

SP - Assignments 1:-

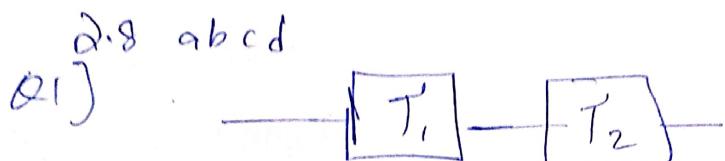
I affirm that I have neither given nor received help or used any means which would make this assignment unfair

2.8(a-d), 2.10, a.13, 2.16 b(s,g), 2.32, 2.35, 2.53

- NitinRamaSaih

~~2023/12/26~~

2023/12/26.



a) if T_1 & T_2 are linear, then T is linear..

let for $\alpha \in \mathbb{R}$, $x_1 \xrightarrow{T_1} y_1$
 $x_2 \xrightarrow{T_1} y_2$

By linearity

$$\alpha x_1 + \beta x_2 \xrightarrow{T_1} \alpha y_1 + \beta y_2 \rightarrow \textcircled{1}$$

Similarly Now if y_1 & y_2 are i/p for T_2

$$\begin{array}{l} y_1 \xrightarrow{T_2} z_1 \\ y_2 \xrightarrow{T_2} z_2 \end{array}$$

By linearity..

$$\alpha y_1 + \beta y_2 \xrightarrow{T_2} \alpha z_1 + \beta z_2$$

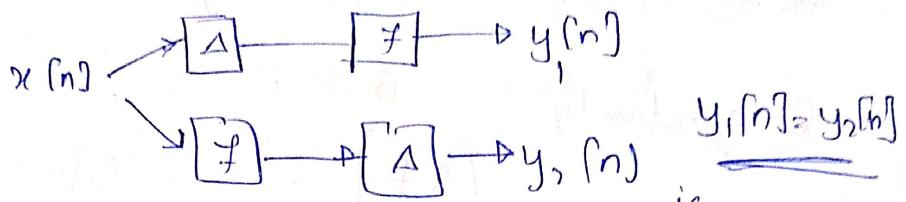
So if T_1 & T_2 is replaced by a black box T
we observe

$$\begin{array}{l} x_1 \xrightarrow{T_1} z_1 \\ x_2 \xrightarrow{T_1} z_2 \end{array} \left(\begin{array}{l} \text{from above} \\ \text{eqns} \end{array} \right) \quad \begin{array}{l} \alpha x_1 + \beta x_2 \xrightarrow{T} \alpha z_1 + \beta z_2 \\ \text{by the above eqns} \end{array}$$

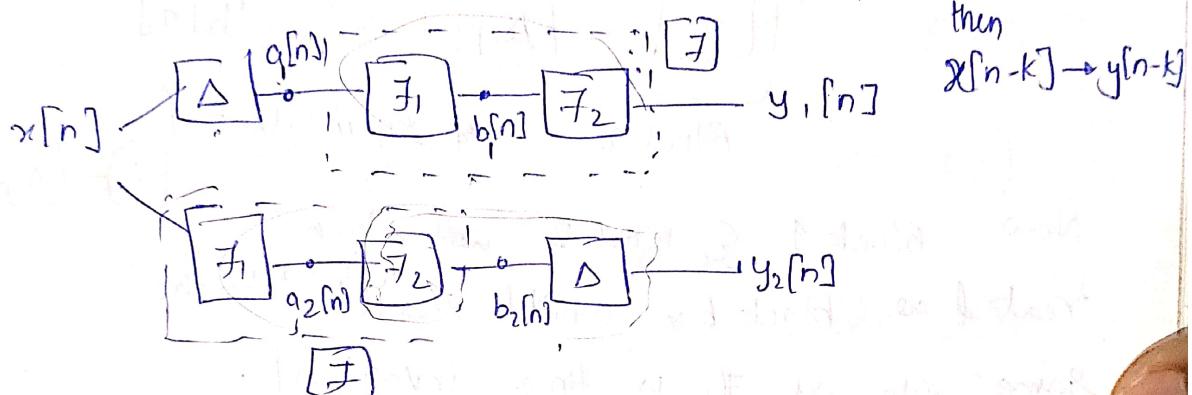
Hence the statement is true! Proved.

b) Given \mathcal{F}_1 , \mathcal{F}_2 are time invariant, then

for a time invariant sys,



We know \mathcal{F}_1 , \mathcal{F}_2 are time invariant, so $x[n] \rightarrow y[n]$



Backtracking relation b/w $y_1[n]$ & $x[n]$

$$y_1[n] \rightarrow x[n-\Delta]$$

since \mathcal{F}_1 is time invariant

$$\text{if } a_1[n] \rightarrow b_1[n]$$

$$\text{then } a_1[n-k] \rightarrow b_1[n-k]$$

if we prove

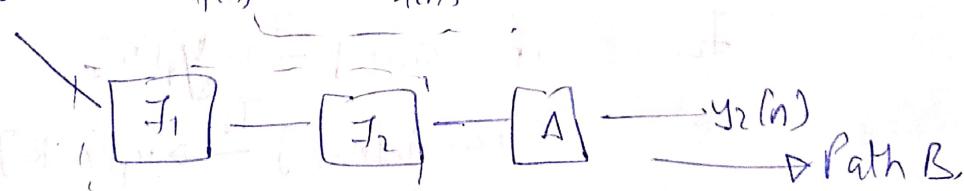
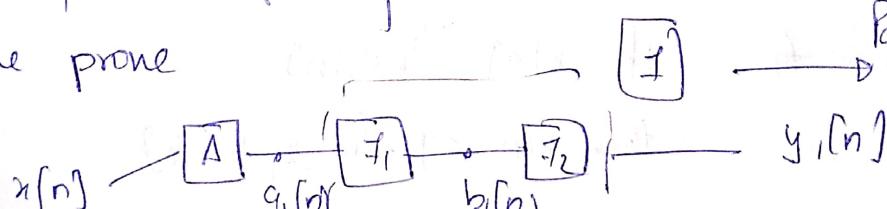
Back tracking relation

