FAST MATERIAL PARAMETRIC SWEEPS BY EXPLOITING A COMBINED DOMAIN-DECOMPOSITION-MODEL-ORDER-REDUCTION TECHNIQUE[1]

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Abstract

A hybrid technique combining Domain Decomposition and Model Order Reduction is presented for obtaining fast parametric sweeps for what concerns material properties within passive microwave devices. The Model Order Reduction technique relies on a multipoint method and the computation of the FEM solution for these points are accelerated by applying the Domain Decomposition method.

1 - INTRODUCTION

Notwithstanding the enormous computing power of modern desktop computers, with multi-core processors and gigabytes of memory, features which were unaffordable only five years ago, numerical analysis and synthesis of electromagnetically large problems via accurate full-wave techniques like the finite element (FE) method can still require too high computing times. This is especially true when such analyses comprehend parametric studies in which some device geometrical or physical parameter is set to vary in a given range. Accelerated and accurate FE analysis are then of paramount importance for applied electromagnetics community and many possible solutions have been proposed in the past. Among these model order reduction techniques (MOR) [1] and domain decomposition methods (DD) [2] represent two different yet somewhat complementary approaches.

The MOR technique performs an efficient characterization of the system over a parameter space by reducing the dimension of the discretized model describing the system. MOR technique has been originally conceived for single-parameter systems only, but it has been extended recently to the multi-parameter case [3], [4]. The reduction of the model order can be performed on the basis of either a solution of the complete discretized system at a single point [3], [5], [6] or at multiple points [4], [7]. Multi-point schemes are more robust but come along with the drawback of requiring a full FE solution at each sampling point.

DD, on the other hand, is a technique for reducing a large problem, which might not be numerically solvable as a whole, to a set of smaller, coupled problems which can be solved one-by-one in a divide-and-conquer scheme [8], [9]. Recently DD has been applied to accelerate parametric sweeps by defining a single sub-domain as containing

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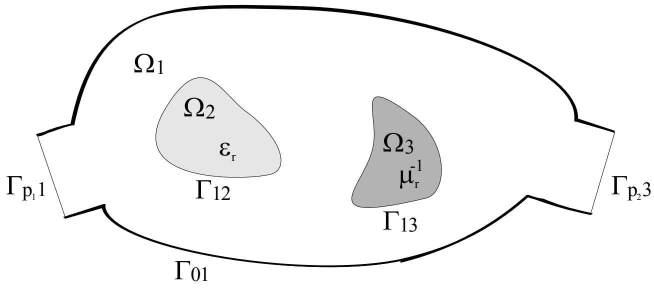

Figure 1 - *Geometry of the problem: a generic 2 port waveguide device subdivided onto three sub-domains.*

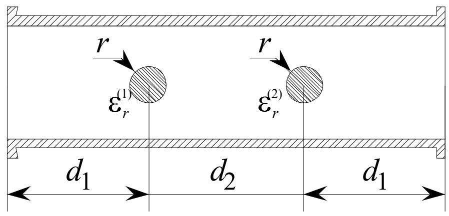

Figure 2 - *Two cylindrical dielectric posts in WR90.*

all the possible materials interested by the parametric sweep and by performing the solution of the *other* sub-domains just once [10] [11].

In this contribution, the two techniques are combined as follows: first, the model is partitioned so that all parameter-dependent features are located in a single sub-domain. Then, the DD technique is employed to reduce the FE system to this sub-domain only, and, finally, a multi-point MOR method is applied to produce a low-dimensional model for this region, which can be evaluated efficiently at any point in parameter space.

As a test case, passive waveguide devices are considered, and, for the sake of simplicity, only devices uniform either along the E-plane or the H-plane, i.e. devices that can be solved by 2D FEM. The next Section briefly presents the methodology, while Section 3 gives some numerical results, and finally Section 4 draws conclusions.

2 – FORMULATION

This work employs a 2D E-field FE formulation for H-plane devices [12]. Fig. 1 sketches a generic 2-port waveguide device in the H-plane, subdivided into an arbitrary number of domains Ω_i which are separated by boundaries Γ_{ij}. Specifically, Γ_{0j} denotes the outer boundary of domain Ω_i, where Dirichlet or Neumann boundary conditions are to be imposed, and $\Gamma_{p_k j}$ represents the portion of the boundary of Ω_j pertaining to port p_k of the device, onto which a suitable modal expansion is to be imposed. It can be shown that the FEM solution of this problem in a DD framework leads to a linear system of equations of the form [11]:

$$\begin{bmatrix} \mathbf{M}_1 & 0 & \cdots & 0 & \mathbf{E}_1 \\ 0 & \mathbf{M}_2 & \cdots & 0 & \mathbf{E}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{M}_N & \mathbf{E}_N \\ \mathbf{E}_1^T & \mathbf{E}_2^T & \cdots & \mathbf{E}_N^T & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \mathbf{c} \end{bmatrix}, \tag{1}$$

in which the matrices $\mathbf{M}_i$ correspond to the internal unknowns $\mathbf{x}_i$ of sub-domain Ω_i, matrix $\mathbf{C}$ is associated with unknowns $\mathbf{y}$ on boundaries, and the matrices $\mathbf{E}_i$ represent interactions between internal and boundary unknowns. The solution of (1) in a DD framework resorts to the Schur complement $\mathbf{S}$

$$\mathbf{S} = \sum_{i=1}^{N} \mathbf{C}_i - \mathbf{E}_i^T \mathbf{M}_i^{-1} \mathbf{E}_i \,, \tag{2}$$

