CHAPTER 16.

FORECASTING WITH NONLINEAR MODELS: AN INTRODUCTION

SOLUTIONS

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Exercise 1

In Figure 1, we plot the quarterly U.S. real gross domestic product from 1947.Q1 to 2013.Q1. The most obvious feature of this time series is the upward trend that has made United States an almost $14 trillions (in real terms) economy. The series is clearly non-stationary. This historical trend is the result of two main factors: population growth and technological progress. However, this trend has been punctuated by episodes of recessions and expansions that we call business cycles. In Figures 2 and 3, we can visualize better the recession times (negative growth) and the expansion times (positive growth). In Figure 2 we compute growth in each quarter as the percentage change from one year ago, and in Figure 3 as the percentage change from the previous quarter, thus the former series is smoother than the latter but the substantial picture is the same. In the three figures, the shaded areas correspond to the National Bureau of Economic Research (NBER) recession dates.

The NBER recession data is available at http://www.nber.org/cycles/cyclesmain.html . Since the post-war years, US has experienced 11 recessions. In Table 1, we report the dates of the recessions from peak to trough and the duration in between. The frequency of the recessions has been uneven over time, with 4 recessions in the 50s and 60s that lasted between 8 and 11 months; the 70s had one the longest recession starting in November 1973 and lasting 16 months; in the 80s we had two almost consecutive recessions in 1980 and 1981, the latter lasting 16 months; in the 90s and 2000s, recessions were sparse (2 recessions) and short (8 months), -this is the time known as the Great Moderation- until the Great Recession that started at the end of 2007 and has been the longest (18 months) and the deepest (with a maximum annual drop of about 5% in GDP) in the modern US history.

From these plots, it is obvious that the dynamics of recessions are very different from those of expansions. The economy tends to bounce back from a recession in a relative short period of time and to stay in expansion mode for longer intervals of time. In short, recessions are shorter and less persistent than expansions. We also observe that the magnitude of the positive growth rates is larger (annual rates above 7% were common from the 1950s to the 1980s) than the magnitude of the negative growth rates (annual negative rates never reached -5%), thus the unconditional density of growth is skewed to the right.

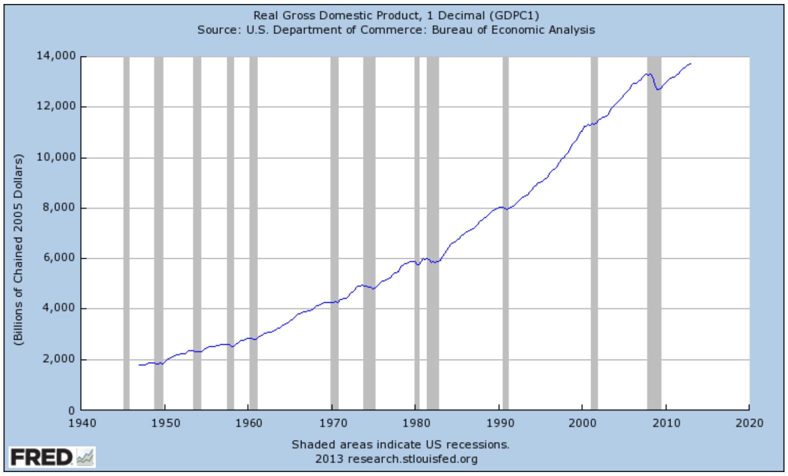

Figure 1: Quarterly U.S. Real Gross Domestic Product

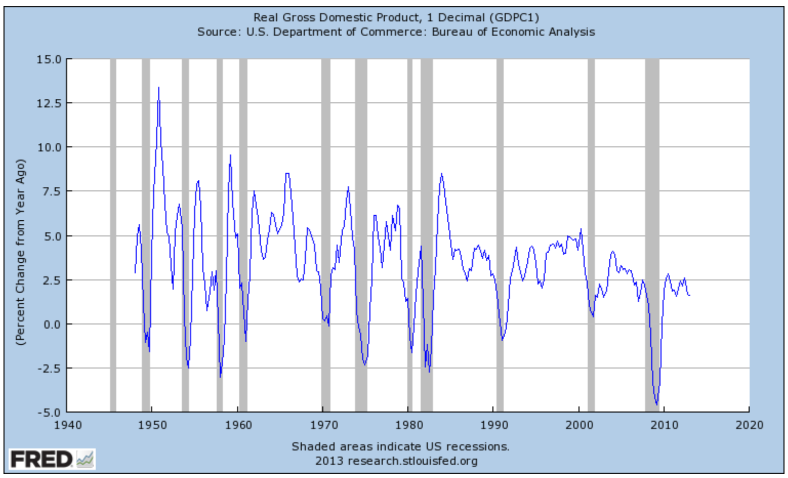

Figure 2: Quarterly U.S. Real Gross Domestic Product (Year-to-Year Changes)

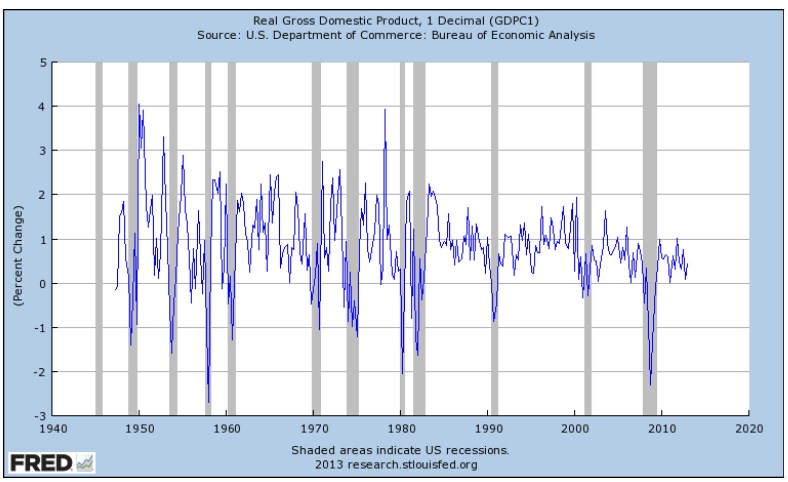

Figure 3: Quarterly U.S. Real Gross Domestic Product (Quarter-to-Quarter Changes)

Peak	Trough	Duration of the contraction
1948-11-01	1949-10-01	11 months
1953-07-01	1954-05-01	10 months
1957-08-01	1958-04-01	8 months
1960-04-01	1961-02-01	10 months
1969-12-01	1970-11-01	11 months
1973-11-01	1975-03-01	16 months
1980-01-01	1980-07-01	6 months
1981-07-01	1982-11-01	16 months
1990-07-01	1991-03-01	8 months
2001-03-01	2001-11-01	8 months
2007-12-01	2009-06-01	18 months

Table 1: U.S. Recession Dates (NBER Dating)

Exercise 2
In Figures 4 and 5, we present the updated time series for percentage changes in U.S. industrial production and 3-month Treasury Bill from 1955 to 2013 (May). These correspond to Figures 16.5 and 16.6 in the textbook. The textbook comments regarding the possibility of entertaining two regimes still apply to the updated series.The updated industrial production series reveals that the recession associated with the financial crisis of 2008 has been the deepest since 1955 (-15% drop in production) and much longer than the two previous recessions of 1991 and 2001. According to NBER the recession started in December 2007 and ended in June 2009, with small positive growth rates of around 2% in the latest years. The updated 3-month Treasury Bill rates series reveals the response of the monetary authority (the Fed) to the 2008 recession. An aggressive monetary policy, cutting interest rates and quantitative easing, brought short term interest rates to basically zero.

