Toán b1 Chương 1 gới hạn hàm số

1.7 giới hạn

a. $\lim\limits_{x\to 2}\frac{x^2-1}{x^2-2x+3}=\frac{2^2-1}{2^2-2*x+3}=1$

b. $\lim\limits_{x\to\sqrt{2}}\frac{x^2-2}{x^4-x^2-2}=$

$\lim\limits_{x\to\sqrt{2}}\frac{x^2-2}{x^4-x^2-2}=\lim\limits_{x\to\sqrt{2}}\frac{x^2-2}{(x^2-2)(x^2+1)}=\frac{1}{3}$

c. $\lim\limits_{x\to+\infty} arctanx=+\frac{\pi}{2}$

d. $\lim\limits_{x\to 3}\frac{2+\sqrt[3]{19-x^2}}{\sqrt{4x-3}}$ thay 3 vào x

e. $\lim\limits_{x\to+\infty}\frac{\sqrt{4x^2+x+5}}{x+1}=\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{4x^2+x+5}{x^2}}}{\frac{x+1}{x}}=$

$\lim\limits_{x\to+\infty}\frac{\sqrt{4+\frac{1}{x}+\frac{5}{x^2}}}{1+\frac{1}{x}}=2$

f. $\lim\limits_{x\to 0}\frac{arcsin2x}{\ln(1-x)}=\lim\limits_{x\to 0}\frac{arcsin2x}{\ln\left(1+(-x)\right)}=\frac{2x}{-x}=2$

g. $\lim\limits_{x\to+\infty}\left(2x-1-\sqrt{4x^2-4x-3}\right)=$

$\lim\limits_{x\to+\infty}\frac{((2x-1)^2-(4x^2-4x-3)}{2x-1+\sqrt{4x^2-4x-3}}=$

$\lim\limits_{x\to+\infty}\frac{4}{2x-1+\sqrt{4x^2-4x-3}}$=0

h. $\lim\limits_{x\to 0}\frac{tan^2x}{\arcsin\left(\frac{x}{2}\right).sin2x}=\lim\limits_{x\to 0}\frac{x^2}{\frac{x}{2}.2x}=1$

i. $\lim\limits_{x\to 2}\frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}}=\lim\limits_{x\to 2}\frac{((x-2)(x+1))^{20}}{((x-2)^2(x+4))^{10}}=$

$\lim\limits_{x\to 2}\frac{(x+1)^{20}}{(x+4)^{10}}=\frac{2^{20}}{6^{10}}=\left(\frac{3}{2}\right)^{10}$

j. $\lim\limits_{x\to 4}\frac{\sqrt{x}-1}{x^2-5x+4}=\lim\limits_{x\to 4}\frac{\sqrt{x}-1}{(x-1)(x-4)}=\lim\limits_{x\to 4}\frac{1}{(\sqrt{x}+1)(x-4)}=$

xét $\lim\limits_{x\to 4^+}\frac{1}{(\sqrt{x}+1)(x-4)}=+\infty$ ($vì\ x-4>0\ với\ mọi\ x>4$)

xét $\lim\limits_{x\to 4^-}\frac{1}{(\sqrt{x}+1)(x-4)}=-\infty$ ($vì\ x-4<0\ với\ mọi\ x<4$

k. $\lim\limits_{x\to 0}\frac{1-cos2x}{2sin^2x+2x.tan3x}=\lim\limits_{x\to 0}\frac{\frac{(2x)^2}{2}}{2x^2+2x.3x}=\frac{1}{4}$

l. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\lim\limits_{x\to+\infty}\frac{x^{\frac{1}{2}}+x^{\frac{1}{3}}+x^{\frac{1}{4}}}{(2x+1)^{\frac{1}{2}}}=$

$\lim\limits_{x\to+\infty}\frac{x^{\frac{1}{2}}}{(2x+1)^{\frac{1}{2}}}=\frac{1}{\sqrt{2}}$

m. $\lim\limits_{x\to+\infty}(1+e^x)e^{-x^2}=\lim\limits_{x\to+\infty}\frac{1+e^x}{e^{x^2}}=0$

n. $\lim\limits_{x\to+\infty}\left(\frac{x+1}{2x-3}\right)^{x+2}=e^{\lim_{x\to+\infty}(\frac{x+1}{2x-3}-1)(x+2)}=$

$$e^{\lim\limits_{x\to+\infty}\left(\frac{-x+4}{2x-3}\right)(x+2)} = e^{-\infty} = 0$$

o.$\lim\limits_{x\to0}\frac{\sqrt{1-cos2x}}{x} = \lim\limits_{x\to0}\frac{\sqrt{\frac{(2x)^2}{2}}}{x} = \sqrt{2}$

p.$\lim\limits_{x\to\frac{\pi}{2}}\frac{1}{1+2^{tanx}}$

xét x->$\frac{\pi}{2}^{-}$ $=> tanx = -\infty$

=> $\lim\limits_{x\to\frac{\pi}{2}}\frac{1}{1+2^{tanx}} = \lim\limits_{x\to\frac{\pi}{2}}\frac{1}{1+2^{-\infty}} = 1$

xét x->$\frac{\pi}{2}^{+}$ $=> tanx = +\infty$

=> $\lim\limits_{x\to\frac{\pi}{2}}\frac{1}{1+2^{tanx}} = \lim\limits_{x\to\frac{\pi}{2}}\frac{1}{1+2^{+\infty}} = 0$

không có giới hạn

q. $\lim\limits_{x\to+\infty}\left(\frac{x+2}{x-3}\right)^{3x-4} =$

$e^{\lim\limits_{x\to+\infty}\left(\frac{x+2}{x-3}-1\right)(3x-4)} = e^{\lim\limits_{x\to+\infty}\left(\frac{2x+5}{x-3}\right)(3x-4)}$=$e^{\infty}$=∞

r.$\lim\limits_{x\to\frac{1}{2}}\frac{arcsin(2x-1)}{4x^2-1} = \lim\limits_{x\to\frac{1}{2}}\frac{2x-1}{4x^2-1} = \lim\limits_{x\to\frac{1}{2}}\frac{1}{2x+1} = 1/2$

s.$\lim\limits_{x\to0}\frac{sinx+tanx}{\sqrt{4x^2+x^3}} = \lim\limits_{x\to0}\frac{x+x}{\sqrt{4x^2+x^3}} = \frac{1}{2}$

t. $\lim_{x\to 1^+}\frac{2x-3}{x-1}$=-∞

vì 2x-3=-1 khi x dần về 1

x-1 >0 với mọi x>1

v. cho y=f(x)=$\begin{cases}\frac{\sin\frac{x}{2}}{x} & khi\ x<0\\ \frac{\sqrt{1+x^2}-1}{x^2} & khi\ x>0\end{cases}$ tính $\lim_{x\to 0} f(x)$

xét $\lim_{x\to 0^+} f(x) =$

$\lim_{x\to 0^+}\frac{\sqrt{1+x^2}-1}{x^2} = \lim_{x\to 0^+}\frac{(1+x^2)-1}{x^2(\sqrt{1+x^2}+1)} =$

$\lim_{x\to 0^+}\frac{1}{(\sqrt{1+x^2}+1)} = \frac{1}{2}$

xét $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{\sin\frac{x}{2}}{x} = \lim_{x\to 0^-}\frac{\frac{x}{2}}{x} = 1/2$

vậy $\lim_{x\to 0} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = \frac{1}{2}$

1.8 tìm giới hạn

a. $\lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8} =$

$\lim_{x\to -2}\frac{(\sqrt[3]{x-6}+2)\left(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4\right)}{(x+2)(x^2+2x+4)\left(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4\right)}$

$$= \lim_{x\to -2} \frac{x-6+8}{(x+2)(x^2+2x+4)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)} =$$

$$\lim_{x\to -2} \frac{1}{(x^2+2x+4)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}$$ thay -2 vào

b. $$\lim_{x\to 0} \frac{sinax-sinbx}{tanx} = \lim_{x\to 0} \frac{ax-bx}{x} = a-b$$

c. $$\lim_{x\to +\infty} \left(\frac{x^2+x+1}{x^2+1}\right)^x =$$

$$e^{\lim\limits_{x\to+\infty}\left(\frac{x^2+x+1}{x^2+1}-1\right)x} = e^{\lim\limits_{x\to+\infty}\left(\frac{x}{x^2+1}\right)x} = e^1 = e$$

d. $$\lim_{x\to 2} \frac{x^2-1}{x} = \frac{2^2-1}{2} = \frac{3}{2}$$

e. $$\lim_{x\to 1^-} \frac{|x^2+2x-3|}{x^2-1} = \lim_{x\to 1^-} \frac{|(x-1)(x+3)|}{(x+1)(x-1)} =$$

$$\lim_{x\to 1^-} -\frac{(x-1)(x+3)}{(x+1)(x-1)} = \lim_{x\to 1^-} -\frac{(x+3)}{(x+1)} = -2$$

f. $$\lim_{x\to 0^+} \frac{3^x-1}{x} = \lim_{x\to 0^+} \frac{3^x ln3}{1} = ln3$$

g.

$$\lim_{x\to 1} \frac{lnx}{x-1} = \lim_{x\to 1} \frac{\ln(1+(x-1))}{x-1} = \lim_{x\to 1} \frac{\mathrm{x}-1}{x-1} = 1$$

