Chapter 4

Oscillators

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Chapter 4

Oscillators

Textbook:

Steven J. Franke, *Wireless Communication Systems*, UIUC
Chapter 5

Outline

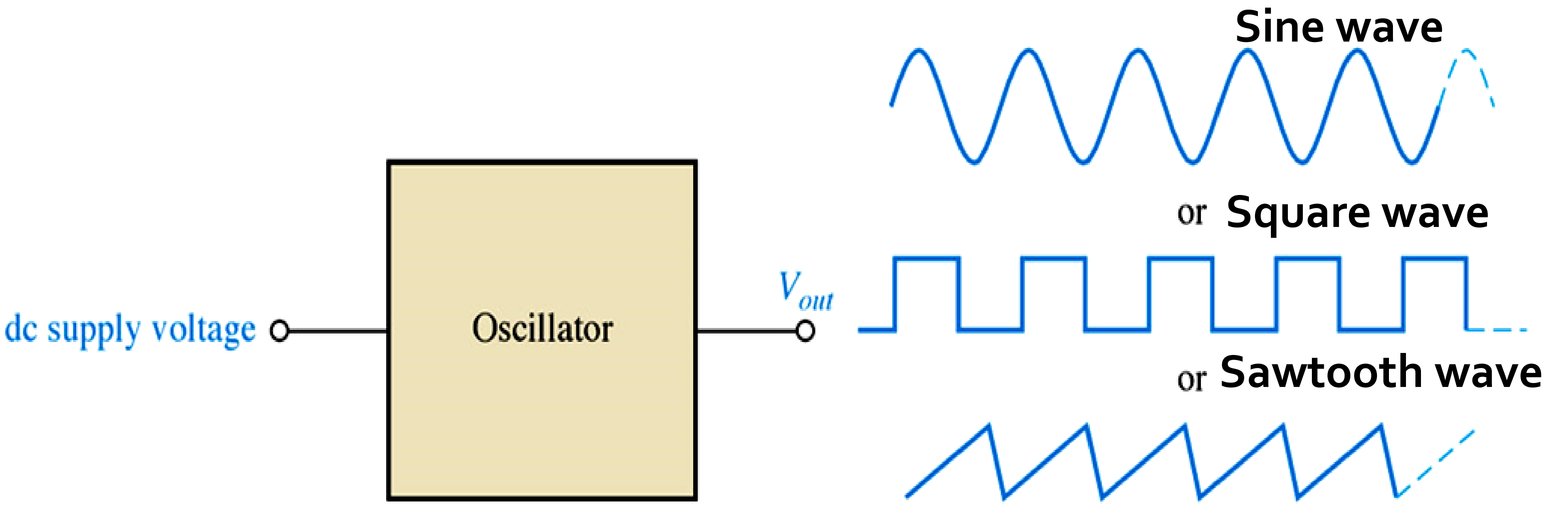
1. Introduction
2. Conditions for Oscillation – Loop Gain
3. Wien-bridge Oscillator
4. Phase-Shift Oscillator
5. Colpitts and Hartley – Oscillators
6. Crystal Oscillator (Reading)
7. Voltage Control Oscillators - VCO

1. Introduction

- Oscillator is an electronic circuit that generates a periodic waveform on its output without an external signal source. It is used to convert dc to ac.
- Oscillators are circuits that produce a continuous signal of some type without the need of an input.
- Oscillators are used in a number of applications in which a reference tone is required. For instance, they can be used as the clock for digital circuits or as the source of the LO signal in transmitters.
- In receivers, oscillator waveforms are used as the reference frequency to mix down the received RF to an IF or to baseband. In most RF applications, sinusoidal references with a high degree of spectral purity (low phase noise) are required.
- Communications systems, digital systems (including computers), and test equipment make use of oscillators

1. Introduction

- An oscillator is a circuit that produces a repetitive signal from a dc voltage.
- The feedback oscillator relies on a positive feedback of the output to maintain the oscillations.
- The relaxation oscillator makes use of an RC timing circuit to generate a nonsinusoidal signal such as square wave

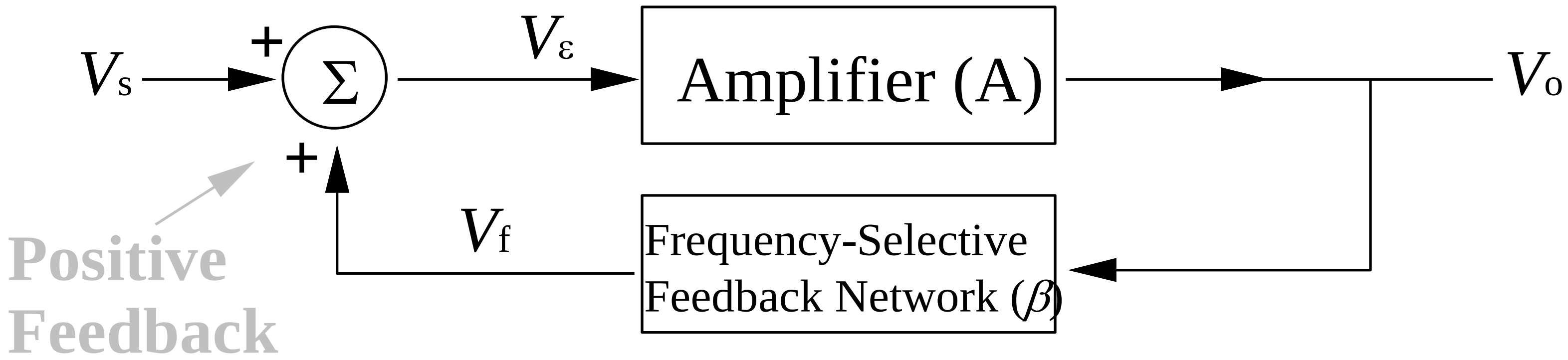


Types of oscillators

1. RC oscillators
   - Wien Bridge
   - Phase-Shift
2. LC oscillators
   - Hartley
   - Colpitts
   - Crystal
3. LC Crossed-couple Oscillator

2. Conditions for Oscillation – Loop Gain

- An oscillator is an amplifier with positive feedback

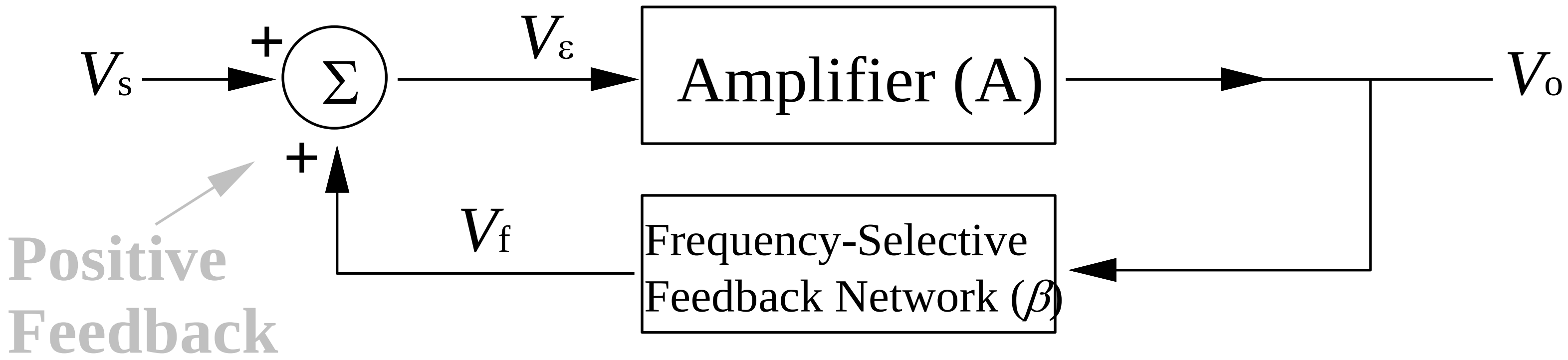


A linear oscillator contains:

- a frequency selection feedback network

- an amplifier to maintain the loop gain at **unity**

- An oscillator is an amplifier with positive feedback



$$V_e = V_s + V_f \quad (1) \qquad V_f = \beta V_o \quad (2)$$

$$V_o = AV_e = A\left(V_s + V_f\right) = A\left(V_s + \beta V_o\right) \quad (3)$$

2. Conditions for Oscillation – Loop Gain

- In general A and β are functions of frequency and thus may be written as;

$$A_f(s) = \frac{V_o}{V_s}(s) = \frac{A(s)}{1 - A(s)\beta(s)}$$

$A(s)\beta(s)$ is known as loop gain

- Replacing *s* with $j\omega$

$$A_f(j\omega) = \frac{A(j\omega)}{1 - T(j\omega)}$$

and

$$T(j\omega) = A(j\omega)\beta(j\omega)$$

2. Conditions for Oscillation – Loop Gain

- At a specific frequency f_0

$$T(j\omega_0) = A(j\omega_0)\beta(j\omega_0) = 1$$

- At this frequency, the closed loop gain;

$$A_f(j\omega_0) = \frac{A(j\omega_0)}{1 - A(j\omega_0)\beta(j\omega_0)}$$

will be infinite, i.e. the circuit will have finite output for zero input signal - oscillation

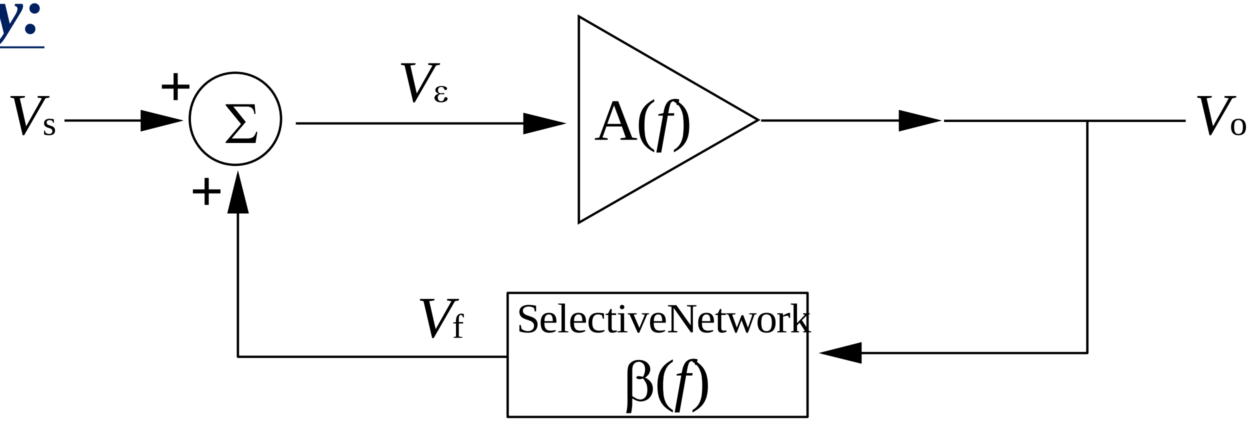
2. Conditions for Oscillation – Loop Gain

- Thus, the condition for sinusoidal oscillation of frequency f_0 is:

$$A(j\omega_0)\beta(j\omega_0) = 1$$

- This is known as Barkhausen criterion.
- The frequency of oscillation is solely determined by the phase characteristic of the feedback loop – the loop oscillates at the frequency for which the phase is zero.

Summary:



$$V_o = AV_\varepsilon = A(V_s + V_f) \quad \text{and} \quad V_f = \beta V_o$$

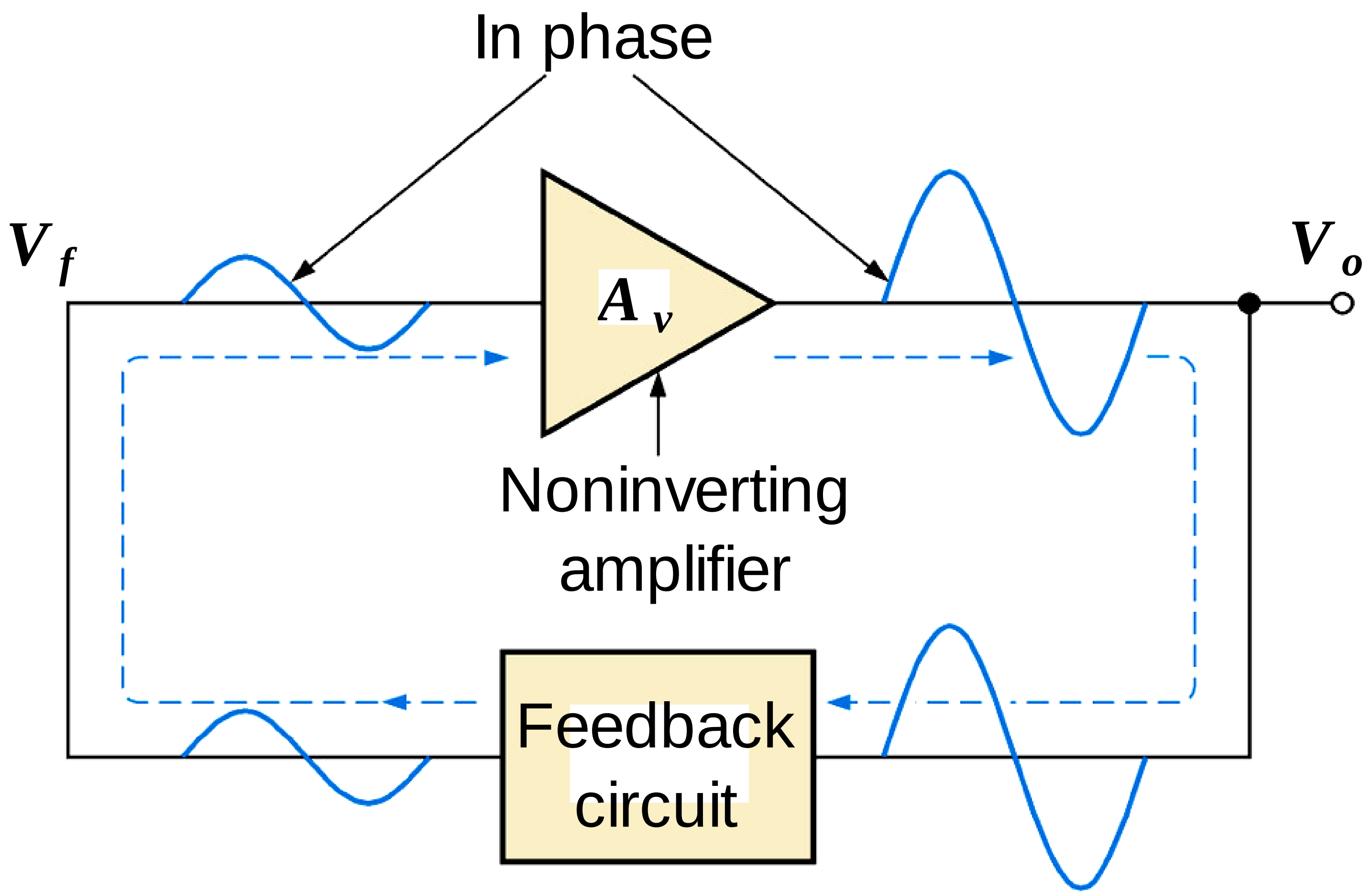
$$\Rightarrow \frac{V_o}{V_s} = \frac{A}{1 - A\beta}$$

If $V_s = 0$, the only way that V_o can be nonzero is that loop gain $A\beta = 1$ which implies that

$$|A\beta| = 1$$

$$\angle A\beta = 0$$

(Barkhausen Criterion)


