Slot Line on a Dielectric Substrate

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IEEE Trans. Microwave Theory and Techniques,
vol. MTT-17, no.10, pp. 768-778, October 1969.

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Nov. 9, ‘04

Outline

- Introduction
- Approximations for Slot Line
- Basis of 2nd-order Solution
- Parameter Formulas for 2nd-order Solution
- Computed Data
- Conclusions and Critics

Introduction - Definition

- A slot in a conductive coating on a dielectric substrate
- To be used as transmission line, $\lambda' < \lambda$such that fields closely confined to the slot.

  (λ' : slot-line mode wavelength

  λ: free-space wavelength)

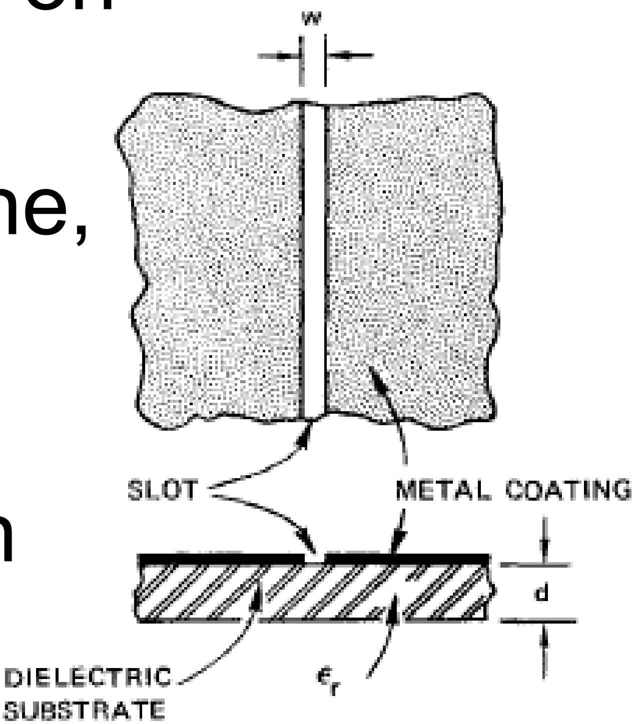
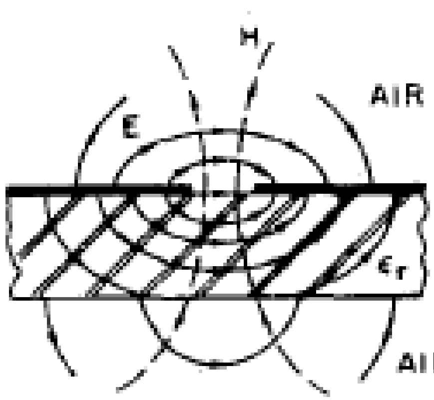
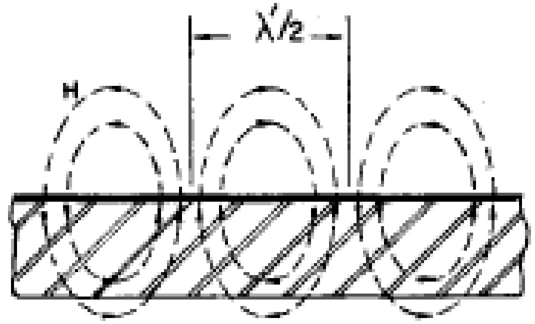
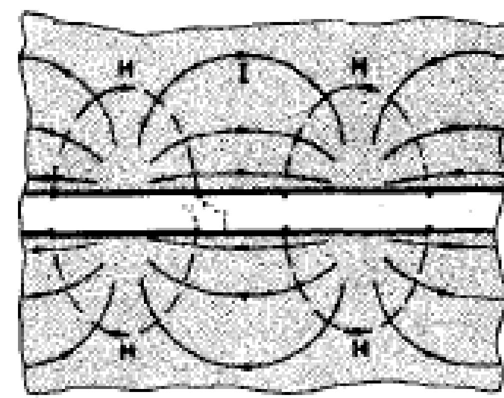


Fig. 1. Slot line on a dielectric substrate.

Introduction
- Field and Current Distribution







- Field distribution in cross section.
- H field in longi-tudinal section.
- Current distribution on metal surface.

