INTEGRAL EQUATION ALGORITHMS FOR EQUIVALENT CURRENTS DISTRIBUTION RETRIEVAL OVER ARBITRARY THREE-DIMENSIONAL SURFACES

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ABSTRACT

The following article describes a method for determination of equivalent currents over arbitrary surfaces from near field measurements. The proposed algorithm solves the electric field integral equations, introducing a mathematical representation of the currents based on their tangential components. This new formulation of the Inverse Radiation Problem reduces the computational cost of the problem. On the other hand, the consideration of both electric and magnetic currents, according to the Equivalence Principles, supposes a better characterization of the electromagnetic Equivalent Problem at all aspect angles. The final goal is the diagnosis and characterization of antennas independently of the complexity of their geometry.

1. INTRODUCTION

The antenna measurement in the Fresnel region requires the application of the near field (NF) to far field (FF) transformations, for the calculation of the radiation pattern. Traditionally, the NF-FF transformations have been based on the modal expansion techniques, which are restricted to the canonical surfaces (planar, cylindrical, spherical). As an alternative, it is possible to characterize the antenna through a model based on a set of equivalent currents [1]-[3].

The methods based on the calculation of the equivalent currents allow the calculation of the radiation pattern, as well as the analysis of the fields and currents distribution on the antenna surface, regarding diagnostic purposes [4]-[8].

The development and optimization of the methods based on the calculation of the equivalent currents, as well as the increase of the computational capabilities, allow the application of these methods for the analysis and characterization of antennas of complex geometry [9].

In the following article, a method for the reconstruction of the equivalent currents is described. The main improvements with respect to other integral equation approaches are based on the usage of the tangential components that reduce the size of the equation system. Apart from this, the consideration of both the electric and magnetic currents supposes a better characterization of the electromagnetic (EM) problem.

2. THE INVERSE RADIATION PROBLEM

The calculation of the equivalent currents given the radiated fields requires the resolution of the integral equations (Inverse Radiation Problem).

Given an homogeneous medium, the Equivalence Principles allow the substitution of a source distribution by an enclosing surface, supposing null fields inside. A set of equivalent currents distribution is calculated from the tangential fields over the closed surface. The equivalent currents radiate the same fields that the original sources distribution.

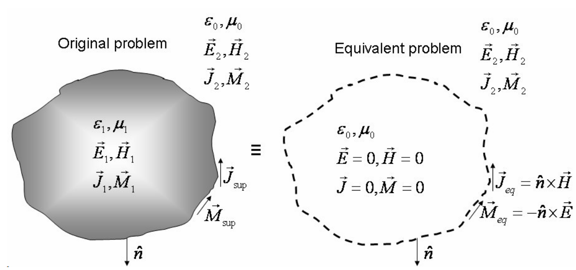

Figure 1. The First Equivalence Principle

Given the radiated fields, as well as the geometric description of the surface that encloses the sources, it is possible to calculate a current distribution that radiates the same fields.

The radiated fields depend on the electric and magnetic currents distribution, so it is necessary the consideration of both types of sources when solving the Inverse Radiation Problem. However, it is possible to consider only the electric currents if the surface can be approximated as a Perfect Electric Conductor (PEC) (e.g., in the case of metallic surfaces). On the other hand, the aperture antennas can be analyzed from the calculation of the equivalent magnetic current distribution over the aperture plane, as defined in the Second Equivalence Principle.

Regarding the antenna diagnosis tasks, the analysis of antennas of complex geometry requires the consideration of a three-dimensional (3D) model as

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similar as possible to the antenna geometry. The equivalent EM currents are reconstructed over the proposed surface.

The analysis of antennas based on the calculation of the equivalent currents requires the resolution of very large equation systems, when considering electrically moderate to large antennas or just antennas of complex geometry.

Consequently, it is necessary the development of algorithms that allows the resolution of large equations systems. The proposed method has been designed in order to reduce memory consumption as well as decreasing the number of unknowns by taking advantage of the formulation of the problem.