1.9 tính các giới hạn sau :

a. $\lim\limits_{x\to 0}\dfrac{cosx - cos2x}{1 - cosx}$

$$= \lim_{x\to 0}\frac{1 - cos2x - (1 - cosx)}{1 - cosx} = \lim_{x\to 0}\frac{2x^2 - \frac{1}{2}x^2}{\frac{1}{2}x^2} = 3$$

b. $\lim\limits_{x\to +\infty} sin\sqrt{x+1}$-sin$\sqrt{x}$= $\lim\limits_{x\to +\infty} sinx$-sin$x$=0 (VCL của $\sqrt{x}$=x)

c. $\lim\limits_{x\to 0}\frac{1-cosx.cos2x.cos3x}{1-cosx} =$

$\lim\limits_{x\to 0}\frac{1-\frac{1}{2}(cos4x+cos2x)cos2x}{1-cosx} = \lim\limits_{x\to 0}\frac{1-\frac{1}{4}(cos6x+cos2x)-\frac{1}{2}cos^2 2x}{1-cosx} =$

$$\lim_{x\to 0}\frac{1 - \frac{1}{4}(cos6x + cos2x) - \frac{1}{4}cos4x - \frac{1}{4}}{1 - cosx}$$

$$= \lim_{x\to 0}\frac{\frac{1}{4}(1 - cos6x + 1 - cos4x) + 1 - cos2x)}{1 - cosx} =$$

$$\lim_{x\to 0}\frac{\frac{1}{4}(18x^2 + 8x^2 + 2x^2)}{\frac{1}{2}x^2} = 14$$

d. $\lim\limits_{x\to 0}\frac{1}{x}ln\sqrt{\frac{1+x}{1-x}}$=$\lim\limits_{x\to 0}\dfrac{\left(\sqrt{\frac{1+x}{1-x}}\right)'}{\sqrt{\frac{1+x}{1-x}}} = \lim\limits_{x\to 0}\dfrac{\left(\frac{1+x}{1-x}\right)'}{2\frac{1+x}{1-x}} = \lim\limits_{x\to 0}\frac{-2}{2(1-x)(1+x)} = -1$

e. $\lim\limits_{x\to 1}\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}$=$\lim\limits_{x\to 1}\frac{(x+x^2+x^3+\cdots+x^n-n)'}{(x-1)'} =$

$\lim\limits_{x\to 1}\frac{1+2x+3x^2+\cdots+nx^{n-1}}{1} = 1 + 2 + 3 + \cdots + n = +\infty$

f. $\lim\limits_{x\to 1}(1 - x)\tan\left(\frac{\pi}{2}x\right) = \lim\limits_{x\to 1}\dfrac{(x-1)\sin\left(\frac{\pi}{2}x\right)}{\cos\left(\frac{\pi}{2}x\right)} =$

$$\lim_{x\to 1}\frac{\sin\left(\frac{\pi}{2}x\right)-x\frac{\pi}{2}\cos\left(\frac{\pi}{2}x\right)+\frac{\pi}{2}\cos\left(\frac{\pi}{2}x\right)}{\frac{\pi}{2}\sin\left(\frac{\pi}{2}x\right)}=\frac{2}{\pi}$$

1.10 xét tính liên tục hàm số :

a). f(x)= $\begin{cases}\sqrt[3]{x^2-1}-2 & khi\ x\neq 3\\ \frac{x^2-4x+3}{3} & khi\ x=3\end{cases}$ tại x_o=3

$$\lim_{x\to 3} f(x)=\lim_{x\to 3}\sqrt[3]{x^2-1}-2=\sqrt[3]{9-1}-2=0$$

f(3)= $\frac{3^2-4*3+3}{3}=0$

do $\lim_{x\to 3} f(x)=f(3)=0$ => hàm số liên tục tại x_o=3

b). f(x)= $\begin{cases}\frac{sinx}{x} & khi\ x\neq 0\\ 1 & khi\ x=0\end{cases}$ tại X_o=0

$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{sinx}{x}=\frac{x}{x}=1$

f(0)=1

do $\lim_{x\to 0} f(x)=f(0)=1$ => hàm số liên tục tại x_o=0

C. $f(x)=\begin{cases}\frac{1-\sqrt{cosx}}{\sin^2 x} & khi\ x\in(-\frac{\pi}{2};\frac{\pi}{2})\{0\}\\ \frac{1}{4} & khi\ x=0\end{cases}$ $tại$ X_o=0

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{1-\sqrt{cosx}}{\sin^2 x}=\lim_{x\to 0}\frac{1-\sqrt{1-(1-cosx)}}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{1-\sqrt{1-\frac{x^2}{2}}}{x^2}=\lim_{x\to 0}\frac{x^2}{x^2\left(1+\sqrt{1-\frac{x^2}{2}}\right)}=\frac{1}{2}$$

$do \lim_{x\to 0} f(x) \neq f(0)$ => hàm số khong liên tục tại x_o=0

d). $f(x) = \begin{cases} \frac{\sqrt{1+\sin^2 x}-cosx}{\sin^2 x} & khi -\frac{\pi}{2} < x < 0 \\ arccosx & khi\ x \geq 0 \end{cases}$ $tại$ X_0=0

$$\lim_{x\to 0} f(x) = \lim_{\substack{x\to 0 \\ x\to 0^-}} f(x) = \lim \frac{\sqrt{1+\sin^2 x}-cosx}{\sin^2 x} =$$

$$\lim_{x\to 0^-} \frac{\sqrt{1+\sin^2 x}+(1-cosx)-1}{\sin^2 x} =$$

$$\lim_{x\to 0^-} \frac{\sqrt{1+x^2}-1+\frac{x^2}{2}}{x^2} = \lim_{x\to 0^-} \frac{1+x^2-1}{x^2(\sqrt{1+x^2}+1)} + \frac{1}{2} = \frac{1}{4}$$

$$\lim_{x\to 0^-} arccosx = arccos0 = 0$$

$do \lim_{x\to 0} f(x) \neq f(0)$ => hàm số khong liên tục tại x_o=0

1.11 tìm a và b để hàm số liên tục

a.$f(x) = \begin{cases} \arctan\frac{1}{x} & khi\ x < 0 \\ a + x & khi\ x \geq 0 \end{cases}$ tại X_0=0

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \arctan\frac{1}{x} = \arctan -\infty = \frac{-\pi}{2}$$

$$\lim_{x\to 0^+} f(x) = a + x = a$$

để hàm số liên tục thì $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \frac{-\pi}{2} => a = \frac{-\pi}{2}$

b.$f(x) = \begin{cases} \frac{tan\pi x}{x-2} & khi\ 2 < x < \frac{5}{2} \\ a + x & khi\ x \leq 2 \end{cases}$ tại X_0=2

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{tan\pi x}{x-2} = \lim_{x\to 2^+} \frac{(tan\pi x)'}{(x-2)'} = \lim_{x\to 2^+} \frac{\pi}{\cos^2 \pi x} = \pi$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (a + x) = 2 + a$$

$$\lim_{x \to 0^+} f(x) = a + x = a$$

để hàm số liên tục thì $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 2 + a = \pi$ =>

a= $\pi - a$

c. $f(x) = \begin{cases} \frac{cosx - \sqrt{cos2x}}{\tan^2 x} & khi\ x \neq 0 \\ a\ \ khi\ x = 0 \end{cases}$ tại X_0=0

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{cosx - \sqrt{cos2x}}{\tan^2 x}$$

$$= \lim_{x \to 0} \frac{1 - cosx + 1 - \sqrt{1 - (1 - cos2x)}}{\tan^2 x}$$

$$= \lim_{x \to 0} \frac{\frac{x^2}{2} + 1 - \sqrt{1 - 2x^2}}{x^2}$$

$$= \lim_{x \to 0} (\frac{1}{2} + \frac{2x^2}{x^2(1 + \sqrt{1 - 2x^2})} = \frac{1}{2} + 1 = \frac{3}{2}$$

f(0)=a

vậy để hàm số liên tục tại x=o thì f(0)= $\lim_{x \to 0} f(x)$ ⇔ a=3/2

d.$f(x) = \begin{cases} a\ \ \ khi\ x = -1 \\ arccosx\ khi - 1 < x \leq 1 \\ b + x\ khi\ x < -1 \end{cases}$ tại X_0=-1

$$\lim_{x \to -1^-} f(x) \quad = \lim_{x \to -1^-} (b + x) = b - 1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to -1^+} (arccosx) = arccos - 1 = -\pi$$

f(-1)=a

để hàm số liên tục tại x=-1 thì f(-1)= $\lim_{x \to -1^-} f(x) =$

$\lim_{x\to 1^+} f(x) \Leftrightarrow \begin{cases} a = b - 1 \\ a = -\pi \\ b - 1 = -\pi \end{cases} => \begin{cases} a = -\pi \\ b = -\pi + 1 \end{cases}$

e.$f(x) = \begin{cases} \frac{\ln(1+x^2)}{\sqrt{1+x^2-1}} & khi\ x \neq 0 \end{cases}$ tại X_0=0
$\quad a \quad khi\ x = 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\ln(1+x^2)}{\sqrt{1+x^2-1}} = \lim_{x\to 0} \frac{\frac{2x}{1+x^2}}{\frac{2x}{2\sqrt{1+x^2}}} = \lim_{x\to 0} \frac{\sqrt{1+x^2}}{1+x^2} = 1$$

f(0)=a

để hàm số liên tục tại X_0=0 thì f(0)= $\lim_{x\to 0} f(x)$=a=1 => a=1

Chương 2 : đạo hàm và vi phân

2.1 đạo hàm các hàm sao:

a.y=$\sqrt{arcsinx}$

y'=$\frac{(arcsinx)'}{2\sqrt{arcsinx}} = \frac{\frac{1}{\sqrt{1-x^2}}}{2\sqrt{arcsinx}} = \frac{1}{2(1-x^2)\sqrt{arcsinx}}$

b.y=$arccos(1-x^2)$

y'=$-\frac{(1-x^2)'}{\sqrt{1-(1-x^2)}} = \frac{2x}{\sqrt{x^2}} = 2\ với\ dk\ x > 0$

c.y=$arcsin(1-x^2)$

y'=$\frac{\left(1-x^2\right)'}{\sqrt{1-(1-x^2)}} = -\frac{2x}{\sqrt{x^2}} = -2\ với\ dk\ x > 0$

d.y=$\frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}}$=$x^{-1} + x^{-\frac{1}{2}} + x^{-\frac{1}{3}}$

y'=$x^{-2} - \frac{1}{2}x^{-\frac{3}{2}} - \frac{1}{3}x^{-\frac{4}{3}}$

e.y=$2^{x^2+1} + \log_2 cosx$

y'=$2(x^2)'ln2.2^{x^2} + \frac{(ln(cosx))'}{ln2}$

y'=4x.ln2.$2^{x^2} + \frac{sinx}{cosx.ln2}$

f. y=$a^x + x^a + x^x$

đặt g=x^x ⇔ lng=xlnx ⇔ $\frac{g'}{g} = lnx + 1 => g' =$
$(lnx + 1).g = (lnx + 1).x^x$