$$b/w y_2[n] \& x[n]$$

$$y_2[n] = b_2[n-\Delta]$$

$$x[n] \rightarrow a_2[n]$$

$$a_2[n] \xrightarrow{\mathcal{F}_2} b_2[n]$$



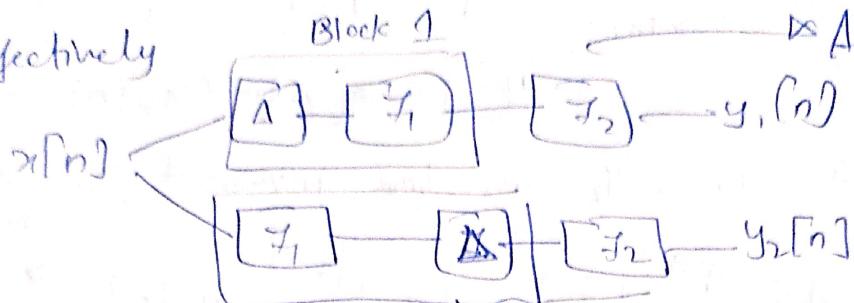
if $y_1[n] = y_2[n] \rightarrow$ it implies \mathcal{F}_1 & \mathcal{F}_2 cascade

is also
time invariant

in Path B,

we can swap b/w \boxed{A} & $\boxed{f_2}$ because
 f_2 is time invariant system as assumed.

so effectively

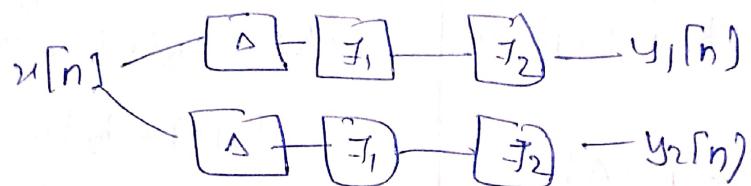


Block 2 $\xrightarrow{\text{up}} \text{equivalent of}$



Now Block 1 & Block 2 ~~will~~ when treated as black box would result into the same o/p as f_1 is time invariant.

So



Since both paths are same, o/p's are also same.
Hence Proved, it is true

Mathematical Proof:-

for f_1 if $x[n] \xrightarrow{f_1} a_1[n]$

then $x[n-k] \xrightarrow{f_1} a_1[n-k]$

f_2 if $a_1[n] \xrightarrow{f_2} y_1[n]$

then $a_1[n-k] \xrightarrow{f_2} y_1[n-k]$

So if $x[n] \xrightarrow{f_1, f_2} y_1[n]$

$x[n-k] \xrightarrow{f_1} a_1[n-k] \xrightarrow{f_2} y_1[n-k]$

$\therefore x[n-k] \xrightarrow{f_1, f_2} y_1[n-k]$

Hence Proved, it is true

c) if \mathcal{F}_1 & \mathcal{F}_2 are causal systems, then \mathcal{F} is causal.

if \mathcal{F}_1 is causal \Rightarrow

$$x[n] \xrightarrow{\mathcal{F}_1} a[n] = f(x[n], x[n-1], x[n-2], \dots)$$

i.e. depends on previous values $\xrightarrow{k \geq 0}$

if \mathcal{F}_2 is causal \Rightarrow $x[n] \xrightarrow{\mathcal{F}_2} b[n] = f(a[n], a[n-1], \dots)$

$$\boxed{x \xrightarrow{\mathcal{F}} y \quad y[n] = f(a[n], a[n-1], \dots, a[n-p])}$$

so if $\mathcal{F}_1, \mathcal{F}_2$ is a system, $a[n] \xrightarrow{\mathcal{F}_1} (n-p) \geq 0$

$$y[n] = f(a[n], a[n-1], \dots, a[n-p])$$

$y[n]$ depends on ~~$a[n-p]$~~

$a[n_2]$ where $n_2 \leq n$

and each $a[n_2]$ depends on $x[n_1]$

where $n_1 \leq n_2$

$n_1 \leq n_2 \leq n$

i.e. $n_1 \leq n \Rightarrow y[n]$ depends on $x[n_1]$ where $n_1 \leq n$

hence the cascaded system is causal.

