Abstract

One of the main difficulties in a class on Sources of Energy and Social Policy is the wide variety of units used by different technologists (BTU's, Barrels of oil, Quads, kWh, etc). As every student eats, I think some of this confusion can be resolved by starting and grounding the class with a discussion of food and food production. A general outline for this introduction is provided and two interesting historical cultural examples, Tenochtitlan and the Irish Potato Famine, are provided. Science and Social Policy classes are full of bespoke units and involve many different contexts. Starting the class with a discussion of food energy is a nice way for everyone to start with the same context. In addition, discussion of Food Energy can lead to interesting historical claims.

Supplemental calculations for How many acres of potatoes does a society need?

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1 Food Energy

To introduce Food Energy, I ask the students to work through a few questions:

1.1 Converting food into body heat

Planning to save money, one college student decides to go to an all-you-can-eat buffet each day at 11am, eg figure 1. If he brings homework and stretches the meal out for a few hours he can get all $3000kcals$ with only one bill. Food is fuel for the human body – could too much fuel make his body feel sick? If his body burned all this food at once, how much warmer would he get? Useful information: the student has a mass of $80kg$ and is made mostly of water. A Calorie heats $1kg$ of water $1°C$.

Here's a possible answer: equate food energy with calorimetric heating and assume human bodies have the same heat capacity as water, about $1\frac{kcal}{kg\cdot°C}$. This allows us to calculate the body's temperature increase.

$$3000kcals = 80kg \cdot 1\frac{kcal}{kg \cdot^\circ C} \cdot \Delta T, \quad (1)$$

$$\Delta T \approx +37.5°C. \quad (2)$$

Students are normally quite surprised at this number. Although wildly unrealistic, $\Delta T \approx +6°C$ is typically fatal, there is a related phenomena of diet-induced thermogenesis, (Caballero, 2003, pp. 5762-7.) known

Figure 1: A proto-college-student at Winona's China King Buffet, dreaming about visiting the steam tables every day.

informally as "the meat sweats". Some students connect this calculation to feeling quite hungry after a cold swim in the pool (a similar effect). On a larger scale, discussing what's wrong with this estimate is useful. The main storage mechanism for storing food energy is fat tissue, which the calculation completely ignores. Infants are generally born with little fat, and an infant sleeping through the night often coincides with the baby developing enough fat tissue to store sufficient kcals to make it though a night without waking up ravenously hungry. A related follow-up is that if a person is stranded in the wilderness, they should immediately start walking downstream (ie, towards civilization) as they likely won't be able to harvest an amount of kcals equivalent to what they already have stored on their hips and abdomen. (USDA ARS, 2019) The contrast of bear hibernation, (North American Bear Center, 2023), and songbirds constantly eating through the winter are related connections to investigate.

1.2 Biophysical Power

A more realistic question to follow up with relates to the average *power* given off by a person over a day. Again, assuming $3000kcal$ is burned over $24hours$, with useful information: $1kcal \approx 4200J$ and $1J/s = 1W$.

$$\frac{3000kcal}{24hours} \cdot \frac{4200J}{1kcal} \cdot \frac{1hour}{3600sec} \approx 145W. \tag{3}$$

Most students still remember $75Watt$ lightbulbs, but given the spread of LED lighting, "A person's body heat is two $75W$ light bulbs" will probably only make sense for a few more years. Desert or cold-weather camping, alone versus with friends, and survival swimming are also examples for students to make sense of this answer. If you can take advantage of other people's waste body heat, you'll sleep more pleasantly and survive longer in cold water.

Another application to discuss is that of "brown fat," a sort of biological space heater that humans and other mammals develop in response to cold weather. This tissue's mitochondria can burn lipids and carbohydrates in a useless proton pumping scheme, which produces metabolic heat. (Cannon & Nedergaard, 2004; Cohen & Spiegelman, 2015; Himms-Hagen, 1984; Shamsi et al., 2021) Most common in rodents and infants, this mechanism can be stimulated by extended exposure to cold temperatures – the original work was done on lumberjacks in Finland. (Huttunen et al., 1981) The idea of a biological space heater that takes a month to turn on and a month to turn off matches the lived experience of college students in Minnesota, who wear down jackets in $4°C$ weather in November, and beachwear in the same $4°C$ weather in March. Additionally, transplants to northern climates often take a few years to "get used to" the colder weather up north. It seems just as easy to say that transplants' bodies take a few years to develop the brown fat cells which allow them to be comfortable in cold weather.

