MAIN CONCEPTS OF SIMULATION

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SIMULATION INTRODUCTION

Varying Inputs

You reach boundary pts reduced cost, shadow price. here data is changing, so far your modelling was based on fixed tationary data but your cost of chair can change.

- Up until this point we have been assuming a rather unrealistic view of the real world – **certainty**.
- In a real world setting – especially the business world – the inputs and coefficients in a problem are rarely fixed quantities.
- Optimization techniques like sensitivity analysis – **reduced cost** and **shadow prices** – are one approach to handling this problem.

Monte Carlo Simulations

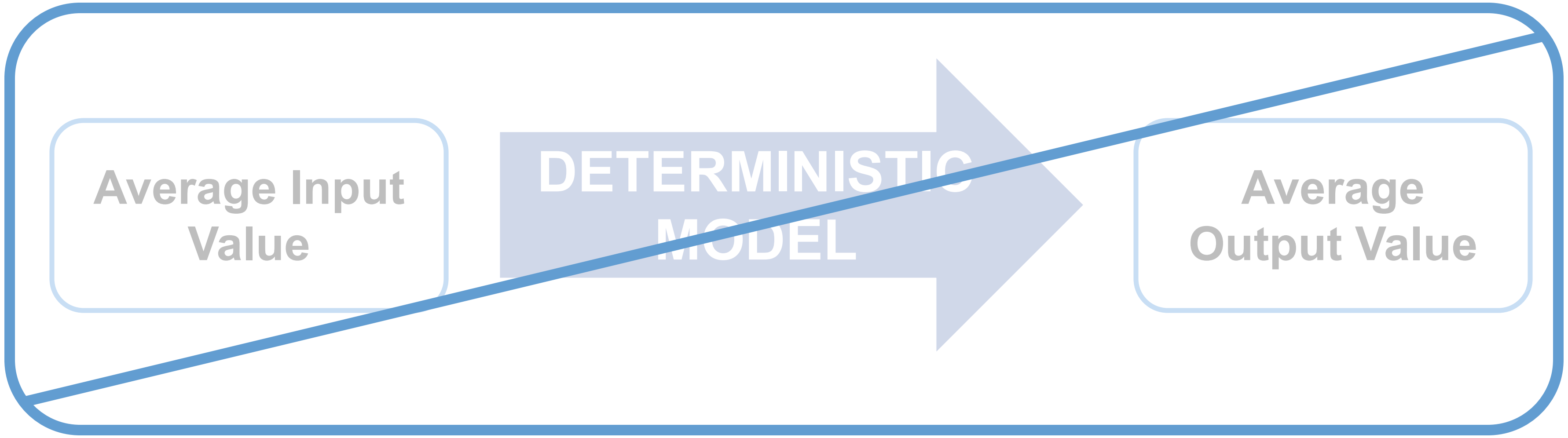
Notion what happens when inputs change. biggest thing used in is Risk Eval. Another use of simualtion is model evaluatin - target shuffling.

- Uncertainty is foundational in Monte Carlo simulations.

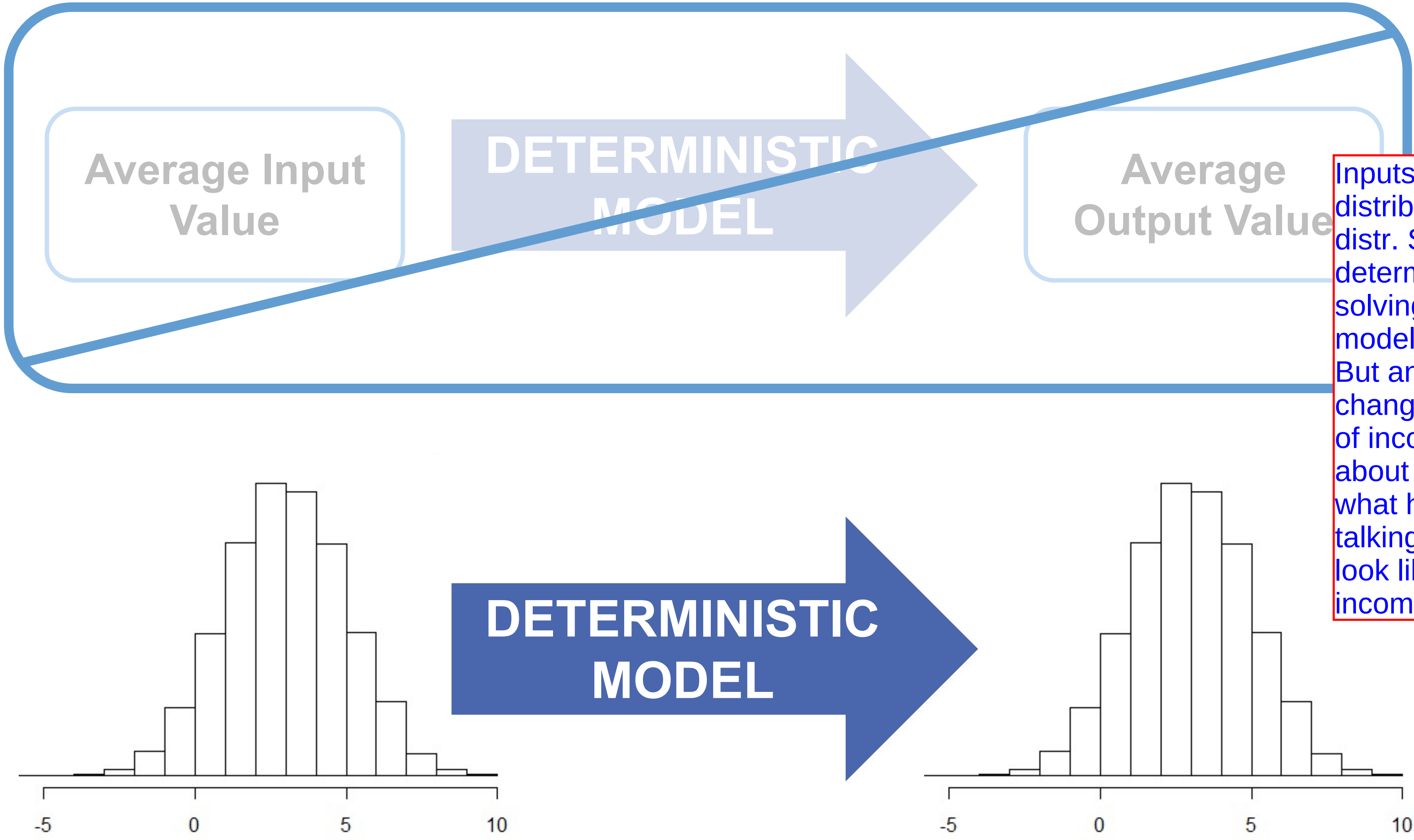
- **Simulations** help us determine not only the full array of outcomes of a given decision, but the probabilities of these outcomes occurring.
- Some examples:
  - Risk analysis – how rare certain outcomes actually are.
  - Model evaluation – how good is our model compared to others.

Monte Carlo Simulations

Monte Carlo Simulations



Monte Carlo Simulations



Inputs are put in as distributions, output are also distr. So far we have done deterministic - one equation solving input to output. All models are just equations. But answer how does churn change with varying levels of income? Not talking about 1 unit incre in income, what happens? We are talking what does churn look like with best or least income (outside of data)?

What-If Analysis

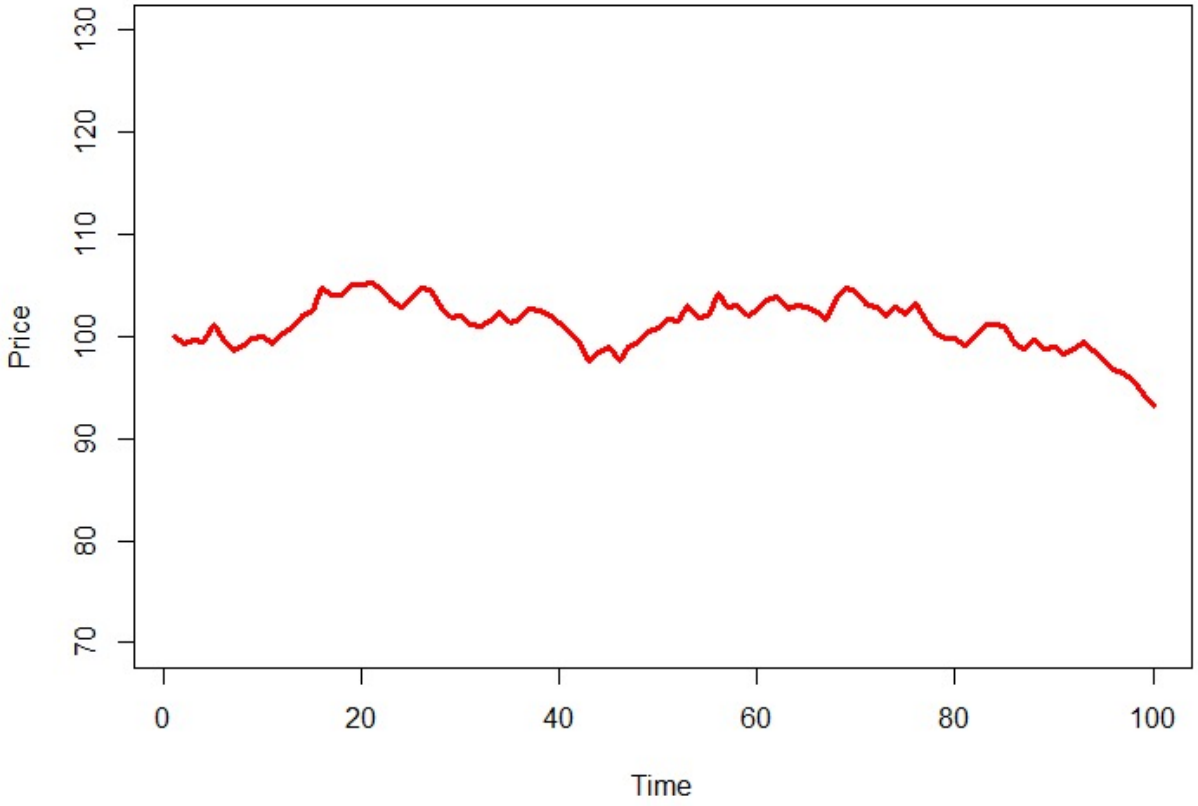
Outputs are entire distributions.

- Each input inside of a model (or process) is assigned a range of possible values – the **probability distribution of the inputs**.
- We then analyze what happens to the decision from our model (or process) under all of these possible scenarios.
- Simulation analysis describes not only the outcomes of certain decisions, but also the **probability distribution of those outcomes** – the probability each of these outcomes occurs.

What-If Analysis

- Assume a stock price is $100.
- Follows a random walk for next 100 days with $\varepsilon_t \sim N(0,1)$.

stock price follows random walk.



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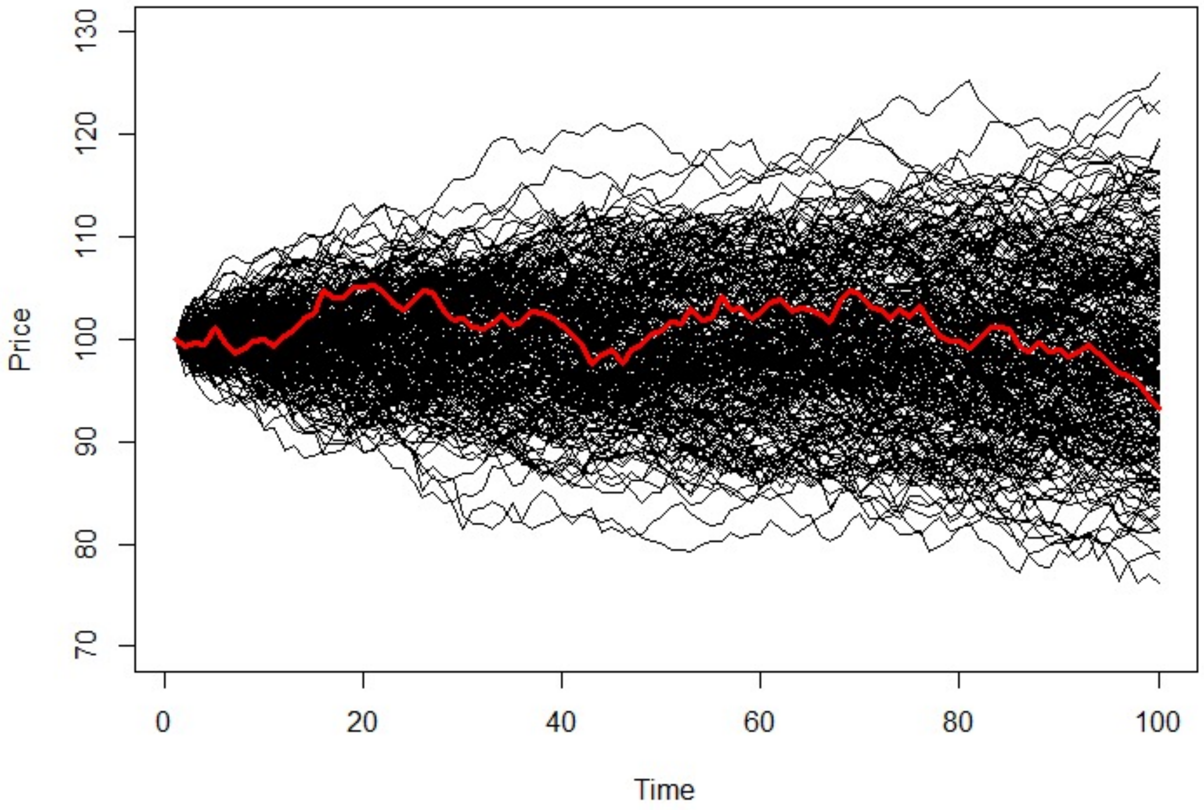

