CHAPTER 12.

FORECASTING THE LONG TERM AND THE SHORT TERM JOINTLY

SOLUTIONS

by
Wei Lin and Yingying Sun
(University of California, Riverside)

Exercise 1
We update the quarterly time series data set of real gross domestic product (GDP) and consumption for U.S. economy up to 2012Q4. We download the data from the FRED website:

http://research.stlouisfed.org/fred2/series/GDPC1
http://research.stlouisfed.org/fred2/series/PCECC96

Figure 1 shows the time series plots of both series. Observe that both series have an upward trend, therefore we run the unit root test to decide the nature of the trend. We run a unit root test (Case III). The ADF statistic and corresponding critical value at the 5% significance level are -1.0849 and -3.4271 for `Log Consumption`, and -1.5015 and -3.4271 for `Log GDP`. Therefore, we fail to reject the null hypothesis of unit root.

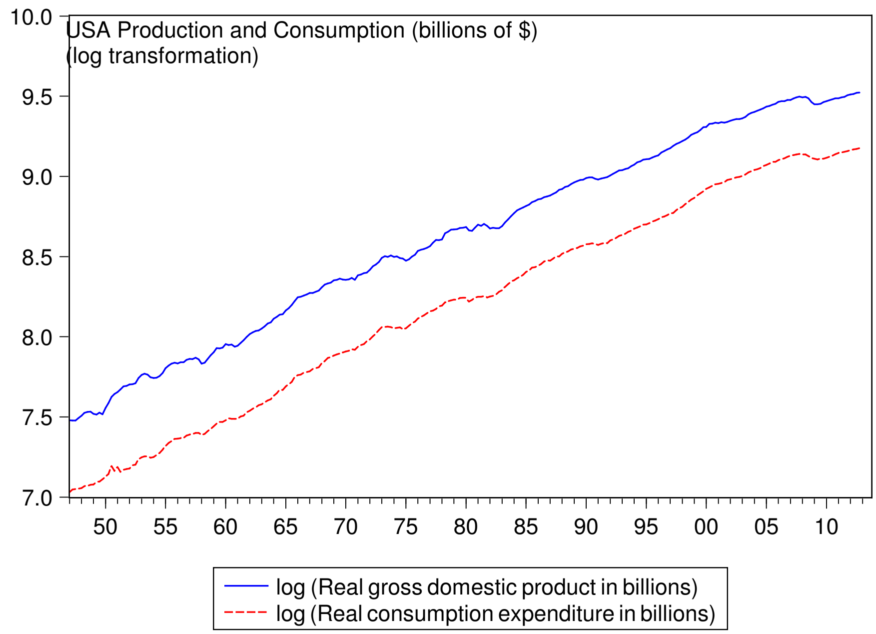


Figure 1: Time Series Plots of Log Transformation of Real GDP and Real Consumption

Figure 2 shows the time series plot for deviations from the equilibrium which is defined as $z_t =$

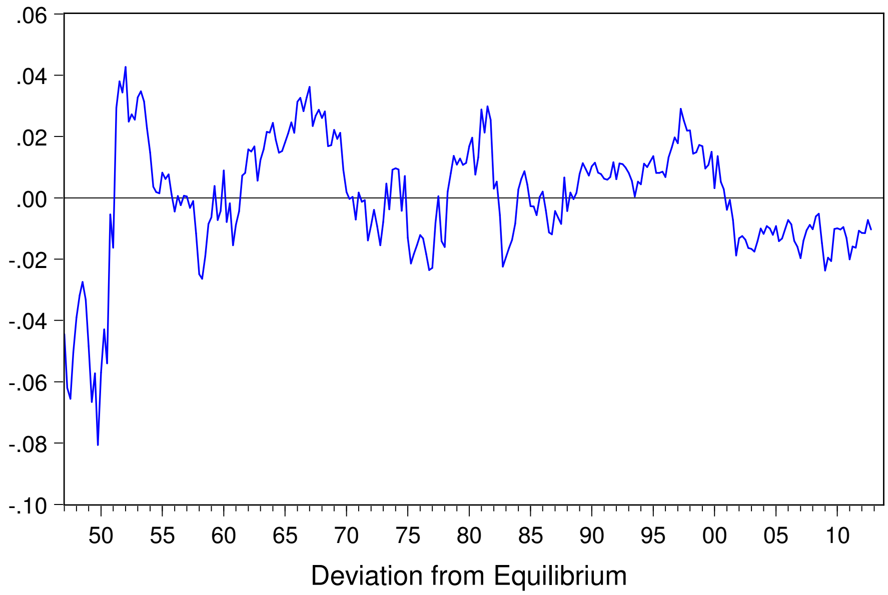


Figure 2: Deviations from equilibrium

$\log Y_t - \log C_t$. To confirm the cointegration relation: $\log Y_t = \alpha_0 + \log C_t + z_t$, we run the unit root test for z_t, which is the disequilibrium error. The ADF statistics for z_t is -3.9243 and we reject the null hypothesis of unit root at the 5% level (using either the ADF critical values directly or those in Table 12.2), which means that the disequilibrium error is stationary. The best vector error correction model is reported in Table 1. Observe that the cointegrating vector is not estimated but imposed on the model. The error correction term is significant in the log DGP equation, confirming that both series are cointegrated. Based on this VEC model, we construct the 1-year-ahead forecast (it is a multistep forecast, 1, 2, 3 and 4 steps, because we have quarterly data) for production and consumption, which is reported in Table 2. Figure 3 shows the multistep forecasts for consumption and production with the 95% confidence bands.

Vector Error Correction Estimates		
Sample (adjusted): 1947Q4 2012Q4		
Included observations: 261 after adjustments		
Standard errors in ( ) & t-statistics in [ ]		
Cointegration Restrictions:		
B(1,1)=-1, B(1,2)=1		
Convergence achieved after 1 iterations.		
Restrictions identify all cointegrating vectors		
LR test for binding restrictions (rank = 1):		
Chi-square(1)	0.015827	
Probability	0.899886	
Cointegrating Eq:	CointEq1	
LCON(-1)	-1.000000	
LGDP(-1)	1.000000	
@TREND(47Q1)	0.000563	
	(5.20E-05)	
	[10.8110]	
C	-0.49994	
Error Correction:	D(LCON)	D(LGDP)
CointEq1	0.01034	-0.10387
	(0.02633)	(0.02878)
	[0.39266]	[-3.60896]
D(LCON(-1))	-0.00301	0.316946
	(0.07654)	(0.08364)
	[-0.03935]	[3.78919]
D(LCON(-2))	0.321831	0.206887
	(0.07706)	(0.08422)
	[4.17633]	[2.45656]
D(LGDP(-1))	0.138471	0.143317
	(0.06887)	(0.07527)
	[2.01051]	[1.90403]
D(LGDP(-2))	-0.08289	0.019977
	(0.06467)	(0.07068)
	[-1.28178]	[0.28265]
C	0.005096	0.002287
	(0.00081)	(0.00089)
	[6.29255]	[2.58391]

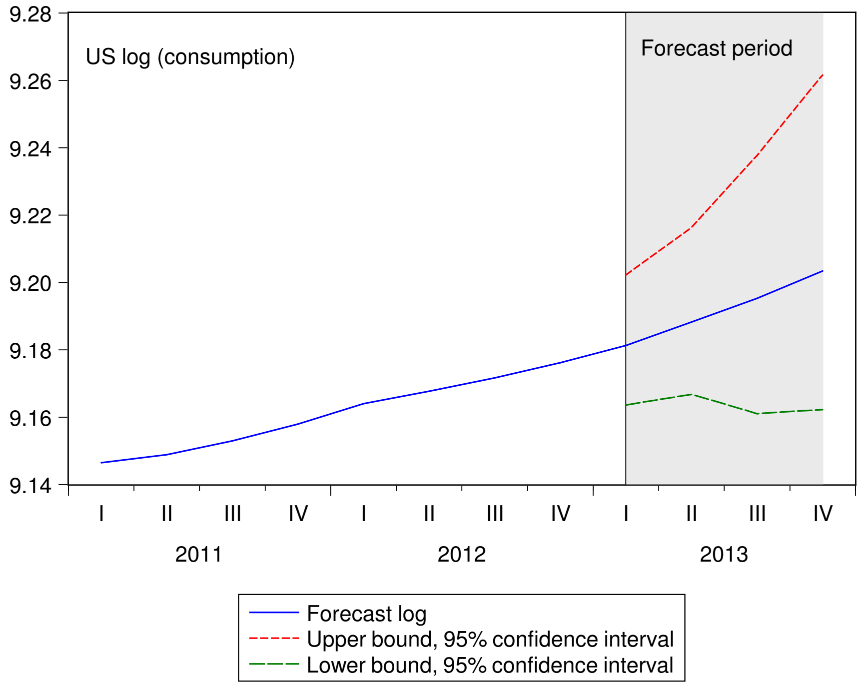
(continued)

