# CHAPTER 5.

# UNDERSTANDING LINEAR DEPENDENCE: A LINK TO ECONOMIC MODELS

## SOLUTIONS

**by**
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**Exercise 1**
We simulate 1000 observations of the process $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ and plot 100 observations. Compare Figures 1, 2, and 3 with Figures 5.2 and 5.3 in the textbook. The price oscillates around an average price of \$14.3 in all these graphs. The time series of the simulated prices in Figures 1, 2 and 3 exhibit smooth dynamics similar to those of the time series in Figure 5.3, in contrast to the zig-zag behavior of the simulated price in Figure 5.2. This is due to the sign of the autoregressive parameter, which is positive, i.e. $\phi = 0.55$. When the variance of the error term $\varepsilon_t$ increases, the time series become noisier and more volatile, so that it tends to 'hide' the time dependence. However, the autocorrelation functions in the three Figures 4, 5 and 6 deliver the same message. The profile of the three ACF and PACFs is the same: a smooth decay of the autocorrelations towards zero in the ACFs, and only a significant spike, partial autocorrelation of order one, in the PACFs. Different variances in the error term do not affect the autocorrelation functions because the effect of the error variance in the numerator and denominator of the autocorrelation coefficients cancel each other out. Observe that these autocorrelation functions are similar to that in Figure 5.3 of the textbook. The main difference with the time series and the ACF and PCF in Figure 5.2 of the textbook is the sign of the autoregressive parameter. In Figure 5.2, the sign is negative ($\phi = -0.6$), which produces the ziz-zag behavior of the time series and the alternating signs of the autocorrelation coefficients.

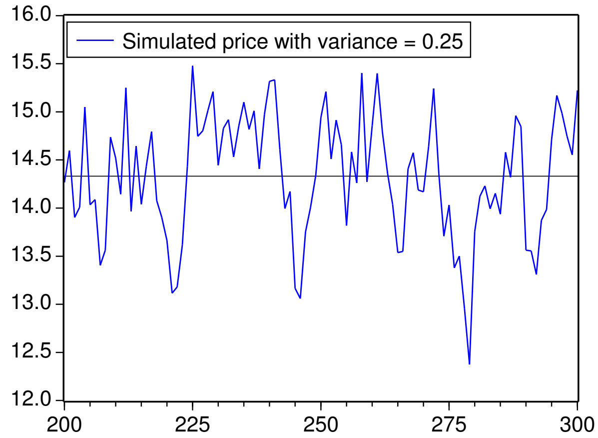



Figure 1: Simulated price $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ with $\sigma^2_\varepsilon = 0.25$

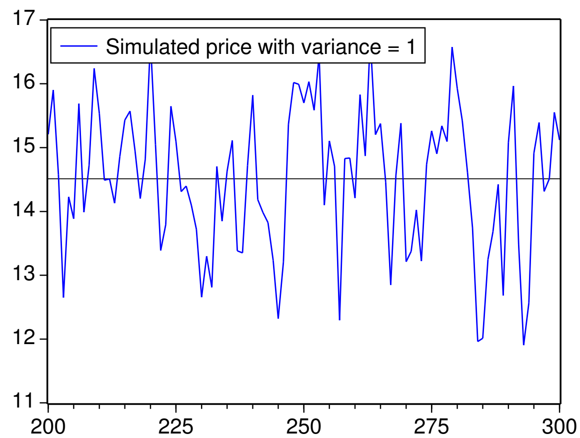



Figure 2: Simulated price $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ with $\sigma^2_\varepsilon = 1$

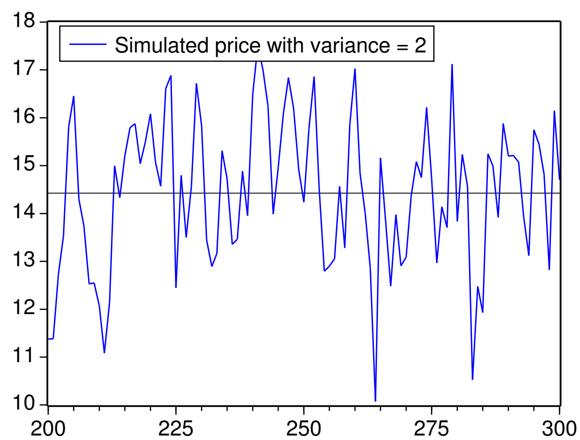



Figure 3: Simulated price $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ with $\sigma^2_\varepsilon = 2$

Sample: 2 1000
Included observations: 999

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.542 | 0.542 | 294.16 | 0.000 |
| | | 2 | 0.262 | -0.045 | 362.76 | 0.000 |
| | | 3 | 0.125 | 0.003 | 378.56 | 0.000 |
| | | 4 | 0.058 | -0.003 | 381.94 | 0.000 |
| | | 5 | 0.054 | 0.040 | 384.93 | 0.000 |
| | | 6 | 0.019 | -0.035 | 385.29 | 0.000 |
| | | 7 | 0.045 | 0.061 | 387.32 | 0.000 |
| | | 8 | 0.073 | 0.038 | 392.65 | 0.000 |

Figure 4: ACF and PACF of $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ with $\sigma_\varepsilon^2 = 0.25$

Sample: 2 1000
Included observations: 999

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.540 | 0.540 | 292.56 | 0.000 |
| | | 2 | 0.279 | -0.019 | 370.51 | 0.000 |
| | | 3 | 0.152 | 0.012 | 393.75 | 0.000 |
| | | 4 | 0.046 | -0.053 | 395.87 | 0.000 |
| | | 5 | -0.013 | -0.024 | 396.04 | 0.000 |
| | | 6 | -0.045 | -0.027 | 398.09 | 0.000 |
| | | 7 | -0.050 | -0.006 | 400.57 | 0.000 |
| | | 8 | -0.037 | 0.003 | 401.97 | 0.000 |

Figure 5: ACF and PACF of $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ with $\sigma_\varepsilon^2 = 1$

