Modeling and Nonlinear Control for a Coaxial Helicopter

A. Dzul°, T. Hamel† and R. Lozano°

°Heudiasyc - UTC UMR 6599, Centre de Recherches de Royallieu
B.P. 20529, 60205 Compiègne Cedex, France
E-mail : dzul@hds.utc.fr, rlozano@hds.utc.fr

†CEMIF, Université d'Evry Val d'Essonne, 40, rue du Pelvoux
CE 1455 Courouronnes, 91020 Evry Cedex, France
E-mail : thamel@cemif.univ-evry.fr

***Abstract*— In this paper a simplified model and a nonlinear control algorithm for a coaxial helicopter are presented. The model of the helicopter dynamics is obtained in hover conditions. By neglecting the small body forces due to the mechanism used to obtain torque control, a dynamic controller, based on backstepping techniques, is developed. Some simulation results are presented to illustrate the performance of such a controller.**

***Keywords*— Unmanned Aerial Vehicle, Helicopter Dynamics, Lyapunov Methods, Recursive Control Algorithms.**

I. Introduction

In the last few years, the construction of small Unmanned Aerial Vehicles (UAV's) has been possible thanks to advances on electronic and computer technologies. We find among the UAV's some complex machines as the helicopters. Their complexity is due to the versatility and manoeuvrability to carry out all type of tasks (i.e. vertical, translational and hover flight, etc.). Several configuration types exist, like a single main rotor and a tail rotor configuration, a tandem rotor configuration, a side-by-side rotors configuration and a coaxial rotors configuration. Given that the most popular configuration (a single main rotor and a tail rotor) focuses the interest of control community [1], [3], [6], our main objective is to obtain a dynamic model of a coaxial helicopter in order to develop a nonlinear control allowing an autonomous flight.

A simplified model of the dynamics of the helicopter for maneuvers close to hover is proposed. This model is based on a consideration of the helicopter as a combination of the dynamics of the two rotors and the rigid body dynamics associated with the helicopter airframe. A new control algorithm based on non-inertial measurements is proposed and analysed

This paper is organized as follows. In section II, we describe the nonlinear dynamic model of the coaxial helicopter system. In section III, we develop a controller based on backstepping techniques. We present, in Section IV some simulation results of the proposed controller. Finally, concluding remarks are given in Section V.

II. Dynamic model for a coaxial helicopter

In this section a nonlinear dynamic model for a coaxial helicopter in hover conditions is proposed. A coaxial helicopter is a twin main rotor helicopter configuration that uses two contrarotating rotors of equal size, loading and with concentric shafts (see figure 1). Some vertical separation of the rotor disks is required to accommodate lateral flapping.

Pitch and roll control is achieved by the cyclic pitch of the swash plate. The height control is achieved by the collective pitch. The yaw control mechanism is more subtle. When a rotor turns, it has to overcome air resistance, so a reactive force acts on the rotor in the direction opposite to the rotation of the rotor. As long as all rotors produce the same torque, they produce the same reactive torque. This torque is mostly a function of rotation speed and rotor blade pitch. Since the sum of the two air resistances is zero, there is no yaw motion. If one of rotors changes its collective pitch, the induced torque will cause the helicopter to rotate in the direction of the induced torque. It is important to note that this operation has not effect on translation in x or y direction in a coaxial helicopter configuration. For the sake of simplicity we present here the dynamic model of a coaxial helicopter in hovering based on the Newton's equations of motion [8] and using the following hypothesis :

1. The main rotor blades are assumed to hinge directly from the hub. That is, the flapping hinge offset is assumed to be zero. The conning angle is assumed to be zero. As a consequence each rotor will always lie in a disk termed the *rotor disk*.
2. The above rotor blades are assumed to rotate in an anti-clockwise direction when viewed from above and the down rotor blades rotate in a clockwise direction, see figure 1.
3. It is assumed that the cyclic lateral tilts are measurable and controllable because the flapping angles will be used as control inputs. Along with the above rotor and down rotor thrusts form the control inputs of the helicopter.
4. The only air resistance modeled are simple drag torques opposing the rotation of the two rotors blades.
5. The operation of two or more rotors in close proximity will modify the flow field at each, and hence the performance of the rotor system will not be the same as for the isolated rotors. We will not consider this phe-

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nomenon to simplify the dynamic model.

