# What is a Raindrop Size Distribution?

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## ABSTRACT

It is commonly understood that the number of drops that one happens to measure as a function of diameter in some sample represents the drop size distribution. However, recent observations show that rain is "patchy" suggesting that such a seemingly "obvious" definition is incomplete. That is, rain consists of patches of elementary drop size distributions over a range of different scales. All measured drop size distributions, then, are statistical mixtures of these patches.

Moreover, it is shown that the interpretation of the measured distribution depends upon whether the rain is *statistically homogeneous* or not. It is argued and demonstrated using Monte Carlo simulations that in *statistically homogeneous* rain, as the number of patches included increases, the observed spectrum of drop sizes approaches a "steady" distribution. On the other hand, it is argued and demonstrated using video disdrometer data that in *statistically inhomogeneous* rain, there is no such steady distribution. Rather as long as one keeps measuring, the drop size distribution continues to change. What is observed, then, depends on when one chooses to stop adding measurements.

Consequently, the distributions measured in statistically inhomogeneous rain are *statistical* entities of *mean* drop concentrations best suited to *statistical* interpretations. In contrast, steady distributions in statistically homogeneous rain are more amenable to deterministic interpretations since they depend upon factors independent of the measurement process.

These findings have implications addressed in two additional questions, namely,

- Are computer-created virtual drop size distributions really the same as those observed?
- What is the appropriate drop size distribution when several measurements used in an algorithm for rain estimations are made at different resolutions?

## 1. Introduction

The meaning of a drop size distribution is one of those things that everyone "knows" yet no one has really ever bothered stating. In part this is because there has been no apparent reason to. In the past, the measurement of drop sizes was a straightforward albeit very laborious task involving sifting for and measuring dried raindrop pellets captured in a box of flour (Laws and Parsons 1943). The drop size distribution, then, was simply what was collected in the box; that is, what you measured is what you got. In search of a faster technique, Marshall et al. (1947) and Marshall and Palmer (1948) switched to dye paper that revealed the impact of each drop on a surface. By involving several students they were able to collect an impressive array of data in several stratiform rain events of different intensities. Because they had an ensemble of measurements they were then able to combine them to form the well-known Marshall–Palmer families of average raindrop size distributions (fit with exponentials) and to perform parametric fits of distribution parameters that correlated with the rainfall rate, $R$. In this sense they were the first to go beyond the "what you see is what you get" philosophy by generating statistical distributions formed by combining many observations of individual spectra.

At the ground, the study of drop size distributions was really revolutionized, however, with the introduction of the electromechanical disdrometer (Joss and Waldvogel 1967), used in a number of fundamental

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studies (e.g., Waldvogel 1974; Joss and Gori 1978; List et al. 1988; Smith et al. 1993; Tokay and Short 1996, and many others) far too numerous to catalog here. Another giant leap occurred with the recent development of the optical disdrometer having high spatial and temporal resolution capabilities without the technical problems associated with mechanical devices (see Sheppard and Joe 1994). A similar revolution in the measurement of rain by aircraft occurred when the cumbersome and limited slide and foil techniques were replaced with the Particle Measuring Systems (PMS) 2D optical probes (Knollenberg 1981) in the 1970s.

Many of these observations inspired innovative attempts to understand distributions physically using experiments in which water was released to fall over long distances thereby allowing the distribution to evolve through drop breakup (Blanchard and Spencer 1970). While cumbersome and incomplete, since the water could never fall a distance sufficient for the drops to achieve "equilibrium," such experiments were a convincing demonstration that the size distributions did indeed approximately evolve through drop breakup and coalescence toward those observed in nature. With the advent of readily available computer technology, however, researchers soon abandoned such difficult physical experiments for numerical studies. But in the transition from three-dimensional real space (excluding time) to one-dimensional virtual space, are such computer-created virtual drop size distributions really the same as those observed in nature?

The question of what a drop size distribution means, however, is not simply of academic interest. For example, the concept is used extensively throughout remote sensing when developing and applying algorithms for estimating rain using radars and radiometers. Measurements are often collected over sampling volumes of vastly different sizes. Moreover, the most advanced techniques combine different measurements using different instruments each having its own "beamwidth" so that, in effect, they are looking at different ensembles of rain patches, that is, at different total drop size distributions. What, then, is the appropriate drop size distribution when the measurements used in an algorithm are made at different resolutions?

