Signal Processing
Assignment- 8

Monsoon 2023

1. Given an analog prototype filter $H(s) = \frac{A}{s+\alpha}$, show that the digital IIR filter obtained using the impulse invariance method is given by $H(z) = \frac{A}{1-e^{-\alpha T}z^{-1}}$. Are these filters causal?

2. The following digital filter was obtained from an analog prototype using the impulse invariance method. What is the corresponding analog filter H(s)?

$$H(z) = \frac{2z}{z - e^{-0.9}} + \frac{3z}{z - e^{-1.2}}$$

3. Aliasing can be exploited to realise interesting frequency response characteristics. An ideal, causal analog LPF has a cutoff frequency Ω_c. Two digital filters (H_1 and H_2) are designed from this using $t = nT; T = \frac{3\pi}{2\Omega_c}$ and $\frac{\pi}{\Omega_c}$ respectively. Assume the filters to be normalized so that $H_1(\Omega=0) = H_2(\Omega=0) = 1$. Consider a new filter G which is constructed by connecting H_1 and H_2 in parallel. Find $G(z)$ and identify if it is LPF/HPF, etc.

4. A first order LPF (normalized) analog filter is given as $H(s) = \frac{1}{s+1}$. Design a digital *bandpass* filter using this analog prototype, *bilinear* and LP to BP filter transformations to meet the following specifications: Passband 200-300 Hz; Sampling frequency 2 kHz; Filter order M = 2.
Draw the pole-zero plot for your design.

5. IIR filters can also be designed by placing zeros and poles at suitable locations as the frequency response will vanish (equal 0) at the frequencies where zeros are located while the response peaks at the locations of the poles
   a. If it is true that closer the pole is to the unit circle, the larger the peak in the response, what is the equivalent statement for zero placement?
   b. Assume sampling frequency to be 500Hz and design a BPF to meet these specifications: total signal blocking at dc and 250 Hz; narrow passband centred at 125 Hz; 3 dB bandwidth of 10 Hz. Determine H(Z) and draw a block diagram of your filter.