# CHAPTER 13. FORECASTING VOLATILITY I

## SOLUTIONS

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#### Exercise 1

The New York Times, in its Sunday edition (April 14, 21, and 28, 2013) ran a series of three columns 'The Housing Haze' written by Professor Robert Shiller on long term forecasting of home prices. The thrust of these articles is that the return on housing as a long-term investment is very uncertain because a long term forecasting of home prices involves the design of a complex model, which needs to consider as main factors construction costs, population dynamics, population preferences, inflation, and dynamics of interest rates and home prices. Professor Shiller concludes that such a forecasting 'is indeed risky'.

As an economist, this article provides the basis to construct a model for house prices. It contains a good explanation of the two sides of the market, the supply and the demand factors. However, as a forecaster, the article does not put much faith on what it can be accomplished because, on one hand, there is not much dependence between the changes in prices that we see today and those ten year down the road (he is referring to autocorrelation coefficients of order 10), and on the other, all factors are subject to so much uncertainty (we should understand that the time series of interest rate changes, house price changes, inflation, etc. must have very large variances) so that confidence intervals for the long term forecast would be extremely wide and thus, of no much use to todays investors.

The business magazine Fortune ran a short article 'Just How Risky Is J.P Morgan Chase?' (April 29, 2013) claiming that, though the banks profits are solid, investors consider the institution risky because of its exposure to the so-called shadow banking system, which is a short-term (intra-day) lending system among financial institutions to fund short-term transactions.

As an economist, we learn that a shadow banking system is a large source of risk and that institutions with a large exposure to it will be in a very fragile situation if a financial crisis develops. J.P. Morgan Chase is considered a systemic important financial institution, which means that a massive failure in their investments decisions (e.g. reckless bets on derivatives) will not be contained within the institution and will propagate through the overall economic system. This is the notion of 'systemic risk'. The financial crisis of 2008 is a prime example. As a forecaster, we learn that the Federal Reserve has estimated that J.P. Morgan Chase will run into a risk loss of \$79 billion should a financial crisis develops; however we do not have information on how this forecast has been calculated, though most likely the Fed has constructed an 'stress scenario' or a set of very adverse conditions in the economy (large drops in the stock market, large drop in house prices, very high unemployment, liquidity drain episodes, etc.) and has simulated the effect of these conditions on the balance sheet of the institution.

We update the time series of the quarterly U.S. Real GDP growth from the webiste http://research.stlouisfed.org/fred2/series/GDPC1

and the monthly S&P500 index from the Yahoo Finance website. The update of Figures 13.3 and 13.4 in the textbook is more difficult because involves ellaborated methodologies described in the corresponding academic articles cited in the textbook.

We plot GDP growth in Figure 1. We observe the large drop in growth in the aftermath of the financial crisis of 2008; in the the fourth quarter of 2008, growth was -9.3% (in annual rates), but with this exception the volatility of the last years seems to be lower than that in the 1970s, 1980s and 1990s. In Figure 2, we plot the updated time series of the monthly returns to S&P500 index from January 1960 to May 2013. The returns from 2008 M08 to 2013 M05 oscillated between -11.65% and 10.23% approximately; in general the volatility of the most recent years is similar to that in the 1980s but larger than that in the 1990s and early 2000s.

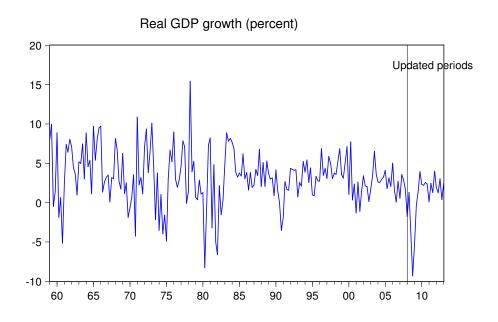


Figure 1: Real GDP Growth

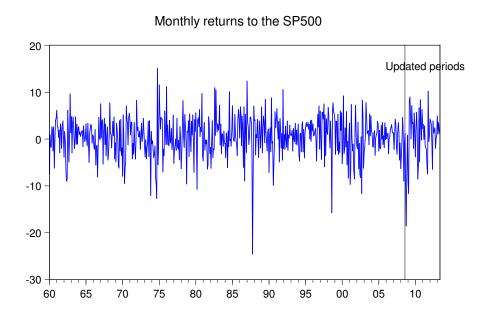


Figure 2: Monthly Returns of the S&P500 Index

We update the time series of the weekly S&P500 index from the Yahoo Finance website, and the daily Yen/US Dollar exchange rate and the daily 10-year Treasury Constant Maturity Rate from following websites respectively:

http://research.stlouisfed.org/fred2/series/DEXJPUS http://research.stlouisfed.org/fred2/series/DGS10

In Figure 3 we plot the three time series. We take log differences of the three series to obtain their corresponding returns, i.e.,  $r_t = 100 \times [\log p_t - \log p_{t-1}]$ . For each return series, we implement MA and EWMA to compute their 1-step-ahead volatility forecasts. For EWMA, we choose the value of  $\lambda$  to be 0.95 for all three series; and for MA, the size of the rolling widow is 4 weeks for weekly data, and 20 trading days for daily data. In Figures 4, 5 and 6 we plot the one-step-ahead volatility forecasts obtained with MA and EWMA specifications. In all three figures, the units are variance units, for instance in Figure 4 (a), the maximum value is a variance of about 140 so that the standard deviation is about 11.83 %. Observe that the forecast of the weekly series is smoother than the daily series and that the EWMA forecasts tend to be smoother (the  $\lambda$  parameter is large) than the MA forecasts. However the volatility profiles from MA and EWMA are very similar. In the three series, the largest volatility shocks corresponds to those triggered by the financial crisis of 2008, in particular for the SP500 and the 10-year Treasure Note; the Yen/\$ exchange rate had also experienced a large volatility increase in the late 1990s.

