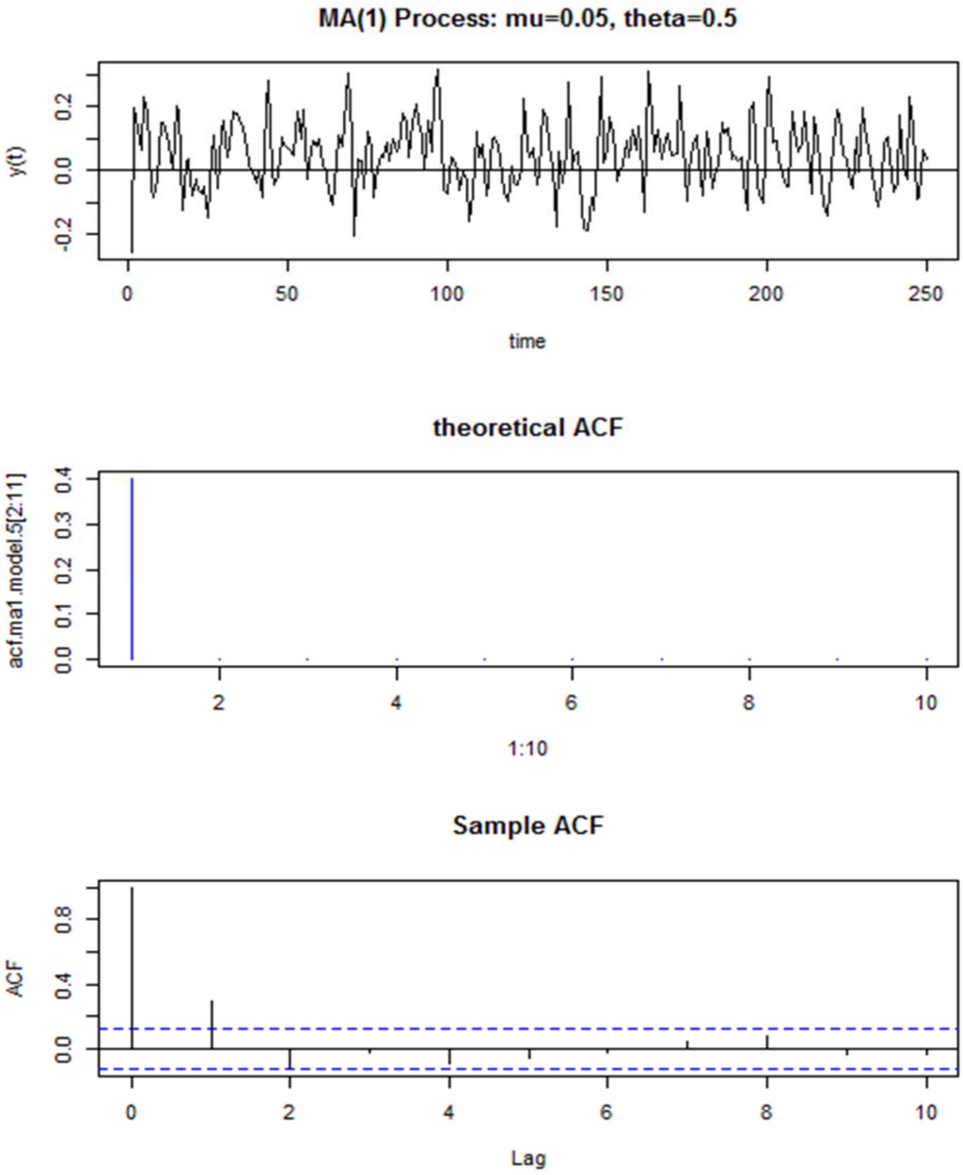
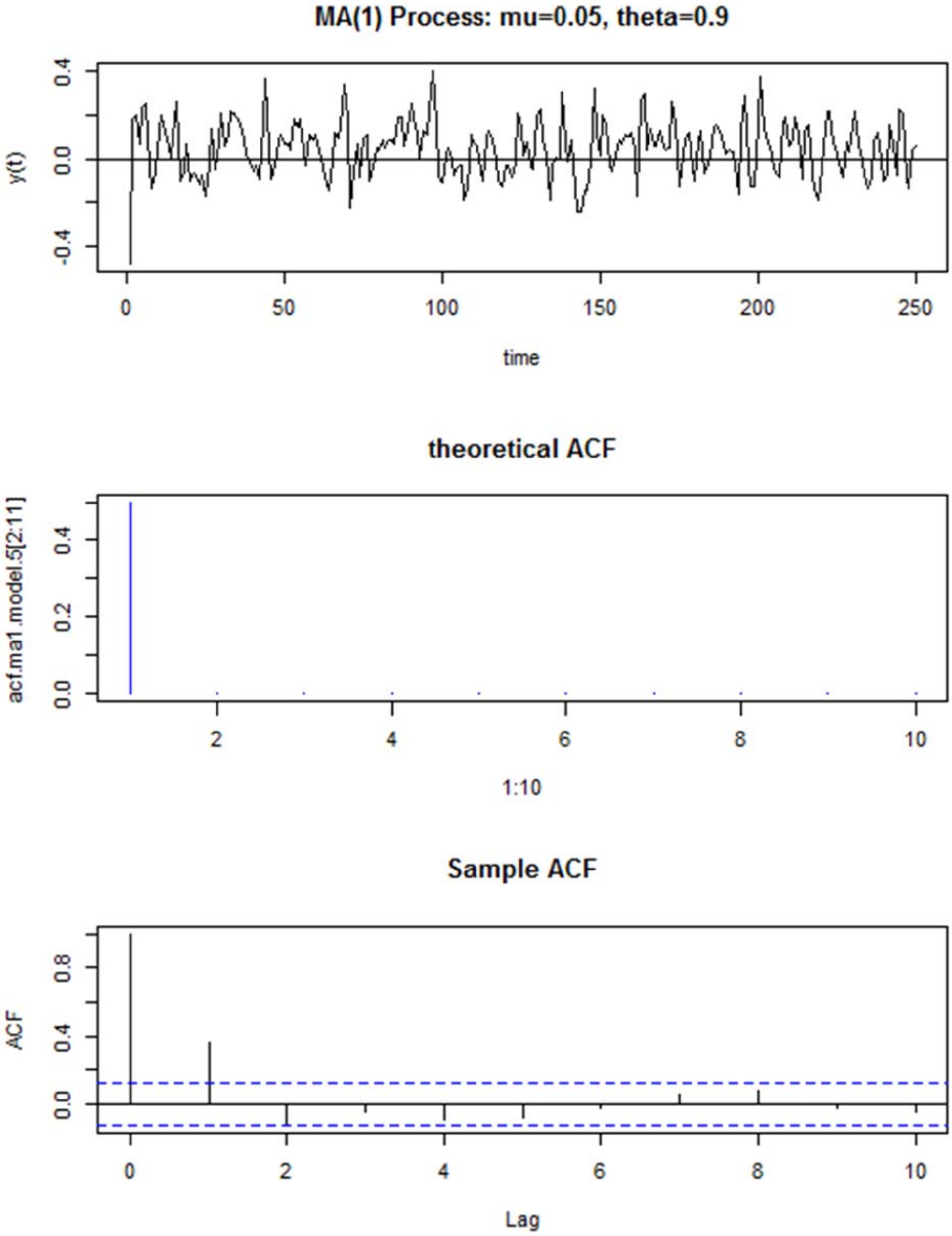
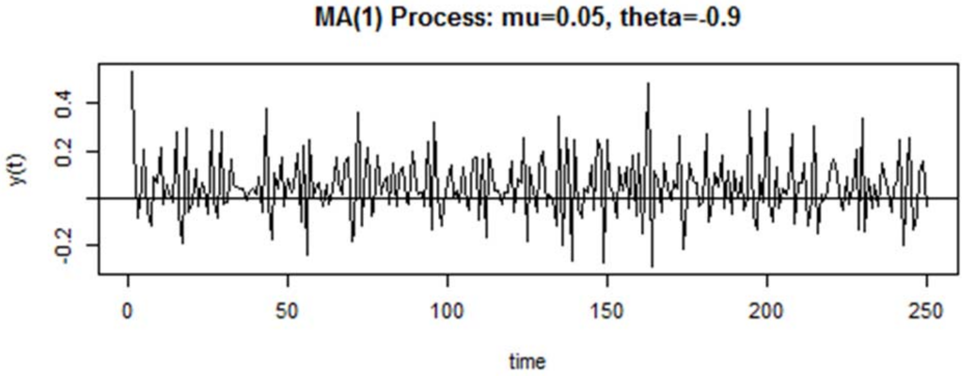
Econ 147 Homework 4 Answer Keys

