Chapter 3

Transceiver, Noise and Nonlinear Distortion

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RADIO COMMUNICATION CIRCUITS

Chapter 3

Transceiver, Noise and Nonlinear Distortion

Textbook:

Steven J. Franke, *Wireless Communication Systems*, UIUC
Chapter 2, 10

References:

B. Razavi, *RF Microelectronics*, Upper Saddle River, Prentice Hall, Second Edition, 2011 (Chapter 2)

Transceiver, Noise and Nonlinear Distortion

Outline

- Introduction and Historical Progression
- RF Transceiver
- Noise
- Nonlinear Distortion

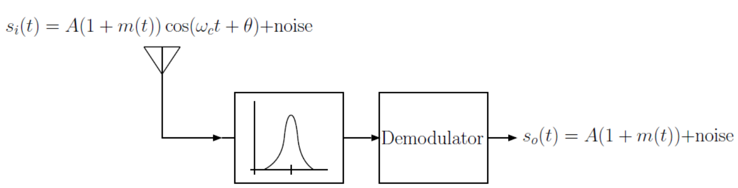
1. Introduction and Historical Progression

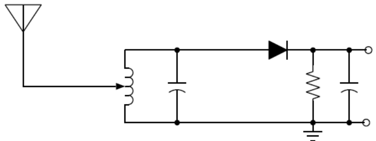
- The history of radio communication begins at the end of the 19'th century, when Italian inventor Guglielmo Marconi developed the first practical radio communication system using spark-gap radio transmitters and a nonlinear circuit element, called a "coherer", as a detector at the receiving end of the link.

- His most famous accomplishment was demonstration of transatlantic wireless communications, in 1902, when the Morse-code letter "S" (dot dot dot) was transmitted from England, and received in Newfoundland.
- Wireless telegraphy transmission using spark-gap transmitters used the broadband damped oscillations generated when the DC current in an LC circuit is suddenly interrupted.

1. Introduction and Historical Progression

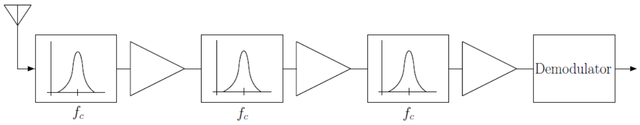
Tuned Detector/Demodulator



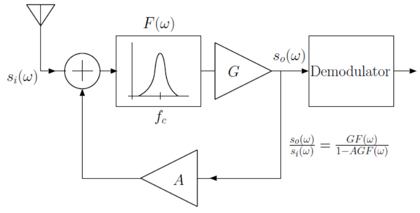


1. Introduction and Historical Progression

Tuned Radio Frequency (TRF) Receiver



- Very popular receiver architecture that was employed in the 1920's for AM broadcast receivers



Require:
- Tunable
- High-gain
- Narrow-bandwidth
- Filter/amplifier

A regenerative receiver employing a regenerative amplifier in front of a demodulator

1. Introduction and Historical Progression

Genesis of the Superheterodyne Receiver

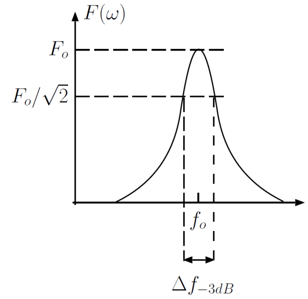
- Superheterodyne receiver was invented by Edwin Armstrong in 1917

- Armstrong became a member of the U.S. Army Signal Corps during World War I. Involved with efforts to find a way to detect enemy airplanes from a distance
- He knew that it might be possible to detect the electromagnetic emissions from the spark plugs in the engine.
- The problem was that the emissions were strongest at the (then) unusually high frequencies above a few MHz.
- Triodes of the day had very little gain at such high frequencies, so Armstrong hit on the idea of employing the heterodyne principle to shift the high-frequency signals to a lower frequency, where they could be more efficiently amplified and filtered.
- Once he decided to incorporate the heterodyne concept into his receiver, it became possible to heterodyne any signal of interest, regardless of its frequency, to a fixed intermediate frequency (IF). Selective filtering, and high-gain amplification could be done at the fixed IF.