Sample: 2 1000
Included observations: 999

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.530 | 0.530 | 281.16 | 0.000 |
| | | 2 | 0.267 | -0.018 | 352.89 | 0.000 |
| | | 3 | 0.139 | 0.006 | 372.27 | 0.000 |
| | | 4 | 0.079 | 0.009 | 378.49 | 0.000 |
| | | 5 | 0.013 | -0.044 | 378.65 | 0.000 |
| | | 6 | -0.032 | -0.032 | 379.72 | 0.000 |
| | | 7 | -0.032 | 0.008 | 380.73 | 0.000 |
| | | 8 | 0.008 | 0.044 | 380.80 | 0.000 |

Figure 6: ACF and PACF of $p_t = 6.43 + 0.55p_{t-1} + \varepsilon_t$ with $\sigma_\varepsilon^2 = 2$

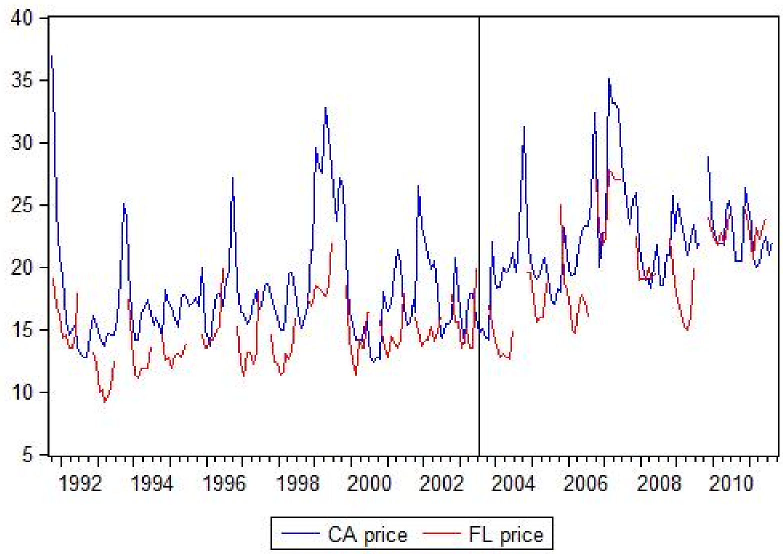



Figure 7: Time Series of Orange Prices in Florida (red) and in California (blue)

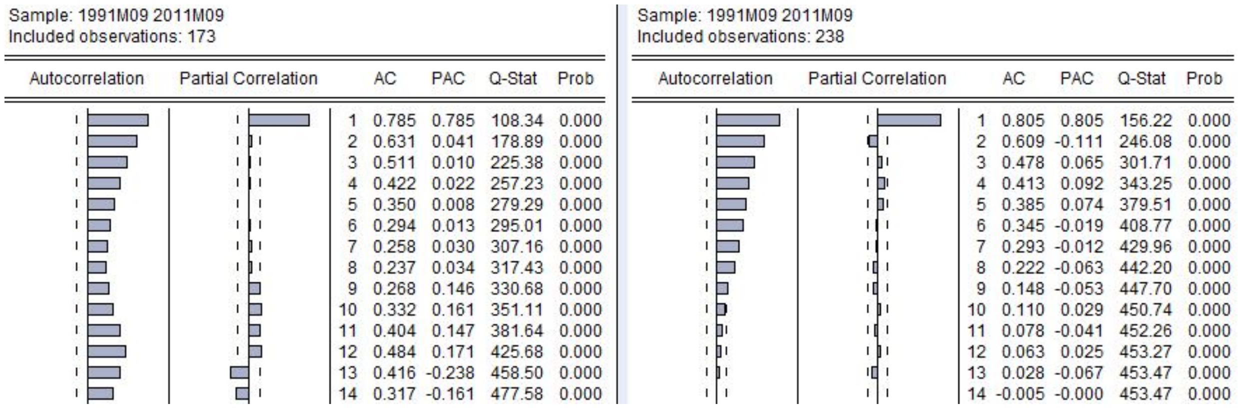
**Exercise 2**
We download agricultural prices from the USDA website:

http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1031

The most recent report on monthly producer prices for oranges is dated 2011. The time series run from September 1991 to September 2011. In Figure 7 we plot the time series of prices for Florida and California. The vertical line on June 2003 separates the sample considered in the textbook from the updated sample. Overall, there seems to be a slight trend in both series; this trend was not obvious with the data ending on June 2003 so that we may suspect a non-stationary behavior in the overall updated series. This trend may be justified either because less land is allocated to this crop, hence reducing the supply, or/and there is more demand (population growth, preferences, substitution effects, etc.). Prices in Florida are lower than those in California, which may be due to supply/demand effects or/and different qualities and different markets. In Florida, the production cycle goes from November to June so there are not prices in the off-season of summer and early fall. On the contrary, California has a continuous cycle with prices peaking up during the off-season in Florida.

In Figure 8, we present the autocorrelation functions of both series. Their profiles are very similar: the ACF decays slowly towards zero, and the PACF shows one very large and significant spike, a partial autocorrelation of order one with a value of approximately 0.80. In the PACF of the California time series, we also observe a second marginally significant spike. Overall, these functions are similar to that in Figure 5.3 of the textbook and, as a first step, we could entertain a model like $p_t = c + \phi p_{t-1} + \varepsilon_t$. The most distinctive feature is in the ACF and PACF of the Florida time series: there are significant spikes for displacements 11, 12, 13, 14. This is an indication of a seasonal cycle (dependence between observations that are 12 months apart, this is to say, December to December prices, January to January prices, etc.), which is very obvious in the Florida series. We do not find such a seasonality in the California series.



