Communications

Planar Near-Field to Far-Field Transformation Using an Array of Dipole Probes

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***Abstract*— A method is presented for computing far-field antenna patterns from measured near-field data measured by an array of planar dipole probes. The method utilizes the near-field data to determine some equivalent magnetic current sources over a fictitious planar surface which encompasses the antenna. These currents are then used to find the far fields. The near-field measurement is carried out by terminating each dipole with 50 Ω load impedances and measuring the complex voltages across the loads. An electric field integral equation (EFIE) is developed to relate the measured complex voltages to the equivalent magnetic currents. The mutual coupling between the array of probes and the test antenna modeled by magnetic dipoles is taken into account. The method of moments with Galerkin's type solution procedure is used to transform the integral equation into a matrix one. The matrix equation is solved with the conjugate gradient–fast Fourier transformation (CG–FFT) method exploiting the block Toeplitz structure of the matrix. Numerical results are presented for several antenna configurations to show the validity of the method.**

I. Introduction

Planar near-field antenna measurements have become widely used in antenna testing since they allow for accurate measurements of antenna patterns in a controlled environment. The earliest works are based on the modal expansion method, in which the fields radiated by the test antenna are expanded in terms of planar wave functions. The measured near fields are then used to determine the coefficients of the wave functions [1]–[4] in the modal expansion. The primary drawback of the mode expansion technique is that when a Fourier transformation is used to compute the far fields, the fields outside the measurement region are assumed to be zero. Consequently, the far fields are accurately determined only over a particular angular sector which is dependent on the measurement configuration and on the characteristics of the antenna [5], [6].

The equivalent current approach which represents an alternate method of computing far fields from measured near fields has been recently explored [7]–[10]. The basic idea for the equivalent current approach is to replace the radiating antenna by an equivalent magnetic and/or electric currents which reside on a fictitious surface and encompass the antenna. Under certain conditions, these equivalent currents produce the correct far fields in all regions in front of the antenna.

For the equivalent magnetic current approach where only magnetic currents are used as equivalent sources, the obtained EFIE equation is a decoupled one with respect to the coordinate axes for the planar scanning case [9]. This means that two simple decoupled EFIE's can be formulated which contain only one component of the measured near fields and equivalent currents and which can be solved separately, in a parallel fashion [13].

In our earlier works, the probe antenna measuring the near fields was always considered to be a single antenna, and the mutual interaction including multiple reflections between the probe antenna and the test antenna was considered. In this work, we replace a single-probe antenna by an array of dipoles separated from each other by approximately a half wavelength. The use of an array of probes is quite advantageous as it eliminates cumbersome precise mechanical movement of the probe antenna over a large planar surface. Secondly, the utilization of a probe array also eliminates the necessity of knowing the physical location of the probes accurately as it moves across the test antenna. These considerations become more important at the millimeter-wave frequencies.

In this new method, the array of dipoles is planar. They are all terminated in a 50 Ω impedance, and a network analyzer or any other noninvasive probing (e.g., optoelectronic devices) may be used to measure the voltages across the 50 Ω loads. Hence, in principle, this method is quite different from [12] where an end-fire array of dipole probes has been used to modulate the response.

In this paper, to compute the mutual interaction between the array of probes and the test antenna, the method of moments with Galerkin's type solution procedure has been used to transform the EFIE to a matrix equation. For this particular case, the matrix has a block Toeplitz structure, and so the conjugate gradient method along with the fast Fourier transform method (CG–FFT) can be utilized to solve such matrix equations without even explicitly forming the normal equations, thereby drastically reducing computation and storage requirements.

The theoretical basis for the equivalent current approach and the formulation of the EFIE are detailed in Section II. The formulation of the matrix equation using Galerkin's method with the application of CG–FFT is presented in Section III. Simple numerical results illustrating this concept are given in Section IV.

II. Theory

Consider an arbitrarily shaped antenna radiating into free space. A planar array of probes consisting of thin dipoles is located in front of the test antenna as shown in Fig. 1. We apply the equivalence principle at an infinite plane in front of the test antenna, so that for $z > 0$, this equivalent source provides the correct fields. The planar array of probes is considered to be parallel to this fictitious plane in front of the test antenna on which some equivalent currents have been applied to predict the correct fields in the region $z > 0$. The distance between the plane placed in front of the test antenna and the planar array is d. The probes consist of x-directed dipoles (typically a half wavelength long), and are uniformly distributed along both the x and y directions. The spacing between the probes is considered to be Δx and Δy in the x and y directions, respectively. The dipoles are terminated by the impedance z_L (typically 50 Ω) to match the input impedance requirement of the measurement device.

