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Economics 144
Economic Forecasting
Spring, 2017

Midterm Exam
May 4, 2017

For full credit on a problem, you need to show all your work and the formula(s) used.

First Name	
Last Name	
UCLA ID #	

Please do not start the exam until instructed to do so.

1.(15%) Suppose that, in a particular monthly dataset, time $t = 10$ corresponds to October 1990.

(a) Name the month and year of the following time: $t + 5$.

March 1991

(b) Suppose that a series of interest follows the simple process $y_t = y_{t-1} + 1$, for $t = 1, 2, 3, \ldots$, meaning that each successive month's value is one higher than the previous month's. Suppose that $y_0 = 0$, and suppose that at present $t = 10$. Calculate the forecast $y_{t+5,t}$, i.e., the forecast made at time t for future time $t + 5$, assuming that $t = 10$ at present.

$y_1 = y_0 + 1 = 1$
$y_2 = y_1 + 1 = 2$
$\ldots, y_{10} = 10$
$\rightarrow y_{t+5,t} = y_{15,10} = 15$

2.(15%) You work for the International Monetary Fund in Washington D. C., monitoring Singapore's real consumption expenditures. Using a sample of real consumption data (measured in billions of 2005 Singapore dollars), y_t, $t = 1990{:}\mathrm{Q1}, \ldots, 2006{:}\mathrm{Q4}$, you estimate the linear consumption trend model, $y_t = \beta_0 + \beta_1 \mathrm{TIME}_t + \varepsilon_t$, where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$, obtaining the estimates $\hat{\beta}_0 = 0.51$, $\hat{\beta}_1 = 2.30$, and $\sigma^2 = 16$. Based on your estimated trend model, construct feasible point, interval (95% where $z = 1.96$), and density forecasts for 2008:Q1.

First we need to find h: 1990:Q1$-$2008:Q1$= 73 \rightarrow h = 73$

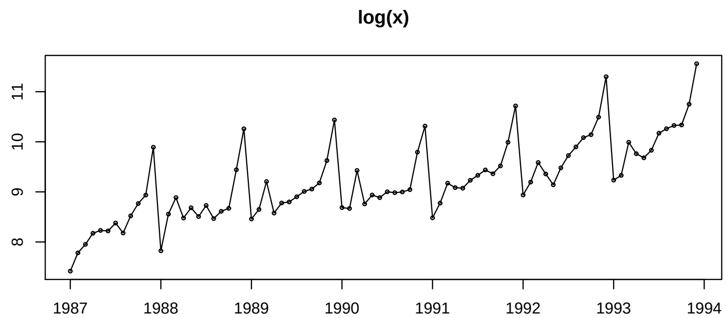
Model: $\hat{y}_{T+h,T} = \hat{\beta}_0 + \hat{\beta}_1 TIME_{T+h}$
$\rightarrow \hat{y}_{T+h,T} = 0.51 + 2.30\ TIME_{T+h}$

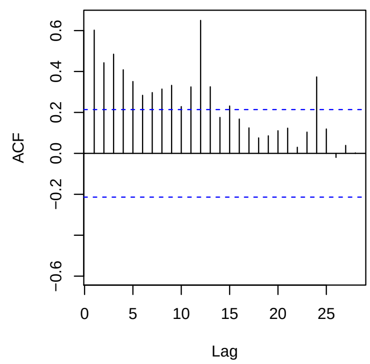
Point Estimate: $\hat{y}_{T+5,T} = 0.51 + 2.30(73) = 168.41$

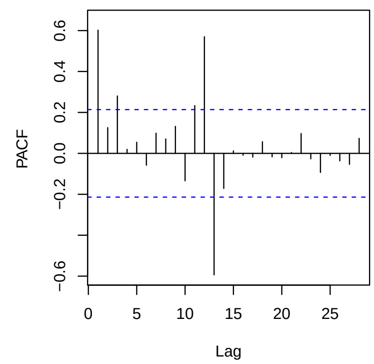
Interval Estimate: $\hat{y}_{T+h,T} \pm z\sigma = 168.41 \pm 1.96(4) = [160.57, 176.25]$

Density Forecast: $\mathcal{N}(\hat{y}_{T+5,T}, \hat{\sigma}^2) = \mathcal{N}(168.41, 16)$

3.(15%) The data below are monthly observations (log of the data) for an unknown series from 1987 to 1994. The ACF and PACF plots are based on the original observations given in the top plot.







(a) Based on the plot of the data alone, what model would you propose for this series in terms of trend and/or seasonality? Does this process appear to be covariance stationary. Justify your answers.