Introduction
- Some Applications

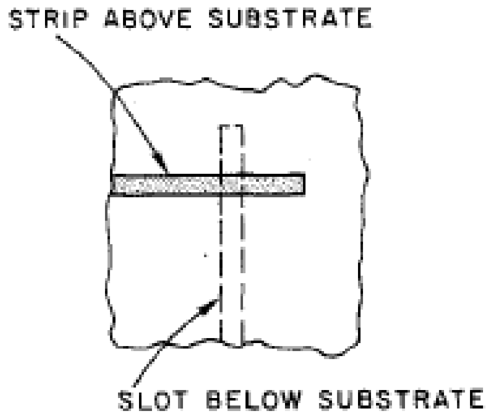


Fig. 3. Simple transition between slot line and microstrip.

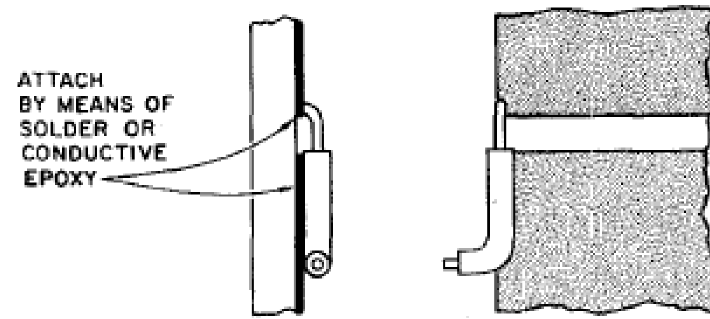


Fig. 4. Broad-band transition between slot line and miniature semirigid coaxial line.

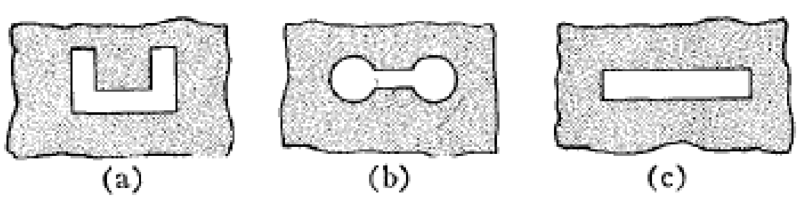


Fig. 5. Resonant slots.

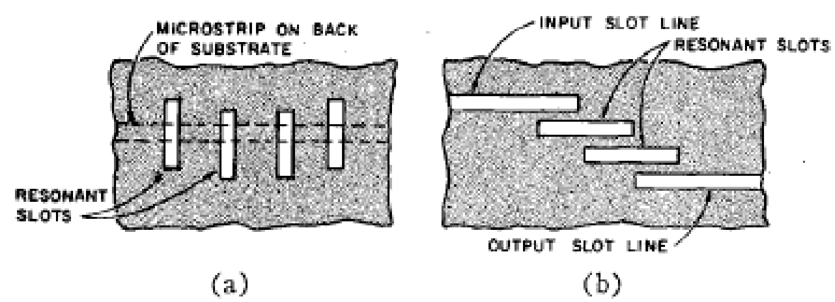


Fig. 6. Filter applications. (a) Bandstop filter. (b) Bandpass filter.

Introduction
- Basic Parameters & Comparison

- Characteristic impedance Z_o
  phase velocity v
- Compared with microstrip lines:
  Non-TEM mode VS. Quasi-TEM mode
- Compared with waveguides:
  No cut-off frequency

Approximations for Slot Line

- Zero-order solution

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{2}{\epsilon_r + 1}} .$$

J. Galejs, "Excitation of slots in a conducting screen above a lossy dielectric half space," *IRE Trans. Antennas and Propagation*, VO1.AP-10, pp. 436-443, July 1962.

- Field components on the air side can be computed as a function of λ, λ', and distance r from the slot

Approximations for Slot Line - Field Components

$$H_z = A H_0^{(1)}(k_c r)$$

Assume $w/\lambda << 1$

$$H_r = -\frac{\gamma_z}{k_c^2}\frac{\partial H_z}{\partial r} = \frac{A}{\sqrt{1-\left(\frac{\lambda'}{\lambda}\right)^2}} H_1^{(1)}(k_c r)$$

$$E_\phi = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial r} = \frac{-\eta(\lambda'/\lambda)A}{\sqrt{1-\left(\frac{\lambda'}{\lambda}\right)^2}} H_1^{(1)}(k_c r)$$

