ECON 147 MIDTERM EXAM

YOUR NAME:

Winter 2018, Feb 13th
3:30pm - 4:45pm

Instruction

- This is a closed book and closed note exam. All necessary information will be provided and all calculation will be done by hands, so no cheat sheet or calculator is needed.
- Try to answer all questions and write all answers **within this exam sheets**. You will hand in this exam sheets. Please write legibly.
- *Examination Rules from Department Policy will be strictly followed.*

1. **(Probability Review; 30 pts)** Consider the following joint distribution of X and Y:

Bivariate pmf

		Y	
		0	1
	0	1/8	1/8
X	1	1/4	1/4
	2	1/8	1/8

1.1. Find the marginal probability mass functions of X and Y. Using these distributions, compute $E[X]$, $Var(X)$, $E[Y]$ and $Var(Y)$. [**8 pts**]

Answer:

marginal pdf's

X	$f(X)$	Y	$f(Y)$
0	1/4	0	1/2
1	1/2	1	1/2
2	1/4		

And $E[X] = 1$, $Var(X) = \frac{1}{2}$, $E[Y] = \frac{1}{2}$ and $Var(Y) = \frac{1}{4}$.

1.2. Compute the conditional pmf $f(x|Y = 0)$ and $f(x|Y = 1)$, and also write down the marginal pmf $f(X)$ from 1.1. [**5 pts**]

Answer:

conditional pdf and marginal pdf

X	$f(X\|Y = 0)$	$f(X\|Y = 1)$	$f(X)$
0	1/4	1/4	1/4
1	1/2	1/2	1/2
2	1/4	1/4	1/4

1.3. Are X and Y independent? Fully justify your answer. (Hint: you will need the answer from 1.2.) **[5 pts]**

Answer: *Yes, since conditional pdf is equal to marginal pdf as in 1.2, they are independent.*

1.4. Compute $E[X|Y=0]$, $Var[X|Y=0]$ and $Cov(X,Y)$. (Hint: use the earlier answers) **[6 pts]**

Answer: *From independence of X and Y,* $E[X|Y=0] = E[X] = 1$ *,* $Var[X|Y=0] = Var(X) = \frac{1}{2}$ *and* $Cov(X,Y) = 0$.

1.5. Now suppose X and Y represent outcomes of two lotteries, and you would like to invest your \$100 to these lotteries. Assume you are extremely risk averse, hence only care about the risk (preferably low variance) of your investment. How do you want to distribute your \$100, i.e., how do you choose your $\alpha \in [0,1]$ for $\alpha X + (1-\alpha)Y$? **[6 pts]**

Answer: The variance of $\alpha X + (1-\alpha)Y$ is

$$\begin{aligned} & \alpha^2 Var(X) + (1-\alpha)^2 Var(Y) \\ = & \frac{\alpha^2}{2} + \frac{\alpha^2 - 2\alpha + 1}{4} \\ = & \frac{3\alpha^2 - 2\alpha + 1}{4} \end{aligned}$$

Since the function $3\alpha^2 - 2\alpha + 1$ is minimized at $\alpha = 1/3$, the variance of $\alpha X + (1-\alpha)Y$ is minimized at $\alpha = 1/3$. Therefore, we should put 1/3 of \$100 on X, and 2/3 of \$100 on Y.

2. **(Return Calculation and VaR; 40 pts)** Consider a 1-month investment in two assets (equities): the Goldman Sachs Group (GS) and American International Group (AIG). Suppose you buy one share of each GS and AIG at the end of September, 2017 for

$$P_{GS,t-1} = 100, \ P_{AIG,t-1} = 20$$

and then sell these shares at the end of October, 2017 for

$$P_{GS,t} = 115, \ P_{AIG,t} = 21.$$

(Note: these are hypothetical prices for your computation.)

2.1. What are the simple 1-month returns for the two investments? [**5 pts**]

Answer:

$$\begin{aligned} R_{GS,t} &= \frac{P_{GS,t}}{P_{GS,t-1}} - 1 = \frac{115}{100} - 1 = 0.15 \\ R_{AIG,t} &= \frac{P_{AIG,t}}{P_{AIG,t-1}} - 1 = \frac{21}{20} - 1 = 0.05 \end{aligned}$$

2.2. What are the continuously compounded (cc) 1-month returns for the two investments? (You will use the approximation formula $\log(1+x) \simeq x$ for this question.) [**5 pts**]

Answer:

$$\begin{aligned} r_{GS,t} &= \log(1+0.15) \simeq 0.15 \\ r_{AIG,t} &= \log(1+0.05) \simeq 0.05 \end{aligned}$$

2.3. Assume you get the same monthly returns from part 2.2. every month for the next year. What are the annualized cc returns? [**6 pts**]