In phase
V f
V o
A v
Noninverting
amplifier
Feedback
circuit

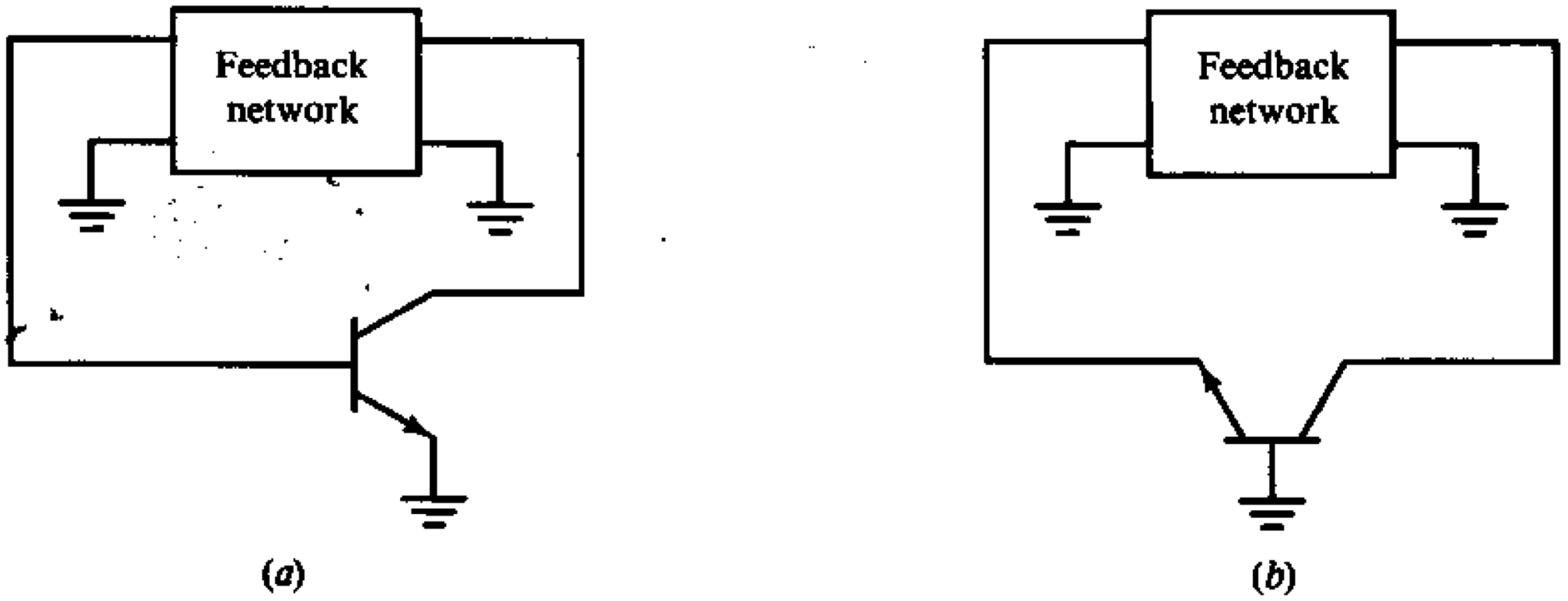
Design Criteria for Oscillators

1. The magnitude of the loop gain must be unity or slightly larger

2. Total phase shift, ϕ of the loop gain must be Nx360° where N=0, 1, 2, …

$$|A\beta| = 1$$

2. Conditions for Oscillation – Loop Gain

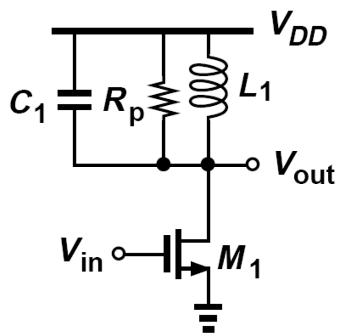


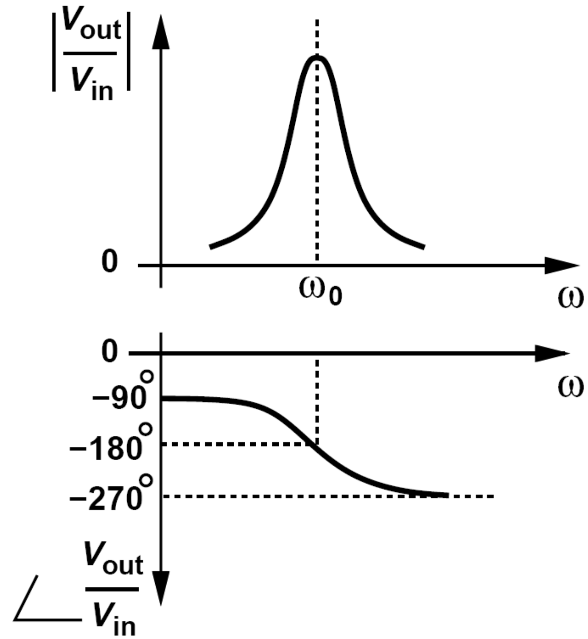
(*a*) A system in which the feedback network must provide 180° phase shift in order for oscillations to occur; (*b*) a system in which the feedback network must provide 0° phase shift for oscillations to occur.

2. Conditions for Oscillation – Loop Gain

Example:

- We wish to build a negative-feedback oscillatory system using "LC-tuned" amplifier stages.





At very low frequencies, L_1 dominates the load and

$$\frac{V_{out}}{V_{in}} \approx -g_m L_1 s$$

$|V_{out}/V_{in}|$ is very small and $\angle(V_{out}/V_{in})$ remains around -90°

At the resonance frequency

$$\frac{V_{out}}{V_{in}} = -g_m R_p$$

The phase shift from the input to the output is thus equal to 180°

At very high frequencies

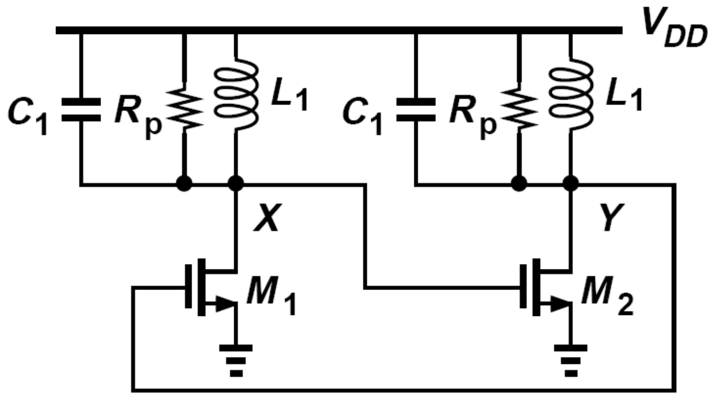
$$\frac{V_{out}}{V_{in}} \approx -g_m \frac{1}{C_1 s}$$

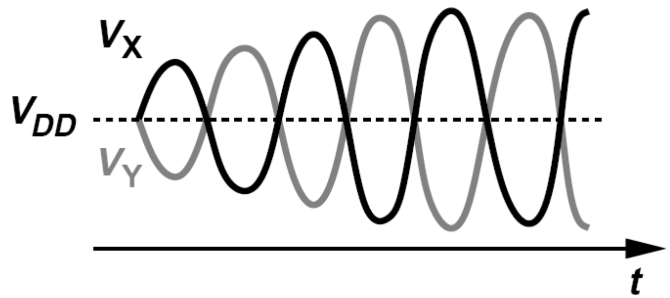
$|V_{out}/V_{in}|$ dinimishes $\angle(V_{out}/V_{in})$ approaches +90°

Can the circuit above oscillate if its input and output are shorted? – No--

2. Conditions for Oscillation – Loop Gain

- **We recognize that the circuit provides a phase shift of 180° with possibly adequate gain (g_mR_p) at ω_0. We simply need to increase the phase shift to 360°.**





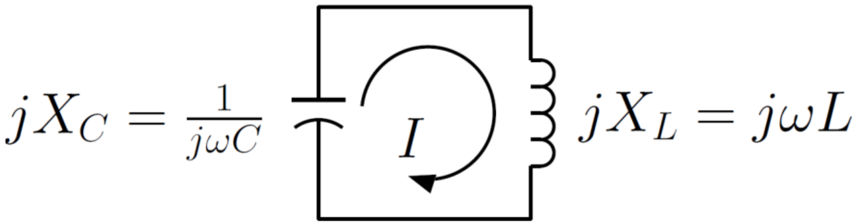
$$(g_m R_p)^2 \geq 1$$

Assuming that the circuit above (left) oscillates, plot the voltage waveforms at *X* and *Y*.

- Wave form is shown above (right). A unique attribute of inductive loads is that they can provide peak voltages above the supply. The growth of V_X and V_Y ceases when M_1 and M_2 enter the triode region for part of the period, reducing the loop gain.

2. Conditions for Oscillation – Negative Resistance

- An alternative perspective views oscillators as two one-port components, namely, a lossy resonator and an active circuit that cancels the loss.
- Consider the lossless LC circuit:

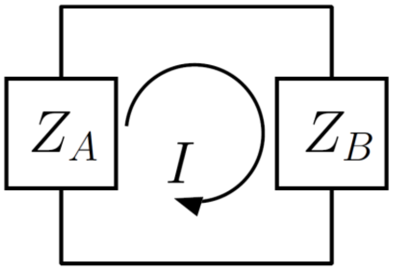


$$I(jXC + jX_L) = 0$$

- Finite current I is allowed if $X_L + X_C = 0$, which occurs at the resonant frequency:

$$\omega_o = \frac{1}{\sqrt{LC}}$$

- Generalizing this idea to a circuit consisting of arbitrary impedances Z_A and Z_B , then finite current, I, is supported if $Z_A + Z_B = 0$.



Resonance condition:

$$\mathbf{Z_A + Z_B = 0}$$

$$R_A + R_B = 0$$

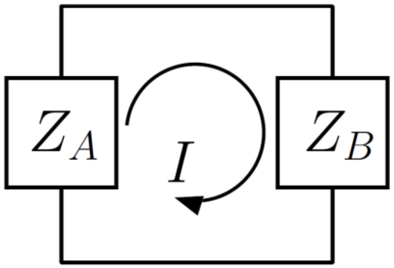
$$X_A + X_B = 0$$

2. Conditions for Oscillation – Negative Resistance

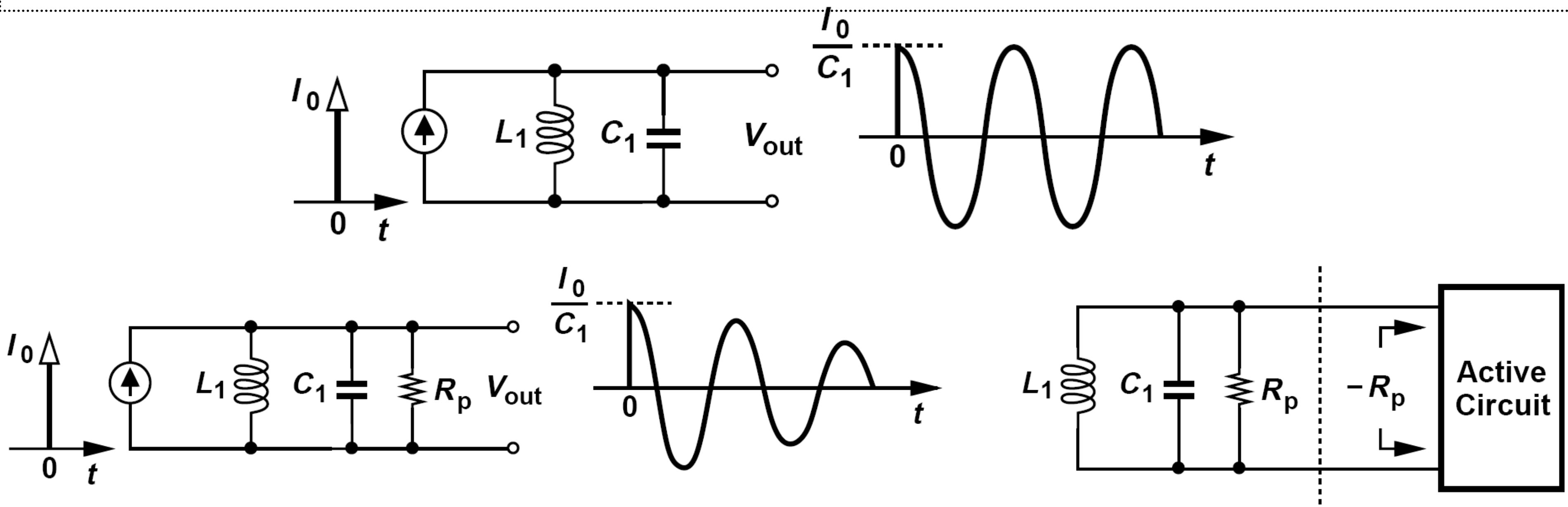
- In practice the negative resistance concept is applied by breaking a candidate oscillator circuit into two parts, which are associated with the impedances Z_A and Z_B.
- Most oscillators consist of a single active device, so when the circuit is divided into two parts, one part will contain only passive components. This part will exhibit an impedance with positive real part.
- In order for oscillations to occur in the circuit, the part containing the active device must have an impedance with a negative real part.

Resonance condition:

$$\mathbf{Z_A + Z_B = 0}$$

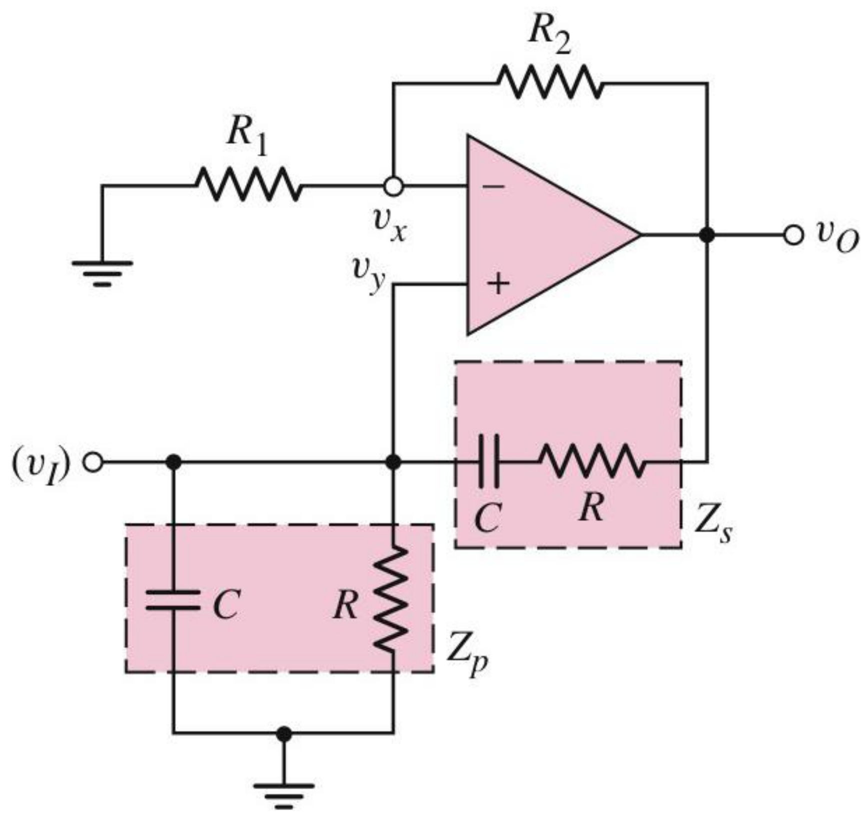


- Under this view, an oscillator is considered as two one-port components, namely, a lossy resonator and an active circuit (with negative input resistance) that cancels the loss.



- If an active circuit replenishes the energy lost in each period, then the oscillation can be sustained.
- In fact, we predict that an active circuit exhibiting an input resistance of -R_p can be attached across the tank to cancel the effect of R_p.

- It is a low frequency oscillator which ranges from a few kHz to 1 MHz.



