# Small Antennas

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*Abstract*—A small antenna is one whose size is a small fraction of the wavelength. It is a capacitor or inductor, and it is tuned to resonance by a reactor of opposite kind. Its bandwidth of impedance matching is subject to a fundamental limitation measured by its "radiation power factor" which is proportional to its "effective volume". These principles are reviewed in the light of a quarter-century of experience. They are related to various practical configurations, including flush radiators for mounting on aircraft. Among the examples, one extreme is a small one-turn loop of wide strip, tuned by an integral capacitor. The opposite extreme is the largest antenna in the world, which is a "small antenna" in terms of its operating wavelength. In each of these extremes, the radiation power factor is much less than one percent.

## I. Introduction

A "SMALL ANTENNA" is here defined as one occupying a small fraction of one radiansphere in space. Typically its greatest dimension is less than $\frac{1}{4}$ wavelength (including any image in a ground plane). Some of its properties and available performance are limited by its size and the laws of nature. An appreciation of these limitations has proved helpful in arriving at practical designs.

The radiansphere is the spherical volume having a radius of $1/2\pi$ wavelength [10]. It is a logical reference here because, around a small antenna, it is the space occupied mainly by the stored energy of its electric or magnetic field.

Some limitations are peculiar to a passive network, where the concepts of efficiency, impedance matching and frequency bandwidth are essential and may be the controlling factors in performance evaluation. This discussion is directed mainly to these limitations in relation to small size. This subject has been on the record for a quarter-century but is still too little taught and appreciated. It centers around the term, "radiation power factor" and its proportionality to volume [2].

As in any area of engineering compromise, there have been some ingenious developments for realizing some qualities at the expense of others. A valid comparison of alternatives requires careful description and evaluation in terms of well defined quantities, especially in the use of terms such as efficiency and impedance matching. Also in the size comparison of circuits qualified for high power or low power [11].

An outline of some of the relevant principles will be followed by a brief reference to the background in the use of an amplifier with a small antenna for reception. Then the principal topic will be introduced in terms of the bandwidth limitations of impedance matching with a resonant circuit, which is a tuned antenna circuit in this discussion.

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TABLE I
Comparison of Topics of Efficiency and Amplification

| EFFICIENCY | TOPIC | AMPLIFICATION |
|---|---|---|
| PASSIVE | LINEAR NETWORK | ACTIVE |
| ESSENTIAL | IMPEDANCE MATCHING | OPTIONAL |
| NO | TOLERANCE OF LOSSES | YES |
| THERMAL | NOISE | AMPLIFIED |
| NO | POWER LIMITING | YES |

The radiation power factor will be reviewed in concept and in some applications to typical antennas in the form of capacitors and inductors. Some special applications will be described for flush mounting and for VLF transmission and reception. In every case, the efficiency and/or bandwidth is seen to be limited ultimately by size.

## II. Principles

Table I shows a comparison between efficiency and amplification, referring to some topics relevant to small antennas. Its purpose is to emphasize the distinction between efficiency and amplification, the former being the basis for this presentation. The relations in this table may help to bring out the accepted meanings of various terms.

Efficiency implies the utilization of the amount of radiated signal power that can be intercepted by the receiver. If the antenna is small, the greatest power transfer to a circuit requires impedance matching. This is achieved in a passive network by tuning the antenna and coupling to the circuit.

Amplification implies the utilization of the intercepted signal, but the excitation of the amplifier may not require impedance matching in the active network. This may facilitate a wideband design, as in one example to be shown. However, the amplifier may add much to the thermal noise generated in the antenna dissipation.

In a linear network, efficiency is associated with a passive network, while amplification is associated with an active network. In a weak-signal receiver, linearity is not a primary problem. In a power transmitter, however, an active network imposes an upper limit.

In general, efficiency is reduced by losses. This is particularly true in a small antenna where the radiation power factor is small and may be far exceeded by the loss power factor. In a weak-signal receiver, an amplifier can make up for losses in respect to signal strength, but only with increasing background of thermal noise. In a power transmitter, the power rating must be increased to cover losses.

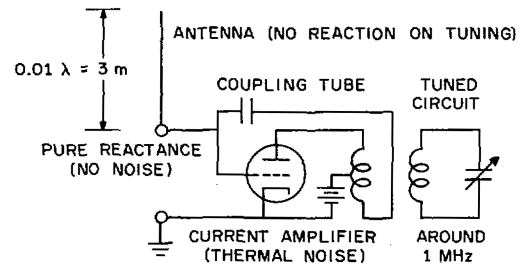

Fig. 1. Small antenna with wideband coupling tube, used in broadcast receivers (1928).

