CHAPTER 14. FORECASTING VOLATILITY II

SOLUTIONS

by
Wei Lin and Yingying Sun
(University of California, Riverside)

Exercise 1
We have generated 5000 observations of the following ARCH(4) process

$$\begin{aligned} Y_t &= 1 + \varepsilon_t \\ \sigma^2_{t|t-1} &= 4 + 0.4\varepsilon^2_{t-1} + 0.25\varepsilon^2_{t-2} + 0.15\varepsilon^2_{t-3} + 0.1\varepsilon^2_{t-4} \end{aligned}$$

which exhibits high persistence because $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0.90$. The unconditional mean of the process is 1 and the unconditional variance is 40, i.e. 4/0.1. See that $\varepsilon_t = \sigma_{t|t-1} z_t$ where z_t is normally distributed with zero mean and variance one.

The EViews program to generate the time series $\{y_t\}$ and its conditional variances is the following:

```
smpl @first @last

series z=nrnd
series vr
series e
series y

vr(1)=4/(1-0.4-0.25-0.15-0.1)

'initialize the first four innovations using the unconditional variance
for !i=1 to 4
e(!i) = @sqrt(vr(1))*z(!i)
y(!i)= 1+e(!i)
next

for !i=5 to 5000
vr(!i)=4+0.4*e(!i-1)*e(!i-1)+0.25*e(!i-2)*e(!i-2)+0.15*e(!i-3)*e(!i-3)+0.1*e(!i-4)*e(!i-4)
e(!i) = @sqrt(vr(!i))*z(!i)
y(!i) = 1+e(!i)
next
```

In Figure 1, we plot the time series of 4500 observations (we have discarded the first 500 observations) and the corresponding conditional standard deviations. Observe the clusters of high volatility across the sample. They tend to last for several periods as a result of the persistence in variance. We present the histogram of the raw series in Figure 2a. The series has an unconditional mean of about 1 and unconditional variance of about 40, which are the theoretical values that correspond to the simulated process. The process is symmetric around the mean and the skewness is about zero. A consequence of the ARCH effects is the large kurtosis or fat tails of the unconditional probability density function of the process. On the contrary, when the process is standardized, i.e. $(y_t - \mu_{t|t-1})/\sigma_{t|t-1}$, the density function of the standardized process is normal with zero mean and variance one; see the histogram in Figure 2b (mean equal to zero, variance equal to one, skewness equal to zero, and kurtosis equal to three).

The autocorrelation function of the raw series, Figure 3a, is basically zero though there is a small spike that makes the Q-statistics large enough to reject the null hypothesis of no autocorrelation. This is just sampling variation and we can safely ignore it. Thus, the process $\{Y_t\}$ is white noise. However, the squared of the process is not, see Figure 3b. The autocorrelation functions clearly show large and significant autocorrelation coefficients, and the profile of both functions is agreement with an autoregressive process of three or four order (the number of significant spikes in the PACF). This is what we should expect given that we have simulated an ARCH(4). Once we model correctly the conditional mean and the conditional variance of the process, the standardized process must be white noise (and because we have simulated the process with conditional normal innovations, the standardized process is also independent). The autocorrelation functions in Figure 4 clearly show that all autocorrelations of the standardized process and the square of the standardized process are statistically zero.

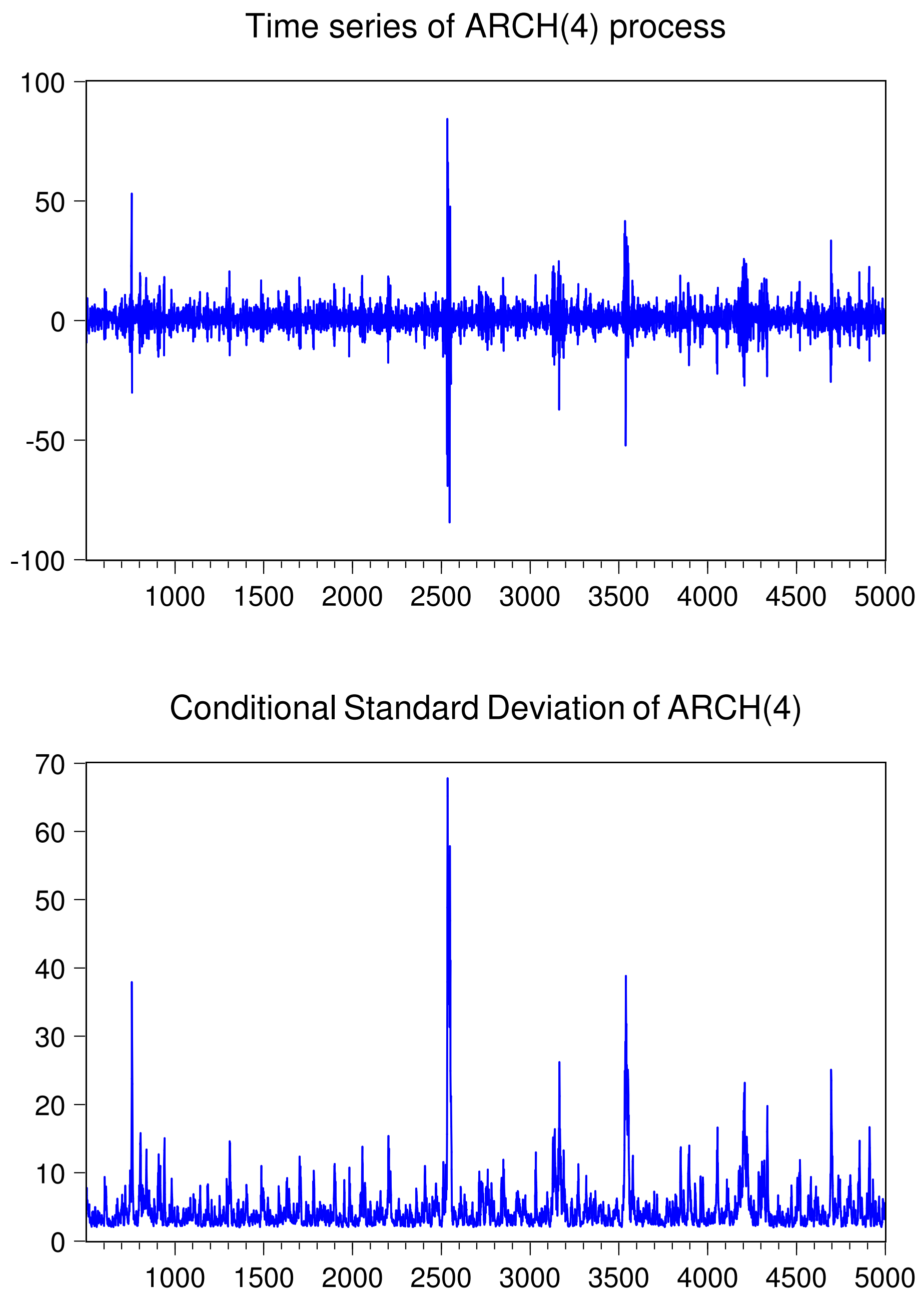
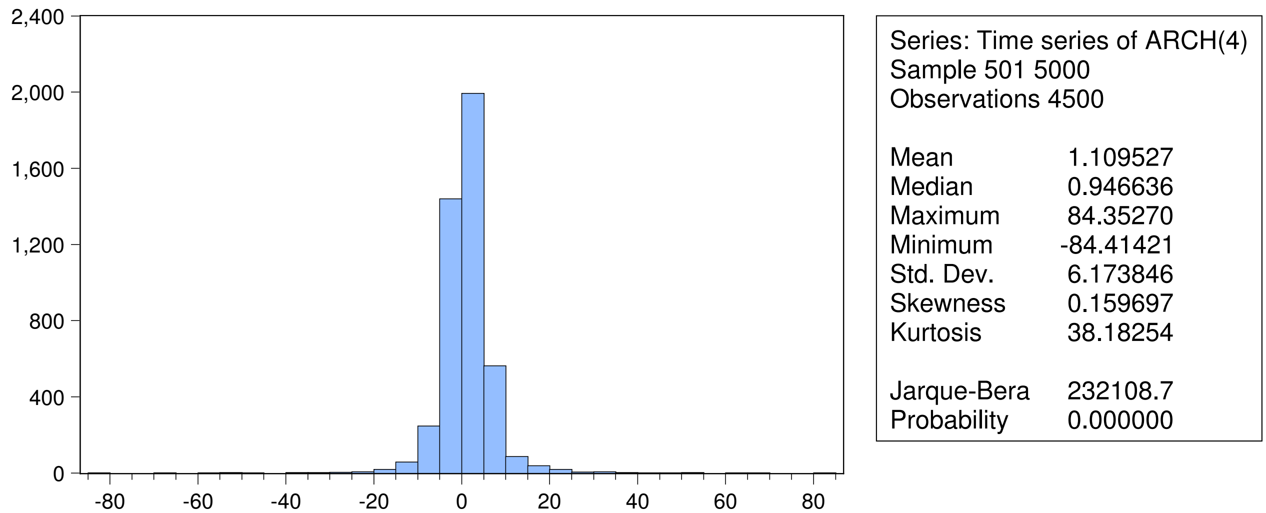
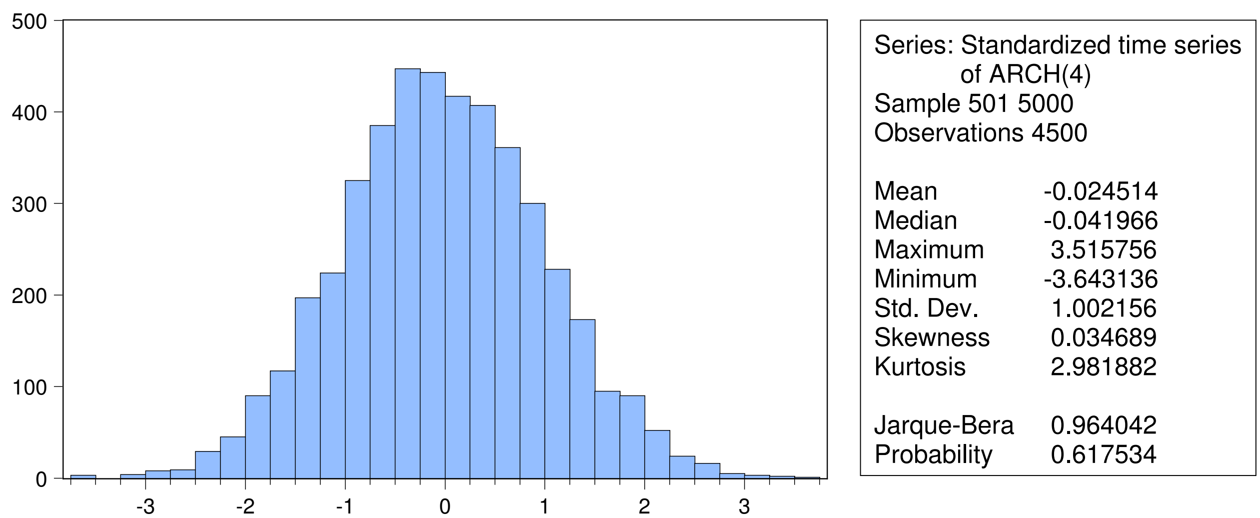


Figure 1: Time Series Plots of an ARCH(4) Process and Conditional Standard Deviation