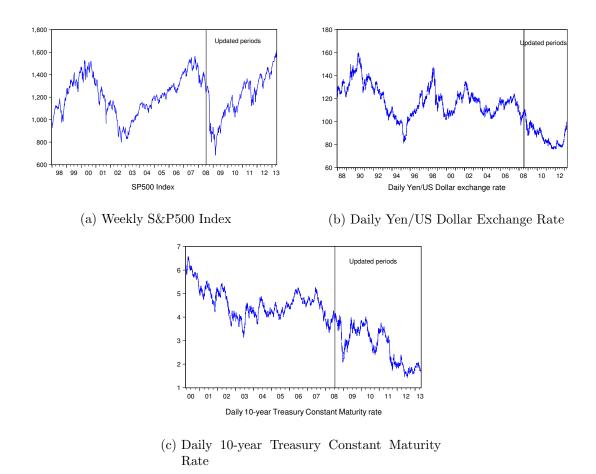


Figure 3: Time Series of S&P500 Index, Yen/\$ Exchange Rate and 10-Year Treasury Note Yield

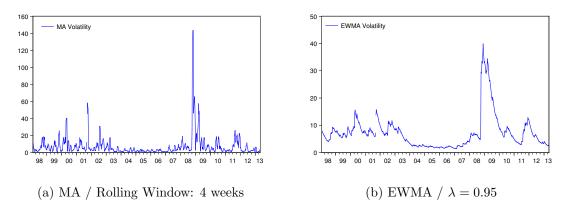


Figure 4: 1-step-ahead Volatility Forecast, Weekly Returns to S&P500 Index

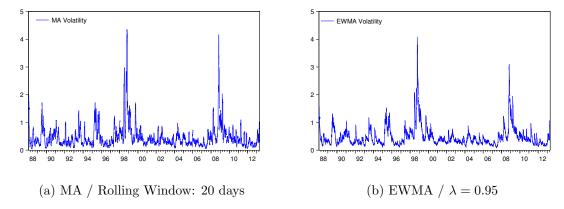


Figure 5: 1-step-ahead Volatility Forecast, Daily Return to Yen/U.S. dollar Exchange Rate Returns

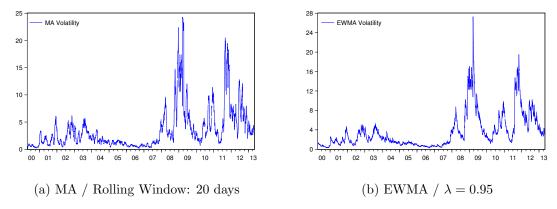


Figure 6: 1-step-ahead Volatility Forecast, Daily Return to 10-year Treasury Note Returns

In Figures 7, 8 and 9, we plot the 95% interval forecast for the returns of each of the three series in Exercise 3. Under the assumption of conditional normality of returns, the forecast interval is  $\mu \pm 1.96\sigma_{t+1|t}$ . Since  $\mu = 0$ , the interval reduces to  $\pm 1.96\sigma_{t+1|t}$ , where the conditional standard deviation is obtained from the MA and EWMA forecasts. With only a few exceptions, observe that the realized returns are contained within the 95% confidence interval.

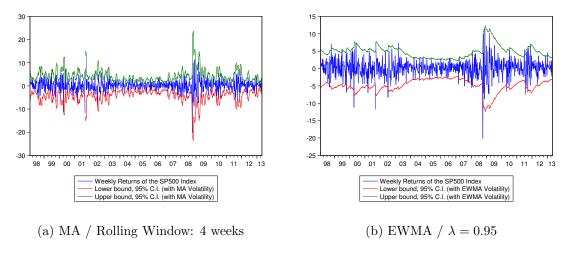


Figure 7: 95% Confidence Interval Forecast, Weekly Returns to S&P500 Index

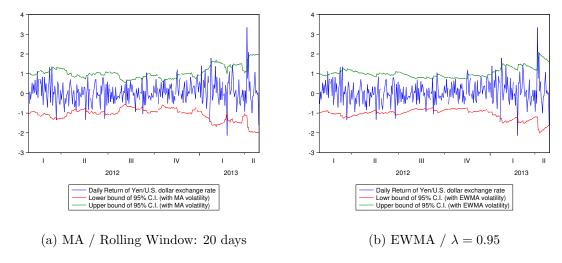


Figure 8: 95% Confidence Interval Forecast, Daily Return to Yen/\$ Exchange Rate Returns

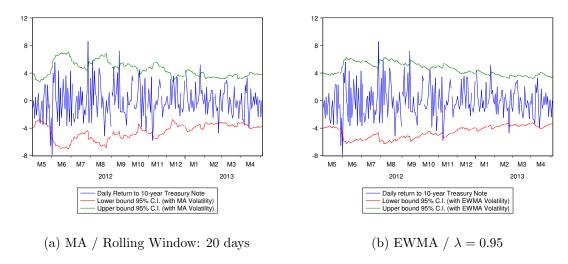


Figure 9: 95% Confidence Interval Forecast, Daily Return to 10-year Treasury Note Returns