1. Introduction and Historical Progression

Characteristics of Practical Filters

- Losses in the components used to implement filter networks set a lower limit on the bandwidth that can be achieved in a bandpass filter.



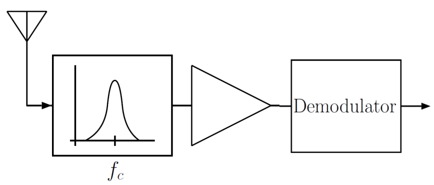
Definition of -3dB bandwidth

- ✓ In many cases, the resonant circuit Q is limited by the Q of the inductor.
- ✓ For miniature inductors, Q's may be limited to values in the range 10-100, whereas inductors implemented using spirals of metallization in integrated circuits typically have very low Q, often <10.
- ✓ Fractional BW is 1/Q
- ✓ Generally speaking, **LC filters** with fractional bandwidths smaller than around 0.01 tend to become impractical.

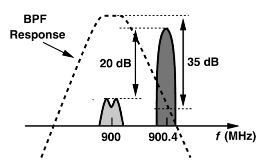
- **Transmission-line and cavity resonator filters** -- > difficult to tune
- **Filters based on piezoelectric devices** - Quartz-Crystal Filter, Ceramic Filter → high Q, but expensive and low frequencies, not tunable
- **SAW filters** → high Q, but expensive, not tunable

1. Introduction and Historical Progression

Filter limitations dictate carrier-frequency conversion



Tuned Radio Frequency (TRF) Receiver

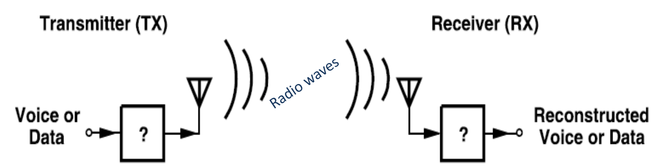


Can we select an RF wanted signal while completely filter the Interferers at RF frequency?

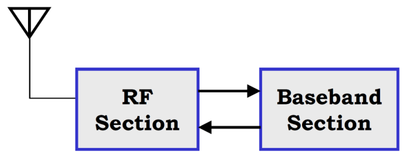
- ✓ First, the filter must provide a very high *Q*
- ✓ Second, the filter would need to have a variable and precise center frequency.
- ✓ Can we select the channel at RF frequency?

- ✓ The bandwidth of CDMA channel is 1.25 MHz, and the center frequency is approximately 1900 MHz. A fractional bandwidth of 1.25/1900 of 0.00066 is required.
- ✓ GSM: 200K/900M

2. RF Transceiver



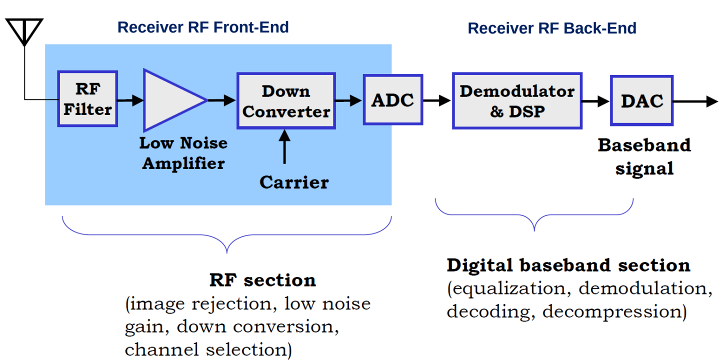
- A Transceiver consists of a Transmitter and a Receiver
- RF/Microwave transceiver can be divided into two sections:



- RF/Microwave Section – analog, high frequencies
- Baseband Section - mostly digital today (DSP), low frequencies