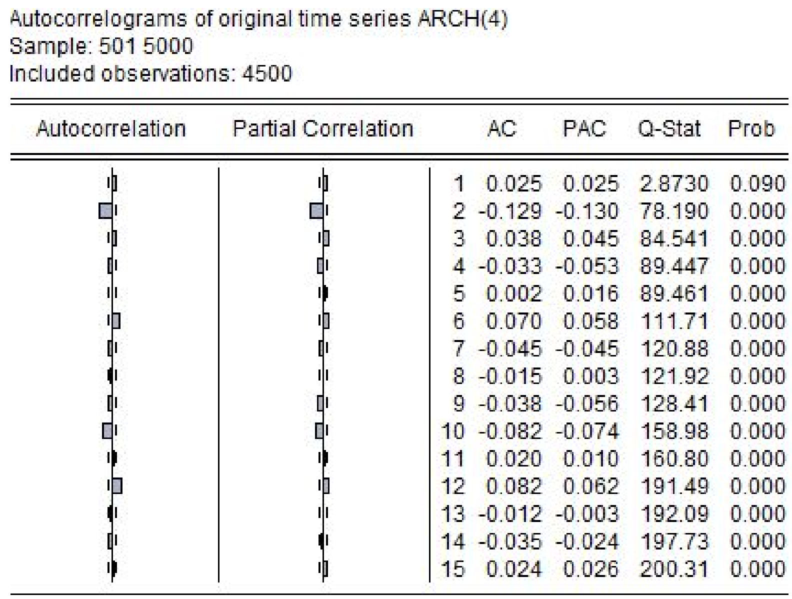


(a) Histogram of ARCH(4) Process



(b) Histogram of Standardized ARCH(4) Process

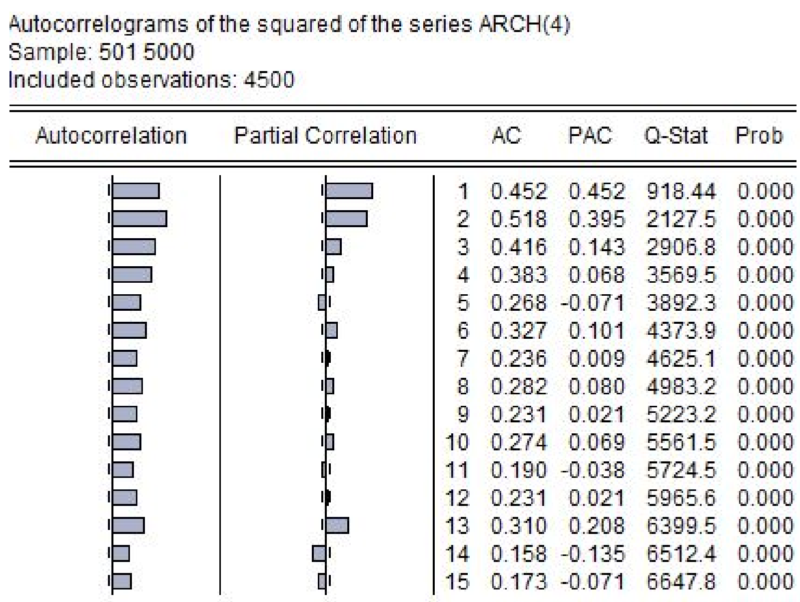
Figure 2: Histograms of ARCH(4) Process



Autocorrelograms of original time series ARCH(4)
Sample: 501 5000
Included observations: 4500

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.025	0.025	2.8730	0.090
		2	-0.129	-0.130	78.190	0.000
		3	0.038	0.045	84.541	0.000
		4	-0.033	-0.053	89.447	0.000
		5	0.002	0.016	89.461	0.000
		6	0.070	0.058	111.71	0.000
		7	-0.045	-0.045	120.88	0.000
		8	-0.015	0.003	121.92	0.000
		9	-0.038	-0.056	128.41	0.000
		10	-0.082	-0.074	158.98	0.000
		11	0.020	0.010	160.80	0.000
		12	0.082	0.062	191.49	0.000
		13	-0.012	-0.003	192.09	0.000
		14	-0.035	-0.024	197.73	0.000
		15	0.024	0.026	200.31	0.000

(a) Autocorrelograms of ARCH(4) Process



Autocorrelograms of the squared of the series ARCH(4)
Sample: 501 5000
Included observations: 4500

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.452	0.452	918.44	0.000
		2	0.518	0.395	2127.5	0.000
		3	0.416	0.143	2906.8	0.000
		4	0.383	0.068	3569.5	0.000
		5	0.268	-0.071	3892.3	0.000
		6	0.327	0.101	4373.9	0.000
		7	0.236	0.009	4625.1	0.000
		8	0.282	0.080	4983.2	0.000
		9	0.231	0.021	5223.2	0.000
		10	0.274	0.069	5561.5	0.000
		11	0.190	-0.038	5724.5	0.000
		12	0.231	0.021	5965.6	0.000
		13	0.310	0.208	6399.5	0.000
		14	0.158	-0.135	6512.4	0.000
		15	0.173	-0.071	6647.8	0.000

(b) Autocorrelograms of Squared ARCH(4) Process

Figure 3: Autocorrelograms of ARCH(4)

Autocorrelograms of standardized series ARCH(4)
Sample: 501 5000
Included observations: 4500

	AC	PAC	Q-Stat	Prob
1	0.018	0.018	1.4591	0.227
2	-0.013	-0.014	2.2627	0.323
3	-0.009	-0.009	2.6689	0.446
4	-0.012	-0.012	3.2942	0.510
5	0.005	0.005	3.4047	0.638
6	0.017	0.017	4.7498	0.576
7	0.007	0.007	4.9916	0.661
8	-0.011	-0.011	5.5386	0.699
9	-0.025	-0.024	8.3730	0.497
10	-0.001	0.000	8.3767	0.592
11	0.007	0.006	8.5874	0.660
12	0.038	0.037	15.254	0.228
13	0.013	0.011	15.978	0.250
14	-0.008	-0.007	16.283	0.296
15	-0.002	0.000	16.301	0.362

(a) Autocorrelograms of Standardized ARCH(4) Process

Autocorrelograms of squared standardized series ARCH(4)
Sample: 501 5000
Included observations: 4500

	AC	PAC	Q-Stat	Prob
1	-0.013	-0.013	0.7943	0.373
2	-0.005	-0.005	0.9175	0.632
3	0.006	0.006	1.0862	0.780
4	-0.011	-0.010	1.5919	0.810
5	-0.026	-0.027	4.7073	0.453
6	0.010	0.009	5.1443	0.525
7	-0.022	-0.022	7.3509	0.393
8	-0.025	-0.026	10.273	0.246
9	0.023	0.021	12.665	0.178
10	-0.006	-0.006	12.845	0.232
11	-0.015	-0.015	13.924	0.237
12	-0.015	-0.018	14.984	0.242
13	0.035	0.034	20.577	0.082
14	0.002	0.004	20.591	0.113
15	0.014	0.013	21.540	0.120

(b) Autocorrelograms of Squared Standardized ARCH(4) Process

Figure 4: Autocorrelograms of Standardized ARCH(4)

Exercise 2

We have generated 20000 observations of two GARCH(1,1), one with low persistence and the other with high persistence. In both cases, we maintain the same conditional mean equation, i.e.,

$$Y_t = 1 + \varepsilon_t$$

low persistence in variance:

$$\sigma^2_{t|t-1} = 4 + 0.3\varepsilon^2_{t-1} + 0.2\sigma^2_{t-1|t-2}$$

high persistence in variance:

$$\sigma^2_{t|t-1} = 4 + 0.3\varepsilon^2_{t-1} + 0.6\sigma^2_{t-1|t-2}$$

In both processes, the unconditional mean is one. In the low persistence, the unconditional variance is $4/(1-0.3-0.2) = 8$ and in the high persistence, $4/(1-0.3-0.6) = 40$. The persistence in the low process is $0.3/(1-0.2) = 0.375$ and in the high $0.3/(1-0.6) = 0.75$. In Table1, we report the descriptive statistics of the time series $\{y_t\}$ for both processes. Observe that the sample statistics correspond to the theoretical values: the unconditional sample mean is about one, the unconditional sample variances are about 8 (low persistence) and about 40 (high persistence). In both cases, the skewness is zero in agreement with the normality assumption of the standardized innovation i.e. $\varepsilon_t/\sigma_{t|t-1} = z_t$; and the kurtosis is higher in the high persistent process as compared with that of the low persistence process. In Figure 5 we plot 5000 observations of both processes and their corresponding conditional standard deviations. The time series have been generated with a similar EViews program to that in page 372 of the textbook. As expected, in a high persistence process, the clusters of high volatility are lasting longer than in a low persistence process. Observe that in both processes the standardized innovation z_t will be the same because it is assumed $N(0,1)$ in both cases; the histogram of z_t will be very similar to the bottom histogram in Figure 2 of Exercise 1, which has sample zero mean, unit variance, zero skewness and kurtosis equal to three as it corresponds to a standard normal random variable.

Descriptive statistics GARCH(1,1)		
Sample: 1 20000		
	Low persistence	High persistence
Mean	0.973192	0.937119
Median	0.970868	0.943278
Maximum	22.69853	49.03081
Minimum	-17.50549	-57.02723
Std. Dev.	2.833394	6.290773
Skewness	0.067403	-0.03742
Kurtosis	4.243694	7.827013

Table 1: Descriptive Statistics of GARCH(1,1) Processes

In Figure 6, we present the autocorrelation functions of the $\{Y_t^2\}$. The autoregressive nature of these two processes is very obvious in both functions: ACFs decaying slowly towards zero and

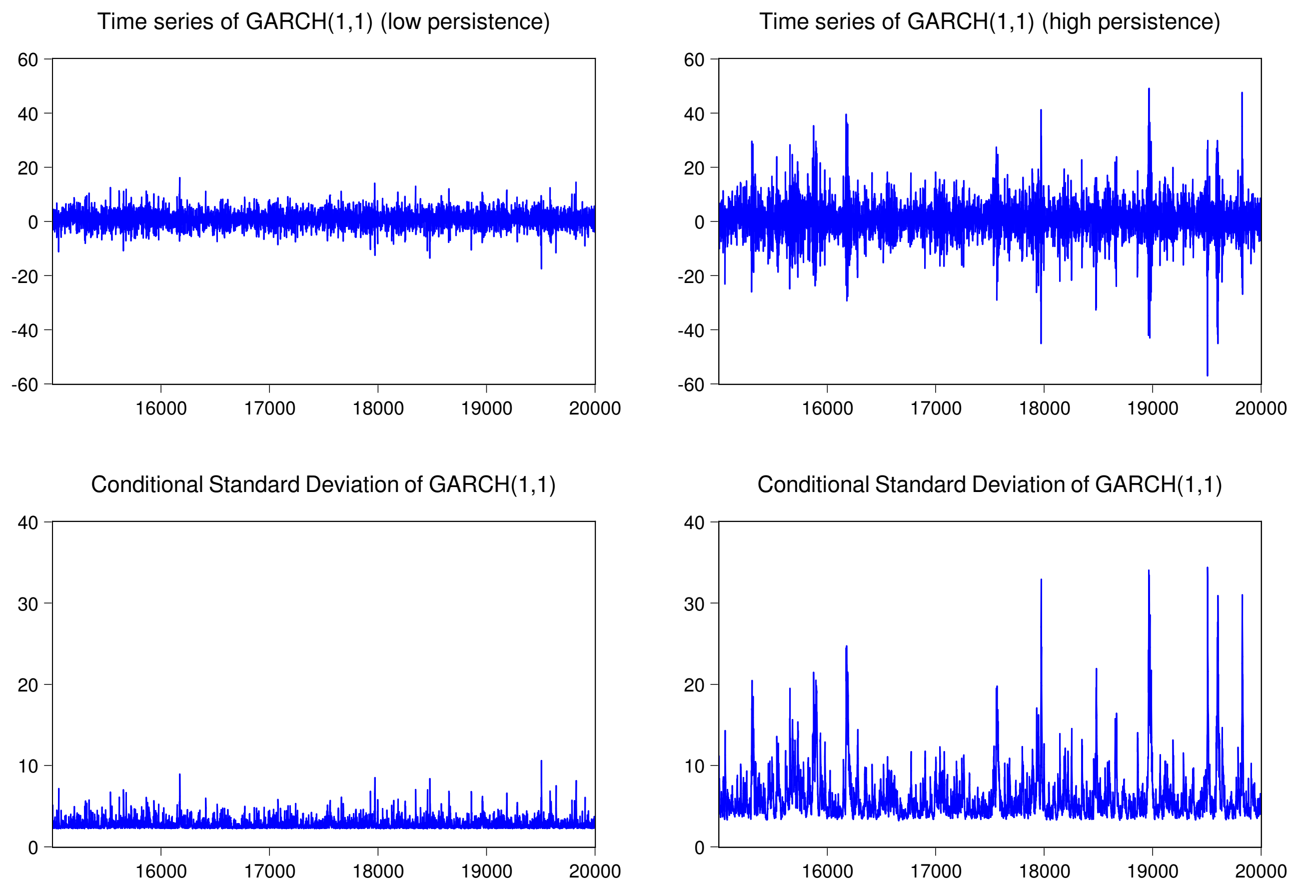
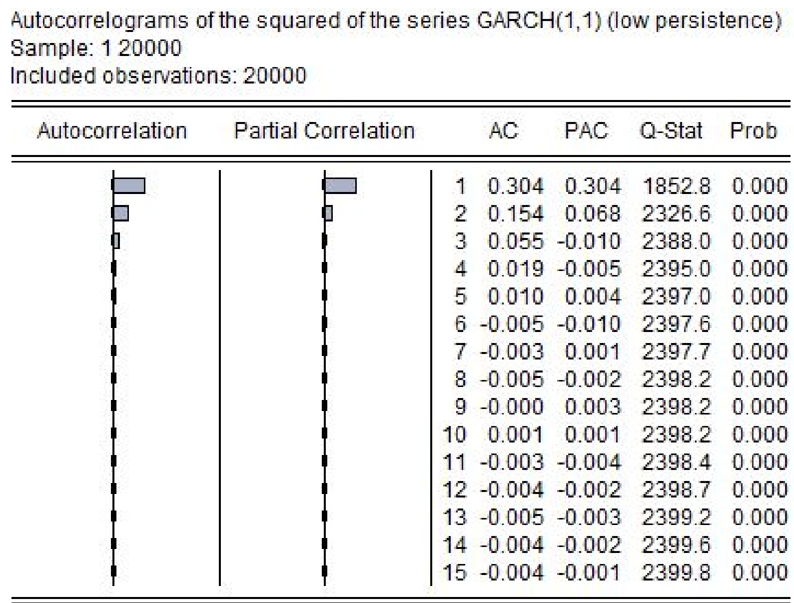