We download the quarterly U.S. inflation rate and real GDP growth rate (growth rate from quarter to quarter) from the following websites:

http://research.stlouisfed.org/fred2/series/CPIAUCSL http://research.stlouisfed.org/fred2/series/GDPC1

The unconditional mean for GDP growth rate is 0.78% and for the inflation rate is 0.88%. In Figure 10 we plot these two time series. We have chosen 4 quarters as the rolling window for the MA specification and  $\lambda=0.8$  for the EWMA. The 1-step-ahead volatility forecasts for GDP growth are plotted in Figures 11a and 11b and those for the inflation rate in Figures 12a and 12b. For both series, observe the 'great moderation' period from mid-1980s to mid- 2000s when the volatility of GDP growth was well under 1% and the volatility of the inflation rate was even lower approaching close to zero. The volatility profiles provided by the MA and EWMA estimators are very similar for both series but the EWMA forecasts tend to be smoother than the MA forecasts. In Figures 13 and 14, we plot the 95% confidence intervals for the naive 1-step-ahead forecast of GDP growth and inflation rate respectively, i.e.,  $\mu \pm 1.96\sigma_{t+1|t}$ , where  $\mu$  is the unconditional mean of the series and  $\sigma_{t+1|t}$  is the squared root of the MA and EWMA volatility forecasts. Overall, the realized values of GDP growth and inflation fall within the bounds of the interval, though for the inflation rate the intervals are extremely wide for practical purposes.

