

How many acres of potatoes does a society need? Using food and historical claims to introduce energy.

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1 Introduction

One regular difficulty in a class on Energy and Social Policy is the wide variety of units used by different technologists (BTU's, Barrels of oil, Quads, kWh, etc). Food is a great source of energy, everyone eats, so I think some of this confusion can be resolved by starting and grounding the class with a discussion of food and food production.

When the United States entered World War One one of the problems they faced was logistics. How much food do you need to ship overseas to Europe to feed a million soldiers? That early work in nutrition led to the 3000 Calorie diet many people remember from secondary Health Education class. A reminder about “Calorie” (uppercase) vs “calorie” (lowercase) units you might remember: 1 *Calorie* = 1 *kilocalorie* = 1*kcal*, and a dietitian might build a 3000*kcal* diet for a 20 year old basketball player. One *calorie* = 0.001*kcal*, the amount of energy it takes to heat a gram of water by a degree Celsius. There are about 4.2 Joules in a single calorie, and a Joule occurs all over introductory physics. If you need to buy a new home furnace, the sales brochure might advertise that it is capable of delivering 100,000 BTU's of heat each hour. What's a BTU? Heat a pound of water by 1°F. Of course Heat Pumps are far more efficient than simply burning methane or propane, but they consume kilo-watt-hours (kWh) of electricity, not BTU's. What's a kWh? Run a 1000 Watt toaster for an hour and you'll have pulled one kWh off the grid, it will cost you about \$0.13 in Minnesota. If you decide to put solar panels in your backyard, they will probably col-

lect about 10% of the 3.5*kWh* the the sun delivers to each square meter of your lawn (in Minnesota) each day.

As the previous paragraph illustrates, there are a frustratingly large number of different units in an “Energy” class. At Winona State, this 3 credit class fulfills a “Science and Social Policy” general education requirement and is taken by students from across the university. Lots of college majors don't require a math class beyond algebra or introductory statistics and the population is largely math-averse. You could jokingly say that one of the main things students learn in the class is unit conversion, but it isn't far off. Nearly every field finds energy a useful representation, and every profession has their own set of units and terminology most well suited for quick calculation. Would a medical lab scientist talk about the fractional acre-foot of urine needed test kidney function? No, but someone in the central valley of California would certainly care about the acre-feet of water necessary to grow almonds! Does a gas station price their gasoline in dollars per kWh? Given the growing electrification of cars, they might soon.

Everyone eats, maybe not 3000*kcal*s per day, but at least something every day. When I teach our energy class, (Kraushaar et al., 2022; Muller, 2010), I spend a few weeks talking about food energy before all other types - an overview of that introduction is given in A. While food production is not central to climate change and wars over oil, food is essential in a way that diesel and gasoline are not. Vehicle fuel makes modern life possible, but we could live, unpleasantly, without it. We can't live without fats and protein.

1.1 Where does food energy come from?

One feature of the aught’s “homesteading” culture¹ is the idea that a person should probably be able to move to the country, eat a lot of peaches, and grow all their own food. Learning that farming labor is *skilled* labor can be brutal and disheartening. Eating 3000*kcal*s each day means planting, weeding, harvesting, and storing more than a million *kcal*s each year. (Haspel, 2015) Where will those Calories come from? Is your backyard enough to homestead in the suburbs?(Madigan, 2009)

At some point between 1920 and 1950, US chemical manufacturers realized that in the post-war period, they could repurpose processes developed for manufacturing munitions and chemical warfare agents, to produce chemicals that would kill insects and increase the nitrogen levels in the soil. As figures 1 and 2 show, the epoch of “Better Living Through Chemistry” produced a dramatic increase in per-acre yields across all commodity food crops, particularly corn and potatoes.

However, if you’re discussing backyard Calorie production it isn’t reasonable to use modern yield estimates for planning. “Roundup Ready” Corn, Soybean, and Sugar Beet seeds are not readily available to the public, nobody wants to put on a respirator to apply Atrazine ten feet from the back door, and the edge effects from deer and insects are much smaller on a 600 acre field than they are in an community garden allotment. In 1917 the USDA published a pamphlet (Cooper & Spillman, 1917) giving detailed Calorie estimates a farmer might expect from a given acre of a crop. A table from this pamphlet is shown in Figure 3. The pamphlet data from pre-war, pre-chemical agriculture, and the yields cited were produced with horses, manure, lime, and large families full of children. If you want to be self sufficient, these yield numbers are probably a good upper bound on what’s realistically possible by a dedicated Luddite.

