A Fast DFT Planar Array Synthesis Tool for Generating Contoured Beams

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***Abstract*—An efficient procedure based on a discrete Fourier transform (DFT) expansion of planar array distributions is described for synthesizing contoured beams in satellite antenna applications. The DFT provides a global basis set that spans over the entire array, and thus produces highly directional beam patterns to serve as corresponding local basis functions for representing the desired far zone radiation field of the array. The unknown coefficients of the DFT expansion are then found via a minimum least square error (MLSE) technique. While the total number of DFT coefficients exactly equals the number of array elements, it is found that only a small fraction of the total unknown DFT coefficients generally remain dominant, and only these need to be solved, thus making the present approach very fast.**

***Index Terms*—Arrays, contoured beams, pattern synthesis.**

I. Introduction

CONTOURED beam forming is one of the most important applications of antenna arrays in satellite communications; with this view in mind, a novel and efficient DFT-based procedure for synthesizing planar array distributions to achieve desired contoured beams is presented in this paper. Contoured beams are utilized to improve antenna efficiency by illuminating only certain coverage areas on earth and to reduce interference to other nearby regions. One is referred to the work of Duan and Rahmat-Samii [1] for a general discussion on the advantages of using contoured beams for satellite antenna applications.

Typical efforts in the past to make the overall pattern synthesis more computationally efficient have focused on the improvement of synthesis algorithms themselves. These synthesis algorithms are based on various iterative optimization processes. The successive projection method (SPM) [2], the steepest descent method (SDM) [3], the genetic algorithm (GA) [4], and minimum least square error (MLSE) [5] are some of the optimization techniques utilized in antenna synthesis problems. It is quite important to note that the improvements to the efficiency of optimization algorithms alone do not really contribute significantly to the improvement in the efficiency of the overall pattern synthesis procedure, especially for large arrays. Indeed, for large arrays, the most time consuming part in the entire array radiation pattern synthesis procedure is the conventional element-by-element field calculation, which via superposition provides the entire array radiation field, and which must be repeated at each step in an iterative optimization procedure. Furthermore, the element by element field superposition method for calculating the entire array radiation pattern essentially utilizes the superposition of relatively broad beams radiated by each element of the array to synthesize the desired relatively narrow contoured beam. Thus, no terms which are dominant in the contoured beam formulation can be identified in the conventional approach, thereby also obscuring the physical picture of the contoured beam synthesis process.

In the present synthesis procedure, where the MLSE-based optimization technique is incorporated, the coefficients of the DFT representation for the unknown array distribution serve as the unknown variables to be synthesized in contrast to the conventional approach that employs the element weights as unknowns. There are three distinguishing advantages in this proposed approach. First of all, each DFT term works as an impressed uniform amplitude and linear phase distribution on the entire planar array, and will generate a directional narrow (or spot) beam, which now automatically serves as a highly local basis function for the radiation field in the contoured beam synthesis; this is in contrast to a very broad beam radiated from each individual array element. The localized radiation basis functions used here tend to behave better in the contoured beam synthesis since only a small number of DFT terms, in which the total DFT components are exactly equal to the total number of array elements, is usually sufficient to cover the confined angular space of the contoured beam, and outside the required beam coverage area the synthesized pattern automatically drops to a very low level. As such, one can easily identify the dominant DFT terms that can contribute significantly to the synthesis. Second, asymptotic UTD-based ray concepts can be employed efficiently to find the near and far fields of the array due to each simple DFT term comprising the array distribution [6], [10], [11]. Third, once the UTD-type ray contributions are identified, they can be incorporated efficiently into any existing UTD-based computer codes to account for the scattering effects of nearby structures if so desired. As will be mentioned again subsequently, the field radiated by each DFT spectral component for a rectangular array distribution reduces in the far zone to the uniform array factor involving the Dirichlet sinc functions. Only the far zone UTD fields are discussed here. Consequently, the far zone contoured beams are synthesized via a sequence of spot beams described by Dirichlet sinc (DIRSINC) functions in the case of rectangular arrays. In this regard, it may appear that the present work is simply akin to numerical interpolation schemes (e.g., [7]) in

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which desired antenna power patterns may be synthesized using a sequence of interpolating sinc-like functions in a manner such that the criterion based on the sampling theorem is satisfied; however, there actually are significant differences between such approaches and the present DFT approach. In particular, the present approach gives rise to DIRSINC functions for the radiated field instead of for the radiated power which is proportional to the square of the field magnitude; thus, it contains the simple relation between the global DFT spectral representation for the array distribution and the resulting far zone array factor pertaining to each DFT spectral component, which can be interpreted physically in terms of just a few UTD rays (see [6]). Furthermore, the present approach can be extended to nonrectangular arrays with piecewise linear array element truncation boundaries in which case a closed-form UTD-based beam array factor can be obtained that is now quite different from the DIRSINC functions for each of the spectral components [12]. In addition, the present method can allow the near field of each DFT component for the relatively general array distribution to also be found in closed form in terms of the UTD rays. The details of these extensions to nonrectangular array boundaries and near field calculations are not presented here due to space limitations. The present work is somewhat related to that in [13]; however, it is also different in that it begins with a rigorous DFT formulation, rather than a physically based formulation [13].

