The Radiansphere Around a Small Antenna*

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Summary—The "radiansphere" is the boundary between the near field and the far field of a small antenna. Its radius is one radianlength ($\lambda/2\pi$), at which distance the three terms of the field are equal in magnitude. A "small"antenna is one somewhat smaller than the radiansphere, but it has a "sphere of influence" occupying the radiansphere. The power that theoretically can be intercepted by a hypothetical isotropic antenna is that which flows through the radiansphere or its cross section, the "radiancircle."

From a small electric dipole, the far field of radiation is identified as a retarded magnetic field. Between two such dipoles, the far mutual impedance is that of mutual inductance, expressed in terms of space properties and the radiansphere.

A small coil wound on a perfect spherical magnetic core is conceived as an ideal small antenna. Its radiation power factor is equal to the ratio of its volume over that of the radiansphere. A fraction of this ratio is obtainable in various forms of small antennas (C or L) occupying a comparable amount of space.

A radiation shield, in the form of a conducting shell the size of the radiansphere, enables separate measurement of radiation resistance and loss resistance.

Introduction

The subject of small antennas deals with the problems of effective radiation and interception by structures whose dimensions are much less than one wavelength. This assumption of small size reduces to simplest terms the antenna properties and the resulting limitations in practical applications. The concepts and rules to be presented are readily appreciated and easily retained for future reference.

The scope of this paper is limited to some principles and viewpoints that are elementary but have not previously been integrated and clearly presented. They come from various sources and have been assembled by the writer in the course of occasional studies and design experience for widely diversified purposes over the past 15 years or so.

Several concepts appear to have been original with the writer, although based on well-known principles, The "radiansphere" is developed to describe the boundary of the transition between near field and far field, and is given significance as the "sphere of influence." The "radiancircle" is the interception area of the hypothetical isotropic radiator. The "radiation power factor," previously introduced by the writer, is formulated for an idealized spherical antenna much smaller than the radiansphere. The "radiation shield," a spherical conductor located at the radian sphere, is presented to enable separate determination of radiation resistance and loss resistance, hence the radiation efficiency. The mutual impedance between small dipoles is simple and useful but seldom stated; here it is analyzed into the three kinds of impedance components (C, R, L), and is formulated directly in terms of the mean radiation resistance of the sending and receiving antennas.

After a list of symbols, the presentation will start with a brief reference to each principal concept, stated in the terminology to be used here.

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Symbols

(MKS units: meters, seconds, watts, volts, amperes, ohms, henries, farads.)

l = length of small dipole ($l \ll \lambda/2\pi$) ($l \ll r$)
r = radial distance ($r \gg l$)
h = height above plane
a = radius of sphere (inductor)
A = area of small loop
A = interception area of antenna
V = volume (of sphere)
λ = wavelength
$\lambda/2\pi$ = radianlength
f = cycle frequency
$\omega = 2\pi f$ = radian frequency
Z = impedance (complex)
R = resistance (radiation)
L = inductance
C = capacitance
I = current
V = voltage
E = electric field
H = magnetic field
P_1 = power radiated from sending antenna
P_2 = power available from receiving antenna
$R_0 = 377$ = wave resistance of square area of plane wave in free space
μ_0 = magnetivity in free space
ϵ_0 = electrivity in free space
k_m = magnetic ratio (in core of inductor)
n = number of turns (in coil of inductor)
$p = R/\omega L$ = power factor (radiation)
g = power ratio of directivity
sub-a = inductor sphere
sub-r = radian sphere
sub-1, 2 = sending, receiving (antennas)
sub-12 = mutual (between antennas)
$*$ = subject to retardation by distance angle

Basic Concepts

Radiansphere

The radiansphere is a hypothetical sphere having a radius of one radianlength from the center of an antenna much smaller than the sphere. Physically, it marks the transition between the "near field" inside and the "far field" outside. While sending, the radiation field comprises stored energy and radiating power, the former predominating in the near field and the latter in the far field. The radiansphere is a measure of the "sphere of influence" of the antenna. It is a convenient reference for all radial distances.

Radiancircle

The radiancircle is the projection of the radiansphere and, conceived as such, is the interception area of the hypothetical isotropic radiator (to be defined) [2].