d) if $\mathcal{F}_1, \mathcal{F}_2$ are linear and time invariant

$$x[n] \xrightarrow{\mathcal{F}_1} y_1[n] \xrightarrow{\mathcal{F}_2} y_2[n]$$

$$\text{if } \mathcal{F} \text{ is LTI} \quad x_1[n] \xrightarrow{\mathcal{F}_1} y_1[n]$$

$$x_2[n] \xrightarrow{\mathcal{F}_2} y_2[n]$$

$$\alpha x_1[n] + \beta x_2[n] \xrightarrow{\mathcal{F}_1} \alpha y_1[n] + y_1[n]$$

$$\alpha x_1[n-k] + \beta x_2[n-k] \xrightarrow{\mathcal{F}_2} \alpha y_2[n-k] + y_2[n-k]$$

so

$$\alpha x_1[n-k] + \beta x_2[n-k] \xrightarrow{\mathcal{F}} \alpha y_1[n-k] + \beta y_2[n-k]$$

same is the case with \mathcal{F}_2 ,

So if $x_1[n] \xrightarrow{f_1} q_1[n]$ and $q_1[n] \xrightarrow{f_2} y_1[n]$
 $x_2[n] \xrightarrow{f_1} q_2[n]$

$$x_1[n-k] \xrightarrow{f_1} q_1[n-k] \xrightarrow{f_2} y_1[n-k]$$

(By time invariance)

$$\alpha x_1[n] \xrightarrow{f_1} \alpha q_1[n] \xrightarrow{f_2} \alpha y_1[n]$$

(By linearity)

$$\alpha x_1[n] + \beta x_2[n] \xrightarrow{f_1} \alpha q_1[n] + \beta q_2[n]$$

$$\xrightarrow{f_2} \alpha y_1[n] + \beta y_2[n]$$

(By linearity)

So

$$x_1[n] \xrightarrow{f_1, f_2} y_1[n]$$

$$\alpha x_1[n] + \beta x_2[n] \xrightarrow{f_1, f_2} \alpha y_1[n] + \beta y_2[n]$$

so and also

$$x[n-k] \xrightarrow{f_1, f_2} y[n-k] \text{ (TI)}$$

$$\alpha x_1[n-k] + \beta x_2[n-k] \xrightarrow{f_1, f_2} \alpha y_1[n-k] + \beta y_2[n-k]$$

So

f_1, f_2 is a LTI system

Hence proved.

Q10.

$$x_1[n] = \begin{pmatrix} 1, 0, 2 \\ y \end{pmatrix} \xrightarrow{f} y_1[n] = \begin{pmatrix} 1, 0, 1, 2 \\ y \end{pmatrix}$$

$$x_2[n] = \begin{pmatrix} 0, 1, 0, 3 \\ y \end{pmatrix} \xrightarrow{f} y_2[n] = \begin{pmatrix} 0, 1, 0, 2 \\ y \end{pmatrix}$$

$$x_3[n] = \begin{pmatrix} 0, 0, 0, 1 \\ y \end{pmatrix} \xrightarrow{f} y_3[n] = \begin{pmatrix} 1, 2, 1 \\ y \end{pmatrix}$$

Since it is time invariant, $x(n) \xrightarrow{f} y(n)$

$$x(n-k) \xrightarrow{f} y(n-k)$$

$$y[n] = \{ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{13}{2}, 5, 2, 0, 0, \dots, 0 \}$$

Q.10)

Q2) Given T is TI

$$x_1[n] = \{ 0, 2y \} \xrightarrow{T} y[n] = \{ 0, 1, 2y \}$$

$$x_2[n] = \{ 0, 0, 3y \} \xrightarrow{T} y[n] = \{ 0, 1, 0, 2y \}$$

$$x_3[n] = \{ 0, 0, 0, 1y \} \xrightarrow{T} y[n] = \{ 1, 2, 1y \}$$

A system is linear if

$$\cancel{x_3 = \alpha x_1 + \beta x_2} \xrightarrow{\text{+}} y_3 = \alpha y_1 + \beta y_2.$$

So we will try to bring one ~~single~~ signal from 2 others

Assume it is linear,

$$\frac{3}{2}x_1 + x_2 \xrightarrow{\text{+}} \frac{3}{2}y_2 - y_1$$

$$\{ 0, \frac{1}{2}, 3, 2y \}$$



delay $\frac{3}{2}x_1 - x_2$ by 3 units

For input $0, 0, 0, \frac{3}{2}y$

$$x_3 = \frac{2}{3}(-\frac{3}{2}x_1 + \frac{1}{2}y) + x_2 \Rightarrow 0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y$$

$$x_3 = \frac{2}{3}(-\frac{3}{2}x_1 + \frac{1}{2}y) + x_2 \Rightarrow 0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y$$

$$(0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y) \rightarrow (0, 0, 0, 0, \frac{1}{3}, -\frac{1}{3})$$

for same I/P we get 2 different O/P,

so it is contradiction

∴ Non-linear

$$(0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y) \rightarrow (0, 0, 0, 0, \frac{1}{3}, -\frac{1}{3})$$

$$(0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y) \rightarrow (0, 0, 0, 0, \frac{1}{3}, -\frac{1}{3})$$

$$(0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y) \rightarrow (0, 0, 0, 0, \frac{1}{3}, -\frac{1}{3})$$

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$$(0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y) \rightarrow (0, 0, 0, 0, \frac{1}{3}, -\frac{1}{3})$$

$$(0, 0, 0, 0, \frac{1}{2}, -\frac{1}{3}y) \rightarrow (0, 0, 0, 0, \frac{1}{3}, -\frac{1}{3})$$



i. Non-linear

Q3] 2.13

Necessary condition:-

if A is true \rightarrow B may be false
 \rightarrow B may be true.

but if B is true \Rightarrow A is true.

Sufficient condition:-

if A is true, B is true, but if B is true
 $A \leftrightarrow B$ A mayn't be true

Given to prove "Necessary & sufficient condition that
relaxed LTI system to be BIBO stable is

$$\sum_{k=-\infty}^{\infty} |h[k]| \leq M_h < \infty$$

Necessary condition.