In this paper, a $(2N+1)\times(2M+1)$ planar rectangular periodic array is considered as an example for demonstrating the fundamental concepts of the proposed procedure. Its extension to more general shapes of arrays is straightforward, and will not be presented for the sake of brevity, as indicated above. The format of this paper is as follows. In Section II, the fundamental concept of the DFT-based synthesis procedure is introduced. A simple synthesis procedure based on DFT-UTD together with the MLSE optimization is described in Section III. Finally, a typical numerical example to generate a continental USA (CONUS) beam is presented in Section IV to demonstrate its validity and efficiency. An $e^{j\omega t}$ time dependence for the fields is assumed and suppressed throughout in this paper.

II. DFT-Based Synthesis

Consider a rectangular, planar periodic array of $(2N+1)\times(2M+1)$ antenna elements, with d_x and d_y being the periods along the x and y coordinates, respectively. It is assumed that the array elements and hence their associated radiation patterns are identical. Let $\overline{E}_{nm}(\theta,\phi)(-N\le n\le N, -M\le m\le M)$ denote the radiation pattern of the (n,m)th array element with unit amplitude excitation and with phase referred to the origin, where

$$\overline{E}_{nm}(\theta,\phi)=\overline{E}_e(\theta,\phi)e^{jk(\sin\theta\cos\phi nd_x+\sin\theta\sin\phi md_y)} \tag{1}$$

in which $\overline{E}_e(\theta,\phi)$ is the radiation pattern of the $(n=0, m=0)$ reference element located at the origin of the coordinate system which is at the center of the planar array. Note that the far zone radiation field of each element is given by $\overline{E}_{nm}(\theta,\phi)\cdot g$, where g is defined as the usual spherical wave factor for each element. The objective here is to find a set of amplitudes, A_{mn}, of the array distribution, so that the superposition of element patterns will approximate the desired radiated field $\overline{E}_d$ as closely as possible within some prescribed error bounds. Thus,

$$\sum_{m=-M}^{M}\sum_{n=-N}^{N}A_{nm}\overline{E}_{nm}(\theta,\phi)\approx\overline{E}_d(\theta,\phi) \tag{2}$$

where the left-hand side denotes the synthesized pattern and the right-hand side indicates the desired pattern. It is noted that most conventional approaches employ a synthesis procedure such as SPM, SDM, or MLSE, to optimize the set A_{nm} until an acceptable pattern is reached. $\overline{E}_{nm}$ is associated with a very broad radiation pattern that usually spans the entire area of coverage, and it is thus equivalently considered as a global radiation basis function in this case. Thus, each $\overline{E}_{nm}$ plays an almost equally important role in the conventional synthesis procedure, and no particularly significant (n,m)th terms can be identified since no single ones dominate. As a result, all A_{nm} terms, whose number is given by $(2N+1)\times(2M+1)$, are simultaneously required in the synthesis procedure, which for a very large array causes a cumbersome and time-consuming computation in the analysis and synthesis of array patterns. On the other hand, the present synthesis procedure, based on a new set of local radiation basis functions for describing the desired radiation pattern, tends to perform better, since only those local radiation basis functions, whose field is now denoted by $\overline{E}^l_{pq}$ [and defined later in (5)], which are confined to primarily within the desired pattern coverage, will contribute significantly in the synthesis; their number is usually only a small portion of the total number of global DFT basis functions described in the previous paragraphs. One obvious reason for the efficiency of the local radiation basis functions is that outside the range of the desired pattern their use becomes unnecessary since in that region the only requirement is to make the power level as low as possible, and this goal can be achieved very easily by removing almost all the local radiation basis functions from that region. The present approach to synthesize the contoured beam in terms of narrow spot beams, or the local radiation basis functions, is to employ the exact, finite global DFT [8] representation for the original amplitudes, A_{nm}, of the array distribution, namely

$$A_{nm}=\sum_{q=-M}^{M}\sum_{p=-N}^{N}B_{pq}e^{-j\frac{2\pi p}{2N+1}n}e^{-j\frac{2\pi q}{2M+1}m} \tag{3}$$

where B_{pq} are the DFT set ($|p|\le N$, $|q|\le M$) of A_{nm}. Substituting (3) into (2) gives

$$\sum_{q=-M}^{M}\sum_{p=-N}^{N}B_{pq}\overline{E}^l_{pq}(\theta,\phi)\approx\overline{E}_d(\theta,\phi) \tag{4}$$

where

$$\overline{E}^l_{pq}(\theta,\phi)=\sum_{m=-M}^{M}\sum_{n=-N}^{N}e^{-j2\pi\left(\frac{pn}{2N+1}+\frac{qm}{2M+1}\right)}\overline{E}_{nm}(\theta,\phi) \tag{5}$$