In this article we attempt to address these questions. In the process we hope to illuminate the increasing subtlety of the concept of drop size distributions and to develop an awareness of what is really being measured. If nothing else we hope at least to instill an appreciation of how the measurement process, the statistical structure of the rain, and the extent of "averaging" all determine the proper interpretation, that is, meaning, of the drop size distribution.

## 2. The problem of sampling

Whether on the ground or in an airplane, the measurements of all drop size distributions reduce to counting drops and placing them into "size categories" or "bins." While simple enough, an immediate question arises. How does one know when to stop counting? When have we "adequately sampled" the distribution?

Until very recently, the answer was to count until you reached some level of confidence based upon some statistical criteria. In physics and other fields, counting is usually treated using Poisson statistics, and it is probably one of the reasons Cornford (1967) applied Poisson statistics to the problem of raindrop counting. Another likely reason Cornford used Poisson statistics is that it has the very useful property that the variance equals the mean. Thus, just by counting drops one can immediately say something about the uncertainty (variance) of the number of drops counted. Hence, the answer to the question of when to stop is simply to continue counting until the number reaches a value consistent with what is needed to achieve a certain level of statistical confidence in the average value. The measurement interval in time or space, then, is simply that required "until the drops are adequately sampled" according to Poisson statistics. (It is worth noting that the search for "adequate samples" of rarer large drops has often greatly extended the measurement interval.) As beautifully simple as this approach is, however, it is, by and large, incorrect.

Why? Because in general, drop counts do not obey Poisson statistics and are correlated from one measurement volume to the next. Consequently, in most cases, if one continues to count even after satisfying the Poisson criteria, the drop size distribution continues to change sometimes substantially even as the number of drops increases. This is illustrated in the example below.

In Fig. 1, the radar reflectivity factor, $Z$, and rainfall rate, $R$, are plotted as functions of sampling pathlength (distance) for video disdrometer observations in a modest 20-min shower using successive 100-L sample volumes. [The sample volume is fixed because remote sensors do not measure over volumes that differ depending upon the size of the drop as is assumed when directly converting drop flux measure-

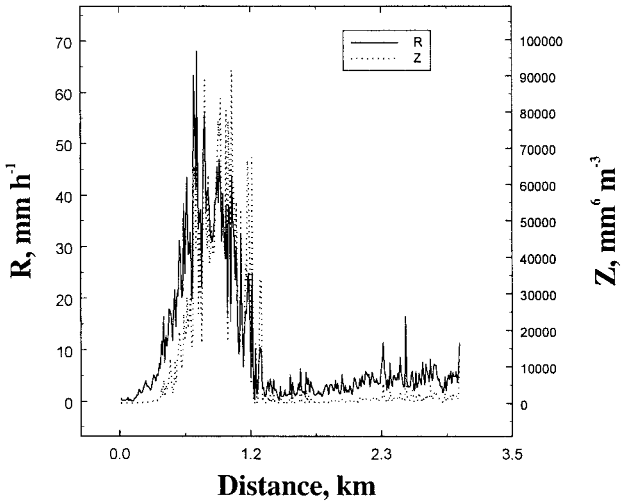


FIG. 1. Profiles of the radar reflectivity factor, $Z$, and rainfall rate, $R$, measured in a shower using a video disdrometer (described in detail by Jameson and Kostinski 1998).