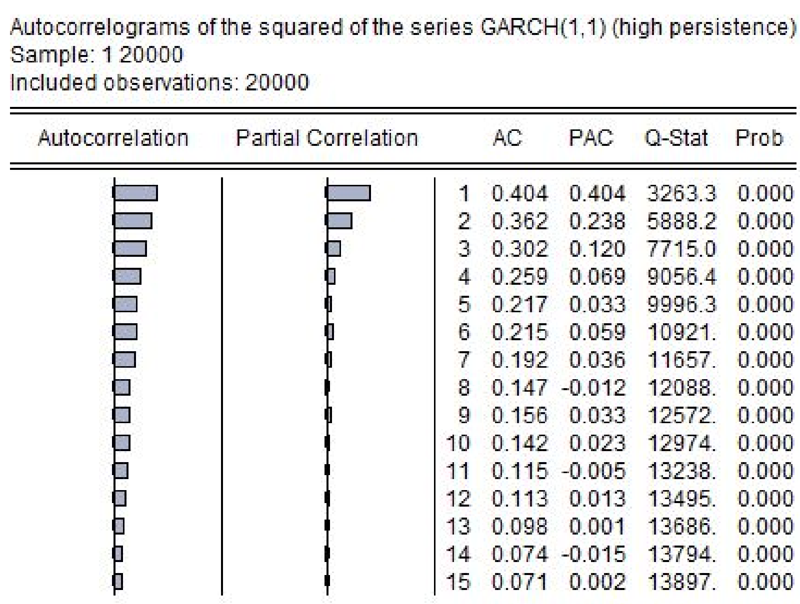
Figure 5: Time Series Plots of GARCH(1,1) Processes and Conditional Standard Deviations



Autocorrelograms of the squared of the series GARCH(1,1) (low persistence)
Sample: 1 20000
Included observations: 20000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.304	0.304	1852.8	0.000
		2	0.154	0.068	2326.6	0.000
		3	0.055	-0.010	2388.0	0.000
		4	0.019	-0.005	2395.0	0.000
		5	0.010	0.004	2397.0	0.000
		6	-0.005	-0.010	2397.6	0.000
		7	-0.003	0.001	2397.7	0.000
		8	-0.005	-0.002	2398.2	0.000
		9	-0.000	0.003	2398.2	0.000
		10	0.001	0.001	2398.2	0.000
		11	-0.003	-0.004	2398.4	0.000
		12	-0.004	-0.002	2398.7	0.000
		13	-0.005	-0.003	2399.2	0.000
		14	-0.004	-0.002	2399.6	0.000
		15	-0.004	-0.001	2399.8	0.000

(a) Autocorrelograms of Squared GARCH(1,1) Process (low persistence)



Autocorrelograms of the squared of the series GARCH(1,1) (high persistence)
Sample: 1 20000
Included observations: 20000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.404	0.404	3263.3	0.000
		2	0.362	0.238	5888.2	0.000
		3	0.302	0.120	7715.0	0.000
		4	0.259	0.069	9056.4	0.000
		5	0.217	0.033	9996.3	0.000
		6	0.215	0.059	10921.	0.000
		7	0.192	0.036	11657.	0.000
		8	0.147	-0.012	12088.	0.000
		9	0.156	0.033	12572.	0.000
		10	0.142	0.023	12974.	0.000
		11	0.115	-0.005	13238.	0.000
		12	0.113	0.013	13495.	0.000
		13	0.098	0.001	13686.	0.000
		14	0.074	-0.015	13794.	0.000
		15	0.071	0.002	13897.	0.000

(b) Autocorrelograms of Squared GARCH(1,1) Process (high persistence)

Figure 6: Autocorrelograms of the squared GARCH(1,1) Processes

PACFs showing only a few significant spikes. However, the profile of the low persistence process is more subdued than that of the high persistence. This is expected because, as we have mentioned above, the clusters of volatility are long lasting (there is more time dependence) compared to those in the low persistence process. The autocorrelation functions of $\{Y_t\}$ and those of the standardized processes $\{z_t\}$ and $\{z_t^2\}$ should be those corresponding to white noise processes like we have seen in Exercise 1 (Figure 3).

Exercise 3
In Figure 7 we plot the daily time series of the SP500 index and its corresponding daily returns from January 3, 2000 to May 6, 2013 for a total of 3355 observations. The most volatile times during this period correspond to the financial crisis of 2008, which generated extreme swings in th index with returns between -10% and +10%, and its aftermath that includes the onset of the sovereign crisis in several European countries in 2011, which also generated large movements in the index with returns between -5 and +5 %. Before the crisis of 2008, times were calmer in particular the period between 2002 and 2007 when the index reached almost the 1600 level.

We proceed to find a model for the conditional mean and the conditional variance of returns. The clusters in volatility point towards a GARCH specification. In Figure 8 we present the autocorrelation functions of the returns and squared returns. The raw returns exhibit very little autocorrelation; the coefficients are very close to zero though they seem to be statistically significant. We propose a low order MA to take care of this autocorrelation. The autocorrelation functions of the squared returns exhibit an autoregressive profile, the ACF function shows high autocorrelations decaying towards zero and the PACF function shows about nine or ten significant spikes. We will entertain an ARCH(9) model for the conditional variance. The estimation results corresponding to a MA(1)-ARCH(9) are presented in Table 2. In the mean equation, the MA(1) term is very statistically significant but the magnitude is too small to be economically significant. This term may be picking up the effect of the bid-ask spread (see Chapter 5). In the variance equation, all terms but ε_{t-1}^2 are very significant and the persistence in variance (sum of all estimates α_i) is very high, about 0.86. It is possible to find a more parsimonious specification by estimating a GARCH process. In Table 3, we present the best GARCH process that is a GARCH(2,1). Together with the MA(1) term in the conditional mean, this specification picks up all the dynamics in the mean and variance. Observe that (Figure 9) the autocorrelation functions of the standardized residuals as well as of the squared standardized innovations are completely clean, all autocorrelations are statistically zero, thus a MA(1)-GARCH(2,1) model seems to be an appropriate specification. It is more parsimonious and it delivers a higher log-likelihood value than the MA(1)-ARCH(9) model.

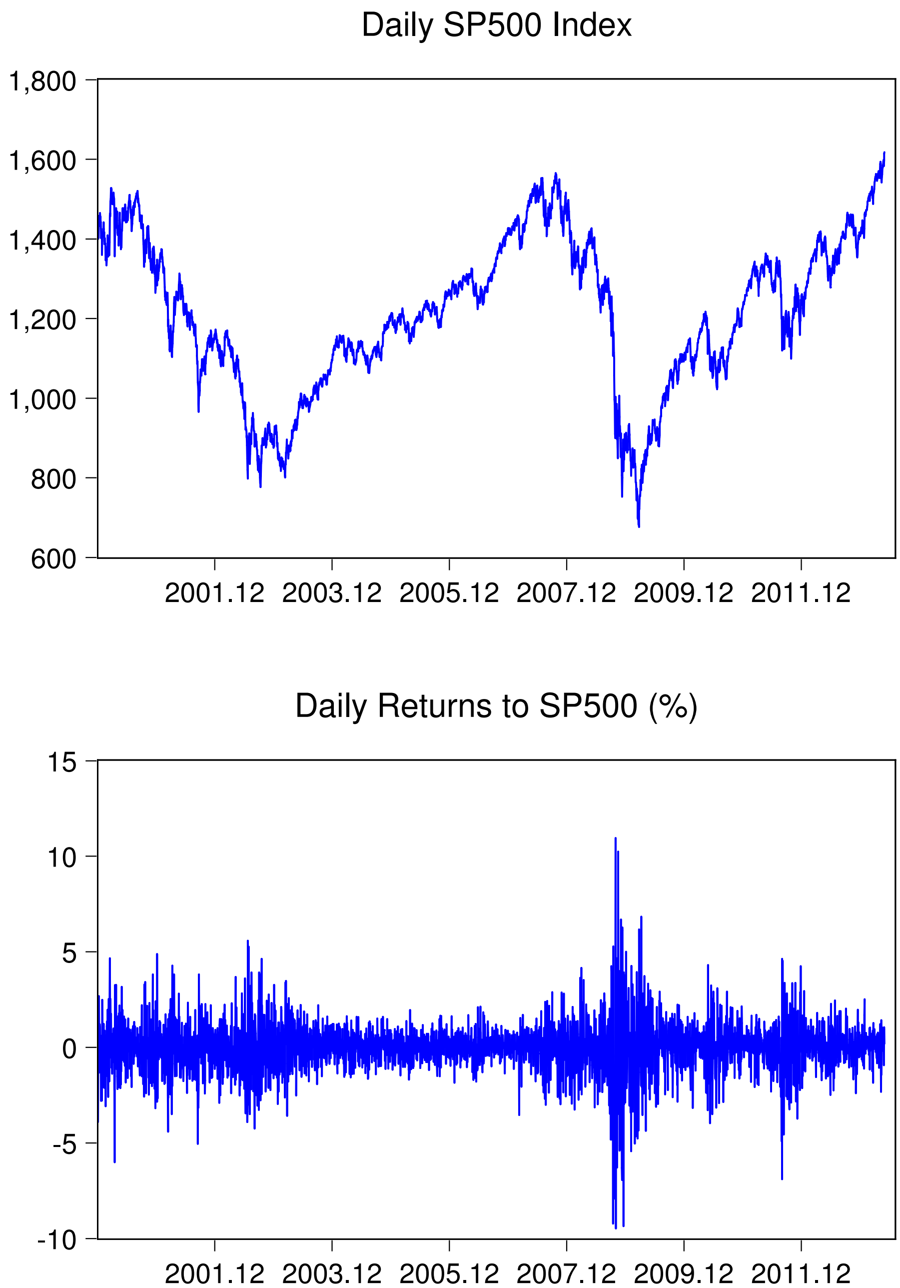
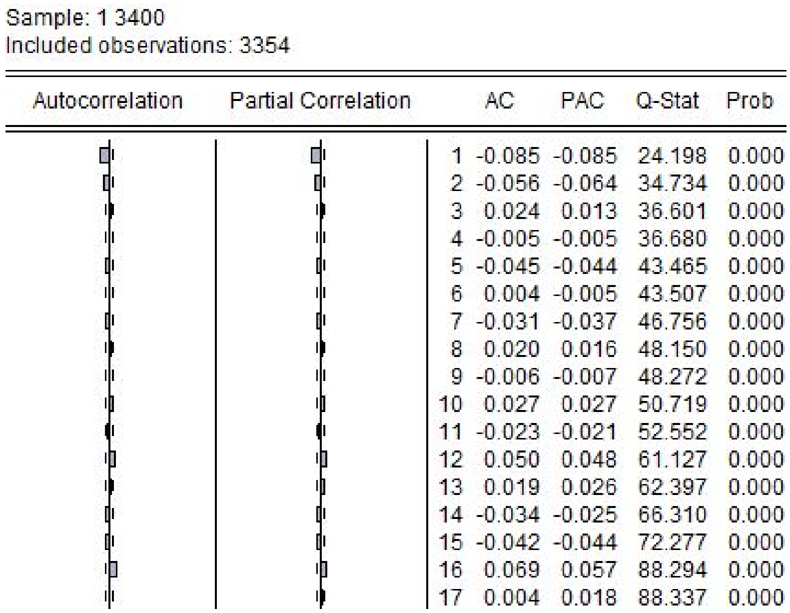