Radianlength

The radianlength is $1/2\pi$ wavelength (denoted $\lambda/2\pi$) which appears in many formulas for antennas and waves. Its principal significance is its role as the radius of the radiansphere and radiancircle. Any length dimension (l) may be eppressed in terms of its ratio over the radianlength ($2\pi l/\lambda$).

Wave Resistance

The wave resistance of free space ($R_0 = 120\pi = 377$ ohms) is the apparent resistance (V/I or E/H) of a square area of a plane wave in free space. It may be included in any impedance formula to provide the required dimension (ohms) in a significant and convenient form. For example, the reactance of an inductor usually includes the factor $\omega\mu_0$, for which may be substituted $R_0(2\pi/\lambda)$; the latter more directly provides the same dimensions, ohms per meter [5].

Small Antenna

A "small" antenna is one which is much smaller than the radiansphere. Conversely, it is one operating at a frequency so low that its sphere of influence is much greater than its size. It is characterized by a small power factor of radiation, meaning that its radiation resistance is much less than the principal component of its self-reactance. A small antenna is usually a simple electric or magnetic dipole. The near field depends on which kind of dipole, while the far field is the same for either kind. The electric dipole is a current element physically realizable, while the magnetic dipole is a flux element simulated by a current loop [5], [6].

Mutual Impedance

Fig. 1 shows the definition of the complex mutual impedance (Z_{12}) between two electric dipoles (as examples of small antennas). It includes the attenuation of amplitude and the retardation of angle with the distance from sending antenna to receiving antenna [2], [3].

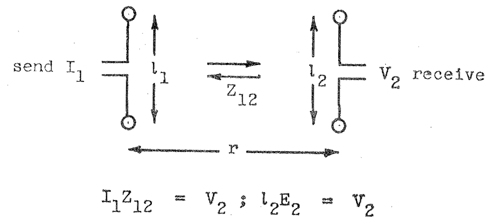

Fig. 1—Definition of mutual impedance between two small electric dipoles.

Efficiency

This is here defined as the maximum efficiency of transmission from a first antenna to a second. It is equal to the ratio of the power available from the second over the power input to the first (P_2/P_1). For present purposes, the associated connection circuits are assumed to be free of dissipation. The second antenna delivers the available power if its resistance is matched to a load resistance while tuning out the reactance of both. If the antennas are separated far enough to give low efficiency, the efficiency may be expressed simply in terms of the radiation resistance of both antennas and the magnitude of mutual impedance therebetween (R_1, R_2, $|Z_{12}|$) [1], [2], [4].

Isotrope

The isotropic radiator or isotrope is one which is conceived to radiate the same in all directions over the sphere in space. It is physically realizable in longitudinal waves (such as sound) but not in transverse waves (such as radio). In any case, it is a helpful concept as a reference for evaluating directivity [2], [4].

Directivity

The usual antennas concentrate their radiated power in some part of the sphere in space. In the direction of greatest concentration, the power ratio of directivity (g) has its maximum value (greater than unity). Inversely, we may say that effectively $1/g$ of the sphere is filled with radiation. The doughnut pattern of a small dipole fills $2/3$ of the sphere, so $g = 3/2$ [2], [4].

Electric Dipole

The electric dipole is one that radiates by virtue of a current flowing in a length of conductor and returning through the capacitance in the surrounding space. By reciprocity, when exposed to an electric field, it receives an induced voltage proportional to its length. It is the simplest type of radiator for theoretical analysis.

Magnetic Dipole

The magnetic dipole is one that radiates by virtue of magnetic flux from the dipole returning through the surrounding space. It is realized by current in a coil of conductor having a certain total area of coaxial turns. It is distinguished from the electric dipole in that the current returns in the conductor and not in the space capacitance. Its radiation may be computed by regarding each small element of conductor as an electric dipole. Some, but not all, of the general properties to be stated for the electric dipole are valid also for the magnetic dipole.

Far Component of Mutual Impedance

At radial distances much greater than the radianlength, the dominant component of mutual impedance is the one caused by the far field of radiation. Its magnitude may be computed from the mutual inductance between current elements. In doing this, we consider only the magnetic field, ignoring the electric field. The only deficiency is the absence of the retardation caused by the interaction of both fields.