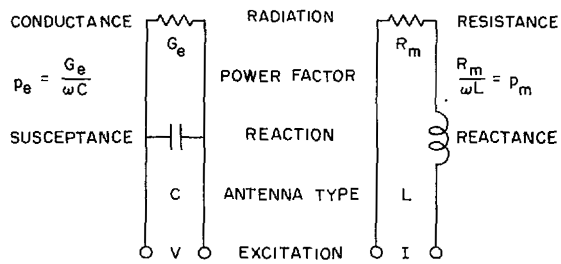

Fig. 2. Radiation power factor of small antenna.

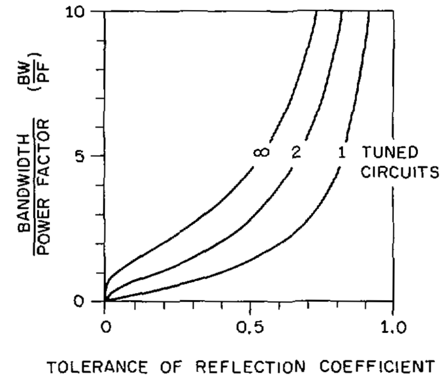

Fig. 3. Bandwidth of matching with tuned circuits.

These relations are emphasized because there have been some invalid ratings of small antennas associated with active devices serving as amplifiers. The greatest confusion has been associated with transmitters, by ignoring the power limitations imposed by small active devices. These limitations are not avoided by any particular relation between the small antenna and the amplifier.

## III. Background

The wideband utilization of a small antenna was accomplished in a receiver about a half-century ago. That history is relevant to the more recent proposals using an amplifier in conjunction with a small antenna [11].

Fig. 1 shows a circuit that was commonly used in radio broadcast receivers about 1928. It operated over a frequency ratio of 1:3. A short wire is simply connected to the grid of the first tube. It bears a striking resemblance to some recent proposals, but using a tube instead of a transistor, and at lower frequencies. It substituted amplification for antenna tuning. It increased the noise threshold and also suffered from crossmodulation of all signals by any one strong signal. Then the pendulum swung and it was superseded by double tuning ahead of the first tube. The tuning yielded efficiency over noise and also preselection against crossmodulation.

## IV. Frequency Bandwidth of Impedance Matching

There are limitations on the frequency bandwidth of impedance matching between a resonant circuit (antenna) and a generator or load. A quarter-century has elapsed since these limitations were developed and clearly stated [5]. In contrast to the history of small antennas, these limitations have been widely taught and appreciated.

The bandwidth of matching, within any specified tolerance of reflection, is proportional to the resonance bandwidth of the resonant circuit. A small bandwidth is logically expressed in terms of the power factor of its reactance, in the manner taught to the writer by Prof. Hazeltine just 50 years ago [1]. Its common expression in terms of $1/Q$ is neither logical nor helpful in clear exposition. The term dissipation factor is numerically equal to power factor but is counter-descriptive of a useful load (as here).

Fig. 2 shows the circuit properties of a small antenna, describing its radiation power factor (PF). The antenna may behave as a capacitor ($C$) or inductor ($L$), and either is to be resonated by a reactor of the opposite kind. Dissipation (other than radiation) is here ignored, because it is treated in the earlier paper [2]. The nominal bandwidth of the resonator is the PF ($p$) times the frequency of resonance, as usual.

Fig. 3 is the bandwidth of matching within any specified tolerance of reflection ($\rho$) as given in 1948 by Fano [5]. It is graphed in the terms of the present discussion. For each graph, the number of tuned circuits includes the antenna circuit and any that are added for increasing the bandwidth of matching. The added circuits are taken to be free of dissipation. Usually double tuning is used, in which case the added circuit can reduce the reflection coefficient to the square of its value for single tuning.

## V. The Radiation Power Factor

The term "radiation power factor" is a natural one introduced by the author in 1947 [2]. It is descriptive of the radiation of real power from a small antenna taking a much larger value of reactive power. It is applicable alike to either kind of reactor and its value is limited by some measure of the size in either kind.

Fig. 4 shows small antennas of both kinds ($C$ and $L$) occupying equal cylindrical spaces [2]. They are here used for introducing the relation between radiation PF and size.