Answer:

$$\begin{aligned} r_{GS,t}(12) &= 1.8 \\ r_{AIG,t}(12) &= 0.6 \end{aligned}$$

2.4. At the end of September, 2017, suppose you have \$10,000 to invest in GS and AIG over the next month. Suppose you purchase \$5,000 worth of GS and the remainder in AIG. What are the portfolio weights in the two assets? Using the results from part 2.1. compute the 1-month *simple* portfolio returns ($R_{p,t}$). **[8 pts]**

Answer:

$$\begin{aligned} x_{GS} &= x_{AIG} = 0.5 \\ R_{p,t} &= x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t} \\ &= 0.5 \times 0.15 + 0.5 \times 0.05 = 0.1 \end{aligned}$$

2.5. Now assume the distributions of the two *simple* 1-month returns as

$$R_{GS,t} \sim iid\ N\left(0.15, \frac{3}{4}\right),\ R_{AIG,t} \sim iid\ N\left(0.05, \frac{1}{4}\right)$$

and suppose they are statistically independent. What is the mean, variance and distribution of the 1-month simple portfolio returns ($R_{p,t}$) from part 2.4.? **[8 pts]**

Answer:

$$R_{p,t} \sim iid\ N\left(0.1, \frac{1}{4}\right),$$

because

$$\begin{aligned} E\left[x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t}\right] &= 0.5 \times 0.15 + 0.5 \times 0.05 = 0.1, \\ Var\left[x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t}\right] &= \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{4}{16} = \frac{1}{4}. \end{aligned}$$

2.6. Using the distribution of $R_{p,t}$ from 2.5., compute the 2.5% monthly Value-at-Risk for the \$10,000 investment in the portfolio returns in 2.4. (You will use $q^Z_{0.025} = -2$, which is the 2.5% quantile of a standard normal random variable.) **[8 pts]**

Answer:

$$\begin{aligned} VaR^{R_{p,t}}_{.025} &= W_0 \times q^{R_{p,t}}_{.025} = W_0 \times \left(0.1 + \frac{1}{2} \times (-2)\right) \\ &= W_0 \times (-0.9) = -\$9,000, \end{aligned}$$

or its absolute value \$9,000

3. **(30 pts, 5pts each)** Indicate whether the following statements are true or false (circle one). Briefly discuss why it is so.

3.1. If r_t is *continuously compounded (cc)* 1-month return, then the annualized cc return is $r_A = \sum_{j=0}^{11} r_{t+j}$.

True False

Why?

Answer:

True

because cc return is additive across time.

3.2. Let $R_{GS,t}$ and $R_{AIG,t}$ be *simple* 1-month returns for Goldman Sachs Group (GS) and American International Group (AIG). If we construct a portfolio using the share $\alpha \in [0,1]$ for GS, the portfolio simple return is $R_{p,t} = \alpha R_{GS,t} + (1-\alpha) R_{AIG,t}$.

True False

Why?

Answer:

True

because simple return is additive across portfolio shares

3.3 In 3.2., if 5% quantile of the portfolio *simple return* is given as $q_{0.05}^{R_p} = -0.5$, then 5% monthly Value-at-Risk for the \$10,000 investment in this portfolio is $\$10,000 \times (-0.5) = -\$5,000$.

True False

Why?

Answer:

True

because by the definition of VaR.

3.4. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ are two different point estimators for θ. If $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$, then confidence interval based on $\hat{\theta}_1$ is more accurate (shorter) so we always prefer to use $\hat{\theta}_1$.

True False

Why?

Answer:

False

because as in class slides, we need to use MSE criteria if bias is huge, it may dominate the smaller variance.

3.5. If $R_{AIG,t} \sim N(0, \sigma^2_{AIG})$ and $R_{GS,t} \sim N(0, \sigma^2_{GS})$ and they are *independent*, the *simple portfolio return* $R_{p,t} = x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t}$ is distributed as $N(0, x^2_{GS}\sigma^2_{GS} + x^2_{AIG}\sigma^2_{AIG})$.

True False

Why?

Answer:

True

because this is directly from property of normal dist'n and variance formula for indep r.v's.

3.6. Assume r_t is *continuously compounded (cc)* 1-month returns with $r_t \sim iid\ N(\mu, \sigma^2)$. Then the sample mean $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r_t$ has a same distribution with r_t hence $\hat{\mu} \sim N(\mu, \sigma^2)$, because of (i) the iid property of r_t, and (ii) the property of the normal distribution.

True False

Why?

Answer:

False

because: $\hat{\mu} \sim N\left(\mu, \frac{\sigma^2}{T}\right)$, as derived in class.