Chapter 2

2 Linear Equations

2.1 Equations In Two Unknowns

Exercises

Solve the following systems of equations for x and y.

1. $2x - y = 3$

   $x + y = 2$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} 2x + x &= 3 + 2 \\ 3x &= 5 \\ x &= \frac{5}{3}. \end{aligned}$$

Using this value for x in the second equation gives us

$$\begin{aligned} \frac{5}{3} + y &= 2 \\ y &= 2 - \frac{5}{3} \\ y &= \frac{1}{3}. \end{aligned}$$

2. $-4x + 7y = -1$

$x - 2y = -4$

Multiplying the second equation by 4 gives us

$$\begin{aligned} -4x + 7y &= -1 \\ 4x - 8y &= -16. \end{aligned}$$

Adding the two equations will cancel out the x, giving us

$$7y - 8y = -1 - 16$$

$$-y = -17$$

$$y = 17.$$

Using this value for y in the second equation gives us

$$x - 2(17) = -4$$

$$x - 34 = -4$$

$$x = 30.$$

3. $3x + 4y = -2$

$-2x - 3y = 1$

Multiplying the first equation by 2 and the second equation by 3 gives us

$$6x + 8y = -4$$

$$-6x - 9y = 3.$$

Adding the two equations will cancel out the x, giving us

$$8y - 9y = -4 + 3$$

$$-y = -1$$

$$y = 1.$$

Using this value for y in the second equation gives us

$$-2x - 3(1) = 1$$

$$-2x - 3 = 1$$

$$-2x = 4$$

$$x = -2.$$

4. $-3x + 2y = -1$

$x - y = 2$

Multiplying the second equation by 3 gives us

$$-3x + 2y = -1$$

$$3x - 3y = 6.$$

Adding the two equations will cancel out the x, giving us

$$2y - 3y = -1 + 6$$

$$-y = 5$$

$$y = -5.$$

Using this value for y in the second equation gives us

$$x - (-5) = 2$$

$$x + 5 = 2$$

$$x = -3.$$

5. $-3x + y = 0$

$x - y = 1$

Adding the two equations will cancel out the y, giving us

$$-3x + x = 0 + 1$$

$$-2x = 1$$

$$x = -\frac{1}{2}.$$

Using this value for x in the second equation gives us

$$\begin{aligned} -\frac{1}{2} - y &= 1 \\ -y &= 1 + \frac{1}{2} \\ -y &= \frac{3}{2} \\ y &= -\frac{3}{2}. \end{aligned}$$

6. $3x + 7y = 0$

$x - y = 0$

Multiplying the second equation by 7 gives us

$$3x + 7y = 0$$

$$7x - 7y = 0.$$

Adding the two equations will cancel out the y, giving us

$$3x + 7x = 0 + 0$$

$$10x = 0$$

$$x = 0.$$

Using this value for x in the second equation gives us

$$\begin{aligned} 0 - y &= 0 \\ -y &= 0 \\ y &= 0. \end{aligned}$$

7. $7x - y = 2$

$2x + 2y = 4$

Multiplying the first equation by 2 gives us

$$\begin{aligned} 14x - 2y &= 4 \\ 2x + 2y &= 4. \end{aligned}$$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} 14x + 2x &= 4 + 4 \\ 16x &= 8 \\ x &= \frac{8}{16} \\ x &= \frac{1}{2}. \end{aligned}$$

Using this value for x in the second equation gives us

$$\begin{aligned} 2\left(\frac{1}{2}\right) + 2y &= 4 \\ 1 + 2y &= 4 \\ 2y &= 3 \\ y &= \frac{3}{2}. \end{aligned}$$

8. $-4x - 7y = 5$

$2x + y = 6$

Multiplying the second equation by 7 gives us

$$\begin{aligned} -4x - 7y &= 5 \\ 14x + 7y &= 42. \end{aligned}$$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} -4x + 14x &= 5 + 42 \\ 10x &= 47 \\ x &= \frac{47}{10}. \end{aligned}$$

Using this value for x in the second equation gives us

$$\begin{aligned} 2\left(\frac{47}{10}\right) + y &= 6 \\ \frac{94}{10} + y &= 6 \\ y &= 6 - \frac{94}{10} \\ y &= \frac{60}{10} - \frac{94}{10} \\ y &= -\frac{34}{10} \\ y &= -\frac{17}{5}. \end{aligned}$$

9. Let a, b, c, d be numbers such that $ad - bc \neq 0$. Solve the following systems of equations for x and y in terms of a, b, c, d.

a) $ax + by = 1$

$cx + dy = 2$

Multiplying the first equation by d and the second equation by $-b$ gives us

$$adx + bdy = d$$

$$-bcx - bdy = -2b.$$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} adx - bcx &= d - 2b \\ x(ad - bc) &= d - 2b \\ x &= \frac{d - 2b}{ad - bc}. \end{aligned}$$

Instead of using this value for x in one of the equations, we can do the same thing as above to find y.