Invoking the surface equivalence principle by an equivalent magnetic current $\overline{M}$ on the plane in front of the test antenna, one can predict the proper electric field for $z > 0$. Utilizing the image theory, one can evaluate the radiated electric field as

$$E_x^s(\boldsymbol{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial g(\boldsymbol{r}, \boldsymbol{r}')}{\partial z'} M_y(\boldsymbol{r}')\, dx'\, dy' \qquad (1)$$

$$E_y^s(\boldsymbol{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial g(\boldsymbol{r}, \boldsymbol{r}')}{\partial z'} M_x(\boldsymbol{r}')\, dy'\, dx' \qquad (2)$$

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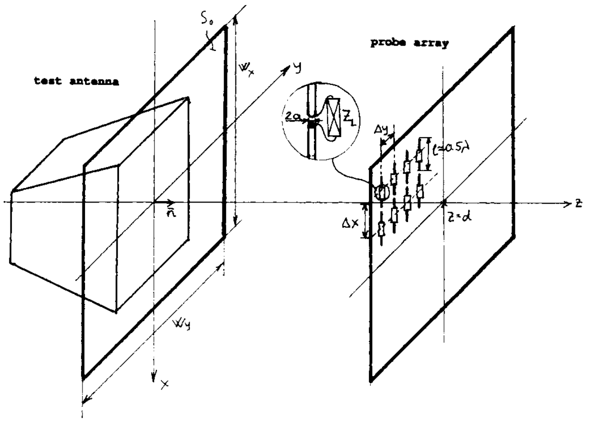

Fig. 1. Planar near-field measurement using phased array as probe antenna.

$$\frac{\partial g(\boldsymbol{r}, \boldsymbol{r})}{\partial z'} = \frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|^2}(z-z')\left[jk_0 + \frac{1}{(\boldsymbol{r}-\boldsymbol{r}')}\right] \quad (3)$$

where $g(\boldsymbol{r}, \boldsymbol{r}')$ is the free-space Green's function, k_0 is the wavenumber, and

$$\boldsymbol{r}' = x'\hat{x} + y'\hat{y} + 0\hat{z}$$

$$r = x\hat{x} + y\hat{y} + d\hat{z} \quad (4)$$

and M_x and M_y are the amplitude of the equivalent x- and y-directed magnetic currents residing on the equivalent surface. The limits of the integrals from $-\infty$ to $+\infty$ can be reduced to $-w_x/2$ to $w_x/2$ and $-w_y/2$ to $w_y/2$. This is because, as the test antenna is of finite size, the truncation error would not be large if the limits of the integration are reduced from the infinite plane to a rectangular plane of size $w_x \times w_y$.

The total electric field in front of the test antenna is produced by the equivalent magnetic current sources M and by the induced electric currents J_{pr} on the probes so

$$E_{\text{total}} = E^s_m(M) + E^s_{pr}(J_{pr}). \quad (5)$$

Enforcing the tangential component of the total electric field to be zero on the probes results in the following EFIE:

$$\{E^s_{pr}(J_{pr}) + E^s_m(M)\}_{\tan} = 0 \text{ on probe dipoles.} \quad (6)$$

In (6), the unknown currents are M and J_{pr}. Since the near-field measurement is carried out by terminating each probe with the load impedances z_L (typically 50 Ω) and we measure the complex voltages across the loads, the current distribution J_{pr} is known. This is because, for the present approach, on every resonant size probe, only one entire domain basis function has been applied for the numerical calculation. Hence, (6) becomes

$$E^s_m(M)_{\tan} = -E^s_{pr}(V_{\text{meas}}) \quad (7)$$

where V_{meas} are the known values of the measured voltages.

To obtain the far fields, we need to find the equivalent source M' on the "test antenna surface" when the probes are not present. If we assume that the same excitation on the test antenna is present with and without the probes, then

$$H(M')_{\tan} = -H^i_{\tan}(V_{\text{mag}}) \text{ on the source plane} \quad (8)$$

where $H^i_{\tan}$ are the known magnetic fields across the excitation gaps of the test antenna produced by the magnetic voltage generators V_{mag}.

