CHAPTER 8.

FORECASTING PRACTICE I

SOLUTIONS

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Note: In Exercises 1, 2, 3 and 4, we work with two different time series downloaded from Freddie Mac's website. In the textbook and Exercise 1, the time series is the Conventional Mortgage Home Price Index (CMHPI), which is a weighted average of nine census region indexes. In February 2011, Freddie Mac discontinued the publication of CMHPI and replaced it with the Freddie Mac House Price Index (FMHPI) `http://www.freddiemac.com/finance/fmhpi`. In Exercises 2, 3 and 4, we work with this new data set.

Exercise 1
The results of the multistep forecasts based on the AR(4), AR(5), and ARMA(2,4) models, under a quadratic loss function, are all very similar. The forecast variances are virtually identical indicating that the extra parameters in the AR(5) and ARMA(2,4) do not contribute much to mitigate the uncertainty of the forecasts. Regarding the point forecasts, those from AR(5) are slightly below those from the ARMA(2,4) but overall they are very similar. There are some differences with the point forecasts based on the AR(4) model as these are more negative than those from the AR(5) or ARMA(2,4). In Figures 1, 2 and Figure 3, we show the multistep forecasts up 2020 Q4 for the three models. The main difference is that the forecasts from the AR(4) model converge to the unconditional mean at a slower pace (higher persistence in the model) that those from the AR(5) and ARMA(2,4) models.

The multistep forecast based on the AR(4) model, reported in the textbook, is

Forecasting Horizon	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2008:4	$f_{t,1} = -4.79\%$	$\hat{\sigma}^2_{t+1\|t} = 2.47^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2009:1	$f_{t,2} = -5.10\%$	$\hat{\sigma}^2_{t+2\|t} = 2.55^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2009:2	$f_{t,3} = -5.21\%$	$\hat{\sigma}^2_{t+3\|t} = 2.67^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

The multistep forecast based on the AR(5) model is

Forecasting Horizon	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2008:4	$f_{t,1} = -4.47\%$	$\hat{\sigma}^2_{t+1\|t} = 2.45^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2009:1	$f_{t,2} = -4.72\%$	$\hat{\sigma}^2_{t+2\|t} = 2.55^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2009:2	$f_{t,3} = -4.66\%$	$\hat{\sigma}^2_{t+3\|t} = 2.69^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

The multistep forecast based on the ARMA(2,4) model is

Forecasting Horizon	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2008:4	$f_{t,1} = -4.20\%$	$\hat{\sigma}^2_{t+1\|t} = 2.44^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2009:1	$f_{t,2} = -4.42\%$	$\hat{\sigma}^2_{t+2\|t} = 2.54^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2009:2	$f_{t,3} = -4.63\%$	$\hat{\sigma}^2_{t+3\|t} = 2.69^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

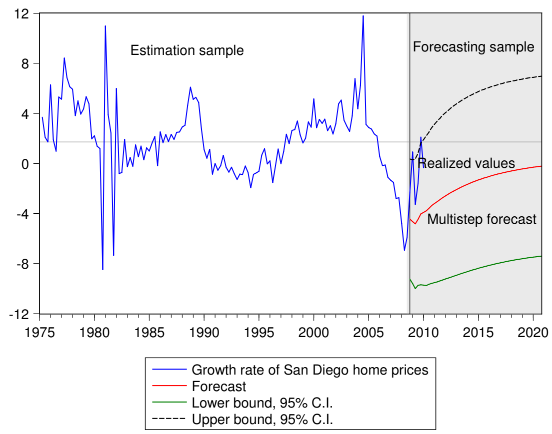


Figure 1: Growth rate of San Diego home prices : AR(4), multistep forecast

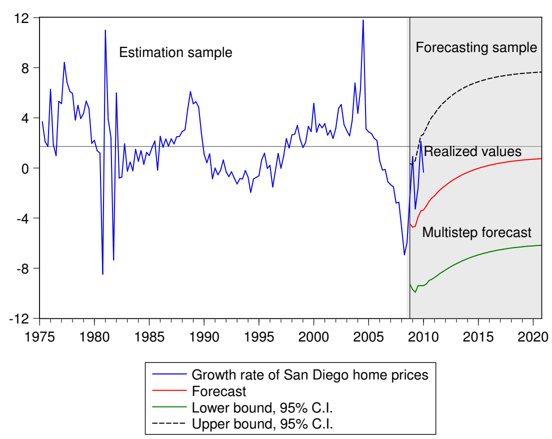


Figure 2: Growth rate of San Diego home prices : AR(5), multistep forecast

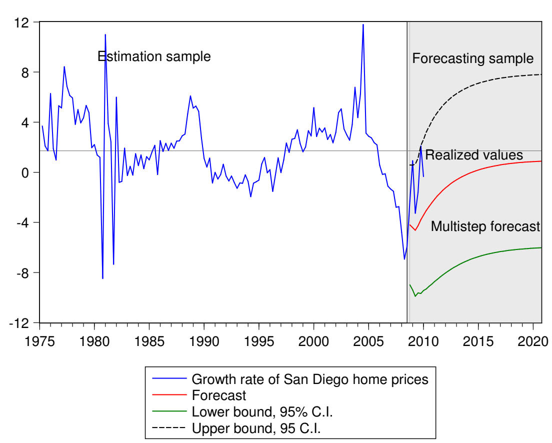
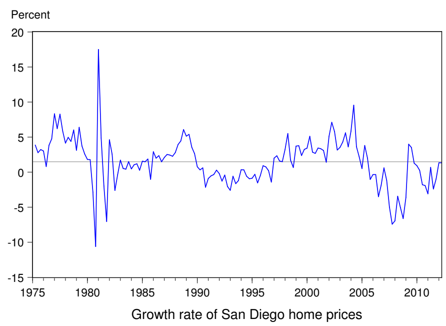


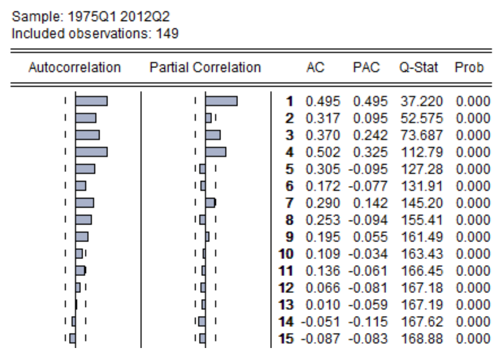
Figure 3: Growth rate of San Diego home prices : ARMA(2,4), multistep forecast

Exercise 2

We update the time series of house prices for San Diego MSA up to 2012.Q2. In Figure 4, we present the updated series and the corresponding autocorrelograms. The autocorrelation functions are very similar to those in the textbook. The AR(4) model seems to hold for the updated series. The estimation results are presented in Table 1. The significant parameters are only those corresponding to the AR(1) and AR(4) terms. The fit of the model is about 35% and the autocorrelation functions of the residuals indicate that these are white noise (see Figure 5). Overall, the AR(4) is a good model to represent the dynamics of this time series.



(a) Time series plot

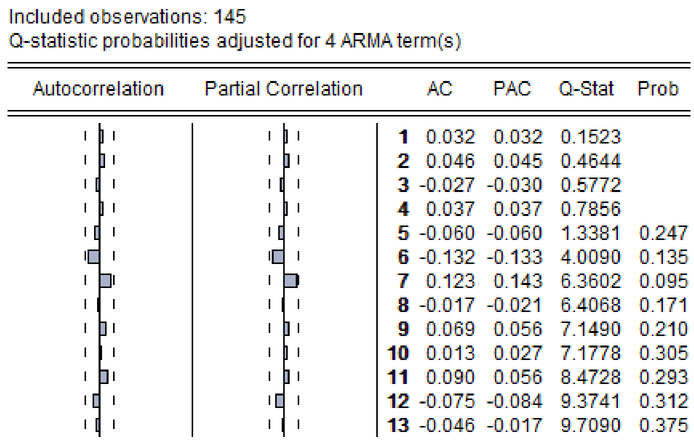


(b) ACF and PACF

Figure 4: Growth rate of San Diego home prices

Dependent Variable: SDG				
Method: Least Squares				
Sample (adjusted): 1976Q2 2012Q2				
Included observations: 145 after adjustments				
Convergence achieved after 3 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.26942	1.004546	1.263675	0.2084
AR(1)	0.342239	0.079772	4.290198	0.0000
AR(2)	-0.01172	0.084389	-0.13891	0.8897
AR(3)	0.10453	0.084422	1.238189	0.2177
AR(4)	0.332168	0.080028	4.150669	0.0001
R-squared	0.372103	Mean dependent var		1.442119
Adjusted R-squared	0.354163	S.D. dependent var		3.490718
S.E. of regression	2.805279	Akaike info criterion		4.934757
Sum squared resid	1101.743	chwarz criterion		5.037403
Log likelihood	-352.77	F-statistic		20.74164
Durbin-Watson stat	1.93163	Prob(F-statistic)		0.00000
Inverted AR Roots	0.91	.04+.76i	.04-.76i	-0.64

Table 1: Estimation results of AR(4) model



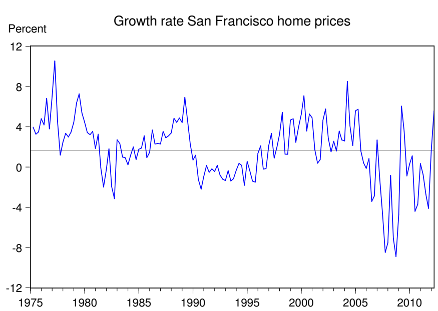
Included observations: 145
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.032	0.032	0.1523	
		2	0.046	0.045	0.4644	
		3	-0.027	-0.030	0.5772	
		4	0.037	0.037	0.7856	
		5	-0.060	-0.060	1.3381	0.247
		6	-0.132	-0.133	4.0090	0.135
		7	0.123	0.143	6.3602	0.095
		8	-0.017	-0.021	6.4068	0.171
		9	0.069	0.056	7.1490	0.210
		10	0.013	0.027	7.1778	0.305
		11	0.090	0.056	8.4728	0.293
		12	-0.075	-0.084	9.3741	0.312
		13	-0.046	-0.017	9.7090	0.375

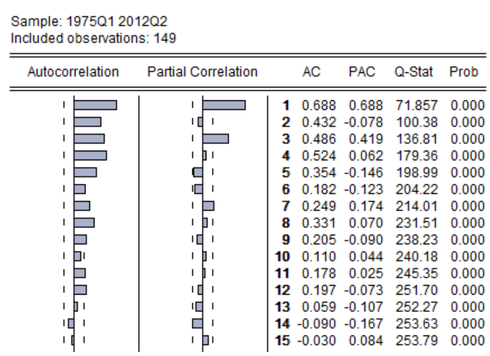
Figure 5: Autocorrelograms of residuals from AR(4) model

Exercise 3

In Figure 6, we plot the time series of the growth rate of San Francisco home prices and the corresponding autocorrelation functions. The overall profile of the series is very similar to that of San Diego with slightly less volatility. The autocorrelograms indicate that an autoregressive process may be adequate to model the dynamics of the series. Since we observe three significant spikes in the PACF, we propose an AR(3). The estimation results are in Table 2. We have excluded the latest three observations for the assessment of the multistep forecast. The three autoregressive parameters are statistically different from zero. The process is very persistent, i.e. $\hat{\phi}_1 + \hat{\phi}_2 + \hat{\phi}_3 = 0.822$. The fit of the model is vey good with an adjusted R-squared of 57%. The residuals seem to be white noise (see the autocorrelograms in Figure 7).



(a) Time series plot



Sample: 1975Q1 2012Q2
Included observations: 149

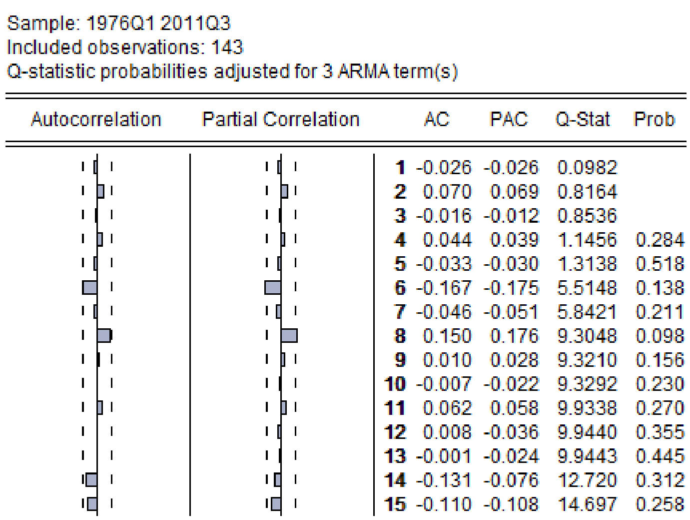
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.688	0.688	71.857	0.000
		2	0.432	-0.078	100.38	0.000
		3	0.486	0.419	136.81	0.000
		4	0.524	0.062	179.36	0.000
		5	0.354	-0.146	198.99	0.000
		6	0.182	-0.123	204.22	0.000
		7	0.249	0.174	214.01	0.000
		8	0.331	0.070	231.51	0.000
		9	0.205	-0.090	238.23	0.000
		10	0.110	0.044	240.18	0.000
		11	0.178	0.025	245.35	0.000
		12	0.197	-0.073	251.70	0.000
		13	0.059	-0.107	252.27	0.000
		14	-0.090	-0.167	253.63	0.000
		15	-0.030	0.084	253.79	0.000

(b) ACF and PACF

Figure 6: Growth rate of San Francisco home prices

Dependent Variable: SFG				
Method: Least Squares				
Sample (adjusted): 1976Q1 2011Q3				
Included observations: 143 after adjustments				
Convergence achieved after 3 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.34279	0.983373	1.365494	0.1743
AR(1)	0.768127	0.0769	9.98859	0.0000
AR(2)	-0.37893	0.095565	-3.96516	0.0001
AR(3)	0.432158	0.076929	5.617616	0.0000
R-squared	0.583331	Mean dependent var		1.61249
Adjusted R-squared	0.574339	S.D. dependent var		3.194205
S.E. of regression	2.083987	Akaike info criterion		4.334016
Sum squared resid	603.6771	Schwarz criterion		4.416893
Log likelihood	-305.882	Hannan-Quinn criter.		4.367693
F-statistic	64.86615	Durbin-Watson stat		2.047844
Prob(F-statistic)	0.000000			
Inverted AR Roots	0.89	-.06+.69i	-.06-.69i	

Table 2: Estimation results of AR(3)



Sample: 1976Q1 2011Q3
Included observations: 143
Q-statistic probabilities adjusted for 3 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	-0.026	-0.026	0.0982	
2	0.070	0.069	0.8164	
3	-0.016	-0.012	0.8536	
4	0.044	0.039	1.1456	0.284
5	-0.033	-0.030	1.3138	0.518
6	-0.167	-0.175	5.5148	0.138
7	-0.046	-0.051	5.8421	0.211
8	0.150	0.176	9.3048	0.098
9	0.010	0.028	9.3210	0.156
10	-0.007	-0.022	9.3292	0.230
11	0.062	0.058	9.9338	0.270
12	0.008	-0.036	9.9440	0.355
13	-0.001	-0.024	9.9443	0.445
14	-0.131	-0.076	12.720	0.312
15	-0.110	-0.108	14.697	0.258

Figure 7: ACF and PACF of residuals, AR(3) model

The forecasting results based on AR(3) are reported in the following table and in Figure 8:

Forecasting Horizon h	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2011:4	$f_{t,1} = -1.42\%$	$\hat{\sigma}^2_{t+1\|t} = 2.11^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2012:1	$f_{t,2} = -0.15\%$	$\hat{\sigma}^2_{t+2\|t} = 2.66^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2012:2	$f_{t,3} = -0.52\%$	$\hat{\sigma}^2_{t+3\|t} = 2.69^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

The model is not able to detect the large uprise in the second quarter of 2012, which somehow was a surprise, but was able to detect a change of direction in the growth rate.