Sample: 1991M09 2011M09
Included observations: 173

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.785 | 0.785 | 108.34 | 0.000 |
| 2 | 0.631 | 0.041 | 178.89 | 0.000 |
| 3 | 0.511 | 0.010 | 225.38 | 0.000 |
| 4 | 0.422 | 0.022 | 257.23 | 0.000 |
| 5 | 0.350 | 0.008 | 279.29 | 0.000 |
| 6 | 0.294 | 0.013 | 295.01 | 0.000 |
| 7 | 0.258 | 0.030 | 307.16 | 0.000 |
| 8 | 0.237 | 0.034 | 317.43 | 0.000 |
| 9 | 0.268 | 0.146 | 330.68 | 0.000 |
| 10 | 0.332 | 0.161 | 351.11 | 0.000 |
| 11 | 0.404 | 0.147 | 381.64 | 0.000 |
| 12 | 0.484 | 0.171 | 425.68 | 0.000 |
| 13 | 0.416 | -0.238 | 458.50 | 0.000 |
| 14 | 0.317 | -0.161 | 477.58 | 0.000 |

Sample: 1991M09 2011M09
Included observations: 238

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.805 | 0.805 | 156.22 | 0.000 |
| 2 | 0.609 | -0.111 | 246.08 | 0.000 |
| 3 | 0.478 | 0.065 | 301.71 | 0.000 |
| 4 | 0.413 | 0.092 | 343.25 | 0.000 |
| 5 | 0.385 | 0.074 | 379.51 | 0.000 |
| 6 | 0.345 | -0.019 | 408.77 | 0.000 |
| 7 | 0.293 | -0.012 | 429.96 | 0.000 |
| 8 | 0.222 | -0.063 | 442.20 | 0.000 |
| 9 | 0.148 | -0.053 | 447.70 | 0.000 |
| 10 | 0.110 | 0.029 | 450.74 | 0.000 |
| 11 | 0.078 | -0.041 | 452.26 | 0.000 |
| 12 | 0.063 | 0.025 | 453.27 | 0.000 |
| 13 | 0.028 | -0.067 | 453.47 | 0.000 |
| 14 | -0.005 | -0.000 | 453.47 | 0.000 |

Figure 8: ACF and PCF of Orange Prices in Florida (left) and in California (right)

**Exercise 3**

In Figure 9, we present the time series of corn and peanuts prices in U.S. from January 2001 to June 2012. The series are non-stationary as we observe an upward trend in both series. The autocorrelation functions in Figure 10 exhibit a profile of an autoregressive process with a very large autoregressive parameter $\phi \sim 1$. This is the profile of a non-stationary process, known as a unit root process to be introduced in Chapter 7. Recall that the cob-web model implies that the prices follow an autoregressive process with *negative* autoregressive parameter (i.e., $\phi < 0$) that will generate autocorrelations of alternating signs. This is not what we observe in the autocorrelograms of corn and peanuts prices, thus the cob-web model does not offer a good explanation of the dynamics of these prices.

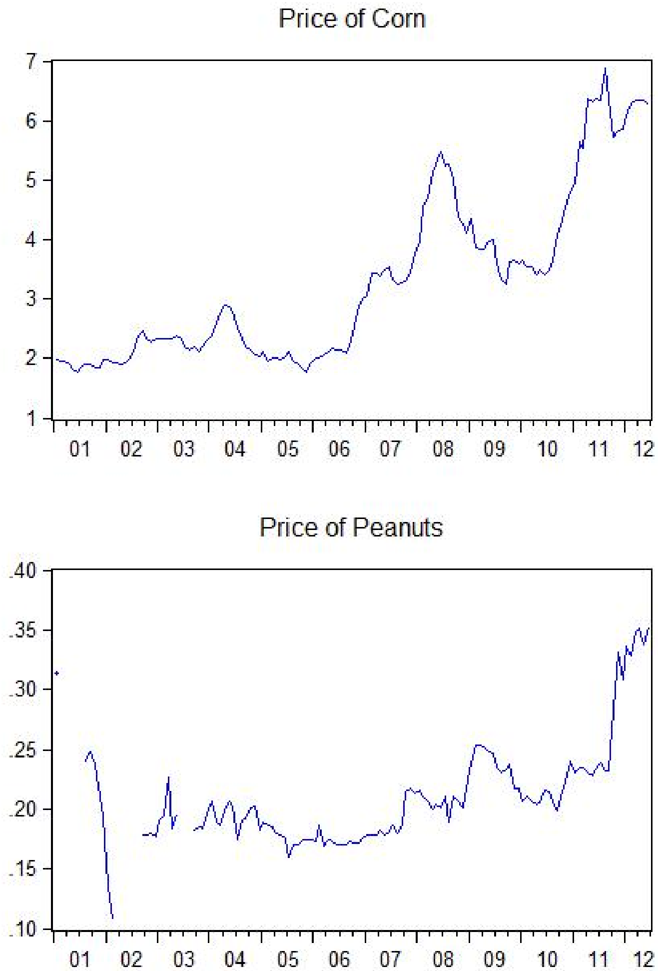
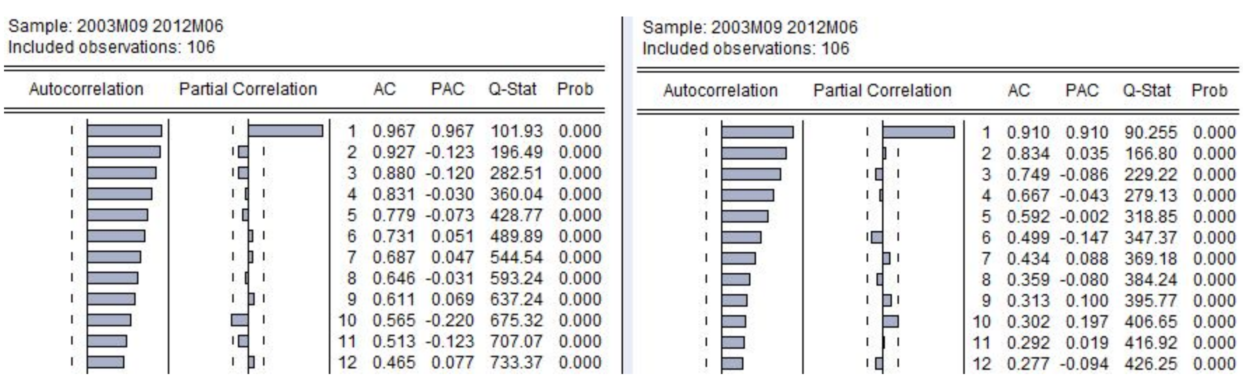



Figure 9: Time Series of Corn and Peanuts Prices in U.S. ($/lbs.)