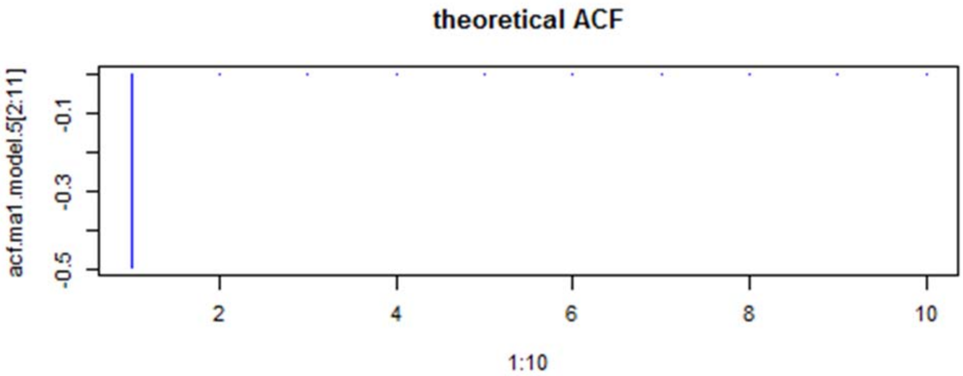
R exercises A

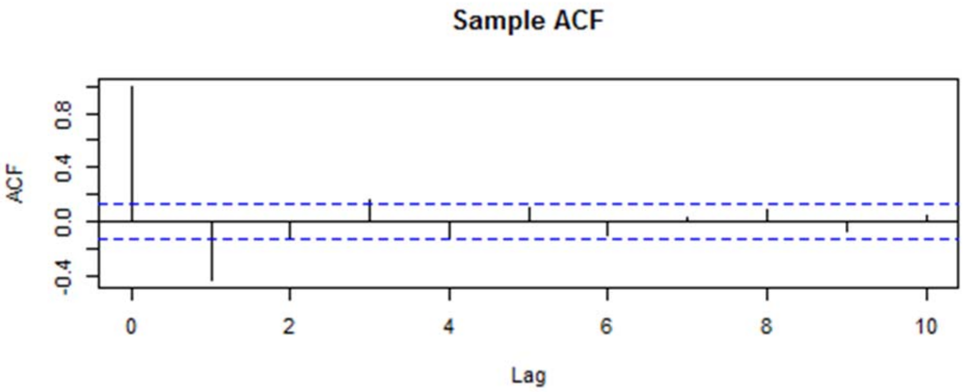
1.a




MA(1) Process: mu=0.05, theta=0.9
y(t)
time
theoretical ACF
acf.ma1.model.5[2:11]
1:10
Sample ACF
ACF
Lag



