Artificial Intelligence

# MACHINE LEARNING BASIC ALGORITHMS

Nguyễn Ngọc Thảo – Nguyễn Hải Minh
{nnthao, nhminh}@fit.hcmus.edu.vn

# Outline

- Introduction to Machine learning
- ID3 Decision tree
- Naïve Bayesian classification

# Acknowledgements

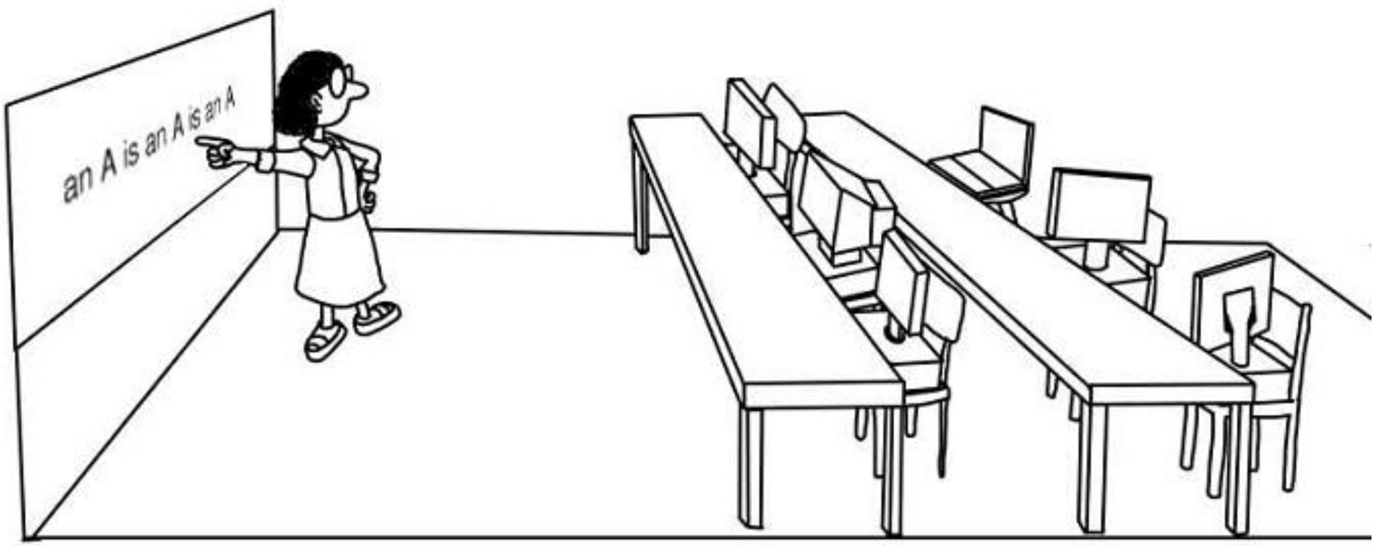

- This slide is mainly based on the textbook AIMA (3rd edition)
- Some parts of the slide are adapted from
  - Maria-Florina Balcan, *Introduction to Machine Learning*, 10-401, Spring 2018, Carnegie Mellon University
  - Ryan Urbanowicz, *An Introduction to Machine Learning*, PA CURE Machine Learning Workshop: December 17, School of Medicine, University of Pennsylvania



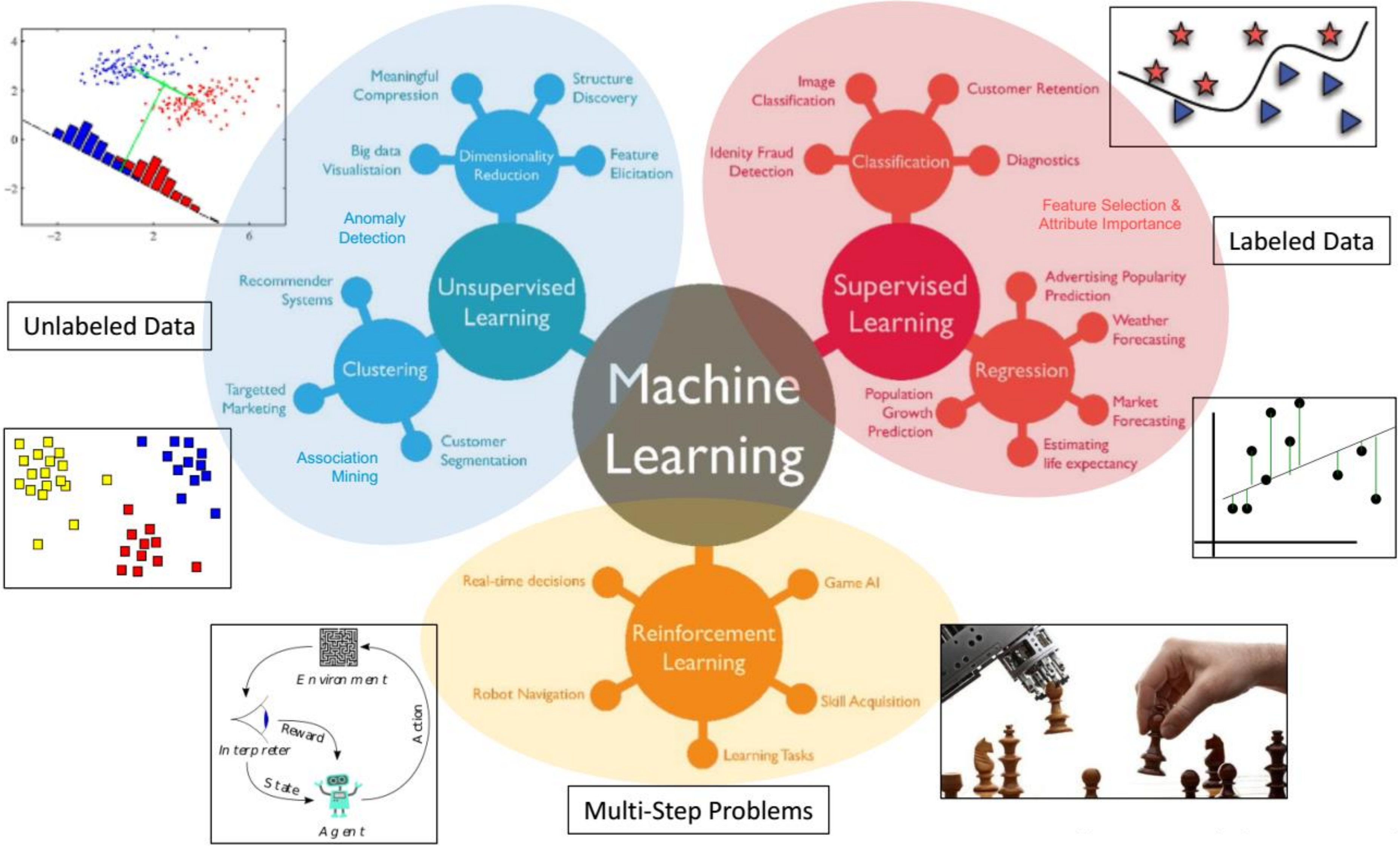
Machine Learning

# What is machine learning?

- **Machine learning** involves adaptive mechanisms that enable computers to learn from experience, learn by example and learn by analogy.

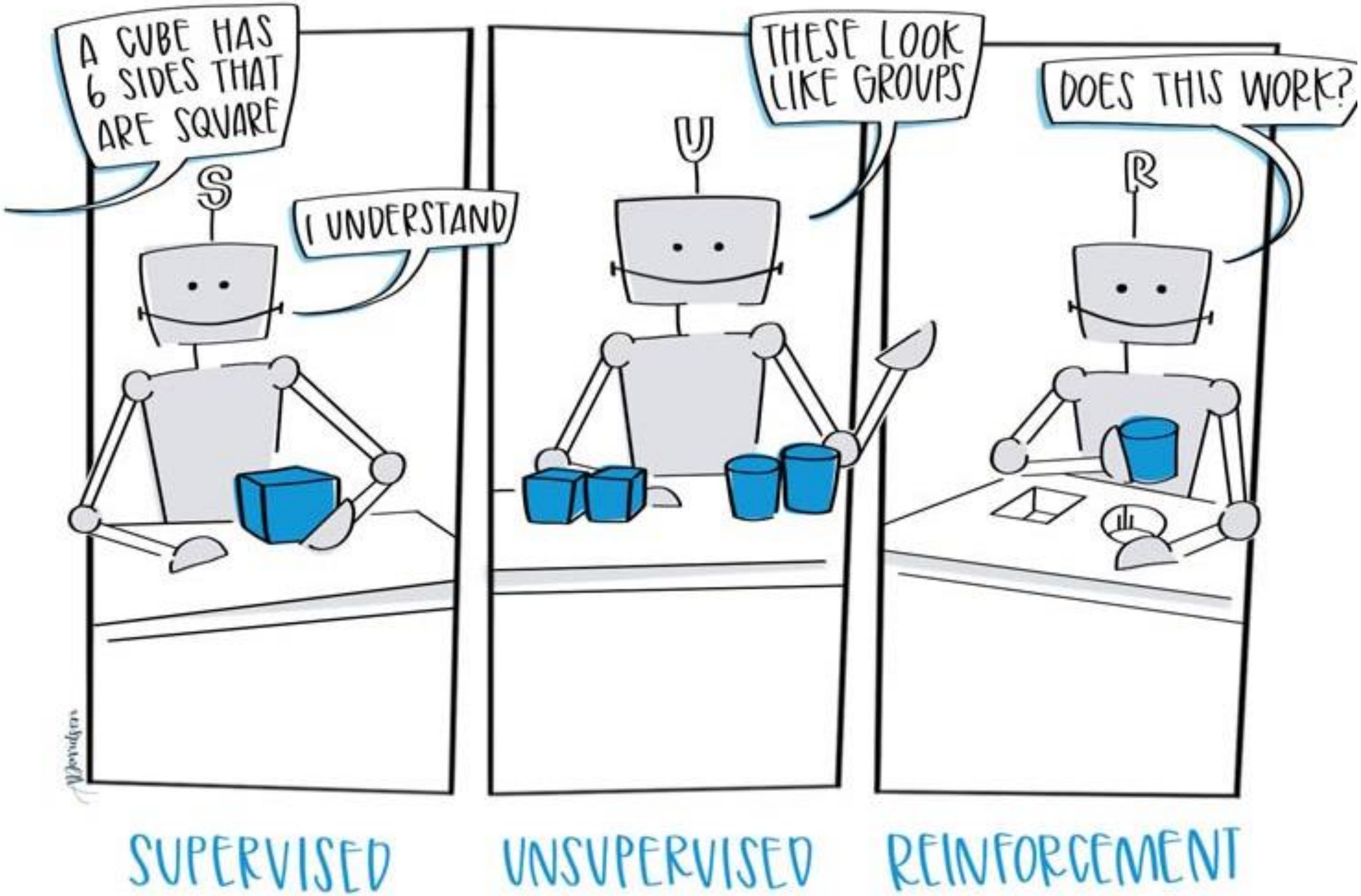


# Types of machine learning



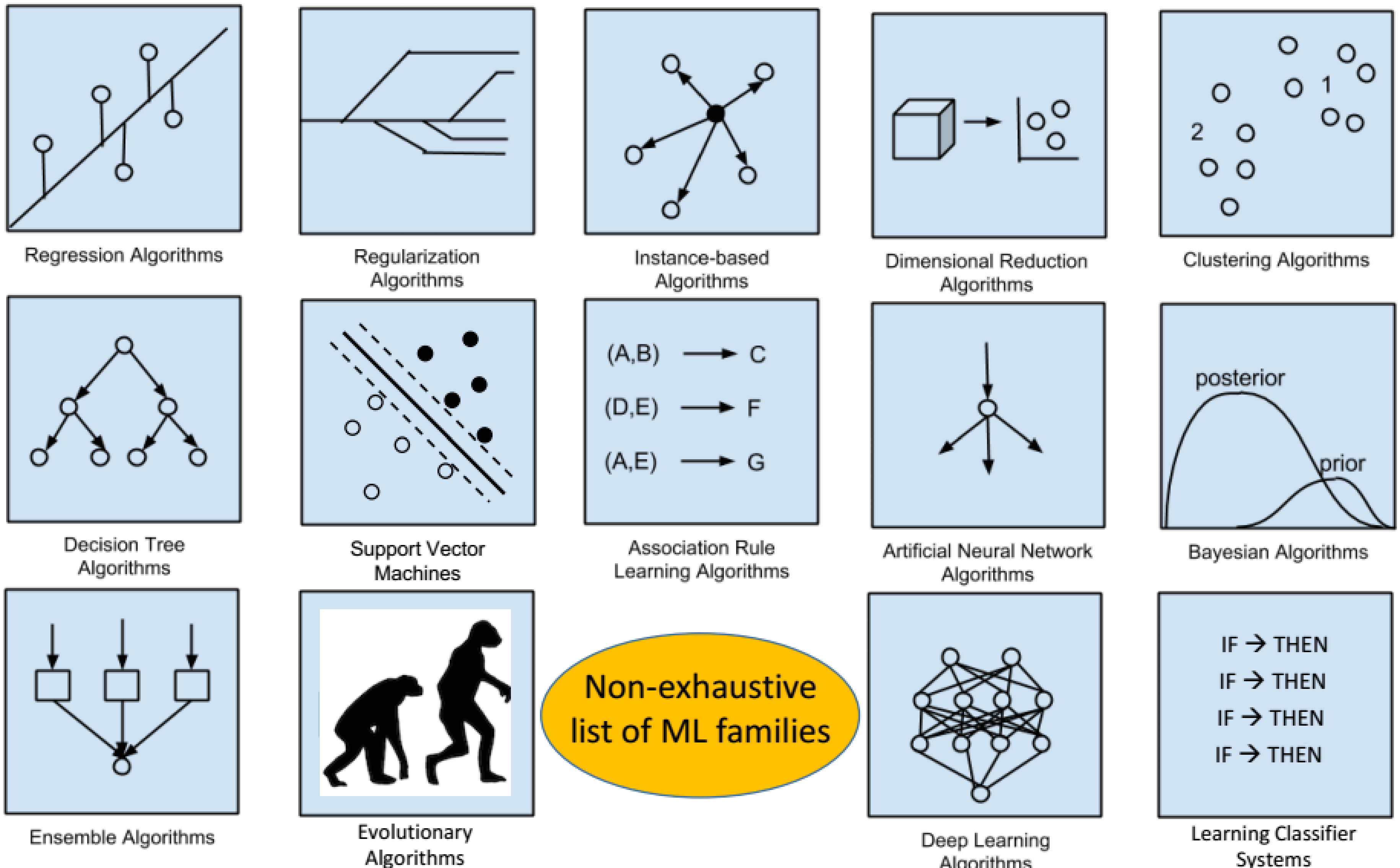

Source: https://ldi.upenn.edu/sites/default/files/Introduction-to-Machine-Learning.pdf

