

# Đề 1

## I. trắc nghiệm:

**câu 1.**  $\lim_{x \rightarrow 0+} x^{\frac{3(\ln x)^2}{4+\ln x}} = e^{\lim_{x \rightarrow 0+} \frac{3(\ln x)^3}{4+\ln x}} = e^{\lim_{x \rightarrow 0+} \frac{9(\ln x)^2}{1}} = e^{\infty} = +\infty$

vì  $\lim_{x \rightarrow 0+} \ln x = -\infty$

**câu 2.**  $\lim_{x \rightarrow 0} \frac{\ln(1-(\sin 2x)^3)}{4x^3} = \lim_{x \rightarrow 0} \frac{-(\sin 2x)^3}{4x^3} = \frac{-2^3}{4} = -2$

**câu 3.**  $\lim_{x \rightarrow \infty} \left(\frac{2x-4}{2x-5}\right)^{1-3x} = e^{\lim_{x \rightarrow \infty} \left(\frac{2x-4}{2x-5}-1\right)(1-3x)} = e^{\lim_{x \rightarrow \infty} \left(\frac{1-3x}{2x-5}\right)} = e^{-3/2}$

**câu 4**  $y = \arcsin\left(\frac{x-3}{2}\right) - \log(4-x)$  TXD

$$\begin{cases} -1 \leq \frac{x-3}{2} \leq 1 \\ 4-x > 0 \end{cases} \begin{cases} 1 \leq x \leq 5 \\ x < 4 \end{cases} \Rightarrow 1 \leq x < 4$$

### **câu 5**

$\lim_{x \rightarrow 0} f(x) =$

$$\lim_{x \rightarrow 0} \frac{2\cos x - 2\sqrt{\cos 2x}}{(\tan x)^2} = \lim_{x \rightarrow 0} \frac{2 - 2(1 - \cos x) - 2\sqrt{1 - (1 - \cos 2x)}}{(\tan x)^2} =$$

$$\lim_{x \rightarrow 0} \frac{2 - x^2 - 2\sqrt{1 - 2x^2}}{x^2} = \lim_{x \rightarrow 0} \frac{-2x - \frac{-4x}{2\sqrt{1 - 2x^2}}}{2x} = -1 + 2 = 1$$

**câu 6**  $y = \frac{1}{2x^2 - 5x + 2} = \frac{1}{(x-2)(2x-1)} = \frac{A}{x-2} - \frac{B}{2x-1} = \frac{1}{x-2} - \frac{2}{2x-1}$

dùng đồng nhất hệ số ta có :  $\begin{cases} 2A - B = 0 \\ -A + 2B = 1 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 2 \end{cases}$

$$y^{(8)} = \frac{8!}{(x-2)^9} - \frac{8!2^9}{(2x-1)^9} \text{ tự qui đồng .}$$

**câu 7**  $y = x^{\frac{1}{x}} \Leftrightarrow \ln y = (1/x) \ln x$

$$\frac{y'}{y} = \left( -\frac{1}{x^2} \ln x + \frac{1}{x^2} \right) \Rightarrow y' = (1 - \ln x) x^{\frac{1}{x}-2}$$

**câu 8**

$$y = \frac{1-x}{e^{-x}} = 1/e^{-x} - x e^{-x}$$

$$y^{(10)} = -C_{10}^0 x e^{-x} - C_{10}^1 - e^{-x} \text{ thay } x=2 \text{ vào } \Rightarrow y^{(10)} = \frac{9}{e^2}$$

**câu 9**

$$\int \frac{-4 \cos^3 x \sin x}{1 + \cos^4 x} dx$$

đặt  $t = 1 + \cos^4 x$  ta có  $t > 0$

$$dt = -4 \cos^3 x \sin x dx \Rightarrow \int \frac{1}{t} dt = \ln(t) + c = \ln(1 + \cos^4 x) + c$$

