ECON 147 Homework 6

No Due Date

Reading

- Please read the course material on the course website.

Review Questions

1. Figure 1 and 2 below show monthly log of stock prices and cc returns on Goldman Sachs Group (GS) and American International Group (AIG). The sample period is from May 2005 to September 2013. (T=100 monthly observations)

Figure 1: log prices on GS and AIG

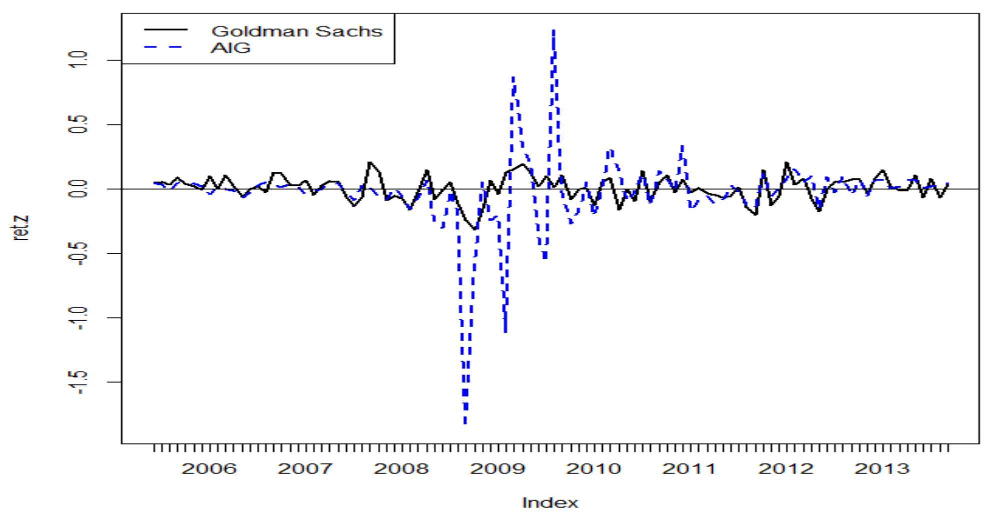

Figure 2: cc returns on GS and AIG

1.1. Do the monthly log of stock prices from AIG look like realizations from a covariance stationary stochastic process? Why or why not?

1.2. Do the monthly cc returns from AIG look like realizations from a covariance stationary stochastic process? Why or why not?

1.3. In late 2008, AIG's credit rating was downgraded and it leads to AIG's liquidity crisis, meanwhile GS was maintaining sizeable profits (relatively). Comment on any common or distinctive features of the two stock prices and return series.

