ECON 147 MIDTERM EXAM

YOUR NAME:

Winter 2018, Feb 13th
3:30pm - 4:45pm

Instruction

- This is a closed book and closed note exam. All necessary information will be provided and all calculation will be done by hands, so no cheat sheet or calculator is needed.
- Try to answer all questions and write all answers **within this exam sheets**. You will hand in this exam sheets. Please write legibly.
- Total points are 100. Use you time wisely according to the assigned points.
- *Examination Rules from Department Policy will be strictly followed.*
- The following results may be useful:

$$\begin{array}{ll} \Pr(Z \leq 0.5) = 0.69, & \Pr(Z \leq 1) = 0.84, \\ \Pr(Z > -1.96) = 0.975, & \Pr(Z > -1.64) = 0.95, \\ \Pr(Z \leq 0.75) = 0.773, & \Pr(Z \leq 0.25) = 0.599 \\ \Pr(Z \leq 0.2) = 0.579 & \Pr(Z \leq 0.6) = 0.726 \end{array}$$

  where Z is a standard normal random variable.

1. **(15 pts)** Let r_t be a cc return. Suppose that $r_1, r_2, \ldots$ are independent and identically distributed normal random variables with mean 0.06 and variance 0.25. Define

$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$$

for any positive integer $k \geq 2$.

1.1. What is the distribution of $r_t(4) = r_t + r_{t-1} + r_{t-2} + r_{t-3}$? **[4 pts]**

Answer: $r_t(4)$ is normally distributed. Since $E[r_t(4)] = 4*0.06 = 0.24$ and $Var[r_t(4)] = 4*0.25 = 1$,

$$r_t(4) \sim N(0.24, 1)$$

1.2. What is $\Pr(r_5(4) < 1.24)$? **[3 pts]**

Answer: Since $r_5(4) \sim N(0.24, 1)$,

$$\begin{aligned} \Pr(r_5(4) < 2) &= \Pr\left(\frac{r_5(4) - 0.24}{1} < \frac{1.24 - 0.24}{1}\right) \\ &= \Pr(Z < 1) = 0.84 \end{aligned}$$

1.3. Find the covariance between $r_3(2)$ and $r_4(2)$? **[4 pts]**

Answer: Note that $r_3(2) = r_3 + r_2$ and $r_4(2) = r_4 + r_3$. Then

$$\begin{aligned} & COV(r_3(2), r_2(2)) \\ = \ & E[r_3(2) r_4(2)] - E[r_3(2)]E[r_4(2)] \\ = \ & E[(r_3 + r_2)(r_4 + r_3)] - E[r_3 + r_2]E[r_4 + r_3] \\ = \ & E[r_3 r_4] + E[r_3^2] + E[r_2 r_4] + E[r_2 r_3] \\ & -E[r_3]E[r_4] - E[r_2]E[r_4] - E[r_3]E[r_3] - E[r_2]E[r_3] \\ = \ & Var(r_3) + COV(r_3, r_4) + COV(r_2, r_4) + COV(r_2, r_3) \\ = \ & Var(r_3) = 0.25 \end{aligned}$$

where $COV(r_3, r_4) = COV(r_2, r_4) = COV(r_2, r_3) = 0$ by independence.

1.4. What is $\Pr(r_9(4) < 1.24 \text{ and } r_5(4) < 1.24)$? [**4 pts**]

Answer: Since $r_9(4) = r_9 + r_8 + r_7 + r_6$ and $r_5(4) = r_5 + r_4 + r_3 + r_2$, we see that $r_9(4)$ and $r_5(4)$ are independent. Moreover

$$r_t(4) \sim N(0.24, 1).$$

Therefore

$$\begin{aligned}
& \Pr(r_9(4) < 1.24 \text{ and } r_5(4) < 1.24) \\
= \; & \Pr(r_9(4) < 1.24) \cdot \Pr(r_9(4) < 1.24) \\
= \; & \Pr(Z < 1) \cdot \Pr(Z < 1) \\
= \; & 0.84^2
\end{aligned}$$

2. **(25 pts)** Consider a 3-year period and that there are 3 mutual funds. The performance of each mutual fund relative to market is random in the sense that each fund has a 50-50 chance of outperforming the market in any year and that performance is independent from year to year, and across each fund. Let $t = 1, ..., 3$ (year) and $i = 1, ..., 3$ (mutual funds), and

$$Y_{it} = \begin{cases} 1, & \text{if fund } i \text{ outperforms the market in year } t \\ 0, & \text{otherwise.} \end{cases}$$