So, another question using this data. If you want

¹See for example, the Discover television show, “Alaska the Last Frontier,” any issue of “Mother Earth News,” or Backyard Chicken feeds on Instagram.

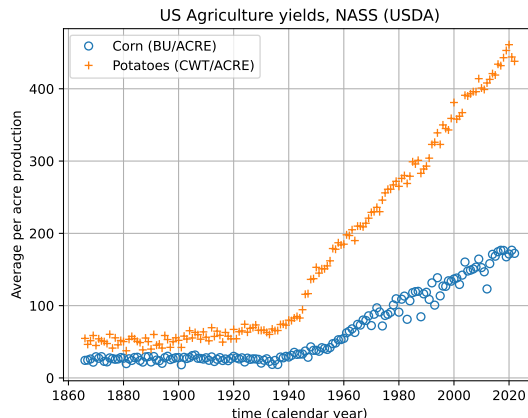


Figure 1: USDA per acre Corn and Potato production figures, plotted over time. Data is given in harvest units, 56*lbs* bushels per acre for field corn and hundred-weight (CWT) for potatoes. By mass, corn is about 4.5 times more calorie dense than potato which results in a nearly equal *kcal/acre* values for both crops in figure 2. Details on the data source and conversions are given in B.

to feed your family of four people potatoes, how much land will you need to cultivate? Here’s an estimate: a family of 4 requires 3000*kcal/person* each day.² If we over-estimate and produce food for the entire year, the family will need about 4.4 million *kcal*s.

$$4 \text{ people} \cdot \frac{3000 \text{ kcal}}{\text{person} \cdot \text{day}} \cdot \frac{365 \text{ days}}{\text{year}} \approx 4.4 \text{ M kcal.} \quad (1)$$

From figure 3 we can estimate 1.9 million *kcal*s per acre of potato production.

$$\frac{4.4 \text{ M kcal}}{\text{family}} \cdot \frac{1 \text{ acre}}{1.9 \text{ M kcal}} \approx 2.3 \text{ acre} \quad (2)$$

What does the answer of 2.3 acres mean? The university’s 91*m* × 49*m* football field has an area of

²Is 3000 $\frac{\text{kcal}}{\text{person} \cdot \text{day}}$ accurate for a family? For soldiers or active athletes it is, but 2000*kcal* is the USDA reference for an “average adult,” e.g. the author, in his 40’s, and 1000 – 1200*kcal* for a senior age (> 60) female. However, weeding the garden all day is physically taxing, mice will probably eat some of the potatoes, and 3000 is a nice round number, so that’s what I’m using.

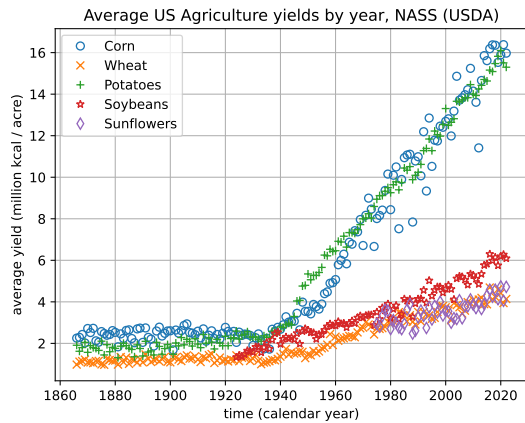


Figure 2: USDA per acre crop production figures, plotted over time. Production data is scaled by estimated dietary kcal content to show that, over all crops, there has been a dramatic increase in kcal production since about 1940. Details of the data source and conversions are given in B. The idea for this plot came from an online blog. (Biegert, 2017) It would be interesting to know if there are patterns of scaling among vegetable families (grains, legumes, tubers, etc) in the same way that there are family classifications for the minimal energy required for transport. (Tucker, 1975)

about 1.1 acres, so you could say that a football field planted in potatoes will probably feed a family through the winter. (Deppe, 2020) Can a person enjoy the benefits of urban living and grow all their own food? The population density of New Jersey is $1,263 \text{ people/mile}^2 \approx 1.97 \text{ people/acre}$ and our 4 person family needs 2.3 acres for their potatoes. Unless the social model is one of a country Dacha or an endless suburb with no duplexes or apartment buildings, urban living and food self-sufficiency seem mutually exclusive.

More emotionally charged conversations can be had about converting the United States to all organic agriculture, which, for corn, typically has a yield penalty of about $20-40 \text{ bu/acre}$ when compared to conventional production. The 1917 data isn't di-

4 FARMERS' BULLETIN 877.

TABLE I.—A comparison of the food produced annually by an acre of land when utilized in the production of various food crops and live-stock products.