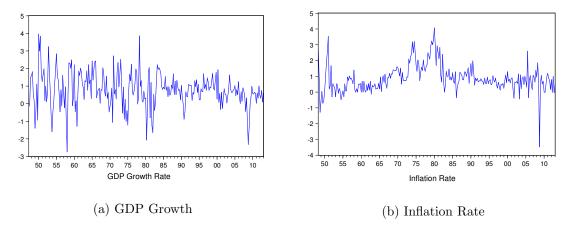


Figure 10: Time Series Plots for GDP Growth and Inflation Rate

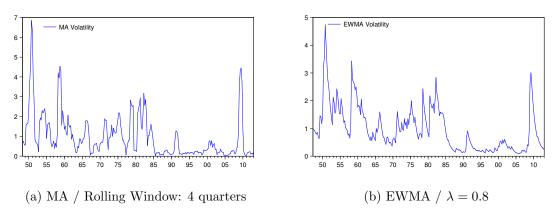


Figure 11: 1-step-ahead Volatility Forecast of GDP Growth

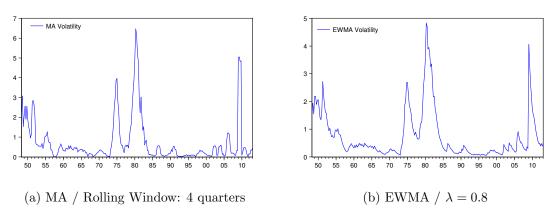


Figure 12: 1-step-ahead Volatility Forecast of Inflation Rate

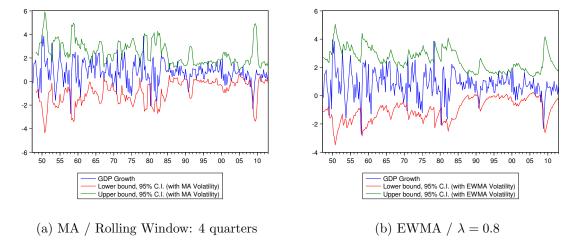


Figure 13: 95% Confidence Interval Forecast (1-step-ahead) for GDP Growth

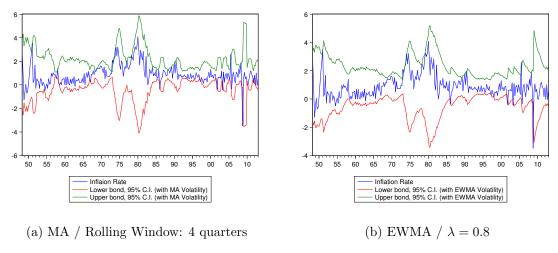


Figure 14: 95% Confidence Interval Forecast (1-step-ahead) for Inflation Rate

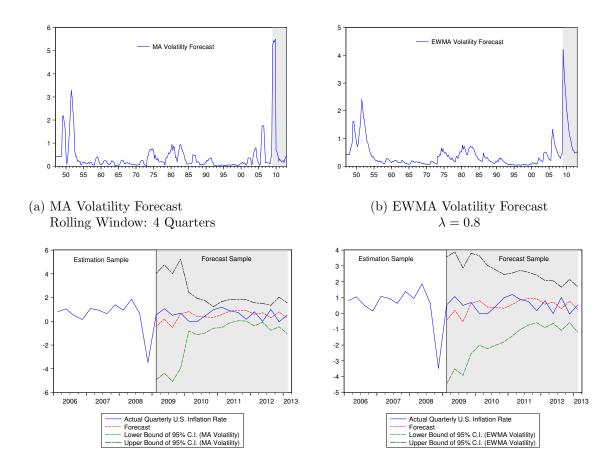
We have selected an AR(3) process for the conditional mean of both the quarterly U.S. inflation rate and real GDP growth in Exercise 5, following the specification strategies that we have studied in Chapter 8. We report the estimation results in Tables 1 and 2, and we reserve the last 17 observations (from 2009Q1 to 2013Q1) to assess the out-of-sample forecast. From each model, we compute the 1-step-ahead forecasts  $\hat{\mu}_{t+1|t}$  and the corresponding forecast errors over the last 17 observations. We need the residuals from the estimation sample and the forecast errors from the forecast sample to calculate the 1-step-ahead MA and EWMA volatility forecast  $\hat{\sigma}_{t+1|t}^2$ . The rolling window for the MA estimates is 4 quarters, and  $\lambda = 0.8$  for the EWMA estimates. Under the normality assumption, the 95% confidence interval forecast is  $[\widehat{\mu}_{t+1|t} - 1.96\widehat{\sigma}_{t+1|t}, \widehat{\mu}_{t+1|t} + 1.96\widehat{\sigma}_{t+1|t}]$ for each of the last 17 observations. In Figures 15a, 15b, 16a and 16b, we plot the MA and EWMA volatility forecasts; the shaded area denotes the last 17 observations in the forecast sample. Observe that the MA and EWMA volatility forecasts based on the conditional mean models are smaller than those forecasts based on the unconditional means (Exercise 5), in particular for the inflation rate. The conditional mean model for the inflation rate is very helpful because it captures very well the dynamics of the series on the mid-1970s to mid-1980s so that the residuals are small and, consequently, the volatility estimates are also small. In Figures 15c, 15d, 16c and 16d we plot the 95% confidence interval forecasts based on MA and EWMA volatility forecasts for the inflation rate and real GDP growth rate respectively. The intervals get narrow over the forecast sample because the shock of 2008 has already been absorbed by 2013. As expected, once the conditional mean of the series is properly modeled, the intervals are narrower than in the unconditional case because the volatility forecasts are smaller. Thus, the conditional mean models are helpful to narrow the uncertainty of the forecasts.

Dependent Variable: INFL								
Method: Least Squares								
Sample (adjusted): 1948Q1 2008Q4								
Included observations: 244 after adjustments								
Convergence achieved after 3 iterations								
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed								
bandwidth = 5.0000)								
Variable	Coefficient	Std. Error	t-Statistic	Prob.				
С	0.788041	0.213799	3.685895	0.0003				
AR(1)	0.370147	0.10452	3.541406	0.0005				
AR(2)	0.106837	0.092009	1.161164	0.2467				
AR(3)	0.326175	0.074952	4.351791	0.0000				
R-squared	0.447286	Mean dependent var		0.901877				
Adjusted R-squared	0.440377	S.D. dependent var		0.874288				
S.E. of regression	0.654037	Akaike info criterion		2.004953				
Sum squared resid	102.6635	Schwarz criterion		2.062283				
Log likelihood	-240.6042	Hannan-Quinn criter.		2.028042				
F-statistic	64.74025	Durbin-Watson stat		1.696113				
Prob(F-statistic)	0.000000							
Inverted AR Roots	0.9	26+.54i	2654i					

Table 1: Estimation Results for Quarterly Inflation Rate

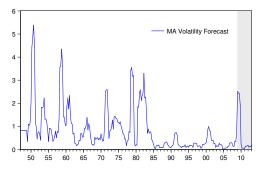
Dependent Variable: GRGDP							
Method: Least Squares							
Sample (adjusted): 1948Q1 2008Q4							
Included observations: 244 after adjustments							
Convergence achieved after 3 iterations							
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed							
bandwidth = 5.0000)							
Variable	Coefficient	Std. Error	t-Statistic	Prob.			
С	0.799274	0.092824	8.610601	0.0000			
AR(1)	0.338882	0.066103	5.126541	0.0000			
AR(2)	0.125637	0.071547	1.756005	0.0804			
AR(3)	-0.11425	0.065457	-1.745403	0.0822			
R-squared	0.141872	Mean dependent var		0.808152			
Adjusted R-squared	0.131146	S.D. dependent var		1.003848			
S.E. of regression	0.935711	Akaike info criterion		2.721237			
Sum squared resid	210.1331	Schwarz criterion		2.778568			
Log likelihood	-327.9909	Hannan-Quinn criter.		2.744327			
F-statistic	13.22621	Durbin-Watson stat		1.991356			
Prob(F-statistic)	0.000000						
Inverted AR Roots	.4029i	.40 + .29i	-0.46				

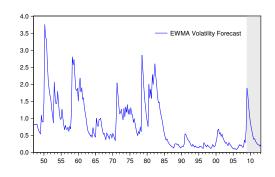
Table 2: Estimation Results for Quarterly Real GDP Growth Rate



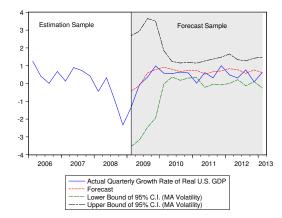
(c) 1-step-ahead 95% Confidence Interval Forecast based on MA Volatility cast based on EWMA Volatility

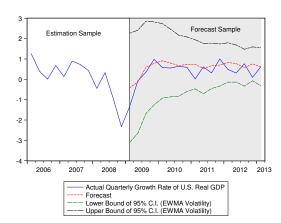
Figure 15: Quarterly U.S. Inflation





- (a) MA Volatility Forecast Rolling Window: 4 Quarters
- (b) EWMA Volatility Forecast  $\lambda = 0.8$





- (c) 1-step-ahead 95% Confidence Interval Fore- (d) 1-step-ahead 95% Confidence Interval Forecast based on MA Volatility
- cast based on EWMA Volatility

Figure 16: Quarterly U.S. Real GDP Growth Rate

We download the time series of the quarterly house price index (value of the index at the end of the quarter) for two MSAs: Chicago MSA (Chicago-Joliet-Naperville IL-IN-WI) and Miami MSA (Miami-Fort Lauderdale-Pompano Beach FL), which are denoted as "ICHI" and "IMIA" respectively.