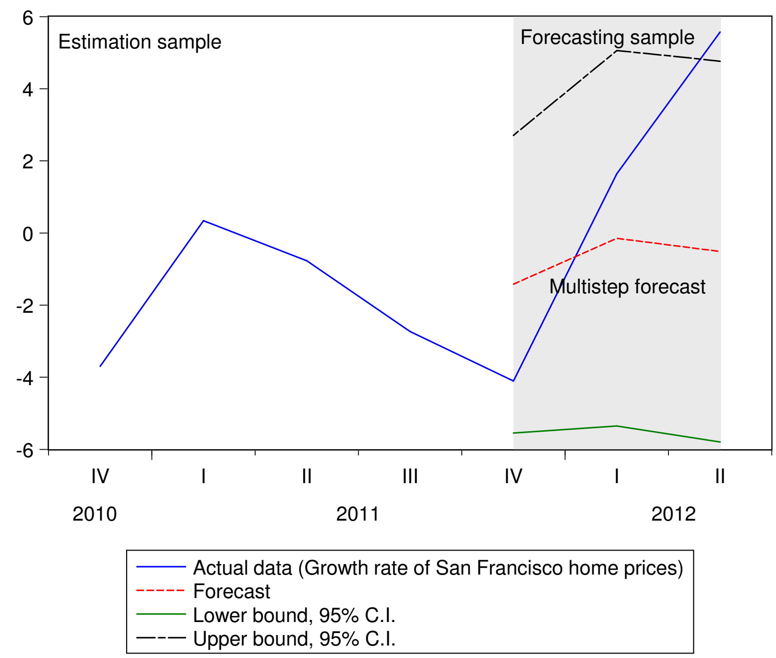


Figure 8: Multistep forecast based on AR(3) model

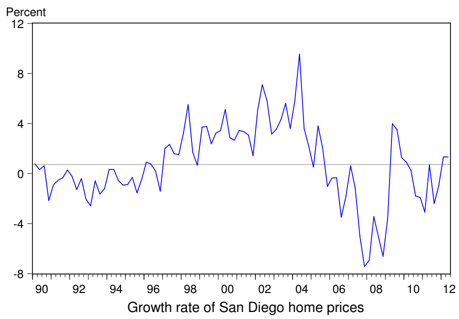
Exercise 4
We download the time series of regional employment for San Diego MSA from the Bureau of Labor Statistics (BLS)

`http://www.bls.gov/data/#employment`

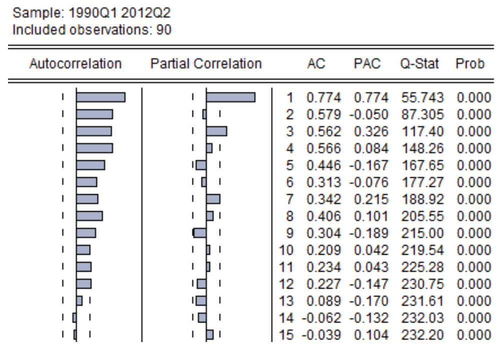
and the time series of the national 30-year fixed mortgage rate from Freddie Mac's website

`http://www.freddiemac.com/pmms/pmms30.htm`

In Figures 9, 10, and 11, we plot the time series and autocorrelation functions of the quarterly growth rate of San Diego home prices, the quarterly percentage change in mortgage rates, and the quarterly growth rate of regional employment respectively, from 1990.Q1 to 2012.Q2.



(a) Time series plot

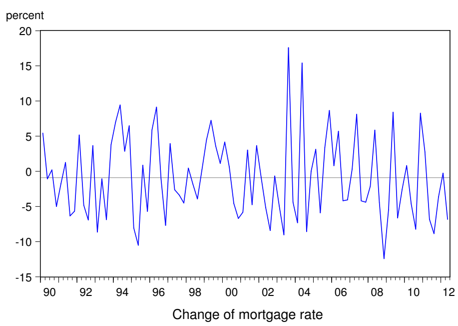


Sample: 1990Q1 2012Q2
Included observations: 90

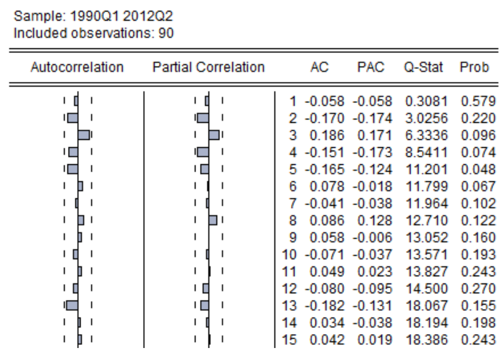
	AC	PAC	Q-Stat	Prob
1	0.774	0.774	55.743	0.000
2	0.579	-0.050	87.305	0.000
3	0.562	0.326	117.40	0.000
4	0.566	0.084	148.26	0.000
5	0.446	-0.167	167.65	0.000
6	0.313	-0.076	177.27	0.000
7	0.342	0.215	188.92	0.000
8	0.406	0.101	205.55	0.000
9	0.304	-0.189	215.00	0.000
10	0.209	0.042	219.54	0.000
11	0.234	0.043	225.28	0.000
12	0.227	-0.147	230.75	0.000
13	0.089	-0.170	231.61	0.000
14	-0.062	-0.132	232.03	0.000
15	-0.039	0.104	232.20	0.000

(b) ACF and PACF

Figure 9: Growth rate of San Diego home prices



(c) Time series plot

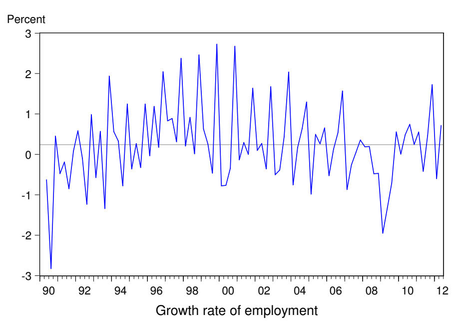


Sample: 1990Q1 2012Q2
Included observations: 90

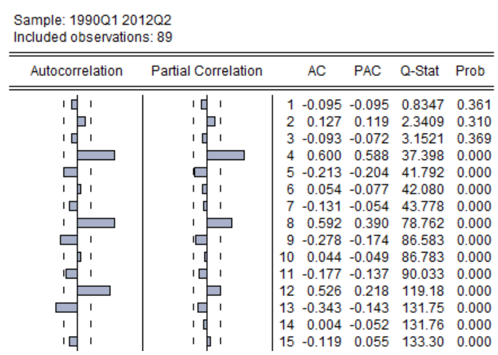
	AC	PAC	Q-Stat	Prob
1	-0.058	-0.058	0.3081	0.579
2	-0.170	-0.174	3.0256	0.220
3	0.186	0.171	6.3336	0.096
4	-0.151	-0.173	8.5411	0.074
5	-0.165	-0.124	11.201	0.048
6	0.078	-0.018	11.799	0.067
7	-0.041	-0.038	11.964	0.102
8	0.086	0.128	12.710	0.122
9	0.058	-0.006	13.052	0.160
10	-0.071	-0.037	13.571	0.193
11	0.049	0.023	13.827	0.243
12	-0.080	-0.095	14.500	0.270
13	-0.182	-0.131	18.067	0.155
14	0.034	-0.038	18.194	0.198
15	0.042	0.019	18.386	0.243

(d) ACF and PACF

Figure 10: Percent changes in 30-year fixed mortgage rates



(a) Time series plot



Sample: 1990Q1 2012Q2
Included observations: 89

	AC	PAC	Q-Stat	Prob
1	-0.095	-0.095	0.8347	0.361
2	0.127	0.119	2.3409	0.310
3	-0.093	-0.072	3.1521	0.369
4	0.600	0.588	37.398	0.000
5	-0.213	-0.204	41.792	0.000
6	0.054	-0.077	42.080	0.000
7	-0.131	-0.054	43.778	0.000
8	0.592	0.390	78.762	0.000
9	-0.278	-0.174	86.583	0.000
10	0.044	-0.049	86.783	0.000
11	-0.177	-0.137	90.033	0.000
12	0.526	0.218	119.18	0.000
13	-0.343	-0.143	131.75	0.000
14	0.004	-0.052	131.76	0.000
15	-0.119	0.055	133.30	0.000

(b) ACF and PACF

Figure 11: Growth rates of employment in San Diego

From the autocorrelation functions, we propose an autoregressive process for the growth rate of San Diego home prices, as we have already analyzed in Exercise 2. The percentage change in mortgage rates looks like a white noise process, and the growth rate of employment is dominated by a seasonal component at the quarterly frequency.

We will consider the following four specifications:

AR(4)	$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4} + \varepsilon_t$
Model i.	$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4} + \alpha_1 \Delta r_{t-1} + \varepsilon_t$
Model ii.	$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4}$ $+ \beta_1 \Delta emp_{t-1} + \beta_2 \Delta emp_{t-2} + \beta_3 \Delta emp_{t-3} + \beta_4 \Delta emp_{t-4} + \varepsilon_t$
Model iii.	$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4}$ $+ \alpha_1 \Delta r_{t-1} + \beta_1 \Delta emp_{t-1} + \beta_3 \Delta emp_{t-3} + \beta_4 \Delta emp_{t-4} + \varepsilon_t$

According to the autocorrelograms of the San Diego price growth, an AR(3) will suffice to model this series in this period of time but we will carry an AR(4) specification to conform to the question in the textbook. As we will see next, the parameter corresponding to the AR(4) term will not be significantly different from zero. We report the estimation results of these four models in Tables 3, 4, 5, and 6 respectively. We summarize all the estimation results in Table 7, and in Figure 12, we present the autocorrelograms of the residuals for each model. For the AR(4) model presented in Table 3, we observe that the order of the process is in fact AR(3) as the autocorrelograms were indicating. The fit of the model is very good with an adjusted R-squares of 64%. The persistence of the model is high around 0.70 and the residuals of the model seem to behave as white noise.

Dependent Variable: SDG				
Method: Least Squares				
Sample (adjusted): 1991Q1 2011Q3				
Included observations: 83 after adjustments				
Convergence achieved after 3 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.609925	1.587885	0.384112	0.7019
AR(1)	0.816748	0.113571	7.191536	0
AR(2)	-0.31411	0.147085	-2.13554	0.0359
AR(3)	0.273653	0.149477	1.830739	0.071
AR(4)	0.092383	0.116249	0.794705	0.4292
R-squared	0.65872	Mean dependent var		0.766382
Adjusted R-squared	0.641218	S.D. dependent var		3.162499
S.E. of regression	1.894287	Akaike info criterion		4.173912
Sum squared resid	279.8892	Schwarz criterion		4.319625
Log likelihood	-168.217	Hannan-Quinn criter.		4.232451
F-statistic	37.63777	Durbin-Watson stat		1.910367
Prob(F-statistic)	0			
Inverted AR Roots	0.92	.06-.66i	.06+.66i	-0.23

Table 3: Estimation results for AR(4)

Dependent Variable: SDG				
Method: Least Squares				
Sample (adjusted): 1991Q2 2012Q2				
Included observations: 85 after adjustments				
Convergence achieved after 7 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.825725	1.422165	0.580611	0.5632
RATE(-1)	-0.05194	0.025287	-2.05397	0.0433
AR(1)	0.879718	0.11459	7.677097	0
AR(2)	-0.41562	0.146794	-2.83128	0.0059
AR(3)	0.34465	0.144578	2.383832	0.0195
AR(4)	0.049311	0.112929	0.436656	0.6636
R-squared	0.667657	Mean dependent var		0.778991
Adjusted R-squared	0.646623	S.D. dependent var		3.126127
S.E. of regression	1.858343	Akaike info criterion		4.145221
Sum squared resid	272.8217	Schwarz criterion		4.317643
Log likelihood	-170.172	Hannan-Quinn criter.		4.214574
F-statistic	31.74126	Durbin-Watson stat		1.972406
Prob(F-statistic)	0			
Inverted AR Roots	0.91	.05-.67i	.05+.67i	-0.12

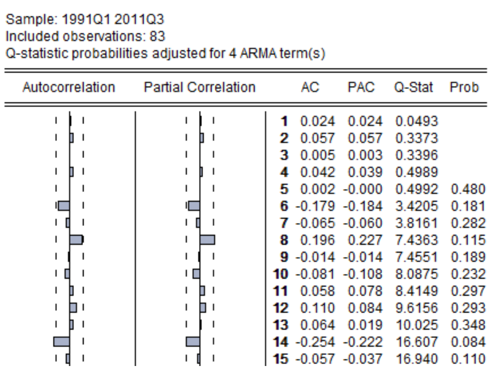
Table 4: Estimation results for Model i.

Dependent Variable: SDG				
Method: Least Squares				
Sample (adjusted): 1992Q2 2011Q3				
Included observations: 78 after adjustments				
Convergence achieved after 12 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.014033	2.025005	0.500756	0.6181
EMPLY(-1)	-0.29906	0.305837	-0.97784	0.3316
EMPLY(-2)	0.101069	0.347995	0.290433	0.7724
EMPLY(-3)	-0.3447	0.345426	-0.99789	0.3218
EMPLY(-4)	-0.65116	0.301618	-2.1589	0.0343
AR(1)	0.807413	0.124294	6.495973	0
AR(2)	-0.20485	0.162057	-1.26408	0.2105
AR(3)	0.221108	0.167293	1.321681	0.1906
AR(4)	0.065753	0.128908	0.510077	0.6116
R-squared	0.68521	Mean dependent var		0.837277
Adjusted R-squared	0.648713	S.D. dependent var		3.24905
S.E. of regression	1.925695	Akaike info criterion		4.256618
Sum squared resid	255.8728	Schwarz criterion		4.528546
Log likelihood	-157.008	Hannan-Quinn criter.		4.365475
F-statistic	18.77425	Durbin-Watson stat		1.945812
Prob(F-statistic)	0			
Inverted AR Roots	0.93	.05-.58i	.05+.58i	-0.21

Table 5: Estimation results for Model ii.

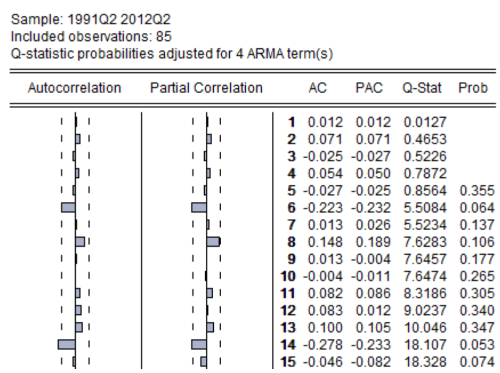
Dependent Variable: SDG				
Method: Least Squares				
Sample (adjusted): 1992Q2 2011Q3				
Included observations: 78 after adjustments				
Convergence achieved after 14 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.998569	2.03613	0.490425	0.6254
RATE(-1)	-0.03551	0.027578	-1.28771	0.2021
EMPLY(-1)	-0.30531	0.225238	-1.35549	0.1797
EMPLY(-3)	-0.34038	0.265503	-1.282	0.2041
EMPLY(-4)	-0.59321	0.282717	-2.09826	0.0395
AR(1)	0.858417	0.127513	6.731984	0
AR(2)	-0.30611	0.170075	-1.79988	0.0763
AR(3)	0.281109	0.172304	1.631473	0.1073
AR(4)	0.059482	0.129124	0.460662	0.6465
R-squared	0.691145	Mean dependent var		0.837277
Adjusted R-squared	0.655336	S.D. dependent var		3.24905
S.E. of regression	1.907456	Akaike info criterion		4.237585
Sum squared resid	251.0489	Schwarz criterion		4.509513
Log likelihood	-156.266	Hannan-Quinn criter.		4.346443
F-statistic	19.30072	Durbin-Watson stat		1.928932
Prob(F-statistic)	0			
Inverted AR Roots	0.93	.05+.62i	.05-.62i	-0.17

Table 6: Estimation results for Model iii.


Sample: 1991Q1 2011Q3
Included observations: 83
Q-statistic probabilities adjusted for 4 ARMA term(s)

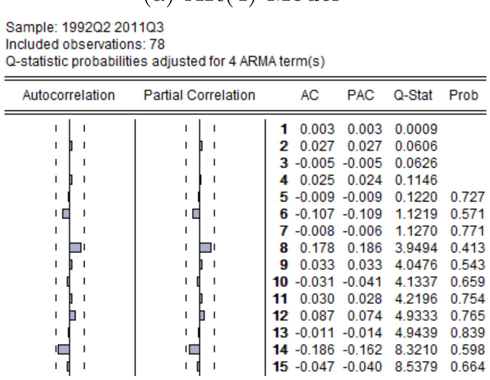
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.024	0.024	0.0493	
		2	0.057	0.057	0.3373	
		3	0.005	0.003	0.3396	
		4	0.042	0.039	0.4989	
		5	0.002	-0.000	0.4992	0.480
		6	-0.179	-0.184	3.4205	0.181
		7	-0.065	-0.060	3.8161	0.282
		8	0.196	0.227	7.4363	0.115
		9	-0.014	-0.014	7.4551	0.189
		10	-0.081	-0.108	8.0875	0.232
		11	0.058	0.078	8.4149	0.297
		12	0.110	0.084	9.6156	0.293
		13	0.064	0.019	10.025	0.348
		14	-0.254	-0.222	16.607	0.084
		15	-0.057	-0.037	16.940	0.110

(a) AR(4) Model


Sample: 1991Q2 2012Q2
Included observations: 85
Q-statistic probabilities adjusted for 4 ARMA term(s)

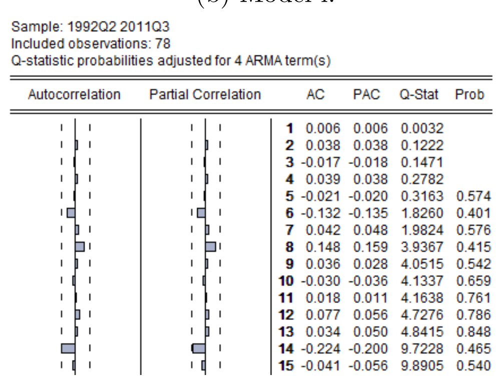
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.012	0.012	0.0127	
		2	0.071	0.071	0.4653	
		3	-0.025	-0.027	0.5226	
		4	0.054	0.050	0.7872	
		5	-0.027	-0.025	0.8564	0.355
		6	-0.223	-0.232	5.5084	0.064
		7	0.013	0.026	5.5234	0.137
		8	0.148	0.189	7.6283	0.106
		9	0.013	-0.004	7.6457	0.177
		10	-0.004	-0.011	7.6474	0.265
		11	0.082	0.086	8.3186	0.305
		12	0.083	0.012	9.0237	0.340
		13	0.100	0.105	10.046	0.347
		14	-0.278	-0.233	18.107	0.053
		15	-0.046	-0.082	18.328	0.074

(b) Model i.