R-squared	0.132162	0.249269
Adj. R-squared	0.115146	0.234549
Sum sq. resids	0.015894	0.018984
S.E. equation	0.007895	0.008628
F-statistic	7.766736	16.93377
Log likelihood	896.3333	873.1522
Akaike AIC	-6.82248	-6.64484
Schwarz SC	-6.74053	-6.5629
Mean dependent	0.008144	0.007839
S.D. dependent	0.008393	0.009862
Determinant resid covariance (dof adj.)		2.96E-09
Determinant resid covariance		2.83E-09
Log likelihood		1827.971
Akaike information criterion		-13.8925
Schwarz criterion		-13.6876

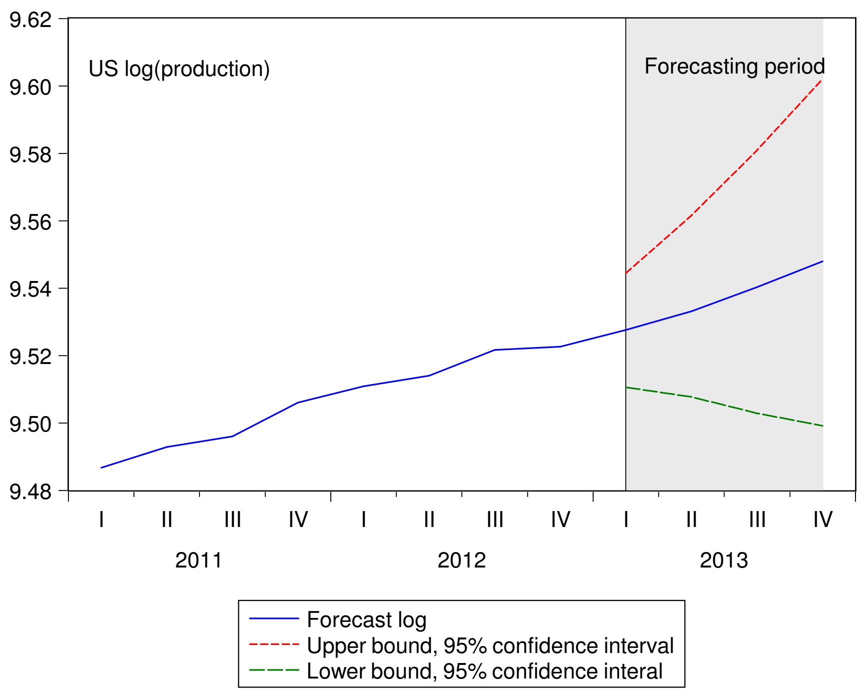
Table 1: Estimation of the VEC Model

Forecast Horizon h	$f^C_{t,h}$	$f^Y_{t,h}$	Consumption growth rate	Production growth rate
2012.Q4	9.176153	9.522622		
h=1, 2013.Q1	9.181287	9.527594	0.51%	0.50%
h=2, 2013.Q2	9.188214	9.533163	0.69%	0.56%
h=3, 2013.Q3	9.195254	9.540331	0.70%	0.72%
h=4, 2013.Q4	9.203381	9.547951	0.81%	0.76%

Table 2: Multistep Forecast for Consumption and Production Growth Rates



(a) Log Consumption



(b) Log Production

Figure 3: Multistep Forecast for Consumption and Production

Exercise 2
To find a potential long-term equilibrium relationship between the two series, we estimate the regression model by OLS

$$\log Y_t = \alpha_0 + \alpha_1 \log C_t + z_t$$

Then, we compute the residuals of the OLS regression, that is, $\widehat{z}_t = \log Y_t - \widehat{\alpha}_0 - \widehat{\alpha}_1 \log C_t$, where $\widehat{\alpha}_0$ and $\widehat{\alpha}_1$ are 0.913478 and 0.940365 reported in Table 3. In We plot the residual series in Figure 4. Finally, we run an ADF test on $\widehat{z}_t$ and use the 5% critical values in Table 12.2 (page 320 of the textbook) to determine whether the residuals have a unit root. The ADF statistic for the residuals is -3.683884, which is *smaller* than 5% critical value -3.3617 with sample size 263, and thus, we reject the null hypothesis that the residual series has a unit root. Thus, we conclude that a cointegration relation does exist between consumption and production. The cointegrating estimate $\widehat{\alpha}_1 = 0.940365$ is slightly different from that in Exercise 1, i.e., $\alpha_1 = 1$. The estimation results of the best VEC model are reported in Table 3. Observe that the estimation results are almost identical to those in Exercise 1, indicating that the cointegrating relations in both exercises are virtually the same. The 1-year-ahead forecasts for production and consumption based on this VEC model are reported in Table 5. In Figure 5 we plot the multistep forecast for consumption and production with the 95% confidence bands. These forecasting results are very similar to those in Exercise 1, as we expect given the estimated cointegrating relation and the VEC estimates.

Dependent Variable: LGDP				
Method: Least Squares				
Sample: 1947Q1 2012Q4				
Included observations: 264				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.913478	0.015638	58.41524	0.000000
LCON	0.940365	0.001902	494.3073	0.000000
R-squared	0.998929	Mean dependent var		8.618744
Adjusted R-squared	0.998925	S.D. dependent var		0.617036
S.E. of regression	0.020233	Akaike info criterion		-4.95546
Sum squared resid	0.107255	Schwarz criterion		-4.92837
Log likelihood	656.1213	Hannan-Quinn criter.		-4.94458
F-statistic	244339.7	Durbin-Watson stat		0.162829
Prob(F-statistic)	0.000000			

Table 3: Estimation of the Cointegrating Relation

Vector Error Correction Estimates		
Sample (adjusted): 1947Q4 2012Q4		
Included observations: 261 after adjustments		
Cointegrating Eq:	CointEq1	
LGDP(-1)	1.000000	
LCON(-1)	-0.934106	
	(0.00637)	
	[-146.669]	
C	-0.965295	
Error Correction:	D(LGDP)	D(LCON)
CointEq1	-0.101758	0.005337
	(0.02747)	(0.02517)
	[-3.70413]	[0.21202]
D(LGDP(-1))	0.139876	0.140142
	(0.07505)	(0.06877)
	[1.86381]	[2.03771]
D(LGDP(-2))	0.020434	-0.080852
	(0.07056)	(0.06466)
	[0.28961]	[-1.25048]
D(LCON(-1))	0.325638	-0.00593
	(0.08311)	(0.07616)
	[3.91809]	[-0.07786]
D(LCON(-2))	0.213999	0.320274
	(0.08396)	(0.07694)
	[2.54876]	[4.16251]
C	0.002181	0.005103
	(0.00088)	(0.00081)
	[2.46754]	[6.29980]
R-squared	0.251213	0.13179
Adj. R-squared	0.236531	0.114767
Sum sq. resids	0.018934	0.015901
S.E. equation	0.008617	0.007897
F-statistic	17.11018	7.741579
Log likelihood	873.4906	896.2774
Akaike AIC	-6.647437	-6.822049
Schwarz SC	-6.565494	-6.740106
Mean dependent	0.007839	0.008144
S.D. dependent	0.009862	0.008393
Determinant resid covariance (dof adj.)		2.97E-09
Determinant resid covariance		2.83E-09
Log likelihood		1827.805
Akaike information criterion		-13.89889
Schwarz criterion		-13.70769