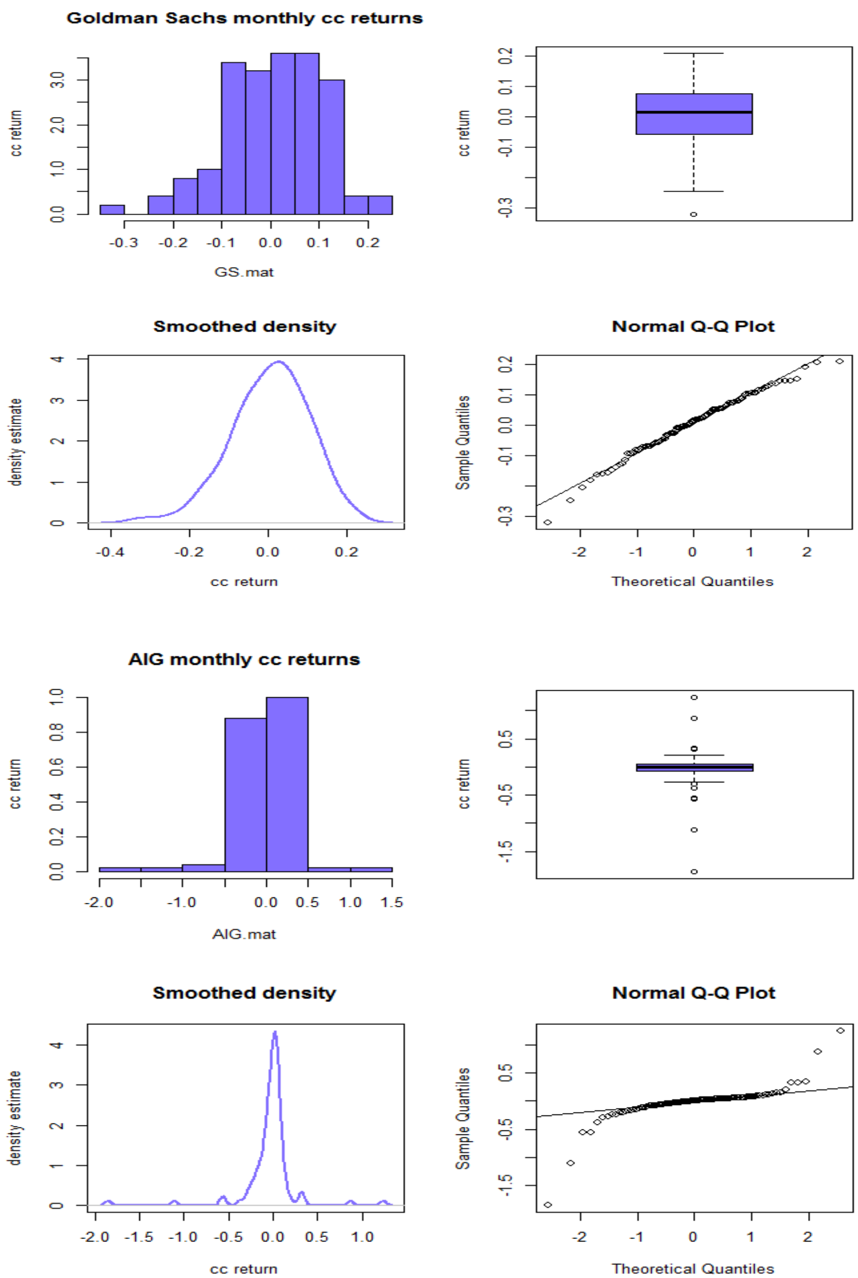

Figure 3: Descriptive Statistics on GS and AIG

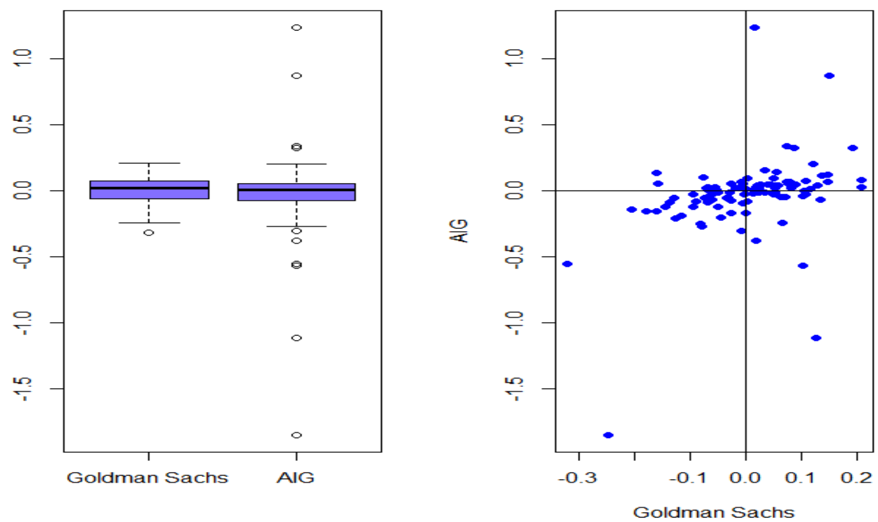

Figure 4: Box Plots and Scatter Plots on GS and AIG

1.4. Based on Figure 3 & 4, do the returns on AIG and GS look normally distributed? Briefly justify your answer.

1.5. Which asset appears to be riskier? Briefly justify your answer.

1.6. Based on the scatterplot of returns, does there appear to be any linear dependence between the returns on GS and AIG? Briefly justify your answer.

2. Figure 5 below shows US quarterly GDP time series data.

Figure 5: US GDP

Many empirical researchers used to model US GDP time series Y_t using the following Unit Root model:

$$Y_t = \mu + Y_{t-1} + \varepsilon_t,\ \varepsilon_t \sim iid\ N(0, \sigma_\varepsilon^2)$$

2.1. Do the US GDP data looks like realizations from a covariance stationary stochastic process? Why or why not?

2.2. When $Y_0 = 0$, using recursive substitution, show that

$$Y_t = \mu t + \sum_{j=1}^{t} \varepsilon_j,$$

and find $E[Y_t]$ and $Var\,(Y_t)$. Is Y_t covariance stationary?

3. Consider the constant expected return model for two stocks (Boeing and Microsoft)

$$R_i = \mu_i + \epsilon_i \text{ for } i = 1, 2 \text{ (Boeing and Msft, respectively)}$$

where $\epsilon_i \sim N(0, \sigma_i^2)$, $\text{cov}(\epsilon_1,\ \epsilon_2) = \sigma_{12}$ and $\text{cor}(\epsilon_1,\ \epsilon_2) = \rho_{12}$.