# Types of machine learning




Source: https://www.ceralytics.com/3-types-of-machine-learning/

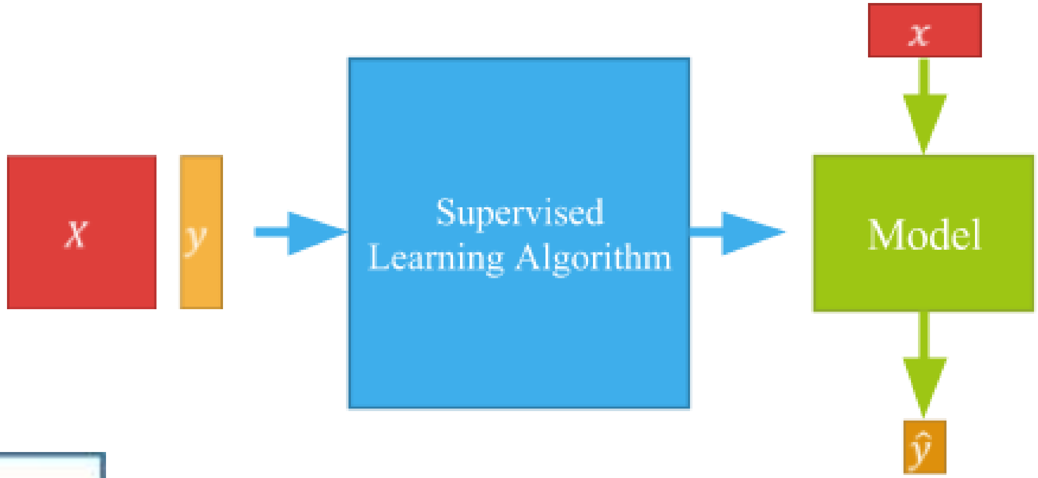
# Machine learning algorithms

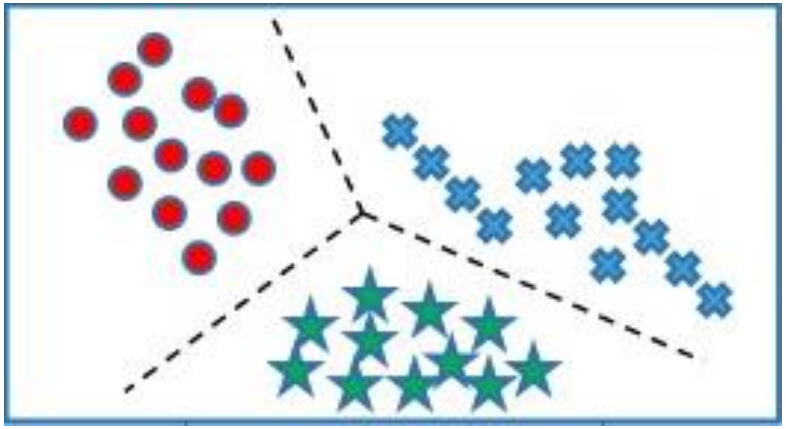



Source: https://ldi.upenn.edu/sites/default/files/Introduction-to-Machine-Learning.pdf

# Supervised learning

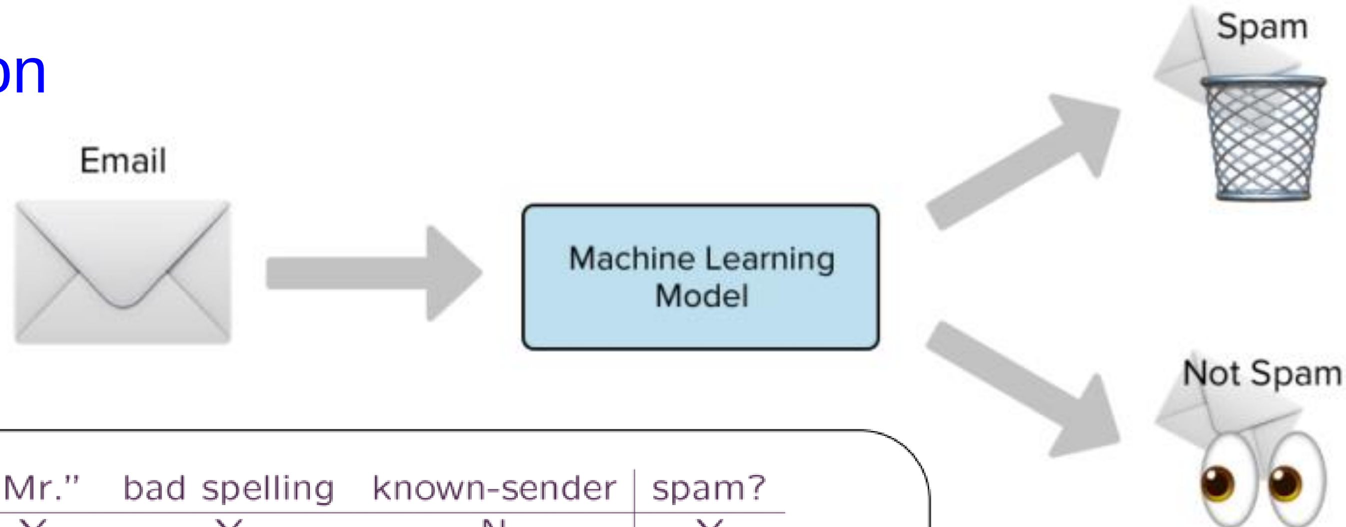
- Learn a function that maps an input to an output based on examples, which are pairs of input-output values.



# Supervised learning: Examples

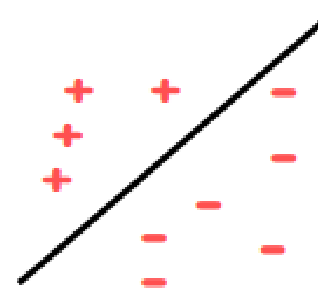
- Spam detection




| | "money" | "pills" | "Mr." | bad spelling | known-sender | spam? | |
|---|---|---|---|---|---|---|---|
| | Y | N | Y | Y | N | Y | |
| | N | N | N | Y | Y | N | |
| | N | Y | N | N | N | Y | |
| example | Y | N | N | N | Y | N | label |
| | N | N | Y | N | Y | N | |
| | Y | N | N | Y | N | Y | |
| | N | N | Y | N | N | N | |

Reasonable RULES

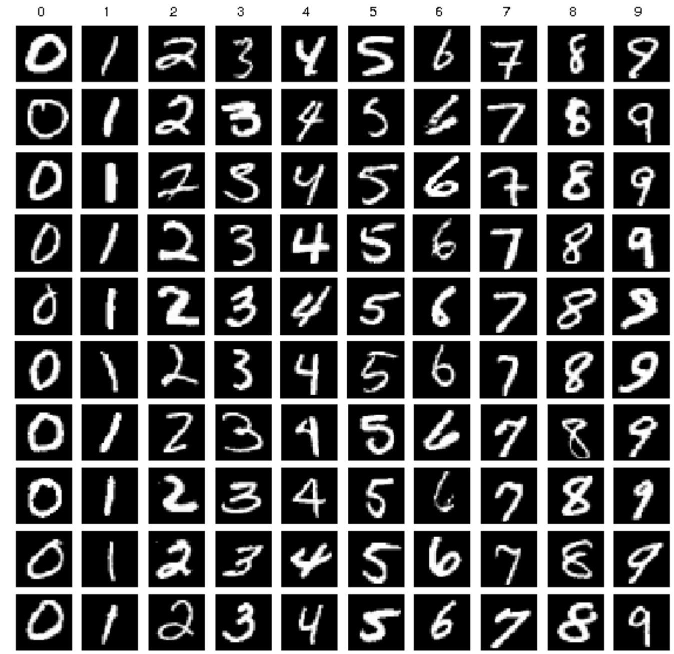
- Predict SPAM if unknown AND (money OR pills)
- Predict SPAM if 2money + 3pills – 5 known > 0



Linearly separable

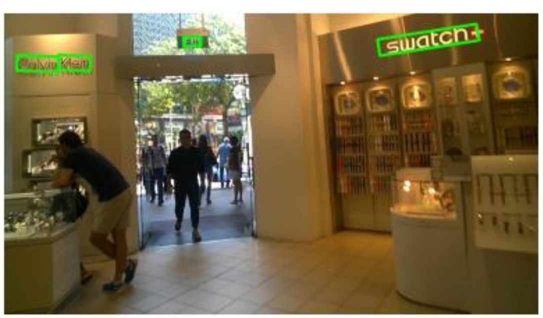
# Supervised learning: Examples

- Object detection


Indoor scene recognition

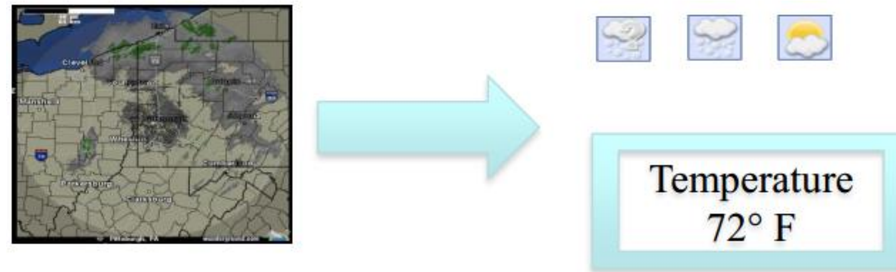
Handwritten digit recognition


Scene text recognition

# Supervised learning: More examples

- Weather prediction: Predict the weather type or the temperature at any given location…

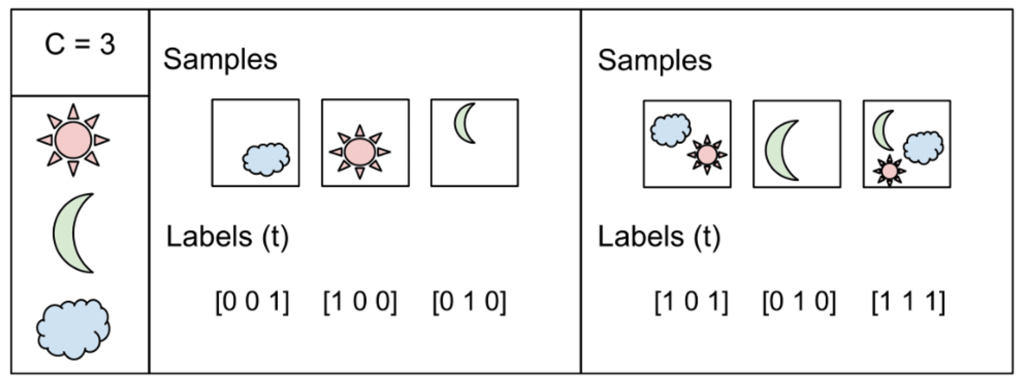


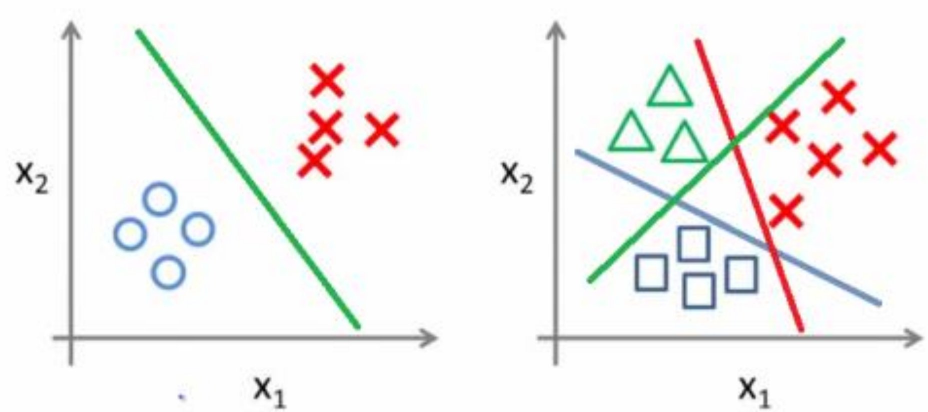

- Medicine: diagnose a disease (or response to chemo drug X, or whether a patient is re-admitted soon?)
  - Input: from symptoms, lab measurements, test results, DNA tests, …
  - Output: one of set of possible diseases, or “none of the above”
  - E.g., audiology, thyroid cancer, diabetes, etc.


- Computational economics:
  - Predict if a user will click on an ad so as to decide which ad to show
  - Predict if a stock will rise or fall (with specific amounts)

# Classification vs. Regression

- Train a model to predict a categorical dependent variable
- Case studies: predicting disease, classifying images, predicting customer churn, buy or won't buy, etc.

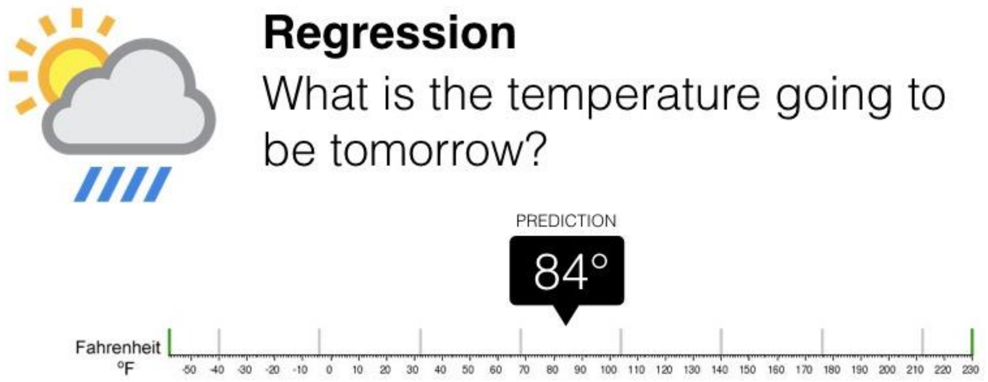


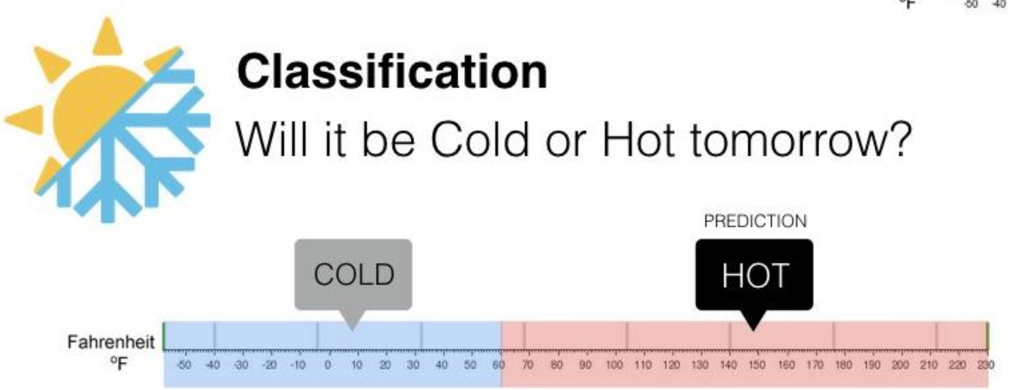




Binary classification
vs.
Multiclass classification
vs.
Multilabel classification

# Classification vs. Regression

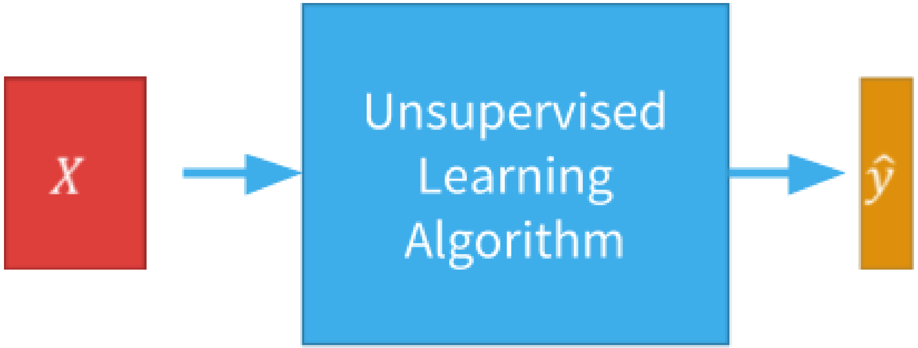
- Train a model to predict a continuous dependent variable
- Case studies: predicting height of children, predicting sales, forecasting stock prices, etc.

