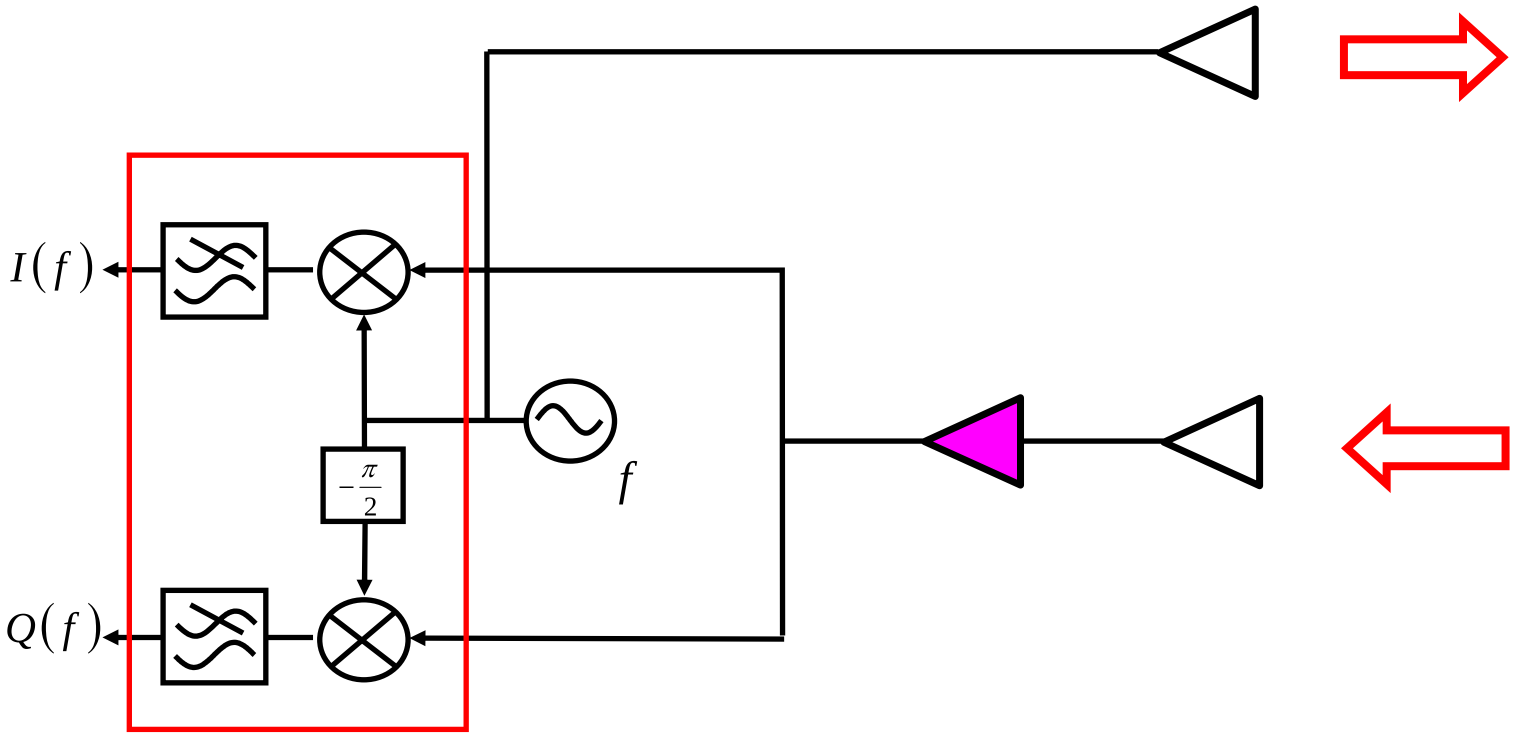


Elettronica

Radar impulsati

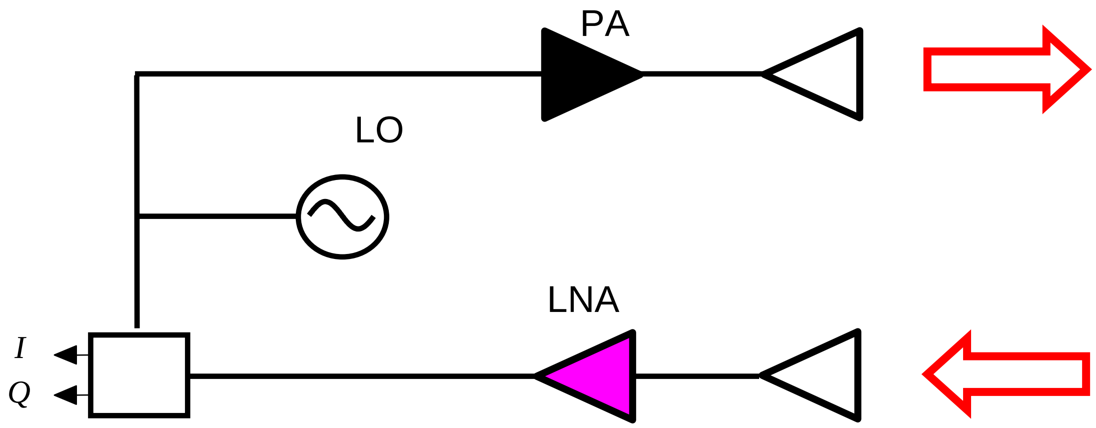