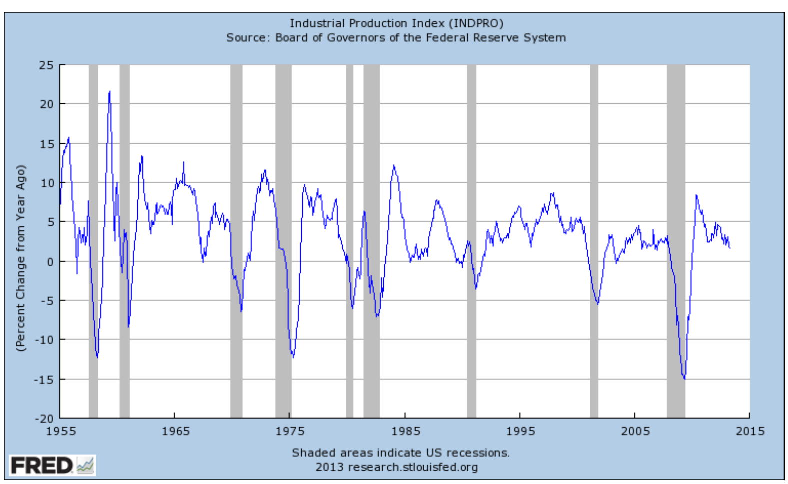

Figure 4: Monthly U.S. Industrial Production (Year-to-Year Changes)

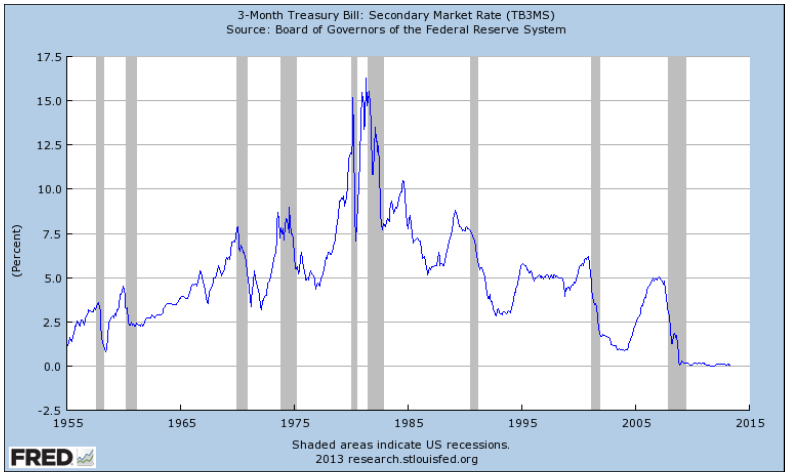

Figure 5: Monthly 3-Month Treasury Bill Rates

Exercise 3

The updated series of 3-month Treasury Bills rates contains an additional 56 observations that do not add much variability to the series; in the recent times the rates have been practically zero. We do not expect much change in the time series properties or in the estimation of the model. An ADF test diagnoses a unit root for the overall series. The estimation results of a nonlinear model that entertains a non-stationary regime when $r_{t-1} \leq 8.5\%$ and a stationary regime when $r_{t-1} > 8.5\%$ (Table 16.2 in the textbook) still hold. In Table 2, we present the updated estimation results. We have added a substantial number of lags in order to whiten out the residuals of the model but the key results i.e., the estimates corresponding to the regressors T3(-1) and T3(-1)*D, still hold.

We experiment with different levels of thresholds. In Table 3, we report the estimation results when the threshold between non-stationary and stationary regimes is 7%. There is not a substantial difference between this model and that in Table 2 but we can see that the evidence weakens. The estimate corresponding to T3(-1) is larger in magnitude (p-value also decreases) and the estimate corresponding to T3(-1)*D is also smaller in magnitude, indicating that the divide between non-stationary and stationarity is fuzzier. Other metrics, like the Adjusted R-squared, AIC, and SIC, are slightly worse than those corresponding to the model in Table 2.

Dependent Variable: DT3				
Method: Least Squares				
Sample (adjusted): 1953M02 2013M05				
Included observations: 940 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.010955	0.017625	0.621585	0.5344
D85	0.762364	0.204188	3.733628	0.0002
T3(-1)	-0.00097	0.00441	-0.218775	0.8269
T3(-1)*D85	-0.06857	0.019228	-3.566226	0.0004
DT3(-1)	0.38033	0.051477	7.388321	0
DT3(-2)	-0.10641	0.046696	-2.278856	0.0229
DT3(-3)	-0.00854	0.04677	-0.1825	0.8552
DT3(-4)	0.065684	0.047359	1.386939	0.1658
DT3(-5)	0.073622	0.047857	1.538396	0.1243
DT3(-6)	-0.09249	0.049864	-1.85482	0.0639
DT3(-7)	0.032522	0.048837	0.665943	0.5056
DT3(-8)	0.031187	0.048083	0.648602	0.5168
DT3(-9)	0.06318	0.047901	1.318973	0.1875
DT3(-10)	-0.05339	0.04826	-1.106295	0.2689
DT3(-11)	0.08431	0.048211	1.748785	0.0807
DT3(-12)	0.038226	0.045378	0.842385	0.3998
DT3(-1)*D85	0.068302	0.066789	1.022645	0.3067
DT3(-2)*D85	-0.1995	0.071012	-2.80934	0.0051
DT3(-3)*D85	0.105559	0.072254	1.460938	0.1444
DT3(-4)*D85	-0.31282	0.070104	-4.462166	0
DT3(-5)*D85	0.182385	0.072075	2.530489	0.0116
DT3(-6)*D85	-0.38187	0.072968	-5.233313	0
DT3(-7)*D85	-0.00491	0.072713	-0.067548	0.9462
DT3(-8)*D85	-0.08555	0.072005	-1.188111	0.2351
DT3(-9)*D85	0.123567	0.069765	1.771185	0.0769
DT3(-10)*D85	-0.12391	0.070577	-1.755622	0.0795
DT3(-11)*D85	0.066674	0.069842	0.954636	0.34
DT3(-12)*D85	-0.3168	0.064634	-4.90147	0
R-squared	0.322468	Mean dependent var		-0.00017
Adjusted R-squared	0.302409	S.D. dependent var		0.377354
S.E. of regression	0.315173	Akaike info criterion		0.557947
Sum squared resid	90.59285	Schwarz criterion		0.702292
Log likelihood	-234.235	Hannan-Quinn criter.		0.612973
F-statistic	16.07633	Durbin-Watson stat		1.929662
Prob(F-statistic)	0			

Table 2: SETAR Model for 3-month Treasury Bills Rates (threshold T3(-1) > 8.5%)

Dependent Variable: DT3				
Method: Least Squares				
Sample (adjusted): 1953M02 2013M05				
Included observations: 940 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.015259	0.018798	0.811713	0.4172
D7	0.434935	0.121635	3.575754	0.0004
TB3(-1)	-0.0032	0.005412	-0.59055	0.555
TB3(-1)*D7	-0.04623	0.01398	-3.3066	0.001
DT3(-1)	0.416282	0.064298	6.474246	0
DT3(-2)	-0.10124	0.070021	-1.4458	0.1486
DT3(-3)	0.20432	0.0701	2.914697	0.0036
DT3(-4)	-0.01905	0.071286	-0.26728	0.7893
DT3(-5)	-0.00222	0.068607	-0.03241	0.9742
DT3(-6)	-0.05642	0.064919	-0.86915	0.385
DT3(-7)	-0.04499	0.063729	-0.70592	0.4804
DT3(-8)	0.103619	0.06246	1.65896	0.0975
DT3(-9)	0.012467	0.06382	0.195353	0.8452
DT3(-10)	0.020912	0.063336	0.330175	0.7413
DT3(-11)	0.049381	0.061574	0.801979	0.4228
DT3(-12)	0.042437	0.058243	0.728614	0.4664
DT3(-1)*D7	0.022086	0.074723	0.295569	0.7676
DT3(-2)*D7	-0.12576	0.081315	-1.54661	0.1223
DT3(-3)*D7	-0.16386	0.081505	-2.01046	0.0447
DT3(-4)*D7	-0.10358	0.082241	-1.25943	0.2082
DT3(-5)*D7	0.229988	0.080366	2.861749	0.0043
DT3(-6)*D7	-0.31331	0.078212	-4.00593	0.0001
DT3(-7)*D7	0.081085	0.077254	1.049592	0.2942
DT3(-8)*D7	-0.07534	0.075848	-0.99328	0.3208
DT3(-9)*D7	0.158455	0.076646	2.067361	0.039
DT3(-10)*D7	-0.19869	0.076445	-2.59916	0.0095
DT3(-11)*D7	0.117909	0.074645	1.579609	0.1145
DT3(-12)*D7	-0.24701	0.070043	-3.5266	0.0004
R-squared	0.304632	Mean dependent var		-0.00017
Adjusted R-squared	0.284045	S.D. dependent var		0.377354
S.E. of regression	0.319295	Akaike info criterion		0.583931
Sum squared resid	92.97769	Schwarz criterion		0.728277
Log likelihood	-246.448	Hannan-Quinn criter.		0.638957
F-statistic	14.79759	Durbin-Watson stat		1.927923
Prob(F-statistic)	0			