What-If Analysis

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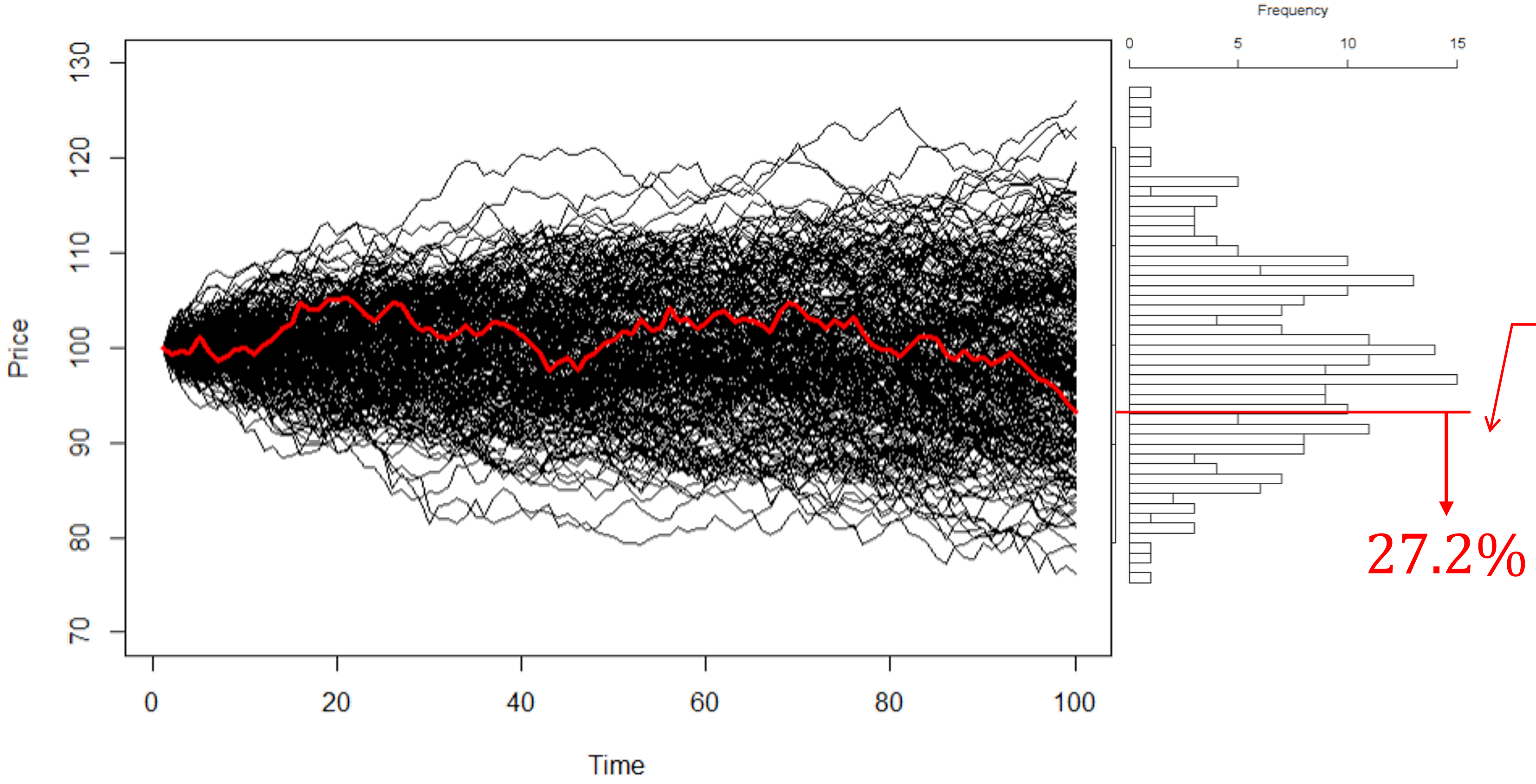

27% chance stock below. P values only in model. here of all the possible things that could happen, what is the prob of stock below based on what oyu see in eval. Investors cared aboutt hoe much $ could i lose not how much make in 10y? Percentiles, quantiles all those things coming back.

Outcome Distribution

- Simulation analysis describes not only the outcomes of certain decisions, but also the **probability distribution of those outcomes** – the probability each of these outcomes occurs.
- After the simulation analysis, the focus then turns to the probability distribution of the outcomes.
- Describe the characteristics of this new distribution – mean, variance, skewness, kurtosis, percentiles, etc.

Example

- You want to invest $1,000 in the US stock market for one year.
- You invest in a mutual fund that tries to produce the same return as the S&P500 Index.

could be +ve or -ve

$$P_1 = P_0 + r_{0,1} * P_0$$

OR

$$P_1 = P_0 * (1 + r_{0,1})$$

return is only input that can change. Now how do we estiamte that return. can give distr as input and output.

Example

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$$P_1 = P_0 * (1 + r_{0,1})$$

Initial Investment Return

Selecting Distributions

- When designing your simulations the biggest choice comes from the decision of the distribution on the inputs that vary.
- 3 Methods:
  1. Common Probability Distribution
  2. Historical (Empirical) Distribution
  3. Hypothesized Future Distribution

Example

- You want to invest \$1,000 in the US stock market for one year.
- You invest in a mutual fund that tries to produce the same return as the S&P500 Index.

$$P_1 = P_0 * \left(1 + r_{0,1}\right)$$

- Assume annual returns follow a Normal distribution with historical mean of 8.79% and std. dev. of 14.75%.

if std dev greater than mean, then it could go negative.

Introduction to Simulation – R

looking at 10000 future 1y returns, under assumption of return mean +-sd. Assumed normal distr.

```
r <- rnorm(n=10000, mean=0.0879, sd=0.1475)
P0 <- 1000
P1 <- P0*(1+r)

hist(P1, breaks=50, main='One Year Value Distribution',
     xlab='Final Value')
abline(v = 1000, col="red", lwd=2)
mtext("Initial Inv.", at=1000, col="red")
```

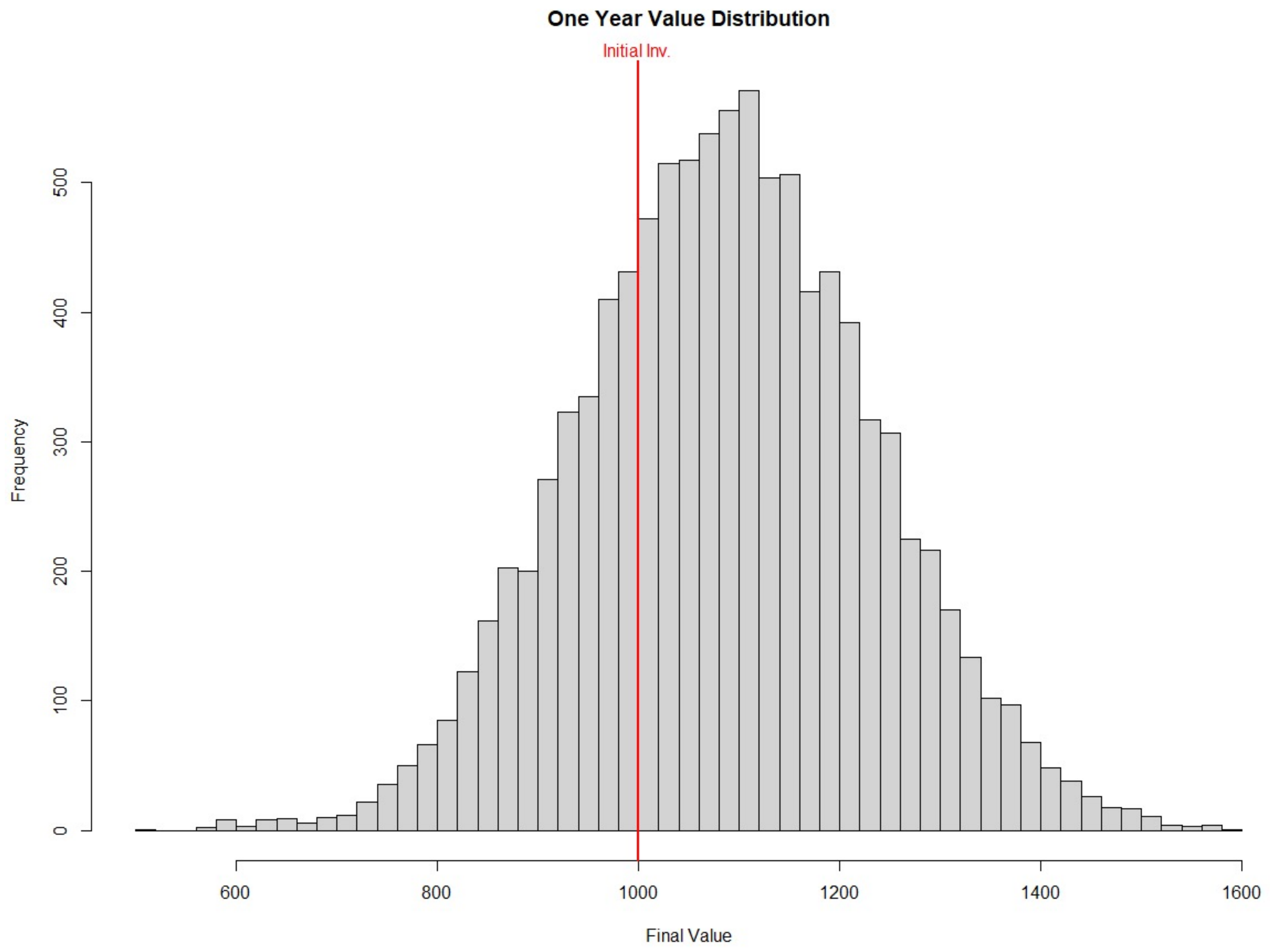
is a vector of 10000 long. vector means can do arithmetic on vector or take values out of vector then do..

all values go up by 1 in vector, then times by 1000

R and Python are vector languages so it helps. Idea is run eqn for a value, get output save that value. R is a vector language it can do loops but can go faster with vectors. THis below is vector way of doing it.
R get all numbers to start, then do an operation on them.
OR you could loop through this,
for (i in 1:10000)
p1[i] <- 1000*eqn
above takes lot longer to run

distributions give you a bunch of answers to qs

Introduction to Simulation – R



DISTRIBUTION SELECTION

Selecting Distributions

- When designing your simulations the biggest choice comes from the decision of the distribution on the inputs that vary.
- 3 Methods:
  1. Common Probability Distribution
  2. Historical (Empirical) Distribution
  3. Hypothesized Future Distribution

letting data do the shape

random

Common Probability Distributions

- Typically, we assume a common probability distribution for inputs that vary in a simulation.
- Common Discrete Distributions:
  1. Uniform Distribution
  2. Poisson Distribution

Common Probability Distributions

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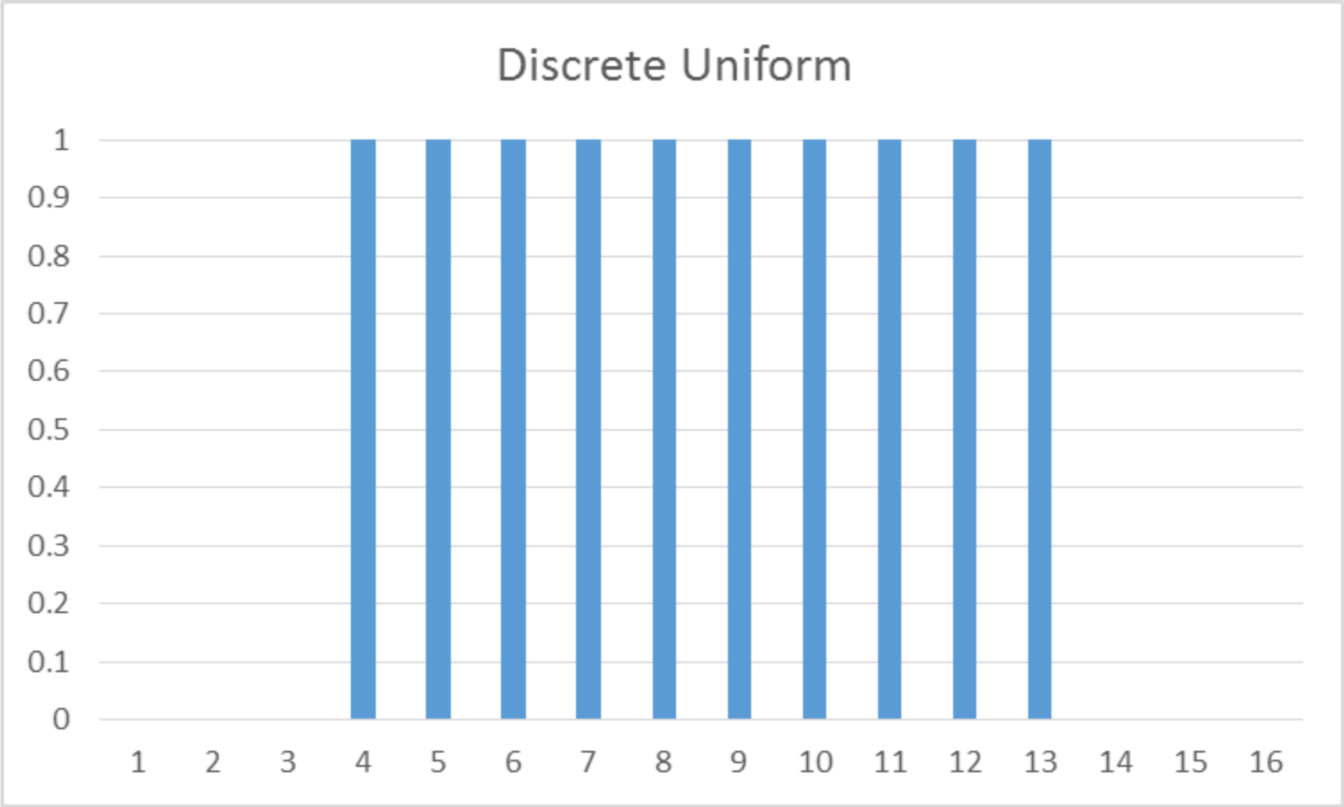
- Common Discrete Distributions:



Common Probability Distributions

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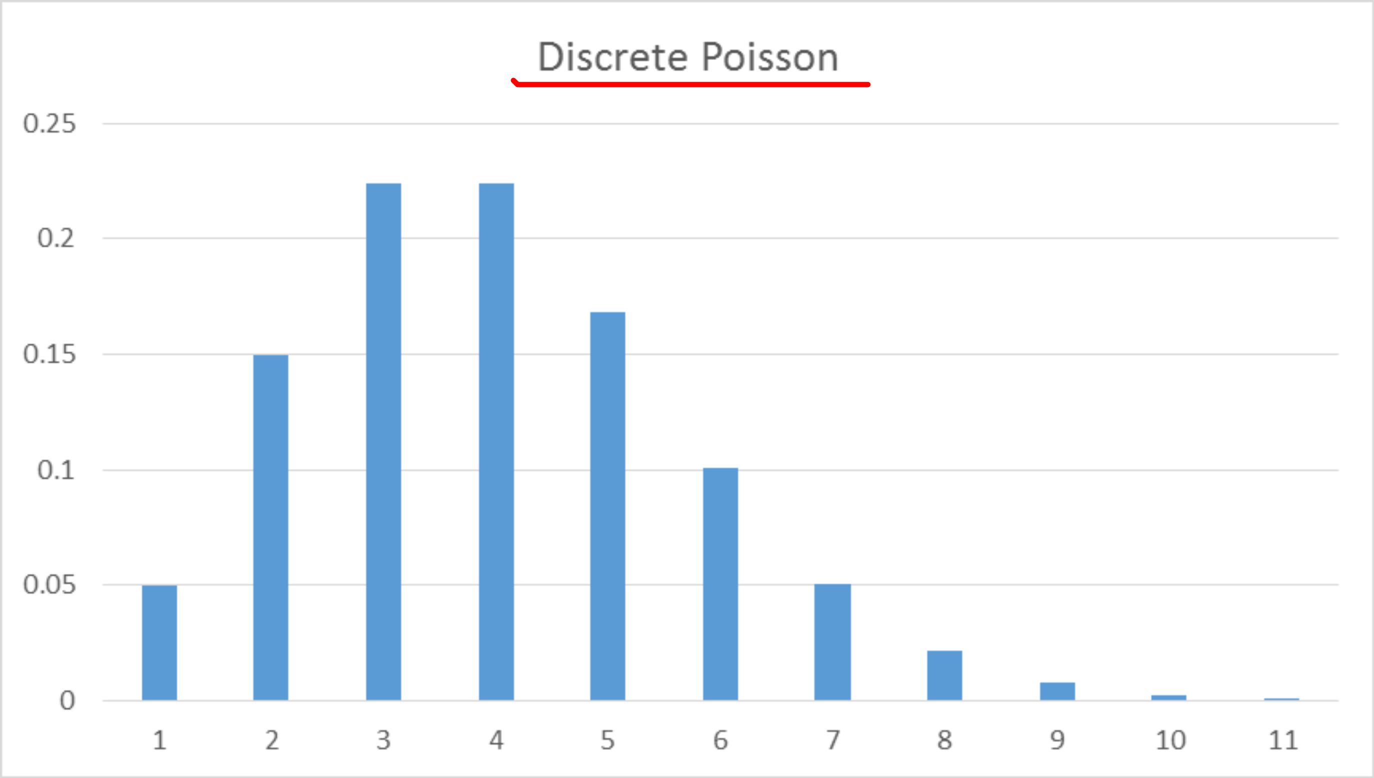

Common Probability Distributions

- Typically, we assume a common probability distribution for inputs that vary in a simulation.
- Common Continuous Distributions:
  1. Continuous Uniform Distribution
  2. Triangular Distribution
  3. Student’s t-Distribution
  4. Lognormal Distribution
  5. Normal Distribution
  6. Exponential Distribution
  7. Chi-Square Distribution
  8. Beta Distribution

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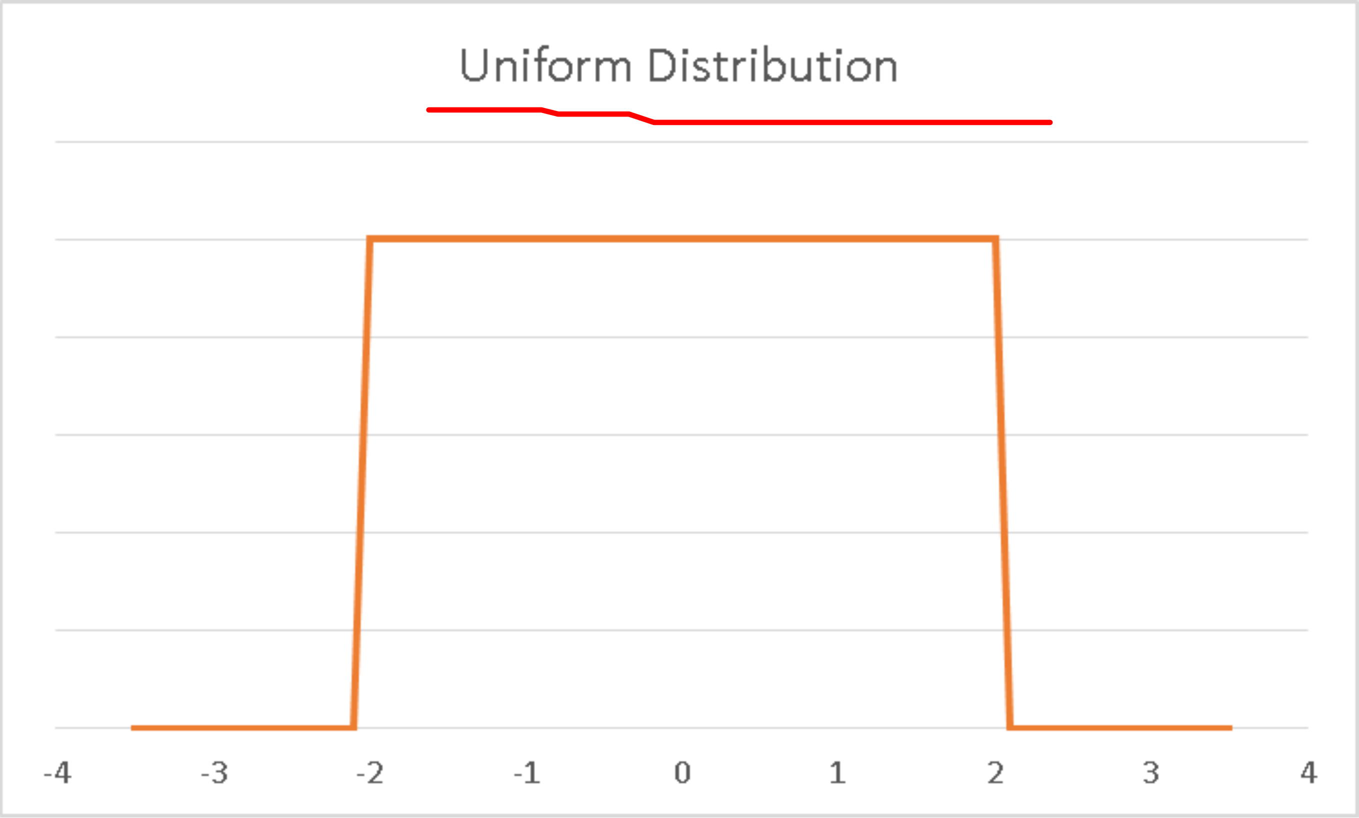

Common Probability Distributions

Something you can always get out of a client - what you think gonna happen (peak), best and wrost case.From those form triangle sim. Oil comapnies build distr on pricing based on 3 values

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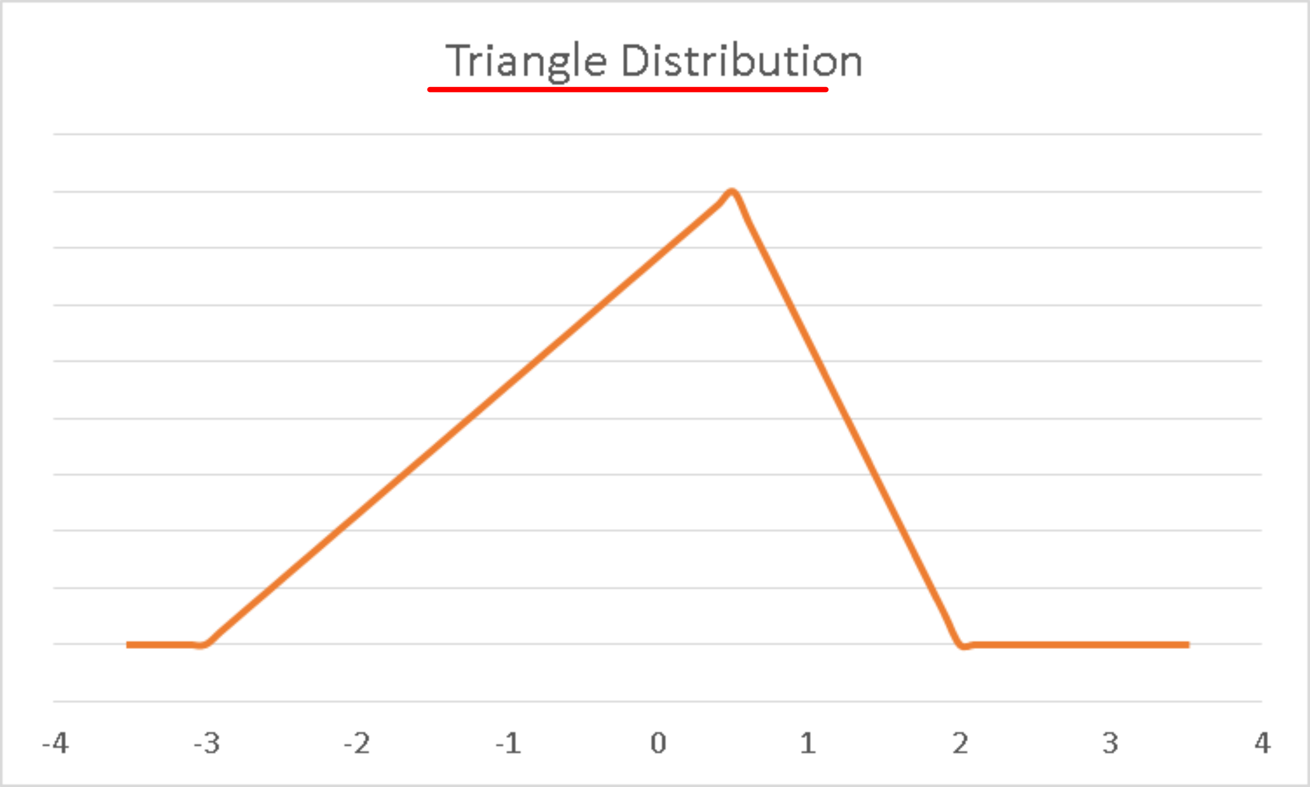
- Common Continuous Distributions:



Common Probability Distributions

t distr more kurtosis, thicker tails, more lee way than normal

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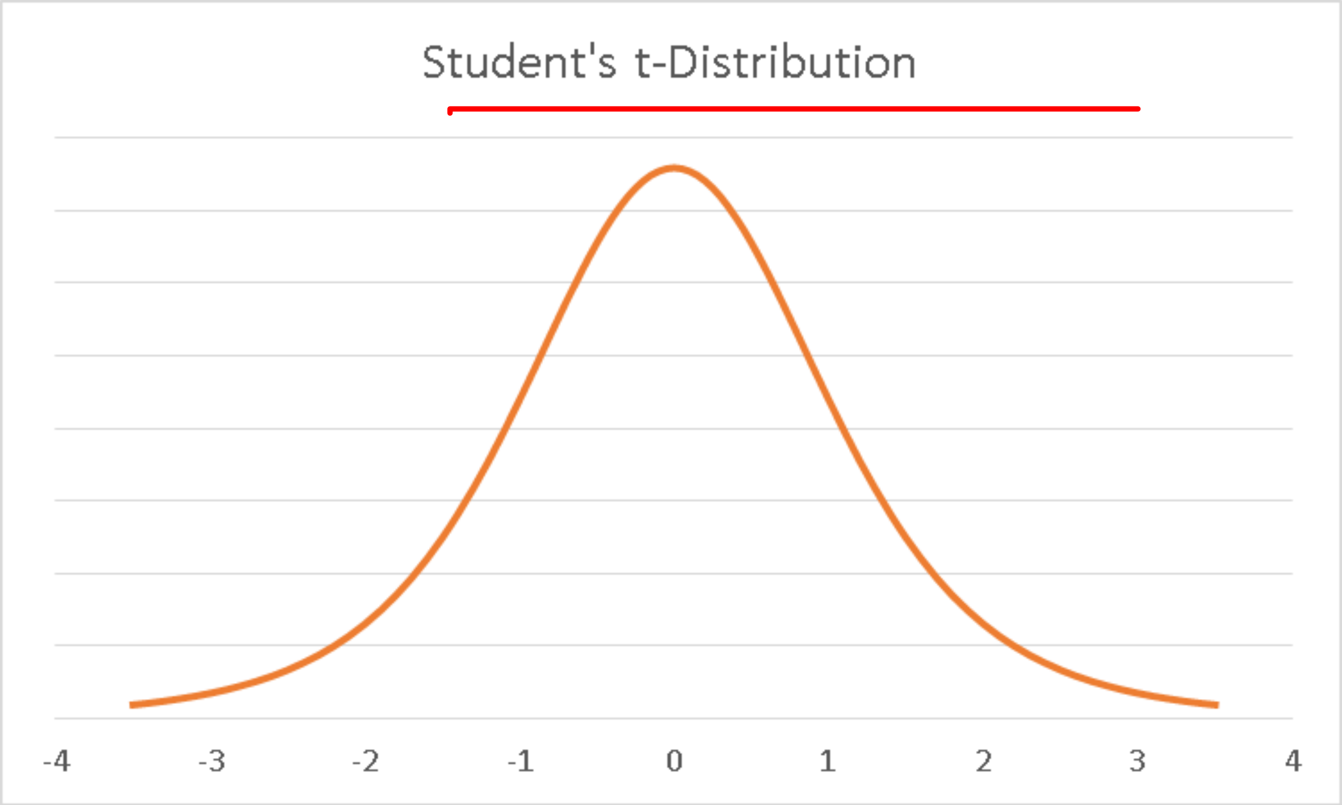

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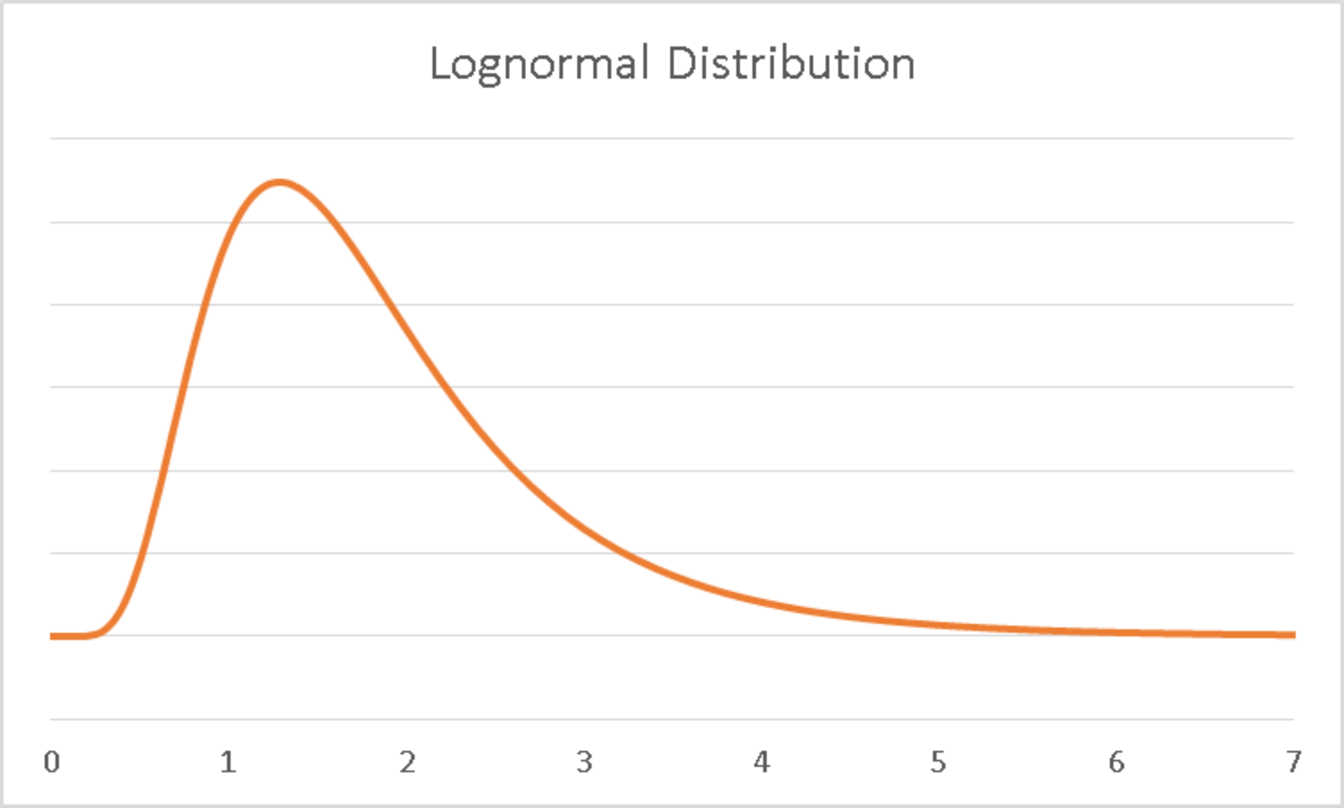

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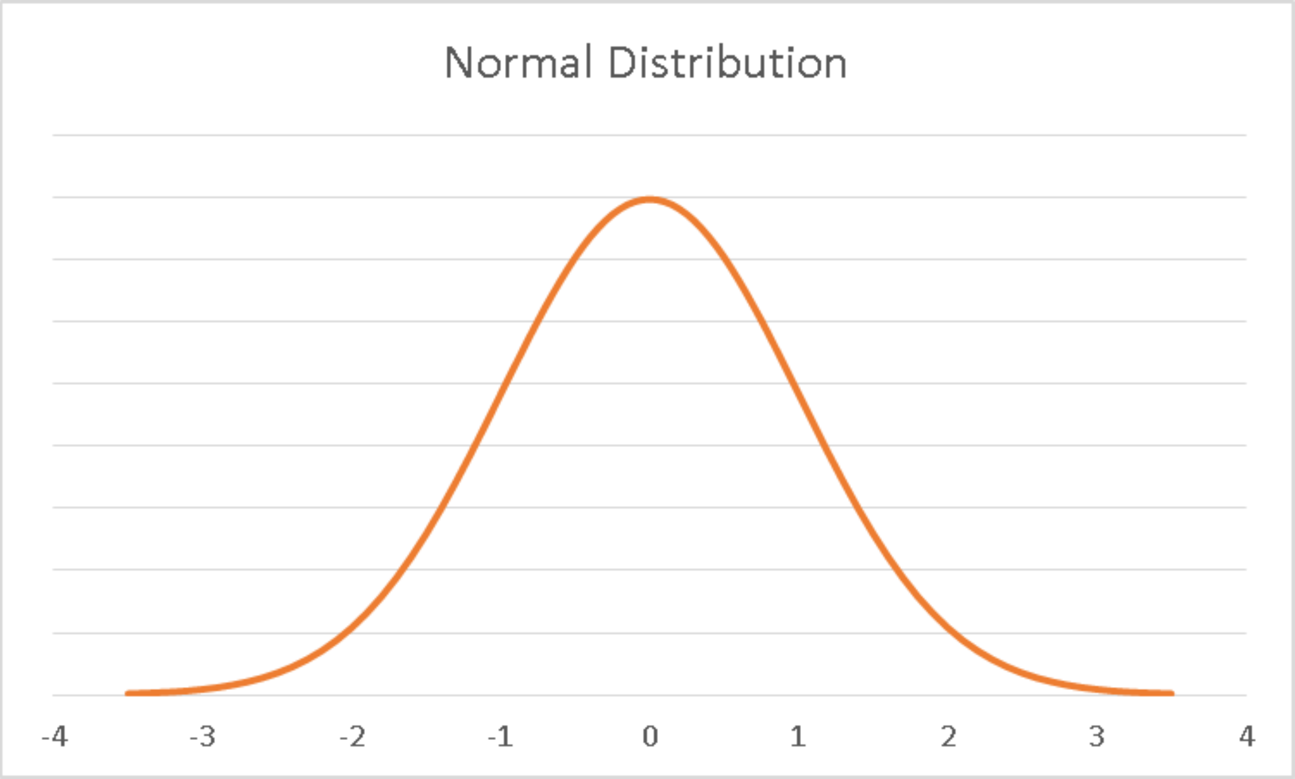

Common Probability Distributions

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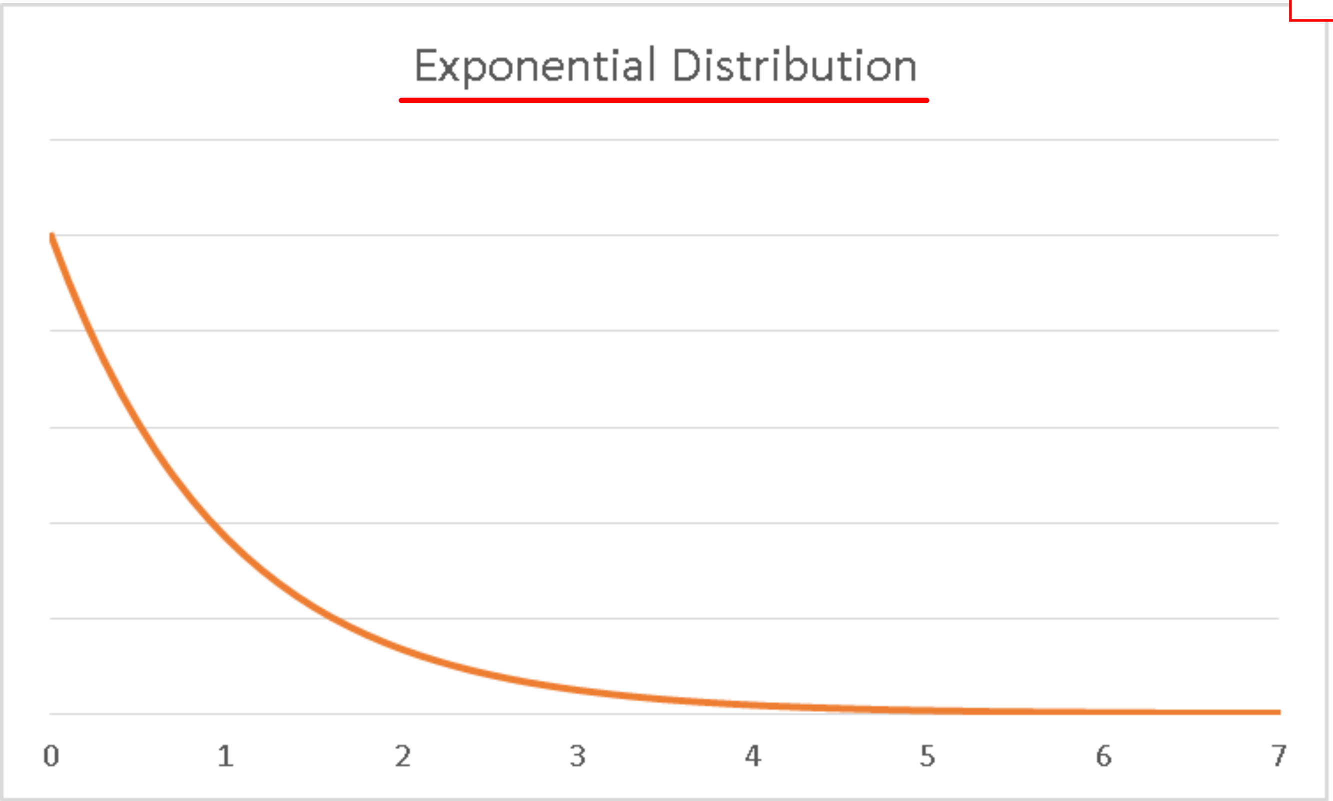
- Common Continuous Distributions:

used in investment, worst case tail scenarios.