Table 4: Estimation of the VEC Model

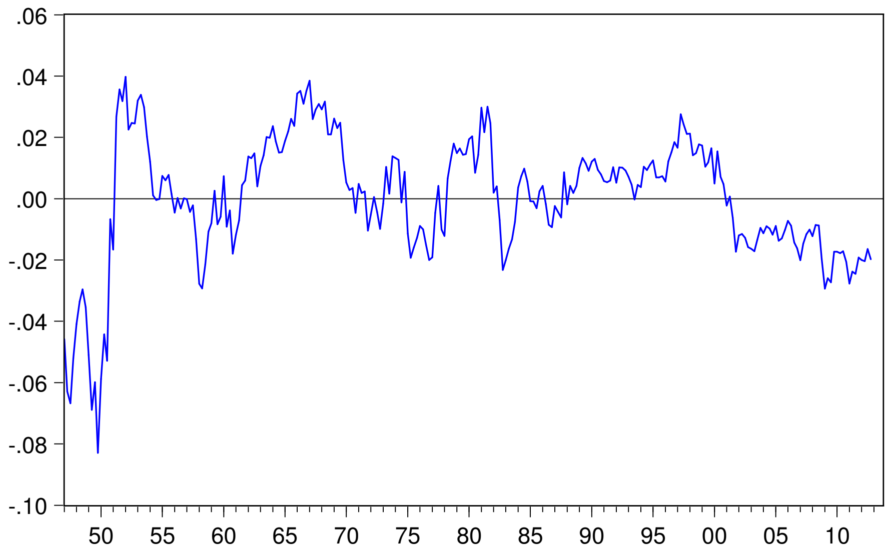
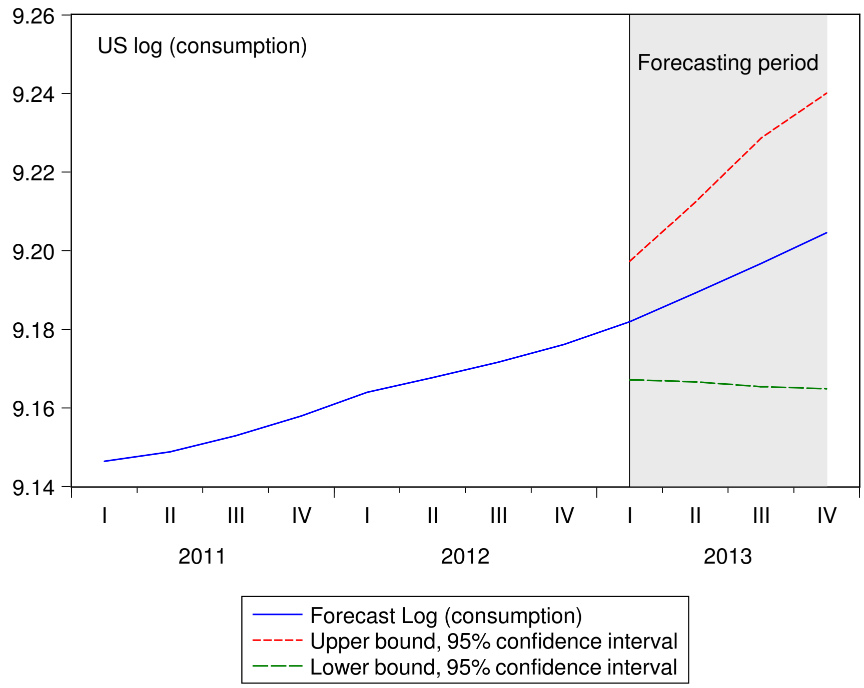


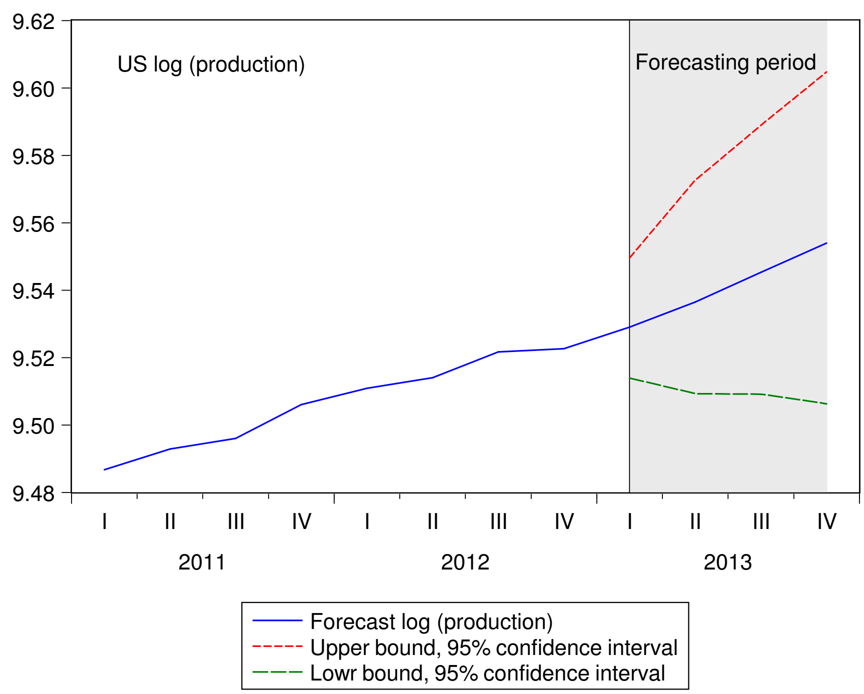
Figure 4: Time Series of the Residuals of the Cointegrating Relation

Forecast Horizon h	$f^C_{t,h}$	$f^Y_{t,h}$	Consumption growth rate	Production growth rate
2012.Q4	9.176153	9.522622		
h=1, 2013.Q1	9.181935	9.529044	0.58%	0.64%
h=2, 2013.Q2	9.189226	9.536498	0.73%	0.75%
h=3, 2013.Q3	9.196707	9.545324	0.75%	0.88%
h=4, 2013.Q4	9.204559	9.553930	0.79%	0.86 %

Table 5: Multistep Forecast for Consumption and Production Growth Rates



(a) Log Consumption



(b) Log Production

Figure 5: Multistep Forecast for Consumption and Production

Exercise 3
We download the monthly consumer price index for United States and Germany from the FRED website. We also download the daily US $/Euro exchange rate to convert the Euro to dollar value. Here are links for each series:

http://research.stlouisfed.org/fred2/series/CPIAUCNS
http://research.stlouisfed.org/fred2/series/DEUCPIALLMINMEI
http://research.stlouisfed.org/fred2/series/EXUSEU

The data sets range from 1999M01 to 2013M01 and we multiply the German price index by the US $/Euro exchange rate to convert the price index into dollar prices. In Figure 6 we plot the three series together, and in Figure 7 we plot both consumer price index for US and Germany in dollar value.

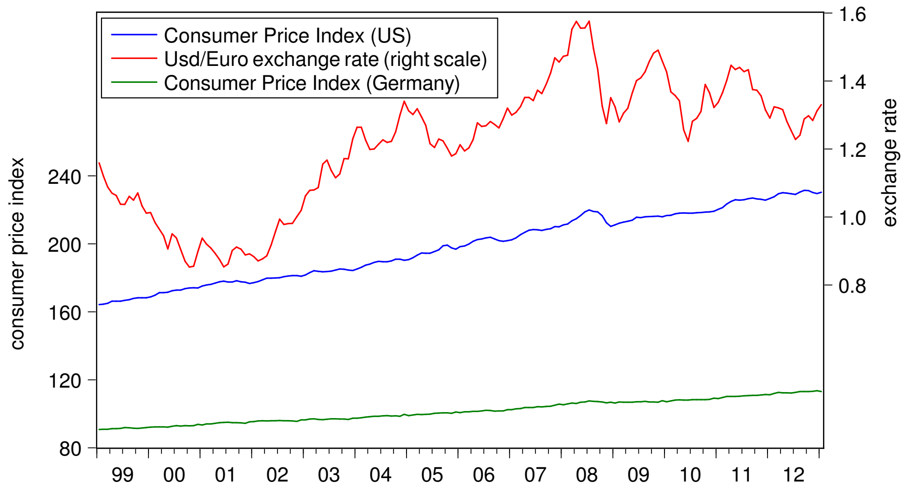


Figure 6: CPI in US, Germany and US $/Euro Exchange Rate

We proceed to test the cointegration relation. First, we test for unit root in the two series, Germany CPI (once it is multiplied by the exchange rate) and US CPI using the ADF statistics. As both series exhibit obvious time trends, we include intercept and trend in the ADF equations (Case III). For the German CPI, the ADF statistic is -2.6687 with a corresponding p-value of 0.25, so that we cannot reject a unit root. For the US CPI, the ADF statistic is -4.3379 with a p-value of 0.0035, so that we reject a unit root in favor of a deterministic trend. A cointegrating relation is properly defined within the context of two unit root processes and here we have a unit root process (Germany CPI) versus a trend-stationary process (US CPI). Nevertheless, we compute the difference between both CPIs ($P_t - P_t^* e_t$) and analyze the resulting process. An ADF test reveals that in fact this difference contains a unit root (ADF is -2.5280 with p-value of 0.11), and conclude that PPP does not hold. Statistically, this result is not very surprising because we already know that a unit root process dominates a stationary process, so that the difference of these two processes must be non-stationary.

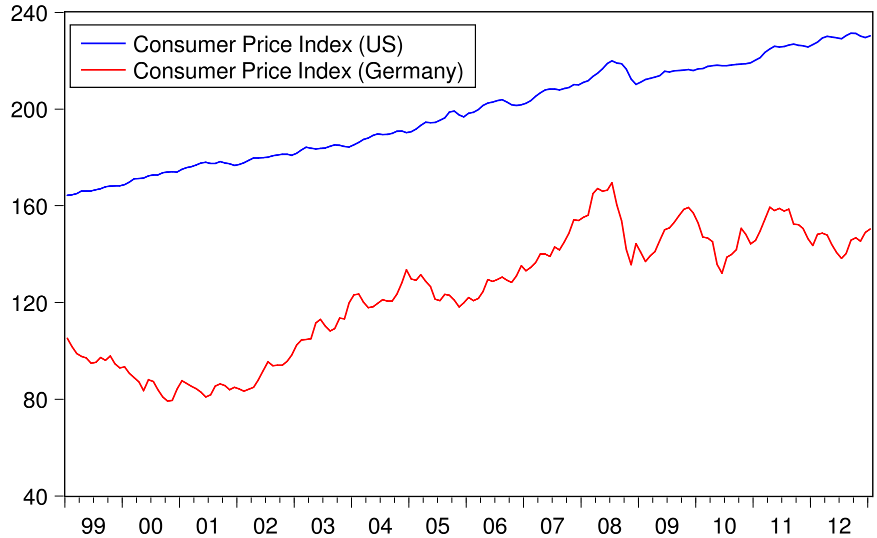


Figure 7: CPI in US and Germany

Exercise 4

We download the quarterly time series of the U.S. consumer price index $\{cpi_t\}$[1] and nominal interest rate $\{i_t\}$[2] from the FRED® website:[3]

http://research.stlouisfed.org/fred2/series/CPILFESL
http://research.stlouisfed.org/fred2/series/TB3MS

The data sets range from 1957Q1 to 2012Q4. As the quarterly nominal interest rate i_t is already annualized, we convert the quarterly CPI to annualized quarterly inflation rate π_t by the following continuously compounded formula

$$\pi_t = \left[\left(\frac{cpi_t}{cpi_{t-1}} \right)^4 - 1 \right] \times 100\%,$$

so that the two time series are comparable to each other. Then, we obtain the real interest rate by taking the difference, i.e., $r_t = i_t - \pi_t$. In Figure 8 we plot the three times series.