For this process I would recommend perhaps a linear trend with a seasonal dummy variable for the month of December to capture the recurring monthly spike around this month. The process does not appear to be covariance stationary given the appearance of a drift away from its lowest value in 1987.

(b) Based on the ACF and PACF plots, ignoring lags of order 10 or higher, what model would you propose for this series in terms of an MA and/or AR process. Justify your answer.

Since the ACF seems to decay to zero at increasing lags, but the PACF has a prominent spike at lag=3, I would suggest an AR(3) model. Also, given that the second spike in the PACF is not statistically significant, for this AR(3) model, I would only keep the coefficients for lags 1 and 3.

(c) Below is summary of a trend and seasonality fit to the data. Based on this output, is this model fit appropriate? Justify your answer.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	7.6059	0.0769	98.94	0.0000
trend	0.0224	0.0008	26.51	0.0000
season2	0.2510	0.0993	2.53	0.0137
season3	0.6952	0.0993	7.00	0.0000
season4	0.3829	0.0994	3.85	0.0003
season5	0.4080	0.0994	4.11	0.0001
season6	0.4470	0.0994	4.50	0.0000
season7	0.6082	0.0995	6.12	0.0000
season8	0.5854	0.0995	5.88	0.0000
season9	0.6663	0.0996	6.69	0.0000
season10	0.7440	0.0996	7.47	0.0000
season11	1.2030	0.0997	12.07	0.0000
season12	1.9581	0.0998	19.63	0.0000

Yes. Based on the statistical significance of the coefficients and the linear trend fit, it seems that combined, they provide a reasonable fit.
However, we need to look at the ACF and PACF of the residuals form this model in order to further confirm how well the model fit the data.

(d) Using the model fit summary from part (c). Interpret the seasonal factors.

There is a strong seasonal component that suggests a steady increase from January to October, and then almost doubling and tripling in November and December respectively.

4.(15%) (a) Describe the difference between autocorrelations and partial autocorrelations. How can autocorrelations at certain displacements be positive while the partial autocorrelations at those same displacements are negative?

Correlation measures linear association between two variables, whereas partial autocorrelation measures linear association between two variables controlling for the effects of one or more additional variables. Hence the two types of correlation, although related, are nevertheless very different, and they may well be of different signs.

(b) Given the following two MA(1) processes, $y_t = \varepsilon_t + 0.32\varepsilon_{t-1}$ and $y_t = \varepsilon_t + 0.87\varepsilon_{t-1}$, where in each case $\varepsilon_t \sim \mathcal{N}(0, 1)$ (you can assume ε_t are iid). Does $\theta = 0.87$ induce much more persistence than $\theta = 0.32$? Explain your answer in detail.

No, it does not. The structure of the MA(1) process, in which only the first lag of the shock appears on the right, forces it to have a very short memory, and hence, weak dynamics, regardless of the parameter value.

5.(15%) Given the time-series below:

(a) Rewrite the following expression without using the lag operator:

$$y_t + 2y_{t-2} = \left(\frac{L}{L-1} + \frac{2L^2}{L-2} - 1\right)\varepsilon_t$$

$$\begin{aligned}
y_t + 2y_{t-2} &= \left(\frac{L}{L-1} + \frac{2L^2}{L-2} - 1\right)\varepsilon_t \\
\rightarrow (L-1)(L-2)[y_t + 2y_{t-2}] &= [L(L-2) + 2L^2(L-1) - (L-1)(L-2)]\varepsilon_t \\
\rightarrow (L^2 - 3L + 2)[y_t + 2y_{t-2}] &= (L^2 - 2L + 2L^3 - 2L^2 - L^2 + 3L - 2)\varepsilon_t \\
\rightarrow y_{t-2} - 3y_{t-1} + 2y_t + 2y_{t-4} - 6y_{t-3} + 2y_t + 4y_{t-2} &= (2L^3 - 2L^2 + L - 2)\varepsilon_t \\
\rightarrow 2y_t - 3y_{t-1} + 5y_{t-2} - 6y_{t-3} + 2y_{t-4} &= 2\varepsilon_{t-3} - 2\varepsilon_{t-2} + \varepsilon_{t-1} - 2\varepsilon_t
\end{aligned}$$

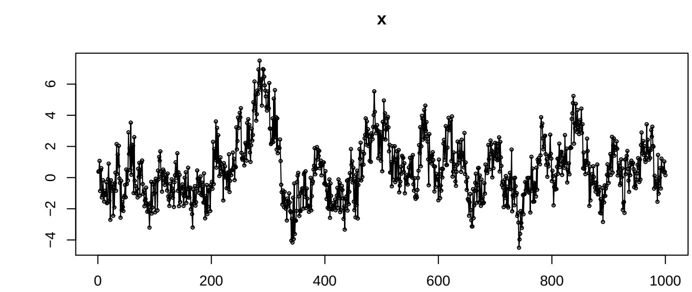
(b) Rewrite the following expression in lag operator form.