3. Wien-bridge Oscillator

- The loop gain for the oscillator is;

$$T(s) = A(s)\beta(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{Z_p}{Z_p + Z_s}\right)$$

- where;

$$Z_p = \frac{R}{1 + sRC}$$

- and;

$$Z_s = \frac{1 + sRC}{sC}$$

3. Wien-bridge Oscillator

- Hence; $$\boldsymbol{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{1}{3 + sRC + (1/sRC)}\right]}$$

- Substituting for *s*;

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{1}{3 + j\omega RC + (1/j\omega RC)}\right]$$

- For oscillation frequency f_0;

$$T(j\omega_0) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{1}{3 + j\omega_0 RC + (1/j\omega_0 RC)}\right]$$

Wien-bridge Oscillator

- Since at the frequency of oscillation, $T(j\omega)$ must be real (for zero phase condition), the imaginary component must be zero;

$$j\omega_0 RC + \frac{1}{j\omega_0 RC} = 0$$

- Which gives us;

$$\omega_0 = \frac{1}{RC}$$

3. Wien-bridge Oscillator

- From the previous equation;

$$T(j\omega_0) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{1}{3 + j\omega_0 RC + (1/j\omega_0 RC)}\right]$$

- the magnitude condition is:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3}\right)$$

or $\frac{R_2}{R_1} = 2$

To ensure oscillation, the ratio R_2/R_1 must be slightly greater than 2.

3. Wien-bridge Oscillator

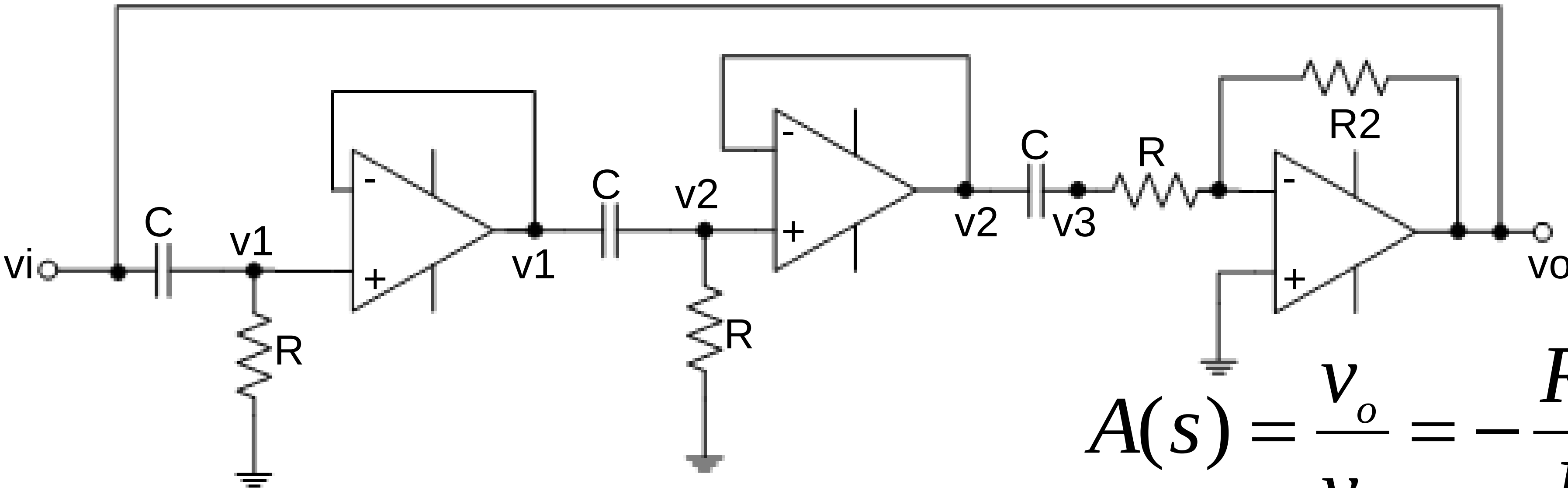
- With the ratio; $$\frac{R_2}{R_1} = 2$$

- then; $$K \equiv 1 + \frac{R_2}{R_1} = 3$$

$K = 3$ ensures the loop gain of unity – oscillation

- $K > 3$: growing oscillations
- $K < 3$: decreasing oscillations

Phase-Shift Oscillator



$$A(s) = \frac{v_o}{v_3} = -\frac{R_2}{R}$$

$$v_1 = \left(\frac{sRC}{1+sRC}\right)v_i \qquad v_3 = \left(\frac{sRC}{1+sRC}\right)^3 v_i$$

$$v_2 = \left(\frac{sRC}{1+sRC}\right)^2 v_i \qquad \frac{v_3}{v_i} = \beta(s) = \left(\frac{sRC}{1+sRC}\right)^3$$

4. Phase-Shift Oscillator

- Loop gain, T(s):

$$T(s) = A(s)\beta(s) = \left(-\frac{R_2}{R}\right)\left(\frac{sRC}{1+sRC}\right)^3$$

- Set s=jw

$$T(j\omega) = \left(-\frac{R_2}{R}\right)\left(\frac{j\omega RC}{1+j\omega RC}\right)^3$$

$$T(j\omega) = \left(\frac{R_2}{R}\right)\frac{(j\omega RC)(\omega RC)^2}{\left[1-3\omega^2R^2C^2\right]+j\omega RC\left[3-\omega^2R^2C^2\right]}$$

Phase-Shift Oscillator

- To satisfy condition $T(jw_o)=1$, real component must be zero since the numerator is purely imaginary.

$$1-3\omega^2R^2C^2=0$$

- the oscillation frequency:

- Apply w_o in equation:

$$\omega_0=\frac{1}{\sqrt{3}RC}$$

- To satisfy condition $T(jw_o)=1$

$$T(j\omega_o)=\left(\frac{R_2}{R}\right)\frac{(j/\sqrt{3})(1/3)}{0+(j/\sqrt{3})\left[3-(1/3)\right]}=\left(\frac{R_2}{R}\right)\left(\frac{1}{8}\right)$$

$$\frac{R_2}{R}=8$$

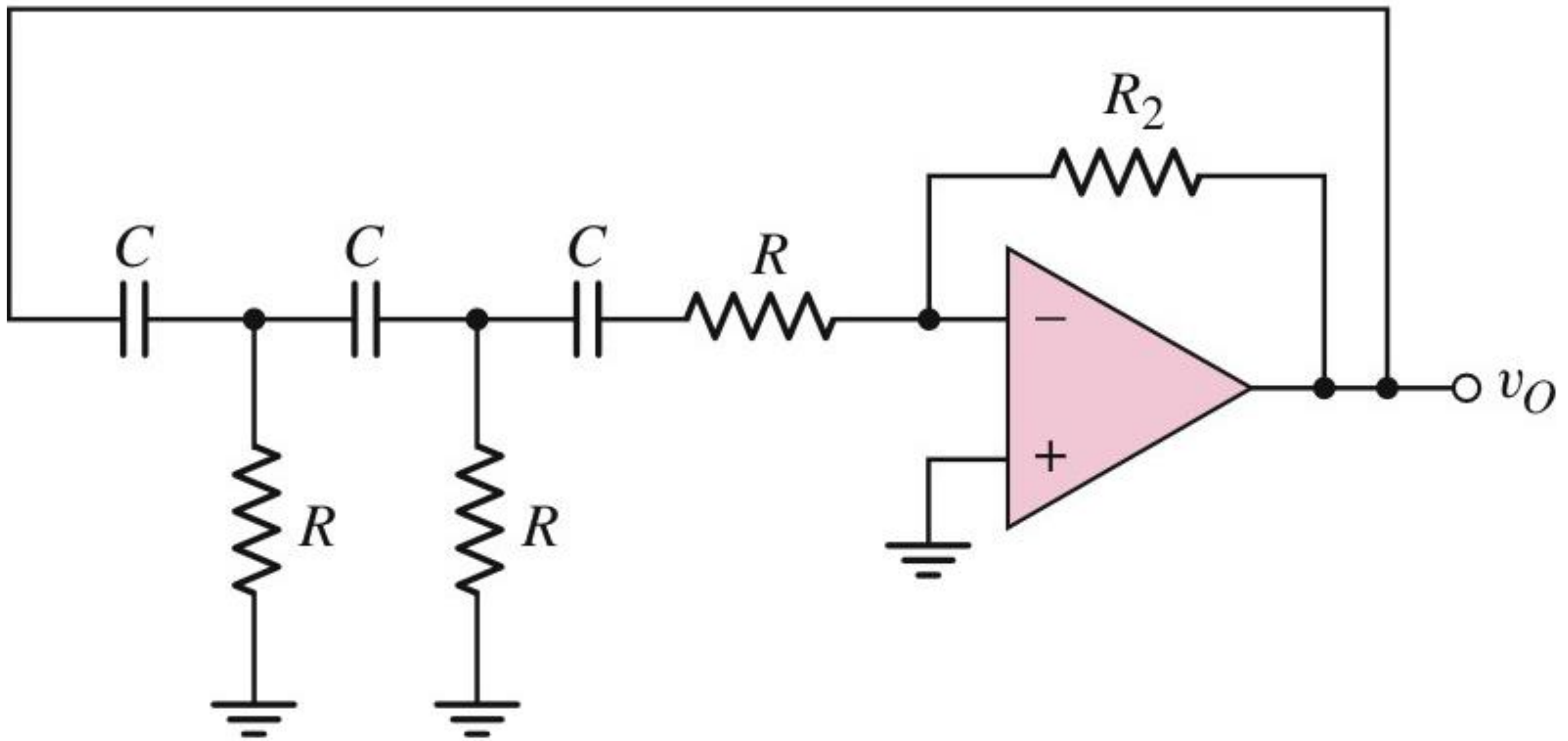
The gain greater than 8, the circuit will spontaneously begin oscillating & sustain oscillations

4. Phase-Shift Oscillator

- The phase shift oscillator utilizes three RC circuits to provide 180º phase shift that when coupled with the 180º of the op-amp itself provides the necessary feedback to sustain oscillations.
- The gain must be at least 29 to maintain the oscillations.
- The frequency of resonance for the this type is similar to any RC circuit oscillator:

$$f_r = \frac{1}{2\pi\sqrt{6}RC}$$

Phase-Shift Oscillator



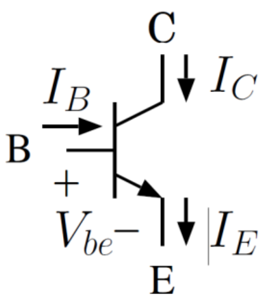
$$f_o = \frac{1}{2\pi\sqrt{6}RC}$$

$$\frac{R_2}{R} = 29$$

The gain must be at least 29 to maintain the oscillations

Circuit Models for BJT and FET

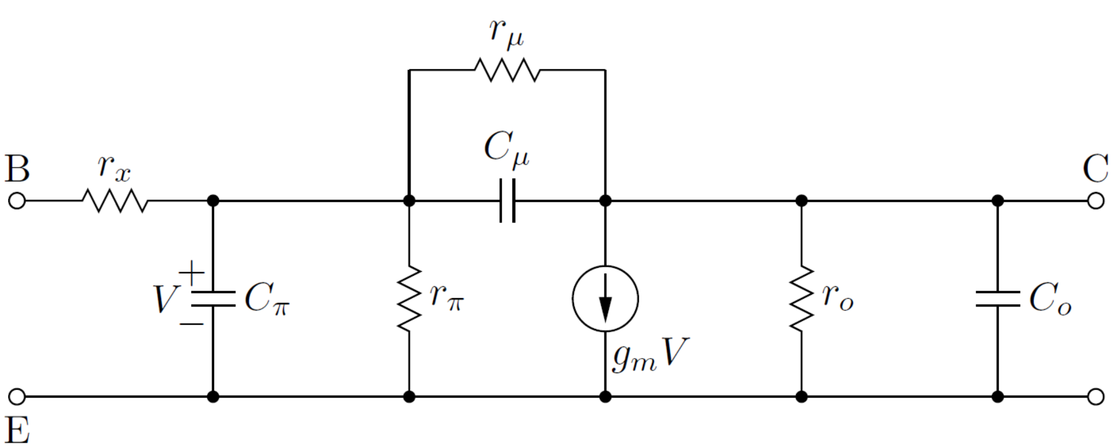
Hybrid-pi equivalent circuit for bipolar junction transistor (BJT)



$$I_C = I_S \exp[\frac{V_{be}}{V_T}] \qquad I_b = \frac{I_C}{\beta}$$

$$V_T = \frac{kT}{q} \simeq 25\text{mV at room temperature}$$

$$r_\pi = \frac{V_T\beta}{I_{CQ}} \qquad g_m = \frac{I_{CQ}}{V_T}$$



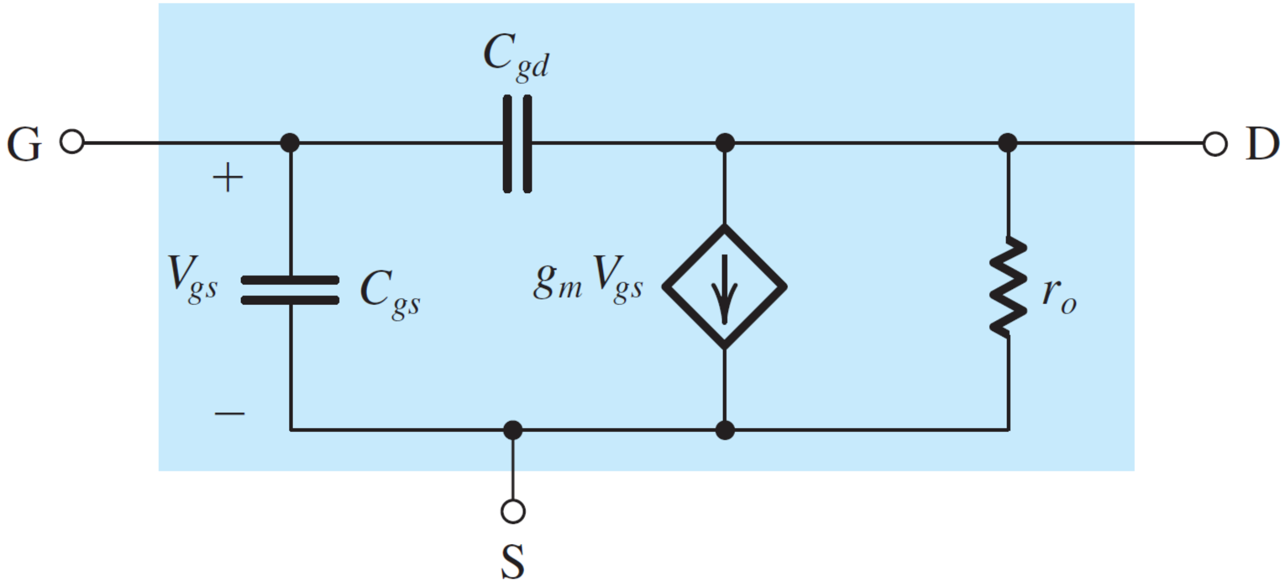
Hybrid-pi small-signal model for BJT

Cut-off Frequency

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_\pi + C_\mu}$$

$$f_\beta = \frac{1}{2\pi}\frac{1}{r_\pi(C_\pi + C_\mu)}$$

❑ Hybrid-pi equivalent circuit for MOSFET transistor



Hybrid-pi small-signal model for MOSFET

$$g_m = \mu_n C_{ox} \frac{W}{L} |V_{OV}| = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{|V_{OV}|}$$

Cut-off Frequency

$$f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$

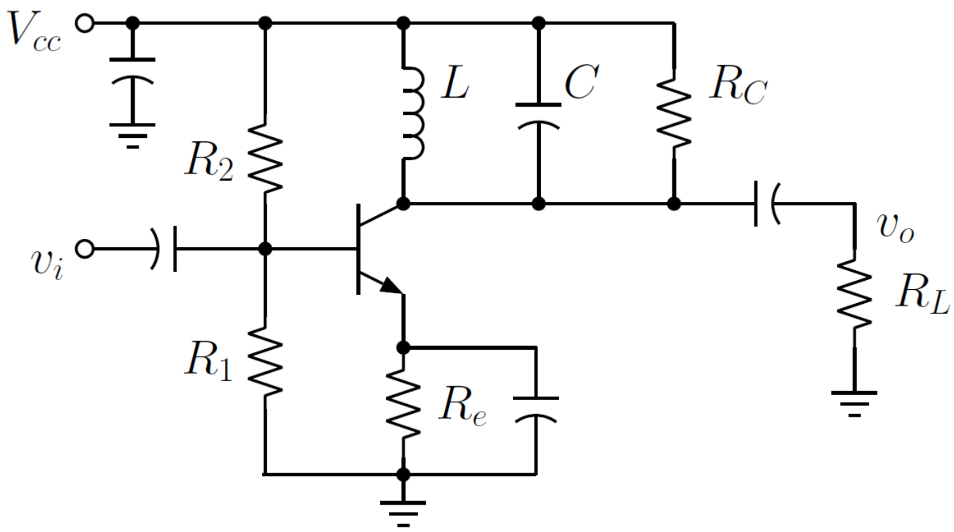
Example: Common-emitter amplifier with tuned output

Consider the voltage amplifier in Figure 5.23, with

V_{cc}=12 V $\quad R_1 = 10\,\text{k}\Omega \quad R_2 = 30\,\text{k}\Omega \quad R_e = 1\,\text{k}\Omega$

$R_C = \infty \quad R_L = 1\,\text{k}\Omega \quad L = 2\,\mu\text{H} \quad C = 50\,\text{pF}$

Capacitors that are not labeled are assumed to be "short circuits" over the frequency range of interest. The transistor's β is large enough so that the bias point does not explicitly depend on its value. You may neglect the transistor parameters r_x, r_μ, C_μ, r_o, and C_o in your analysis for parts 1b, 1c, and 1d.