Sufficient condition:-

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \quad (\forall [k] h[n-k] \in \mathbb{R})$$

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} x[k] h[n-k] \right|$$

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} x[k] h[n-k] \right| \leq \sum_{k=-\infty}^{\infty} |x[k]| |h[n-k]|$$

Since $|x[n]| \leq M_x < \infty$ as it is BIBO stable

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} M_x |h[n-k]| \quad (\text{if } a \leq c k b \leq b)$$

$$|y[n]| \leq M_x \sum_{k=-\infty}^{\infty} |h[n-k]|$$

we want this ~~last~~ term to be bounded

$$\text{ie } \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

This is a sufficient condition as we have used various inequalities.

Necessary condition:-

To see whether necessary or not.

Given $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$, then we to show

that $y[n]$ may be unbounded.

so if we show one condition is satisfied
that $y[n]$ is bounded, we can show it is
necessary for BIBO stability.



$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k] h[k]$$

~~x[n-k]~~

$$y[0] = \sum_{k=-\infty}^{\infty} x[-k] h[k]$$

$$x(-k) = +1 \quad h[-k] \geq 0$$

$$x[-k] = -1 \quad h[-k] < 0$$

$$|y[0]| = \sum_{k=-\infty}^{\infty} |h[k]| \rightarrow \infty$$

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \quad \text{necessary \& sufficient condition}$$

which is a condition for causality.

\Rightarrow if $\sum_{k=-\infty}^{\infty} |h[n-k]| \leq M_h < \infty \Rightarrow$ relaxed LTI system

\hookrightarrow Sufficient condition ($A \Rightarrow B$) is BIBO stable.
Necessary

Q4] 2.16, if $y[n] = x[n] * h[n] \Rightarrow \sum_y = \sum_x \sum_h$

$$\begin{aligned}
 \sum_y &= \sum_{n=-\infty}^{\infty} y[n] = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k] h[n-k] \\
 &= \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} h[n-k] \\
 &= x[1] \sum_{n=-\infty}^{\infty} h[n-1] + x[2] \sum_{n=-\infty}^{\infty} h[n-2] \dots \\
 &\quad (\sum_{k=-\infty}^{\infty} x[k]) \sum_{n=-\infty}^{\infty} h[n]
 \end{aligned}$$



$$b \rightarrow 8) \quad x[n] = \begin{cases} 1, 1, 2 \\ \downarrow \end{cases} \quad h[n] = u[n]$$

$$y[n] = x[n] * (\sum_{k=-\infty}^n u[k])$$

$$u[n] = \sum_{k=0}^n u[k]$$

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[n-k] u[k] \\ &= \sum_{k=0}^{\infty} x[n-k] \quad (u[k]=0 \text{ if } k<0) \end{aligned}$$

$x[n-k]=0 \wedge n-k \leq 0$

$$y[0] = \sum_{k=0}^{\infty} x[0-k] \Rightarrow \begin{cases} k \geq 0 \\ -k \leq 0 \\ \text{So} \end{cases} \quad \underline{n < k}$$

$$y[0] = 1$$

$$y[1] = \sum_{k=0}^{\infty} x[1-k] \Rightarrow \begin{cases} k \geq 0 \\ k-1 \geq (-1) \\ 1-k \leq 1 \end{cases}$$

$$= x[1] + x[1-1] + \sum_{k=0}^{\infty} x[1-k]$$

$$= 1 + 1 = 2$$

$$y[2] = x[0] + x[1] + x[2] + \sum_{k=3}^{\infty} x[2-k]$$

$$= 4$$

$$y[3] = \cancel{x[0]} + x[0] + x[1] + x[2] + \sum_{k=4}^{\infty} x[3-k]$$

$$y[0] = 1$$

$$y[1] = 2$$

$$y[n] = 4 \wedge n \geq 2$$

= 0 else if $n < 0$

Testing through $\sum y = \sum_x \sum_h$

$$\begin{aligned} \text{LHS: } \sum_{k=-\infty}^{\infty} y[k] &= \underbrace{\sum_{k=-\infty}^{-1} y[k]}_0 + y[1] + \sum_{k=2}^{\infty} y[k] + y[0] \\ &= 32 + \sum_{k=2}^{\infty} (4) \rightarrow \infty \end{aligned}$$

$$\begin{aligned}
 \text{RHS: } & \sum_{k_1=-\infty}^{\infty} x[k_1] \sum_{k_2=-\infty}^{\infty} h[k_2] = \left(\sum_{k=0}^{\infty} 0 + 2 + \cancel{\sum_{k=0}^{\infty} 1} \right) \\
 & \quad \left(\sum_{k=0}^{\infty} 1 \right) \\
 & = \left(\sum_{k=0}^{\infty} 0 + 1 + 1 + 2 + \sum_{k=3}^{\infty} 0 \right) \left(\sum_{k=0}^{\infty} 1 \right) \rightarrow 0 \\
 \text{So } & \underline{\text{LHS} = \text{RHS}}
 \end{aligned}$$

(Qb \rightarrow q) $x[n] = \begin{cases} 1, 1, 0, 1, 1 \end{cases}^T$ $y[n] = \begin{cases} 1, -2, -3, 4 \end{cases}^T$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$h[n-k]$ exists from $[-3, 0]$