Food products.	Yield per acre.		Calories per pound.	Pounds protein per acre (digestible).	Calories per acre.
	Bushels.	Pounds.			
Food crops:					
Corn.....	35	1,960	1,594	147.0	3,124,240
Sweet potatoes.....	110	^a 5,940	480	53.5	2,851,200
Irish potatoes.....	100	6,000	318	66.0	1,908,000
Rye.....	20	1,200	1,506	118.8	1,807,200
Wheat.....	20	1,200	1,490	110.4	1,788,000
Rice, unpolished.....	40	1,154	1,460	55.4	1,684,840
Rice, polished.....	16	1,086	1,456	50.0	1,581,216
Soy beans.....	16	960	1,598	294.7	1,534,000
Peanuts.....	34	524	2,416	126.2	1,265,018
Oats.....	35	^b 784	1,600	89.4	1,254,400
Beans.....	14	840	1,337	157.9	1,123,080
Cowpeas.....	10	600	1,421	116.4	852,600
Buckwheat.....	24	^c 600	1,252	34.5	751,800
Dairy products:					
Milk.....	2,190		325	72.3	711,750
Cheese.....	219		1,950	56.7	427,050
Butterfat.....	98.55		3,605	1.0	355,273
	Live (pounds).	Dressed (pounds).			
Meat:					
Pork.....	350	273	2,465	22.7	672,945
Mutton.....	205	113	1,215	14.7	137,295
Beef.....	216	125	1,040	18.5	130,000
Poultry: ^d					
Meat.....	103	66	1,045	12.7	68,970
	Dozen.	Pounds.			
Eggs.....	73.8	110.7	720	14.8	79,704
Total.....				27.5	148,674
	Live (pounds).	Dressed (pounds).			
For poultry meat alone.....	267	171	1,045	33.0	178,695
	Dozen.	Pounds.			
For eggs alone.....	122.4	183.6	720	24.6	132,192

Figure 3: A table from a USDA booklet giving 1917 yields for various farm products. The amounts listed were almost certainly produced via only animal and human power with only manure and lime available as chemical soil amendments. Accordingly, they are probably a reasonable upper bound on what's possible in a modern "back to the land" backyard garden.

rectly applicable, but it relates. At 180 bu/acre conventional corn requires $\approx 24 \text{ million acres}$ (half of Wisconsin, or all of Indiana) to feed the US population (350 million people) corn for a year. The remainder of the corn belt can be devoted to animal feed, ethanol, and export. If the corn belt was devoted to producing organic corn at lower yield, (Delate et al., 1998) we probably wouldn't starve, but cheap meat and ethanol vehicle fuel would likely disappear.

2 Example: How big could Tenochtitlan have been?

The questions described thus far have largely been centered within a physics context. The paper closes with two more examples that leverage this food energy picture to make historical claims. The first example relates to the pre-Colombian capital of the Aztec Empire, Tenochtitlan, now known as Mexico City. Tenochtitlan was built on and around a endorheic lake, Texcoco. Crops were grown in shallow parts of the lake via chinampas, (Borunda & Rodriguez, 2022) floating patches of decaying vegetation and soil. Given the proximity to water and decaying vegetation, these fields were very fertile (Coe, 1964; Ebel, 2019) and some continue to be used in the present day.³

Estimates of Tenochtitlan's population in 1500CE vary widely, from 40,000 (Evans, 2013, p549) to more than 400,000 inhabitants (Britannica, 2022), comparable in size to Paris at that time. These estimates come from oral and written records and estimates of archaeological building density and land area. While cannibalism was part of Aztec religious ritual and practice, (Ortiz de Montellano, 1978), the staple Calorie sources for the Aztecs were corn and beans.

Few if any Native American cultures made use of draft animals for food or power before the Colombian Exchange. This means that the food that fed Tenochtitlan must have been brought to the city center by foot or canoe. How much land must have been devoted to chinampas to feed the population, or conversely, how many people could be supported by the land within walking or paddling distance from the city center?

A 1964 paper in Scientific American (Coe, 1964) gives a general outline of the chinampas in the area of Tenochtitlan in 1500CE. This map, shown in figure 9 seems to be the basis for the similar figure in Wikipedia. (File:Lake Texcoco c 1519.png, 2016) Descriptions of chinampas agriculture indicate that as many as 7 successive crops could be grown and har-

vested from the same plot of soil each year, two of which could be maize (corn). This is truly amazing productivity, given that in the midwest United States corn is normally grown, at most, every other year because of its extreme nutrient demands on the soil.

