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Economics 144
Economic Forecasting
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Midterm Exam
February 13, 2014

For full credit on a problem, you need to show all your work and the formula(s) used.

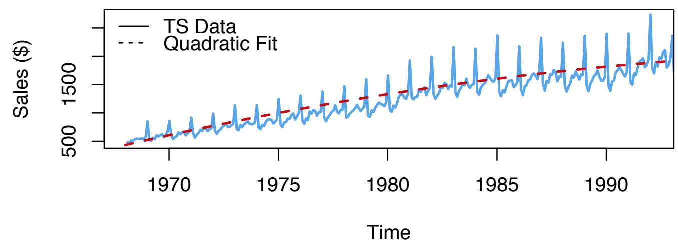
First Name	
Last Name	
UCLA ID #	

Please do not start the exam until instructed to do so.

1.(15%) The figure below shows the monthly U.S. current-dollar liquor sales between 1968 and 1993. We fit a quadratic trend to the data of the form $T_t = \beta_0 + \beta_1 TIME_t + \beta_2 TIME_t^2$. The summary table for the fit is also provided below.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4776540.5571	883279.5188	-5.41	0.0000
t	4765.6277	891.9577	5.34	0.0000
t2	-1.1882	0.2252	-5.28	0.0000

Table 1: Summary statistics for the quadratic fit where t represents the linear term and $t2$ the quadratic term respectively.



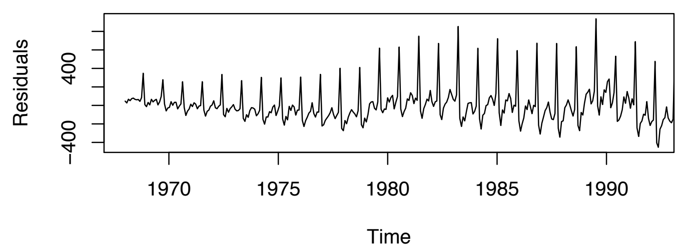


Fig. 1.— The figure in the top row shows the time series plot of the data (solid curve) and the respective quadratic fit (dashed curve). The figure in the bottom row shows the residuals from the quadratic fit.

(a) Based on the summary statistics table, what can you conclude about the quadratic fit?
$\Rightarrow$ Based on the low p-values for all the parameters, they are all statistically significant and therefore should be included in the model.

(b) From the residuals plot, what would you conclude about the quadratic fit?
$\Rightarrow$ The residuals plot shows a strong seasonality component in addition to a growing amplitude after ∼1978. There is also a hint of some higher order pattern (maybe a hint of cycles being present as well).

(c) Suppose that you decide to improve your fit, and therefore, decide to look at the 'stl' plot of data. Briefly sketch what you expect to see for the (i) seasonal and (ii) trend components of the plot.

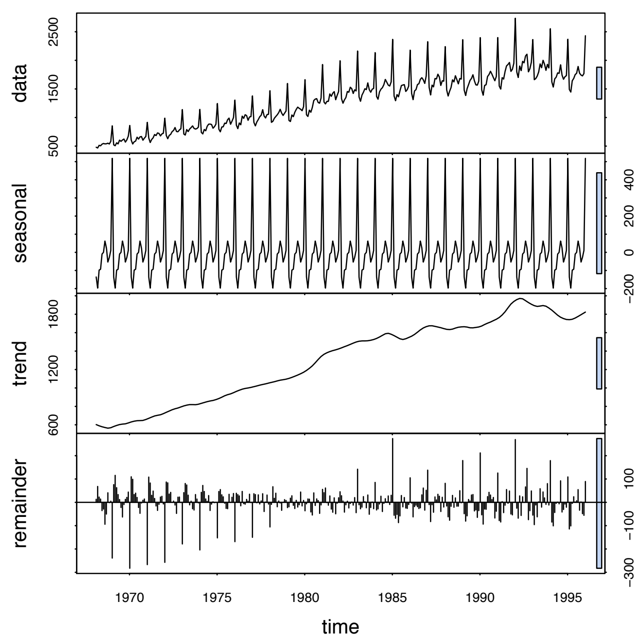


Fig. 2.— As expected, there is a strong seasonality component and a somewhat linear positive trend.