Table 3: SETAR Model for 3-month Treasury Bills Rates (threshold T3(-1) > 7%)

Exercise 4

We download monthly data of the 10-year Treasury Constant Maturity Rate (annualized) (T10) and the Consumer Price Index (CPI) from the FRED database. The data goes from April 1953 to May 2013 for a total of 722 observations. We construct the monthly inflation rate (INFL) as the percentage change in the CPI over the last twelve months, i.e. $INFL = 100 \times (CPI - CPI(-12))/CPI(-12)$. Note that on constructing the monthly series of INFL, we are inducing autocorrelation of high order in the series because there are overlapping observations between successive rates. We plot both series in Figure 6. Observe that the profile of both series is very similar, Treasury rates tracking inflation rates and reaching maximum values during the Volcker's years in the late 70s and early 80s. The maximum Treasury rate was 15.3% in September 1981 and the maximum inflation rate 14.6% in March 1980. As soon as inflation was under control, the Treasury rates started to come down. The correlation coefficient between both series is 0.66 but we need to interpret correlation with care when we deal with unit root process. The ADF test for T10 is -1.30, so that we cannot reject the unit root at any standard significance level but for INFL the evidence for unit root is weaker: the ADF test for INFL is -2.95 and we cannot reject a unit root at the 1% level, but at the 5% level we do reject unit root marginally. We will consider that INF has a unit root to proceed with our analysis.

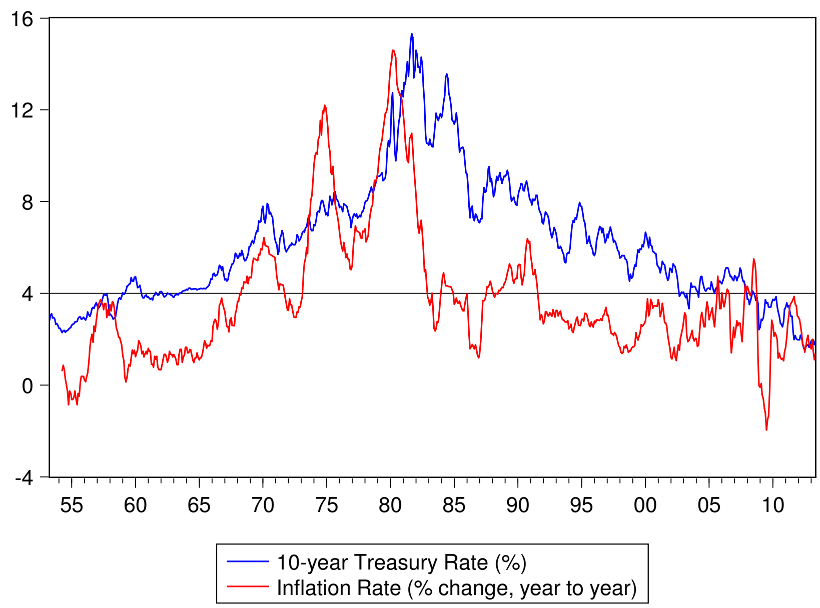

Figure 6: Monthly 10-year Treasury Rate and Inflation Rate (%)

We need to understand whether the series are cointegrated or not in order to construct either a VEC model or a VAR in the first differences of the series. The evidence for cointegration is weak. By implementing an Engle-Granger cointegration test, we cannot reject a unit root in the residuals of the cointegrating relation, i.e. regressing T10 on INFL. The ADF test is -2.37 with a p-vlaue of 33.6%, so that the series are not cointegrated. However, on running a VEC model, we find that the cointegration relation is statistically significant in both equations, thus cointegration exists. The estimation results of the VEC are presented in Table 4. We need a substantial number of lags (13) to capture the autocorrelation of the INFL series, though these are not all necessary in the T10 equation. The equation of interest in the VEC is that concerning T10. It tells us that changes in interest rates respond to changes in inflation mainly through the cointegrating equation. We simplify further the dynamics of the T10 equation by running several tests on exclusion restrictions and we arrive to a more parsimonious model for changes in T10. The estimation results for the best linear model are reported in Table 5. Observe that the AIC and SIC are lower for this model than for the unrestricted VEC equation and, even though we have trimmed the number of regressors, the goodness of fit remains the same. In conclusion, changes in T10 responds positively to changes in inflation, there is a direct effect through DINFL(-1) and indirectly through the cointegrating relation that push interest rates towards the long run equilibrium whenever the gap between inflation and interest rates grows too large.

Vector Error Correction Estimates
Sample (adjusted): 1955M06 2013M05
Included observations: 696 after adjustments
t-statistics in [ ]

Cointegrating Eq:	CointEq1	
T10(-1)	1	
INFL(-1)	-1.504132	
	[-5.70272]	
C	-0.417154	
	[-0.34655]	
Error Correction:	**D(T10)**	**D(INFL)**
CointEq1	-0.01065	0.009883
	[-2.91088]	[2.27626]
D(T10(-1))	0.38273	0.101399
	[9.87039]	[2.20358]
D(T10(-2))	-0.259864	0.0387
	[-6.27169]	[0.78705]
D(T10(-3))	0.100374	0.066778
	[2.36314]	[1.32482]
D(T10(-4))	-0.069903	0.04091
	[-1.63833]	[0.80795]
D(T10(-5))	0.089342	0.053296
	[2.08783]	[1.04951]
D(T10(-6))	-0.094695	-0.088735
	[-2.21135]	[-1.74613]
D(T10(-7))	-0.055699	0.019585
	[-1.29653]	[0.38416]
D(T10(-8))	0.074205	0.019208
	[1.73241]	[0.37789]
D(T10(-9))	-0.010678	0.056502
	[-0.24936]	[1.11189]
D(T10(-10))	0.01703	0.026845
	[0.39847]	[0.52928]
D(T10(-11))	0.083985	-0.065218
	[1.97265]	[-1.29083]
D(T10(-12))	-0.077394	-0.001856
	[-1.86599]	[-0.03770]
D(T10(-13))	-0.006862	-0.075047
	[-0.17611]	[-1.62293]

(continued)

		(continued)
D(INFL(-1))	0.079266	0.359687
	[2.43100]	[9.29559]
D(INFL(-2))	-0.025595	0.058808
	[-0.85468]	[1.65479]
D(INFL(-3))	-0.021718	-0.074554
	[-0.72781]	[-2.10528]
D(INFL(-4))	-0.025109	0.076232
	[-0.83979]	[2.14846]
D(INFL(-5))	0.000127	0.031687
	[0.00424]	[0.89119]
D(INFL(-6))	-0.034995	-0.002842
	[-1.16773]	[-0.07990]
D(INFL(-7))	0.021289	0.090119
	[0.71481]	[2.54975]
D(INFL(-8))	0.017796	0.009712
	[0.59676]	[0.27442]
D(INFL(-9))	-0.034577	0.040396
	[-1.15907]	[1.14104]
D(INFL(-10))	-0.019762	0.104751
	[-0.66238]	[2.95864]
D(INFL(-11))	0.016627	0.124719
	[0.55573]	[3.51257]
D(INFL(-12))	-0.031584	-0.504114
	[-1.04822]	[-14.0985]
D(INFL(-13))	0.04365	0.126973
	[1.34522]	[3.29744]
R-squared	0.203042	0.392792
Adj. R-squared	0.172069	0.369193
Sum sq. resids	43.13385	60.74571
S.E. equation	0.25392	0.301332
F-statistic	6.555454	16.64477
Log likelihood	-19.77878	-138.9299
Akaike AIC	0.134422	0.47681
Schwarz SC	0.31075	0.653138
Mean dependent	-0.001193	0.002854
S.D. dependent	0.279061	0.379399
Determ. resid covar.(dof adj.)		0.005726
Determ. resid covar.		0.00529
Log likelihood		-150.9941
Akaike criterion		0.597684
Schwarz criterion		0.969933