Multiplying the first equation by $-c$ and the second equation by a gives us

$$\begin{aligned} -acx - bcy &= -c \\ acx + ady &= 2a. \end{aligned}$$

Adding the two equations will cancel out the x, giving us

$$\begin{aligned} -bcy + ady &= -c + 2a \\ y(-bc + ad) &= -c + 2a \\ y &= \frac{2a - c}{ad - bc}. \end{aligned}$$

b) $ax + by = 3$

$cx + dy = -4$

Multiplying the first equation by d and the second equation by $-b$ gives us

$$adx + bdy = 3d$$

$$-bcx - bdy = 4b.$$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} adx - bcx &= 3d + 4b \\ x(ad - bc) &= 3d + 4b \\ x &= \frac{3d + 4b}{ad - bc}. \end{aligned}$$

Instead of using this value for x in one of the equations, we can do the same thing as above to find y.

Multiplying the first equation by $-c$ and the second equation by a gives us

$$-acx - bcy = -3c$$

$$acx + ady = -4a.$$

Adding the two equations will cancel out the x, giving us

$$\begin{aligned} -bcy + ady &= -3c - 4a \\ y(-bc + ad) &= -3c - 4a \\ y &= \frac{-4a - 3c}{ad - bc}. \end{aligned}$$

c) $ax + by = -2$

$cx + dy = 3$

Multiplying the first equation by d and the second equation by $-b$ gives us

$$adx + bdy = -2d$$

$$-bcx - bdy = -3b.$$

Adding the two equations will cancel out the y, giving us

$$adx - bcx = -2d - 3b$$

$$x(ad - bc) = -2d - 3b$$

$$x = \frac{-2d - 3b}{ad - bc}.$$

Instead of using this value for x in one of the equations, we can do the same thing as above to find y.

Multiplying the first equation by $-c$ and the second equation by a gives us

$$-acx - bcy = 2c$$

$$acx + ady = 3a.$$

Adding the two equations will cancel out the x, giving us

$$-bcy + ady = 2c + 3a$$

$$y(-bc + ad) = 2c + 3a$$

$$y = \frac{3c + 3a}{ad - bc}.$$

d) $ax + by = 5$

$cx + dy = 7$

Multiplying the first equation by d and the second equation by $-b$ gives us

$$adx + bdy = 5d$$

$$-bcx - bdy = -7b.$$

Adding the two equations will cancel out the y, giving us

$$adx - bcx = 5d - 7b$$

$$x(ad - bc) = 5d - 7b$$

$$x = \frac{5d - 7b}{ad - bc}.$$

Instead of using this value for x in one of the equations, we can do the same thing as above to find y.

Multiplying the first equation by $-c$ and the second equation by a gives us

$$-acx - bcy = -5c$$

$$acx + ady = 7a.$$

Adding the two equations will cancel out the x, giving us

$$-bcy + ady = -5c + 7a$$

$$y(-bc + ad) = -5c + 7a$$

$$y = \frac{7a - 5c}{ad - bc}.$$

10. Making the same assumptions as in Exercise 9, show that the solution of the system

$$ax + by = 0,$$

$$cx + dy = 0$$

must be $x = 0$ and $y = 0$.

Multiplying the first equation by d and the second equation by $-b$ gives us

$$adx + bdy = 0$$

$$-bcx - bdy = 0.$$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} adx - bcx &= 0 \\ x(ad - bc) &= 0 \\ x &= \frac{0}{ad - bc} \\ x &= 0. \end{aligned}$$

We can't use this value for x in the second equation (or the first, but it's the same situation anyway). Let's see what happens.

$$\begin{aligned} c(0) + dy &= 0 \\ 0 + dy &= 0 \\ dy &= 0 \end{aligned}$$

We can't divide both sides by d because we don't know if d is equal to 0 or not.

Instead, we'll multiply both equations and cancel out the x. Multiplying the first equation by $-c$ and the second equation by a gives us

$$\begin{aligned} -acx - bcy &= 0 \\ acx + ady &= 0. \end{aligned}$$

Adding the two equations will cancel out the x, giving us

$$\begin{aligned} -bcy + ady &= 0 \\ y(-bc + ad) &= 0 \\ y &= \frac{0}{ad - bc} \\ y &= 0. \end{aligned}$$

11. Let a, b, c, d, u, v be numbers and assume that $ad - bc \neq 0$. Solve the following system of equations for x and y in terms of a, b, c, d, u, v:

$$\begin{aligned} ax + by &= u \\ cx + dy &= v. \end{aligned}$$

Verify that the answer you get is actually a solution.

