ECON 147 Homework 3

Answer Keys

Review Questions

1. Let Y_1, Y_2, Y_3 and Y_4 be iid (μ, σ^2). Let $\bar{Y} = \frac{1}{4}\sum_{t=1}^{4} Y_t$.

   - What are the expected value and variance of $\bar{Y}$? **Answers:** μ and $\frac{\sigma^2}{4}$.
   - Now, consider a different estimator of μ:

     $$W = \frac{1}{8}Y_1 + \frac{1}{8}Y_2 + \frac{1}{4}Y_3 + \frac{1}{2}Y_4,$$

     What are the expected value and variance of W? **Answers:** still μ, and variance is $\frac{11}{32}\sigma^2$.
   - Which estimator of μ do you prefer? Fully justify your answer. **Answers:** since both estimators are unbiased, we just choose the one with lower variance (recall MSE criteria), so $\bar{Y}$.

2. Let $Y_1, Y_2, Y_3, \ldots, Y_n \sim (\mu, \sigma^2)$ and let $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$

   - Define the class of *linear estimator* of μ by

     $$W_a = \sum_{i=1}^{n} a_i Y_i$$

     where $a_i's$ are constants. What restriction on the $a_i's$ is need for W_a to be an unbiased estimator of μ? **Answer:** We need $\sum_{i=1}^{n} a_i = 1$
   - Find $Var(W_a)$. **Answer:** $\left(\sum_{i=1}^{n} a_i^2\right)\sigma^2$.

- For any numbers a_i, $i = 1, ..., n$, the following inequality holds

$$\left(\sum_{i=1}^{n} a_i\right)^2 \leq n \sum_{i=1}^{n} a_i^2.$$

Use this (and above results) to show $\bar{Y}$ is the *best linear unbiased estimator* (BLUE). **Answer:** for any linear unbiased estimator,

$$Var\left(\bar{Y}\right) = \frac{\sigma^2}{n} \leq \left(\sum_{i=1}^{n} a_i^2\right) \sigma^2 = Var\left(W_a\right)$$

so $\bar{Y}$ is the *best* in MSE sense.

3. Consider the constant expected return model

$$r_{it} = \mu_i + \epsilon_{it} \quad t = 1, \cdots, T; \quad i = 1 \text{ (GS)}, \ 2 \text{ (AIG)},$$
$$\epsilon_{it} \sim \text{iid } N(0, \sigma_i^2), \ \text{cov}(\epsilon_{1t}, \ \epsilon_{2t}) = \sigma_{12}, \ \text{cor}(\epsilon_{1t}, \ \epsilon_{2t}) = \rho_{12}$$

for the monthly cc returns on GS (Goldman Sachs) and AIG (American International Group). The estimates (rounded for computations) are given ($T = 100$ months):

	GS	AIG	GS&AIG	
$\hat{\mu}_i$	0.01	-0.03	$\hat{\sigma}_{12}$	0.01
$\hat{\sigma}_i$	0.1	0.3	$\hat{\rho}_{12}$	0.4

- For both GS and AIG cc returns, compute (asymptotic) 95% CI for μ_i and σ_i^2. **Answers:** use the formula:

$$\hat{\mu}_i \pm 1.96 \frac{\hat{\sigma}_i}{\sqrt{T}} \text{ and } \hat{\sigma}_i^2 \pm 1.96 \frac{\sqrt{2}\hat{\sigma}_i^2}{\sqrt{T}}$$

- Compute (asymptotic) 95% confidence interval for ρ_{12} (You will use $SE(\hat{\rho}_{12}) = \sqrt{\frac{1-\hat{\rho}_{12}^2}{T}}$). **Answer:** use $\hat{\rho}_{12} \pm 1.96\sqrt{\frac{1-\hat{\rho}_{12}^2}{T}}$.

	GS	AIG
μ	$(-0.0096, 0.0296)$	$(-0.0888, 0.0288)$
σ^2	$(0.0072, 0.0127)$	$(0.0651, 0.1149)$
ρ	$(0.2204, 0.5796)$	$(0.2204, 0.5796)$

- Test the hypothesis (significance tests) for $i = 1, 2$, with 5% confidence level,

$$H_0 : \mu_i = 0 \quad \text{v.s. } H_1 : \mu_i \neq 0.$$

  Are expected returns of these assets (statistically) different from zero? Justify your answer. **Answers:** First calculate the test statistics:

$$\left| \frac{\hat{\mu}_i - 0}{\hat{\sigma}_i/\sqrt{T}} \right| = \begin{cases} 1, & \text{GS} \\ 1, & \text{AIG} \end{cases}.$$

  Since the critical value is $q_{0.975}^{T(99)} = 1.9842$, the alternative hypothesis H_1 is rejected for both GS and AIG.

- Test the hypothesis for $i = 1, 2$, with 5% confidence level,

$$H_0 : \sigma_i^2 = 0.0225 \quad \text{v.s.} \quad H_1 : \sigma_i^2 \neq 0.0225.$$

  Answers: First calculate the test statistics:

$$\left| \frac{\hat{\sigma}_i^2 - 0.0225}{\sqrt{2}\hat{\sigma}_i^2/\sqrt{T}} \right| = \begin{cases} 8.8388, & \text{GS} \\ 5.3033, & \text{AIG} \end{cases}.$$

  Since the critical value is $q_{0.975}^{Z} = 1.96$, the null hypothesis H_0 is rejected for both GS and AIG.