3. DESCRIPTION OF THE METHOD

The fields radiated by a sources distribution can be decomposed, by the superposition principle, into a magnetic contribution and electric contribution (see Eq. 1).

$$\vec{E} = \vec{E}_J + \vec{E}_M \qquad (1)$$

The Eq. 2 relates the electric field with the electric sources, and the Eq. 3 relates the electric field with the magnetic sources.

$$\vec{E}_J = -\frac{j\eta}{4\pi\beta}\oint_{S'}\left\{\beta^2 \vec{J}_s(\vec{r}\,')\frac{e^{-j\beta R}}{R} + \nabla\left(\nabla\cdot\left(\vec{J}(\vec{r}\,')\frac{e^{-j\beta R}}{R}\right)\right)\right\}dS' \quad (2)$$

$$\vec{E}_M = -\frac{1}{4\pi}\nabla\times\int_{S'}\vec{M}_s(\vec{r}\,')\frac{e^{-j\beta R}}{R}dS' \quad (3)$$

The equivalent EM currents can be expressed into their tangential components (J_t, M_t), being null the normal component (J_n=0, M_n=0). Consequently, the radiated field can be expressed as a function of the tangential components (Eq. 4).

$$\vec{E} = \int_{S'}\left(\vec{J}_t(\vec{r}\,')G_J(\vec{r},\vec{r}\,') + \vec{M}_t(\vec{r}\,')G_M(\vec{r},\vec{r}\,')\right)dS' \quad (4)$$

Thus, the resolution of the Inverse Radiation Problem over an arbitrary surface requires the determination of the equivalent currents given the radiated fields, forcing the normal component to zero (Eq. 5).

$$\begin{aligned}&\vec{E} = \int_{S'}\left(\vec{J}(\vec{r}\,')G_J(\vec{r},\vec{r}\,') + \vec{M}(\vec{r}\,')G_M(\vec{r},\vec{r}\,')\right)dS'\\&\vec{J}_n(\vec{r}\,') = 0\\&\vec{M}_n(\vec{r}\,') = 0\end{aligned} \quad (5)$$

In the case of the canonical surfaces, it is possible to define two tangential components in an easy way. For instance, the tangential components θ, φ can be defined over an spherical surface.

However, for complex 3D surfaces, the definition of the tangential components is more difficult. An alternative to the usage of tangential components is the consideration of the cartesian components. But in this case, the number of equations of the systems is increased, because three components are considered instead of the two tangential components, increasing the computational cost of the problem. Apart from this, due to the additional condition of forcing the normal component to zero, the convergence of the algorithm when considering the cartesian components decreases.

Regarding the formulation based on the tangential components, it is necessary to define a local coordinate system on each facet in which the 3D surface has been discretized. The Fig. 2 represents the local coordinate system on two arbitrary facets.

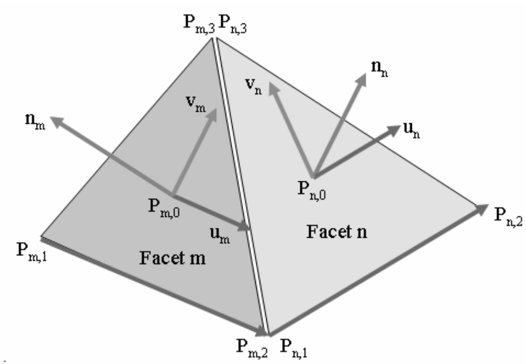

Figure 2. Definition of the coordinate system over each facet.

The consideration of the local coordinate system (*u,v*) is equivalent to the usage of the cartesian coordinates. The Eq. 6 relates the tangential coordinates with the cartesian system.

$$J_x\hat{x} + J_y\hat{y} + J_z\hat{z} = J_u\hat{u} + J_v\hat{v} \qquad (6)$$

The equation system that relates the tangential components of the equivalent currents with the spherical components of the radiated fields is shown in Eq. 7.

$$\begin{pmatrix} E_\theta \\ E_\varphi \end{pmatrix} = \begin{pmatrix} Z_{Ju,\theta} & Z_{Jv,\theta} & Z_{Mu,\theta} & Z_{Mv,\theta} \\ Z_{Ju,\varphi} & Z_{Jv,\varphi} & Z_{Mu,\varphi} & Z_{Mv,\varphi} \end{pmatrix} \begin{pmatrix} J_u \\ J_v \\ M_u \\ M_v \end{pmatrix} \quad (7)$$

Where the Z-matrix are the coefficients that depend on the position of the points where near field is measured, the geometry of the surface where sources are reconstructed, and the propagation conditions.