**câu 10** diện tích bởi  $y = \frac{\ln x}{x^2}$  và  $x = e$  và trục hoành  $y = 0$

phương trình hoành độ giao điểm  $\frac{\ln x}{x^2} = 0 \Rightarrow x = 1$

$$\int_1^e \frac{\ln x}{x^2} dx = 1 - 2/e$$

**câu 11** tìm tích phân suy rộng Hội tụ :

$$\int_0^{+\infty} x^2 dx = \lim_{b \rightarrow \infty} \int_0^b x^2 dx = \lim_{b \rightarrow \infty} \frac{1}{3}(b^3 - 0^3) = +\infty \text{ PK}$$

$$\int_{-\infty}^0 e^x dx = \lim_{b \rightarrow -\infty} \int_{-\infty}^0 e^x dx = \lim_{b \rightarrow -\infty} (e^b - e^0) = +\infty \text{ PK}$$

$$\int_1^{+\infty} \frac{1}{x \ln^3 x} dx = \int_1^{+\infty} \frac{1}{\ln^3 x} d(\ln x) = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{\ln^3 x} d(\ln x) =$$

$$\lim_{b \rightarrow \infty} \left( \frac{-1}{4} \frac{1}{\ln^4 b} + 1/4 * 1/0 \right) \text{ tới đây tạm bí làm câu D.}$$

$$\int_e^{+\infty} \frac{\ln^3 x}{x} dx = \int_e^{+\infty} \ln^3 x d(\ln x) = \lim_{b \rightarrow \infty} \int_e^b \ln^3 x d(\ln x) =$$

$$\lim_{b \rightarrow \infty} 1/4 (\ln^4 b - 1) = +\infty \text{ PK}$$

=> câu C hội tụ .

### **câu 12**

$$\sum_{n=2}^{\infty} 2 \cdot 3^{-n} [1.5^n - (-2)^n] = \sum_{n=2}^{\infty} 2 \cdot \left(\frac{1}{2}\right)^n - 8 \left(-\frac{2}{3}\right)^n$$

vì n=2

$$\Rightarrow S = 2 \frac{\left(\frac{1}{2}\right)^2}{1 - 1/2} - 8 \frac{\left(-\frac{2}{3}\right)^2}{1 + 2/3} = -17/15$$

### **câu 13**

$$\sum_{n=1}^{\infty} \frac{(\sqrt{n+1})^2}{n^2+1} \text{ xét } \lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} \frac{(\sqrt{n+1})^2}{n^2+1} = \lim_{n \rightarrow \infty} 1/n \text{ dãy này}$$

PK

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right) \text{ xét } \lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} n \sin\left(\frac{1}{n}\right) = n(1/n) = 1 \text{ PK}$$

$$\sum_{n=1}^{\infty} \frac{n^{\sqrt{e}+1}}{n^2+1} \text{ xét } \lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} \frac{e^{1/n}+1}{n^2+1} = \lim_{n \rightarrow \infty} \frac{1/n}{n^2+1} = \lim_{n \rightarrow \infty} \left(\frac{1}{n^3}\right) = 0$$

HT vì  $3 > 1$

**câu 14**

$$\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n \text{ Dùng } \lim_{n \rightarrow \infty} \sqrt[n]{U_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(-\frac{1}{3}\right)^n}$$

$$= -1/3 < 1 \text{ HT}$$

$$\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n \text{ Dùng } \lim_{n \rightarrow \infty} \sqrt[n]{U_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n} = 1/2 < 1 \text{ HT}$$

$$\sum_{n=1}^{\infty} \sqrt{n} - \sqrt{n-1}$$

ta có

$\sqrt{n} - \sqrt{n-1} > 0$  dãy số luôn tăng với mọi  $n$

$\Rightarrow \lim_{n \rightarrow \infty} U_n > 0 \forall n > 1 \Rightarrow$  dãy PK chọn D

**câu 15** bán kính hội tụ :

$$\sum_{n=3}^{\infty} \left(\frac{n}{n-2}\right)^{n^2} \text{ bán kính hội tụ là : } \rho = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n}{n-2}\right)^{n^2}} =$$

$$\lim_{n \rightarrow \infty} \left(\frac{n}{n-2}\right)^n = e^{\lim_{n \rightarrow \infty} \left(\frac{n}{n-2} - 1\right)n}$$

$$= e^{\lim_{n \rightarrow \infty} \left(\frac{2n}{n-2}\right)} = e^2$$

$$R = 1/\rho = e^{-2}$$

Đề 2

trắc nghiệm

Câu 1

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos 2x}}{(\sin x)^2} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - (1 - \cos 2x)}}{(\sin x)^2} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - (2x^2)}}{(2x)^2}$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{4x}{\sqrt{1 - (2x^2)}}}{4x} = -1$$

câu 2

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\ln(\cos x)}{(x)^2} &= \lim_{x \rightarrow 0} \frac{\ln(1 + (-(1 - \cos x)))}{(x)^2} \\ &= \lim_{x \rightarrow 0} \frac{(-(x^2/2))}{(x)^2} = -\frac{1}{2} \end{aligned}$$