Sample: 1992Q2 2011Q3
Included observations: 78
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.003	0.003	0.0009	
		2	0.027	0.027	0.0606	
		3	-0.005	-0.005	0.0626	
		4	0.025	0.024	0.1146	
		5	-0.009	-0.009	0.1220	0.727
		6	-0.107	-0.109	1.1219	0.571
		7	-0.008	-0.006	1.1270	0.771
		8	0.178	0.186	3.9494	0.413
		9	0.033	0.033	4.0476	0.543
		10	-0.031	-0.041	4.1337	0.659
		11	0.030	0.028	4.2196	0.754
		12	0.087	0.074	4.9333	0.765
		13	-0.011	-0.014	4.9439	0.839
		14	-0.186	-0.162	8.3210	0.598
		15	-0.047	-0.040	8.5379	0.664

(c) Model ii.


Sample: 1992Q2 2011Q3
Included observations: 78
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.006	0.006	0.0032	
		2	0.038	0.038	0.1222	
		3	-0.017	-0.018	0.1471	
		4	0.039	0.038	0.2782	
		5	-0.021	-0.020	0.3163	0.574
		6	-0.132	-0.135	1.8260	0.401
		7	0.042	0.048	1.9824	0.576
		8	0.148	0.159	3.9367	0.415
		9	0.036	0.028	4.0515	0.542
		10	-0.030	-0.036	4.1337	0.659
		11	0.018	0.011	4.1638	0.761
		12	0.077	0.056	4.7276	0.786
		13	0.034	0.050	4.8415	0.848
		14	-0.224	-0.200	9.7228	0.465
		15	-0.041	-0.056	9.8905	0.540

(d) Model iii.

Figure 12: Correlograms of the residuals for the four models

	AR(4)	Model i.	Model ii.	Model iii.
θ (t-ratio) ϕ (t-ratio) α (t-ratio) β (t-ratio)	$\hat{\phi}_1 = 0.82(7.19)$ $\hat{\phi}_2 = -0.31(-2.14)$ $\hat{\phi}_3 = 0.27(1.83)$ $\hat{\phi}_4 = 0.09(0.79)$	$\hat{\phi}_1 = 0.88(7.68)$ $\hat{\phi}_2 = -0.42(-2.83)$ $\hat{\phi}_3 = 0.34(2.38)$ $\hat{\phi}_4 = 0.05(0.44)$ $\hat{\alpha}_1$=-0.05 (-2.05)	$\hat{\phi}_1 = 0.81(6.50)$ $\hat{\phi}_2 = -0.20(-1.26)$ $\hat{\phi}_3 = 0.22(1.32)$ $\hat{\phi}_4 = 0.07(0.51)$ $\hat{\beta}_1$=-0.30 (-0.98) $\hat{\beta}_2$=0.10 (0.29) $\hat{\beta}_3$=-0.34 (-1.00) $\hat{\beta}_4$=-0.65 (-2.16)	$\hat{\phi}_1 = 0.86(6.73)$ $\hat{\phi}_2 = -0.31(-1.80)$ $\hat{\phi}_3 = 0.28(1.63)$ $\hat{\phi}_4 = 0.06(0.46)$ $\hat{\alpha}_1$=-0.04 (-1.29) $\hat{\beta}_1$=-0.31 (-1.36) $\hat{\beta}_3$=-0.34(-1.28) $\hat{\beta}_4$=-0.59 (-2.09)
Convariance stationary	yes	yes	yes	yes
Invertibility	yes	yes	yes	yes
White noise residuals? Q-statistics (p-value)	yes $Q_5 = 0.50(0.48)$ $Q_8 = 7.44(0.12)$	yes $Q_5 = 0.86(0.36)$ $Q_8 = 7.63(0.11)$	yes $Q_5 = 0.12(0.73)$ $Q_10 = 4.13(0.66)$	yes $Q_5 = 0.32(0.57)$ $Q_12 = 0.47(0.79)$
Residual variance	3.57	3.46	3.72	3.65
Adjusted R-squared	0.64	0.65	0.65	0.66
AIC	4.17	4.15	4.26	4.24
SIC	4.32	4.32	4.53	4.51

Table 7: Summary of model estimation and selection

When we add information about mortgage rates, Model i., these are marginally significant but the goodness of fit and the dynamics of the model remain the same. When we add employment, Model ii., the statistical significance of the AR terms substantially decreases but the overall fit of the model remains the same. This may be a sign of multicollinearity between employment and the dynamics of the series. In Model iii. , when we add mortgage rates and employment jointly, we do not find any major improvement in the fit of the model and, with the exception of the AR(1) term, the regressors become insignificant or marginally significant. In the summary Table 7, we see that the most successful models are the AR(4) model and Model i.; these two models have the lowest residual variance and the lowest AIC and SIC. In conclusion, employment information does not provide additional value on modeling the growth rate of home prices once we have modeled properly the dynamics of the series. Mortgages rates though seem to add marginal value.

We calculate the optimal forecasts based on these four models, which are shown in the following tables. We plot the multistep forecasts and their confidence bands in Figures 13, 14, 15 and 16.

The multistep forecast based on AR(4) model is:

Forecasting Horizon h	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2011:4	$f_{t,1} = -3.12\%$	$\hat{\sigma}^2_{t+1\|t} = 1.99^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2012:1	$f_{t,2} = -1.81\%$	$\hat{\sigma}^2_{t+2\|t} = 2.57^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2012:2	$f_{t,3} = -1.01\%$	$\hat{\sigma}^2_{t+3\|t} = 2.62^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

The multistep forecast based on model i. is:

Forecasting Horizon h	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2011:4	$f_{t,1} = -3.22\%$	$\hat{\sigma}^2_{t+1\|t} = 1.95^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2012:1	$f_{t,2} = -1.63\%$	$\hat{\sigma}^2_{t+2\|t} = 2.59^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2012:2	$f_{t,3} = -0.85\%$	$\hat{\sigma}^2_{t+3\|t} = 2.66^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

The multistep forecast based on model ii. is:

Forecasting Horizon h	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2011:4	$f_{t,1} = -3.07\%$	$\hat{\sigma}^2_{t+1\|t} = 2.08^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2012:1	$f_{t,2} = -2.09\%$	$\hat{\sigma}^2_{t+2\|t} = 2.68^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2012:2	$f_{t,3} = -0.39\%$	$\hat{\sigma}^2_{t+3\|t} = 2.82^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

The multistep forecast based on model iii. is:

Forecasting Horizon h	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2011:4	$f_{t,1} = -3.25\%$	$\hat{\sigma}^2_{t+1\|t} = 2.05^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2012:1	$f_{t,2} = -2.17\%$	$\hat{\sigma}^2_{t+2\|t} = 2.71^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2012:2	$f_{t,3} = -0.57\%$	$\hat{\sigma}^2_{t+3\|t} = 2.85^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

Though the point forecasts are not very different across models, the forecast uncertainty is larger for Models ii. and iii. than for Model i. and AR(4) model. The plots show that all forecasts are very similar across models. The forecast is picking up the actual uprise in the growth rate with the actual values falling well within the 95% forecasting bands.

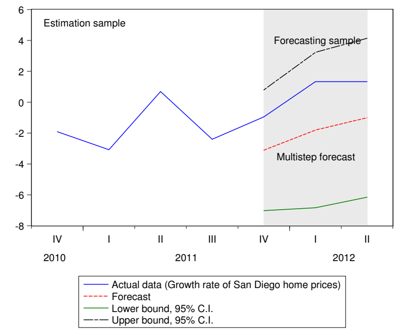


Figure 13: Multistep forecast from AR(4)

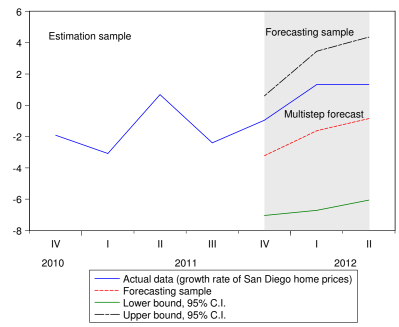


Figure 14: Multistep forecast from Model i.

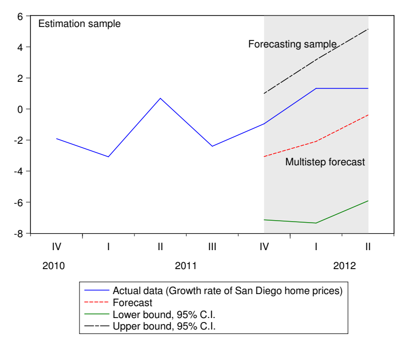


Figure 15: Multistep forecast from Model ii.

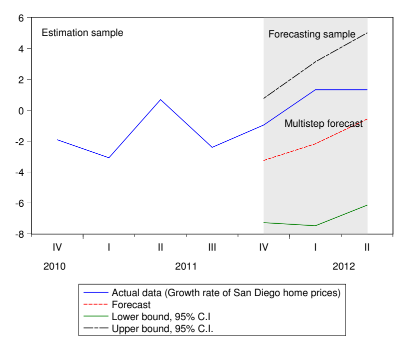


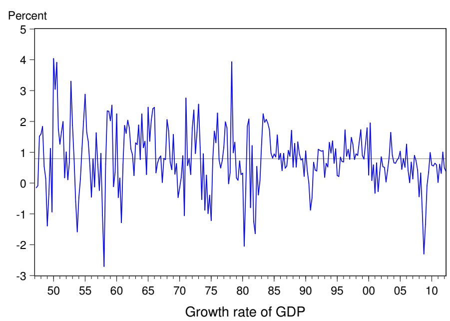
Figure 16: Multistep forecast from Model iii.

Exercise 5

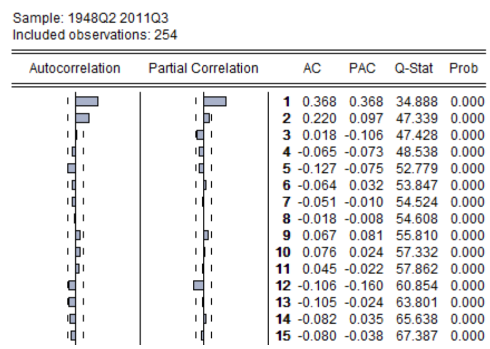
We download the time series of quarterly U.S. gross domestic product (GDP) growth from the St. Louis Federal Reserve Bank

`http://research.stlouisfed.org/fred2/series/GDPC1`

We plot the time series and the autocorrelation functions of GDP growth in Figure 17. The series runs from 1948.Q1 to 2012.Q2. From the profile of the autocorrelation functions, we suggest an autoregressive process, either AR(3) or potentially AR(4), AR(5). The ACF shows a weak sinusoidal decay towards zero but the autocorrelations are very small for displacements larger than two. For this reason, we would like to propose an ARMA model of low order.



(a) Time series plot



	AC	PAC	Q-Stat	Prob
1	0.368	0.368	34.888	0.000
2	0.220	0.097	47.339	0.000
3	0.018	-0.106	47.428	0.000
4	-0.065	-0.073	48.538	0.000
5	-0.127	-0.075	52.779	0.000
6	-0.064	0.032	53.847	0.000
7	-0.051	-0.010	54.524	0.000
8	-0.018	-0.008	54.608	0.000
9	0.067	0.081	55.810	0.000
10	0.076	0.024	57.332	0.000
11	0.045	-0.022	57.862	0.000
12	-0.106	-0.160	60.854	0.000
13	-0.105	-0.024	63.801	0.000
14	-0.082	0.035	65.638	0.000
15	-0.080	-0.038	67.387	0.000

(b) ACF and PACF

Figure 17: Growth rate of U.S. GDP

We select the following two specifications:

ARMA(2,2)	$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t$
AR(3)	$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \varepsilon_t$

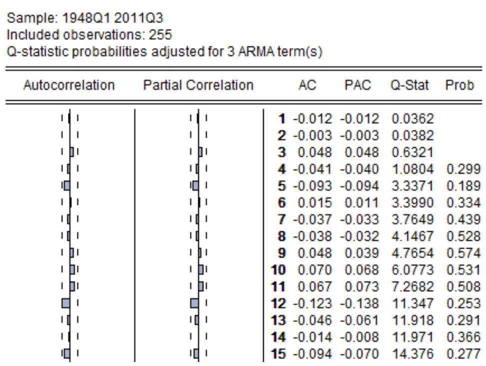
The estimation results are presented in Tables 8 and 9, and in the summary Table 10. The autocorrelograms of the residuals of these two models are shown in Figure 18.

Dependent Variable: GGDP Method: Least Squares Sample (adjusted): 1948Q1 2011Q3 Included observations: 255 after adjustments Convergence achieved after 3 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.791277	0.091413	8.656024	0.0000
AR(1)	0.348385	0.062547	5.569954	0.0000
AR(2)	0.131566	0.065708	2.002289	0.0463
AR(3)	-0.114	0.062473	-1.8248	0.0692
R-squared	0.157672	Mean dependent var		0.794036
Adjusted R-squared	0.147605	S.D. dependent var		1.002464
S.E. of regression	0.925527	Akaike info criterion		2.698656
Sum squared resid	215.0069	Schwarz criterion		2.754205
Log likelihood	-340.079	Hannan-Quinn criter.		2.721
F-statistic	15.66125	Durbin-Watson stat		2.021771
Prob(F-statistic)	0.00000			
Inverted AR Roots	.41-.28i	.41+.28i	-0.46	

Table 8: Estimation results for AR(3)

Dependent Variable: GGDP				
Method: Least Squares				
Sample (adjusted): 1947Q4 2011Q3				
Included observations: 256 after adjustments				
Convergence achieved after 27 iterations				
MA Backcast: 1947Q2 1947Q3				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.787851	0.073108	10.77657	0.0000
AR(1)	1.376195	0.095306	14.43978	0.0000
AR(2)	-0.78564	0.075403	-10.4192	0.0000
MA(1)	-1.1282	0.106917	-10.552	0.0000
MA(2)	0.651329	0.09056	7.192253	0.0000
R-squared	0.177228	Mean dependent var		0.796861
Adjusted R-squared	0.164116	S.D. dependent var		1.001517
S.E. of regression	0.915653	Akaike info criterion		2.68098
Sum squared resid	210.4436	Schwarz criterion		2.750222
Log likelihood	-338.165	Hannan-Quinn criter.		2.708829
F-statistic	13.51659	Durbin-Watson stat		1.861735
Prob(F-statistic)	0.000000			
Inverted AR Roots	.69+.56i	.69-.56i		
Inverted MA Roots	.56+.58i	.56-.58i		

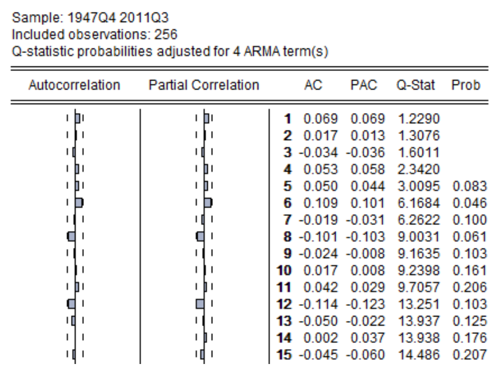
Table 9: Estimation results for ARMA(2,2)