=> y'=(lna)$a^x + ax^{a-1} + (lnx + 1).x^x$

h. bỏ không lien quan ct học

i.y=$arccot\sqrt{x^2 - 1}$

y'=-$\frac{\left(\sqrt{x^2-1}\right)'}{1+\left(\sqrt{x^2-1}\right)^2} = \frac{\frac{\left(x^2-1\right)'}{2\left(\sqrt{x^2-1}\right)}}{x^2} = \frac{2x}{2x^2\left(\sqrt{x^2-1}\right)} = \frac{1}{x\left(\sqrt{x^2-1}\right)}$

j.y=$\log_x(2x + 1)$

y'=$\left(\frac{ln(2x+1)}{lnx}\right)' = \frac{\frac{2}{2x+1}lnx-\frac{1}{x}ln(2x+1)}{(lnx)^2}$

2.2 tìm vi phân cấp 1:

a.y=$ln\left|\frac{x+1}{x-2}\right|$

y'=$(ln|x+1| - ln|x-2|)' = \frac{1}{|x+1|} - \frac{1}{|x-2|}$

ta có dy=y'dx => dy=$\left(\frac{1}{|x+1|} - \frac{1}{|x-2|}\right)dx$

b.y=e^{x^2} $y' = (x^2)'e^{x^2} = 2xe^{x^2} => dy = \left(2xe^{x^2}\right)dx$

c.y=sin^2x $y' = 2(sinx)'sinx = 2cosxsinx = sin2x => dy = (sin2x)dx$

d.y=$arctan^2x$ $y' = 2(arctanx)'.arctanx = 2\frac{1}{1+x^2}arctanx$

dy=$(2\frac{1}{1+x^2}arctanx)dx$

2.3 tính đạo hàm cấp cap tương ứng :

a.y=$e^{sinx}.arcsin(cosx)$

$$y^{(2)} = C_2^0(sinx)^{(0)}\big(arcsin(cosx)\big)^{(2)} + C_2^1(sinx)^{(1)}\big(arcsin(cosx)\big)^{(1)} + C_2^2(sinx)^{(2)}\big(arcsin(cosx)\big)^{(0)}$$

$$y^{(2)} = sinx.\big(arcsin(cosx)\big)^{(2)} + 2cosx.\frac{1}{1-cos^2x} - sinx.arcsin(cosx)$$

$$= sinx.\big(arcsin(cosx)\big)^{(2)} + 2cosx.sin^{-2}x - sinx.arcsin(cosx)$$

$$= \frac{-2\,cosx}{sin^2x} + 2cosx.sin^{-2}x - sinx.arcsin(cosx)$$

b.y=$\frac{x^2}{x+1}$ $tính\ \ y^{(8)}$

$$y^{(8)} = C_8^0\left(x^2\right)^{(0)}\left(\frac{1}{x+1}\right)^{(8)} + C_8^1\left(x^2\right)^{(1)}\left(\frac{1}{x+1}\right)^{(7)} + C_8^2\left(x^2\right)^{(2)}\left(\frac{1}{x+1}\right)^{(6)}$$

vì x^2 đạo hàm tới 3 lần ra không nên không cần làm đầy đủ

rút ra đa thức(x^{α})thì k chạy tới a là dừng .

$$y^{(8)} = x^2\frac{8!}{(x+1)^9} + 8.2x\frac{-7!}{(x+1)^8} + 28.2\frac{6!}{(x+1)^7}$$

C.$y = x^2e^{2x}$ $tính\ y^{(10)}$

$y^{(10)}$=$C_{10}^0\left(x^2\right)^{(0)}\left(e^{2x}\right)^{(10)} + C_{10}^1\left(x^2\right)^{(1)}\left(e^{2x}\right)^{(9)} + C_{10}^2\left(x^2\right)^{(2)}\left(e^{2x}\right)^{(8)}$

$$y^{(10)} = x^2 2^{10}e^{2x} + 20x.2^9e^{2x} + 90e^{2x}$$

d.y=x^2cosx $\quad tính\ y^{(10)}$

$$y^{(10)} = C_{10}^0(x^2)^{(0)}(cosx)^{(10)} + C_{10}^1(x^2)^{(1)}(cosx)^{(9)} + C_{10}^2(x^2)^{(2)}cosx)^{(8)}$$

$$y^{(10)} = x^2cos\left(x+\frac{10\pi}{2}\right) + 20xcos\left(x+\frac{9\pi}{2}\right) + 90cos(x+\frac{8\pi}{2})$$

c.y=$\frac{1}{x^2-3x+2} = \frac{1}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{1}{x-2}$

$$y^{(10)} = \left(\frac{-1}{x-1} + \frac{1}{x-2}\right)^{(10)} = \left(\frac{-1}{x-1}\right)^{(10)} + \left(\frac{2}{x-2}\right)^{(10)} = \frac{2.10!}{(x-2)^{-11}} - \frac{1.10!}{(x-1)^{-11}}$$

2.4 tính vi phân cấp cao tương ứng :

a.y=$\sqrt{1-x^2}$ tính $d^2y = ?$

y'=$\frac{-x}{\sqrt{1-x^2}}$

$$y''=\frac{-\sqrt{1-x^2}-\frac{x^2}{\sqrt{1-x^2}}}{1-x^2}=\frac{\frac{-\left(1-x^2\right)-x^2}{\sqrt{1-x^2}}}{1-x^2}=\frac{-1}{(1-x^2)\sqrt{1-x^2}}$$

$$\Rightarrow d^2y=\frac{-dx}{(1-x^2)\sqrt{1-x^2}}$$

b. $y=\frac{lnx}{x}$ $\quad$ *tính* d^5y

$$y^{(5)}=C_5^0(lnx)^{(0)}\left(\frac{1}{x}\right)^{(5)}+C_5^1(lnx)^{(1)}\left(\frac{1}{x}\right)^{(4)}+$$
$$C_5^2(lnx)^{(2)}\left(\frac{1}{x}\right)^{(3)}+C_5^3(lnx)^{(3)}\left(\frac{1}{x}\right)^{(2)}+C_5^4(lnx)^{(4)}\left(\frac{1}{x}\right)^{(1)}+$$
$$C_5^5(lnx)^{(5)}\left(\frac{1}{x}\right)^{(0)}$$

$$y^{(5)}=\frac{-5!\,lnx}{x^6}+\frac{5.4!}{x^6}+\frac{-10.3!}{x^6}+\frac{10.2!.2!}{x^6}+\frac{5.3!}{x^6}+\frac{4!}{x^6}$$

$$d^5y=\left(\frac{-5!lnx}{x^6}+\frac{5.4!}{x^6}+\frac{-10.3!}{x^6}+\frac{10.2!.2!}{x^6}+\frac{5.3!}{x^6}+\frac{4!}{x^6}\right)(dx)^5$$

c. $y=xsin^2x$ $\;$ *tính* $y^{(10)}$

$$y^{(10)}=C_{10}^0(x)^{(0)}\left(sin^2x\right)^{(10)}+C_{10}^1(x)^{(1)}\left(sin^2x\right)^{(9)}$$

ta có $\left(sin^2x\right)'=sin2x$

=>

$$y^{(10)}=C_{10}^0(x)^{(0)}(sin2x)^{(9)}+C_{10}^1(x)^{(1)}(sin2x)^{(8)}$$

$$y^{(10)}=2^9xsin\left(2x+\frac{\pi 9}{2}\right)+2^8sin\left(2x+\frac{\pi 8}{2}\right)$$

d, $y=\frac{1}{x+1}$ $\quad$ *tính* d^ny

$$d^n y=\frac{(-1)^n n!}{(x)^{(n+1)}}(dx)^n$$

e.y=$x^n e^x$ $\quad$ *tính* $d^n y$

$$y^{(n)}=C_n^0(x^n)^{(0)}(e^x)^{(n)}+C_n^1(x^n)^{(1)}(e^x)^{(n-1)}+\cdots+C_n^n(x^n)^{(n)}(e^x)^{(0)}$$