In order to obtain the final dynamic equations, we have separated the aerodynamic forces in two parts. The first part is composed by the total translational forces applied to the helicopter and the second part is related to the sum of the rotational torques. More details on coaxial helicopter dynamics can be found in [5].

A. Translational forces.

Let $\xi = (x, y, z)$ denote the position vector of the centre of mass of the helicopter airframe relative to the right-hand inertial frame $\mathcal{I}$. The body-fixed frame is represented by $CG = (E_1, E_2, E_3)$ as showed in figure 1. Denote by T_A and T_D the thrusts generated by the above (A) and down (D) rotors respectively. Then, the thrust vectors are defined by

$$T_A = T_A^1 E_1 + T_A^2 E_2 - T_A^3 E_3 \quad (1)$$
$$T_D = T_D^1 E_1 + T_D^2 E_2 - T_D^3 E_3 \quad (2)$$

It is desirable to express the thrust components in terms of the cyclic tilt angles a and b which form the system inputs (hypothesis 2.3). Define β as the measure of the tilt of the rotor disk (see figure 2). It represents the angle between the axe E_3 and the actual thrust vector. By simple analysis we obtain that

$$\tan^2\beta = tan^2 a + tan^2 b \quad (3)$$
$$\cos\beta = \frac{\cos a \cdot \cos b}{\sqrt{1 - \sin^2 a \cdot \sin^2 b}} \quad (4)$$

Thus, the thrust components can be directly computed by the projection of T_A or T_D onto the axis of the body fixed frame by :

$$T_A = G(a,b) \cdot |T_A| \quad (5)$$
$$T_D = G(a,b) \cdot |T_D| \quad (6)$$

where

$$G(a,b) = \frac{1}{\sqrt{1 - \sin^2 a \cdot \sin^2 b}} \begin{pmatrix} -\sin a \cdot \cos b \\ \sin b \cdot \cos a \\ -\cos a \cdot \cos b \end{pmatrix} \quad (7)$$

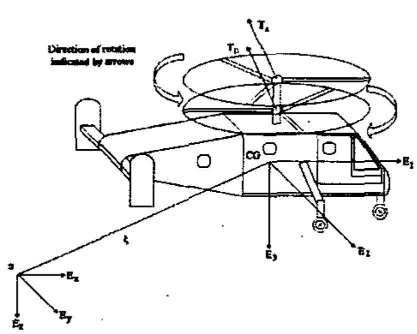

Fig. 1. Coaxial helicopter configuration.

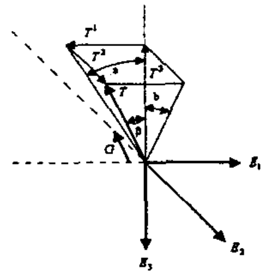

Fig. 2. Thrust vector.

For the purpose of this calculation it is assumed that the thrust vectors are non-negative and positive in a direction away from the helicopter. This assumption is valid for maneuvers close to hover. To simplify notation take the values of a and b sufficiently small. In this case, we can approximate the expression obtained for $G(a,b)$ by

$$G(a,b) = \begin{pmatrix} G^1(a,b) \\ G^2(a,b) \\ G^3(a,b) \end{pmatrix} \approx \begin{pmatrix} -a \\ b \\ -1 \end{pmatrix} \quad (8)$$