from which the solution for the boundary unknowns can be obtained as $\mathbf{Sy} = \mathbf{c}$. Provided that all structures with parameter-dependent material properties ε_r and μ_r are placed in domain *N*, the first *N*-1 components of the Schur complement can be computed and stored once for all. For the last domain, the decomposition $\mathbf{M}_N = \mathbf{M}_N^0 + \varepsilon_r \mathbf{M}_N^\varepsilon + \mu_r^{-1} \mathbf{M}_N^\mu$ leads to a parameterized linear system of equations of the form

$$\left(\mathbf{A}_0 + \varepsilon_r \mathbf{A}_1 + \mu_r^{-1} \mathbf{A}_2\right)\mathbf{z} = \mathbf{b}\,, \tag{3}$$

where $\dim \mathbf{A}_i = \dim \mathbf{C} + \dim \mathbf{M}_N = n$, $i \in \{0,1,2\}$. The reduced order model (ROM) is constructed by applying to (3) a Galerkin projection with a suitable orthonormal matrix $\mathbf{V} \in \mathbb{C}^{n\times m}$, $m \ll n$. Here, $\mathbf{V}$ is chosen so that $\operatorname{colsp}\mathbf{V} = \operatorname{span}\{\mathbf{z}_1,\ldots,\mathbf{z}_m\}$, where the $\mathbf{z}_k$ are the solution of the DD reduced system (3) at the chosen interpolation points $p_k = \left(\varepsilon_r^k, \mu_r^k\right)$. Since (3) stems from a single subdomain, it is of much lower dimension than the original system (1). Hence solution vectors and also the final ROM based on (3) can be constructed more efficiently [13].

3 – NUMERICAL RESULTS

The geometry of the test case is depicted in Fig. 2. It comprises a WR90 waveguide segment, exhibiting two posts, each of radius 4mm placed on the waveguide median line as shown in Fig. 2 ($d_1 = d_2 = 20mm$). Sub-domain Ω_1 encompasses the empty space, while the two posts both belong to sub-domain Ω_2. The relative permittivity of the posts varies, independently, in the range $\varepsilon_r^{(1)} = \varepsilon_r^{(2)} = (1\ldots46) - j0.01$ and the frequency is equal to 12 GHz. The problem is discretized using 75424 first order triangular elements and by expanding the field on each port into one mode, yielding a total of 37865 degrees of freedom. By applying the DD technique to the outer domains once, a reduced FE system for the core region (3) of dimension 6990 is obtained. The latter is solved at 6×6 equidistant points in the $\left(\varepsilon_r^{(1)}, \varepsilon_r^{(2)}\right)$ space to construct the MOR projection matrix $\mathbf{V}$. Fig. 3 reports the contour lines relative to the numerically computed reflection coefficient. It is worth mentioning that the contour plots are generated over a grid of 201×201 values, thus requiring 40401 computations. Fig. 4 demonstrates the high accuracy of the order reduction approach by showing the absolute error between the reflection coefficients computed by the full FE model and the ROM. Tab. I reports the speed-up attainable via the proposed methodology. It can be seen that the evaluation of the ROM is by three orders of magnitude faster than the full FE model. By incorporating the DD approach, the time for ROM generation is reduced by 35%.

Table I - *CPU times in seconds.*

EVALUATION

	PER SAMPLE	TOTAL
FEM	1.318	5.323×10^4
FEM+DD	0.7839	3.167×10^4
MOR	1.045×10^{-3}	42.20

ROM GENERATION

	TOTAL
FEM	53.914
FEM+DD	34.699

4 – CONCLUSIONS

A combined DD-MOR method extending existing approaches by incorporating the DD in the generation process of the ROM has been presented. The DD method allows the

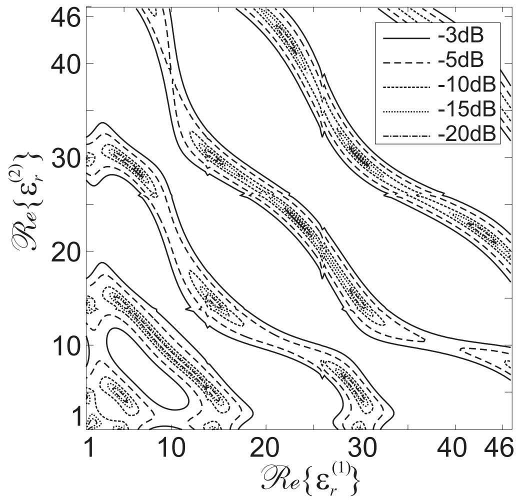

Figure 3 $|S_{11}|$ *as a function of* $\varepsilon_r^{(1)}$ *and* $\varepsilon_r^{(2)}$.

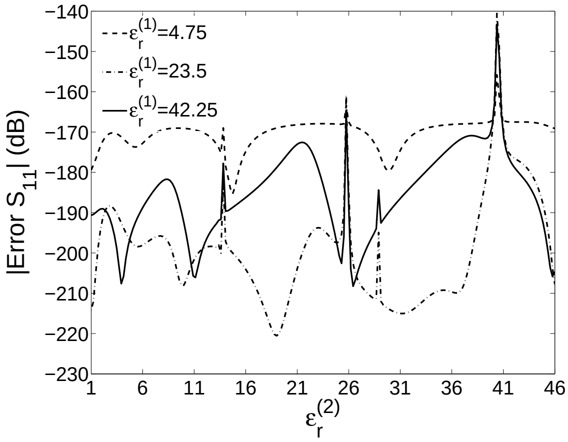

Figure 4 *Absolute error between the FEM and the FEM+DD+MOR computed reflection coefficients.*

efficient use of a multipoint MOR procedure, since only the subdomain containing parameter-dependent features has to be solved at the ROM interpolation points.

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