Referring to Fig. 1, the mutual inductance between the two short current elements is given by the Neumann formula (Ramo-Whinnery), [9]:

$$L_{12} = \mu_0 \frac{l_1 l_2}{4\pi r}. \tag{1}$$

From this is computed the mutual impedance, expressed in terms of wave quantities:

$$|Z_{12}| = \omega L_{12} = \frac{R_0}{4\pi} \frac{l_1 l_2}{r(\lambda/2\pi)} = R_0 \frac{l_1 l_2}{2r\lambda} = 60\pi \frac{l_1 l_2}{r\lambda}. \tag{2}$$

The 4π in the denominator appears when formulating a spherical problem in terms of rationalized (cylindrical) units. It is notable that all length dimensions appear in ratios, while the impedance dimension is provided by R_0.

The phase angle of inductive reactance and the retardation by distance are easily added to this formula, as will be shown below in a complete formula.

In radial directions different from Fig. 1, the magnetic-field coupling is opposed in some degree by electric-field coupling to give the characteristic doughnut pattern.

Radiation Resistance

Since the radiation resistance is determined by the radiated power in the far field, it can be computed from the simple formula for mutual impedance. We use also the concept that the doughnut pattern fills only $\frac{2}{3}$ of the sphere. The radiation field is

$$|E_2| = \frac{V_2}{l_2} = \frac{|Z_{12} I_1|}{l_2} = \frac{R_0}{4\pi} \frac{l_1}{r(\lambda/2\pi)} |I_1|. \tag{3}$$

The radiated power, which determines the radiation resistance (R_1), is computed as the product of $\frac{2}{3}$ the area of the distance sphere times the power density of radiation outward through this sphere.

$$P_1 = R_1 |I_1|^2 = \frac{2}{3}(4\pi r^2)|E_2|^2/R_0$$
$$= \frac{2}{3}\frac{R_0}{4\pi}\left(\frac{2\pi l_1}{\lambda}\right)^2 |I_1|^2. \tag{4}$$

The radiation resistance is therefore

$$R_1 = \frac{2}{3}\frac{R_0}{4\pi}\left(\frac{2\pi l_1}{\lambda}\right)^2 = 20\left(\frac{2\pi l_1}{\lambda}\right)^2. \tag{5}$$

In this formula, the length of the dipole is expressed as a fraction of the radianlength $(2\pi l_1/\lambda)$.

Four such small dipoles may form the basis for computing the radiation resistance of a small square loop (of area $A_1 = l_1^2$). In a direction parallel to one pair of sides, only the other pair radiate and they nearly cancel each other. The residual far field is $2\pi l_1/\lambda$ of that of one side because this is the angle of the difference of their distance and retardation. The directive pattern is that of a small magnetic dipole which, like the small electric dipole, fills $\frac{2}{3}$ of the sphere in space. Therefore the radiation resistance of the loop is that of one side, multiplied by the power ratio $(2\pi l_1/\lambda)^2$.

$$R_1 = \frac{2}{3}\frac{R_0}{4\pi}\left(\frac{2\pi l_1}{\lambda}\right)^4 = 20\left(\frac{2\pi l_1}{\lambda}\right)^4$$
$$= 20\left[\frac{A_1}{(\lambda/2\pi)^2}\right]^2. \tag{6}$$

The strength of the equivalent magnetic dipole is proportional to the area (A_1). If there are several parallel turns carrying the same current (I_1), the effective area is the total area of all turns.

Efficiency in Terms of Interception Area

The available-power efficiency (if small) is simply formulated from the radiation quantities:

$$\frac{P_2}{P_1} = \frac{|Z_{12}|^2}{4R_1R_2}. \tag{7}$$

Substituting for these quantities in terms of length dimensions, and generalizing each R by changing from $\frac{2}{3}$ to $1/g$:

$$\frac{P_2}{P_1} = \frac{1}{4} g_1 g_2 \left(\frac{\lambda}{2\pi r}\right)^2 = g_1 g_2 \frac{\pi(\lambda/2\pi)^2}{4\pi r^2}$$
$$= g_1 g_2 \frac{\text{area of radian circle}}{\text{area of distance sphere}}. \tag{8}$$

The last two forms were discovered by the writer [2]; the first form was published by Friis [4].

Fig. 2 illustrates this rule for the basic simple case of two isotropes, while Fig. 3 does the same for the more general case, exemplified by two small dipoles.

