Assignment-4

Q1. Let the signals $x_1[n] = a^n u[n]$ and $x_2[n] = (-1)^n a^n u[n]$ then,

1. Compute DTFT $X_1(\omega)$ and $X_2(\omega)$ of the signals.
2. Plot the magnitude and phase spectrums of $X_1(\omega)$ and $X_2(\omega)$.
3. Compare the spectrums of $X_1(\omega)$ and $X_2(\omega)$ and obtain a mathematical relationship between $X_1(\omega)$ and $X_2(\omega)$.
4. From the relationship, please deduce a property of DTFT.

Hint: Express $(-1)^n$ in terms of complex exponential signal form.

Q2. Prove the following DTFT properties.

- **Time shifting:** $DTFT(x[n-k]) = e^{-j\omega k} X(\omega)$
- **Frequency shifting**: $DTFT\left(e^{-j\omega_0 n} x[n]\right) = X(\omega + \omega_0)$
- **Convolution**: $DTFT(x_1 * x_2[n]) = X_1(\omega) X_2(\omega)$
- **Multiplication**: $DTFT(x_1[n] x_2[n]) = X_1(\omega) * X_2(\omega)$
- **Differentiation in frequency**: $DTFT(nx[n]) = j\frac{dX(\omega)}{d\omega}$
- **Parseval's relation**: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(\omega)|^2 \, d\omega$
- **Down sampling:** $x[2n] = \frac{1}{2}[X\left(\frac{\omega}{2}\right) + X(\frac{\omega}{2} - \pi)]$
- **Accumulation property:** $\sum_{k=-\infty}^{n} x[k] = \frac{X(\omega)}{1-e^{-j\omega}} + \pi X(0) \sum_{l=-\infty}^{\infty} \delta(\omega + 2\pi l)$

Q3. Let $\delta(\omega) = 0 \; for \; \omega \neq 2\pi k, k \; is \; an \; integer$ and $\int_{2\pi} \delta(\omega) d\omega = 1$ then prove the following properties.

- $\int_{2\pi} \delta(\omega) X(\omega) d\omega = X(0)$
- $\int_{2\pi} \delta(\omega - \omega_0) X(\omega) d\omega = X(\omega_0)$
- $\delta(\omega - \omega_0) X(\omega) = X(\omega_0) \delta(\omega - \omega_0)$

Q4. Let $x[n] = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-1)$ with DTFT as $X(\omega) = X_R(\omega) + jX_I(\omega)$ then

- Compute $\int_{-\pi}^{\pi} X(\omega) \, d\omega$
- Find the signal whose DTFT is $X_R(\omega) e^{j2\omega} + jX_I(\omega)$