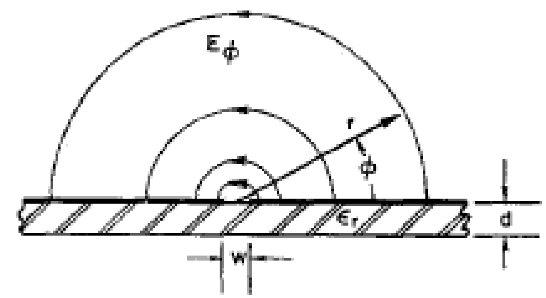


Fig. 7. Cylindrical coordinates with axis on center line of slot.

where

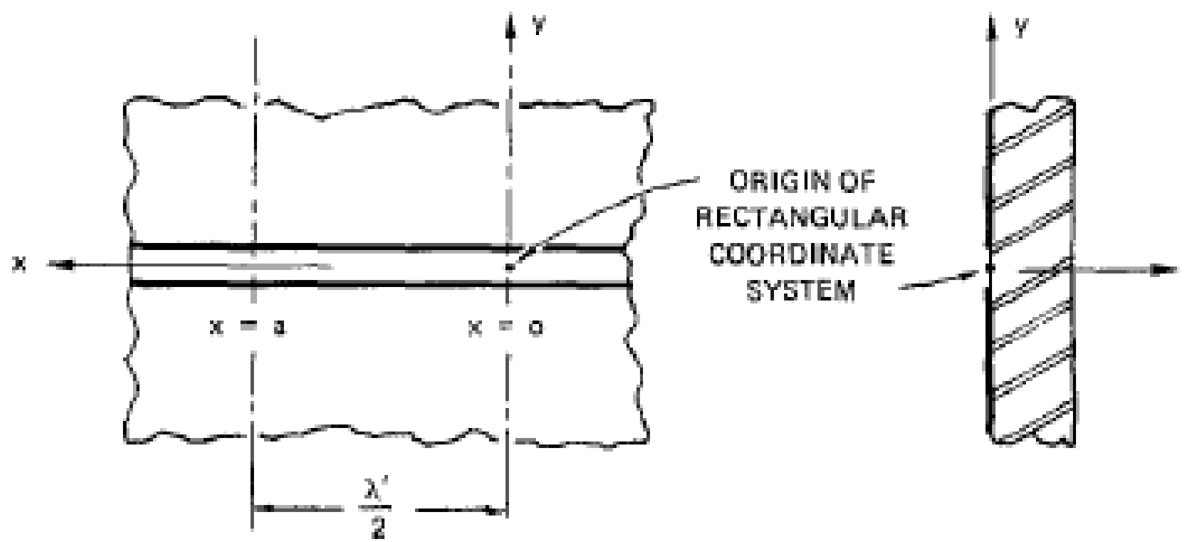
$$k_c = \sqrt{\gamma_z^2 + k^2} = j\frac{2\pi}{\lambda}\sqrt{\left(\frac{\lambda}{\lambda'}\right)^2 - 1} = j\frac{2\pi}{\lambda}\sqrt{\frac{\epsilon_r - 1}{2}}$$

$H_n^{(1)}(x)$ Is the Hankel function of first kind, order n, and argument x

Approximations for Slot Line - Derived Results

- A relative wavelength ratio less than unity is needed to ensure decay of the slot-mode field with radial distance
- A solution for circular-polarization does not exist, but elliptical-polarization occurs for all r
- A wall or other perturbing objects can be as close as 0.5 inch with little effect on Z_o and λ'