In particular, the radiation pattern in the far-zone of each spot beam of $\overline{E}^l_{pq}$ (5) can be shown to result in a product of two DIRSINC functions by [9]

$$\overline{E}^l_{pq}(\theta,\phi) = \overline{E}_e(\theta,\phi)\left(\frac{\sin\left(\frac{2N+1}{2}\phi_x\right)}{\sin\left(\frac{\phi_x}{2}\right)}\right)\left(\frac{\sin\left(\frac{2M+1}{2}\phi_y\right)}{\sin\left(\frac{\phi_y}{2}\right)}\right) \tag{6}$$

or

$$\begin{aligned}\overline{E}^l_{pq}(\theta,\phi) = &\left(\frac{\overline{E}_e(\theta,\phi)}{4\sin\frac{\phi_x}{2}\sin\frac{\phi_y}{2}}\right)\\ &\cdot\Big(e^{j\frac{2N+1}{2}\phi_x}e^{j\frac{2M+1}{2}\phi_y} - e^{j\frac{2N+1}{2}\phi_x}e^{-j\frac{2M+1}{2}\phi_y}\\ &- e^{-j\frac{2N+1}{2}\phi_x}e^{j\frac{2M+1}{2}\phi_y}\\ &+ e^{-j\frac{2N+1}{2}\phi_x}e^{-j\frac{2M+1}{2}\phi_y}\Big)\end{aligned} \tag{7}$$

which represents the contribution to the pattern of rays emanating in the (θ,ϕ) direction from each of the four corners of the rectangular array. Also

$$\begin{cases}\phi_x = (k\sin\theta\cos\phi)d_x - \frac{2\pi p}{2N+1}\\ \phi_y = (k\sin\theta\sin\phi)d_y - \frac{2\pi q}{2M+1}\end{cases} \tag{8}$$

In (6) identical array elements as well as their associated radiation patterns are assumed, and (1) has also been included due to the phase reference at the array center. It is noted that once the coefficients, B_{pq}, of each DFT term (or the local radiation basis function $\overline{E}^l_{pq}$) are found, the overall radiation pattern of the synthesized array can be found via the left-hand side of (4) by

$$\overline{E}(\theta,\phi) = \sum_p\sum_q B_{pq}\overline{E}^l_{pq}(\theta,\phi) \tag{9}$$

where the total number of B_{pq} are significantly less than $(2N+1)\times(2M+1)$. Also, $\overline{E}^l_{pq}(\theta,\phi)$ in (9) is given for the far zone case by (6) or (7).

Thus, each global DFT component of the actual array distribution radiates a narrow spotlike beam in a direction dictated by its corresponding linear phase distribution over the array. Each resulting spot beam serves as the local basis function in the far zone field pattern synthesis. Now the coefficients, B_{pq}, in (4) or (9) will serve as the unknown variables to be optimized in the present synthesis procedure. After a set of optimized DFT coefficients, B_{pq}, are found, the actual element excitation coefficients, A_{nm}, can then be rapidly obtained via the inverse DFT from (3). The present synthesis scheme is interpreted in Fig. 1 where only those spot beams (local radiation basis set) confined within the desired hypothetical pattern coverage region are employed in the synthesis procedure. Furthermore, it is not necessary to place spot beams outside the desired pattern coverage area since the low sidelobe levels of the spot beams automatically makes the synthesized contoured beam drop to very low levels outside the coverage area. It is noted, however, that one may also employ extra spot beams (local radiation basis) in the neighborhood of the contoured boundary in order to achieve the requirements of ultralow-pattern levels in this region; this will not significantly increase the total number of local radiation basis functions. An MLSE optimization-based synthesis algorithm is used to determine B_{pq}, and then employ (3) to find the element amplitudes A_{nm} for the actual array distribution. The MLSE procedure will be briefly described in Section III.

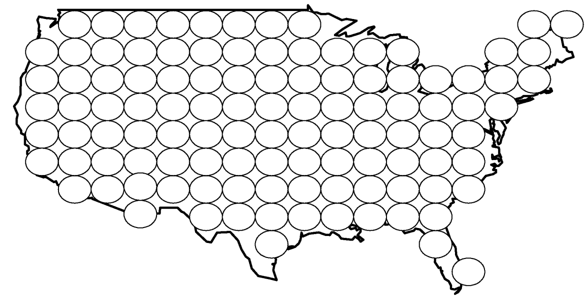

Fig. 1. Contoured beam may be thought of as being comprised of a set of narrow $\overline{E}^l_{pq}$ beams in the angular space. It is noted that only those spotlike beams confined within the desired contoured region are shown. The spot beams refer to the pattern of each local radiation basis function $\overline{E}^l_{pq}$ related to the DFT approach.