Since sending and receiving are reciprocal functions, it is natural to identify the interception area of each one. This is diagramed in Fig. 3, showing the area each presents to the other. Letting this area be $A = g\pi(\lambda/2\pi)^2 = g$ radiancircles, for each antenna, the efficiency becomes [2], [4]:

$$\frac{P_2}{P_1} = \frac{A_1 A_2}{(4\pi r^2)\pi(\lambda/2\pi)^2} = \frac{A_1 A_2}{r^2\lambda^2}$$
$$= \frac{(\text{sending area})(\text{receiving area})}{(\text{distance sphere})(\text{radiancircle})}. \tag{9}$$

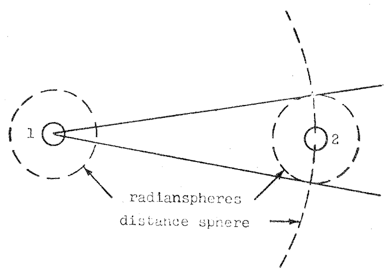

Fig. 2—Area of interception for two isotropes.

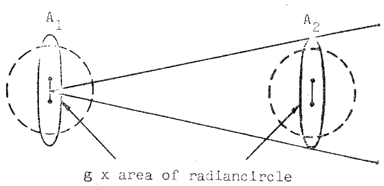

Fig. 3—Area of interception for two small dipoles.

By the simple formulas, two isotropes at a distance of one radianlength have a coupling efficiency of $\frac{1}{4}$. This is approximately valid, being in the transition between high and low efficiency. In general, this occurs at a distance of $\sqrt{g_1g_2}$ radianlengths, by (8). At lesser distances, the interaction complicates the formula for efficiency.

The mutual impedance may be expressed in terms of the values of radiation resistance by rearranging (7) and (8).

$$|Z_{12}| = 2\sqrt{R_1R_2}\sqrt{P_2/P_1} = \frac{\lambda}{2\pi r}\sqrt{g_1g_2R_1R_2}. \tag{10}$$

This is a corollary to the theorem of interception area [2]. It was independently discovered by Huntoon at NBS during the war while studying the problem of proximity fuzes [3].

The receiving antenna reradiates an amount of power equal to the available power it delivers to the matched load. If instead the antenna is tuned without adding any resistance, the received current is doubled. The second antenna then reradiates four times its available power. This rule is limited to a small antenna.

A large flat array with a reflector can be designed to intercept all the power incident on its area. Its power ratio of directivity, by comparison with the isotope, is then

$$g = \frac{\text{area}}{\pi(\lambda/2\pi)^2} = \frac{4\pi(\text{area})}{\lambda^2} = \frac{\text{area}}{\text{radiancircle}}. \tag{11}$$

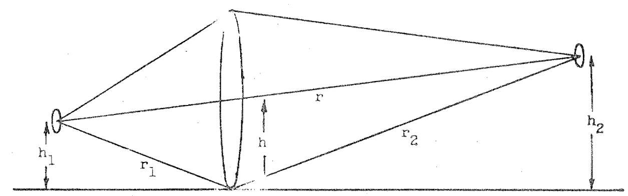

Fig. 4—Intermediate area of interception for two antennas over plane ground.

If the area is covered by many dipoles pitched $\frac{1}{2}$ wavelength in a rectangular array, the power ratio of directivity is seen to be π times the number of dipoles.

Two antennas may be located above the ground at heights so low that there is near-cancellation of direct and reflected waves. Fig. 4 shows the geometry of such a case. It is assumed that the path difference is less than one radianlength ($h_1h_2 < r\lambda/4\pi$) and that the ground is a flat surface with a reflection coefficient of minus one (which is typical of imperfect conductors near grazing incidence). It can be shown that

$$|Z_{12}| = \frac{2h_1h_2}{r^2}\sqrt{g_1g_2R_1R_2}; \qquad \frac{P_2}{P_1} = g_1g_2\left(\frac{h_1h_2}{r^2}\right)^2. \tag{12}$$

Taking the partial distances as shown,

$$r_1 = r\frac{h_1}{h_1+h_2}; \qquad r_2 = r\frac{h_2}{h_1+h_2}. \tag{13}$$

The intermediate height (h) is that of the direct line over the point of reflection:

$$h = \frac{2h_1h_2}{h_1+h_2}; \qquad 1/h = \tfrac{1}{2}(1/h_1 + 1/h_2). \tag{14}$$

This is taken as the radius of an intermediate circular area. It is found that the transmission efficiency is the product of two values, one computed by (9) from the first antenna to the intermediate circle, and the other from this circle to the second antenna. The proximity of the ground has the effect of an intermediate aperture as shown.