Multiplying the first equation by $-c$ and the second equation by a gives us

$$\begin{aligned} -acx - bcy &= -cu \\ acx + ady &= av. \end{aligned}$$

Adding the two equations will cancel out the x, giving us

$$
\begin{aligned}
-bcy + ady &= -cu + av \\
y(-bc + ad) &= -cu + av \\
y &= \frac{-cu + av}{ad - bc}.
\end{aligned}
$$

Multiplying the first equation by d and the second equation by $-b$ gives us

$$
\begin{aligned}
adx + bdy &= du \\
-bcx - bdy &= -bv.
\end{aligned}
$$

Adding the two equations will cancel out the y, giving us

$$
\begin{aligned}
adx - bcx &= du - bv \\
x(ad - bc) &= du - bv \\
x &= \frac{du - bv}{ad - bc}.
\end{aligned}
$$

To verify that x, y are actually solutions, we plug them into the two equations.

Starting with the first equation, we get

$$\begin{aligned}
a\left(\frac{du - bv}{ad - bc}\right) + b\left(\frac{-cu + av}{ad - bc}\right) &= u \\
\frac{adu - abv}{ad - bc} + \frac{-bcu + abv}{ad - bc} &= u \\
\frac{adu - abv - bcu + abv}{ad - bc} &= u \\
\frac{adu - bcu}{ad - bc} &= u \\
\frac{u(ad - bc)}{ad - bc} &= u \\
u &= u.
\end{aligned}$$

For the second equation, we get

$$\begin{aligned}
c\left(\frac{du - bv}{ad - bc}\right) + d\left(\frac{-cu + av}{ad - bc}\right) &= v \\
\frac{cdu - bcv}{ad - bc} + \frac{-cdu + adv}{ad - bc} &= v \\
\frac{cdu - bcv - cdu + adv}{ad - bc} &= v \\
\frac{-bcv + adv}{ad - bc} &= v \\
\frac{v(ad - bc)}{ad - bc} &= v \\
v &= v.
\end{aligned}$$

2.2 Equations In Three Unknowns

Exercises

Solve the following equations for x, y, z.

1. $2x - 3y + z = 0$

$$x + y + z = 1$$

$$x - 2y - 4z = 2$$

We will reduce this to a system of two equations in two unknowns by eliminating the z.

We start by looking at the first and second equations. Multiplying the first equation by -1 gives us

$$-2x + 3y - z = 0 \tag{1}$$

$$x + y + z = 1 \tag{2}$$

$$x - 2y - 4z = 2 \tag{3}$$

Adding (1) and (2) will cancel out the z, giving us

$$-2x + x + 3y + y = 0 + 1 \tag{4}$$

$$-x + 4y = 1 \tag{5}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the z, this time looking at the first and third equations. Multiplying the first equation by 4 gives us

$$8x - 12y + 4z = 0 \tag{6}$$

$$x + y + z = 1 \tag{7}$$

$$x - 2y - 4z = 2 \tag{8}$$

Adding (6) and (8) will cancel out the z, giving us

$$8x + x - 12y - 2y = 0 + 2 \tag{9}$$

$$9x - 14y = 2 \tag{10}$$

Now we have a system of two equations (from (5) and (10))

$$-x + 4y = 1 \tag{11}$$

$$9x - 14y = 2 \tag{12}$$

Multiplying (11) by 9 gives us

$$-9x + 36y = 9$$

$$9x - 14y = 2$$

Adding the two equations will cancel out the x, giving us

$$36y - 14y = 9 + 2$$

$$22y = 11$$

$$y = \frac{11}{22}$$

$$y = \frac{1}{2}$$

Using this value for y in (11) gives us

$$-x + 4\left(\frac{1}{2}\right) = 1$$

$$-x + 2 = 1$$

$$-x = -1$$

$$x = 1$$

Using these values for x and y in the second equation (from the beginning) gives us

$$1 + \frac{1}{2} + z = 1$$

$$\frac{1}{2} + z = 0$$

$$z = -\frac{1}{2}$$

2. $2x - y + z = 1$

   $4x + y + z = 2$

   $x - y - 2z = 0$

We will reduce this to a system of two equations in two unknowns by eliminating the y.