$$y^{(n)}=C_n^0.x^n e^x+C_n^1.n.x^{n-1}.e^x+\cdots+C_n^n.n!.e^x$$

=>$d^n y$ =($C_n^0.x^n e^x+C_n^1.n.x^{n-1}.e^x+\cdots+C_n^n.n!.e^x)(dx)^n$

2.5 tính gần đúng

a.$\sqrt[3]{1.02}$

chọn $f(x)=\sqrt[3]{x}$ $\quad$ có $x_0=1$ và $\Delta x=0.02$

f'(x)=$\frac{x^{-\frac{2}{3}}}{3}$

f'(1)=1/3 f(1)=1

ta có $f(x)=f(x_0)+f'(x_0).\Delta x=1+\frac{1}{3}0.02\approx 1.0066667$

b.$\sqrt[4]{16.1}$

chọn $f(x)=\sqrt[4]{x}$ $\quad$ chọn $x_0=16$ và $\Delta x=0.1$

$$f'(x)=\frac{x^{-\frac{3}{4}}}{4}$$

$f'(16)$=$\frac{1}{32}$ $\quad$ $f(16)=2$

ta có $f(x)=f(x_0)+f'(x_0).\Delta x=2+\frac{0.1}{32}=2.003125$

C. $cos31$

chọn $f(x)=cosx$ $\quad$ chọn $x_0=\frac{\pi}{6}$ và $\Delta x=\frac{\pi}{180}$

$f'(x)=-sinx$

$f'(\frac{\pi}{6})$=$-\frac{1}{2}$ $\quad$ $f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

ta có $f(x)=f(x_0)+f'(x_0).\Delta x=\frac{\sqrt{3}}{2}-\frac{1}{2}.\frac{\pi}{180}$

d.arcsin0.54

chọn $f(x)=arcsix$ $\quad$ $chọn\ x_0=\frac{1}{2}\ \ và\ \Delta x=0.04$

$f'(x)=\frac{1}{\sqrt{1-x^2}}$

$f'(\frac{1}{2})$=$\frac{2\sqrt{3}}{3}$ $\quad$ $f\left(\frac{1}{2}\right)=\frac{\pi}{6}$

ta có $f(x)=f(x_0)+f'(x_0).\Delta x=\frac{2\sqrt{3}}{3}+\frac{\pi}{150}$

e. arctan1.05

chọn $f(x)=arctanx$ $\quad$ $chọn\ x_0=1\ \ và\ \Delta x=0.05$

$f'(x)=\frac{1}{1+x^2}$

$f'(1)$=$\frac{1}{2}$ $\quad$ $f\left(\frac{1}{2}\right)=\frac{\pi}{4}$

ta có $f(x)=f(x_0)+f'(x_0).\Delta x=\frac{1}{2}+\frac{0.05\pi}{4}$

2.6 bỏ

2.7 áp dụng L'Hospital:

a. $\lim_{x\to 0^+} x^{\frac{3}{4+lnx}} = e^{\lim_{x\to 0^+}\frac{3lnx}{4+lnx}} = e^{\lim_{x\to 0^+}\frac{(3lnx)'}{(4+lnx)'}} = e^3$

b.$\lim_{x\to\frac{\pi}{2}}(tanx)^{2cosx}$=$e^{\lim_{x\to\frac{\pi}{2}}(tanx-1)2.cosx} = e^{\lim_{x\to\frac{\pi}{2}} sinx-1} = 1$

c. $\lim_{x\to+\infty}(x+e^x)^{\frac{1}{x}} = e^{\lim_{x\to+\infty}\frac{ln(x+e^x)}{x}} = e^{\lim_{x\to+\infty}\frac{1+e^x}{x+e^x}} = e^{\lim_{x\to+\infty}\frac{e^x}{1+e^x}} =$
$e^{\lim_{x\to+\infty}\frac{e^x}{e^x}} = e$

d. $\lim_{x\to0}(\frac{1}{x}-\frac{1}{e^x-1}) = \lim_{x\to0}\frac{e^x-1-x}{x(e^x-1)} = \lim_{x\to0}\frac{e^x-1}{e^x+xe^x-1} =$
$\lim_{x\to0}\frac{e^x}{e^x+e^x+xe^x} = 1/2$

e. $\lim_{x\to+\infty}(\pi-2arctanx)x = \lim_{x\to+\infty}\frac{\pi-2arctanx}{\frac{1}{x}} = \lim_{x\to+\infty}\frac{\frac{-2}{1+x^2}}{-\frac{1}{x^2}} =$
$\lim_{x\to+\infty}\frac{-2}{-\frac{1+x^2}{x^2}} = -2$

f. $\lim_{x\to0}\frac{e^x-e^{-x}-2}{tanx} = \lim_{x\to0}\frac{e^x+e^{-x-1}}{\frac{1}{cos^2x}} = \frac{1+e^\wedge-1}{1} = 1+\frac{1}{e}$

Chương3 :tíchphânxácđịnh

3.1

1. $\int\frac{(2x+1)^2}{x}dx = \int\frac{4x^2+4x+2}{x}dx =$
$\int\left(4x+4+\frac{2}{x}\right)dx = \frac{4}{2}x^2+4x+2lnx+c$

2. $\int\frac{cos2x}{sin^2x.cos^2x}dx = \int\frac{cos^2x-sin^2x}{sin^2x.cos^2x}dx =$
$\int\left(\frac{cos^2x}{sin^2x.cos^2x}-\frac{sin^2x}{sin^2x.cos^2x}\right)dx = \int\left(\frac{1}{sin^2x}-\right.$

$\frac{1}{cos^2x}\Big) dx = -cotx - tanx + C$

3. $\int sinx.cos^2xdx = I$

trìnhbàipp 1: đặt t=cosx

=>dt=-sinxdx =>sinxdx=-dt

=> $\int sinx.cos^2xdx = \int -t^2dt = -\frac{1}{3}t^3 +$

$c \quad => I = -\frac{1}{3}cos^3x + c$

4. $\int \frac{2x-5}{\sqrt{x^2-5x+6}} dx$**=I**

đặt t=$\sqrt{x^2 - 5x + 6}$ **=>**$t^2 = x^2 - 5x = 6 =>$

$2tdt = (2x - 5)dx$

=> I=$\int \frac{2tdt}{t} = \int 2dt = 2t + c => I =$

$2\sqrt{x^2 - 5x + 6} + c$

5. $\int \frac{dt}{cos^2tsin^2t}$**=**$\int \frac{cos^2t+sin^2t}{cos^2t.sin^2t} dt = \int \Big(\frac{cos^2t}{cos^2t.sin^2t} -$

$\frac{sin^2t}{cos^2t.sin^2t}\Big) dt = \int \frac{1}{sin^2t} dt + \int \frac{1}{cos^2t} dt =$

$-cott + tant + c$

6. $\int \frac{x^9}{x^{10}-1} dx$ đặt $t = x^{10} - 1 => dt =$

$9\boldsymbol{x^9dx} \Rightarrow \boldsymbol{x^9dx = \frac{dt}{9}}$

=> $\boldsymbol{\int \frac{x^9}{x^{10}-1}dx = \int \frac{dt}{9t} = \frac{1}{9}lnt + c}$

7.$\boldsymbol{\int \frac{x^4}{x^2+2}dx}$**=I**

Ta có :$\boldsymbol{\frac{x^4}{x^2+2}} = \mathrm{x}^2 - 1 + \frac{2}{x^2+2}$

=>I=$\int \left(\mathrm{x}^2 - 1 + \frac{2}{x^2+2}\right)dx = \frac{x^3}{3} - x +$
$\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right) + c$

8.$\int 3^{2x}(2^{1-x} + 1)dx = \int \left(9^x\left(\frac{2}{2^x} + 1\right)\right)dx =$
$\int \left(2.\left(\frac{9}{2}\right)^x + 9^x\right)dx = \frac{2}{\ln 4.5}4.5^x + \frac{1}{ln9}9^x + c$

9.$\int \frac{dx}{\cos^2 x.\sqrt{1+tanx}}$ đặt $t = \sqrt{1 + tanx} \Rightarrow t^2 =$
$1 + tanx \Rightarrow 2tdt = \frac{1}{\cos^2 x}dx$

=> I=$\int \frac{2tdt}{t} = 2t + c = 2\sqrt{1 + tanx} + c$

10.$\int (x + 1)\sin^2 xdx = P = \int (x +$
$1)\left(\frac{1-cos2x}{2}\right)dx = \frac{1}{2}\int (x - xcos2x + 1 -$

$cos2x)dx = \frac{1}{2}\left(\frac{x^2}{2} + x + \frac{1}{2}sin2x\right) + I\ với\ I =$
$\int xcos2xdx$

=>$I = \int xcos2xdx$đặt$\begin{cases} u = x \\ dv = cos2xdx \end{cases} =>$

$\begin{cases} du = dx \\ v = \frac{1}{2}sin2x \end{cases}$

=>I=$\frac{1}{2}xsin2x - \int \frac{1}{2}sin2xdx = \frac{1}{2}xsin2x +$
$\frac{1}{4}cos2x + c$

=>P=$\frac{1}{2}\left(\frac{x^2}{2} + x + \frac{1}{2}sin2x\right) + \frac{1}{2}\left(\frac{x^2}{2} + x +\right.$
$\left.\frac{1}{2}sin2x\right)$

11.$\int tanxdx = \int \frac{sinx}{cosx}dx = \int \frac{d(cosx)}{cosx} =$
$\ln(cosx) + c$

12.$\int\left(\sqrt[3]{3x^2} - 4\sqrt[5]{x^2}\right)x^3dx = \int(\sqrt[3]{3}\,x^{\frac{2}{3}+3} -$
$4x^{\frac{2}{5}+3})dx = \frac{\sqrt[3]{3}}{\frac{14}{3}}x^{\frac{14}{3}} + \frac{4}{\frac{22}{5}}x^{\frac{22}{5}} + c$

13.$\boldsymbol{\int x^2\sqrt{x+1}\,dx}$đặt $t = \sqrt{x+1} => t^2 =$

$x+1 => 2tdt = dx \ => t^2 - 1 = x$

$=> \int (t^2-1)t.2t.dt = \int (2t^4 - 2t^2)dt =$

$\frac{2}{5}t^5 - \frac{2}{3}t^3 + c = \frac{2}{5}\left(\sqrt{x+1}\right) + \frac{2}{3}\sqrt{x+1} + c$