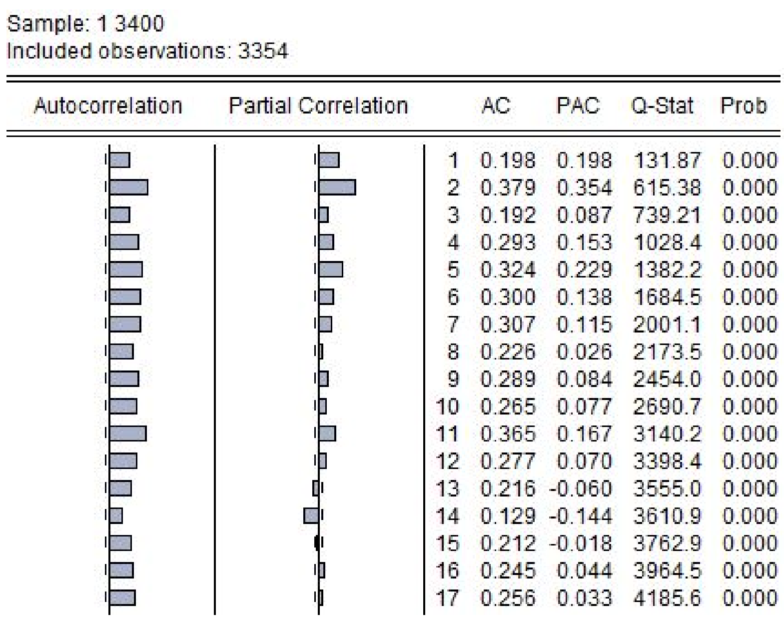
Figure 7: Time Series Plots of Daily SP500 Index and its Daily Returns (%)



Sample: 1 3400
Included observations: 3354

Lag	AC	PAC	Q-Stat	Prob
1	-0.085	-0.085	24.198	0.000
2	-0.056	-0.064	34.734	0.000
3	0.024	0.013	36.601	0.000
4	-0.005	-0.005	36.680	0.000
5	-0.045	-0.044	43.465	0.000
6	0.004	-0.005	43.507	0.000
7	-0.031	-0.037	46.756	0.000
8	0.020	0.016	48.150	0.000
9	-0.006	-0.007	48.272	0.000
10	0.027	0.027	50.719	0.000
11	-0.023	-0.021	52.552	0.000
12	0.050	0.048	61.127	0.000
13	0.019	0.026	62.397	0.000
14	-0.034	-0.025	66.310	0.000
15	-0.042	-0.044	72.277	0.000
16	0.069	0.057	88.294	0.000
17	0.004	0.018	88.337	0.000

(a) Autocorrelograms of Returns to SP500 Index



Sample: 1 3400
Included observations: 3354

Lag	AC	PAC	Q-Stat	Prob
1	0.198	0.198	131.87	0.000
2	0.379	0.354	615.38	0.000
3	0.192	0.087	739.21	0.000
4	0.293	0.153	1028.4	0.000
5	0.324	0.229	1382.2	0.000
6	0.300	0.138	1684.5	0.000
7	0.307	0.115	2001.1	0.000
8	0.226	0.026	2173.5	0.000
9	0.289	0.084	2454.0	0.000
10	0.265	0.077	2690.7	0.000
11	0.365	0.167	3140.2	0.000
12	0.277	0.070	3398.4	0.000
13	0.216	-0.060	3555.0	0.000
14	0.129	-0.144	3610.9	0.000
15	0.212	-0.018	3762.9	0.000
16	0.245	0.044	3964.5	0.000
17	0.256	0.033	4185.6	0.000

(b) Autocorrelograms of Squared Returns to SP500 Index

Figure 8: Autocorrelograms of Returns and Squared Returns to SP500 Index

Dependent Variable: RET				
Method: ML - ARCH (Marquardt) - Normal distribution				
Sample (adjusted): 2 3355				
Included observations: 3354 after adjustments				
Convergence achieved after 16 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
MA Backcast: 1				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*RESID(-3)^2 + C(7)*RESID(-4)^2 + C(8)*RESID(-5)^2 + C(9)*RESID(-6)^2 +C(10)*RESID(-7)^2 + C(11)*RESID(-8)^2 + C(12)*RESID(-9)^2				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.046456	0.01445	3.214902	0.0013
MA(1)	-0.063632	0.017005	-3.74203	0.0002
Variance Equation				
C	0.249484	0.033686	7.406114	0.0000
RESID(-1)^2	0.011852	0.023306	0.508522	0.6111
RESID(-2)^2	0.146163	0.034374	4.252107	0.0000
RESID(-3)^2	0.111259	0.023406	4.753472	0.0000
RESID(-4)^2	0.112858	0.023526	4.7972	0.0000
RESID(-5)^2	0.11165	0.021675	5.1511	0.0000
RESID(-6)^2	0.071941	0.019116	3.763477	0.0002
RESID(-7)^2	0.092215	0.02441	3.777717	0.0002
RESID(-8)^2	0.140069	0.026384	5.308825	0.0000
RESID(-9)^2	0.061963	0.020694	2.994283	0.0028
R-squared	0.006057	Mean dependent var		0.003152
Adjusted R-squared	0.005761	S.D. dependent var		1.338504
S.E. of regression	1.334643	Akaike info criterion		2.970091
Sum squared resid	5970.824	Schwarz criterion		2.991979
Log likelihood	-4968.842	Hannan-Quinn criter.		2.977919
Durbin-Watson stat	2.045756			
Inverted MA Roots	0.06			

Table 2: Estimation Results MA(1)-ARCH(9). Daily SP500 Returns

Dependent Variable: RET				
Method: ML - ARCH (Marquardt) - Normal distribution				
Sample (adjusted): 2 3355				
Included observations: 3354 after adjustments				
Convergence achieved after 14 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
MA Backcast: 1				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.045074	0.014343	3.142586	0.0017
MA(1)	-0.06995	0.016959	-4.12483	0.0000
Variance Equation				
C	0.022835	0.00689	3.314165	0.0009
RESID(-1)^2	0.003275	0.019691	0.166338	0.8679
RESID(-2)^2	0.109632	0.024122	4.544896	0.0000
GARCH(-1)	0.872056	0.014667	59.45666	0.0000
R-squared	0.006449	Mean dependent var		0.003152
Adjusted R-squared	0.006153	S.D. dependent var		1.338504
S.E. of regression	1.33438	Akaike info criterion		2.954369
Sum squared resid	5968.472	Schwarz criterion		2.965313
Log likelihood	-4948.48	Hannan-Quinn criter.		2.958283
Durbin-Watson stat	2.033991			
Inverted MA Roots	0.07			

Table 3: Estimation Results MA(1)-GARCH(2,1). Daily SP500 Returns

Sample: 2 3355
Included observations: 3354
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.008	0.008	0.2160	
		2	-0.027	-0.027	2.7152	0.099
		3	-0.007	-0.006	2.8755	0.237
		4	0.005	0.004	2.9600	0.398
		5	-0.037	-0.038	7.5983	0.107
		6	-0.028	-0.027	10.169	0.071
		7	-0.002	-0.004	10.183	0.117
		8	-0.004	-0.006	10.229	0.176
		9	-0.007	-0.008	10.418	0.237
		10	0.017	0.015	11.335	0.253
		11	0.005	0.003	11.435	0.325
		12	0.045	0.045	18.265	0.076
		13	0.019	0.018	19.478	0.078
		14	-0.012	-0.011	19.961	0.096
		15	-0.026	-0.023	22.210	0.074
		16	0.028	0.029	24.833	0.052
		17	0.008	0.009	25.043	0.069

(a) Autocorrelograms of Standardized Residuals MA(1)-GARCH(2,1)

Sample: 2 3355
Included observations: 3354
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.003	0.003	0.0250	
		2	0.021	0.021	1.4892	0.222
		3	-0.013	-0.013	2.0381	0.361
		4	-0.002	-0.003	2.0542	0.561
		5	-0.001	-0.000	2.0554	0.726
		6	-0.019	-0.019	3.3203	0.651
		7	-0.003	-0.003	3.3459	0.764
		8	0.018	0.019	4.4458	0.727
		9	0.001	0.000	4.4486	0.815
		10	0.029	0.028	7.2608	0.610
		11	0.006	0.006	7.3904	0.688
		12	0.001	-0.000	7.3940	0.766
		13	-0.016	-0.016	8.3075	0.761
		14	0.006	0.007	8.4208	0.815
		15	-0.028	-0.027	11.044	0.683
		16	0.002	0.002	11.058	0.748
		17	-0.010	-0.009	11.416	0.783

(b) Autocorrelograms of Squared Standardized Residuals MA(1)-GARCH(2,1)

Figure 9: Autocorrelograms of Standardized Residuals MA(1)-GARCH(2,1). SP500 Returns

Exercise 4

We proceed to construct the 1-day and 2-day-ahead forecasts for SP500 returns based on the MA(1)-GARCH(2,1) model estimated in Exercise 3. The model is

$$\begin{aligned} Y_t &= c + \theta\varepsilon_{t-1} + \varepsilon_t \\ \sigma^2_{t|t-1} &= \omega + \alpha_1\varepsilon^2_{t-1} + \alpha_2\varepsilon^2_{t-2} + \beta\sigma^2_{t-1|t-2} \end{aligned}$$

For $h = 1$,

$$\begin{aligned} Y_{t+1} &= c + \theta\varepsilon_t + \varepsilon_{t+1} \\ \sigma^2_{t+1|t} &= \omega + \alpha_1\varepsilon^2_t + \alpha_2\varepsilon^2_{t-1} + \beta\sigma^2_{t|t-1} \end{aligned}$$

Assuming a quadratic loss function, the 1-day ahead forecast for the returns is

$$f_{t,1} = c + \theta\varepsilon_t$$

and the forecast for the 1-day-ahead conditional variance of the returns is

$$\sigma^2_{t+1|t} = \omega + \alpha_1\varepsilon^2_t + \alpha_2\varepsilon^2_{t-1} + \beta\sigma^2_{t|t-1}$$

Observe that $Var(Y_{t+1|t}) = Var(\varepsilon_{t+1|t}) = \sigma^2_{t+1|t}$.