Table 4: Vector Error Correction Model

Dependent Variable: DT10				
Method: Least Squares				
Sample (adjusted): 1954M06 2013M05				
Included observations: 708 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINT(-1)	-0.008723	0.003073	-2.83873	0.0047
DT10(-1)	0.386337	0.037798	10.22113	0.0000
DT10(-2)	-0.262125	0.040126	-6.53256	0.0000
DT10(-3)	0.096403	0.041342	2.331818	0.0200
DT10(-4)	-0.07875	0.041636	-1.8914	0.0590
DT10(-5)	0.086289	0.041437	2.08239	0.0377
DT10(-6)	-0.098667	0.041292	-2.38949	0.0171
DT10(-7)	-0.053171	0.04021	-1.32233	0.1865
DT10(-8)	0.064153	0.037438	1.713597	0.0870
DT10(-11)	0.088148	0.036691	2.402451	0.0165
DT10(-12)	-0.089057	0.036736	-2.42426	0.0156
DINFL(-1)	0.048908	0.025846	1.89233	0.0589
R-squared	0.191174	Mean dependent var		-0.00062
Adjusted R-squared	0.17839	S.D. dependent var		0.276771
S.E. of regression	0.250873	Akaike info criterion		0.089061
Sum squared resid	43.8042	Schwarz criterion		0.16639
Log likelihood	-19.52756	Hannan-Quinn criter.		0.118938
Durbin-Watson stat	1.993054			

Table 5: Dynamics of Changes in T10 (DT10) as a Function of Changes in Inflation (DINFL)

Exercise 5

In Figure 6, we observe that potentially the data may contain two regimes: high and low treasury and inflation rates. With the exception of the period from early 70s to early 80, inflation rates have been mostly below 4%; and in the recent years the average inflation rate has been about 2 %. We would like to analyze the response of interest rates to high inflation rates and we choose an inflation threshold of 4%. We create a dummy variable $D4_t = 1$ if INFL $> 4\%$ and 0 otherwise. We maintain the same cointegration relation as in Exercise 4. We could explore whether there are different cointegration relations in different regimes (it will require econometrics techniques beyond OLS) but the notion of cointegration relates to long run equilibrium, thus it may not be justifiable to search for a cointegration relation within only a few months of data. Our starting model is the best linear model that we found in Exercise 4 (details in Table 5). We proceed to specify two regimes according to the threshold established by the dummy variable $D4_t$. We start by augmenting the linear model with new regressors that interact the dummy variable with each regressor of the linear model, i.e.,

$$\{COINT(-1), DT10(-1), DT10(-2), ..., DT10(-12), DINFL(-1), D4(-1) * COINT(-1),$$

$$D4(-1) * DT10(-1), D4(-1) * DT10(-2), ..., D4(-1) * DT10(-12), D4(-1) * DINFL(-1)\}$$

so that we allow for different dynamics in the high and low rates regimes.

Within this general model, we proceed to search for a more parsimonious model by testing the exclusion of the least relevant regressors. The final estimation results are shown in Table 6. We observe that in the low regime (when the dummy is zero and inflation is below 4%), there is not cointegration between inflation and interest rates but changes in interest rates are still positively driven by previous changes in inflation rates. In the high regime (when the dummy is one and inflation is above 4%), the cointegration relation is most relevant and it ensures that interest rates and inflation rates do not drift apart. In addition, the dynamics of DT10 are also different as many other lags of DT10 come into play.

Comparing the linear model of Exercise 4 with the above nonlinear model (TAR), we see that the goodness of fit is better, with an adjusted R-squared of 20% in TAR versus 17% in the linear model. The information criteria, AIC and SIC, are also substantially lower in TAR than in the linear model, and value of the log-likelihood function is much improved in TAR. Thus, the nonlinear model seems to be a better model than the linear model to understand the empirical relation between interest rates and inflation rates.

Dependent Variable: DT10				
Method: Least Squares				
Sample (adjusted): 1954M06 2013M05				
Included observations: 708 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
DT10(-1)	0.386261	0.037571	10.28091	0.0000
DT10(-2)	-0.26302	0.039437	-6.669347	0.0000
DT10(-3)	0.100041	0.038851	2.575009	0.0102
DT10(-8)	0.071084	0.035412	2.007324	0.0451
DT10(-11)	0.081246	0.036351	2.235009	0.0257
DT10(-12)	-0.06243	0.036313	-1.719349	0.0860
DINF(-1)	0.048263	0.025376	1.901958	0.0576
D4(-1)*COINT(-1)	-0.01325	0.003709	-3.57308	0.0004
D4(-1)*DT10(-4)	-0.18344	0.05483	-3.34555	0.0009
D4(-1)*DT10(-5)	0.203103	0.055994	3.627243	0.0003
D4(-1)*DT10(-6)	-0.17201	0.05673	-3.032035	0.0025
D4(-1)*DT10(-7)	-0.10694	0.05416	-1.97443	0.0487
R-squared	0.215804	Mean dependent var		-0.00062
Adjusted R-squared	0.20341	S.D. dependent var		0.276771
S.E. of regression	0.247023	Akaike info criterion		0.058135
Sum squared resid	42.47026	Schwarz criterion		0.135465
Log likelihood	-8.57995	Hannan-Quinn criter.		0.088012
Durbin-Watson stat	1.989124			

Table 6: Threshold Model for Changes in T10

Exercise 6

The updated time series of quarterly growth of U.S. industrial production contains 19 additional observations and includes the severe drop (more than 6%) associated with the 2008 financial crisis. In Figure 7, we present the updated series.

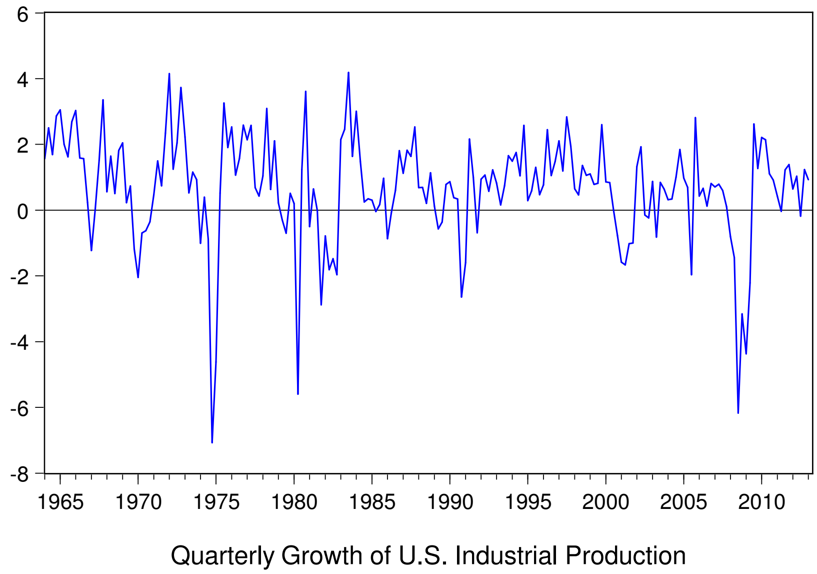

Figure 7: Quarterly Growth of U.S. Industrial Production (%)

The best linear model for the updated series remains the same as that provided in section 16.2 of the textbook, i.e. AR(8) model. The estimation results are provided in Table 7. The estimates of the autoregressive parameters are statistically equivalent producing a similar cycle (see the inverted roots of the autoregressive characteristic equation). The fit has marginally improved with an adjusted R-squared of 26%.