Sample: 1948Q1 2011Q3
Included observations: 255
Q-statistic probabilities adjusted for 3 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.012	-0.012	0.0362	
		2	-0.003	-0.003	0.0382	
		3	0.048	0.048	0.6321	
		4	-0.041	-0.040	1.0804	0.299
		5	-0.093	-0.094	3.3371	0.189
		6	0.015	0.011	3.3990	0.334
		7	-0.037	-0.033	3.7649	0.439
		8	-0.038	-0.032	4.1467	0.528
		9	0.048	0.039	4.7654	0.574
		10	0.070	0.068	6.0773	0.531
		11	0.067	0.073	7.2682	0.508
		12	-0.123	-0.138	11.347	0.253
		13	-0.046	-0.061	11.918	0.291
		14	-0.014	-0.008	11.971	0.366
		15	-0.094	-0.070	14.376	0.277

(a) AR(3) Model



Sample: 1947Q4 2011Q3
Included observations: 256
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.069	0.069	1.2290	
		2	0.017	0.013	1.3076	
		3	-0.034	-0.036	1.6011	
		4	0.053	0.058	2.3420	
		5	0.050	0.044	3.0095	0.083
		6	0.109	0.101	6.1684	0.046
		7	-0.019	-0.031	6.2622	0.100
		8	-0.101	-0.103	9.0031	0.061
		9	-0.024	-0.008	9.1635	0.103
		10	0.017	0.008	9.2398	0.161
		11	0.042	0.029	9.7057	0.206
		12	-0.114	-0.123	13.251	0.103
		13	-0.050	-0.022	13.937	0.125
		14	0.002	0.037	13.938	0.176
		15	-0.045	-0.060	14.486	0.207

(b) ARMA(2,2) Model

Figure 18: Autocorrelograms of the residuals

	AR(3)	ARMA(2,2)
θ (t-ratio) ϕ (t-ratio)	$\hat{\phi}_1 = 0.35(5.57)$ $\hat{\phi}_2 = 0.13(2.00)$ $\hat{\phi}_3 = -0.11(-1.82)$	$\hat{\phi}_1 = 1.38(14.4)$ $\hat{\phi}_2 = -0.79(-10.4)$ $\hat{\theta}_1 = -1.13(-10.55)$ $\hat{\theta}_2 = 0.65(7.19)$
Convariance stationary Invertibility	yes yes	yes yes
White noise residuals? Q-statistics (p-value)	yes $Q_4 = 1.08(0.299)$ $Q_7 = 3.76(0.44)$	yes $Q_6 = 6.17(0.05)$ $Q_9 = 9.16(0.10)$
Residual variance	0.86	0.85
Adjusted R-squared	0.15	-
AIC	2.70	2.68
SIC	2.75	2.75

Table 10: Summary of model estimation and selection for GDP growth rate

Both specifications enjoy parameters that are statistically different from zero and deliver residuals that are white noise. The residual variances and the AIC and SIC are also very similar, thus from the estimation point of view, both models are almost equivalent with ARMA(2,2) providing only a marginal advantage. We calculate the optimal forecast based on the ARMA(2,2) model, which is reported in the following table (the student may also construct the forecast based on the AR(3) model). The multistep forecast is plotted in Figure 19.

Forecasting Horizon h	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, 2011:4	$f_{t,1} = 0.75\%$	$\hat{\sigma}^2_{t+1\|t} = 0.92^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, 2012:1	$f_{t,2} = 0.91\%$	$\hat{\sigma}^2_{t+2\|t} = 0.95^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, 2012:2	$f_{t,3} = 0.98\%$	$\hat{\sigma}^2_{t+3\|t} = 0.97^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

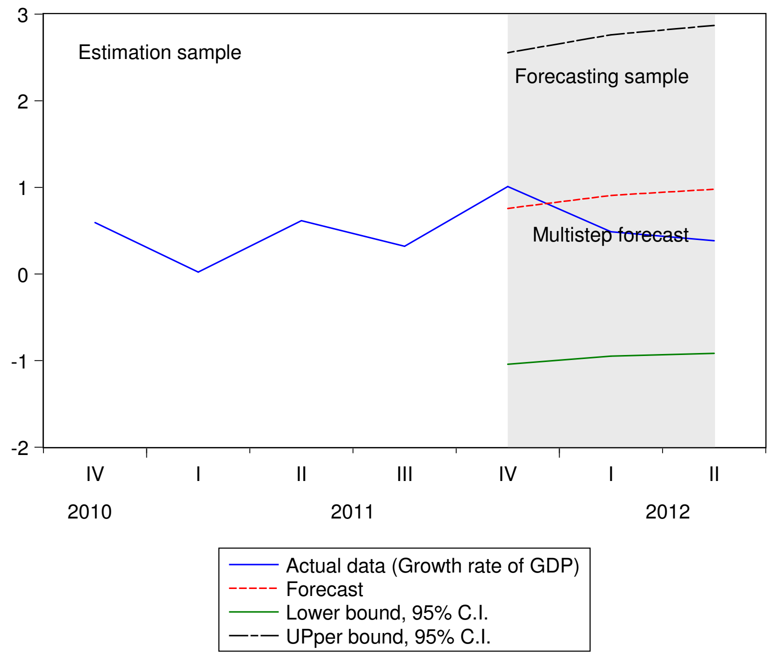
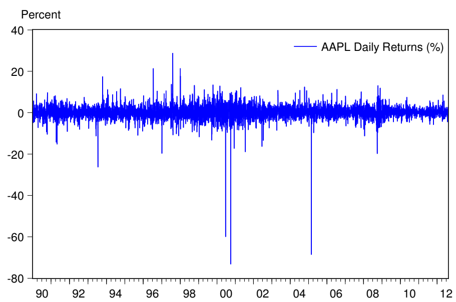


Figure 19: Multistep forecast from ARMA(2,2) model

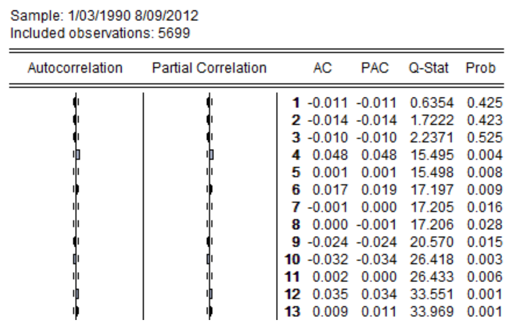
Exercise 6

We download the daily and monthly stock prices of Apple Inc. (AAPL) from YAHOO!® Finance website.

The corresponding daily and monthly returns (in percentage changes) are obtained by taking the log-difference of the stock prices. In Figures 20a and 20b we plot the time series of daily and monthly returns respectively, dating from January 1990 to August 2012. In Figures 20b and 21b we report the autocorrelation functions for both series. The time series are covariance stationary and show very little dependence both in daily and monthly frequency. At the daily frequency, the unconditional mean is basically zero, i.e. 0.05%, and the unconditional standard deviation is about 3.34%. At the monthly frequency, the unconditional mean is 1.05%, and the unconditional standard deviation is 14.90%. A test on the unconditional mean reveals that it is statistically zero. The most striking feature is the high volatility of the series with a few but very large negative and positive returns.



(a) Time Series Plot

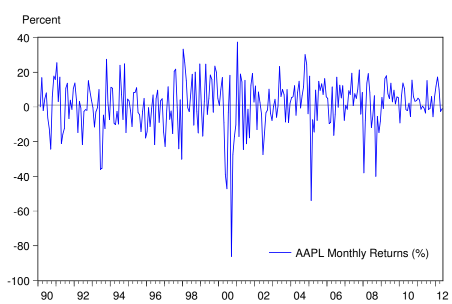


Sample: 1/03/1990 8/09/2012
Included observations: 5699

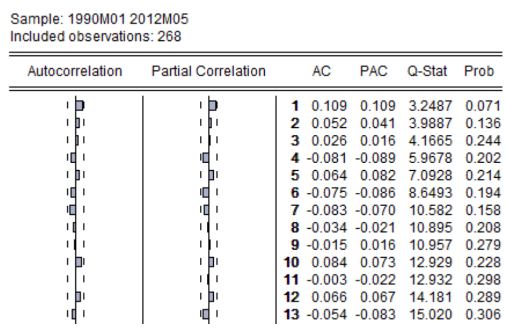
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.011	-0.011	0.6354	0.425
		2	-0.014	-0.014	1.7222	0.423
		3	-0.010	-0.010	2.2371	0.525
		4	0.048	0.048	15.495	0.004
		5	0.001	0.001	15.498	0.008
		6	0.017	0.019	17.197	0.009
		7	-0.001	0.000	17.205	0.016
		8	0.000	-0.001	17.206	0.028
		9	-0.024	-0.024	20.570	0.015
		10	-0.032	-0.034	26.418	0.003
		11	0.002	0.000	26.433	0.006
		12	0.035	0.034	33.551	0.001
		13	0.009	0.011	33.969	0.001

(b) Correlogram

Figure 20: AAPL daily returns



(a) Time series plot



Sample: 1990M01 2012M05
Included observations: 268

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.109	0.109	3.2487	0.071
		2	0.052	0.041	3.9887	0.136
		3	0.026	0.016	4.1665	0.244
		4	-0.081	-0.089	5.9678	0.202
		5	0.064	0.082	7.0928	0.214
		6	-0.075	-0.086	8.6493	0.194
		7	-0.083	-0.070	10.582	0.158
		8	-0.034	-0.021	10.895	0.208
		9	-0.015	0.016	10.957	0.279
		10	0.084	0.073	12.929	0.228
		11	-0.003	-0.022	12.932	0.298
		12	0.066	0.067	14.181	0.289
		13	-0.054	-0.083	15.020	0.306

(b) Autocorrelogram

Figure 21: AAPL monthly returns

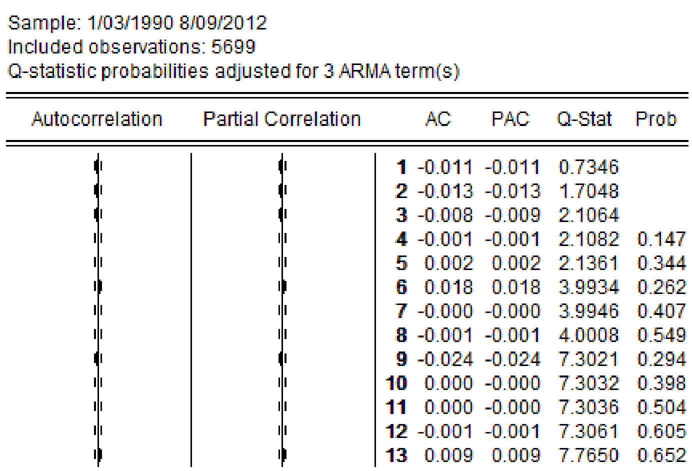
The profiles of the ACF and PACF point towards white noise processes especially at the monthly frequency. Thus, any exercise aiming to search for a linear model of dependence will be unsuccessful. However, to illustrate that the estimation stage is consistent with the message of the autocorrelation functions, we will estimate an MA(12) model for the daily returns since it seems that the autocorrelations, though very small, are statistically significant (see Q-statistics and their p-values). At the monthly frequency, there are not significant autocorrelations, and the best forecast that we can provide is the unconditional mean of the monthly returns.

In Table 11 we report the estimation results of an MA(12) model.

Dependent Variable: RETURN				
Method: Least Squares				
Sample (adjusted): 1/03/1990 8/09/2012				
Included observations: 5699 after adjustments				
Convergence achieved after 5 iterations				
MA Backcast: 12/18/1989 1/02/1990				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.049305	0.046567	1.058792	0.2897
MA(4)	0.050282	0.01322	3.803528	0.0001
MA(10)	-0.03351	0.013223	-2.53418	0.0113
MA(12)	0.036459	0.013224	2.757113	0.0059
R-squared	0.004795	Mean dependent var		0.049364
Adjusted R-squared	0.004271	S.D. dependent var		3.345096
S.E. of regression	3.337944	Akaike info criterion		5.249289
Sum squared resid	63452.97	Schwarz criterion		5.253955
Log likelihood	-14953.9	Hannan-Quinn criter.		5.250914
F-statistic	9.14712	Durbin-Watson stat		2.022694
Prob(F-statistic)	0.000005			
Inverted MA Roots	.70+.17i	.70-.17i	.57+.52i	.57-.52i
	.22-.75i	.22+.75i	-.22-.75i	-.22+.75i
	-.57+.52i	-.57-.52i	-.70+.17i	-.70-.17i

Table 11: Estimation results of MA(12) model

The three moving average parameters are statistically significant at the 5% level but their values are very small to be economically significant. Observe that the residual standard deviation (S.E. of regression) is about the same as the unconditional standard deviation of the daily returns. Thus, the model is not contributing much to explain the variation of the daily returns. However, this fitting has "cleaned up" the autocorrelation functions, and now the residuals are white noise process, see the autocorrelograms in Figure 22.



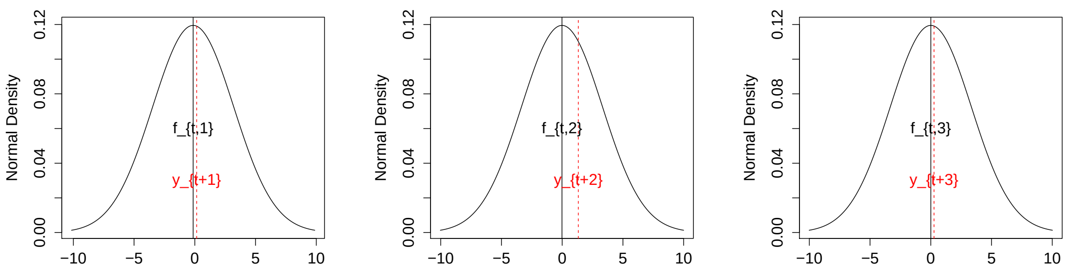
	AC	PAC	Q-Stat	Prob
1	-0.011	-0.011	0.7346	
2	-0.013	-0.013	1.7048	
3	-0.008	-0.009	2.1064	
4	-0.001	-0.001	2.1082	0.147
5	0.002	0.002	2.1361	0.344
6	0.018	0.018	3.9934	0.262
7	-0.000	-0.000	3.9946	0.407
8	-0.001	-0.001	4.0008	0.549
9	-0.024	-0.024	7.3021	0.294
10	0.000	-0.000	7.3032	0.398
11	0.000	-0.000	7.3036	0.504
12	-0.001	-0.001	7.3061	0.605
13	0.009	0.009	7.7650	0.652

Figure 22: Correlograms of the residuals for daily returns, MA(12) model

If we were to calculate a forecast based on this model, we would obtain values around the unconditional mean of zero. In the following table, we present the forecast for the next three days:

Forecasting Horizon	Actual	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, August, 10, 2012	0.1561	$f_{t,1} = -0.1345$	$\hat{\sigma}^2_{t+1\|t} = 3.3389^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, August, 13, 2012	1.3262	$f_{t,2} = -0.0177$	$\hat{\sigma}^2_{t+2\|t} = 3.3383^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, August, 14, 2012	0.2679	$f_{t,3} = 0.0033$	$\hat{\sigma}^2_{t+3\|t} = 3.3386^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$

In Figure 23 we plot the density forecast for the three days ahead. The vertical black lines are the optimal point forecasts $f_{t,h}$ (conditional expectations), and the vertical red lines are the actual daily return y_{t+h}. Observe that the three densities are basically the unconditional density of the daily returns.



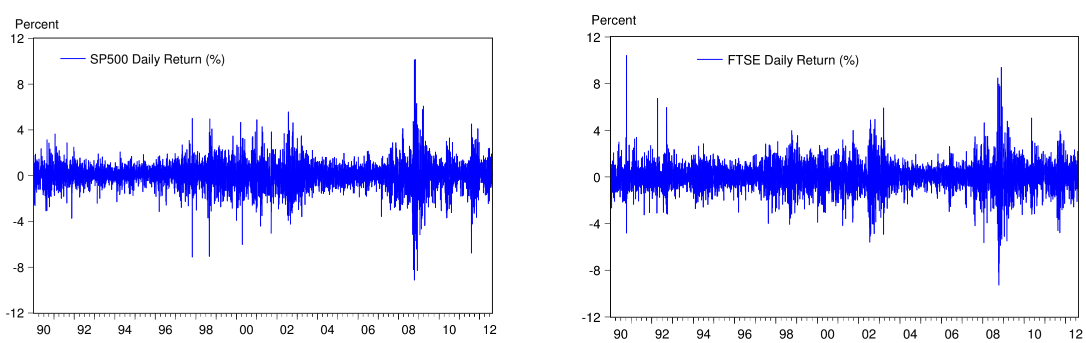
(a) $h = 1$ (b) $h = 2$ (c) $h = 3$

Figure 23: h-step-ahead density forecast of AAPL daily returns

Exercise 7

We download the SP500 and FTSE daily indexes from YAHOO!® Finance website.