A small antenna of either kind is basically a reactor with some small value of PF associated with useful radiation. The latter depends primarily on its size relative to the wavelength ($\lambda$), as discovered by the writer [2]. The size may be stated relative to the radianlength ($\lambda/2\pi$) in terms of either of two values of reference volume:

$$\text{radiancube} = V_c = \left(\frac{\lambda}{2\pi}\right)^3 = \frac{3}{4\pi} V_s \qquad (1)$$

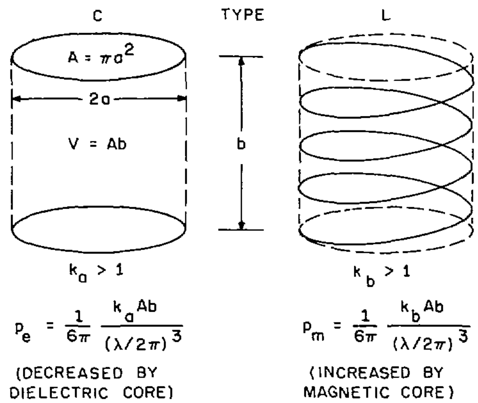


Fig. 4. Radiation power factor in terms of equivalent volume.

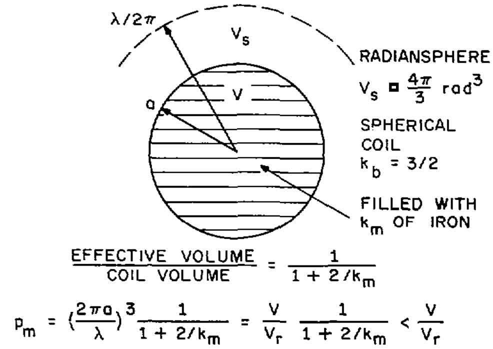


Fig. 5. Spherical coil with magnetic core.

or

$$\text{radiansphere} = V_s = \frac{4\pi}{3}\left(\frac{\lambda}{2\pi}\right)^3 = \frac{4\pi}{3} V_c. \tag{2}$$

The former was used in the writer's first paper. The latter is particularly significant in radiation because it defines the space in which the reactive power density exceeds the radiation power density [10]. Also the latter is convenient if the antenna is spherical [9] or its effective volume is expressed as a sphere.

In either type of antenna, the radiation PF is found to be proportional to volume and also to a shape factor. The cylindrical volume ($V = Ab$) is here multiplied by a shape factor ($k_a$ or $k_b > 1$) to give the effective volume ($V' = k_aAb$ or $k_bAb$). Then the general formula is

$$\text{rad PF} = p = \frac{1}{6\pi}\frac{V'}{V_c} = \frac{2}{9}\frac{V'}{V_s}. \tag{3}$$

The effective volume may be stated as a sphere of radius ($a'$), in which case

$$V' = \frac{4\pi}{3}a'^3, \quad p = \frac{2}{9}\left(\frac{2\pi a'}{\lambda}\right)^3, \quad a' = \frac{\lambda}{2\pi}\left(\frac{9}{2}p\right)^{1/3}. \tag{4}$$

It is noted in passing that a certain shape of self-resonant coil radiates equally as both $C$ and $L$, in which case the total radiation PF is double either one [3].

There is one theoretical case of a small coil which has the greatest radiation PF obtainable within a spherical volume. Fig. 5 shows such a coil and its relation to the radiansphere ($V_s$) [9], [10]. The effective volume of an empty spherical coil has a shape factor 3/2. Filling with a perfect magnetic core ($k_m = \infty$) multiplies the effective volume by 3:

$$p_m = \frac{2}{9}\frac{(3)(3/2)V}{V_s} = \frac{V}{V_s} = \left(\frac{2\pi a}{\lambda}\right)^3. \tag{5}$$

This is indicated by the shaded sphere ($a$).

This idealized case depicts the physical meaning of the radiation PF that cannot be exceeded. Outside the sphere occupied by the antenna, there is stored energy or reactive power that conceptually fills the radiansphere [10], but there is none inside the antenna sphere. The reactive power density, which is dominant in the radiation within the radiansphere, is related to the real power density, which is dominant in the radiation outside.

In a rigorous description of the electromagnetic field from a small dipole of either kind, the radiation of power in the far-field is accompanied by stored energy which is mostly located in the near-field (within the radiansphere) [4], [10]. The small spherical inductor in Fig. 5 is conceptually filled with perfect magnetic material, so there is no stored energy inside the sphere. This removes the "avoidable" stored energy, leaving only the "unavoidable" amount outside the inductor but mostly inside the radiansphere. This unavoidable stored energy is what imposes a fundamental limitation on the obtainable radiation PF.

One of the fallacies in some studies has been the provision of dielectric or magnetic material outside of the space occupied by the antenna conductors, without including that material in rating the size of the antenna. The fundamental limitations are based on the size of all the material structure which forms the antenna. Likewise, such material would naturally be included in a practical evaluation of the size. Fig. 5 shows the empty space outside the antenna but inside the radiansphere ($V_s$) which space is filled with stored energy and therefore reduces the radiation PF of the antenna.