câu 3

$\lim_{x \rightarrow 2} \cos x^{\cot x}$  = bấm máy thẳng x=2 ra câu C

câu 4 TXĐ

$$y = \frac{\arccos\left(\ln\left(\frac{x}{e}\right)\right)}{4\arctan x - \pi}$$

$$\begin{cases} -1 \leq \ln\left(\frac{x}{e}\right) \leq 1 \\ x \neq \arctan\left(\frac{\pi}{4}\right) \end{cases} \Leftrightarrow \begin{cases} e^{-1} \leq \left(\frac{x}{e}\right) \leq e \\ x \neq 1 \end{cases} \Leftrightarrow \begin{cases} 1 \leq (x) \leq e^2 \\ x \neq 1 \end{cases}$$

đáp án C

câu 5

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{e^{2x} - 2x - 1}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{2e^{2x} - 2}{\sin 2x} = \lim_{x \rightarrow 0} \frac{4e^{2x}}{2\cos 2x} = 2$$

với  $f(0) = 3a - 1$

$$\text{ta có } \lim_{x \rightarrow 0} f(x) = f(0) = 3a - 1 = 2 \Rightarrow a = 1$$

câu 6

$$y = 2x \cos^2 x \text{ ta có } (\cos^2 x)' = -\sin 2x$$

$$y^{(20)} = -C_{20}^0 2x(-2^{19}) \left( \sin \left( 2x + \frac{19}{2\pi} \right) \right) - \\ C_{20}^1 2(-2^{19}) \left( \sin \left( 2x + \frac{18}{2\pi} \right) \right)$$

$$\text{câu 7 } y = \arctan\left(\frac{\ln x}{3}\right)$$

$$y' = \frac{\left(\frac{\ln x}{3}\right)'}{1 + \left(\frac{\ln x}{3}\right)^2} = \frac{3}{x(9 + \ln^2 x)} \Rightarrow dy = \frac{3}{x(9 + \ln^2 x)} dx$$

câu 8

$$\text{xét } f(x) = \sqrt[3]{x} \Rightarrow f'(x) = \frac{1}{3}(x)^{-\frac{2}{3}}$$

$$\text{với } x_0 = 1 \quad \Delta x = 0.02$$

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0) \Delta x = 1 + \frac{1}{3}(1) * 0.02 = 1 + 0.02/3$$

câu 9

$$\int \frac{dx}{\sqrt{1-x^2} \arccos^2 x}$$

$$\text{đặt } t = \arccos x$$

$$dt = -1/(\sqrt{1-x^2}) dx$$

$$\Rightarrow \int -\frac{dt}{t^2} = 1/t + c = 1/\arccos x + c$$

câu 10

tính  $y^2 = 4x$  và  $x - y - 1 = 0$  xem lại đề đã .

ta có pt tung độ giao điểm :

$$\frac{y^2}{4} - y - 1 = 0$$

$$I = \int_{2-\sqrt{2}}^{2+\sqrt{2}} -\left(\frac{y^2}{4} - y - 1\right) dy$$

câu 11

$$A. \int_{-\infty}^0 \frac{e^x + 1}{e^x} dx = \lim_{b \rightarrow -\infty} \int_b^0 (1 + e^{-x}) dx = \lim_{b \rightarrow -\infty} (0 - e^{-b}) \Big|_b^0 = -\infty \text{ PK}$$

$$B. \int_0^{\infty} e^x dx = \lim_{b \rightarrow \infty} \int_0^b e^x dx = \lim_{b \rightarrow \infty} (e^b - e^0) = +\infty \text{ PK}$$

C. Tích phân suy rộng loại 2 không thi bỏ

$\Rightarrow D$

câu 12

ta có

$$U_1 = \frac{1}{3} = \frac{1}{1.3} = \frac{1}{2} (1 - 1/3)$$

$$U_1 = \frac{1}{8} = \frac{1}{2.4} = \frac{1}{2} (1/2 - 1/4)$$

$$U_3 = \frac{1}{15} = \frac{1}{5.3} = \frac{1}{2} (1/3 - 1/5)$$

$$U_n = \frac{1}{n.(n+2)} = 1/2 (1/n - 1/(n+2))$$

ta có  $S_n = U_1 + U_2 + U_3 + \dots + U_n$

$$S_n = \frac{1}{2} \left( 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \dots \dots + \frac{1}{n} - \frac{1}{n+2} \right)$$