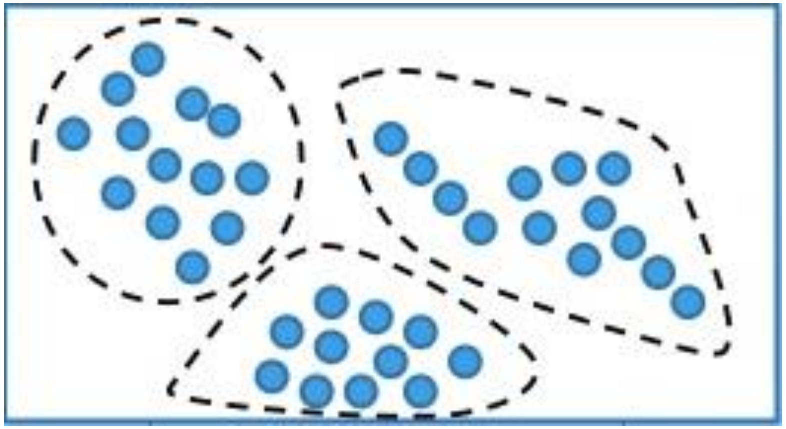


# Unsupervised learning

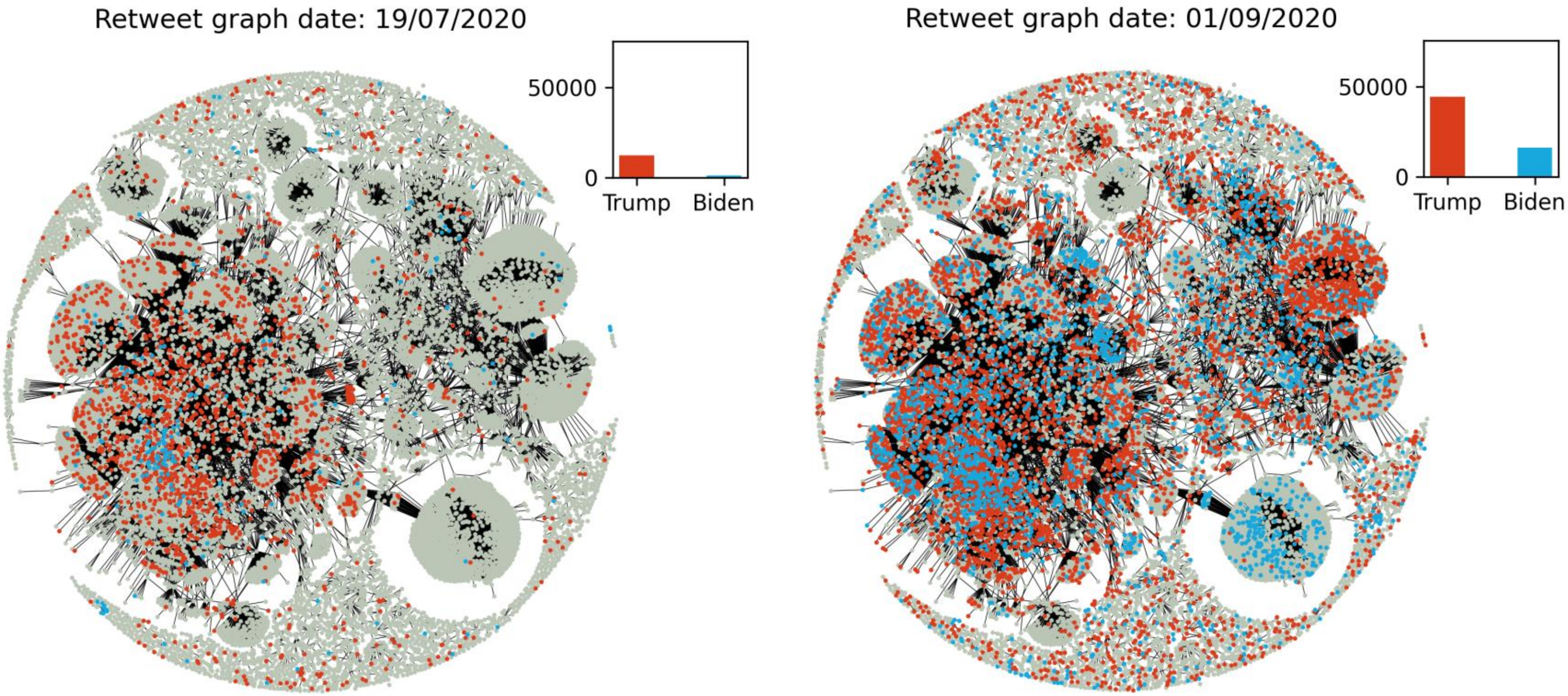
- Infer a function to describe hidden structure from "unlabeled" data
  - A classification (or categorization) is not included in the observations.



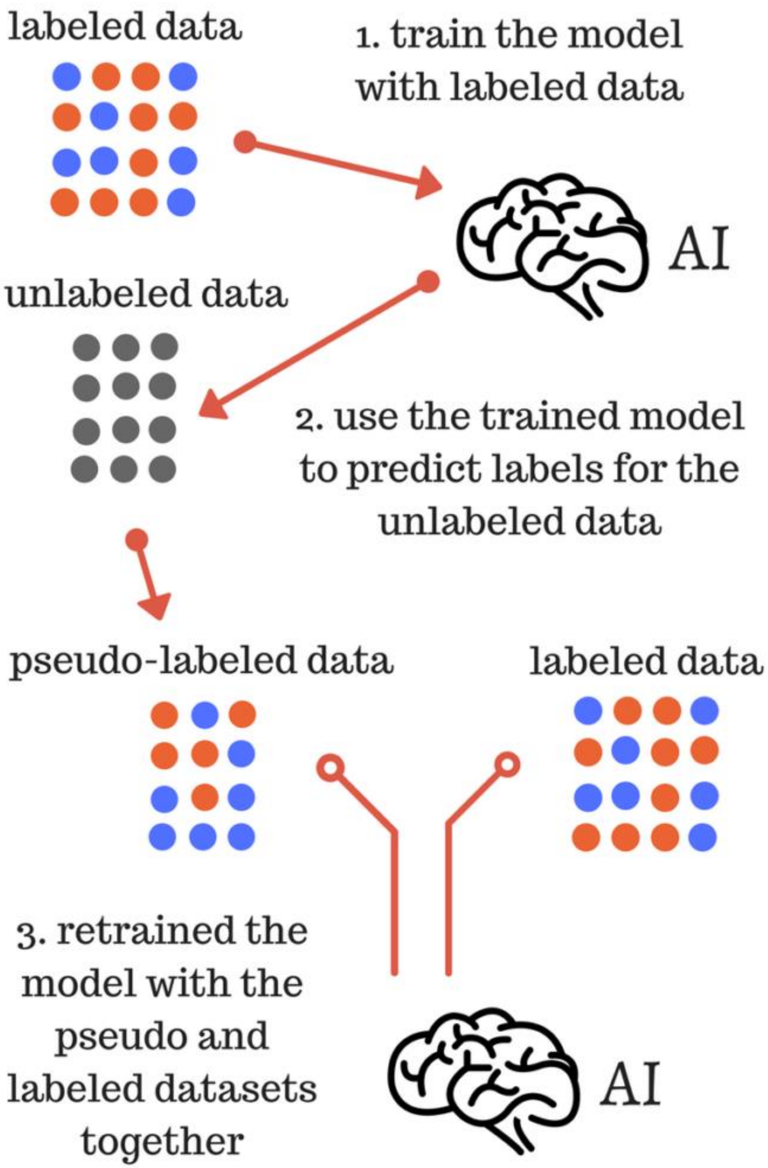
# Unsupervised learning: Examples

- Social network analysis: cluster users of social networks by interest (community detection)

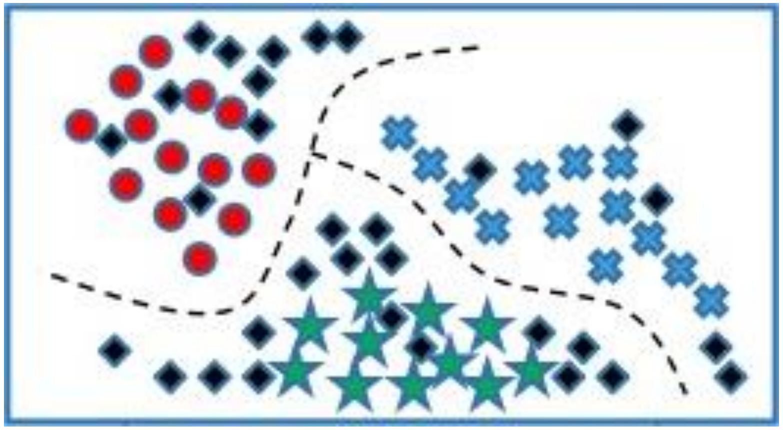



Ref: Shevtsov, Alexander, et al. "Analysis of Twitter and YouTube during US elections 2020." *arXiv e-prints* (2020): arXiv-2010.

# Semi-supervised learning

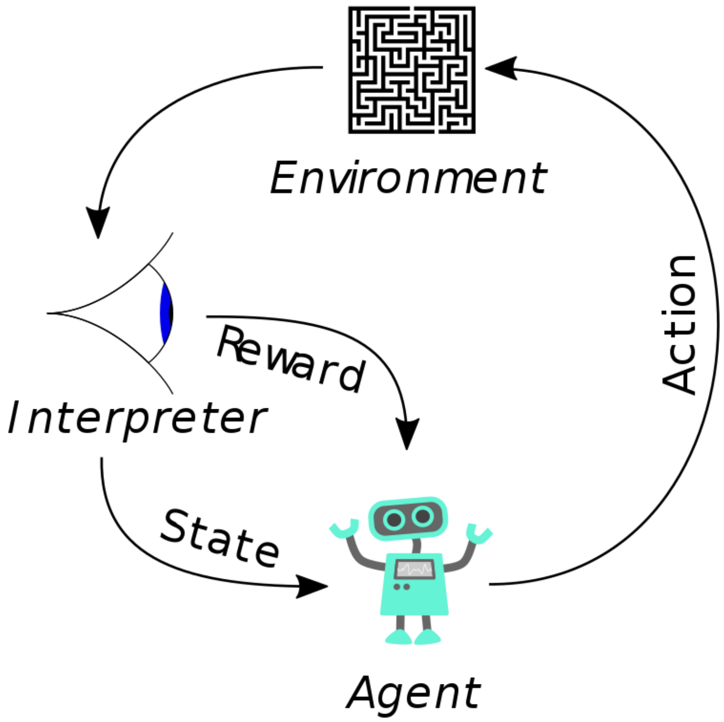



- The model is initially trained with a small amount of labeled data and a large amount of unlabeled data.



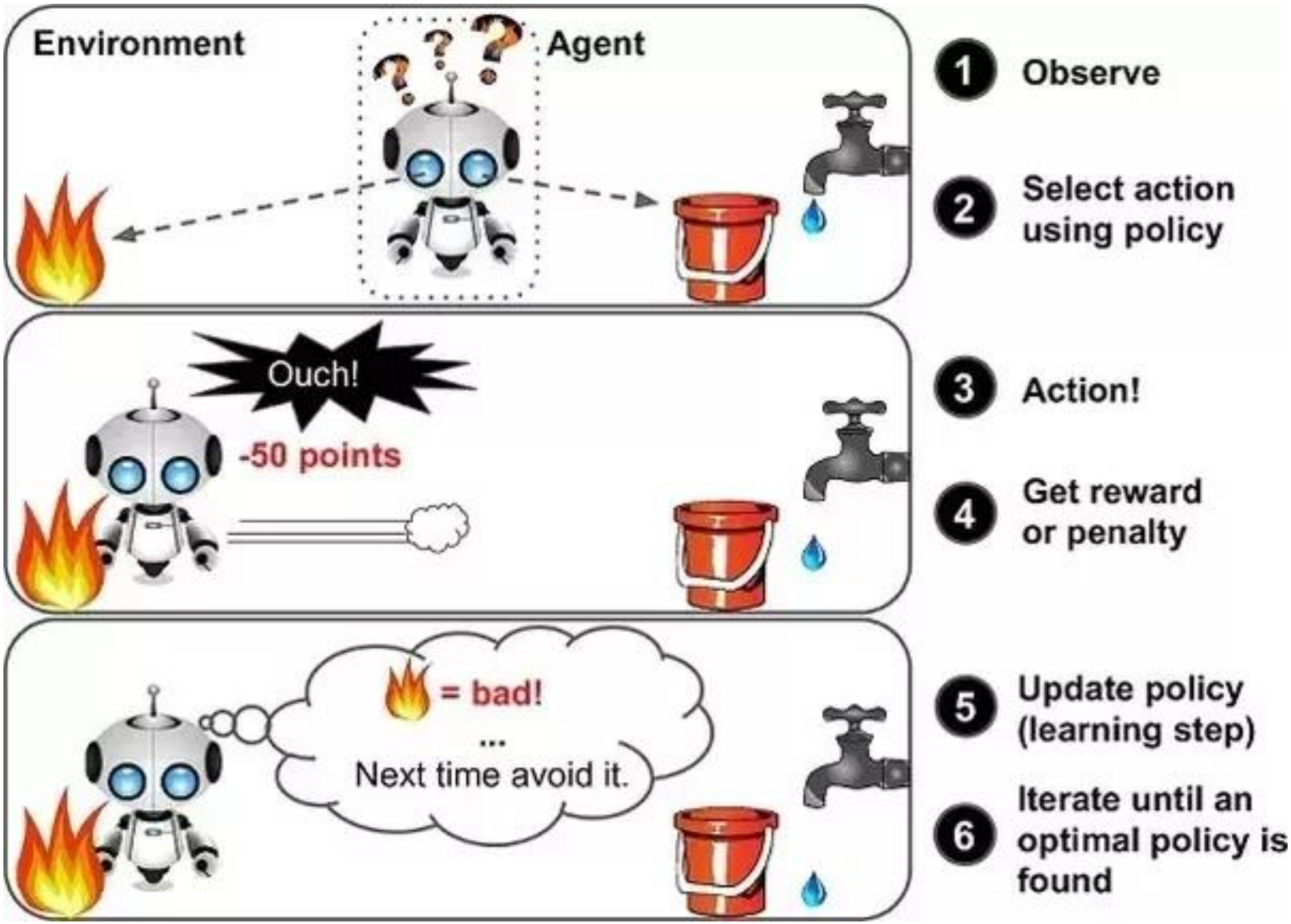
# Reinforcement learning

- The agent learns from the environment by interacting with it and receives rewards for performing actions.




Learning to ride a bike requires trial and error, much like reinforcement learning. (Video courtesy of Mark Harris, who says he is “learning reinforcement” as a parent.)