Regarding stationarity, we perform ADF unit root tests on the three series. All three series do not exhibit any trend. Thus, we choose to include only an intercept in the ADF test equations (Case II of Table 10.4). In Table 6 we report the results. Note that the equilibrium restriction $r_t = i_t - \pi_t$ is directly imposed, so that we will use the ADF critical value and p-value directly. At 5% significance level, for both inflation and nominal interest rates, we fail to reject the null hypotheses that the series have a unit root. On the contrary, we reject the hypothesis of a unit root for real interest rate with p-value (0.0008) much smaller than 5%. We conclude that inflation and Treasury Bill rate are non-stationary and have a cointegration relation $r_t = i_t - \pi_t$, which is stationary.

[1] Consumer Price Index for All Urban Consumers: All Items Less Food & Energy (CPILFESL)
[2] 3-Month Treasury Bill: Secondary Market Rate (TB3MS)
[3] In the download links, we choose "End of Period" in the "Aggregation Method" option.

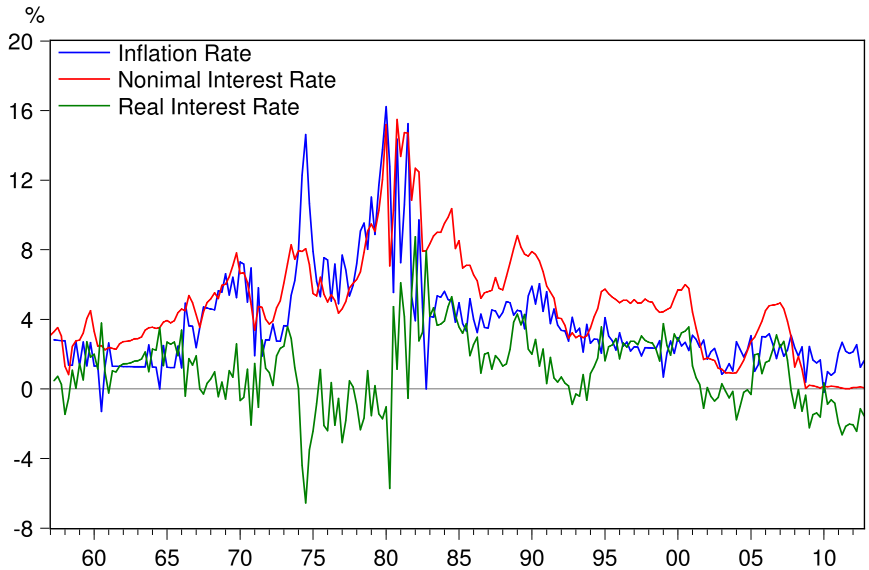


Figure 8: Time Series Plots of Three Rates

	ADF Statistic	p-value
nominal interest rate: i_t	-2.058233	0.2620
inflation rate: π_t	-2.474342	0.1232
real interest rate: $r_t = i_t - \pi_t$	-4.227173	0.0008

Table 6: ADF Tests on the Three Series

Exercise 5

We download the monthly short-term rates (3-month Treasury bills (3TB)) and long-term rates (10-year Treasury bonds (10TB)) from FRED website. Here are links for each series:

http://research.stlouisfed.org/fred2/series/TB3MS
http://research.stlouisfed.org/fred2/series/GS10

The data sets range from 1953M04 to 2013M09. In Figure 9 we plot both series. Observe that these two series tend to move together. We proceed to test the cointegration relation. First, we test for unit root in the two series using ADF statistics. We include an intercept in the test equations (Case II). The ADF statistics and corresponding p-value are -2.2558 and 0.1869 for 3-month Treasury bills, and -1.2893 and 0.6362 for 10-year Treasury bonds. Therefore, the hypothesis of unit root cannot be rejected for both series at the 5% significance level and both, the short-and long-term rates, are non-stationary. The spread is constructed as the difference between long-term and short-term rates: $spread = 10TB_t - 3TB_t$, which we plot in Figure 10. Since we impose the cointegrating relation, we test for the stationarity of the spread using the ADF critical values directly. With an intercept in the ADF equation (Case II), the value of the ADF statistic is -4.7182 with p-value of 0.0001. Therefore, the hypothesis of unit root is rejected for the spread series at 5% significance level and we conclude that the spread is stationary and it is the cointegrating relation between short and long term interest rates.

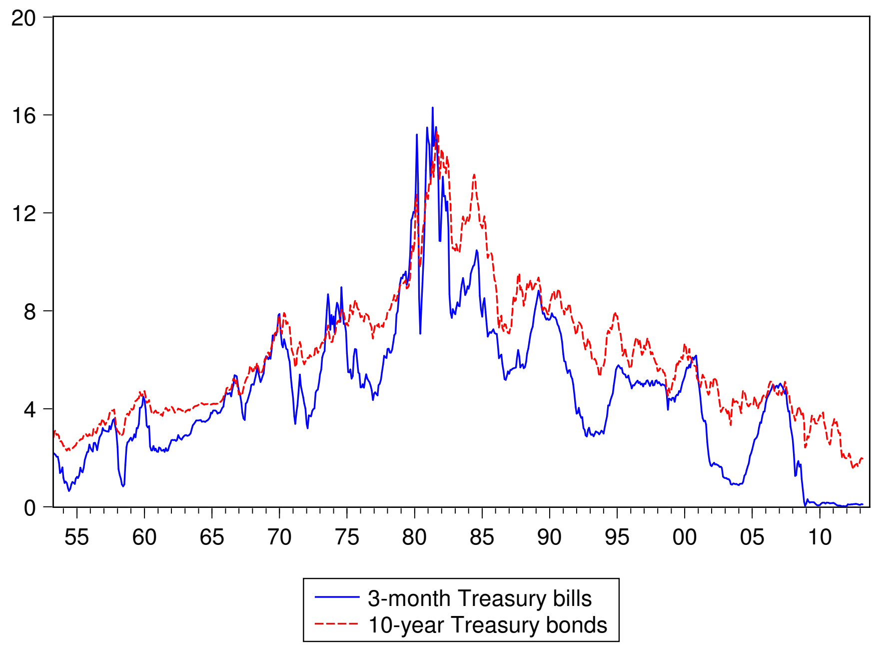


Figure 9: Time Series of 3-month Treasury bills and 10-year Treasury bonds

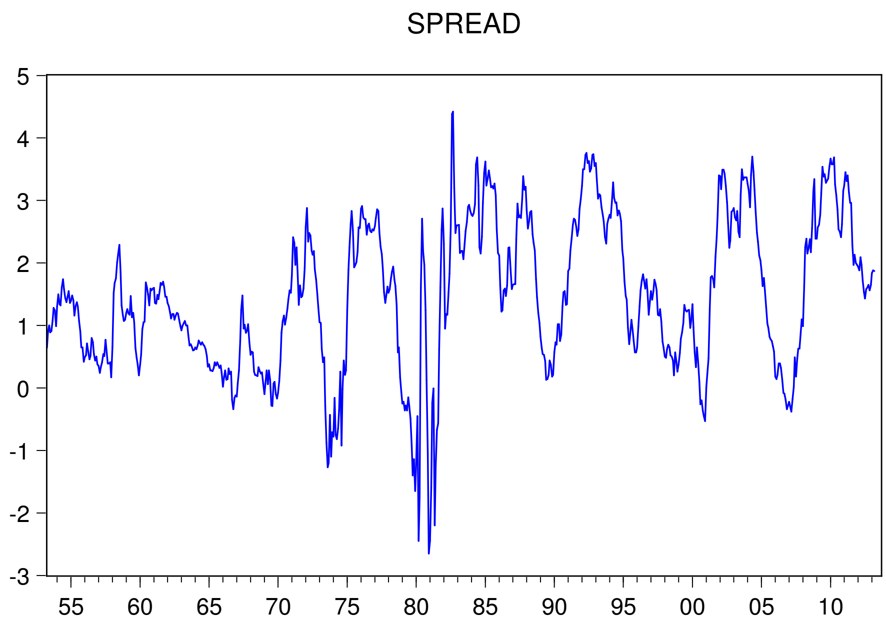


Figure 10: Time Series of Spread

Exercise 6
Since the series are cointegrated, we proceed to estimate a vector error correction model. The estimation results are reported in Table 7. Observe the strong statistical significance of the error correction terms as well as the importance of past changes in interest rates. Based on this VEC model, we construct the multistep forecasts for the short term rate and long term rates, which is reported in Table 8.