All Components of Mutual Impedance

In Fig. 1, the complex mutual impedance of two small dipoles has three terms at distances much greater than the dimension of the dipoles but not necessarily greater than the radianlength. These components are readily derived from the formula for the transverse electric field given in textbooks:

$$Z_{12} = \frac{R_0}{4\pi}\frac{j2\pi l_1}{\lambda}\frac{j2\pi l_2}{\lambda}\left[\left(\frac{2}{j2\pi r}\right)^3 + \left(\frac{\lambda}{j2\pi r}\right)^2 + \left(\frac{\lambda}{j2\pi r}\right)\right]\exp -\frac{j2\pi r}{\lambda}. \tag{15}$$

(ohms) (sphere) — (length angles) — (C) — (R) — (L) — (retard) — (distance angle)

The coefficient in front of the brackets [ ] is equal to $-\frac{3}{2}\sqrt{R_1R_2}$, in terms of radiation resistance. In Ramo-Whinnery [9] is found an expression which emphasizes the significance of the three components (C, R, L); this expression is revised as follows to give the three components the dimensions of impedance:

$$Z_{12} = \frac{l_1 l_2}{4\pi r^2}\,(1/j\omega\epsilon_0 r + R_0 + j\omega\mu_0 r)\exp - j2\pi r/\lambda. \quad (16)$$

This form is instructive and is also useful for evaluating the equivalent circuit elements (C, R, L). The preceding form (15) is the ultimate in dimensional simplicity.

Fig. 5 shows the network equivalent to two small dipoles, giving a breakdown of the three components of mutual impedance, and their variation with distance (r). They are marked (*) to denote that they are subject to retardation with distance.

Fig. 6 shows the variation of the three components with distance. At a distance of one radianlength, the three components are equal in magnitude, so that first and third cancel, leaving only the resistance. At lesser distances, the capacitive coupling predominates; at greater distances, the inductive coupling predominates, as derived above for the far field.

In any other direction, these components are modified. The far component disappears if either dipole is in line with the radial distance.

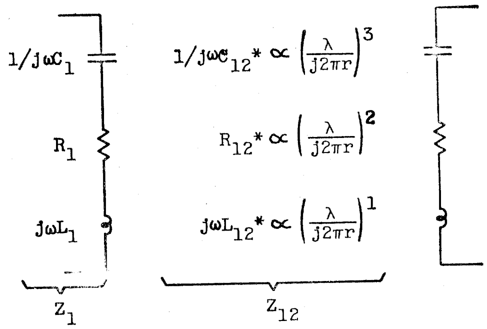

Fig. 5—Network equivalent to two small electric dipoles.

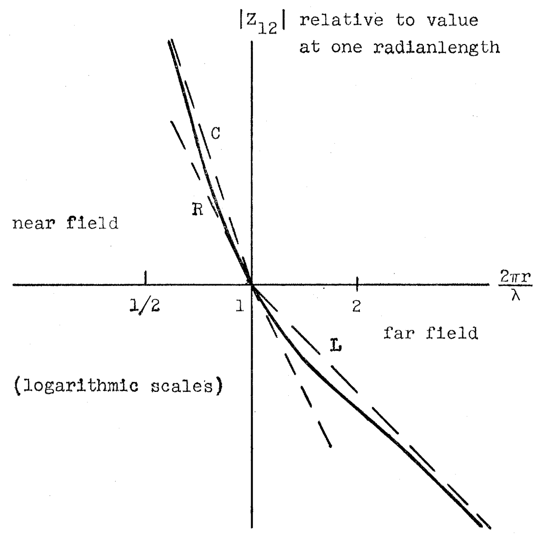

Fig. 6—Variation of components of mutual impedance.

Spherical Small Antenna

In relation to a spherical wave and the radiansphere, the ideal shape of a small antenna might be spherical. There is one such antenna that is significant. It is a "magnetic dipole" simulated by a spherical inductor [10], [12].