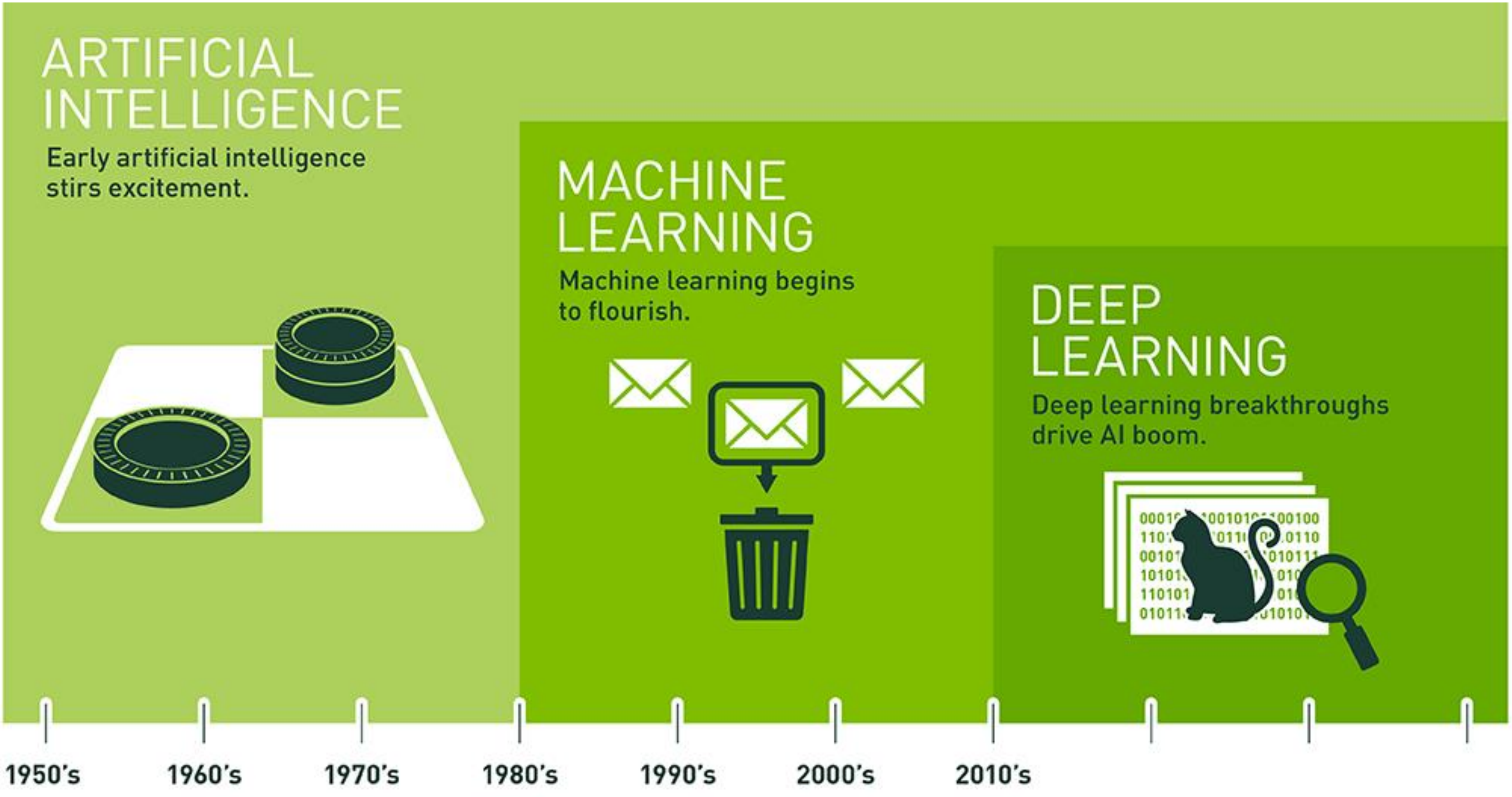
# Reinforcement learning: Example



# Reinforcement learning: Examples

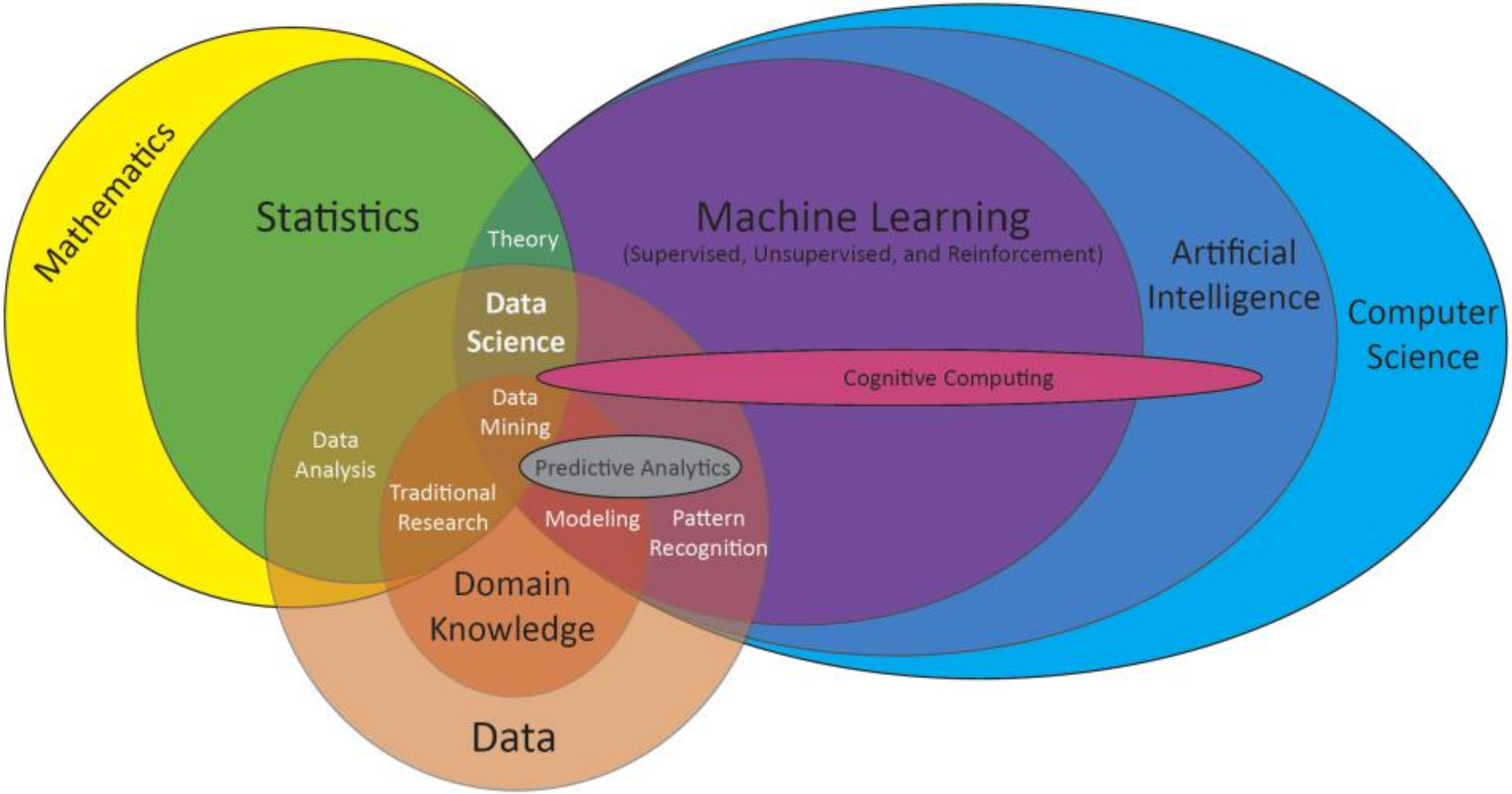

https://www.youtube.com/watch?v=gn4nRCC9TwQ

# Machine learning and related concepts

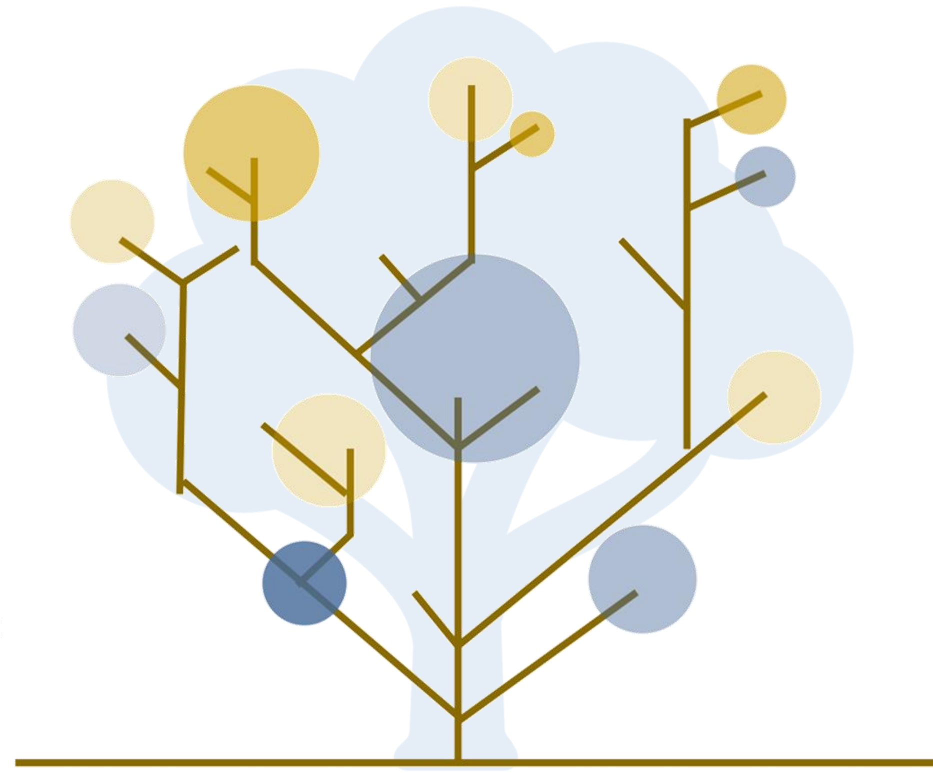



Source: https://blogs.nvidia.com/blog/2016/07/29/whats-difference-artificial-intelligence-machine-learning-deep-learning-ai/

# Machine learning and related concepts



# ID3
# Decision Tree



# Learning agents – Why learning?

- Unknown environments
  - A robot designed to navigate mazes must learn the layout of each new maze it encounters.
- Environment changes over time
  - An agent designed to predict tomorrow’s stock market prices must learn to adapt when conditions change from boom to bust.
- No idea how to program a solution
  - The task to recognizing the faces of family members

# Learning element

- Design of a learning element is affected by
  - Which *components* is to be improved
  - What *prior knowledge* the agent already has
  - What *representation* is used for the components
  - What ***feedback*** is available to learn these components

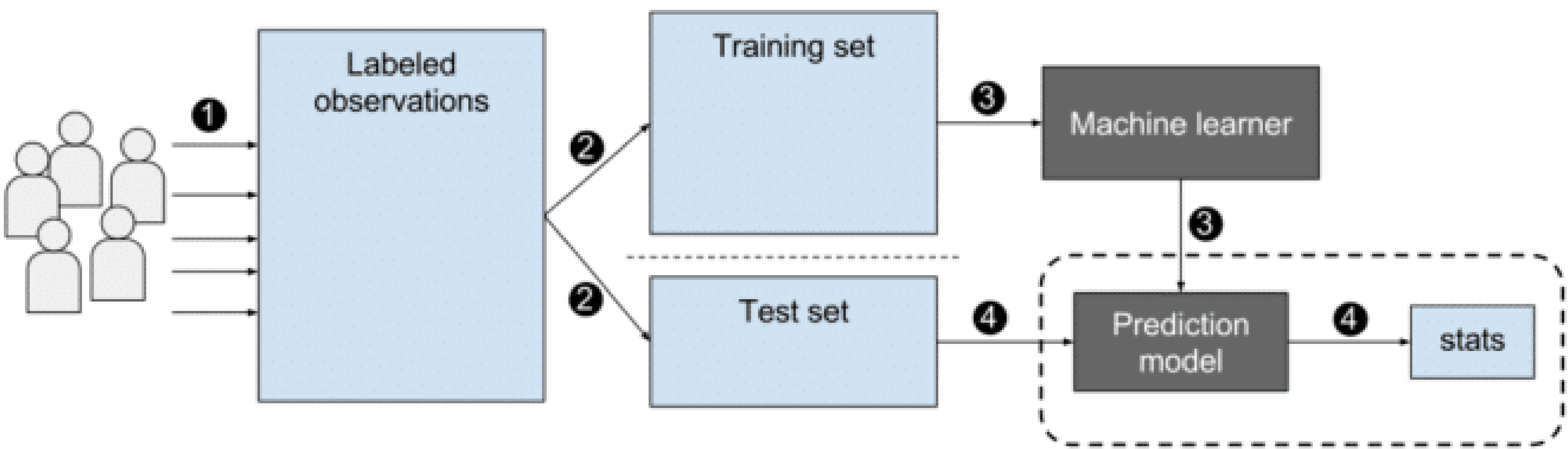
- Type of feedback
  - Supervised learning: correct answers for each example
  - Unsupervised learning: correct answers not given
  - Reinforcement learning: occasional rewards

# Supervised learning

- Simplest form: learn a function from examples
- Given a **training set** of $N$ example input-output pairs

$$(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$$

  - where each $y_j$ was generated by an unknown function $y = f(x)$
- Find a hypothesis $\boldsymbol{h}$ such that $\boldsymbol{h} \approx \boldsymbol{f}$
- To measure the accuracy of a hypothesis, give it a **test set** of examples that are different with those in the training set.



# Supervised learning

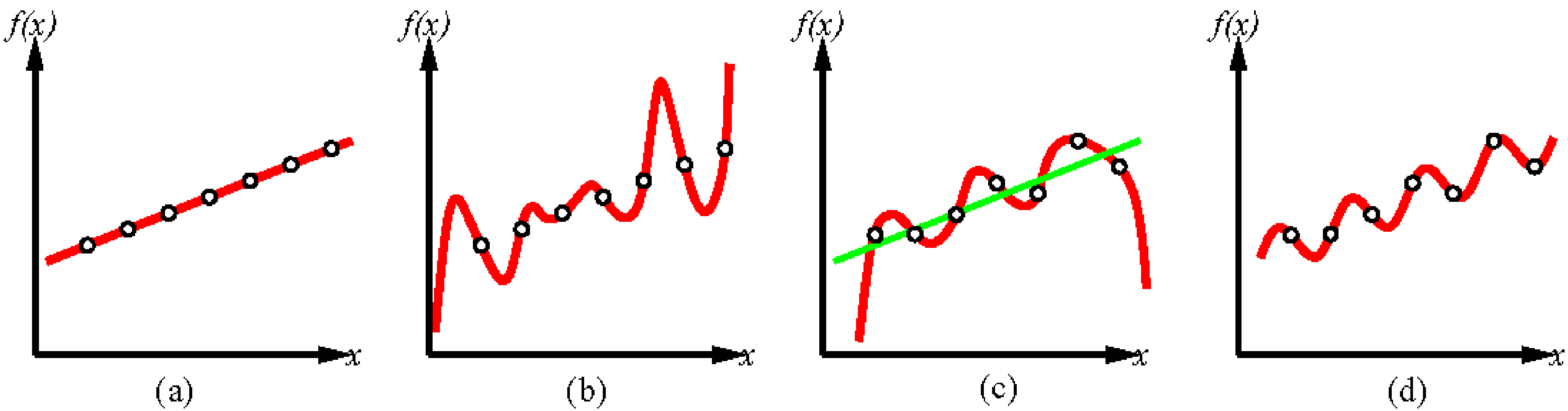
- Construct $h$ so that it agrees with $f$.
- The hypothesis $h$ is **consistent** if it agrees with $f$ on all observations.
- **Ockham's razor:** Select the simplest consistent hypothesis.

Consistent linear fit

Consistent 7th order polynomial fit

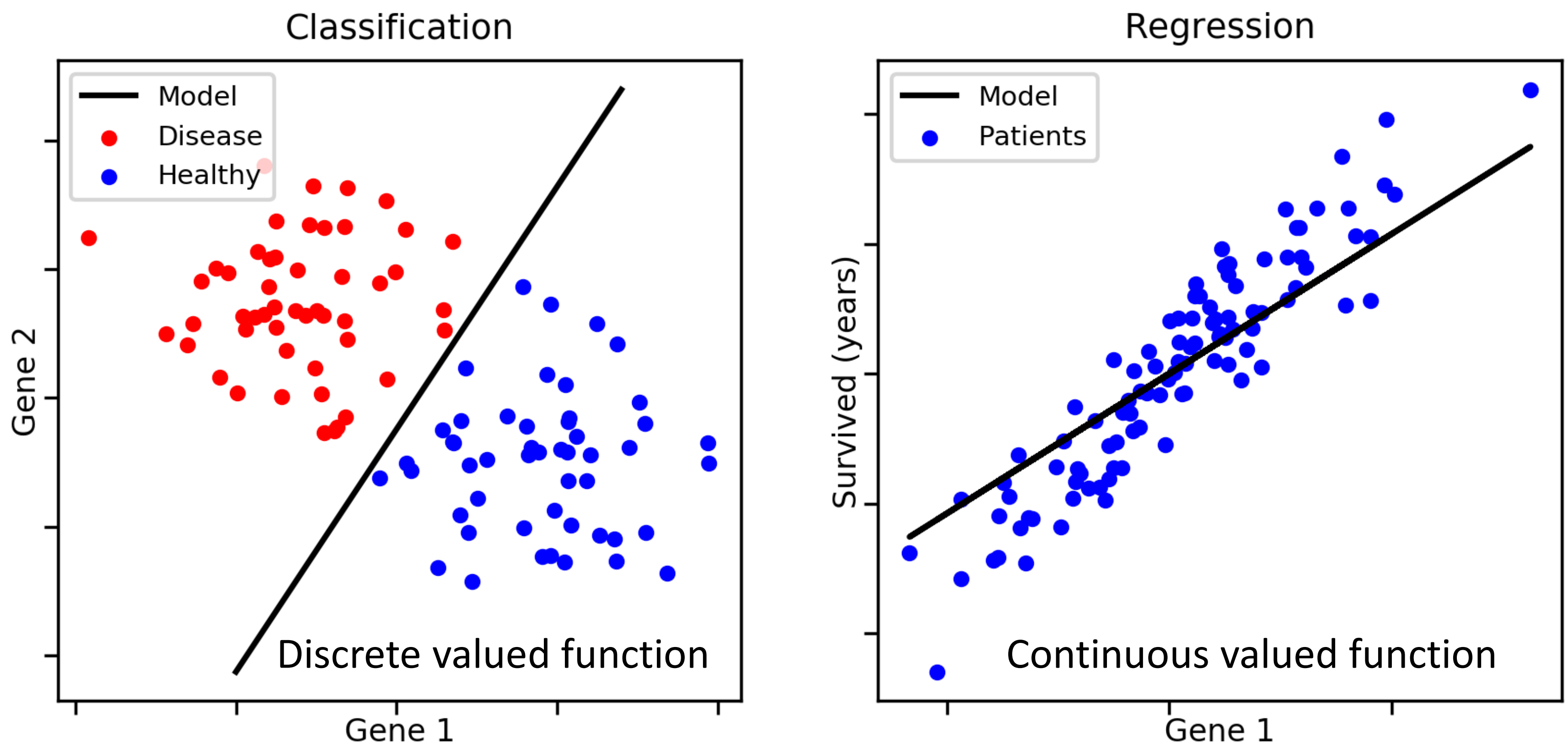
Inconsistent linear fit. Consistent 6th order polynomial fit.

