

Name: _____

Class: _____

Class #: _____

Section #: _____

Instructor: Nathaniel Stevens

Assignment: Quiz 11

Question 1: (2 points)

Suppose that a 2^{5-2} fractional factorial design is used to explore the influence of $K = 5$ factors A, B, C, D, E on a response variable y . The complete aliasing structure for the design is

$$I = ABD = BCE = ACDE$$

$$A = BD = ABCE = CDE$$

$$B = AD = CE = ABCDE$$

$$C = ABCD = BE = ADE$$

$$D = AB = BCDE = ACE$$

$$E = ABDE = BC = ACD$$

$$AC = BCD = ABE = DE$$

$$AE = BDE = ABC = CD$$

Suppose that we fit a regression model with the following linear predictor:

$$\beta_0 + \beta_A x_A + \beta_B x_B + \beta_C x_C + \beta_D x_D + \beta_E x_E + \beta_{AC} x_A x_C + \beta_{AE} x_A x_E$$

where the x 's are ± 1 depending on whether the factor is at its low or high level.

Suppose that we find β_C is significantly different from 0. Why might this be the case?

- (a) The main effect of Factor C is significant
- (b) The ABCD interaction effect is significant
- (c) The BE interaction effect is significant
- (d) The ADE interaction effect is significant
- (e) None of the aforementioned effects is significant, but their aggregate effect is.
- ☒ (f) Any of the explanations above are possible reasons for why β_C is significantly different from 0.

Suppose that we find β_A is not significantly different from 0. Why might this be the case?

- (a) None of the effects for A, BD, ABCE, or CDE are significant
- (b) The aggregate effects of A, BD, ABCE, and CDE cancel each other out, independent of their individual significance.
- ☒ (c) Either of the explanations above are possible reasons for why β_A is not significantly different from 0.

Question 2: (1 point)

Suppose that a 2^{4-1} fractional factorial design is used to explore the influence of $K = 4$ factors A, B, C, D on a response variable y . The complete aliasing structure for the design is

$$I = ABCD$$

$$A = BCD$$

$$B = ACD$$

$$C = ABD$$

$$D = ABC$$

$$AB = CD$$

$$AC = BD$$

$$AD = BC$$

Suppose that we fit a regression model with the following linear predictor:

$$\beta_0 + \beta_A x_A + \beta_B x_B + \beta_C x_C + \beta_D x_D + \beta_{AB} x_A x_B + \beta_{AC} x_A x_C + \beta_{AD} x_A x_D$$

where the x 's are ± 1 depending on whether the factor is at its low or high level.

Suppose that we find β_C is significantly different from 0. In light of the principle of effect sparsity, what is *most likely* reason for this?

- ☒ (a) The main effect of Factor C is significant
- ☐ (b) The ABD interaction effect is significant
- ☐ (c) None of the aforementioned effects is significant, but their aggregate effect is.
- ☐ (d) Both of the aforementioned effects is significant, and so their aggregate effect also is.

Question 3: (1 point)

Suppose that interest lies in investigating the influence of three factors A, B, C on a response variable y . Ideally, a full 2^3 factorial design would be used and the following linear predictor would be fit:

$$\beta_0 + \beta_A x_A + \beta_B x_B + \beta_C x_C + \beta_{AB} x_A x_B + \beta_{AC} x_A x_C + \beta_{BC} x_B x_C + \beta_{ABC} x_A x_B x_C$$

However, only $m = 4$ experimental conditions may be performed and so a 2^{3-1} fractional factorial design with the following complete aliasing structure is performed.

$$I = ABC$$

$$A = BC$$

$$B = AC$$

$$C = AB$$

With the data collected from this experiment the following linear predictor may be estimated.

$$\beta_0^* + \beta_A^* x_A + \beta_B^* x_B + \beta_C^* x_C$$

Which of the following statements concerning the relationship between the regression coefficients in these two linear predictors is true?