Vector Error Correction Estimates		
Sample (adjusted): 1953M07 2013M03		
Included observations: 717 after adjustments		
Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
TB10(-1)	1.000000	
TB3(-1)	-1.01937	
	(0.07784)	
	[-13.0957]	
C	-1.36364	
Error Correction:	D(TB10)	D(TB3)
CointEq1	-0.01994	0.028399
	(0.00809)	(0.01237)
	[-2.46533]	[2.29649]
D(TB10(-1))	0.391977	0.33364
	(0.04381)	(0.06696)
	[8.94808]	[4.98255]
D(TB10(-2))	-0.26736	-0.20039
	-(0.04437)	(0.06783)
	[-6.02535]	[-2.95428]
D(TB3(-1))	-0.02504	0.305485
	(0.02877)	(0.04398)
	[-0.87013]	[6.94559]
D(TB3(-2))	0.049867	-0.10931
	(0.02875)	(0.04395)
	[1.73427]	[-2.48704]
C	-0.00129	-0.00202
	(0.00951)	(0.01454)
	[-0.13528]	[-0.13910]
R-squared	0.149672	0.190326
Adj. R-squared	0.143692	0.184632
Sum sq. resids	46.09972	107.7184
S.E. equation	0.254633	0.389233
F-statistic	25.02963	33.42616
Log likelihood	-33.5585	-337.822
Akaike AIC	0.110344	0.959058
Schwarz SC	0.14863	0.997343
Mean dependent	-0.0016	-0.00282
S.D. dependent	0.275169	0.431055
Determinant resid covariance (dof adj.)		0.006679
Determinant resid covariance		0.006568
Log likelihood		-233.096
Akaike information criterion		0.689249
Schwarz criterion		0.778581

Table 7: Estimation of the VEC Model

Forecast Horizon h	$f_{t,h}^{Short}$	$f_{t,h}^{Long}$
h=1, 2013.M4	0.098283	1.928246
h=2, 2013.M5	0.123093	1.911224
h=3, 2013.M6	0.144571	1.911280
h=4, 2013.M7	0.163826	1.905903
h=5, 2013.M8	0.174682	1.889032
h=6, 2013.M9	0.160632	1.871077

Table 8: Multistep Forecast for Short- and Long-Term Rates

Exercise 7

We update the time series of the quarterly house price index for the two MSAs (1975Q1–2012Q4) used in Chapter 11: Los Angeles MSA (Los Angeles-Long Beach-Santa Ana) and Riverside MSA (Riverside-San Bernardino-Ontario) in California from the Freddie Mac website:

http://www.freddiemac.com/finance/fmhpi/

Let `ILA` and `IRV` denote Los Angeles and Riverside house price index series respectively. In Figure 11, we plot the time series.

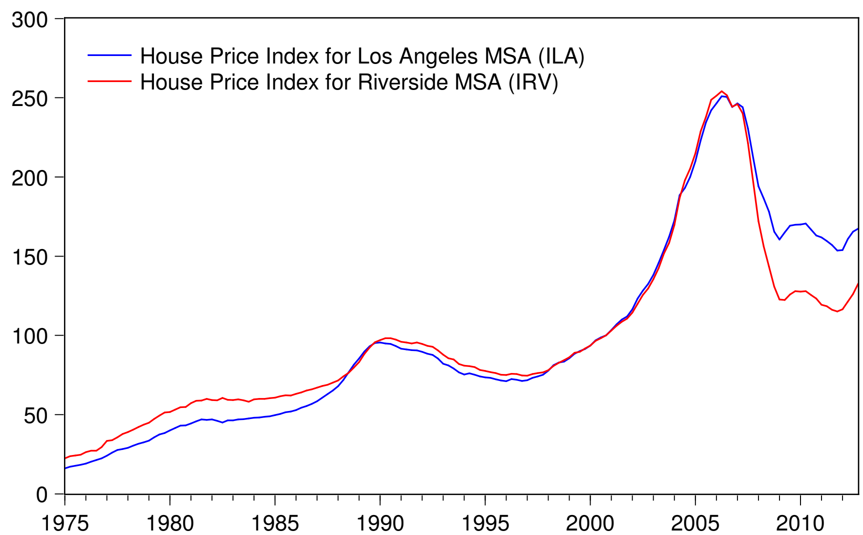


Figure 11: Time Series Plot for L.A. and Riverside House Price Index (`ILA`, `IRV`)

We investigate whether there is a long-term equilibrium for house prices in both locations. First, we test for unit root in the two series using ADF statistics. We include an intercept in the ADF equations (Case II). The ADF statistics and corresponding p-value are -1.671915 and 0.4434 for `ILA`, and -2.360857 and 0.1547 for `IRV`. Therefore, we fail to reject the null hypothesis of unit root at 5% significance level. Second, we estimate the regression model

$$IRV_t = \alpha_0 + \alpha_1 ILA_t + z_t,$$

by OLS to find a potential long-term equilibrium relationship between the two series. Third, compute the residual of the OLS regression, that is, $\widehat{z}_t = IRV_t - \widehat{\alpha}_0 - \widehat{\alpha}_1 ILA_t$, where $\widehat{\alpha}_0$ and $\widehat{\alpha}_1$ are 13.5977 and 0.852673 reported in Table 9. In Figure 11 we plot the residual series, which exhibits strong persistence.

Dependent Variable: IRV				
Method: Least Squares				
Sample: 1975Q1 2012Q4				
Included observations: 152				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	13.5977	1.948322	6.979188	0.0000
ILA	0.852673	0.016858	50.57877	0.0000
R-squared	0.944613	Mean dependent var		97.11954
Adjusted R-squared	0.944244	S.D. dependent var		53.98616
S.E. of regression	12.74763	Akaike info criterion		7.941639
Sum squared resid	24375.32	Schwarz criterion		7.981427
Log likelihood	-601.5646	Hannan-Quinn criter.		7.957802
F-statistic	2558.212	Durbin-Watson stat		0.025837
Prob(F-statistic)	0.000000			

Table 9: Estimation of the Cointegration Relation

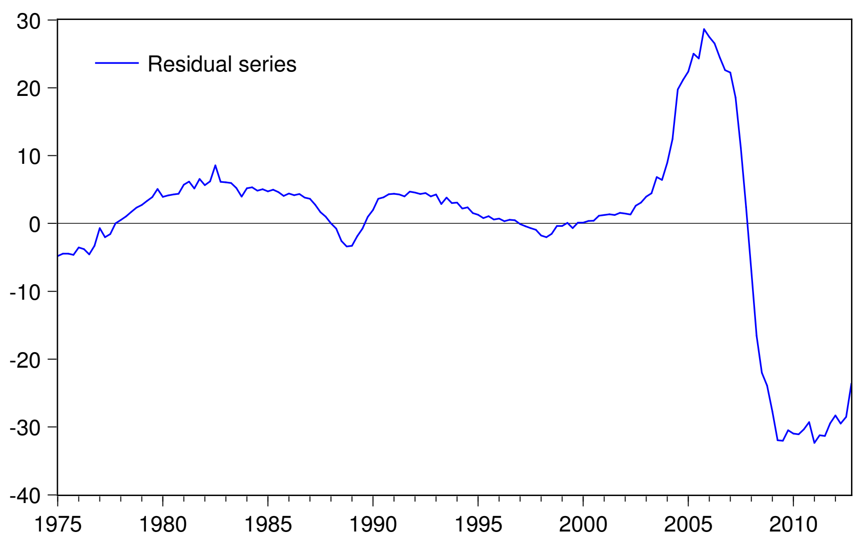


Figure 12: Time Series of Residuals

Finally, we run an ADF test on $\widehat{z}_t$, and use 5% critical values in Table 12.2 (on page 320 of the textbook) to determine whether the residual series has a unit root. The ADF statistic for the residuals is -3.065946, which is *larger* than 5% critical value -3.3982 for a sample size of 100, and we fail to reject the null hypothesis that the residual series has a unit root. Thus, we conclude that no cointegration relation exists between Los Angeles and Riverside house prices and the model estimated in Chapter 11 would be the correct approach to forecast these prices.

