2025 NTPU Signal and System Final term exam

1. (10%) Find CTFT of the given function

$$x(t) = \begin{cases} 2t+4 & -2 \le t < 2 \\ 8 & 2 \le t < \infty \\ 0 & \text{otherwise} \end{cases}$$

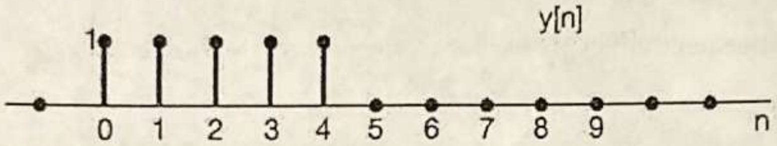
2. Consider the finite-length signal

$$x[n] = \begin{cases} e^{j\Omega_0 n}, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

   a. (10%) Derive the DTFT of $x[n]$

   b. (10%) Derive the N-point DFT of $x[n]$

3. (10%) Given the finite-length signal $y[n]$ as



Find DTFT of

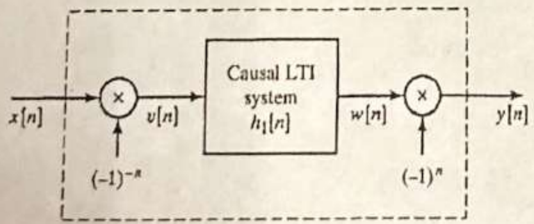
$$x[n] = y_2[n] + y_2[n-1]$$

where

$$y_k[n] = \begin{cases} y[n/k], & \text{if } n \text{ is a multiple of } k \\ 0, & \text{if } n \text{ is not a multiple of } k \end{cases}$$

4. (10%) The following system is one kind of digital filter(dash line). For the given LTI causal system $h_1(t)$ with

$$H_1(\Omega) = \begin{cases} 1, & |\Omega| < \Omega_c \\ 0, & \Omega_c < |\Omega| \le \pi \end{cases}$$



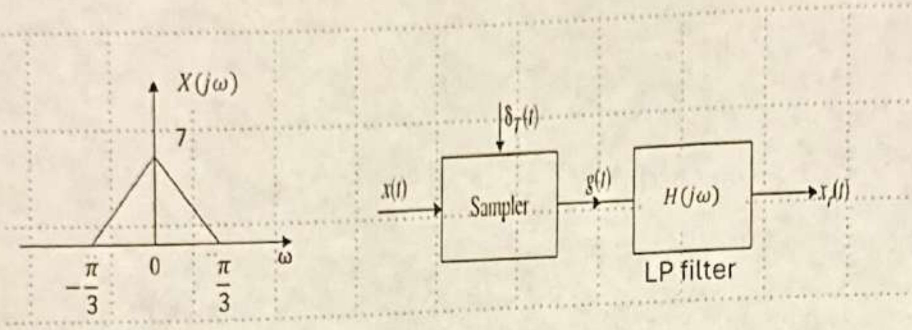
Hints: $e^{j\pi n} = (-1)^n$

Suppose $x[n] = \frac{sin(\Omega_x n)}{\pi n}$ with $\pi > \Omega_x > \pi - \Omega_c > 0$. Draw the magnitude spectrum of $Y(\Omega)$.

5. (10%) Use **Modulo Operation method** to computer DFT of

$$x[n] = [1,\ \ j,\ \ -1,\ \ j] \quad \text{for } N = 4.$$

6. For the given system



a. (5%) Determine the minimum allowable sampling angular frequency.

b. (10%) Draw the spectrum of $g(t)$ with the minimum allowable sampling angular frequency.

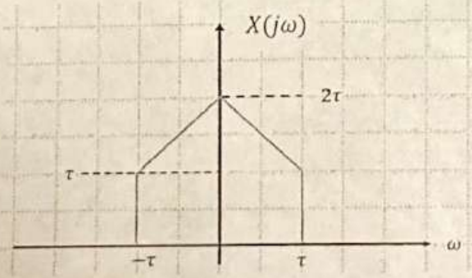
c. (10%) Suppose $x(t)$ can be recovered exactly from $g(t)$ using an ideal low-pass filter $h(t)$. Determine $h(t)$

7. (10%) Let

$$x(t) = e^{-at^2} \text{ with } a > 0.$$

Calculate the Fourier Transform of $x(t)$.

8. (5%) Use the fact that $\mathcal{F}\{\delta(t)\} = 1$ and the properties of CTFT to determine $x(t)$ if its spectrum $X(j\omega)$ is



Note that: $\mathcal{F}\{\cdot\}$ denotes the CTFT operator.