We start by looking at the first and second equations. Adding the first and second equation will cancel out the y, giving us

$$2x + 4x + z + z = 1 + 2 \tag{1}$$

$$6x + 2z = 3 \tag{2}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the y, this time looking at the second and third equations. Adding the second and third equation will cancel out the y, giving us

$$4x + x + z - 2z = 2 + 0 \tag{3}$$

$$5x - z = 2 \tag{4}$$

Now we have a system of two equations (from (2) and (4))

$$6x + 2z = 3 \tag{5}$$

$$5x - z = 2 \tag{6}$$

Multiplying (6) by 2 gives us

$$6x + 2z = 3$$

$$10x - 2z = 4$$

Adding the two equations will cancel out the z, giving us

$$6x + 10x = 3 + 4$$

$$16x = 7$$

$$x = \frac{7}{16}$$

Using this value for x in (6) gives us

$$5\left(\frac{7}{16}\right) - z = 2$$

$$\frac{35}{16} - z = 2$$

$$-z = 2 - \frac{35}{16}$$

$$-z = \frac{32}{16} - \frac{35}{16}$$

$$-z = -\frac{3}{16}$$

$$z = \frac{3}{16}$$

Using these values for x and z in the first equation (from the beginning) gives us

$$\begin{aligned}
2\left(\frac{7}{16}\right) - y + \frac{3}{16} &= 1 \\
\frac{14}{16} - y + \frac{3}{16} &= 1 \\
-y + \frac{17}{16} &= 1 \\
-y &= 1 - \frac{17}{16} \\
-y &= \frac{16}{16} - \frac{17}{16} \\
-y &= -\frac{1}{16} \\
y &= \frac{1}{16}
\end{aligned}$$

3. $$\begin{aligned}
x + 4y - 4z &= 1 \\
x + 2y + z &= 2 \\
4x - 3y - 2z &= 1
\end{aligned}$$

We will reduce this to a system of two equations in two unknowns by eliminating the x.

We start by looking at the first and second equations. Multiplying the first equation by -1 gives us

$$-x - 4y + 4z = -1 \tag{1}$$

$$x + 2y + z = 2 \tag{2}$$

$$4x - 3y - 2z = 1 \tag{3}$$

Adding (1) and (2) will cancel out the x, giving us

$$-4y + 2y + 4z + z = -1 + 2 \tag{4}$$

$$-2y + 5z = 1 \tag{5}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the x, this time looking at the second and third equations. Multiplying the second equation by -4 gives us

$$x + 4y - 4z = 1 \tag{6}$$

$$-4x - 8y - 4z = -8 \tag{7}$$

$$4x - 3y - 2z = 1 \tag{8}$$

Adding (7) and (8) will cancel out the x, giving us

$$-8y - 3y - 4z - 2z = -8 + 1 \tag{9}$$

$$-11y - 6z = -7 \tag{10}$$

Now we have a system of two equations (from (5) and (10))

$$-2y + 5z = 1 \tag{11}$$

$$-11y - 6z = -7 \tag{12}$$

Multiplying (11) by 6 and (12) by 5 gives us

$$-12y + 30z = 6$$

$$-55y - 30z = -35$$

Adding the two equations will cancel out the z, giving us

$$\begin{aligned} -12y - 55y &= 6 - 35 \\ -67y &= -29 \\ y &= \frac{-29}{-67} \\ y &= \frac{29}{67} \end{aligned}$$

Using this value for y in (11) gives us

$$
\begin{aligned}
-2\left(\frac{29}{67}\right) + 5z &= 1 \\
-\frac{58}{67} + 5z &= 1 \\
5z &= 1 + \frac{58}{67} \\
5z &= \frac{67}{67} + \frac{58}{67} \\
5z &= \frac{125}{67} \\
\frac{5z}{1} &= \frac{125}{67} \\
5z \cdot 67 &= 125 \cdot 1 \\
335z &= 125 \\
z &= \frac{125}{335} \\
z &= \frac{25}{67}
\end{aligned}
$$

Using these values for y and z in the second equation (from the beginning) gives us

$$\begin{aligned} x + 2\left(\frac{29}{67}\right) + \frac{25}{67} &= 2 \\ x + \frac{58}{67} + \frac{25}{67} &= 2 \\ x + \frac{83}{67} &= 2 \\ x &= 2 - \frac{83}{67} \\ x &= \frac{134}{67} - \frac{83}{67} \\ x &= \frac{51}{67} \end{aligned}$$

4. $$\begin{aligned} x + y + z &= 0 \\ x - y + z &= 0 \\ 2x - y - z &= 0 \end{aligned}$$

Normally, we would want to reduce this to a system of two equations in two unknowns. However, we can quickly find the value of x by looking at the first and third equations. Adding the first and third equation will cancel out the y (and the z), giving us

$$x + 2x = 0 + 0$$

$$3x = 0$$

$$x = 0$$

Now we need an equation in two unknowns (with one of the unknowns being x), so we go back to the beginning, this time looking at the second and third equations.