Exercise 8

We download the daily stock prices of Banco Santander (SAN), which is traded in the United States (New York Stock Exchange, ADR) from the Yahoo Finance website, and Spain (Madrid Stock Exchange). Also, we download the daily US \$/Euro exchange rate to convert the Euro stock price to dollar value. Here are links for SAN in the Madrid market and the euro exchange rate:

Stock Price in Madrid Stock Exchange:
http://www.tr4der.com/download/historical-prices/SAN.MC/
US \$/Euro exchange rate:
http://research.stlouisfed.org/fred2/series/DEXUSEU

The data sets range from 01/02/2003 to 2/28/2013. Due to different holiday days in the US and Europe, the trading days in the United States and Spain do not match perfectly. We overcome this problem by imputing missing values with yesterday's values. Then, we multiply the stock prices in Spain by the US \$/Euro exchange rate to convert the prices into dollars. We expect that the two stock prices should be very similar because otherwise the investors will find arbitrage opportunities.

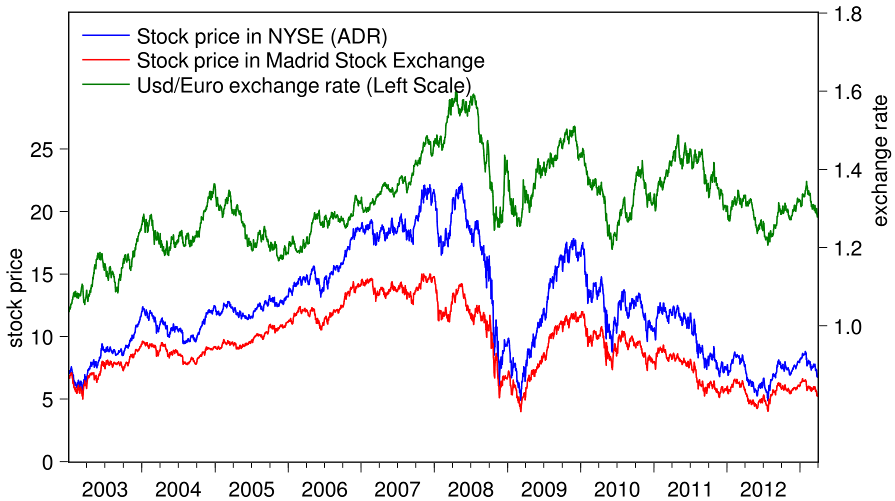
In Figure 13a we plot the three series together, and in Figure 13b we plot both stock prices in NYSE and in Madrid Stock Exchange in dollars. Note that after the conversion to dollars, the two stock prices in the United States and Spain are almost identical, which confirms our initial expectation and also suggests the existence of a cointegrating relation.

We proceed to test for cointegration. First, we test for unit root in the two series using ADF statistics. We include an intercept in the ADF equations (Case II). The ADF statistics and corresponding p-values are -1.607177 and 0.4787 for the ADR price in NYSE, and -1.659348 and 0.4520 for the price in Madrid Stock Exchange. Therefore, we fail to reject the hypothesis of a unit root at the 5% significance level. Second, we estimate the regression model

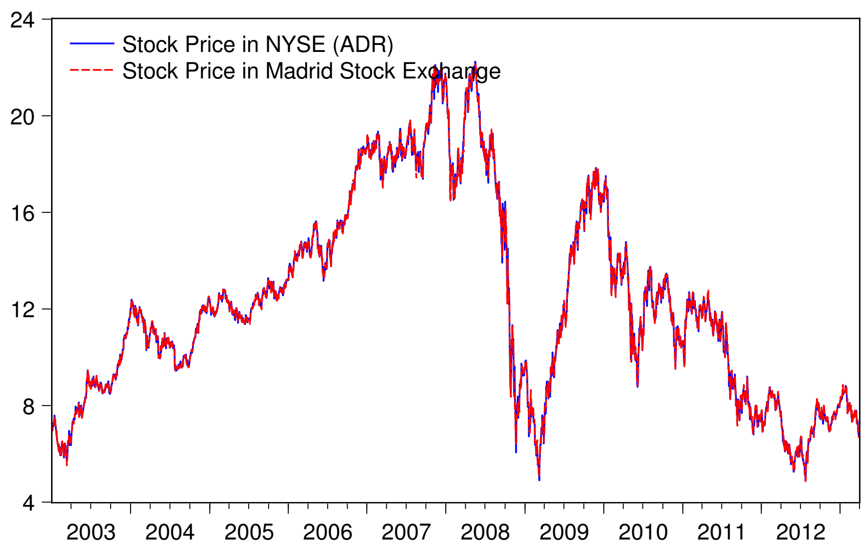
$$ADR_t = \alpha_0 + \alpha_1 SANEX_t + z_t,$$

by OLS to find a potential long-term equilibrium between the two series, where ADR_t and $SANEX_t$ stand for the stock price in the United States and Spain respectively. Third, compute the residuals of the OLS regression, that is, $\widehat{z}_t = ADR_t - \widehat{\alpha}_0 - \widehat{\alpha}_1 SANEX_t$, where $\widehat{\alpha}_0$ and $\widehat{\alpha}_1$ are -0.012396 and 1.001086 reported in Table 9. In Figure 11, we plot the residual series.

Finally, we run an ADF test on $\widehat{z}_t$, and use the 5% critical values in Table 12.2 (page 320 of the textbook) to determine whether the residuals are stationary. The ADF statistic for the residuals is -50.00975, which is much *smaller* than the 5% critical value -3.3496 with a sample size of 500, and we reject the null hypothesis that the residual series has a unit root. Thus, we conclude that a cointegration relation does exist between the stock prices in NYSE and Madrid Stock Exchange. We estimate a VEC model and the estimation results are reported in Table 11. Note that the error correction terms, which is "CointEq1" in the table, are very statistically significant in both equations with t-statistics 10.7790 and -3.41399. It shows that the stock prices in both places do adjust to absorb any disequilibrium error z_t, so arbitrage opportunities disappear rather quickly. Other than the error correction terms, the remaining regressors are not very significant and both R-squared are very low, though 11% R-squared for SANEX should not be underestimated. We do not expect great forecasting ability from this model but given the significance of the error correction terms, we may find a bit of help in forecasting prices in the very short term. As in the previous exercises, we can construct the forecasts based on VEC. We leave it as an exercise for the student.



(a) Stock Prices in Both Markets and US $/Euro Exchange Rate



(b) Stock Prices in Both Markets in Dollar

Figure 13: Time Series of Stock Prices and US $/Euro Exchange Rate

Dependent Variable: ADR				
Method: Least Squares				
Sample (adjusted): 1/02/2003 2/28/2013				
Included observations: 2651 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.012396	0.008765	-1.414263	0.1574
SANEX	1.001086	0.000672	1490.482	0.0000
R-squared	0.998809	Mean dependent var		12.38135
Adjusted R-squared	0.998809	S.D. dependent var		4.134397
S.E. of regression	0.142708	Akaike info criterion		-1.055273
Sum squared resid	53.9487	Schwarz criterion		-1.050835
Log likelihood	1400.764	Hannan-Quinn criter.		-1.053666
F-statistic	2221537	Durbin-Watson stat		1.942466
Prob(F-statistic)	0.000000			

Table 10: Estimation of the Cointegration Relation

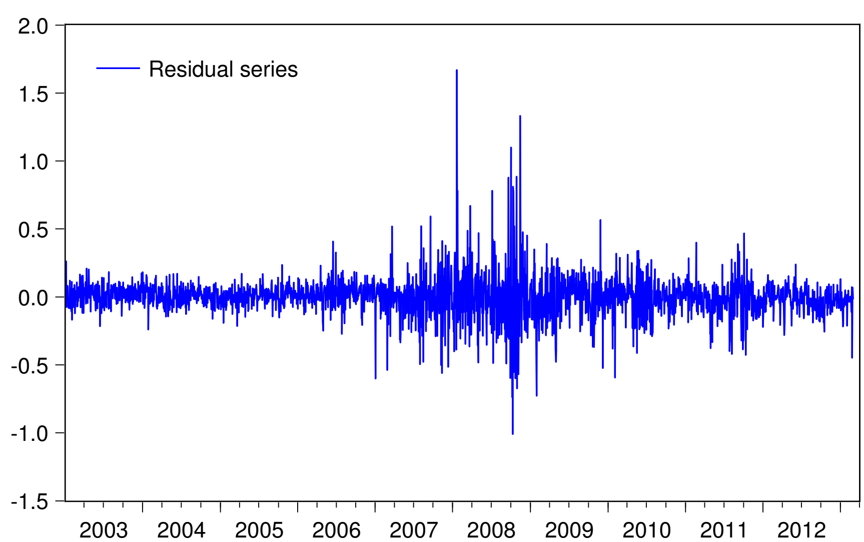


Figure 14: Time Series of Residuals

Vector Error Correction Estimates		
Sample (adjusted): 1/07/2003 3/29/2013		
Included observations: 2669 after adjustments		
Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
ADR(-1)	1.000000	
SANEX(-1)	-1.001729	
	(0.00072)	
	[-1392.29]	
C	0.020149	
Error Correction:	D(ADR)	D(SANEX)
CointEq1	-0.22585	0.641749
	(0.06615)	(0.05954)
	[-3.41399]	[10.7790]
D(ADR(-1))	-0.110087	0.009371
	(0.05688)	(0.05119)
	[-1.93534]	[0.18306]
D(ADR(-2))	-0.072631	-0.026923
	(0.04128)	(0.03715)
	[-1.75951]	[-0.72472]
D(SANEX(-1))	0.078041	0.009271
	(0.05591)	(0.05032)
	[1.39573]	[0.18424]
D(SANEX(-2))	0.024355	0.030414
	(0.03947)	(0.03552)
	[0.61701]	[0.85616]
C	-0.000175	-0.000101
	(0.0055)	(0.00495)
	[-0.03183]	[-0.02046]
R-squared	0.028425	0.116185
Adj. R-squared	0.026601	0.114526
Sum sq. resids	214.9308	174.082
S.E. equation	0.284095	0.255677
F-statistic	15.58201	70.015
Log likelihood	-425.3506	-144.0519
Akaike AIC	0.32323	0.112441
Schwarz SC	0.33647	0.12568
Mean dependent	-0.000165	-0.000104
S.D. dependent	0.287951	0.271709
Determinant resid covariance (dof adj.)		0.001292
Determinant resid covariance		0.001286
Log likelihood		1308.337
Akaike information criterion		-0.969904
Schwarz criterion		-0.939011