Consistent sinusoidal fit



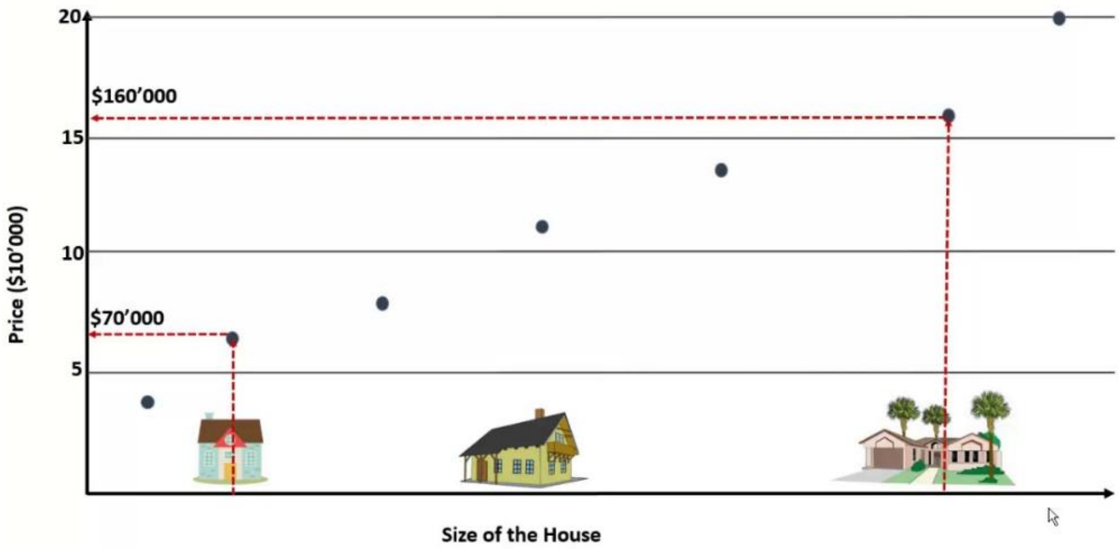
# Supervised learning problems

- $\boldsymbol{h(x)} = \textit{the predicted output value for the input } \boldsymbol{x}$



# Regression vs. Classification

- Estimating the price of a house




- Will you pass or fail the exam?
    - 2 classes: Fail/Pass


- Is this an apple, an orange or a tomato?
    - 3 classes: Apple / Orange / Tomato

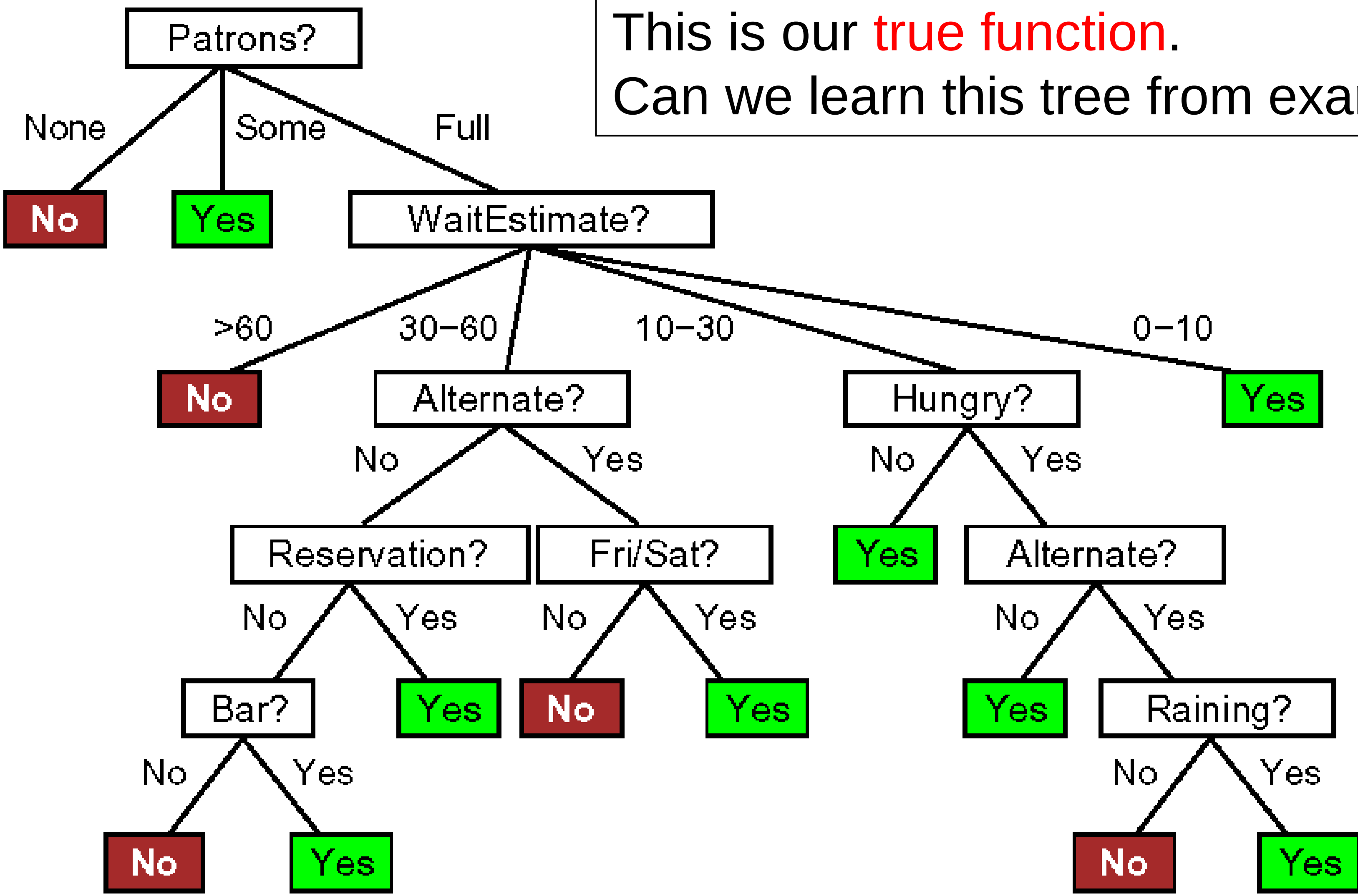
# The wait@restaurant problem

Predicting whether a certain person will wait to have a seat in a restaurant.

# The wait@restaurant problem

- The decision is based on the following attributes
    1. **Alternate:** is there an alternative restaurant nearby?
    2. **Bar:** is there a comfortable bar area to wait in?
    3. **Fri/Sat:** is today Friday or Saturday?
    4. **Hungry:** are we hungry?
    5. **Patrons:** number of people in the restaurant (None, Some, Full)
    6. **Price:** price range ($, $$, $$$)
    7. **Raining:** is it raining outside?
    8. **Reservation:** have we made a reservation?
    9. **Type:** kind of restaurant (French, Italian, Thai, Burger)
    10. **WaitEstimate:** estimated waiting time (0-10, 10-30, 30-60, >60)

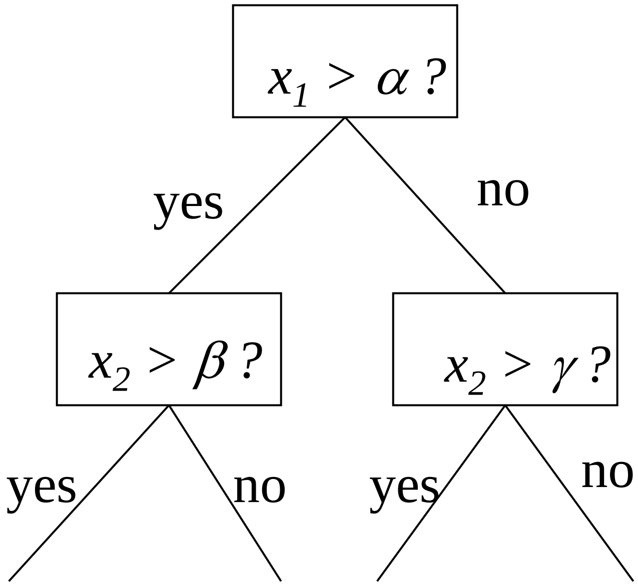
# The wait@restaurant decision tree




This is our true function.
Can we learn this tree from examples?

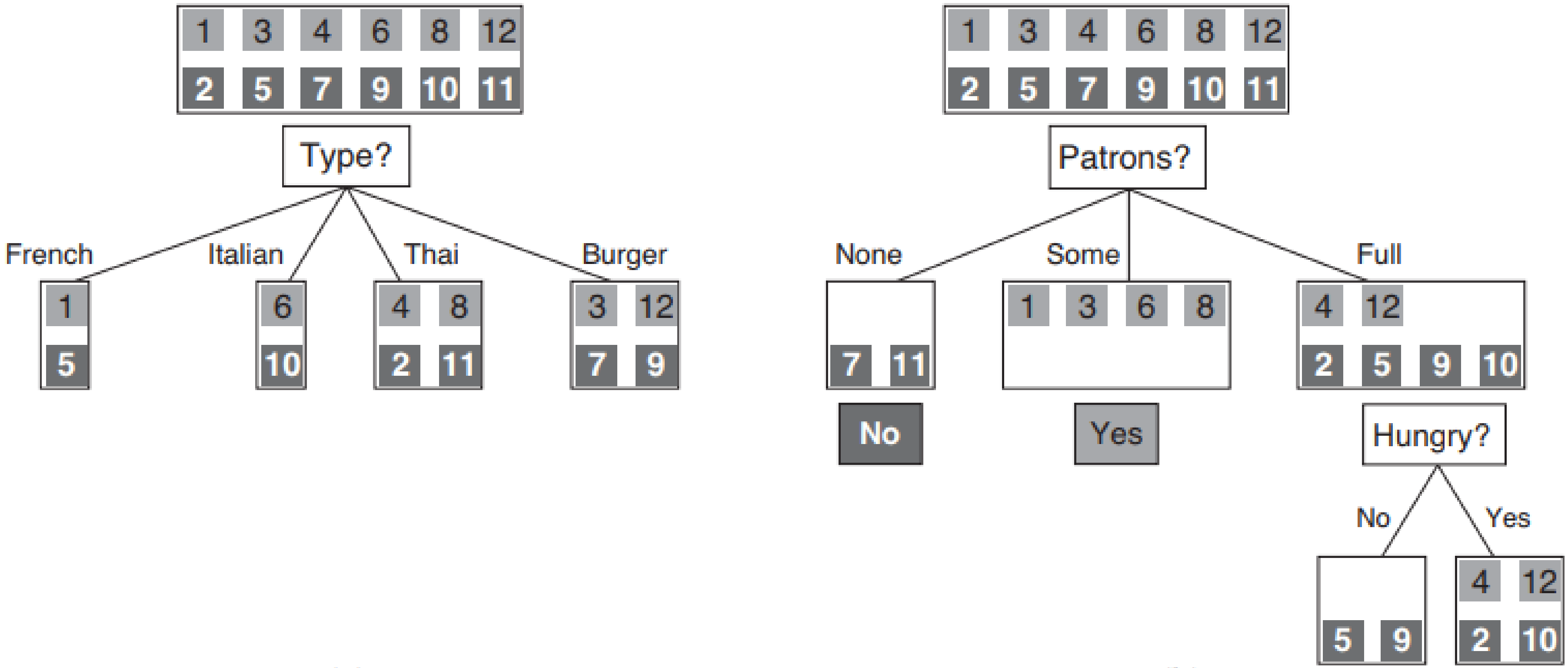
# Learning decision trees

- **Divide and conquer:** Split data into smaller and smaller subsets
- Splits are usually on a single variable




- After splitting up, each outcome is a new decision tree learning problem with fewer examples and one less attribute.

# Learning decision trees




(a) (b)

Splitting the examples by testing on attributes

# ID3 Decision tree algorithm

1. The remaining examples are **all** positive (or **all** negative), → DONE, it is possible to answer Yes or No.
   - E.g., in Figure (b), None and Some branches
2. There are **some** positive and **some** negative examples → choose the **best** attribute to split them
   - E.g., in Figure (b), Hungry is used to split the remaining examples

# ID3 Decision tree algorithm

3. **No** examples left at a branch $\rightarrow$ return a default value.
   - No example has been observed for a combination of attribute values
   - The default value is calculated from the plurality classification of all the examples that were used in constructing the node's parent.
   - These are passed along in the variable parent examples
4. **No** attributes left but both positive and negative examples $\rightarrow$ return the plurality classification of remaining ones.
   - Examples of the same description, but different classifications
   - Usually an error or noise in the data, nondeterministic domain, or no observation of an attribute that would distinguish the examples.

# ID3 Decision tree: Pseudo-code

**function** DECISION-TREE-LEARNING(*examples*, *attributes*, *parent examples*)
**returns** a tree

  **if** *examples* is empty

    **then return** PLURALITY-VALUE(*parent examples*)

  **else if** all *examples* have the same classification

    **then return** the classification

  **else if** *attributes* is empty

    **then return** PLURALITY-VALUE(*examples*)

  **else**

   ...

No examples left

remaining examples are all pos/all neg

No attributes left but examples are still pos & neg

# ID3 Decision tree: Pseudo-code

**function** DECISION-TREE-LEARNING($examples, attributes, parent\ examples$)
**returns** a tree

**...**

**else**

$A \leftarrow argmax_{a \in attributes}$ IMPORTANCE($a, examples$)

$tree \leftarrow$ a new decision tree with root test $A$

**for each** value $v_k$ of $A$ **do**

$exs \leftarrow \{e : e \in examples$ **and** $e.A = v_k\}$

$subtree \leftarrow$ DECISION-TREE-LEARNING($exs, attributes - A, examples$)

add a branch to $tree$ with label $(A = v_k)$ and subtree $subtree$

**return** $tree$

# Decision tree: Inductive learning

- **Simplest:** Construct a decision tree with one leaf for every example

  $\rightarrow$ memory based learning

  $\rightarrow$ *worse generalization.*

- **Advanced:** Split on each variable so that the purity of each split increases (i.e. either only yes or only no)
  - E.g., using Entropy to measure the purity of data

# A purity measure with entropy

- **Entropy** is a measure of the uncertainty of a random variable $V$ with values $v_k$.

