# Luminosity Distance and Angular Diameter

1. **Luminosity Distance $D_L$**

2. **Angular Diameter**

# Luminosity Distance $D_L$

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- The observed flux $F$ from a supernova explosion can be used to infer its *luminosity distance*, once the comoving coordinate $\chi$ is known.
- In a static Euclidean space the flux–luminosity relation is simply

$$F = \frac{L}{4\pi\,\chi^2},$$

where $L$ is the absolute luminosity and $\chi$ is the distance to the source.

# Why Flux Dims by $(1 + z)^2$

- Cosmological effects (expansion, red-shift, curvature) modify this basic relation, leading to the definition of $D_L(z)$ in an FRW universe.

- In FRW space–time, the observed flux suffers two red-shift factors of $1 + z$.

- When the light reaches us at $t = t_0$, it has spread over the proper area of a sphere

$$A = 4\pi\, D_M^2,$$

where $D_M$ is the transverse comoving distance.

# Why Flux Dims by $(1+z)^2$

- If the source radiates luminosity $L$, the naive Euclidean flux would be

$$F_{\text{Eucl}} = \frac{L}{4\pi\, D_M^2}.$$

- Cosmological corrections
  The arrival rate of photons is slowed by a factor $1/(1+z)$.
  Each photon's energy is red-shifted by another $1/(1+z)$.

# Why Flux Dims by $(1+z)^2$

- Therefore the observed flux is

$$F_{\rm obs} = \frac{L}{4\pi\, D_M^2}\,\frac{1}{(1+z)^2}.$$

- The luminosity distance is defined so that $F_{\rm obs} = L/[4\pi\, D_L^2]$, implying

$$D_L = D_M\,(1+z)$$

which is the definition of *Luminosity Distance*

# Angular Diameter

# Angular-Diameter Distance $D_A$

- Idea: use "standard rulers" of known physical size $D$ to infer distance from their apparent angular size $\theta$.
- In Euclidean geometry the relation is simply

$$D_A = \frac{D}{\theta}.$$

- In an expanding FRW universe the object's physical size at the time of emission is smaller relative to today by the factor $a(t_{\mathrm{e}}) = 1/(1+z)$.

# Angular-Diameter Distance $D_A$

- Spatial metric on a constant-time slice:

$$dl^2 = a^2(t)\,\left[d\chi^2 + S_K^2(\chi)\,d\Omega^2\right].$$

- The transverse proper size of the ruler at emission is $D = a(t_{\mathrm{e}})\,S_K(\chi)\,\Delta\theta$, leading to

$$D_A(z) = \frac{S_K(\chi)}{1+z},$$

where $\chi = \chi(z)$ is the comoving radial distance.

# Angular-Diameter Distance $D_A$

- Proper transverse size at emission (radial coordinate $\chi$):

$$D = a(t_{\mathrm{e}})\, S_K(\chi)\, \Delta\theta = \frac{S_K(\chi)\, \Delta\theta}{1+z}.$$

- Angular-diameter distance defined by $D = D_A\, \Delta\theta$:

$$D_A(z) = \frac{S_K(\chi)}{1+z} = \frac{D_M}{1+z},$$

where $D_M = S_K(\chi)$ is the transverse comoving distance.

# Angular-Diameter Distance $D_A$ and the Etherington Relation

- Etherington distance–duality relation:

$$D_A = \frac{D_M}{1+z} = \frac{D_L}{(1+z)^2}, \qquad D_L = (1+z)^2 D_M.$$

- Interpretation:
  $D_A$: how far the object was *when it emitted the light*.
  $D_L$: how dim the object appears relative to its intrinsic luminosity.

# Thank You!!!!!!!!!!!!

Thank you so much for reading this slide:)))))