However, when the probes are present, (8) becomes

$$\{H(M) + H(J_{pr})\}_{\tan} = -H^i_{\tan}(V_{\text{mag}}) \text{ on the source plane} \quad (9)$$

where $H(J_{pr})$ is the magnetic field produced by the electric currents on the probes.

Combining (8) with (9), the following equation is obtained for the unknown magnetic current distribution M':

$$H(M')_{\tan} = \{H(M) + H(J_{pr})\}_{\tan} \text{ on the source plane} \quad (10)$$

where M is the magnetic current distribution when the probes are present and J_{pr} is the known electric current distribution on the probes. So the two decoupled integral equations to be solved are (7) and (10).

It is important to note that the numerical effort required to solve (7) is the same for [10, eq. (2)]. This is because the computation of the term $E^s_{pr}(V_{\text{meas}})$ requires only a simple preprocessing of the measured voltage data. The solution of (10) is a well-behaved one because the moment matrix has dominant diagonal elements, and therefore does not cause any numerical difficulty.

It is important to point out that this formulation takes all mutual couplings into account, i.e., the mutual coupling between the test antenna and the dipole probes, and the mutual coupling between the dipole probes.

III. Formulation of the Matrix Equations

For use of computation, on the planar equivalent surface of dimension $w_x \times w_y$, we put an array of P magnetic dipoles in the x directions and Q in the y directions so the total number of dipoles is PQ. This is a good approximation if d is greater than a few wavelengths [6], [9]. Also, this assumption does away with the numerical integration required in the evaluation of the matrix elements. Therefore,

$$\Delta x = \frac{w_x}{P} \text{ and } \Delta y = \frac{w_y}{Q} \quad (11)$$

and the centers of the dipoles are located at

$$x_{ij} = -\frac{w_x}{2} + (i-1)\Delta x, \qquad i = 1, \cdots, P \quad (12)$$

$$y_{ij} = -\frac{w_y}{2} + (j-1)\Delta y, \qquad j = 1, \cdots, Q. \quad (13)$$

For the dipole probes, we consider the entire domain sinusoidal basis functions as

$$f_{ij}(x') = \begin{cases} \frac{\sin\{k_0[x - x_c + h/2]\}}{\sin(k_0 h/2)} & \text{for } x_c - h/2 \le x \le x_c \\ \frac{\sin\{k_0[x_c + h/2 - x]\}}{\sin(k_0 h/2)} & \text{for } x_c \le x \le x_c + h/2 \end{cases} \quad (14)$$

where h and a are the length and radius of the probe dipoles, and x_c, y_c are the coordinates of the center of a dipole probe.

Applying the discretization procedure to (7) and enforcing Galerkin's method, one obtains

$$HM_y = \frac{1}{z_L}(G^j_{xx} + \langle z_L \rangle)V \quad (15)$$

where matrix H represents the interaction between the probes and the magnetic current elements. The explicit expression for matrix H is given by

$$H_{(i,j),(k,l)} = -\int_{p_{kl}} \frac{\partial g(\boldsymbol{r}_{kl}, r'_{ij})}{\partial z'} f_{kl}(x)\, dx. \quad (16)$$

Here, r_{kl} defines the surface of the (rk)th probe antenna of the phased array and r'_{ij} defines the (ij)th magnetic dipole on the source plane.

M_y is the unknown vector containing the y component of the equivalent magnetic source current elements. z_L is the load impedance terminating the probes. The matrix G_{xx} represents the interaction between the probes and is given by

$$G_{(ij),(kl)} = \frac{1}{j\omega\epsilon_0} \int_{p_{ij}} \int_{p_{kl}} \left\{ \left[k_0^2 f_{ij}(x') f_{kl}(x) - \frac{df_{ij}(x')}{dx'} \frac{df_{kl}(x)}{dx} \right] \cdot g(x, x', y_l, y_j') \right\} dx\, dx'. \quad (17)$$

Here, p_{kl} defines the surface of the (kl)th probe antenna.

Matrix $\langle z_L \rangle$ is a diagonal matrix containing the values of the load impedances z_L. Vector V contains the complex values of the measured voltages which are measured across the loads.

A detailed explanation for application the of CG–FFT method to solve (17) with a block Toeplitz structure can be found in [10]. For the case of a rectangular matrix, a least squares solution for the currents is found without explicitly computing the normal form of the equation.

