ECON 147 Homework 2

Due: 12:30 pm, April 30th (Monday)

Reading and Program Downloads

- Please read the course material in course webpage.
  - Slides and lecture note for probability review
- For some questions, the R code (econ147lab2_Hint.r) will be helpful. Make sure you modify the codes to have the correct answers.

Review Questions

1. Suppose X is a uniform random variable over [0,1] (i.e., $X \backsim U[0,1]$) and Y is a Bernoulli random variable with a success probability $\Pr(Y = 1) = 0.5, i.e.,$

$$Y = \begin{cases} 1, & \text{with probability } 0.5 \\ 0, & \text{with probability } 0.5 \end{cases}.$$

   Compute the following (You will not need any program for this problem, just solve by hands after your readings.)

   - $\Pr(X < 0.1)$ and $\Pr(Y < 0.1)$:
   - $E(X), Var(X),\ E(Y)$ and $Var(Y)$:
   - $E(0.3X + 0.7Y)$ and $E(0.5X + 0.5Y)$:
   - For any $\alpha \in [0,1],\ E(\alpha X + (1-\alpha)Y)$:
   - Now suppose X and Y represent (statistically independent) outcomes of two lotteries, and you would like to invest your $100 to these lotteries. Assume you only care about the mean (preferably high) and variance (preferably low) of your investment. How do you want to distribute your $100, i.e., how do you choose your $\alpha \in [0,1]$ for $\alpha X + (1-\alpha)Y$? Justify your answer.

2. Suppose X is a normally distributed random variable with mean 0 and variance 1 (i.e., standard normal). Compute the following (Hint: you can use the R functions *pnorm* and *qnorm* to answer these questions).

   - $Pr(X < -1.96)$
   - $Pr(X > 1.64)$
   - $Pr(-0.5 < X < 0.5)$
   - 1% quantile, $q_{.01}$ and 99% quantile, $q_{.99}$:
   - 5% quantile, $q_{.05}$ and 95% quantile, $q_{.95}$:

3. Let X denote the monthly return on Microsoft Stock and let Y denote the monthly return on Starbucks stock. Assume that $X \sim N(0.05, (0.10)^2)$ and $Y \sim N(0.025, (0.05)^2)$.

- Using a grid of values between –0.25 and 0.35, plot the normal curves for X and Y. Make sure that both normal curves are on the same plot.
- Comment on the risk-return tradeoffs for the two stocks. Which one do you want to invest?

4. Let R denote the simple monthly return on Microsoft stock and let W_0 denote initial wealth to be invested over the month. Assume that $R \sim N(0.05, (0.12)^2)$ and that $W_0 = \$100{,}000$.

    - Determine the 1% and 5% value-at-risk (VaR) over the month on the investment. That is, determine the loss in investment value that may occur over the next month with 1% probability and with 5% probability.

5. Let r denote the continuously compounded monthly return on Microsoft stock and let W_0 denote initial wealth to be invested over the month. Assume that $r \sim iid\ N(0.05, (0.12)^2)$ and that $W_0 = \$100{,}000$.

    - Determine the 1% and 5% value-at-risk (VaR) over the month on the investment. That is, determine the loss in investment value that may occur over the next month with 1% probability and with 5% probability. (Hint: compute the 1% and 5% quantile from the Normal distribution for r and then convert continuously compounded return quantile to a simple return quantile using the transformation $R = e^r - 1$.)
    - Determine the 1% and 5% value-at-risk (VaR) over the year on the investment. (Hint: to answer this question, you must determine the normal distribution that applies to the annual (12 month) continuously compounded return.)

6. In this question, you will examine the chi-square and Student's t distributions.

    - On the same graph, plot the probability curves of chi-squared distributed random variables with 1, 2, 4 and 10 degrees of freedom. Use different colors and line styles for each curve.

- On the same graph, plot the probability curves of Student's t distributed random variables with 1, 2, 4 and 10 degrees of freedom (d.f). Also include the probability curve for the standard normal distribution. Use different colors and line styles for each curve.
- Without doing any actual calculation, which one do you expect is greater (in absolute values) among the followings:

5% VaR of standard normal distribution
Versus
5% VaR of t distribution with d.f 2

7. Consider the following joint distribution of X and Y:

Bivariate pdf

		$Y=0$	$Y=1$
X	0	1/8	0
	1	1/8	1/8
	2	1/8	3/8
	3	0	1/8

- Find the marginal distributions of X and Y. Using these distributions, compute E[X], Var(X), SD(X), E[Y], Var(Y) and SD(Y).
- Compute Cov(X,Y) and Corr(X,Y)
- Are X and Y independent? Justify your answer.
- Compute the conditional distributions $f(X|Y = 0)$ and $f(Y|X = 2)$.
- Compute $E[X|Y = 0]$ and $E[Y|X = 2]$.
- Compute $Var[X|Y = 0]$ and $Var[Y|X = 2]$.