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HELICOPTER'S NONLINEAR CONTROL VIA BACKSTEPPING TECHNIQUES.

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Abstract

In this paper a Lyapunov control algorithm for a small scale autonomous model helicopter is obtained using robust backstepping techniques. The controller design does not consider the small body forces. The proposed controller is based on dynamic order reduction for the nonlinear dynamical model. Some simulation results are presented to illustrate the performance of such controller in the presence of the small body forces.

1 Introduction

In the last years, the control of flying machines has attracted a major interest. Different approaches have been proposed to control airplanes, helicopters, rockets, satellites, etc. [13][22]. Each one of these aircrafts has a specific model describing its behavior. Helicopters are among the most complex flying objects because its flight dynamics is inherently nonlinear and they have strong couplings of all the variables. The helicopter has however the ability to hover which is required in some applications.
The first attempt to control helicopters was done by using linear techniques [1], [6], [15], [19] and [20]. Nevertheless, the nonlinear nature of helicopters has to be taken into account in the controller design if one wishes to improve its performance. The Backstepping techniques were used for the design of a non-linear control law [3], [5], [9] and [10]. The first of these works [5] uses this technique to stabilize altitude and the pitch angle (two degrees of freedom used for take-off and landing). The control strategies developed in [3], [9] and [10] deal with a six-degree of freedom helicopter model. In [9] and [10] the proposed controllers are based on Euler angle configuration while [3] uses a quaternion representation to avoid singularities.
This paper presents a controller synthesis for a six-degree of freedom model obtained using Newton's equations. To simplify the analysis we have neglected the small body forces (associated with the torque control) acting on the helicopter. We use backstepping techniques to obtain the control law. This control strategy achieves a desired helicopter position and orientation in hover and achieves also tracking of a desired trajectory. The proposed approach is based on dynamic order reduction of the controller [17]. Simulations have shown that the proposed controller is robust with respect to small body forces. The outline of the paper is as follows. Section 2 describes the helicopter dynamical model. Section 3 presents the backstepping controller design. Section 4 is devoted to present some simulation results. Finally, we present the conclusion of this work.

2 Dynamic Model of Autonomous Helicopter

In this section the nonlinear dynamical model proposed in [8] is used. Let $\xi = (x, y, z)$ denote the posi-

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tion vector of the centre of mass of the helicopter airframe relative to the right-hand inertial frame $\mathcal{I}$ (cf. Figure 1). Denote v the velocity of the centre of the mass expressed in the inertial frame. Let $R : \mathcal{A} \to \mathcal{I}$ be the rotation matrix representing the orientation (η) of the body fixed frame $\mathcal{A}$ with respect to $\mathcal{I}$. The angular velocity of $\mathcal{A}$ is represented by a vector $\Omega \in \mathcal{A}$ in the body fixed frame.

In addition to gravitational effect, a helicopter is acted by a number of forces. These forces seen in the Equations (2)-(5) are explained in the following list :

uRe_3 : The principal force for sustaining the helicopter in flight.

$\hat{\Omega}$: The skew symmetric matrix of the angular velocity.

$$\hat{\Omega} = \begin{pmatrix} 0 & -\Omega^3 & \Omega^2 \\ \Omega^3 & 0 & -\Omega^1 \\ -\Omega^2 & \Omega^1 & 0 \end{pmatrix} \quad (1)$$

$|Q_M|, |Q_T|$: Air resistance on the main and tail rotor result in forces applied directly to the helicopter airframe $|Q_M|, |Q_T|$ oriented around the axis of respective rotor.

$P(u,w)$: Torque control for the airframe, obtained by a combination of the rigidity of the rotor blades with the induced torques obtained from the deformation of the main rotor disk and consequent tilting of force vector representing the combined lift generated by the main rotor blades. The vector $w \in \Re^3$ represents the contribution to the torque from the deformation of the main rotor disk, the principal mechanism used to obtain control, while the positive definite matrix $P(u,w) > I_3$ provides the modification for the relative rigidity of the rotor blades.