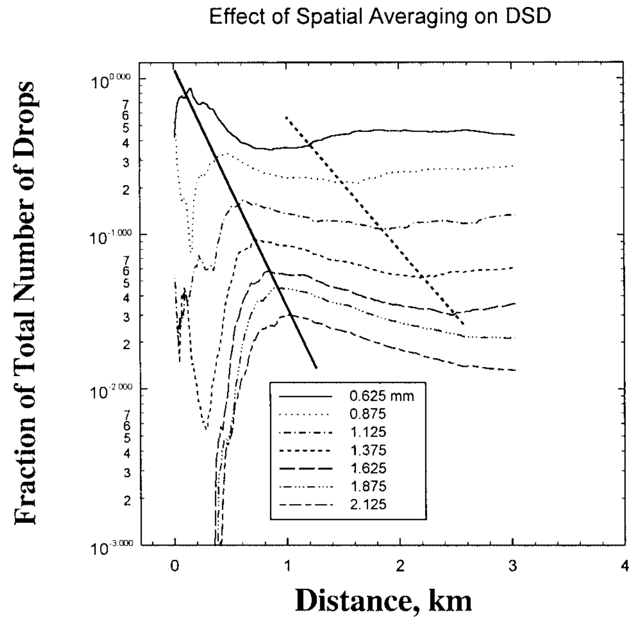


FIG. 2. Fractions of the total number of drops as functions of increasing sample pathlength (sample size) for the seven indicated drop size categories corresponding to Fig. 1 as discussed in the text. Note that the distributions never become steady (i.e., simultaneously horizontal at all sizes.)

ments into estimates of $Z$ and $R$. Consequently, the time series of moments of arrival for each drop is used to place the drops into proper relative positions in sample space. This space is then resampled using 100-L volumes (10 m × 100 cm$^2$, the sample area of the video disdrometer), as discussed in greater detail in Jameson and Kostinski 2001.][1]

In order to see how the net distribution changes with increasing sampling (volume), these samples are then combined one by one and the resulting fraction of the total number of observed drops is calculated and plotted for the different drop sizes in Fig. 2. In spite of increases in the total volume sampled, at no time do the contributions at any of the sizes become "steady." At first with increasing distance the maximum in the fractional contribution peaks at increasing sizes (solid line). The contributions from the larger drops then decrease while at still greater distances (sampling pathlengths) there is a subsequent resurgence in the fractional contributions at the smaller sizes to the right of the dashed line.

This can be seen as well by looking at some selected size distributions as shown in Fig. 3. As the sampling distance (volume) increases, the diameter distribution keeps changing even though the total number of observed drops in all seven size bins increases from 116 for the 100-m distribution, to 16 324 drops for the 3-km distribution.

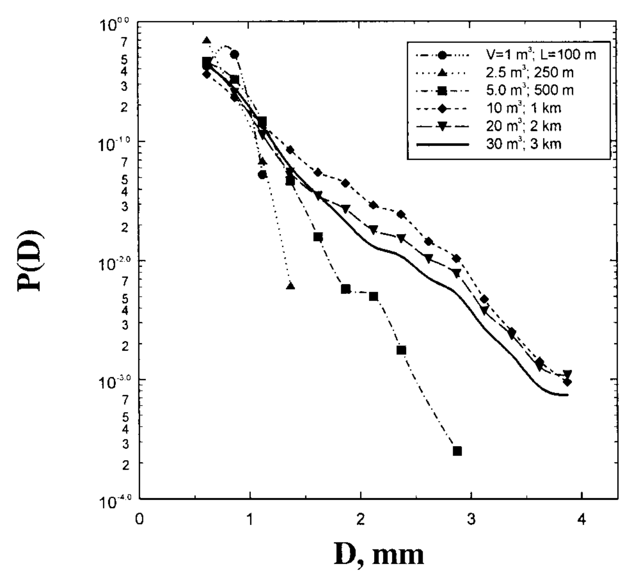


FIG. 3. Observed distributions of diameter corresponding to the indicated sample volumes ($V$) and corresponding sample pathlengths ($L$). As in Fig. 2, the distributions never become steady in spite of a 30-fold increase in sampling.

[1]Note that this approach ignores drop transformations due to coalescence and breakup. However, with respect to averaging, this approach is not substantially different from using flux measurements when computing average drop size distributions with increasing sample duration (volume). At no point in this work do we claim nor is it important to this discussion to have recreated the "true" distributions for which no information is available anyway.

While this result may seem almost "trivial" given the variability evident in Fig. 1, this is precisely how many drop size distributions are actually measured. Yet, such distributions are often then misinterpreted as though they were steady. They are not, as we discuss below.