There are many ways to approach this estimation problem. We could assume a Tenochtitlan population of 100,000 people has a $3000kcal/day$ diet that comes completely from corn. Assuming that corn's density and nutritional content haven't changed in the 4 centuries preceding the 1917 data in figure 3, we could assume $1lbs$ of corn contains $\approx 1594kcal$ of food energy. Looking at the map with ImageJ, (Schneider et al., 2012) it seems like the recorded area devoted to chinampas might be about 16,000 *acres*. With these assumptions, we could equate the corn energy production from chinampas with the population's yearly food need. Note, in this version of the story, the corn productivity, $P \frac{bu}{acre}$ is treated as an unknown variable.

$$\begin{aligned} \text{Food production} &= 16,000 \text{ acres} \cdot \frac{2 \text{ corn crops}}{\text{year}} \cdot \\ &P \frac{bu \text{ of corn}}{acre}. \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Population requires} &= 100,000 \text{ people} \cdot \frac{3000kcal}{\text{person} \cdot \text{day}} \\ &\cdot \frac{365 \text{ days}}{\text{year}} \cdot \frac{1 \text{ lbs corn}}{1594kcal} \cdot \frac{1 \text{ bu}}{56 \text{ lbs}} \end{aligned} \quad (4)$$

$$P \approx 38 \frac{bu}{acre}. \quad (5)$$

This crop productivity is in remarkable agreement with the 1917 USDA yields, $35bu/acre$, which seems to validate the assumed 100,000 person population of Tenochtitlan. Some references (Coe, 1964) describe an extensive tribute system that Aztec government required of its subjects, which certainly would have been necessary to support populations on the upper end of historical estimates. (Britannica, 2022)

³Chinampas are still visible in satellite imagery. See for example *latitude* = 19.268, *longitude* = -99.087.

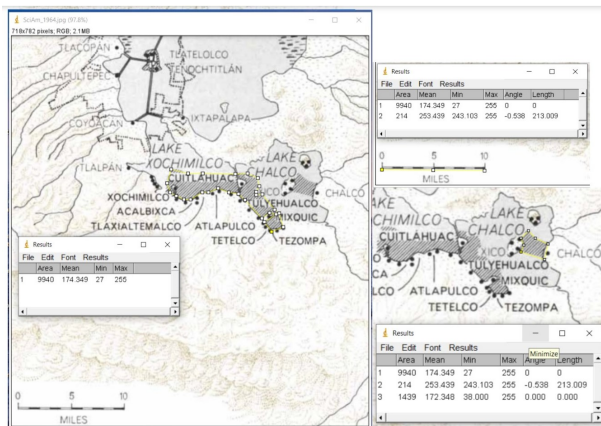


Figure 4: I used ImageJ, a free software program developed by the National Institutes of Health for photo analysis (Schneider et al., 2012) to measure a calibration scale in a map and the area of two polygons that I drew on the map. While there were certainly other regions devoted to chimanpas agriculture, after unit conversions, the portion visible near the Aztec capital seems to be about 25.1 miles^2 or $16,000 \text{ acres}$. The image being analyzed is available online. (Coe, 1964)

3 Example: Was the Irish Potato Famine a Natural Disaster?

In contrast to native cultures of the Americas, Ireland's population boomed with the Colombian Exchange and the introduction of the potato. (Fagan, 2001; Salaman & Hawkes, 1985) Figure 5 shows that from about 1700 onward there was a dramatic growth in the island's population. There's never just one reason for historical events, but unlike grains, potatoes thrived in Ireland's cool damp climate. Potatoes, kale, and milk form a nutritionally complete diet that greatly reduced hunger-related mortality among the poor working-class in Ireland. If you look closely at the data in figure 5 you might believe that there were *two* weather and potato related famines, the most obvious 1845-49 and the second, with much smaller effect on population in 1740-1. Both famines were precipitated by poor weather, but an important difference is that in 1740, Ireland was a sovereign state but by 1845 the island was effectively an economic colony of the British Empire. (Fagan, 2001)

As the story goes, the two main commodity crops in Ireland were potatoes (for humans), and oats, which as horse feed, were something like gasoline in today's economy. A sovereign government can halt the export of food to feed English horses, which is what happened in 1741 (and 1782). The grain was diverted back as relief to starving people in Ireland, reducing the famine's mortality. However, by 1845 most of Irish farmland was economically controlled by foreign (English) markets, and grain traders typically refused to divert oats (horse feed) as famine relief for the sake of their investment income.