Another force applied to the coaxial helicopter is the gravitational force given by $f_g = mgE_z$ where m is the complete mass of the helicopter and g is the gravitational constant. The f_g expression is defined in the inertial frame $\mathcal{I}$. In terms of the body fixed frame, it is necessary to multiply f_g by the inverse of the rotation matrix $\mathcal{R}(\eta)$ that represents the orientation of the body fixed frame CG with respect to $\mathcal{I}$. The orientation vector η (yaw, pitch, roll) is defined by

$$\eta = [\phi, \theta, \psi]^T \quad (9)$$

and the rotation matrix

$$\mathcal{R}(\eta) = \begin{pmatrix} c_\theta c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\psi s_\theta s_\phi - s_\psi c_\phi \\ -s_\theta & s_\psi c_\theta & c_\psi c_\theta \end{pmatrix} \quad (10)$$

where the following shorthand notation is used :

$$c_\alpha = \cos\alpha, \quad and \quad s_\alpha = \sin\alpha$$

Denote by F the total translational force applied to the helicopter and expressed in the body-fixed frame CG

$$F = G(a,b)(|T_A| + |T_D|) + mg\mathcal{R}^T E_z \quad (11)$$
$$F = G^3(a,b)(|T_A| + |T_D|)E_z + mg\mathcal{R}^T E_z + (|T_A| + |T_D|) \begin{pmatrix} G^1(a,b) \\ G^2(a,b) \\ 0 \end{pmatrix} \quad (12)$$
$$F = -uE_z + mg\mathcal{R}^T E_z + \sigma \quad (13)$$

The variable u represent the lift of the helicopter ($u > 0$). The negative sign indicate the thrust direction with respect to E_3 axis

$$-uE_3 = (|T_A| + |T_D|)G^3(a,b) \approx -(|T_A| + |T_D|)E_3 \quad (14)$$

The vector σ gathers the thrust components given in E_1 and E_2 directions. This forces are called *small body forces* because their magnitudes are very slight in comparison to the lift magnitude.

$$\sigma = (|T_A| + |T_D|) \begin{pmatrix} G^1(a,b) \\ G^2(a,b) \\ 0 \end{pmatrix} \tag{15}$$

B. Torques and anti-torques.

The torques generated by the thrust vectors T_A and T_D are due to separation between the center of mass CG and the rotor hubs (represented by τ_A and τ_D). The gravitational force does not generate a torque since the helicopter is free to rotate around its center of mass.

Define the measured distances between the center of mass to the hubs of the two rotors as l_A for the above rotor and l_D for the down rotor. The torques applied to the airframe by the thrust vectors are defined by

$$\tau_A = l_A \times T_A \tag{16}$$
$$\tau_D = l_D \times T_D \tag{17}$$

The total torque generated by the above and down rotors is given by the sum of the equations (16) and (17), it is given by :

$$\begin{aligned} \tau^1_{AD} &= -(l^3_A|T_A| + l^3_D|T_D|)G^2(a,b) \\ &\quad +(l^2_A|T_A| + l^2_A|T_D|)G^3(a,b) \qquad (18) \\ \tau^2_{AD} &= (l^3_A|T_A| + l^3_D|T_D|)G^1(a,b) \\ &\quad -(l^1_A|T_A| + l^1_D|T_D|)G^3(a,b) \qquad (19) \\ \tau^3_{AD} &= -(l^2_A|T_A| + l^2_D|T_D|)G^1(a,b) \\ &\quad +(l^1_A|T_A| + l^1_D|T_D|)G^2(a,b) \qquad (20) \end{aligned}$$

The aerodynamic drags on the rotors generate some pure torques acting through the rotor hubs. Evoking the hypothesis 2.4, the anti-torques are defined by

$$Q_A = |Q_A|E_3 \tag{21}$$
$$Q_D = -|Q_D|E_3 \tag{22}$$

Finally, the total torque applied to the coaxial helicopter, expressed in the body fixed frame, is given by

$$\tau = \tau_{AD} + |Q_A|E_3 - |Q_D|E_3. \tag{23}$$

C. Complete Model

For the translational motion of the helicopter let $V = \mathcal{R}^T\dot{\xi}$ denote the velocity of its center of mass expressed in the body-fixed frame. The Newton's equations [8] yields $m\dot{V} = -\Omega \times mV + F$, where F is the external translational force applied to airframe (13).