We perform a linearity test by running auxiliary regressions as those in Tables 16.4 and 16.5 of the textbook. We choose as transition variables IPG(-1) and IPG(-8), and we run F-tests. The results are displayed in Tables 8 and 9. In both instances, we reject the null hypothesis of linearity at the customary significance level of 5%. However, the case for IPG(-1) as a transition variable seems to be stronger as the F-test is much larger and its p-value is zero. Thus, we proceed by constructing a logistic transition function with transition variable IPG(-1), i.e.,

$$G(IPG(-1), \gamma, 0) = \frac{1}{1 + exp\{-\gamma IPG(-1)\}}$$

for $\gamma > 0$. We choose the parameter γ by minimizing the sum of the squared residuals of the regression that contains the nonlinear terms (see Table 16.6 in the textbook). The results are displayed in Table 10. It seems that the optimal value of γ is around 2.

Dependent Variable: IPG				
Method: Least Squares				
Sample: 1964Q1 2013Q1				
Included observations: 197				
Convergence achieved after 3 iterations				
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.671296	0.159071	4.220111	0
AR(1)	0.482137	0.081421	5.921504	0
AR(8)	-0.13968	0.073927	-1.88946	0.0603
R-squared	0.268606	Mean dependent var		0.667281
Adjusted R-squared	0.261066	S.D. dependent var		1.656858
S.E. of regression	1.424257	Akaike info criterion		3.560288
Sum squared resid	393.5303	Schwarz criterion		3.610286
Log likelihood	-347.688	Hannan-Quinn criter.		3.580528
F-statistic	35.62344	Durbin-Watson stat		1.972085
Prob(F-statistic)	0			
Inverted AR Roots	.80-.29i	.80+.29i	.36-.70i	.36+.70i
	-.25-.71i	-.25+.71i	-.67+.30i	-.67-.30i

Table 7: Linear Model for Industrial Production Growth

Wald Test:			
Test Statistic	Value	df	Probability
F-statistic	5.878444	(6, 188)	0
Chi-square	35.27066	6	0
Null Hypothesis: C(4)=C(5)=C(6)=C(7)=C(8)=C(9)=0			
Null Hypothesis Summary:			
Normalized Restriction (= 0)		Value	Std. Err.
C(4)		0.099079	0.061074
C(5)		0.129245	0.123347
C(6)		-0.03833	0.011539
C(7)		-0.03152	0.016544
C(8)		-0.00623	0.002787
C(9)		-0.00374	0.00546
Restrictions are linear in coefficients.			

Table 8: Linearity Test with Transition Variable IPG(-1)

Wald Test:			
Test Statistic	Value	df	Probability
F-statistic	2.886306	(6, 188)	0.0103
Chi-square	17.31784	6	0.0082
Null Hypothesis: C(4)=C(5)=C(6)=C(7)=C(8)=C(9)=0			
Null Hypothesis Summary:			
Normalized Restriction (= 0)		Value	Std. Err.
C(4)		0.055997	0.074589
C(5)		-0.09888	0.052483
C(6)		0.035434	0.027689
C(7)		-0.01181	0.014178
C(8)		0.000242	0.007246
C(9)		-0.0005	0.001818
Restrictions are linear in coefficients.			

Table 9: Linearity Test with Transition Variable IPG(-8)

γ	0.5	1	1.5	2	2.5	3	5
SSR	379.11	375.98	374.70	374.68	375.32	376.23	379.76

Table 10: Choice of the Parameter γ

The estimation results corresponding to the LSTAR model with function $G(IPG(-1), 2, 0)$ are displayed in Table 11. The term IPG(-1)*G(-1,2) is omitted because its t-statistics was practically zero. With these results, we observe that the lower regime (recession), i.e. G =0, has an unconditional mean of -0.37% and the upper regime (expansion), i.e. G=1, of 1.29%.

The LSTAR model that we have found with the updated time series is not very different from that in Section 16.2.1 of the textbook. The dynamics are similar in that the most prominent lags to explain dependence are IPG(-1) and IPG(-8). The transition function is slightly smoother with the updated time series but the unconditional quarterly growth rates in the extreme regimes have not changed much. Overall, we can say that the last Great Recession of 2008 has not brought new features to the time series and it seems to conform to the average dynamics of previous recessions.

Dependent Variable: IPG
Method: Least Squares
Sample: 1964Q1 2013Q1
Included observations: 197
Convergence achieved after 3 iterations
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.44459	0.594199	-0.74822	0.4552
IPG(-1)	0.164934	0.142802	1.15498	0.2495
IPG(-8)	-0.36614	0.349144	-1.04868	0.2956
G(-1,2)	1.607012	0.840448	1.912091	0.0574
IPG(-8)*G(-1,2)	0.298453	0.384395	0.776423	0.4385

R-squared	0.303526	Mean dependent var	0.667281
Adjusted R-squared	0.289017	S.D. dependent var	1.656858
S.E. of regression	1.39706	Akaike info criterion	3.53167
Sum squared resid	374.7412	Schwarz criterion	3.615
Log likelihood	-342.87	Hannan-Quinn criter.	3.565403
F-statistic	20.91863	Durbin-Watson stat	1.940374
Prob(F-statistic)	0		

Table 11: LSTAR Model with Transition Variable IPG(-1) for Industrial Production Growth

Exercise 7
For the time series of U.S. GDP growth rates (Exercise 1), we find that the best dynamic linear model is an AR(12). In Table 12, we report the estimation results.

Dependent Variable: Y				
Method: Least Squares				
Sample (adjusted): 1950Q2 2009Q4				
Included observations: 239 after adjustments				
Convergence achieved after 3 iterations				
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.801643	0.063643	12.59587	0.0000
AR(1)	0.392093	0.066899	5.861015	0.0000
AR(5)	-0.120549	0.051707	-2.331386	0.0206
AR(12)	-0.158756	0.066231	-2.397005	0.0173
R-squared	0.201631	Mean dependent var		0.797685
Adjusted R-squared	0.191439	S.D. dependent var		0.980997
S.E. of regression	0.882113	Akaike info criterion		2.603602
Sum squared resid	182.8591	Schwarz criterion		2.661786
Log likelihood	-307.1305	Hannan-Quinn criter.		2.627049
F-statistic	19.78332	Durbin-Watson stat		2.109864
Prob(F-statistic)	0.000000			
Inverted AR Roots	.86+.24i	.86-.24i	.67+.59i	.67-.59i
	.24+.81i	.24-.81i	-.20+.84i	-.20-.84i
	-.56+.60i	-.56-.60i	-.81-.21i	-.81+.21i

Table 12: Quarterly GDP Growth Rate: Estimation of a AR(12) Model

Before constructing a nonlinear model, we implement a linearity test against the STAR model and choose Y_{t-1}, Y_{t-5}, and Y_{t-12} as the potential transition variables. In Tables 13 – 15 and Tables 16 – 18, we report the results of the auxiliary regressions and their corresponding F tests (Wald tests) for linearity. Given the customary 5% significance level, the tests reject linearity in favor of a nonlinear STAR model with Y_{t-1} and Y_{t-5} as the transition variables, and they fail to reject linearity when Y_{t-12} is the transition variable. Note that the rejection of linearity is the strongest (p-values are zero) when the transition variable is Y_{t-5}, and thus we proceed to model nonlinear features based on Y_{t-5} as the transition variable. In addition, a logistic transition function seems to be an appropriate choice because, in the auxiliary regression, the statistical significance of the odd powers of the transition variable is very high (p-values around 3%). The results of this F and Wald tests are reported in Table 19.