(a) Find the quiescent collector current, I_{CQ}. Express your result in mA.

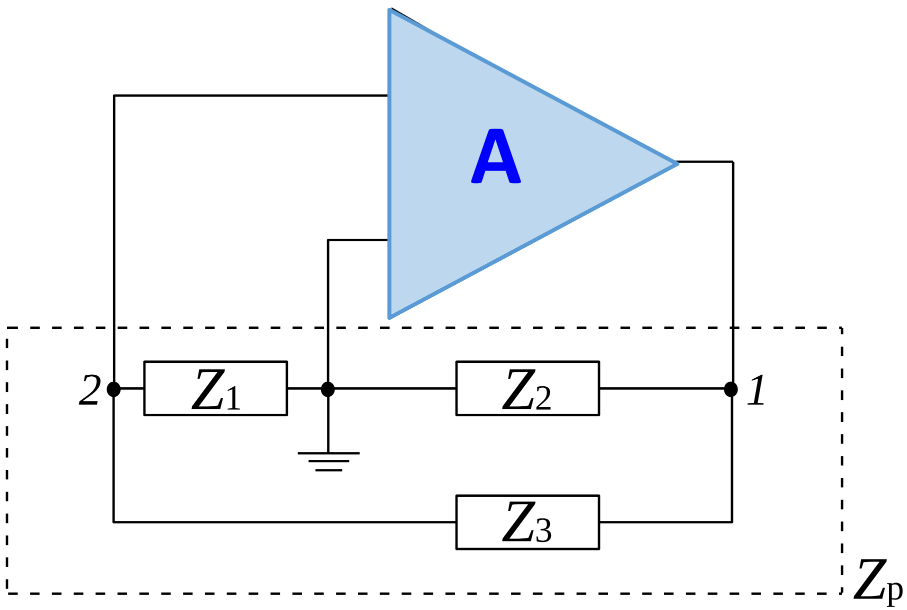
(b) Find the resonant frequency of the amplifier. The voltage gain will be largest at this frequency. Express your result in MHz.

(c) Find the voltage gain at resonance.

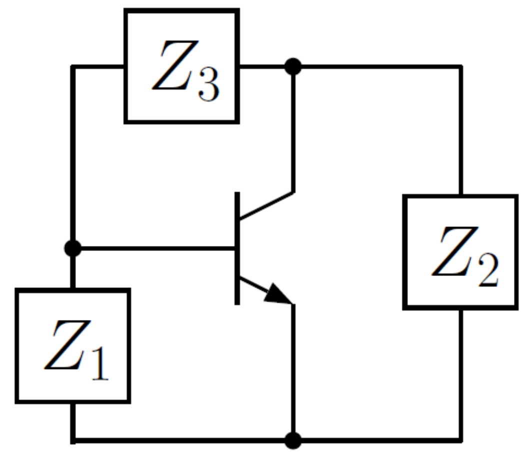
(d) Find the 3 dB bandwidth of the amplifier. Express your result in MHz.

5. Colpitts and Hartley – Oscillators

- RF oscillators are usually made using one amplifier/transistor accompanying with a feedback network consisting of 3 elements Z_1, Z_2, Z_3.

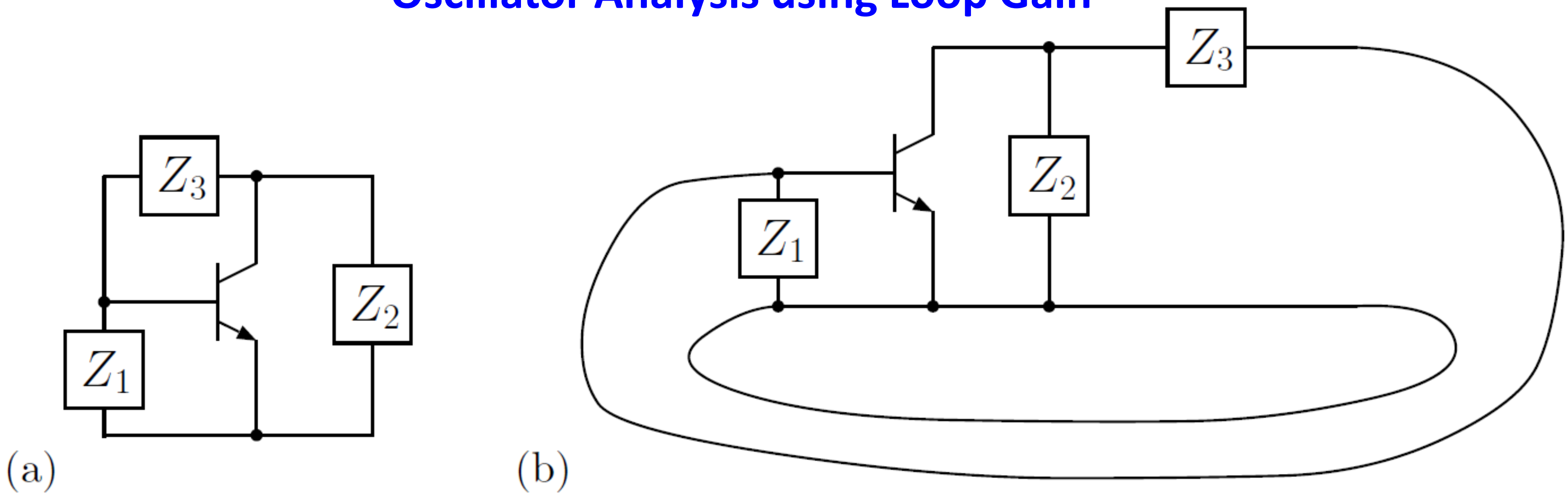


- The feedback network (Z_1, Z_2 and Z_3) provides a phase shift of 180^o
- The amplifier provides an addition shift of 180^o
- The amplifier is realized using a transistor (BJT or FET)
- The ground of the oscillator can be any at B, C or E terminal
- These oscillators can be analysed using Loop-Gain or Negative-Resistance methods.

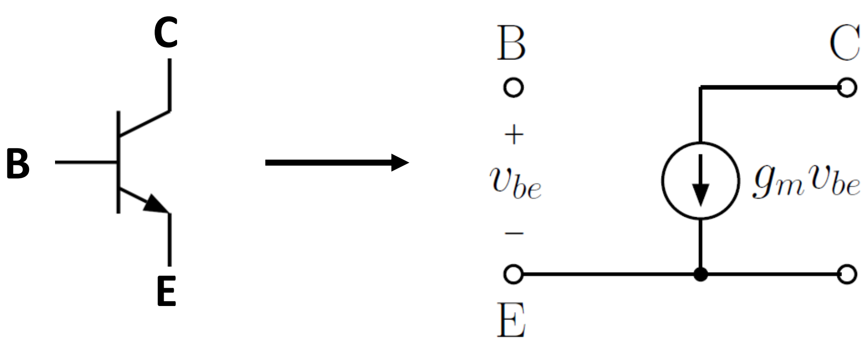


- Two well-known Oscillators:
  - Colpitts Oscillator
  - Harley Oscillator

Oscillator Analysis using Loop Gain



(a) Topology of one class of oscillator circuits. (b) Same as (a), redrawn to show the feedback path from output to input through Z_3.



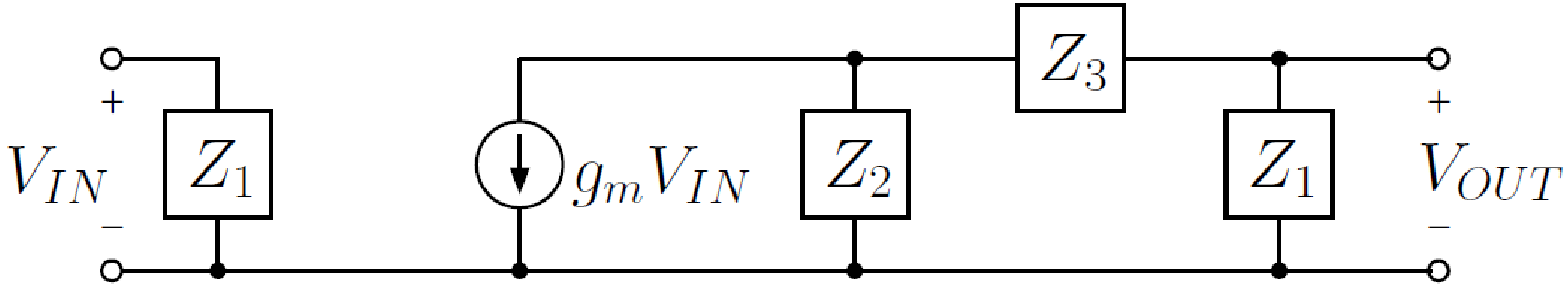
Simplified hybrid-pi model of transistor

Note: For small-signal analysis we can model the transistor as shown in this figure. Note that this is a simplified version of the hybrid-pi model for the transistor. The passive elements of the model (e.g., r_π, C_π, C_μ, etc.) can be lumped into the external impedances Z_1, Z_2, Z_3.

To find the open loop gain of this circuit, we break the loop at a convenient point and terminate that point in the impedance that it sees when the loop is closed.

Oscillator Analysis using Loop Gain

- Since the output of Z_3 normally looks into Z_1 when the loop is closed, we terminate the loop with Z_1 as shown:



- Compute the loop gain:

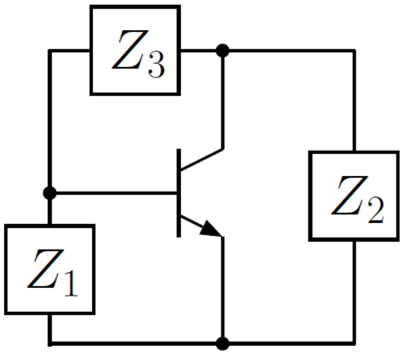
$$A_{lo} = \frac{V_{OUT}}{V_{IN}} = \frac{-g_m Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

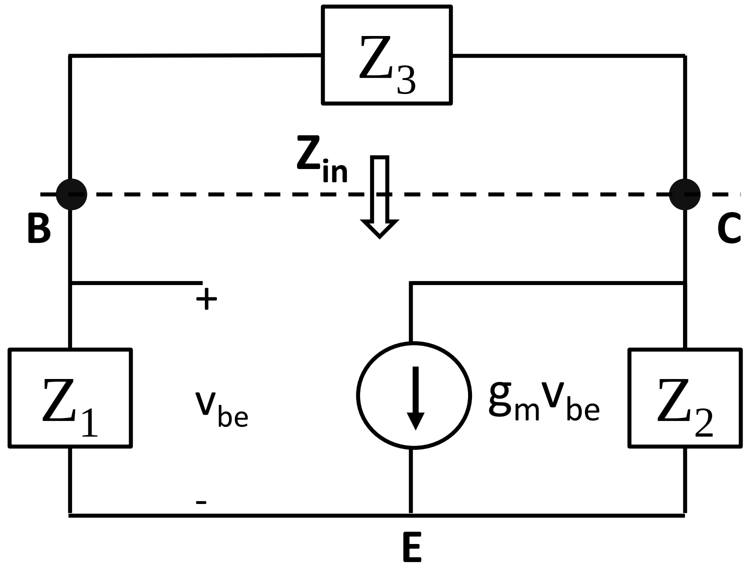
- **Oscillation condition:**

$$A_{lo} = 1$$

$$\boxed{\mathbf{Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0}}$$

Oscillator Analysis using Negative Resistance





$$Z_{in} = Z_1 + Z_2 + g_m Z_1 Z_2$$

- **Oscillation condition:** $$Z_3 + Z_{in} = 0$$

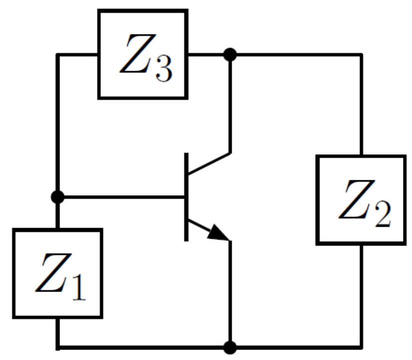
which leads to

$$\boxed{\mathbf{Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0}}$$

- It is the same as the condition resulting from the loop gain method !

Colpitts, Hartley Oscillatior Analysis

- Oscillation condition:



$$\boxed{A_{lo} = \frac{V_{OUT}}{V_{IN}} = \frac{-g_m Z_1 Z_2}{Z_1 + Z_2 + Z_3}}$$

$$\boxed{\mathbf{Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0}}$$

- Some useful insights can be gained if we assume for this moment that:

  Z_1 and Z_2 are purely reactive, i.e., $Z_1 = jX_1$, $Z_2 = jX_2$.

  We allow Z_3 to have a non-zero (positive) real part ($Z_3 = R + jX_3$).