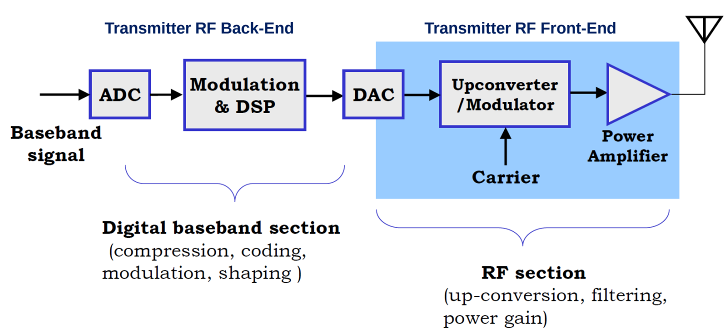
2. RF Transceiver

- Typical Digital Receiver

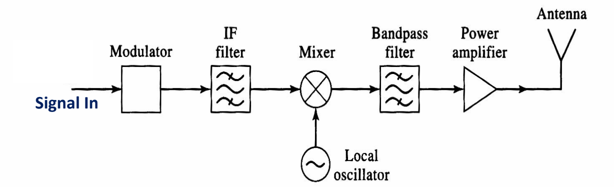


2. RF Transceiver

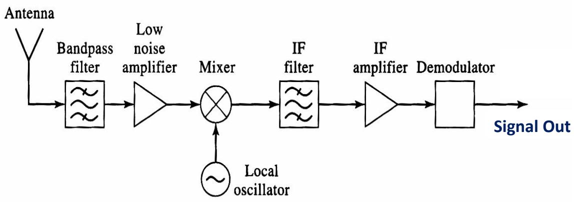
- Typical Digital Transmitter



❑ What are the functions of each building block ?



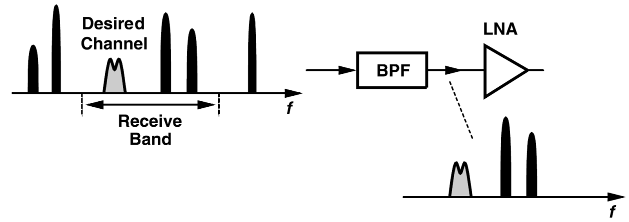
(a) **Transmitter**



(b) **Superheterodyne Receiver**

Heterodyne: combine (a high-frequency signal) with another to produce a lower frequency in this way

❑ Channel Selection and Band Selection

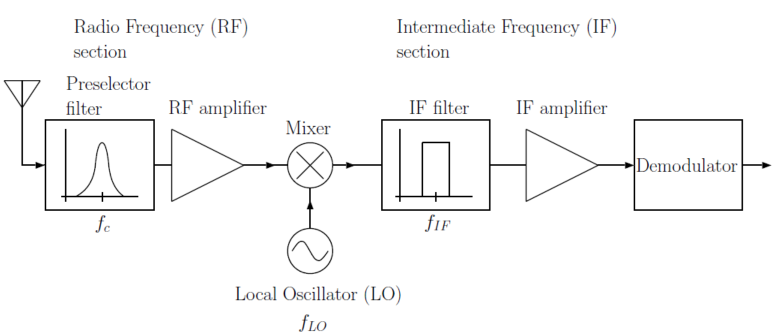


- It is extremely difficult to select channel at RF frequency due to high-Q RF filter.
- Channel selection must be deferred to some other point where center frequency is lower and hence required *Q* is more reasonable
- Most receiver front ends do incorporate a "band-select" filter

- Constant IF: LO frequency is variable, all RF channels within the band of interest translated to a single value of IF.

Superheterodyne Receiver

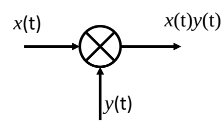
- Conversion to a fixed frequency takes advantage of the fact that it is much easier to realize narrow-band filters and stable, high-gain amplifiers if the frequency of operation doesn't change.
- The IF frequency is fixed, the LO frequency is adjusted to select different channel.