Dependent Variable: Y				
Method: Least Squares				
Sample (adjusted): 1950Q2 2009Q4				
Included observations: 239 after adjustments				
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.553099	0.14831	3.729338	0.0002
Y(-1)	0.493578	0.16038	3.077551	0.0023
Y(-5)	0.030547	0.106413	0.287056	0.7743
Y(-12)	-0.149331	0.125145	-1.193256	0.234
Y(-1)*Y(-1)	0.064759	0.110321	0.587003	0.5578
Y(-5)*Y(-1)	0.050625	0.080906	0.625731	0.5321
Y(-12)*Y(-1)	-0.021206	0.089537	-0.236844	0.813
Y(-1)*Y(-1)^2	-0.053832	0.053173	-1.012401	0.3124
Y(-5)*Y(-1)^2	-0.116339	0.063541	-1.830915	0.0684
Y(-12)*Y(-1)^2	-0.011593	0.070742	-0.163871	0.87
Y(-1)*Y(-1)^3	0.008747	0.010431	0.838589	0.4026
Y(-5)*Y(-1)^3	0.013201	0.015504	0.851423	0.3954
Y(-12)*Y(-1)^3	0.009182	0.022778	0.403112	0.6872
R-squared	0.231456	Mean dependent var		0.797685
Adjusted R-squared	0.190649	S.D. dependent var		0.980997
S.E. of regression	0.882544	Akaike info criterion		2.640842
Sum squared resid	176.0278	Schwarz criterion		2.829939
Log likelihood	-302.5806	Hannan-Quinn criter.		2.717043
F-statistic	5.671884	Durbin-Watson stat		2.108717
Prob(F-statistic)	0.000000			

Table 13: Testing for linearity: Auxiliary Regression with Transition Variable Y_{t-1}

Dependent Variable: Y				
Method: Least Squares				
Sample (adjusted): 1950Q2 2009Q4				
Included observations: 239 after adjustments				
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.491798	0.148266	3.316988	0.0011
Y(-1)	0.417692	0.11123	3.755212	0.0002
Y(-5)	-0.093473	0.14588	-0.640751	0.5223
Y(-12)	-0.021262	0.114488	-0.185715	0.8528
Y(-1)*Y(-5)	-0.018335	0.083978	-0.218327	0.8274
Y(-5)*Y(-5)	0.206261	0.070552	2.923548	0.0038
Y(-12)*Y(-5)	-0.144647	0.06544	-2.210363	0.0281
Y(-1)*Y(-5)^2	-0.023616	0.040669	-0.580671	0.562
Y(-5)*Y(-5)^2	-0.045043	0.034839	-1.292898	0.1974
Y(-12)*Y(-5)^2	-0.076323	0.05591	-1.36511	0.1736
Y(-1)*Y(-5)^3	0.004797	0.016221	0.295697	0.7677
Y(-5)*Y(-5)^3	-0.002161	0.007628	-0.283283	0.7772
Y(-12)*Y(-5)^3	0.038233	0.017994	2.124727	0.0347
R-squared	0.240222	Mean dependent var		0.797685
Adjusted R-squared	0.19988	S.D. dependent var		0.980997
S.E. of regression	0.877497	Akaike info criterion		2.629371
Sum squared resid	174.0201	Schwarz criterion		2.818467
Log likelihood	-301.2098	Hannan-Quinn criter.		2.705572
F-statistic	5.95461	Durbin-Watson stat		2.042379
Prob(F-statistic)	0.000000			

Table 14: Testing for linearity: Auxiliary Regression with Transition Variable Y_{t-5}

Method: Least Squares				
Sample (adjusted): 1950Q2 2009Q4				
Included observations: 239 after adjustments				
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.507394	0.176021	2.882578	0.0043
Y(-1)	0.476607	0.104957	4.540993	0.0000
Y(-5)	-0.057499	0.109342	-0.525869	0.5995
Y(-12)	-0.134734	0.306177	-0.440052	0.6603
Y(-1)*Y(-12)	-0.098428	0.095715	-1.028351	0.3049
Y(-5)*Y(-12)	0.036329	0.138381	0.262529	0.7932
Y(-12)*Y(-12)	0.180741	0.101051	1.788609	0.075
Y(-1)*Y(-12)^2	-0.037131	0.065403	-0.56773	0.5708
Y(-5)*Y(-12)^2	-0.038259	0.051194	-0.747334	0.4556
Y(-12)*Y(-12)^2	0.011647	0.062815	0.185419	0.8531
Y(-1)*Y(-12)^3	0.027565	0.016757	1.644973	0.1014
Y(-5)*Y(-12)^3	-0.017117	0.029365	-0.582912	0.5605
Y(-12)*Y(-12)^3	-0.022754	0.017538	-1.297432	0.1958
R-squared	0.23724	Mean dependent var		0.797685
Adjusted R-squared	0.196739	S.D. dependent var		0.980997
S.E. of regression	0.879217	Akaike info criterion		2.633289
Sum squared resid	174.7032	Schwarz criterion		2.822385
Log likelihood	-301.678	Hannan-Quinn criter.		2.709489
F-statistic	5.857691	Durbin-Watson stat		2.165071
Prob(F-statistic)	0.000000			

Table 15: Testing for linearity: Auxiliary Regression with Transition Variable Y_{t-12}

Wald Test:			
Equation: POWER1			
Test Statistic	Value	df	Probability
F-statistic	2.621177	(9, 226)	0.0067
Chi-square	23.59059	9	0.0050
Null Hypothesis: C(5)=0,C(6)=0,C(7)=0,C(8)=0,C(9)=0,C(10)=0,C(11)=0,C(12)=0,C(13)=0			
Null Hypothesis Summary:			
Normalized Restriction (= 0)		Value	Std. Err.
C(5)		0.064759	0.110321
C(6)		0.050625	0.080906
C(7)		-0.021206	0.089537
C(8)		-0.053832	0.053173
C(9)		-0.116339	0.063541
C(10)		-0.011593	0.070742
C(11)		0.008747	0.010431
C(12)		0.013201	0.015504
C(13)		0.009182	0.022778
Restrictions are linear in coefficients.			

Table 16: F Test and Wald Test for Linearity (Transition Variable: Y_{t-1})

Wald Test:			
Equation: POWER2			
Test Statistic	Value	df	Probability
F-statistic	6.481092	(9, 226)	0.0000
Chi-square	58.32982	9	0.0000
Null Hypothesis: C(5)=0,C(6)=0,C(7)=0,C(8)=0,C(9)=0,C(10)=0,C(11)=0,C(12)=0,C(13)=0			
Null Hypothesis Summary:			
Normalized Restriction (= 0)		Value	Std. Err.
C(5)		-0.018335	0.083978
C(6)		0.206261	0.070552
C(7)		-0.144647	0.06544
C(8)		-0.023616	0.040669
C(9)		-0.045043	0.034839
C(10)		-0.076323	0.05591
C(11)		0.004797	0.016221
C(12)		-0.002161	0.007628
C(13)		0.038233	0.017994
Restrictions are linear in coefficients.			

Table 17: F Test and Wald Test for Linearity (Transition Variable: Y_{t-5})

Wald Test:			
Equation: POWER3			
Test Statistic	Value	df	Probability
F-statistic	1.67172	(9, 226)	0.0969
Chi-square	15.04548	9	0.0897
Null Hypothesis: C(5)=0,C(6)=0,C(7)=0,C(8)=0,C(9)=0,C(10)=0,C(11)=0,C(12)=0,C(13)=0			
Null Hypothesis Summary:			
Normalized Restriction (= 0)		Value	Std. Err.
C(5)		-0.098428	0.095715
C(6)		0.036329	0.138381
C(7)		0.180741	0.101051
C(8)		-0.037131	0.065403
C(9)		-0.038259	0.051194
C(10)		0.011647	0.062815
C(11)		0.027565	0.016757
C(12)		-0.017117	0.029365
C(13)		-0.022754	0.017538
Restrictions are linear in coefficients.			