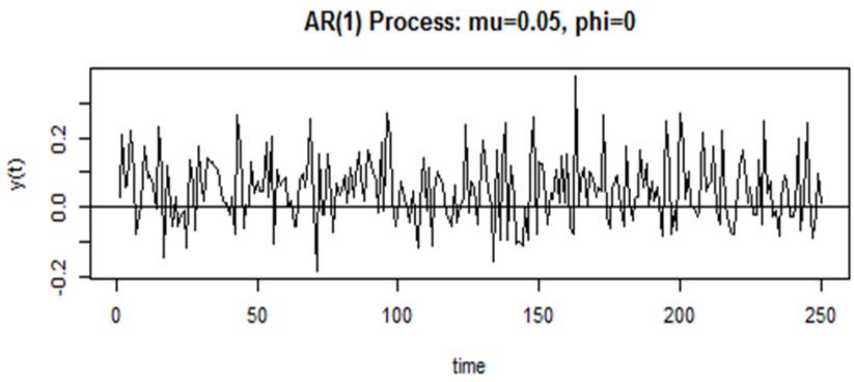


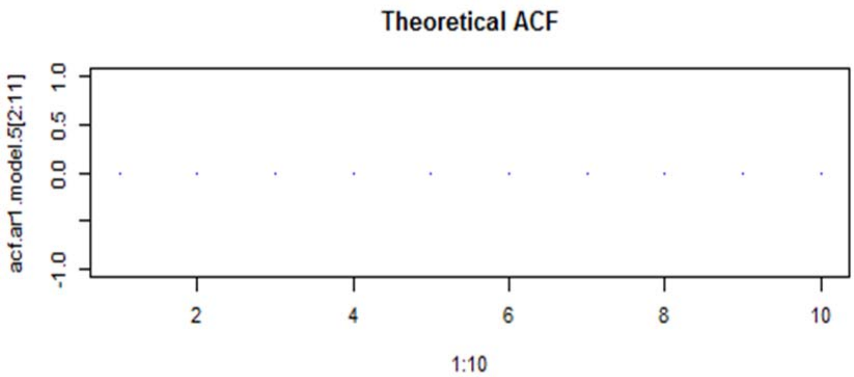


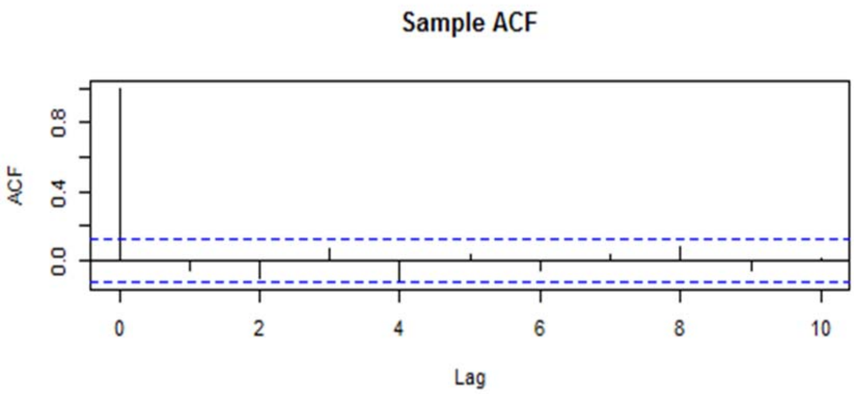
1.b

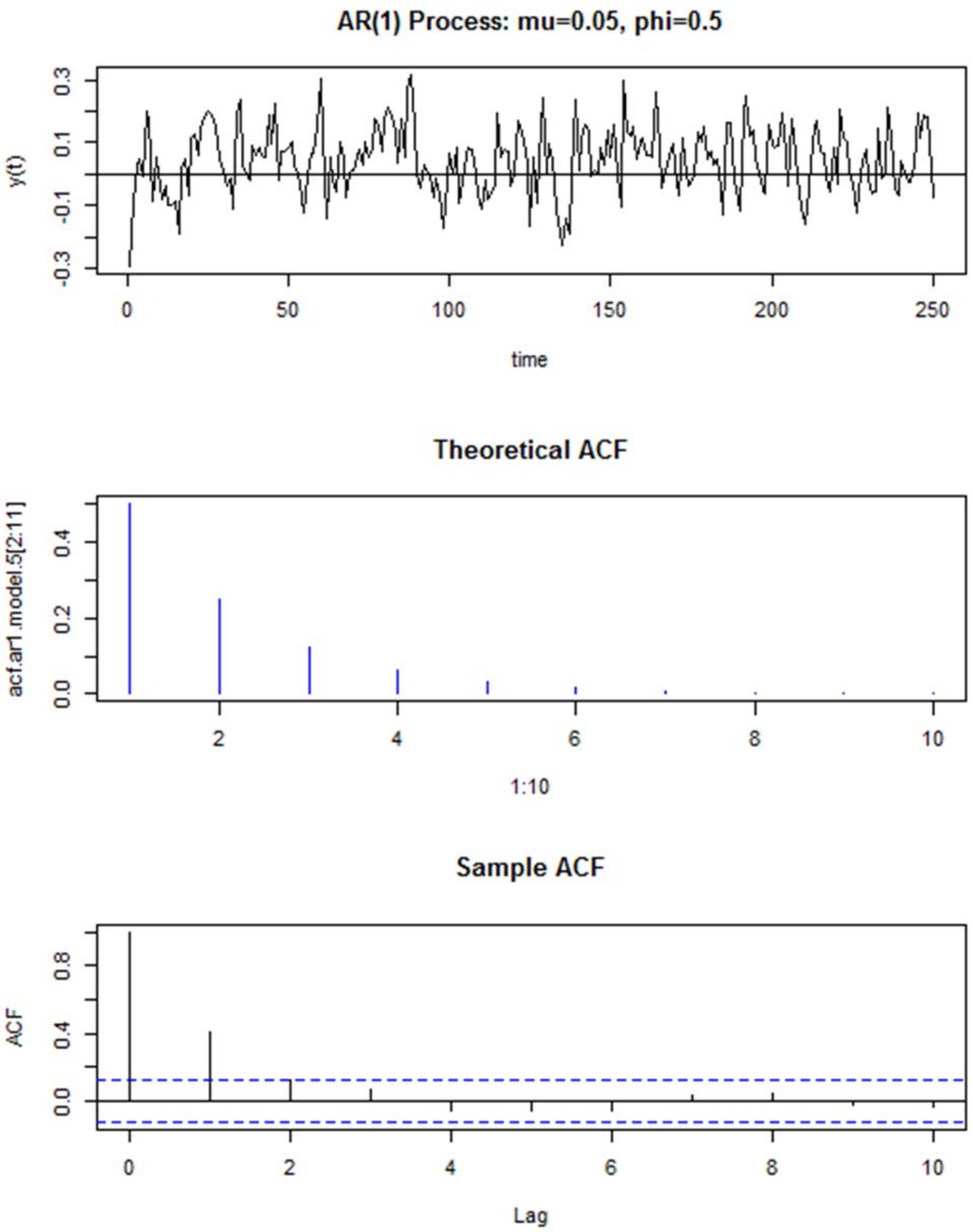
The MA(1) process shows one period dependence. The direction of the correlation of two adjacent random variables, say Y_{t-1} and Y_t, depends on the sign of the θ parameter. Increase the magnitude of θ does not greatly increase the correlation between Y_{t-1} and Y_t.