## VI. Application to Typical Antennas

The radiation PF may be evaluated for any kind of small antenna. From its value, we may state the effective volume of the antenna, as formulated (4):

$$V' = 6\pi p V_c = \tfrac{9}{2} p V_s, \quad a' = \frac{\lambda}{2\pi}\left(\tfrac{9}{2}p\right)^{1/3}.$$

This is a useful quantity which can be shown on a space drawing. It gives a direct comparison of the bandwidth capability of different structures. It will be shown for $C$ and $L$ antennas of elementary configurations. It will be drawn as a dashed circle the size of the spherical effective volume.

Fig. 6 shows some examples of an electric dipole with a linear axis of symmetry. A thin wire (a) and a thick conical conductor (b) differ greatly in the occupied volume, but much less in effective volume. The latter is influenced most by length and less by the smaller transverse dimensions.

Fig. 6(c) shows a pair of separated discs [2], which is found to approach the greatest effective volume for some shapes within limited length and diameter. However, any

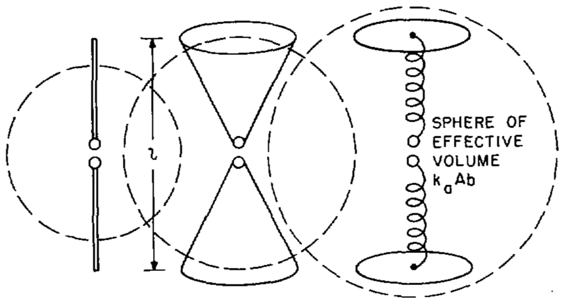

Fig. 6. Effective volume of axial electric dipole.

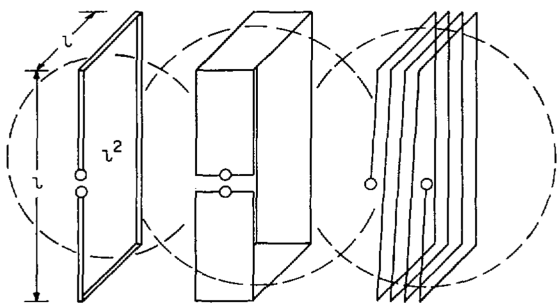

Fig. 7. Effective volume of square loop.

intermediate connecting wires would detract from this rating. The full value of the radiation PF can be realized by the use of a tuning inductor distributed along the axial line between the discs. It is proportioned to conform to the natural pattern of electric potential, thereby contributing no extra amount to the stored electric energy. A coil of small diameter may be used to avoid extra (cross-polarized) radiation therefrom. The spherical effective volume may extend beyond the length between the discs, as shown. This occurs if the disc diameter exceeds $\frac{1}{4}$ the length ($2a > b/4$), as in the example shown. This may be interpreted as a "sphere of influence" extending beyond the antenna structure.

In further reference to Fig. 6(c), there is a pair of end electrodes which will give the greatest radiation PF within a cylindrical boundary. At each end, a hollow cup is connected with its open end toward the center. Its depth is proportioned to maximize the radiation PF. No greater value can be obtained by simple conductors subject to the stated constraints.

Fig. 7 shows some examples of a loop inductor on a square frame. A thin wire (a) and a wide strip (b) differ rather little in effective volume, because it is influenced most by the size of the square. A multiturn loop (c) has nearly the same effective volume as one turn occupying the same space. This is one of the principal conclusions presented in the writer's first paper [2]. It superseded some incomplete evaluations based on the concept of "effective height" of a number of turns, irrespective of their width and spacing.

Referring again to Fig. 4, the shape factors are related to the shape in opposite ways in the two kinds ($C$ and $L$).

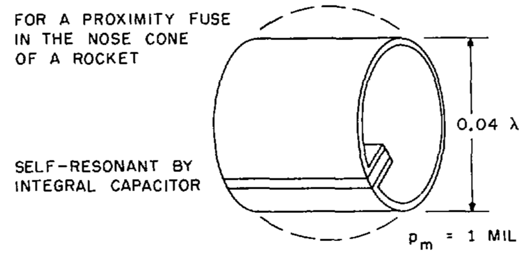

Fig. 8. One-turn loop of wide strip.

