Econ 147 (Spring 2018)
Department of Economics, UCLA

R Lab 1: Simulating Probability and Expectation

Probability of a random variable and its mean and variance can be approximated by simulation. The justification for the accuracy of approximation via simulation is based on the law of large numbers. We will explore the main idea of the simulation approximation in this session.

Problem 1. Suppose that X is a normal random variable with mean 0.1 and variance 1. What is the probability that $X \geq 0.5$?

Using R, we can directly calculate the above probability

$$1 - \mathbf{pnorm}(0.5,\ 0.1,\ 1) = 0.3446.$$

To answer this question through simulation, we will run a random experiment 10,000 times. In the tth trial ($t = 1, \ldots, 10,000$), we generate 1 normal random variable X_t with mean 0.1 and variance 1. Let $p = \Pr(X_t \geq 0.5)$. Note that p is the value we try to approximate through computer. We define a new random variable Y_t such that

$$Y_t = \begin{cases} 1, & \text{if } X_t \geq 0.5 \\ 0, & \text{if } X_t < 0.5 \end{cases}.$$

By definition, Y_t is a Bernoulli random variable with

$$\Pr(Y_t = 1) = \Pr(X_t \geq 0.5) = p$$

and

$$\Pr(Y_t = 1) = \Pr(X_t < 0.5) = 1 - p.$$

It is clear that

$$E[Y_t] = 1 \times p + 0 \times (1 - p) = p.$$

Therefore, we can approximate/simulate $E[Y_t]$ by

$$\overline{Y} = \frac{1}{10,000} \sum_{t=1}^{10,000} Y_t.$$

The law of large numbers tells us that $\overline{Y}$ should be close to $E[Y_t]$ since we have $10,000$ observations on Y_t. The R code for this question is listed below.

```
N_Exp = 10000                          # replication number of the experiment
Y_v = rep(0, N_Exp)                    # outcome variables (default value = 0)
for (s in 1:N_Exp)                     # loop for replication of the experiment
{
X = rnorm(1, mean = 0.1, sd = 1)       # generate 1 normal random variable
if (X >= 0.5) { Y_v[s] = 1}            # save the outcome result
}
mean(Y_v)                              # estimate the probability
```

Problem 2. Now we use simulation to study a non-trivial problem. Consider the following game that you play with your friend. There is a referee who will flip a coin. You and your friend will guess the outcome (Head or Tail) of this random experiment. If your guess is the same as your friend, you lose \$3 if the guess is Head and you lose \$1 if your guess is Tail. If your guess is different from your friend, you win \$2. Suppose that both you and your friend think that the probability of observing Head is 1/2. What is the expectation of the money you will win from this game? (Hint: the R function to generate a Bernoulli random variable is **rbinom**(1,1,p)).

Problem 3. Consider the game described in Problem 2. But now, you play a different strategy. Instead of guessing Head or Tail with probability 1/2, you guess Head with probability 0.36, and Tail with probability 0.64. Suppose that your friend still guess Head and Tail with probability 1/2. What is the expectation of the money you will win from this game?

Problem 4. Consider the game described in Problem 2. Mathematically, we can show that your strategy in Problem 3 guarantees that you will win money from your friend regardless the guess of your friend if you play the game many times with your friend. That is we can show that the expectation of the money you get from the game is positive regardless of you friend's guess. Check this by simulation!