2.a

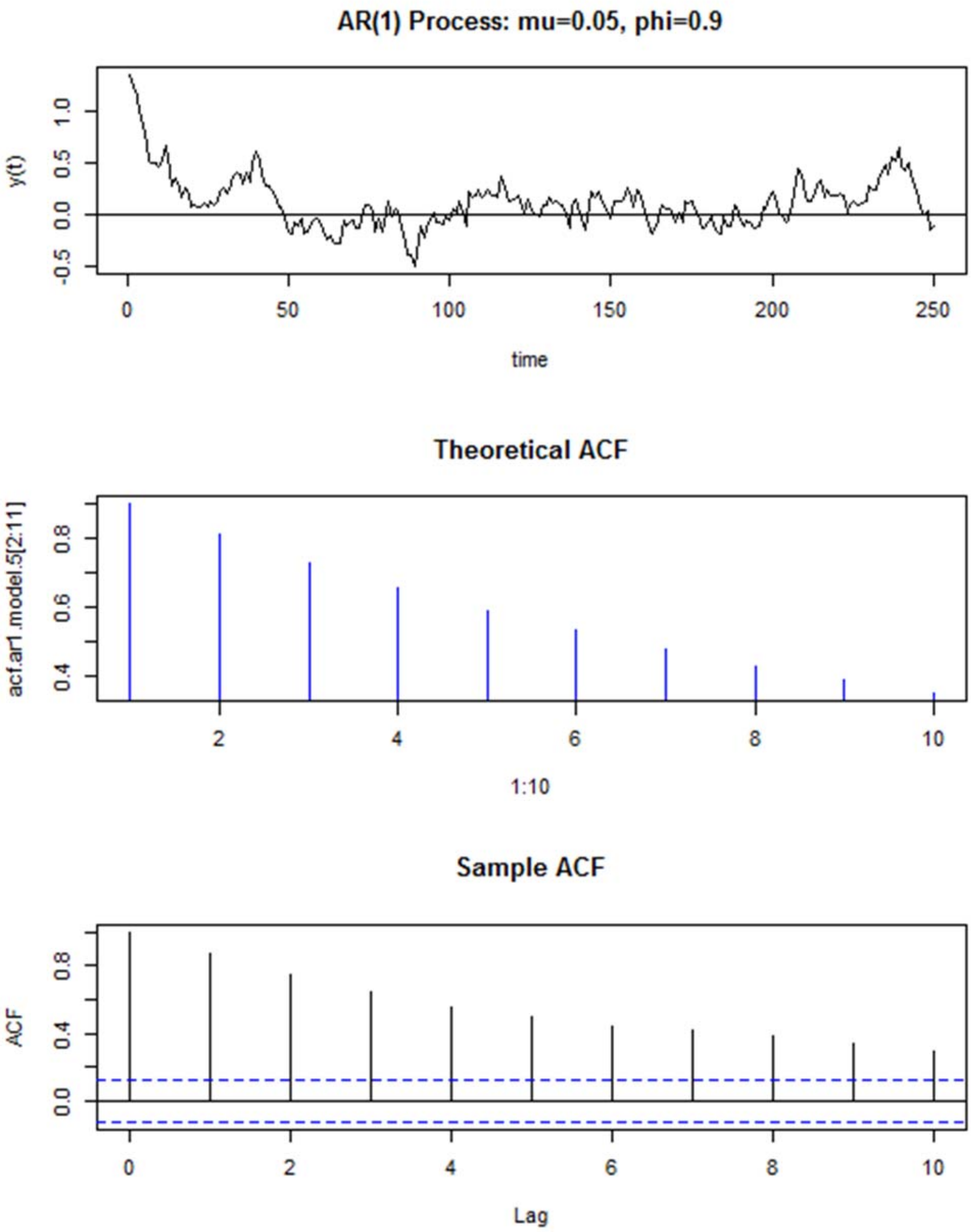

AR(1) Process: mu=0.05, phi=0
y(t)
time
0
50
100
150
200
250
-0.2
0.0
0.2

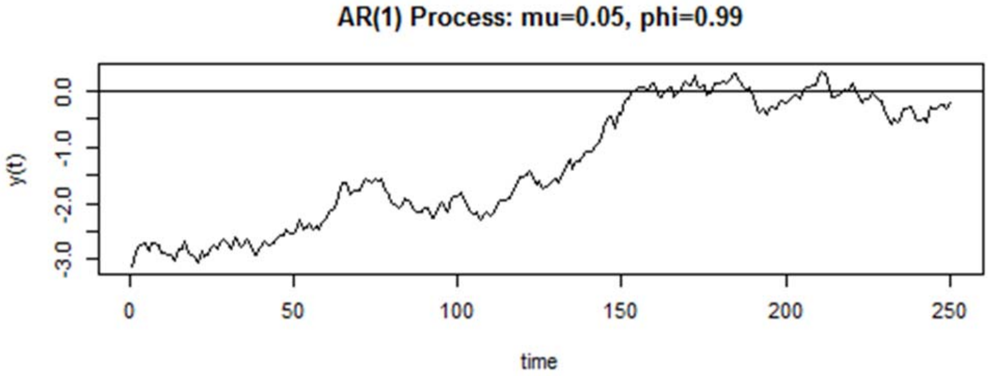

Theoretical ACF
acf.ar1.model.5[2:11]
1:10
2
4
6
8
10
-1.0
0.0
0.5
1.0

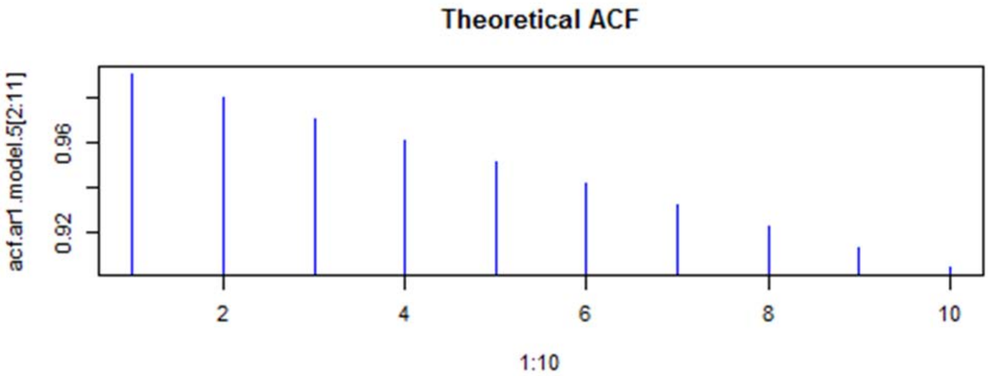

Sample ACF
ACF
Lag
0
2
4
6
8
10
0.0
0.4
0.8

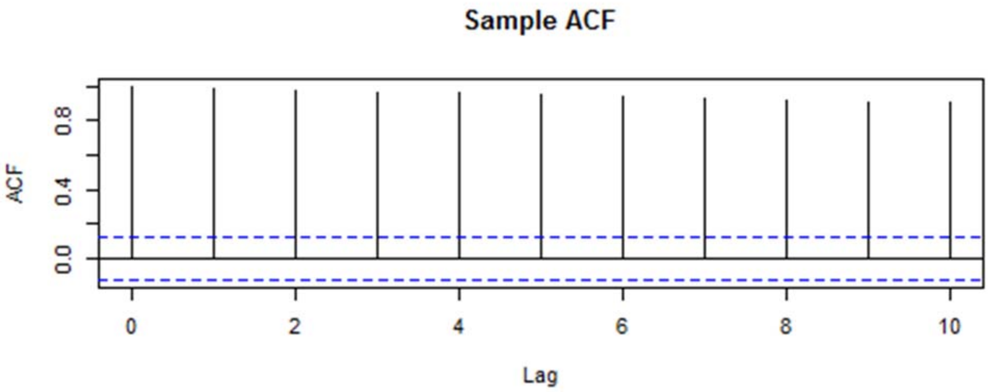


AR(1) Process: mu=0.05, phi=0.5
y(t)
time
Theoretical ACF
acf.ar1.model.5[2:11]
1:10
Sample ACF
ACF
Lag


AR(1) Process: mu=0.05, phi=0.9
y(t)
time
Theoretical ACF
acf.ar1.model.5[2:11]
1:10
Sample ACF
ACF
Lag