Table 18: F Test and Wald Test for Linearity (Transition Variable: Y_{t-12})

Wald Test:			
Equation: POWER2			
Test Statistic	Value	df	Probability
F-statistic	2.336639	(6, 226)	0.0329
Chi-square	14.01984	6	0.0294
Null Hypothesis: C(5)=0, C(6)=0, C(7)=0, C(11)=0, C(12)=0, C(13)=0			
Null Hypothesis Summary:			
Normalized Restriction (= 0)		Value	Std. Err.
C(5)		-0.018335	0.083978
C(6)		0.206261	0.070552
C(7)		-0.144647	0.06544
C(11)		0.004797	0.016221
C(12)		-0.002161	0.007628
C(13)		0.038233	0.017994
Restrictions are linear in coefficients.			

Table 19: F Test and Wald Test for Functional Form (Transition Variable: Y_{t-5})

We proceed to search for the optimal value of γ by implementing a grid procedure. In Table 20, we report the SSRs of the conditional regression model given transition variable Y_{t-5} and different values of γ. The optimal γ is 10because it delivers the smallest SSR.

γ	1	6	7	8	9	10
SSR	176.1115	174.9898	174.8387	174.6972	174.5668	**174.4484**

Table 20: Selection of γ

We present the estimation of the LSTAR model, conditioning of a value of $\hat{\gamma} = 10$, in Table 21. After some specification testing, we have excluded the two variables, Y(-12) and G(-5,10)*Y(-1) because their t-ratios were close to zero. The notation $G(-5, 10)$ means the logistic transition function

$$G(Y_{t-5}, 10, 0) = \frac{1}{1 + \exp\{-10 \times Y_{t-5}\}}.$$

Comparing the linear model (Table 12) with the LSTAR model (Table 21), we observe that the goodness of fit has improved with an Adj.R-squared moving from 19% to 22%. The information criteria are also smaller in the LSTAR model. Thus, the modeling of nonlinearity produces some additional gains to understand the dynamics of the data. We also calculate the dynamics in the extreme upper ($G = 1$) and lower ($G = 0$) regimes. The dynamics of the intermediate regimes are bounded between these two and can be calculated in a similar fashion for different values of the function G.

When $G = 0$, the autoregressive process in the lower regime (low growth/recessions) is

$$Y_t = 0.26 + 0.38Y_{t-1} - 0.70Y_{t-5} + \varepsilon_t,$$

for which the unconditional mean is $\mu_L = 0.26/(1 - 0.38 + 0.70) = 0.20\%$. This regime is stationary (all inverted AR roots are inside unit circle).

When $G = 1$, the autoregressive process in the high regime (high growth/expansions) is

$$Y_t = 0.59 + 0.38Y_{t-1} - 0.19Y_{t-12} + \varepsilon_t,$$

for which the unconditional mean is $\mu_U = 0.59/(1 - 0.38 + 0.19) = 0.73\%$. The regime is also stationary and it is more persistent than the lower regime, as we expected given that expansions are longer than recessions (Exercise 1).

Dependent Variable: Y
Method: Least Squares
Sample (adjusted): 1950Q2 2009Q4
Included observations: 239 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.258285	0.32263	0.800561	0.4242
Y(-1)	0.380616	0.065847	5.780319	0.0000
Y(-5)	-0.700247	0.226623	-3.089926	0.0022
G(-5,10)	0.329953	0.368988	0.894211	0.3721
G(-5,10)*Y(-5)	0.703677	0.216301	3.253228	0.0013
G(-5,10)*Y(-12)	-0.186676	0.066421	-2.810486	0.0054

R-squared	0.23682	Mean dependent var	0.797685
Adjusted R-squared	0.220442	S.D. dependent var	0.980997
S.E. of regression	0.866148	Akaike info criterion	2.575262
Sum squared resid	174.7994	Schwarz criterion	2.662537
Log likelihood	-301.7438	Hannan-Quinn criter.	2.610431
F-statistic	14.46027	Durbin-Watson stat	2.085444
Prob(F-statistic)	0.000000		

Table 21: Estimation of STAR Model (Transition Variable: Y_{t-5}, $\gamma = 10$)

Exercise 8
In Figure 8, we plot the logistic transition function and the corresponding time series of the values of the transition function calculated in Exercise 7. Observe that the time series of the transition function captures very faithfully the 11 recessions that we have documented in Table 1 (Exercise 1). The logistic function is very steep as $\gamma = 10$ is quite large value. Therefore, the proposed LSTAR model in Exercise 7 seems to be very similar to a threshold model SETAR.

We estimate the following two-regime SETAR model, whose specification is driven by our findings regarding the LSTAR model in Exercise 7,

$$Y_t = \begin{cases} c_1 + \phi_{11} Y_{t-1} + \phi_{12} Y_{t-5} + \phi_{13} Y_{t-12} & \text{if } Y_{t-5} > 0\% \\ c_2 + \phi_{21} Y_{t-1} + \phi_{22} Y_{t-5} & \text{if } Y_{t-5} \leq 0\% \end{cases}$$

This SETAR model discriminates between economic recessions (growth below zero) and expansions (growth above zero). To proceed with the estimation, we construct a dummy variable IND as

$$IND \equiv I(Y_{t-5} > 0\%) = \begin{cases} 1 & \text{if } Y_{t-5} > 0\% \text{ (expansion)} \\ 0 & \text{if } Y_{t-5} \leq 0\% \text{ (recession)} \end{cases}$$

The dummy variable IND is similar to the logistic smooth transition function $G(\cdot)$ of LSTAR model in Exercise 7 because the γ parameter is quite large. The difference is that the transition function $G(-5, 10)$ in the LSTAR model is smooth while IND is a step function in the SETAR model. In Figure 9, we plot the step function IND and the its time series values. Observe that there is not much difference with the logistic smooth transition function.

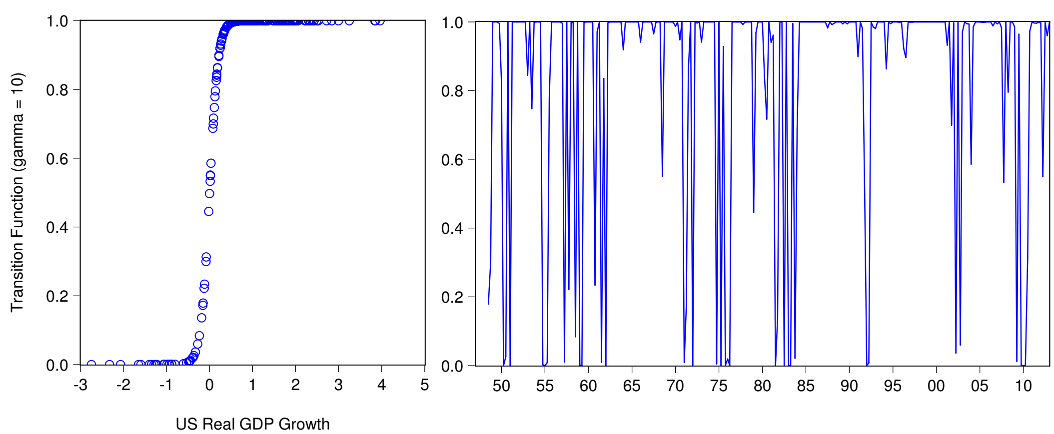

Figure 8: Logistic Transition Function for U.S. Real GDP Growth

In Table 22 we present the estimation results of the SETAR model. The results are very similar to those of the LSTAR model confirming the equivalence of both models. We calculate the dynamics in the expansion (IND=1) and recession (IND=0) regimes. When IND=0, the process is

$$Y_t = 0.13 + 0.38 Y_{t-1} - 0.80 Y_{t-5} + \varepsilon_t,$$

for which the unconditional mean is $\mu_L = 0.13/(1 - 0.38 + 0.80) = 0.09\%$.