Sample: 2003M09 2012M06
Included observations: 106

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.967 | 0.967 | 101.93 | 0.000 |
| 2 | 0.927 | -0.123 | 196.49 | 0.000 |
| 3 | 0.880 | -0.120 | 282.51 | 0.000 |
| 4 | 0.831 | -0.030 | 360.04 | 0.000 |
| 5 | 0.779 | -0.073 | 428.77 | 0.000 |
| 6 | 0.731 | 0.051 | 489.89 | 0.000 |
| 7 | 0.687 | 0.047 | 544.54 | 0.000 |
| 8 | 0.646 | -0.031 | 593.24 | 0.000 |
| 9 | 0.611 | 0.069 | 637.24 | 0.000 |
| 10 | 0.565 | -0.220 | 675.32 | 0.000 |
| 11 | 0.513 | -0.123 | 707.07 | 0.000 |
| 12 | 0.465 | 0.077 | 733.37 | 0.000 |

Sample: 2003M09 2012M06
Included observations: 106

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.910 | 0.910 | 90.255 | 0.000 |
| 2 | 0.834 | 0.035 | 166.80 | 0.000 |
| 3 | 0.749 | -0.086 | 229.22 | 0.000 |
| 4 | 0.667 | -0.043 | 279.13 | 0.000 |
| 5 | 0.592 | -0.002 | 318.85 | 0.000 |
| 6 | 0.499 | -0.147 | 347.37 | 0.000 |
| 7 | 0.434 | 0.088 | 369.18 | 0.000 |
| 8 | 0.359 | -0.080 | 384.24 | 0.000 |
| 9 | 0.313 | 0.100 | 395.77 | 0.000 |
| 10 | 0.302 | 0.197 | 406.65 | 0.000 |
| 11 | 0.292 | 0.019 | 416.92 | 0.000 |
| 12 | 0.277 | -0.094 | 426.25 | 0.000 |

Figure 10: ACF and PACF of Corn (left) and Peanuts (right) Prices in U.S.

Sample: 1 4167
Included observations: 4166

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.038 | 0.038 | 6.1242 | 0.013 |
| | | 2 | -0.037 | -0.038 | 11.698 | 0.003 |
| | | 3 | 0.011 | 0.013 | 12.158 | 0.007 |
| | | 4 | -0.009 | -0.011 | 12.482 | 0.014 |
| | | 5 | -0.004 | -0.002 | 12.547 | 0.028 |
| | | 6 | -0.002 | -0.002 | 12.561 | 0.051 |
| | | 7 | -0.034 | -0.034 | 17.406 | 0.015 |
| | | 8 | 0.006 | 0.009 | 17.562 | 0.025 |
| | | 9 | 0.016 | 0.013 | 18.628 | 0.029 |
| | | 10 | 0.021 | 0.021 | 20.454 | 0.025 |

Sample: 1 5675
Included observations: 5674

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.059 | -0.059 | 19.647 | 0.000 |
| | | 2 | -0.042 | -0.046 | 29.849 | 0.000 |
| | | 3 | 0.003 | -0.002 | 29.914 | 0.000 |
| | | 4 | -0.003 | -0.005 | 29.970 | 0.000 |
| | | 5 | -0.044 | -0.044 | 40.761 | 0.000 |
| | | 6 | -0.001 | -0.007 | 40.772 | 0.000 |
| | | 7 | -0.037 | -0.042 | 48.494 | 0.000 |
| | | 8 | 0.012 | 0.007 | 49.299 | 0.000 |
| | | 9 | 0.000 | -0.003 | 49.299 | 0.000 |
| | | 10 | 0.032 | 0.031 | 55.121 | 0.000 |

Figure 11: ACF and PACF of AMEX (left) and SP500 (right) Returns

**Exercise 4**
We update the two datasets: for the AMEX returns the sample runs from December 27, 1995 to July 25, 2012 for a total of 4167 observations; and for the SP500 returns, the sample runs from January 22, 1990 to July 25, 2012 for a total of 5675 observations.

The autocorrelation functions of the updated samples are shown in Figure 11. Both sets of autocorrelograms are very similar in that the autocorrelations are virtually zero; though the Q-statistics show that they are statistically significant from zero, their values are not economically meaningful. The main difference with Figure 5.6 in the textbook is in the autocorrelograms of the AMEX returns. The first order autocorrelation is now much smaller, 0.038 versus 0.142, which indicates that the nontrading frequency is basically zero if we interpret the 0.038 autocorrelation as the nontrading probability by following the claim of models of non-synchronous trading.

**Exercise 5**
We consider the following three portfolios: Dow Jones Industrial Average (DJIA), IBEX and N225, which are traded in U.S., Spain and Japan respectively. Figures 12, 13, and 14 report the autocorrelation functions of their respective returns. The DJIA and IBEX consists only of a few large companies; there are 30 industrial firms in the DJIA and the 35 largest corporations of Spain in IBEX. The Japanese Index N225 offers a much broader spectrum of the Japanese economy. Large firms tend to trade very frequently. The autocorrelations of N225 returns are economically and statistically zero; the process is clearly a white noise process, and following the models of non-synchronous trading the probability of nontrading is zero. The DJIA and IBEX returns have also very small autocorrelations that for all practical effects are considered zero. In the DJIA returns we observe a small negative autocorrelation of order one, which given the small number of large firms in the index may be due to bid-ask bounce effects. All firms in DJIA, IBEX, and N225 are traded every day. More trading implies less autocorrelation, and this is practically zero in the three indexes considered.