$n-k \in [-3, 0]$
 $k-n \in [0, 3]$

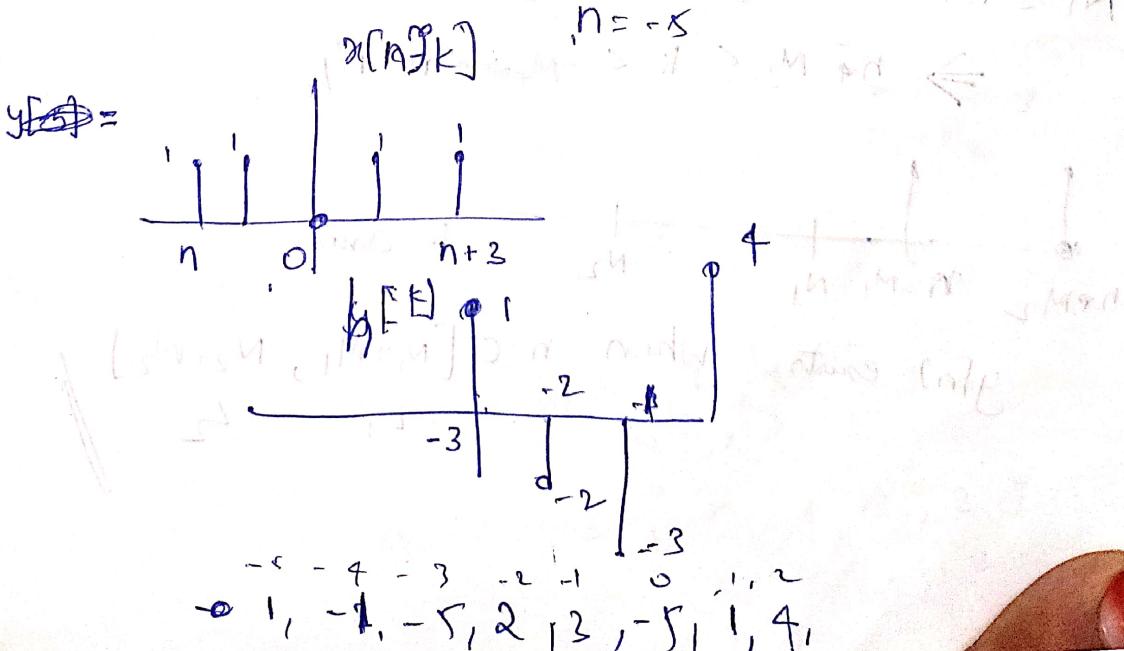
$x[k]$ exists when $k \in [-2, 2]$

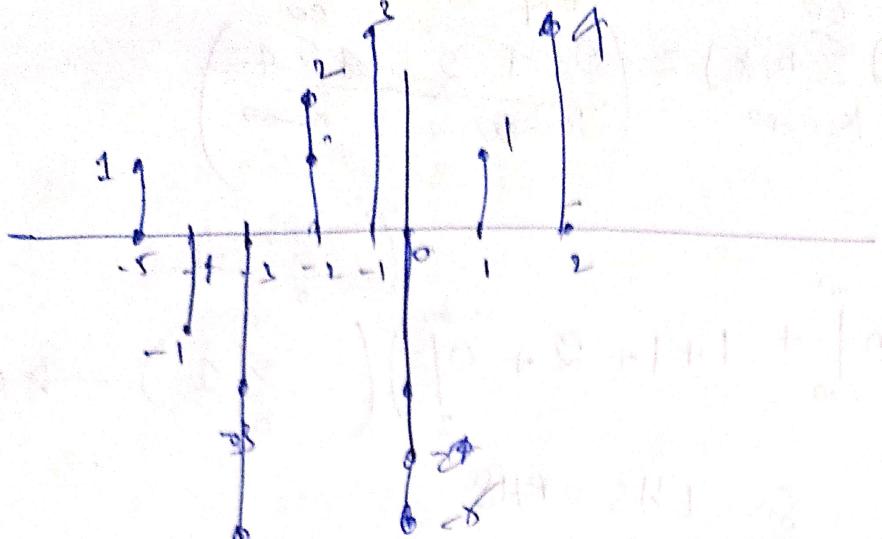
$$\text{yet } n+k \in [-2, 2] \Rightarrow k \in [-n, n+3]$$



Range of k : $n+3 \geq -2 \Rightarrow n \geq -5$

$y[n]$ exists when $n+3 = -2 \Rightarrow n = -5$





Testing $\sum_{n=-\infty}^{\infty} y[n] = 0$

$$\text{RHS} \sum_{k=-\infty}^{\infty} x[k] = 4 \quad \left(\sum_{k=-\infty}^{\infty} h[k] = 0 \right) = 0$$

$LHS = RHS$

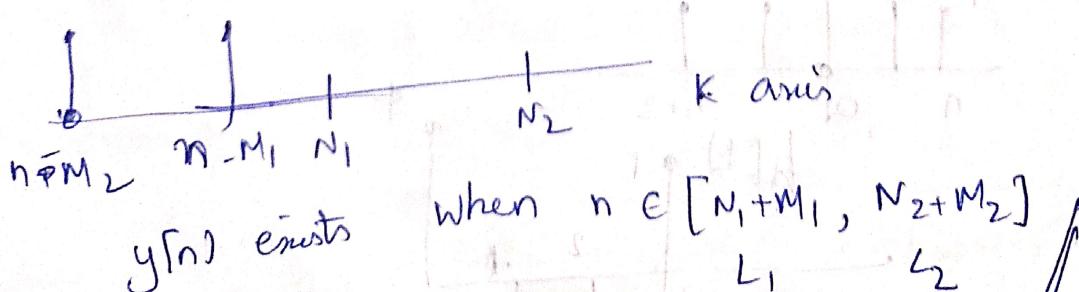
Hence Proved

Q5) 2.32
 a) $x[n]$ exists when $N_1 \leq n \leq N_2$ and y finite
 $h[n]$ " " $M_1 \leq n \leq M_2$