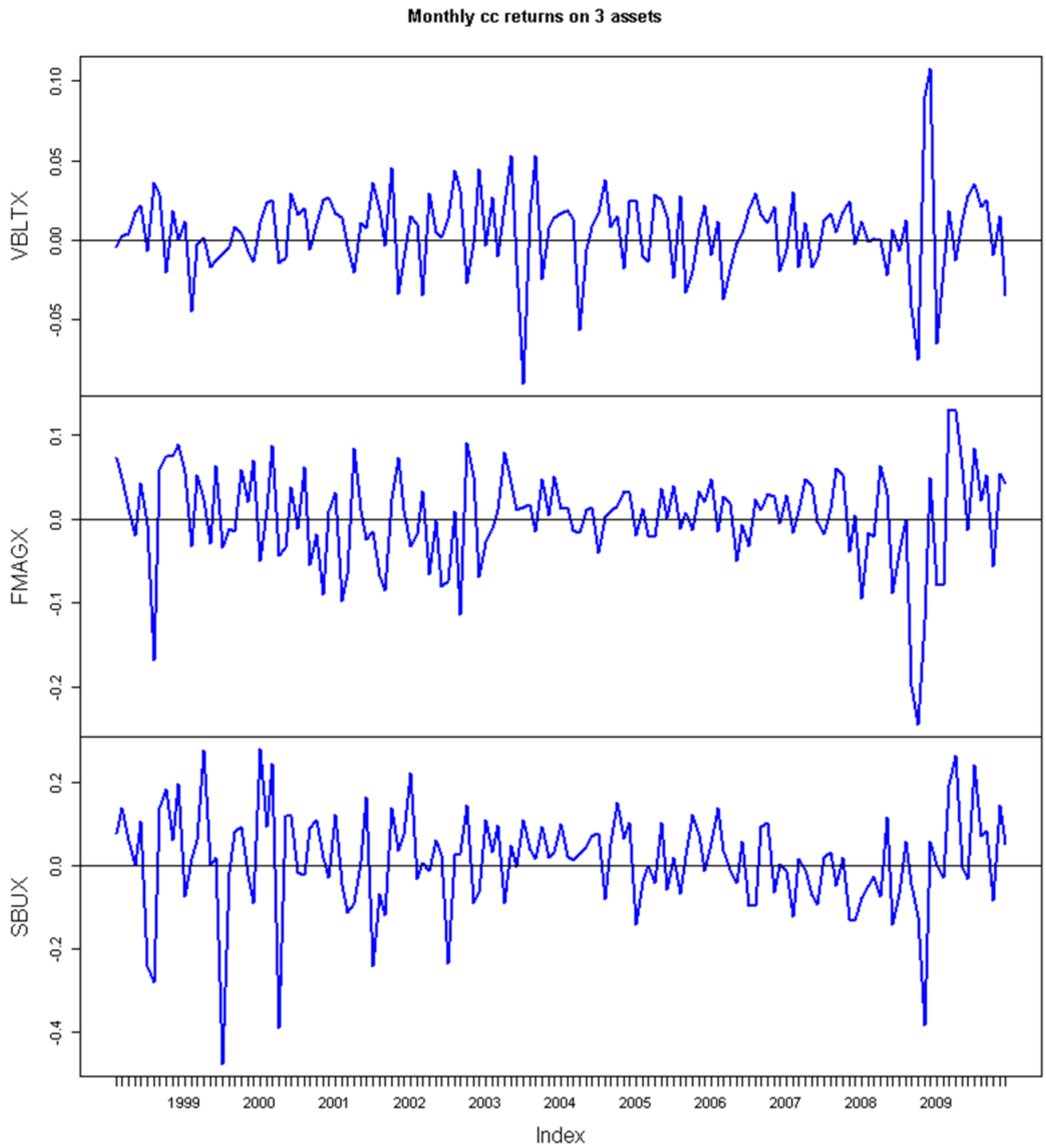
2.b

The magnitude of the parameter ϕ determines the strength of the dependence in the AR(1) process. When ϕ=0, the AR(1) process is an i.i.d. N(0.05,0.01) process and hence is a uncorrelated time series. When the absolute value of ϕ becomes close to 1, the AR(1) process becomes very much like a nonstationary process, i.e., the random walk. This can be clearly seen from the picture with ϕ = 0.99.

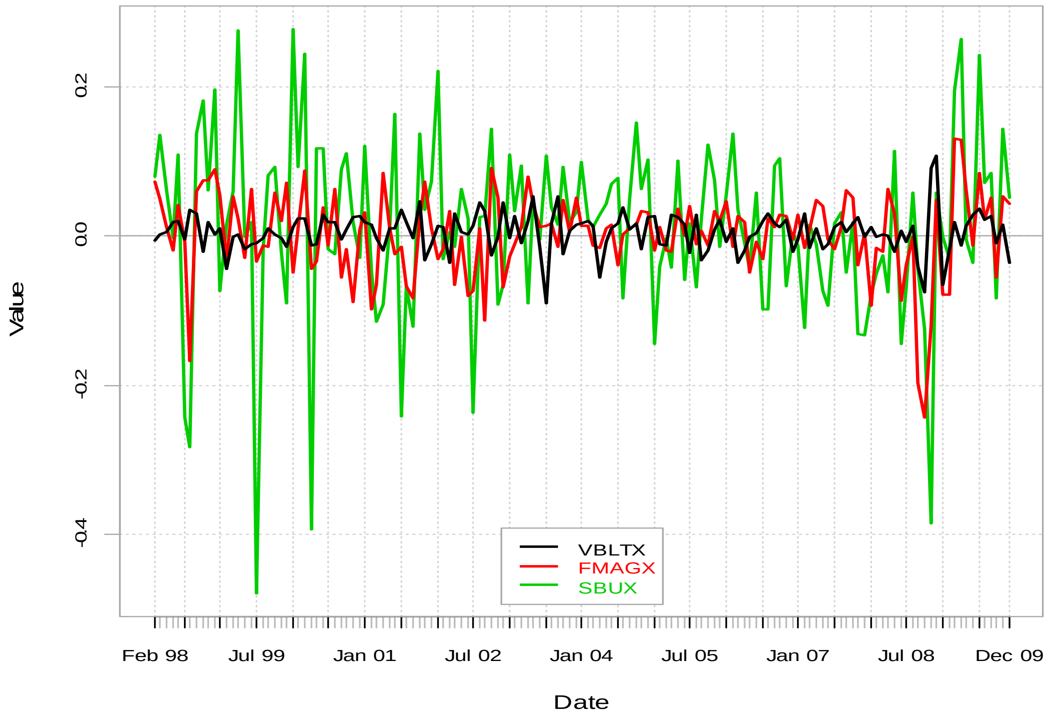
R exercises B

Descriptive Statistics

a. Here is a time plot of the return data in this assignment

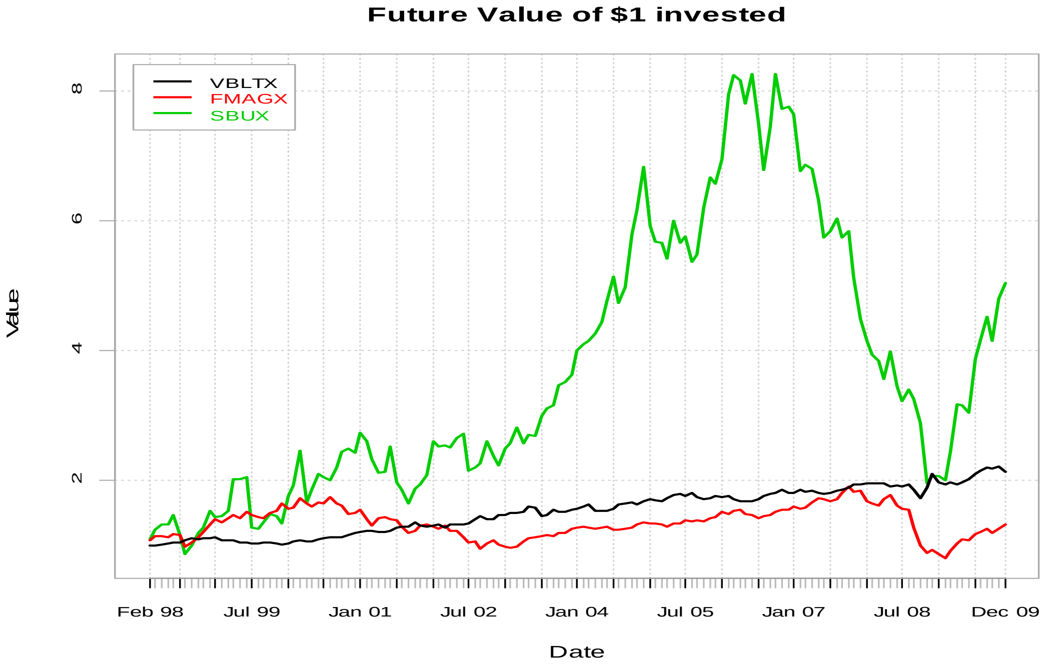


Notice, in particular, the scale differences in the returns. The bond returns, VBLTX, have much lower volatility than the other return series and SBUX has the most volatilie series. To see co-movements and scale differences better, put all three series on one plot (see below). Notice how FMAGX and SBUX seem to move a bit together (positive correlation), whereas VBLTX does not seem to move with FMAGX or SBUX (zero correlation). At the end of 2008, the bond yields went up but the stock returns went down.