$$\lim S_n = \frac{1}{2} * \left( 1 + \frac{1}{2} \right) = 3/4$$

câu 13

$$A \sum_{n=1}^{\infty} \frac{e^n n!}{n^n}$$

$$\text{xét } \lim_{n \rightarrow \infty} \frac{U(n+1)}{U_n} = \lim_{n \rightarrow \infty} \frac{e^{n+1}(n+1)!}{(n+1)^{n+1}} * \frac{n^n}{e^n n!} = \lim_{n \rightarrow \infty} e \cdot \frac{n+1}{n+1} = e > 1 \text{ PK}$$

$$B. \sum_{n=1}^{\infty} (-1)^n \left( \frac{n}{n-1} \right)^n$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{(-1)^n \left( \frac{n}{n-1} \right)^n} = \lim_{n \rightarrow \infty} - \frac{n}{n-1} = -1 < 1 \text{ HT}$$

câu 14

$$\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{2^n}}$$

$$\text{xét } \lim_{n \rightarrow \infty} \frac{U(n+1)}{U_n} = \frac{(n+1)^3}{2^{\left(\frac{1}{2}\right)(n+1)}} \frac{2^{n/2}}{n^3} = \frac{1}{\sqrt{2}} < 1 \text{ HT}$$

câu B đã làm HT vì có  $S_n = 3/4$

câu C

$$\sum_{n=1}^{\infty} \frac{\sqrt[n]{3} + 1}{n\sqrt{n} - 1}$$

$$\text{xét } \lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} \frac{3^{1/n} + 1}{n\sqrt{n} - 1} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} \ln 3 + 2}{n\sqrt{n} - 1} = \lim_{n \rightarrow \infty} \frac{-\frac{1}{n^2} \ln 3}{3/2\sqrt{n}} = 0 \text{ HT}$$

=> câu | D PK



câu 15  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} (2x)^n$

$$\rho = \lim_{n \rightarrow \infty} \frac{U(n+1)}{U_n} = \lim_{n \rightarrow \infty} \frac{((n+1)!)^2 (2n)!}{(2n+2)!(n!)^2} = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = 1/4$$

$$R=1/\rho=4$$

đề 3

câu 1

$$\lim_{x \rightarrow 0^-} \frac{2^x - \cos 2x}{x}$$

$$\lim_{x \rightarrow 0^-} \frac{\ln 2 \cdot 2^x + \sin 2x}{1} = \ln 2$$

câu 2

$$\lim_{x \rightarrow \infty} (1 + e^x)^{\frac{1}{x^2}} = e^{\lim_{x \rightarrow \infty} (1+e^x-1) \frac{1}{x^2}} = e^{\lim_{x \rightarrow \infty} \frac{e^x}{2x}} = e^{\lim_{x \rightarrow \infty} \frac{e^x}{2}} = +\infty$$

câu 3

$$\lim_{x \rightarrow \infty} \frac{x^2-2}{x^4-x^2-2} = \lim_{x \rightarrow \infty} \frac{x^2-2}{(x^2-2)(x^2+1)} = \frac{1}{\infty} = 0$$

câu 4

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \arctan \frac{1}{x^2-4x+4} = \lim_{x \rightarrow 2^-} \arctan \frac{1}{(x-2)^2} = \frac{\pi}{2}$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{x^2 - a^2}{x^2 + 1} = \frac{4 - a^2}{5}$$

ta có  $\frac{4 - a^2}{5} = \frac{\pi}{2} \Rightarrow$  đáp án khác.

câu 5  $y = \arccos x$

$$dy = y' dx = \frac{-1}{\sqrt{1-x^2}} dx \Rightarrow dy(1/2) = -2/\sqrt{3}$$

câu 6

vi phân  $y = (4x)^x$

$$\ln y = x \ln 4x$$

$$y'/y = \ln 4x + 1$$

$$y' = (\ln 4x + 1)(4x)^x$$

câu 7 ko có đề

câu 8

$$\begin{cases} x = 2e^t \Rightarrow x = 2 \Rightarrow t = 0 \\ y = 1 + t^2 \end{cases}$$

$$y'(x) = \frac{Y'(t)}{x'(t)} = \frac{2t}{2e^t} y'(t=0) = 0$$

câu 9

$$\int \frac{2e^x}{\sqrt{2+2e^x+e^{2x}}} dx = \int \frac{2e^x}{\sqrt{1+(e^x+1)^2}} dx$$