14. $\int \frac{x+1}{(x+2)^2} dx = \int \frac{x+2-1}{(x+2)^2} dx = \int \left(\frac{1}{x+2} - \frac{1}{(x+2)^2}\right) dx = ln|x+2| - (x+2)^{-1} + c$

15. $\int \frac{dx}{\cos 2x + \sin^2 x} = \int \frac{dx}{\cos^2 x - \sin^2 x + \sin^2 x} = \int \frac{dx}{\cos^2 x} = tanx + c$

16. $\int e^u \sqrt{4+e^u} du$ đặt $t = \sqrt{4+e^u} => t^2 = 4 + e^u => 2tdt = e^u du$

$=> \int e^u \sqrt{4+e^u} du = \int 2t.t.dt = \frac{2}{3}t^3 + c$

17. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx = \int \frac{1+\cos^2 x}{1+2\cos^2 x - 1} dx = \int \frac{1+\cos^2 x}{2\cos^2 x} dx = \int \left(\frac{1}{2\cos^2 x} + \frac{1}{2}\right) dx = \frac{1}{2} tanx + \frac{1}{2}x + c$

18. $I = \int e^{2x}.cosxdx$ đặt $\begin{cases} u = cosx \\ dv = e^{2x}dx \end{cases} =>$

$$\begin{cases} du = -sinxdx \\ v = \frac{1}{2}e^{2x} \end{cases}$$

=>I=$cosx.\frac{1}{2}e^{2x} - \int -\frac{1}{2}e^{2x}sinxdxdx =$

$cosx.\frac{1}{2}e^{2x} + \frac{1}{2}\int \frac{1}{2}e^{2x}sinxdxdx$

đăt I1=$\int \frac{1}{2}e^{2x}sinxdxdx$

đặt$\begin{cases} u = sinx \\ dv = e^{2x}dx \end{cases} => \begin{cases} du = cosxdx \\ v = \frac{1}{2}e^{2x} \end{cases}$

=>I1=$sinx.\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}.cosxdx =$

$sinx.\frac{1}{2}e^{2x} - \frac{1}{2}I$

=>$I = cosx.\frac{1}{2}e^{2x} + sinx.\frac{1}{2}e^{2x} - \frac{1}{2}I$

=>$\frac{3}{2}I = \frac{1}{2}e^{2x}(cosx + sinx) => I = \frac{e^{2x}(cosx+sinx)}{3}$

19.$\int \frac{x}{\sqrt[3]{2x-3}}dx$ đặt $t = \sqrt[3]{2x-3} => t^3 = 2x - 3 => 3t^2dt = 2dx => dx = \frac{3}{2}t^2dt =>$

$x = \frac{t^3+3}{2}$

$$=>\int \frac{x}{\sqrt[3]{2x-3}}dx = \int \frac{\frac{t^3+3}{2}}{t}\frac{3}{2}t^2dt = \frac{3}{4}\int \frac{t^3+3}{t}t^2dt =$$

$$\frac{3}{4}\int (t^4 + 3t)dt = \frac{3}{4.5}t^5 + \frac{3.3t^2}{4.2} + c$$

$$20. \int \frac{1}{2^x+1}dx = \int \frac{2^x-2^x+1}{2^x+1}dx = \int \left(-\frac{2^x}{2^x+1} + 1\right)dx = x - \int \frac{2^x}{2^x+1}dx$$

$=>\int \frac{2^x}{2^x+1}dx$ đặt $t = 2^x + 1 => dt = (ln2)2^x dx => \frac{dt}{ln2} = 2^x dx$

$$=>\int \frac{2^x}{2^x+1}dx = \int \frac{\frac{dt}{ln2}}{t} = \frac{lnt}{ln2} + c = \frac{ln(2^x+1)}{ln2} + C$$

$$=>\int \frac{1}{2^x+1}dx = x + \frac{ln(2^x+1)}{ln2} + C$$

3.2

$1. \int \frac{x^3}{\sqrt{1-x^8}}dx$ đặt t=$x^4 => dt = 4x^3dx => \frac{dt}{4} = x^3dx$

$$\int \frac{x^3}{\sqrt{1-x^8}}dx$$

$$= \int \frac{\frac{dt}{4}}{\sqrt{1-t^2}} = \frac{1}{4}.arcsint + c = arcsinx^4 + C$$

$2.\int \frac{x^2-4}{x^3-5x^2+6x}dx = \int \frac{(x-x)(x+2)}{x(x-3)(x-2)}dx = \int \frac{x+2}{x(x-3)}dx =$
$\int (\frac{1}{x-3} + \frac{2}{x(x-3)})dx$

với$\frac{2}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3} = \frac{A(x-3)}{x} + \frac{Bx}{x-3} = \frac{Ax-3A+Bx}{x(x-3)}$

=>$\begin{cases} A + B = 0 \\ -3A = 2 \end{cases}$ => $\begin{cases} A = -2/3 \\ B = 2/3 \end{cases}$

=>$\int (\frac{1}{x-3} + \frac{2}{x(x-3)})dx = \int \left(\frac{1}{x-3} - \frac{2}{3x} + \frac{2}{3(x-3)}\right) dx =$
$ln|x-3| - \frac{2}{3}ln|x| + \frac{2}{3}ln|x-3| + C$

$3.\int \frac{sinx+\sin^3 x}{3+cos2x}dx \int \frac{sinx(1+\sin^2 x)}{2+2\cos^2 x}dx = \int \frac{(2-\cos^2 x)sinx}{2+2\cos^2 x}dx$

đặt t= cosx =>dt=-sinxdx =>sinxdx=-dt

=>$\int \frac{t^2-2}{2t^2+2}dt = \int \frac{1}{2}(1 + \frac{-3}{t^2+1})dt = \frac{1}{2}t - 3arctant +$
$C = \frac{1}{2}cosx - 3arc(cost) + C$

$4.\int \frac{(1+x)^2}{x(1+x^2)}dx$=$\int \frac{x^2+2x+1}{x(1+x^2)}dx = \int (\frac{x}{1+x^2} + \frac{2}{1+x^2} +$
$\frac{1}{x(1+x^2)})dx = I1 + 2arctanx + I2 + C$

với$I1 = \int \frac{x}{1+x^2}dx = \int \frac{1}{2}\frac{d(x^2)}{1+x^2} = \frac{1}{2}\ln(1+x^2) + C1$

$$I2 = \int \frac{1}{x(1+x^2)} dx$$

$= \int \frac{x}{x^2(1+x^2)} dx$ đặt $t = x^2 + 1 => x^2$

$= t - 1 => 2xdx = dt => xdx = \frac{dt}{2}$

$=> I2 = \int \frac{\frac{dt}{2}}{(t-1).t} = \frac{1}{2}\int \left(\frac{A}{t} + \frac{B}{t-1}\right) dt = \frac{1}{2}\int \left(-\frac{1}{t} + \frac{1}{t-1}\right) dt = \frac{1}{2}(-lnt + \ln(t+1)) + C2 = \frac{1}{2}(-\ln(x^2 + 1) + \ln(x^2+1) + C2$

$=> \int \frac{x^2+2x+1}{x(1+x^2)} dx = 2arctanx + \frac{1}{2}\ln(1+x^2) + \frac{1}{2}(-\ln(x^2+1) + \ln(x^2+1) + C$

5. $\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$ ta đặt $t^6 = x => 5t^5 dt = dx => \sqrt{x} = t^3$ và $\sqrt[3]{x} = t^2$

$\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}} = \int \frac{5t^5 dt}{t^3 - t^2} = \int \frac{5t^3 dt}{t-1} = 5\int (At^2 + Bt + C + \frac{D}{t-1})dt = 5\int (t^2 + t + 1 + \frac{1}{t-1})dt = 5\left(\frac{t^3}{3} + \frac{t^2}{2} + t + ln|t-1|\right) + C$

6. $\int \cos(lnx)\, dx$ đặt $\begin{cases} u = \cos(lnx) \\ dv = dx \end{cases} =>$

$\begin{cases} du = -\frac{1}{x}\sin(lnx) \\ v = x \end{cases}$

=>I=$\cos(lnx).x-\int -\sin(lnx)\,dx$=$\cos(lnx).x+\int \sin(lnx)\,dx = x.\cos(lnx)+I1$

$I1=\int \sin(lnx)\,dx$đặt$\begin{cases} u=\sin(lnx) \\ dv=dx \end{cases}$ =>

$\begin{cases} du=\frac{1}{x}cos(lnx) \\ v=x \end{cases}$

$$I1=x.\sin(lnx)-\int \cos(lnx)=xsin(lnx)-I$$

=>$I=x.\cos(lnx)+xsin(lnx)-I$

=>$I=\frac{x(\cos(lnx)+\sin(lnx))}{2}$

7.$\int \frac{x^2}{\sqrt{9+x^2}}dx$đặt x=3tant =>$dx=\frac{3dt}{\cos^2 t}=3(1+\tan^2 t)dt$

=>$\int \frac{9\tan^2 t}{\sqrt{9(1+\tan^2 t)}}dt=\int \frac{9\tan^2 t}{3\sqrt{\frac{1}{\cos^2 t}}}dt=\int \frac{\frac{3\sin^2 t}{\cos^2 t}}{\cos t}dt=$

$\int \frac{3\sin^2 t}{cost}dt=\int \frac{3\sin^2 t.costdt}{\cos^2 t}=\int \frac{3\sin^2 t.cost}{1-\sin^2 t}dt$

đặt u=sint => du=costdt

=>$\int \frac{3u^2}{1-u^2}du=3\int (\frac{u^2-1+1}{1-u^2})du=3\int \left(-1-\frac{1}{u^2-1}\right)du=$