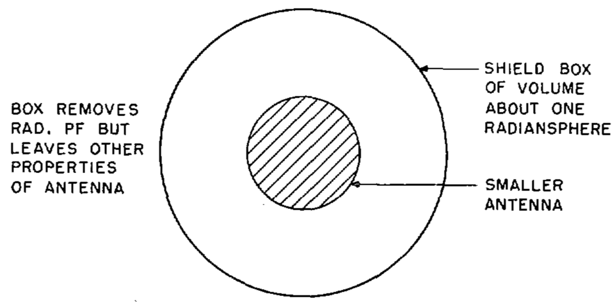

Fig. 9. Radiation shield for use in measuring radiation power factor.

With greater ratio of length/diameter ($b/2a$), one factor ($k_a$ for $C$) is greater and the other ($k_b$ for $L$) is smaller. Therefore the utilization of volume is greater for the $C$ type made of a long wire or for the $L$ type made of a "short coil" or loop. These are exemplified in Figs. 6 and 7. Each of these has large and small dimensions, and the smaller dimensions may be less significant in a practical allocation of space.

In the writer's experience, the concept of radiation PF was first applied to the design of a very small loop antenna for coaxial location in the nose cone of a small rocket. Fig. 8 shows the resulting one turn of wide strip. It superseded some attempts to design a multiturn loop. It is resonated by an integral capacitor made of a ceramic slab metallized on both faces. It proved superior in performance, simplicity, and ruggedness. It may have been the smallest antenna then known to realize about 50 percent radiation efficiency, the size being rated in fractions of the wavelength. Its diameter and length were about 0.04 wavelength so its radius was about 0.12 radianlength. It was measured by a method to be described here.

For efficiency of radiation, a small antenna of one kind is resonated by a reactor of the opposite kind. Then

$$\text{radiation efficiency} = \frac{\text{radiation PF}}{\text{rad PF} + \text{loss PF}}. \qquad (6)$$

In a very small antenna, the radiation and loss power factors may be so small that their ratio is difficult to measure. In any case, how would they be separated in measurement? Direct measurement of radiated power is laborious. Another method was developed, using a "radiation shield" [10].

Fig. 9 shows the concept of the radiation shield. Its purpose is to avoid radiation of power while leaving the inherent dissipation in the resonant circuit of the small antenna. The shield is a box with conductive walls for preventing radiation. Its size and shape are noncritical,

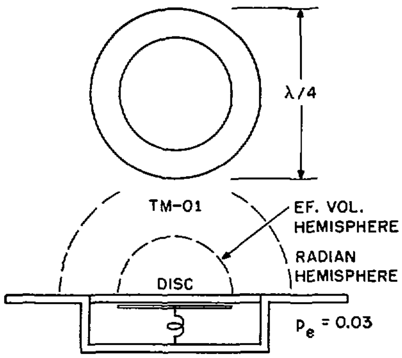


Fig. 10. Flush disc capacitor.

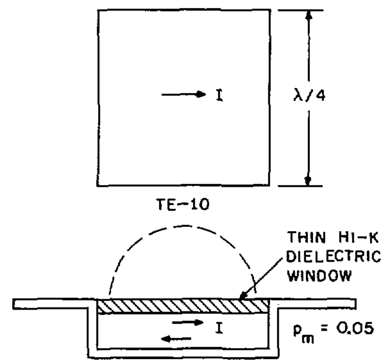


Fig. 11. Flush cavity inductor with dielectric window.

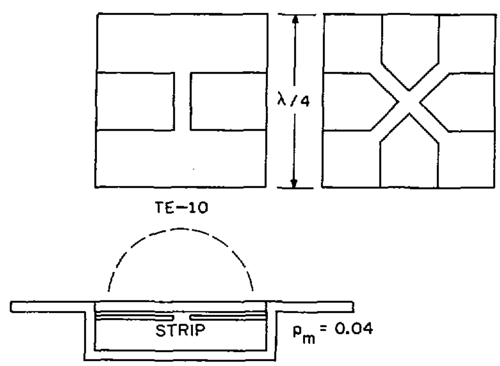


Fig. 12. Flush strip inductor.

but the theoretical ideal is a radiansphere as indicated. It should be much larger than the antenna to be shielded, so as to retain substantially the reactance and loss PF of the antenna. Then the PF is measured with and without the shield, for evaluating the power efficiency of the useful radiation [10]. In the design shown in Fig. 8, the circuit was included in an oscillator, so the effect of the shield on the amplitude of oscillation could be interpreted in terms of radiation efficiency.

## VII. Flush Antennas

A useful family of small antennas comprises those that are recessed in a shield surface, such as a ground plane or the skin of an aircraft. Some may be inherently flush designs, while others may be suited for operation adjacent to a shield surface, whether recessed or not. The antenna may be $C$ or $L$ type, either one radiating in a polarization compatible with the shield surface.