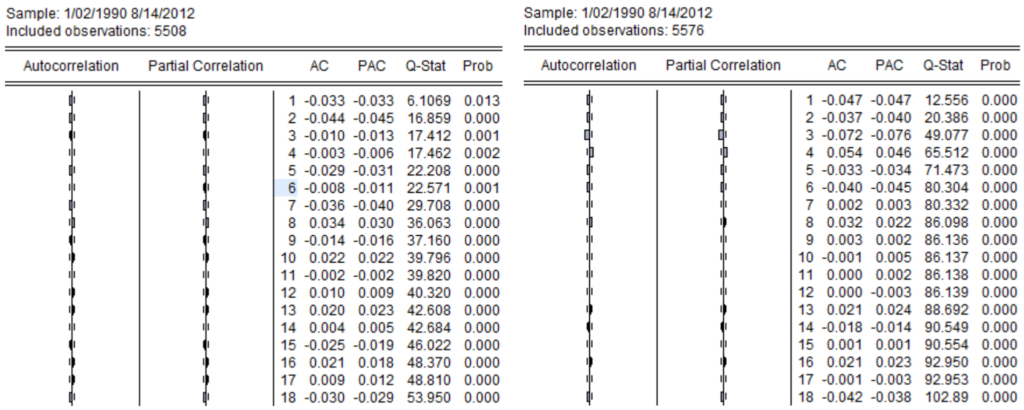
In Figure 24 we plot the time series of SP500 and FTSE daily returns, dating from January 2, 1999 to August 14, 2012. In Figure 25 we report the autocorrelation functions of the two series. For ACF and PACF, both time series have very small autocorrelations to be economically significant but the Q-statistics report statistically significant autocorrelations.



(a) SP500 daily returns (%) (b) FTSE daily returns (%)

Figure 24: Time series plots

Because of the time difference between Europe and North America, when the SP500 index begins trading in New York, the FTSE has been trading in London. To evaluate whether FTSE returns



Sample: 1/02/1990 8/14/2012
Included observations: 5508

	AC	PAC	Q-Stat	Prob
1	-0.033	-0.033	6.1069	0.013
2	-0.044	-0.045	16.859	0.000
3	-0.010	-0.013	17.412	0.001
4	-0.003	-0.006	17.462	0.002
5	-0.029	-0.031	22.208	0.000
6	-0.008	-0.011	22.571	0.001
7	-0.036	-0.040	29.708	0.000
8	0.034	0.030	36.063	0.000
9	-0.014	-0.016	37.160	0.000
10	0.022	0.022	39.796	0.000
11	-0.002	-0.002	39.820	0.000
12	0.010	0.009	40.320	0.000
13	0.020	0.023	42.608	0.000
14	0.004	0.005	42.684	0.000
15	-0.025	-0.019	46.022	0.000
16	0.021	0.018	48.370	0.000
17	0.009	0.012	48.810	0.000
18	-0.030	-0.029	53.950	0.000

(a) SP500 returns

Sample: 1/02/1990 8/14/2012
Included observations: 5576

	AC	PAC	Q-Stat	Prob
1	-0.047	-0.047	12.556	0.000
2	-0.037	-0.040	20.386	0.000
3	-0.072	-0.076	49.077	0.000
4	0.054	0.046	65.512	0.000
5	-0.033	-0.034	71.473	0.000
6	-0.040	-0.045	80.304	0.000
7	0.002	0.003	80.332	0.000
8	0.032	0.022	86.098	0.000
9	0.003	0.002	86.136	0.000
10	-0.001	0.005	86.137	0.000
11	0.000	0.002	86.138	0.000
12	0.000	-0.003	86.139	0.000
13	0.021	0.024	88.692	0.000
14	-0.018	-0.014	90.549	0.000
15	0.001	0.001	90.554	0.000
16	0.021	0.023	92.950	0.000
17	-0.001	-0.003	92.953	0.000
18	-0.042	-0.038	102.89	0.000

(b) FTSE returns

Figure 25: Autocorrelograms

can help to forecast SP500 returns at the daily frequency, we propose and compare two models:

$$\text{AR(8):} \qquad (1 - \sum_{i=1}^{8} \phi_i L^i) Y_t = \mu + \varepsilon_t$$

$$\text{ARX(8):} \qquad (1 - \sum_{i=1}^{8} \phi_i L^i) Y_t = \mu + \beta_1 W_t + \beta_2 W_{t-1} + \varepsilon_t$$

where Y_t and W_t are the SP500 and the FTSE returns based on open prices, which are obtained by taking the log difference of the SP500 and FTSE daily indexes. Note that in the ARX(8), although the variable W_t has a time subscript t, it is already realized and known *prior to* Y_t. The choice of eight lags is triggered by looking at the largest autocorrelation coefficients and it will be judged according to the results in the estimation stage.

In Tables 12 and 13, we report the estimation results of the two models. As expected, in the AR(8) model, the autoregressive parameters are very small and most of them are not statistically different from zero. The adjusted R-squared is very small 0.7% so that the fit is very poor. Note that the standard error of the regression is the same as the unconditional standard deviation of the series. On contrast, the ARX(8) offers a much better fit due to the inclusion of the FTSE returns; the corresponding parameters are very significant indicating that FTSE returns provide information to the opening trading on the SP500 index. They move in the same direction so that if the FTSE index has been moving upwards (downwards), the SP500 will move upwards (downwards) at the opening. The adjusted R-squared has increased dramatically up to 32.6% and the standard error of the regression is smaller than the unconditional standard deviation of the series.

Dependent Variable: R_SP500_OPEN				
Method: Least Squares				
Sample (adjusted): 1/15/1990 7/31/2012				
Included observations: 4034 after adjustments				
Convergence achieved after 3 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.020793	0.016166	1.286197	0.1984
AR(1)	-0.02192	0.015729	-1.39359	0.1635
AR(2)	-0.04454	0.01581	-2.81687	0.0049
AR(3)	-0.02194	0.015809	-1.38778	0.1653
AR(4)	-0.00265	0.015988	-0.16557	0.8685
AR(5)	-0.03332	0.016018	-2.08038	0.0376
AR(6)	-0.0019	0.01605	-0.11832	0.9058
AR(7)	-0.0585	0.015931	-3.67186	0.0002
AR(8)	0.04314	0.015785	2.733041	0.0063
R-squared	0.009016	Mean dependent var		0.021333
Adjusted R-squared	0.007046	S.D. dependent var		1.176262
S.E. of regression	1.172111	Akaike info criterion		3.157718
Sum squared resid	5529.723	Schwarz criterion		3.171779
Log likelihood	-6360.12	Hannan-Quinn criter.		3.162701
F-statistic	4.577356	Durbin-Watson stat		2.034184
Prob(F-statistic)	0.000014			
Inverted AR Roots	0.52	.52+.41i	.52-.41i	.05+.70i
	.05-.70i	-.47+.54i	-.47-.54i	-0.75

Table 12: Estimation results of AR(8) model

Dependent Variable: R_SP500_OPEN				
Method: Least Squares				
Sample (adjusted): 1/16/1990 7/31/2012				
Included observations: 3572 after adjustments				
Convergence achieved after 6 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017511	0.010645	1.645034	0.1001
R_FTSE_OPEN	0.58359	0.014252	40.94912	0.0000
R_FTSE_OPEN(-1)	0.030105	0.01417	2.124511	0.0337
AR(1)	-0.27333	0.017743	-15.4048	0.0000
AR(2)	-0.10302	0.018305	-5.62777	0.0000
AR(3)	-0.06421	0.017737	-3.62027	0.0003
AR(4)	-0.00213	0.017591	-0.12096	0.9037
AR(5)	-0.03286	0.017525	-1.87477	0.0609
AR(6)	-0.01188	0.017562	-0.67666	0.4987
AR(7)	-0.07341	0.017487	-4.19818	0.0000
AR(8)	0.026785	0.016604	1.613181	0.1068
R-squared	0.327869	Mean dependent var		0.024044
Adjusted R-squared	0.325982	S.D. dependent var		1.18858
S.E. of regression	0.975808	Akaike info criterion		2.791972
Sum squared resid	3390.786	Schwarz criterion		2.811006
Log likelihood	-4975.46	Hannan-Quinn criter.		2.798758
F-statistic	173.7076	Durbin-Watson stat		2.046315
Prob(F-statistic)	0			
Inverted AR Roots	.52+.37i	.52-.37i	0.32	.06+.71i
	.06-.71i	-.48+.54i	-.48-.54i	-0.78

Table 13: Estimation results of ARX(8) model

We compare the out-of-sample forecasting performance of the two models by calculating the multistep-period forecast for the latest observations (August 1st, 2012 to August 14th, 2012), which were excluded in the estimation step. The following tables report the forecasting results from the two models:

Multistep Forecast of the AR(8)				
Forecasting Horizon	Actual	Point Forecast	Forecast Uncertainty	Density Forecast
8/01/2012	-0.4304	$f_{t,1} = 0.0086$	$\hat{\sigma}^2_{t+1\|t} = 1.1732^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
8/02/2012	-0.3042	$f_{t,2} = -0.0115$	$\hat{\sigma}^2_{t+2\|t} = 1.1735^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
8/03/2012	-0.7064	$f_{t,3} = -0.0190$	$\hat{\sigma}^2_{t+3\|t} = 1.1745^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$
8/06/2012	1.8568	$f_{t,4} = -0.0796$	$\hat{\sigma}^2_{t+4\|t} = 1.1747^2$	$N(f_{t,4}, \hat{\sigma}^2_{t+4\|t})$
8/07/2012	0.2456	$f_{t,5} = -0.0709$	$\hat{\sigma}^2_{t+5\|t} = 1.1746^2$	$N(f_{t,5}, \hat{\sigma}^2_{t+5\|t})$
8/08/2012	0.4843	$f_{t,6} = -0.0113$	$\hat{\sigma}^2_{t+6\|t} = 1.1752^2$	$N(f_{t,6}, \hat{\sigma}^2_{t+6\|t})$
8/09/2012	0.0735	$f_{t,7} = 0.1135$	$\hat{\sigma}^2_{t+7\|t} = 1.1749^2$	$N(f_{t,7}, \hat{\sigma}^2_{t+7\|t})$
8/10/2012	0.0228	$f_{t,8} = 0.0216$	$\hat{\sigma}^2_{t+8\|t} = 1.1763^2$	$N(f_{t,8}, \hat{\sigma}^2_{t+8\|t})$
8/13/2012	0.2343	$f_{t,9} = 0.0224$	$\hat{\sigma}^2_{t+9\|t} = 1.1776^2$	$N(f_{t,9}, \hat{\sigma}^2_{t+9\|t})$
8/14/2012	-0.1075	$f_{t,10} = 0.0230$	$\hat{\sigma}^2_{t+10\|t} = 1.1776^2$	$N(f_{t,10}, \hat{\sigma}^2_{t+10\|t})$

Multistep Forecast of the ARX(8)				
Forecasting Horizon	Actual	Point Forecast	Forecast Uncertainty	Density Forecast
8/01/2012	-0.4304	$f_{t,1} = -0.4766$	$\hat{\sigma}^2_{t+1\|t} = 0.9766^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
8/02/2012	-0.3042	$f_{t,2} = 0.7286$	$\hat{\sigma}^2_{t+2\|t} = 1.0125^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
8/03/2012	-0.7064	$f_{t,3} = -0.3851$	$\hat{\sigma}^2_{t+3\|t} = 1.0128^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$
8/06/2012	1.8568	$f_{t,4} = 1.1832$	$\hat{\sigma}^2_{t+4\|t} = 1.0134^2$	$N(f_{t,4}, \hat{\sigma}^2_{t+4\|t})$
8/07/2012	0.2456	$f_{t,5} = 0.2684$	$\hat{\sigma}^2_{t+5\|t} = 1.0137^2$	$N(f_{t,5}, \hat{\sigma}^2_{t+5\|t})$
8/08/2012	0.4843	$f_{t,6} = 0.3015$	$\hat{\sigma}^2_{t+6\|t} = 1.0140^2$	$N(f_{t,6}, \hat{\sigma}^2_{t+6\|t})$
8/09/2012	0.0735	$f_{t,7} = 0.1957$	$\hat{\sigma}^2_{t+7\|t} = 1.0138^2$	$N(f_{t,7}, \hat{\sigma}^2_{t+7\|t})$
8/10/2012	0.0228	$f_{t,8} = 0.0241$	$\hat{\sigma}^2_{t+8\|t} = 1.0158^2$	$N(f_{t,8}, \hat{\sigma}^2_{t+8\|t})$
8/13/2012	0.2343	$f_{t,9} = -0.0100$	$\hat{\sigma}^2_{t+9\|t} = 1.0178^2$	$N(f_{t,9}, \hat{\sigma}^2_{t+9\|t})$
8/14/2012	-0.1075	$f_{t,10} = -0.1472$	$\hat{\sigma}^2_{t+10\|t} = 1.0180^2$	$N(f_{t,10}, \hat{\sigma}^2_{t+10\|t})$

Observe that the forecasts from the AR(8) model are basically the unconditional mean of the series, which is 0%, and the forecast variance is the unconditional variance of the series, which is 1.17^2. This is to say that the SP500 returns are not predictable based on their past history. On the contrary, we find some predictability when we include information about the FTSE returns. In Figure 26, we show the multistep point forecasts and interval forecasts of the two models for the SP500 daily returns. Observe that the predicted returns based on the ARX(8) model with FTSE information successfully pick up the ups and downs of the actual SP500 returns, while the predicted returns from the AR(8) model without FTSE information fail to do so. Based on this comparison, we conclude that the FTSE returns indeed help to predict SP500 returns at the daily frequency.

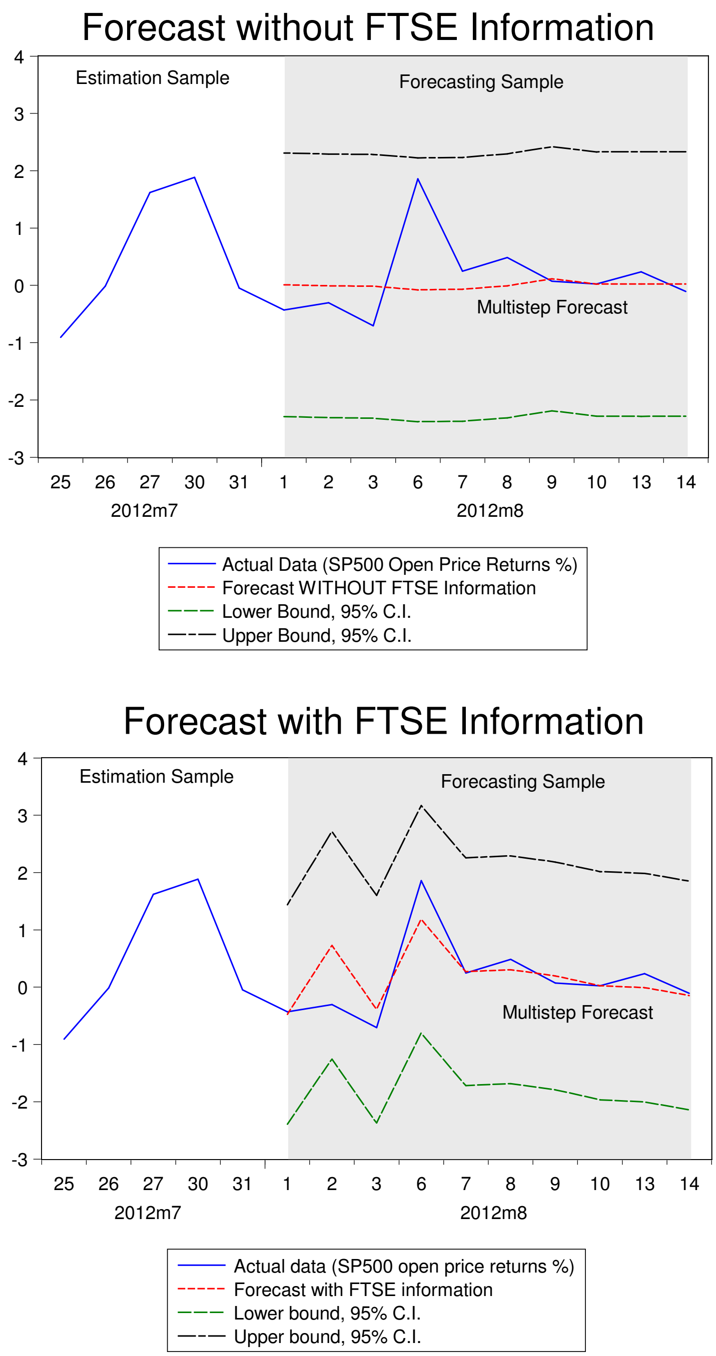
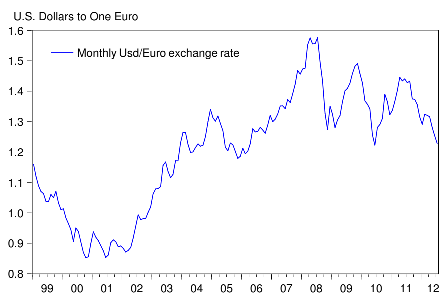


Figure 26: SP500 Daily Return, Multi-step Forecast

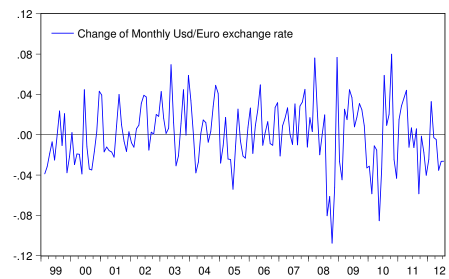
Exercise 8
We download the monthly U.S. Dollar/Euro Foreign Exchange Rate (in U.S. Dollars to one Euro) from FRED® Economic Data (St. Louis Fed) website:

`http://research.stlouisfed.org/fred2/series/DEXUSEU`

In Figures 27a and 27b, we plot the time series of monthly U.S. Dollar/Euro foreign exchange rate in both, levels and monthly changes, respectively. The data set dates from January 1999 to July 2012. The time series in levels has strong time dependence, suggesting likely non-stationarity in the series. In this exercise we build focus on models for the monthly *changes* of the exchange rate, which is stationary. In Figure 28a, we report the autocorrelation functions of the series. The ACF has one significant spike at displacement one and the PACF decays quickly to zero in an alternating fashion, thus an MA(1) model seems an appropriate specification. In Table 14, we report the estimation results. The moving average parameter is statistically significant and the autocorrelogram of the residuals, in Figure 28b, does not show any linear dependence, so that the residuals are white noise. Thus, an MA(1) captures the dynamics of the series correctly.