Mixer Fundamentals

If

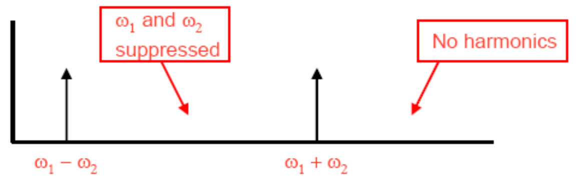
$$x(t) = A\cos\omega_1 t$$
$$y(t) = B\cos\omega_2 t$$



Then the output is

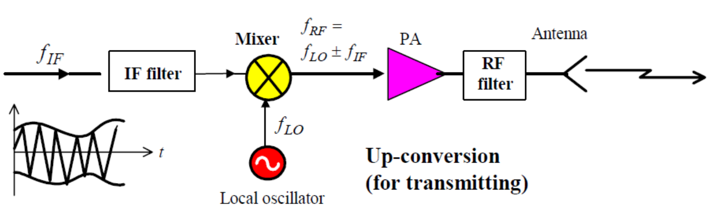
$$A\cos\omega_1 t \cdot B\cos\omega_2 t = \frac{AB}{2}\cos(\omega_1 - \omega_2)t + \frac{AB}{2}\cos(\omega_1 + \omega_2)t$$

down convert   up convert

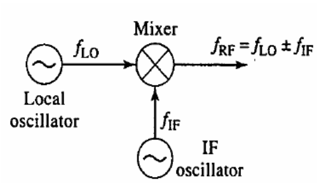


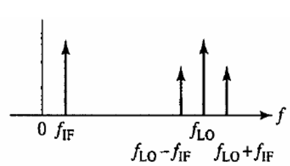
❍ **Mixer in a Transmitter (Up-conversion Mixer)**

- In a transmitter, a mixer is used to **mix with IF signal** to **up-convert** the signal frequency for **efficient radio-wave transmission from antenna**.

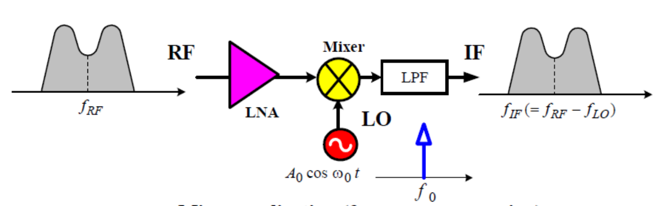


Up-conversion (for transmitting)

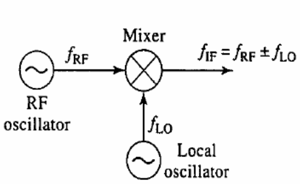


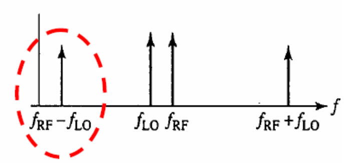


❍ **Mixer in a Receiver (Down-conversion Mixer)**



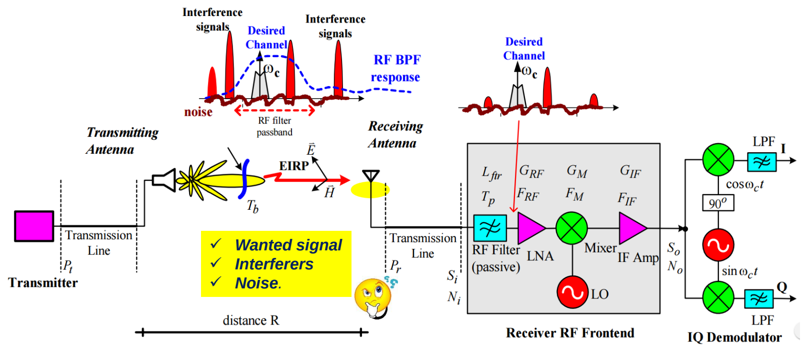
Mixer application (frequency conversion)





General Considerations

- The main function of a receiver is the demodulation of a wanted signal (channel) in the presence of **undesired interferers** and **noise**.
- Due to the strong attenuation during air transmission, the RF signal has to be amplified and recovered.
- Taking into account scenarios with varying attenuation, a wide dynamic range is required for the detection of signals with high data-rates.



BPF First or LNA First?