$y[n] = \sum x[k] h[n-k]$ exists when both statements are true

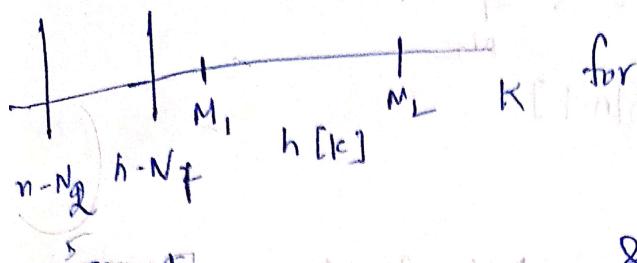
$$N_1 \leq k \leq N_2 \quad M_1 \leq n-k \leq M_2$$

$$\Rightarrow n-M_2 \leq k \leq n-N_1$$



When $n \in [N_1 + M_1, N_2 + M_2]$

b) if $h[n]$ has shorter duration
ie $|M_2 - N_1| \leq |N_2 - N_1|$



for extreme partial overlap from left

$$n - N_1 = M_2 - 1 \quad (\text{last partial left overlap})$$

$$n - N_1 = M_1 + 1 \quad (\text{first partial left overlap})$$

$$\text{ie } n \in [M_1 + N_1, N_1 + M_2 - 1] \quad (\text{left overlap})$$

for complete overlap

$$\text{first complete overlap, } n - N_1 = M_2$$

$$\text{last complete overlap } n - N_2 = M_1$$

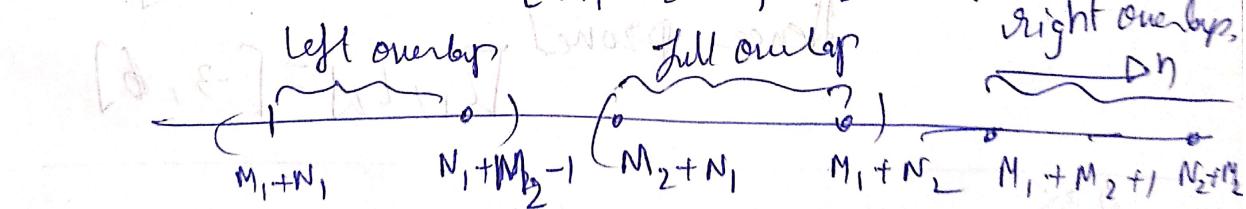
$$\text{ie } n \in [M_2 + N_1, M_1 + N_2]$$

for partial right overlap,

$$\text{first partial right } \Rightarrow n - N_2 = M_1 + 1$$

$$\text{last partial right } \Rightarrow n - N_2 = M_2$$

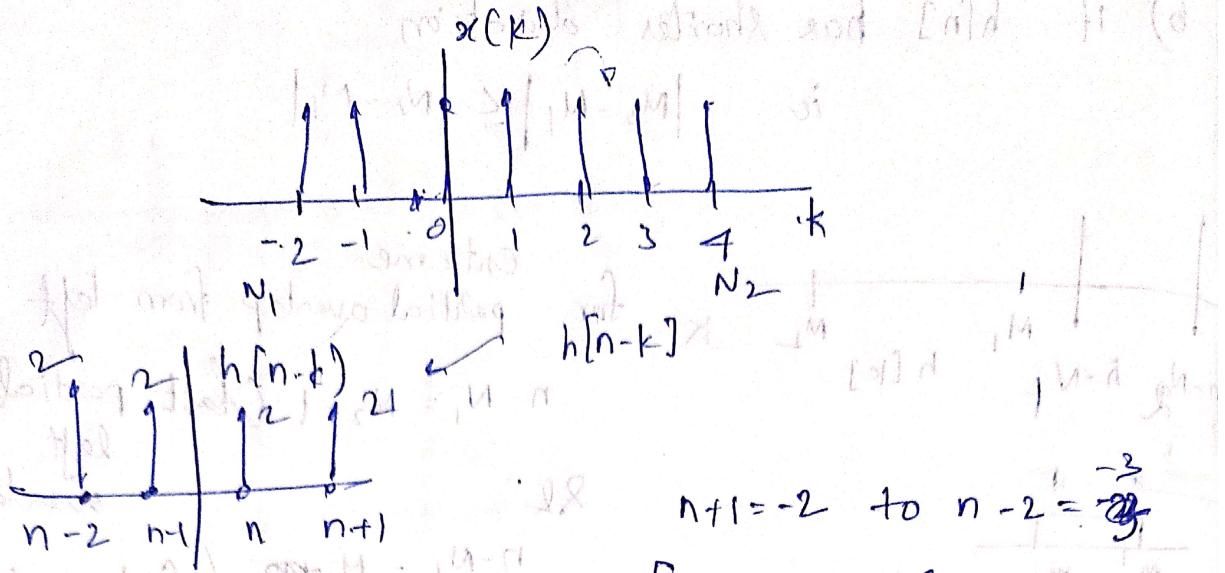
$$\text{ie } n \in [M_1 + N_2 + 1, N_2 + M_2]$$



$$c) x[n] = \begin{cases} 1 & -2 \leq n \leq 4 \\ 0 & \text{else} \end{cases}$$

$$= \{1, 1, 1, 1, 1, 1, 1\}$$

$$h[n] = \begin{cases} 2 & -1 \leq n \leq 2 \\ 0 & \text{else} \end{cases} = \{2, 2, 2, 2\}$$