## 3. A brief statistical characterization of rain and its relation to the meaning of raindrop size distributions

Obviously from the discussion above, the measurement of a drop size distribution is very much a statistical process. A Poisson process is characterized by three assumptions (e.g., Ochi 1990), namely, 1) that the probability of detecting more than one drop in a given volume $\delta V$ is vanishingly small for sufficiently small $\delta V$, 2) that drop counts in nonoverlapping volumes are statistically independent random variables (at any length scale), and 3) that the process is statistically homogeneous. With regard to rain, the first point can usually be satisfied. The second assumption, however, is usually found not to be true for single size bins (Kostinski and Jameson 1997) nor for several size bins either (Jameson and Kostinski 1998; Jameson et al. 1999). That is, the presence of a drop enhances (or in some cases decreases) the likelihoods that there are other drops of the same or different size in the neighboring volume. In other words, the drop counts in neighboring volumes are correlated. Such correlations do not exist for a Poisson process since counts in all neighboring volumes at all separations are statistically independent. Thus, natural rain cannot normally be described using Poisson statistics.

The correlations in natural rain arise because rain appears to consist of "patches" of different dimensions. That is, there are locations rich in drops interspersed with regions where drops are scarcer (see discussion in Jameson and Kostinski 1999, p. 3921). Before we explore the nature of these patches below, let us first return once more to the discussion of Poisson statistics. Assumption 3 means that Poisson statistics can never apply in statistically inhomogeneous conditions that likely often exist in nature. As we will see, however, the entire topic of statistical homogeneity is subtle and has implications for the meaning of drop size distributions well beyond concerns about Poisson statistics. It is, therefore, worth dwelling here briefly on this topic.

Statistical homogeneity means that the expected value (mean) and, in its broader sense, the variance of a random variable remain constant throughout the domain of observations. Unfortunately, the term homogeneity seems to generate a great deal of confusion, because it is often assumed that the random variable is then also *physically homogeneous* and has no apparent structures of any significant size. This is simply not the case because fluctuations can be correlated. That is, a fluctuation from the mean in one volume may be correlated with a fluctuation in a neighboring volume so that taken together larger-scale "structures" or "features" can appear. That is, *statistical homogeneity does not imply spatial homogeneity.* This is illustrated in Fig. 4 when in both cases the rainfall rates are statistically homogeneous, but correlated fluctuations obviously introduce significant "spatial" structures. Nevertheless, the presence of such features on scales less than the correlation length in statistically homogeneous rain should not be misinterpreted as evidence of statistical inhomogeneity [see Wunsch (1999) and the appendix in Jameson and Kostinski (2000) as well as Jameson and Kostinski (2001) for more extensive discussions].

There are two effects of correlated fluctuations in statistically homogeneous rain. One is to increase the variance, as Fig. 4 illustrates. The second occurs because the presence of correlation acts to reduce the effective number of independent samples thereby slowing the convergence toward the expected value

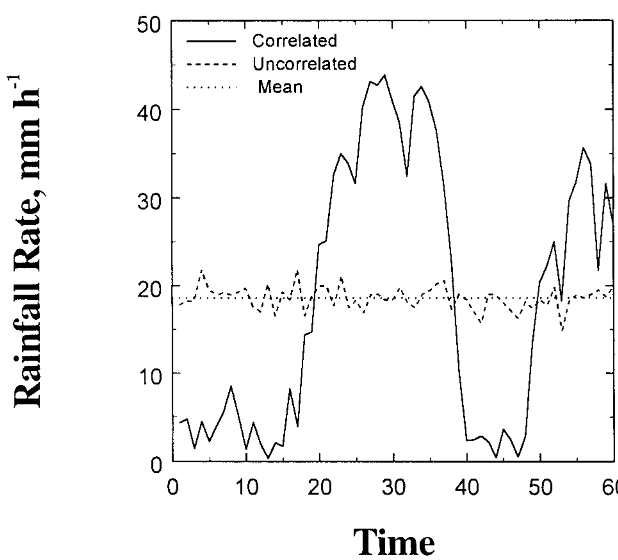


FIG. 4. Rain rate profiles for a Monte Carlo simulation of correlated and uncorrelated (Poisson) rain plotted as a function of dimensionless time. The horizontal line represents the mean for both types of simulated rain. Note the considerably larger fluctuations associated with the correlated rain (from Jameson and Kostinski 1999).