Table 11: Estimation of the VEC Model

Exercise 9

We download the quarterly rent index and the owner's equivalent rent of primary residence (OER) and housing price index in Dallas-Fort Worth, TX from the FRED® website:

http://research.stlouisfed.org/fred2/series/CUURA316SEHA
http://research.stlouisfed.org/fred2/series/CUURA316SEHC01
http://research.stlouisfed.org/fred2/series/DAXRSA

The data sets range from 1986Q4 to 2012Q4. In Figure 15 we plot the three series together. The three series exhibit a clear upward trend. We test for unit root including an intercept and time trend in the ADF equations (Case III). The ADF statistics and corresponding p-values are -3.548943 and 0.0396 for rent series, -1.744979 and 0.7239 for owner's equivalent rent series, -2.595938 and 0.2831 for house price index. Therefore, at 5% significance level, we marginally reject the null hypothesis of unit root for the rent index, and fail to reject the unit root for both owner's equivalent rent and house price index.

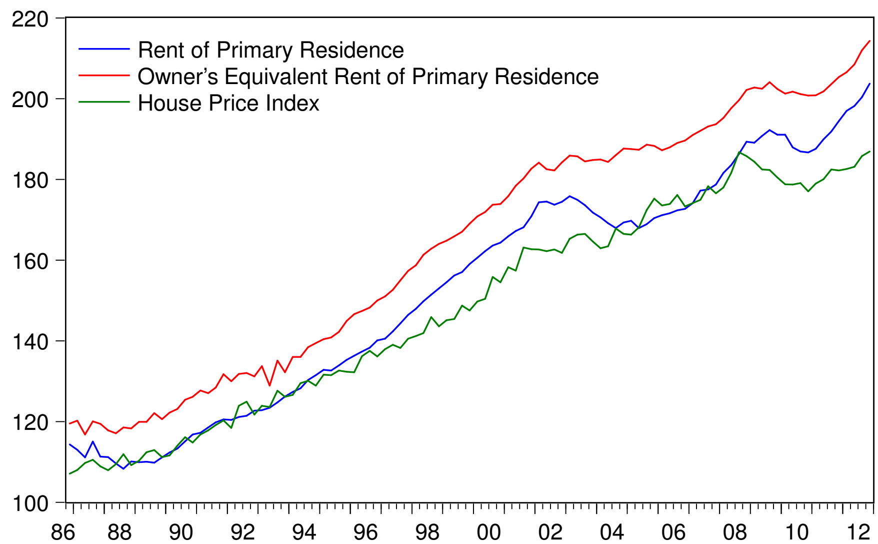


Figure 15: Time Series of Rents and House Price Indices in Dallas-Fort Worth, TX

We proceed by testing for cointegration between the two non-stationary series and estimate the regression model by OLS

$$EQRENT_t = \alpha_0 + \alpha_1 PRICE_t + z_t$$

where $EQRENT_t$ and $PRICE_t$ stand for the owner's equivalent rent of primary residence and house price index in Dallas-Fort Worth, TX respectively. We calculate the residual of the OLS regression, that is $\widehat{z}_t = EQRENT_t - \widehat{\alpha}_0 - \widehat{\alpha}_1 PRICE_t$, where $\widehat{\alpha}_0$ and $\widehat{\alpha}_1$ are -9.737565 and 1.169781 reported in Table 12. In Figure 16 we plot the residual series. Finally, we run an ADF test on $\widehat{z}_t$ and use 5% critical values in Table 12.2 (on page 320 of the textbook) to determine whether the residual series has a unit root. The ADF statistic for the residuals is -3.566577, which is *smaller* than the 5% critical values -3.3982 with a sample size of 100, and we reject the null hypothesis that the residual series has a unit root. Thus, we conclude that a cointegrating relation does exist between the owner's equivalent rent of primary residence and the house price index in Dallas-Fort Worth, TX.

Dependent Variable: EQRENT				
Method: Least Squares				
Sample: 1986Q4 2012Q4				
Included observations: 105				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-9.737565	1.924448	-5.059925	0.0000
PRICE	1.169781	0.012765	91.63619	0.0000
R-squared	0.987883	Mean dependent var		163.9927
Adjusted R-squared	0.987765	S.D. dependent var		30.61022
S.E. of regression	3.385859	Akaike info criterion		5.295956
Sum squared resid	1180.796	Schwarz criterion		5.346508
Log likelihood	-276.0377	Hannan-Quinn criter.		5.316441
F-statistic	8397.192	Durbin-Watson stat		0.451735
Prob(F-statistic)	0.000000			

Table 12: Estimation of the Cointegration Relation

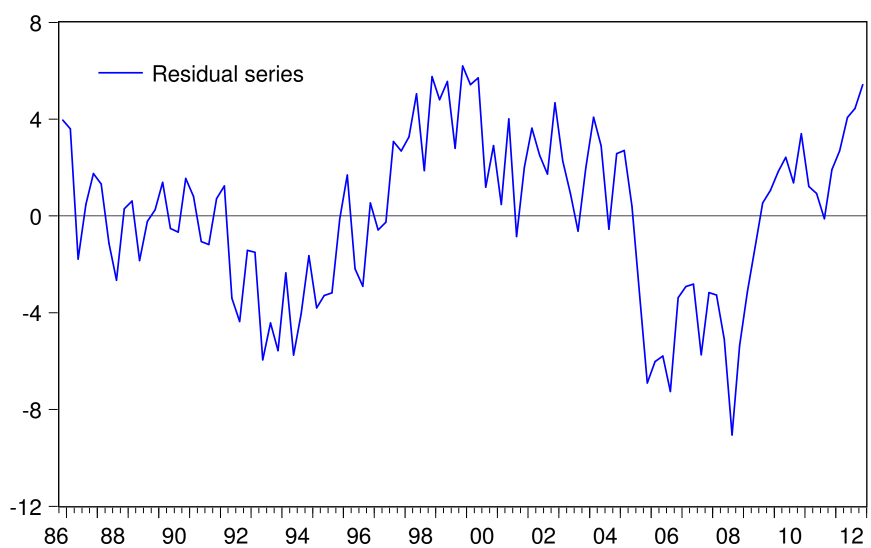


Figure 16: Time Series of Residuals

In Table 13 we report the estimation results of a VEC model for the two series. Note that the error correction term, which is "CointEq1" in the table, is statistically significant only in the price equation with a t-statistic 2.13375. Thus, rents are helpful to forecast house prices.

Vector Error Correction Estimates		
Sample (adjusted): 1987Q3 2012Q4		
Included observations: 102 after adjustments		
Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
EQRENT(-1)	1.000000	
PRICE(-1)	-1.206959	
	(0.05468)	
	[-22.0727]	
C	15.40216	
Error Correction:	D(EQRENT)	D(PRICE)
CointEq1	0.011678	0.13139
	(0.04415)	(0.06158)
	[0.26451]	[2.13375]
D(EQRENT(-1))	-0.240642	-0.051303
	(0.10423)	(0.14537)
	[-2.30871]	[-0.35290]
D(EQRENT(-2))	0.229538	0.14269
	(0.10271)	(0.14325)
	[2.23489]	[0.99612]
D(PRICE(-1))	0.104484	-0.139784
	(0.08188)	(0.1142)
	[1.27605]	[-1.22403]
D(PRICE(-2))	0.11502	-0.100736
	(0.07901)	(0.11019)
	[1.45581]	[-0.91418]
C	0.806791	0.859942
	(0.19391)	(0.27045)
	[4.16070]	[3.17972]
R-squared	0.17517	0.111688
Adj. R-squared	0.13221	0.065422
Sum sq. resids	186.7231	363.2186
S.E. equation	1.394644	1.945129
F-statistic	4.077534	2.414025
Log likelihood	-175.5691	-209.5034
Akaike AIC	3.560178	4.225556
Schwarz SC	3.714588	4.379966
Mean dependent	0.955853	0.756716
S.D. dependent	1.497119	2.012058
Determinant resid covariance (dof adj.)		5.918611
Determinant resid covariance		5.242783
Log likelihood		-373.9629
Akaike information criterion		7.607116
Schwarz criterion		7.967407

Table 13: Estimation of the VEC Model

Exercise 10
We download the adjusted close prices[4] and dividends of Alcoa stock from Yahoo Finance website.