Sample: 1/04/2005 7/16/2012
Included observations: 1896

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.119 | -0.119 | 27.094 | 0.000 |
| | | 2 | -0.063 | -0.079 | 34.747 | 0.000 |
| | | 3 | 0.058 | 0.041 | 41.063 | 0.000 |
| | | 4 | -0.020 | -0.013 | 41.825 | 0.000 |
| | | 5 | -0.066 | -0.064 | 50.043 | 0.000 |
| | | 6 | 0.016 | -0.005 | 50.522 | 0.000 |
| | | 7 | -0.032 | -0.039 | 52.503 | 0.000 |
| | | 8 | 0.018 | 0.016 | 53.092 | 0.000 |
| | | 9 | -0.007 | -0.011 | 53.199 | 0.000 |
| | | 10 | 0.044 | 0.044 | 56.877 | 0.000 |

Figure 12: ACF and PACF of DJIA Returns

Sample: 1/04/2005 7/17/2012
Included observations: 1922

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.009 | 0.009 | 0.1431 | 0.705 |
| | | 2 | -0.061 | -0.061 | 7.2378 | 0.027 |
| | | 3 | -0.052 | -0.051 | 12.372 | 0.006 |
| | | 4 | 0.013 | 0.010 | 12.714 | 0.013 |
| | | 5 | -0.050 | -0.056 | 17.477 | 0.004 |
| | | 6 | -0.014 | -0.015 | 17.869 | 0.007 |
| | | 7 | -0.017 | -0.023 | 18.458 | 0.010 |
| | | 8 | 0.014 | 0.007 | 18.851 | 0.016 |
| | | 9 | -0.029 | -0.032 | 20.425 | 0.015 |
| | | 10 | -0.021 | -0.024 | 21.248 | 0.019 |

Figure 13: ACF and PACF of IBEX Returns

Sample: 1/05/2005 7/17/2012
Included observations: 1847

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.040 | -0.040 | 2.9048 | 0.088 |
| | | 2 | -0.021 | -0.023 | 3.7515 | 0.153 |
| | | 3 | -0.024 | -0.026 | 4.8560 | 0.183 |
| | | 4 | -0.013 | -0.016 | 5.1790 | 0.269 |
| | | 5 | -0.041 | -0.044 | 8.3672 | 0.137 |
| | | 6 | 0.010 | 0.006 | 8.5710 | 0.199 |
| | | 7 | 0.008 | 0.006 | 8.7038 | 0.275 |
| | | 8 | 0.001 | -0.000 | 8.7058 | 0.368 |
| | | 9 | -0.036 | -0.037 | 11.140 | 0.266 |
| | | 10 | 0.041 | 0.037 | 14.201 | 0.164 |

Figure 14: ACF and PACF of N225 Returns

**Exercise 6**
In Figures 15, 17, 19, and 21 we plot 100 observations of the time series of a simulated MA(1) process for different values of the parameter $\theta$, and in Figures 16, 18, 20, and 22 , their corresponding autocorrelograms. When $\theta < 0$, the time series is zig-zagging around zero (which is the unconditional mean of the process) more often than when $\theta > 0$. When $\theta < 0$, the first order autocorrelation is also negative; and when $\theta > 0$, the first order autocorrelation is also positive. The common feature to the four autocorrelograms is that there is only one significant spike in the ACF, and that the spikes in the PACF decay towards zero, in an alternating fashion when $\theta > 0$, or exponentially when $\theta < 0$. The magnitude of $\theta$ relates to the strength of the autocorrelations: the larger $\theta$ is, the larger the autocorrelations are.

Comparing these simulated processes with that of Figure 5.7 in the textbook, we observe that the simulated processes with $\theta < 0$ have the same autocorrelation profiles as that of Figure 5.7.

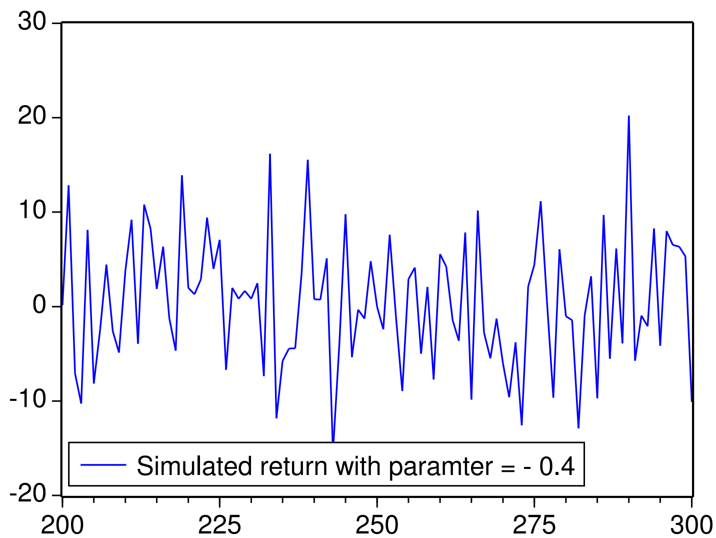
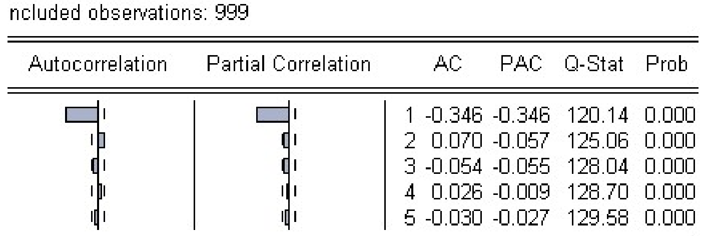



Figure 15: Simulated return $\Delta p_t = \theta\varepsilon_{t-1} + \varepsilon_t$ with $\theta = -0.4$



Sample: 1 1000
ncluded observations: 999

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.346 | -0.346 | 120.14 | 0.000 |
| | | 2 | 0.070 | -0.057 | 125.06 | 0.000 |
| | | 3 | -0.054 | -0.055 | 128.04 | 0.000 |
| | | 4 | 0.026 | -0.009 | 128.70 | 0.000 |
| | | 5 | -0.030 | -0.027 | 129.58 | 0.000 |

Figure 16: ACF and PACF of simulated return with $\theta = -0.4$

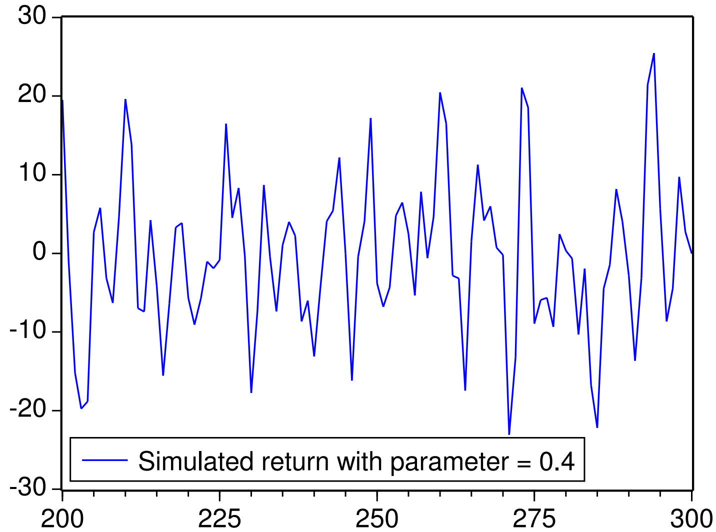



Figure 17: Simulated return $\Delta p_t = \theta\varepsilon_{t-1} + \varepsilon_t$ with $\theta = 0.4$