Fig. 10 shows a flush disc capacitor. (It is sometimes termed an "annular slot.") This capacitor in the flush mounting may be compared with the same capacitor just above the surface. The recessing somewhat reduces the radiation PF. The remaining effective volume is that of a hemisphere indicated by the dashed semicircle. Its size is comparable with that of the disc. The cylindrical walls may be regarded as a short length of waveguide beyond cutoff, operating in the lowest TM mode (circular TM-01, as shown, or rectangular TM-11). The capacitor may be resonated by an integral inductor as shown. In any cavity, there is a size and shape of disc that can yield the greatest radiation PF. The primary factor is the size of the cavity.

The evaluation of a flush antenna includes the shield surface. It is necessary first to evaluate the radiation PF by some method of computation. Then it can be stated in terms of a volume ratio. Here we consider the half-space of radiation and show the hemisphere of $\frac{1}{2}V'$ which may then be compared with the half-radiansphere, $\frac{1}{2}V_s$. The radii are retained ($a'$ and $\lambda/2\pi$). An antenna located on the surface (not recessed) could be considered with its image to yield the complete sphere of $V'$ to be compared with the radiansphere $V_s$. Then $\frac{1}{2}$ of each may be shown above the shield plane, as for the flush antenna.

The disc capacitor radiates in the same mode as a small vertical electric dipole, by virtue of vertical electric flux from the disc. This is vertical polarization on the plane of the shield, with omnidirective radiation. The other examples of a flush antenna, to be shown here, radiate as a small horizontal magnetic dipole, by virtue of magnetic flux leaving the cavity on one side and returning on the other side. This is vertical polarization but directive in a figure-eight pattern. Omnidirective radiation can be provided by quadrature excitation of two crossed modes in the same cavity. The radiation PF of either kind is reduced by recessing, but the magnetic dipole suffers less reduction.

Fig. 11 shows an idealized cavity resonator which radiates as an inductor. The cavity is covered by a thin window of high-$k$ dielectric which serves two purposes. It completes the current loop indicated by the arrows ($I$). Also it provides, in effect, series capacitance which resonates the current loop. The cylindrical walls and the aperture excitation may be regarded as the lowest (cutoff) TE mode (circular TE-11), or rectangular TE-10 or TE-01, as shown). Each of these modes has two crossed orientations, of which one is indicated by the current loop. The continuous dielectric sheet on a square (or circular) cavity resonates the two crossed modes. Because each resonance is in the lowest mode, it involves the smallest amount of stored energy relative to radiated power, and therefore the greatest value of radiation PF.

Fig. 12 shows some practical designs which yield nearly the same performance by the use of conductive strips on ordinary (low-$k$) dielectric windows. (High-$k$ dielectric is not required.) Here the radiating inductor (strip) and the resonating series capacitor (gap) are apparent. The two

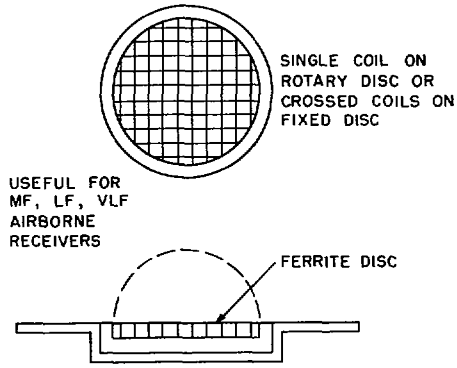


Fig. 13. Flush inductor on thin ferrite disc.

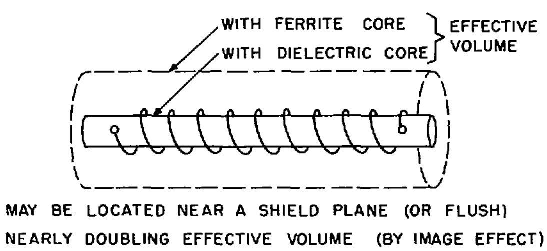


Fig. 14. Long coil on ferrite rod.

alternatives are shown, one mode or a pair of crossed modes. Practical designs about $\lambda/4$ square have been made with radiation PF about 0.04. This is about the largest size that follows the rules of a small antenna.

The required coupling with any of the resonant antennas in Figs. 10–12 may be provided by another (smaller) resonator located within the cavity. This enables the bandwidth of matching shown by the intermediate graph in Fig. 3. Each of these is suited for self-resonance, and requires some depth of activity to hold down the extra amount of energy storage in this nonradiating space.