This inflammatory claim, which is certainly a simplified version of history, serves as a useful evaluation example for students. Specifically, in years that the potato crop failed because of weather or late blight, could the amount of oats produced (and exported) have fed the Irish population? More broadly, was the Great Famine due to weather and disease, natural causes "we can't do anything about," or was the depth of the tragedy a result of political choices?

Some estimates follow: Ireland's population in

1845 was about 8.5 million people. The island has an area of about $84,400\text{km}^2$ ⁴ and you might estimate that 64% of the land ($54,000\text{km}^2$) is arable for agriculture. (Ask about Ireland, 2023) It seems reasonable to use the 1917 productivity, figure 3, to make calculations for Ireland in 1845. Reminder, in 1917, potatoes produced $1.908 \times 10^6 \text{kcal/acre}$ and oats $1.254 \times 10^6 \text{kcal/acre}$. With students, evaluation of the claim could be approached as a series of questions:

How much food does the island need?

$$\begin{aligned} \text{food needed per year} &= 8.5 \times 10^6 \text{ people} \cdot \frac{3000}{\text{person} \cdot \text{day}} \cdot \frac{365 \text{ days}}{\text{year}} \\ &\approx 9.3 \times 10^{12} \text{kcal/s}. \end{aligned} \quad (6)$$

How much land area, sown in potatoes, would produce this food?

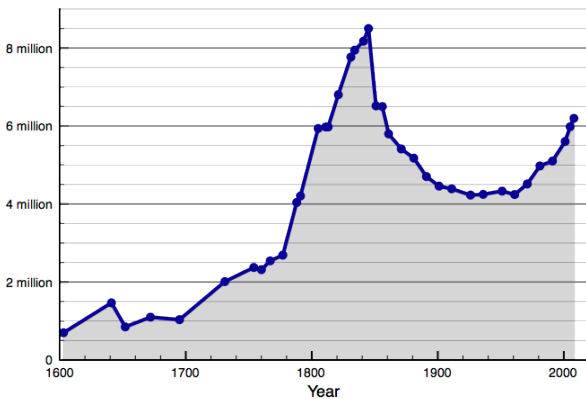
$$\begin{aligned} 9.3 \times 10^{12} \text{kcal/s} / \left(1.908 \times 10^6 \frac{\text{kcal}}{\text{acre}} \right) &= 4.87 \times 10^6 \text{acres}, \\ &\approx 19,700 \text{km}^2. \end{aligned} \quad (7)$$

How much land area, sown in oats, would produce this food?

$$\begin{aligned} 9.3 \times 10^{12} \text{kcal/s} / \left(1.254 \times 10^6 \frac{\text{kcal}}{\text{acre}} \right) &= 7.41 \times 10^6 \text{acres}, \\ &\approx 30,000 \text{km}^2. \end{aligned} \quad (8)$$

Summed, $49,700\text{km}^2$, these two areas devoted to oats and potatoes are roughly equivalent to the amount of arable land estimated above for Ireland, $54,000\text{km}^2$. (Ask about Ireland, 2023) What do the numbers mean? Did there have to be a famine? If all of the potato crop failed because of late blight, there would likely have been enough oats to feed the population a 2000kcal ration of oats with leftover to spare. Like the Holodomor or the Great Leap Forward, the numbers suggest that large-scale suffering wasn't a natural disaster, but rather a human disaster resulting from poor government policy insensitive to the value of human life.

⁴Currently, Northern Ireland and the Republic of Ireland are separate countries. Together, their land area is about $84,400\text{km}^2$.



sions with Diane Dahle-Koch, John Deming, Carl Ferkinhoff, Larry Moore, and Sarah Taber.

Figure 5: The population of Ireland over time, file from Wikipedia. (File:Population of the island of Ireland since 1600.png, 2010) The humble potato, kale, and milk were part of an amazing population boom. Note that there were two weather-related “potato” famines in Ireland, in about 1740 and 1850. Government policy response to the famines could explain the drastic difference in subsequent population following each of the two famines. The population of Ireland finally re-reached its 1851 peak in 2021. (Carroll, 2021)

4 Conclusion

A class about Energy and Social Policy and the author hasn’t mentioned climate change, coal, or solar panels even once! What is he thinking?