For $h = 2$,

$$\begin{aligned} Y_{t+2} &= c + \theta\varepsilon_{t+1} + \varepsilon_{t+2} \\ \sigma^2_{t+2|t} &= \omega + \alpha_1 E(\varepsilon^2_{t+1|t}) + \alpha_2\varepsilon^2_t + \beta\sigma^2_{t+1|t} = \omega + \alpha_2\varepsilon^2_t + (\alpha_1 + \beta)\sigma^2_{t+1|t} \end{aligned}$$

The 2-day ahead forecast for the returns is

$$f_{t,1} = c$$

and the forecast for the 2-day-ahead conditional variance of the returns is

$$Var(Y_{t+2|t}) = Var(\theta\varepsilon_{t+1|t} + \varepsilon_{t+2|t}) = \theta^2\sigma^2_{t+1|t} + \sigma^2_{t+2|t} + 2\theta cov(\varepsilon_{t+1|t}, \varepsilon_{t+2|t})$$

It is reasonable to assume that the covariance term in the last expression is zero, and because the estimate of θ is very small, the 2-day-ahead forecast of the conditional variance of the returns will be dominated by $\sigma^2_{t+2|t}$. By plugging in the above expressions the estimates obtained in Table 3, we have

	$h = 1$ May 7, 2013	$h = 2$ May 8, 2013
$f_{t,h}$	0.0297%	0.0451%
$\sigma^2_{t+h\|t}$	0.7789	0.7099
$\sigma_{t+h\|t}$	0.8825	0.8425
$Var(Y_{t+h\|t})$	0.7789	0.7137
$St.Dev(Y_{t+h\|t})$	0.8825	0.8448
95% Confidence Interval	0.0297 ± 1.96 × 0.8825 [-1.7%, 1.76%]	0.0451 ± 1.96 × 0.8448 [-1.61%, 1.70%]

Exercise 5

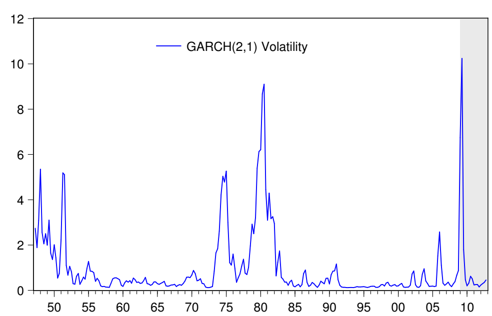
We use the same time series of U.S. CPI and GDP as those in Exercise 5 of Chapter 13. We construct the inflation rate and GDP growth. Both series range from 1947Q1 to 2013Q1 and we reserve the last 17 observations (2009Q1 to 2013Q1) for out-of-sample forecast. The unconditional means are 0.92% for quarterly inflation and 0.80% for quarterly real GDP growth. We do not entertain any ARMA model for the conditional mean with the exception of an intercept. The best model for the conditional variance is an ARCH(2) for the inflation rate and a GARCH(1,1) for GDP growth. These specifications ensure that the squared of the standardized residuals are white noise. However, a check on the autocorrelation function of the standardized residuals reveal that they are not yet white noise and a model for the conditional mean is needed. We will perform this analysis in the next Exercise 6. Now we focus in the (G)ARCH modelling. We present the estimation results in Tables 4 and 5. Observe that in both cases, inflation and GDP growth, the conditional variance is an integrated process, i.e. the sum of $\alpha_1 + \alpha_2 = 1$ for inflation, and $\alpha + \beta = 1$ for GDP growth. Based on these estimates, we compute and plot the (G)ARCH volatility in Figures 10a and 11a. Comparing the conditional variances generated by (G)ARCH and those generated by EWMA (Exercise 5, Chapter 13), we see that the profile of the time series is identical, though EWMA estimates are smaller in magnitude, in particular for the inflation series. The corresponding 95% interval forecast for the sample forecast (last 17 observations) based on (G)ARCH estimates are plotted in Figures 10b and 11b. Observe that the realized values of the series are well contained within the 95% bands; inflation and GDP growth have been fluctuating very close around their unconditional means; and the bands are getting narrower over time because volatility has been lower in the recent years.

Dependent Variable: INFL				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1947Q2 2008Q4				
Included observations: 247 after adjustments				
Convergence achieved after 22 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.731922	0.032329	22.63981	0
Variance Equation				
C	0.143958	0.038534	3.73587	0.0002
RESID(-1)^2	0.355762	0.134973	2.635802	0.0084
RESID(-2)^2	0.658954	0.271575	2.426411	0.0152
R-squared	-0.043045	Mean dependent var		0.916073
Adjusted R-squared	-0.043045	S.D. dependent var		0.88939
S.E. of regression	0.908331	Akaike info criterion		2.15108
Sum squared resid	202.9659	Schwarz criterion		2.207913
Log likelihood	-261.6584	Hannan-Quinn criter.		2.173961
Durbin-Watson stat	0.755856			

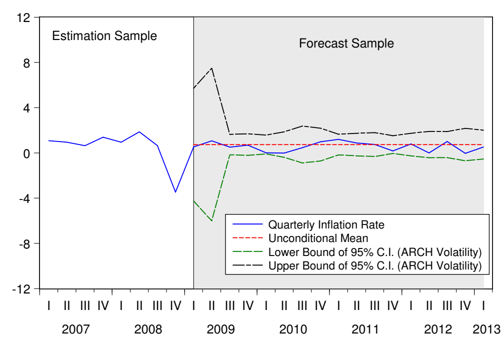
Table 4: Quarterly Inflation Rate: Estimation of a ARCH(2) Model

Dependent Variable: GRGDP				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1947Q2 2008Q4				
Included observations: 247 after adjustments				
Convergence achieved after 13 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.86186	0.036244	23.77947	0.0000
Variance Equation				
C	0.03178	0.022775	1.395362	0.1629
RESID(-1)^2	0.264902	0.095018	2.787922	0.0053
GARCH(-1)	0.746756	0.085113	8.773718	0.0000
R-squared	-0.003408	Mean dependent var		0.803471
Adjusted R-squared	-0.003408	S.D. dependent var		1.002184
S.E. of regression	1.00389	Akaike info criterion		2.692518
Sum squared resid	247.9178	Schwarz criterion		2.74935
Log likelihood	-328.526	Hannan-Quinn criter.		2.715399
Durbin-Watson stat	1.265964			

Table 5: Quarterly Real GDP Growth Rate: Estimation of a GARCH(1,1) Model

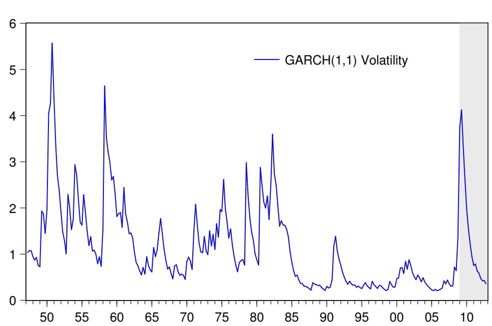


(a) ARCH(2) Volatility

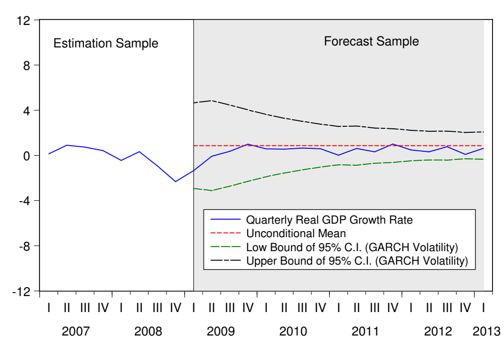


(b) Interval Forecast with ARCH Volatility

Figure 10: Quarterly Inflation Rate



(a) GARCH(1,1) Volatility



(b) Interval Forecast with GARCH Volatility

Figure 11: Quarterly Growth Rate of U.S. Real GDP

Exercise 6

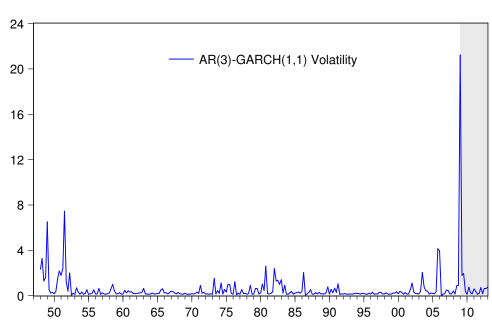
We continue with the same time series as in Exercise 5. The estimation sample goes from 1947Q1 to 2008Q4, and the forecast sample from 2009Q1 to 2013Q1. We estimated an AR(3) for the conditional mean of inflation and GDP growth in Exercise 6 of Chapter 13. Now we search for the best model for the conditional mean and variance jointly. We find that, for inflation, an AR(3)-ARCH(2) and, for GDP growth, an AR(3)-GARCH(1,1) are the best specifications. The models guarantee that the standardized residuals and the square of the standardized residuals are white noise, so that we are capturing the dynamics of the series very well. We report the estimation results in Tables 6 and 7. Observe that the parameter estimates in the conditional mean are very similar to those in Exercise 6 of Chapter 13. However, the estimates in the conditional variance of inflation are quite different from those in Exercise 5 of this chapter. This is not surprising because the residuals of the conditional mean equation, once we model the dynamics, are different from the residuals of the model in Exercise 5, which were basically the demeaned inflation rates. Note that the R-squared of the regression has jumped to 44%. For GDP growth, the parameter estimates in the conditional variance are similar to those in Exercise 5. In this case, the modeling of the conditional mean has not had much impact in the residuals, see that the R-squared is only about 12%. In both series, the process of the conditional variance remains integrated. Based on the estimation of the conditional mean and variance, we compute the (G)ARCH volatility for both series, see Figures 12a and 13a. For inflation, with the exception of the 2008 volatility shock, the AR(3)-ARCH(2) volatility is smaller than the C-ARCH(2) volatility of Exercise 5; and for GDP growth, the AR(3)-GARCH(1,1) volatility is smaller than the C-GARCH(1,1) volatility of Exercise 5. As a consequence, the 95% interval forecasts are narrower than those in Exercise 5, see Figures 12b and 13b.

Dependent Variable: INFL				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1948Q1 2008Q4				
Included observations: 244 after adjustments				
Convergence achieved after 25 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.808238	0.155995	5.181185	0.0000
AR(1)	0.331519	0.094537	3.506775	0.0005
AR(2)	0.199526	0.100025	1.994761	0.0461
AR(3)	0.279157	0.052608	5.306338	0.0000
Variance Equation				
C	0.09964	0.028222	3.530555	0.0004
RESID(-1)^2	0.722277	0.235675	3.064712	0.0022
RESID(-2)^2	0.356381	0.206339	1.727163	0.0841
R-squared	0.443031	Mean dependent var		0.901877
Adjusted R-squared	0.436069	S.D. dependent var		0.874288
S.E. of regression	0.65655	Akaike info criterion		1.676077
Sum squared resid	103.4538	Schwarz criterion		1.776405
Log likelihood	-197.4814	Hannan-Quinn criter.		1.716484
Durbin-Watson stat	1.634498			
Inverted AR Roots	0.9	-.28+.48i	-.28-.48i	

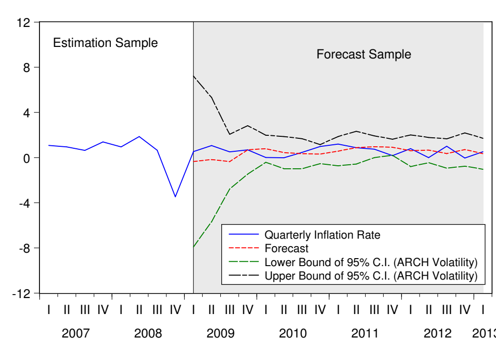
Table 6: Quarterly Inflation Rate: Estimation of a AR(3)-ARCH(2) Model

