ECON 147 Homework 3

Due: 12:30 pm, May 7th (Mon)

Reading and Program Downloads

- Please read the course material in course website.
- The R code (econ147lab3_Hint.r) in the course website will be helpful. Make sure you modify the codes to have the correct answers.

Review Questions

1. Let Y_1, Y_2, Y_3 and Y_4 be iid(μ, σ^2). Let $\bar{Y} = \frac{1}{4}\sum_{t=1}^{4} Y_t$.

   - What are the expected value and variance of $\bar{Y}$?
   - Now, consider a different estimator of μ:

     $$W = \frac{1}{8}Y_1 + \frac{1}{8}Y_2 + \frac{1}{4}Y_3 + \frac{1}{2}Y_4,$$

     What are the expected value and variance of W?

   - Which estimator of μ do you prefer? Fully justify your answer.

2. Let $Y_1, Y_2, Y_3, , , Y_n$ be iid(μ, σ^2) and let $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$

   - Define the class of *linear estimator* of μ by

     $$W_a = \sum_{i=1}^{n} a_i Y_i$$

     where $a_i's$ are constants. What restriction on the $a_i's$ is need for W_a to be an unbiased estimator of μ?
   - Find $Var\,(W_a)$.
   - For any numbers a_i, $i = 1, ..., n$, the following inequality holds

     $$\left(\sum_{i=1}^{n} a_i\right)^2 \leq n\sum_{i=1}^{n} a_i^2.$$

     Use this (and above results) to show $\bar{Y}$ is the *best linear unbiased estimator* (BLUE).

3. Consider the constant expected return model

$$r_{it} = \mu_i + \epsilon_{it} \quad t = 1, \cdots, T; \quad i = 1 \text{ (GS)},\ 2 \text{ (AIG)},$$
$$\epsilon_{it} \sim \text{iid } N(0, \sigma_i^2),\ \text{cov}(\epsilon_{1t},\ \epsilon_{2t}) = \sigma_{12},\ \text{cor}(\epsilon_{1t},\ \epsilon_{2t}) = \rho_{12}$$

for the monthly cc returns on GS (Goldman Sachs) and AIG (American Internation Group). The estimates (rounded for computations) are given ($T = 100$ months):

	GS	AIG	GS&AIG	
$\hat{\mu}_i$	0.01	-0.03	$\hat{\sigma}_{12}$	0.01
$\hat{\sigma}_i$	0.1	0.3	$\hat{\rho}_{12}$	0.4

- For both GS and AIG cc returns, compute (asymptotic) 95% CI for μ_i and σ_i^2.
- Compute (asymptotic) 95% confidence interval for ρ_{12} (You will use $SE(\hat{\rho}_{12}) = \sqrt{\frac{1-\hat{\rho}_{12}^2}{T}}$).
- Test the hypothesis (significance tests) for $i = 1, 2$, with 5% confidence level,

  $$H_0 : \mu_i = 0 \quad \text{v.s.} \quad H_1 : \mu_i \neq 0.$$

  Are expected returns of these assets (statistically) different from zero? Justify your answer.
- Test the hypothesis for $i = 1, 2$, with 5% confidence level,

  $$H_0 : \sigma_i^2 = 0.0225 \quad \text{v.s.} \quad H_1 : \sigma_i^2 \neq 0.0225.$$

R Exercises

The following questions require R. On our course website there is the R script file 147lab3_Hint.r. The file contains hints for completing this R exercises. Copy and paste all statistical results and graphs into a MS Word document (or your favorite word processor) while you work, and add any comments and answer all questions in this document.

Start MS Word and open a blank document. You will save all of your work in this document.

1. Let X and Y be distributed bivariate normal with

$$\mu_X = 0.01,\ \mu_Y = 0.05,\ \sigma_X = 0.25,\ \sigma_Y = 0.15.$$

   (a) Using R package function *rmvnorm()*, simulate 100 observations from the bivariate distribution with $\rho_{XY} = 0.99$. Using the *plot()* function create a scatterplot of the observations and comment on the direction and strength of the linear association. Using the function *pmvnorm()*, compute the joint probability $P\left(X \leq 0, Y \leq 0\right).$

   (b) Do the same exercise with $\rho_{XY} = 0.9$.

   (c) Do the same exercise with $\rho_{XY} = 0.5$.

   (d) Do the same exercise with $\rho_{XY} = 0$.

   (e) Do the same exercise with $\rho_{XY} = -0.9$.