(see discussions in Jameson and Kostinski 1998, p. 284; Kostinski and Jameson 1999, 114–116; Kostinski and Jameson 2000, p. 914). This contrasts sharply with random variables obeying a statistically homogeneous Poisson process having no correlation in which *every* single sample corresponding to the smallest observation volume is independent so that the convergence is quite rapid.

The logical opposite of statistical homogeneity is *statistical inhomogeneity* in which the expected value and, in its broader sense, the variance of $R$ change from location to location within the observation volume. Statistically inhomogeneous rain is also patchy as well. So, then, just what does all this mean with regard to drop size distributions?

To address this question, we first return to rain patches. When the drops in patches are combined, they add up differently depending upon whether the rain is statistically homogeneous or inhomogeneous. Statistically homogeneous rain by definition means that the rainfall rate, $R$, is statistically homogeneous; that is, expected value of $R$, $E(R)$, and the variance, $\mathrm{var}(R)$, remain constant with regard to shifts in the origin. But since $R$ is the sum of the number of drops over each of the different sizes times their mass times their terminal fall speeds, the statistical homogeneity of $R$ implies that the expected number of drops $E(n)$ for each drop size is also fixed throughout the observation volume. If this were not so, then $E(R)$ and/or $\mathrm{var}(R)$ would change so that $R$ could not be statistically homogeneous, in conflict with the initial assumption.[2] This means that in statistically homogeneous rain, there is a steady drop size distribution independent of the measurement process. That is, when the drops in these patches are combined, they converge to an overall, steady drop size distribution as illustrated in Fig. 5.

In statistically inhomogeneous rain, however, this is not the case since the mean and variance of $R$ change throughout the observation volume. Thus $E(n)$ changes for each drop size from patch to patch so that adding patches together does not yield a steady drop size distribution. Rather the addition of each new patch changes the net $E(n)$ for each drop size. This is precisely what is happening in Figs. 2–3.

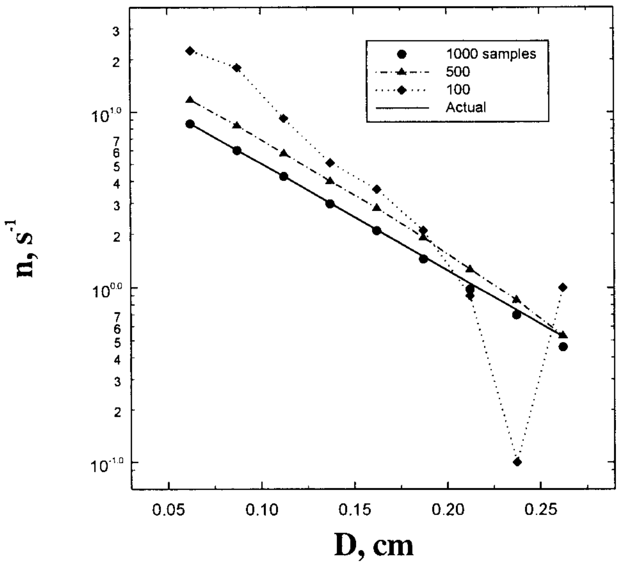


FIG. 5. The convergence of observed distributions of drop fluxes across a 100 cm$^2$ surface (video disdrometer) in a Monte Carlo simulation of correlated but statistically homogeneous rain (see Jameson and Kostinski 1999 for details) as a function of increasing number of samples. In statistically homogeneous rain, the measured distributions converge toward a steady function.