http://www.freddiemac.com/finance/fmhpi/

The house price indices range from 1975Q1 to 2012Q4 and are plotted in Figure 17a. We take the log differences to obtain the growth rates of house price index, denoted as "GCHI" and "GMIA" respectively and plotted in Figure 17b. The housing bubble of 2008 was more pronounced in the Miami market than in Chicago and the subsequent decline in prices much more severe, though it seems that by the end of 2012, Miami is recovering at a stronger pace than Chicago. Observe that both markets exhibit strong seasonality intra-year; on average, prices tend to increase in spring and summer and to decrease in fall and winter. For each market, we calculate the 1-step-ahead volatility forecast by implementing MA with a rolling window of 4 quarters (this window will smooth the quarterly seasonality out of the variance) and EWMA with  $\lambda=0.6$ . In Figure 17c and 17d, we plot the MA and EWMA volatility forecast for the growth rates of the two MSA house price index. The episodes of large volatility are common to both markets, mainly mid-1970s to mid-1980s and the 2008 crisis with its aftermath but, in general, the Miami market is much more volatile that the Chicago market.

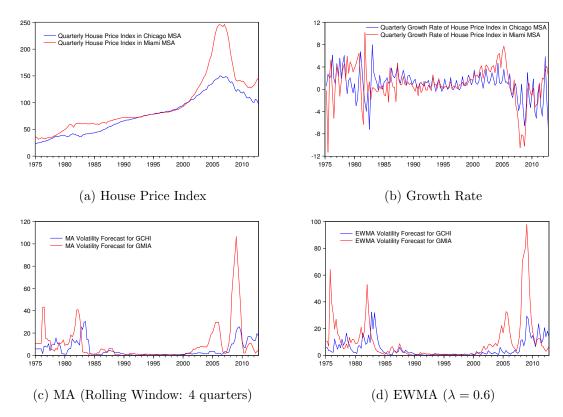


Figure 17: House Price Index, Growth Rates, and Volatility Forecasts of Two MSAs

We download the S&P500 stock price index at the daily, weekly, and monthly frequencies from Yahoo Finance website. The index prices are adjusted close prices for dividends and splits, and range from 2003/1/2 to 2013/4/29. The high frequency (daily) data are converted into low frequency (weekly or monthly) data by taking the last observation corresponding to each low frequency time interval (each week or each month). We take log differences of the three series to obtain the index returns at the three frequencies:  $r_t = 100 \times [\log p_t - \log p_{t-1}]$ . In Figure 18, we plot the three index returns series. The profiles of the time series are very similar across frequencies but as the frequency becomes lower, the series become less noisy and smoother because the very short term fluctuations are smoothed out of the data.

For each return series, we compute their 1-step-ahead MA and EWMA volatility forecasts. For EWMA, we choose the value of  $\lambda$  to be 0.95 for all series; and for MA, the sizes of rolling widow are 20 trading days, 4 weeks and 3 months for daily, weekly, and monthly data respectively. In Figures 19, 20, and 21, we plot the one-step-ahead MA and EWMA volatility forecasts for the three frequencies. There are two large episodes of volatility in the last decade, the global shock of 2008-2009 and the European sovereign crisis of 2010-2011, the former being about three order of magnitude larger than the latter. The MA volatility estimates have similar profiles across frequencies and become smoother as the frequency decreases. The EWMA volatility estimates are also smoother than the MA estimates because the smoothing parameter  $\lambda$  is very large. However, for the monthly frequency, the smoothing parameter is too large, it smooths the data too much and, as a result, we are only able to observe the 2008-2009 volatility episode.