To transform (10) into a matrix equation, the same procedure is followed as for (7). Using the method of moments with Galerkin's type solution procedure, the following system of linear equations is obtained:

$$G_{yy}^m M_y' = G_{yy}^m M_y - \frac{1}{z_L} HV. \quad (18)$$

Here, M_y is the unknown vector containing the y component of the equivalent magnetic source current elements when the probes are not present. M_y is the known vector obtained by solving (15). Matrix M and vector V are the same as defined in (15). Matrix G_{yy}^m represents the interaction between the magnetic current sources and is given by

$$G_{yy(ij),(kl)}^m = \frac{1}{j\omega\mu_0} \int_{s_{ij}} \int_{s_{kl}} \left\{ \left[k_0^2 f_{ij}(x') f_{kl}(x) - \frac{df_{ij}(x')}{dx'} \frac{df_{kl}(x)}{dx} \right] g(x, x', y_l, y_j') \right\} dx\, dx'. \quad (19)$$

Equation (19) is very similar to (17), but here the integration is carried out on the surfaces of the magnetic current elements s_{ij} and s_{kl}.

For the other polarization, the array of probes can be rotated 90°, and the same equations may be solved as for the 0° case. In this case, equivalent magnetic current elements only have an x component (M_x, M_x') and the electric currents induced on the probes only have a y component $(J_{pr,y})$.

IV. Numerical Results

In order to illustrate the methodology described in this paper, typical numerical results are presented.

As a first example, consider an array of four y-directed magnetic Hertzian dipoles located at the corners for a $4\lambda \times 4\lambda$ surface. The dipoles are located on the "equivalent test antenna plane" (x–y plane) of dimensions $5.61\lambda \times 5.61\lambda$. The near fields are measured by an 11×11 array of 0.5λ dipoles distributed uniformly along the x and y directions with distances $\Delta x = \Delta y = 0.51\lambda$. Each of the elements in the 11×11 dipole array is terminated by an impedance, $z_L = 50\ \Omega$. The probe array is situated 3.0λ away from the "equivalent plane" for the test antenna. The complex voltage values measured across the impedance z_L are the measured near-field data. The data will be used to calculate the far field.

On the equivalent magnetic current source surface, we consider 11×11 uniformly distributed magnetic dipoles with separation distances $\Delta x = \Delta y = 0.51\lambda$. Such a choice of magnetic dipoles on the equivalent source plane allows us to use CG–FFT for the numerical solution of the integral equation. The near-field data of voltages across z_L are used to find the amplitude of the current on the probes. Since only one entire domain basis function has been chosen for the probes, the currents on the probes are all known. From this, we solve (7) and (10) to find M', the current on the magnetic

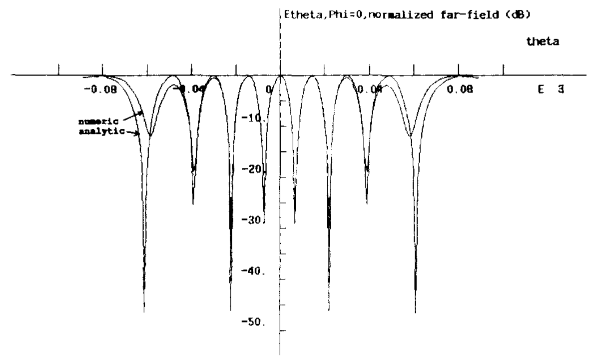

Fig. 2. Comparison of exact and computed far fields for $\Phi = 0$ cut for 2×2 magnetic dipoles on a $4\lambda \times 4\lambda$ surface.

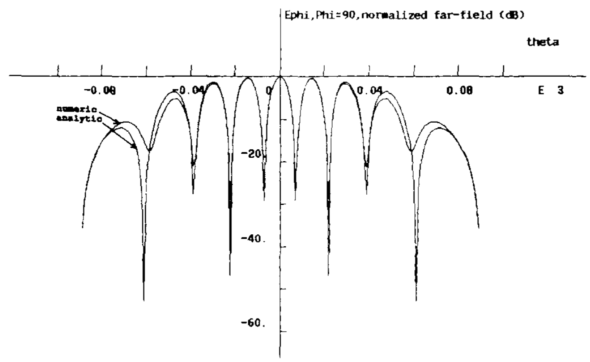

Fig. 3. Comparison of exact and computed far fields for $\Phi = 90$ cut for 2×2 magnetic dipoles on a $4\lambda \times 4\lambda$ surface.

dipoles in the absence of the planar probe array. Once M' is obtained, the far field can be calculated from M'. Figs. 2 and 3 present the normalized absolute value in decibels of the theta component of the electric far field $[E_\theta]$ for $\phi = 0°$ and $[E_\theta]$ for $\theta = 90°$, respectively, as a function of θ. The calculated far fields are accurate upto $\pm 45°$ from $\theta = 0°$ and acceptable for all values of θ.