One other distinction to emphasize is the difference between power and energy. A graph of a human body's "kcal content" over the course of a day can be a useful illustration. When sedentary, this graph probably has the slope of $-150W \approx -125\frac{kcals}{hour}$. If the $3000kcal$ meal at the buffet takes an hour, this period corresponds to an energy-time slope of $+3000\frac{kcal}{hour} \approx +3500W$.

In medicine, these slopes known as "Metabolic Equivalent of Task" (METS), a common measure in cardiology and exercise physiology. METS is power normalized by mass, $1METS = 1\frac{kcal}{kg \cdot hour}$, and METS levels are available for many different physical activities. (Jetté & Blümchen, 1990) For example, doing the dishes is $2.1METS$, folkdancing is $4.8METS$, and the fun part of human reproduction reportedly ranks at $5.8METS$. (Frappier et al., 2013)

1.3 Burning off food energy

Imagine that after eating a $600kcal$ bacon-maple long-john (donut), you decide to go for a hike to "work off" the Calories. Winona State is in a river valley bounded by $200m$ tall bluffs. How high up the bluff would you have to hike to burn off the donut? Useful information: human muscle is about 1/3 efficient, and on Earth's surface, gravitational energy has a slope of about $10\ \frac{Joules}{kg \cdot m}$.

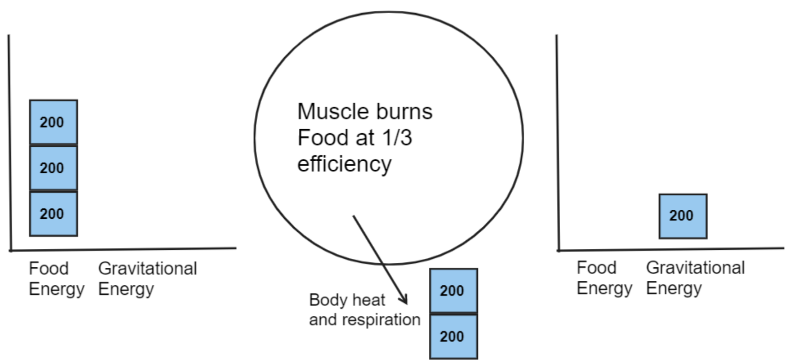

Figure 2: An Energy Bar Chart to illustrate the 1/3 efficient student hiking up a bluff to burn off the morning's donut. The initial state (left) is the hiker at the bottom of the hill, with donut in stomach. The final state (right) is the hiker at the top of the bluff with 2/3 of the energy removed to the atmosphere by sweat and exhalation of warm air. 1/3 of the donut's energy is stored in elevation. The system for this diagram includes the earth, the hiker, and the donut. The system does not include the atmosphere around the hiker.

One way to approach this problem is by using Energy Bar Charts (Brewe, 2011) to illustrate how the energy held in food changes form as it is used. An approximation for this question is shown in figure 2. In this story, the "system" is taken to be the earth, food, and hiker. The hiker's body is assumed to be 1/3 efficient, which means one of the food energy blocks of energy is transformed into gravitational energy (elevation) at the end of the hike. The other 2 blocks of energy are transformed into heat and leave the hiker's body, most likely by mechanisms of respiration and sweat evaporation. The purpose of a bar chart like this is to provide a pictorial and mathematical representation of the energy conservation equation given in 4.

$$\frac{1}{3} \cdot 600kcal \cdot \frac{4200J}{1kcal} = 80kg \cdot 10 \frac{Joules}{kg \cdot m} \cdot height, \tag{4}$$

$$height \approx 1000m. \tag{5}$$

This estimate is again surprising to students. Five trips up the bluff to burn off $2 of saturated fat, sugar, and flour! A nice followup calculation is to imagine a car that can burn a $100kcal$ piece of toast in the engine: from rest, what speed will the toast propel it to? If (again) the engine converts 1/3 of the energy into motion (kinetic energy), a $1300kg$ Honda Civic will reach a speed of about $15\frac{m}{s} \approx 33mph$!

The point of these energy calculations is not to give students an eating disorder. Rather, the numbers show food's amazing power. A single slice of toast will bring a car up to the residential speed limit! A day's food, $3000kcal$, will power you up an $5000m$ mountain peak! The body-work food allows us to do is astonishing, and increases in food production have made modern comforts, unimaginable 150 years ago, possible to the point of being taken for granted.

1.4 Creating the historical kcal/acre figure from USDA data

The United States Department of Agriculture (USDA) provides historical crop information via the National Agricultural Statistics Service. (USDA NASS, 2020) Data was downloaded in spreadsheet csv format and then combined and plotted via a Python Jupyter notebook.