$R\sigma w$: Due to the mechanism used to obtain torque control and consequent tilting of the force vector associated with the main rotor lift, the torque control inputs w are coupled directly to small body forces which affect the translational dynamics of the helicopter airframe. Hence σ is a constant matrix depending on the geometric parameters of the helicopter [8]. It's norm $\|\sigma\|_2$ represents the offset between the center of mass of the helicopter and the center of the rotor disk.

For more details on the modeling the interested reader is referred to the paper [8]. Newton equations of motion for helicopter subject to the forces outlined above are :

$$\begin{aligned} \dot{\xi} &= v && (2) \\ m\dot{v} &= -uRe_3 + mge_3 + R\sigma w && (3) \\ \dot{R} &= R\hat{\Omega} && (4) \\ I\dot{\Omega} &= -\Omega \times I\Omega + |Q_M|\, e_3 - |Q_T|\, e_2 \\ &\quad +P(u,w)w && (5) \end{aligned}$$

A similar model has been used by *Koo and Sastry* for the motion of a helicopter. Note that, in practice such a model is difficult to control due to the presence of the small body forces which couple torques inputs to translational dynamics (3). To avoid this problem in the control design we will thus base our control design on the above model considering $\sigma = 0$, and then select the controller parameters to take into account the effect of the small body forces ($\sigma \neq 0$). In this study case, we first consider a backstepping algorithm of the approximate model. Finally, recognizing that the model includes the small body forces, we select the controller parameters to insure the stability of the original model.

Compared with the dynamic extension design used in [8], or with a dynamic reduction based on a singular perturbation which has been developed for the VTOL by [17], our control algorithm represents the average of the existing control designs as it can be seen in the next section.

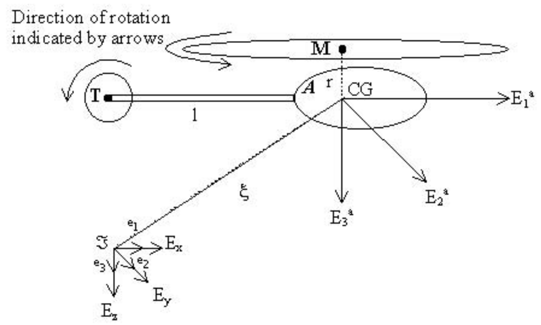

Figure 1: Geometry of model helicopter.

3 Backstepping based tracking control design.

In this section a backstepping control design is proposed for tracking a desired smooth trajectory $\xi^d = (x^d, y^d, z^d)$. Note that, the dynamic perturbation $R\sigma w$ in Eq. 3 is linked to weakly non-minimum phase zero dynamics [6] and results in a structure similar to that encountered for the original investigation of the VTOL [4]. Unlike the VTOL [12] the system is not differentially flat [21], [7], [11]. As a consequence (to the best of the authors knowledge) there is no control design available that deals with the general model and full dynamics of the system. The approach taken in previous control algorithms [2], [16], [14], [18], [20] and [3] is to design a robust controller for the system where $\sigma \equiv 0$ and analyze the robustness of the closed loop system with respect to the perturbation $R\sigma w$. We take an analogous approach in the present paper. The focus of the present investigation, is on dynamic order reduction of the controller which has not been considered in previous works. For this reason, and in the sake of a less complicated presentation, we analyze only the dynamic model Eqn's 2-5 where $\sigma \equiv 0$ in detail and leave the discussion on the dynamic perturbation σ to Section 4.

The control problem considered is to find a control law (u, w^1, w^2, w^3) depending only on the measurable states $(\xi, \dot{\xi}, \eta, \dot{\eta})$ and arbitrary many derivatives of the smooth trajectory (ξ^d) such that the tracking error

$$\delta_1 := \xi - \xi^d \tag{6}$$

is asymptotically stable for all initial conditions.