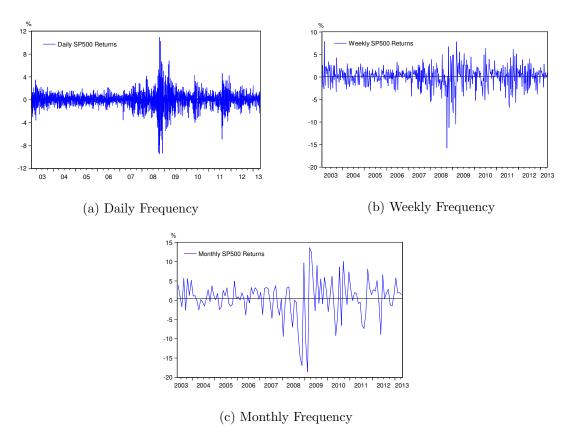


Figure 18: Returns to S&P500 Index at Three Different Frequencies

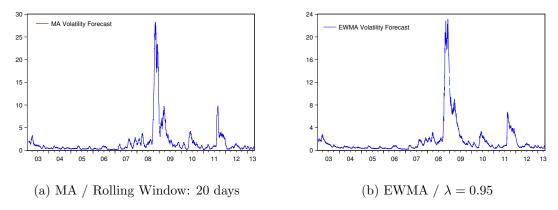


Figure 19: 1-step-ahead Volatility Forecast at Daily Frequency

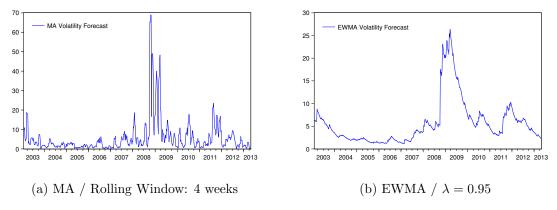


Figure 20: 1-step-ahead Volatility Forecast at Weekly Frequency

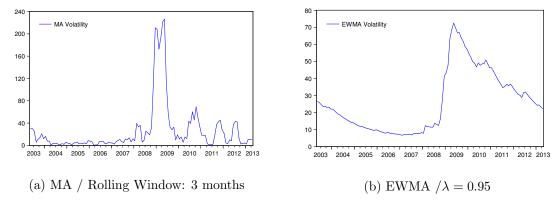
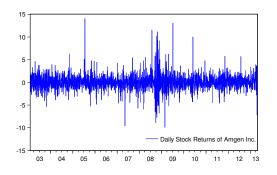
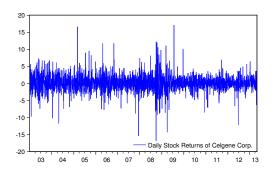


Figure 21: 1-step-ahead Volatility Forecast at Monthly Frequency

We download the daily stock prices of two biotechnology companies from Yahoo Finance website. The two companies are Amgen Incorporation (AMGN) and Celgene Corporation (CELG), both components of the S&P500 index.

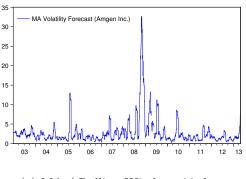
The stock prices are adjusted close prices for dividends and splits, and range from 2003/1/2 to 2013/4/29. The daily stock returns are obtain by taking log differences as  $r_t = 100 \times [\log p_t - \log p_{t-1}]$ , and are plotted in Figure 22a and 22b respectively. For each series, we compute the 1-step-ahead MA and EWMA volatility forecasts; for MA we choose a rolling window of 20 trading days and for EWMA we choose  $\lambda = 0.95$ . The volatility forecasts are shown in Figures 23 and 24. Though the volatility profiles of both companies are similar, CELG is more volatile than AMGN, and CELG was more exposed to the 2008 shock than AMGN. In comparison with the daily volatility forecasts of S&P500 returns, 19, both companies are more volatile, they have their own idiosyncratic shocks and they only share with the market the 2008 global volatility shock but not the 2011 episode. The profile of AMGEN volatility is more similar to the SP500 volatility than that of CELG; the former has a larger weight in the index than the latter.

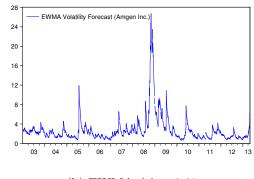




- (a) Amgen Inc. (AMGN), Thousand Oaks, CA
- (b) Celgene Corp. (CELG), Summit, NJ

Figure 22: Daily Stock Returns of Two Biotechnological Companies





(a) MA / Rolling Window: 20 days

(b) EWMA /  $\lambda = 0.95$ 

Figure 23: 1-step-ahead Volatility Forecast for AMGN

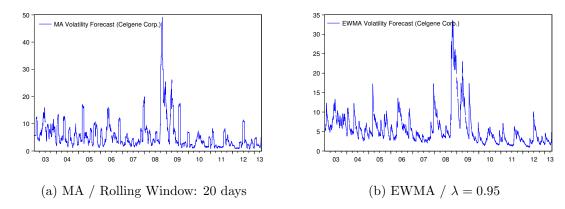


Figure 24: 1-step-ahead Volatility Forecast for CELG

We compute and plot the squared values, the absolute values, and the range (for stock returns) of the variables in Exercises 6 to 9 and their corresponding autocorrelograms in Figures 25 - 33.

We find substantial time dependence in the squared and absolute values of the quarterly U.S. inflation rate in Figure 25, indicating that the the conditional variance is time varying. The profiles of the ACF and PACF point towards an autoregressive structure in volatility. In contrast, there is very little dependence in squared and absolute growth rates of real GDP in Figure 26.

The absolute values of the quarterly growth rate of the house price indexes in Miami and Chicago exhibit more dependence than the squared values. There is a more pronounced seasonality in the absolute values of the Chicago series than in those of Miami, though Miami shows a more time dependent process that points to an autoregressive model in volatility being measured by the absolute value of the series. See Figures 27 and 28.

For the returns to the SP500 series, Figures 29, 30, and 31, observe that the time dependence in volatility fades as a function of the frequency so that the lower the frequency the weaker the dependence is. There are two commonalities across the three frequencies, first, the range is more dependent than the absolute and squared values, and second, all the profiles of the ACF and PACF seem to point out towards an autoregressive model in volatility. We also observe the same common behavior in the two individual stocks, AMGN and CELG, in Figures 32 and 33.

