ECON 147 Homework 4

May 23th, 12:30pm

Readings

- Course slides and lecture notes.
- Using the provided R coding, you will be able to simulate AR and MA processes. I highly recommend you to understand the plots, depending on the different values of θ and ϕ.
- Using the provided R coding, you will also be able to compute the descriptive statistics of real financial returns. Make sure you understand the output rather than just following the hint.

Review Questions

1. One popular concept on stock market price is called "martingale pricing", which is relevant to "rational expectations" or "efficient market hypothesis" in economic theories. A rough statement on this is that, a (log of) stock market price p_t (e.g., S&P 500 index) is supposed to reflect all the relevant information available up to time t, i.e.,

$$E\left[p_t|\mathcal{F}_{t-1}\right] = p_{t-1},$$

therefore $p_t = \log P_t$ is a martingale.

   (a) Under the martingale pricing, show that continuously compounded (cc) return for stock market price is mds.

   (b) In what follows, we assume cc return r_t is a covariance stationary process. Prove the following statements:

      i. If $r_t \sim iid\ (0, \sigma^2)$ (or independent white noise), then $r_t \sim mds\ (0, \sigma^2)$.

      ii. If $r_t \sim mds\ (0, \sigma^2)$, then $r_t \sim WN\ (0, \sigma^2)$ (or weak white noise).

   (c) Prove the following statements:

      i. If $\{r_t\}$ is iid, then it is strictly stationary.

      ii. If $\{r_t\}$ is strictly stationary and $E[r_1^2] < \infty$, then it is covariance stationary.

2. Consider the AR(1) model

$$\begin{aligned} Y_t &= 5 - 0.55Y_{t-1} + \varepsilon_t, \\ \varepsilon_t &\sim mds(0, 1.2). \end{aligned}$$

   (a) Is this process stationary? Why or why not?

   (b) What is the mean of this process?

   (c) What is the variance of this process?

   (d) What is the auto-covariance function of this process?

R Exercises A

The following questions require R. On the course website, you can find the R script file *econ147lab4A_Hint.r*. The file contains hints for completing this R exercises.

1. Consider the MA(1) model

$$\begin{aligned} Y_t &= 0.05 + \varepsilon_t + \theta\varepsilon_{t-1}, \ \ |\theta| < 1, \\ \varepsilon_t &\sim iid\ N\left(0, (0.10)^2\right), \end{aligned}$$

   (a) Using the R function *arima.sim()*, simulate and plot 250 observations of the MA(1), theoretical ACF (autocorrelation function) and sample ACF with $\theta = 0.5$, $\theta = 0.9$ and $\theta = -0.9$.

   (b) Briefly comment on the behavior of the simulated data series.

2. Consider the AR(1) model

$$\begin{aligned} Y_t - 0.05 &= \phi\left(Y_{t-1} - 0.05\right) + \varepsilon_t, \ \ |\phi| < 1, \\ \varepsilon_t &\sim iid\ N\left(0, (0.10)^2\right), \end{aligned}$$

   (a) Using the R function *arima.sim()*, simulate and plot 250 observations of the AR(1) with $\phi = 0$, $\phi = 0.5$, $\phi = 0.9$ and $\phi = 0.99$.

   (b) Comment on the behavior of the simulated data series. Which series is close to nonstationary (or persistent) time series?

R Exercises B

The following questions require R. On the course website, you can find the R script file *econ147lab4B_Hint.r*. The file contains hints for completing this R exercises. Copy and paste all statistical results and graphs into a MS Word document (or your favorite word processor) while you work, and add any comments and answer all questions in this document. Start MS Word and open a blank document. You will save all of your work in this document.

In this lab, you will analyze continuously compounded monthly return data on the Vanguard long term bond index fund (VBLTX), Fidelity Magellan stock mutual fund (FMAGX), and Starbucks stock (SBUX). I encourage you to go to finance.yahoo.com and research these assets. The script file *econ147lab4B_Hint.r* walks you through all of the computations for the lab. You do not need to show the R commands in your lab write up. You will use the get.hist.quote() function from the tseries package to automatically load this data into R. You will also use several functions from the PerformanceAnalytics package. Remember to install packages before you load them into R.

0. Briefly discuss what these assets are. You can search the Ticker symbol VBLTX, FMAGX and SBUX from finance.yahoo.com.

1. **(Descriptive Statistics)** Do the following replication exercises.

   (a) Make time plots of the return data using the R command plot() as illustrated in the script file *econ147lab4B_Hint.r*. Comment on any relationships between the returns suggested by the plots. Pay particular attention to the behavior of returns toward the end of 2008 at the beginning of the financial crisis.

   (b) Make a cumulative return plot (future of $1 invested in each asset) and comment. Which assets gave the best and worst future values over the investment horizon?

   (c) For each return series, make a four panel plot containing a histogram, density plot, boxplot and normal QQ-plot. Do the return series look normally distributed? Briefly compare the return distributions.

   (d) Compute numerical descriptive statistics for all assets using the R functions summary(), mean(), var(), stdev(), skewness() (in

package PerformanceAnalytics) and kurtosis() (in package PerformanceAnalytics). Compare and contrast the descriptive statistics for the three assets. Which asset appears to be the riskiest asset?

(e) Using the mean monthly return for each asset, compute an estimate of the annual continuously compounded return (i.e., recall the relationship between the expected monthly cc return and the expected annual cc return). Convert this annual continuously compounded return into a simple annual return. Are there any surprises?

(f) Using the estimate of the monthly return standard deviation for each asset, compute an estimate of the annual return standard deviation. Briefly comment on the magnitude of the annual standard deviations.

(g) Use the R pairs() function to create all pair-wise scatterplots of returns. Comment on the direction and strength of the linear relationships in these plots.

(h) Use the R functions var() and cor() to compute the sample covariance matrix and sample correlation matrix of the returns. Comment on the direction and strength of the linear relationships suggested by the values of the covariances and correlations.

(i) Use the R function acf() to compute and plot the sample autocorrelation functions of each return. Do the returns appear to be uncorrelated over time?

2. (**IID Normal Model)** Consider the IID normal model:

$$\begin{aligned} R_{it} &= \mu_i + \varepsilon_{it},\ t = 1, ..., T, \\ \varepsilon_{it} &\sim iid\ N\left(0, \sigma_i^2\right), \\ Cov\left(\varepsilon_{it}, \varepsilon_{jt}\right) &= \sigma_{ij} \end{aligned}$$

where R_{it} denotes the continuously compounded return on asset i, i =Vanguard long term bond index fund (VBLTX), Fidelity Magellan stock mutual fund (FMAGX), and Starbucks stock (SBUX).

(a) Using sample descriptive statistics, give estimates for the model parameters $\mu_i, \sigma_i^2, \sigma_i, \sigma_{ij}$ and ρ_{ij} (correlation coefficient). Arrange these estimates nicely in a table. Briefly comment.

(b) For each estimate of the above parameters (except σ_{ij}) compute the estimated standard error. That is, compute $SE\left(\hat{\mu}_i\right)$, $SE\left(\hat{\sigma}_i^2\right)$, $SE\left(\hat{\sigma}_i\right)$ and $SE\left(\hat{\rho}_{ij}\right)$. Briefly comment on the precision of the estimates.

(c) For each parameter $\mu_i, \sigma_i^2, \sigma_i$ and ρ_{ij} compute 95% and 99% confidence intervals. Briefly comment on the width of these intervals.

(d) Using the estimated values of μ_i, σ_i^2 for each mutual fund, compute the 1% and 5% monthly value-at-Risk (VaR) based on an initial \$100,000 investment. Which fund has the lowest VaR?