In order to ensure a well-conditioned control algorithm a fourth objective is added to make the problem one block of input/output regulation. The choice made is to stabilize the yaw speed of the helicopter to the most natural orientation with respect to the tracked trajectory.

Define a first storage function

$$S_1 = \frac{1}{2}\delta_1^T\delta_1 = \frac{1}{2}\left|\delta_1\right|^2 \tag{7}$$

Taking the time derivative of S_1 yields

$$\dot{S}_1 = \delta_1^T\dot{\delta}_1 = \delta_1^T(v - v^d). \tag{8}$$

where v^d represents the velocity of tracked trajectory.

Let v^v be the virtual control for this first stage of this procedure and be chosen as

$$v^v = v^d - k_1\delta_1. \tag{9}$$

Introducing the above into (8) we get

$$\dot{S}_1 = -k_1|\delta_1|^2 + \frac{1}{m}\delta_1^T\delta_2. \tag{10}$$

Where δ_2 is defined as

$$\delta_2 = mv - mv^v. \tag{11}$$

Differentiating δ_2 and using equation (3) without the small body forces, it follows

$$\dot{\delta}_2 = -uRe_3 + mge_3 - m\dot{v}^v. \tag{12}$$

Define a second storage function associated with the second error term δ_2 like the equation 7. Therefore, from (12)

$$\dot{S}_2 = \delta_2^T(-uRe_3 + mge_3 - m\dot{v}^v).$$

At this stage of the procedure the developed idea in [8] considers the vector input (uRe_3) to the translational dynamics as a vectorial virtual control. In contrast, the idea developed in [17] considers u as an actual control input and $Re_3 = \alpha$, the vector such that $(u\alpha)$ stabilizes the translational dynamics. The problem behind this idea is that Re_3 can not be considered as an actual control, because R is a state variable and for the simple case 'VTOL', [17] has proposed a high gain control for Re_3 near α. In our approach, we propose the case of (uRe_3) as a virtual control as

$$X = (uRe_3)^v = mge_3 - m\dot{v}^v + \frac{1}{m}\delta_1 + k_2\delta_2. \tag{13}$$

In this case the derivative of the second storage function becomes

$$\dot{S}_2 = -\frac{1}{m}\delta_2^T\delta_1 - k_2|\delta_2|^2 + \delta_2^T\delta_3, \tag{14}$$

where δ_3 is the third error used in this procedure. It is defined as follows:

$$\delta_3 = X - uRe_3 \tag{15}$$

Consider the new storage function S_3 with δ_3, taking the time derivative of S_3 (like the equations (7) and (10)) and recalling (4), (15) and (1), one has

$$\begin{aligned} \dot{S}_3 &= \delta_3^T(\dot{X} - \dot{u}Re_3 - uR\hat{\Omega}e_3) && (16) \\ &= \delta_3^T\left(\dot{X} - R\begin{bmatrix} u\Omega_2 \\ -u\Omega_1 \\ \dot{u} \end{bmatrix}\right) && (17) \end{aligned}$$

$\dot{u}$ can be assigned directly via the following control law

$$\dot{u} = e_3^TR^T(\dot{X} + \delta_2 + k_3\delta_3), \tag{18}$$

Note that if we assume that the measurements of $(\xi, \dot{\xi}, \eta, u)$ are available, then one can estimate the value of the derivative of the thrust $\dot{u}$. When the measurement of u is not available, but the measurement of $\ddot{\xi}$ is available, then one can estimate the value of u, using the following relation

$$|u| = m\left|\ddot{\xi} - ge_3\right|$$

Now, define the following virtual input :

$$\left(\begin{array}{c} u\Omega_2 \\ -u\Omega_1 \\ 0 \end{array}\right)^v = [I - e_3e_3^T]R^T(\dot{X} + \delta_2 + k_3\delta_3) = Y.$$

To proceed we introduce the error variable :

$$\delta_4 = Y - \left(\begin{array}{c} u\Omega_2 \\ -u\Omega_1 \\ 0 \end{array}\right),$$