Basis of Second-order Solution - Key Feature

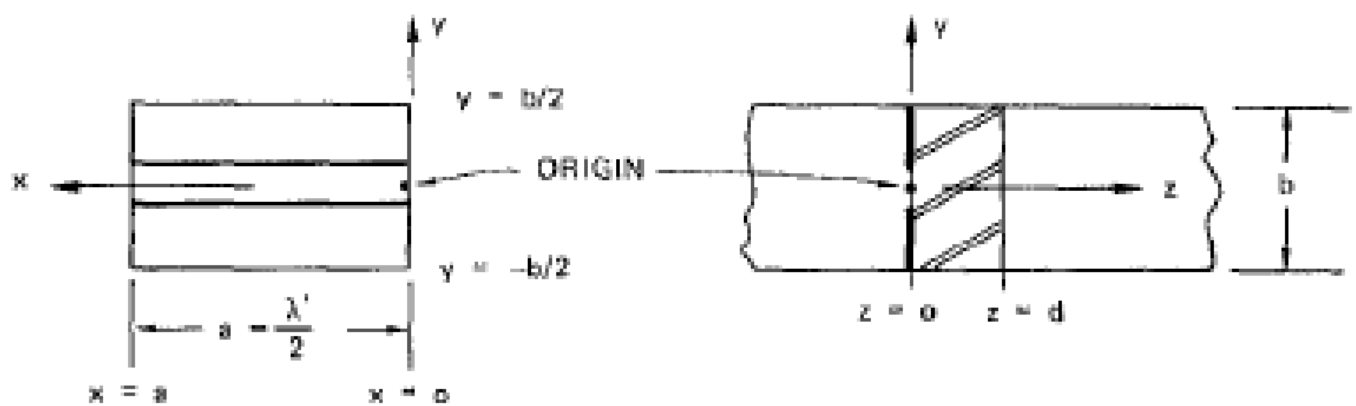


(a)

- Conversion into a waveguide problem by introducing boundary walls.
- Simple rectangular-waveguide modes instead of cylindrical modes embodying Hankel functions.

Basis of Second-order Solution - Development of Waveguide Models

- Assume slot waves of equal amplitude traveling in the $+x$ and $-x$ direction
- Transverse E-field and normal H-field cancel to zero at planes spaced by $\lambda'/2$
- A resonant slot-wave mode



(b)

Basis of Second-order Solution - Applying Waveguide Modes

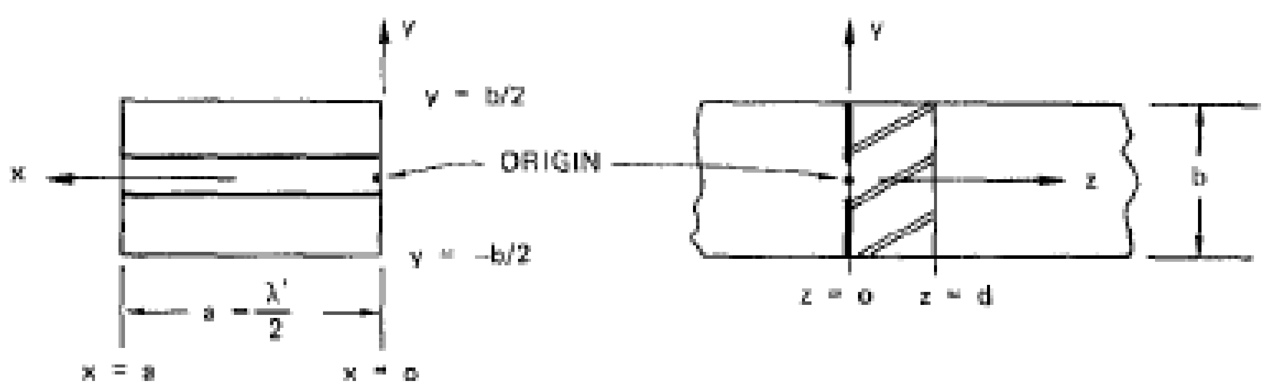
- All modes must have:

  1. $\lambda'/2$ variation in the x direction

  2. E-field maximum at the center of the slot

- Possible modes satisfying the B.C.s:

  TE_{10}, TE_{12}, TE_{14},…, TM_{12}, TM_{14},…

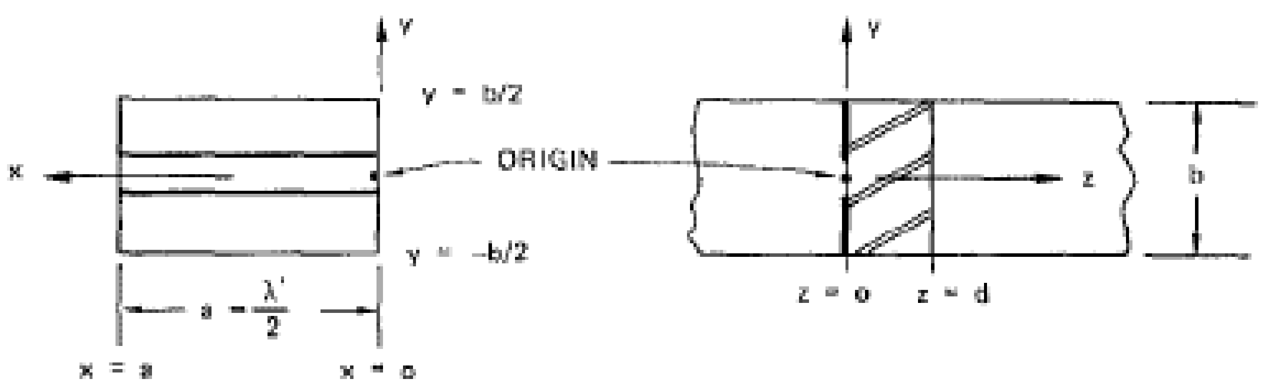


(b)