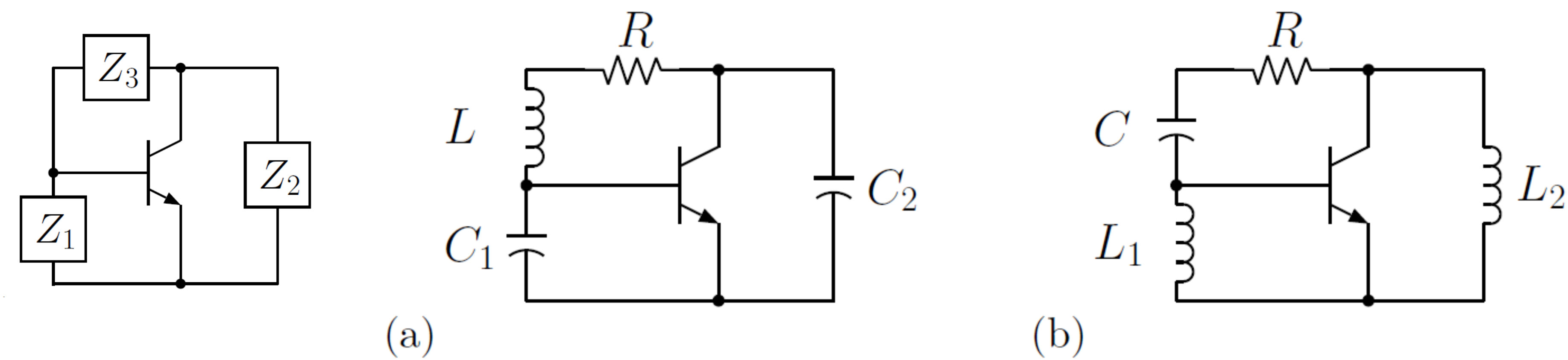
Then:

$$\mathbf{R + j(X_1 + X_2 + X_3) - g_m X_1 X_2 = 0}$$

$$\mathbf{X_1 + X_2 + X_3 = 0}$$
$$\mathbf{R - g_m X_1 X_2 = 1}$$

Colpitts, Hartley Oscillatior Analysis

- For oscillation to occur: $$A_{lo} = \frac{g_m X_1 X_2}{R + j(X_1 + X_2 + X_3)} = 1$$
  - $|A_{lo}| = 1$
  - The phase angle of Alo must be zero at some frequency
- It indicates that:
  - $X_1 + X_2 + X_3 = 0$ and $\frac{g_m X_1 X_2}{R} = 1$
  - At least one reactance must be capacitance (negative)
  - X_1 and X_2 must be of same type and X_3 must be of opposite type



(a) $X_1 < 0,\ X_2 < 0 \Rightarrow$ Colpitts, (b) $X_1 > 0,\ X_2 > 0 \Rightarrow$ Hartley

Colpitts Oscillator

- The Colpitts oscillator is a type of oscillator that uses an LC circuit in the feed-back loop.
- The feedback network is made up of a pair of *tapped capacitors* (C_1 *and* C_2) and *an inductor L* to produce a feedback necessary for oscillations.

- The Hartley oscillator is almost identical to the Colpitts oscillator.
- The primary difference is that the feedback network of the Hartley oscillator uses *tapped inductors* (L_1 *and* L_2) and *a single capacitor C*.

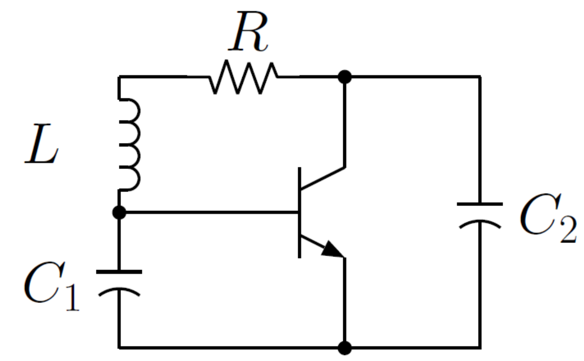
Colpitts, Hartley Oscillatior Analysis

Colpitts:

$$\omega_o = \frac{1}{\sqrt{L\frac{C_1C_2}{C_1+C_2}}}$$

$$|A_{lo}|_{\omega=\omega_o} = \frac{g_m}{R\omega_o^2C_1C_2} = 1$$

$$\text{or,} \quad g_m = \frac{R(C_1+C_2)}{L}$$



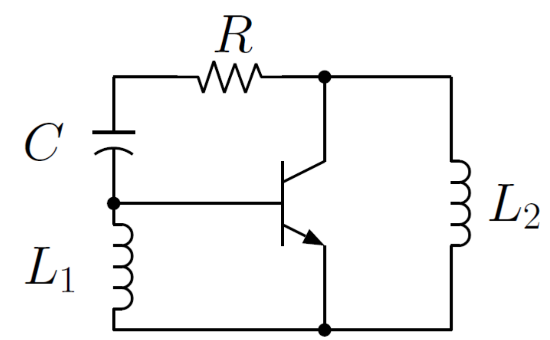
g_m are the values necessary for the circuit to support steady-state oscillations.

Hartley:

$$\omega_o = \frac{1}{\sqrt{C(L_1+L_2)}}$$

$$|A_{lo}|_{\omega=\omega_o} = \frac{g_m\omega_o^2L_1L_2}{R} = 1$$

$$\text{or,} \quad g_m = \frac{RC(L_1+L_2)}{L_1L_2}$$



These frequencies of oscillation are simply the resonant frequencies of the networks that result when the transistor is removed from the circuits.

Colpitts, Hartley Oscillatior Analysis

- The values of g_m obtained in previous equations are the values necessary for the circuit to support steady-state oscillations. We will denote these values by $g_{m,ss}$.
- In practical applications the transistor is biased to set the transconductance to a value somewhat larger, e.g. a factor of 2 to 5 larger, than $g_{m,ss}$.
- Setting $g_m > g_{m,ss}$ causes the loop gain at ω_o to be larger than 1 by the factor $g_m/g_{m,ss}$. One reason for doing this is to ensure that oscillations start reliably even if component values change slightly.
- Setting A_{lo} (at ω_o) > 1 means that the oscillations will not be maintained at a steady state; rather, they will grow in amplitude.

Colpitts, Hartley Oscillatior Analysis

- Growth will proceed until the active device is no longer operating in the “small-signal” mode. As the oscillation grows, eventually the amplitude of the oscillation will be limited by nonlinear effects.
- In large signal condition, nonlinear operation reduce the gain of the active device. This is called gain saturation, and the effect can be modeled as a decrease in the transconductance, gm, and hence a decrease in the loop gain.
- As the amplitude of the oscillation grows, the transconductance is decreased to the point where the magnitude of the loop gain is 1.
- At this point steady-state oscillation will be maintained.

Colpitts, Hartley Oscillatior Analysis

- Note that previous considerations did not specify which of the transistor terminals was at RF ground.
- Thus they apply without modification to common base, common emitter or common collector circuits.
- Also, it was assumed that Z_1 and Z_2 were purely reactive. If this is not the case, one must go back to the general expression for oscillation condition:

$$A_{lo} = \frac{-gmZ1Z2}{Z_1+Z2+Z3} = 1 \quad \text{or}$$

$$\boxed{Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0}$$

- The oscillating frequency and g_m may be different with those derived.

Colpitts, Hartley Oscillatior Analysis

- If: $\boxed{Z_1 = R_1 + jX_1\,,\ \ Z_2 = R_2 + jX_2\,,\ \ Z_3 = R\ \ + jX_3}$

$$Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0$$

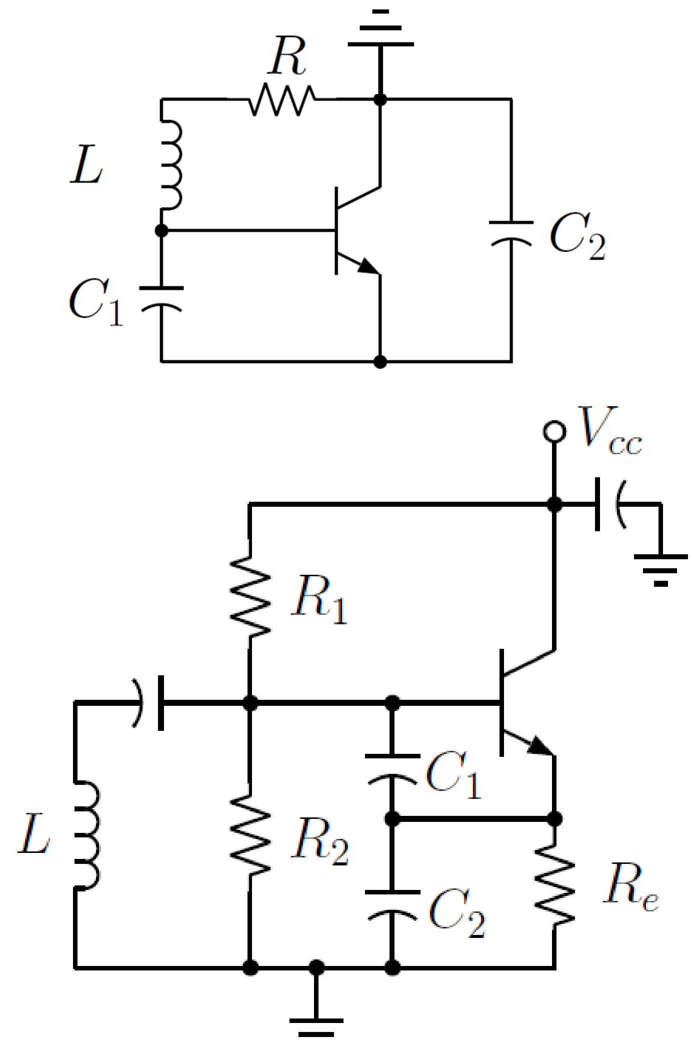
leads to:

$$R_1 + R_2 + R_3 + g_m(R_1 R_2 - X_1 X_2) = 0$$

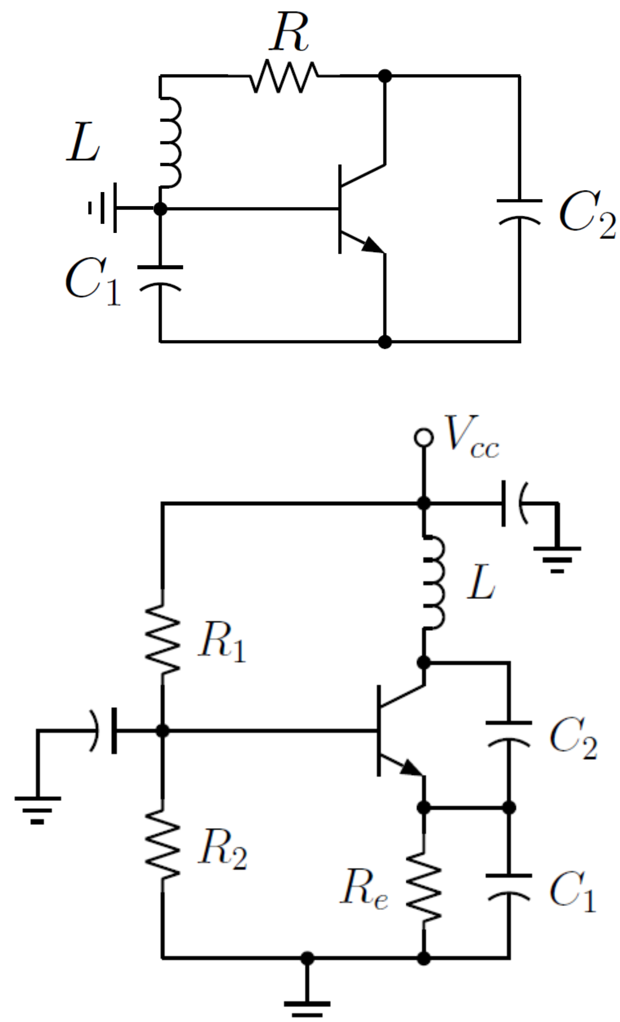
$$X_1 + X_2 + X_3 + g_m(R_1 X_2 + R_2 X_1) = 0$$

- The oscillating frequency and g_m are obtained from those .

Common-Collector, Common-Base Colpitts Oscillator

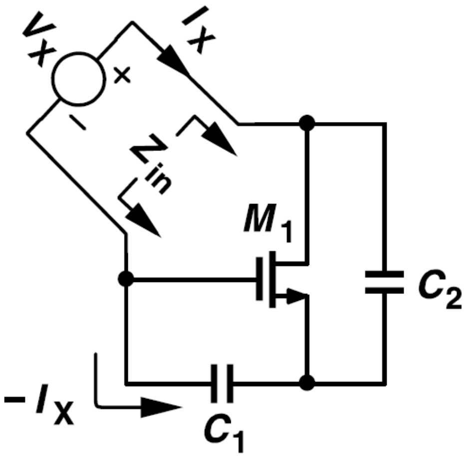


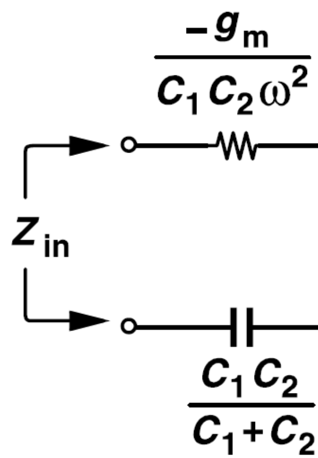
Common-collector Colpitts oscillator



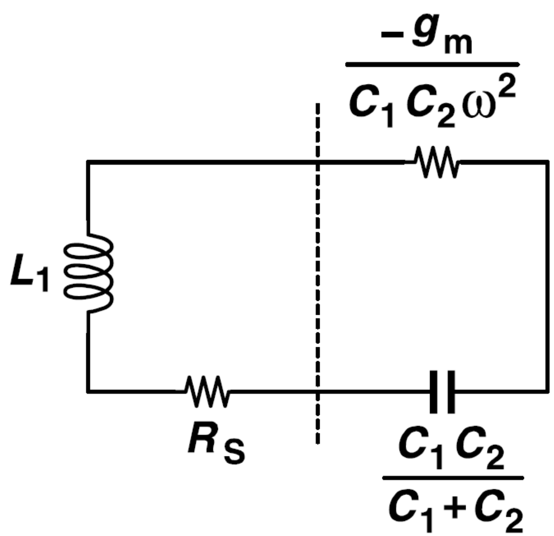
Common-base Colpitts oscillator circuit.

MOSFET Colpitts Oscillator





$$\mathbf{Z_{in}} = \frac{1}{jC_1\omega} + \frac{1}{jC_2\omega} - \frac{g_m}{C_1C_2\omega^2}$$



$$\omega_{osc} = \frac{1}{\sqrt{L_1 \frac{C_1C_2}{C_1+C_2}}}$$

$$R_S = \frac{g_m}{C_1C_2\omega^2}$$

Inductor with parallel $\mathrm{R_p}$

$$\frac{L_1\omega}{R_S} \approx \frac{R_p}{L_1\omega}.$$

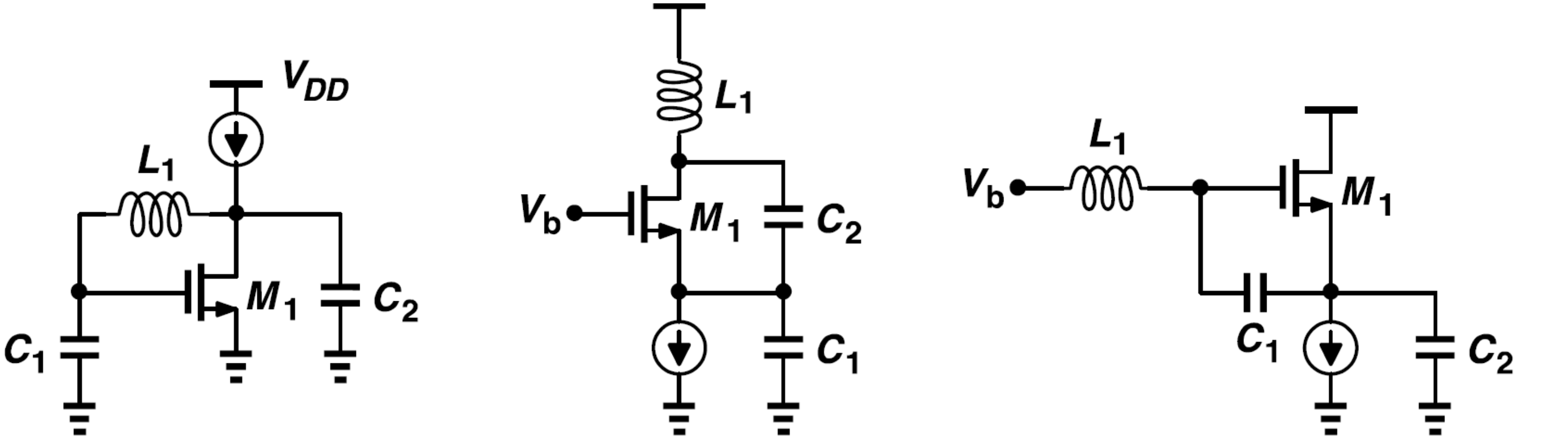
$$\frac{L_1^2\omega^2}{R_p} = \frac{g_m}{C_1C_2\omega^2}$$

$$g_mR_p = \frac{(C_1+C_2)^2}{C_1C_2}$$

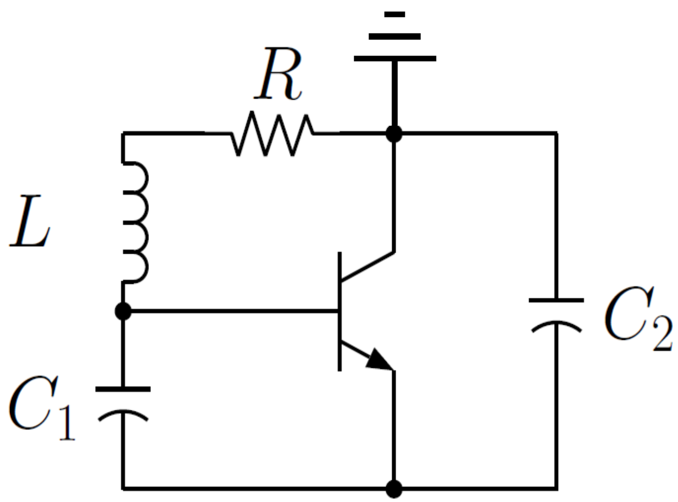
$$= \frac{C_1}{C_2} + \frac{C_2}{C_1} + 2$$