Newton's equations show that the rotational component of motion in a non-inertial frame is given by $\mathbf{I}\dot{\mathbf{\Omega}} = -\mathbf{\Omega} \times \mathbf{I}\mathbf{\Omega} + \tau$, where Ω is the angular velocity expressed in the non-inertial frame; I denote the inertia of the helicopter around its center of mass with respect to the body fixed frame and τ is the applied external torque in the body fixed frame. Finally, recalling (13) and (23) the full dynamic model is given by

$$\dot{\xi} = \mathcal{R}V \tag{24}$$
$$m\dot{V} = -uE_3 + mg\mathcal{R}^T E_z + \sigma - \Omega \times mV \tag{25}$$
$$\dot{\mathcal{R}} = \mathcal{R}\hat{\Omega} \tag{26}$$
$$\mathbf{I}\dot{\mathbf{\Omega}} = -\mathbf{\Omega} \times \mathbf{I}\mathbf{\Omega} + |\mathbf{Q_A}|\mathbf{E_3} - |\mathbf{Q_D}|\mathbf{E_3} + \tau_{\mathbf{AD}} \tag{27}$$

where $\hat{\Omega}$ ($\Omega \in \mathbb{R}^3$) represents the skew-symmetric matrix of the vector Ω

$$\hat{\Omega} = \begin{pmatrix} 0 & -\Omega^3 & \Omega^2 \\ \Omega^3 & 0 & -\Omega^1 \\ -\Omega^2 & \Omega^1 & 0 \end{pmatrix}. \tag{28}$$

III. Backstepping based tracking control design.

In this section, we propose a nonlinear control based on backstepping techniques [7] for the nonlinear dynamic model developed in the previous section. The control problem considered is to find a control law $(u, \gamma^1, \gamma^2, \gamma^3)$ depending only on the measurable states $(\xi, \dot{\xi}, \eta, \dot{\eta})$ and arbitrary many derivatives of the smooth trajectory (position ξ^d, yaw ϕ^d) such that the tracking error

$$\mathcal{E} := (\xi(t) - \xi^d(t), \phi(t) - \phi^d(t)) \in \mathbb{R}^4 \tag{29}$$

is asymptotically stable for all initial conditions.

The design of such control law was carried out for the classical helicopter (a main rotor and a tail rotor) by using a dynamic model representation in the inertial frame [2], [3]. This requires the use of expensive sensor such as a DGPS for the measure of the position. We justify the proposed control strategy by the fact that it is more important to regulate the error between the desired and actual position of the helicopter than the stabilization of the helicopter around the desired position in the inertial frame. Another motivation is the economical expense because the DGPS can be replaced by not-inertial sensors (rangefinders, camera, etc.). The backstepping controller is based on a approximate model ($\sigma = 0$) where the neglecting part is analyzed in section IV. We start the backstepping procedure by the definition of the position error in the body fixed frame

$$\delta_1 = \mathcal{R}^T(\xi_r - \xi) \tag{30}$$

where its time derivative is given by

$$\dot{\delta}_1 = -\Omega \times \delta_1 + \mathcal{R}^T\dot{\xi}_r - \mathcal{R}^T\dot{\xi} \tag{31}$$

We can rewrite (31) as follows :

$$\dot{\delta}_1 = -\Omega \times \delta_1 + \delta_2 \tag{32}$$

where δ_2 represents the speed error defined as

$$\delta_2 = \mathcal{R}_e V_r - V \tag{33}$$

with

$$\mathcal{R}_e = \mathcal{R}^T\mathcal{R}_r \tag{34}$$
$$V = \mathcal{R}^T\dot{\xi} \tag{35}$$
$$V_r = \mathcal{R}^T_r\dot{\xi}_r \tag{36}$$

Differentiating δ_2 we get

$$\dot{\delta}_2 = -\Omega \times \delta_2 + \mathcal{R}^T \ddot{\xi}_r - \mathcal{R}^T \ddot{\xi} \quad (37)$$

Now, consider the variables associated to the forces applied to airframe by the equations :

$$\frac{F}{m} = \mathcal{R}^T \ddot{\xi} \quad (38)$$

$$\frac{F_r}{m} = \mathcal{R}_r^T \ddot{\xi}_r \quad (39)$$

Using these equations, $\dot{\delta}_2$ becomes

$$\dot{\delta}_2 = -\Omega \times \delta_2 + \frac{\mathcal{R}_e F_r}{m} - \frac{F}{m} \quad (40)$$