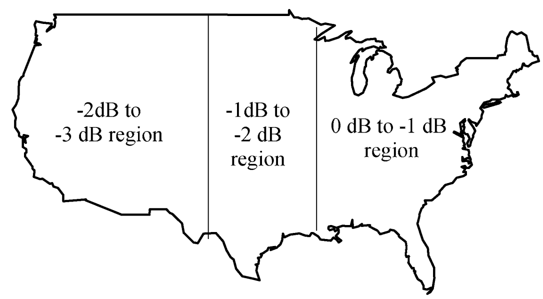

Fig. 2. An example of a desired CONUS beam requirement. The pattern magnitude in the region outside the United States coverage area is required to be ≤ -15 dB, whereas that inside the coverage area is as shown in the figure.

III. Radiation Pattern Synthesis Based on MLSE

Consider the radiation field pattern expansion for the entire array in terms of the local radiation basis set as in (4) or (9), which is required to synthesize the desired contoured beam pattern, $\overline{E}_d$, as closely as possible.

Next, one takes the inner product of (4) with $\overline{E}^*_e(\theta,\phi)/(|\overline{E}_e(\theta,\phi)|)$, where $(*)$ denotes the complex conjugate, and then solves for the coefficients, B_{pq}, using the standard MLSE technique [5] so that the desired radiation coverage beam can be formed.

IV. Numerical Examples

In this section, the DFT combined with the MLSE algorithm is employed to synthesize a CONUS coverage contoured beam. An array of simple antenna elements consisting of 201×201 Huygen's sources with spacing of 0.4λ between them are employed to generate the contoured pattern. Only those spot beams corresponding to the local radiation basis functions, $\overline{E}^l_{pq}$, which are located within the desired contoured beam region, are selected according to the rule suggested in Section II. Fig. 2 indicates the desired contoured pattern with the predefined power levels. It is noted that only 61 local basis set (or spot beams)

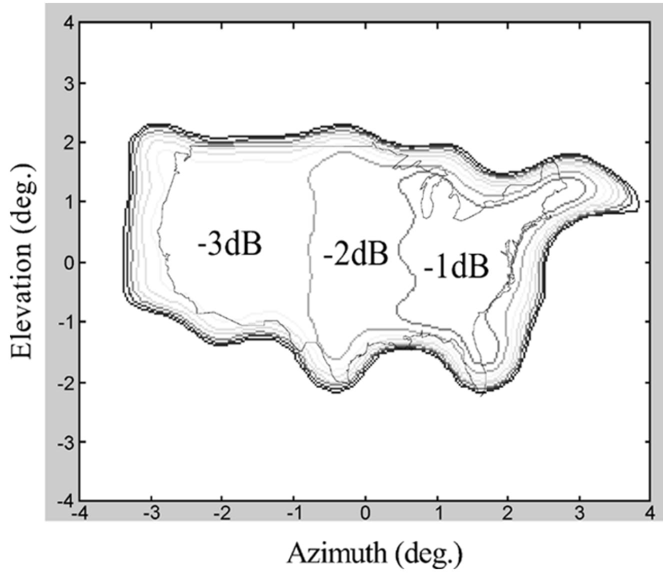

Fig. 3. MLSE synthesized CONUS pattern for the requirement defined in Fig. 2.

are employed here in contrast to 40 401 global (element pattern) basis set in the conventional approach. The MLSE algorithm optimizes the power pattern. The MLSE synthesized contoured pattern is shown in Fig. 3. The ripples inside the coverage area are <0.3 dB, and the side lobe level is < -30 dB. It is noted that 0.16 s of CPU time (PIII 550 MHz PC) were required to find the spot beams in the coverage area with 608 sample points in the MLSE-based synthesis technique. The MLSE took 0.33 s to find B_{pq}, and it took 4.5 s to find the array distribution coefficients A_{nm} from B_{pq}. Finally, 5.33 s were taken to compute the final contoured pattern at 14 400 sample points.

V. Conclusion

In this letter, a simple DFT-based method combined with a standard synthesis algorithm is utilized in satellite antenna planar array applications for generating a contoured beam. The approach employs the DFT representation for the actual array amplitude distribution; each DFT global basis set for the array distribution radiates a local radiation basis function (spot beam) which can be expressed in closed form and interpreted in terms of the UTD ray concept. The advantages of the approach are that only a few spot beams are sufficient to synthesize a contoured pattern. Numerical results have demonstrated its efficiency and validity.

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