The amount of data involved in the electromagnetic problem increases the size of the Z-matrix. In fact, this is the main problem regarding the resolution of the matrix system. For instance, if the field were measured in 64,800 points (assuming an spherical acquisition system) and the complex geometry surface were discretized into 20,000 facets, then, the number of elements that conforms the system matrix were 23 billions. Consequently, it is difficult to storage this amount of data in the physical memory of the computers, due to the required 340 GB.

The resolution of the proposed equation system is performed through an iterative method based on the conjugated gradient (CG). Given a linear system of equations, it is possible to solve the system without storing the Z-matrix (which is usually larger than the fields and currents matrices) if there is a theoretical expression for calculating their elements. When solving the Inverse Radiation Problem, the elements are determined from the integral equations, being a function of the position of the sources as well as the points where fields are calculated. Consequently, only a small part of the Z-matrix is stored at each iteration. For the calculation of the field in another point, the stored values are replaced by the required elements of the Z matrix.

The main advantage of using this method is the possibility of including numerical techniques that avoids the storage of the full Z-matrix. The implemented algorithm only works with a few elements of the matrix at each iteration, reducing the memory consumption. On the other hand, computational time required to perform the calculation is slightly increased.

4. RESULTS

4.1. Analysis of the equivalent currents as an application of the Equivalence Principles

The first example is an application of the Equivalence Principles, in which an antenna array will be replaced by an equivalent sources distribution (see Fig. 3).

The antenna array is a 4-element array of Hertz dipoles. The separation between the elements is 0.5 λ. This array is enclosed by an sphere of 2 λ diameter.

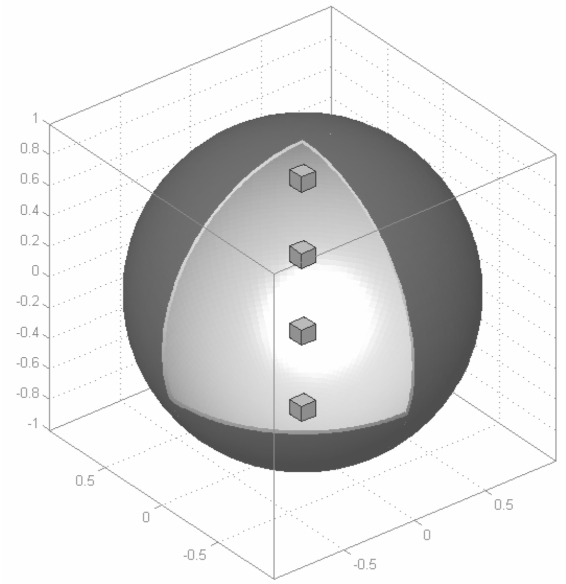

Figure 3. Position of the hertz dipoles (represented by the cubes) and the enclosing sphere where the equivalent currents are reconstructed.

Given the excitation law of the antenna array (with linear progressive phase), the NF is calculated at 2.5 λ. From the theoretical NF, the equivalent EM currents are reconstructed over the spherical surface. Finally, the far field of the array and the equivalent currents are calculated, comparing the calculated radiation patterns.

The tangential components of the equivalent magnetic currents over the spherical surface are represented in the Fig. 4 and Fig. 5. By the application of the Equivalence Principles, the equivalent magnetic currents are the tangential electric fields over this surface. In the figures, the phase and quadrature components are represented.