left overlap when $n \in [-3, -1]$

right full overlap when $n \notin n-2 = -2 \quad n+1 = 4$
 $n \in [0, 3]$

right overlap when $n+1 = 5 \quad n-2 = 4$

$$N_1 = -2 \quad | \quad M_1 = -1 \dots n \quad \text{overlap stop} \quad n \in [4, 5]$$

$$N_2 = 4 \quad | \quad M_2 = 2 \dots$$

Left overlap, when $n \in [-3, -1]$

right full $M_1 + N_1, M_1 + M_2, M_1 + N_2$
 $n \in [0, 3]$

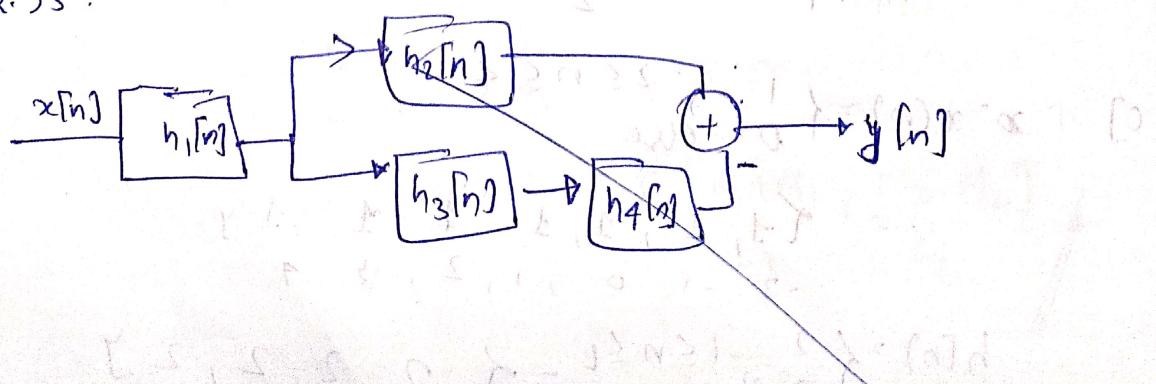
right $n \in [M_1 + N_2 + 1, M_2 + M_1]$

$$[4, 6]$$

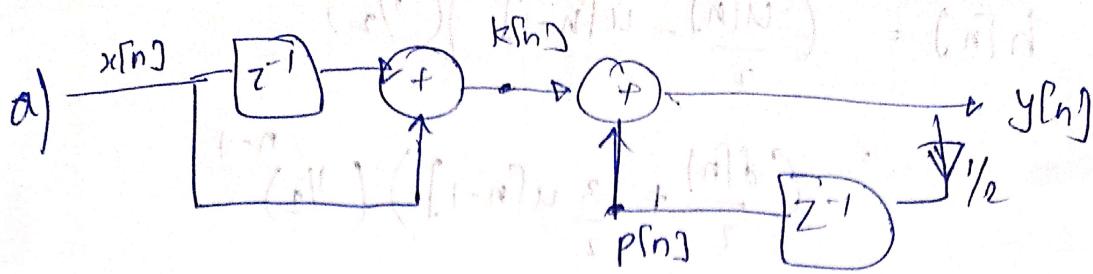
Hence prove

$$[L_1, L_2] = [-3, 6]$$

Q6) 2.35.



Q7) \star



b) Show $h[n] = h_1[n] * h_2[n]$

where $h_1[n] = \delta[n] + \delta[n-1]$, $h_2[n] = (\frac{1}{2})^n u(n)$

$$h[n] = ((\frac{1}{2})^n u(n)) * (\delta[n] + \delta[n-1])$$

$$= (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$$

$$= (\cancel{\frac{1}{2}})^n \left[\frac{u[n] + u[n-1]}{2} \right] (\frac{1}{2})^{n-1}$$

~~h[n]~~

$$h[n] = \left(\frac{u[n] + u[n-1]}{2} \right) \left(\frac{1}{2}\right)^{n-1}$$

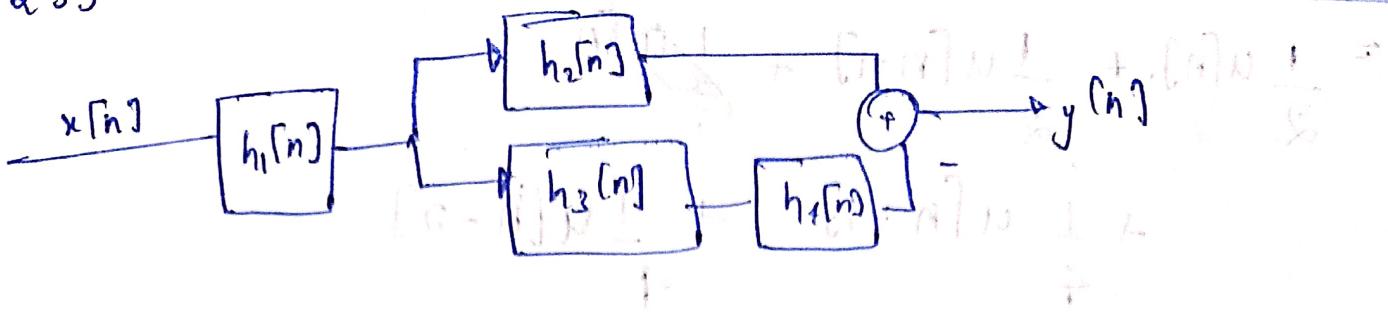
$$= \left(\frac{d[n]}{2} + \frac{3}{2} u[n-1] \right) \left(\frac{1}{2}\right)^{n-1}$$

$$= \frac{d[n]}{2^n} + \frac{3}{2^n} u[n-1]$$

$$\boxed{h[n] = d[n] + \frac{3}{2^n} u[n-1]}$$

$$2 \left((x[n] * h_1[n]) + h_2[n] \right) + \left(\frac{1}{2} u[n] \right) * \left((x[n] * h_1[n]) + h_2[n] \right)$$