$3\left(-u-ln\left|\frac{u-1}{u+1}\right|\right)+C=3(-sint-ln\left|\frac{sint-1}{sint+1}\right|+c=$

$3(-\sin(\arctan\left(\frac{x}{3}\right))-ln\left|\frac{sin(\arctan\left(\frac{x}{3}\right))-1}{sin(\arctan\left(\frac{x}{3}\right))+1}\right|$+C

8.$\int \frac{3x^2+1}{(x^2-1)^2}dx$

$=\int \frac{3(x^2-1)+4}{(x^2-1)^2}dx$

$$\frac{4}{(x-1)^2(x+1)^2}$$

$$=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x-1)^2}+\frac{D}{(x+1)^2}$$

$=>\frac{A(x-1)(x+1)^2+B(x-1)^2(x+1)+c(x+1)^2+D(x-1)^2}{(x-1)^2(x+1)^2}$

$\frac{A(x^3+2x^2+x-x^2-2x-1)+B(x^3-2x^2+x+x^2-2x+1)+C(x^2+2x+1)+D(x^2-2x+1)}{(x-1)^2(x+1)^2}$

$=>\begin{cases}\begin{cases}A+b=0\\A-B+C+D=0\end{cases}\\\begin{cases}-A-B+2C-2D=0\\-A+B+c+D=4\end{cases}\end{cases}=>\begin{cases}\begin{cases}A=-b\\-B-B+C+D=0\end{cases}\\\begin{cases}B-B+2C-2D=0\\B+B+c+D=4\end{cases}\end{cases}$

$=>\begin{cases}\begin{cases}A=-\boldsymbol{B}\\-2B+2C=0\end{cases}\\\begin{cases}C=D\\2B+2C=4\end{cases}\end{cases}$=>C=D=1 và B=1 A=-1

$=>\int \frac{3(x^2-1)+4}{(x^2-1)^2}dx=\int(\frac{3}{x^2-1}+\frac{4}{(x-1)^2(x+1)^2})dx$

$=\int(\frac{3}{x^2-1}+\frac{-1}{x-1}+\frac{1}{x+1}+\frac{1}{(x-1)^2}+\frac{1}{(x+1)^2})dx => tự\ tính$

9. $\int \frac{dx}{1+\sqrt{x+1}}$

=> đạt $t=1+\sqrt{x+1}=>t-1=\sqrt{x+1}=>$

$(t-1)^2=x+1=>2(t-1)dt=dx$

$=>\int \frac{2(t-1)dt}{t}=2\int\left(1-\frac{1}{t}\right)dt=2(t-ln|t|)+C=>$

$2\left(1+\sqrt{x+1}-ln\left|1+\sqrt{x+1}\right|\right)+C$

10.$\int \frac{x}{x^4-4x^2+3}dx$

=> đặt t=x^2 $=> dt = 2xdx => xdx = \frac{dt}{2}$

=>$\int \frac{\frac{dt}{2}}{t^2-4t+3}dt = \frac{1}{2}\int \frac{dt}{(t-1)(t-3)}$ $với$ $\frac{1}{(t-1)(t-3)} = \frac{A}{t-1} + \frac{B}{t-3}$

=>$\begin{cases} A+B=0 \\ -3A-B=1 \end{cases}$ $=> \begin{cases} A=-\frac{1}{2} \\ B=\frac{1}{2} \end{cases}$

=>$\frac{1}{2}\int \frac{dt}{(t-1)(t-3)} = \frac{1}{4}\int \left(\frac{1}{t-3}-\frac{1}{t-1}\right)dt = \frac{1}{4}ln|t-3| + \frac{1}{4}ln|t-1| + c => I = \frac{1}{4}ln|x^2-3| + \frac{1}{4}ln|x^2-1| + c$

11.$\int \frac{x^2}{(1+x^2)^2}dx$=I

=>$I = \frac{1}{2}\int \frac{(x^2+1)+(x^2-1)}{(1+x^2)^2}dx$=$\frac{1}{2}\int \left(\frac{1}{(1+x^2)} + \frac{\left(1-\frac{1}{x^2}\right)}{\left(\frac{1}{x}+x\right)^2}\right)dx = \frac{1}{2}(I1+I2)$

I2=$\int \frac{\left(1-\frac{1}{x^2}\right)}{\left(\frac{1}{x}+x\right)^2}dx$ => đặt t=$\frac{1}{x}+x => dt = \left(1-\frac{1}{x^2}\right)dx$

=>I2=$\int \frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{\frac{1}{x}+x} + C$

I1=$\int \frac{1}{(1+x^2)}dx = arctanx$+C

=>I=$-\frac{1}{\frac{1}{x}+x}+\ arctanx$

12.$\int \frac{2x-\sqrt{arcsinx}}{\sqrt{1-x^2}}dx$

=>I=$\int\left(\frac{2x}{\sqrt{1-x^2}}-\frac{\sqrt{arcsinx}}{\sqrt{1-x^2}}\right)dx=I1-I2$

=> $I1=\int\frac{2x}{\sqrt{1-x^2}}dx$ đặt t=$\sqrt{1-x^2}$ => $t^2=1-x^2$ =>

$2tdt=-2xdx => 2xdx=-2tdt$

=> $I1=\int\frac{2x}{\sqrt{1-x^2}}dx=\int\frac{-2tdt}{t}=-2t+c=$

$-2\sqrt{1-x^2}+C$

=> $I2=\int\frac{\sqrt{arcsinx}}{\sqrt{1-x^2}}dx$ đặt t=$\sqrt{arccsinx}$ => $t^2=$

$arcsinx => 2tdt=\frac{1}{\sqrt{1-x^2}}dx$

=>I2=$\int\frac{\sqrt{arcsinx}}{\sqrt{1-x^2}}dx=\int 2t.tdt=\frac{2}{3}t^3+c=$

$\frac{2}{3}(\sqrt[3]{arcsinx})+C$

=>I=$-2\sqrt{1-x^2}+\frac{2}{3}\sqrt[3]{arcsinx}+C$

13.$\int\frac{dx}{x(x^2+2)}$

=>$\int\frac{xdx}{x^2(x^2+2)}$ đặt t=$x^2+2 => dt=2xdx => xdx=\frac{dt}{2}$

=>$\int\frac{\frac{dt}{2}}{(t-2)t}=\frac{1}{2}\int\frac{dt}{t(t-2)}$ $với\ \frac{1}{t(t-2)}=\frac{A}{t}+\frac{B}{t-2} => A=$

$-\frac{1}{2}\ B=\frac{1}{2}$

$=>\frac{1}{2}\int\frac{dt}{t(t-2)}=\frac{1}{4}\int\left(-\frac{1}{t}+\frac{1}{t-2}\right)dt=\frac{1}{4}(-ln|t|+$

$ln|t-2|+C => I=\frac{1}{4}(-ln|x^2+2|+ln|x^2|+C$

14.$\int\sin^2 x.\cos^4 xdx=I$

$=>\int\frac{1}{4}Sin^2 2x.\cos^2 x\,dx=\frac{1}{4}\int\frac{1-cos4x}{2}\frac{1+cos2x}{2}dx$

$=>\frac{1}{16}\int(1+cos2x-cos4x-cos4x.cos2x)dx$

$=>\frac{1}{16}\int(1+cos2x-cos4x+\frac{1}{2}(cos6x+cos2x))dx$

$=>\frac{1}{16}\int(1+\frac{3}{2}cos2x-cos4x+\frac{1}{2}cos6x)dx =>$ *tự tính*

15. Như bài 3.1 số 18

16.$\int\frac{dx}{1+tanx}$

đặt t=tan$\frac{x}{2} => dt=\frac{dx}{\cos^2\frac{x}{2}}=\frac{dx}{\frac{1+cosx}{2}}$ *mà* $cosx=\frac{1-t^2}{1+t^2}$

$=> dt=\frac{2dx}{1+\frac{1-t^2}{1+t^2}} => dt=\frac{2(1+t^2)dx}{2} => dx=\frac{dt}{1+t^2}$

tanx=$\frac{2t}{1-t^2}$

=> I=$\int\frac{dt}{(1+t^2)(1+\frac{2t}{1-t^2})}=\int\frac{(1-t^2)dt}{(1+t^2)(2t+1-t^2)}$

$=>\int\frac{dx}{1+\frac{sinx}{cosx}}=\int\frac{cosxdx}{cosx+sinx}=\int\frac{\cos\left(x+\frac{\pi}{4}-\frac{\pi}{4}\right)dx}{\sqrt{2}\sin(x+\frac{\pi}{4})}=$

$\int\frac{\cos\left(x+\frac{\pi}{4}\right).\frac{\sqrt{2}}{2}+\sin\left(x+\frac{\pi}{4}\right).\frac{\sqrt{2}}{2}}{\sqrt{2}\sin(x+\frac{\pi}{4})}dx=\frac{1}{2}\int\left(\frac{\cos\left(x+\frac{\pi}{4}\right)}{\sin\left(x+\frac{\pi}{4}\right)}+1\right)dx$

$=> \frac{1}{2}\left(ln\left|\sin\left(x+\frac{\pi}{4}\right)\right|+x\right)+C$

17. $\int \frac{dx}{x^4+1}$

$=> 2I = \int \frac{2dx}{x^4+1} = \int \left(\frac{x^2-1}{x^4+1}+\frac{1+x^2}{x^4+1}\right)dx = I1 + I2$

$=> I1 = \int \frac{x^2-1}{x^4+1}dx = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}dx$

=> đat t= $x+\frac{1}{x} => dt = \left(1-\frac{1}{x^2}\right)dx$

$=> I1 = \int \frac{dt}{t^2-2} = ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C = ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + C$

