Homework 3

Noah Kawasaki

05/07/2018

R Exercises

Question 1.

Let X and Y be distributed bivariate normal with:

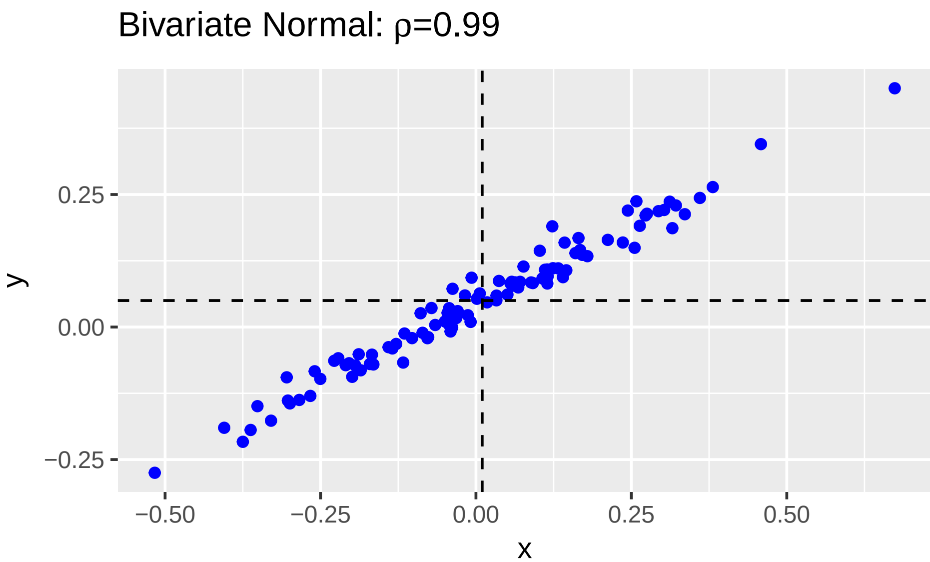
$$\mu_X = 0.01,\ \mu_Y = 0.05,\ \sigma_X = 0.25,\ \sigma_Y = 0.15$$

```
# Initial parameters and variables
mu_x <- 0.01
mu_y <-  0.05
sig_x <-  0.25
sig_y <-  0.15

n <-  100
set.seed(123)
```

Using R package function rmvnorm(), simulate 100 observations from the bivariate distribution. Using the plot() function create a scatterplot of the observations and comment on the direction and strength of the linear association. Using the function pmvnorm(), compute the joint probability: $P(X \leq 0, Y \leq 0)$:

a) $\rho_{XY} = 0.99$

```
rho <- 0.99
sig_xy <-  rho*sig_x*sig_y
Sigma_xy <-  matrix(c(sig_x^2, sig_xy, sig_xy, sig_y^2), 2, 2, byrow=TRUE)
xy_vals <-  data.frame(rmvnorm(n, mean=c(mu_x, mu_y), sigma=Sigma_xy))

ggplot(xy_vals, aes(X1, X2)) +
  geom_point(color='blue') +
  geom_hline(yintercept=mu_y, linetype='dashed', color='black') +
  geom_vline(xintercept=mu_x, linetype='dashed', color='black') +
  ggtitle(expression(paste("Bivariate Normal: ", rho, "=0.99"))) +
  xlab('x') +
  ylab('y')
```



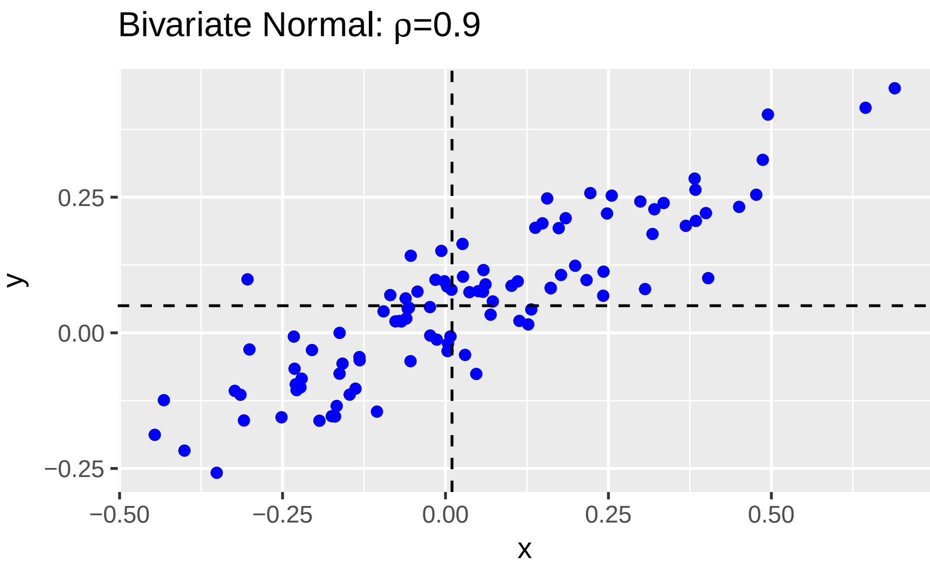

```
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu_x, mu_y), sigma=Sigma_xy)[1]
```

```
## [1] 0.3690556
```