For this particular example, the conventional modal expansion provides accurate results only up to $\theta = \tan^{-1}(1.61/6) \approx 15°$ [5]. Hence, this approach, utilizing an array of dipole probes, has a larger region of validity than conventional approaches utilizing a single-probe antenna.

As a second example, consider a half-wave dipole antenna of radius 0.007λ located at the center of the coordinate system as shown in Fig. 4. The planar near fields are measured by an array of probes as shown in Fig. 4. In this example, the input impedance of the half-wave dipole is calculated from the measured near fields. The input impedance directly depends on the current of the source dipole in free space. The number of probes is varied from $n = 1$ to 961 $(= 31 \times 31)$. The input impedance of a half-wave dipole is about 73 Ω in free space. If the present methodology of this paper is meaningful, then even when the near fields are measured by an array of 961 probes (which significantly affects the current distribution on the test dipole), one should still be able to obtain the input impedance of the dipole in isolation. This would illustrate that it is possible to deembedd the effects of the probe array and obtain the proper current (or input

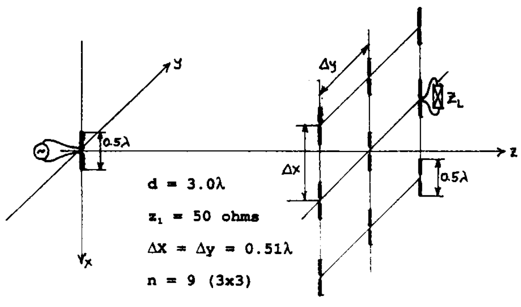

Fig. 4. Planar near-field measurement configuration. The test antenna is a resonant size electric dipole. The probe antenna is a phased array ($n = 9$).

TABLE I
INPUT IMPEDANCE OF THE TEST ANTENNA AS A FUNCTION OF THE NUMBER OF ELEMENTS OF PHASED ARRAY PROBE

n	Z_{in}
1	$73.402 + j41.654$
9	$74.398 + j39.747$
25	$71.644 + j39.339$
49	$73.218 + j40.535$
81	$72.113 + j39.696$
121	$73.149 + j39.705$
169	$72.874 + j40.454$
225	$72.350 + j40.256$
441	$72.948 + j39.716$
961	$72.776 + j40.387$
Single	$73.129 + j41.797$

impedance) on the test antenna in isolation. Once the currents are known, the far fields can easily be computed.

Table I contains the calculated value of the input impedance of the test antenna in isolation as a function of $n = 1$–961. The largest value of the error is 2.4% in the real part and 5.6% in the imaginary part of the input impedance.

It has been our experience that, by utilizing this new method, if $d > 3\lambda$, then instead of solving two equations (7) and (10), it is possible to drop (10) and just solve (7) with M replaced by M'. Our limited experimentation has proved that this is valid irrespective of the size of the planar probe array.

V. CONCLUSION

A method is described for computing the far fields of a test antenna from measured near-field data obtained by a planar phased array of probe dipoles. In this approach, there is no need to move the probe mechanically, nor is there any necessity to measure the spatial probe position accurately as it is carrying out the measurements. This method could be very useful, particularly at millimeter waves where it may be difficult to move a probe mechanically with millimeter accuracy. The surface equivalence principle along with the method of moments is utilized to solve the integral equation for equivalent unknown magnetic currents on a planar surface located in front of the test antenna. A numerically very efficient CG–FFT scheme has been implemented to solve the integral equation for the magnetic currents, thereby drastically reducing both computation time and storage requirements.

This procedure takes into account not only the mutual interaction between dipoles in the planar array probe, but also between the test antenna and the planar array probe. However, it has been our experience that if $d > 3\lambda$, then the second mutual interaction in this procedure can be neglected without affecting the final results significantly.

The validity of this approach has been shown through limited examples, and this method has a wider range of validity than that of the contemporary modal expansion method.

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