Define by S_1 the first storage function of the backstepping procedure :

$$S_1 = \frac{1}{2}|\delta_1 + \lambda\delta_2|^2 + \frac{1}{2}|\delta_1|^2 \quad (41)$$

where λ is defined as a positive constant. The time derivative of S_1 is given by

$$\dot{S}_1 = (\delta_1 + \lambda\delta_2)^T \left(\delta_2 + \lambda\frac{\mathcal{R}_e F_r}{m} - \lambda\frac{F}{m}\right) + \delta_1^T \delta_2 \quad (42)$$

Consider the vector thrust F as a control input of the system and denote F^v as a virtual force that represents the desired value of the actual force:

$$\left(\frac{\lambda F}{m}\right)^v = 2\delta_2 + \frac{\lambda \mathcal{R}_e F_r}{m} + (\delta_1 + \lambda\delta_2) \quad (43)$$

Thus, the equation (42) becomes

$$\dot{S}_1 = (\delta_1 + \lambda\delta_2)^T[-\delta_2 - (\delta_1 + \lambda\delta_2) + \delta_3] + \delta_1^T \delta_2 \quad (44)$$

where δ_3 represents the error between the virtual force and the actual force applied to the airframe :

$$\delta_3 = \left(\frac{\lambda F}{m}\right)^v - \frac{\lambda F}{m} \quad (45)$$

Finally, the time derivative of S_1 can be rewritten as

$$\dot{S}_1 = -|\delta_1 + \lambda\delta_2|^2 - \lambda|\delta_2|^2 + (\delta_1 + \lambda\delta_2)^T \delta_3 \quad (46)$$

Define a second storage function given by

$$S_2 = \frac{1}{2}|\delta_3|^2 + \frac{1}{2}|\epsilon_2|^2 \quad (47)$$

where ϵ_2 is a term that penalizes the yaw error :

$$\epsilon_2 = \phi - \phi_r \quad (48)$$

The yaw component of the error term is introduced at this stage of the backstepping procedure in order that the relative degree of δ_3 and ϵ_2 with respect to the controls u and γ match. Indeed, the relative degree of each control with respect either is two. Differentiating the expression (47), one has :

$$\begin{aligned}\dot{S}_2 &= \delta_3^T[-\Omega \times \delta_3 - \frac{\lambda+2}{\lambda}\delta_1 - \frac{\lambda^2+3\lambda+4}{\lambda}\delta_2 \\ &\quad + \frac{\lambda+2}{\lambda}\delta_3 + \frac{\lambda}{m}R^T \xi_r^{(3)} + \frac{\lambda \dot{u} E_3}{m} + \frac{\lambda u \hat{E}_3 \Omega}{m}] \\ &\quad + \epsilon_2(\dot{\phi} - \dot{\phi}_r) \quad (49)\end{aligned}$$

Note that the vectors $\dot{u}E_3$ and $\hat{E}_3\Omega$ are completely decoupled. This enables us to affect directly the input $\dot{u}$. Furthermore, we continue the controller design only with the rotation dynamic. While choosing for $\dot{u}$ and the virtual inputs $\hat{E}_3\Omega$ and $\dot{\phi}$ the following expressions

$$\begin{aligned}\dot{u} &= \frac{mE_3^T}{\lambda}[\frac{\lambda+2}{\lambda}\delta_1 + \frac{\lambda^2+3\lambda+4}{\lambda}\delta_2 \\ &\quad - \frac{\lambda+2}{\lambda}\delta_3 - \frac{\lambda}{m}\mathcal{R}^T \xi_r^{(3)} \\ &\quad - (\delta_1 + \lambda\delta_2) - k_1\delta_3] \quad (50) \\ \left(\frac{\lambda u \hat{E}_3 \Omega}{m}\right)^v &= (\mathbf{I} - E_3 E_3^T)[-\frac{\lambda+2}{\lambda}\delta_1 \\ &\quad - \frac{\lambda^2+3\lambda+4}{\lambda}\delta_2 + \frac{\lambda+2}{\lambda}\delta_3 \\ &\quad + \frac{\lambda}{m}\mathcal{R}^T \xi_r^{(3)} + (\delta_1 + \lambda\delta_2) \\ &\quad + k_1\delta_3] \quad (51) \\ \dot{\phi}^v &= \dot{\phi}_r - k_2\epsilon_2 \quad (52)\end{aligned}$$