So what are these patches? Using plots of accumulated numbers of drops at different sizes (see discussion in Jameson and Kostinski 2000, p. 378) as illustrated in Fig. 6 (from that paper), it appears that the patches are characterized by quite steady fluxes at the different drop sizes. That is, they are apparently associated with well-defined, local drop size distributions (see Fig. 11 in Jameson and Kostinski 2000) over a spectrum of dimensions likely ranging from tens of centimeters to several tens or hundreds of meters. They even appear in Monte Carlo simulations of statistically homogeneous rain (Fig. 7), apparently the result of "stochastic accidents."[3] It appears, then, that they are what may be called *basic or elementary drop size distributions* from which other measured distributions are constructed. In the case of statistically ho-

[2]It can be proven mathematically that these remarks apply to all drop size distributions described using exponential and gamma functions. For more complex drop size distributions, "steadiness" requires that statistical homogeneity extend to higher moments of the distribution of $R$ as well. In the limiting case of strict sense homogeneity, the entire distribution of $R$ is invariant with respect to origin (Feller 1971, p. 88), and there is always an accompanying steady drop size distribution, regardless of form.

[3]This may explain why these elementary distributions often appear to be approximately exponential as well if such "accidents" were the result of a "memoryless" process. It is also worth noting that the summation of exponentially distributed random variables is itself gamma distributed, a function often used to describe observed drop size distributions.

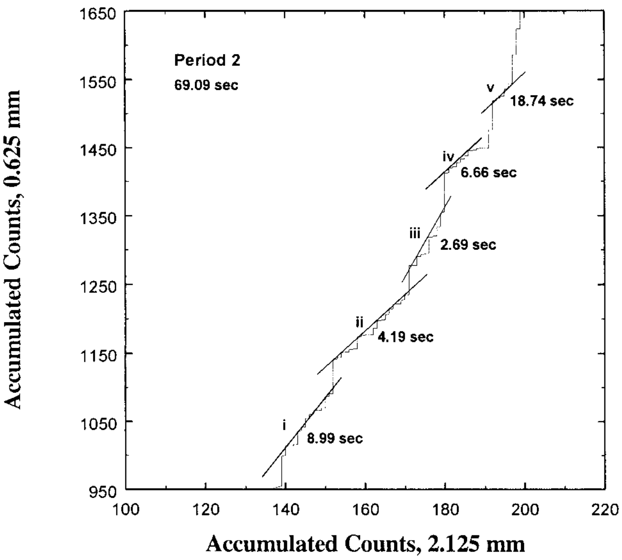


FIG. 6. Accumulated counts of 0.625-mm-diameter drops are plotted as a function of accumulated counts of those at 2.125-mm diameter for video disdrometer observations in rain. There are several regions or patches of linear relations between the two counts consistent with the presence of steady drop size distributions. Also note the slopes of the lines change indicating that the "slopes" of the distributions themselves are different for each patch (from Jameson and Kostinski 2000).

mogeneous rain, the measured distributions approach the overall, steady distribution as more and more patches are combined. Since such distributions exist independently of the measurement process, they have intrinsic and presumably deterministic meanings. In statistically inhomogeneous rain, however, the drop size distributions continually change as more and more data are added so that the final drop size distributions depend upon where one stops. *Thus, unlike drop size distributions in statistically homogeneous rain, the net distribution in statistically inhomogeneous rain depends critically upon the measurement process.* Consequently, such distributions are statistical mixtures of several elementary drop size distributions, and they represent "mean conditions." They are statistical entities that, unlike the steady distributions in statistically homogeneous rain, no longer have well-defined intrinsic, deterministic meanings as the addition of more data demonstrates.

So what does this mean? In a real sense, the most basic or elementary drop size distributions that can be observed are of those found in patches. However, sometimes it is really more useful to have "representative" distributions over larger dimensions. In statistically homogeneous rain this is easy since the more one measures, the more one converges to the overall, steady distribution that exists outside the measurement process. In statistically inhomogeneous rain, however, the drop size distributions should be viewed as statistical mixtures or, alternatively, as distributions of mean concentrations that depend critically upon where and how the measurements were made. Yet, a review of the literature reveals that often the distributions likely measured in statistically inhomogeneous rain are treated *as though* the measurements were made in statistically homogeneous rain and are often misinterpreted as if they were steady distributions having intrinsic, deterministic meanings independent of the measurement process. They are not. This has some important implications as discussed below.

## 4. Some implications

We are now in a position to respond to the questions originally posed: namely,

1) How does scaling affect what one means by a drop size distribution?
2) Are computer-created virtual drop size distributions really the same as those observed?
3) What, then, is the appropriate drop size distribution when the measurements used in an algorithm are made at different resolutions?