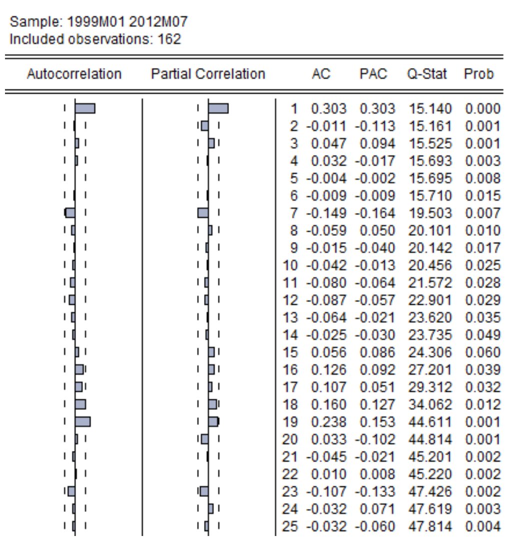


(a) Level



(b) Change

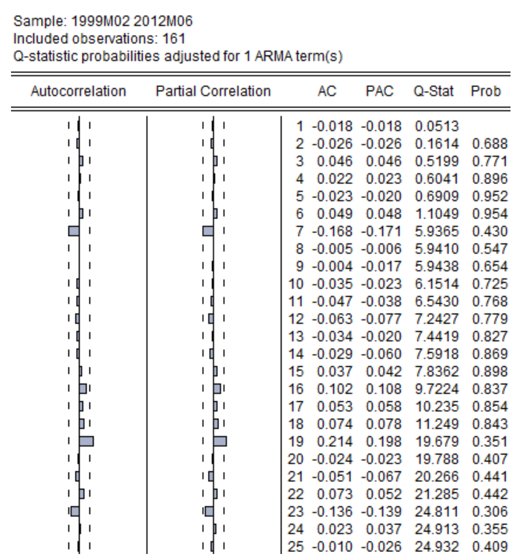
Figure 27: Monthly U.S. $/Euro exchange rate (U.S. Dollars in one Euro)



Sample: 1999M01 2012M07
Included observations: 162

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.303	0.303	15.140	0.000
		2	-0.011	-0.113	15.161	0.001
		3	0.047	0.094	15.525	0.001
		4	0.032	-0.017	15.693	0.003
		5	-0.004	-0.002	15.695	0.008
		6	-0.009	-0.009	15.710	0.015
		7	-0.149	-0.164	19.503	0.007
		8	-0.059	0.050	20.101	0.010
		9	-0.015	-0.040	20.142	0.017
		10	-0.042	-0.013	20.456	0.025
		11	-0.080	-0.064	21.572	0.028
		12	-0.087	-0.057	22.901	0.029
		13	-0.064	-0.021	23.620	0.035
		14	-0.025	-0.030	23.735	0.049
		15	0.056	0.086	24.306	0.060
		16	0.126	0.092	27.201	0.039
		17	0.107	0.051	29.312	0.032
		18	0.160	0.127	34.062	0.012
		19	0.238	0.153	44.611	0.001
		20	0.033	-0.102	44.814	0.001
		21	-0.045	-0.021	45.201	0.002
		22	0.010	0.008	45.220	0.002
		23	-0.107	-0.133	47.426	0.002
		24	-0.032	0.071	47.619	0.003
		25	-0.032	-0.060	47.814	0.004

(a) Change



Sample: 1999M02 2012M06
Included observations: 161
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.018	-0.018	0.0513	
		2	-0.026	-0.026	0.1614	0.688
		3	0.046	0.046	0.5199	0.771
		4	0.022	0.023	0.6041	0.896
		5	-0.023	-0.020	0.6909	0.952
		6	0.049	0.048	1.1049	0.954
		7	-0.168	-0.171	5.9365	0.430
		8	-0.005	-0.006	5.9410	0.547
		9	-0.004	-0.017	5.9438	0.654
		10	-0.035	-0.023	6.1514	0.725
		11	-0.047	-0.038	6.5430	0.768
		12	-0.063	-0.077	7.2427	0.779
		13	-0.034	-0.020	7.4419	0.827
		14	-0.029	-0.060	7.5918	0.869
		15	0.037	0.042	7.8362	0.898
		16	0.102	0.108	9.7224	0.837
		17	0.053	0.058	10.235	0.854
		18	0.074	0.078	11.249	0.843
		19	0.214	0.198	19.679	0.351
		20	-0.024	-0.023	19.788	0.407
		21	-0.051	-0.067	20.266	0.441
		22	0.073	0.052	21.285	0.442
		23	-0.136	-0.139	24.811	0.306
		24	0.023	0.037	24.913	0.355
		25	-0.010	-0.026	24.932	0.409

(b) Residuals of MA(1)

Figure 28: Autocorrelograms

Dependent Variable: D(EXRATE)				
Method: Least Squares				
Sample (adjusted): 1999M02 2012M06				
Included observations: 161 after adjustments				
Convergence achieved after 6 iterations				
MA Backcast: 1999M01				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000525	0.00319	0.16445	0.8696
MA(1)	0.366008	0.073701	4.966095	0.0000
R-squared	0.109526	Mean dependent var		0.00059
Adjusted R-squared	0.103926	S.D. dependent var		0.031342
S.E. of regression	0.029669	Akaike info criterion		-4.18509
Sum squared resid	0.139959	Schwarz criterion		-4.14681
Log likelihood	338.8997	Hannan-Quinn criter.		-4.16955
F-statistic	19.55662	Durbin-Watson stat		2.025069
Prob(F-statistic)	0.000018			
Inverted MA Roots	-0.37			

Table 14: Estimation Results of MA(1) Model

Based on the MA(1) model and assuming a quadratic loss function, we obtain the optimal one-month-ahead forecast for the month of July 2012 (this observation was excluded in the estimation step). A positive (negative) change means that the dollar will depreciate (appreciate). The following table report the forecasting results:

Forecasting Horizon	$h = 1$, July 2012
Actual Change	$y_{t+1} = -0.026300$ (appreciation)
Point Forecast	$f_{t,1} = -0.004507$ (appreciation)
Forecast Uncertainty	$\hat{\sigma}^2_{t+1\|t} = 0.029857^2$
Density Forecast	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$

In Figure 29, we show the one-month-ahead density forecast for the monthly change of U.S. dollar/Euro exchange rate. In Figure 29, the vertical thick black line is the optimal point forecast $f_{t,1}$ (conditional expectations), and the vertical red line is the actual change y_{t+1}. Based on this model, we can also predict the probability of appreciation and depreciation of U.S. dollars. Note that negative (positive) change indicates appreciation (depreciation) of U.S. dollar against Euro. Since the density forecast for July 2012 is $N(-0.0045, 0.0299^2)$, the predicted probability of *appreciation* of U.S. dollar is,

$$\Pr(\Delta e_{\text{Jul}} < 0) = \Phi\left(\frac{0-(-0.0045)}{0.0299}\right) \approx 0.56$$

which corresponds to the pink region of Figure 29, and $\Phi(\cdot)$ is the cumulative distribution function of standard normal density $N(0, 1)$. Similarly, the predicted probability of *depreciation* of U.S. dollar is,

$$\Pr(\Delta e_{\text{Jul}} > 0) = 1 - \Phi\left(\frac{0-(-0.0045)}{0.0299}\right) \approx 0.44$$

which corresponds to the red region of Figure 29.

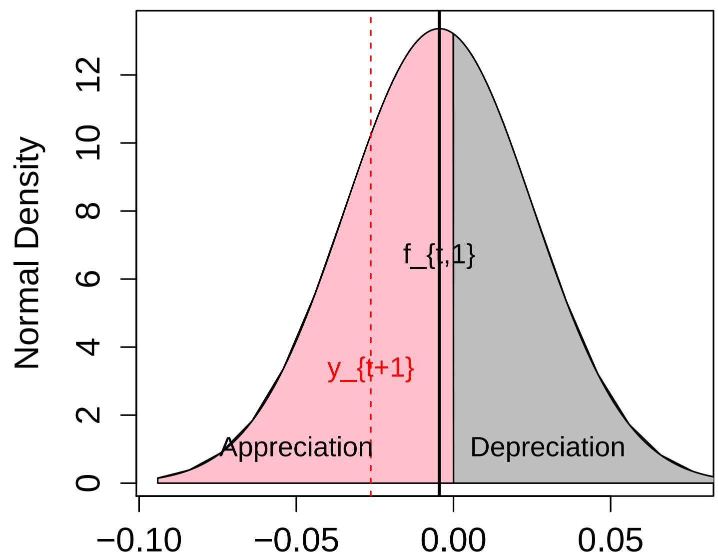


Figure 29: Monthly U.S. $/Euro exchange rate, density forecast for July 2012

Exercise 9
We download the monthly changes in U.S. residential construction (in millions of $) from FRED® Economic Data website: `http://research.stlouisfed.org/fred2/series/TLRESCON`.

In Figure 30 we plot the updated time series of monthly changes in residential construction, dating from January 2002 to June 2012. As explained in Chapter 7 (Section 7.3.3), the time series is covariance stationary, shows strong time dependence, and exhibits a clear seasonal cycle. In Figure 31a we report the autocorrelation functions of the series. Following the SAR(1)-AR(1) specification proposed in Chapter 7 Section 7.3.3,

$$\text{SAR(1)-AR(1):} \quad (1-\gamma_1 L^{12})(1-\phi_1 L)Y_t = \mu + \varepsilon_t$$

we estimate the model and report the result in Table 15.

The estimation results with the updated sample are very similar to those in the textbook. The non-seasonal and seasonal autoregressive parameters are very statistically significant with p-values of 0%. In Figure 31b, we plot the autocorrelograms of the residuals. All spikes in the ACFs and PACFs are within the dashed lines (95% confidence interval). The Q-statistics deliver the same message when the autocorrelations are tested jointly: all Q-statistics have p-values larger than 5%. Therefore, we conclude that white noise is a reasonable characterization of the residuals of the SAR(1)-AR(1) model. Note that one of the inverted AR roots is very close to one (0.99) so that it is likely that the seasonal cycle is not stationary, in fact, the seasonal autoregressive estimate is very large (0.92). However, at this stage, we will carry on with these estimates, and once we analyze non-stationarity in Chapter 10 this estimation could be revised.

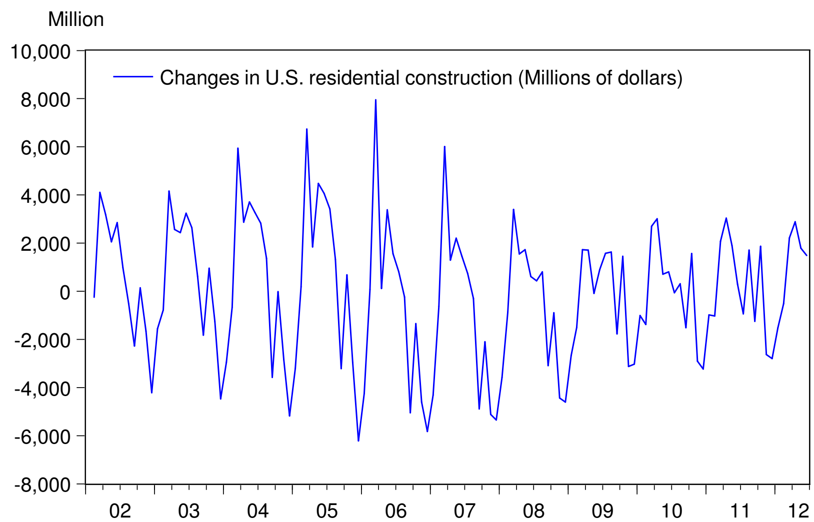
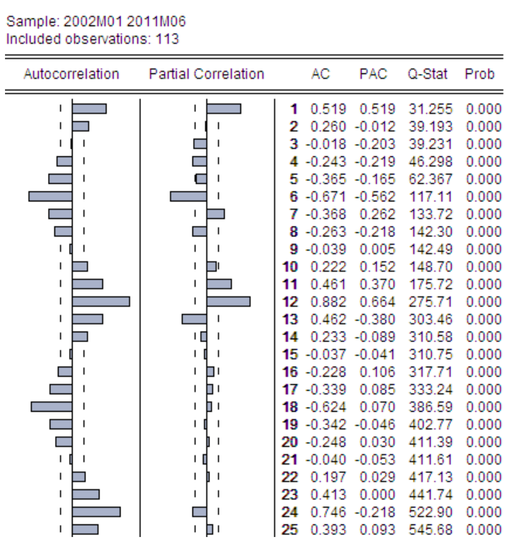


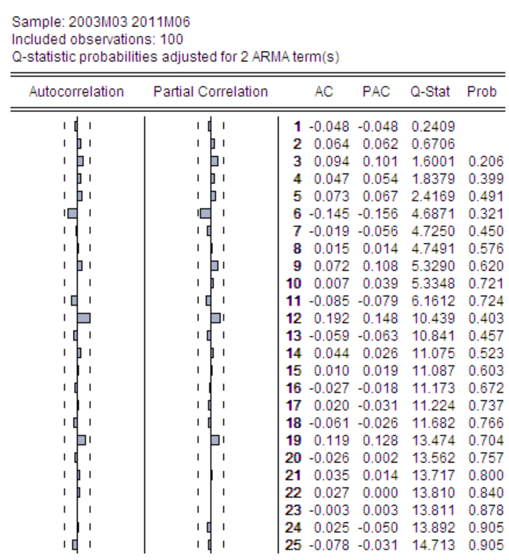
Figure 30: Monthly changes in U.S. residential construction (millions of dollars)



Sample: 2002M01 2011M06
Included observations: 113

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.519	0.519	31.255	0.000
		2	0.260	-0.012	39.193	0.000
		3	-0.018	-0.203	39.231	0.000
		4	-0.243	-0.219	46.298	0.000
		5	-0.365	-0.165	62.367	0.000
		6	-0.671	-0.562	117.11	0.000
		7	-0.368	0.262	133.72	0.000
		8	-0.263	-0.218	142.30	0.000
		9	-0.039	0.005	142.49	0.000
		10	0.222	0.152	148.70	0.000
		11	0.461	0.370	175.72	0.000
		12	0.882	0.664	275.71	0.000
		13	0.462	-0.380	303.46	0.000
		14	0.233	-0.089	310.58	0.000
		15	-0.037	-0.041	310.75	0.000
		16	-0.228	0.106	317.71	0.000
		17	-0.339	0.085	333.24	0.000
		18	-0.624	0.070	386.59	0.000
		19	-0.342	-0.046	402.77	0.000
		20	-0.248	0.030	411.39	0.000
		21	-0.040	-0.053	411.61	0.000
		22	0.197	0.029	417.13	0.000
		23	0.413	0.000	441.74	0.000
		24	0.746	-0.218	522.90	0.000
		25	0.393	0.093	545.68	0.000

