ECON 147 MIDTERM EXAM

YOUR NAME:

Winter 2018, Feb 13th
3:30pm - 4:45pm

Instruction

- This is a closed book and closed note exam. All necessary information will be provided and all calculation will be done by hands, so no cheat sheet or calculator is needed.
- Try to answer all questions and write all answers **within this exam sheets**. You will hand in this exam sheets. Please write legibly.
- *Examination Rules from Department Policy will be strictly followed.*

1. **(Probability Review; 30 pts)** Consider the following joint distribution of X and Y:

Bivariate pmf

		Y	
		0	1
	0	1/8	1/8
X	1	1/4	1/4
	2	1/8	1/8

1.1. Find the marginal probability mass functions of X and Y. Using these distributions, compute $E[X]$, $Var(X)$, $E[Y]$ and $Var(Y)$. **[8 pts]**

1.2. Compute the conditional pmf $f(x|Y = 0)$ and $f(x|Y = 1)$, and also write down the marginal pmf $f(X)$ from 1.1. **[5 pts]**

1.3. Are X and Y independent? Fully justify your answer. (Hint: you will need the answer from 1.2.) [**5 pts**]

1.4. Compute $E[X|Y=0]$, $Var[X|Y=0]$ and $Cov(X,Y)$. (Hint: use the earlier answers) [**6 pts**]

1.5. Now suppose X and Y represent outcomes of two lotteries, and you would like to invest your \$100 to these lotteries. Assume you are extremely risk averse, hence only care about the risk (preferably low variance) of your investment. How do you want to distribute your \$100, i.e., how do you choose your $\alpha \in [0,1]$ for $\alpha X + (1-\alpha)Y$? **[6 pts]**

2. **(Return Calculation and VaR; 40 pts)** Consider a 1-month investment in two assets (equities): the Goldman Sachs Group (GS) and American International Group (AIG). Suppose you buy one share of each GS and AIG at the end of September, 2017 for

$$P_{GS,t-1} = 100, \;\; P_{AIG,t-1} = 20$$

and then sell these shares at the end of October, 2017 for

$$P_{GS,t} = 115, \;\; P_{AIG,t} = 21.$$

(Note: these are hypothetical prices for your computation.)

2.1. What are the simple 1-month returns for the two investments? [**5 pts**]

2.2. What are the continuously compounded (cc) 1-month returns for the two investments? (You will use the approximation formula $\log(1+x) \simeq x$ for this question.) [**5 pts**]

2.3. Assume you get the same monthly returns from part 2.2. every month for the next year. What are the annualized cc returns? [**6 pts**]

2.4. At the end of September, 2017, suppose you have \$10,000 to invest in GS and AIG over the next month. Suppose you purchase \$5,000 worth of GS and the remainder in AIG. What are the portfolio weights in the two assets? Using the results from part 2.1. compute the 1-month *simple* portfolio returns ($R_{p,t}$). [**8 pts**]

2.5. Now assume the distributions of the two *simple* 1-month returns as

$$R_{GS,t} \sim iid\ N\left(0.15, \frac{3}{4}\right),\ R_{AIG,t} \sim iid\ N\left(0.05, \frac{1}{4}\right)$$

and suppose they are statistically independent. What is the mean, variance and distribution of the 1-month simple portfolio returns ($R_{p,t}$) from part 2.4.? [**8 pts**]

2.6. Using the distribution of $R_{p,t}$ from 2.5., compute the 2.5% monthly Value-at-Risk for the $10,000 investment in the portfolio returns in 2.4. (You will use $q^Z_{0.025} = -2$, which is the 2.5% quantile of a standard normal random variable.) [**8 pts**]

3. **(30 pts, 5pts each)** Indicate whether the following statements are true or false (circle one). Briefly discuss why it is so.

3.1. If r_t is *continuously compounded (cc)* 1-month return, then the annualized cc return is $r_A = \sum_{j=0}^{11} r_{t+j}$.

True False

Why?

3.2. Let $R_{GS,t}$ and $R_{AIG,t}$ be *simple* 1-month returns for Goldman Sachs Group (GS) and American International Group (AIG). If we construct a portfolio using the share $\alpha \in [0, 1]$ for GS, the portfolio simple return is $R_{p,t} = \alpha R_{GS,t} + (1 - \alpha) R_{AIG,t}$.

True False

Why?

3.3. In 3.2., if 5% quantile of the portfolio *simple return* is given as $q_{0.05}^{R_p} = -0.5$, then 5% monthly Value-at-Risk for the \$10,000 investment in this portfolio is $\$10,000 \times (-0.5) = -\$5,000$.

True False

Why?

3.4. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ are two different point estimators for θ. If $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$, then confidence interval based on $\hat{\theta}_1$ is more accurate (shorter) so we always prefer to use $\hat{\theta}_1$.

True False

Why?

3.5. If $R_{AIG,t} \sim N\left(0, \sigma^2_{AIG}\right)$ and $R_{GS,t} \sim N\left(0, \sigma^2_{GS}\right)$ and they are *independent*, the *simple portfolio return* $R_{p,t} = x_{GS}R_{GS,t} + x_{AIG}R_{AIG,t}$ is distributed as $N\left(0, x^2_{GS}\sigma^2_{GS} + x^2_{AIG}\sigma^2_{AIG}\right)$.

True False

Why?

3.6. Assume r_t is *continuously compounded (cc)* 1-month returns with $r_t \sim$ *iid* $N(\mu, \sigma^2)$. Then the sample mean $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r_t$ has a same distribution with r_t hence $\hat{\mu} \sim N\left(\mu, \sigma^2\right)$, because of (i) the iid property of r_t, and (ii) the property of the normal distribution.

True False

Why?