AI/ML

Maths

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Linear Algebra

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(1) Basics of Linear Algebra :

(1.1) scalars, vectors, Matrices, Tensors

→ scalars :

scalar is a just single number.
(single value with no direction, no dimension)

eg: Age = 21
Tempreture = 47°c
Price = 69 ₹
any number like 2/7, 6.1 etc.

→ Vectors :

vector is a list of numbers (1D Array).
(A line of numbers - 1D (one direction))(row or co

eg: Juhil's marks in three subject = [80, 85, 82]
position in 2D space [x = 5, y = 7]
Daily temprature for week : [30, 29, 41, 20, 35, 32, 31].

→ Matrix :

Matrix is combination of rows and columns (2D)
(2D grid like excel sheet)

eg: Marks of 3 students in 4 subjects

[[80, 85, 90, 75],
[70, 72, 92, 81],
[60, 78, 85, 80]] (3×4)

→ Tensor :

Tensor is a multi-dimensiona array (more than 2D).

(can be 3D, 4D ---nD).

eg: - colored image have 3D structure (Height x width x 3 colors-RGB)

eg : shape : [100, 100, 3]

It's mean 100 X 100 pixels and 3 color channels

- video data : [Frames, height, width, channels].

eg with deep explain :

one Box, its length = 5

width = 4

Depth = 2

than it can be simply write like [5, 4, 2] but

in tensor it represent like (2 depth, each with 5 rows & 4 cols)

```
tensor = [
   [ [1, 2, 3, 4],
     [5, 6, 7, 8],          depth 1
     [9, 10, 11, 12],
     [13, 14, 15, 16],
     [17, 18, 19, 20] ](5,4)
   [ [21, 22, 23, 24],
     [25, 26, 27, 28],      depth 2
     [29, 30, 31, 32],
     [33, 34, 35, 36],
     [37, 38, 39, 40] ]
                      (5,4)
](5,4,2)
```

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(1.2) Indexing and Slicing

→ scalar :
scalar is a single value - so no indexing or slicing needed.
eg: x = 10

→ Vector (1D) :
Indexing :

```
v = [10, 20, 30, 40]
      0   1   2   3
Print(v[0])      o/p : 10
Print(v[2])      o/p : 30
```

slicing : Extract a part (range) from vector

```
Print(v[1:3])    o/p : [20, 30]
Print(v[:2])     o/p : [10, 20]
print(v[2:])     o/p : [30, 40]
```

→ Matrix (2D) :
Matrix is a list of lists
Indexing :

```
m = [ [1, 2, 3],
      [4, 5, 6],
      [7, 8, 9]
    ](3x3)
Print(m[0])      o/p : [1, 2, 3]
Print(m[1][2])   o/p : 6
       ↑  ↑
     row column
```

Slicing :

```
Print(m[:2])     o/p : First two rows
Print(m[0][:2])  o/p : [1, 2]
```

→ tensor (3D or more):
It's like a cube (list of matrices)

```
tensor = [
    [ # depth 0
        [1,2],
        [3,4]
    ],
    [ # depth 1
        [5,6],
        [7,8]
    ]
]
```

Indexing:

print(t[0]) o/p: First depth matrix
[1,2]
[3,4]

Print (t[1][0][1]) o/p: 6

↑ Depth ↑ row ↑ col

Slicing:

Print (t[1][1][0:1] o/p: 7

Depth row col

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(2) <u>Vector</u> <u>Operation's</u> :

(2.1) vector addition and subtraction
Note: vectors must be the same size (same no. of dimensions).

- vector Addition :

$\vec{a} = [2, 4, 6]$, $\vec{b} = [1, 3, 5]$

$\vec{a} + \vec{b} = [2+1, 4+3, 6+5]$

$\vec{a} + \vec{b} = [3, 7, 11]$

- vector subtraction :

$\vec{a} - \vec{b} = [2-1, 4-3, 6-5]$

$\vec{a} - \vec{b} = [1, 1, 1]$

(2.2) scalar multiplication.

- multiplication of vector by single number (scalar).

if: $\vec{v} = [v_1, v_2, v_3, \ldots, v_n]$

and: k = scalar

then: $k \cdot \vec{v} = [k \cdot v_1, k \cdot v_2, k \cdot v_3, \ldots k \cdot v_n]$

<u>eg</u> : $\vec{v} = [2, 4, 6]$

$k = 3$

$3 \cdot [2, 4, 6] = [6, 12, 18]$.

Means that :
The length (magnitude) of the vector increases or decreases.

if $k > 1$: vector gets longer
if $0 < k < 1$: vector gets shorter
if $k < 0$: vector reverse direction.

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(2.3) Dot product (Inner product)

- The dot product of two vectors is a single number (scalar) you get by multiplying their corresponding elements and adding the results.

if : $\vec{a} = [a_1, a_2, a_3]$, $\vec{b} = [b_1, b_2, b_3]$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\therefore \vec{a} \cdot \vec{b} = \sum_{i=1}^{N} a_i b_i$

eg : $\vec{a} = [2, 3, 4]$, $\vec{b} = [5, 6, 7]$

$\vec{a} \cdot \vec{b} = 2 \times 5 + 3 \times 6 + 4 \times 7$

$\vec{a} \cdot \vec{b} = 10 + 18 + 28$

$\vec{a} \cdot \vec{b} = 56$

usecase :

- Measure how much two vectors align with each other.
- If dot product = 0 than vectors are perpendicular.

