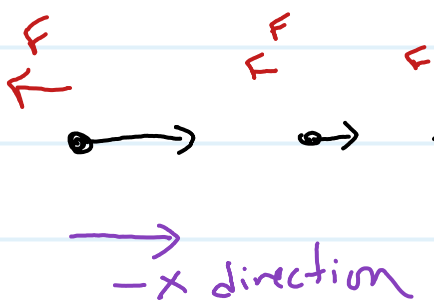
08-28 Excel Homework

1. Sometimes, the force of fluid "air" resistance can be described as $F = -b \cdot v \cdot \frac{v}{|v|} = -b \cdot v \cdot \hat{v}$, where v is velocity and F is force. This description is often correct for things that are small and slow-moving (ie, not cars or rockets, maybe butterflies)
   a. If you're working in meters, seconds, and Newtons, what are the units of b?
   b. If the velocity of a 10 gram acorn is initially -10m/s (downward, up is +y), the non-relativistic version of Newton's second law $\frac{dP}{dt} = \frac{d\,mv}{dt} = F_{ext} = -b \cdot v \cdot \hat{v}$ is reliable. In terms of initial velocity v0, initial position x0, b, and t, what equations will describe the position and velocity of the acorn?
   c. If the acorn slows from -10m/s to -6m/s in 2 seconds, what's the value of b?

   f. What average speed does an integral of the equation in b give for the first 2 seconds of the acorn's motion?



$$F = ma = m\frac{dv}{dt}$$

and $F = -bv\hat{v}$

So in this problem $m\frac{dv}{dt} = -bv$

or

$$\frac{dv}{dt} = -\frac{b}{m} \cdot v$$

Solve by integrating?

$$\int_1^2 \frac{dv}{v} = \int_1^2 -\frac{b}{m} \cdot dt$$

$$\ln\left(\frac{v_2}{v_1}\right) = -\frac{b}{m}(t_2 - t_1)$$

Call $t_1 = 0$ and then
$t_2 = t$
$V_1 = V_0$
$V_2 = v(t)$

$$V(t) = V_0 \exp\left(\frac{-b}{m} \cdot t\right)$$

based on this

units are $[t] = [m/b]$

b/c $e^{\#}$ must be number!
not 100cm vs 1m

"tau"

call $\tau = m/b$ with $v(t) = V_0 \exp\left[-t/\tau\right]$

$\boxed{a}$

So, b has units of kg/second

cool! same answer!

or $F = -b \cdot v \cdot \hat{v}$ $\quad [b] = [F/v]$ $\quad \hat{v}$ is unitless

$$[b] = \frac{\text{Newton} \cdot \text{sec}}{m} = \frac{kg \cdot m}{s^2} \cdot \frac{s}{m}$$

---

if $v(t) = V_0 \exp\left[-t/\tau\right]$

and $v = \frac{dx}{dt}$ then $dx = v\,dt$

or $\int_1^2 dx = \int_1^2 v\,dt$

$$x_2 - x_1 = -\tau \int_1^2 \frac{-dt}{\tau} \; V_0 \cdot \exp\left[-t/\tau\right]$$

$x_1 = 0$

$x_2 = x(t)$

$t_1 = 0$

$t_2 = t$

so

$$x(t) = -\tau \cdot V_0 \cdot \exp\left[-t/\tau\right] \Big|_0^t$$

$$x(t) = V_0 \cdot \tau \cdot \left(e^0 - e^{-t/\tau}\right)$$

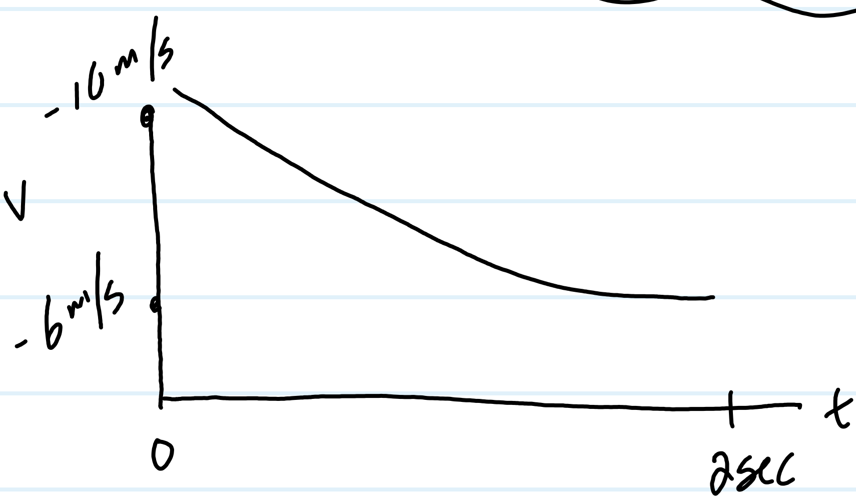
$\boxed{b}$

$$x(t) = V_0 \cdot \tau \left(1 - e^{-t/\tau}\right)$$

and from before

$$V(t) = V_0 \exp\left(-t/\tau\right)$$

$$\text{w/} \quad \tau = m/b$$



$V(t=0) = -10 \text{ m/s}$

$V(t=2s) = -6 \text{ m/s}$

use velocity eqn $\quad v(t=0) = V_0 = -10 m/s$

and $\quad v(t=2sec) = -6 m/s$

or $\quad v(2sec) = -10 m/s \cdot \exp(-2sec/\tau) = -6sec$

or $\quad \dfrac{-6 m/s}{-10 m/s} = \exp(-2sec/\tau)$

or $\quad \ln(0.6) = \ln(\exp(-2/\tau))$

or $\quad \tau = -2/\ln(0.6) \approx$ (-2) / ln(0.6) = 3.91523037794

So $\tau \approx 3.92$ seconds $\quad$ and $\tau = m/b$ so

$$b = m/\tau = \frac{0.01 kg}{3.92 sec}$$

$\boxed{C}$

$$b \approx 0.0025 kg/sec$$

f. what's avge velocity (in time)

$$\langle v \rangle = \frac{\int v \cdot dt}{\int dt} = \frac{1}{T}\int_0^T V_0 \exp[-t/\tau]\, dt$$

$$= \frac{V_0}{T}(-\tau)\int_0^T -\frac{dt}{\tau} \exp\left[\frac{-t}{\tau}\right]$$

$$= \frac{V_0 \cdot \tau}{T} \exp[-t/\tau]\Big|_T^0$$

$$\langle v \rangle = \frac{V_0 \tau}{T}\left(1 - \exp[-T/\tau]\right)$$

averaging from $t = \emptyset$ to $t = T$