國立臺北大學 112 學年度第 2 學期期 中 考試試卷
National Taipei University Student's Answer Paper

系級/Department & Grade 電機系2年級　　科目/Course Title 工程數學

4.(a)寫出非週期輸入信號 $x(t)=A\Pi\left(\frac{t-\tau/2}{\tau}\right)$ 的傅利葉轉換，(b)畫出 $X(f)$振幅頻譜及相位頻譜，(c)透過反傅利葉轉換算出一階 RC 低通濾波器的脈衝響應 $h(t)$，(d)求輸出響應 $y(t)$，(e)畫出 $x(t)$及 $y(t)$波形。(f)如果輸入 $f(t)=10\cos 3t$，輸出 $z(t)$為何? (30%)

(a) $A\pi\left(\frac{t-\frac{\tau}{2}}{\tau}\right)=A\left(u\left(\frac{1}{2}\right)-u\left(\frac{\tau}{\tau}\right)\right)$

$A\int_0^\infty x(t)\cdot e^{-j\omega t}dt$

$=A\left(\int_{\frac{1}{2}}^{\infty}e^{j\omega t}dt-\int_{\frac{\tau}{2}}^{\infty}e^{-j\omega t}dt\right)$

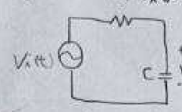
$=A\left(-\frac{1}{j\omega}e^{-j\omega t}\Big|_{\frac{1}{2}}^{\infty}+\frac{1}{j\omega}e^{-j\omega t}\Big|_{\frac{\tau}{2}}^{\infty}\right)$

$=\frac{A}{j\omega}\left(e^{-\frac{1}{2}j\omega}-e^{-\frac{\tau}{2}j\omega}\right)$

$=\frac{A}{j\omega}\left(\left(\cos\frac{\omega}{2}-j\sin\frac{\omega}{2}\right)-\left(\cos\frac{\tau\omega}{2}-j\sin\frac{\tau\omega}{2}\right)\right)$

(b)

(c) $\frac{V_o(j\omega)}{V_i(j\omega)}=H(j\omega)$



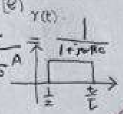
$=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}$

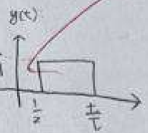
$V_i(t)=Ri(t)+\frac{i(t)}{j\omega C}$

$h(t)=\frac{1}{R+\frac{1}{j\omega C}}$

$=\frac{j\omega C}{j\omega RC+1}$

(e)





(d) $y(t)=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}\cdot X(t)=\frac{\frac{1}{RC}}{j\omega+\frac{1}{RC}}\cdot X(t)$

(f) $f(t)=10\cos 3t$

$z(t)=\frac{h(t)}{j\omega C}f(t)$

5. (a). $\overrightarrow{F}=4\cos\phi\overrightarrow{a_r}+6r\overrightarrow{a_\phi}+9\overrightarrow{a_z}$，以直角座標表示$\overrightarrow{F}$?，(b)畫出球座標$(r,\theta,\phi)$示意圖，寫出$(r,\theta,\phi)$ V.S. (x,y,z)關係式，(c) 球座標 $\overrightarrow{a_\theta}dS$ 如何由$\overrightarrow{a_r}$，$\overrightarrow{a_\phi}$ 之單位長度來計算?(d) $\overline{A}=2\overline{a_x}-6\overline{a_y}-3\overline{a_z}$，$\overline{B}=4\overline{a_x}+3\overline{a_y}-\overline{a_z}$，垂直$\overline{A}$及$\overline{B}$的單位向量為? (20%)

(a) $\vec{F}=(4\cos\phi, 6r, 9)$

(d) $A=(2,-6,-3)$

$B(4,3,-1)$

$\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\frac{(15,-10,30)}{35}$

$=\left(\frac{3}{7},-\frac{2}{7},\frac{6}{7}\right)$

+1

6.(a) $\nabla\phi(x,y)=\overrightarrow{F}=\frac{1}{(x+y)^2}\overrightarrow{a_x}+\frac{1}{(x+y)^2}\overrightarrow{a_y}$，$\phi(x,y)$=? (8%), (b)磁位向量$\overrightarrow{A}(x,y,z)$，證明$\nabla\cdot\left(\nabla\times\overrightarrow{A}\right)=0$ (7%)