Fig. 7 shows such an inductor. Its winding is pitched uniformly in the axial direction. Its core may be filled with magnetic material (k_m).

If the length of wire is much less than the resonant length, the magnetic field inside is uniform, and outside has the same pattern as that of a small magnetic dipole. (Such an inductor is mentioned by Maxwell but is seldom found in the more recent literature; the writer made use of this concept about 1941.)

The inductance of this sphere is

$$L = \frac{2\pi}{3}\,\mu_0 a n^2\,\frac{1}{1 + 2/k_m}\,. \quad (17)$$

Its radiation resistance is

$$R = \frac{2\pi}{3}\,R_0 n^2\left(\frac{2\pi a}{\lambda}\right)^4\left(\frac{1}{1 + 2/k_m}\right)^2. \quad (18)$$

Its inductive reactance is

$$\omega L = \frac{2\pi}{3}\,R_0 n^2\,\frac{2\pi a}{\lambda}\,\frac{1}{1 + 2/k_m}\,. \quad (19)$$

Therefore the radiation power factor is

$$p = R/\omega L = \left(\frac{2\pi a}{\lambda}\right)^3\frac{1}{1 + 2/k_m}$$
$$= \frac{\text{volume of inductor sphere}}{\text{volume of radiansphere}}\,\frac{1}{1 + 2/k_m}\,. \quad (20)$$

This relation reaches the ultimate simplicity for the ideal case of a perfect magnetic core ($k_m = \infty$) so that there is no stored energy inside the coil. This limiting case is represented in Fig. 8.

The radiansphere may be regarded as a hypothetical inductor whose internal energy is the stored energy of the magnetic field, and whose external energy is the radiating power. The small antenna radiates by virtue of its coefficient of coupling with the radiansphere; the above ratio (20) is proportional to the square of this coefficient of coupling.

Practical Small Antennas

In a previous paper, the writer has treated the topic of practical small antennas [5]. Special emphasis was placed on the role of the volume occupied by the antenna in determining its radiation power factor. A cylindrical volume was taken as a basis for comparing electric dipoles with magnetic dipoles (air-core coils).

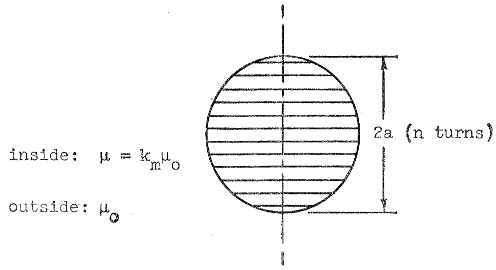

Fig. 7—Spherical inductor.

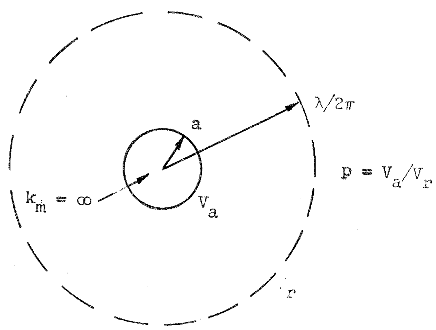

Fig. 8—Spherical inductor in radiansphere.

Some of the most common forms of small antennas are of such shape that the occupied volume is no longer significant. This is true of an electric dipole made of a straight wire or rod or tower. It is also true of a magnetic dipole made of a thin loop of wire. Either of these would require a certain size of sphere to contain it. Since this volume is only partially utilized, it is presumptive that the antenna would have a radiation power factor much less than the theoretical upper limit for this size of sphere.

The opposite extreme is a flat capacitor with air-dielectric or a long inductor with air core. In these cases, the radiation power factor approaches a lower limit of 2/9 the value for an ideal sphere (with perfect magnetic core) of the same volume.

It is interesting to compare two small antennas of opposite kinds occupying the same circular-cylindrical space, namely the disk capacitor (C) and the solenoid inductor (L), each having an air core. The shape chosen is a cylinder of equal diameter and height. As compared with an ideal sphere of the same volume (having no energy stored inside), each kind has a power factor less than the ideal by the following factors:

$$C: \quad \frac{8}{\pi}\,\frac{2}{9} = 0.57$$

$$L: \quad \left(1 + \frac{4}{3\pi}\right)\frac{2}{9} = 0.32.$$

In this rating, the power factor of the capacitor is about twice that of the inductor. However, the latter can be increased by a factor of two or more by inserting an iron core, while the former cannot be increased by any known materials.