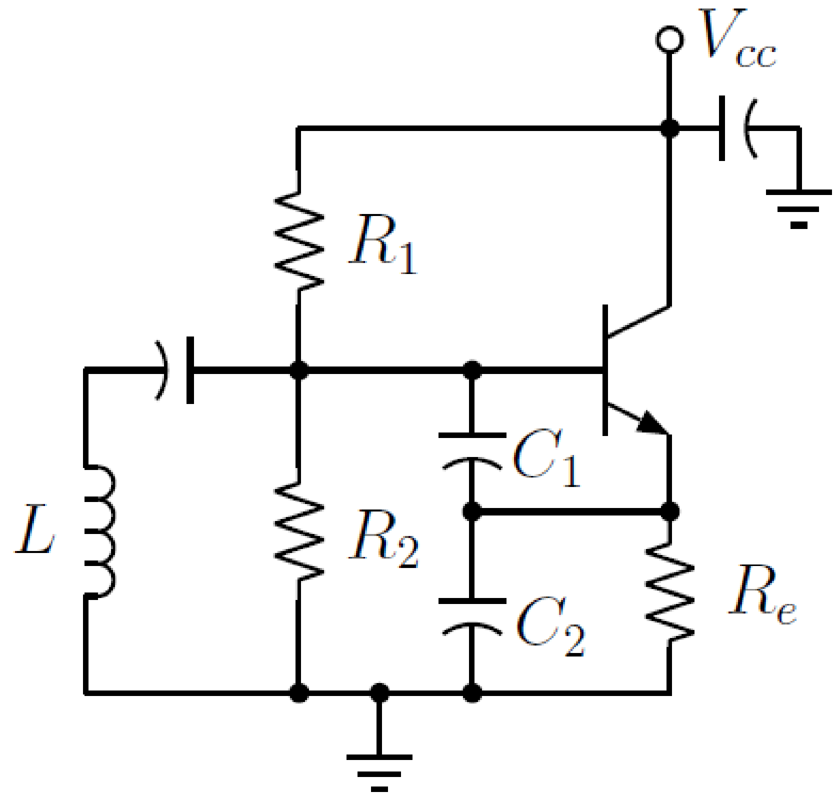
if $C_1 = C_2$. That is, $g_mR_p \geq 4$

MOSFET Colpitts Oscillator

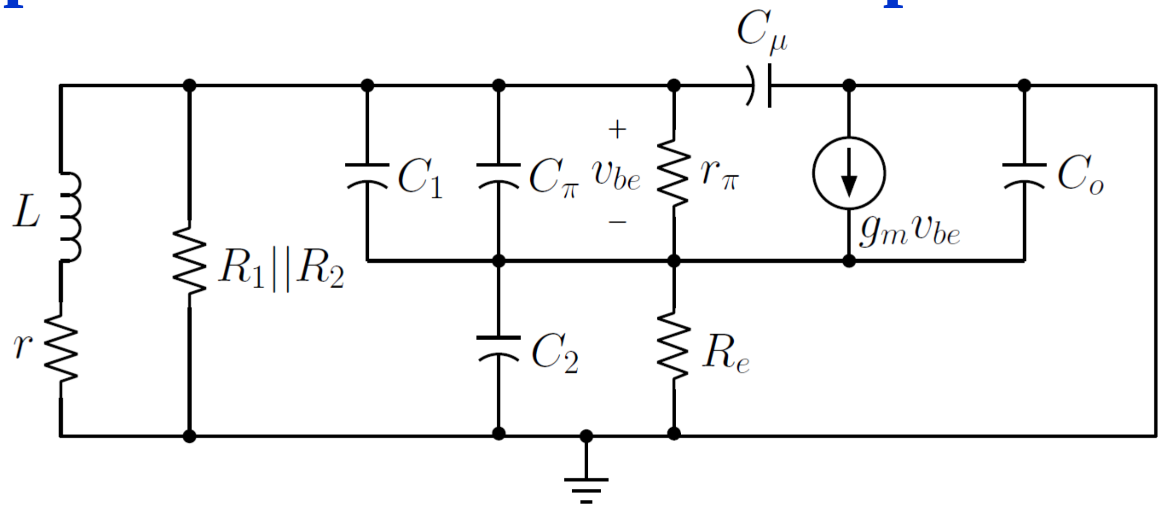


Example - Common-collector Colpitts Oscillator

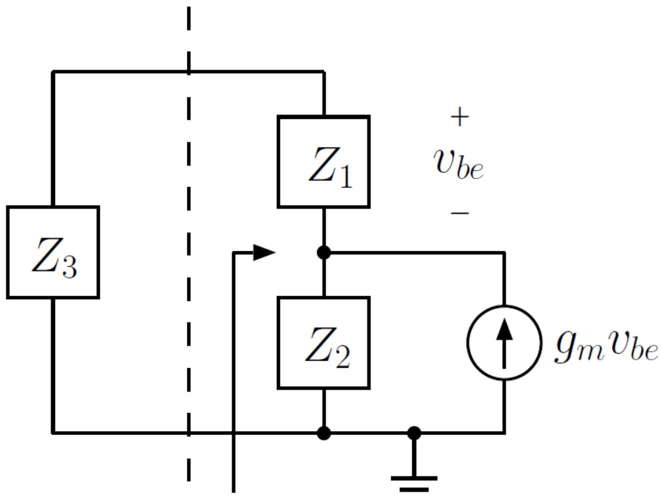




Example - Common-collector Colpitts Oscillator



Small-signal equivalent circuit for common-collector Colpitts.
The resistance, r represents the series resistance of the inductor.



$$Z_{in} = Z_1 + Z_2 + g_m Z_1 Z_2$$

$$Z_1 = \frac{1}{j\omega(C_1 + C_\pi)}||r_\pi$$

$$Z_2 = \frac{1}{j\omega(C_2 + C_o)}||R_e$$

$$Z_3 = R_1||R_2||(r + j\omega L)||\frac{1}{j\omega C_\mu}$$

Example - Common-collector Colpitts Oscillator

- In practice it is useful to choose C1 >> Cπ, C2 >> Co.
- This ensures that the external components swamp the internal capacitances of the transistor, thereby minimizing the circuit's dependence on variations in the internal transistor capacitances.
- It is also useful to choose C1 and C2 to be large enough so that $1/(\omega C_1) \ll r_\pi$, $1/(\omega C_2) \ll R_e$. This causes Z_1 and Z_2 to be dominated by the external capacitances, thereby minimizing dependence on r_π (which depends on bias current and transistor β) and losses in r_π and R_e.
- In view of these considerations, in the following analysis we shall make the following replacements:

$$C_1 + C_\pi \rightarrow C_1 \text{ and } C_2 + Co \rightarrow C_2.$$

Example - Common-collector Colpitts Oscillator

- It is now useful to make some approximations in order to simplify the analysis.
- We assume that the impedances of the capacitances C_1 and C_2 are small compared to r_π and R_e, respectively. In other words, define $Q_1 = \omega C_1 r_\pi$ and $Q_2 = \omega C_2 R_e$. We assume that $Q_1 >> 1$ and $Q_2 >> 1$.
- Then Z_1 and Z_2 can be transformed using a high-Q parallel to series
- transformation, i.e.:

$$Z_1 \simeq \frac{r_\pi}{Q_1^2} + \frac{1}{j\omega C_1} = \frac{1}{\omega^2 C_1^2 r_\pi} + \frac{1}{j\omega C_1}, \quad Q_1 \gg 1$$

$$Z_2 \simeq \frac{R_e}{Q_2^2} + \frac{1}{j\omega C_2} = \frac{1}{\omega^2 C_2^2 R_e} + \frac{1}{j\omega C_2}, \quad Q_2 \gg 1.$$

$$Z_3 \simeq r + j\omega L.$$

Example - Common-collector Colpitts Oscillator

- The condition for steady-state oscillation is $Z_{in} + Z_3 = 0$, or

$$\boxed{\mathbf{Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0}}$$

- The real part of this equation is

$$\frac{1}{\omega^2 C_1^2 r_\pi} + \frac{1}{\omega^2 C_2^2 R_e} + r + g_m\left(\frac{1}{\omega^4 C_1^2 C_2^2 r_\pi R_e} - \frac{1}{\omega^2 C_1 C_2}\right) = 0.$$

  - $r_\pi = \beta / g_m$
  - The term involving ω^{-4} may be neglected provided that:

    $\omega^2 C_1 C_2 r_\pi R_e >> 1$, or if $Q_1 Q_2 >> 1$ ☺.
  - The steady-state transconductance can be written as follows:

$$g_{m,ss} = \frac{\omega^2 C_1 C_2 r + \frac{C_1}{C_2 R_e}}{1 - \frac{C_2}{C_1 \beta}}.$$

Example - Common-collector Colpitts Oscillator

- The steady-state transconductance can be written as follows:

$$g_{m,ss} = \frac{\omega^2 C_1 C_2 r + \frac{C_1}{C_2 R_e}}{1 - \frac{C_2}{C_1 \beta}}.$$

- If Re is allowed to approach infinity (so that Z2 becomes a pure reactance), and if $\beta >> C_2/C_1$, then $g_{m,ss}$ reduces to $g_{m,ss} = \omega^2 C_1 C_2\, r$, the same as the result which was derived by assuming that Z_1 and Z_2 were pure reactances.
- Practically $C_1 > C_2$ and $\beta >>1$, the second term in the denominator can be neglected, in which case:

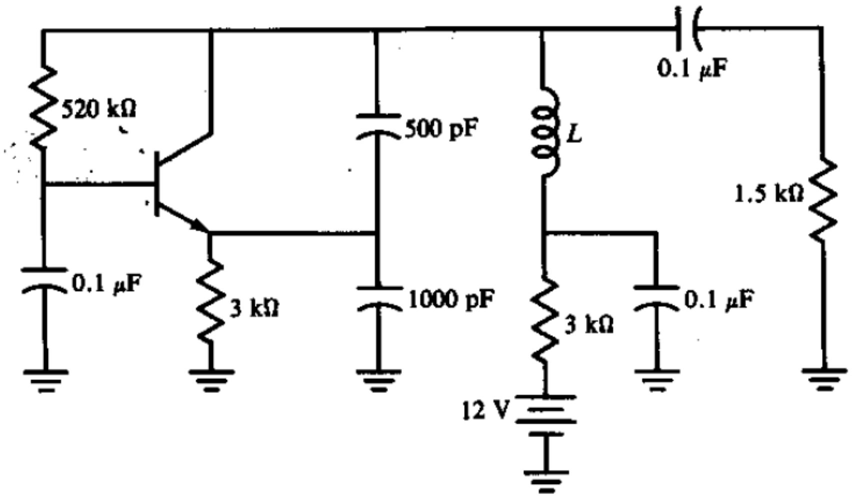
$$g_{m,ss} \simeq \omega^2 C_1 C_2 r + \frac{C_1}{C_2}\frac{1}{R_e}.$$

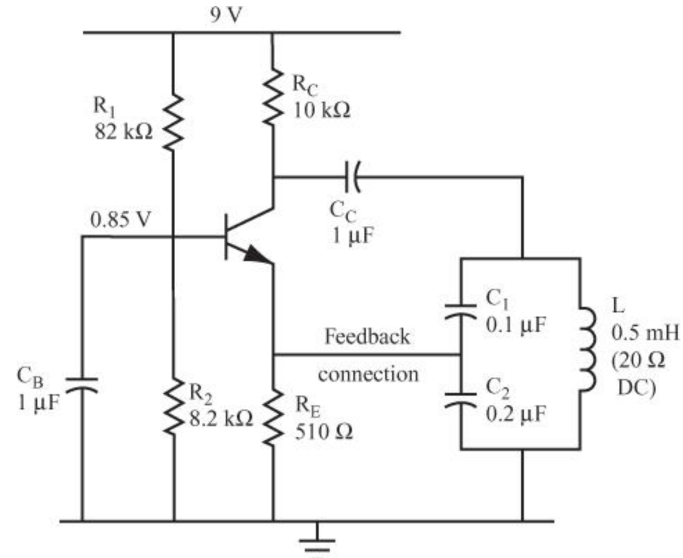
which leads to

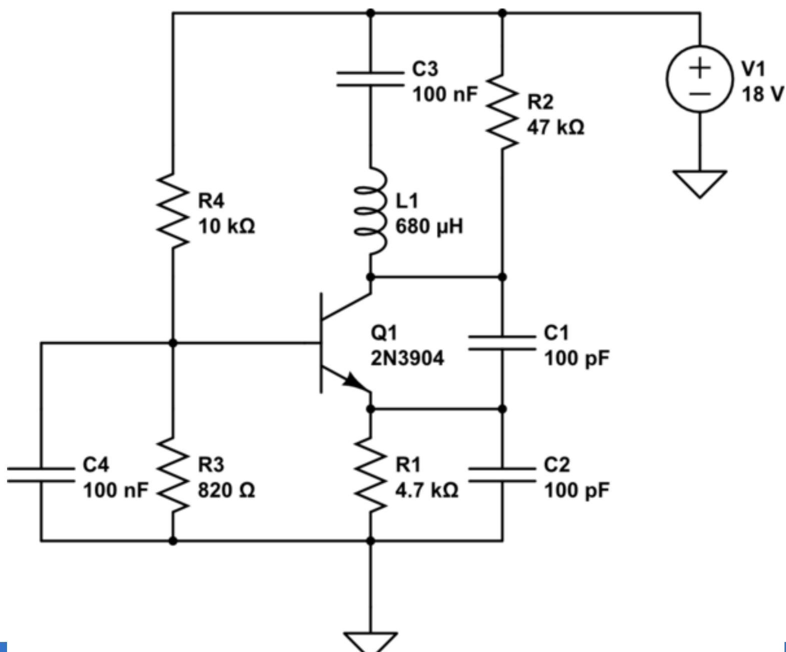
$$g_{m,ss} \simeq \frac{\omega_o (C_1 + C_2)}{Q_L} + \frac{C_1}{C_2}\frac{1}{R_e}$$ with $$\omega_o = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$$