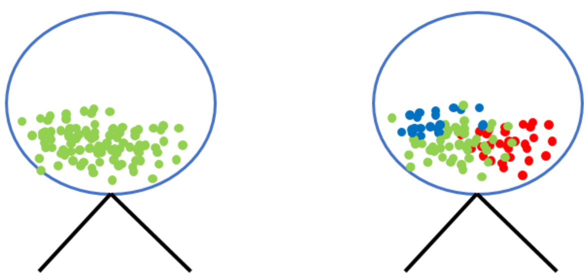
An indicator of how messy your data is

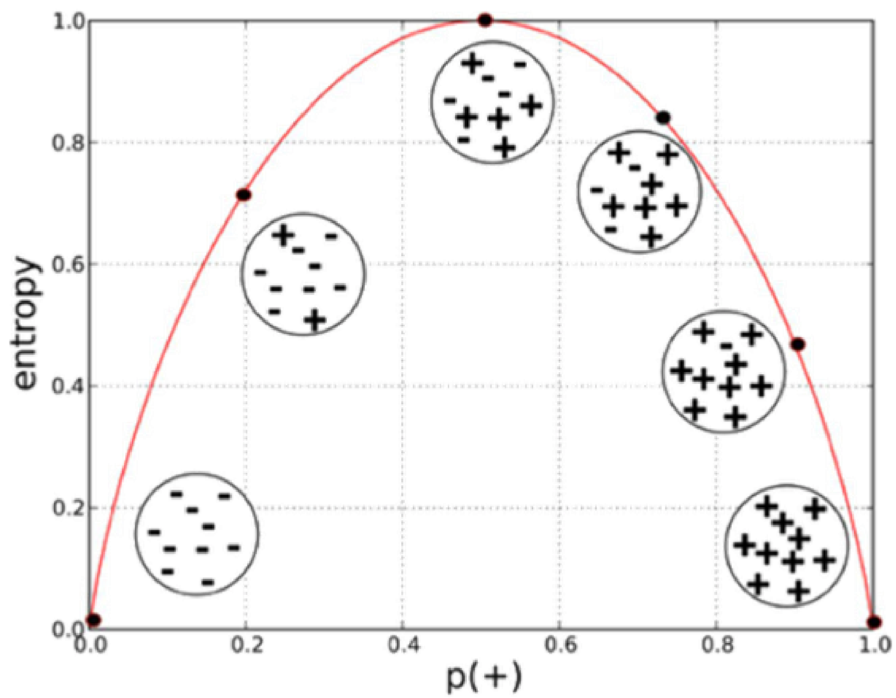
$$H(V) = \sum_k P(v_k)\, log_2 \frac{1}{P(v_k)} = -\sum_k P(v_k)\, log_2 P(v_k)$$

  - $v_k$ is a class in $V$ (e.g., yes/no in binary classification)
  - $P(v_k)$ is the proportion of the number of elements in class $v_k$ to the number of elements in $V$

# A purity measure with entropy

- Entropy is maximal when all possibilities are equally likely.
- Entropy is zero in a pure "yes" (or pure "no") node.




Provost, Foster; Fawcett, Tom. Data Science for Business: What You Need to Know about Data Mining and Data-Analytic Thinking

- Decision tree aims to **decrease** the entropy in each node.

# The wait@restaurant training data

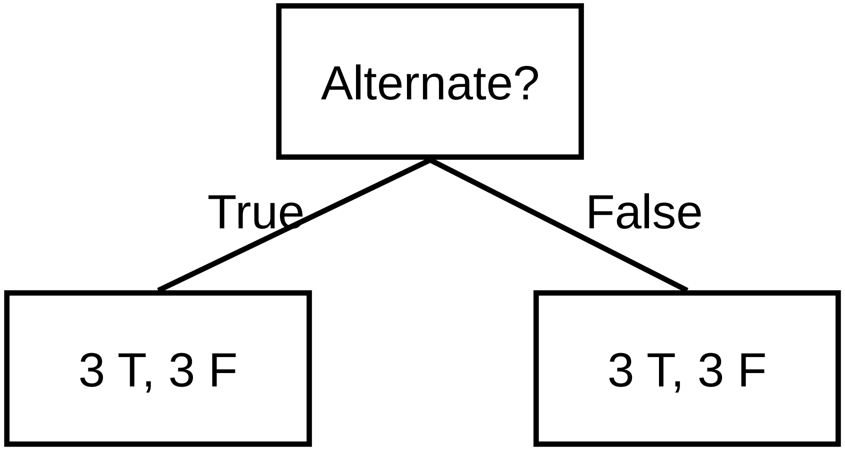
T = True, F = False

| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | *T* | *F* | *F* | *T* | *Some* | *$$$* | *F* | *T* | *French* | *0–10* | *T* |
| $X_2$ | *T* | *F* | *F* | *T* | *Full* | *$* | *F* | *F* | *Thai* | *30–60* | *F* |
| $X_3$ | *F* | *T* | *F* | *F* | *Some* | *$* | *F* | *F* | *Burger* | *0–10* | *T* |
| $X_4$ | *T* | *F* | *T* | *T* | *Full* | *$* | *F* | *F* | *Thai* | *10–30* | *T* |
| $X_5$ | *T* | *F* | *T* | *F* | *Full* | *$$$* | *F* | *T* | *French* | *>60* | *F* |
| $X_6$ | *F* | *T* | *F* | *T* | *Some* | *$$* | *T* | *T* | *Italian* | *0–10* | *T* |
| $X_7$ | *F* | *T* | *F* | *F* | *None* | *$* | *T* | *F* | *Burger* | *0–10* | *F* |
| $X_8$ | *F* | *F* | *F* | *T* | *Some* | *$$* | *T* | *T* | *Thai* | *0–10* | *T* |
| $X_9$ | *F* | *T* | *T* | *F* | *Full* | *$* | *T* | *F* | *Burger* | *>60* | *F* |
| $X_{10}$ | *T* | *T* | *T* | *T* | *Full* | *$$$* | *F* | *T* | *Italian* | *10–30* | *F* |
| $X_{11}$ | *F* | *F* | *F* | *F* | *None* | *$* | *F* | *F* | *Thai* | *0–10* | *F* |
| $X_{12}$ | *T* | *T* | *T* | *T* | *Full* | *$* | *F* | *F* | *Burger* | *30–60* | *T* |

$$H(S) = -\left({}^{6}/_{12}\right)\log_2\left({}^{6}/_{12}\right) - \left({}^{6}/_{12}\right)\log_2\left({}^{6}/_{12}\right) = 1$$

6 True,
6 False

# ID3 Decision tree: An example



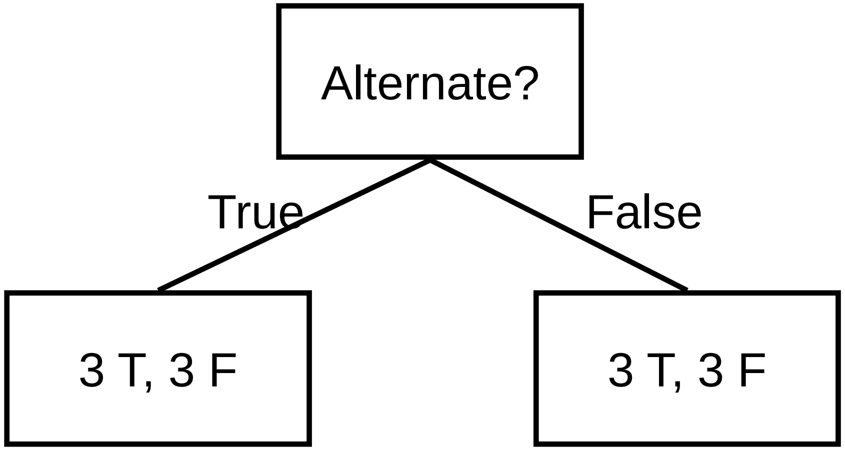

| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

- Calculate Average Entropy of attribute Alternate

$$AE_{Alternate} = P(Alt = T) \times H(Alt = T) + P(Alt = F) \times H(Alt = F)$$

$$AE_{Alternate} = \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right) - \left(\frac{3}{6}\log_2\frac{3}{6}\right)\right] + \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right) - \left(\frac{3}{6}\log_2\frac{3}{6}\right)\right] = 1$$

# ID3 Decision tree: An example

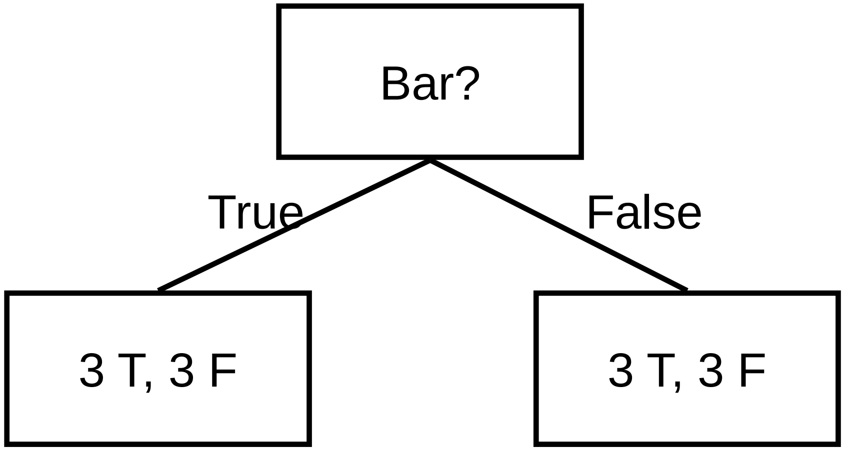



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | \$\$\$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | \$ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | \$ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | \$ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | \$\$\$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | \$\$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | \$ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | \$\$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | \$ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | \$\$\$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | \$ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | \$ | F | F | Burger | 30–60 | T |

- **Information Gain** is the difference in entropy from before to after the set $S$ is split on the selected attribute.

$$IG(Alternate, S) = H(S) - AE_{Alternate} = 1 - 1 = 0$$

# ID3 Decision tree: An example

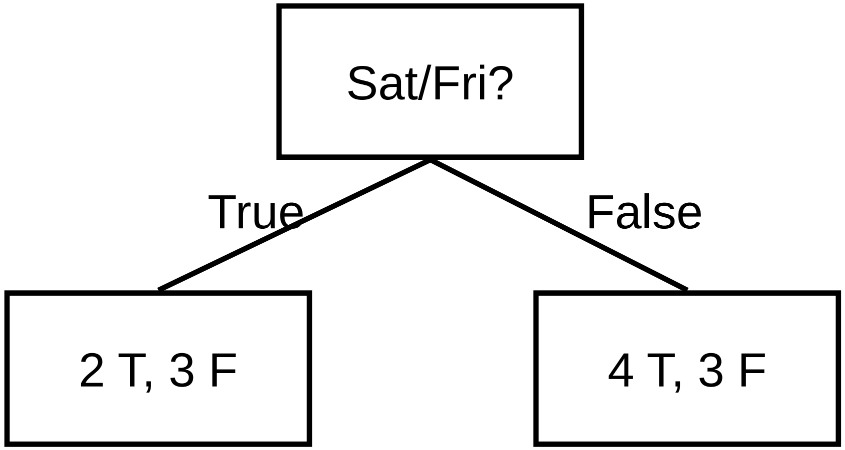



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

$$AE_{Bar} = \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right]+\frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right]=1$$

$$IG(Bar,S) = H(S) - AE_{Bar} = 1 - 1 = {\color{red}0}$$

# ID3 Decision tree: An example

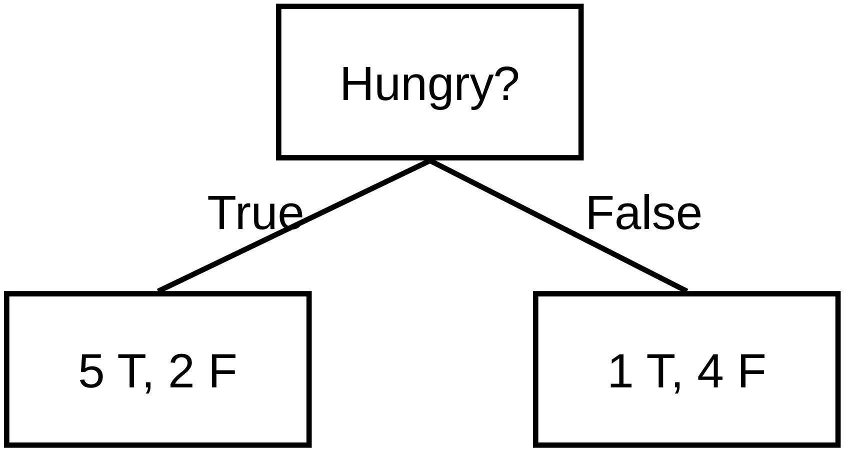



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | \$\$\$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | \$ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | \$ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | \$ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | \$\$\$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | \$\$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | \$ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | \$\$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | \$ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | \$\$\$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | \$ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | \$ | F | F | Burger | 30–60 | T |

$$AE_{Sat/Fri?} = \frac{5}{12}\left[-\left(\frac{2}{5}\log_2\frac{2}{5}\right)-\left(\frac{3}{5}\log_2\frac{3}{5}\right)\right]+\frac{7}{12}\left[-\left(\frac{4}{7}\log_2\frac{4}{7}\right)-\left(\frac{3}{7}\log_2\frac{3}{7}\right)\right]$$
$$= 0.979$$

$$IG(Sat/Fri?, S) = H(S) - AE_{Sat/Fri?} = 1 - 0.979 = 0.021$$

# ID3 Decision tree: An example

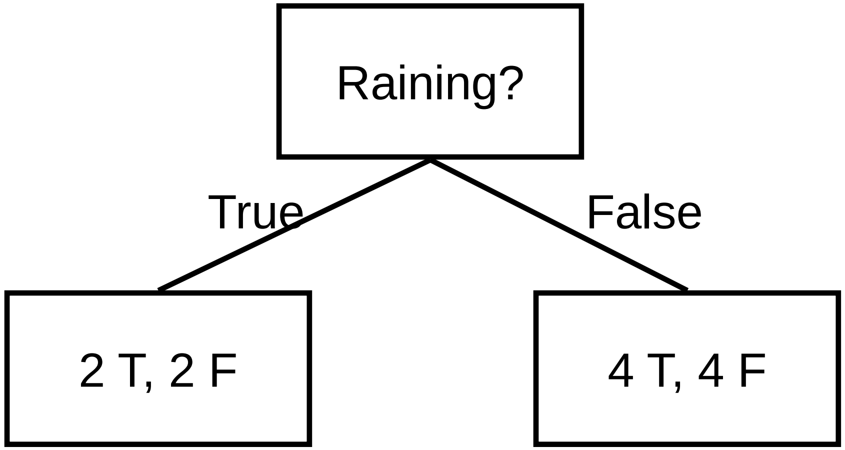



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | *T* | *F* | *F* | *T* | *Some* | *$$$* | *F* | *T* | *French* | *0–10* | *T* |
| $X_2$ | *T* | *F* | *F* | *T* | *Full* | *$* | *F* | *F* | *Thai* | *30–60* | *F* |
| $X_3$ | *F* | *T* | *F* | *F* | *Some* | *$* | *F* | *F* | *Burger* | *0–10* | *T* |
| $X_4$ | *T* | *F* | *T* | *T* | *Full* | *$* | *F* | *F* | *Thai* | *10–30* | *T* |
| $X_5$ | *T* | *F* | *T* | *F* | *Full* | *$$$* | *F* | *T* | *French* | *>60* | *F* |
| $X_6$ | *F* | *T* | *F* | *T* | *Some* | *$$* | *T* | *T* | *Italian* | *0–10* | *T* |
| $X_7$ | *F* | *T* | *F* | *F* | *None* | *$* | *T* | *F* | *Burger* | *0–10* | *F* |
| $X_8$ | *F* | *F* | *F* | *T* | *Some* | *$$* | *T* | *T* | *Thai* | *0–10* | *T* |
| $X_9$ | *F* | *T* | *T* | *F* | *Full* | *$* | *T* | *F* | *Burger* | *>60* | *F* |
| $X_{10}$ | *T* | *T* | *T* | *T* | *Full* | *$$$* | *F* | *T* | *Italian* | *10–30* | *F* |
| $X_{11}$ | *F* | *F* | *F* | *F* | *None* | *$* | *F* | *F* | *Thai* | *0–10* | *F* |
| $X_{12}$ | *T* | *T* | *T* | *T* | *Full* | *$* | *F* | *F* | *Burger* | *30–60* | *T* |

$$AE_{Hungry} = \frac{7}{12}\left[-\left(\frac{5}{7}\log_2\frac{5}{7}\right)-\left(\frac{2}{7}\log_2\frac{2}{7}\right)\right]+\frac{5}{12}\left[-\left(\frac{1}{5}\log_2\frac{1}{5}\right)-\left(\frac{4}{5}\log_2\frac{4}{5}\right)\right]$$
$$= 0.804$$