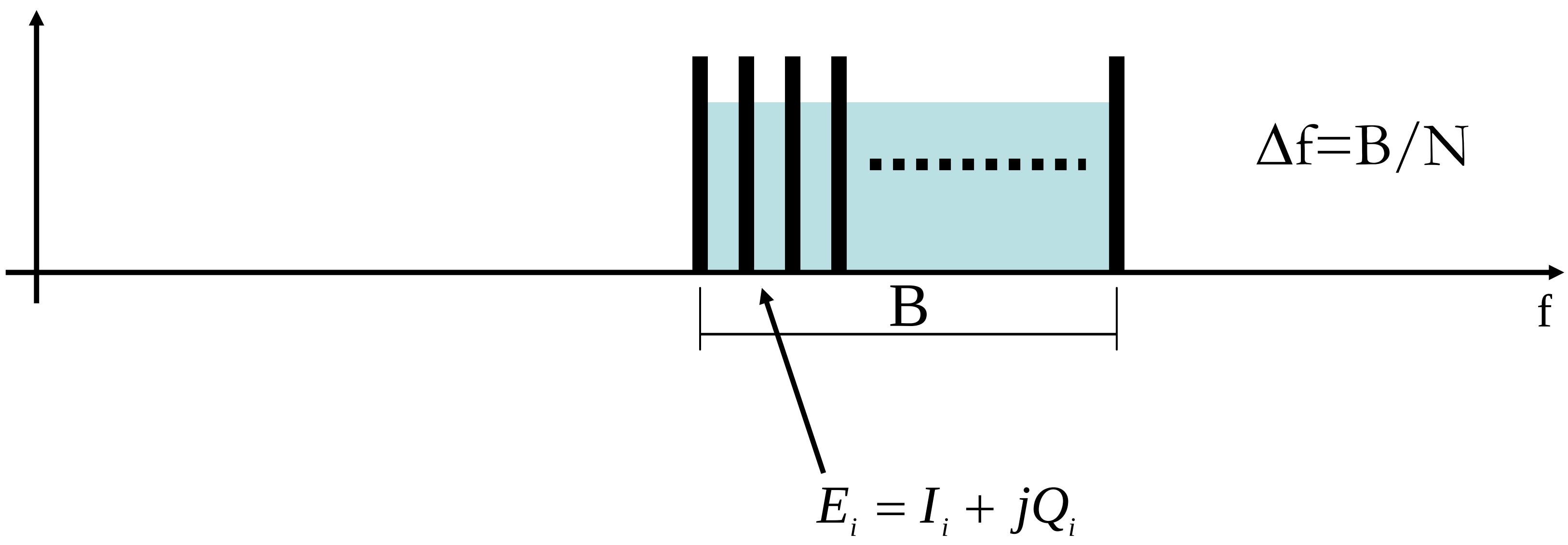
Radar ad onda continua



Radar ad onda continua a passi di frequenza

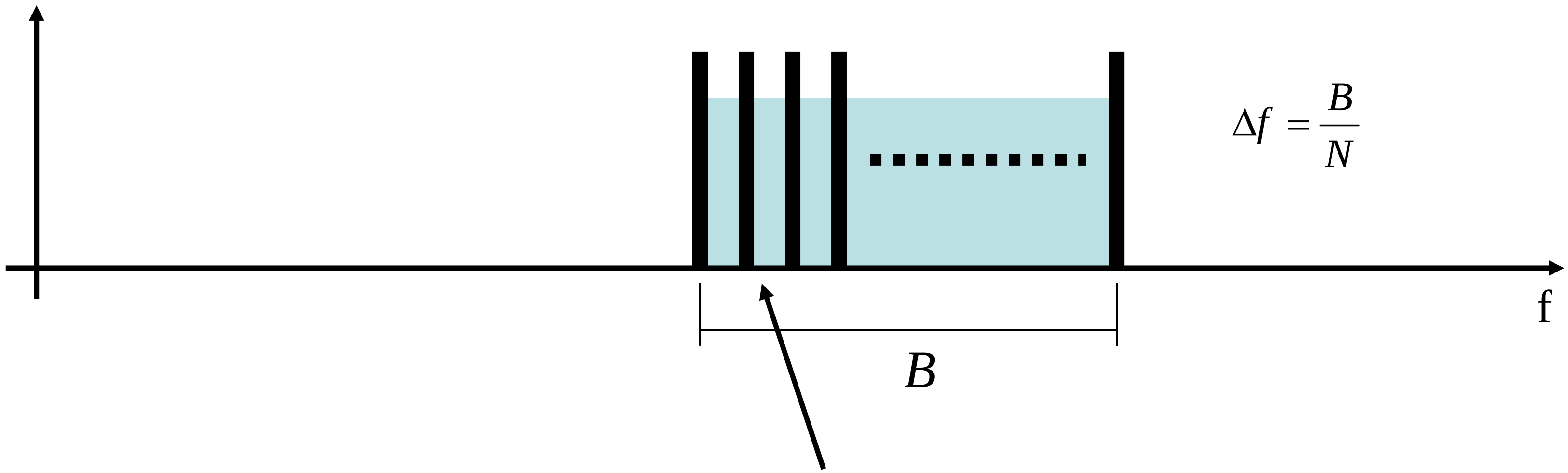
(SF-CW: Step Frequency Continuous Wave)





Radar ad onda continua a passi di frequenza

(SF-CW: Step Frequency Continuous Wave)