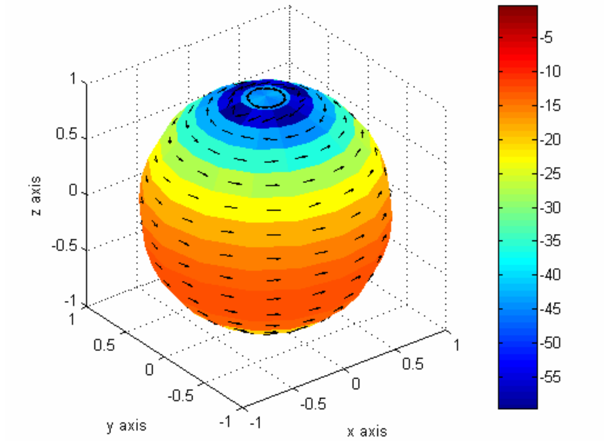

Figure 4. Equivalent magnetic currents over the surface of the sphere. Normalized amplitude (in dB) of the phase (real) component.

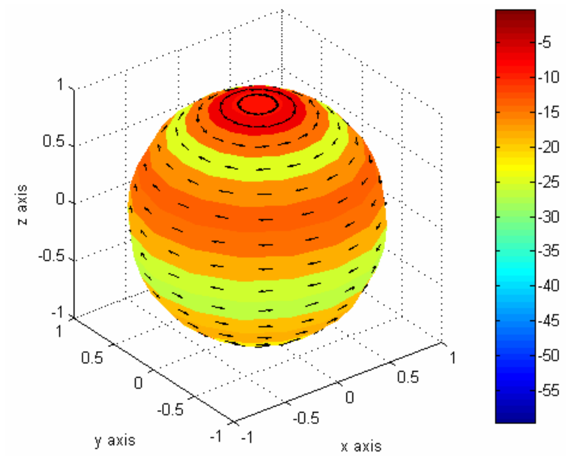

Figure 5. Equivalent magnetic currents over the surface of the sphere. Normalized amplitude (in dB) of the quadrature (imaginary) component.

In the Fig. 6, a comparison between the NF radiated by the array and the NF radiated by the EM equivalent currents is plotted. The NF has been calculated at 5 λ. The Fig. 7 represents the far field radiated by the antenna array and the equivalent currents over the sphere surface. It is possible to see the agreement between the fields radiated by the array and the equivalent currents. Consequently, the results show that it is possible to replace the antenna array by the reconstructed equivalent currents in order to calculate and analyze the radiated fields.

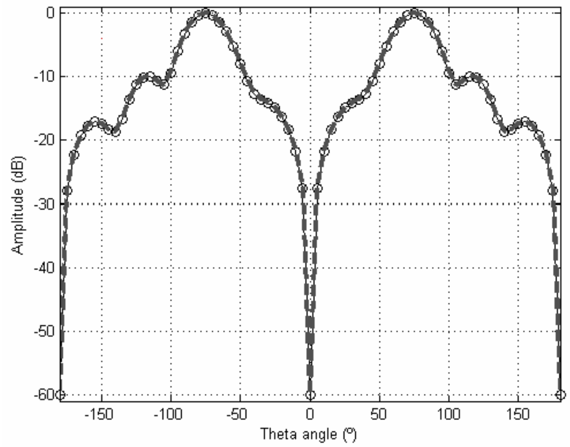

Figure 6. Normalized amplitude (in dB) of the near field versus theta angle (in deg.). The o-marked line represents the NF radiated by the array. The dashed grey line represents the NF radiated by the equivalent EM currents.

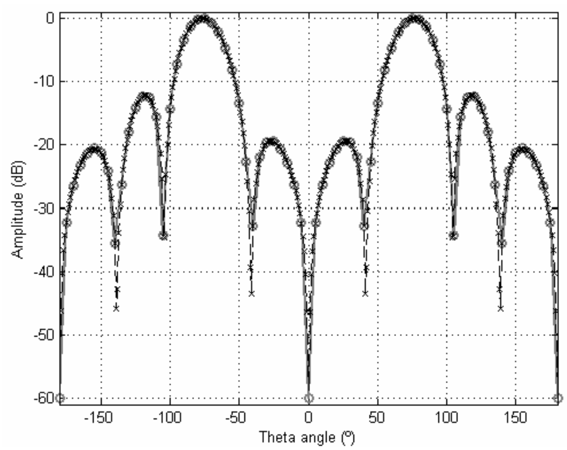

Figure 7. Normalized amplitude (in dB) of the far field versus theta angle (in deg.). The o-marked line represents the FF radiated by the array. The dashed grey line represents the FF radiated by the equivalent EM currents.

