# 1.

La $P = \{x_0, x_1, ..., x_n\}$ være en partisjon av intervallet $[1, 2]$ med $n$ like store delintervaller, der $x_0 = 1$ og $x_n = 2$, og la $f(x) = x^2 - 2x + 3$.

## 1a.

Finn et uttrykk for $\sum_{i=1}^{n} f(x_i)$. (Vink: $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$.)

$$\begin{aligned}
\sum_{i=1}^{n} f(x_i) &= \left(\left(1+\frac{1}{n}\right)^2 - 2\left(1+\frac{1}{n}\right) + 3\right) + ... + \left(\left(1+\frac{n}{n}\right)^2 - 2\left(1+\frac{n}{n}\right) + 3\right) \\
&= \left(\left(1+\frac{2}{n}+\frac{1}{n^2}\right) - 2 - \frac{2}{n} + 3\right) + \left(\left(1+\frac{4}{n}+\frac{4}{n^2}\right) - 2 - \frac{4}{n} + 3\right) + ... \\
&= \left(2+\frac{1}{n^2}\right) + \left(2+\frac{4}{n^2}\right) + ... + 3 \\
&= 2 \cdot n + \frac{n(n+1)(2n+1)}{6n^2} \\
&= \frac{12n^2}{6n} + \frac{(n+1)(2n+1)}{6n} \\
&= \frac{12n^2 + (n+1)(2n+1)}{6n} \\
&= \frac{12n^2 + 2n^2 + 3n + 1}{6n} \\
&= \underline{\underline{\frac{14n^2 + 3n + 1}{6n}}}
\end{aligned} \tag{1}$$

## 1b.

Bestem

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(x_i) \tag{2}$$

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(x_i) \\
&\qquad = \lim_{n\to\infty} \frac{1}{n} \frac{14n^2 + 3n + 1}{6n} \\
&\qquad = \lim_{n\to\infty} \frac{14n^2 + 3n + 1}{6n^2} \\
&\qquad = \lim_{n\to\infty} \frac{28n + 3}{12n} \\
&\qquad = \lim_{n\to\infty} \frac{28}{12} \\
&\qquad = \frac{28}{12} = \frac{14}{6} = \underline{\underline{\frac{7}{3}}}
\end{aligned} \tag{3}$$

## 1c.

Regn ut

$$\int_1^2 f(x)\,\mathrm{d}x \tag{4}$$

$$\int x^2 - 2x + 3\,\mathrm{d}x = \frac{1}{3}x^3 - x^2 + 3x + C \tag{5}$$

$$\begin{aligned}
\int_1^2 f(x)\,\mathrm{d}x &= \left[\frac{1}{3}x^3 - x^2 + 3x\right]_1^2 \\
&= \left(\frac{8}{3} - 4 + 6\right) - \left(\frac{1}{3} - 1 + 3\right) \\
&= \left(\frac{8}{3} + \cancel{2}\right) - \left(\frac{1}{3} + \cancel{2}\right) \\
&= \underline{\underline{\frac{7}{3}}}
\end{aligned} \tag{6}$$

# 2.

Regn ut

$$\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\sqrt{2+\frac{2i}{n}} \tag{7}$$

Velger $x_i = \frac{i}{n} \Rightarrow \Delta x_i = \frac{1}{n}$

$$\sqrt{2+\frac{2i}{n}} = \sqrt{2+2x_i} \tag{8}$$

$$\frac{2}{n}\sum_{i=1}^{n}\sqrt{2+\frac{2i}{n}} \Rightarrow f(x_i) = 2\sqrt{2+2x_i} \tag{9}$$

$$\begin{aligned}
&\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\sqrt{2+\frac{2i}{n}} \\
&= \int_0^1 2\sqrt{2+2x}\,\mathrm{d}x \\
&= 2\int_0^1 \sqrt{2+2x}\,\mathrm{d}x \\
&= 2\int_0^1 (2+2x)^{\frac{1}{2}}\,\mathrm{d}x \\
&= 2\left[\frac{1}{\frac{3}{2}\cdot 2}(2+2x)^{\frac{3}{2}}\right]_0^1 \\
&= 2\left(\frac{1}{3}\sqrt{4^3} - \frac{1}{3}\sqrt{2^3}\right) \\
&= 2\left(\frac{8-2\sqrt{2}}{3}\right) \\
&= \underline{\underline{\frac{16-4\sqrt{2}}{3}}}
\end{aligned} \tag{10}$$

# 3.

## 3a.

Finn alle løsningene av ligningen $\cos(2x) = \sin(x)$ for $x \in [0, \pi]$.

(Vink: $\cos(2x) = 1 - 2\sin^2(x)$.)

$$\cos(2x) = \sin(x)$$

$$\Updownarrow$$

$$1 - 2\sin^2(x) = \sin(x)$$

$$2\sin^2(x) + \sin(x) - 1 = 0$$

$$\sin(x) = \frac{-1 \pm \sqrt{1^2 - (4 \cdot 2 \cdot (-1))}}{2 \cdot 2}$$

$$\sin(x) = \frac{-1 \pm \sqrt{9}}{4} \tag{11}$$

$$\sin(x) = \frac{-1 \pm 3}{4} \qquad \text{me forkastar den negative løysinga}$$

$$\sin(x) = \frac{1}{2}$$

$$x = \arcsin\left(\frac{1}{2}\right)$$

$$\underline{\underline{x = \frac{\pi}{6}}}$$

## 3b.