Thus, equation (16) becomes

$$\dot{S}_3 = -\delta_3^T\delta_2 - k_3|\delta_3|^2 + \delta_3^T R\delta_4. \tag{19}$$

Select a fourth storage function and design a vector control law for (w_1, w_2) in the subsystem (5). Define S_4 as follows

$$S_4 = \frac{1}{2}\delta_4^T\delta_4 = \frac{1}{2}|\delta_4|^2, \tag{20}$$

Taking the derivative of S_4, we have

$$\dot{S}_4 = \delta_4^T\left(\dot{Y} - \frac{d}{dt}\left[\begin{array}{c} u\Omega_2 \\ -u\Omega_1 \\ 0 \end{array}\right]\right)$$

and the next equations describe the control inputs that achieve the desired dynamics of the closed-loop system. Note that

$$\frac{d}{dt}\left(u\left[\begin{array}{c} \Omega_2 \\ -\Omega_1 \\ 0 \end{array}\right]\right) = \dot{u}\left[\begin{array}{c} \Omega_2 \\ -\Omega_1 \\ 0 \end{array}\right] + u\hat{e}_3\dot{\Omega}$$

where $\hat{e}_3 = S(e_3)$ which is the skew-symetric matrix of e_3 (see (1))

Considering the following control input transformation :

$$\tilde{w} = \dot{\Omega}, \tag{21}$$

and introducing (21) into (3), it yields

$$\frac{d}{dt}\left(u\left[\begin{array}{c} \Omega_2 \\ -\Omega_1 \\ 0 \end{array}\right]\right) = \dot{u}\hat{\Omega}e_3 + u\hat{e}_3\tilde{w} = \dot{Y} + R^T\delta_3 + k_4\delta_4.$$

Now, to achieve the desired control, we choose :

$$\hat{e}\tilde{w} = \left(\begin{array}{c} \tilde{w}_2 \\ -\tilde{w}_1 \\ 0 \end{array}\right) = (\dot{Y} + R^T\delta_3 + k_4\delta_4 - \dot{u}\hat{\Omega}e_3)/u$$

Introducing (3) into (3) we get

$$\dot{S}_4 = -\delta_4^T R^T\delta_3 - k_4|\delta_4|^2. \tag{22}$$

The choice considered to stabilize the yaw speed is a control law given by

$$\tilde{w}_3 = -k_5\Omega_3. \tag{23}$$

Finally, we can define the control signals equations as follows (including (18) equation) :

$$\begin{aligned} \dot{u} &= e_3^T R^T(\dot{X} + \delta_2 + k_3\delta_3), && (24) \\ \tilde{w}_1 &= -e_2^T(\dot{Y} + R^T\delta_3 + k_4\delta_4 - \dot{u}\Omega_1)/u, && (25) \\ \tilde{w}_2 &= e_1^T(\dot{Y} + R^T\delta_3 + k_4\delta_4 - \dot{u}\Omega_2)/u, && (26) \\ \tilde{w}_3 &= -k_5\Omega_3. && (27) \end{aligned}$$

Note that, the Backstepping process showed before, accomplish now the monotonic decrease of the following Lyapunov function

$$\begin{aligned} V &= S_1 + S_2 + S_3 + S_4. \\ &= \frac{1}{2}\sum_{i=1}^{4}|\delta_i|^2 \end{aligned}$$

The function V is a positive definite function of $\xi, v, \phi, \theta, \Omega_1, \Omega_2$. Deriving V, one obtain

$$\dot{V} = \dot{S}_1 + \dot{S}_2 + \dot{S}_3 + \dot{S}_4,$$

Using (10), (14), (19) and (22) we obtain

$$\dot{V} = -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2 - k_4|\delta_4|^2.$$

Let $k = \frac{1}{2}\min\{k_1, k_2, k_3, k_4\}$. The full procedure leads to

$$\dot{V} \leq kV$$

It follows from classical Lyapunov theory that the errors δ_i converges exponentially to zero for $i = 1, ..., 4$.