Each crop has its own custom units, for example potatoes are sold by hundredweight (CWT) but sugar beets are measured by the ton. Every imaginable agricultural product seems to be tracked in the NASS site, for example Maple Syrup production is tracked and given in gallons of syrup per tap! Conversion factors used are summarized in Table 1. Calorie (kcal) density for each crop was taken from the USDA's Food Data Central.(USDA ARS, 2023) Within this database, foods are identified by an FDC ID.

An example calculation (implemented in a Jupyter notebook) follows for Corn. In 2022 the USDA reported an average production of 172.3 bushels of corn per acre of farmland.

$$172.3 \frac{bu}{acre} \cdot \frac{56lbs\ corn}{bu} \cdot \frac{453.6\ grams}{lbs} \cdot \frac{365\ kcal}{100\ grams} = 15,974,657 \frac{kcal}{acre}. \tag{6}$$

Obviously the result is only reasonable to two significant figures!

Raw data from the USDA NASS is plotted in figure 6. The scaling described in equation 6 produces figure 4 earlier in the paper.

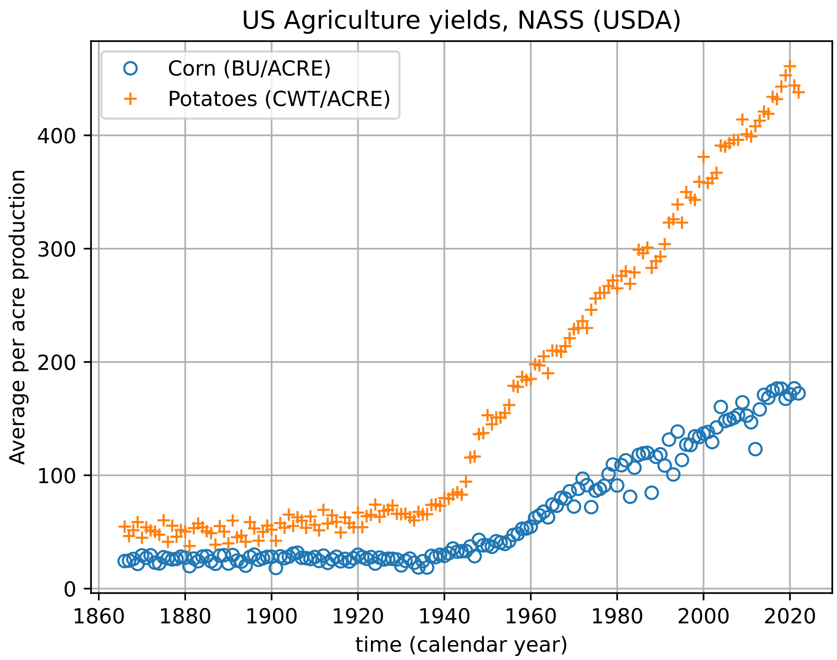

Figure 3: USDA per acre Corn and Potato production figures, plotted over time. Data is given in harvest units, $56lbs$ bushels per acre for field corn and hundred-weight (CWT) for potatoes. By mass, corn is about 4.5 times more calorie dense than potato which results in a nearly equal $kcal/acre$ values for both crops in figure 4. Details on the data source and conversions are given in 1.4.

1.5 Estimating land area devoted to chinampas with ImageJ

ImageJ is a free software program developed by the National Institutes of Health for photo analysis. (Schneider et al., 2012) I used the program to measure a calibration scale in a map and the area of two polygons that I drew on the map. Both areas and the calibration length are shown in figure 7.

Specifically, to find the area of the two large chinampas areas near Tenochtitlan, I took a screenshot from the 1964 paper (Coe, 1964) and saved it in jpg format. Then, I opened the image in the Windows-Java edition of ImageJ. The length of the 10 mile distance scale was 213 pixels. The long chinampas area at the south end of the lake was