Sample: 1 1000
Included observations: 999

| Lag | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.365 | 0.365 | 133.76 | 0.000 |
| 2 | 0.005 | -0.148 | 133.79 | 0.000 |
| 3 | -0.055 | -0.004 | 136.83 | 0.000 |
| 4 | -0.008 | 0.020 | 136.90 | 0.000 |
| 5 | -0.008 | -0.023 | 136.96 | 0.000 |

Figure 18: ACF and PACF of simulated return with $\theta = 0.4$

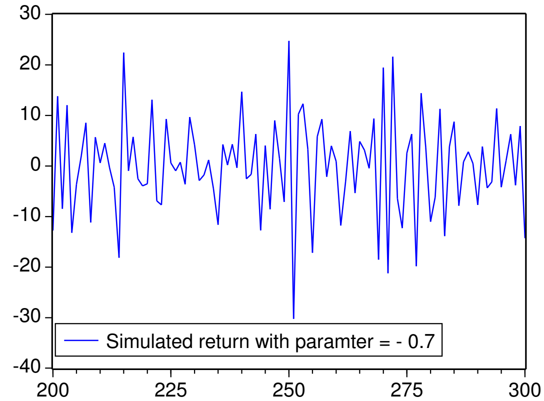



Figure 19: Simulated return $\Delta p_t = \theta\varepsilon_{t-1} + \varepsilon_t$ with $\theta = -0.7$

Sample: 1 1000
Included observations: 999

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.494 | -0.494 | 244.77 | 0.000 |
| | | 2 | 0.012 | -0.307 | 244.92 | 0.000 |
| | | 3 | 0.056 | -0.127 | 248.12 | 0.000 |
| | | 4 | -0.073 | -0.137 | 253.46 | 0.000 |
| | | 5 | 0.018 | -0.112 | 253.77 | 0.000 |

Figure 20: ACF and PACF of simulated return with $\theta = -0.7$

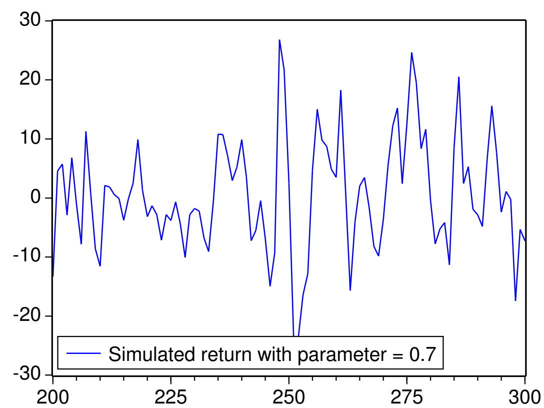



Figure 21: Simulated return $\Delta p_t = \theta\varepsilon_{t-1} + \varepsilon_t$ with $\theta = 0.7$

Sample: 1 1000
Included observations: 999

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.519 | 0.519 | 270.17 | 0.000 |
| | | 2 | 0.096 | -0.238 | 279.45 | 0.000 |
| | | 3 | 0.020 | 0.118 | 279.84 | 0.000 |
| | | 4 | -0.056 | -0.148 | 282.97 | 0.000 |
| | | 5 | -0.049 | 0.078 | 285.41 | 0.000 |

Figure 22: ACF and PACF of simulated return with $\theta = 0.7$

**Exercise 7**
We update the monthly returns to BBVA for a sample running from January 1999 to July 2012. The plot of the time series is in Figure 23 and the autocorrelation function in Figure 24.

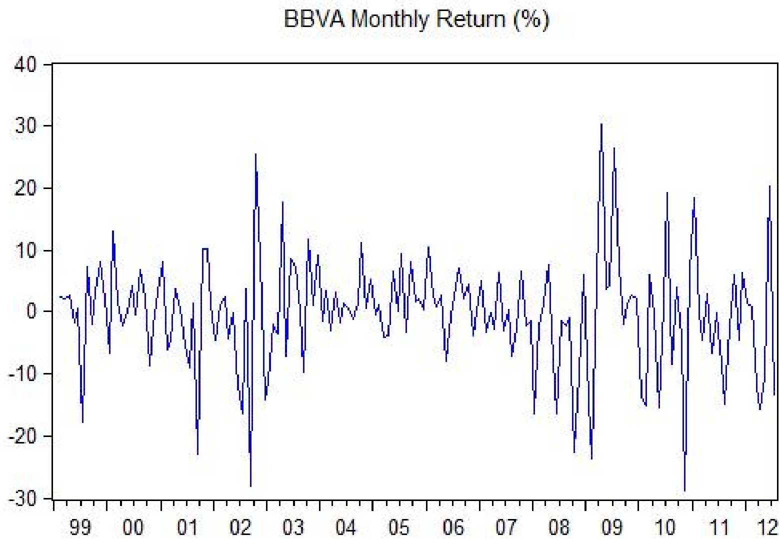



Figure 23: Time Series of Monthly BBVA Returns

Sample: 1999M01 2012M07
Included observations: 162

| Lag | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.017 | -0.017 | 0.0499 | 0.823 |
| 2 | -0.142 | -0.142 | 3.3998 | 0.183 |
| 3 | 0.075 | 0.071 | 4.3304 | 0.228 |
| 4 | 0.085 | 0.069 | 5.5557 | 0.235 |
| 5 | -0.090 | -0.069 | 6.9202 | 0.227 |
| 6 | 0.020 | 0.035 | 6.9895 | 0.322 |
| 7 | -0.081 | -0.116 | 8.1107 | 0.323 |
| 8 | 0.047 | 0.061 | 8.4972 | 0.386 |
| 9 | 0.024 | 0.007 | 8.5983 | 0.475 |
| 10 | -0.107 | -0.095 | 10.604 | 0.389 |

Figure 24: ACF and PACF of Monthly BBVA Returns

The updated time series seems to be more volatile that the series in Figure 5.7 of the textbook, mainly because of the financial events of 2008 onwards. The autocorrelation functions are virtually and statistically zero. Overall they are not very different from the ACF and PACF in Figure 5.7; the first two autocorrelation coefficients are negative, which is consistent with the predictions of a bid-ask bounce model (see Exercise 10 for an extended bid-ask bounce model with a time dependent indicator), but the values are very small to be economically relevant.

