Mixed-Potential Integral Equation Technique for the Characterization of Microstrip Antennas Printed on Uniaxial Substrates

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Introduction

Many substrate materials commonly used for planar antennas exhibit uniaxial anisotropy. It is well-known also that anisotropic substrates exercise a non negligible influence on the radioelectric performances of planar circuits and antennas [1] [2]. Hence it is really important to quantify this influence in order to evaluate its effects on these performances. Antennas printed on uniaxial substrates have already been studied in the literature using integral equations associated to Green's functions in spectral domain [2]. In the present work, we develop the mixed-potential formulation to analyse circuits and antennas using anisotropic materials. The Green's functions for a multilayer structure with both electric and magnetic anisotropism are analytically determined in space domain rather than spectral domain. The main advantage of this formulation is to get the current distribution directly without using an inverse Fourier transform which is time consumed. As an application the effect of the anisotropic properties of an uniaxial substrate on the characteristics of a microstrip antenna is calculated and compared to the isotropic case.

The Mixed Potential Formulation

a) Electric Field Integral Equation

The general structure to be analysed is shown in fig. 1. Substrate materials are supposed uniaxial and characterized by permittivity and permeability tensors of the form :

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_r & 0 \\ 0 & 0 & \varepsilon_{r_z} \end{bmatrix} \qquad \bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu_r & 0 & 0 \\ 0 & \mu_r & 0 \\ 0 & 0 & \mu_{r_z} \end{bmatrix} \tag{1}$$

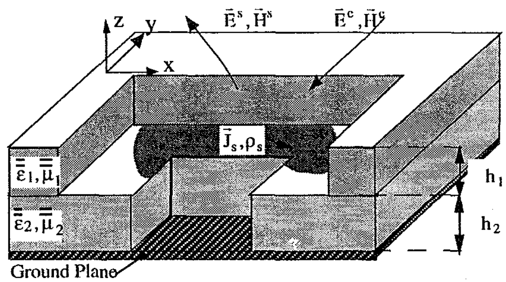


fig n°1 - *Configuration of the planar structure printed on a uniaxial substrate*

The substrate and the ground plane are assumed to be transversally infinite, the ohmic losses are taken into account for the upper conductor. Dielectric and magnetic losses are also taken into account by the introduction of a complex dielectric and magnetic constant.

$$\varepsilon_u = \varepsilon_0 \varepsilon_{r_u}(1 - j\mathrm{tg}\delta_e) \qquad \mu_u = \mu_0 \mu_{r_u}(1 - j\mathrm{tg}\delta_m) \qquad \text{with } u=x,y,z \tag{2}$$

To establish integral equations for the current and the charge distributions, we start with the boundary condition of the tangential electric field which expresses the equality of the diffracted $\vec{E}^s$ and the feeding field $\vec{E}^e$. For a multilayer structure, if the field is derived from mixed potentials with the associated Green's functions $\overline{\overline{G}}_A$ and G_V, the following equation is obtained at each metallic interface :

$$\vec{e}_z \times \vec{E}^e = \vec{e}_z \times \left(j\omega \int_s \overline{\overline{G}}_A(\vec{r}/\vec{r}').\vec{J}_s(\vec{r}')ds' + \nabla \int_s G_V(\vec{r}/\vec{r}')q_s(\vec{r}')ds' + Z_s\vec{J}_s \right) \tag{3}$$

where charge and current densities are related through the continuity equation.

$$\nabla.\vec{J}_s + j\omega q_s = 0 \tag{4}$$

and Z_s is the surface impedance of the conductor.

b) Dyadic Green's Function for the Vector and the Scalar Potentials

The both components of dyadic Green's functions are found by solving the Helmholtz equation with the appropriate boundary conditions, which are expressed in terms of Sommerfeld integrals as in the isotropic case.

For the x-directed horizontal electric dipole (HED), the dyadic Green's function can be reduced to $\overline{\overline{G}}_{A_i} = G^{xx}_{A_i}\vec{e}_x\vec{e}_x + G^{zx}_{A_i}\vec{e}_z\vec{e}_x$ where $G^{xx}_{A_i}$ and $G^{zx}_{A_i}$ must satisfy the following Helmholtz equations :

$$\begin{aligned}
&\left[\nabla_t^2 + \frac{\mu_{z_i}}{\mu_i}\frac{\partial^2}{\partial z^2} + \omega^2\varepsilon_i\mu_{z_i}\right]G^{xx}_{A_i} = \begin{cases} 0 & i \neq j \\ -\overline{\overline{\mu}}_j.\delta(\vec{r} - \vec{r}') & i = j \end{cases} \\
&\left[\nabla_t^2 + \frac{\mu_{z_i}}{\mu_i}\frac{\partial^2}{\partial z^2} + \omega^2\varepsilon_{z_i}\mu_i\right]G^{zx}_{A_i} = j\omega\left(\varepsilon_{z_i}\mu_i - \varepsilon_i\mu_{z_i}\right)\frac{\partial G_{V_{i_{dip}}}}{\partial z}, \quad i = 0,1,2
\end{aligned} \tag{5}$$

The vector and scalar potentials are related by the Lorentz gauge which is obtained from the radial component of the Helmholtz equation for the vector potential

$$\left[\nabla_t + \vec{e}_z\frac{\mu_z}{\mu}\frac{\partial}{\partial z}\right].\overline{\overline{G}}_A + j\omega\varepsilon\mu_z G_{V_{dip}} = 0 \tag{6}$$

The boundary conditions result from the requirement that the tangential electric field is continuous across each interface and null on the ground plane. We also applied the radiation condition and the "jump" condition of the magnetic field.