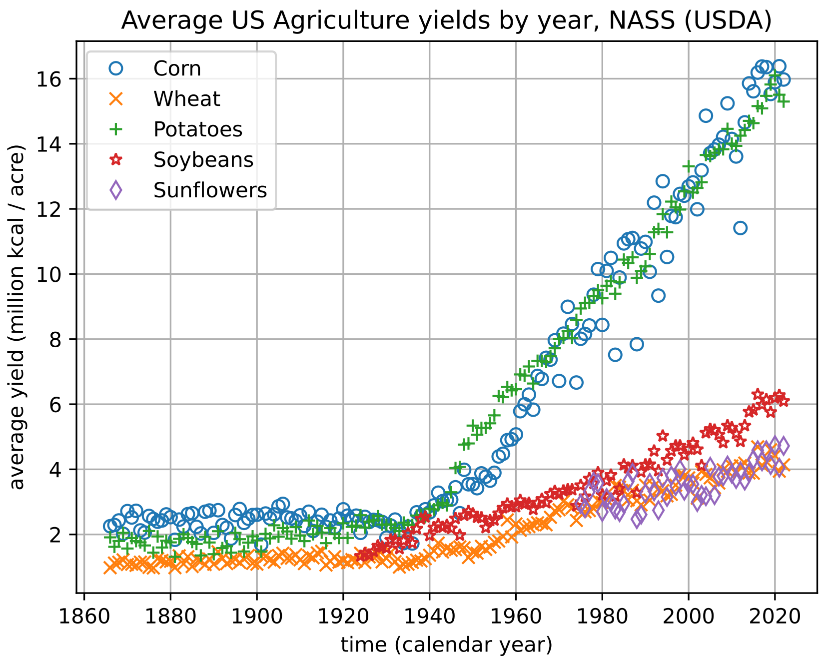

Figure 4: USDA per acre crop production figures, plotted over time. Production data is scaled by estimated dietary kcal content to show that, over all crops, there has been a dramatic increase in kcal production since about 1940. Details of the data source and conversions are given in 1.4. The idea for this plot came from an online blog. (Biegert, 2017) It would be interesting to know if there are patterns of scaling among vegetable families (grains, legumes, tubers, etc) in the same way that there are family classifications for the minimal energy required for transport. (Tucker, 1975)

measured with a Polygon selection via the Measure tool to have an area of 9940 $pixel^2 \approx 21.9miles^2$. The smaller region near Chalco had an area of about 1439 $pixel^2 \approx 3.2miles^2$. While there were certainly other regions devoted to chimanpas agriculture, the portion visible near the Aztec capital seems to be about $25.1miles^2$ or $16,000acres$.

Table 1: A summary of units and conversions used to create figure 4 from USDA NASS data. $1cwt$ is a hundred pounds of potatoes. A bushel, $1bu$, is a volume unit of about 35liters and corresponds to about 60lbs of grain. Calorie content per 100 gram (mass) of food is taken from the USDA's "Food Data Central" database. For context, typical serving sizes are included. It isn't clear from any of these resources if lb is pound-force (lbf) or pound-mass (lbm) and so I am treating them as "grocery store units" where $1lbs \approx 453.6grams$.

Crop	per acre unit	production unit	kcals per 100gram	typical portion	FDC ID
Corn	bu/acre	$1bu = 56lbs$	365	1 cup is 166g	170288
Potatoes	cwt/acre	$1CWT = 100lbs$	77	0.5 cup is 75g	170026
Soybeans	bu/acre	$1bu = 60lbs$	446	1 cup is 186g	174270
Sunflowers	lbs/acre		584	1 cup is 140g	170562
Wheat	bu/acre	$1bu = 60lbs$	327	1 cup is 192g	168890

4 FARMERS' BULLETIN 877.

TABLE I.—*A comparison of the food produced annually by an acre of land when utilized in the production of various food crops and live-stock products.*

Food products.	Yield per acre.		Calories per pound.	Pounds protein per acre (digestible).	Calories per acre.
	Bushels.	Pounds.			
Food crops:					
Corn	35	1,960	1,594	147.0	3,124,240
Sweet potatoes	110	[a] 5,940	480	53.5	2,851,200
Irish potatoes	100	6,000	318	66.0	1,908,000
Rye	20	1,200	1,506	118.8	1,807,200
Wheat	20	1,200	1,490	110.4	1,788,000
Rice, unpolished	40	1,154	1,460	55.4	1,684,840
Rice, polished		1,086	1,456	50.0	1,581,216
Soy beans	16	960	1,598	294.7	1,534,000
Peanuts	34	524	2,416	126.2	1,265,018
Oats	35	[b] 784	1,600	89.4	1,254,400
Beans	14	840	1,337	157.9	1,123,080
Cowpeas	10	600	1,421	116.4	852,600
Buckwheat	24	[c] 600	1,252	34.5	751,800
Dairy products:					
Milk		2,190	325	72.3	711,750
Cheese		219	1,950	56.7	427,050
Butterfat		98.55	3,605	1.0	355,273
	Live (pounds).	Dressed (pounds).			
Meat:					
Pork	350	273	2,465	22.7	672,945
Mutton	205	113	1,215	14.7	137,295
Beef	216	125	1,040	18.5	130,000
Poultry: [d]					
Meat	103	66	1,045	12.7	68,970
	Dozen.	Pounds.			
Eggs	73.8	110.7	720	14.8	79,704
Total				27.5	148,674
	Live (pounds).	Dressed (pounds).			
For poultry meat alone	267	171	1,045	33.0	178,695
	Dozen.	Pounds.			
For eggs alone	122.4	183.6	720	24.6	132,192

Figure 5: A table from a USDA booklet giving 1917 yields for various farm products. The amounts listed were almost certainly produced via only animal and human power with only manure and lime available as chemical soil amendments. Accordingly, they are probably a reasonable upper bound on what's possible in a modern "back to the land" backyard garden.