3.1. Write down the optimization problem to determine the global minimum variance portfolio. For $i = 1, 2$, let m_i denote the portfolio weight in the global minimum variance portfolio on asset i.

3.2. Write down the optimization problem used to determine the tangency portfolio when the risk free rate is given by r_f. For $i = 1, 2$, let t_i denote the portfolio weight in the tangency portfolio on asset i. What does the following ratio represent,

$$\frac{t_1\mu_1 + t_2\mu_2 - r_f}{(t_1^2\sigma_1^2 + t_2^2\sigma_2^2 + 2t_1t_2\sigma_{12})^{1/2}}$$

in financial economics?

3.3. State Mutual Fund Separation Theorem and draw a portfolio frontier based on the Theorem. Discuss the resulting weights determination (x_f for r_f and $x_{\tan}$ for $R_{\tan}$) according to an investor's risk preferences.

3.4. Suppose that

$$r_f = 0.1,\ \mu_1 = 0.2,\ \mu_2 = 0.4,\ \sigma_1^2 = 1,\ \sigma_2^2 = 4 \text{ and } \sigma_{12} = -2. \quad (1)$$

Find the global minimum variance portfolio and the tangency portfolio.

3.5. Suppose we have $W_0 = 1000$. Find the value of risk at $\alpha = 0.05$ if we invest all the money in the global minimum variance portfolio found in 3.4. Find the value of risk at $\alpha = 0.05$ if we invest all the money in the tangency portfolio found in 3.4.

4. Let $\{X_t\}_t$ be a time series generated by

$$X_t = u_t + \theta_1 u_{t-1}, \text{ where } \{u_t\}_t \sim iid(0, \sigma^2),$$

where θ_1 is a finite real number. Let $\gamma(j)$ and $\rho(j)$ denote the auto-covariance function and the auto-correlation function of $\{X_t\}_t$ respectively.

4.1. Find the auto-covariance function $\gamma(j)$.

4.2. Find the auto-correlation function $\rho(j)$.

4.3. What are the largest and smallest possible values for $\rho(1)$?

4.4. Suppose that $\sigma^2 = 1$. Find the value of θ_1 such that $\gamma(0) = 2$, $\gamma(1) < 0$ and $\gamma(h) = 0$ for $|h| > 1$.

5 One of the recent Nobel laureate in Economics, Eugene Fama, has been advocating the efficient-market hypothesis. One testable implication of this hypothesis is called martingale (MG) pricing:

$$E\left[p_t|\mathcal{F}_{t-1}\right] = p_{t-1} \text{ for all } t,$$

where p_t is a log of stock price index (e.g., S&P 500 index) and $\mathcal{F}_{t-1}$ represents all available information up to time $t-1$. Answer the following questions.

5.1. Briefly discuss why MG pricing is one supporting evidence for efficient-market hypothesis.

5.2. Under MG pricing, show cc return r_t is mds, i.e., $E\left[r_t|\mathcal{F}_{t-1}\right] = 0$.

5.3. The definition of mds allows a flexible specification of the second moment. For example, $r_t \sim mds\left(0, \sigma_t^2\right)$ where σ_t^2 can be time varying (hence nonstationary). Briefly discuss the benefit of this specification over CER model.

6. Indicate whether the following statements are true or false (circle one). Briefly discuss why it is so.

6.1. If $\varepsilon_t \sim mds\ (0, \sigma_\varepsilon^2)$, then $\varepsilon_t \sim iid\ (0, \sigma_\varepsilon^2)$.

True False

Why?

6.2. If $\varepsilon_t \sim mds\ (0, \sigma_\varepsilon^2)$, then $\varepsilon_t \sim WN\ (0, \sigma_\varepsilon^2)$.

True False

Why?

6.3. If $\{\varepsilon_t\}_t$ is strictly stationary, then it is also covariance stationary.

True False

Why?

6.4. Let Y_1 and Y_2 be iid r.v's from (μ, σ^2), and let $\hat{\mu}_1 = Y_1$ and $\hat{\mu}_2 = \frac{Y_1+Y_2}{2}$ are two different point estimators for μ. From MSE criteria, we prefer to use $\hat{\mu}_1$ rather than $\hat{\mu}_2$.

True False

Why?

6.5. MA(1) process $Y_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$, $\varepsilon_t \sim mds(0, \sigma_\varepsilon^2)$ is *not covariance stationary* when $|\theta| = 1$.

True False

Why?