Dependent Variable: GRGDP				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1948Q1 2008Q4				
Included observations: 244 after adjustments				
Convergence achieved after 50 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.862595	0.068677	12.5602	0.0000
AR(1)	0.316155	0.071557	4.418218	0.0000
AR(2)	0.218858	0.073751	2.967545	0.0030
AR(3)	-0.124296	0.076177	-1.631667	0.1027
Variance Equation				
C	0.024216	0.019151	1.26449	0.2061
RESID(-1)^2	0.261482	0.10339	2.529075	0.0114
GARCH(-1)	0.754282	0.088096	8.562014	0.0000
R-squared	0.133352	Mean dependent var		0.808152
Adjusted R-squared	0.122519	S.D. dependent var		1.003848
S.E. of regression	0.940345	Akaike info criterion		2.570542
Sum squared resid	212.2196	Schwarz criterion		2.670871
Log likelihood	-306.6062	Hannan-Quinn criter.		2.610949
Durbin-Watson stat	1.94319			
Inverted AR Roots	.42-.23i	.42+.23i	-0.53	

Table 7: Quarterly GDP Growth Rate: Estimation of a AR(3)-GARCH(1,1) Model

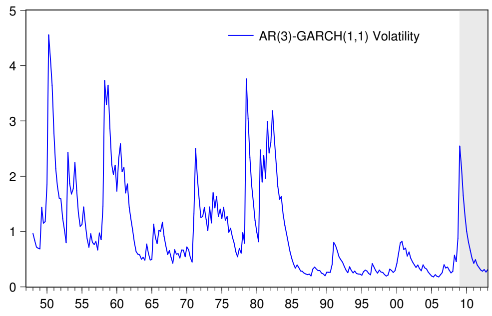


(a) AR(3)-ARCH(2) Volatility

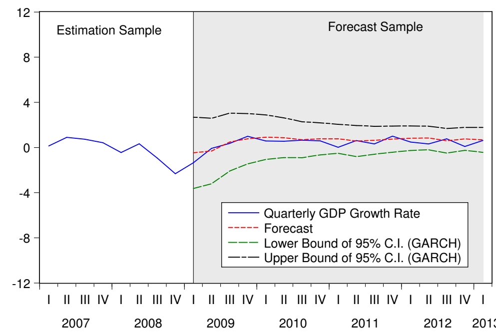


(b) Interval Forecast with ARCH Volatility

Figure 12: Quarterly Inflation Rate



(a) AR(3)-GARCH(1,1) Volatility



(b) Interval Forecast with GARCH Volatility

Figure 13: Quarterly Growth Rate of U.S. Real GDP

Exercise 7

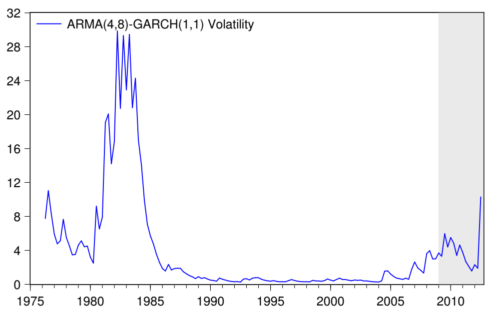
We work with the same time series as in Exercise 7 of Chapter 13: quarterly house prices of Chicago MSA (Chicago-Joliet-Naperville IL-IN-WI) and Miami MSA (Miami-Fort Lauderdale-Pompano Beach FL). Both series range from 1975Q1 to 2012Q4 and we reserve the last 16 observations (2009Q1 to 2012Q4) for out-of-sample forecast. The best models are an ARMA(4,8)-GARCH(1,1) for Chicago and AR(4)-GARCH(1,1) for Miami. These specifications capture the dynamics of the conditional mean and conditional variance very well because they generate standardized residuals and squared standardized residuals that are white noise processes. We report the estimation results in Tables 8 and 9. The persistence in conditional mean is similar and very high in both markets, the models have a root very close to one; in addition the process for the conditional variance is integrated for both markets. Overall, the model fit for the Miami market is better than for the Chicago market (see the adjusted R-squared), though Miami has been more volatile. In Figures 14 and 15, we plot the GARCH volatilities computed from the estimated models for Chicago and Miami house price growth. We also plot the MA and EWMA volatility estimates (Exercise 7, Chapter 13) for comparison with the GARCH volatilities. These are very similar to MA and EWMA volatilities except for the period between 2009Q1 and 2012Q4, in which GARCH volatility is much lower. For the full sample, the correlation between GARCH and MA volatility is 0.70 for Chicago and 0.39 for Miami, and the correlation between GARCH and EWMA volatility is 0.69 for Chicago and 0.54 for Miami. We report the contemporaneous correlations in Tables 10 and 9. The correlation drops substantially to values below 0.4 in the period 2009Q1 to 2012Q4. The main reason is that for GARCH volatility we have modeled the conditional mean while for MA and EWMA volatility we did not specify any dynamics in the conditional mean. It seems that the 2008 housing crisis is better captured in the dynamics of the conditional mean than in the conditional variance. The important message here is that we need to model the conditional mean first and extract the right dynamics, then we model the conditional variance because it is based on the residuals of the conditional mean equation.

Dependent Variable: GCHI				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1976Q2 2008Q4				
Included observations: 131 after adjustments				
Convergence achieved after 216 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
MA Backcast: 1974Q2 1976Q1				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(7) + C(8)*RESID(-1)^2 + C(9)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.543394	1.038689	1.485905	0.1373
AR(1)	0.252416	0.078641	3.209723	0.0013
AR(2)	-0.13849	0.07625	-1.81626	0.0693
AR(3)	0.238462	0.086595	2.753771	0.0059
AR(4)	0.548326	0.086627	6.329772	0.0000
MA(8)	0.335595	0.067143	4.998224	0.0000
		Variance Equation		
C	0.055317	0.045997	1.202609	0.2291
RESID(-1)^2	0.29673	0.097244	3.051398	0.0023
GARCH(-1)	0.692475	0.08239	8.404877	0.0000
R-squared	0.156563	Mean dependent var		1.21493
Adjusted R-squared	0.122826	S.D. dependent var		2.106592
S.E. of regression	1.972983	Akaike info criterion		3.311006
Sum squared resid	486.5829	Schwarz criterion		3.508539
Log likelihood	-207.871	Hannan-Quinn criter.		3.391272
Durbin-Watson stat	1.421692			
Inverted AR Roots	0.97	-.01-.90i	-.01+.90i	-0.69
Inverted MA Roots	.81+.33i	.81-.33i	.33+.81i	.33-.81i
	-.33-.81i	-.33+.81i	-.81-.33i	-.81+.33i

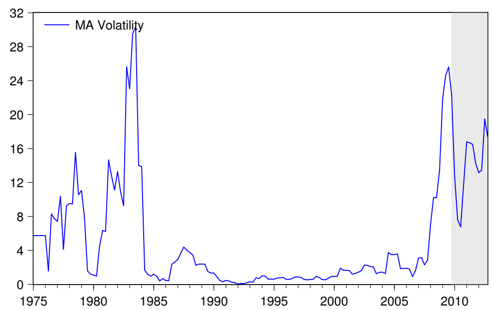
Table 8: Growth Rate of House Price Index for Chicago: Estimation of ARMA(4,8)-GARCH(1,1)

Dependent Variable: GMIA				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1976Q2 2008Q4				
Included observations: 131 after adjustments				
Convergence achieved after 194 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	4.273292	5.463588	0.78214	0.4341
AR(1)	0.385896	0.087921	4.38914	0.0000
AR(2)	0.275829	0.070428	3.916456	0.0001
AR(3)	0.161659	0.087566	1.846144	0.0649
AR(4)	0.142232	0.064441	2.20715	0.0273
		Variance Equation		
C	0.182666	0.086018	2.123582	0.0337
RESID(-1)^2	0.81439	0.18956	4.296213	0.0000
GARCH(-1)	0.320825	0.090498	3.545103	0.0004
R-squared	0.469819	Mean dependent var		1.124302
Adjusted R-squared	0.452988	S.D. dependent var		3.147334
S.E. of regression	2.327778	Akaike info criterion		3.714098
Sum squared resid	682.7371	Schwarz criterion		3.889683
Log likelihood	-235.273	Hannan-Quinn criter.		3.785446
Durbin-Watson stat	1.79827			
Inverted AR Roots	0.98	-.04+.52i	-.04-.52i	-0.53

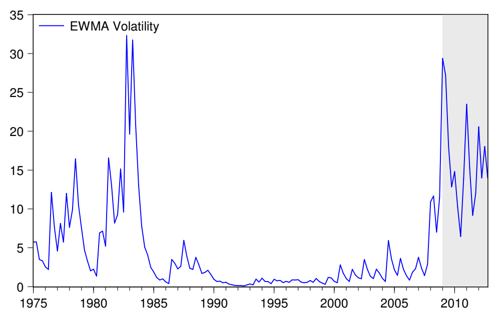
Table 9: Growth Rate of House Price Index for Miami: Estimation of AR(4)-GARCH(1,1)



(a) ARMA(4,8)-GARCH(1,1)



(b) MA (Rolling Window: 4 quarters)



(c) EWMA ($\lambda = 0.6$)

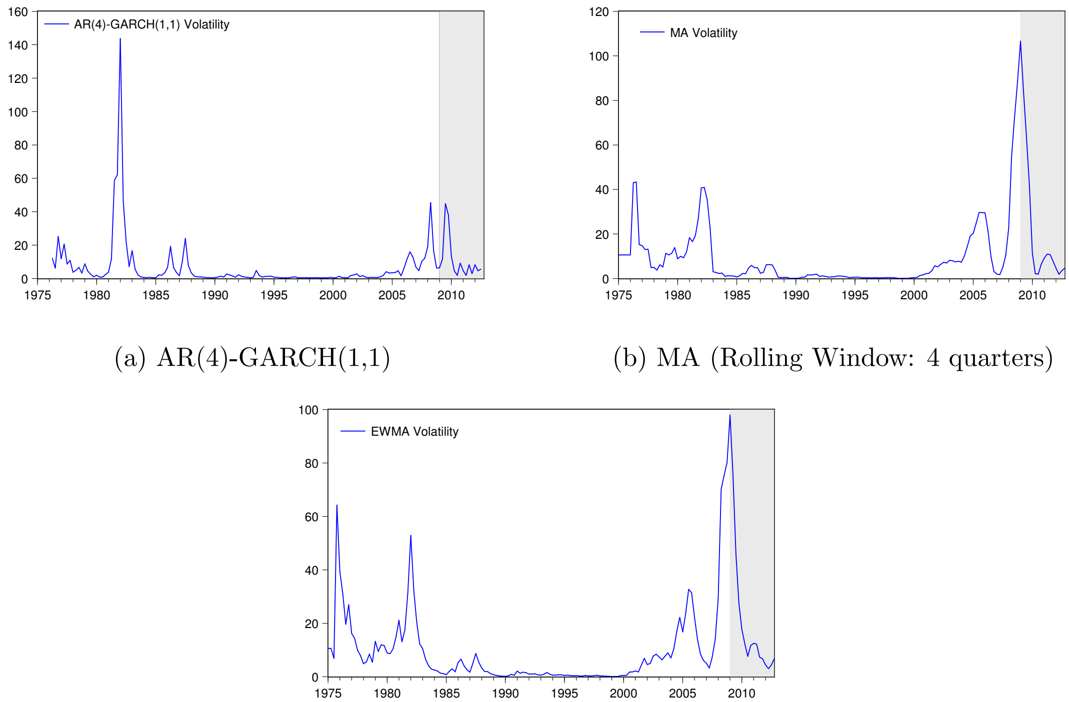
Figure 14: Volatility Forecast of House Price Growth Rate for Chicago

	MA	EWMA	GARCH
MA	1.000000	0.9396	0.700249
EWMA	0.9396	1.000000	0.690485
GARCH	0.700249	0.690485	1.000000

Table 10: Full Sample Correlation of Different Volatility Estimates of House Price Index Growth Rate for Chicago