Adding the first and second equation will cancel out the y, giving us

$$x + x + z + z = 0 + 0$$

$$2x + 2z = 0$$

Using the value for x we got above gives us

$$2(0) + 2z = 0$$

$$0 + 2z = 0$$

$$2z = 0$$

$$z = 0$$

Using these values for x and z in the first equation (from the beginning) gives us

$$\begin{aligned} 0 + y + 0 &= 0 \\ y &= 0 \end{aligned}$$

5. $5x + 3y - z = 0$

   $x + 2y + 2z = 1$

   $x - 2y - 2z = 0$

Normally, we would want to reduce this to a system of two equations in two unknowns. However, we can quickly find the value of x by looking at the second and third equations. Adding the second and third equation will cancel out the y and z, giving us

$$\begin{aligned} x + x &= 1 + 0 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Now we need an equation in two unknowns (with one of the unknowns being x), so we go back to the beginning, this time looking at the first and second equations.

Multiplying the first equation by 2 gives us

$$10x + 6y - 2z = 0 \tag{1}$$

$$x + 2y + 2z = 1 \tag{2}$$

$$x - 2y - 2z = 0 \tag{3}$$

Adding (1) and (2) will cancel out the z, giving us

$$10x + x + 6y + 2y = 0 + 1$$

$$11x + 8y = 1$$

Using the value for x we got above gives us

$$11\left(\frac{1}{2}\right) + 8y = 1$$

$$\frac{11}{2} + 8y = 1$$

$$8y = 1 - \frac{11}{2}$$

$$8y = \frac{2}{2} - \frac{11}{2}$$

$$8y = -\frac{9}{2}$$

$$\frac{8y}{1} = -\frac{9}{2}$$

$$8y \cdot 2 = -9 \cdot 1$$

$$16y = -9$$

$$y = -\frac{9}{16}$$

Using these values for x and y in the third equation (from the beginning) gives us

$$\begin{aligned}
\frac{1}{2} - 2\left(-\frac{9}{16}\right) - 2z &= 0 \\
\frac{1}{2} + \frac{18}{16} - 2z &= 0 \\
\frac{8}{16} + \frac{18}{16} - 2z &= 0 \\
\frac{26}{16} - 2z &= 0 \\
-2z &= -\frac{26}{16} \\
\frac{-2z}{1} &= -\frac{26}{16} \\
-2z \cdot 16 &= -26 \cdot 1 \\
-32z &= -26 \\
z &= \frac{-26}{-32} \\
z &= \frac{13}{16}
\end{aligned}$$

6. $$\begin{aligned} 2x + 2y - 3z &= 0 \\ x - 3y + z &= 3 \\ 2x + y - 4z &= 0 \end{aligned}$$

We will reduce this to a system of two equations in two unknowns by eliminating the x.

We start by looking at the first and third equations. Multiplying the first equation by -1 gives us

$$-2x - 2y + 3z = 0 \tag{1}$$

$$x - 3y + z = 3 \tag{2}$$

$$2x + y - 4z = 0 \tag{3}$$

Adding (1) and (3) will cancel out the x, giving us

$$-2y + y + 3z - 4z = 0 + 0 \tag{4}$$

$$-y - z = 0 \tag{5}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate x, this time looking at the second and third equations. Multiplying the second equation by -2 gives us

$$2x + 2y - 3z = 0 \tag{6}$$

$$-2x + 6y - 2z = -6 \tag{7}$$

$$2x + y - 4z = 0 \tag{8}$$

Adding (7) and (8) will cancel out the x, giving us

$$6y + y - 2z - 4z = -6 + 0 \tag{9}$$

$$7y - 6z = -6 \tag{10}$$

Now we have a system of two equations (from (5) and (10))

$$-y - z = 0 \tag{11}$$

$$7y - 6z = -6 \tag{12}$$

Multiplying (11) by 7 gives us

$$-7y - 7z = 0$$

$$7y - 6z = -6$$

Adding the two equations will cancel out the y, giving us

$$\begin{aligned} -7z - 6z &= 0 - 6 \\ -13z &= -6 \\ z &= \frac{-6}{-13} \\ z &= \frac{6}{13} \end{aligned}$$

Using this value for z in (5) gives us

$$\begin{aligned} -y - \frac{6}{13} &= 0 \\ -y &= \frac{6}{13} \\ y &= -\frac{6}{13} \end{aligned}$$

Using these values for x and y in the second equation (from the beginning) gives us

$$\begin{aligned}
x - 3\left(-\frac{6}{13}\right) + \frac{6}{13} &= 3 \\
x + \frac{18}{13} + \frac{6}{13} &= 3 \\
x + \frac{24}{13} &= 3 \\
x &= 3 - \frac{24}{13} \\
x &= \frac{39}{13} - \frac{24}{13} \\
x &= \frac{15}{13}
\end{aligned}$$

7. $4x - 2y + 5z = 1$

$$x + y + z = 0$$

$$-x + y - 2z = 2$$

We will reduce this to a system of two equations in two unknowns by eliminating the x.