(a) $\phi(x,y)=\int\frac{1}{(x+y)^2}dx=\int\frac{1}{(x+y)^2}dy$

$=-\frac{1}{x+y}$ +C

+7

(b) $\frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\ \frac{\partial}{\partial z}$

$\vec{A}=(a_x,a_y,a_z)$

$\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$ $\nabla\times\vec{A}=\left(\frac{\partial}{\partial y}a_z-\frac{\partial}{\partial z}a_y,\ \frac{\partial}{\partial z}a_x-\frac{\partial}{\partial x}a_z,\ \frac{\partial}{\partial x}a_y-\frac{\partial}{\partial y}a_x\right)$

$\nabla\cdot(\nabla\times\vec{A})=\frac{\partial^2}{\partial x\partial y}a_z-\frac{\partial^2}{\partial x\partial z}a_y+\frac{\partial^2}{\partial y\partial z}a_x-\frac{\partial^2}{\partial y\partial x}a_z+\frac{\partial^2}{\partial z\partial x}a_y-\frac{\partial^2}{\partial z\partial y}a_x$

$=0$ #

+7

x^2+y^2　$\frac{1}{x^2+1}$　$x^2+1=u^2$

$2u\,du=2x\,dx$

記分 Score	教師簽章 Instructor Signature
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系級/Department & Grade 電機系2年級　　科目/Course Title 工程數學

(該科目所屬系級)/Course Given Department 電機

☑學士班 Bachelor Program　☐碩士班 Master Program　☐博士班 Ph.D. Program

學號/Student ID 4[redacted]　　姓名/Student Name [redacted]

1. 求週期函數 $f(x)$ 的傅利葉級數(以複數型來表示)；$f(x) = x^2$,　$-\pi \le x < \pi$,　$f(x) = f(x+2\pi)$ (15%)

$f(x)$為偶函數, $b_n = 0$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{3\pi} x^3 \Big|_{-\pi}^{\pi}$

$= \frac{2}{3}\pi^2$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx \, dx$

$= \frac{2}{\pi}\left(\frac{x^2}{n}\sin nx - \frac{1}{n}\int \sin nx \cdot 2x \, dx\right)$

$= \frac{2}{\pi}\left(\frac{x^2}{n}\sin nx - \frac{2}{n}\left(x\left(-\frac{1}{n}\cos nx\right) + \frac{1}{n}\int \cos nx \, dx\right)\right)$

$= \frac{2}{\pi}\left(\frac{x^2}{n}\sin nx + \frac{2x}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right)\Big|_0^{\pi}$

$= \frac{2}{\pi}\cdot\frac{2\pi}{n}\cos n\pi = \frac{4}{n}\cos n\pi \rightarrow \begin{cases} n為奇數: -\frac{4}{n} \\ n為偶數: \frac{4}{n} \end{cases}$

$x^2 = u$, $du = 2x\,dx$　　$\cos nx\,dx = dv$, $v = \frac{1}{n}\sin nx$

$x = m$, $dx = dm$　　$\sin nx\,dx = dv$, $v = -\frac{1}{n}\cos nx$

$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n}\cos n\pi \cdot \cos nx$

$= \frac{\pi^2}{3} + 4\left(-\cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + \frac{1}{4}\cos 4x - \dots\right)$

+5

2. 求 $f(t) = e^{-t}u(t)$ 的傅利葉餘弦積分式. (10%)

$f(t) = \begin{cases} e^{-t}, & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases}$

$F_c(w) = \int_0^{\infty} e^{-t} \cos wx \, dw$

$= \frac{1}{1+w^2}$　　$\mathcal{L}[\cos wt], s=1$

$f(t) = \frac{2}{\pi}\int_0^{\infty} \frac{\cos wx}{1+w^2} dx$

$= \frac{2}{\pi}\left(\frac{1}{(1+w^2)w} \sin wx\right)\Big|_0^{\infty}$

$= \frac{1}{2}\left(\frac{1}{(1+w^2)w}\right)$

+6

3. 給 $\overline{E} = E_x(x,y,z)\overline{a_x} + E_y(x,y,z)\overline{a_y} + E_z(x,y,z)\overline{a_z}$，證明 $\nabla\cdot\overline{E} = \lim_{\Delta v \to 0} \frac{\oint_s \overline{E}\cdot\overline{dS}}{\Delta v} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$ (10%)

$\int_3 \sqrt{1+9}$