	MA	EWMA	GARCH
MA	1.000000	0.922637	0.388065
EWMA	0.922637	1.000000	0.544507
GARCH	0.388065	0.544507	1.000000

Table 11: Full Sample Correlation of Different Volatility Estimates of House Price Index Growth Rate for Miami



(a) AR(4)-GARCH(1,1)

(b) MA (Rolling Window: 4 quarters)

(c) EWMA ($\lambda = 0.6$)

Figure 15: Volatility Forecast of House Price Growth Rate for Miami

Exercise 8

For the S&P500 index at daily, weekly, and monthly frequencies of Exercise 8 in Chapter 13, we compute the 1-step-ahead volatility forecasts. For the daily frequency, MA(1)-GARCH(2,1) is the best specification, for weekly and monthly a GARCH(1,1) specification suffices in order to whiten out the standardized residuals and squared standardized residuals. We present the estimation results in Tables 12, 13 and 14. At the daily frequency, the MA(1) term in the conditional mean captures the bid-ask spread effect, which is small but significant. At the weekly and monthly frequencies such an effect does not exist. The conditional variance exhibits high persistence at all frequencies rendering the process integrated or almost integrated. In Figure 16, we show the comparison of the GARCH, MA and EWMA conditional variances at the daily frequency. The profiles are very similar across the three plots. In Table 15, we report the contemporaneous correlation among the three series, which as expected it is very high between 0.94 and 0.97. In Figure 17, we show the comparison of the GARCH, MA and EWMA conditional variances at the weekly frequency. While the profile of MA and GARCH variances is almost identical, the EWMA variances are too smooth indicating that the smooth parameter λ is large and it should be reduced. The contemporaneous correlation between GARCH and EWMA is only 0.65, see Table 16. In Figure 18, we plot the GARCH, MA and EWMA conditional variances at the monthly frequency. The EWMA variance is too smooth compared to GARCH and MA variances, the correlation between EWMA and GARCH variances is 0.72, much smaller than the correlation between MA adn GARCH, which is 0.91, see Table 17. In EWMA, the smoothing parameter λ should be adjusted according to the frequency. At the lower frequencies, we find less dependence in conditional variance (see exercise10 in Chapter 13), thus λ should be also reduced.

Dependent Variable: R				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1/06/2003 4/29/2013				
Included observations: 2596 after adjustments				
Convergence achieved after 18 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
MA Backcast: 1/03/2003				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.055739	0.014898	3.741388	0.0002
MA(1)	-0.08432	0.018288	-4.610652	0.0000
Variance Equation				
C	0.025353	0.007891	3.212911	0.0013
RESID(-1)^2	-0.019646	0.004289	-4.580237	0.0000
RESID(-2)^2	0.132596	0.016768	7.907815	0.0000
GARCH(-1)	0.864522	0.018326	47.17383	0.0000
R-squared	0.012374	Mean dependent var		0.021643
Adjusted R-squared	0.011993	S.D. dependent var		1.297732
S.E. of regression	1.289927	Akaike info criterion		2.784546
Sum squared resid	4316.184	Schwarz criterion		2.798094
Log likelihood	-3608.34	Hannan-Quinn criter.		2.789455
Durbin-Watson stat	2.071221			
Inverted MA Roots	0.08			

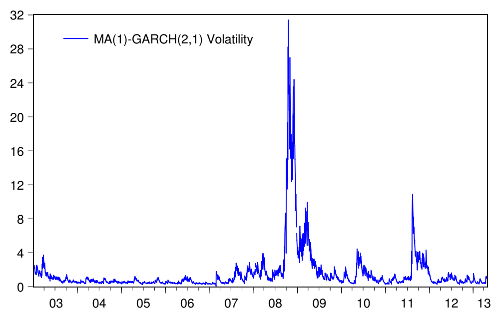
Table 12: Daily S&P500 Price Index: Estimation of a MA(1)-GARCH(2,1) Model

Dependent Variable: R				
Method: ML - ARCH (BHHH) - Normal distribution				
Sample (adjusted): 1/06/2003 4/29/2013				
Included observations: 539 after adjustments				
Convergence achieved after 15 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.298366	0.08626	3.458914	0.0005
Variance Equation				
C	0.520958	0.212965	2.446211	0.0144
RESID(-1)^2	0.262892	0.107604	2.443148	0.0146
GARCH(-1)	0.654367	0.108535	6.029096	0.0000
R-squared	-0.00562	Mean dependent var		0.106649
Adjusted R-squared	-0.00562	S.D. dependent var		2.559832
S.E. of regression	2.567014	Akaike info criterion		4.350678
Sum squared resid	3545.184	Schwarz criterion		4.382513
Log likelihood	-1168.508	Hannan-Quinn criter.		4.36313
Durbin-Watson stat	2.124698			

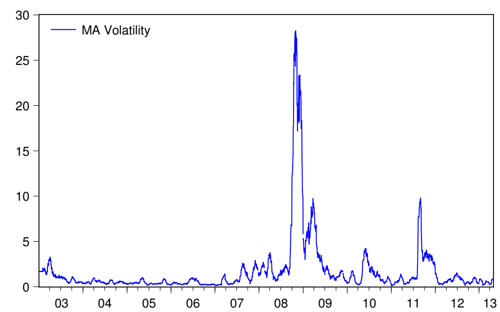
Table 13: Weekly S&P500 Price Index: Estimation of a GARCH(1,1) Model

Dependent Variable: R				
Method: ML - ARCH (Marquardt) - Normal distribution				
Sample (adjusted): 2003M02 2013M05				
Included observations: 124 after adjustments				
Convergence achieved after 269 iterations				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.801447	0.314831	2.545641	0.0109
Variance Equation				
C	0.576465	0.597382	0.964985	0.3346
RESID(-1)^2	0.215494	0.067613	3.187143	0.0014
GARCH(-1)	0.761786	0.076505	9.957378	0.0000
R-squared	-0.004606	Mean dependent var		0.511945
Adjusted R-squared	-0.004606	S.D. dependent var		4.282947
S.E. of regression	4.2928	Akaike info criterion		5.501541
Sum squared resid	2266.66	Schwarz criterion		5.592518
Log likelihood	-337.0955	Hannan-Quinn criter.		5.538498
Durbin-Watson stat	1.524545			

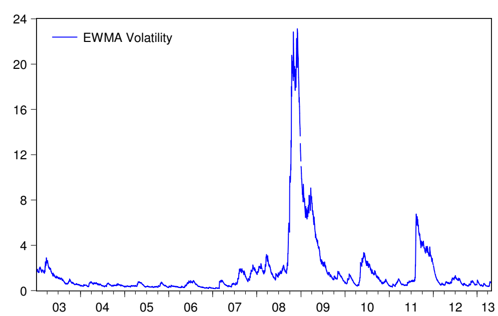
Table 14: Monthly S&P500 Price Index: Estimation of a GARCH(1,1) Model



(a) MA(1)-GARCH(2,1)

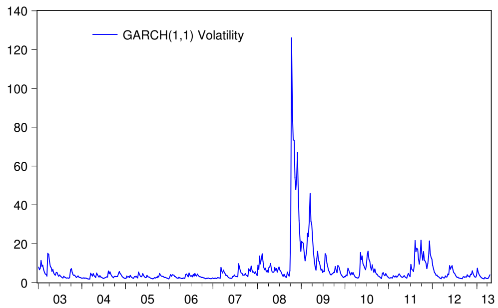


(b) MA / Rolling Window: 20 days

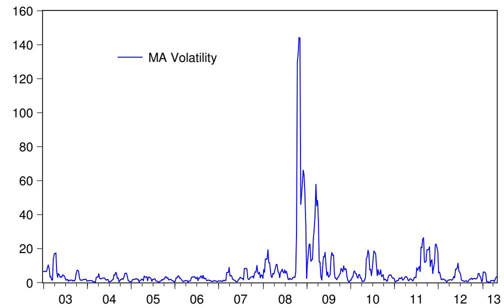


(c) EWMA / $\lambda = 0.95$

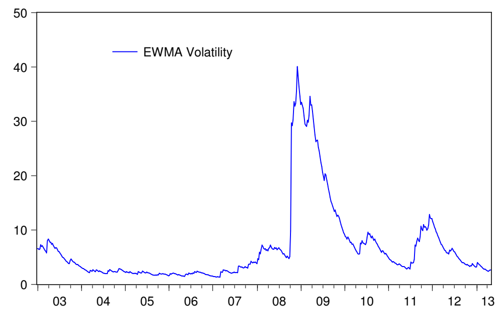
Figure 16: Volatility Forecast of Daily Returns to S&P500



(a) GARCH(1,1)



(b) MA / Rolling Window: 4 weeks



(c) EWMA / $\lambda = 0.95$

Figure 17: Volatility Forecast of Weekly Returns to S&P500

	MA	EWMA	GARCH
MA	1.000000	0.969919	0.964278
EWMA	0.969919	1.000000	0.942787
GARCH	0.964278	0.942787	1.000000

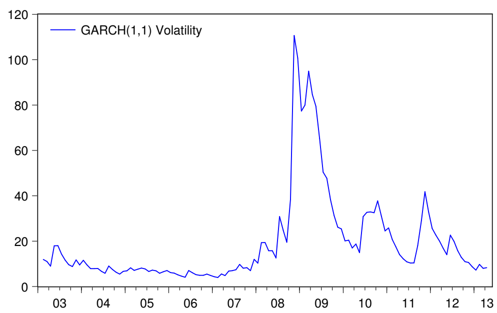
Table 15: Correlation Among Three Different Models of Daily SP500 Conditional Variance

	MA	EWMA	GARCH
MA	1.000000	0.653904	0.952383
EWMA	0.653904	1.000000	0.697963
GARCH	0.952383	0.697963	1.000000

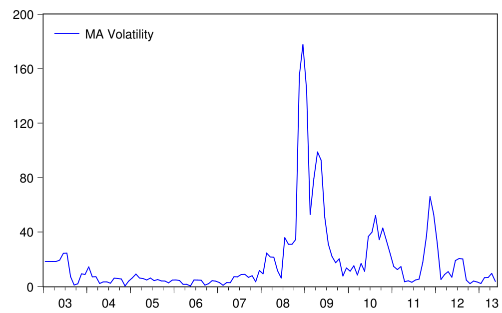
Table 16: Correlation Among Three Different Models of Weekly SP500 Conditional Variance

	MA	EWMA	GARCH
MA	1.000000	0.548432	0.912816
EWMA	0.548432	1.000000	0.724712
GARCH	0.912816	0.724712	1.000000

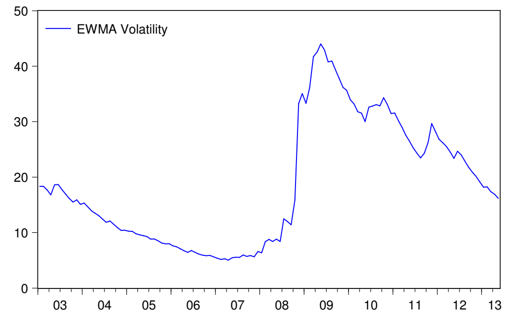
Table 17: Correlation Among Three Different Models of Monthly SP500 Conditional Variance



(a) GARCH(1,1)



(b) MA / Rolling Window: 3 months



(c) EWMA / $\lambda = 0.95$

Figure 18: Volatility Forecast of Monthly Returns to S&P500

Exercise 9

We work with the daily stock prices of two biotechnology companies, Amgen Incorporation (AMGN) and Celgene Corporation (CELG), analyzed in Chapter 13 Exercise 9. We compute the 1-step-ahead volatility forecast of their stock returns. For AMGN, the best model is a GARCH(1,1), and for CELG, a MA(2)-GARCH(1,1). These specifications whiten out the standardized residuals and the square of the standardized residuals, so that they capture correctly the dynamics of the series. In Tables 18 and 20 we report the estimation results. As expected, the conditional mean contains just a constant or a constant and a MA term to capture the bid-ask effect. We have already seen in many instances that stock returns are not predictable based on their past histories. However, the conditional variance is predictable and we find reach dynamics in both stocks. The conditional variance is highly persistent in both cases. In Figure 19, we show the comparison of the GARCH, MA, and EWMA estimates of the conditional variance of AMGN returns. The profiles are very similar across the three plots with EWMA estimates being smoother than the other two. The contemporaneous correlation among the three series is very high, around 0.9, see Table 19. We find similar results for the conditional variance of CELG returns. See Figure 20 for a comparison of the three estimates of the conditional variance, and Table 21 for the contemporaneous correlation among the three estimates. In addition, the comparison among Figures 16, 19, and 20 shows that the stock returns for these two individual companies are more volatile than those for the daily index.