(a) Original time series



Sample: 2003M03 2011M06
Included observations: 100
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.048	-0.048	0.2409	
		2	0.064	0.062	0.6706	
		3	0.094	0.101	1.6001	0.206
		4	0.047	0.054	1.8379	0.399
		5	0.073	0.067	2.4169	0.491
		6	-0.145	-0.156	4.6871	0.321
		7	-0.019	-0.056	4.7250	0.450
		8	0.015	0.014	4.7491	0.576
		9	0.072	0.108	5.3290	0.620
		10	0.007	0.039	5.3348	0.721
		11	-0.085	-0.079	6.1612	0.724
		12	0.192	0.148	10.439	0.403
		13	-0.059	-0.063	10.841	0.457
		14	0.044	0.026	11.075	0.523
		15	0.010	0.019	11.087	0.603
		16	-0.027	-0.018	11.173	0.672
		17	0.020	-0.031	11.224	0.737
		18	-0.061	-0.026	11.682	0.766
		19	0.119	0.128	13.474	0.704
		20	-0.026	0.002	13.562	0.757
		21	0.035	0.014	13.717	0.800
		22	0.027	0.000	13.810	0.840
		23	-0.003	0.003	13.811	0.878
		24	0.025	-0.050	13.892	0.905
		25	-0.078	-0.031	14.713	0.905

(b) Residuals from SAR(1)-AR(1) model

Figure 31: Autocorrelograms

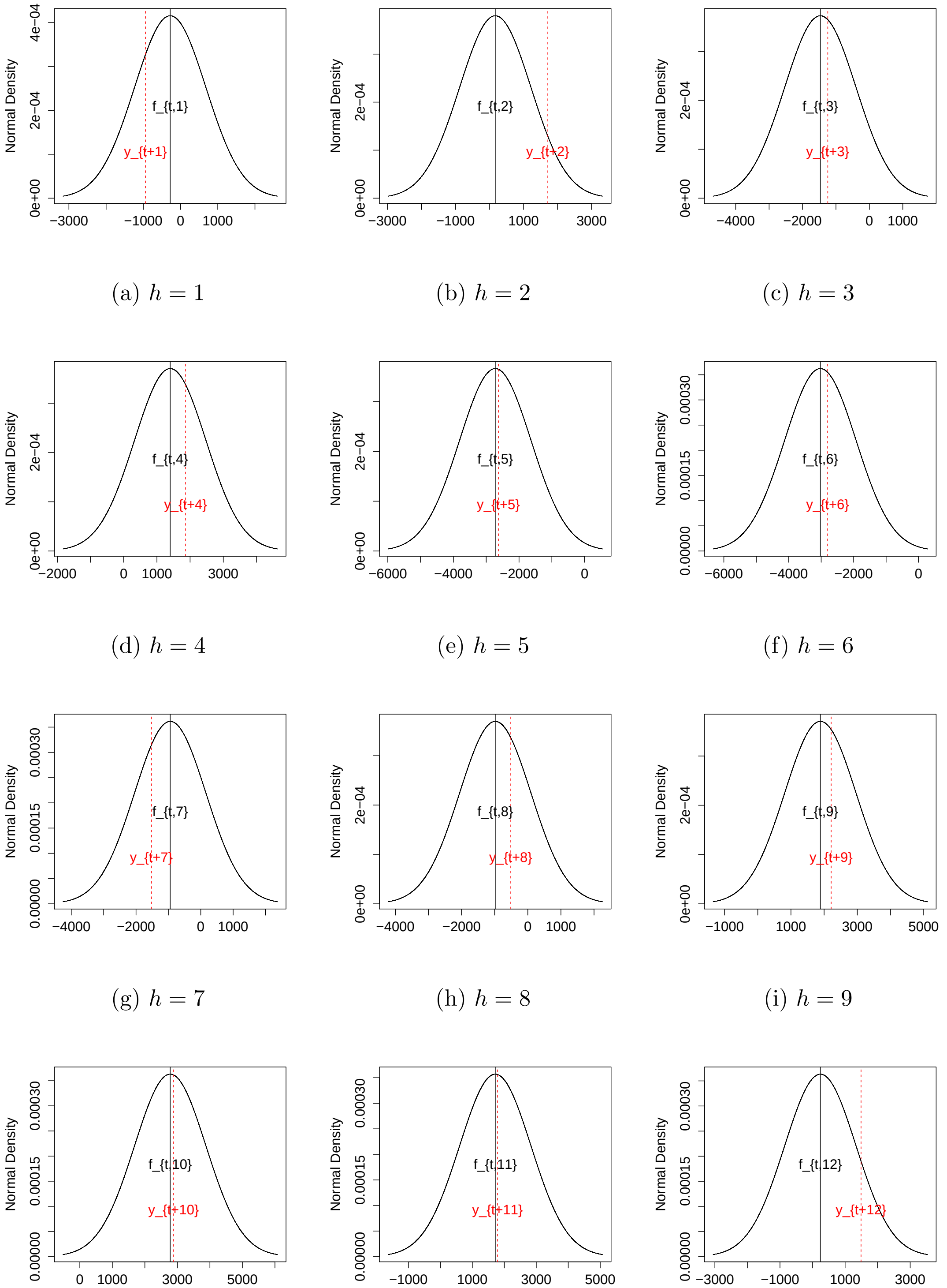
Dependent Variable: CONST				
Method: Least Squares				
Sample (adjusted): 2003M03 2011M06				
Included observations: 100 after adjustments				
Convergence achieved after 6 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-396.442	2305.319	-0.171968	0.8638
AR(1)	0.446749	0.090799	4.920223	0.0000
SAR(12)	0.92498	0.036815	25.12535	0.0000
R-squared	0.901164	Mean dependent var		-53.93
Adjusted R-squared	0.899126	S.D. dependent var		3002.758
S.E. of regression	953.6949	Akaike info criterion		16.58811
Sum squared resid	88224786	Schwarz criterion		16.66626
Log likelihood	-826.405	Hannan-Quinn criter.		16.61974
F-statistic	442.2124	Durbin-Watson stat		2.081229
Prob(F-statistic)	0.000000			
Inverted AR Roots	0.99	.86-.50i	.86+.50i	.50-.86i
	.50+.86i	0.45	.00+.99i	-.00-.99i
	-.50+.86i	-.50-.86i	-.86+.50i	-.86-.50i
	-0.99			

Table 15: Estimation results of SAR(1)-AR(1) model

With this model, we calculate the optimal forecast, assuming a quadratic loss function, for the latest 12 months, which were excluded in the estimation stage. In the following table, we report the forecasting results:

Forecasting Horizon	Actual	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, Jul, 2011	-941	$f_{t,1} = -276.5173$	$\hat{\sigma}^2_{t+1\|t} = 960.5119^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, Aug, 2011	1712	$f_{t,2} = 169.0893$	$\hat{\sigma}^2_{t+2\|t} = 1050.342^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, Sep, 2011	-1245	$f_{t,3} = -1466.858$	$\hat{\sigma}^2_{t+3\|t} = 1069.485^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$
$h = 4$, Oct, 2011	1864	$f_{t,4} = 1401.044$	$\hat{\sigma}^2_{t+4\|t} = 1077.195^2$	$N(f_{t,4}, \hat{\sigma}^2_{t+4\|t})$
$h = 5$, Nov, 2011	-2627	$f_{t,5} = -2723.391$	$\hat{\sigma}^2_{t+5\|t} = 1089.291^2$	$N(f_{t,5}, \hat{\sigma}^2_{t+5\|t})$
$h = 6$, Dec, 2012	-2801	$f_{t,6} = -3025.405$	$\hat{\sigma}^2_{t+6\|t} = 1101.426^2$	$N(f_{t,6}, \hat{\sigma}^2_{t+6\|t})$
$h = 7$, Jan, 2012	-1526	$f_{t,7} = -943.2698$	$\hat{\sigma}^2_{t+7\|t} = 1103.938^2$	$N(f_{t,7}, \hat{\sigma}^2_{t+7\|t})$
$h = 8$, Feb, 2012	-515	$f_{t,8} = -982.2148$	$\hat{\sigma}^2_{t+8\|t} = 1079.493^2$	$N(f_{t,8}, \hat{\sigma}^2_{t+8\|t})$
$h = 9$, Mar, 2012	2212	$f_{t,9} = 1886.518$	$\hat{\sigma}^2_{t+9\|t} = 1077.420^2$	$N(f_{t,9}, \hat{\sigma}^2_{t+9\|t})$
$h = 10$, Apr, 2012	2884	$f_{t,10} = 2780.213$	$\hat{\sigma}^2_{t+10\|t} = 1099.081^2$	$N(f_{t,10}, \hat{\sigma}^2_{t+10\|t})$
$h = 11$, May, 2012	1788	$f_{t,11} = 1718.411$	$\hat{\sigma}^2_{t+11\|t} = 1117.737^2$	$N(f_{t,11}, \hat{\sigma}^2_{t+11\|t})$
$h = 12$, Jun, 2012	1490	$f_{t,12} = 240.3262$	$\hat{\sigma}^2_{t+12\|t} = 1097.860^2$	$N(f_{t,12}, \hat{\sigma}^2_{t+12\|t})$

In Figures 32 and 33 , we plot the multistep density forecasts and point forecasts with 95% confidence bands, respectively, for the changes in U.S. residential construction. Observe that the point forecasts follow very closely the actual changes, which are well within the values of the 95% interval forecast. In Figure 32, the vertical black lines are the optimal point forecasts $f_{t,h}$ (conditional expectations), and the vertical red lines are the actual changes y_{t+h}.



(a) $h = 1$ (b) $h = 2$ (c) $h = 3$

(d) $h = 4$ (e) $h = 5$ (f) $h = 6$

(g) $h = 7$ (h) $h = 8$ (i) $h = 9$

(j) $h = 10$ (k) $h = 11$ (l) $h = 12$

Figure 32: h-step-ahead density forecast of changes in U.S. residential construction

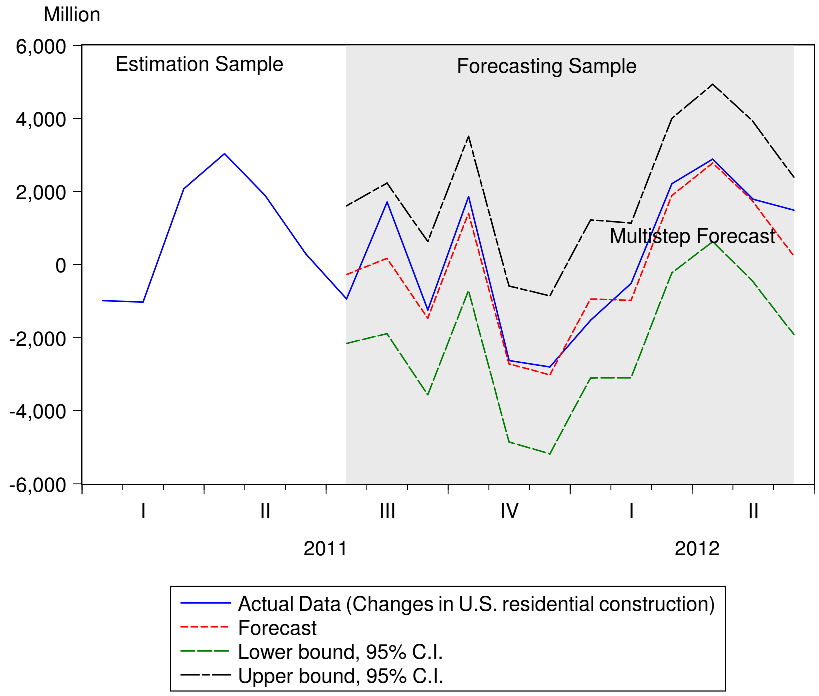


Figure 33: Changes in U.S. residential construction, multistep forecast

Exercise 10

We download the monthly sales of new cars in U.S. (in thousands of cars) from the Bureau of Economic Analysis (BEA) website: `http://www.bea.gov/national/index.htm`.

In Figure 34 we plot the time series of monthly new car sales, dating from January 1967 to July 2012. The series shows a strong seasonal cycle with sales rising in the first half of the year and falling in the second half. In Figure 35 we report the autocorrelation functions of the series. The profile of the autocorrelograms indicates that the process is autoregressive in both non-seasonal and seasonal cycles. In the PACF, we observe a very significant spike at displacement one followed by several marginally significant spikes; the seasonal cycle has relevant spikes around 12 months and marginal spikes at 24, 36, and 48 months. We identify the following two potential specifications:

SAR(1)-AR(5)	$(1-\gamma_1 L^{12})(1-\sum_{i=1}^{5}\phi_i L^i)Y_t = \mu + \varepsilon_t$
SAR(4)-AR(5)	$(1-\gamma_1 L^{12}-\gamma_2 L^{24}-\gamma_3 L^{36}-\gamma_4 L^{48})(1-\sum_{i=1}^{5}\phi_i L^i)Y_t = \mu + \varepsilon_t$

We report the estimation results in Tables 16 and 17. Comparing both models, the SAR(4)-AR(5) seems to provide a better fit than the SAR(1)-AR(5) because it has a larger adjusted R-squared as well as lower residual variance and lower AIC and SIC. In Figure 36, we plot the autocorrelograms of the residuals. We observe that the residuals of the SAR(1)-AR(5) model have some dependence for displacement 12 and above indicating that the seasonal cycle has not been fully captured by the model. In contrast, the residuals of the SAR(4)-AR(5) are whitened out, so that this model is preferred. We also note that in both specifications there is one root very close to one, which indicates that the seasonal cycle may be non-stationary. We could take seasonal differences but at this stage we carry the potential root in the estimation and forecasting exercises.

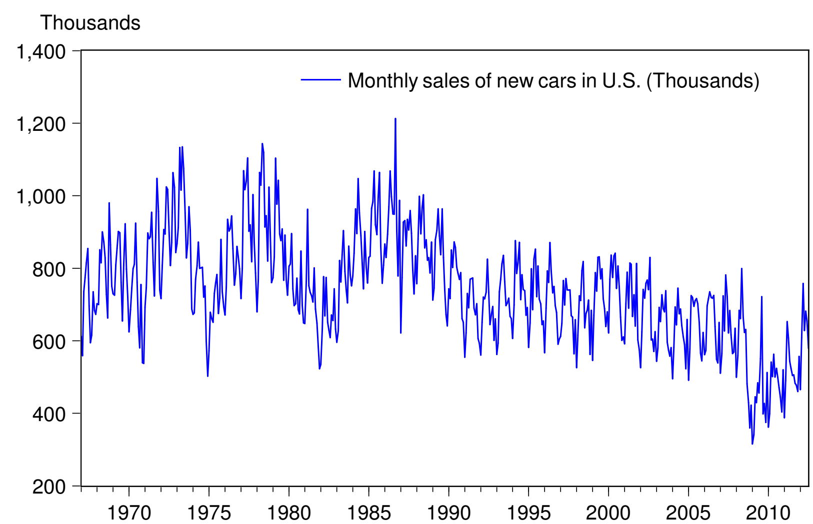
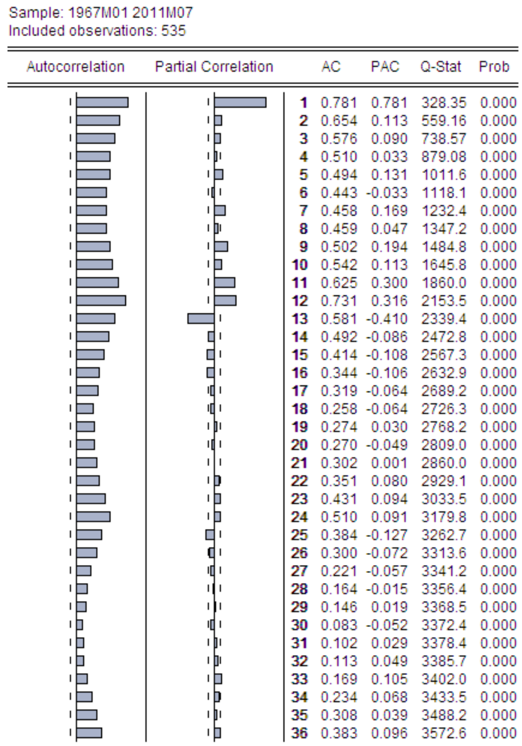


Figure 34: Monthly new car sales in the U.S. (thousands)