Q6) 2.35



$$\begin{aligned} x[n] &\xrightarrow{\text{h}_1[n]} \text{h}_2[n] \xrightarrow{(x[n]*\text{h}_1[n]) * \text{h}_2[n]} \\ &\quad (x[n]*\text{h}_1[n]) \\ &\quad \downarrow \\ &\quad \text{h}_3[n] \xrightarrow{(x[n]*\text{h}_1[n]) * \text{h}_2[n]} \\ &\quad \quad \quad = x[n] * (\underbrace{\text{h}_1[n] * \text{h}_2[n]}_{\text{h}_{\text{eff}}[n]}) \end{aligned}$$

a) $\text{h}_{\text{eff}}[n] = (h_2[n] - h_3[n] * h_4[n]) * h_1[n]$

b) $h_1[n] = \frac{1}{2}, \frac{1}{4}, \frac{1}{2} y = \frac{1}{2} d[n] + \frac{1}{4} d[n-1] + \frac{1}{2} d[n-2]$

$$h_2[n] = h_3[n] = (n+1) u[n]$$

$$h_4[n] = d[n-2]$$

$$k[n]$$

$$h[n] = ((n+1)u[n] - (n+1)u[n]) * d(n-a) * h_1[n]$$

$$= ((n+1)u[n] - (n+1)u[n-2]) * h_1[n]$$

$$= (n[u[n] - u[n-2]] + u[n] + u[n-2]) * h_1[n]$$

$$n(u[n] - u[n-2]) = \underbrace{u[n]}_{\delta[n]} (\delta[n] + \delta[n-1]) \\ = \cancel{n \delta[n]} + n \underbrace{\delta[n-1]}_{\delta[n-1]} \\ = \delta[n-1] \quad (1)$$

$$h[n] = (\delta[n-1] + u[n] + u[n-2]) * h_1[n] \\ = (u[n] + u[n-1]) * h_1[n] \\ = (u[n] + u[n-1]) * \left(\frac{1}{2} \delta[n] + \frac{1}{4} \delta[n-1] + \frac{1}{2} \delta[n-2] \right) \\ = \frac{1}{2} u[n] + \frac{1}{2} u[n-1] + \cancel{\frac{1}{2} u[n]} \\ + \frac{1}{4} u[n-1] + \frac{1}{4} u[n-2]$$

$$= \frac{1}{2} u[n-2] + \frac{1}{2} u[n-3] \\ = \frac{1}{2} u[n-2] + \frac{1}{2} u[n-3] \\ = \frac{1}{2} u[n-2] + \frac{3}{4} u[n-1] + \frac{3}{4} u[n-2] + \frac{1}{2} u[n-3] \\ = \frac{1}{2} u[n-2] + \left(\frac{3}{4} u[n-1] + \frac{3}{4} u[n-2] \right) + \frac{1}{2} u[n-3] \\ = \frac{1}{2} u[n-2] + \left(\frac{3}{4} u[n-1] + \frac{3}{4} u[n-2] \right) + \frac{1}{2} u[n-3] \\ = \frac{1}{2} u[n-2] + \left(\frac{3}{4} u[n-1] + \frac{3}{4} u[n-2] \right) + \frac{1}{2} u[n-3]$$

$$c) x[n] = \delta[n+2] + 3 \delta[n-1] + 3 \delta[n-3] + (1/2) \delta[n]$$

$$x[n] * h[n] = \left(\frac{1}{2} u[n] + \frac{3}{4} u[n-1] + \frac{3}{4} u[n-2] + \frac{1}{2} u[n-3] \right) * \left(\cancel{\delta[n]} + \delta[n+2] + 3 \delta[n-1] \right)$$

$$= \left(\cancel{\delta[n]} + \delta[n+2] + 3 \delta[n-1] \right) * \left(\frac{1}{2} u[n] + \frac{3}{4} u[n-1] + \frac{3}{4} u[n-2] + \frac{1}{2} u[n-3] \right) \\ = 4 \delta[n-3]$$

$$y[n] = \frac{1}{2}u[n+2] + \frac{3}{4}u[n+1] + \frac{3}{4}u[n] + \frac{1}{4}u[n-1]$$

$$= \frac{3}{4}u[n-1] + \frac{9}{4}u[n-2] \\ + \frac{9}{4}u[n-3] \\ - \frac{3}{2}u[n-4] - 3u[n-5] \\ + 3\frac{1}{2}u[n-6]$$

$$y[n] = \frac{1}{2}u[n+2] + \frac{3}{4}u[n+1] + \frac{3}{4}u[n] + u[n-1] + \frac{9}{4}u[n-2] + \frac{1}{4}u[n-3]$$

$$- 3\frac{1}{2}u[n-4] - 3u[n-5] - 2u[n-6]$$

$$y[n] = \left\{ \frac{1}{2}, \frac{5}{4}, 2, 4, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \dots \right\}$$

Q.10)



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