$=> I2 = \int \frac{1+x^2}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx =$

$\frac{1}{\sqrt{2}} arctan\frac{x^2-1}{x\sqrt{2}} + C$

$=> I = \frac{ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right|+\frac{1}{\sqrt{2}}arctan\frac{x^2-1}{x\sqrt{2}}}{2} + C$

18. $\int \sqrt{x^2-4}dx$

$=> I = \frac{x}{2}\sqrt{x^2-4} + \frac{4}{2}ln\left|x+\sqrt{x^2-4}\right| + C$

19. $\int \frac{1+2x^2}{x^3(1+x^2)}dx$

$=> \int \left(\frac{1+x^2}{x^3(1+x^2)}+\frac{x^2}{x^3(1+x^2)}\right)dx = \int \left(\frac{1}{x^3}+\frac{x}{x^2(1+x^2)}\right)dx =$

$I1 + I2$

=> $I2 = \int \frac{x}{x^2(1+x^2)} dx$

đặt t=$x^2 + 1 => dt = 2xdx => xdx = \frac{dt}{2}$

=>$\frac{1}{2}\int \frac{dt}{t(t-1)} = \frac{1}{2}\int \left(-\frac{1}{2t} + \frac{1}{2(t-1)}\right) dt = \frac{1}{4}(- ln|t| +$

$ln|t-1| + C$=$\frac{1}{4}(-ln|x^2+1| + ln|x^2| + C)$

=>I=$\frac{-1}{2x^2} + \frac{1}{4}(-ln|x^2+1| + ln|x^2| + C)$

20.$\int \frac{\sin^3 x}{\sqrt[3]{cosx}} dx$

=>đặt $t = \sqrt[3]{cosx} => t^3 = cosx => 3t^2 = -sinxdx$

=> I=$\int \frac{(1-\cos^2 x)sinx}{\sqrt[3]{cosx}} => I = \int \frac{-(1-t^6)3t^2}{t} dx =$

$\int(-3t + 3t^7)dt = -\frac{3t^2}{2} + \frac{3}{8}t^8 + C = -\frac{3\sqrt[3]{cosx}^2}{2} +$

$\frac{3}{8}\sqrt[3]{cosx}^8 + C$

21.$\int \frac{dx}{e^x+e^{-x}}$

=>I=$\int \frac{dx}{e^x+\frac{1}{e^x}} = \int \frac{e^x}{(e^x)^2+1} dx$

đăt t=$e^x => dt = e^x dx$

=>I=$\int \frac{dt}{t^2+1} = arctant + C = arctane^x + C$

22.$\int \frac{x}{\sqrt{x^2+2}} dx$

đặt t=$\sqrt{x^2+2} => t^2 = x^2 + 2 => tdt = xdx$

=>I=$\int \frac{tdt}{t} = t + C = \sqrt{x^2+2} + C$

23.$\int \cos^5 x dx$

=>I=$\int (1-\sin^2 x)^2\, cosxdx$

đăt t=sinx => dt=cosxdx

=>I=$\int (1-t^2)^2 dt = \int (t^4 - 2t^2 + 1)dt = \frac{1}{5}t^5 - \frac{2}{3}t^3 + t + C$

24.$\int \sin^5 x.\cos^3 xdx$

=> $I = \frac{1}{2}\int (1-\cos^2 x)^2 \left(\frac{1+cos2x}{2}\right) sin2xdx$

=$\frac{1}{2}\int (1 - \frac{1+cos2x}{2})^2 \left(\frac{1+cos2x}{2}\right) sin2xdx$

đăt t=cos2x => dt=-2sin2xdx=>sinxdx=-dt/2

$$= \frac{1}{2} - \int (1 - \frac{1+t}{2})^2 \left(\frac{1+t}{2}\right) dt => tự\ tính$$

25.$\int \frac{dx}{\sqrt{x^2+2x}}$

=>I=$\int \frac{dx}{\sqrt{(x+1)^2-2}} = \frac{1}{\sqrt{2}} ln\left|x + 1 + \sqrt{(x+1)^2 - \sqrt{2}}\right|$+C

26.$\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}} dx$

=>I=$\int \left(\frac{x}{\sqrt[3]{1+x}} + (x+1)^{\frac{1}{6}}\right) dx = I1 + I2$

=>$I1 = \int \frac{x}{\sqrt[3]{1+x}} dx$ đặt t=$\sqrt[3]{1+x} => 3t^2 dt = dx$

=> $I1 = \int \frac{(t^3-1)3t\text{^}2}{t} dt = \frac{3}{5}t^5 - \frac{3}{2}t^2 + C$

=>=$\frac{3}{5}t^5-\frac{3}{2}t^2+C=\frac{3}{5}(\sqrt[3]{1+x})^5-\frac{3}{2}(\sqrt[3]{1+x})^2+C$

=>I=$\frac{3}{5}(\sqrt[3]{1+x})^5-\frac{3}{2}\left(\sqrt[3]{1+x}\right)^2+\frac{6}{7}(x+1)^{\frac{7}{6}}+C$

27.$\int\frac{dx}{1+sinx}$

=>I=$\int\frac{dx}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\left(\frac{x}{2}\right).\cos(\frac{x}{2})}=\int\frac{dx}{(\cos\frac{x}{2}+\sin(\frac{x}{2}))^2}=$

$\int\frac{dx}{2.\cos^2(\frac{x}{2}+\frac{\pi}{4})}=\frac{2}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)+C$

28.$\int e^{\sqrt{x}}$dx

=> đặt t=$\sqrt{x}$ $=>2tdt=dx$

=>I=$\int 2te^t\,dt$

=> đặt $\begin{cases} u=t \\ dv=e^tdt\end{cases}=>\begin{cases} du=dt \\ v=e^t\end{cases}$

=>I=$t.e^t-\int e^tdt=t.e^t-e^t+C=\sqrt{x}.e^{\sqrt{x}}-e^{\sqrt{x}}+C$

29.$\int\frac{\cos^3x+\cos^5x}{sin^2x+sin^4x}dx$

=>I=$\int\frac{(\cos^2x+\cos^4x)cosx}{sin^2x+sin^4x}dx=\int\frac{(\cos^2x+\cos^4x)cosx}{sin^2x+sin^4x}$dx

=>I=$\int\frac{((1-\sin^2x)+(1-\sin^2x)^2)cosx}{\sin^2x+\sin^4x}dx\ t=sinx=>dt=cosx\,dx$

=>I=$\int\frac{((1-t^2)+(1-t^2)^2}{t^4+t^2}dt=\int\frac{t^4-t^2+2}{t^4+t^2}dt=\int\left(1-\frac{1}{t^2+1}+\frac{2}{t^4+t^2}\right)dt=t-arctant+I1$

$=> I1 = \int \frac{2}{t^4+t^2} dt$ đặt $t = tanu => dt = (1+t^2)du$

$=> I1 = \int \frac{2du}{\tan^2 u} = 2\int \left(\frac{\cos^2 u}{\sin^2 u}\right) du = 2\int \left(\frac{1}{\sin^2 u} - 1\right) dx$

$=> I1 = 2(-cotu - u) + C =>$ *tra biền về*

30.$\int \frac{dx}{\sqrt{x-x^2}}$

=>I=$\int \frac{dx}{\sqrt{\left(\frac{1}{2}-x\right)^2 - \frac{1}{4}}}$

đăt t=$\frac{1}{2} - x => dx = -dt$

=>I=$\int \frac{-dt}{\sqrt{t^2-\frac{1}{4}}} = 2ln\left|t + \sqrt{t - \frac{1}{2}}\right| + C = 2ln\left|\frac{1}{2} - x + \sqrt{-x}\right| + C$

31.$\int \sqrt{\frac{x+1}{x-1}dx}$

=>I=$\int \sqrt{1 + \frac{2}{x-1}} dx$ đăt $t = \sqrt{1+\frac{2}{x-1}} => t^2 = 1 + \frac{2}{x-1}$

=>$t^2 - 1 = \frac{2}{x-1} => x - 1 = \frac{2}{t^2-1} => dx = \frac{4t}{t^2-1} dt$

=>I=$\int \frac{4t^2}{t^2-1} dt = \int \left(4 + \frac{4}{t^2-1}\right) dt = 4(t + arctant) + C$

=4$\left(\sqrt{1+\frac{2}{x-1}} + arctan\sqrt{1+\frac{2}{x-1}}\right) + C$

32.$\int \frac{dx}{x\sqrt{x^2+1}}$

=>I=$\int \frac{xdx}{x^2\sqrt{x^2+1}}$ đặt $t = \sqrt{x^2+1} => t^2 = x^2 + 1$

=>$tdt = xdx => I = \int \frac{tdt}{(t^2-1)t} = \frac{1}{2} ln \left|\frac{t-1}{t+1}\right| + C$

=> $I = \frac{1}{2} ln \left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right| + C$

33.$\int e^{arccosx} dx$

đặt $\begin{cases} u = e^{arccosx} \\ dv = dx \end{cases} => \begin{cases} du = -\frac{e^{arccosx}}{\sqrt{1-x^2}} \\ v = x \end{cases}$

=>I= $xe^{arccosx} + \int \frac{xe^{arccosx}}{\sqrt{1-x^2}}$

=>$I2 = \int \frac{xe^{arccosx}}{\sqrt{1-x^2}}$ đặt $\begin{cases} u = e^{arccosx} \\ dv = \frac{x}{\sqrt{1-x^2}} dx \end{cases} =>$

$\begin{cases} du = -\frac{e^{arccosx}}{\sqrt{1-x^2}} \\ v = -2\sqrt{1-x^2} \end{cases}$