the equation (49) can be rewritten by

$$\dot{S}_2 = -k_1|\delta_3|^2 - \delta_3^T(\delta_1 + \lambda\delta_2) + \delta_3^T\delta_4 - k_2|\epsilon_2|^2 + \epsilon_2\epsilon_3 \quad (53)$$

where δ_4 defines the new vector error associated with the speed rotations (only in roll and pitch), k_1 and k_2 are positive constants and ϵ_3 represents of the yaw speed error

$$\begin{aligned}\delta_4 &= \frac{\lambda u \hat{E}_3 \Omega}{m} - \left(\frac{\lambda u \hat{E}_3 \Omega}{m}\right)^v \quad (54) \\ \epsilon_3 &= \dot{\phi} - \dot{\phi}^v \quad (55) \\ & \quad (56)\end{aligned}$$

The last storage function associated with the backstepping procedure is given by

$$S_3 = \frac{1}{2}|\delta_4|^2 + \frac{1}{2}|\epsilon_4|^2, \quad (57)$$

Taking the time derivative of S_3, it yields

$$\begin{aligned}\dot{S}_3 &= \delta_4^T\left(\frac{d}{dt}\left(\frac{\lambda u \hat{E}_3 \Omega}{m}\right)^v - \frac{\lambda \dot{u} \hat{E}_3 \Omega}{m} - \frac{\lambda u \hat{E}_3 \dot{\Omega}}{m}\right) \\ &\quad + \epsilon_3(\ddot{\phi} - \ddot{\phi}^v) \quad (58)\end{aligned}$$

Note that the torque vector is represented in the previous equation via $\dot{\Omega}$ and $\ddot{\phi}$. If we consider $\dot{\Omega} = [\gamma^1, \gamma^2, \gamma^3]^T$ as the control input of the system instead of the τ vector and if we choose the next control vectors to achieve the desired control:

$$\begin{aligned}\frac{\lambda u \hat{E}_3 \dot{\Omega}}{m} &= \frac{d}{dt}\left(\frac{\lambda u \hat{E}_3 \Omega}{m}\right)^v - \frac{\lambda \dot{u} \hat{E}_3 \Omega}{m} \\ &\quad + \delta_3 + k_3\delta_4 \quad (59) \\ \ddot{\phi} &= \ddot{\phi}^v - k_4\epsilon_3 - \epsilon_2 \quad (60)\end{aligned}$$

the expression of $\dot{S}_3$ (58) becomes :

$$\dot{S}_3 = -\delta_4^T\delta_3 - k_3|\delta_4|^2 - \epsilon_3\epsilon_2 - k_4|\epsilon_3|^2 \quad (61)$$

where k_3 and k_4 are defined as positive constants. The control inputs γ^1 and γ^2 can be obtained by the equation (59). The control input γ^3 is defined by using the second derivative of η [4] :

$$\ddot{\eta} = -W_\eta^{-1}\dot{W}_\eta W_\eta^{-1}\Omega + W_\eta^{-1}\dot{\Omega} \tag{62}$$

where W_η is defined as follows

$$W_\eta = \begin{pmatrix} -s_\theta & 0 & 1 \\ c_\theta s_\psi & c_\psi & 0 \\ c_\theta c_\psi & -s_\psi & 0 \end{pmatrix} \tag{63}$$

Taking the yaw component of equation (62), it yields

$$\ddot{\phi} = -E_x^T(W_\eta^{-1}\dot{W}_\eta W_\eta^{-1}\Omega + W_\eta^{-1}\dot{\Omega}) \tag{64}$$