4.2. Analysis and characterization of a horn antenna

In the second example, the distribution of the electric currents over the metallic walls of a horn antenna is calculated (see Fig. 8). The field radiated by the horn has been measured at the spherical scanning facility anechoic chamber of the University of Oviedo. The distance between the probe antenna and the antenna under test is 4.856 m, being the angular resolution of 1 deg. both in theta and phi angles. The working frequency is 2.5 GHz.

From the physical dimensions of the horn antenna (0.4 m x 0.3 m x 0.55 m), a 3D model has been made by using a CAD software. In the aperture plane, both the electric and magnetic currents distribution have been considered. The metallic walls have been considered as PEC surfaces, so the electric currents are considered.

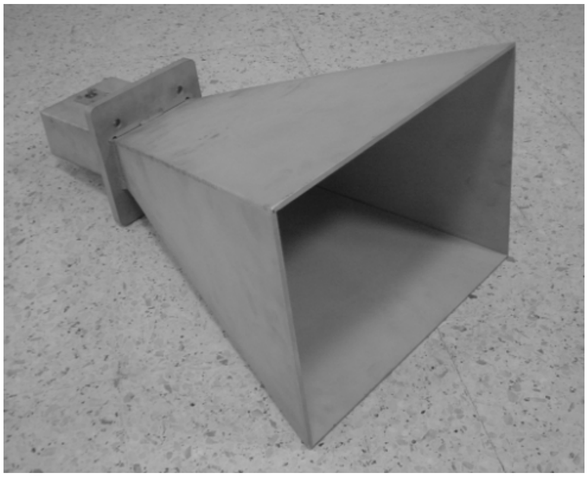

Figure 8. Photography of the horn antenna.

From the measured fields, the equivalent EM currents are reconstructed over the surface of the horn antenna. The equivalent electric current distribution is represented in the Fig. 9 and in the Fig. 10. In the case of the aperture plane, the electric equivalent currents are the magnetic fields, as the application of the Equivalence Principles.

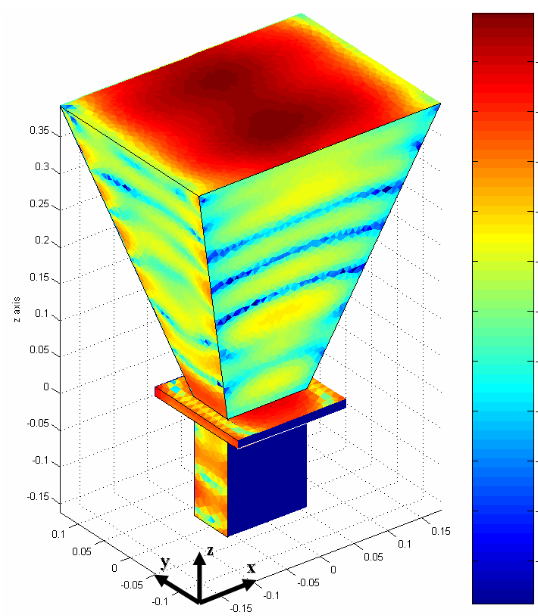

Figure 9. Equivalent electric currents. Amplitude (in dBA/m) of the y-component.