We start by looking at the second and third equations. Adding the second and third equation will cancel out the x, giving us

$$y + y + z - 2z = 0 + 2 \tag{1}$$

$$2y - z = 2 \tag{2}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the x, this time looking at the first and third equations. Multiplying the third equation by 4 gives us

$$4x - 2y + 5z = 1 \tag{3}$$

$$x + y + z = 0 \tag{4}$$

$$-4x + 4y - 8z = 8 \tag{5}$$

Adding (3) and (5) will cancel out the x, giving us

$$-2y + 4y + 5z - 8z = 1 + 8 \tag{6}$$

$$2y - 3z = 9 \tag{7}$$

Now we have a system of two equations (from (2) and (7))

$$2y - z = 2 \tag{8}$$

$$2y - 3z = 9 \tag{9}$$

Multiplying (8) by -1 gives us

$$-2y + z = -2$$

$$2y - 3z = 9$$

Adding the two equations will cancel out the y, giving us

$$z - 3z = -2 + 9$$

$$-2z = 7$$

$$z = -\frac{7}{2}$$

Using this value for z in (8) gives us

$$2y - \left(-\frac{7}{2}\right) = 2$$

$$2y + \frac{7}{2} = 2$$

$$2y = 2 - \frac{7}{2}$$

$$2y = \frac{4}{2} - \frac{7}{2}$$

$$2y = \frac{-3}{2}$$

$$\frac{2y}{1} = \frac{-3}{2}$$

$$2y \cdot 2 = -3 \cdot 1$$

$$4y = -3$$

$$y = \frac{-3}{4}$$

Using these values for y and z in the second equation (from the beginning) gives us

$$\begin{aligned} x - \frac{3}{4} - \frac{7}{2} &= 0 \\ x - \frac{3}{4} - \frac{14}{4} &= 0 \\ x - \frac{17}{4} &= 0 \\ x &= \frac{17}{4} \end{aligned}$$

8. $x + y + z = 0$

   $x - y - z = 1$

   $x + y - z = 1$

Normally, we would want to reduce this to a system of two equations in two unknowns. However, we can quickly find the value of x by looking at the first and second equations. Adding the first and second equation will cancel out the y and z, giving us

$$\begin{aligned} x + x + z - z &= 0 + 1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Now we need an equation in two unknowns (with one of the unknowns being x), so we go back to the beginning, this time looking at the first and third equations.

Adding the first and third equation will cancel out the z, giving us

$$x + x + y + y = 0 + 1$$

$$2x + 2y = 1$$

Using the value for x we got above gives us

$$2\left(\frac{1}{2}\right) + 2y = 1$$

$$1 + 2y = 1$$

$$2y = 0$$

$$y = 0$$

Using these values for x and y in the first equation (from the beginning) gives us

$$\frac{1}{2} + 0 + z = 0$$

$$z = -\frac{1}{2}$$

In the next exercises, you will find it easiest to clear denominators before solving.

9. $\dfrac{1}{2}x + y - \dfrac{3}{4}z = 1$

$\dfrac{2}{3}x - \dfrac{1}{3}y + z = 2$

$x - \dfrac{1}{5}y + 2z = 1$

To clear denominators, we multiply the first equation by 4, the second equation by 3, and the third equation by 5, which gives us

$$2x + 4y - 3z = 4 \qquad (1)$$

$$2x - y + 3z = 6 \qquad (2)$$

$$5x - y + 10z = 5 \qquad (3)$$

We will reduce this to a system of two equations in two unknowns by eliminating the y.

We start by looking at (1) and (2). Multiplying (2) by 4 gives us

$$2x + 4y - 3z = 4 \qquad (4)$$

$$8x - 4y + 12z = 24 \qquad (5)$$

$$5x - y + 10z = 5 \qquad (6)$$

Adding (4) and (5) will cancel out the y, giving us

$$2x + 8x - 3z + 12z = 4 + 24 \tag{7}$$

$$10x + 9z = 28 \tag{8}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the y, this time looking at (2) and (3). Multiplying (2) by -1 gives us

$$2x + 4y - 3z = 4 \tag{9}$$

$$-2x + y - 3z = -6 \tag{10}$$

$$5x - y + 10z = 5 \tag{11}$$

Adding (10) and (11) will cancel out the y, giving us

$$-2x + 5x - 3z + 10z = -6 + 5 \tag{12}$$

$$3x + 7z = -1 \tag{13}$$

Now we have a system of two equations (from (8) and (13))