2.1. What is the probability that at least one fund outperforms the market in all three years? [**8 pts**]

Answer: The probability that fund i outperforms the market in all three years is $(0.5)^3 = \frac{1}{8}$. The probability that fund i fails to outperform the market in all three years is $1-\frac{1}{8} = 7/8$. Therefore, none of the funds outperform the market in all three years is $(7/8)^3$ and the probability that at least one fund outperforms the market in all three years is

$$1 - (7/8)^3 = 0.33.$$

2.2. Find the $E[Y_{it}]$ and $Var(Y_{it})$. **[3 pts]**

Answer:

$$\begin{aligned} E[Y_{it}] &= 1 * \frac{1}{2} + 0 * \frac{1}{2} = \frac{1}{2}; \\ Var(Y_{it}) &= E\left[Y_{it}^2\right] - (E[Y_{it}])^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \end{aligned}$$

2.3. Now assume that $P(Y_{it} = 1) = p_i$ is unknown. Given a random sample $\{Y_{i1}, Y_{i2}, Y_{i3}\}$, we may want to use $\hat{p}_i = \frac{1}{3}\sum_{t=1}^{3} Y_{it}$ as an estimator for p_i. Show that $\hat{p}_i$ is an unbiased estimator for p_i.**[4 pts]**

Answer: By the linearity of the expectation operator:

$$E[\hat{p}_i] = \frac{1}{3}\sum_{t=1}^{3} E[Y_{it}] = \frac{1}{3}\sum_{t=1}^{3} p_i = p_i.$$

2.4. Suppose we are interested in estimating the odds ratio $\gamma = \frac{p_i}{1-p_i}$, then a natural estimator is $\hat{\gamma} = \frac{\hat{p}_i}{1-\hat{p}_i}$. Is $\hat{\gamma}$ an unbiased estimator for γ? Explain why or why not. **[6 pts]**

Answer: Biased estimator.

$$\hat{\gamma} = \frac{\hat{p}_i}{1-\hat{p}_i} = \frac{1}{1-\hat{p}_i} - 1.$$

Since $(1-x)^{-1}$ is a nonlinear function of x,

$$E\left[\frac{1}{1-\hat{p}_i}\right] \neq \frac{1}{1-E[\hat{p}_i]}$$

where $E[\hat{p}_i] = p_i$ as we show in 2.3. Therefore,

$$E[\hat{\gamma}] = E\left[\frac{1}{1-\hat{p}_i}\right] - 1 \neq \frac{1}{1-E[\hat{p}_i]} - 1 = p$$

2.5. Now assume that $t = 1, ..., 100$ (years) which is large enough (sample period is $T = 100$), and that $\hat{p}_i = 0.5$. Construct an 95% asymptotic confidence interval for p_i. (You can use the fact $SE(\hat{p}_i) = \sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{100}}$ and $q^Z_{0.025} = -2$). [**4 pts**]

Answer: Using the formula

$$\hat{p}_i \pm q^Z_{0.975} \cdot SE(\hat{p}_i)$$

we get

$$0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{100}} = 0.5 \pm 0.1$$

3. **(20 pts)** Consider the following (actual) monthly adjusted closing price data for Delta Airline stock over the period December 2009 through December 2011:

End of Month Price Data for Delta Airline Stock	
December, 2009	\$10.72
January, 2010	\$11.52
February, 2010	\$12.17
March, 2010	\$13.74
April, 2010	\$11.38
May, 2010	\$12.79
June, 2010	\$11.06
July, 2010	\$11.19
August, 2010	\$9.85
September, 2010	\$10.96
October, 2010	\$13.08
November, 2010	\$12.88
December, 2010	\$11.87

3.1. Using the data in the table, what is the simple monthly return between the end of December, 2009 and the end of January 2010? If you invested \$10,000 in Delta Airline at the end of December 2009, how much would the investment be worth at the end of January 2010? [**5 pts**]

Answer: This is a one month investment. The simple return between December and January is

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{\$11.52 - \$10.72}{\$10.72} = \frac{0.8}{10.72} = 0.0746.$$