We also download monthly returns for Intel Corporation (INTC) from January 2001 to July 2012. The time series plot and the corresponding autocorrelation functions are in Figures 25 and 26. Though INTC, a technology company, is very different from BBVA, a banking company, their stock returns have a similar ACF and PACFs. The autocorrelations are economically and statistically zero, so that there is not linear dependence in the returns that could be exploited to predict future returns. Note that the first two autocorrelations are negative, consistent with a bid-ask bounce model, but the values are virtually zero.

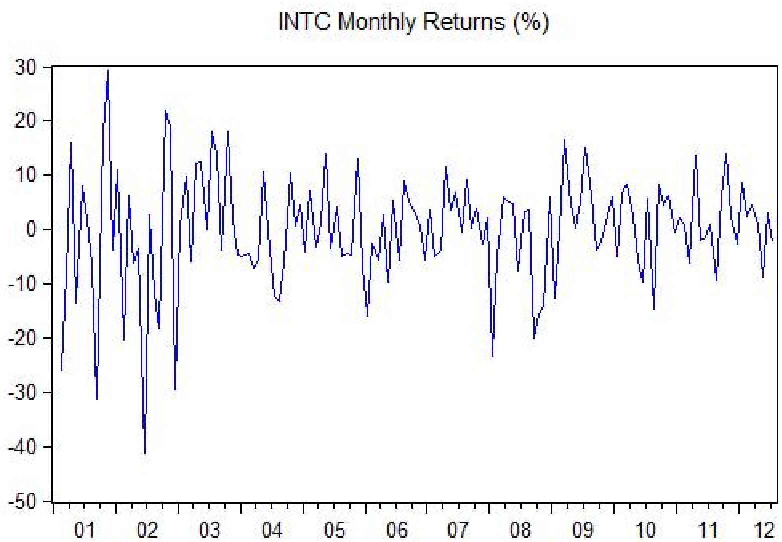
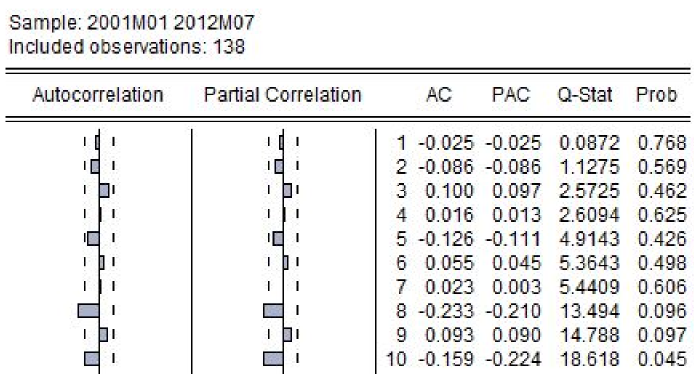



Figure 25: Time Series of Monthly INTC Returns



Sample: 2001M01 2012M07
Included observations: 138

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | -0.025 | -0.025 | 0.0872 | 0.768 |
| | | 2 | -0.086 | -0.086 | 1.1275 | 0.569 |
| | | 3 | 0.100 | 0.097 | 2.5725 | 0.462 |
| | | 4 | 0.016 | 0.013 | 2.6094 | 0.625 |
| | | 5 | -0.126 | -0.111 | 4.9143 | 0.426 |
| | | 6 | 0.055 | 0.045 | 5.3643 | 0.498 |
| | | 7 | 0.023 | 0.003 | 5.4409 | 0.606 |
| | | 8 | -0.233 | -0.210 | 13.494 | 0.096 |
| | | 9 | 0.093 | 0.090 | 14.788 | 0.097 |
| | | 10 | -0.159 | -0.224 | 18.618 | 0.045 |

Figure 26: ACF and PACF of Monthly INTC Returns

**Exercise 8**

Under the assumption stated in Section 5.2, we compute the third order autocorrelation coefficient of the portfolio returns:

$$\begin{aligned}
\gamma_3 &\equiv cov(R_t, R_{t-3}) = E(R_t R_{t-3}) - E(R_t)E(R_{t-3}) = E(R_t R_{t-3}) = \\
&= \frac{1}{4}E((R_t^i + R_t^j)(R_{t-3}^i + R_{t-3}^j)) = \frac{1}{4}(E(R_t^i R_{t-3}^i) + E(R_t^j R_{t-3}^i) + E(R_t^i R_{t-3}^j) + \\
&\quad + E(R_t^j R_{t-3}^j)) = \frac{1}{4}(E(R_t^j R_{t-3}^i) + E(R_t^i R_{t-3}^j))
\end{aligned}$$

$E(R_t^i R_{t-3}^i) = E(R_t^j R_{t-3}^j) = 0$ because of the assumption of no autocorrelation in assets $i$ and $j$. The terms $E(R_t^j R_{t-3}^i)$ and $E(R_t^i R_{t-3}^j)$ will be zero because there is no common news between the returns $R_t^j$ and $R_{t-3}^i$ as well as between the returns $R_t^i$ and $R_{t-3}^j$. For instance, the return $R_4^i$ contains the news for days 2, 3 and 4 while $R_1^j$ contains only the news for day one. They do not share any day with common news. We would like to have a longer time series of returns to understand how news develop beyond day 4 but with only the information in Figure 5.5 of the textbook, we could state that $\gamma_3 = 0$ and $\rho_3 = \gamma_3/\gamma_0 = 0$.