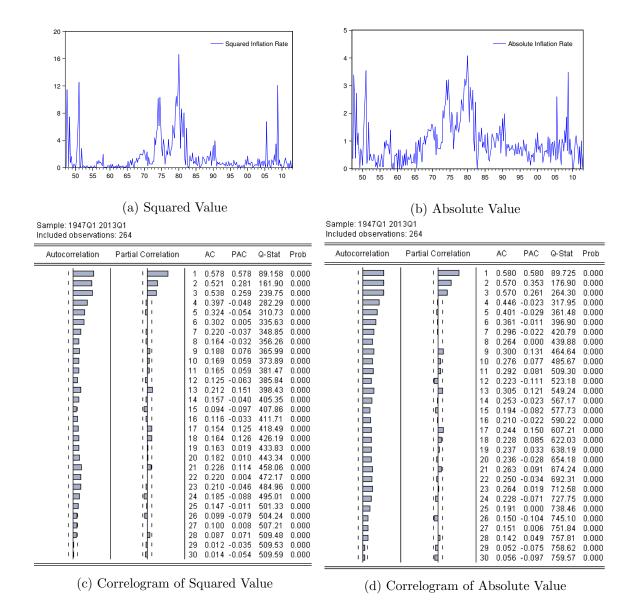


Figure 25: Quarterly U.S. Inflation Rate

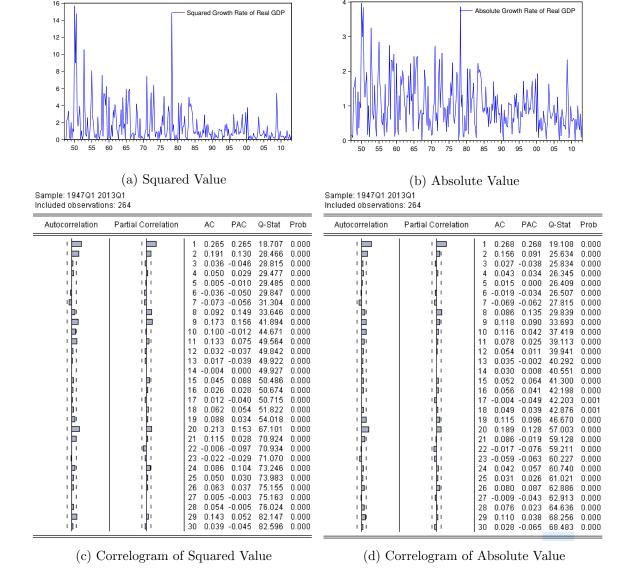


Figure 26: Quarterly Growth Rate of U.S. Real GDP

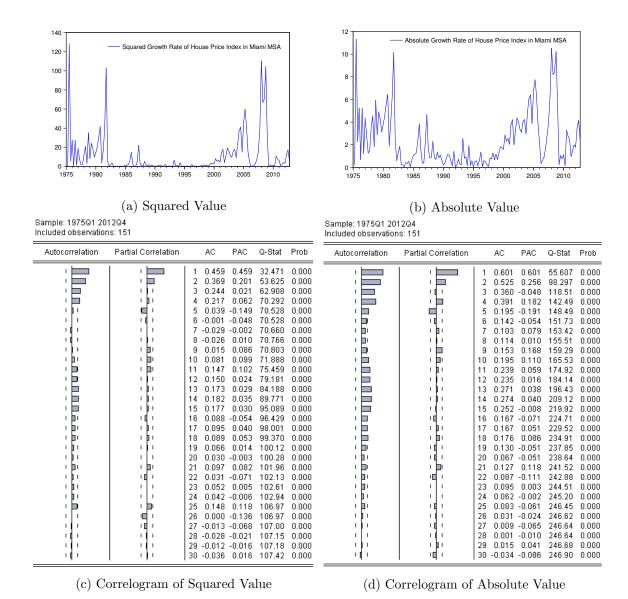


Figure 27: Quarterly Growth Rate of House Price Index in Miami MSA

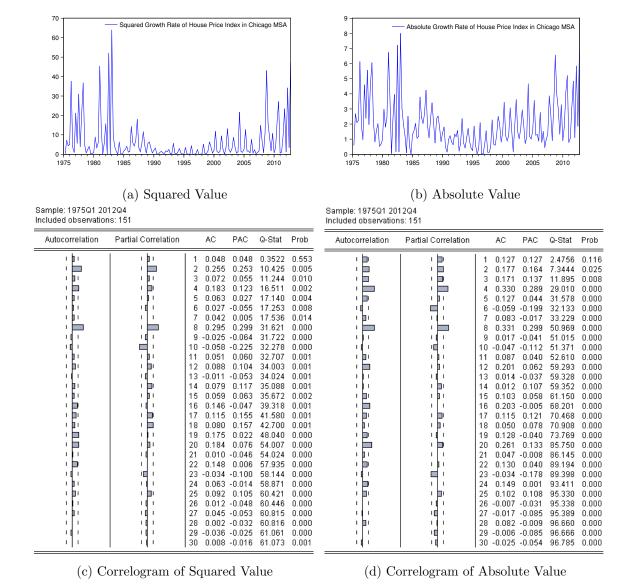


Figure 28: Quarterly Growth Rate of House Price Index in Chicago MSA