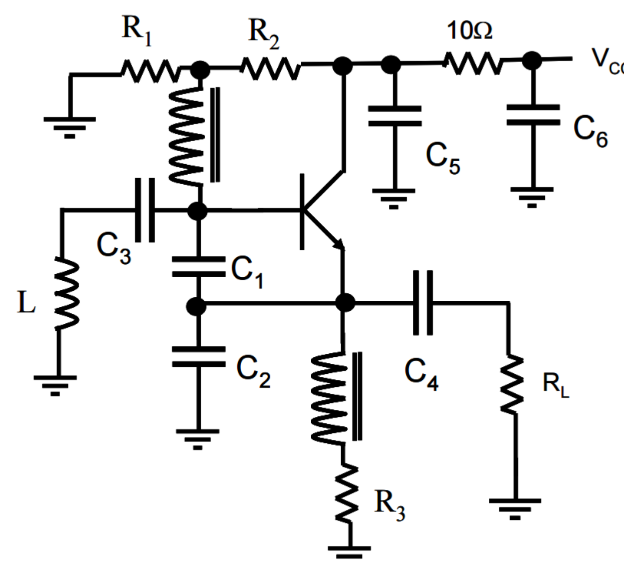
5. Colpitts and Hartley – Oscillators

Some Practical Colpitts Oscillator

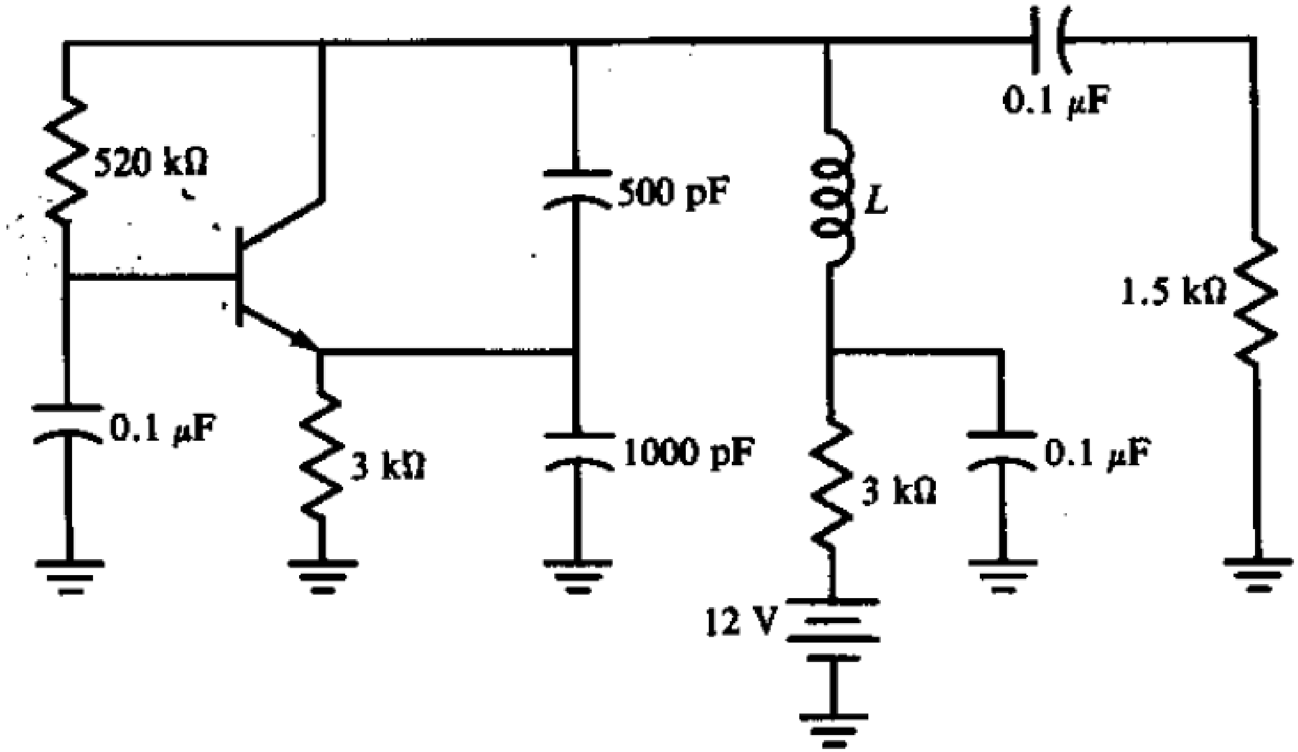








Some Practical Colpitts Oscillator

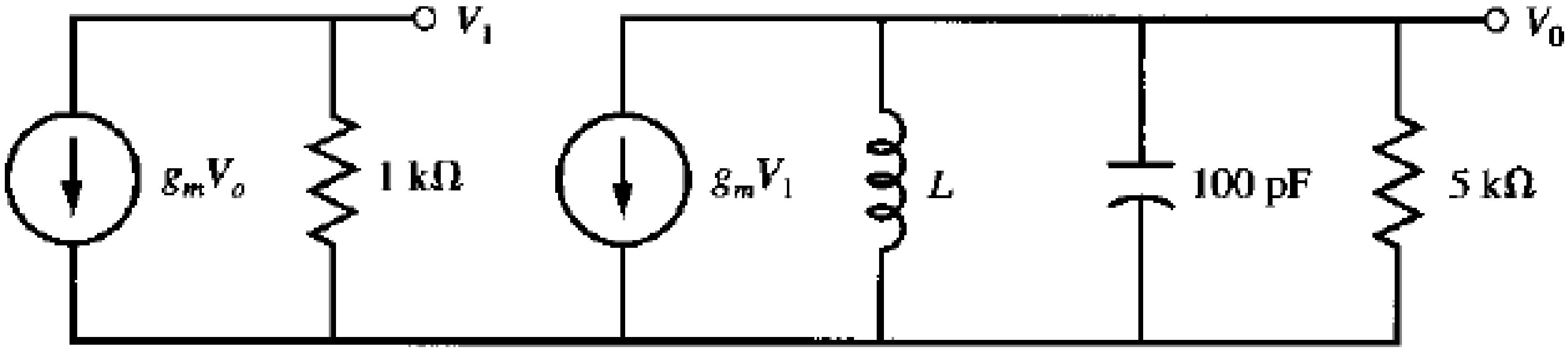


a/. Assume that it oscillates, Find oscillating frequency

b/. Check to see whether this circuit oscillate or not. If not give solution

Some Practical Colpitts Oscillator

The circuit of Fig. P7.1 represents the small-signal equivalent circuit of a two-stage amplifier with feedback from output to input. Determine the value of L required for the circuit to oscillate at 10 MHz. What is the minimum value of g_m required for the circuit to oscillate at this frequency?



Some Practical Colpitts Oscillator

Determine the value of inductance L and the turns ratio N_1/N_2 so that the circuit illustrated in Fig. P7.3 will oscillate at 5 MHz. The loop gain should initially be approximately equal to 3. Assume the transistor input impedance is sufficiently large so that it does not load down the autotransformer. (The transistor $\beta = 100$.)

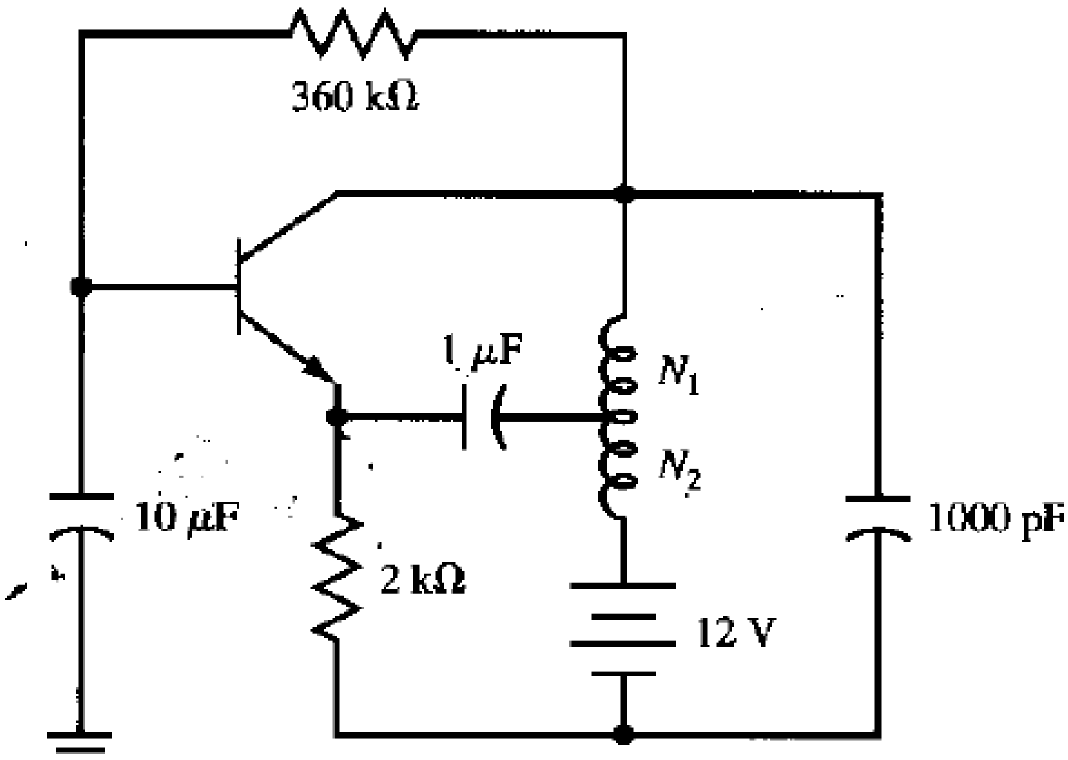


FIGURE P7.3
A Hartley oscillator.

Some Practical Colpitts Oscillator

The FET in the circuit of Fig. P7.4 is biased so that $g_m = 5$ mS. Determine capacitors C_1 and C_2 so that the circuit will oscillate at 10 MHz. The open-loop gain should be at least 2.5 to ensure that oscillations begin. The unloaded inductor $Q_u = 100$.

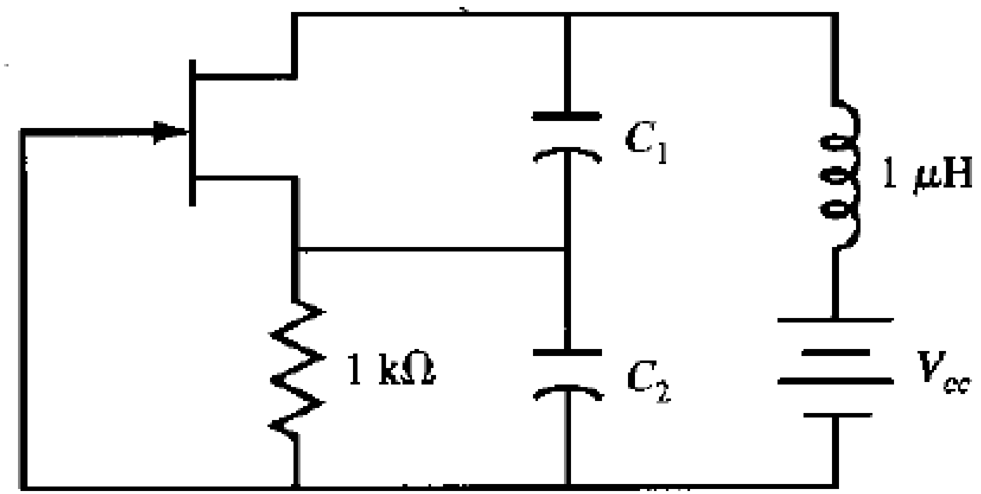
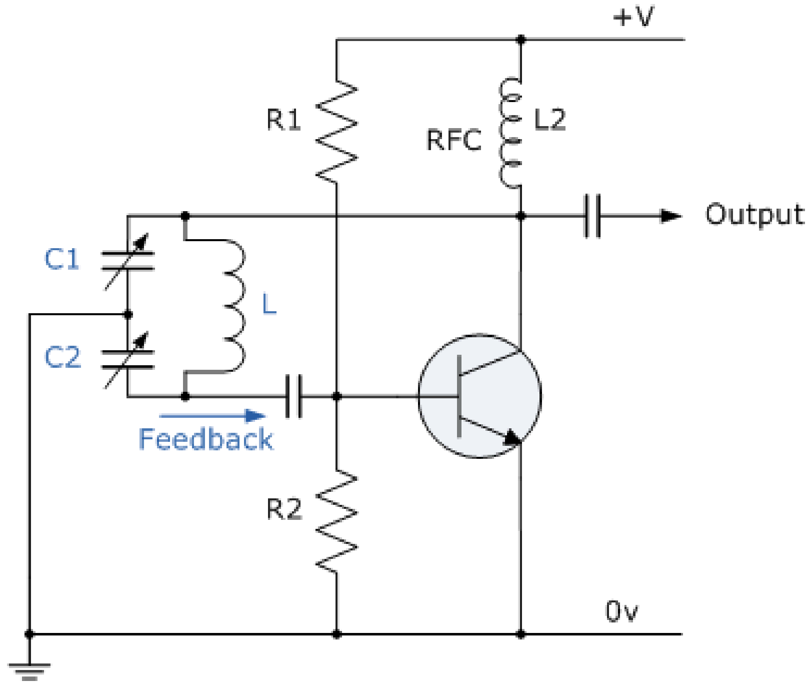
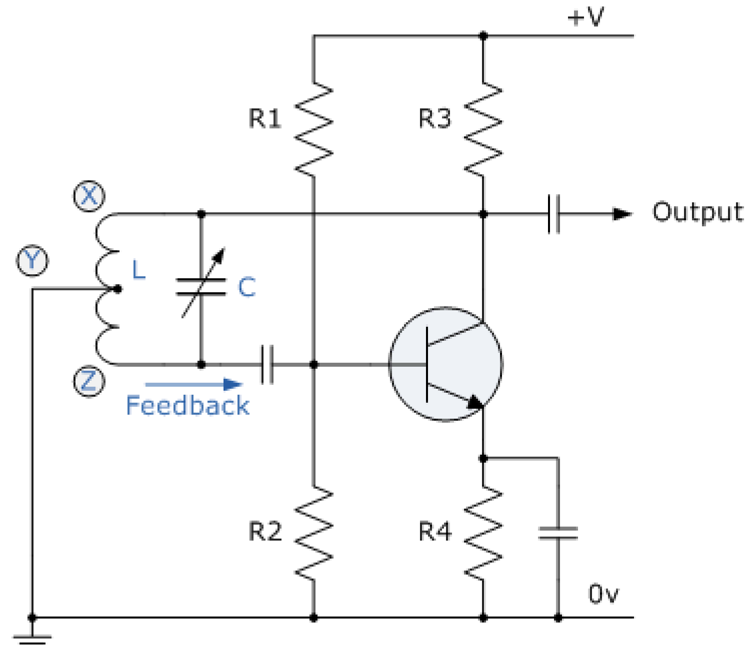


FIGURE P7.4
A common-gate Colpitts oscillator.

Some Practical Colpitts Oscillator





Example of Colpitts circuit (with bias), oscillating frequency:

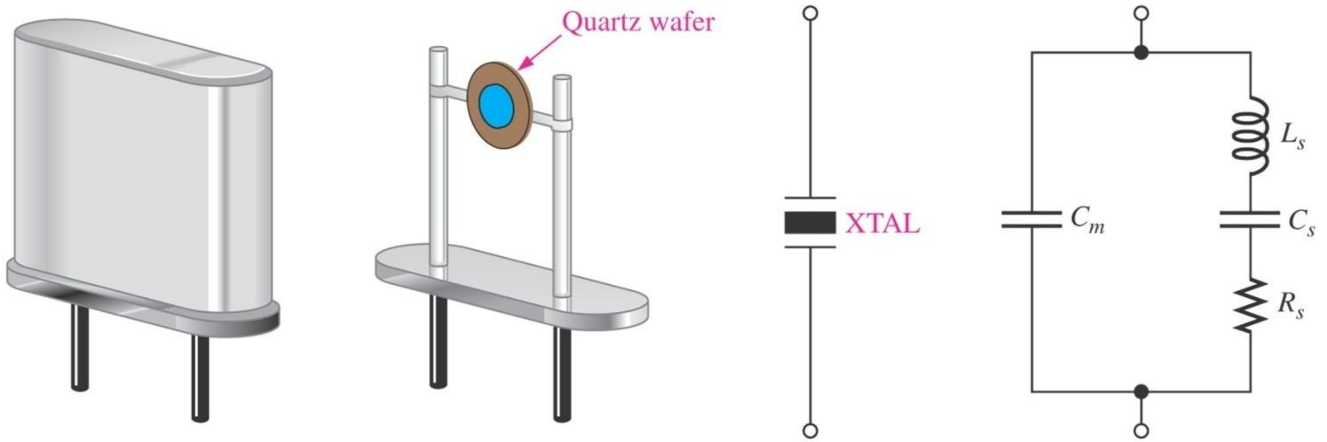
$$f = \frac{1}{2\pi\sqrt{L\frac{C_1 C_2}{C_1 + C_2}}}$$

Example of Hartley circuit (with bias), oscillating frequency:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

- Most communications and digital applications require the use of oscillators with extremely stable output. Crystal oscillators are invented to overcome the output fluctuation experienced by conventional oscillators.