In the z-component of the Faraday's equation, after replacing the electric field by its expression with potentials, one can see that the horizontal component of the dyadic Green's function G^{xx}_A depends only on TE waves (i.e. u_a), while G^{zx}_A depends on both TE and TM waves (i.e. u_a and u_b given later) as expected. The horizontal component can be found, using (5) and boundary conditions :

$$\left.\begin{matrix} G^{xx}_{A_0} \\ G^{xx}_{A_1} \\ G^{xx}_{A_2} \end{matrix}\right\} = \frac{\mu_0}{4\pi}\int_c H_0^{(2)}(\lambda\rho)\lambda d\lambda \left\{ \begin{matrix} \dfrac{\mu_r \sinh u_a h_2}{DTE} e^{-u_0 z} \\ \dfrac{\mu_r}{u_a}\left\{ \sinh u_a h_2 \cosh u_a(z+h) - \dfrac{N^{xx}_{TE_1}}{DTE}\sinh u_a(z+h) \right\} \\ \dfrac{\mu_r}{u_a}\dfrac{N^{xx}_{TE_2}}{DTE}\sinh u_a(z+h) \end{matrix} \right\} \tag{7}$$

From the Lorentz gauge, and the scalar Green's function equation due to an x-directed dipole, the Green's function corresponding to the scalar potential G_V can finally be obtained from :

$$G_{V_{i_{dip}}} = \frac{-1}{j\omega}\frac{\partial G_{V_i}}{\partial x} \tag{8}$$

$$
\left.\begin{matrix} G_{V_0} \\ G_{V_1} \\ G_{V_2} \end{matrix}\right\} = \frac{1}{4\pi\varepsilon_0} \int_c H_0^{(2)}(\lambda\rho)\lambda d\lambda \left\{ \begin{matrix} \left\{ \omega^2 \varepsilon_0 \mu_0 \mu_r \dfrac{\sinh u_a h_2}{\lambda^2 DTE} + u_0 u_b \dfrac{\sinh u_b h_2}{\lambda^2 DTM} \right\} . e^{-u_0 z} \\ \dfrac{1}{\lambda^2} \left\{ \dfrac{u_b}{\varepsilon_r} \left[\sinh u_b h_2 \cosh u_b (z+h) - \dfrac{N^V_{TM_1}}{DTM} \sinh u_b (z+h) \right] \right. \\ \left. + \dfrac{\omega^2 \varepsilon_0 \mu_0 \mu_r}{u_a} \left[\sinh u_a h_2 \cosh u_a (z+h) - \dfrac{N^V_{TE_1}}{DTE} \sinh u_a (z+h) \right] \right\} \\ \left\{ \dfrac{\omega^2 \varepsilon_0 \mu_0 \mu_r}{u_a} \dfrac{N^V_{TE_2}}{DTE} \sinh u_a (z+h) + \dfrac{u_b}{\varepsilon_r} \dfrac{N^V_{TM_2}}{DTM} \sinh u_b (z+h) \right\} \end{matrix} \right\} \quad (9)
$$

$$
u_b^2 = \frac{\varepsilon}{\varepsilon_z}\lambda^2 - \omega^2 \mu\varepsilon = \left(jk_z^{TM}\right)^2 \qquad u_a^2 = \frac{\mu}{\mu_z}\lambda^2 - \omega^2 \mu\varepsilon = \left(jk_z^{TE}\right)^2 \quad (10)
$$

with $\lambda^2 = k_x^2 + k_y^2$, $\quad u_0^2 = \lambda^2 - \omega^2 \mu_0 \varepsilon_0 = \left(jk_{z_0}\right)^2$ and $h = h_1 + h_2$.

$$
DTE = u_0 \mu_r \sinh u_a (h_1 + h_2) + u_a \cosh u_a (h_1 + h_2) \quad (11)
$$

$$
DTM = \varepsilon_r u_0 \cosh u_b (h_1 + h_2) + u_b \sinh u_b (h_1 + h_2) \quad (12)
$$

DTE and DTM represent respectively transverse electric and magnetic surface waves equations.

Axis x and y are interchangeable in planar structure for uniaxial substrates. Consequently, the components of dyadic Green's function created by a HED along y axis can be obtained using : $G_A^{yy} = G_A^{xx}$ and $G_A^{zy} = \tan\phi . G_A^{zx}$.