The future value of $\$10,000$ is then

$$FV = \$10,000 \times (1 + R_t) = \$10,000 \times (1 + 0.0746) = \$10746.27.$$

3.2. Using the data in the table, what is the continuously compounded monthly return between December, 2009 and January 2010? Convert this continuously compounded return to a simple return. [**5 pts**]

Answer: *The continuously compounded return is defined as*

$$r_t = \ln(1 + R_t) = \ln(P_t/P_{t-1})$$

Using $R_t = 0.0746$ *gives*

$$r_t = \ln(1 + 0.0746) = 0.0719$$

3.3. Assuming that the *continuously compounded* monthly return you computed in part 2 is the same for 12 months, what is the continuously compounded annual return? [**5 pts**]

Answer: *The annual continuously compounded return assuming* $r_t = 0.0719$ *every month for a year is*

$$r_t(12) = r_A = 12 \times r_t = 12 \times (0.0719) = 0.8628$$

3.4. Using the data in the table, compute the actual annual continuously compounded return between December 2009 and December 2010. Compare with your result in part 3 and discuss your finding(s). **[5 pts]**

Answer: *The annual continuously compounded return is defined as*

$$r_A = r_t(12) = \ln(P_t/P_{t-12})$$

Using $P_t/P_{t-12} = 11.87/10.72 = 1.1073$ *gives*

$$r_A = \ln(1.1073) = 0.1019.$$

4. **(15 pts)** Let Y_1, Y_2, Y_3, Y_4 and Y_5 be iid (μ, σ^2). Let $\bar{Y} = \frac{1}{5}\sum_{t=1}^{5} Y_t$.

4.1. What are the expected value and variance of $\bar{Y}$? **[5 pts]**

Answer: $E[\bar{Y}] = \mu$ and $Var(\bar{Y}) = \sigma^2/5$

4.2. Now, consider a different estimator of μ:

$$W = \frac{1}{8}Y_1 + \frac{1}{8}Y_2 + \frac{1}{4}Y_3 + \frac{1}{4}Y_4 + \frac{1}{4}Y_5,$$

What are the expected value and variance of W? **[5 pts]**

Answer: $E[W] = \mu$ and $Var(W) = \frac{7}{32}\sigma^2$

4.3. Which estimator of μ do you prefer? Fully justify your answer. **[5 pts]**

Answer: $E[W] = E[\bar{Y}]$ but $Var(W) = \frac{7}{32}\sigma^2 > \sigma^2/5 = Var(\bar{Y})$, so prefer $\bar{Y}$ since it has smaller MSE.

5. **(25 pts, 5pts each)** Indicate whether the following statements are true or false (circle one). Briefly discuss why it is so.

5.1. If r_t is *continuously compounded (cc)* 1-month return, then the annualized cc return is $r_A = \sum_{j=0}^{11} r_{t+j}$.

True False

Why?

Answer:

True

because cc return is additive across time.

5.2. Let $r_{GS,t}$ and $r_{AIG,t}$ be *cc* 1-month returns for Goldman Sachs Group (GS) and American International Group (AIG). If we construct a portfolio using the share $\alpha \in [0, 1]$ for GS, the portfolio cc return is $r_{p,t} = \alpha r_{GS,t} + (1 - \alpha) r_{AIG,t}$.

True False

Why?

Answer:

False

because cc return is not additive across portfolio shares

5.3. In 5.2., if 5% quantile of the portfolio *simple return* is given as $q_{0.05}^{R_p} = -0.5$, then 5% monthly Value-at-Risk for the \$10,000 investment in this portfolio is $\$10,000 \times (-0.5) = -\$5,000$.

Answer:

True

because by the definition of VaR.

5.4. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ are two different point estimators for θ. If $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$, then confidence interval based on $\hat{\theta}_1$ is more accurate (shorter) so we always prefer to use $\hat{\theta}_1$.

True False

Why?

Answer:

False

because as in class slides, we need to use MSE criteria if bias is huge, it may dominate the smaller variance.

5.5. If the simple returns $R_{AIG,t} \sim N(0, \sigma^2_{AIG})$ and $R_{GS,t} \sim N(0, \sigma^2_{GS})$ and they are *independent*, the *simple portfolio return* $R_{p,t} = x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t}$ is distributed as $N(0, x^2_{GS}\sigma^2_{GS} + x^2_{AIG}\sigma^2_{AIG})$.

True False

Why?

Answer:

True

because this is directly from property of normal dist'n and variance formula for independent r.v's.