How many tons of carbon does your car release in a year? How many shiploads of iron oxide will we have to dump into the ocean for phytoplankton to eat up the equivalent about of carbon? Every question in a class like this is, to at least some extent, informed by numerical calculation and it is pretty arrogant to assume that “those students” don’t need to (or can’t) do the math. If you’re going to have success talking about numerical calculations, you might as well start with examples that everyone can relate to, and everyone eats! Along the way you might find fascinating historical questions to investigate.

The work was influenced and improved by discus-

A Food Energy

To introduce Food Energy, I ask the students to work through a few questions:



Figure 6: A proto-college-student at Winona’s China King Buffet, dreaming about visiting the steam tables every day.

A.1 Converting food into body heat

Planning to save money, one college student decides to go to an all-you-can-eat buffet each day at 11am, eg figure 6. If he brings homework and stretches the meal out for a few hours he can get all 3000kcal s with only one bill. Food is fuel for the human body – could too much fuel make his body feel sick? If his body burned all this food at once, how much warmer would he get? Useful information: the student has a mass of 80kg and is made mostly of water. A Calorie heats 1kg of water 1°C .

Here’s a possible answer: equate food energy with calorimetric heating and assume human bodies have the same heat capacity as water, about $1\frac{\text{kcal}}{\text{kg}\cdot^\circ\text{C}}$. This allows us to calculate the body’s temperature increase.

$$3000\text{kcal} = 80\text{kg} \cdot 1\frac{\text{kcal}}{\text{kg}\cdot^\circ\text{C}} \cdot \Delta T, \quad (9)$$

$$\Delta T \approx +37.5^\circ\text{C}. \quad (10)$$

Students are normally quite surprised at this number. Although wildly unrealistic, $\Delta T \approx +6^\circ\text{C}$ is typically fatal, there is a related phenomena of diet-induced thermogenesis, (Caballero, 2003, pp. 5762-7.) known informally as “the meat sweats”. Some students connect this calculation to feeling quite hungry after a cold swim in the pool (a similar effect). On a larger scale, discussing what’s wrong with this estimate is useful. The main storage mechanism for storing food energy is fat tissue, which the calculation completely ignores. Infants are generally born with little fat, and an infant sleeping through the night often coincides with the baby developing enough fat tissue to store sufficient kcals to make it through a night without waking up ravenously hungry. A related follow-up is that if a person is stranded in the wilderness, they should immediately start walking downstream (ie, towards civilization) as they likely won’t be able to harvest an amount of kcals equivalent to what they already have stored on their hips and abdomen. (USDA ARS, 2019) The contrast of bear hibernation, (North American Bear Center, 2023), and songbirds constantly eating through the winter are related connections to investigate.

A.2 Biophysical Power

A more realistic question to follow up with relates to the average *power* given off by a person over a day. Again, assuming 3000kcal is burned over 24hours , with useful information: $1\text{kcal} \approx 4200\text{J}$ and $1\text{J/s} = 1\text{W}$.

$$\frac{3000\text{kcal}}{24\text{hours}} \cdot \frac{4200\text{J}}{1\text{kcal}} \cdot \frac{1\text{hour}}{3600\text{sec}} \approx 145\text{W}. \quad (11)$$

Most students still remember 75Watt lightbulbs, but given the spread of LED lighting, “A person’s body heat is two 75W light bulbs” will probably only make sense for a few more years. Desert or cold-weather camping, alone versus with friends, and survival swimming are also examples for students to make sense of this answer. If you can take advantage of other people’s waste body heat, you’ll sleep more pleasantly and survive longer in cold water.

Another application to discuss is that of “brown fat,” a sort of biological space heater that hu-

mans and other mammals develop in response to cold weather. This tissue's mitochondria can burn lipids and carbohydrates in a useless proton pumping scheme, which produces metabolic heat. (Canon & Nedergaard, 2004; Cohen & Spiegelman, 2015; Himms-Hagen, 1984; Shamsi et al., 2021) Most common in rodents and infants, this mechanism can be stimulated by extended exposure to cold temperatures – the original work was done on lumberjacks in Finland. (Huttunen et al., 1981) The idea of a biological space heater that takes a month to turn on and a month to turn off matches the lived experience of college students in Minnesota, who wear down jackets in 4°C weather in November, and beachwear in the same 4°C weather in March. Additionally, transplants to northern climates often take a few years to “get used to” the colder weather up north. It seems just as easy to say that transplants’ bodies take a few years to develop the brown fat cells which allow them to be comfortable in cold weather.