Here we observe a very strong positive linear association.

b) $\rho_{XY} = 0.9$

```
rho <- 0.9
sig_xy <-  rho*sig_x*sig_y
Sigma_xy <-  matrix(c(sig_x^2, sig_xy, sig_xy, sig_y^2), 2, 2, byrow=TRUE)
xy_vals <-  data.frame(rmvnorm(n, mean=c(mu_x, mu_y), sigma=Sigma_xy))

ggplot(xy_vals, aes(X1, X2)) +
  geom_point(color='blue') +
  geom_hline(yintercept=mu_y, linetype='dashed', color='black') +
  geom_vline(xintercept=mu_x, linetype='dashed', color='black') +
  ggtitle(expression(paste("Bivariate Normal: ", rho, "=0.9"))) +
  xlab('x') +
  ylab('y')
```



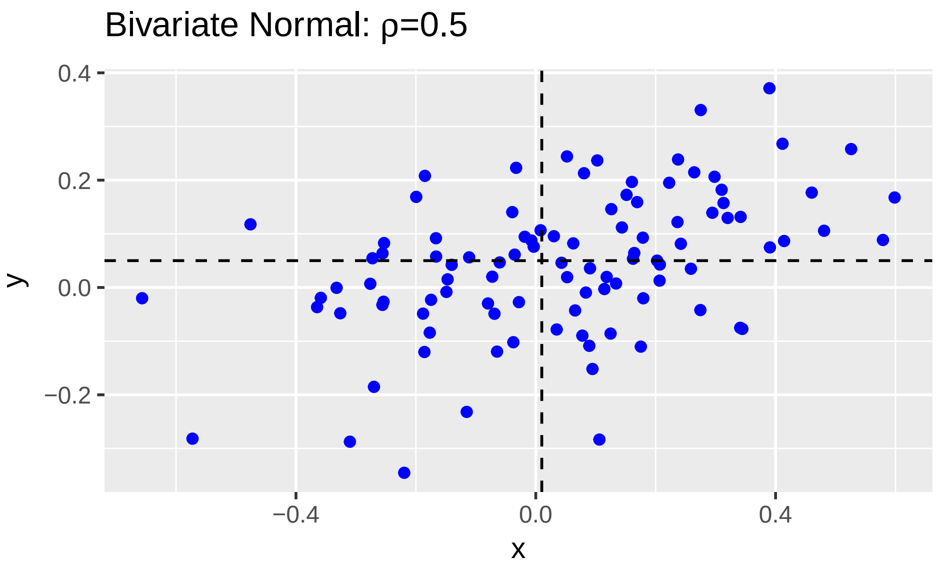

```
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu_x, mu_y), sigma=Sigma_xy)[1]
```

```
## [1] 0.3420967
```

Here we still observe a clear positive linear association, but it is more sparse.

c) $\rho_{XY} = 0.5$

```
rho <- 0.5
sig_xy <-  rho*sig_x*sig_y
Sigma_xy <-  matrix(c(sig_x^2, sig_xy, sig_xy, sig_y^2), 2, 2, byrow=TRUE)
xy_vals <-  data.frame(rmvnorm(n, mean=c(mu_x, mu_y), sigma=Sigma_xy))

ggplot(xy_vals, aes(X1, X2)) +
  geom_point(color='blue') +
  geom_hline(yintercept=mu_y, linetype='dashed', color='black') +
  geom_vline(xintercept=mu_x, linetype='dashed', color='black') +
  ggtitle(expression(paste("Bivariate Normal: ", rho, "=0.5"))) +
  xlab('x') +
  ylab('y')
```