2.(10%) Suppose that, in a particular monthly dataset, time $t = 9$ corresponds to August 1982.

(a) Name the month and year of each of the the following times:

i. $t + 6$
$\Rightarrow$ February 1983

ii. $t - 3$
$\Rightarrow$ May 1982

iii. $t + 2$
$\Rightarrow$ October 1982

(b) Suppose that a series of interest follows the simple process $y_t = 2y_{t-1} + 1$, $t = 1, 2, 3, \ldots, n$. Suppose that $y_0 = 0$, and that at the present, $t = 2$. Calculate the forecast $y_{t+2,t}$, i.e., the forecast made at time t for future time $t + 2$, assuming that $t = 2$ at present.
$y_1 = 2y_0 + 1 = 2(0) + 1 = 1$
$y_2 = 2y_1 + 1 = 2(1) + 1 = 3$ (present)
$y_{t+1,t} = y_{3,2} = 2y_2 + 1 = 2(3) + 1 = 7$
$\rightarrow y_{t+2,t} = y_{4,2} = 2y_3 + 1 = 2(7) + 1 = 15$

3.(15%) For this problem we will look at the quarterly earnings per share for Johnson and Johnson from 1960:Q1 to 1980:Q4. A 'linear trend + seasonality' model is fit to the data (the $y-$intercept is not included). The table below shows the summary statistics from the fit. In the figure below we show relevant diagnostic plots.

	Estimate	Std. Error	t value	Pr(>|t|)
trend	0.1637	0.0073	22.39	0.0000
season1	-1.9211	0.4640	-4.14	0.0001
season2	-1.9105	0.4688	-4.08	0.0001
season3	-1.7861	0.4736	-3.77	0.0003
season4	-3.0156	0.4785	-6.30	0.0000

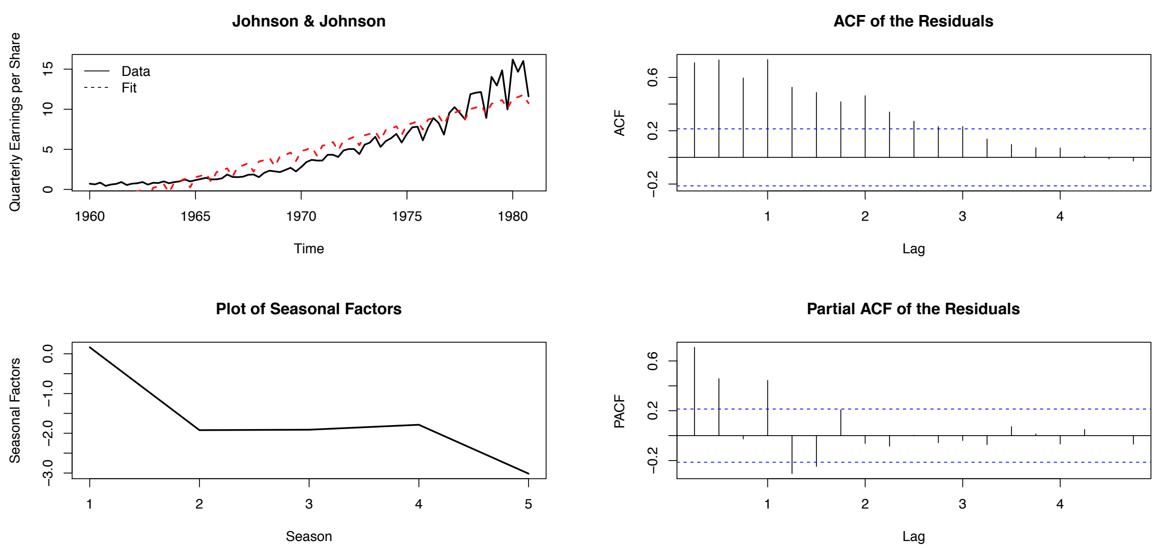


Fig. 3.— The figure shows the data(top-left), fit(top-left), seasonal factors(bottom-left), ACF of the residuals(top-right) and partial ACF of the residuals (bottom-right).

(a) Discuss the results from the summary statistics table.

$\Rightarrow$ Based on the low p-values, both the linear trend and quarterly seasonal components are statistically significant.

(b) Interpret the seasonal factors plot.

