ECON 147 MIDTERM EXAM

YOUR NAME:

Winter 2018, Feb 13th
3:30pm - 4:45pm

Instruction

- This is a closed book and closed note exam. All necessary information will be provided and all calculation will be done by hands, so no cheat sheet or calculator is needed.
- Try to answer all questions and write all answers **within this exam sheets**. You will hand in this exam sheets. Please write legibly.
- Total points are 100. Use your time wisely!
- *Examination Rules from Department Policy will be strictly followed.*
- The following results may be useful:

$$
\begin{array}{ll}
\Pr(Z \leq 0.5) = 0.69, & \Pr(Z \leq -1.1) = 0.136, \\
\Pr(Z > -1.96) = 0.975, & \Pr(Z \leq -0.6) = 0.274, \\
\Pr(Z \leq -1.64) = 0.05, & \Pr(Z \leq -0.70) = 0.242, \\
\Pr(Z \leq -0.35) = 0.363, & e^{-2.3} = 0.100, \\
e^{-1.2} = 0.300, & e^{-1.9} = 0.150, \\
e^{-3.2} = 0.041, & e^{-2.9} = 0.055,
\end{array}
$$

  where Z is a standard normal random variable.

1. **(10 pts)** The daily cc returns r_t on a stock are independent and normally distributed with mean 0.001 and standard deviation 0.01. Suppose you buy \$1000 worth of this stock.

1.1. What is the probability that after one trading day your investment is worth less than \$990? **[5 pts]** [**Hint:** use the fact that $\log(e^{r_t}) = r_t$ is a normal random variable to transfer the event in the question to an event about the standard normal random. You should use $\log(1+x) \simeq x$ for small x.]

1.2. What is the probability that after four trading days your investment is worth less than \$990? [**5 pts**]

2 **(5 pts)** Let r_t be a montly cc return. Suppose that $r_1, r_2, ...$ are independent and identically distributed normal random variables with mean 0.06 and variance 0.49. Let $W_0 = \$100$. Determine the 5% value-at-risk (VaR) over 9 months on the investment.

3. **(30 pts)** Let Y_1, Y_2, Y_3, Y_4 and Y_5 be iid (μ, σ^2).

3.1. Let $W_1 = \frac{1}{8}Y_1 + \frac{1}{8}Y_2 + \frac{1}{2}Y_3 + \frac{1}{8}Y_4 + \frac{1}{8}Y_5$. Find the mean and variance of W_1? **[4 pts]**

3.2. Let $W_2 = \frac{1}{4}Y_1 + \frac{1}{4}Y_2 + \frac{1}{5}Y_3 + \frac{1}{4}Y_4 + \frac{1}{4}Y_5$. Find the mean and variance of W_2? **[4 pts]**

3.3. Which estimator of μ (W_1 or W_2) do you prefer? Fully justify your answer. **[9 pts]**

3.4. An estimator W_a is called a linear estimator of μ if it takes the following form

$$W = a_1Y_1 + a_2Y_2 + a_3Y_3 + a_4Y_4 + a_5Y_5.$$

Among all possible unbiased linear estimators of μ based on Y_1, Y_2, Y_3, Y_4 and Y_5, does there exist a best (unbiased) estimator which has smallest mean square error? Fully justify your answer. [Hint: the inequality $\left(\sum_{i=1}^5 a_i\right)^2 \leq 5\sum_{i=1}^5 a_i^2$ will be useful.] **[9 pts]**

4. **(30 pts)** Suppose X is a uniform random variable over [-1,1] (i.e., $X \backsim U[-1,1]$) and Y is a Bernoulli random variable with a success probability $\Pr(Y=1) = 0.6$, *i.e.*,

$$Y = \begin{cases} 1, & \text{with probability } 0.6 \\ -\frac{3}{2}, & \text{with probability } 0.4 \end{cases}.$$

Compute the following.

4.1. $\Pr(X < 0.1)$ and $\Pr(Y < 0.1)$. **[6 pts]**

4.2. $E(X)$, $Var(X)$, $E(Y)$ and $Var(Y)$. **[10 pts]**

4.3. For any $\alpha \in [0,1]$, $E(\alpha X + (1-\alpha)Y)$. **[4 pts]**

4.4. Now suppose X and Y represent (statistically independent) outcomes of two lotteries, and you would like to invest your \$100 to these lotteries. Assume you only care about the mean (preferably high) and variance (preferably low) of your investment. How do you want to distribute your \$100, i.e., how do you choose your $\alpha \in [0, 1]$ for $\alpha X + (1 - \alpha)Y$? Justify your answer. **[10 pts]**

5. **(25 pts)** Consider the constant expected return model

$$r_{it} = \mu_i + \epsilon_{it} \quad t = 1, \cdots, T; \quad i = 1 \text{ (GS)}, \ 2 \text{ (AIG)},$$
$$\epsilon_{it} \sim \text{iid } N(0, \sigma_i^2), \ \text{cov}(\epsilon_{1t}, \ \epsilon_{2t}) = \sigma_{12}, \ \text{cor}(\epsilon_{1t}, \ \epsilon_{2t}) = \rho_{12}$$

for the monthly cc returns on GS (Goldman Sachs) and AIG (American International Group). The estimates (rounded for computations) are given ($T = 100$ months):

	GS	AIG	GS&AIG	
$\hat{\mu}_i$	0.03	-0.01	$\hat{\sigma}_{12}$	0.048
$\hat{\sigma}_i$	0.2	0.4		

5.1. For both GS and AIG cc returns, compute (asymptotic) 95% CI for μ_i and σ_i^2. [Hint: $q^Z_{0.975} \simeq 2$ and $\sqrt{2} \simeq 1.4$.] **[8 pts]**

5.2. Compute (asymptotic) 95% confidence interval for ρ_{12} (You will use $SE(\hat{\rho}_{12}) = \sqrt{(1 - \hat{\rho}_{12}^2)/T}$). [Hint: $q^Z_{0.975} \simeq 2$] **[5 pts]**

5.3. Test the hypothesis (significance tests) for $i = 1, 2$, with 5% confidence level,

$$H_0 : \mu_i = 0 \quad \text{v.s.} \quad H_1 : \mu_i \neq 0.$$

Are expected returns of these assets (statistically) different from zero? Justify your answer. [Hint: $q_{0.975}^{T(99)} \simeq 2$.] **[8 pts]**

5.4. Test the hypothesis for $i = 1, 2$,

$$H_0 : \sigma_i^2 = 0.0225 \quad \text{v.s.} \quad H_1 : \sigma_i^2 \neq 0.0225$$

with 5% confidence level. **[4 pts].**