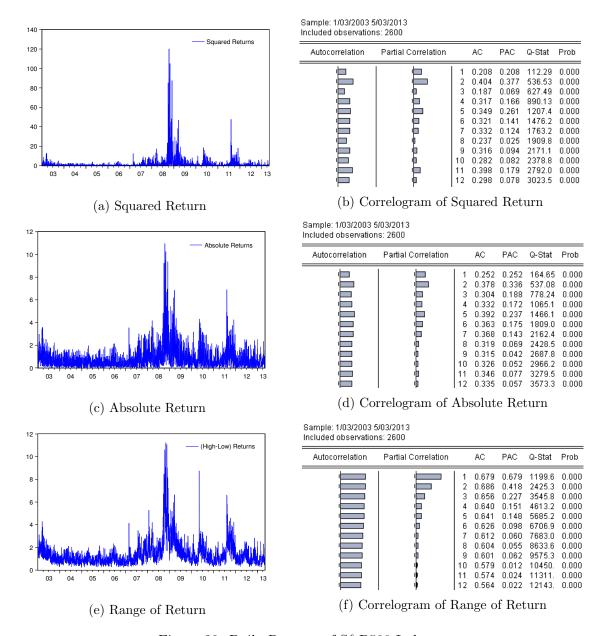


Figure 29: Daily Returns of S&P500 Index

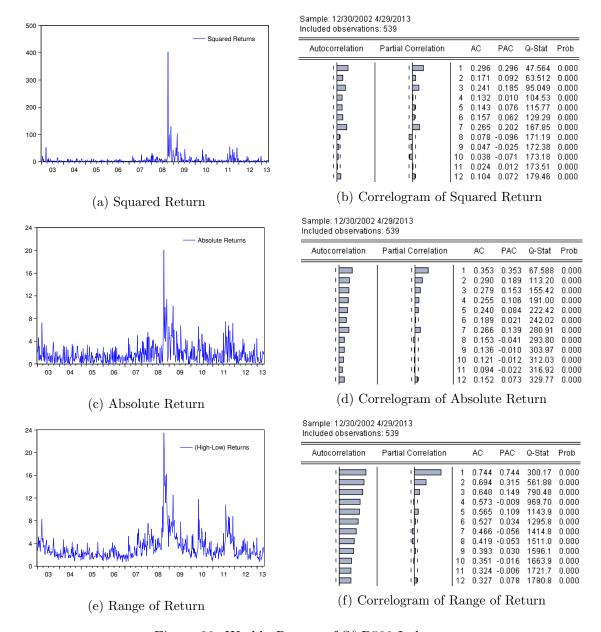


Figure 30: Weekly Return of S&P500 Index

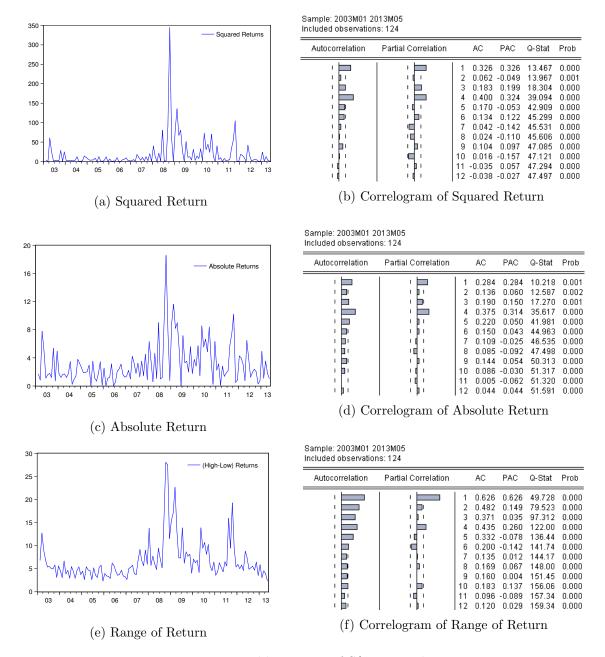


Figure 31: Monthly Return of S&P500 Index

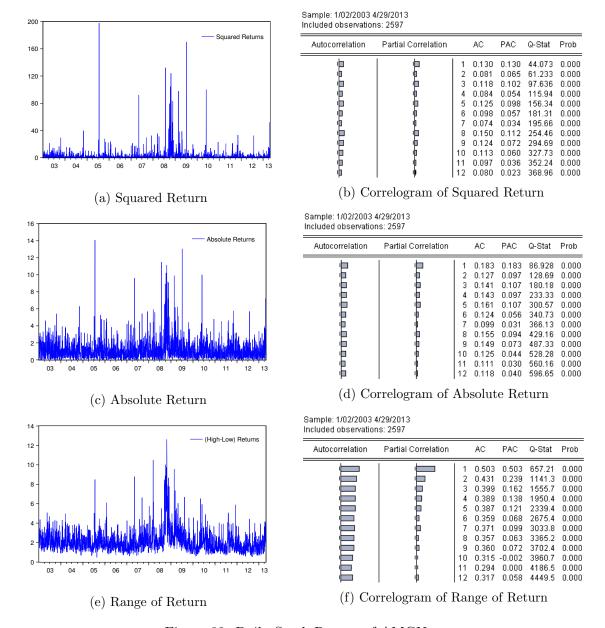


Figure 32: Daily Stock Return of AMGN

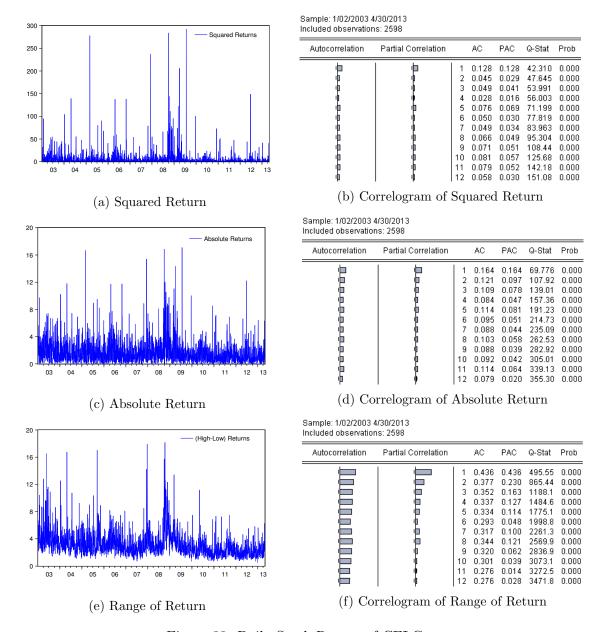


Figure 33: Daily Stock Return of CELG