$$IG(Hungry, S) = H(S) - AE_{Hungry} = 1 - 0.804 = 0.196$$

# ID3 Decision tree: An example

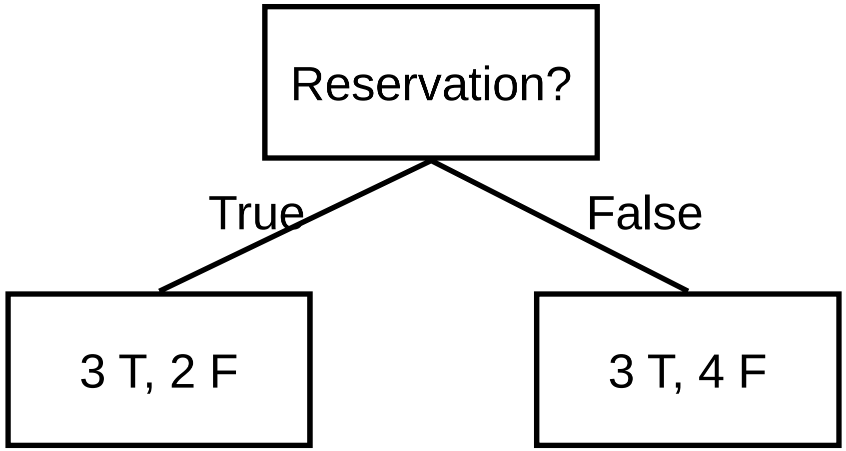



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

$$AE_{Raining} = \frac{4}{12}\left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right)-\left(\frac{2}{4}\log_2\frac{2}{4}\right)\right]+\frac{8}{12}\left[-\left(\frac{4}{8}\log_2\frac{4}{8}\right)-\left(\frac{4}{8}\log_2\frac{4}{8}\right)\right]=1$$

$$IG(Raining, S) = H(S) - AE_{Hungry} = 1 - 1 = {\color{red}0}$$

# ID3 Decision tree: An example

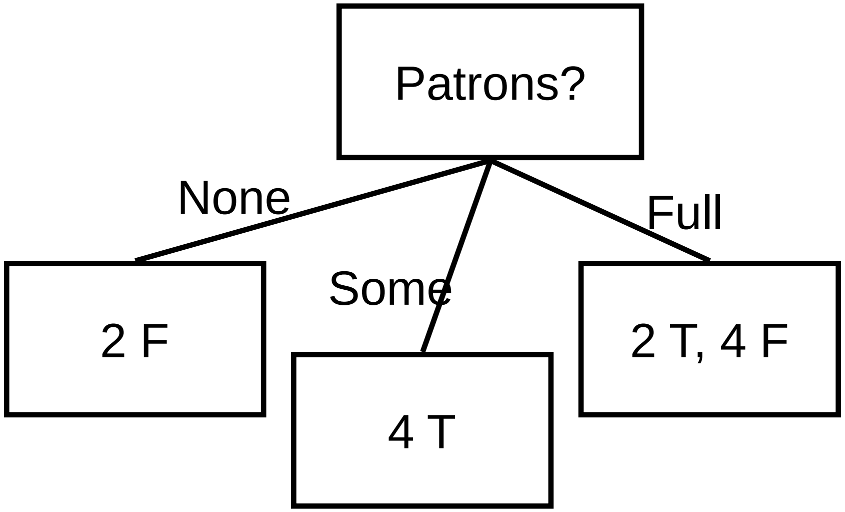



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | \$\$\$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | \$ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | \$ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | \$ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | \$\$\$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | \$\$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | \$ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | \$\$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | \$ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | \$\$\$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | \$ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | \$ | F | F | Burger | 30–60 | T |

$$AE_{Reservation} = \frac{5}{12}\left[-\left(\frac{3}{5}\log_2\frac{3}{5}\right)-\left(\frac{2}{5}\log_2\frac{2}{5}\right)\right]+\frac{7}{12}\left[-\left(\frac{3}{7}\log_2\frac{3}{7}\right)-\left(\frac{4}{7}\log_2\frac{4}{7}\right)\right]$$
$$= 0.979$$

$$IG(Reservation, S) = H(S) - AE_{Reservation} = 1 - 0.979 = \color{red}{0.021}$$

# ID3 Decision tree: An example

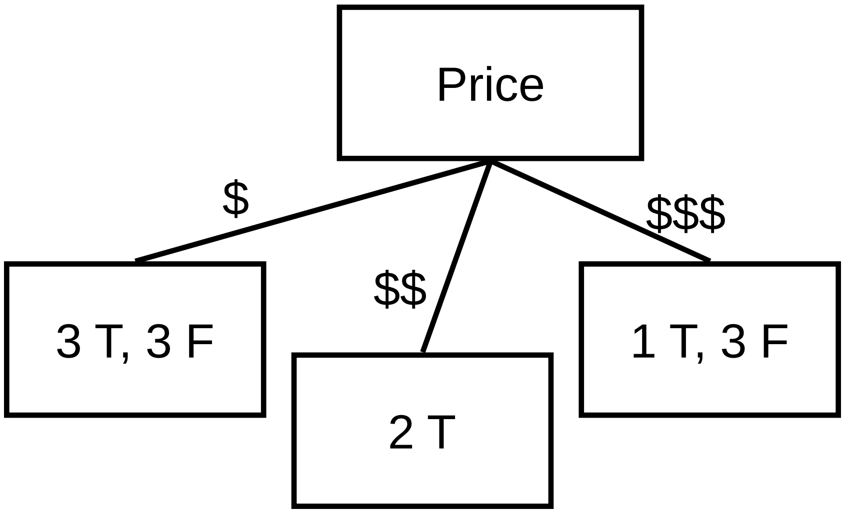



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

$$AE_{Patron} = \frac{2}{12}\left[-\left(\frac{0}{2}\log_2\frac{0}{2}\right)-\left(\frac{2}{2}\log_2\frac{2}{2}\right)\right]+\frac{4}{12}\left[-\left(\frac{4}{4}\log_2\frac{4}{4}\right)-\left(\frac{0}{4}\log_2\frac{0}{4}\right)\right]+\frac{6}{12}\left[-\left(\frac{2}{6}\log_2\frac{2}{6}\right)-\left(\frac{4}{6}\log_2\frac{4}{6}\right)\right]=0.541$$

$$IG(Patron,S)=H(S)-AE_{Patron}=1-0.541=0.459$$

# ID3 Decision tree: An example

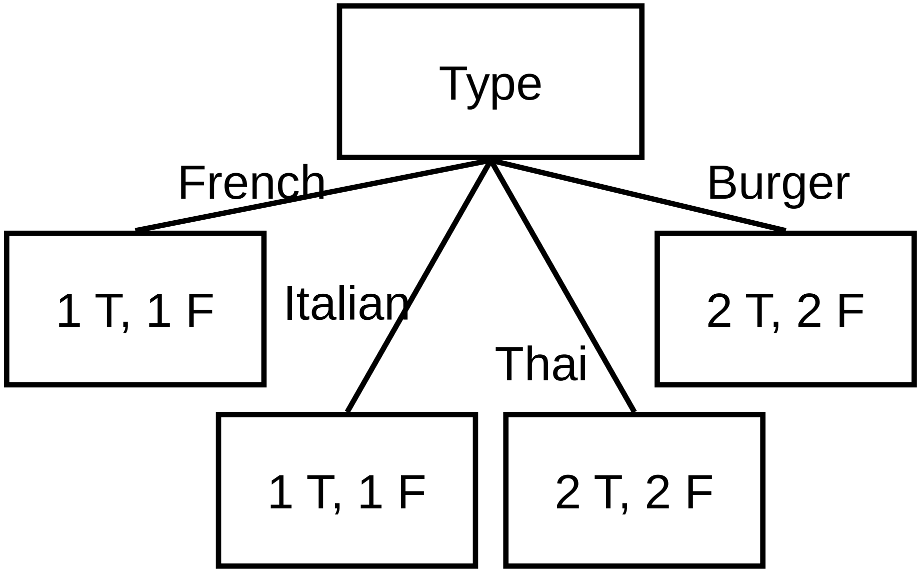



| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

$$AE_{Price}$$
$$= \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right]+\frac{2}{12}\left[-\left(\frac{2}{2}\log_2\frac{2}{2}\right)-\left(\frac{0}{2}\log_2\frac{0}{2}\right)\right]$$
$$+\frac{4}{12}\left[-\left(\frac{1}{4}\log_2\frac{1}{4}\right)-\left(\frac{3}{4}\log_2\frac{3}{4}\right)\right]=0.770$$

$$IG(Price,S)=H(S)-AE_{Price}=1-0.770=0.23$$

# ID3 Decision tree: An example




| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

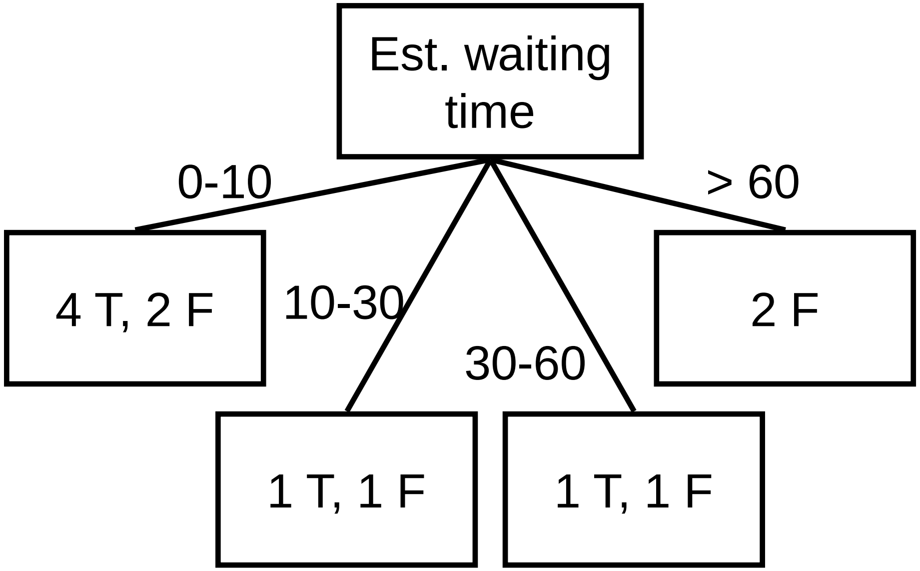
$$AE_{Type}$$

$$= \frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]+\frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]$$

$$+\frac{4}{12}\left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right)-\left(\frac{2}{4}\log_2\frac{2}{4}\right)\right]+\frac{4}{12}\left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right)-\left(\frac{2}{4}\log_2\frac{2}{4}\right)\right]=1$$

$$IG(Type,S) = H(S) - AE_{Type} = 1 - 1 = 0$$

# ID3 Decision tree: An example




| Example | Attributes | | | | | | | | | | Target |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $X_1$ | T | F | F | T | Some | $$$ | F | T | French | 0–10 | T |
| $X_2$ | T | F | F | T | Full | $ | F | F | Thai | 30–60 | F |
| $X_3$ | F | T | F | F | Some | $ | F | F | Burger | 0–10 | T |
| $X_4$ | T | F | T | T | Full | $ | F | F | Thai | 10–30 | T |
| $X_5$ | T | F | T | F | Full | $$$ | F | T | French | >60 | F |
| $X_6$ | F | T | F | T | Some | $$ | T | T | Italian | 0–10 | T |
| $X_7$ | F | T | F | F | None | $ | T | F | Burger | 0–10 | F |
| $X_8$ | F | F | F | T | Some | $$ | T | T | Thai | 0–10 | T |
| $X_9$ | F | T | T | F | Full | $ | T | F | Burger | >60 | F |
| $X_{10}$ | T | T | T | T | Full | $$$ | F | T | Italian | 10–30 | F |
| $X_{11}$ | F | F | F | F | None | $ | F | F | Thai | 0–10 | F |
| $X_{12}$ | T | T | T | T | Full | $ | F | F | Burger | 30–60 | T |

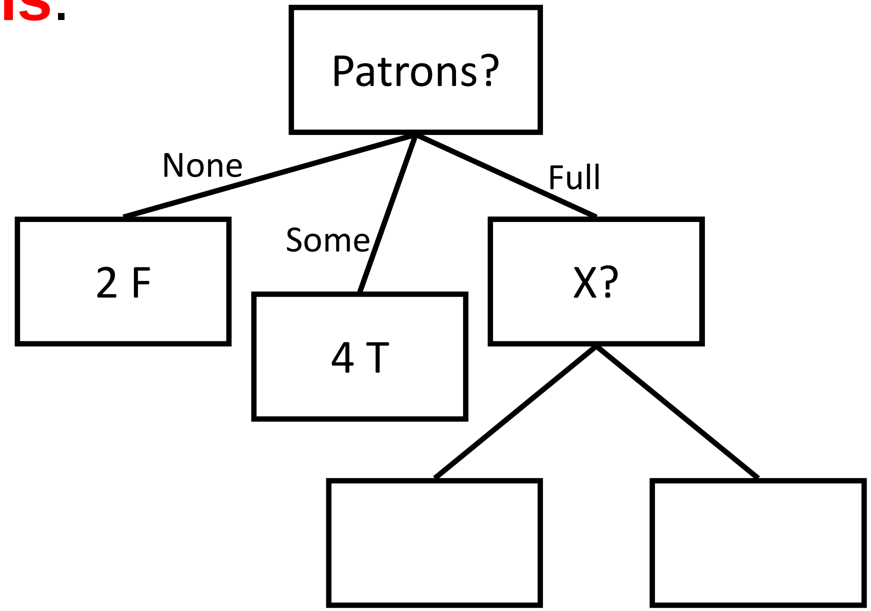
$$AE_{Est.waiting\ time}$$

$$= \frac{6}{12}\left[-\left(\frac{4}{6}\log_2\frac{4}{6}\right)-\left(\frac{2}{6}\log_2\frac{2}{6}\right)\right]+\frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]$$
$$+\frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]+\frac{2}{12}\left[-\left(\frac{0}{2}\log_2\frac{0}{2}\right)-\left(\frac{2}{2}\log_2\frac{2}{2}\right)\right]=0.792$$

$$IG(Est.waiting\ time, S) = H(S) - AE_{Est.waiting\ time} = 1 - 0.792$$
$$= 0.208$$

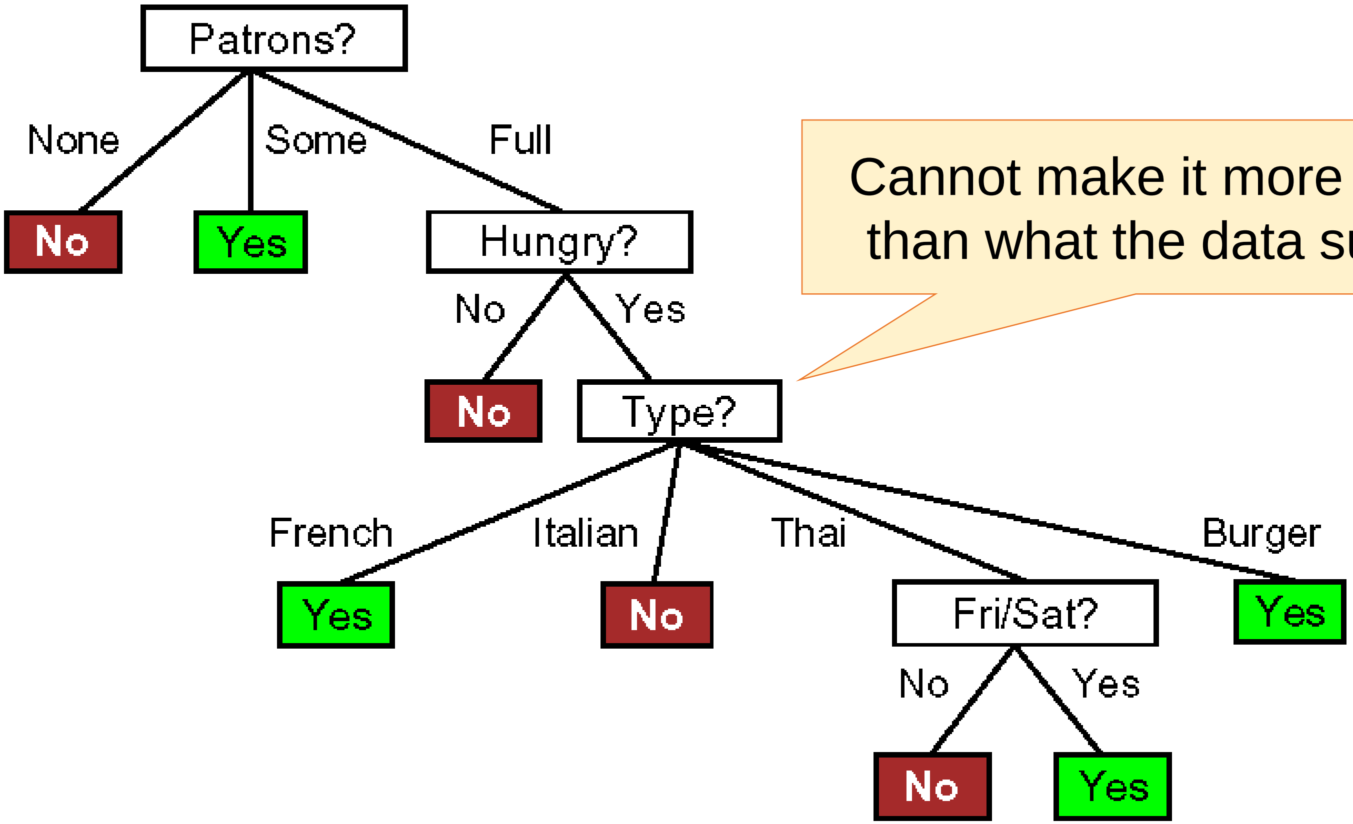
# ID3 Decision tree: An example

- Largest Information Gain (0.459) / Smallest Entropy (0.541) achieved by splitting on **Patrons**.