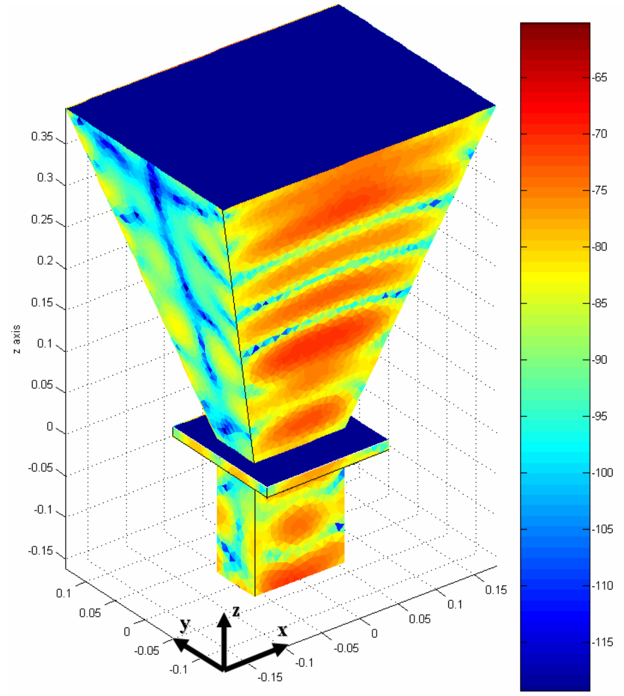

Figure 10. Equivalent electric currents. Amplitude (in dBA/m) of the z-component.

Regarding the verification of the results, a comparison between the measured fields at the anechoic chamber and the fields radiated by the equivalent EM currents is plotted in the Fig. 11 and Fig. 12.

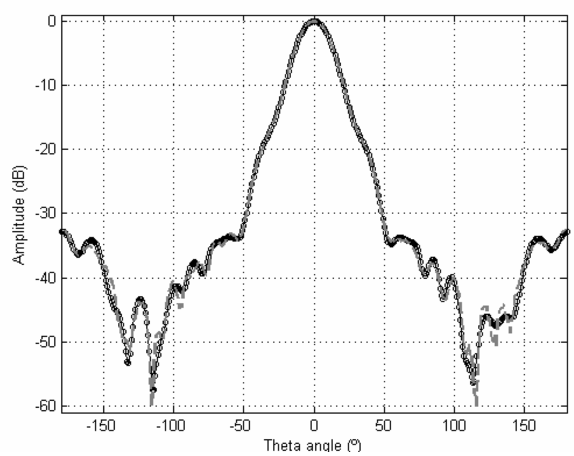

Figure 11. Normalized amplitude (in dB) of the field versus theta angle (in deg.), phi = 0 deg. The dashed grey line represents the measured field at the anechoic chamber. The o-marked line represents the field radiated by the equivalent EM currents.

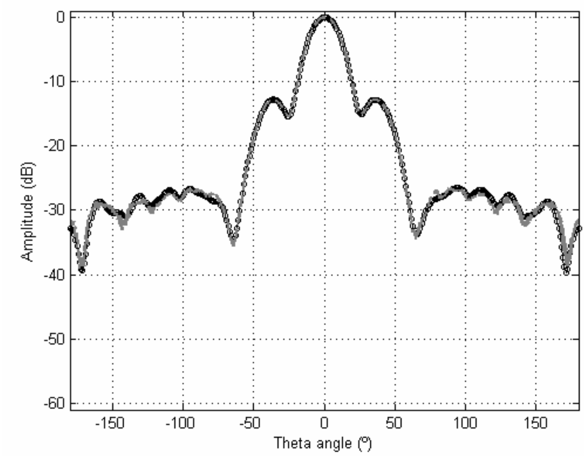

Figure 12. Normalized amplitude (in dB) of the field versus theta angle (in deg.), phi = 90 deg. The dashed grey line represents the measured field at the anechoic chamber. The o-marked line represents the field radiated by the equivalent EM currents.

5. CONCLUSSIONS

As it is shown in the results, the described algorithm is very efficient when reconstructing the equivalent currents over arbitrary three-dimensional surfaces of radiating structures. Both the First and the Second Equivalence Principles can be applied in order to characterize the electric and magnetic currents over PEC surfaces, or electric fields on aperture antennas. From the reconstructed equivalent currents, it is possible to calculate the far field radiation pattern of such antennas (for example, in the NF to FF transformations). The integration of this algorithm in a software tool for the analysis and representation of radiated fields and currents distribution, can be very useful tool for the diagnosis and characterization of most types of antennas.

6. ACKNOWLEDGEMENT

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