$$\text{đặt } t = e^x + 1 \Rightarrow dt = e^x dx$$

$$\int \frac{2dt}{\sqrt{1+t^2}} = 2\arcsin t + c = 2\arcsin(e^x + 1) + C$$

câu 10

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{\cos x \sin x} = \int_{\pi/4}^{3\pi/4} \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} dx = \int_{\pi/4}^{3\pi/4} (\tan x + \cot x) dx = -\ln(\cos x) + \ln(\sin x) \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \ln(\tan x) \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

câu 11

$$\int_0^{+\infty} \frac{dx}{(1+x)^2}$$

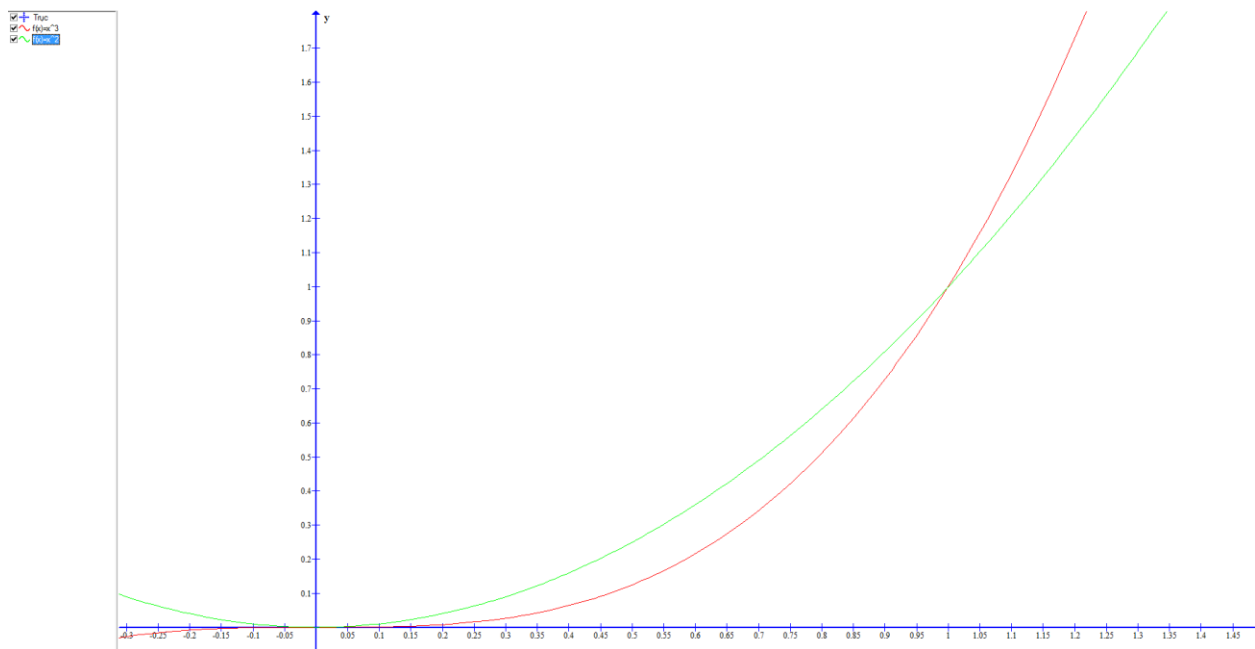
$$\lim_{b \rightarrow +\infty} \int_0^b \frac{dx}{(1+x)^2} = \lim_{b \rightarrow +\infty} \left( -\frac{1}{1+b} + \frac{1}{1+0} \right) = 1 \Rightarrow \text{HT}$$

câu 12

pt hoành độ giao điểm  $y = x^3$  và  $y = x^2$

có 2 nghiệm là  $x=0$  và  $x=1$

$$\int_0^1 (x^2 - x^3) dx = 1/12$$



câu 13

$$V = \pi \int_{\frac{3}{4}}^2 x dx = \pi \left( \frac{2^2}{2} - \frac{9}{8} \right)$$

câu 14

$$\sum_{n=0}^{\infty} \frac{2 + (-1)^n}{3^n}$$

$$S_n = 2 \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{\left(-\frac{1}{3}\right)^0}{1 - \frac{1}{3}} = \frac{15}{4}$$

Câu 15

$$A \sum_{n=1}^{\infty} \frac{n+1}{n(\sqrt{n}+1)}$$

$$\text{xét lim } U_n = \lim_{n \rightarrow \infty} \frac{n+1}{n(\sqrt{n}+1)} = \frac{1}{\sqrt{n}} \text{ do } \alpha = 1/2 < 1 \Rightarrow \text{PK}$$