Basis of Second-order Solution - Slot wave condition

- For slot wave, $\lambda' < \lambda$

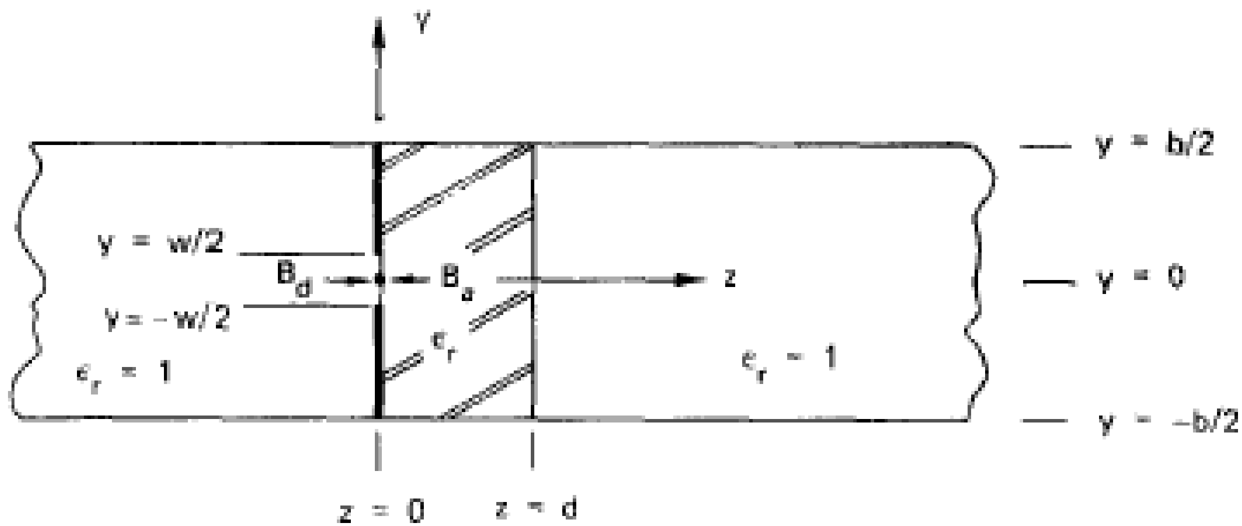
  i.e., $a < \lambda/2$
- All modes cut off in the air region
- Energy of resonant slot-mode energy trapped near the slot



(b)

Basis of Second-order Solution - An equivalent condition

- Transverse resonance occur at z=0 plane
- The sum of the susceptance B_t equals 0 at z=0 plane
- E.g., at z=0, $B_t = B_d + B_a = 0$



Parameter Formulas for 2nd-order solution - Solution Goal

- Define $p = \lambda / 2a$
- At transverse-resonance frequency

  $a = \lambda'/2 \rightarrow p = \lambda / \lambda'$
- Goal : to solve $p = \lambda / \lambda'$ as the solution for Bt = 0 as given a set of parameters

  (ε_r, w, d, b, and a)

Parameter Formulas for 2nd-order solution - Formulations

- The formula for B_t is given by

$$\eta B_t = \frac{a}{2b}\left[-v + u\tan\left(\frac{\pi d u}{ap} - \tan^{-1}\frac{v}{u}\right)\right]$$

$$+ \frac{1}{p}\left\{\left(\frac{\epsilon_r+1}{2} - p^2\right)\ln\frac{2}{\pi\delta} + \frac{1}{2}\right.$$

$$\left.\cdot \sum_{n=1,2,3,\ldots}\left[v^2\left(1-\frac{1}{F_n}\right) + M_n\right]\frac{\sin^2(\pi n\delta)}{n(\pi n\delta)^2}\right\}$$

where $\eta = \sqrt{\mu_0/\epsilon_0} = 376.7$ ohms, $\delta = w/b$, and

$$u = \sqrt{\epsilon_r - p^2}, \qquad v = \sqrt{p^2 - 1}$$

$$F_n = \sqrt{1 + \left(\frac{b}{2an}\cdot\frac{v}{p}\right)^2},$$

$$F_{n1} = \sqrt{1 - \left(\frac{b}{2an}\cdot\frac{u}{p}\right)^2}.$$

Parameter Formulas for 2nd-order solution - Formulations (cont'd)