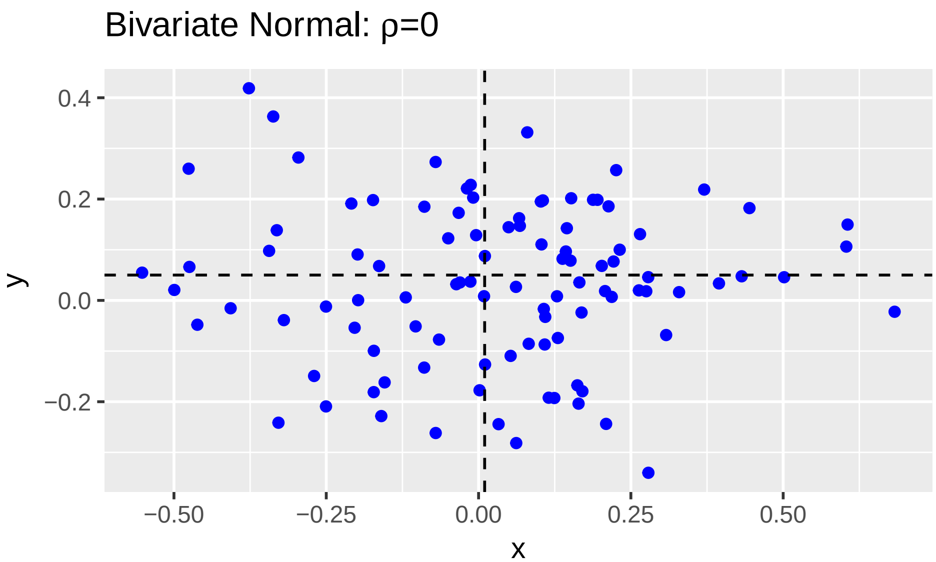

```r
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu_x, mu_y), sigma=Sigma_xy)[1]
```

```
## [1] 0.2574488
```

Here it is difficult to tell, but there is still a weak linear positive association.

d) $\rho_{XY} = 0$

```r
rho <- 0
sig_xy <-  rho*sig_x*sig_y
Sigma_xy <-  matrix(c(sig_x^2, sig_xy, sig_xy, sig_y^2), 2, 2, byrow=TRUE)
xy_vals <-  data.frame(rmvnorm(n, mean=c(mu_x, mu_y), sigma=Sigma_xy))

ggplot(xy_vals, aes(X1, X2)) +
  geom_point(color='blue') +
  geom_hline(yintercept=mu_y, linetype='dashed', color='black') +
  geom_vline(xintercept=mu_x, linetype='dashed', color='black') +
  ggtitle(expression(paste("Bivariate Normal: ", rho, "=0"))) +
  xlab('x') +
  ylab('y')
```



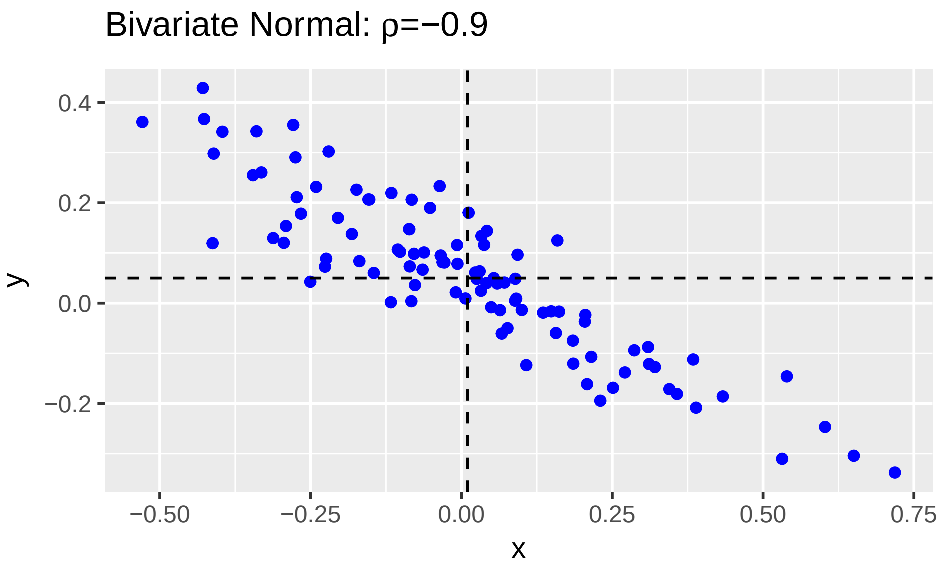

```
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu_x, mu_y), sigma=Sigma_xy)[1]
```

```
## [1] 0.1788268
```

Here we observe no correlation, the dots look randomly scattered.

e) $\rho_{XY} = -0.9$

```
rho <- -0.9
sig_xy <-  rho*sig_x*sig_y
Sigma_xy <-  matrix(c(sig_x^2, sig_xy, sig_xy, sig_y^2), 2, 2, byrow=TRUE)
xy_vals <-  data.frame(rmvnorm(n, mean=c(mu_x, mu_y), sigma=Sigma_xy))

ggplot(xy_vals, aes(X1, X2)) +
  geom_point(color='blue') +
  geom_hline(yintercept=mu_y, linetype='dashed', color='black') +
  geom_vline(xintercept=mu_x, linetype='dashed', color='black') +
  ggtitle(expression(paste("Bivariate Normal: ", rho, "=-0.9"))) +
  xlab('x') +
  ylab('y')
```




```r
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu_x, mu_y), sigma=Sigma_xy)[1]
```

```
## [1] 0.02024602
```

This plot shows a clear negative linear association.