With all assets on the same scale, we see very clearly the lower volatility for the bond fund and the higher volatility for SBUX.

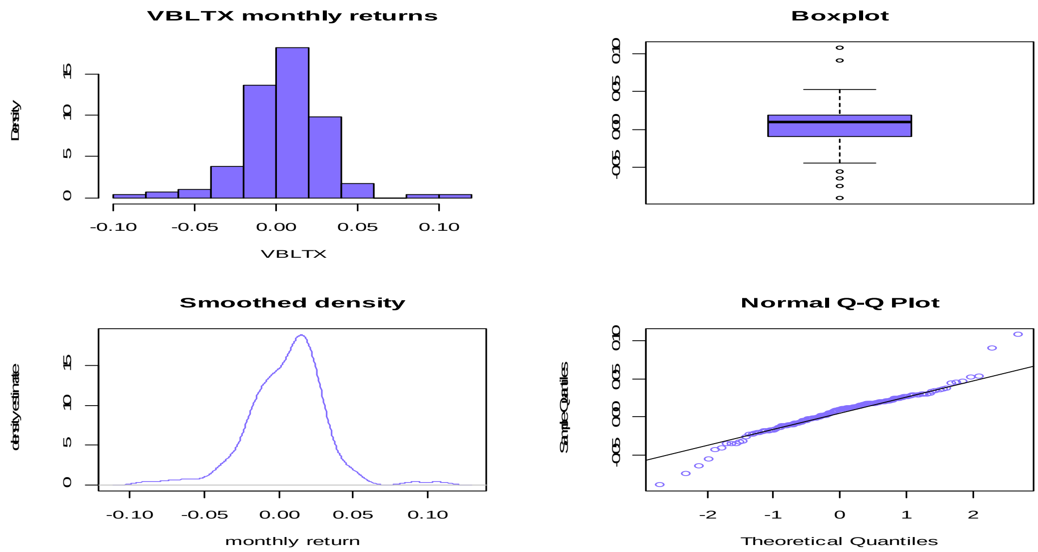
b.



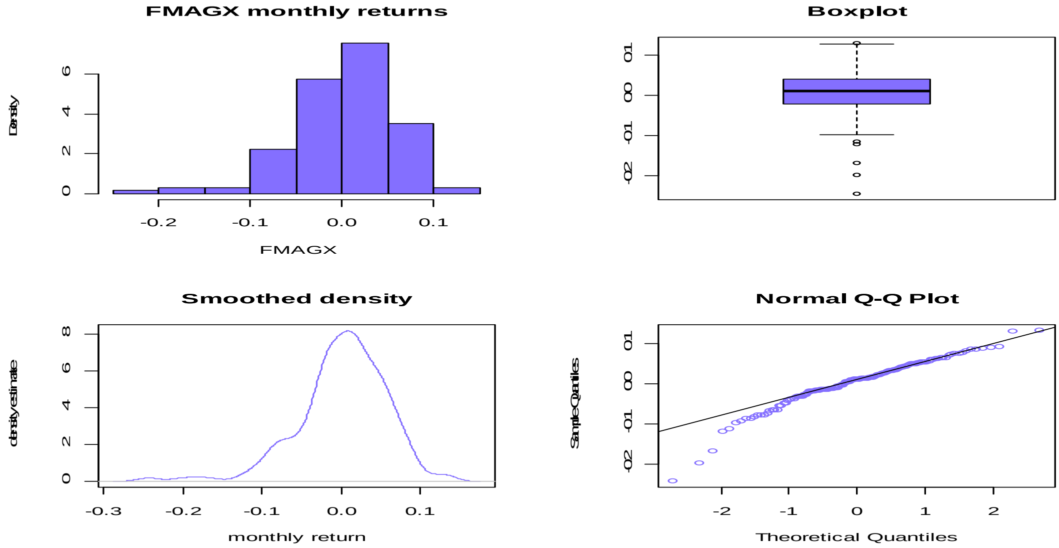
The growth of $1 plot above shows that SBUX produced the highest end of period wealth, but with quite a lot of volatility. The bond fund gave a steady growth of $1 with very little volatility. FMAGX did very poorly indeed.

c.

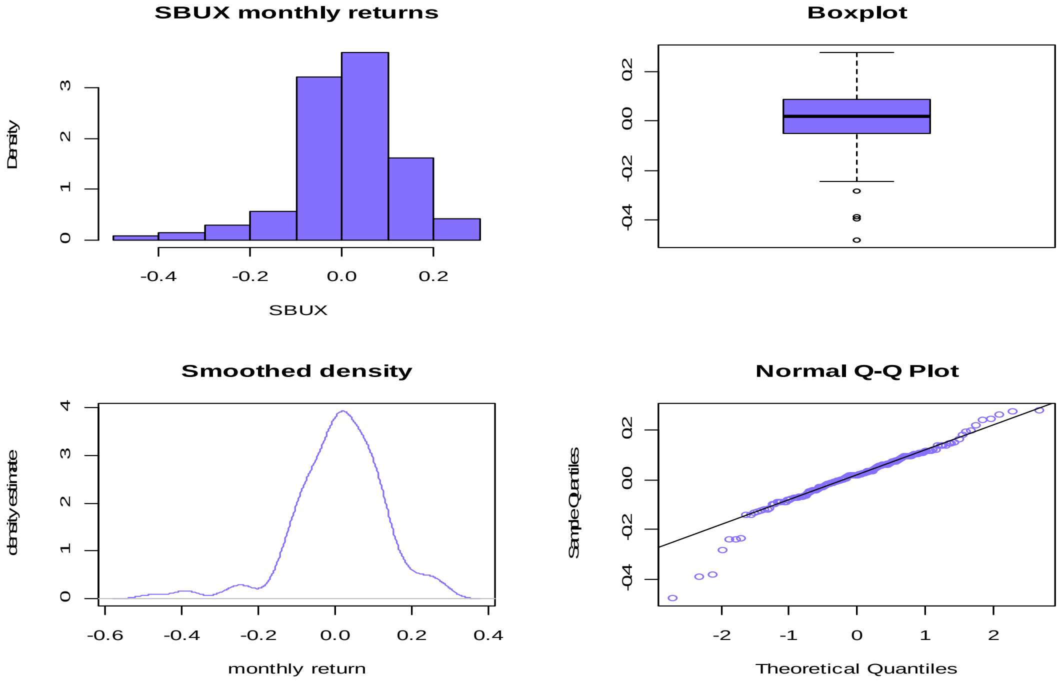
Four panel diagnostic plots for each return are given below



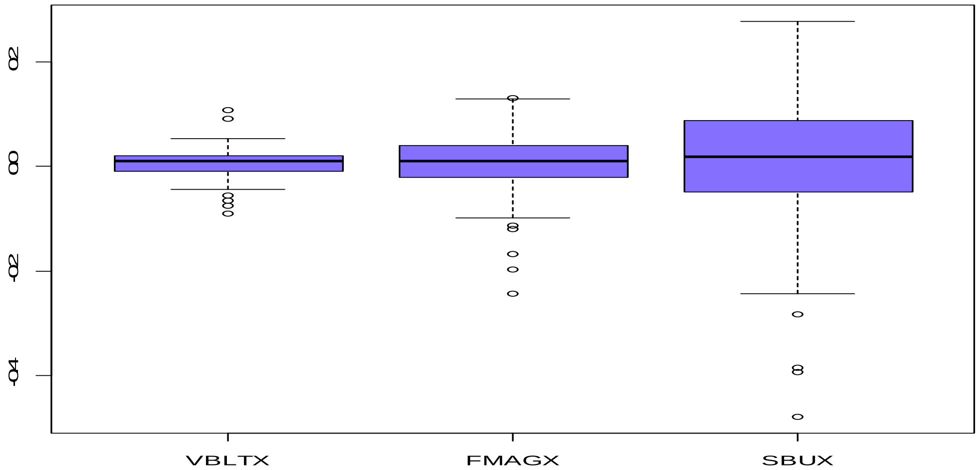
The distribution of returns on VBLTX look symmetric but show fatter tails than the normal. The histogram and boxplot indicate a nice symmetric distribution centered near zero with a SD around 0.025 and with a few positive and negative outliers. The QQ-plot is nice and linear in the middle but deviate in the tails.