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution
Sample (adjusted): 1/03/2003 4/29/2013
Included observations: 2597 after adjustments
Convergence achieved after 15 iterations
Bollerslev-Wooldridge robust standard errors & covariance
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

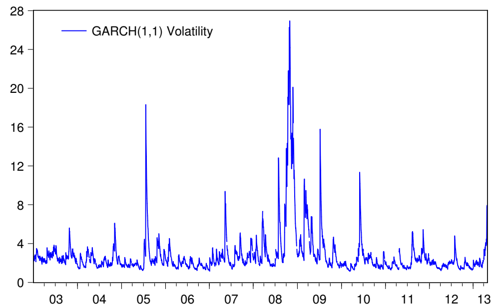
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.022152	0.029228	0.757901	0.4485
Variance Equation				
C	0.109687	0.02865	3.828486	0.0001
RESID(-1)^2	0.080332	0.01515	5.302407	0.0000
GARCH(-1)	0.882578	0.02073	42.57466	0.0000

R-squared	-2.9E-05	Mean dependent var	0.031046
Adjusted R-squared	-2.9E-05	S.D. dependent var	1.661529
S.E. of regression	1.661553	Akaike info criterion	3.715505
Sum squared resid	7166.927	Schwarz criterion	3.724534
Log likelihood	-4820.58	Hannan-Quinn criter.	3.718777
Durbin-Watson stat	2.11872		

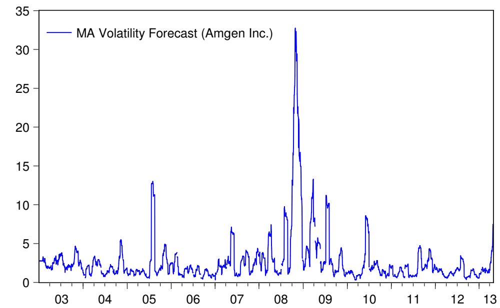
Table 18: Daily Stock Returns to AMGN: Estimation of GARCH(1,1) Model

	MA	EWMA	GARCH
MA	1.000000	0.947451	0.932776
EWMA	0.947451	1.000000	0.932847
GARCH	0.932776	0.932847	1.000000

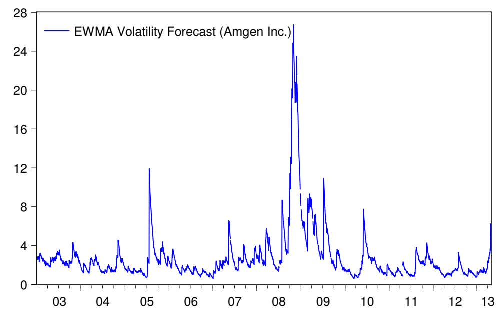
Table 19: Correlation Among Three Different Estimates of the Conditional Variance of AMGN Daily Returns



(a) GARCH(1,1)



(b) MA / Rolling Window: 4 weeks

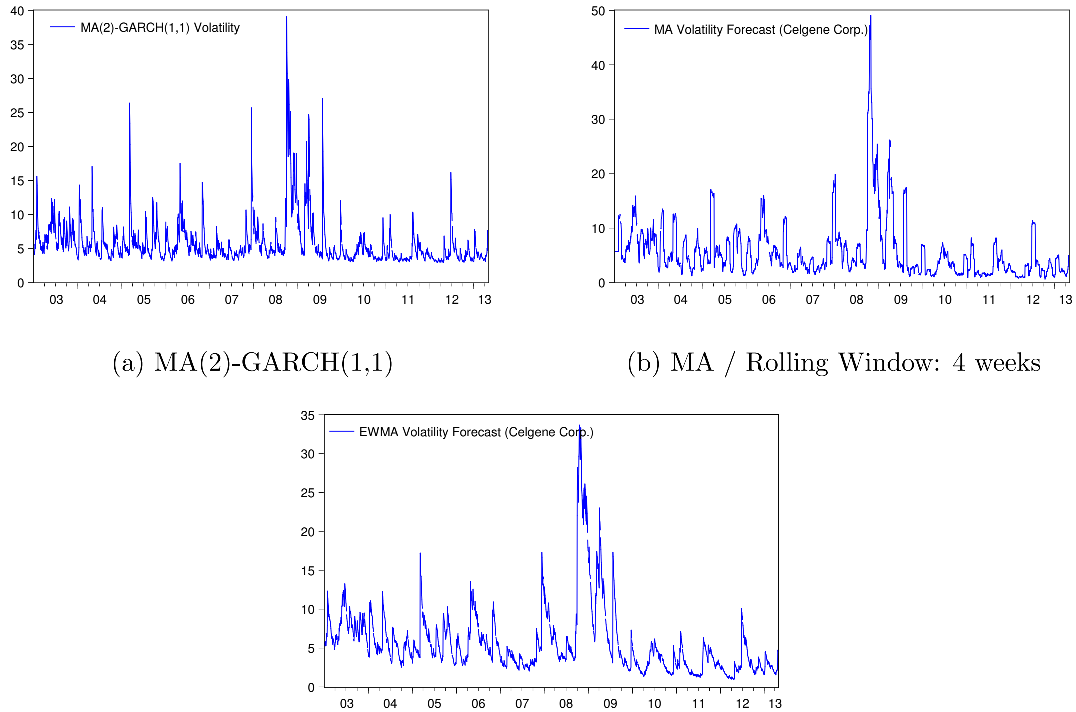


(c) EWMA / $\lambda = 0.95$

Figure 19: Volatility Forecast of Daily Stock Returns to AMGN

Dependent Variable: R				
Method: ML - ARCH (Marquardt) - Normal distribution				
Sample (adjusted): 1/03/2003 4/30/2013				
Included observations: 2598 after adjustments				
Convergence achieved after 24 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
MA Backcast: 1/01/2003 1/02/2003				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.146111	0.041129	3.552529	0.0004
MA(2)	-0.05714	0.022411	-2.54963	0.0108
Variance Equation				
C	0.382655	0.141135	2.71126	0.0067
RESID(-1)^2	0.082526	0.021975	3.755353	0.0002
GARCH(-1)	0.852898	0.037588	22.69067	0.0000
R-squared	0.0046	Mean dependent var		0.118245
Adjusted R-squared	0.004216	S.D. dependent var		2.394876
S.E. of regression	2.389822	Akaike info criterion		4.496844
Sum squared resid	14826.41	Schwarz criterion		4.508127
Log likelihood	-5836.4	Hannan-Quinn criter.		4.500932
Durbin-Watson stat	2.143404			
Inverted MA Roots	0.24	-0.24		

Table 20: Daily Stock Returns to CELG: Estimation of MA(2)-GARCH(1,1) Model



(a) MA(2)-GARCH(1,1) (b) MA / Rolling Window: 4 weeks

(c) EWMA / $\lambda = 0.95$

Figure 20: Volatility Forecast of Daily Stock Returns to CELG

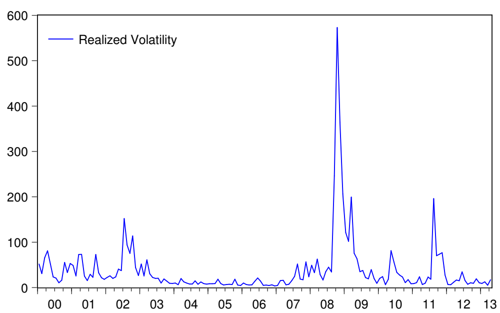
	MA	EWMA	GARCH
MA	1.000000	0.936089	0.858863
EWMA	0.936089	1.000000	0.882303
GARCH	0.858863	0.882303	1.000000

Table 21: Correlation Among Three Different Estimates of the Conditional Variance of CELG Daily Returns

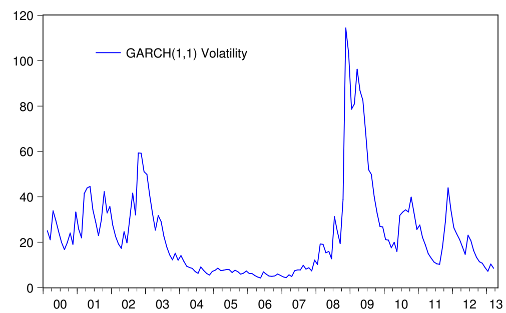
Exercise 10

We use the updated time series of the daily S&P500 index in Exercise 3, and construct its monthly realized volatility (RV) by aggregating daily squared returns. In Figure 21a, we plot the monthly realized volatility; and in Figures 21b, 21c, and 21d, we plot the monthly volatility estimates based on a GARCH(1,1), MA with a rolling window of 3 months, and EWMA with $\lambda = 0.6$.

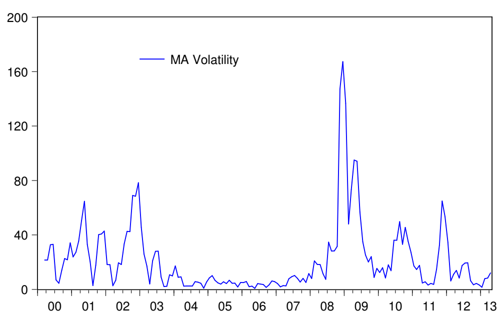
The profiles of the four series is very similar. The contemporaneous correlation is very strong, around 0.9, among the GARCH, MA and EWMA volatility estimates. However, the correlations with the realized volatility measure is much weaker, around 0.5, see Table 22. The main difference among the series is the magnitude of the volatility. While MA and EWMA estimates are of similar magnitude, and the GARCH estimates are slightly lower, the RV measure is substantially larger, and in some instances like October 2008, RV is about 5 times larger than the GARCH estimates. There is an important conceptual difference: RV is an estimate of the integrated variance, which only makes sense in a continuous time environment. The RV estimate will be better when the recording time of prices shrinks toward zero. In this exercise, we are aggregating daily trading records, which is a much lower frequency than the 5 minutes trading frequency that we have studied in the textbook, and so the continuous time assumption is difficult to justify. Nevertheless, the profile of the RV estimates has valid information to detect high volatility episodes and their relative magnitude within a time series.



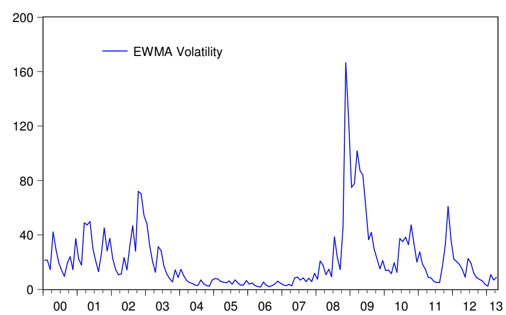
(a) Realized Volatility



(b) GARCH(1,1) Volatility



(c) MA Volatility, Rolling Window: 3 months



(d) EWMA Volatility, $\lambda = 0.6$

Figure 21: Four Estimates of the Monthly Conditional Variance of S&P500 Index Returns

Pairwise Correlation Matrix				
	RV	GARCH	MA	EWMA
RV	1.000000	0.518274	0.503965	0.571106
GARCH	**0.518274**	1.000000	0.914632	0.971132
MA	**0.503965**	0.914632	1.000000	0.936632
EWMA	**0.571106**	0.971132	0.936632	1.000000

Table 22: Correlation Among Four Different Estimates of the Monthly S&P500 Conditional Variance