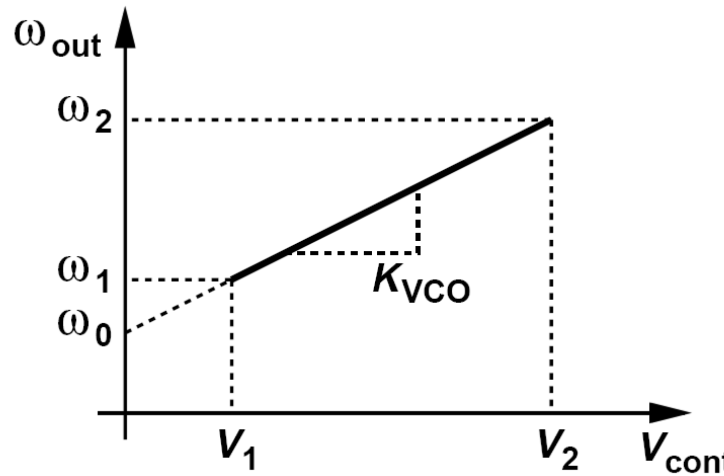
- Crystals used in electronic applications consist of a quartz wafer held between two metal plates and housed in a package as shown in Fig. (a) and (b).



(a) Typical packaged crystal (b) Basic construction (without case) (c) Symbol (d) Electrical equivalent

7. Voltage Control Oscillators - VCO

- VCO is an electronic oscillator specifically designed to be controlled in oscillation frequency by a voltage input using a varactor or varicap.
- The frequency of oscillation, is varied with an applied DC voltage.

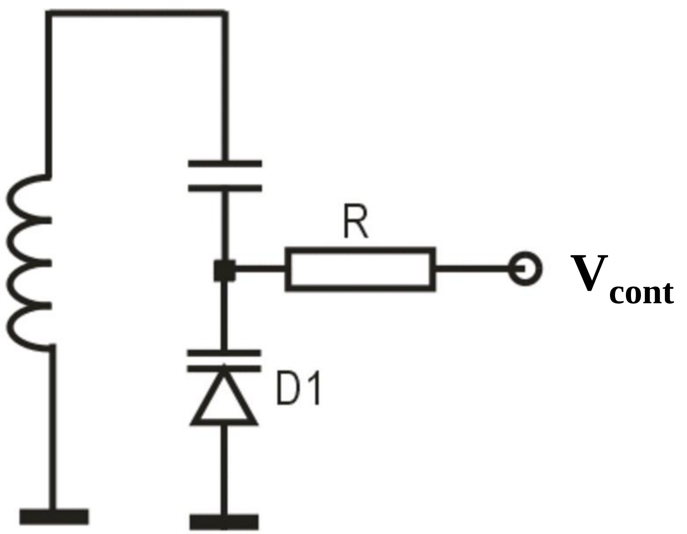


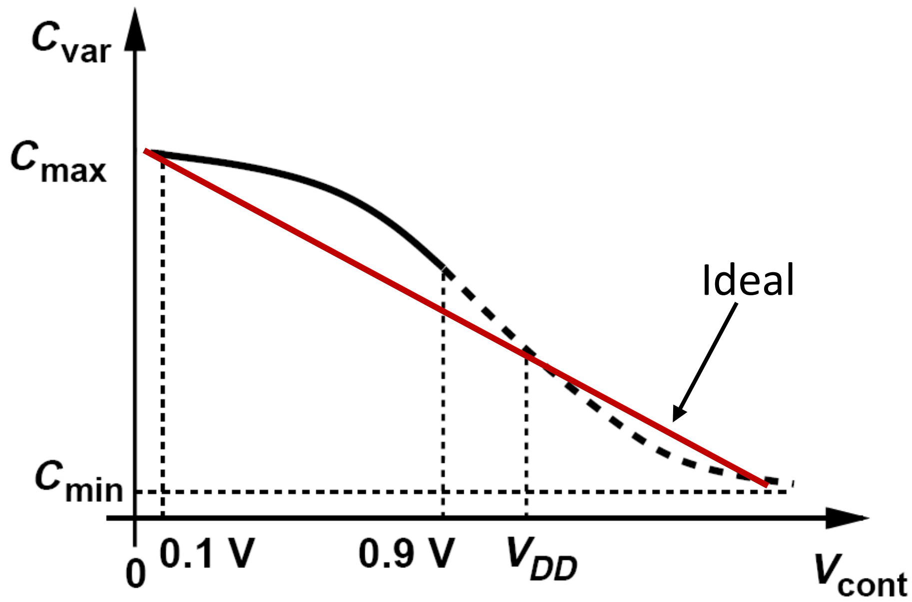
$$\omega_{out} = K_{VCO} V_{cont} + \omega_0$$

- The output frequency varies from ω_1 to ω_2 (the required tuning range) as the control voltage, V_{cont}, goes from V_1 to V_2.
- The slope of the characteristic, K_{VCO}, is called the "gain" or "sensitivity" of the VCO and expressed in rad/Hz/V.

7. Voltage Control Oscillators - VCO

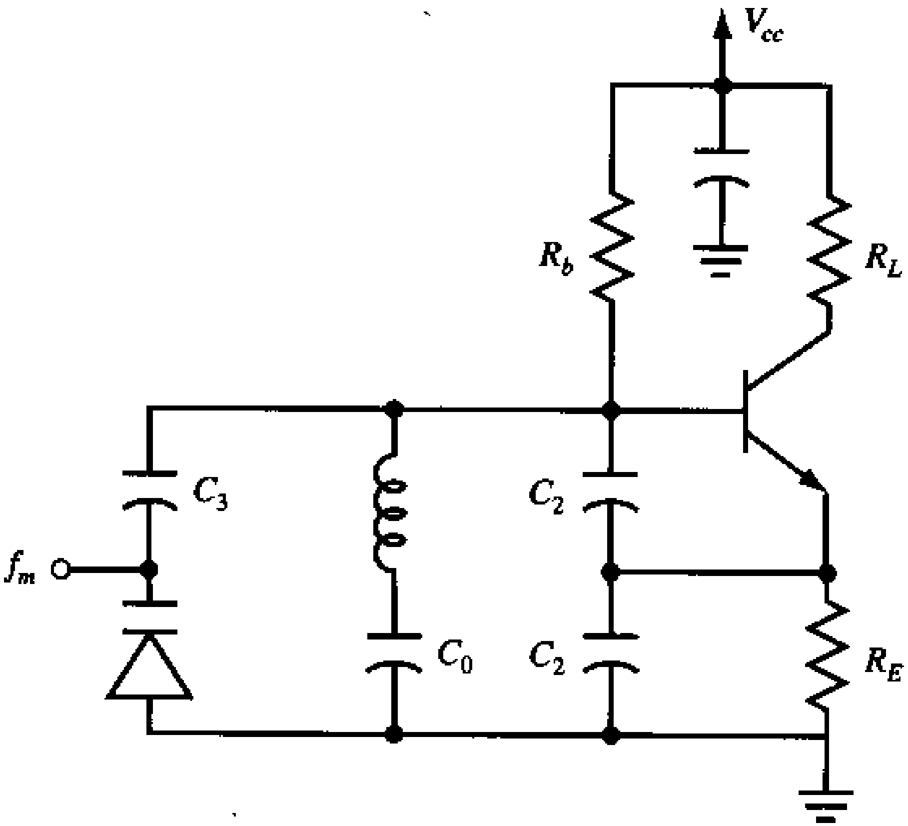
➢ Oscillation frequency by a voltage input using a varactor or varicap

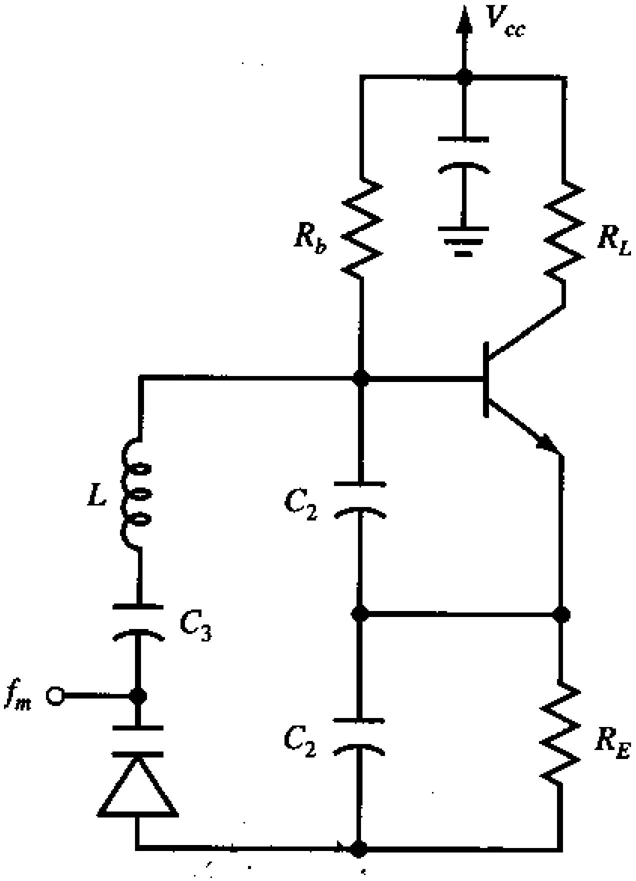




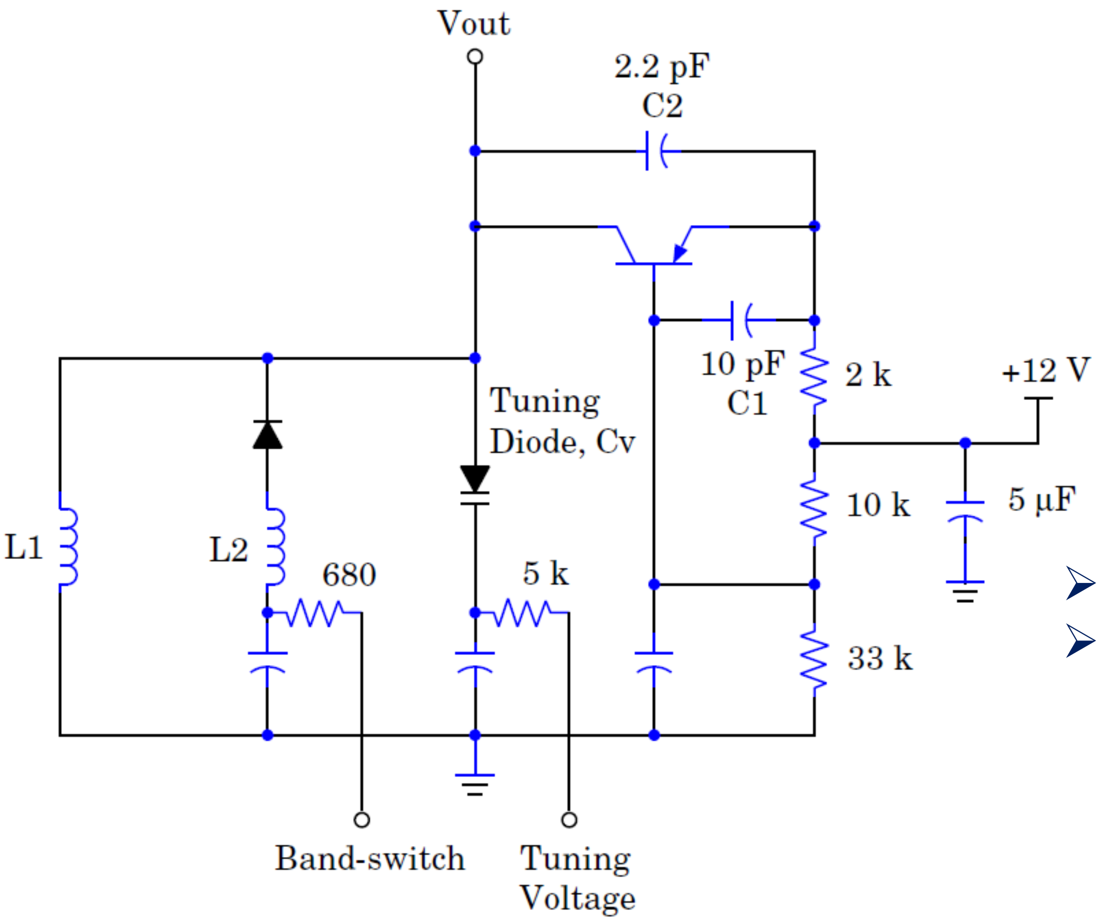
Varactor characteristic

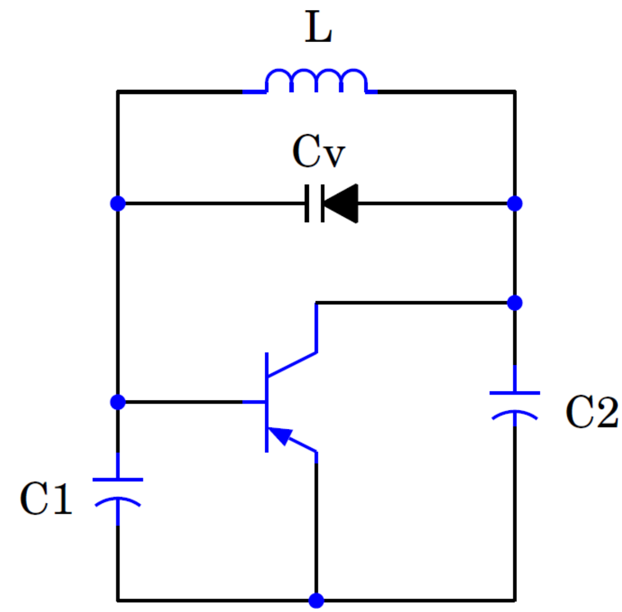
Colpitts VCO





Example: VHF VCO for TV tuner





- The L-Cv combination looks inductive.
- The frequency of oscillation for this circuit will be the frequency where C1, C2 and the L-Cv combination are resonant.

$$\frac{1}{j\omega_o C_S} + \frac{j\omega_o L \frac{1}{j\omega_o C_v}}{j\omega_o L + \frac{1}{j\omega_o C_v}} = 0$$

$$C_S = \frac{C_1 C_2}{C_1 + C_2}$$

$$\omega_o = \frac{1}{\sqrt{L(C_S + C_v)}}$$

Oscillators

Design a Colpitts oscillator operating at 200 MHz using an FET in a common gate configuration, including the effect of a lossy inductor. First derive equations for the resonant frequency and condition required for sustaining oscillation for an inductor with loss. Use these results to find the required capacitances, assuming an inductor of 15 nH with a Q of 50, and a transistor with gm = 20 mS and $Ro = 1/Go = 200$.

Determine the minimum value of the inductor Q required to sustain oscillations.

Oscillators

HW: 1, 2, 3, 5, 6, 8, 11, 16, 17