ECON 147 MIDTERM EXAM

YOUR NAME:

Winter 2018, Feb 13th
3:30pm - 4:45pm

Instruction

- This is a closed book and closed note exam. All necessary information will be provided and all calculation will be done by hands, so no cheat sheet or calculator is needed.
- Try to answer all questions and write all answers **within this exam sheets**. You will hand in this exam sheets. Please write legibly.
- Total points are 100. Use you time wisely according to the assigned points.
- *Examination Rules from Department Policy will be strictly followed.*
- The following results may be useful:

$$\begin{array}{ll} \Pr(Z \leq 0.5) = 0.69, & \Pr(Z \leq 1) = 0.84, \\ \Pr(Z > -1.96) = 0.975, & \Pr(Z > -1.64) = 0.95, \\ \Pr(Z \leq 0.75) = 0.773, & \Pr(Z \leq 0.25) = 0.599 \\ \Pr(Z \leq 0.2) = 0.579 & \Pr(Z \leq 0.6) = 0.726 \end{array}$$

  where Z is a standard normal random variable.

1. **(15 pts)** Let r_t be a cc return. Suppose that $r_1, r_2, ...$ are independent and identically distributed normal random variables with mean 0.06 and variance 0.25. Define

$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$$

for any positive integer $k \geq 2$.

1.1. What is the distribution of $r_t(4) = r_t + r_{t-1} + r_{t-2} + r_{t-3}$? [**4 pts**]

1.2. What is $\Pr(r_5(4) < 1.24)$? [**3 pts**]

1.3. Find the covariance between $r_3(2)$ and $r_4(2)$? [**4 pts**]

1.4. What is $\Pr(r_9(4) < 1.24 \text{ and } r_5(4) < 1.24)$? [**4 pts**]

2. **(25 pts)** Consider a 3-year period and that there are 3 mutual funds. The performance of each mutual fund relative to market is random in the sense that each fund has a 50-50 chance of outperforming the market in any year and that performance is independent from year to year, and across each fund. Let $t = 1, ..., 3$ (year) and $i = 1, ..., 3$ (mutual funds), and

$$Y_{it} = \begin{cases} 1, & \text{if fund } i \text{ outperforms the market in year } t \\ 0, & \text{otherwise.} \end{cases}$$

2.1. What is the probability that at least one fund outperforms the market in all three years? [**8 pts**]

2.2. Find the $E[Y_{it}]$ and $Var(Y_{it})$. [**3 pts**]

2.3. Now assume that $P(Y_{it} = 1) = p_i$ is unknown. Given a random sample $\{Y_{i1}, Y_{i2}, Y_{i3}\}$, we may want to use $\hat{p}_i = \frac{1}{3}\sum_{t=1}^{3} Y_{it}$ as an estimator for p_i. Show that $\hat{p}_i$ is an unbiased estimator for p_i.[**4 pts**]

2.4. Suppose we are interested in estimating the odds ratio $\gamma = \frac{p_i}{1-p_i}$, then a natural estimator is $\hat{\gamma} = \frac{\hat{p}_i}{1-\hat{p}_i}$. Is $\hat{\gamma}$ an unbiased estimator for γ? Explain why or why not. **[6 pts]**

2.5. Now assume that $t = 1, ..., 100$ (years) which is large enough (sample period is $T = 100$), and that $\hat{p}_i = 0.5$. Construct an 95% asymptotic confidence interval for p_i. (You can use the fact $SE\left(\hat{p}_i\right) = \sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{100}}$ and $q^Z_{0.025} = -2$). **[4 pts]**

3. **(20 pts)** Consider the following (actual) monthly adjusted closing price data for Delta Airline stock over the period December 2009 through December 2011:

End of Month Price Data for Delta Airline Stock	
December, 2009	\$10.72
January, 2010	\$11.52
February, 2010	\$12.17
March, 2010	\$13.74
April, 2010	\$11.38
May, 2010	\$12.79
June, 2010	\$11.06
July, 2010	\$11.19
August, 2010	\$9.85
September, 2010	\$10.96
October, 2010	\$13.08
November, 2010	\$12.88
December, 2010	\$11.87

3.1. Using the data in the table, what is the simple monthly return between the end of December, 2009 and the end of January 2010? If you invested \$10,000 in Delta Airline at the end of December 2009, how much would the investment be worth at the end of January 2010? [**5 pts**]

3.2. Using the data in the table, what is the continuously compounded monthly return between December, 2009 and January 2010? Convert this continuously compounded return to a simple return. [**5 pts**]

3.3. Assuming that the *continuously compounded* monthly return you computed in part 2 is the same for 12 months, what is the continuously compounded annual return? [**5 pts**]

3.4. Using the data in the table, compute the actual annual continuously compounded return between December 2009 and December 2010. Compare with your result in part 3 and discuss your finding(s). [**5 pts**]

4. **(15 pts)** Let Y_1, Y_2, Y_3, Y_4 and Y_5 be iid (μ, σ^2). Let $\bar{Y} = \frac{1}{5}\sum_{t=1}^{5} Y_t$.

4.1. What are the expected value and variance of $\bar{Y}$? **[5 pts]**

4.2. Now, consider a different estimator of μ:

$$W = \frac{1}{8}Y_1 + \frac{1}{8}Y_2 + \frac{1}{4}Y_3 + \frac{1}{4}Y_4 + \frac{1}{4}Y_5,$$

What are the expected value and variance of W? **[5 pts]**

4.3. Which estimator of μ do you prefer? Fully justify your answer. **[5 pts]**

5. **(20 pts, 4pts each)** Indicate whether the following statements are true or false (circle one). Briefly discuss why it is so.

5.1. If r_t is *continuously compounded (cc)* 1-month return, then the annualized cc return is $r_A = \sum_{j=0}^{11} r_{t+j}$.

True　　　　False

Why?

5.2. Let $r_{GS,t}$ and $r_{AIG,t}$ be *cc* 1-month returns for Goldman Sachs Group (GS) and American International Group (AIG). If we construct a portfolio using the share $\alpha \in [0,1]$ for GS, the portfolio cc return is $r_{p,t} = \alpha r_{GS,t} + (1-\alpha)\, r_{AIG,t}$.

True　　　　False

Why?

5.3. In 5.2., if 5% quantile of the portfolio *simple return* is given as $q_{0.05}^{R_p} = -0.5$, then 5% monthly Value-at-Risk for the \$10,000 investment in this portfolio is $\$10,000 \times (-0.5) = -\$5,000$.

5.4. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ are two different point estimators for θ. If $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$, then confidence interval based on $\hat{\theta}_1$ is more accurate (shorter) so we always prefer to use $\hat{\theta}_1$.

True False

Why?

5.5. If the simple returns $R_{AIG,t} \sim N\left(0, \sigma_{AIG}^2\right)$ and $R_{GS,t} \sim N\left(0, \sigma_{GS}^2\right)$ and they are *independent*, the *simple portfolio return* $R_{p,t} = x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t}$ is distributed as $N\left(0, x_{GS}^2\sigma_{GS}^2 + x_{AIG}^2\sigma_{AIG}^2\right)$.

True False

Why?