For F_{n1} real, M_n is

$$M_n = \frac{\epsilon_r \tanh r_n - p^2 F_{n1}{}^2 \coth q_n}{\left[1 + \left(\frac{b}{2an}\right)^2\right] F_{n1}} - u^2$$

where

$$r_n = \frac{2\pi n d F_{n1}}{b} + \tanh^{-1}\left(\frac{F_{n1}}{\epsilon_r F_n}\right)$$

$$q_n = \frac{2\pi n d F_{n1}}{b} + \coth^{-1}\left(\frac{F_n}{F_{n1}}\right).$$

For F_{n1} imaginary, M_n is

$$M_n = \frac{\epsilon_r \tan r_n' - p^2 \left| F_{n1} \right|^2 \cot q_n'}{\left[1 + \left(\frac{b}{2an}\right)^2\right] \left| F_{n1} \right|} - u^2$$

where

$$r_n' = \frac{2\pi n d \left| F_{n1} \right|}{b} + \tan^{-1}\left(\frac{\left| F_{n1} \right|}{\epsilon_r F_n}\right)$$

$$q_n' = \frac{2\pi n d \left| F_{n1} \right|}{b} + \cot^{-1}\left(\frac{F_n}{\left| F_{n1} \right|}\right).$$

Parameter Formulas for 2nd-order solution - v/vg and Z_o as Functions of $p=\lambda/\lambda'$

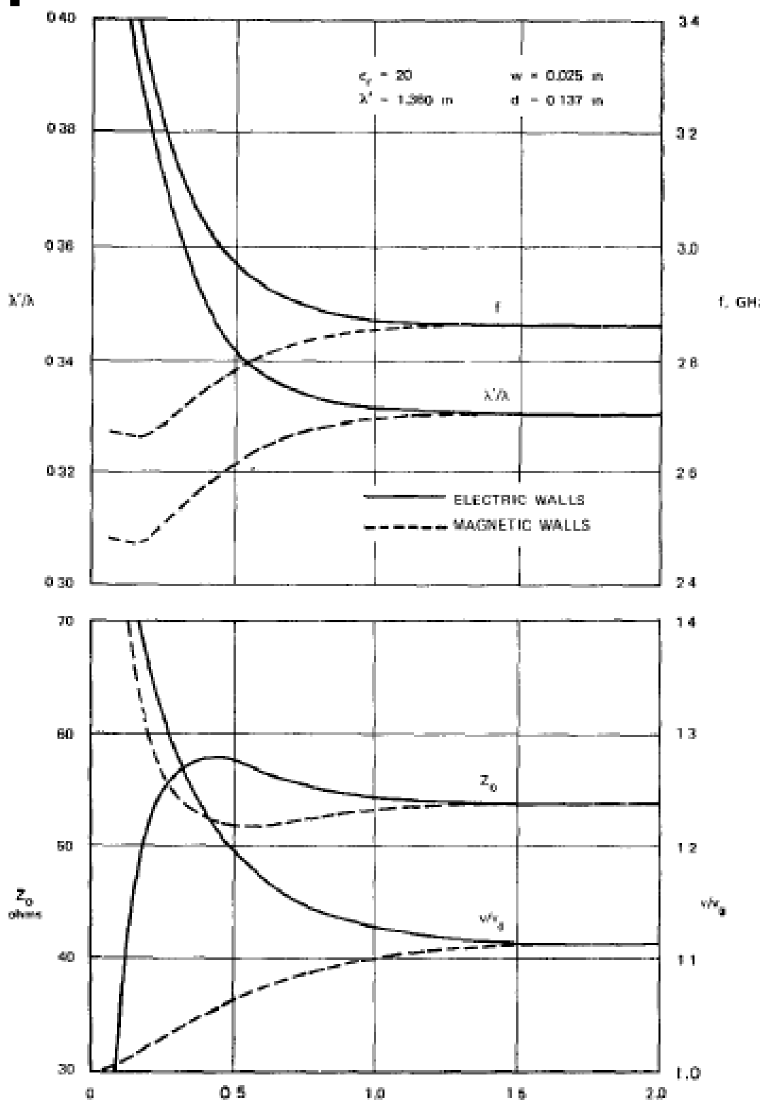
- $vg = d\omega/d\beta$

$$\frac{v}{v_g} = 1 + \frac{f}{\lambda'/\lambda} \cdot \frac{-\Delta(\lambda'/\lambda)}{\Delta f} = 1 + \frac{f}{\lambda/\lambda'} \cdot \frac{\Delta(\lambda/\lambda')}{\Delta f}$$