One other distinction to emphasize is the difference between power and energy. A graph of a human body’s “kcal content” over the course of a day can be a useful illustration. When sedentary, this graph probably has the slope of $-150\text{W} \approx -125 \frac{\text{kcal}}{\text{hour}}$. If the 3000kcal meal at the buffet takes an hour, this period corresponds to an energy-time slope of $+3000 \frac{\text{kcal}}{\text{hour}} \approx +3500\text{W}$.

In medicine, these slopes known as “Metabolic Equivalent of Task” (METs), a common measure in cardiology and exercise physiology. METs is power normalized by mass, $1\text{MET} = 1 \frac{\text{kcal}}{\text{kg} \cdot \text{hour}}$, and METs levels are available for many different physical activities. (Jetté & Blümchen, 1990) For example, doing the dishes is 2.1METs , folkdancing is 4.8METs , and the fun part of human reproduction reportedly ranks at 5.8METs . (Frappier et al., 2013)

A.3 Burning off food energy

Imagine that after eating a 600kcal bacon-maple long-john (donut), you decide to go for a hike to “work off” the Calories. Winona State is in a river valley bounded by 200m tall bluffs. How high up the bluff would you have to hike to burn off the donut? Useful information: human muscle is about $1/3$ ef-

ficient, and on Earth’s surface, gravitational energy has a slope of about $10 \frac{\text{Joules}}{\text{kg} \cdot \text{m}}$.

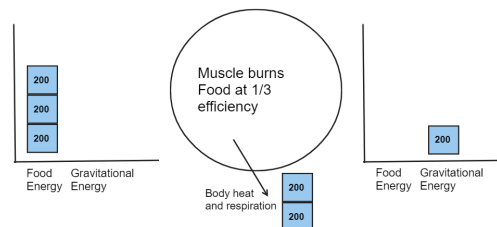


Figure 7: An Energy Bar Chart to illustrate the $1/3$ efficient student hiking up a bluff to burn off the morning’s donut. The initial state (left) is the hiker at the bottom of the hill, with donut in stomach. The final state (right) is the hiker at the top of the bluff with $2/3$ of the energy removed to the atmosphere by sweat and exhalation of warm air. $1/3$ of the donut’s energy is stored in elevation. The system for this diagram includes the earth, the hiker, and the donut. The system does not include the atmosphere around the hiker.

One way to approach this problem is by using Energy Bar Charts (Brewer, 2011) to illustrate how the energy held in food changes form as it is used. An approximation for this question is shown in figure 7. In this story, the “system” is taken to be the earth, food, and hiker. The hiker’s body is assumed to be $1/3$ efficient, which means one of the food energy blocks of energy is transformed into gravitational energy (elevation) at the end of the hike. The other 2 blocks of energy are transformed into heat and leave the hiker’s body, most likely by mechanisms of respiration and sweat evaporation. The purpose of a bar chart like this is to provide a pictorial and mathematical representation of the energy conservation equation given in 12.

$$\frac{1}{3} \cdot 600\text{kcal} \cdot \frac{4200\text{J}}{1\text{kcal}} = 80\text{kg} \cdot 10 \frac{\text{Joules}}{\text{kg} \cdot \text{m}} \cdot \text{height} \quad (12)$$

$$\text{height} \approx 1000\text{m}. \quad (13)$$

This estimate is again surprising to students. Five

trips up the bluff to burn off \$2 of saturated fat, sugar, and flour! A nice followup calculation is to imagine a car that can burn a $100kcal$ piece of toast in the engine: from rest, what speed will the toast propel it to? If (again) the engine converts $1/3$ of the energy into motion (kinetic energy), a $1300kg$ Honda Civic will reach a speed of about $15\frac{m}{s} \approx 33mph$!

The point of these energy calculations is not to give students an eating disorder. Rather, the numbers show food's amazing power. A single slice of toast will bring a car up to the residential speed limit! A day's food, $3000kcal$, will power you up an $5000m$ mountain peak! The body-work food allows us to do is astonishing, and increases in food production have made modern comforts, unimaginable 150 years ago, possible to the point of being taken for granted.

B Creating the historical kcal/acre figure from USDA data

The United States Department of Agriculture (USDA) provides historical crop information via the National Agricultural Statistics Service.[35] Data was downloaded in spreadsheet csv format and then combined and plotted via a Python Jupyter notebook.

Each crop has its own custom units, for example potatoes are sold by hundredweight (CWT) but sugar beets are measured by the ton. Every imaginable agricultural product seems to be tracked in the NASS site, for example Maple Syrup production is tracked and given in gallons of syrup per tap! Conversion factors used are summarized in Table 1. Calorie (kcal) density for each crop was taken from the USDA’s “Food Data Central”.[34] Within this database, foods are identified by an FDC ID.