Beginning with the third question, if the rain is statistically inhomogeneous, it is now clear that different remote sensing instruments, even if pointed at the same target, will see different total drop size distributions simply because the beamwidths are not the same. (There are other more sophisticated reasons such as differences in illumination functions as well but that is not the thrust of this paper.) Yet, almost all algorithms assume that different instruments are viewing the same set of drops. Consequently, this disparity introduces errors into subsequent estimates of rain parameters. Moreover, these errors vary in a complex and unknown fashion depending upon the spatial variability of the drop size distributions themselves. Such errors, then, are not equivalent to white noise so that even abundant averaging will not reduce the biases nor variances associated with estimates of the rain parameters. It is no wonder, then, that there is often substantial scatter in such estimates. It is disappointing, though, that the associated variances are routinely ignored or are estimated using inappropriate statistics. At the very least, results from multi-sensor programs such as NASA's Tropical Rainfall Measuring Mission (TRMM) should be viewed with healthy skepticism. *One important lesson is that if observed drop size distributions are to be used in algorithms, it is important that the scales over which they are measured match the scales of the remote sensors to be used.* In particular, one must then question the use of Marshall–Palmer distributions, derived for a particular set of statistically inhomogeneous rain events, in the development of general retrieval algorithms that are subsequently applied to locations and to scales of observations that are inconsistent with the original Marshall–Palmer data.

It is also just as important to match aircraft observations to the beamwidths of remote sensing devices when such observations are to be used to develop and test the relevant algorithms. Of particular concern here are analyses of aircraft measurements in which it is assumed (often implicitly) that the observed distributions of raindrops are steady. However, as we have just seen, that can only happen when the rain in statistically homogeneous. Yet, aircraft distributions are usually collected over long traverses using instruments having small cross-sectional areas that act to maximize the effects of raindrop clustering and statistical mixing of many distributions. Hence, it is highly likely that most if not all aircraft drop size distributions are statistical mixtures in inhomogeneous rain. Therefore, they are most likely *not* steady but, rather, are only distributions of mean concentrations that change continually as more and more data are added. Hence, they have little if any intrinsic meaning independent of the measurement process and should only be interpreted statistically, not deterministically.

These same comments apply when using Doppler radar profilers to calculate drop size distributions by converting observed Doppler vertical velocity spectra into distributions of fall speeds after accounting for air velocities. (The most sophisticated method is to use the air velocity spectrum observed at a lower frequency to extract the fall speed distribution using deconvolution techniques. However, the air and precipitation observations are usually made over significantly different sampling volumes. The standard assumption, often likely invalid because the different beam dimensions imply different cutoffs of observed scales of air motion, is that the air velocity spectrum applies as well to the smaller sampling volume of the profiler observing the precipitation.) At the very least, the sampling volumes and times associated with the precipitation spectra are usually quite large so that the deduced drop

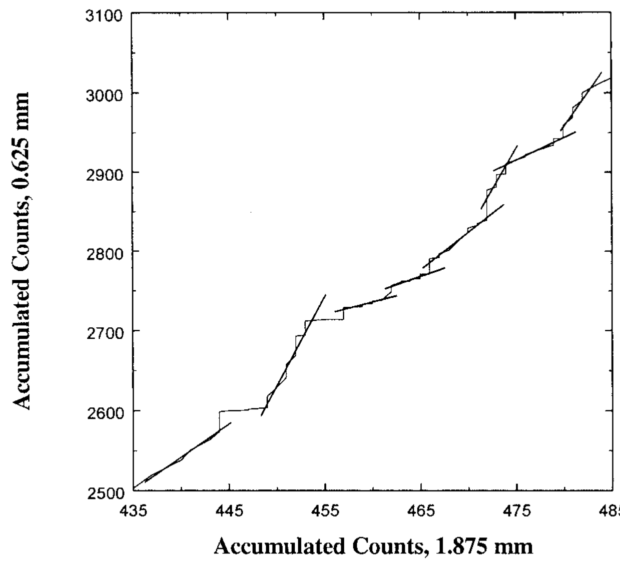


FIG. 7. Similar to Fig. 6 except that the counts correspond to video disdrometer "measurements" in a Monte Carlo simulation of statistically homogeneous rain as described in Jameson and Kostinski (1999). Drop size distribution patches are clearly identifiable.

size distributions are often most likely samples from statistically inhomogeneous rain. Such spectra should then be interpreted as statistical distributions of mean values that depend on the measurement processes.