- $$Z_0 = \frac{\pi}{\omega(dB_t/d\omega)} \cdot \frac{v}{v_g}$$

$$Z_0 = 376.7 \frac{v}{v_g} \frac{\pi}{p} \cdot \frac{\Delta p}{-\Delta\eta B_t} \text{ ohms.}$$

Computed Data
- Slot-line parameters VS. b



Computed Data
- Slot-line parameters VS. frequency

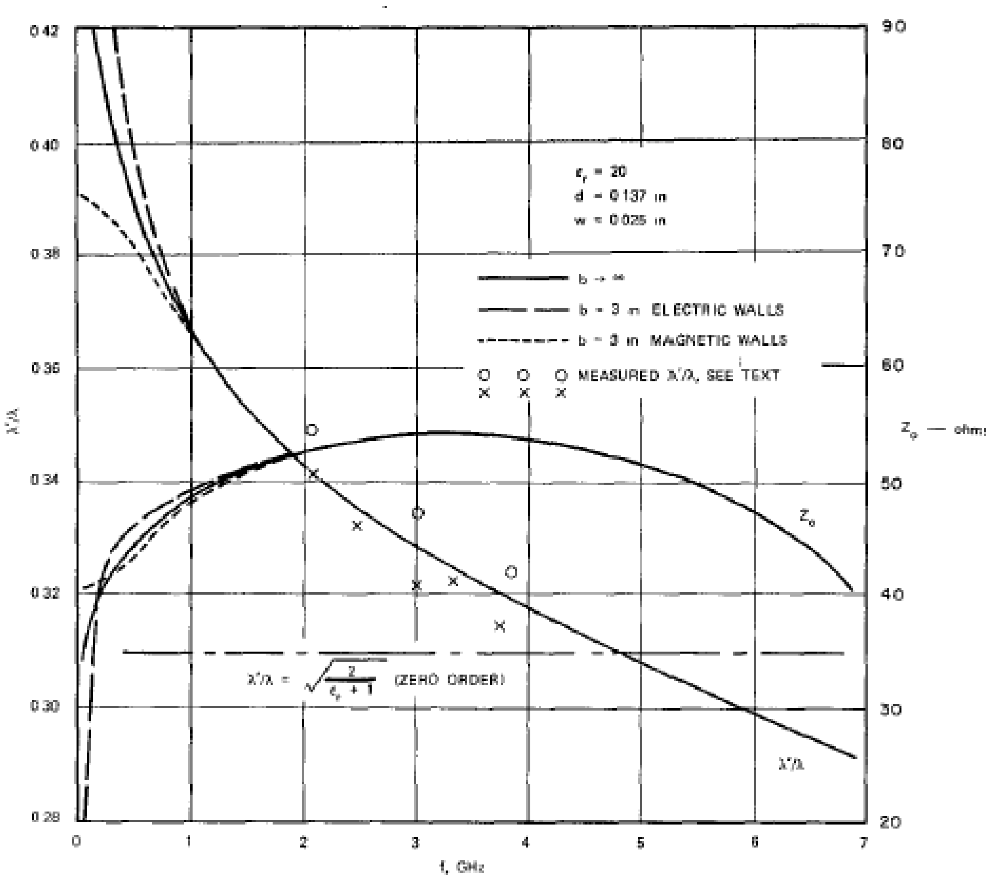


Fig. 10. Graph of λ'/λ and Z_0 versus frequency.