Fig. 13 shows a flush inductor made of crossed coils on a thin magnetic disc. At medium or low frequencies (MF, LF, VLF) the available ferrite materials [12] can provide a magnetic core which is a return path nearly free of extra energy storage, even in the thin disc; also which adds very little dissipation. The required depth of cavity is then only sufficient to take the disc thickness with some margin. Relative to the wavelength at the lower frequencies, the antenna is too small to enable high efficiency, even at its frequency of resonance, so it is useful only for reception. A rotary coil or crossed coils can be used for a direction finder or omnidirectional reception. The principal application is on the skin of an aircraft.

Fig. 14 shows the ferrite-rod inductor which is the antenna most commonly used in small broadcast receivers (MF, around 1 MHz). The ferrite rod greatly increases the effective volume of a thin coil, as indicated. The effective volume is then determined primarily by the length, rather than the diameter, of the coil. Like the ferrite disc, this can be used close (parallel) to a shield surface or recessed in the surface.

Here we may note that a long coil, with its small shape factor ($k_b \to 1$), can have its effective volume greatly

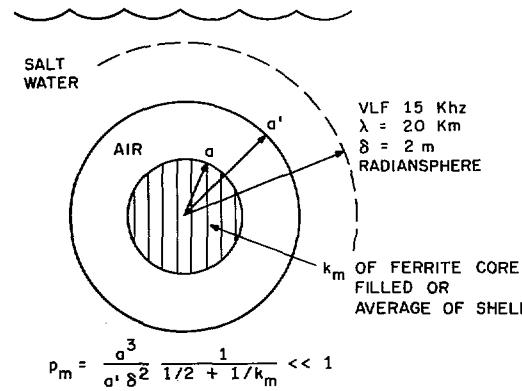


Fig. 15. Inductor in radome submerged in sea water.

increased by a ferrite core. On the other hand, a parallel-plate capacitor, with its small shape factor ($k_a \to 1$), can only have its effective volume decreased by a dielectric core. This is one respect in which the inductor offers more opportunity in design. In another respect, the number of turns can be used to set the impedance level, a freedom that may be desired but is unavailable in a simple capacitor.

If a long coil as a magnetic dipole were filled with perfect magnetic material, its effective volume would be comparable with that of an equally long conductor as an electric dipole. If the coil had many turns, they could theoretically be distributed (crowded toward the ends) to give an effective volume greater than that of a pair of discs far apart, Fig. 6(a). If the coil is not too thin, this result can be approximated at the lower frequencies with many turns on a ferrite core.

## VIII. Antennas for VLF

The greater the wavelength, the more relevant may be the concept of a small antenna. Current activities go as low as 10 kHz with a wavelength of 30 km. Even the largest of transmitter antennas is small in terms of this wavelength, or its radianlength of 5 km. For underwater reception, however, the radianlength or skin depth in salt water is only a few meters, so a small antenna may occupy a substantial fraction of this size. The latter will be discussed first, as another example of a small inductor.

For submarine reception of VLF signals in salt water, an inductor in a hollow cavity (radome) is the preferred type [8]. As compared with a capacitor, its efficiency is greater because the conductivity of the water causes near-field losses in response to electric field but not magnetic field. Also there is no need for conductive contact with the water.

Fig. 15 shows an idealized small antenna in a submarine cavity [8], [9]. It is a spherical coil with a magnetic core, as shown in Fig. 5. In the water, the radianlength is equal to the skin depth ($\delta$). At 15 kHz, this is about 2 m. The size of the cavity is much less, and the coil still less, so it is a small antenna in this environment. The radiation PF indicates two qualities, the desired coupling to the medium and the undesired dissipation in the medium. The former is proportional to the coil volume, and is increased by the magnetic core. The latter is decreased by increasing the

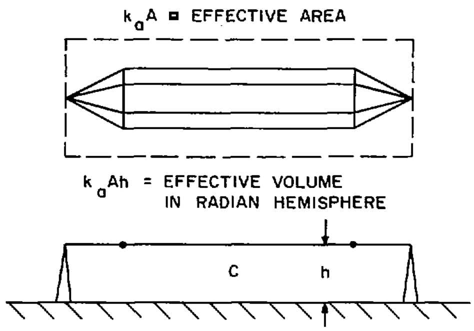


Fig. 16. Large flat-top capacitor which is still small relative to wavelength.