With regard to the first question, in some sense the meaning remains the same, namely, what you measure *is* the drop size distribution. Yet, in a very real sense the definition is different now because the interpretation and meaning of what you measure depend upon the match between the measurement scales and the stochastic structure of the rain itself. Measurements of distributions in rain patches are likely the most elementary. Most other measurements involve statistical mixtures of these distributions. In the case of statistically homogeneous rain, these mixed distributions converge toward a steady spectrum that exists independently of the measurement process. In statistically inhomogeneous rain, however, such mixtures do not converge and simply represent a distribution of mean values that depend heavily on the measurement process, that is, the location and sampling volume. *It is no longer appropriate always to assume that such measurements represent a steady distribution having intrinsic, deterministic meaning like those in statistically homogeneous rain.*

With regard to the second question, one should not use numerical studies blindly. In the atmosphere, there are no "control volumes" in which the same drops in-

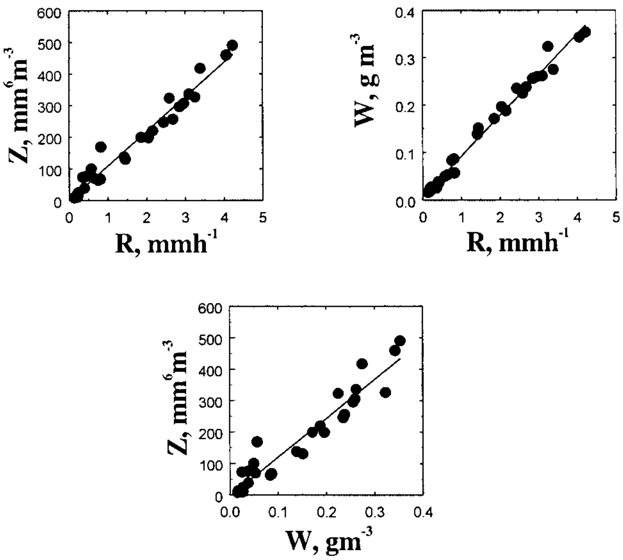


FIG. 8. Scatterplots between different average powers of the diameter (zero moments) observed during the first 29 samples in Fig. 1. Linearity between such moments may be useful as an indicator of statistical homogeneity.

teract sufficiently to reach an equilibrium between coalescence and drop breakup. More importantly, such virtual distributions do not include three-dimensional spatial variability, an important characteristic of real rain. *Consequently, while such studies no doubt serve the purpose of establishing the relevance of different physical mechanisms responsible for the formation of a drop size distribution, direct comparison to observed drop size spectra in real, three-dimensional space characterized by drop advection and drop clustering is likely to be misleading.*

Obviously an important characteristic of rain is whether or not it is statistically homogeneous. Yet, determining this characteristic is not trivial. One promising approach may be to use the observation that in statistically homogeneous rain, since the drop size distribution is steady, averages of powers of the diameter (so-called zero moments of the size distribution) are linearly related (see the discussion in Jameson and Kostinski 2001). Consequently, one expedient approach for identifying statistically homogeneous rain may be to use scatterplots along the lines illustrated in Fig. 8 corresponding to the first 29 samples in Fig. 1. Where such near linearity exists, there is at least a chance that the data may be statistically homogeneous.

In summary, then, with the exception of statistically homogeneous rain, we really are back to the result that in many cases, the drop size distribution is just what you measure. But it must be remembered that, unlike size spectra in statistically homogeneous rain, these are statistical distributions of mean concentrations that should be interpreted in a statistically appropriate manner, not as steady distributions having intrinsic, deterministic meanings independent of the measurement process. At a minimum it behooves those making observations to report intervals (time–distance) and sample volumes of the measurements so that they may subsequently be compared meaningfully to the distributions observed by others.

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