Common Probability Distributions

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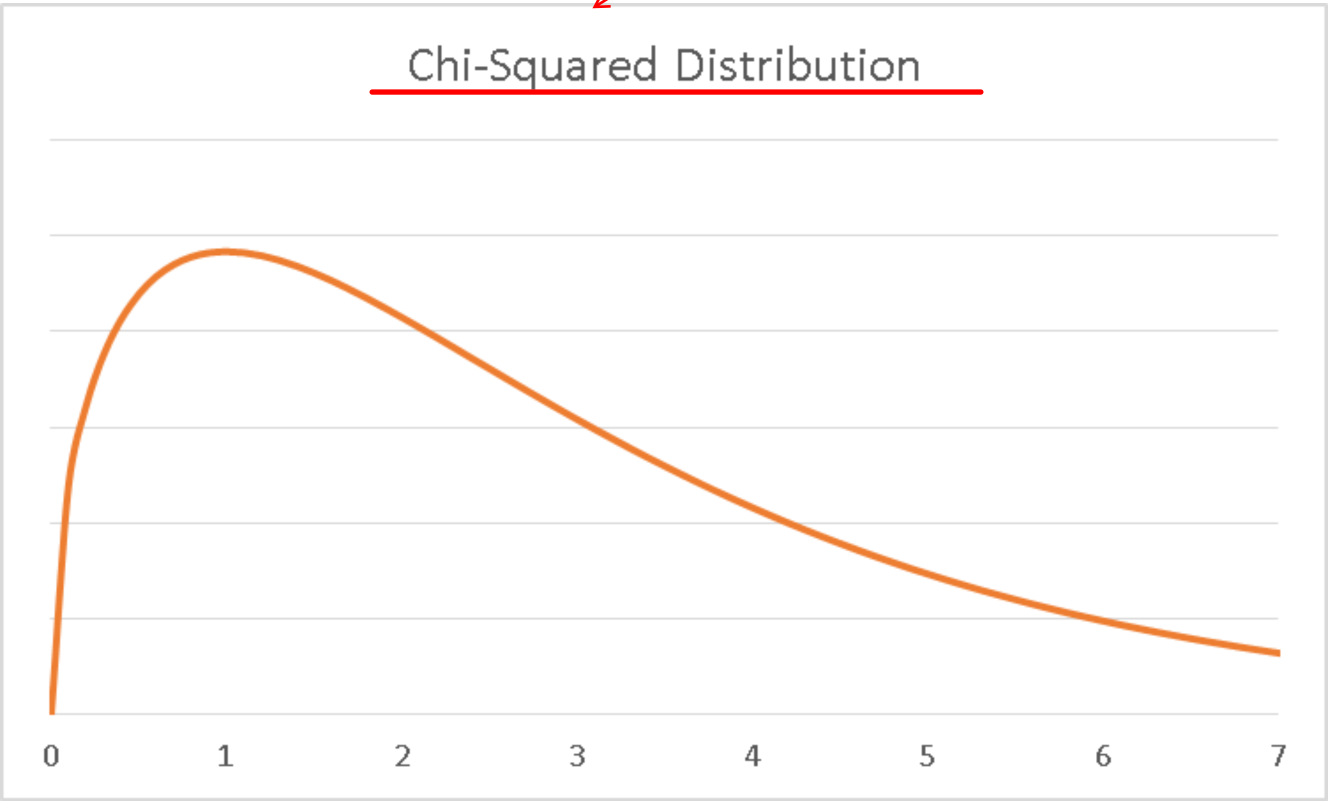
- Common Continuous Distributions:

if hump clsoer to 0



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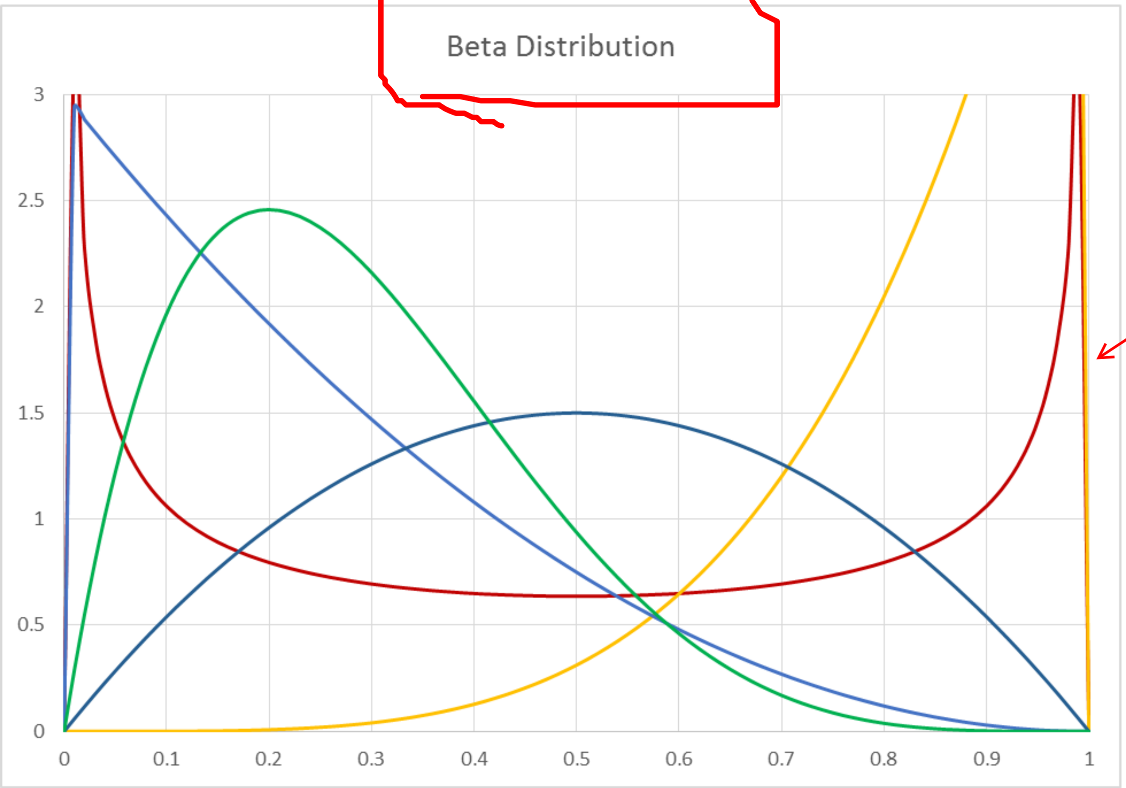

Beta distr =bathtub distribution. disr that can peak at 0 or 1. reallly high chance of sth happeneing lower chance of middle. Extremes are more likey to happen than middle

Historical (Empirical) Distributions

- If you are unsure of the distribution of the data you are trying to simulate, you can estimate it using **kernel density estimation**.
- Kernel density estimation is a non-parametric method of estimating distributions of data through smoothing out of data values.

Historical (Empirical) Distributions

- The Kernel density estimator is as follows:

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} \kappa\left(\frac{x - x_i}{h}\right)$$

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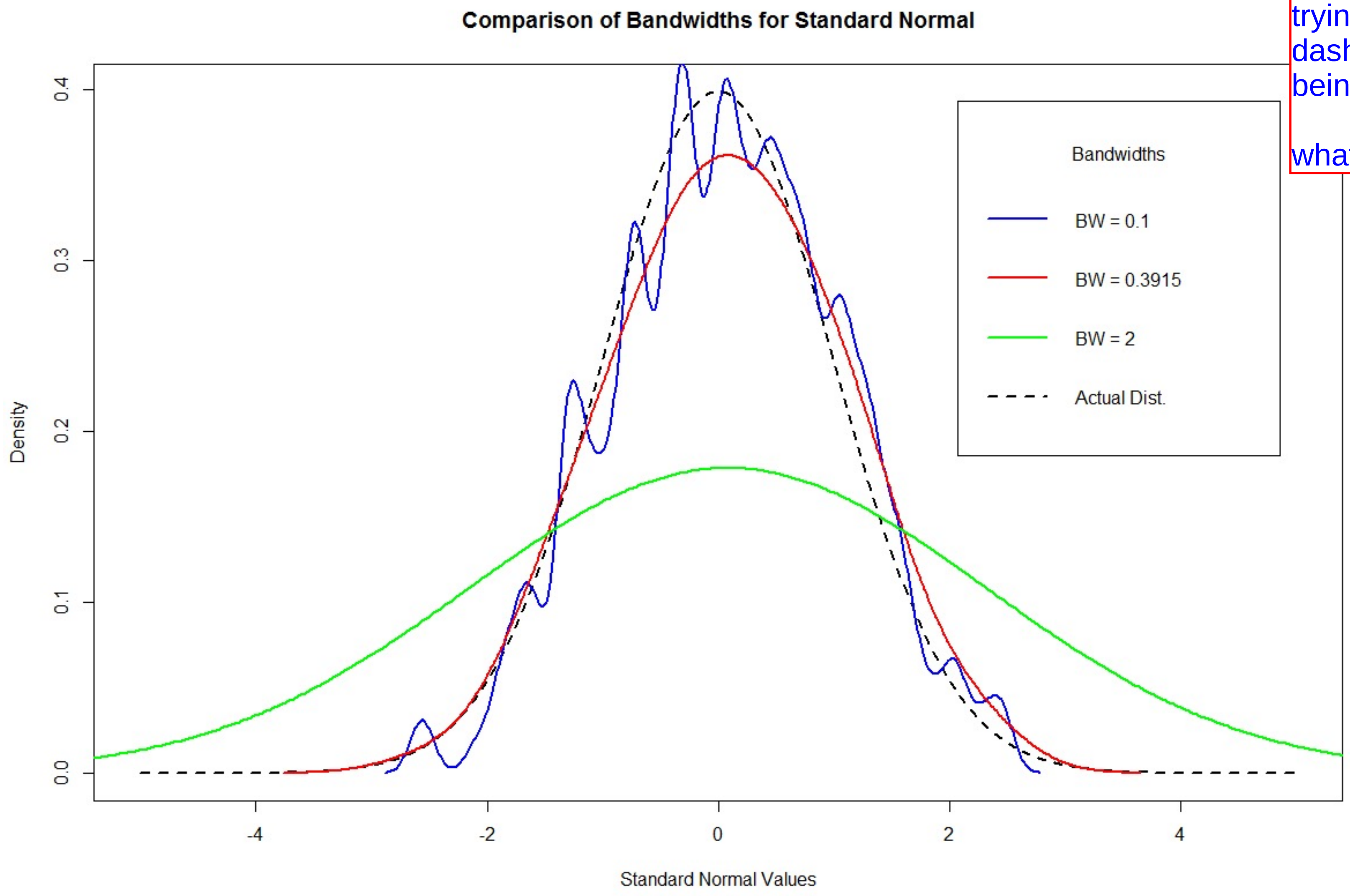
Bandwidth

Bandwidth Comparison

as bandwidth gets bigger , it smooths it out. every nook and cranny estimated with bandwidth too small. Red line is trying to get close est to dashed oline without being too wiggly.

what is this variability of?



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Kernel Function

underlying distr of what you building smooth function off of. Kernel fucntion based on little distr you build off of

- Typical Kernel functions:
  1. Normal
  2. Quadratic
  3. Triangular
  4. Epanechnikov

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SAS

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SAS

R

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SAS – default

R – default

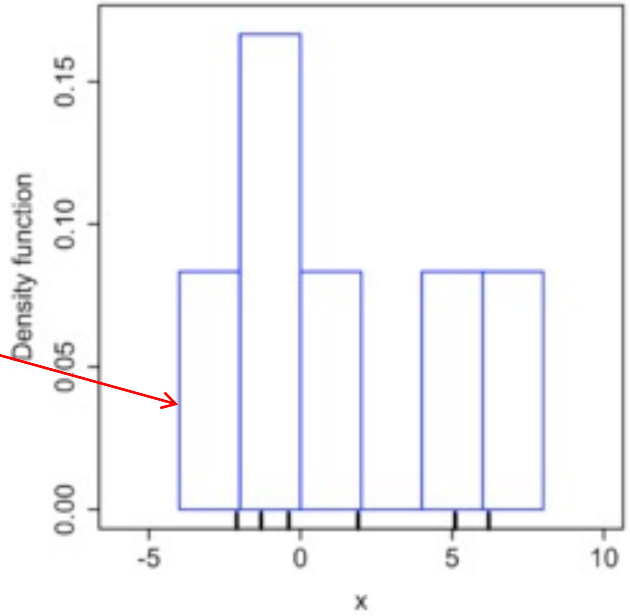
Historical (Empirical) Distributions

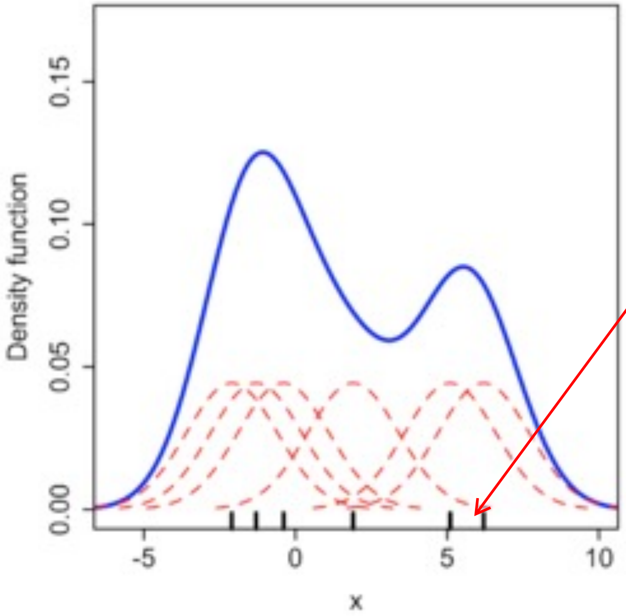
- The Kernel density estimator is as follows:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} \kappa\left(\frac{x - x_i}{h}\right)$$

- Assume Normal Kernel function:

only 6 obs here, black lines, they form hist on left hand side.





each black line has normal distr built on it (mean and std dev needed). Each norm distr is centered around the data point.

let computer estimate optimal bandwidth for us. add those norm distr. See dual peaked distr on right. If you change underlying kernel, you change the underlying distr little ones

GOal is trying to build density curve.

