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Electromagnetics Midterm Examination

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Note: Do not use any electronic devices.

*** Follow the notations used in the textbook** ((x,y,z), (r,ϕ,z), (R,θ,ϕ):position in Cartesian, cylindrical, and spherical coordinates, respectively. ϵ: permittivity, ρ: charge density. **E**, **D**: electric field, electric flux density. E_{1n}, E_{1t}: the normal component and tangential component of **E** in medium 1. $\mathbf{a}_n$: the unit normal vector pointing from the boundary of medium 2 to medium 1 when regarding boundary conditions).

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A. Selections (50%)

(2)1. Given vectors $\mathbf{A} = \mathbf{a}_x + \mathbf{a}_y 2 - \mathbf{a}_z 3$ and $\mathbf{B} = \mathbf{a}_x 5 + \mathbf{a}_z 2$. What is the component of **A** in the direction of **B**? [1.]$\frac{2}{\sqrt{29}}$; [2.]$\frac{-1}{\sqrt{29}}$; [3.]$\frac{6}{\sqrt{29}}$; [4.]$\frac{-1}{\sqrt{14}}$; [5.] None of the above.

(3)2. Two point charges, Q_1 and Q_2, are located at $(1,2,0)$ and $(2,0,0)$, respectively. Find the relation between Q_1/Q_2 such that the total force on a test charge at the point $P(-1,1,0)$ will have no x-component. $Q_1/Q_2 =$ [1.] $\frac{3}{4\sqrt{2}}$; [2.] $\frac{1}{2\sqrt{2}}$; [3.]$-\frac{3}{4\sqrt{2}}$;[4.]$-\frac{1}{2\sqrt{2}}$; [5.] None of the above.

(3)3. The position of a point P in spherical coordinates is $(8, 120°, 330°)$. The position vector $\overrightarrow{OP}$ in cylindrical coordinates is: [1.]$\mathbf{a}_r 4\sqrt{3} - \mathbf{a}_z 4$; [2.]$\mathbf{a}_r 4 - \mathbf{a}_z 4\sqrt{3}$; [3.]$\mathbf{a}_r 4\sqrt{3} + \mathbf{a}_\phi \frac{11}{6} - \mathbf{a}_z 4$; [4.]$\mathbf{a}_r 4 + \mathbf{a}_\phi \frac{11}{6} - \mathbf{a}_z 4\sqrt{3}$; [5.] None of the above.

(5)4. In the cylindrical coordinate system, $\int_0^\pi \mathbf{a}_r d\phi =$ [1.] 0 ; [2.] $2\mathbf{a}_y$; [3.] $\pi \mathbf{a}_r$; [4.] $-2\mathbf{a}_y$; [5.] None of the above.

(1)5. In the cylindrical coordinate system, $\frac{\partial \mathbf{a}_\phi}{\partial \phi} =$ [1.] $-\mathbf{a}_r$; [2.]0 ; [3.]$\mathbf{a}_\phi$; [4.]$\mathbf{a}_r$; [5.] None of the above.

(3)6. Given that in free space a point charge q is placed at the position $P_0: (x,y,z) = (1,2,3)$, what is the resultant **E** at point $P_1: (x,y,z) = (1,4,3)$? [1.]$\mathbf{a}_R \frac{q}{16\pi\epsilon_0}$; [2.]$-\mathbf{a}_z \frac{q}{12\pi\epsilon_0}$; [3.]$\mathbf{a}_y \frac{q}{16\pi\epsilon_0}$; [4.] $-\mathbf{a}_y \frac{q}{12\pi\epsilon_0}$; [5.] None of the above.

(4) 7.Assuming that the electric field intensity is $\mathbf{E} = \mathbf{a}_x 100x$ (V/m), find the total electric charge contained inside a cubical volume 100(mm) on a side centered symmetrically at the origin. [1.]$0.01\epsilon_0$; [2.]$10\epsilon_0$; [3.]$100\epsilon_0$; [4.]$0.1\epsilon_0$; [5.] None of the above. (C)

(5)8. Given a scalar function $V = \left(\sin\frac{\pi}{2}x\right)\left(\sin\frac{\pi}{3}y\right)e^{-z}$, find $\nabla \times (\nabla V)$ at the point $P: (x,y,z) = (1,2,3)$. [1.]$\left(-\mathbf{a}_y \frac{\pi}{6} - \mathbf{a}_z \frac{\sqrt{3}}{2}\right)e^{-3}$; [2.]$\frac{1}{\sqrt{14}}(-\mathbf{a}_x - \mathbf{a}_y 2 - \mathbf{a}_z 3)$; [3.]$-\mathbf{a}_x - \mathbf{a}_y 2 - \mathbf{a}_z 3$; [4.]$\mathbf{a}_x 5 + \mathbf{a}_z 2$; [5.] None of the above.

(4) 9.An infinite planar charge, with a uniform surface charge density ρ_s, lies on the xy-plane. Determine the electric field intensity **E** everywhere. $\mathbf{E} =$ [1.]$\mathbf{a}_z \frac{\rho_s}{2\epsilon_0}$; [2.]$-\mathbf{a}_z \frac{\rho_s}{2\epsilon_0}$; [3.]$\mathbf{a}_z \frac{\rho_s}{\epsilon_0}\frac{z}{|z|}$; [4.]$\mathbf{a}_z \frac{\rho_s}{2\epsilon_0}\frac{z}{|z|}$; [5.] None of the above.