Sample: 1967M01 2011M07
Included observations: 535

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.781	0.781	328.35	0.000
		2	0.654	0.113	559.16	0.000
		3	0.576	0.090	738.57	0.000
		4	0.510	0.033	879.08	0.000
		5	0.494	0.131	1011.6	0.000
		6	0.443	-0.033	1118.1	0.000
		7	0.458	0.169	1232.4	0.000
		8	0.459	0.047	1347.2	0.000
		9	0.502	0.194	1484.8	0.000
		10	0.542	0.113	1645.8	0.000
		11	0.625	0.300	1860.0	0.000
		12	0.731	0.316	2153.5	0.000
		13	0.581	-0.410	2339.4	0.000
		14	0.492	-0.086	2472.8	0.000
		15	0.414	-0.108	2567.3	0.000
		16	0.344	-0.106	2632.9	0.000
		17	0.319	-0.064	2689.2	0.000
		18	0.258	-0.064	2726.3	0.000
		19	0.274	0.030	2768.2	0.000
		20	0.270	-0.049	2809.0	0.000
		21	0.302	0.001	2860.0	0.000
		22	0.351	0.080	2929.1	0.000
		23	0.431	0.094	3033.5	0.000
		24	0.510	0.091	3179.8	0.000
		25	0.384	-0.127	3262.7	0.000
		26	0.300	-0.072	3313.6	0.000
		27	0.221	-0.057	3341.2	0.000
		28	0.164	-0.015	3356.4	0.000
		29	0.146	0.019	3368.5	0.000
		30	0.083	-0.052	3372.4	0.000
		31	0.102	0.029	3378.4	0.000
		32	0.113	0.049	3385.7	0.000
		33	0.169	0.105	3402.0	0.000
		34	0.234	0.068	3433.5	0.000
		35	0.308	0.039	3488.2	0.000
		36	0.383	0.096	3572.6	0.000

Figure 35: Autocorrelogram of monthly new car sales

Based on the SAR(4)-AR(5) model, we calculate the optimal forecasts for the latest 12 observations (one year), which were excluded in the estimation step. In the following table, we report the forecasting results:

Forecasting Horizon	Actual	Point Forecast	Forecast Uncertainty	Density Forecast
$h = 1$, Aug, 2011	505.700	$f_{t,1} = 540.7620$	$\hat{\sigma}^2_{t+1\|t} = 64.4421^2$	$N(f_{t,1}, \hat{\sigma}^2_{t+1\|t})$
$h = 2$, Sep, 2011	483.200	$f_{t,2} = 508.1813$	$\hat{\sigma}^2_{t+2\|t} = 72.0882^2$	$N(f_{t,2}, \hat{\sigma}^2_{t+2\|t})$
$h = 3$, Oct, 2011	477.400	$f_{t,3} = 472.1717$	$\hat{\sigma}^2_{t+3\|t} = 75.1388^2$	$N(f_{t,3}, \hat{\sigma}^2_{t+3\|t})$
$h = 4$, Nov, 2011	459.800	$f_{t,4} = 456.3227$	$\hat{\sigma}^2_{t+4\|t} = 77.7311^2$	$N(f_{t,4}, \hat{\sigma}^2_{t+4\|t})$
$h = 5$, Dec, 2011	557.100	$f_{t,5} = 538.4947$	$\hat{\sigma}^2_{t+5\|t} = 80.3092^2$	$N(f_{t,5}, \hat{\sigma}^2_{t+5\|t})$
$h = 6$, Jan, 2012	465.117	$f_{t,6} = 434.3530$	$\hat{\sigma}^2_{t+6\|t} = 83.8508^2$	$N(f_{t,6}, \hat{\sigma}^2_{t+6\|t})$
$h = 7$, Feb, 2012	608.753	$f_{t,7} = 506.4524$	$\hat{\sigma}^2_{t+7\|t} = 86.1561^2$	$N(f_{t,7}, \hat{\sigma}^2_{t+7\|t})$
$h = 8$, Mar, 2012	758.210	$f_{t,8} = 627.7875$	$\hat{\sigma}^2_{t+8\|t} = 87.8220^2$	$N(f_{t,8}, \hat{\sigma}^2_{t+8\|t})$
$h = 9$, Apr, 2012	628.090	$f_{t,9} = 600.8603$	$\hat{\sigma}^2_{t+9\|t} = 89.4718^2$	$N(f_{t,9}, \hat{\sigma}^2_{t+9\|t})$
$h = 10$, May, 2012	681.854	$f_{t,10} = 617.9337$	$\hat{\sigma}^2_{t+10\|t} = 91.0243^2$	$N(f_{t,10}, \hat{\sigma}^2_{t+10\|t})$
$h = 11$, Jun, 2012	654.398	$f_{t,11} = 566.1156$	$\hat{\sigma}^2_{t+11\|t} = 92.1763^2$	$N(f_{t,11}, \hat{\sigma}^2_{t+11\|t})$
$h = 12$, Jul, 2012	578.800	$f_{t,12} = 544.3256$	$\hat{\sigma}^2_{t+12\|t} = 93.0229^2$	$N(f_{t,12}, \hat{\sigma}^2_{t+12\|t})$

In Figures 37 and 38 we show the multistep forecast with the 95% bands, and the density forecasts, respectively, for the monthly new car sales from Aug. 2011 to July 2012. Observe that the actual new car sales are very close to the point forecasts and well within the values of the 95% interval forecast. In Figure 38, the vertical black lines are the optimal point forecasts $f_{t,h}$ (conditional expectations), and the vertical red lines are the actual new car sales y_{t+h}.

Dependent Variable: CAR_TOTAL				
Method: Least Squares				
Date: 08/27/12 Time: 16:33				
Sample (adjusted): 1968M06 2011M07				
Included observations: 518 after adjustments				
Convergence achieved after 6 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	726.362	45.96403	15.80283	0.0000
AR(1)	0.548335	0.044326	12.37058	0.0000
AR(2)	0.064696	0.050243	1.28768	0.1984
AR(3)	0.066734	0.050231	1.328535	0.1846
AR(4)	0.024318	0.05025	0.483947	0.6286
AR(5)	0.105112	0.04405	2.386167	0.0174
SAR(12)	0.649342	0.034152	19.01309	0.0000
R-squared	0.783037	Mean dependent var		743.2749
Adjusted R-squared	0.780489	S.D. dependent var		149.176
S.E. of regression	69.89191	Akaike info criterion		11.3452
Sum squared resid	2496173	Schwarz criterion		11.40263
Log likelihood	-2931.406	Hannan-Quinn criter.		11.3677
F-statistic	307.3731	Durbin-Watson stat		1.996375
Prob(F-statistic)	0.000000			
Inverted AR Roots	0.96	0.9	.84+.48i	.84-.48i
Other roots are omitted.				

Table 16: Estimation results of SAR(1)-AR(5) model

Dependent Variable: CAR_TOTAL				
Method: Least Squares				
Sample (adjusted): 1971M06 2011M07				
Included observations: 482 after adjustments				
Convergence achieved after 7 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	655.3094	118.1898	5.544552	0
AR(1)	0.50368	0.045967	10.95739	0
AR(2)	0.075193	0.05132	1.465173	0.1435
AR(3)	0.0947	0.051326	1.845075	0.0657
AR(4)	0.102019	0.051306	1.98843	0.0473
AR(5)	0.089844	0.045905	1.957146	0.0509
SAR(12)	0.481789	0.037948	12.69619	0.0000
SAR(48)	0.332413	0.038188	8.704623	0.0000
R-squared	0.82522	Mean dependent var		741.1618
Adjusted R-squared	0.822639	S.D. dependent var		151.5134
S.E. of regression	63.80872	Akaike info criterion		11.16611
Sum squared resid	1929916	Schwarz criterion		11.23546
Log likelihood	-2683.034	Hannan-Quinn criter.		11.19337
F-statistic	319.7119	Durbin-Watson stat		1.997991
Prob(F-statistic)	0			
Inverted AR Roots	0.99	.97+.11i	.97-.11i	0.93
Other roots are omitted.				

Table 17: Estimation results of SAR(4)-AR(5) model

Sample: 1968M06 2011M07
Included observations: 518
Q-statistic probabilities adjusted for 6 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.001	0.001	0.0006	
		2	-0.002	-0.002	0.0027	
		3	-0.005	-0.005	0.0137	
		4	-0.010	-0.010	0.0672	
		5	-0.011	-0.011	0.1333	
		6	-0.016	-0.016	0.2717	
		7	-0.029	-0.029	0.7103	0.399
		8	0.001	0.000	0.7104	0.701
		9	0.076	0.076	3.7692	0.287
		10	-0.006	-0.007	3.7884	0.435
		11	0.068	0.068	6.2278	0.285
		12	-0.130	-0.132	15.268	0.018
		13	0.013	0.015	15.353	0.032
		14	0.082	0.084	18.939	0.015
		15	0.068	0.073	21.434	0.011
		16	-0.013	-0.011	21.529	0.018
		17	0.001	-0.001	21.529	0.028
		18	-0.028	-0.035	21.965	0.038
		19	0.019	0.018	22.152	0.053
		20	-0.002	-0.007	22.155	0.075
		21	-0.046	-0.019	23.317	0.078
		22	-0.028	-0.036	23.747	0.095
		23	0.070	0.078	26.437	0.067
		24	0.024	-0.006	26.757	0.084
		25	-0.015	-0.022	26.875	0.108

(a) SAR(1)-AR(5)

Sample: 1971M06 2011M07
Included observations: 482
Q-statistic probabilities adjusted for 7 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.001	0.001	0.0002	
		2	0.003	0.003	0.0034	
		3	-0.001	-0.001	0.0036	
		4	-0.008	-0.008	0.0332	
		5	-0.006	-0.006	0.0487	
		6	-0.020	-0.020	0.2395	
		7	-0.052	-0.052	1.5445	
		8	-0.008	-0.008	1.5789	0.209
		9	0.061	0.061	3.4242	0.180
		10	-0.016	-0.017	3.5535	0.314
		11	0.049	0.048	4.7534	0.314
		12	-0.059	-0.061	6.4951	0.261
		13	-0.047	-0.048	7.5735	0.271
		14	0.059	0.058	9.3172	0.231
		15	0.066	0.069	11.475	0.176
		16	-0.005	-0.001	11.490	0.244
		17	0.022	0.021	11.723	0.304
		18	0.007	0.005	11.747	0.383
		19	0.025	0.021	12.064	0.441
		20	0.040	0.032	12.865	0.458
		21	-0.047	-0.031	14.003	0.449
		22	-0.054	-0.047	15.487	0.417
		23	0.080	0.082	18.708	0.284
		24	-0.054	-0.057	20.176	0.265
		25	-0.054	-0.064	21.648	0.248

(b) SAR(4)-AR(5)

Figure 36: Autocorrelograms of the residuals

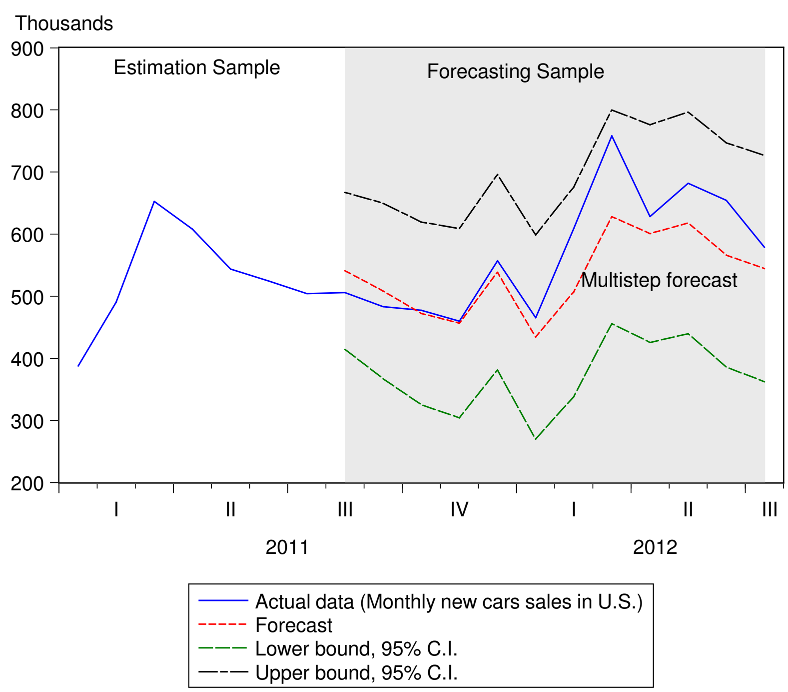
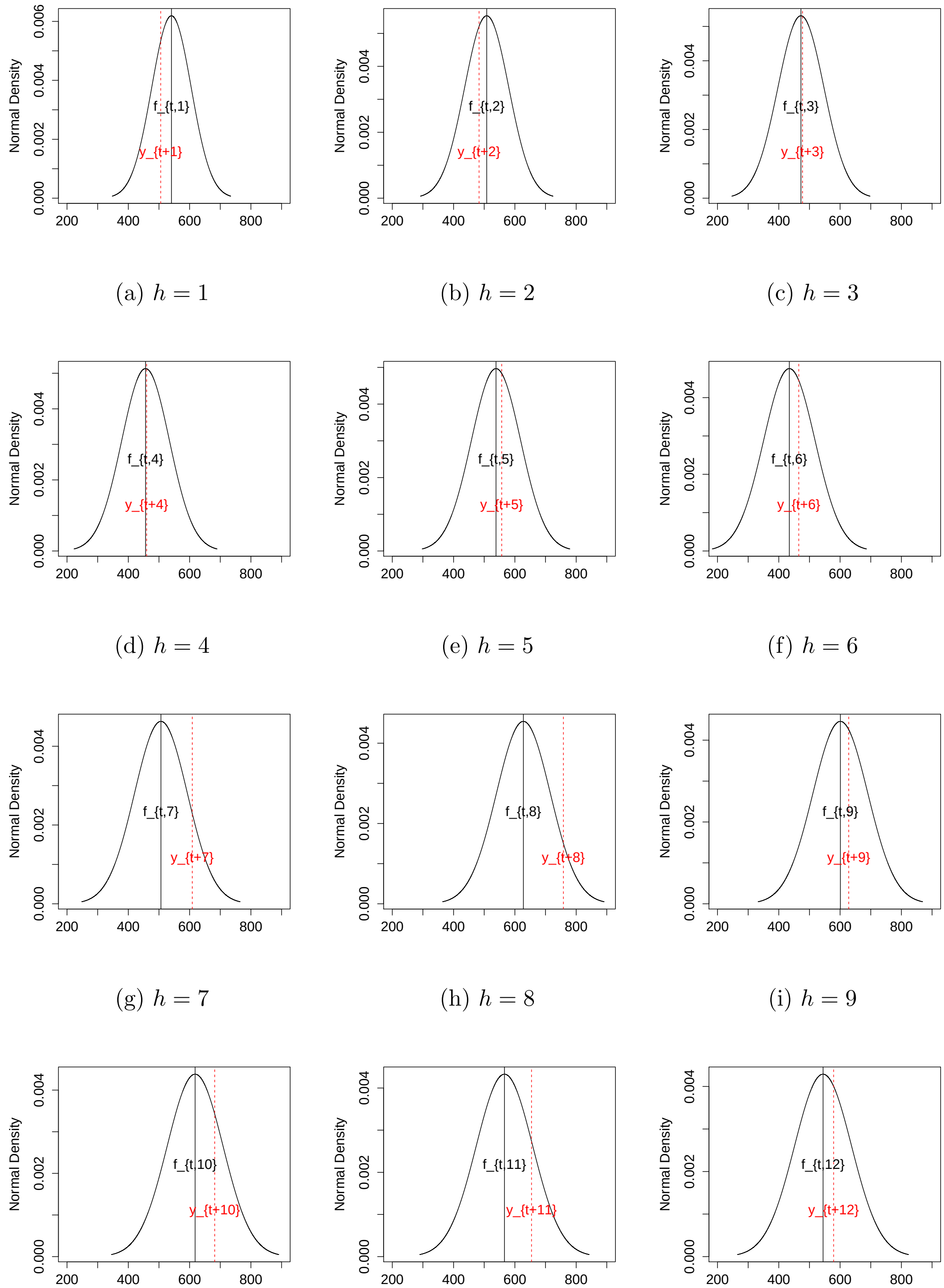


Figure 37: Monthly new car sales in the U.S., multistep forecast



(a) $h = 1$ (b) $h = 2$ (c) $h = 3$

(d) $h = 4$ (e) $h = 5$ (f) $h = 6$

(g) $h = 7$ (h) $h = 8$ (i) $h = 9$

(j) $h = 10$ (k) $h = 11$ (l) $h = 12$

Figure 38: h-step-ahead density forecast of new car sales