The comparison with air cores brings out a basic difference between the two kinds. The external (useful) stored energy of the capacitor is about $\frac{2}{3}$ of the total, while that of the inductor is about $\frac{1}{3}$. This is because the inside and outside flux paths differ in impedance in a ratio of about two to one; these paths are in parallel for the capacitor and in series for the inductor. Decreasing the effective length of internal flux path by inserting some material has the effect of increasing the stored energy in the capacitor but decreasing it in the inductor. The latter is advantageous.

If a small antenna is restricted in its maximum dimension but not in its occupied volume, the radiation power factor is increased by utilizing as much as possible of the volume of a sphere whose diameter is equal to this dimension. The cylinder discussed above is a good practical compromise. The practical limitations of capacitor and inductor are only slightly different, so the choice may be determined by other considerations (such as wave polarization, loss power factor, associated circuits, construction, and environment).

A special case is a small antenna operating underground or underwater. These mediums are dissipative toward a electric field but not a magnetic field. Therefore the loop antenna is much to be preferred for efficiency of radiation in either of these environments [11].

Radiation Shield

For purposes of measurement, it may be desired to remove the radiation resistance of a small antenna while retaining its other properties (loss resistance, capacitance, inductance). This can be accomplished to a close approximation by enclosing the antenna in a radiation shield which ideally is a perfectly conducting spherical shell whose inner surface is located at the radiansphere. (See Fig. 8, for example.) This prevents the radiation while causing little disturbance of the near field. In practice, the size, shape, and material are not critical. A cylinder with one or both ends open may suffice.

The writer devised this test for a very small loop antenna operating at a frequency such that the radiansphere had a convenient size; the loop was in an oscillating circuit so the radiation shield caused an increase in the amplitude of oscillation. The increase in amplitude was a measure of the radiation efficiency. In general, the radiation shield enables the separate determination of loss resistance and radiation resistance.

Conclusion

The radiansphere around a small antenna is logically regarded as the boundary between the near field of stored energy and the far field of radiating power. There is not a definite boundary but rather a transition, since the terms associated with the near field predominate inside and those associated with the far field predominate outside. The interception area defined for the hypothetical isotropic antenna is the area of the radiancircle, a projection of the radiansphere, so the latter is logically regarded as the sphere of influence of such an antenna. An idealized small spherical antenna is found to have a radiation power factor equal to the ratio of its volume over that of the radiansphere. A radiation shield is described whose ideal location is at the radiansphere. All of these concepts are helpful in visualizing and remembering the rules governing small antennas, especially their near field and far field.

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Voltage Breakdown Characteristics of Microwave Antennas*

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***Summary*—The problem of voltage breakdown and its effects is discussed for a pulse antenna system. Voltage breakdown occurs at power levels when the pressure is reduced. The minimum breakdown potential occurs approximately at the pressure where the frequency of collision between electrons and gas atoms is equal to the frequency of the applied field. Experiments were made to determine the power levels required to produce breakdown and the effect of breakdown on the VSWR, pulse shape, radiation pattern, and radiated power. It is shown that all four of the quantities vary with pulse width and peak power.**

Introduction

At low pressures, antennas are susceptible to voltage breakdown. In the case of antennas on high-altitude vehicles, there are indications that very low power is sufficient to initiate and maintain breakdown.[1–3] When voltage breakdown occurs, the effect is fourfold: the input impedance is altered; the pulse shape is modified; the total radiated power is decreased; and the radiation pattern is changed.

Any evaluation of a system that is to perform at high altitude must consider the voltage breakdown effect. Therefore, an experimental investigation of antenna breakdown and the problems associated with it was initiated. A brief description of the voltage breakdown problem is presented here, followed by a report on a series of measurements made on a particular antenna type to determine its electrical characteristics under breakdown conditions.

Voltage Breakdown Phenomena

The theory of high-frequency gas-discharge breakdown has been well covered in the literature for nonradiating structures.[4–6] It has been shown that primary

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