4 Simulation results.

In this section and with the aim to validate the proposed control design, simulations for two experiments are presented. The first experiment contains a simulation of the approximate dynamics used to derive the control law. The second concerns the behavior of the complete helicopter dynamics (ie. with the small body forces). The parameters used for the dynamic model are based on preliminary measurements for a VARIO 23cc scale model helicopter. The values used are $m = 9.6$, $I = (0.4, 0.56, 0.22)$ and $g = 9.8$. The air resistances are estimated by the following constants ($|Q_M| = 0.02, |Q_T| = 0.002$). The form of σ used is:

$$\sigma = \begin{pmatrix} 0 & -2.2 & 0 \\ 2.2 & 0 & 0.7 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

This coupling between the roll (pitch) dynamics with lateral (longitudinal) linear dynamics corresponds to the form used in contemporary works [3] and [6]. The magnitude of the initial force input is chosen to be $u_0 = gm \approx 94$ corresponding to the fact that the helicopter is initially in hover flight. The initial position is

$$\xi_0 = (2, 4, 5)^T, \ \phi_0 = 0,$$

while the desired position is $\check{\xi} = [0, 0, 0]^T$. The results obtained when small body forces are present in the helicopter model are shown in Figure 2. Figure 3 represents the simulation that does not take into account small body forces. In both simulations, the desired position is achieved.

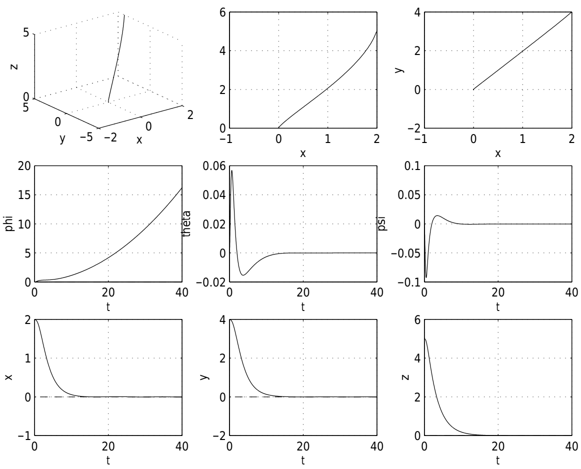

Figure 2: Position regulation of the helicopter dynamics in the presence of small body forces.

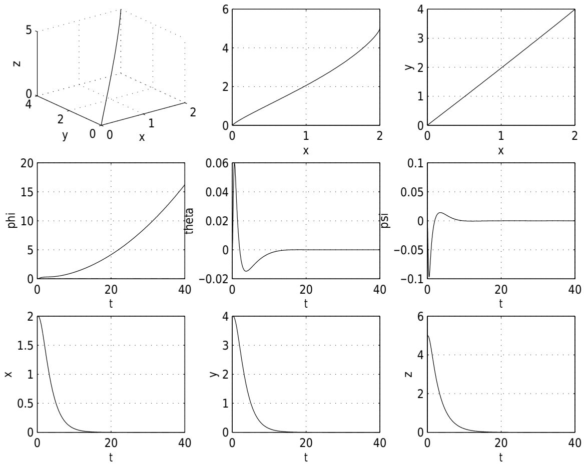

Figure 3: Position regulation of the helicopter dynamics in absence of small body forces.

5 Conclusion

In this paper, we have proposed a nonlinear controller that guarantees a position regulation for an autonomous model helicopter. We have shown that neglecting the small body forces in the backstepping process does not alter significantly the system behavior. Simulation results show that the control law produces the desired performance. The advantage with respect to the work of [9] is the suppression of one stage in the backstepping process (dynamic reduction [17]) and therefore some sensors are not longer required in the practical implementation. In future work, we will implement and test the control law on the scale model helicopter.

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