Historical (Empirical) Distributions

- The Kernel density estimator is as follows:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} \kappa\left(\frac{x - x_i}{h}\right)$$

- Once you have the Kernel density function, you can sample from this density function.

Historical (Empirical) Distributions – R

goes back to 2007

convert to daily or yearly etc returns

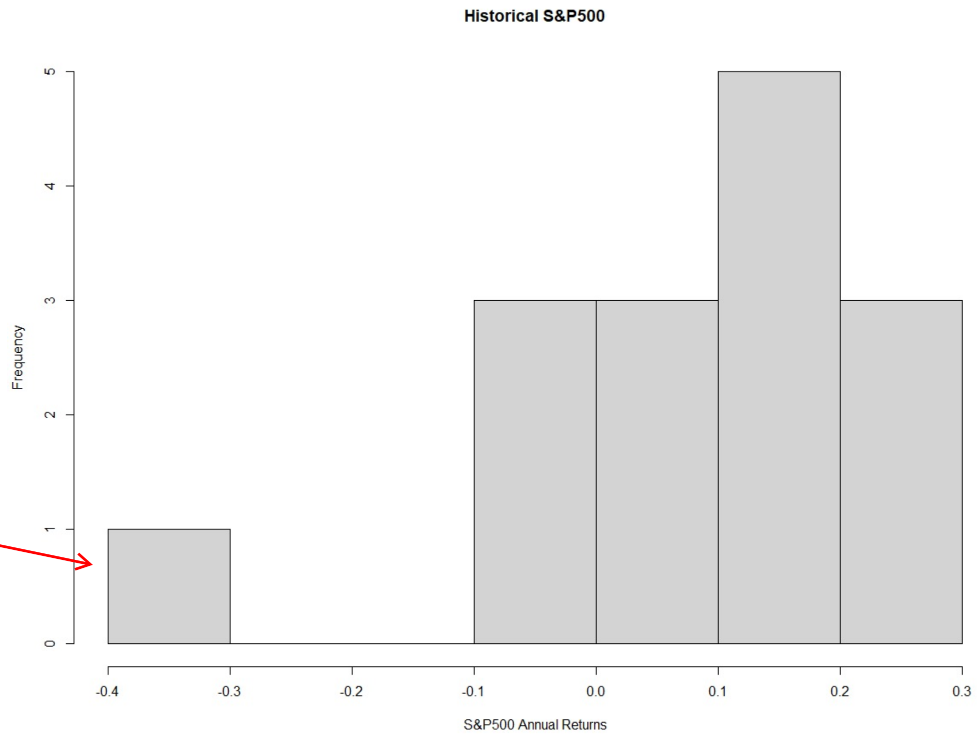
```
tickers = "^GSPC"
getSymbols(tickers)

gspc_r <- periodReturn(GSPC$GSPC.Close, period = "yearly")

hist(gspc_r, main='Historical S&P500', xlab='S&P500 Annual Returns')
```

Frequency VS Annual Returns (-0.4 to)

recession



Historical (Empirical) Distributions – R

Steps: Have my data annual returns, step 2 density function onthose returns to get 1 # bandwidth (optimal bandwidth)

```
Density.GSPC <- density(gspc_r)
Density.GSPC
```

```
## Call:
##  density.default(x = gspc_r)
##
## Data: gspc_r (15 obs.);  Bandwidth 'bw' = 0.06908
##
##        x                   y
##  Min.   :-0.59211   Min.   :0.004325
##  1st Qu.:-0.31827   1st Qu.:0.123180
##  Median :-0.04442   Median :0.378304
##  Mean   :-0.04442   Mean   :0.911823
##  3rd Qu.: 0.22942   3rd Qu.:1.795512
##  Max.   : 0.50326   Max.   :2.620657
```

Historical (Empirical) Distributions – R

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##  3rd Qu.: 0.22942   3rd Qu.:1.795512
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```

random kernel density est

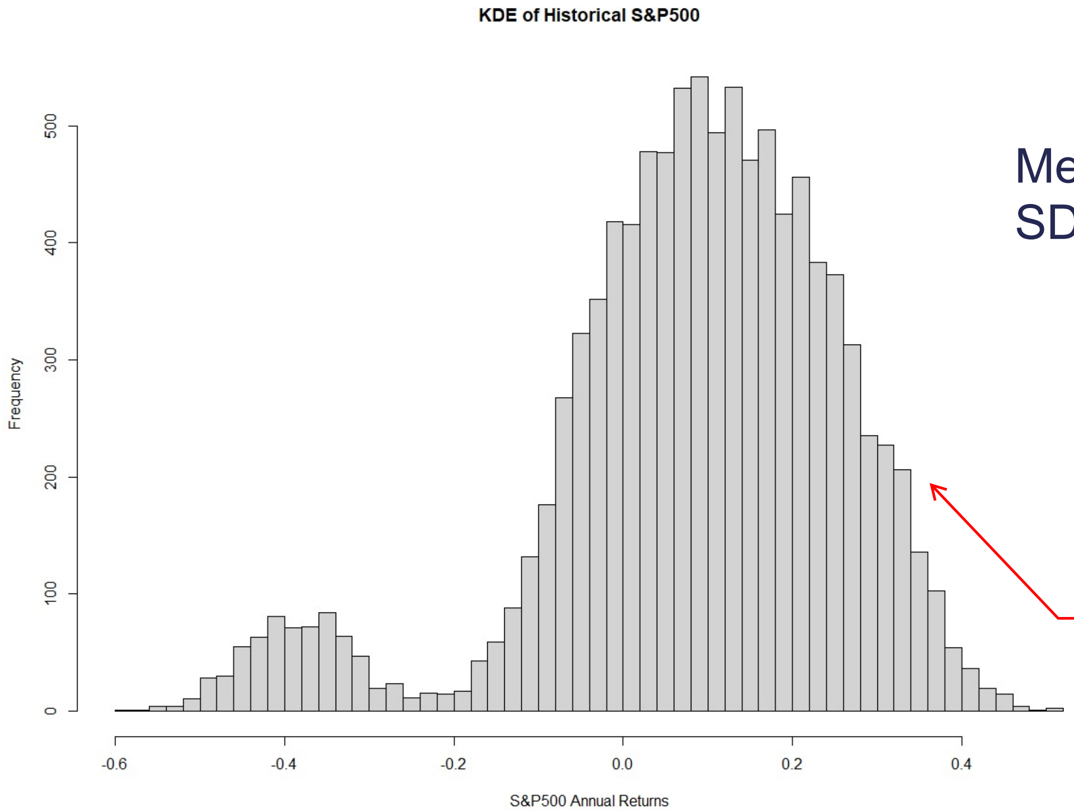
```
Est.GSPC <- rkde(fhat=kde(gspc_r, h=0.06908), n=1000)
```

draw 1000 obs from that kernel density estimate

Historical (Empirical) Distributions – R

```
Est.GSPC <- rkde(fhat=kde(gspc_r, h=0.06908), n=1000)

hist(Est.GSPC, breaks=50, main='KDE of Historical S&P500’,
     xlab='S&P500 Annual Returns')
```

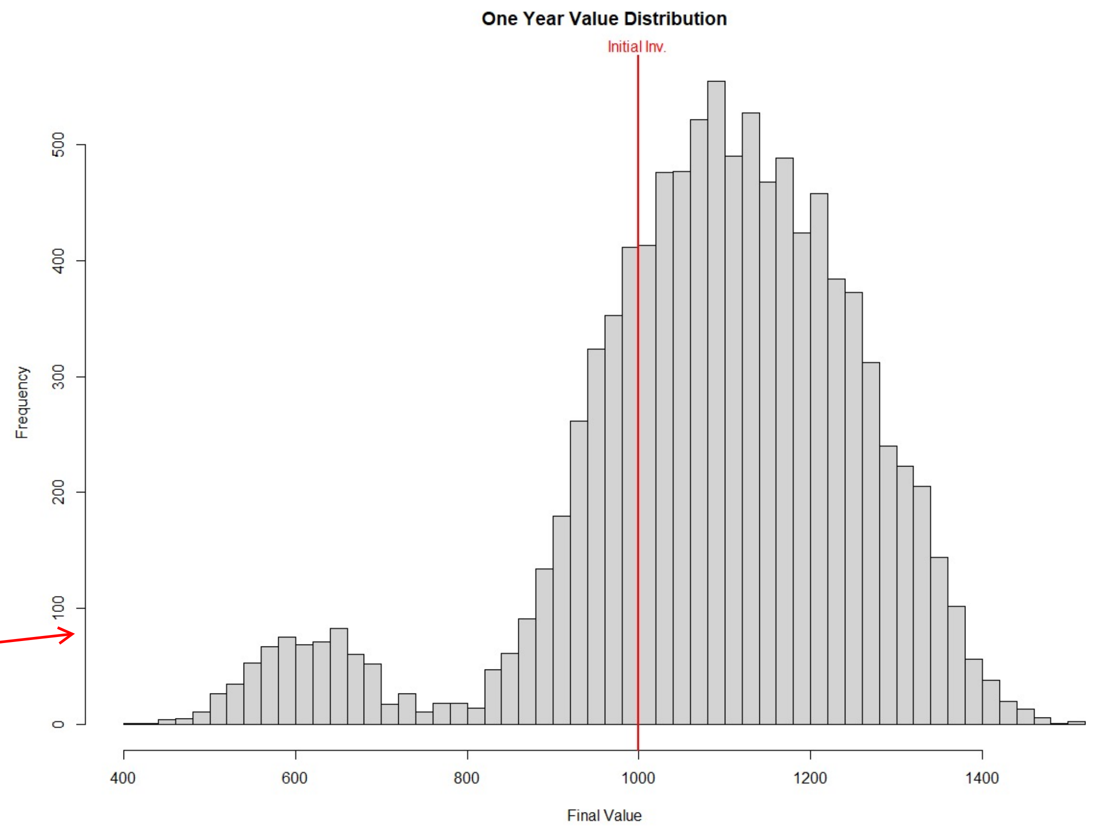


Mean = 8.22%
SD = 17.74%

this is what returns would look like with kernel density estimation built on top of histogram oriingal.

Historical (Empirical) Distributions – R

```
r <- Est.GSPC
P0 <- 1000
P1 <- P0*(1+r)
```



this bump too high due to smaller data size

Historical (Empirical) Distributions

bandwidth can be smaller with large n. you need 1000s of obs to get a good kernel density estiamte. to really trust it.

- The Kernel density estimator is as follows:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}\kappa\left(\frac{x - x_i}{h}\right)$$

- Once you have the Kernel density function, you can sample from this density function.

- WARNING: Sample size matters!
  1. If you have large sample sizes, your bandwidth can be smaller and your estimates more accurate.
  2. If you have small sample sizes, your bandwidth increases and estimates are more smoothed.

Hypothesized Future Distribution

- Maybe you know of an upcoming change that will occur in your variable so that the past information is not going to be the future distribution.
- Example:
  - The volatility of the market is forecasted to increase, so instead of a standard deviation of 14.75% it is 18.25%.
- In these situations, you can select any distribution of choice.

COMPOUNDING AND CORRELATIONS

Multiple Input Probability Distributions

- Complication arises when you are now simulating multiple inputs changing at the same time.
- Even when the distributions of these inputs are the same, the final result can still be hard to mathematically calculate – benefit of simulation.

Multiple Input Probability Distributions

normail+normal=normal is rare exception

- General Facts:
  1. When a constant is added to a **random variable** (the variable with the distribution) then the distribution is the same, only shifted by the constant.
  2. The addition of many distributions that are the same is rarely the same shape of distribution – exception would be INDEPENDENT Normal distributions.
  3. The product of many distributions that are the same is rarely the same shape of distribution – exception would be INDEPENDENT lognormal distributions (popular in finance for this reason).

simulation is for be able to find what kind of distr comes out

Example

Over 30 y we will see we wont get normal distr on th eother side.

