Homework 2

Question 2

Suppose X is a normally distributed random variable with mean 0 and variance 1 (i.e., standard normal). Compute the following (Hint: you can use the R functions pnorm and qnorm to answer these questions).

$Pr(X < -1.96)$

```
pnorm(-1.96)
```

```
## [1] 0.0249979
```

$Pr(X > 1.64)$

```
pnorm(-1.64)
```

```
## [1] 0.05050258
```

$Pr(-0.5 < X < 0.5)$

```
pnorm(0.5) - pnorm(-0.5)
```

```
## [1] 0.3829249
```

$q_{.01}$

```
qnorm(0.01)
```

```
## [1] -2.326348
```

$q_{.99}$

```
qnorm(0.99)
```

```
## [1] 2.326348
```

$q_{.05}$

```
qnorm(0.05)
```

```
## [1] -1.644854
```

$q_{.95}$

```
qnorm(0.95)
```

```
## [1] 1.644854
```

Question 3

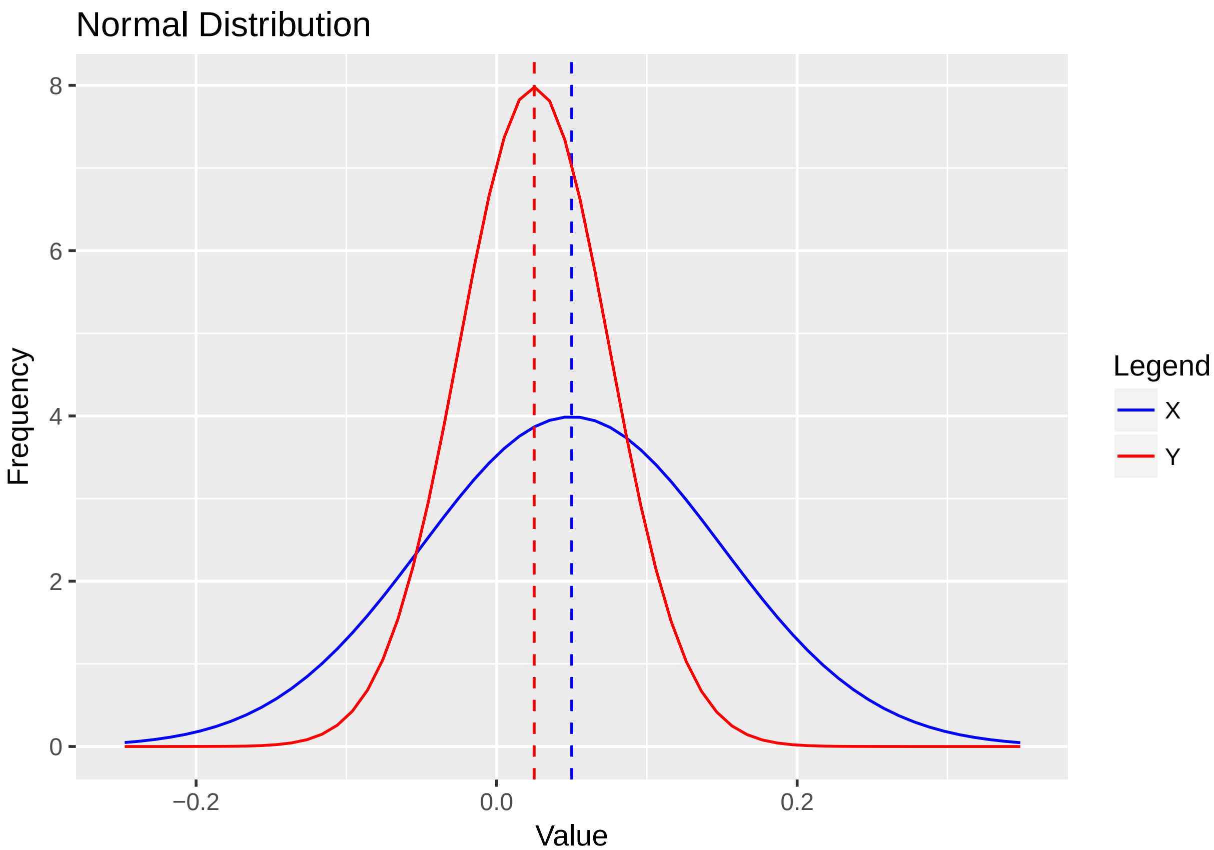
Let X denote the monthly return on Microsoft Stock and let Y denote the monthly return on Starbucks stock. Assume that X~N(0.05, $(0.10)^2$) and Y~N(0.025, $(0.05)^2$).