(2.4) cross product (only in 3D)

- The cross product of two 3D vectors is a new vector (not a scalar) that is perpendicular to both original vectors.

if : $\vec{a} = [a_1, a_2, a_3]$, $\vec{b} = [b_1, b_2, b_3]$

$\vec{a} \times \vec{b} = [(a_2 b_3 - a_3 b_2), (a_3 b_1 - a_1 b_3), (a_1 b_2 - a_2 b_1)]$

eg : $\vec{a} = [1, 2, 3]$, $\vec{b} = [4, 5, 6]$

$\vec{a} \times \vec{b} = [(2 \times 6 - 3 \times 5), (3 \times 4 - 1 \times 6), (1 \times 5 - 2 \times 4)]$

$\vec{a} \times \vec{b} = [-3, 6, -3]$

usecase :
- Finding a vector that perpendicular to two vectors.
- used in physics & 3D graphics (not much in deep learning)

(2.5) (L1, L2 Norms / magnitude / length)

→ L1 Norms
This is the sum of absolute values of the vector elements

$$\|\vec{v}\|_1 = |v_1| + |v_2| + \dots + |v_n|$$

eg: $\vec{v} = [3, -4]$

$$\|\vec{v}\|_1 = |3| + |-4| = 7$$

→ L2 Norms (Euclidean Norm)
This is the most common norm - It tells you how long the vector is (like distance from origin)

$$\|\vec{v}\|_2 = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$$

eg: $\vec{v} = [3, 4]$

$$\|\vec{v}\|_2 = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$$

(2.6) Unit vector :
A unit vector is a vector with a length of 1, pointing the same direction as the original.

Formula : $\hat{v} = \dfrac{\vec{v}}{\|\vec{v}\|_2}$

eg : $\vec{v} = [3, 4]$

$\|\vec{v}\|_2 = 5$

$\therefore \hat{v} = \left[\frac{3}{5}, \frac{4}{5}\right]$

$\therefore \hat{v} = [0.6, 0.8]$

$\rightarrow \|\hat{v}\| = \sqrt{0.6^2 + 0.8^2} = 1$

(3) Matrix operations :

(3.1) Matrix addition & subtraction

- You can add or subtract two matrices only if they have same dimensions.

eg : $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$A+B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

(3.2) Scalar multiplication

- Multiply every element of matrix by scalar value.

eg $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $K = 3$

$$\therefore K \cdot A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot 3 = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$$

(3.3) Matrix Multiplication (Dot product):

Rule:

- you can multiply two matrices $A(m \times n)$ and $B(n \times p)$ if:
- The number of columns in A = The number of rows in B.
- Multiply row of A with column of B and then sum the products.

eg: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{(2\times2)}$, $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{(2\times2)}$

$$A \cdot B = \begin{bmatrix} (1\times5)+(2\times7) & (1\times6)+(2\times8) \\ (3\times5)+(4\times7) & (3\times6)+(4\times8) \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

(3.4) Transpose of matrix:

- flip rows into columns (and vice versa)

eg:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{(2\times3)}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{(3\times2)}$$

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(3.5) Element wise operation in Matrix:

(i) Element wise Addition

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\therefore A+B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

(ii) Element wise subtraction

$$\therefore A-B = \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

(iii) Element wise Multiplication
(Hadamard product)

$$\therefore E = A \circ B$$

$$\therefore A \circ B = \begin{bmatrix} 1\times5 & 2\times6 \\ 3\times7 & 4\times8 \end{bmatrix} = \begin{bmatrix} 5 & 12 \\ 21 & 32 \end{bmatrix}$$

(iv) Element wise division

$$\therefore A|B = \begin{bmatrix} 1/5 & 2/6 \\ 3/7 & 4/8 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.33 \\ 0.42 & 0.5 \end{bmatrix}$$

It's all operations useful in:

- Image processing : Adjust Brightness etc
- ML : Apply activation Funct^n, loss Funct
- Data Analysis
- Scientific computation.

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(4) Special matix and properties :

(4.1) – Identity matrix and zero matrix
- main diagonal (from top left → bottom right) are 1, and other elements are 0.

eg : $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Zero matrix have all elements are 0.

$O_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Properties : → $A+O = A$ & $O+A = A$
→ $AO = O$ & $OA = O$

(4.2) Diagonal matrix and Orthogonal matrix
- main diagonal have different values and other elements are 0.

eg : $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

- orthogonal matrix :
properties : $A^T \cdot A = I$ OR $A^T = A^{-1}$
(Transpose × matrix = Identity)

eg : $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A \cdot A^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

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(4.3) Symmetric Matrix & Inverse Matrix

- symmetric matrix :
  A square matrix where
  $\therefore A = A^T$

  eg : $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

  $\therefore$ This is symmetric matrix

- Inverse matrix :
  $\therefore A \cdot A^{-1} = I$ & $A^{-1} \cdot A = I$
  How to find Inverse matrix
  $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

  Now Interchange 1 & 4 and change sign of 2 & 3.
  $\therefore A^{-1} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

  Not divide by It's determinant
  $\therefore A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

(4.4) Rank of matrix & Trace OF matrix

- Rank OF matrix :
  The rank of matrix is the number of linearly independent rows or colu
- Independent = "not a copy or combination of other"
- Rank tell us how much actual information is in the matrix.