- You want to invest $1,000 in the US stock market for **thirty** years.
- You invest in a mutual fund that tries to produce the same return as the S&P500 Index.

$$P_t = P_0 * (1 + r_{0,1})(1 + r_{1,2})(1 + r_{2,3}) \ldots (1 + r_{t-1,t})$$

Example

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Annual Returns

Example

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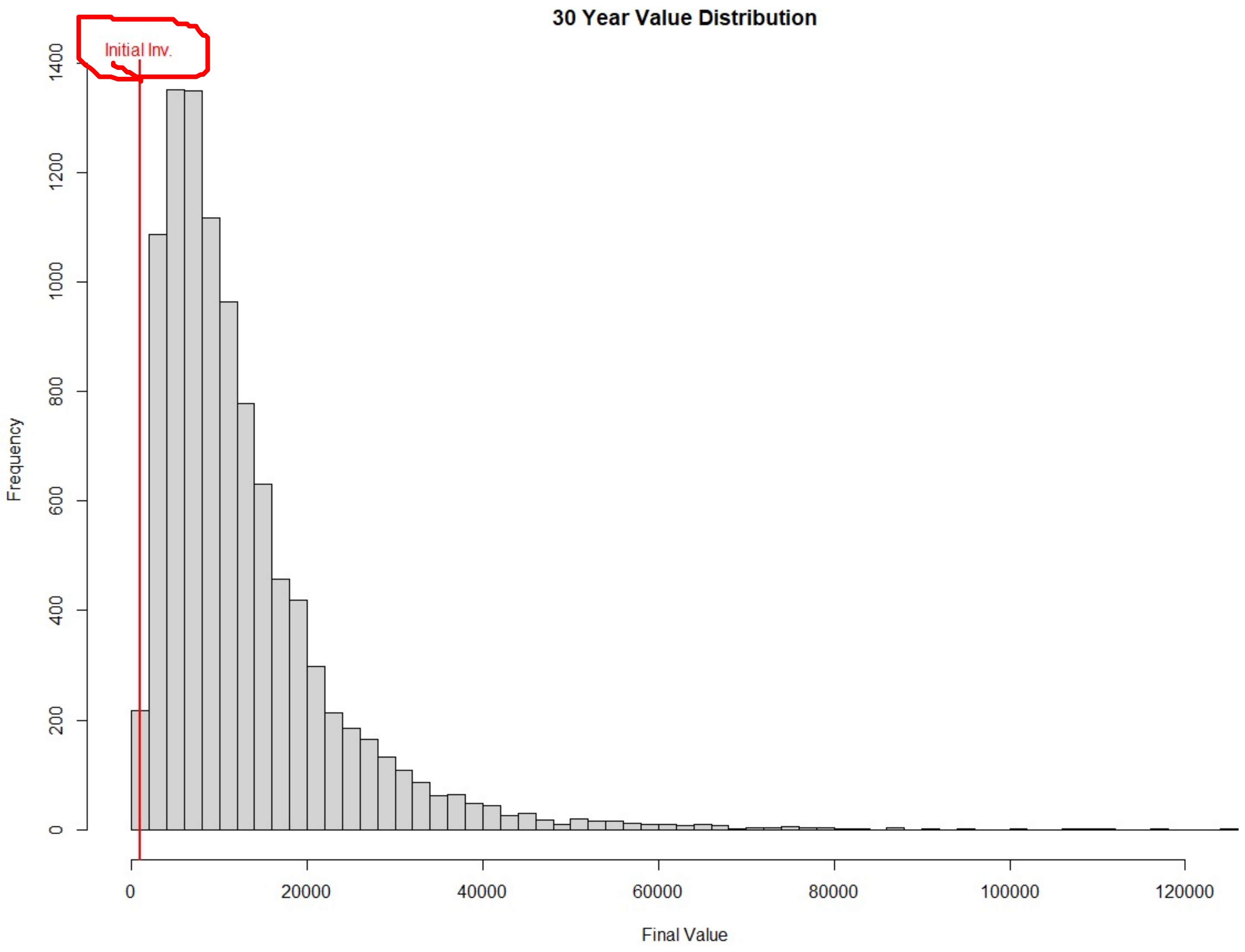
$$P_t = P_0 * (1 + r_{0,1})(1 + r_{1,2})(1 + r_{2,3}) \ldots (1 + r_{t-1,t})$$

- Assume annual returns follow a Normal distribution with historical mean of 8.79% and std. dev. of 14.75% every year.

Example

Product of normal distr is not Normal distr. It is chi sq distr.
You would hope this was not a normal distr cuz why would anyone invest in it. You want long tail on right

Frequency VS Portfolio final value - distr based on simulation



Multiple Input Prob. Distribution – R

```
P30 <- rep(0,10000)
for(i in 1:10000){
  P0 <- 1000
  r <- rnorm(n=1, mean=0.0879, sd=0.1475)

  Pt <- P0*(1 + r)

  for(j in 1:29){
    r <- rnorm(n=1, mean=0.0879, sd=0.1475)
    Pt <- Pt*(1+r)
  }
  P30[i] <- Pt
}

hist(P30, breaks=50, main='30 Year Value Distribution',
     xlab='Final Value')
```

this is loop way of doing it not vectorize. wanted to show you.

single value draw out

Never start with for loop. Always calc once first, then build for loop on top of it.

Correlated Inputs

- Not all inputs are independent of each other.
- Having correlations between your input variables adds even more complication to the simulation and final distribution.
- May need to simulate random variables that have correlation with each other.

Example

- You want to invest $1,000 in the US stock market **or US Treasury bonds** for **thirty** years.
- You invest a certain percentage in a mutual fund that tries to produce the same return as the S&P500 Index and the rest in US Treasury bonds.

$$P_{t,S} = P_{0,S} * (1 + r_{0,1})(1 + r_{1,2})(1 + r_{2,3}) \dots (1 + r_{t-1,t})$$

$$P_{t,B} = P_{0,B} * (1 + r_{0,1})(1 + r_{1,2})(1 + r_{2,3}) \dots (1 + r_{t-1,t})$$

$$P_t = P_{t,S} + P_{t,B}$$

Example

- You want to invest \$1,000 in the US stock market **or US Treasury bonds** for **thirty** years.
- You invest a certain percentage in a mutual fund that tries to produce the same return as the S&P500 Index and the rest in US Treasury bonds.
- Treasury bonds perceived as safer investment so when stock market does poorly more people invest in bonds – negatively correlated.

- Assume mutual fund Normal(8.79%, 14.75%).
- Assume Treasury Bond Normal(4.00%, 7.00%).
- Assume correlation of -0.2.

Adding Correlation

- One way to "add" correlation to data is to multiply the correlation into the data through matrix multiplication (linear algebra!).
- One variable example:
  - X ~ N(mean = 3, var = 2)
  - Want to have a variance of 4
  - What can we do?

Centrer around 0, cuz any number times 0 will be 0. It will alsways be centered.

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- One way to "add" correlation to data is to multiply the correlation into the data through matrix multiplication (linear algebra!).
- One variable example:
  - X ~ N(mean = 3, var = 2)
  - Want to have a variance of 4
  - What can we do?
    1. Standardize X → $\frac{X-3}{\sqrt{2}}$ → Z ~ N(mean = 0, var = 1)
    2. Multiply Z by $\sqrt{4}$ → $\sqrt{4}Z$ → Y ~ N(mean = 0, var = 4)
    3. Convert Y back → $Y + 3$ → Y ~ N(mean = 3, var = 4)

Same mean as X, but now has larger variance!

mutliply cuz you want it to be more spread out 4 instead of variance of 2.
Multiply by sq rt of 4 cuz you want same scale.
variance is swaured
have a distr, move it to 0, stretch it out, then move it back. Thats how you play with distr

Adding Correlation

doing correlation matrix, because bond returns and stock returns not independent. One goes up other goes down. year to year investment is assumed independent here but that could be correalted too. This is problem banks did when stress testing didnt account for correlation. Housing market carashes ppl lose jobs, all bad happeneding same time. cuz all correlated with it self. IN good, things are not correlated, in bad times things are correltaed all go down.

linear algerbra cuz its regular algebra on a matrix

- For multiple variables at the same time, we can use the variance matrix instead:
  - **X** has 2 columns with correlation matrix $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

    no correaltion bet x and y

  - Want to have a variance matrix of $\Sigma^* = \begin{bmatrix} 1 & -0.2 \\ -0.2 & 1 \end{bmatrix}$

    correlation of var with itself

    change spread is same as changing correaltion bet x and y. Gonna do it with matrix instead. Same three steps now applied to a matrix instead of single number.

    no such thing as sq rt of matrix. Instead it is cholesky decomp

  - What can we do?

    all columns of data matrix

    1. Standardize each column of **X** → means = 0, variances = 1 in **Z**
    2. Multiply **Z** by "square root" of Σ^* (Cholesky Decomposition)

       of variance

    3. Convert **Z** back → means and variances back to what they were before to get **Y**

function in every language for this

Cholesky Decomposition

- What is the square root of a number?
  - The square root is a number(s) that when multiplied by itself gives you the original value.
  - Ex: Square root of 4 is either -2 or 2 since both of those numbers when multiplied by themselves equal 4.
- What is the square root of a matrix?
  - The “square root” of a matrix is a matrix that when multiplied by itself gives you the original matrix.
  - This is called a **Cholesky decomposition**.
  - Ex: Cholesky decomp of $\Sigma^* = \begin{bmatrix} 1 & -0.2 \\ -0.2 & 1 \end{bmatrix}$ is $L = \begin{bmatrix} 1 & 0 \\ -0.2 & 0.98 \end{bmatrix}$ since $L \times L^T = \begin{bmatrix} 1 & -0.2 \\ -0.2 & 1 \end{bmatrix}$

Cholesky Decomposition

works well with normal distr, bends the second column to be more correalted with the first. Normal normal fine. Any other distr bell curve and expo distr, how can they be correalted with each other. CHolesky not perfect.

take log of something right tailed to make it more normal. then delog it

- How does it work in idea?
  - Takes the first column and leaves it alone. "Bends" the second column to be more correlated with the first.
- Cholesky decomposition works best when variables are normally distributed.
- It will be OK if they are symmetric and unimodal.
- If not either, put the column you want unchanged the most first.

Correlated Inputs – R

```
Value.r <- rep(0,10000)
R <- matrix(data=cbind(1,-0.2, -0.2, 1), nrow=2)
U <- t(chol(R))
Perc.B <- 0.5
Perc.S <- 0.5
Initial <- 1000

standardize <- function(x){
  x.std = (x - mean(x))/sd(x)
  return(x.std)
}

destandardize <- function(x.std, x){
  x.old = (x.std * sd(x)) + mean(x)
  return(x.old)
}
```

create corr matrix. off diagnols are your correlations.

transpose

returning vector of standardized

takes 2 inputs

Correlated Inputs – R

returns for stocks
returns for bonds
grabbing 30 of those cuz watching for 30y each come from their separate normal dist
at this point 2 distr random normal that are not correalted

how do we correalte them? std, multiply, destrandardize.

std version of those, both.r is a matrix

these 2 lines of code, are making long wide so you can multiply

cholesky decomp multiply with

3rd step destandardize

percentrage in bonds

inv in bonds

```
for(j in 1:10000){

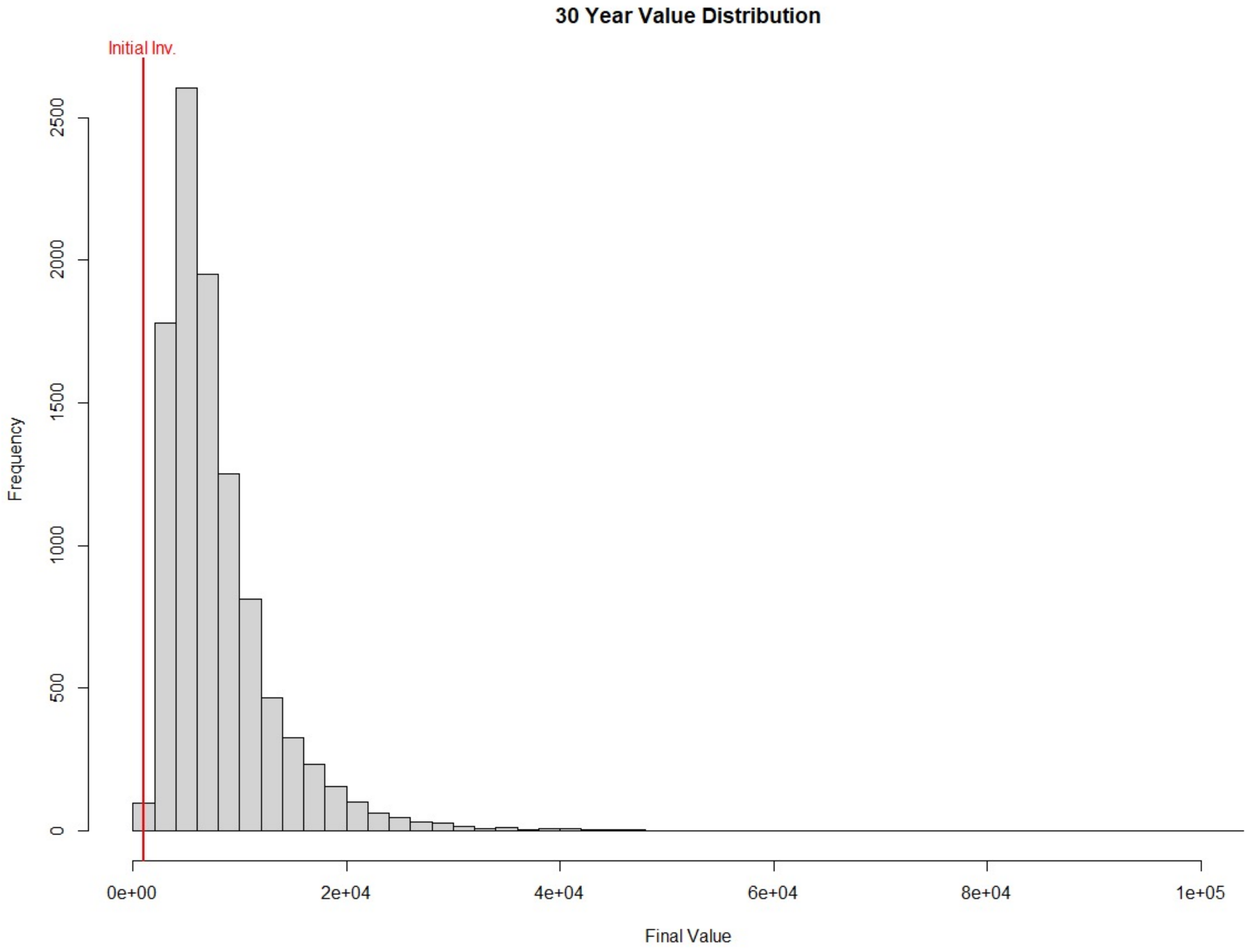
  S.r <- rnorm(n=30, mean=0.0879, sd=0.1475)
  B.r <- rnorm(n=30, mean=0.04, sd=0.07)
  Both.r <- cbind(standardize(S.r), standardize(B.r))
  SB.r <- U %*% t(Both.r)
  SB.r <- t(SB.r)

  final.SB.r <- cbind(destandardize(SB.r[,1], S.r),
                      destandardize(SB.r[,2], B.r))

  Pt.B <- Initial*Perc.B
  Pt.S <- Initial*Perc.S
  for(i in 1:30){
    Pt.B <- Pt.B*(1 + final.SB.r[i,2])
    Pt.S <- Pt.S*(1 + final.SB.r[i,1])
  }
  Value.r[j] <- Pt.B + Pt.S
}
```

Correlated Inputs – R

how much money in stocks or bonds? Can do simualation or just optimization method.