$\Rightarrow$ The seasonal factors plot suggests that the earnings per share during the first, second, and third quarters were rather stagnant followed by a sharp decrease in the fourth quarter.

(c) Interpret the ACF of the residuals plot.

$\Rightarrow$ The persistency observed between lags 1 and 3 suggests that cycles might be present. In addition, it appears that the data might be consistent with covariance stationarity.

(d) Is the series covariance stationary? Explain your answer.

$\Rightarrow$ The ACF of the residuals plot suggest that the data are covariance stationary over the long run (i.e., it converges to zero as the lag increases). However, the PACF provides further confirmation.

(e) If you wanted to investigate more closely any spikes in the PACF plot, what statistic(s) would you need to look at?

$\Rightarrow$ We would need to look at the Box-Pierce and Ljung-Box Q-Statistics at the lag locations of the spikes.

4.(10%) Briefly describe the following terms/concepts.

(a) *Event Outcome Forecast*

$\Rightarrow$ Event is certain, outcome is unknown, timing known.

(b) *Event Timing Forecast*

$\Rightarrow$ Event is certain, outcome is known, timing unknown.

(c) *Time Series Forecast*

$\Rightarrow$ Project the future value of a time series (most commonly encountered).

(d) *Parsimony Principle*

$\Rightarrow$ All other things being equal, simpler models are better than complex ones.

(e) *Shrinkage Principle*

$\Rightarrow$ Imposing restrictions on forecasting models often improves forecast performance.

5.(10%) Given the time-series below:

(a) Rewrite the following expression without using the lag operator.
$y_t = \left(3 + \frac{L^2}{L+1} - 2L^2\right)\varepsilon_t$

$$
\begin{aligned}
y_t &= \left(3 + \frac{L^2}{L+1} - 2L^2\right)\varepsilon_t \\
\rightarrow (L+1)y_t &= [3(L+1) + L^2 - 2L^2(L+1)]\varepsilon_t \\
\rightarrow y_{t-1} + y_t &= [-2L^3 - L^2 + 3L + 3]\varepsilon_t \\
\rightarrow y_{t-1} + y_t &= -2\varepsilon_{t-3} - \varepsilon_{t-2} + 3\varepsilon_{t-1} + 3\varepsilon_t
\end{aligned}
$$

(b) Rewrite the following expression in lag operator form.
$y_t + y_{t-1} + \cdots + y_{t-N} = \alpha + 4\varepsilon_{t-2} + 8\varepsilon_{t-4}$, where α is a constant.

$$
\begin{aligned}
y_t + y_{t-1} + \cdots + y_{t-N} &= \alpha + 4\varepsilon_{t-2} + 8\varepsilon_{t-4} \\
\rightarrow (L^0 + L^1 + \cdots L^N)y_t &= \alpha + 4L^2\varepsilon_t + 8L^4\varepsilon_t \\
\rightarrow \sum_{i=0}^{N} L^i y_t &= \alpha + (4L^2 + 8L^4)\varepsilon_t
\end{aligned}
$$

6.(10%) You work for the international Monetary Fund in Washington D. C., monitoring Singapore's real consumption expenditures. Using a sample of real consumption data (measured in billions of 2005 Singapore dollars), y_t, t = 1990:Q1,..., 2006:Q4, you estimate the linear consumption trend model, $y_t = \beta_0 + \beta_1 \text{TIME}_t + \varepsilon_t$, where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$, obtaining the estimates $\hat{\beta}_0 = 0.51$, $\hat{\beta}_1 = 2.30$, and $\sigma^2 = 16$. Based on your estimate trend model, construct feasible point, interval (95% where $z = 1.96$), and density forecasts for 2007:Q3.

First we need to find h: 2007:Q3−1990:Q1= 71 $\rightarrow h = 71$

Model: $\hat{y}_{T+h,T} = \hat{\beta}_0 + \hat{\beta}_1 TIME_{T+h}$
$\rightarrow \hat{y}_{T+h,T} = 0.51 + 2.30\ TIME_{T+h}$

Point Estimate: $\hat{y}_{T+5,T} = 0.51 + 2.30(71) = 163.81$

Interval Estimate: $\hat{y}_{T+h,T} \pm z\sigma = 163.81 \pm 1.96(4) = [155.97, 171.65]$

Density Forecast: $\mathcal{N}(\hat{y}_{T+67,T}, \hat{\sigma}^2) = \mathcal{N}(163.81, 16)$

7.(15%) After fitting the polynomial model, $\hat{y}_t \sim \beta_0 + \beta_1 TIME + \beta_2 TIME^2$ to a time-series y_t (assuming only trend), the following residuals were obtained: $e_t = \{-2, -1, 0, 1, 2\}$.

