Assignment-4

Q1. Let the signals $x_1[n] = a^n u[n]$ and $x_2[n] = (-1)^n a^n u[n]$ then,

1. Compute DTFT $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ of the signals.
2. Plot the magnitude and phase spectrums of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$.
3. Compare the spectrums of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ and obtain a mathematical relationship between $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$.
4. From the relationship, please deduce a property of DTFT.

Hint: Express $(-1)^n$ in terms of complex exponential signal form.

Q2. Prove the following DTFT properties.

- **Time shifting:** $DTFT(x[n-k]) = e^{-j\omega k} X(e^{j\omega})$
- **Frequency shifting**: $DTFT\left(e^{-j\omega_0 n} x[n]\right) = X(e^{j(\omega+\omega_0)})$
- **Convolution**: $DTFT(x_1 * x_2[n]) = X_1(e^{j\omega}) X_2(e^{j\omega})$
- **Multiplication**: $DTFT(x_1[n] x_2[n]) = X_1(e^{j\omega}) * X_2(e^{j\omega})$
- **Differentiation in frequency**: $DTFT(nx[n]) = j\frac{dX(e^{j\omega})}{d\omega}$
- **Parseval's relation**: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2\, d\omega$

Q3. Let $\delta(\omega) = 0\ for\ \omega \neq 2\pi k, k\ is\ an\ integer$ and $\int_{2\pi} \delta(\omega) d\omega = 1$ then prove the following properties.

- $\int_{2\pi} \delta(\omega) X(\omega) d\omega = X(0)$
- $\int_{2\pi} \delta(\omega - \omega_0) X(\omega) d\omega = X(\omega_0)$
- $\delta(\omega - \omega_0) X(\omega) = X(\omega_0)\delta(\omega - \omega_0)$

Q4. Can the DTFT of arbitrary causal signal result zero phase spectrum? Please explain it briefly.

Q5. Let $x[n] = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-1)$ with DTFT as $X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$ then

- Compute $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$
- Find the signal whose DTFT is $X_R(e^{j\omega}) e^{j2\omega} + jX_I(e^{j\omega})$