The monthly cc returns on FMAGX don’t look particularly normally distributed. There is a strong negative skewness (large negative returns) that shows up in the histogram and smoothed density that appears to be driven by several large negative values. The QQ-plot indicates a fat left tail but normal right tail.



The distribution of SBUX's return is asymmetric with a long left tail. The QQ-plot indicates that the left tail is fatter than the normal but that the right tail looks fairly normal.



The side-by-side boxplot is a very good way to quickly compare distributions. Here we see that the distribution of the bond fund, VBLTX, is the most concentrated about its mean (smallest SD) and SBUX has the widest distribution.

d.

```
> summary(ret.mat)
     VBLTX                FMAGX                 SBUX
 Min.   :-0.09010   Min.   :-0.24335   Min.   :-0.4792
 1st Qu.:-0.00980   1st Qu.:-0.02047   1st Qu.:-0.0488
 Median : 0.00868   Median : 0.00993   Median : 0.0183
 Mean   : 0.00530   Mean   : 0.00189   Mean   : 0.0113
 3rd Qu.: 0.01991   3rd Qu.: 0.03974   3rd Qu.: 0.0864
 Max.   : 0.10761   Max.   : 0.13020   Max.   : 0.2785
> apply(ret.mat, 2, mean)
   VBLTX    FMAGX     SBUX
0.005299 0.001889 0.011312
> apply(ret.mat, 2, var)
    VBLTX     FMAGX      SBUX
0.0006811 0.0032067 0.0142561
> apply(ret.mat, 2, sd)
  VBLTX   FMAGX    SBUX
0.02610 0.05663 0.11940
> apply(ret.mat, 2, skewness)
  VBLTX   FMAGX    SBUX
-0.1457 -1.0434 -0.9032
```

```
> apply(ret.mat, 2, kurtosis)
VBLTX FMAGX  SBUX
2.867 2.704 2.690
```

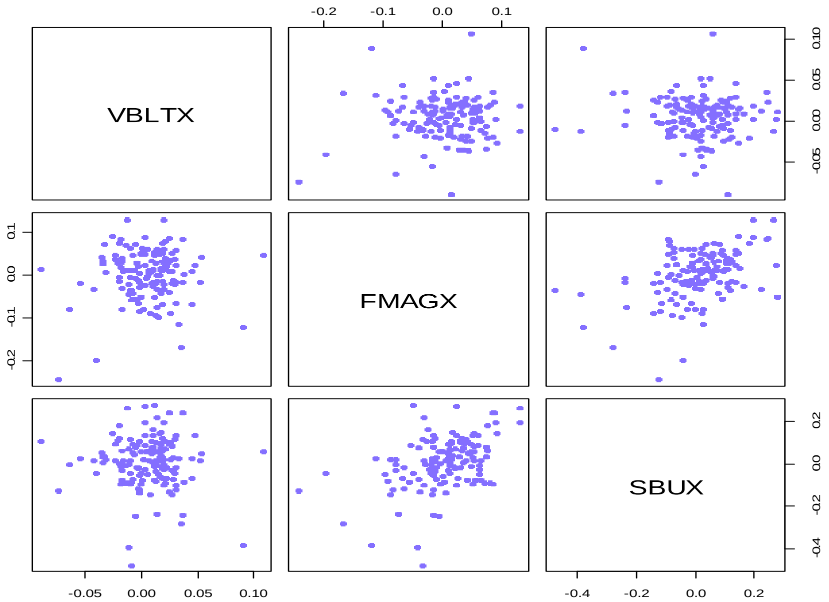
The mean and median values for all assets are positive, with the median greater than the mean (indicating negative skewness). SBUX has the highest monthly mean at 1.1%. The monthly SD value for VBLTX is the smallest at 2.6%, which makes sense because short terms bonds are thought to be quite safe. The SD for FMAGX is about 5.6% and the SD for SBUX is 12%. These results are typical: individual stocks have the highest SD, portfolios have smaller SDs than individual stocks, and bonds have the smallest SDs. All distributions are negatively skewed, and FMAGX and SBUX have larger negative returns than positive returns. Also, the excess kurtosis measures (R reports excess kurtosis, not regular kurtosis) for all are positive (around 2.5) indicating fatter tails than the normal.

e and f. Because the returns are continuously compounded, an estimate of the annual cc return is just 12 times the monthly cc return, and an estimate of the cc annual SD is the square root of 12 times the monthly SD. For the three assets we have

```
> # annualized cc mean
> 12*apply(ret.mat, 2, mean)
  VBLTX   FMAGX    SBUX
0.06359 0.02267 0.13575
>
> # annualized simple mean
> exp(12*apply(ret.mat, 2, mean)) - 1
  VBLTX   FMAGX    SBUX
0.06566 0.02293 0.14539
>
> # annualized sd values
> sqrt(12)*apply(ret.mat, 2, sd)
 VBLTX  FMAGX   SBUX
0.0904 0.1962 0.4136
```

The annual means are: 6.6% for bonds, 2.3% for the mutual fund and 14.5% for SBUX. Notice the large annual SD for SBUX at 41% and the small annual SD for the bond fund at 9%.

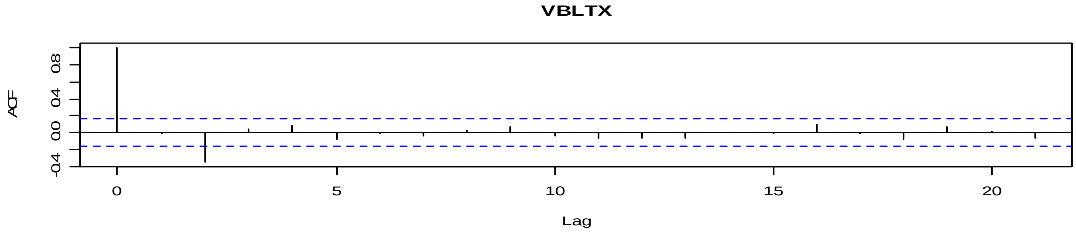
g. The pairs plot shows all possible pair-wise scatterplots. The top panel shows all pairwise scatterplots between VBLTX and the other assets. There is no clear linear association between VBLTX and the other assets. The 2nd row shows all pairwise scatterplots between FMAGX and the other assets. Here we also see positive linear association between FMAGX and SBUX.