Note that the original time series has daily frequency. However, dividends are paid quarterly. Therefore, we convert the price frequency from daily to quarterly. For the stock prices, we take the last observation (adjusted close price) in each quarter; and for the dividends, we take the sum of all dividends paid in each quarter. After the conversion, the two series range from 1986Q4 to 2012Q4. In addition, the values of stock prices and dividends are very different in magnitude, so we index both series by taking the stock price in 2007Q2 (\$36.61) and dividend in 2006Q4 (\$0.30) as 100. In Figure 17, we plot the two index series.

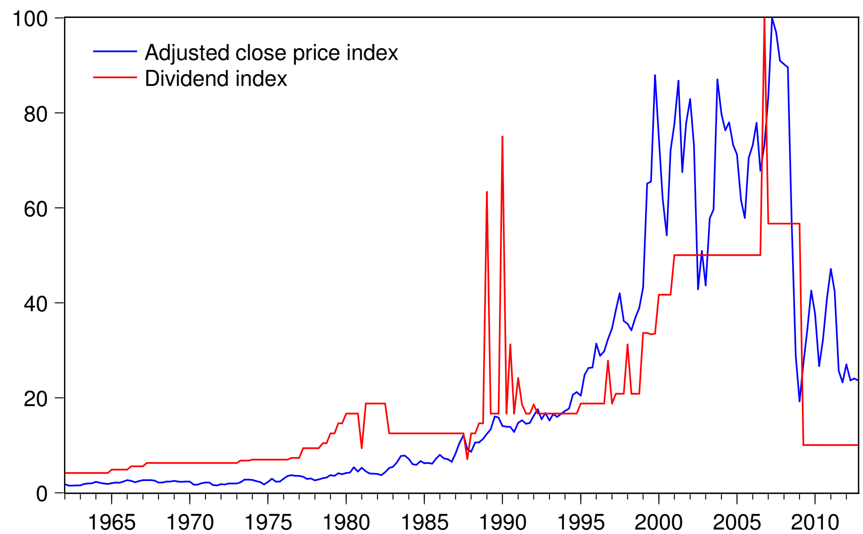


Figure 17: Time Series of Adjusted Close Price and Dividend Indices

We test for unit root in the two series using ADF statistics. The ADF statistics and corresponding p-values are -1.491469 and 0.5361 for stock price index, -1.570210 and 0.4960 for dividend index. Therefore, at 5% significance level, we fail to reject the null hypotheses of unit root for both stock price and dividend indices. Next, we estimate the following regression model by OLS

$$PRICE_t = \alpha_0 + \alpha_1 DIVID_t + z_t$$

where $PRICE_t$ and $DIVID_t$ stand for stock price index and dividend index for Alcoa Incorporation respectively. We calculate the residuals from the OLS regression, that is $\widehat{z}_t = PRICE_t - \widehat{\alpha}_0 - \widehat{\alpha}_1 DIVID_t$, where $\widehat{\alpha}_0$ and $\widehat{\alpha}_1$ are -2.297702 and 1.258916 reported in Table 14. We plot the residuals series in Figure 18. We run an ADF test on $\widehat{z}_t$ and use 5% critical values in Table 12.2 (on page 320 of the textbook) to determine whether the residual has a unit root. The ADF statistic for the residuals is -3.587239, which is *smaller* than 5% critical values -3.3617 with sample size 250, so we reject the null hypothesis that the residual series has a unit root. Thus, we conclude that a cointegrating relation does exist between the two series. In Table 13 we report the estimation

[4] It is the close price adjusted for stock dividends and splits.

results of a VEC model for the two series. Note that the error correction terms, which is "CointEq1" in the table, are statistically significant in both equations, stock price and dividend indices, with t-statistics -2.15956 and 3.07891 respectively. Thus, stock prices help to forecast dividends and, in a smaller measure, dividends may also have some ability to forecast prices in the short run.

Dependent Variable: PRICE				
Method: Least Squares				
Sample: 1962Q1 2012Q4				
Included observations: 204				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.297702	1.589651	-1.445413	0.1499
DIVID	1.258916	0.059795	21.05378	0.0000
R-squared	0.686949	Mean dependent var		22.73099
Adjusted R-squared	0.685399	S.D. dependent var		26.87379
S.E. of regression	15.07332	Akaike info criterion		8.273486
Sum squared resid	45895.43	Schwarz criterion		8.306016
Log likelihood	-841.8956	Hannan-Quinn criter.		8.286645
F-statistic	443.2616	Durbin-Watson stat		0.882806
Prob(F-statistic)	0.000000			

Table 14: Estimation of the Cointegration Relation

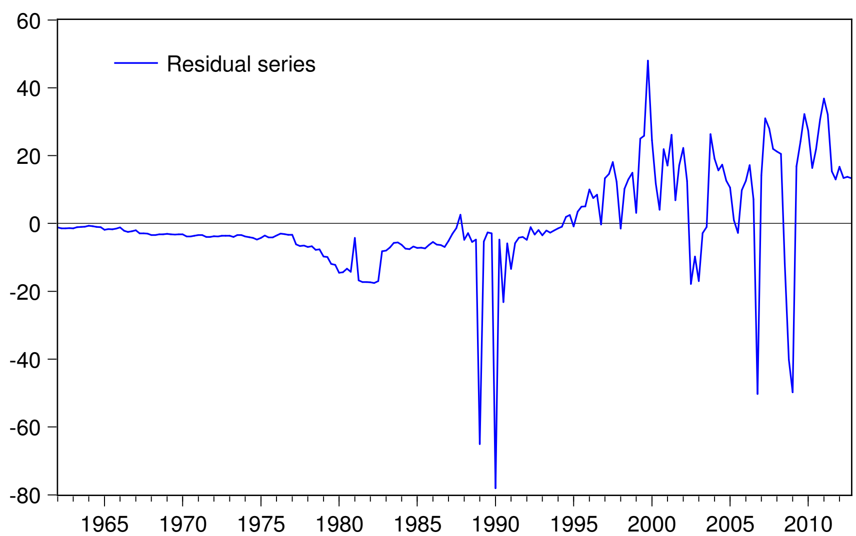


Figure 18: Time Series of Residuals

Vector Error Correction Estimates		
Sample (adjusted): 1963Q2 2012Q4		
Included observations: 199 after adjustments		
Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
PRICE(-1)	1.000000	
DIVID(-1)	-1.605488	
	(0.165)	
	[-9.73022]	
C	9.354094	
Error Correction:	D(PRICE)	D(DIVID)
CointEq1	-0.088129	0.154678
	(0.04081)	(0.05024)
	[-2.15956]	[3.07891]
D(PRICE(-1))	0.234115	-0.121275
	(0.07768)	(0.09562)
	[3.01397]	[-1.26824]
D(PRICE(-2))	0.082152	0.073468
	(0.07275)	(0.08956)
	[1.12920]	[0.82030]
D(PRICE(-3))	-0.285134	0.205939
	(0.07313)	(0.09003)
	[-3.89876]	[2.28737]
D(PRICE(-4))	0.187394	0.162629
	(0.07717)	(0.095)
	[2.42827]	[1.71183]
D(DIVID(-1))	-0.07766	-0.466045
	(0.07644)	(0.09411)
	[-1.01589]	[-4.95224]
D(DIVID(-2))	-0.013118	-0.239891
	(0.07325)	(0.09018)
	[-0.17908]	[-2.66017]
D(DIVID(-3))	0.015564	-0.176576
	(0.06556)	(0.0807)
	[0.23741]	[-2.18798]
D(DIVID(-4))	-0.003094	0.160621
	(0.05463)	(0.06725)
	[-0.05664]	[2.38825]
C	0.094099	0.012061
	(0.42997)	(0.52931)
	[0.21885]	[0.02279]
		(continued)

R-squared	0.167562	0.457498
Adj. R-squared	0.127922	0.431665
Sum sq. resids	6941.402	10519.72
S.E. equation	6.06028	7.460555
F-statistic	4.227096	17.70954
Log likelihood	-635.7882	-677.1551
Akaike AIC	6.490334	6.906081
Schwarz SC	6.655827	7.071574
Mean dependent	0.111044	0.029313
S.D. dependent	6.489556	9.896207
Determinant resid covariance (dof adj.)		2036.838
Determinant resid covariance		1837.274
Log likelihood		-1312.583
Akaike information criterion		13.4129
Schwarz criterion		13.77698

Table 15: Estimation of the VEC Model