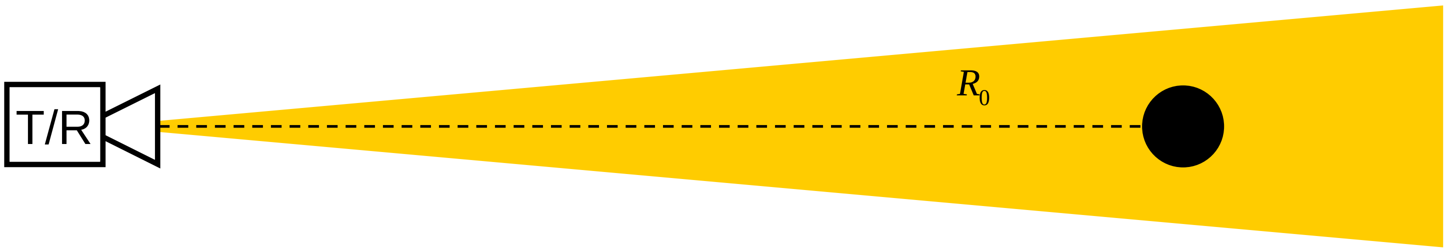
$$\Delta f = \frac{B}{N}$$

$$E_i = I_i + jQ_i$$

$$\boxed{f(t) = \sum_i E_i e^{j\omega_i t}}$$

Formula per la sintesi dell'impulso

Radar ad onda continua a passi di frequenza



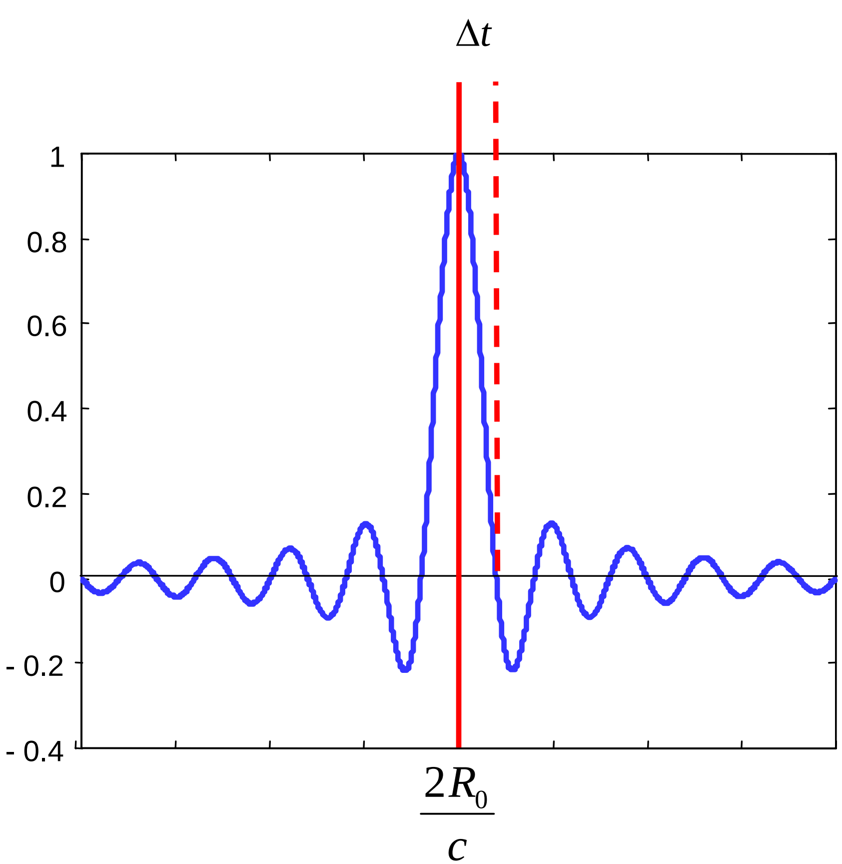
$$E(f) = e^{-j\frac{2\pi}{\lambda}(2R_0)}$$

$$\tilde{E}^{-1}(t) = \frac{1}{2\pi(f_2 - f_1)} \int_{f_1}^{f_2} E(f) e^{j2\pi ft} df = \frac{1}{2\pi(f_2 - f_1)} \int_{f_1}^{f_2} e^{-j\frac{2\pi}{c} f (2R_0)} e^{j2\pi ft} df =$$

$$= \frac{1}{2\pi(f_2 - f_1)} \int_{f_1}^{f_2} e^{-j2\pi f\left(\frac{2R_0}{c}\right)} e^{j2\pi ft} df = \ldots = e^{j2\pi\frac{f_1+f_2}{2}\left(\frac{2R_0}{c}\right)} \frac{\sin\left[2\pi\left(\frac{f_2 - f_1}{2}\right)\left(t - \frac{2R_0}{c}\right)\right]}{2\pi\left(\frac{f_2 - f_1}{2}\right)\left(t - \frac{2R_0}{c}\right)}$$

Radar ad onda continua a passi di frequenza

$$\tilde{E}^{-1}(t) = e^{j2\pi f_c\left(\frac{2R_0}{c}\right)} \frac{\sin\left[2\pi\left(\frac{B}{2}\right)\left(t-\frac{2R_0}{c}\right)\right]}{2\pi\left(\frac{B}{2}\right)\left(t-\frac{2R_0}{c}\right)}$$