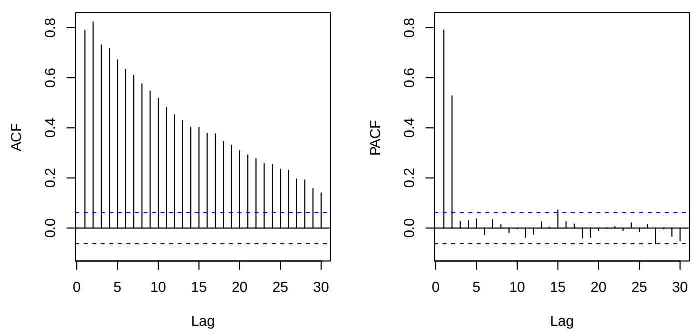
$$10 = 2y_{t-3} + y_t - 5y_{t-5} + 6 - 3\varepsilon_t + 4\varepsilon_{t-5} + 4\varepsilon_{t-2}$$

$$\begin{aligned}
10 &= 2y_{t-3} + y_t - 5y_{t-5} + 6 - 3\varepsilon_t + 4\varepsilon_{t-5} + 4\varepsilon_{t-2} \\
\rightarrow 4 + 3\varepsilon_t - 4\varepsilon_{t-5} - 4\varepsilon_{t-2} &= 2y_{t-3} + y_t - 5y_{t-5} \\
\rightarrow 4 + (3 - 4L^5 - 4L^2)\varepsilon_t &= (2L^3 + 1 - 5L^5)y_y
\end{aligned}$$

Questions 6-10 are multiple choice. Please select one answer per question only.

6.(5%) What model would you propose for the given series below?





(a) MA(2)

(b) AR(2)

(c) MA(∞)

(d) AR(∞)

(e) None of the above

7.(5%) Given following potential loss function, $L(e) = \begin{cases} \sqrt{e} & \text{if } e > 0 \\ |e| & \text{if } e \leq 0 \end{cases}$, we can see that $L(e)$ is

(a) symmetric and satisfies the criteria for a loss function.

(b) asymmetric and satisfies the criteria for a loss function.

(c) symmetric but does not satisfy the criteria for a loss function.

(d) asymmetric but does not satisfy the criteria for a loss function.

(e) None of the above

8.(5%) After fitting the polynomial model, $\hat{y}_t \sim \beta_0 + \beta_2 TIME^2$ to a time-series y_t (assuming only trend), the following residuals were obtained: $e_t = \{-2, -1, 0, 1, 2\}$. Compute the MSE.

(a) 0.00

(b) 4.45

(c) 2.00
MSE$= 1/T \sum_{t=1}^{T} e_t^2 = 1/5((-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2) = 2$

(d) 1.67

(e) None of the above

9.(5%) What process does the series $y_t = 0.95\varepsilon_t$ represent?

(a) Trend only

(b) AR(1)

(c) MA(1)

(d) White noise

(e) None of the above

10.(5%) The table below summarizes the estimation results from fitting an MA(1) model to an unknown economic variable (monthly frequency). Based on the results, compute the 3-step ahead density forecast, $f(Y_{t+3}|_t) = \mathcal{N}(\mu_{t+3|t}, \sigma^2_{t+3|t})$. Assume your estimate of ε_t is 8.25.

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.800	1.290	0.620	0.5400
MA(1)	2.425	0.17234	14.07	0.0000
Std. Error. Regression	5.361			
Std. Dev. y	4.248			
R^2	0.217			

(a) $\mathcal{N}(0.80, 197.7)$
$f_{t,1} = \hat{\mu} + \widehat{\theta}\varepsilon_t = 0.80 + 2.425(8.25) = 20.8$ and $\sigma^2_{t+1|t} = \widehat{\sigma}^2_\varepsilon = 5.361^2 = 28.74$
$f_{t,3} = f_{t,2} = f_{t,1} = 20.8$, and $\sigma^2_{t+3|t} = \sigma^2_{t+2|t} = \widehat{\sigma}^2_\varepsilon(1 + \widehat{\theta}^2) = 197.7$

(b) $\mathcal{N}(20.8, 5.36)$

(c) $\mathcal{N}(0.80, 28.74)$

(d) $\mathcal{N}(20.8, 197.7)$

(e) None of the above