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Bài 1:

a, $A = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$ $A.P = \lambda.P$

$\Leftrightarrow (\lambda.I - A).P = 0$

$\Rightarrow \det(\lambda I - A) = 0$

$\Leftrightarrow \begin{vmatrix} \lambda+1 & -3 \\ 2 & \lambda-4 \end{vmatrix} = 0 \Leftrightarrow (\lambda+1)(\lambda-4)+6=0$

$\Leftrightarrow (\lambda-1)(\lambda-2)=0$

$\Rightarrow \left[\begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 2 \end{array}\right.$

* Với $\lambda = 1 \Rightarrow (\lambda I - A) = \begin{pmatrix} 2 & -3 \\ 2 & -3 \end{pmatrix} \xrightarrow{d_2 - d_1} \begin{pmatrix} 2 & -3 \\ 0 & 0 \end{pmatrix}$

$\xrightarrow{\cancel{d_2 - d_1}}$ $\Rightarrow 2x_1 - 3x_2 = 0$

$\Leftrightarrow \begin{cases} x_1 = \frac{3}{2}t \\ x_2 = t \end{cases} \to v_1 = \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix} t \quad t \in R$

* Với $\lambda = 2 \Rightarrow \lambda I - A = \begin{pmatrix} 3 & -3 \\ 2 & -2 \end{pmatrix} \xrightarrow{d_2 - \frac{2}{3}d_1} \begin{pmatrix} 3 & -3 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow 3x_1 - 3x_2 = 0 \Rightarrow x_1 = x_2$

$\Rightarrow \begin{cases} x_1 = t \\ x_2 = t \end{cases} (t \in R) \Rightarrow v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} t \quad (t \in R)$

$\Rightarrow$ $\lambda = 2$ là trị riêng của A với vector riêng $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$

Ta có $P = \begin{pmatrix} \frac{3}{2} & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow P^{-1}.A.P = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = D$

$P^{-1} = \begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}$

$\Rightarrow A^k = P.D^k.P^{-1} = \begin{pmatrix} \frac{3}{2} & 1 \\ 1 & 1 \end{pmatrix} . \begin{pmatrix} 1^k & 0 \\ 0 & 2^k \end{pmatrix} \begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{2} & 2^k \\ 1 & 2^k \end{pmatrix} \begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3 - 2^{k+1} & -3 + 3.2^k \\ 2 - 2^{k+1} & -2 + 3.2^k \end{pmatrix}$

b, $B = \begin{pmatrix} 5 & 2 \\ 9 & 2 \end{pmatrix} \Rightarrow \det(\lambda I - B) = \begin{vmatrix} \lambda - 5 & -2 \\ -9 & \lambda - 2 \end{vmatrix}$

$\det(\lambda I - B) = 0$

$\Leftrightarrow (\lambda - 5)(\lambda - 2) - 18 = 0$

$\Leftrightarrow \lambda = -1 \; ; \; \lambda = 8.$

Xét $\lambda = -1$

$\Rightarrow \lambda I - B = \begin{pmatrix} -6 & -2 \\ -9 & -3 \end{pmatrix} \xrightarrow[d_2 \to \frac{d_2}{3}]{d_1 \to \frac{-d_1}{2}} \begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix} \xrightarrow{d_2 - d_1} \begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow 3x_1 + x_2 = 0$

$\Leftrightarrow \begin{cases} x_1 = t \\ x_2 = -3t \end{cases} \; (t \in \mathbb{R}) \Rightarrow$ VTR $u_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

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Xét $\lambda = 8 \Rightarrow \lambda I - B = \begin{pmatrix} 3 & -2 \\ -9 & 6 \end{pmatrix} \xrightarrow{d_2 + d_1} \begin{pmatrix} 3 & -2 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow 3x_1 - 2x_2 = 0$

$\Leftrightarrow \begin{cases} x_1 = t \\ x_2 = \frac{3}{2} t \end{cases}$ $(t \in \mathbb{R} \setminus \{0\})$ VTR $u_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ Chọn $t = 2$

$\Rightarrow$ Ma trận khả nghịch $P = \begin{pmatrix} 1 & 2 \\ -3 & 3 \end{pmatrix} \Rightarrow P^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{-2}{9} \\ \frac{1}{3} & \frac{1}{9} \end{pmatrix}$

Ma trận đường chéo $D = P^{-1}.B.P = \begin{pmatrix} -1 & 0 \\ 0 & 8 \end{pmatrix}$

$\Rightarrow B^k = P^{-1}.D^k.P^{-1} = \begin{pmatrix} 1 & 2 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} -1^k & 0 \\ 0 & 8^k \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{-2}{9} \\ \frac{1}{3} & \frac{1}{9} \end{pmatrix}$

$= \begin{pmatrix} -1^k & 2.8^k \\ (-3).(-1)^k & 3.8^k \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{-2}{9} \\ \frac{1}{3} & \frac{1}{9} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}(-1 + 2.8^k) & \frac{1}{9}((-2).(-1)^k + 2.8^k) \\ (3.(-1)^{k-1} + 8^k) & \frac{1}{3}(2.(-1)^k - 8^k) \end{pmatrix}$