$$2\pi\left(\frac{B}{2}\right)\Delta t = \pi$$

$$\Delta t = \frac{1}{B}$$

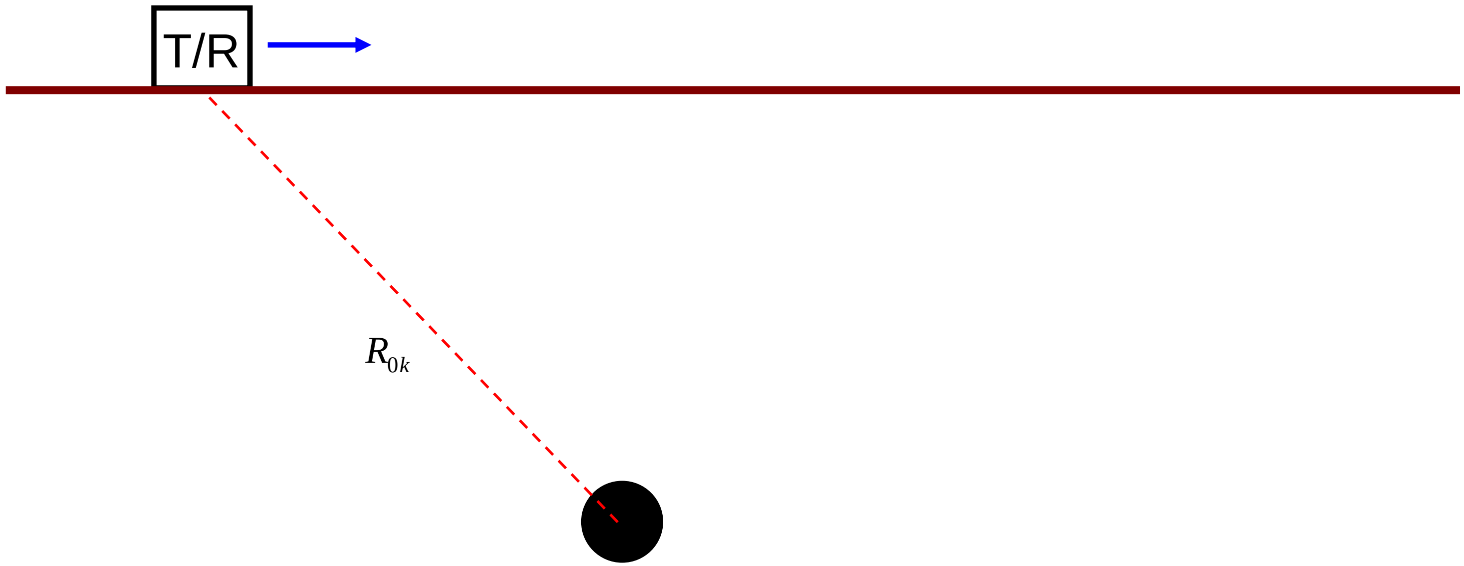
Impulso sintetico

$$\boxed{\Delta R = \frac{c}{2B}}$$

Radar ad onda continua a passi di frequenza

Focalizzazione

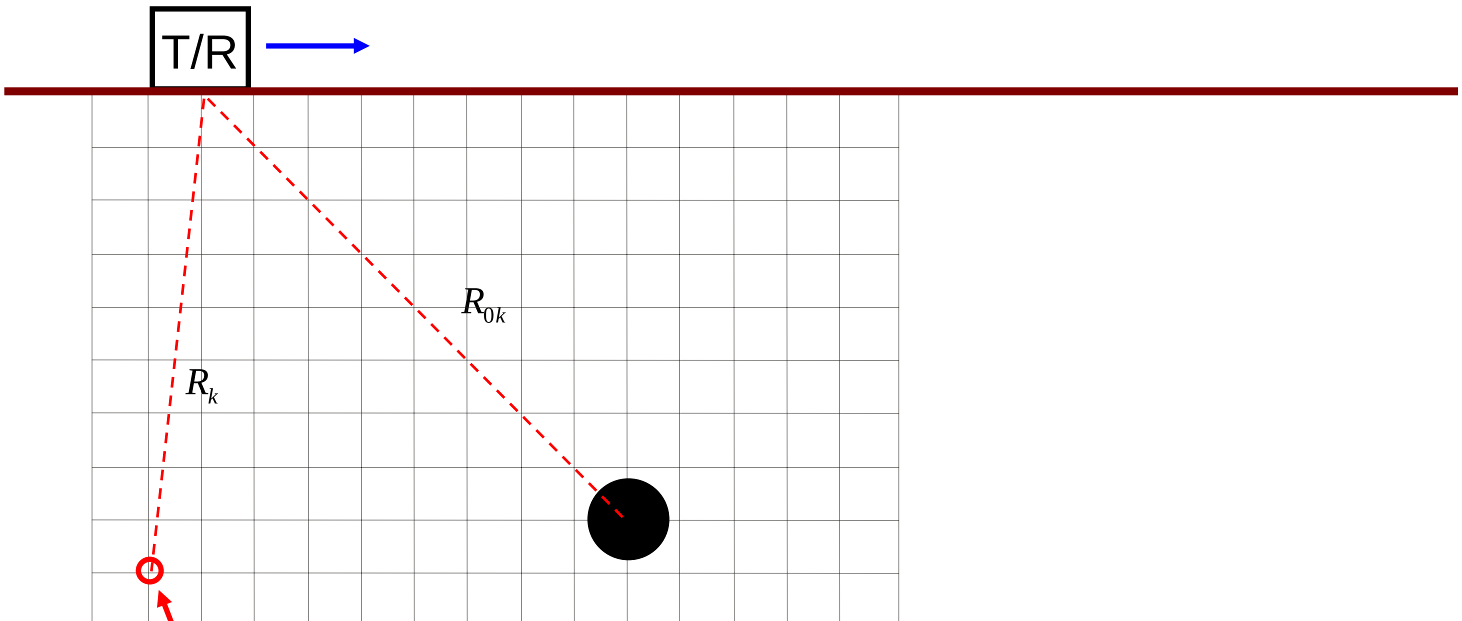
$$E_{i,k} = e^{-j\frac{2\pi f_i}{v}(2R_{0k})}$$