$$10x + 9z = 28 \tag{14}$$

$$3x + 7z = -1 \tag{15}$$

Multiplying (14) by 3 and (15) by -10 gives us

$$30x + 27z = 84$$

$$-30x - 70z = 10$$

Adding the two equations will cancel out the z, giving us

$$27z - 70z = 84 + 10$$

$$-43z = 94$$

$$z = -\frac{94}{43}$$

Using this value for z in (15) gives us

$$\begin{aligned}
3x + 7\left(-\frac{94}{43}\right) &= -1 \\
3x - \frac{658}{43} &= -1 \\
3x &= -1 + \frac{658}{43} \\
3x &= \frac{43}{43} + \frac{658}{43} \\
3x &= \frac{615}{43} \\
\frac{3x}{1} &= \frac{615}{43} \\
3x \cdot 43 &= 615 \cdot 1 \\
129x &= 615 \\
x &= \frac{615}{129} \\
x &= \frac{205}{43}
\end{aligned}$$

Using these values for x and z in (2) gives us

$$\begin{aligned}
2\left(\frac{205}{43}\right) - y + 3\left(-\frac{94}{43}\right) &= 6 \\
\frac{410}{43} - y - \frac{282}{43} &= 6 \\
-y + \frac{128}{43} &= 6 \\
-y &= 6 - \frac{128}{43} \\
-y &= \frac{258}{43} - \frac{128}{43} \\
-y &= \frac{130}{43} \\
y &= -\frac{130}{43}
\end{aligned}$$

10. $$\begin{aligned}
\frac{1}{2}x - y + z &= 1 \\
x + \frac{1}{3}y - \frac{2}{3}z &= 2 \\
x + y - z &= 3
\end{aligned}$$

To clear denominators, we multiply the first equation by 2 and the second equation by 3, which gives us

$$x - 2y + 2z = 2 \tag{1}$$

$$3x + y - 2z = 6 \tag{2}$$

$$x + y - z = 3 \tag{3}$$

Normally, we would want to reduce this to a system of two equations in two unknowns. However, we can quickly find the value of x by looking at (1) and (3). Multiplying (3) by 2 gives us

$$x - 2y + 2z = 2 \tag{4}$$

$$3x + y - 2z = 6 \tag{5}$$

$$2x + 2y - 2z = 6 \tag{6}$$

Adding (4) and (6) will cancel out the y and z, giving us

$$x + 2x = 2 + 6$$

$$3x = 8$$

$$x = \frac{8}{3}$$

Now we need an equation in two unknowns (with one of the unknowns being x), so we go back to the beginning, this time looking at (1) and (2). Adding (1) and (2) will cancel out the z, giving us

$$x + 3x - 2y + y = 2 + 6 \tag{7}$$

$$4x - y = 8 \tag{8}$$

Using the value we found for x in (8) gives us

$$\begin{aligned}
4\left(\frac{8}{3}\right) - y &= 8 \\
\frac{32}{3} - y &= 8 \\
-y &= 8 - \frac{32}{3} \\
-y &= \frac{24}{3} - \frac{32}{3} \\
-y &= -\frac{8}{3} \\
y &= \frac{8}{3}
\end{aligned}$$

Using these values for x and y in (3) gives us

$$\begin{aligned}
\frac{8}{3} + \frac{8}{3} - z &= 3 \\
\frac{16}{3} - z &= 3 \\
-z &= 3 - \frac{16}{3} \\
-z &= \frac{9}{3} - \frac{16}{3} \\
z &= \frac{7}{3}
\end{aligned}$$

11. $\frac{3}{4}x - y + z = 1$

    $x - \frac{1}{2}y + z = 0$

    $x + y - \frac{1}{3}z = 1$

To clear denominators, we multiply the first equation by 4, the second equation by 2, and the third equation by 3, which gives us

$$3x - 4y + 4z = 4 \tag{1}$$

$$2x - y + 2z = 0 \tag{2}$$

$$3x + 3y - z = 3 \tag{3}$$

We will reduce this to a system of two equations in two unknowns by eliminating the z.

We start by looking at (2) and (3). Multiplying (3) by 2 gives us

$$3x - 4y + 4z = 4 \tag{4}$$

$$2x - y + 2z = 0 \tag{5}$$

$$6x + 6y - 2z = 6 \tag{6}$$

Adding (5) and (6) will cancel out the z, giving us

$$2x + 6x - y + 6y = 0 + 6 \tag{7}$$

$$8x + 5y = 6 \tag{8}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the z, this time looking at (1) and (3). Multiplying