Computed Data
- Slot-line parameters VS. slot width

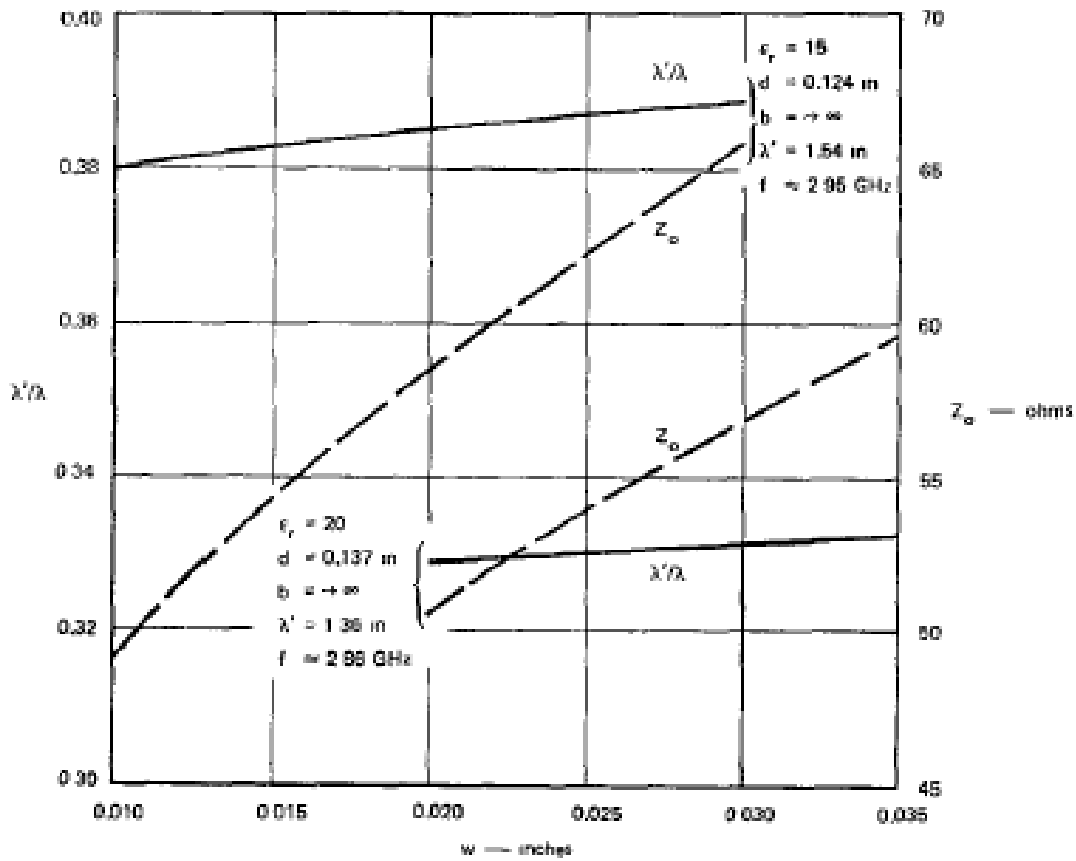


Fig. 11. Graph of λ'/λ and Z_0 versus slot width.

Computed Data - Verification of Z_o

- For magnetic-wall case

  The cross-section approximates a TEM-mode transmission when *b/λ' <<1*

- Z_o for TEM line

$$Z_0 = \frac{591.7}{\sqrt{\epsilon_r'}\ln\left(\frac{8b}{\pi w}\right)} = \frac{591.7(\lambda'/\lambda)}{\ln\left(\frac{8b}{\pi w}\right)} \text{ ohms} \qquad \begin{array}{l} b/w > 3 \\ b/\lambda' \to 0. \end{array}$$

F. Oberhettinger and W. Magnus, ***Anwendung der Ellipfischen Fanktionen in Physik und Technik.*** Berlin: Springer, 1949, pp. 63, 114-116.

Computed Data
- Verification of Zo (cont'd)

TABLE I

COMPARISON OF SECOND-ORDER AND STATIC SOLUTIONS FOR Z_0

Second-Order Solution			Eq (22)
b (inches)	λ'/λ	Z_0 (Ω)	Z_0 (Ω)
0.10	0.30752	78.38	78.40
0.14	0.30707	68.35	68.37
0.20	0.30763	60.45	60.39
0.30	0.31230	54.60	54.04
0.40	0.31640	52.42	50.50
0.60	0.32384	51.77	46.59

- Good agreement for $b \leq 0.20$ *inch* or $b/\lambda' \leq 0.15$

(ε_r=20, *w*=0.025 *in*, *d*=0.137 *in*, λ'=1.36 *in*, *f*=2.67GHz)

Conclusion

- A second-order analysis yields formulas for slot-line wavelength, phase velocity, characteristic impedance, and effect of adjacent electric and magnetic walls.

Critics

- The assumption that the discussed section of slot-line in resonant mode may fail when considering the dielectric substrate and conductor loss
- By introducing electric or magnetic walls, images of infinite parallel slots should be taken into account
- Insufficient verification
- Even and odd mode analysis