- Using a grid of values between -0.25 and 0.35, plot the normal curves for X and Y. Make sure that both normal curves are on the same plot.
- Comment on the risk-return trade-offs for the two stocks. Which one do you want to invest?

```
x <- seq(-0.5, 0.5, length=100)
xd <- dnorm(x, 0.05, 0.10)
```

```r
y <- seq(-0.5, 0.5, length=100)
yd <- dnorm(y, 0.025, 0.05)

cols <- c('X'='blue', 'Y'='red')
ggplot() +
  geom_line(aes(x=x, y=xd, color='X'), na.rm=TRUE) +
  geom_vline(xintercept=0.05, color='blue', linetype='dashed') +
  geom_line(aes(x=y, y=yd, color='Y'), na.rm=TRUE) +
  geom_vline(xintercept=0.025, color='red', linetype='dashed') +
  xlim(-0.25, 0.35) +
  ggtitle('Normal Distribution') +
  xlab('Value') +
  ylab('Frequency') +
  scale_color_manual('Legend', values=cols)
```



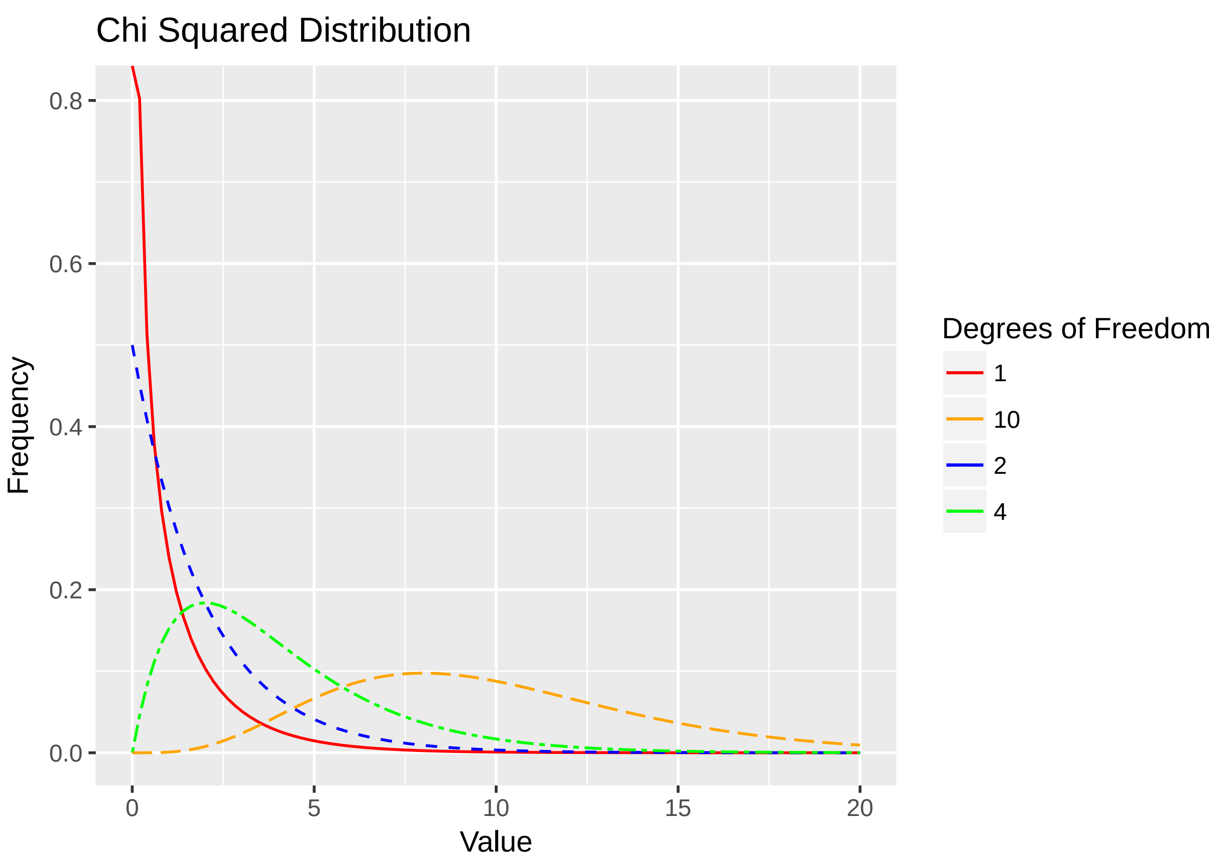
From this distribution plot we can see that X has a higher expected value than Y, but also a higher variance. So the decision on which stock to invest in will depend on the risk averse level of the investor

Question 6

```r
xc = seq(0, 20, length=100)
xcd_1 <- dchisq(xc, df=1)
xcd_2 <- dchisq(xc, df=2)
xcd_4 <- dchisq(xc, df=4)
xcd_10 <- dchisq(xc, df=10)
```

```
df_chi <- data.frame(xc, xcd_1, xcd_2, xcd_4, xcd_10)

cols2 <- c('1'='red', '2'='blue', '4'='green', '10'='orange')
ggplot(df_chi, aes(x=xc)) +
  geom_line(aes(y=xcd_1, color='1'), linetype='solid') +
  geom_line(aes(y=xcd_2, color='2'), linetype='dashed') +
  geom_line(aes(y=xcd_4, color='4'), linetype='twodash') +
  geom_line(aes(y=xcd_10, color='10'), linetype='longdash') +
  xlim(0,20) +
  ggtitle('Chi Squared Distribution') +
  xlab('Value') +
  ylab('Frequency') +
  scale_color_manual('Degrees of Freedom', values=cols2)
```