(3) by 4 gives us

$$3x - 4y + 4z = 4 \tag{9}$$

$$2x - y + 2z = 0 \tag{10}$$

$$12x + 12y - 4z = 12 \tag{11}$$

Adding (9) and (11) will cancel out the z, giving us

$$3x + 12x - 4y + 12y = 4 + 12 \tag{12}$$

$$15x + 8y = 16 \tag{13}$$

Now we have a system of two equations (from (8) and (13))

$$8x + 5y = 6 \tag{14}$$

$$15x + 8y = 16 \tag{15}$$

Multiplying (14) by -8 and (15) by 5 gives us

$$-64x - 40y = -48$$

$$75x + 40y = 80$$

Adding the two equations will cancel out the y, giving us

$$-64x + 75x = -48 + 80$$

$$11x = 32$$

$$x = \frac{32}{11}$$

Using this value for x in (14) gives us

$$8\left(\frac{32}{11}\right) + 5y = 6$$

$$\frac{256}{11} + 5y = 6$$

$$5y = 6 - \frac{256}{11}$$

$$5y = \frac{66}{11} - \frac{256}{11}$$

$$5y = -\frac{190}{11}$$

$$\frac{5y}{1} = -\frac{190}{11}$$

$$5y \cdot 11 = -190 \cdot 1$$

$$55y = -190$$

$$y = \frac{-190}{55}$$

$$y = -\frac{38}{11}$$

Using these values for x and y in (3) gives us

$$
\begin{aligned}
3\left(\frac{32}{11}\right) + 3\left(-\frac{38}{11}\right) - z &= 3 \\
\frac{96}{11} - \frac{114}{11} - z &= 3 \\
-\frac{18}{11} - z &= 3 \\
-z &= 3 + \frac{18}{11} \\
-z &= \frac{33}{11} + \frac{18}{11} \\
-z &= \frac{51}{11} \\
z &= -\frac{51}{11}
\end{aligned}
$$

12. $$
\begin{aligned}
\frac{1}{2}x - \frac{2}{3}y + z &= 1 \\
x - \frac{1}{5}y + z &= 0 \\
2x - \frac{1}{3}y + \frac{2}{5}z &= 1
\end{aligned}
$$

To clear denominators, we multiply the first equation by 6, the second equation by 5, and the third equation by 15, which gives us

$$3x - 4y + 6z = 6 \tag{1}$$

$$5x - y + 5z = 0 \tag{2}$$

$$30x - 5y + 6z = 15 \tag{3}$$

We will reduce this to a system of two equations in two unknowns by eliminating the y.

We start by looking at (1) and (2). Multiplying (2) by -4 gives us

$$3x - 4y + 6z = 6 \tag{4}$$

$$-20x + 4y - 20z = 0 \tag{5}$$

$$30x - 5y + 6z = 15 \tag{6}$$

Adding (4) and (5) will cancel out the y, giving us

$$3x - 20x + 6z - 20z = 6 + 0 \tag{7}$$

$$-17x - 14z = 6 \tag{8}$$

Now we need another equation in two unknowns, so we go back to the beginning and eliminate the y, this time looking at (2) and (3). Multiplying (2) by -5 gives us

$$3x - 4y + 6z = 6 \tag{9}$$

$$-25x + 5y - 25z = 0 \tag{10}$$

$$30x - 5y + 6z = 15 \tag{11}$$

Adding (10) and (11) will cancel out the y, giving us

$$-25x + 30x - 25z + 6z = 0 + 15 \tag{12}$$

$$5x - 19z = 15 \tag{13}$$

Now we have a system of two equations (from (8) and (13))

$$-17x - 14z = 6 \tag{14}$$

$$5x - 19z = 15 \tag{15}$$

Multiplying (14) by 5 and (15) by 17 gives us

$$-85x - 70z = 30$$

$$85x - 323z = 255$$

Adding the two equations will cancel out the x, giving us

$$-70z - 323z = 30 + 255$$

$$-393z = 285$$

$$z = \frac{285}{-393}$$

$$z = \frac{95}{-131}$$

Using this value for z in (14) gives us

$$\begin{aligned}
-17x - 14\left(-\frac{95}{131}\right) &= 6 \\
-17x + \frac{1330}{131} &= 6 \\
-17x &= 6 - \frac{1330}{131} \\
-17x &= \frac{786}{131} - \frac{1330}{131} \\
-17x &= -\frac{544}{131} \\
-\frac{17x}{1} &= -\frac{544}{131} \\
-17x \cdot 131 &= -544 \cdot 1 \\
-2227x &= -544 \\
x &= \frac{-544}{-2227} \\
x &= \frac{32}{131}
\end{aligned}$$

Using these values for x and z in (2) gives us

$$\begin{aligned}
5\left(\frac{32}{131}\right) - y + 5\left(-\frac{95}{131}\right) &= 0 \\
\frac{160}{131} - y - \frac{475}{131} &= 0 \\
-y - \frac{315}{131} &= 0 \\
-y &= \frac{315}{131} \\
y &= -\frac{315}{131}
\end{aligned}$$