Signal Processing (EC5.201)

Assignment – 06
Released – 25th October 2024
Due date – 4th November 2024 (11:59 pm IST)

- The handwritten assignments are to be submitted individually. Compile all your scans in the right order into a single pdf file. Moodle submission portal will be opened soon.

- Student discussions are allowed but copying and plagiarism will NOT be tolerated and will attract strict penalties.

- At the beginning of the assignment, each student must declare the honor code:
  "I affirm that I have neither given nor received help or used any means which would make this assignment unfair."
  - YOUR Signature

- Assignments submitted without an honor code and signature will have a 10% penalty.

- Late submission: 10% penalty per day (will be accepted up to at most 3 days after deadline).

- Solve and submit solutions to the following problems:

- **Q1.** Let the inputs to a discrete-time LTI system be sum of two sinusoids with frequencies ω_1 and ω_2. Assume that the system has some arbitrary but even *magnitude response*. Answer the following questions by doing the analysis in frequency domain only.
  (a) Show that if the system has *linear phase*, the two sinusoids experience equal amounts of delay at the output.
  (b) Show that if the system has *quadratic phase*, the two sinusoids experience un-equal amounts of delay at the output. For $\omega_1 = \pi/6$ and $\omega_2 = 5\pi/6$, what is the difference in delays?

- **Q2.**

[**Marks]** A discrete-time LTI system is described by the difference equation

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

where $x[n]$ and $y[n]$ are input and output signals respectively. Assume that this system is stable.

(a) Find expression for its frequency response.

(b) Given the constants a_1 and a_2, identify the constraints on b_0, b_1 and b_2 such that this system has constant magnitude response.

(c) Identify conditions on the constants a_i and b_i for this system to have linear phase.

- Q3.

[**Marks**] An audio compact disc (CD) stores audio signals sampled at 44.1 kHz and quantized using 32 bits. Assume there are two audio channels. Answer the following with explanations:

(a) If the CD contains 5 songs of duration 5 mins each, what is the amount of memory used up in storing the files in the CD?

(b) If it is known that the stored audio signals have a maximum frequency of 10 kHz, could you suggest a way to reduce the amount of memory used in the CD without any loss in audio quality using concepts from the course? How much memory can you save?

- Q4.

[**Marks**] An LTI system with impulse response $h(t)$ produces output $y(t) = h(t) \star x(t)$ when the input is $x(t)$, here $\star$ denotes continuous-time convolution. The bandlimited signal $x(t)$ is sampled at rate $f_s = \frac{1}{T_s}$ (which is above its Nyquist rate) to obtain the discrete-time signal $x[n] = x(nT_s)$.

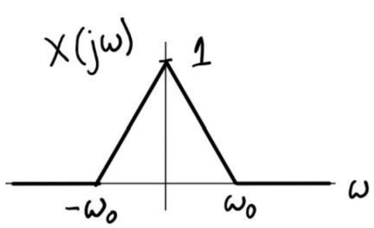
(a) Show that $y(t)$ can also be sampled at rate f_s without any loss of information.

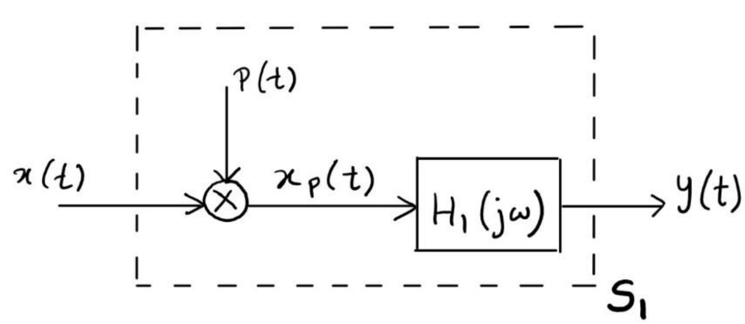
Let the samples be $y[n] = y(nT_s)$.

(b) Find the impulse response $h[n]$ of a discrete-time LTI system such that $y[n] = h[n] \star x[n]$, here $\star$ denotes discrete-time convolution. Justify whether this impulse response is unique.

- Q5.

[ ] A system S_1 is designed to perform impulse train sampling followed by filtering with $H_1(j\omega)$ as shown below. The input to this system is a *bandlimited* continuous-time signal $x(t)$ with Fourier transform $X(j\omega)$ as shown, $X(j\omega) = 0$ for $|\omega| > \omega_0$.

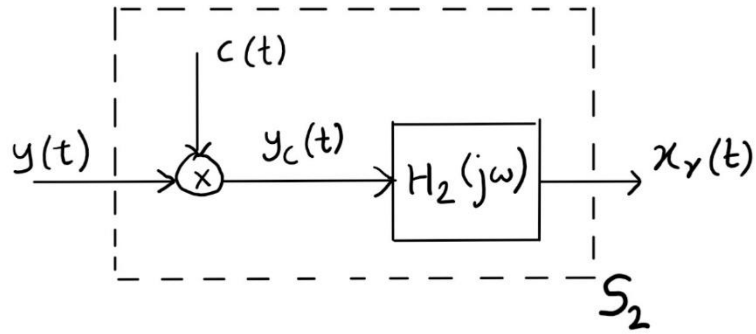




(a) Write expression for the Fourier transform of the sampled signal $x_p(t) = x(t)p(t)$ and plot it. It is given that impulse train $p(t) = \sum_{-\infty}^{\infty} \delta(t - nT)$ and sampling period $T = \frac{2\pi}{3\omega_0}$.

(b) Due to an error in system design, $H_1(j\omega)$ was chosen to be an ideal high pass filter with cutoff frequency $\frac{3\omega_0}{2}$. Plot the Fourier transform of the output $y(t)$ of this filter.

To correct for the error, a system S_2 is designed with input $y(t)$ as shown below. The system S_2 consists of a multiplication step giving $y_c(t) = y(t)c(t)$ where $c(t) = e^{-j\omega_1 t}$, followed by application of a ideal low pass filter $H_2(j\omega)$ with cutoff frequency $\frac{3\omega_0}{2}$ and gain T.



(c) Give expression for Fourier transform of $y_c(t)$ and plot it.

(d) Find all possible values of frequency ω_1 so that the reconstructed signal $x_r(t)$ is exactly equal to the original signal $x(t)$.

(e) For *any* arbitrary input $x(t)$, is the system S_2 inverse of the system S_1 ? Explain.