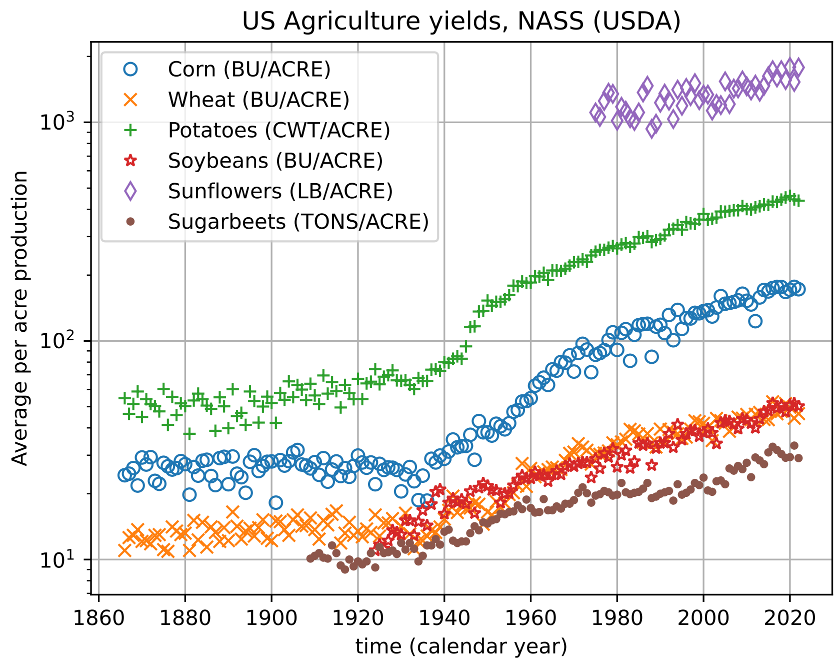

Figure 6: Average USDA per acre yields for a number of commodity crops over time. This "raw" data (in custom harvest units) was scaled to produce the data in figure 4 earlier in the paper.

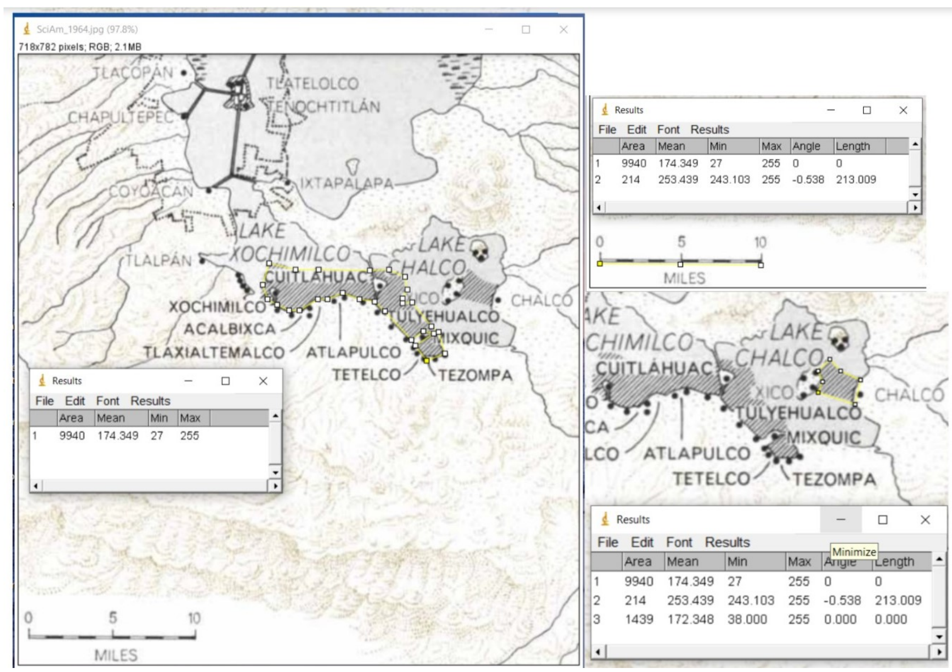

Figure 7: Three screen captures showing chinampa areas and the calibration stick used to convert pixel-squared area into $miles^2$. The image being analyzed is available online. (Coe, 1964)

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