(4) 10.A straight line charge with uniform density ρ_ℓ is placed along the z-axis in the region $0 \le z \le 2L$. At point $P: (r,\phi,z) = \left(3, \frac{\pi}{4}, L\right)$, find the electric field intensity **E**. $\mathbf{E} =$ [1.] $\mathbf{a}_r \frac{\rho_\ell L}{12\pi\epsilon_0}\frac{1}{\sqrt{9+\left(\frac{L}{2}\right)^2}}$; [2.] $-\mathbf{a}_r \frac{\rho_\ell L}{6\pi\epsilon_0}$; [3.] $\mathbf{a}_r \frac{\rho_\ell L}{6\pi\epsilon_0}$; [4.] $\mathbf{a}_r \frac{\rho_\ell L}{6\pi\epsilon_0(3+L)}$; [5.] None of the above.

B. Calculations (Please clearly write down the calculation process. Answers alone will get zero points.)

11. [10%] Determine the **E** field caused by a spherical cloud of electrons with a volume charge density $\rho = -\rho_0$ for $0 \le R \le b$ (both ρ_0 and b are positive) and $\rho = 0$ for $R > b$.

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$0 \le R \le b$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E \oint d\vec{s} = \frac{-\frac{4}{3}\pi R^3 \rho_0}{\varepsilon_0}$$

$$\Rightarrow E \cdot 4\pi R^2 = \frac{-4\pi R^3 \rho_0}{3\varepsilon_0}$$

$$\Rightarrow E = \frac{-R\rho_0}{3\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{-R\rho_0}{3\varepsilon_0}\vec{a}_R \quad ※$$

$b < R$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E \oint ds = \frac{-\frac{4}{3}\pi b^3 \rho_0}{\varepsilon_0}$$

$$\Rightarrow E \cdot 4\pi R^2 = \frac{-\frac{4}{3}\pi b^3 \rho_0}{\varepsilon_0}$$

$$\Rightarrow E = \frac{-b^3\rho_0}{3\varepsilon_0 R^2}$$

$$\Rightarrow \vec{E} = \frac{-b^3\rho_0}{3\varepsilon_0 R^2}\vec{a}_R \quad ※$$

12. [10%] A line charge of uniform density ρ_ℓ in free space forms a semicircle of radius b. At the center of the semicircle, determine (a) the electric potential, V, and (b) the electric field intensity $\mathbf{E}$.

V: $dV = \frac{\rho_\ell r d\phi}{4\pi\varepsilon_0 b}$

+7 $V = \int_0^\pi \frac{\rho_\ell b d\phi}{4\pi\varepsilon_0 b} = \frac{\rho_\ell}{4\pi\varepsilon_0}\int_0^\pi d\phi = \frac{\rho_\ell}{4\varepsilon_0}$ # (5)

E: $\vec{R} = \cos\phi \vec{a}_x + \sin\phi \vec{a}_y$, $|\vec{R}| = 1$

x軸方向上會被抵消，故只需要算y方向

$d\vec{E}_y = -\frac{\rho_\ell b d\phi}{4\pi\varepsilon_0 b^2} \sin\phi \vec{a}_y$ (2)

$\vec{E}_y = \int_0^\pi \frac{\rho_\ell b \sin\phi}{4\pi\varepsilon_0} d\phi \vec{a}_y = \frac{-\rho_\ell b}{4\pi\varepsilon_0}\int_0^\pi \sin\phi \, d\phi \vec{a}_y$

$= \frac{-\rho_\ell b}{4\pi\varepsilon_0}\left[-\cos\phi\right]_0^\pi \vec{a}_y = \frac{-\rho_\ell b \times 2}{4\pi\varepsilon_0}\vec{a}_y$

$= \frac{-\rho_\ell b}{2\pi\varepsilon_0}\vec{a}_y$ #

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13. [15%] Two infinitely long coaxial cylindrical surfaces, $r = a$ and $r = b$ $(b > a)$, carry surface charge densities ρ_{sa} and ρ_{sb}, respectively. (a)Determine $\mathbf{E}$ everywhere. [12%] (b) What must be the relation between b and a in order that $\mathbf{E}$ vanishes for $r > b$? [3%]

(a) $r < a \Rightarrow \vec{E} = 0$ # (2)，令L為長度

$a < r < b$，令L為長度

$\Rightarrow \oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$

$\Rightarrow E\oint d\vec{s} = \frac{Q}{\varepsilon_0}$

$\Rightarrow E \, \cancel{2\pi r L} = \frac{\rho_{sa} \times \cancel{2\pi a L}}{\varepsilon_0}$

$\Rightarrow E = \frac{\rho_{sa} a}{\varepsilon_0 r}$ (5)

$\Rightarrow \vec{E} = \frac{\rho_{sa} a}{\varepsilon_0 r}\vec{a}_r$ #

$b < r$，令L為長度

$\Rightarrow \oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$

$\Rightarrow E\oint d\vec{s} = \frac{Q}{\varepsilon_0}$

$\Rightarrow E \, \cancel{2\pi r L} = \frac{\cancel{2\pi a L}\rho_{sa} + \cancel{2\pi b L}\rho_{sb}}{\varepsilon_0}$

$\Rightarrow E = \frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r}$ (5)

$\Rightarrow \vec{E} = \frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r}\vec{a}_r$ #

(b) 若$\vec{E} = 0$，則 $a\rho_{sa} + b\rho_{sb} = 0$

$\Rightarrow a\rho_{sa} = -b\rho_{sb}$ (3)

$\Rightarrow \frac{b}{a} = \frac{-\rho_{sa}}{\rho_{sb}}$ #

14.[15%] A circular ring in the xy-plane (radius b, centered at the origin) carries a uniform line charge density ρ_ℓ. Find the electric potential $V(R,\theta)$ at a distant point $P:(R,\theta,\phi)$. Use proper approximations, if necessary, but keep V as a function of both R and θ.

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令 $R = R\vec{a}_R + \theta\vec{a}_\theta + \phi\vec{a}_\phi$

$R' = b\cos\theta$