It remains to define the control signals equations as

$$\begin{aligned}
\dot{u} &= \frac{mE_3^T}{\lambda}[\frac{\lambda+2}{\lambda}\delta_1 + \frac{\lambda^2+3\lambda+4}{\lambda}\delta_2 - \frac{\lambda+2}{\lambda}\delta_3 \\
&\quad -\frac{\lambda}{m}\mathcal{R}^T\xi_r^{(3)} - (\delta_1+\lambda\delta_2) - k_1\delta_3] \qquad (65)\\
\gamma^1 &= -\frac{mE_2^T}{\lambda u}(\frac{d}{dt}\left(\frac{\lambda u\hat{E}_3\Omega}{m}\right)^v - \frac{\lambda\dot{u}\hat{E}_3\Omega}{m} \\
&\quad +\delta_3 + k_3\delta_4) \qquad (66)\\
\gamma^2 &= \frac{mE_1^T}{\lambda u}(\frac{d}{dt}\left(\frac{\lambda u\hat{E}_3\Omega}{m}\right)^v - \frac{\lambda\dot{u}\hat{E}_3\Omega}{m} \\
&\quad +\delta_3 + k_3\delta_4) \qquad (67)\\
\gamma^3 &= \frac{\cos\theta}{\cos\psi}(\ddot{\phi}^v - k_4\epsilon_3 - \epsilon_2 + E_1^T W_\eta^{-1}\dot{W}_\eta W_\eta^{-1}\Omega \\
&\quad -\frac{\sin\psi}{\cos\theta}\gamma^2). \qquad (68)
\end{aligned}$$

The Backstepping procedure showed before, accomplish the monotonic decrease of the following Lyapunov function

$$\begin{aligned}
L &= S_1 + S_2 + S_3 \\
&= \frac{1}{2}|\delta_1+\lambda\delta_2|^2 + \frac{1}{2}|\delta_1|^2 + \frac{1}{2}|\delta_3|^2 + \frac{1}{2}|\delta_4|^2 \\
&\quad + \frac{1}{2}|\epsilon_2|^2 + \frac{1}{2}|\epsilon_3|^2
\end{aligned}$$

One can directly verify that

$$\begin{aligned}
\dot{L} &= -|\delta_1+\lambda\delta_2|^2 - \lambda|\delta_2|^2 - k_1|\delta_3|^2 - k_2|\delta_4|^2 \\
&\quad -k_2|\epsilon_2|^2 - k_4|\epsilon_3|^2
\end{aligned}$$

Note that δ_1 and ϵ_3 together form the original tracking error that we wish to minimize. Then the Lyapunov function V is monotonically decreasing and thus the control objective is achieved. Using simple calculations, we can recover the control inputs T_A, T_D, a and b using the equations (14), (18) - (23) and by knowing the relation which binds T_A and T_D with Q_A and Q_D respectively.

IV. Simulation results.

In this section, we present the simulation of the behavior of the complete helicopter dynamics and the approximative dynamics used to obtain the control law. The experiment considers the case of stabilization of the tandem helicopter dynamics to a stationary configuration. The parameters used in the coaxial helicopter model are based on a modified Kalt Whisper miniature helicopter :

$$\begin{aligned}
Mass &= 7,5\,kg \\
g &= 9,8\,ms^{-2} \\
l_A &= [0,0,-0,35]\,m \\
l_D &= [0,0,-0,25]\,m \\
\mathbf{I} &\approx \begin{pmatrix} 0,21 & 0 & 0 \\ 0 & 0,288 & 0 \\ 0 & 0 & 0,278 \end{pmatrix} kgm^2
\end{aligned} \tag{69}$$

The anti-torques Q_A and Q_D are supposed to be proportional to the respective thrusts :

$$\begin{aligned}
|Q_A| &= 0,02|T_A| \\
|Q_D| &= 0,02|T_D|
\end{aligned}$$

The values of the gains are : $\lambda = 2$, $k_1 = 1$, $k_2 = 1$, $k_3 = 10$ and $k_4 = 3$. The initial condition adopted for the control is $u = gm \approx 74$. It is exactly equal to the force required for sustaining the helicopter in stationary flight. The initial and desired position chosen are

$$\xi_0 = \begin{pmatrix} 0 \\ 5 \\ -5 \end{pmatrix} m, \quad \phi_0 = 0^\circ \tag{70}$$

$$\xi^d = \begin{pmatrix} 5 \\ 10 \\ -15 \end{pmatrix} m, \quad \phi^d = 20^\circ \tag{71}$$

Figures 3 and 4 show the helicopter system that does not take into consideration the small body forces. The results obtained when small body forces are present in the helicopter model are shown in figures 5 and 6. In both the ideal case and when the small body forces are present the simulations indicate that the position regulations is achieved.