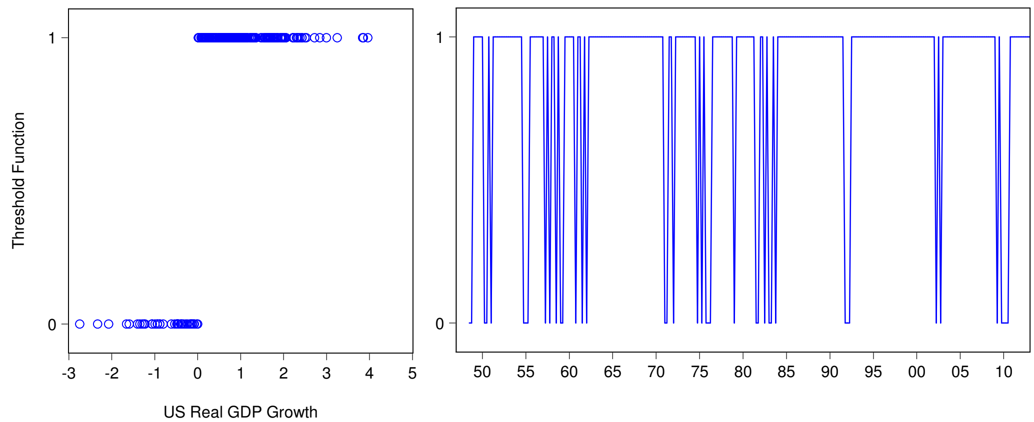

Figure 9: Threshold Function for U.S. Real GDP Growth

When IND=1, the autoregressive process is

$$Y_t = 0.59 + 0.38Y_{t-1} - 0.01Y_{t-5} - 0.18Y_{t-12} + \varepsilon_t,$$

for which the unconditional mean is $\mu_U = 0.59/(1 - 0.38 + 0.01 + 0.18) = 0.73\%$. These are in agreement with the lower and upper regimes provided by the LSTAR model.

Dependent Variable: Y				
Method: Least Squares				
Sample (adjusted): 1950Q2 2009Q4				
Included observations: 239 after adjustments				
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.133842	0.302805	0.442006	0.6589
Y(-1)	0.382067	0.065988	5.789922	0.0000
Y(-5)	-0.798734	0.219596	-3.637285	0.0003
IND	0.464653	0.326479	1.423227	0.1560
IND*Y(-5)	0.792644	0.221076	3.585388	0.0004
IND*Y(-12)	-0.182788	0.062904	-2.905833	0.0040
R-squared	0.239928	Mean dependent var		0.797685
Adjusted R-squared	0.223617	S.D. dependent var		0.980997
S.E. of regression	0.864382	Akaike info criterion		2.571181
Sum squared resid	174.0875	Schwarz criterion		2.658456
Log likelihood	-301.2561	Hannan-Quinn criter.		2.60635
F-statistic	14.70995	Durbin-Watson stat		2.076802
Prob(F-statistic)	0.000000			

Table 22: Estimation of Self-excited Threshold Autoregressive Model

Exercise 9

Based on the linear model in Table 12, the STAR model in Table 21, and the SETAR mdoel in Table 22, we calculate a sequence of 1-step-ahead optimal forecasts for GDP growth rates assuming a quadratic loss function. The forecast sample runs from 2010Q1 to 2013Q1. Forecast results are presented in Table 23 and, in the last line of the table, we calculate the MSE of the three models. Among the three, the linear model has the best performance because it delivers the smallest MSE. This is an instance in which nonlinear models seem to bring a better fit to the data but they do not deliver in an out-of-sample exercise. In general, linear models are difficult to beat.Observe that the three models overestimate the pace of the recovery from the recession that ended in 2009.Q2. Their forecasts are substantially higher than the actual growth rates. This indicates that the recovery from the Great Depression does not fit the average recovery path of the previous U.S. recessions.

		Linear Model		STAR Model		SETAR Model	
Date	Actual	Forecasts	Errors	Forecasts	Erorrs	Forecasts	Errors
2009Q3	0.359168						
2009Q4	0.986745						
2010Q1	0.577058	1.357184	-0.780126	2.263784	-1.686726	2.370017	-1.792959
2010Q2	0.554547	0.95775	-0.403202	1.42183	-0.867283	1.430983	-0.876436
2010Q3	0.642346	0.82264	-0.180294	0.567867	0.074478	0.408578	0.233767
2010Q4	0.59122	0.852696	-0.261475	0.741555	-0.150335	0.764475	-0.173255
2011Q1	0.019723	0.894773	-0.87505	0.899708	-0.879985	0.899754	-0.880031
2011Q2	0.611757	0.59719	0.014567	0.534239	0.077518	0.542405	0.069352
2011Q3	0.317633	1.032351	-0.714719	0.993661	-0.676029	0.999377	-0.681745
2011Q4	1.002704	1.127863	-0.125159	1.143882	-0.141179	1.141407	-0.138703
2012Q1	0.485391	1.247108	-0.761717	1.220852	-0.735461	1.224386	-0.738995
2012Q2	0.311219	0.911663	-0.600444	0.626105	-0.314885	0.798212	-0.486993
2012Q3	0.764681	0.702488	0.062194	0.640219	0.124462	0.648023	0.116658
2012Q4	0.094444	0.816112	-0.721669	0.681381	-0.586938	0.708354	-0.61391
2013Q1	0.44029	0.535772	-0.095482	0.519861	-0.079571	0.522992	-0.082702
MSE		**0.279922**		**0.453006**		**0.500877**	

Table 23: 1-step-ahead Optimal Forecasts and Forecast Errors under Quadratic Loss Function

Exercise 10

We calculate the 2-step-ahead forecast for GDP growth rates based on the LSTAR model (Exercise 7) with a forecast sample from 2010Q1 to 2013Q1. Since the transition variable is the lagged dependent variable Y_{t-5} and we wish to calculate a forecast with a horizon of 2 quarters, there is no need for implementing a numerical forecast. Formally, given the LSTAR model in Table 21, we have,

$$\begin{aligned}
f_{t,2} &= \mu_{t+2|t} = E(Y_{t+2}|I_t) \\
&= E(c_0 + \alpha_1 Y_{t+1} + \alpha_5 Y_{t-3} + G(Y_{t-3}, 10, 0)(c_1 + \beta_5 Y_{t-3} + \beta_{12} Y_{t-10}) + \varepsilon_{t+2}|I_t) \\
&= c_0 + \alpha_1 E(Y_{t+1}|I_t) + \alpha_5 Y_{t-3} + G(Y_{t-3}, 10, 0)(c_1 + \beta_5 Y_{t-3} + \beta_{12} Y_{t-10}) \\
&= c_0 + \alpha_1 f_{t,1} + \alpha_5 Y_{t-3} + G(Y_{t-3}, 10, 0)(c_1 + \beta_5 Y_{t-3} + \beta_{12} Y_{t-10})
\end{aligned}$$

where $f_{t,1}$ is the 1-step-ahead forecast from 2009Q4 to 2012Q4. By plugging in the estimates of the coefficients (Table 21), we calculate the 2-quarter-ahead forecasts of GDP growth, which are reported in Table 24.

Date	Actual	$f_{t,1}$	$f_{t,2}$
2009Q3	0.359168		
2009Q4	0.986745	1.05037	
2010Q1	0.577058	2.266811	0.756617
2010Q2	0.554547	1.424228	1.360552
2010Q3	0.642346	0.568861	1.217779
2010Q4	0.591220	0.741526	0.743452
2011Q1	0.019723	0.899706	1.043716
2011Q2	0.611757	0.534321	1.366655
2011Q3	0.317633	0.993632	1.043316
2011Q4	1.002704	1.143924	0.819694
2012Q1	0.485391	1.220749	0.957593
2012Q2	0.311219	0.626718	0.855066
2012Q3	0.764681	0.640244	0.72284
2012Q4	0.094444	0.681339	0.725779
2013Q1	0.440290	0.519861	0.710439

Table 24: Two-step-ahead Optimal Forecasts of STAR Model under Quadratic Loss Function