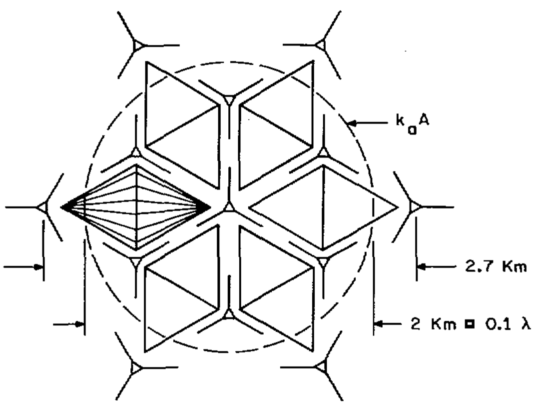


Fig. 17. Large VLF antenna (plan view).

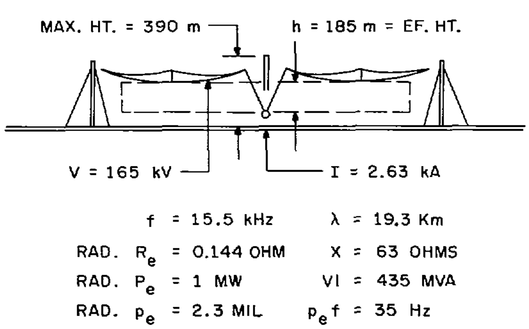


Fig. 18. Large VLF antenna (elevation view).

cavity radius. The coil is in the vertical plane for vertical polarization. Crossed coils may be used for omnidirective reception and direction finding.

For efficient transmission at the lower frequencies, one of the early simple types is the one shown in Fig. 16 [7]. It is a "flat-top" grid of wires forming a capacitor with ground as the lower conductor. In the terms of small antennas, it may be described in the manner indicated. The effective height ($h$) is related to the radiation resistance. The capacitance enables the statement of an effective area ($k_aA$) as noted. The effective volume ($k_aAh$) in half-space is compared with $\frac{1}{2}$ radiansphere to determine the radiation PF. It is notable that the grid of many wires may provide an effective area greater than that of the grid, in spite of the much smaller area of conductor.

As an extreme example, we shall consider the later one of the two largest antennas in the world. They are the Navy transmitters located at Cutler, Me., (NAA) and Northwest Cape, Australia, (NWC). The latter was commissioned in 1967. It is taken as an example because it is the simpler. Figs. 17 and 18 show the plan and elevation views of the structure. It operates down to about 15 kHz, a wavelength of 20 km.

The lowest "specification" frequency determines the required size. At this frequency, the following statistics are relevant:

| | |
|---|---|
| frequency | 15.5 kHz |
| wavelength | $\lambda = 19.3$ km |
| extreme diameter | 2.7 km $= \frac{1}{7}\lambda$ |
| center-tower height | 390 m |
| effective height | 185 m $= \frac{1}{104}\lambda$ |
| capacitance | 0.163 $\mu$F |
| effective area | 3.4 (km)$^2$ |
| effective volume | $V' = 0.63$ (km)$^3$ |
| radiation resistance | $R_e = 0.144\ \Omega$ |
| reactance | $X_e = 63\ \Omega$ |
| radiation PF | $p_e = 2.3$ mils |
| loss PF | $< 2.3$ mils |
| efficiency | $> 0.50$ |
| resonance bandwidth | 134 Hz |
| radiated power | 1 MW |
| input power | 2 MW |
| reactive power | 435 MVA |
| voltage | 165 kV |
| current | 2.63 kA. |

Particularly spectacular are the reactive power of 435 MVA in the air dielectric, and the real power of 2 MW delivered to a resistance of about 0.3 Ω. Less than half of this resistance is budgeted to all losses, including the ground connection and the tuning inductor. The small value of radiation PF (2.3 mils) well qualifies this structure as a "small antenna." The choice of a capacitor (rather than an inductor) was influenced by the need for omnidirective coverage.

The effective volume is diagramed in the form of a cylinder bounded by the dashed lines. Fig. 17, the effective area is a circle including more area than the grid of wires. In Fig. 18, the effective height is reduced by two practical considerations. The top level is lower than the top wires by the effect of the downleads (48 wires around the central tower). The bottom level is higher than the ground, by the effect of the grounded towers and guy wires (each tower having 3 at each of 4 or 5 levels). The resulting effective height is about $\frac{1}{2}$ the average height of the 13 towers. The radiation PF is related to this effective volume by (3) adapted to half-space above ground. (The effective volume is compared with $\frac{1}{2}$ radiansphere.)

## IX. Conclusion

The principles of small antennas can be described in simple terms, both mathematically and pictorially. They are helpful in the understanding and design of practical antennas in either type, capacitor or inductor. While the two types have a common rating in terms of effective volume, there are differences that may give either an advantage in size or other practical considerations. For any configuration, the efficiency and/or bandwidth is ultimately limited by size relative to the wavelength.


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