(a) Compute the penalty factor.
The model has $k = 3$ parameters and there are $T = 5$ residuals, therefore
Penalty Factor$= T/(T-k) = 5/(5-3) = 2.5$

(b) Compute the MSE.
MSE$= 1/T \sum_{t=1}^{T} e_t^2 = 1/5((-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2) = 2$

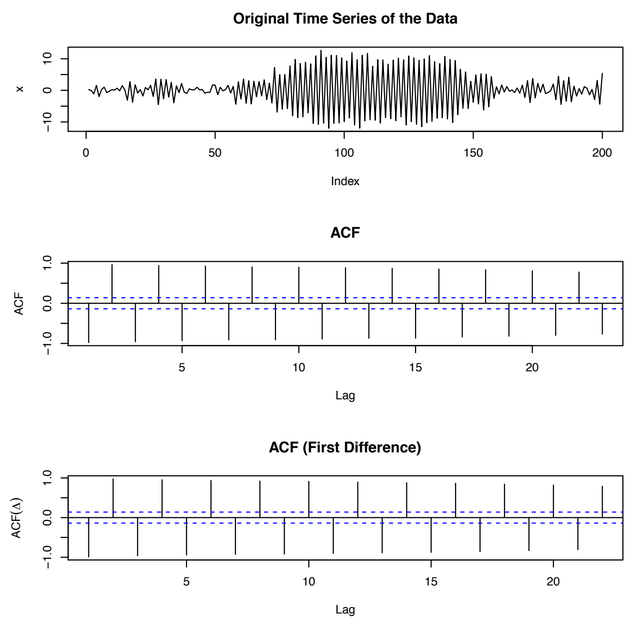
(c) Compte the s^2
$s^2 = T/(T-k)$ MSE$= 2.5 \times 2 = 5$

(d) Compute the AIC
AIC$= exp(2k/T)$MSE$= 3.32 \times 2 = 6.64$

(e) Compute the SIC
AIC$= T^{k/T}$MSE$= 5^{3/5} \times 2 = 5.25$

Questions 8-12 are multiple choice. Please select one answer per question only.

8.(3%) The figure below shows a time series plot of an unknown data set. The respective ACF, and ACF of the First Difference of the time series are also shown as a function of their lag. Which one of the statements below is true based on the 3 plots?



(a) The figure suggests that there is some dynamic structure in the data in the form of cycles.

(b) The figure suggests that the data are consistent with white noise.

(c) The figure suggests that if we detrend the series, then the data are consistent with white noise.

(d) The figure suggests there is a strong seasonality component in the data but no cycles.

9.(3%) Seasonal factors

(a) characterize the seasonally adjusted time series.

(b) characterize the detrended time series.

(c) summarize the seasonal pattern over the year.

(d) are variables used to indicate which season we are in.

10.(3%) From the list of four autocovariance functions below, where $\alpha > 0$, which autocovariance function(s) are consistent with covariance stationarity?
I. $\gamma(t, \tau) = \alpha\tau$
II. $\gamma(t, \tau) = e^{-\alpha\tau}$
III. $\gamma(t, \tau) = \alpha/\tau^2$
IV. $\gamma(t, \tau) = \alpha\tau + 3$

(a) I and II
(b) II and III
(c) I and III
(d) I and IV

11.(3%) Which one of the following statements is *not* true.

(a) SIC: is consistent
(b) AIC: is not asymptotically efficient
(c) SIC: is not asymptotically efficient
(d) MSE: is inconsistent

12.(3%) In the regression framework, the *in-sample forecast errors* are given by

(a) $e_t = y_t - \hat{y}_t$
(b) $\varepsilon \sim WN(0, \sigma^2)$
(c) $e_T = y_{T+h} - \hat{y}_T$
(d) $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, 1)$