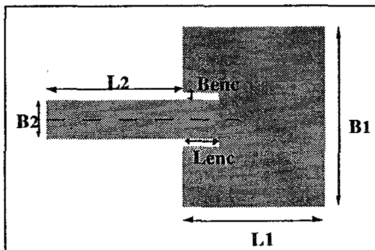
The Green's function of the isotropic case [3] [4], ($\varepsilon = \varepsilon_z$ and $\mu = \mu_z$), can be derived, without any problems, from the present Green's functions of anisotropic substrate.

c) Method of Moments

The integral equation is solved numerically by the Method of Moments using 2D rectangular discretization. In this approach, the current is expanded as a set of a complete basis functions. By taking the inner product of resulting equations with a set of test one gets a linear system equations which provides the current distribution on the conductor and then the characteristics of the antenna such as the input impedance or radiating patterns.

Results

To investigate the impedance, the microstrip antenna printed on a dielectric uniaxial substrate is consider as shown in fig. 2. Fig. 3 presents the magnitude and the phase of the reflection coefficient versus frequency. The desired resonance is observed at 3.28 GHz. The antenna is matched to 50Ω microstrip line thanks to the inserted coplanar line between the patch and the feeding line. To compare this results to isotropic case, the magnitude and the phase are presented in the same figure (fig. 3). One can see that the effect of the anisotropy is similar to the well-known effect of a radome on the resonance length of a microstrip antenna. In other words the anisotropy is the cause of a frequency shift of 40 MHz lower than the isotropic one.



L1 = 14.1 mm **B2 = 1.14 mm**
B1 = 20.2 mm **H2 = 1.27 mm**
H1 = 0 mm **Lenc = 4.98 mm**
$\mu_r = \mu_{r_z} = 1$ **Benc = 1.21 mm**

$$
\bar{\bar{\varepsilon}}_{r_{anisotropic}} = \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 10.2 \end{bmatrix} \qquad \bar{\bar{\varepsilon}}_{r_{isotropic}} = \begin{bmatrix} 10.2 & 0 & 0 \\ 0 & 10.2 & 0 \\ 0 & 0 & 10.2 \end{bmatrix}
$$

fig. 2 *: Geometry of the microstrip antenna*

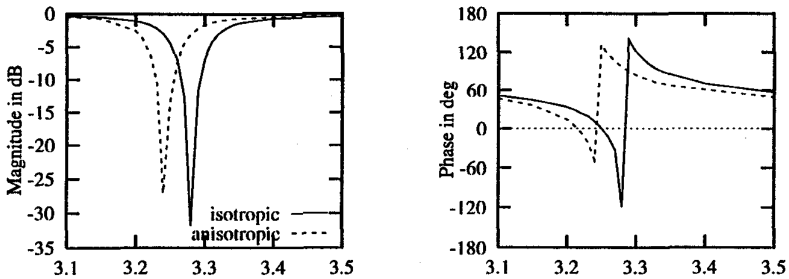


fig. 3 : *Magnitude and Phase of the reflection coefficient*

It means that the natural anisotropy due to the discontinuity between the substrate and the space can be enforced by the adjonction of a radome or by the nature of the substrate (anisotropic uniaxial substrate).

The use of the uniaxial material makes possible to reduce the antenna's dimensions.

Radiating patterns of both configurations (isotropic and anisotropic) are also investigated. Nevertheless, we did not obtained any particular effect created by the anisotropy. The obtained results will be presented.

Conclusion

An analysis based on the Mixed-Potential Integral Equation to study planar structures printed on uniaxially substrates with both electric and magnetic anisotropism is presented. The Green's function for the corresponding structure are obtained analytically in space domain. The presented results show that the corresponding substrate allows to reduce the antenna size.

Appendix

Expression of the terms mentioned in different Green's function.

$$N_{TE_1}^{xx} = \left(u_a \sinh u_a (h_1 + h_2) + u_0 \mu_r \cosh u_a (h_1 + h_2)\right) \sinh u_a h_2$$

$$N_{TE_2}^{xx} = u_a \cosh u_a h_1 + u_0 \mu_r \sinh u_a h_1$$

$$N_{TE_1}^{V} = \left(u_a \sinh u_a (h_1 + h_2) + u_0 \mu_r \cosh u_a (h_1 + h_2)\right) \sinh u_a h_2$$

$$N_{TM_1}^{V} = \left(\varepsilon_r u_0 \sinh u_b (h_1 + h_2) + u_b \cosh u_b (h_1 + h_2)\right) \sinh u_b h_2$$

$$N_{TE_2}^{V} = \frac{\left(u_a \cosh u_a h_1 + u_0 \mu_r \sinh u_a h_1\right)}{\lambda^2}$$

$$N_{TM_2}^{V} = \frac{\left(\varepsilon_r u_0 \cosh u_b h_1 + u_b \sinh u_b h_1\right)}{\lambda^2}$$

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