Radar ad onda continua a passi di frequenza

Focalizzazione

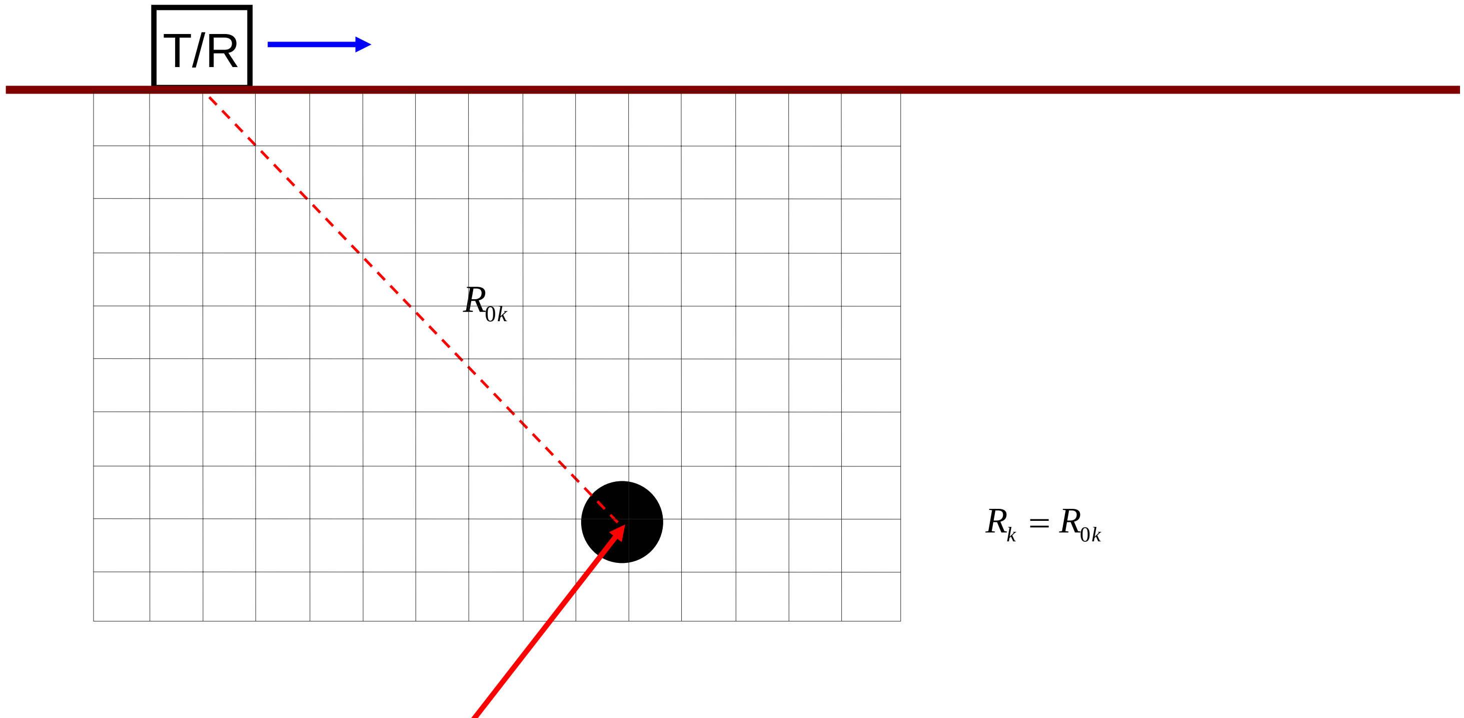
$$E_{i,k} = e^{-j\frac{2\pi f_i}{v}(2R_{0k})}$$