=> $I2 = -2\sqrt{1-x^2}e^{arccosx} - 2\int e^{arccosx} dx$

=> $I = xe^{arccosx} - 2\sqrt{1-x^2}e^{arccosx} - 2I$

=> $I = \frac{xe^{arccosx} - 2\sqrt{1-x^2}e^{arccosx}}{3}$

34.$\int \sin^5 x dx$

$$=> I = \int (1-\cos^2 x)^2 sinxdx$$

=> $I = \int -(1-\cos^2 x)^2 d(cosx) = \frac{-\cos^5 x}{5} + 2\frac{\cos^3 x}{3} +$
$x + C$

35. $\int \frac{dx}{1+cosx}$

$$=> I = \int \frac{dx}{1 + (2\cos^2\frac{x}{2} - 1)} = \int \frac{dx}{2\cos^2\frac{x}{2}} = \tan\frac{x}{2} + C$$

36. $\int \frac{\sqrt{x+1}+2}{(x+1)^2-\sqrt{x+1}} dx$

đặt t= $\sqrt{x+1} => 2tdt = xdx$

$=> I = \int \frac{t+2}{t^3-1} dt = \int \left(1 + \frac{3}{t^3-1}\right) dt$

$$\frac{1}{t^3-1} = \frac{A}{t-1} + \frac{Bt+C}{t^2+t+1}$$

$$= \frac{A(t^2+t+1) + B(t+C)(t-1)}{t^3-1}$$

$$=> \begin{cases} A = \frac{1}{3} \\ B = -\frac{1}{3} \\ C = -\frac{2}{3} \end{cases}$$

$=> I = \int \left(1 + \frac{1}{t-1} - \frac{t+2}{t^2+t+1}\right) = \int \left(1 + \frac{1}{t-1} - \frac{t+1}{t^2+t+1} - \frac{2}{\left(t+\frac{1}{2}\right)^2 - \frac{1}{4}}\right) dt = t + ln|t-1| - \frac{1}{2} ln|t^2+t+1| - 2ln\left|\frac{t}{t+1}\right| + C => trả\ biến\ về$

37. $\int \frac{dx}{1+\sin^2 x}$

$$=>I = \int \frac{dx}{\cos^2 x(\frac{1}{\cos^2 x}+\tan^2 x)}$$ đặt $t = tanx => dt = \frac{dx}{\cos^2 x}$

$$=> I = \int \frac{dt}{1+2t^2} = \frac{1}{2}\int \frac{dt}{t^2+\frac{1}{2}} = \frac{1}{2}.\frac{1}{\sqrt{2}} \arctan(t\sqrt{2}) + C$$

38. $\int ln|cosx|dx$

từng phần : $\begin{cases} u = ln|cosx| \\ dv = dx \end{cases} => \begin{cases} du = -\frac{\sin x}{|cosx|} \\ v = x \end{cases}$

=>I= $x\, ln|cosx| + \int x\, tanxdx$

Xét H = $\int x.tanxdx$

Ta có : sử dụng công thức Taylor :

$$x.tanx = \sum_{k=1}^{\infty} \frac{2^{2k}\left(2^{2k}-1\right)B_k}{(2k)!}x^{2n}$$

$$\text{nên } H = \sum_{k=1}^{\infty} \frac{2^{2k}\left(2^{2k}-1\right)B_k}{(2k)!}x^{2k+1} + C$$

$$\text{Vậy } I = x.ln\left(cosx\right) + \sum_{k=1}^{\infty} \frac{2^{2k}\left(2^{2k}-1\right)B_k}{(2k)!}x^{2k+1} + C$$

Bài này không có nguyên hàm sơ cấp.

Chương 4

4.2

a. $\int_0^1 \frac{xdx}{(x^2+1)}$

=>đặt t=$x^2+1 => xdx = \frac{dt}{2}$ =>x=1=>t=2

x=0=>t=1

=>I=$\int_1^2 \frac{\frac{dt}{2}}{t} = \frac{1}{2}lnt|_1^2 = \frac{1}{2}ln2$

b. $\int_1^{e^3} \frac{dx}{x\sqrt{1+lnx}}$

đạt t=$\sqrt{1+lnx} => 2tdt = \frac{dx}{x}$

với x=1=>t=1 x=e^3 =>t=2

=>I=$\int_1^2 \frac{2tdt}{t} = 2t|_1^2 = 4-2 = 2$

C. $\int_1^2 \frac{dx}{x+x^3}$

=>I=$\int_1^2 \frac{xdx}{x^2+x^4}$ đặ$t\ t = x^2 => xdx = \frac{dt}{2}$

với x=1=> t=1 và x=2=>t=4

=>I=$\int_1^4 \frac{\frac{dt}{2}}{t+t^2} = \frac{1}{2}\int_1^4 (\frac{1}{t} - \frac{1}{t+1})\, dt = \frac{1}{2} ln \left|\frac{t}{t+1}\right|\Big|_1^4 =$

$\frac{1}{2} ln\left(\frac{\frac{4}{5}}{\frac{1}{2}}\right) =$

D.$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{1+cosx}$

=> y như bài 35. Chương 3 =>

I=$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{1+cosx} = tan\frac{x}{2}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1 - -1 = 2$

E.$\int_0^{\pi} \sqrt{1 + cos2x}dx$

=>I=$\int_0^{\pi} \sqrt{2\cos^2 x}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cosx\, dx -$ $\int_{\frac{\pi}{2}}^{\pi} \sqrt{2}cosxdx = \sqrt{2}\big(0 - 1 - (-1 + 0)\big) = 0$

f.$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{xdx}{\sin^2 x}$

$$=> \begin{cases} u = x \\ dv = \dfrac{dx}{\sin^2 x} \end{cases} => \begin{cases} du = dx \\ v = cotx \end{cases}$$

=>I=$xcotx\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} cotxdx = xcotx\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} -$

$ln|sinx|\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} => bấm\ máy$

g.$\int_3^8 \frac{xdx}{\sqrt{1+x}}$

đat t=$\sqrt{1+x} => 2tdt = dx => x = t^2 - 1$

với x=3 =>t=2 x=8 =>t=3

=>I=$\int_2^3 \frac{2t(t^2-1)dt}{t} = 2\left(\frac{t^3}{3} - t\right)\Big|_2^3 = 2(\frac{27}{3} - 1 - \frac{8}{3})$

h.$\int_0^1 x^2\left(\sqrt{1-x^2}\right)dx$

đăt x=sint =>dx=costdt với x=1 =>t=$\frac{\pi}{2}$ $vã\ x = 0 => t = 0$

=>I=$\int_0^{\frac{\pi}{2}} \sin^2 t\sqrt{\cos^2 t}\, dt = \int_0^{\frac{\pi}{2}} \sin^2 t cos^2 t dt = \int_0^{\frac{\pi}{2}} \frac{1}{4}\sin^4 2tdt = \int_0^{\frac{\pi}{2}} \frac{1}{4}\left(\frac{1-cos4t}{2}\right)^2 dt =>$ $triển\ khai\ làm\ tiếp$.

i.$\int_0^{\sqrt[5]{2}} \frac{x^9}{(1+x^5)^3} dx$

đăt t=$1 + x^5 => dt = 5x^4dx => t - 1 = x^5$

với x=0=>t=1 với $x = \sqrt[5]{2} => t = 3$

=>I=$\int_0^3 \frac{\frac{dt}{5}(t-1)}{t^3} = \frac{1}{5}\int_0^3 \left(\frac{1}{t^2} - \frac{1}{t^3}\right) dt =$

$\frac{1}{5}(\frac{1}{2t^2}-\frac{1}{t})\Big|_0^3 => tự\ tính$

J. $\int_0^1 \sqrt{2x+x^2}dx$

=>I= $\int_0^1 \sqrt{(x+1)^2-1}dx =$
$(\frac{x+1}{2}\sqrt{(x+1)^2-1}+\frac{1}{2}ln\left|x+1+\sqrt{(x+1)^2-1}\right|)\Big|_0^1$

tích phân suy rộng chỉ làm loại I

4.15

a. $\int_1^{+\infty}\frac{dx}{\sqrt{x}}$

đặt t=$\sqrt{x}$=>2tdt=dx

=> $\int_1^{+\infty} 2dt = \lim_{b\to+\infty}(2b-2) = +\infty => Phân\ Kỳ$

b. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)+1}$

=> $\int_{-\infty}^{0} \frac{dx}{(x^2+1)+1} + \int_{0}^{+\infty} \frac{dx}{(x^2+1)+1}$

=>I= $\lim_{a\to-\infty} (\arctan(1) - \arctan(a+1)) + \lim_{b\to+\infty} (\arctan(b+1) - \arctan(1) = -\frac{\pi}{2} - \frac{\pi}{2} = -\pi =>$ $hội\ tụ$

c. $\int_{0}^{+\infty} \frac{xdx}{\sqrt{x-1}}$

đặt t= $\sqrt{x-1} => 2tdt = dx$

=>I= $\int_{0}^{+\infty} \frac{t^2-1}{t} 2tdt =$

$$\lim_{a\to+\infty} 2\left(\frac{a^3}{3} - a\right) = +\infty =>$$
PK

h. $\int_1^{+\infty} \frac{1+x^2}{x^3} dx$

=>I= $\lim_{b\to+\infty} \int_1^b \left(\frac{1}{x^3} + \frac{1}{x}\right) dx$

=>I= $\lim_{b\to+\infty} \left(\left(-\frac{1}{2b^2} + lnb\right) - (1 + 0)\right) = \lim_{b\to+\infty} \frac{-1+2.b^2.\ln\frac{b}{e}}{2.b^2} =$

$0 + \infty = +\infty => Phân\ kỳ$