- Continue making new splits, always purifying nodes

# ID3 Decision tree algorithm

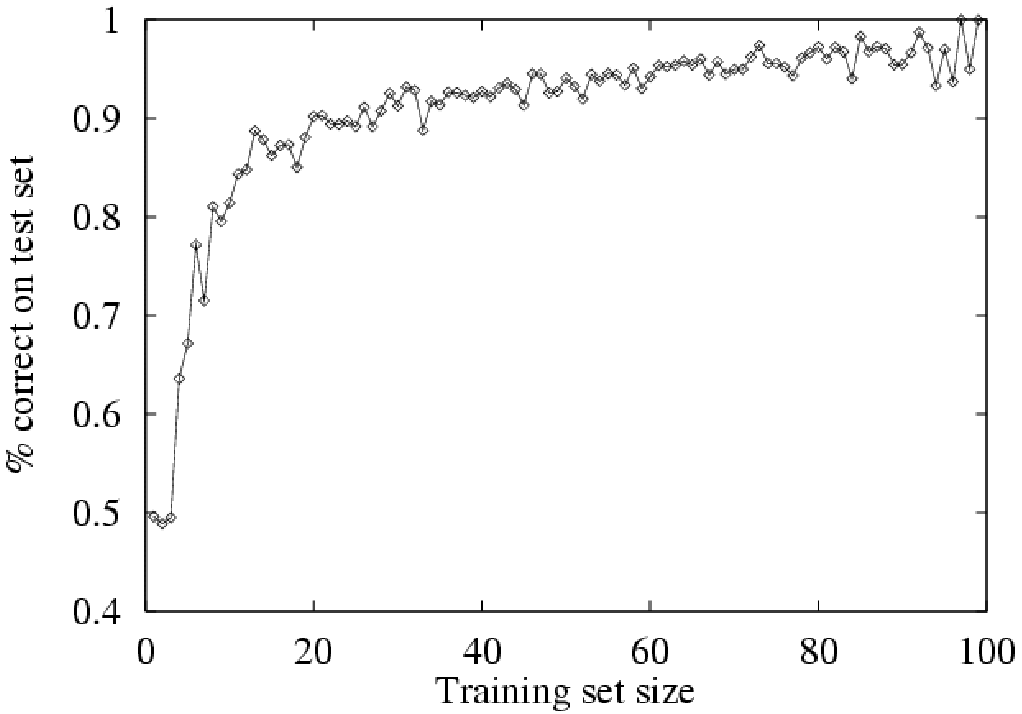



Cannot make it more complex than what the data supports.

Induced tree (from examples)

# Performance measurement

- How do we know that $h \approx f$?
  1. Use theorems of computational or statistical learning theory
  2. Try $h$ on a new test set of examples
     - Use the same distribution over example space as training set




Learning curve = % correct on test set as a function of training set size

# Quiz 01: ID3 decision tree

- The data represent files on a computer system. Possible values of the class variable are “infected”, which implies the file has a virus infection, or “clean” if it doesn't.
- Derive decision tree for virus identification.

| No. | Writable | Updated | Size | Class |
|---|---|---|---|---|
| 1 | Yes | No | Small | Infected |
| 2 | Yes | Yes | Large | Infected |
| 3 | No | Yes | Med | Infected |
| 4 | No | No | Med | Clean |
| 5 | Yes | No | Large | Clean |
| 6 | No | No | Large | Clean |

# Naïve Bayesian classification

# Bayesian classification

- A statistical classifier performs probabilistic prediction, i.e., predicts class membership probabilities
- Foundation: Based on **Bayes' Theorem**

$$\underbrace{P(c \mid x)}_{\text{Posterior Probability}} = \frac{\overbrace{P(x \mid c)}^{\text{Likelihood}}\,\overbrace{P(c)}^{\text{Class Prior Probability}}}{\underbrace{P(x)}_{\text{Predictor Prior Probability}}}$$

$$P(c \mid X) = P(x_1 \mid c) \times P(x_2 \mid c) \times \cdots \times P(x_n \mid c) \times P(c)$$

# Bayesian classification

- Performance
  - A simple Bayesian classifier (e.g., naïve Bayesian), has comparable performance with decision tree and selected neural networks.
- Incremental
  - Each training example can incrementally increase/decrease the probability that a hypothesis is correct
  - That is, prior knowledge can be combined with observed data.
- Standard
  - Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

# The buying computer dataset

| age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|
| <=30 | high | no | fair | no |
| <=30 | high | no | excellent | no |
| 31…40 | high | no | fair | yes |
| >40 | medium | no | fair | yes |
| >40 | low | yes | fair | yes |
| >40 | low | yes | excellent | no |
| 31…40 | low | yes | excellent | yes |
| <=30 | medium | no | fair | no |
| <=30 | low | yes | fair | yes |
| >40 | medium | yes | fair | yes |
| <=30 | medium | yes | excellent | yes |
| 31…40 | medium | no | excellent | yes |
| 31…40 | high | yes | fair | yes |
| >40 | medium | no | excellent | no |

# Bayes' Theorem

- Total Probability Theorem: $P(B) = \sum_{i=1}^{M} P(B|A_i)P(A_i)$

- Let $\mathbf{X}$ be a data sample ("evidence") with unknown class label and $H$ be a hypothesis that $\mathbf{X}$ belongs to class $C$

- **Bayes' Theorem:** $\boldsymbol{P(H \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}}$

- Classification is to determine $P(H \mid \mathbf{X})$, the probability that the hypothesis $H$ holds given the observed data sample $\mathbf{X}$.

# Bayes' Theorem

- $P(H)$ (prior probability): the initial probability
  - E.g., $\mathbf{X}$ will buy computer, regardless of age, income, …
- $P(\mathbf{X})$ : the probability that sample data is observed
  - E.g., $\mathbf{X}$ is 31..40 and has a medium income, regardless of the buying
- $P(\mathbf{X} \mid H)$ (likelihood): the probability of observing the sample $\mathbf{X}$, given that the hypothesis holds
  - E.g., given that $\mathbf{X}$ will buy computer, the probability that $\mathbf{X}$ is 31..40 and has a medium income
- $P(H \mid \mathbf{X}) = \dfrac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}$ (posterior probability)
  - E.g., given that $\mathbf{X}$ is 31..40 and has a medium income, the probability that $\mathbf{X}$ will buy computer

# Bayes' Theorem

- Informally, $P(H \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}$ can be viewed as

$$posteriori \; = \; likelihood * prior \, / \, evidence$$

- $\mathbf{X}$ belongs to $C_i$ iff the probability $P(C_i | \mathbf{X})$ is the highest among all the $P(C_k | \mathbf{X})$ for all the $k$ classes

- **Practical difficulty**
  - Require initial knowledge of many probabilities
  - Significant computational cost involved

# Classification with Bayes' Theorem

- Let $D$ be a training set of tuples and associated class labels
- Each tuple is represented by a $n$-attribute $\mathbf{X} = (x_1, x_2, \dots, x_n)$
- Suppose there are $m$ classes $C_1, C_2, \dots, C_m$
- Classification is to derive the maximum posteriori $P(C_i | \mathbf{X})$ from **Bayes' theorem**

$$P(C_i \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C_i) P(C_i)}{P(\mathbf{X})}$$

- $P(X)$ is constant for all classes, only $P(\mathbf{X} \mid C_i) P(C_i)$ needs to be maximized.

# Naïve Bayesian classification

- Class-conditional independence: There are no dependence relationships **among the attributes**
- The **naïve Bayesian classification** formula is written as

$$P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i) = P(x_1 \mid C_i) \times P(x_2 \mid C_i) \times \cdots \times P(x_n \mid C_i)$$

  - $A_k$ is categorical: $P(x_k \mid C_i)$ is the number of tuples in $C_i$ having value $x_k$ for $A_k$ divided by $|C_{i,D}|$ (# of tuples of $C_i$ in $D$)
  - $A_k$ is continuous: $P(x_k \mid C_i) = g\left(x_k, \mu_{C_i}, \sigma_{C_i}\right)$ with the Gaussian distribution $g(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- Count class distributions only $\rightarrow$ computation cost reduced

# Naïve Bayesian classification: An example

| | |
|---|---|
| P(buys_computer = "yes") | 9/14 |
| P(buys_computer = "no") | 5/14 |

| | buys_computer = "yes" | buys_computer = "no" |
|---|---|---|
| **age = "<=30"** | 2/9 | 3/5 |
| **age = "31…40"** | 4/9 | 0/5 |
| **age = ">40"** | 3/9 | 2/5 |
| **income = "low"** | 3/9 | 1/5 |
| **income = "medium"** | 4/9 | 2/5 |
| **income = "high"** | 2/9 | 2/5 |
| **student = "yes"** | 6/9 | 1/5 |
| **student = "no"** | 3/9 | 4/5 |
| **credit_rating = "fair"** | 6/9 | 2/5 |
| **credit_rating = "excellent"** | 3/9 | 3/5 |

# Naïve Bayesian classification: An example

| age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|
| <=30 | medium | yes | fair | ? |

- $P(\mathbf{X}|\,C_i)$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}yes\text{”})\ =\ 2/9\ *\ 4/9\ *\ 6/9\ *\ 6/9\ =\ 0.044$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}no\text{”})\ =\ 3/5\ *\ 2/5\ *\ 1/5\ *\ 2/5\ =\ 0.019$
- $P(\mathbf{X}|\,C_i) * P(C_i)$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}yes\text{”}) * P(buys\_computer\ = \text{“}yes\text{”}) = 0.028$
  - $P(\mathbf{X}|\,buys\_computer\ =\ \text{“}no\text{”}) * P(buys\_computer\ =\ \text{“}no\text{”}) = 0.007$
- $P(C_i\,|\,\mathbf{X})$
  - $P(buys\_computer\ =\ \text{“}yes\text{”}\,|\,\mathbf{X}) = 0.8$
  - $P(buys\_computer\ =\ \text{“}no\text{”}\,|\,\mathbf{X}) = 0.2$

**Therefore, $\mathbf{X}$ belongs to class (“buys_computer = yes”)**

# Avoiding the zero-probability issue

- The naïve Bayesian prediction requires each conditional probability be **non-zero**.

$$P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i)$$

- Otherwise, the predicted probability will be zero
- For example,

| age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|
| 31…40 | medium | yes | fair | ? |

  - $P(\mathbf{X} \mid buys\_computer = \text{“}no\text{”}) = 0 * 2/5 * 1/5 * 2/5 = 0$
  - Therefore, the conclusion is always **yes** regardless the value of $P(\mathbf{X} \mid buys\_computer = \text{“}yes\text{”})$

# Avoiding the zero-probability issue

- **Laplacian correction** (or Laplacian estimator)

$$P(C_i) = \frac{|C_i| + 1}{|D| + m} \qquad P(x_k \mid C_i) = \frac{|x_k \cup C_i| + 1}{|C_i| + r}$$

  - where $m$ is the number of classes, $|x_k \cup C_i|$ denotes the number of tuples contains both $A_k = x_k$ and $C_i$, and $r$ is the number of values of attribute $A_k$

- The “corrected” probability estimates are close to their “uncorrected” counterparts

# Naïve Bayesian classification: An example

| | |
|---|---|
| P(buys_computer = “yes”) | 10/16 |
| P(buys_computer = “no”) | 6/16 |

| | buys_computer = “yes” | buys_computer = “no” |
|---|---|---|
| **age = “<=30”** | 3/12 | 4/8 |
| **age = “31…40”** | 5/12 | 1/8 |
| **age = “>40”** | 4/12 | 3/8 |
| **income = “low”** | 4/12 | 2/8 |
| **income = “medium”** | 5/12 | 3/8 |
| **income = “high”** | 3/12 | 3/8 |
| **student = “yes”** | 7/11 | 2/7 |
| **student = “no”** | 4/11 | 5/7 |
| **credit_rating = “fair”** | 7/11 | 3/7 |
| **credit_rating = “excellent”** | 4/11 | 4/7 |

# Naïve Bayesian classification: An example

| age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|
| 31..40 | medium | yes | fair | ? |

- $P(\mathbf{X}|\ C_i)$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}yes\text{”})\ = 5/12 * 5/12 * 7/11 * 7/11\ =\ 0.070$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}no\text{”})\ = 1/8\ * 3/8\ * 2/7 * 3/7 =\ 0.006$
- $P(\mathbf{X}|\ C_i) * P(C_i)$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}yes\text{”}) * P(buys\_computer\ = \text{“}yes\text{”}) = 0.044$
  - $P(\mathbf{X}|\,buys\_computer\ =\ \text{“}no\text{”}) * P(buys\_computer\ =\ \text{“}no\text{”}) = 0.002$
- $P(C_i\ |\ \mathbf{X})$
  - $P(buys\_computer\ =\ \text{“}yes\text{”}\,|\,\mathbf{X}) = 0.953$
  - $P(buys\_computer\ =\ \text{“}no\text{”}\,|\,\mathbf{X}) = 0.047$

**Therefore, $\mathbf{X}$ belongs to class (“buys_computer = yes”)**

# Handling missing values

- If the values of some attributes are missing, these attributes are omitted from the product of probabilities
- As a result, the estimation is less accurate
- For example,

| age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|
| ? | medium | yes | fair | ? |

# Naïve Bayesian classification: Evaluation

- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Class conditional independence $\rightarrow$ loss of accuracy
  - Practically, dependencies exist among variables, which cannot be modeled by Naïve Bayes
    - E.g., in medical records, patients' profile (age, family history, etc.), symptoms (fever, cough etc.), disease (lung cancer, diabetes, etc.)
- *How to deal with these dependencies?*
  - Bayesian Belief Networks

# Quiz 02: Naïve Bayesian classification

- The data represent files on a computer system. Possible values of the class variable are “infected”, which implies the file has a virus infection, or “clean” if it doesn't.
- Derive naïve Bayesian probabilities for virus identification in either cases, with or without Laplacian correction.

| No. | Writable | Updated | Size | Class |
|---|---|---|---|---|
| 1 | Yes | No | Small | Infected |
| 2 | Yes | Yes | Large | Infected |
| 3 | No | Yes | Med | Infected |
| 4 | No | No | Med | Clean |
| 5 | Yes | No | Large | Clean |
| 6 | No | No | Large | Clean |


# THE END