Evaluating Decisions

- Careful about only using summary statistics to evaluate the decisions to be made from simulations.
- Need to look at whole picture – whole distribution.
- Example:
  - Which is “better” – 50/50 stocks/bonds (Strategy A) or 30/70 stocks/bonds (Strategy B)?

Evaluating Decisions – R

```
Perc.B <- 0.7
Perc.S <- 0.3
for(j in 1:10000){

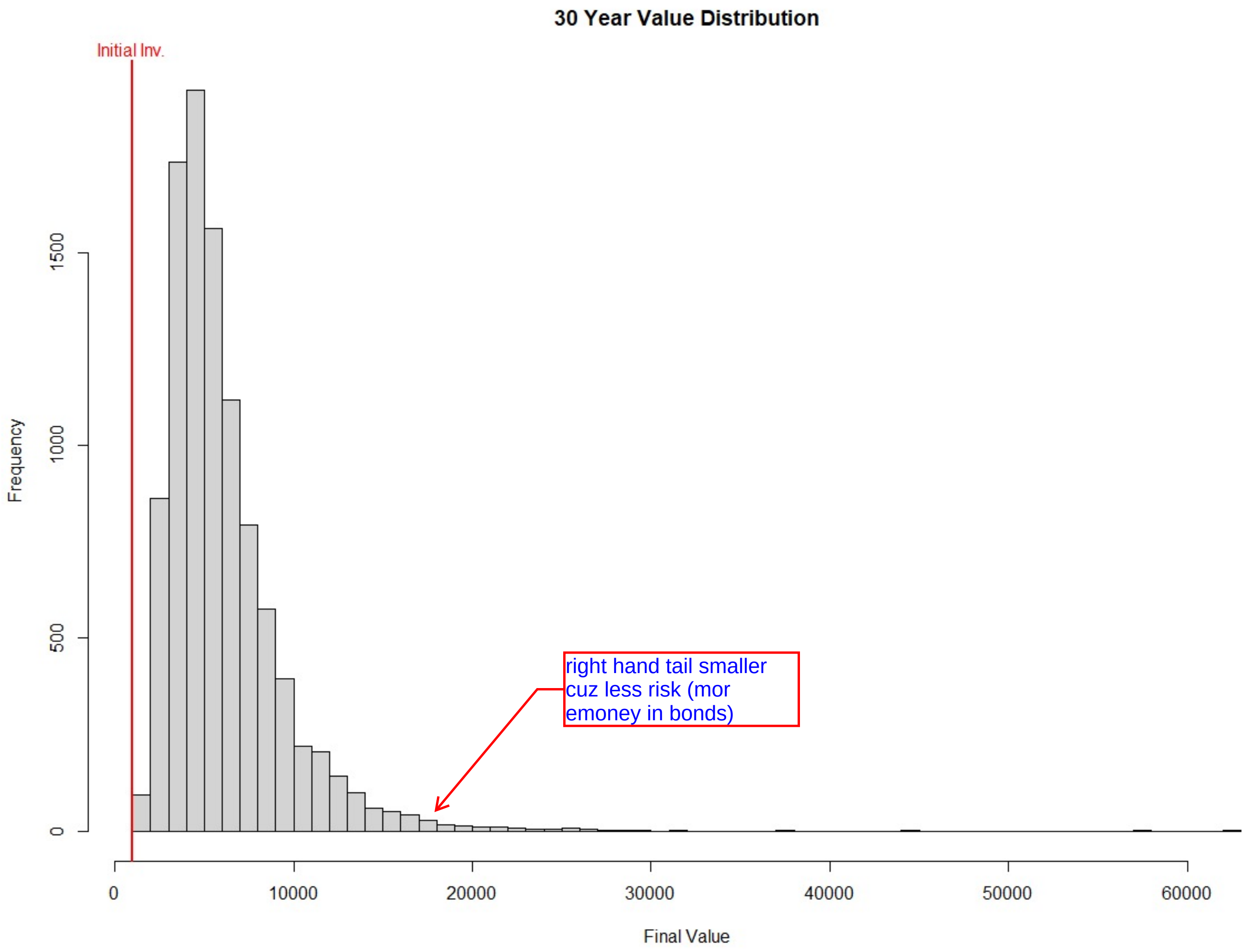
  S.r <- rnorm(n=30, mean=0.0879, sd=0.1475)
  B.r <- rnorm(n=30, mean=0.04, sd=0.07)
  Both.r <- cbind(standardize(S.r), standardize(B.r))
  SB.r <- U %*% t(Both.r)
  SB.r <- t(SB.r)

  final.SB.r <- cbind(destandardize(SB.r[,1], S.r),
                      destandardize(SB.r[,2], B.r))

  Pt.B <- Initial*Perc.B
  Pt.S <- Initial*Perc.S
  for(i in 1:30){
    Pt.B <- Pt.B*(1 + final.SB.r[i,2])
    Pt.S <- Pt.S*(1 + final.SB.r[i,1])
  }
  Value.r[j] <- Pt.B + Pt.S
}
```

same as before now just change % in bons and stocks

Evaluating Decisions – R



watch video of this.

Evaluating Decisions

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- Example:
  - Which is "better" – 50/50 stocks/bonds (Strategy A) or 30/70 stocks/bonds (Strategy B)?
  - Mean return of Strategy A – $7,904
  - Mean return of Strategy B – $6,042
  - C.V. of returns for Strategy A – 66.51%
  - C.V. of returns for Strategy B – 52.35%

bigger number for CV
means more spread out

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  - Mean return of Strategy A – \$7,904
  - Mean return of Strategy B – \$6,042
  - C.V. of returns for Strategy A – 66.51%
  - C.V. of returns for Strategy B – 52.35%
  - Strategy A has higher return but APPEARS riskier.

Evaluating Decisions

if investing in sth, want spread of distr to be on the right side. Symettry is good when comparing means and std dev. When dont have symmetry then dont compare mean and std dev. A is more spread out but more spread out in your favor. They both have same chance same 5th percentile but strategy A has 95th percentile of making more more money. if tail is where benefit is , then you want that.

- Careful about only using summary statistics to evaluate the decisions to be made from simulations.
- Need to look at whole picture – whole distribution.
- Example:
  - Which is "better" – 50/50 stocks/bonds (Strategy A) or 30/70 stocks/bonds (Strategy B)?
  - 5th Percentile of Strategy A – \$2,944
  - 5th Percentile of Strategy B – \$2,839
  - 95th Percentile of Strategy A – \$17,558
  - 95th Percentile of Strategy B – \$11,719
  - Strategy A has **less** downside, but **higher** upside.

Evaluating Decisions

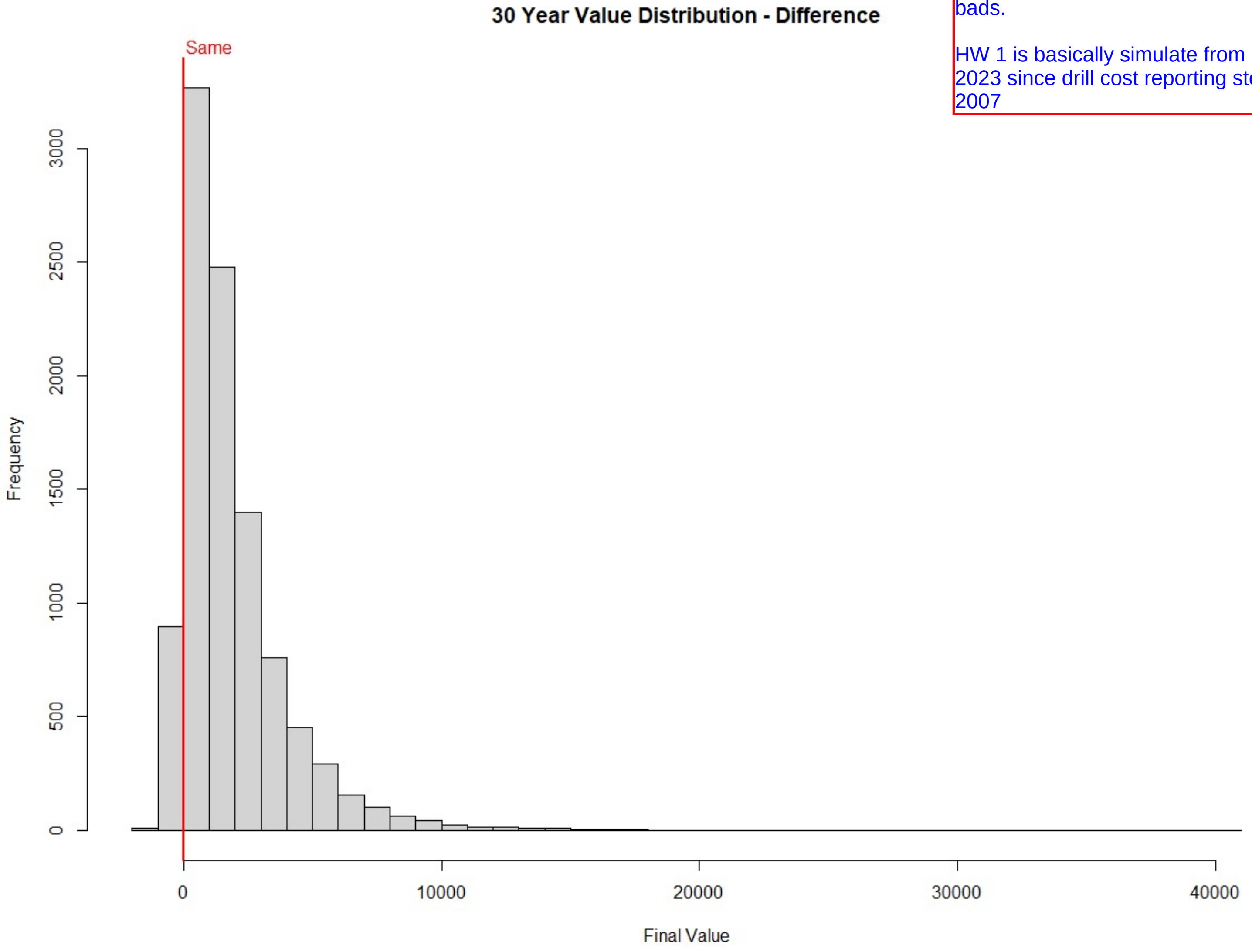
- Careful about only using summary statistics to evaluate the decisions to be made from simulations.
- Need to look at whole picture – whole distribution.
- Standard deviation is not always a good measure of riskiness.
- Higher standard deviation not necessarily bad if the largest deviations from the mean are on the upside!

Difference (A – B) – R

```
Value.r.diff <- Value.r.bal - Value.r.unbal

hist(Value.r.diff, breaks=50,
     main='30 Year Value Distribution - Difference',
     xlab='Final Value')
abline(v = 0, col="red", lwd=2)
mtext("Same", at = 0, col = "red")
```

Difference (A – B) – R



A always beats..

More complesx you have more more samples n run.
if worst case modelling, then need even more samples more good to see few bads.

HW 1 is basically simulate from 2007 to 2023 since drill cost reporting stopped in 2007

HOW MANY AND HOW LONG?

Accuracy vs. Time

- The possible number of outcomes for a simulation output variable is basically infinite.
- We need to get a “sampling” of these values.
- Accuracy of the estimates depends on the number of simulated values.
- **How many simulations do you need to run?**

Accuracy vs. Time

- **How many simulations do you need to run?**
- Confidence interval theory in statistics helps reveal the relationship between accuracy and number of simulations.
- Imagine you are interested in the mean value of the output distribution from your simulation.
- We know the margin of error of the mean!

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Number of simulated values

Standard deviation from simulated values

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- To **double the accuracy** we need to approximately **quadruple the number of scenarios**.

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- To **double the accuracy** we need to approximately **quadruple the number of scenarios**.