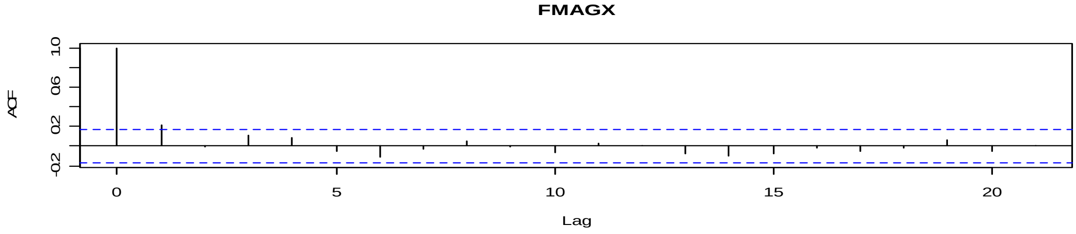


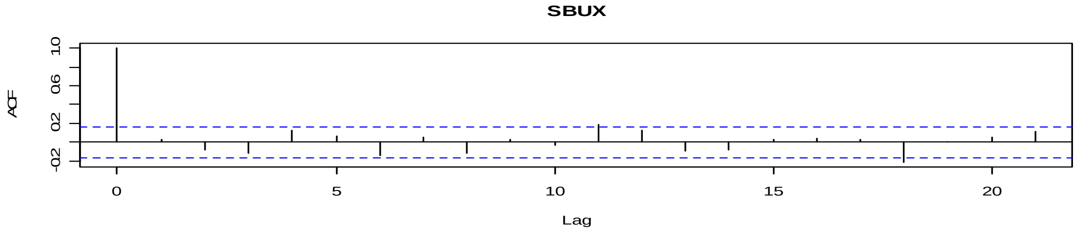

```
h. > var(ret.mat)
            VBLTX     FMAGX       SBUX
VBLTX  6.811e-04 8.004e-05 -0.0001659
FMAGX  8.004e-05 3.207e-03  0.0030332
SBUX  -1.659e-04 3.033e-03  0.0142561
> cor(ret.mat)
          VBLTX   FMAGX     SBUX
VBLTX  1.00000 0.05416 -0.05323
FMAGX  0.05416 1.00000  0.44862
SBUX  -0.05323 0.44862  1.00000
```

The covariances between VBLTX and FMAGX, and between FMAGX and SBUX are positive, indicating positive linear associations, and the covariance between VBLTX and SBUX is negative indicating a negative linear association. The correlations indicate a moderately strong linear association is between FMAGX and SBUX, but very weak associations between VBLTX and the other assets.

i. The sample autocorrelation (SACF) plots are given below:







The SACF plots for the three assets give the sample autocorrelation estimates for lags 1 through 20. None of the autocorrelation estimates are very big and the plots suggest that the month cc returns are essentially uncorrelated over time (at the monthly frequency).

CER model

a. The CER model estimates are (use courier font for computer output)

```
> cbind(muhat.vals,sigma2hat.vals,sigmahat.vals)
      muhat.vals sigma2hat.vals sigmahat.vals
VBLTX   0.005293      0.0006754       0.02599
FMAGX   0.001888      0.0032058       0.05662
SBUX    0.011313      0.0142488       0.11937
> cbind(covhat.vals,rhohat.vals)
            covhat.vals rhohat.vals
VBLTX,FMAGX   8.004e-05     0.05416
VBLTX,SBUX   -1.659e-04    -0.05323
FMAGX,SBUX    3.033e-03     0.44862
```

SBUX has the highest average return and FMAGX has the lowest. SBUX has the highest SD and VBLTX has the lowest. This makes sense because VBLTX is a government bond index, and its return should not vary too much. SBUX is an individual stock, whereas FMAGX is a highly diversified portfolio.

The bond index is essentially uncorrelated with the returns on SBUX and FMAGX. SBUX and FMAGX are moderately positively correlated.

b. The SE values for the estimates are given below

```
> cbind(muhat.vals,se.muhat)
      muhat.vals se.muhat
VBLTX   0.005299 0.002182
FMAGX   0.001889 0.004735
SBUX    0.011312 0.009985

> cbind(sigma2hat.vals,se.sigma2hat)
      sigma2hat.vals se.sigma2hat
VBLTX      0.0006811    8.054e-05
FMAGX      0.0032067    3.792e-04
SBUX       0.0142561    1.686e-03

> cbind(sigmahat.vals,se.sigmahat)
      sigmahat.vals se.sigmahat
VBLTX       0.02610    0.001543
FMAGX       0.05663    0.003348
SBUX        0.11940    0.007060

> cbind(rhohat.vals,se.rhohat)
            rhohat.vals se.rhohat
VBLTX,FMAGX     0.05416   0.08338
VBLTX,SBUX     -0.05323   0.08339
FMAGX,SBUX      0.44862   0.06679
```

The mean values are not estimated very precisely, since the SE values are almost as large or larger than the mean estimates. The variance and SD values are estimated more precisely. The SE values for the SD are much smaller than the SD estimates. The correlations between VBLTX and the other assets are also not estimated too precisely, whereas the correlation between FMAGX and SBUX is estimated fairly precisely.

c. The 95% confidence intervals for the true values are given below

```
> cbind(mu.lower,mu.upper)
        mu.lower mu.upper
VBLTX  0.0009348 0.009664
FMAGX -0.0075819 0.011360
SBUX  -0.0086568 0.031282
```

Notice that the 95% confidence intervals for the mean contain both positive and negative values for FMAGX and SBUX. This is further evidence that the means for these assets are not estimated very well.

```
> cbind(sigma2.lower,sigma2.upper)
      sigma2.lower sigma2.upper
VBLTX    0.0005157    0.0008352
FMAGX    0.0024475    0.0039640
SBUX     0.0108786    0.0176190
```

```
> cbind(sigma.lower,sigma.upper)
      sigma.lower sigma.upper
VBLTX     0.02301     0.02918
FMAGX     0.04993     0.06332
SBUX      0.10528     0.13352
```

The 95% confidence intervals for the SD are fairly narrow, indicating that the SD values are estimated fairly precisely.

```
> cbind(rho.lower,rho.upper)
            rho.lower rho.upper
VBLTX,FMAGX   -0.1126    0.2209
VBLTX,SBUX    -0.2200    0.1135
FMAGX,SBUX     0.3150    0.5822
```

The 95% confidence intervals for the correlations involving VBLTX are quite wide. In fact, the first two intervals contain both positive and negative values. The third interval contains only positive values and is fairly narrow.

d. Value at Risk Calculations

A simple R function to compute value at risk for a given probability and initial wealth is

```
Value.at.Risk = function(x,p=0.05,w=100000) {
   q = colMeans(x) + colStdevs(x)*qnorm(p)
   VaR = (exp(q) - 1)*w
   VaR
}
```

The required argument `x` represents the continuously compounded return data (which may be a matrix), the argument `p` represents the loss probability, and the argument `w` represents the initial wealth. Using the function, the 5% and 1% VaR estimates based on an initial investment of $100,000 are

```
> Value.at.Risk(ret.mat,p=0.05,w=100000)
 VBLTX  FMAGX   SBUX
 -3693  -8722 -16896
```

```
> Value.at.Risk(ret.mat,p=0.01,w=100000)
 VBLTX  FMAGX   SBUX
 -5390 -12177 -23391
```

SBUX has the largest VaR estimates followed by FMAGX and VBLTX. Not surprising, the bond fund VBLTX has the lowest VaR values.