Similarly, we can compute the fourth order autocorrelation coefficient of the portfolio returns:

$$\begin{aligned}
\gamma_4 &\equiv cov(R_t, R_{t-4}) = E(R_t R_{t-4}) - E(R_t)E(R_{t-4}) = E(R_t R_{t-4}) = \\
&= \frac{1}{4}E((R_t^i + R_t^j)(R_{t-4}^i + R_{t-4}^j)) = \frac{1}{4}(E(R_t^i R_{t-4}^i) + E(R_t^j R_{t-4}^i) + E(R_t^i R_{t-4}^j) + \\
&+ E(R_t^j R_{t-4}^j)) = \frac{1}{4}(E(R_t^j R_{t-4}^i) + E(R_t^i R_{t-4}^j))
\end{aligned}$$

As before, Figure 5.5 does not have enough information on returns of asset $i$, i.e. $R_5^i, R_6^i, .....$ . Make your own assumptions and reason on whether or not there is common news between the corresponding returns.

**Exercise 9**

Under the assumption of Section 5.3, we compute the third order autocorrelation coefficient of the transaction price:

$$\begin{aligned}
\gamma_3 &\equiv cov(\Delta p_{t+3}, \Delta p_t) = E(\Delta p_{t+3}\Delta p_t) - E(\Delta p_{t+3})E(\Delta p_t) \\
&= E(\Delta p_{t+3}\Delta p_t) = E\left[(\Delta p_{t+3}^* + (I_{t+3} - I_{t+2})\frac{s}{2})(\Delta p_t^* + (I_t - I_{t-1})\frac{s}{2})\right] \\
&= E\left[\Delta p_{t+3}^*\Delta p_t^* + \frac{s}{2}\Delta p_t^*(I_{t+3} - I_{t+2}) + \frac{s}{2}\Delta p_{t+3}^*(I_t - I_{t-1}) + \frac{s^2}{4}(I_t - I_{t-1})(I_{t+3} - I_{t+2})\right] \\
&= \frac{s^2}{4}E(I_t I_{t+3} - I_{t-1}I_{t+3} - I_{t+2}I_t + I_{t-1}I_{t+2}) = 0
\end{aligned}$$

The autocorrelation coefficient is $\rho_3 \equiv \frac{\gamma_3}{\gamma_0} = \frac{0}{\sigma^2 + s^2/2} = 0$.

By the same arguments, we can compute the fourth order autocorrelation coefficient:

$$\begin{aligned}
\gamma_4 &\equiv cov(\Delta p_{t+4}, \Delta p_t) = \\
&= \frac{s^2}{4}E(I_t I_{t+4} - I_{t-1}I_{t+4} - I_{t+3}I_t + I_{t-1}I_{t+3}) = 0
\end{aligned}$$

The autocorrelation coefficient is $\rho_4 \equiv \frac{\gamma_4}{\gamma_0} = \frac{0}{\sigma^2 + s^2/2} = 0$.

**Exercise 10**

If the binary indicator $I_t$ is positively autocorrelated of order one ($\rho_1 > 0$), we compute the first order autocorrelation function of price changes as:

$$\begin{aligned}
\gamma_1 &\equiv cov(\Delta p_{t+1}, \Delta p_t) = E(\Delta p_{t+1}\Delta p_t) - E(\Delta p_{t+1})E(\Delta p_t) \\
&= \frac{s^2}{4}E(I_t I_{t+1} - I_{t-1}I_{t+1} - I_t I_t + I_{t-1}I_t) \\
&= \frac{s^2}{4}E(I_t I_{t+1} - I_t I_t + I_{t-1}I_t) \\
&= \frac{s^2}{4}(2E(I_t I_{t-1}) - 1)
\end{aligned}$$

Note that the first order autocovariance $E(I_t I_{t-1})$ is equal to the first order autocorrelation of the indicator because the variance of the indicator is one. We will write the autocorrelation of the indicator as $\rho_1^I \equiv E(I_t I_{t-1})$ . In addition, the variance of the transaction price is also affected by the autocorrelation of the indicator: $var(\Delta p_{t+1}) = \sigma^2 + \frac{s^2}{2}(1 - \rho_1^I)$. Then, the autocorrelation function of transaction price changes is

$$\rho_1 \equiv \frac{\gamma_1}{\gamma_0} = \frac{\frac{s^2}{4}(2\rho_1^I - 1)}{\sigma^2 + \frac{s^2}{2}(1 - \rho_1^I)}$$

so that

$$\begin{cases} \rho_1 > 0 & \text{when } \rho_1^I > \frac{1}{2} \\ \rho_1 \leq 0 & \text{when } 0 < \rho_1^I \leq \frac{1}{2} \end{cases}$$

Under the same assumptions,we compute the second order autocovariances:

$$\begin{aligned}
\gamma_2 &\equiv cov(\Delta p_{t+2}, \Delta p_t) = E(\Delta p_{t+2}\Delta p_t) - E(\Delta p_{t+2})E(\Delta p_t) \\
&= \frac{s^2}{4}E(I_t I_{t+2} - I_{t-1}I_{t+2} - I_t I_{t+1} + I_{t-1}I_{t+1}) \\
&= -\frac{s^2}{4}E(I_t I_{t-1})
\end{aligned}$$

and the autocorrelation coefficient will be $\rho_2 \equiv \frac{\gamma_2}{\gamma_0} = \frac{\frac{-s^2}{4}\rho_1^I}{\sigma^2 + \frac{s^2}{2}(1-\rho_1^I)} < 0$ given that $I_t$ is positively autocorrelated of order 1 (i.e., $\rho_1^I > 0$).

Applying the same arguments, we can compute higher order autocovariances:

$$\begin{aligned}
\gamma_3 &\equiv cov(\Delta p_{t+3}, \Delta p_t) = \\
&= \frac{s^2}{4}E(I_t I_{t+3} - I_{t-1}I_{t+3} - I_{t+2}I_t + I_{t-1}I_{t+2}) = 0
\end{aligned}$$

so that $\rho_3 = 0$ and similarly for $\rho_4 = \rho_5 = ..... = \rho_k = 0$.