An example calculation (implemented in the Jupyter notebook) follows for Corn. In 2022 the USDA reported an average production of 172.3 bushels of corn per acre of farmland.

$$172.3 \frac{bu}{acre} \cdot \frac{56lbs \text{ corn}}{bu} \cdot \frac{453.6 \text{ grams}}{lbs} \cdot \frac{365 \text{ kcal}}{100 \text{ grams}} = 15,9 \quad (14)$$

Obviously the result is only reasonable to two significant figures!

Raw data from the USDA NASS is plotted in figure 8. The scaling described in equation 14 produces figure 2 earlier in the paper.

Table 1: A summary of units and conversions used to create figure 2 from USDA NASS data. 1cwt is a hundred pounds of potatoes. A bushel, 1bu, is a volume unit of about 35liters and corresponds to about 60lbs of grain. Calorie content per 100 gram (mass) of food is taken from the USDA’s “Food Data Central” database. For context, typical serving sizes are included. It isn’t clear from any of these resources if lb is pound-force (lbf) or pound-mass (lbm) and so I am treating them as “grocery store units” where 1lbs \approx 453.6grams.

Crop	per acre unit	production unit	kcal per 100gram
Corn	bu/acre	1bu = 56lbs	365
Potatoes	cwt/acre	1CWT = 100lbs	77
Soybeans	bu/acre	1bu = 60lbs	446
Sunflowers	lbs/acre		584
Wheat	bu/acre	1bu = 60lbs	327

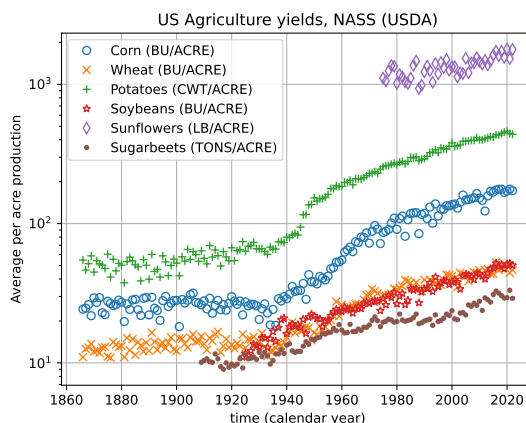


Figure 8: Average USDA per acre yields for a number of commodity crops over time. This “raw” data (in custom harvest units) was scaled to produce the data in figure 2 earlier in the paper.

C Estimating land area devoted to chinampas with ImageJ

ImageJ is a free software program developed by the National Institutes of Health for photo analysis.[30] I used the program to measure a calibration scale in a map and I also used the program to measure the area of two polygons that I drew on the map. The length and both areas are shown in figure 9.

Specifically, to find the area of the two large chinampas areas near Tenochtitlan, I took a screenshot from the 1964 paper,[9] and saved it in jpg format. Then, I opened the image in the Windows-Java edition of ImageJ.[30] The length of the 10 mile distance scale was 213 pixels. The long chinampas area at the south end of the lake was measured with a Polygon selection via the Measure tool to have an area of $9940 \text{ pixel}^2 \approx 21.9 \text{ miles}^2$. The smaller region near Chalco had an area of about $1439 \text{ pixel}^2 \approx 3.2 \text{ miles}^2$. While there were certainly other regions devoted to chinampas agriculture, the portion visible near the Aztec capital seems to be about 25.1 miles^2 or $16,000 \text{ acres}$.

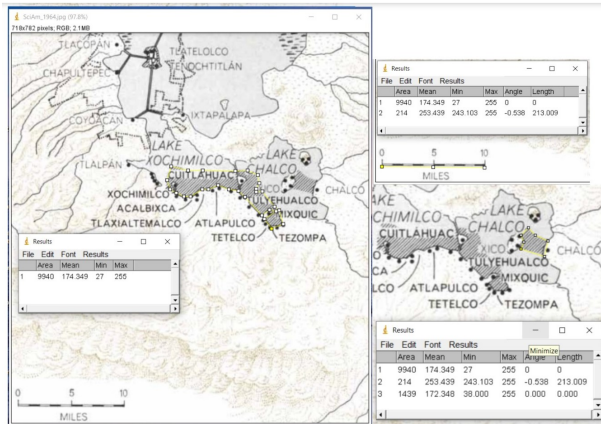


Figure 9: Three screen captures showing chinampa areas and the calibration stick used to convert pixel-squared area into miles^2 . The image being analyzed is available online. [9]

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