$$\sum_{i,k} E_{i,k} e^{j\frac{2\pi f_i}{v}(2R_k)} = \sum_{i,k} e^{-j\frac{2\pi f_i}{v}(2R_{0k})} e^{j\frac{2\pi f_i}{v}(2R_k)} = \sum_{i,k} e^{j\frac{4\pi f_i}{v}(R_k - R_{0k})} \approx 0$$

Radar ad onda continua a passi di frequenza

$$E_{i,k} = e^{-j\frac{2\pi f_i}{v}(2R_{0k})}$$

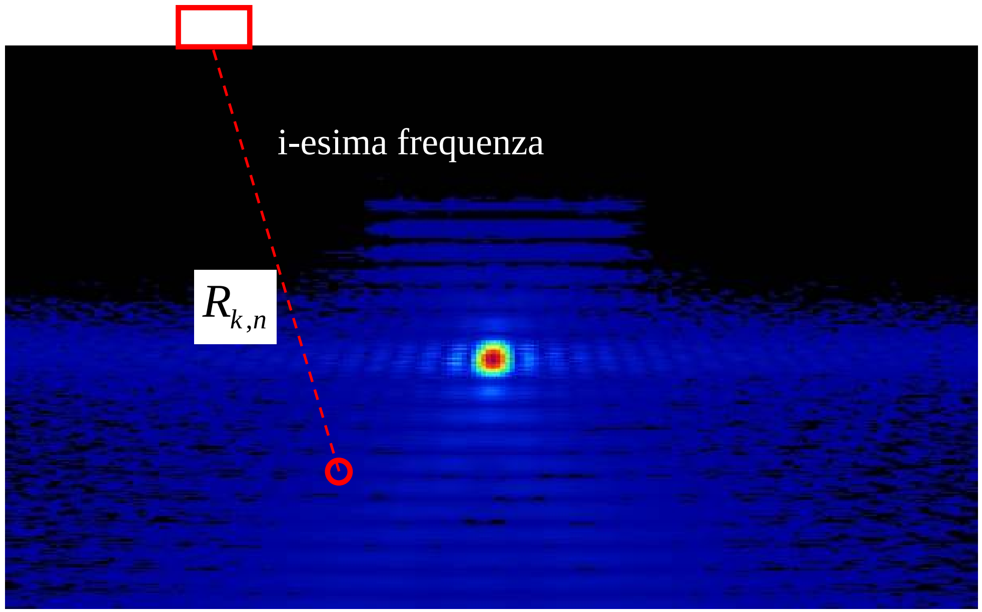


$$R_k = R_{0k}$$

$$\sum_{i,k} E_{i,k} e^{j\frac{2\pi f_i}{v}(2R_k)} = \sum_{i,k} e^{-j\frac{2\pi f_i}{v}(2R_{0k})} e^{j\frac{2\pi f_i}{v}(2R_k)} = \sum_{i,k} 1 = N_f N_p$$

Impulso sintetico e apertura sintetica

k-esima posizione



Impulso sintetico e apertura sintetica

Diversità in frequenza → Risoluzione in range

Diversità spaziale → Risoluzione angolare (cross-range)