☒ (a) $\beta_A^* = \beta_A + \beta_{BC}$

☐ (b) $\beta_A^* = \beta_A - \beta_{BC}$

☐ (c) $\beta_A^* = \beta_A \times \beta_{BC}$

☐ (d) There is no relationship between β_A^* and β_A and β_{BC}

Question 4: (1 point)

In the context of experimental design, the acronym RSM stands for:

☐ (a) Regional Sales Manager

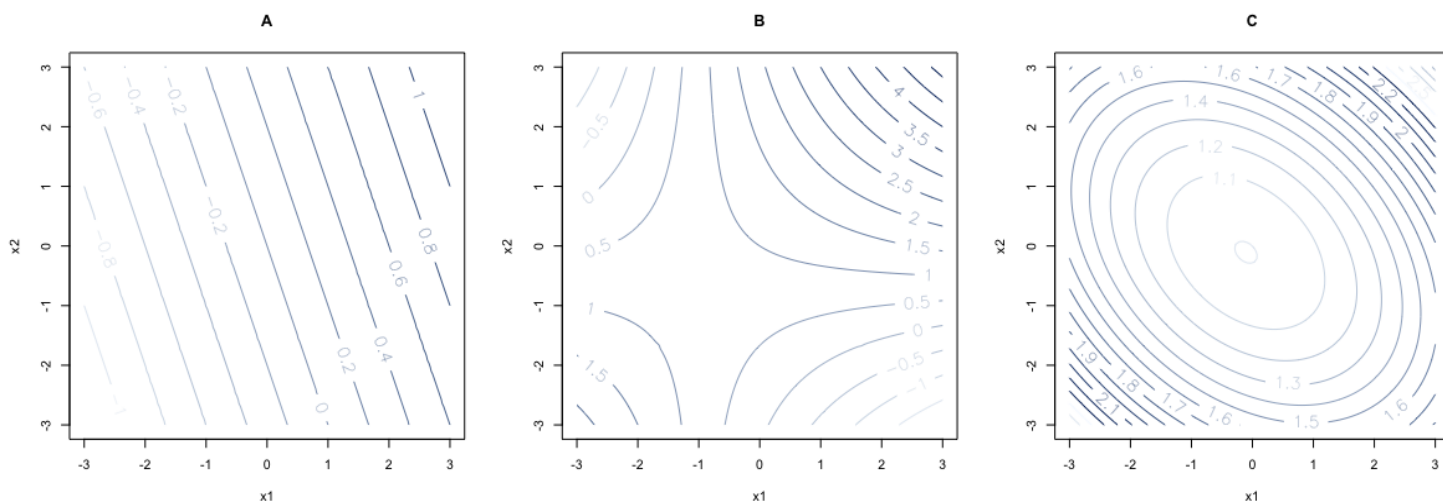
☒ (b) Response Surface Methodology

☐ (c) Rusty Scrap Metal

☐ (d) Really Small Manatee

Question 5: (1 point)

Contour plots for three different types of response surface models are shown below.



Which model would be *most useful* for purposes of response optimization?

(a) A

(b) B

(c) C

Question 6: (2 points)

Suppose an experiment involving the numeric factor *discount amount* is performed. In the context of this experiment, *low* and *high* values of discount amount are respectively 20% and 60%, translating to -1 and $+1$ in coded units.

(a) What is 40% in coded units? Where necessary, round your answers to four decimals.

_____ ↗
$$\left[40 - \left(\frac{60+20}{2} \right) \right] \div \left(\frac{60-20}{2} \right) = 0$$

(b) Suppose that it is found that 0.75 is the optimal discount amount in coded units. What is the optimal discount amount in natural units? Where necessary, round your answer to two decimals. Note an answer of say 60.25% should be entered in the box as 60.25.

_____ ↗
$$(0.75) \times \left(\frac{60-20}{2} \right) + \left(\frac{60+20}{2} \right) = 55$$

Question 7: (2 points)

Suppose that a response surface design is performed to investigate the relationship between a response variable and two factors, and the resulting data yields the following estimated second order linear predictor:

$$5 - 2x_1 + 8x_2 + x_1^2 + 4x_2^2$$

Find the *stationary point* of this surface and input the x_1 and x_2 coordinates in the numeric input boxes below.

- x_1 : _____ ▽ → 1
- x_2 : _____ ▽ → -1

$$b = \begin{bmatrix} -2 \\ 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$x_s = -\frac{1}{2} B^{-1} b$$

$$= -\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$