Robustness of the proposed controller has been tested in the case where the inertia matrix $\mathbf{I}$ is not exactly known. Figures 7 and 8 illustrate the behavior of the cyclic angles for $\mathbf{I}$ equal to (69), $\mathbf{I} = 1.25\mathbf{I}$ and $\mathbf{I} = 0.75\mathbf{I}$. Note that the magnitude of the control inputs is acceptable.

V. Conclusion

In this paper a simple model for the dynamics of a coaxial helicopter in close to hover conditions is presented. A new controller based on backstepping design was implemented. It is based on the approximation of the system dynamics obtained by neglecting the small body forces associated with the torque control. Simulations showed good performances of the proposed controller.

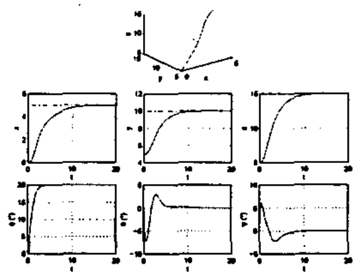

Fig. 3. Position regulation of the helicopter dynamics in absence of small body forces.

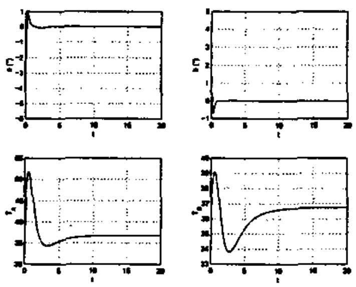

Fig. 4. Cyclic angles and thrust vectors of the helicopter dynamics in absence of small body forces.

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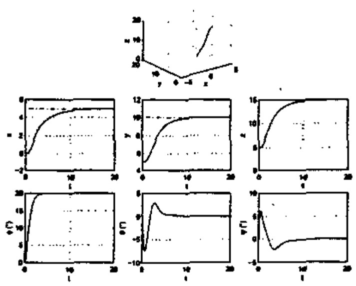

Fig. 5. Position regulation of the helicopter dynamics in presence of small body forces.

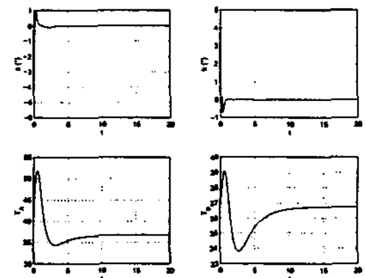

Fig. 6. Cyclic angles and thrust vectors of the helicopter dynamics in presence of small body forces.

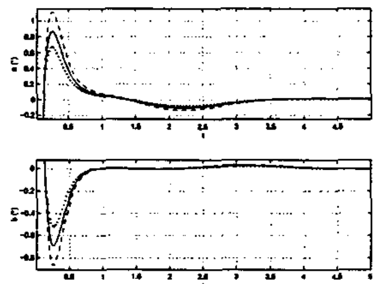

Fig. 7. Behavior of cyclic angles in absence of small body forces. Solid for I, dashed for 1.25I and dotted for 0.75I.

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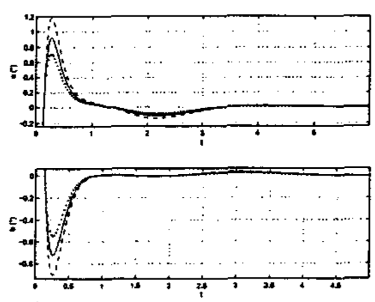

Fig. 8. Behavior of cyclic angles in presence of small body forces. Solid for I, dashed for 1.25I and dotted for 0.75I.