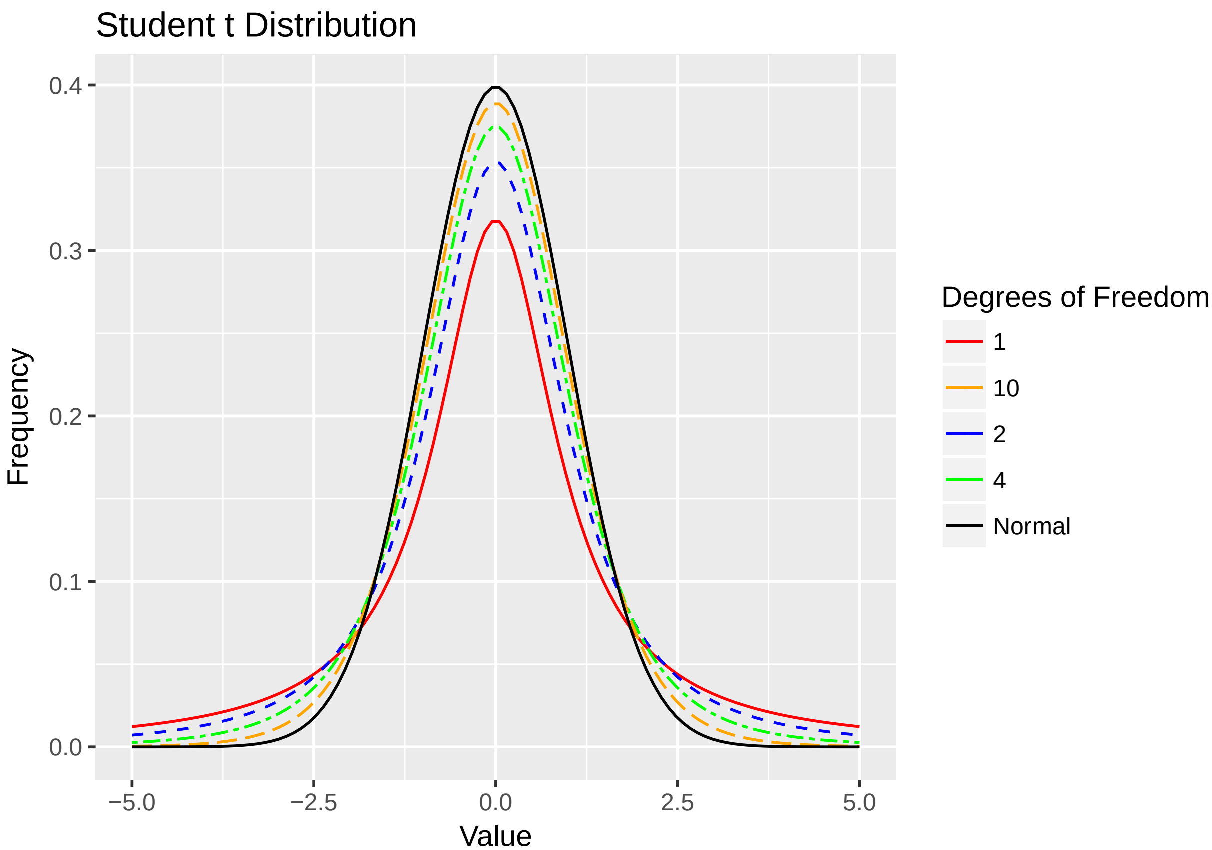



```
xt = seq(-5, 5, length=100)
xtn <- dnorm(xt)
xt_1 <- dt(xt, df=1)
xt_2 <- dt(xt, df=2)
xt_4 <- dt(xt, df=4)
xt_10 <- dt(xt, df=10)

df_t <- data.frame(xt, xt_1, xt_2, xt_4, xt_10, xtn)

cols2 <- c('1'='red', '2'='blue', '4'='green', '10'='orange', 'Normal'='black')
ggplot(df_t, aes(x=xt)) +
  geom_line(aes(y=xt_1, color='1'), linetype='solid') +
```

```r
  geom_line(aes(y=xt_2, color='2'), linetype='dashed') +
  geom_line(aes(y=xt_4, color='4'), linetype='twodash') +
  geom_line(aes(y=xt_10, color='10'), linetype='longdash') +
  geom_line(aes(y=xtn, color='Normal')) +
  xlim(-5,5) +
  ggtitle('Student t Distribution') +
  xlab('Value') +
  ylab('Frequency') +
  scale_color_manual('Degrees of Freedom', values=cols2)
```



The 5% VaR of the t distribution with 2 degrees of freedom will be greater in absolute value because the tails are fatter than the normal distribution. Assuming normality may lead to underestimation of Value at Risk.