- Trade-off between suppressing inter-modulation products due to interferers vs. noise figure
- BPF first: better interferer rejection, but higher noise figure due to the insertion loss of the filter
- LNA first: better noise figure, receiver can be desensitized due to interferers
- Interferers are bigger problem, so BPF first is adapted in all receivers

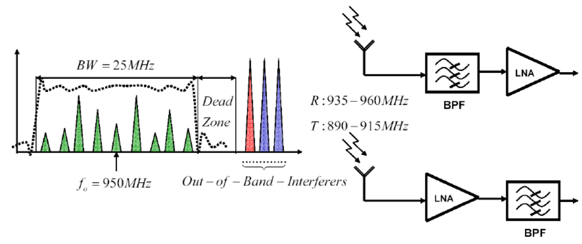
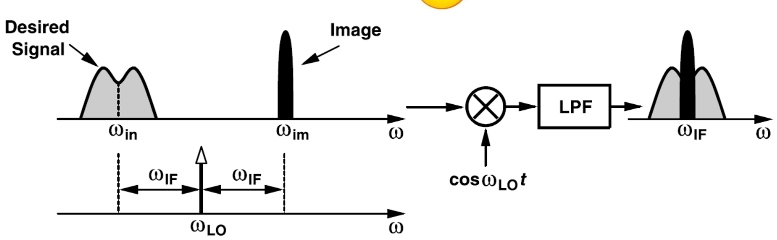


Image Frequency Problem



$$A\cos\omega_{IF}t = A\cos(\omega_{in}-\omega_{LO})t$$
$$= A\cos(\omega_{LO}-\omega_{in})t$$
$$\omega_{im} = \omega_{in} + 2\omega_{IF} = 2\omega_{LO} - \omega_{in}$$

- **Two spectra located symmetrically around ω_{LO} are downconverted to the IF**

Superheterodyne Receiver

- Using Image Reject Filter

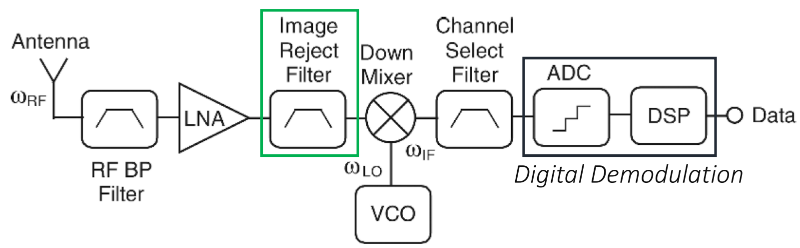
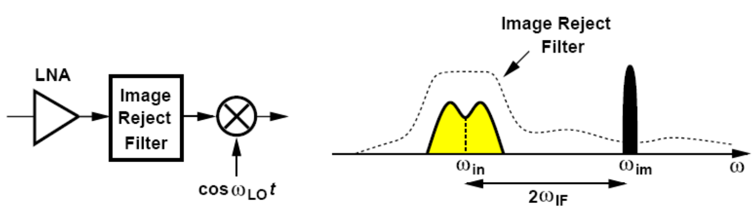
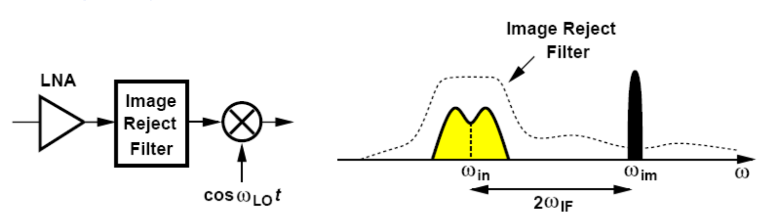


Image Rejection



- **The most common approach is to precede the mixer with an "image-reject filter"**
- **The IRF should exhibits the low loss in the desired band and high attenuation in the image band.**
- **A filter with high image rejection typically appears between the LNA and the mixer so that the gain of the LNA lowers the filter's contribution to the receiver noise figure.**
- **The linearity and selectivity required of the image-reject filter have dictated passive, off-chip implementations.**

Image Rejection

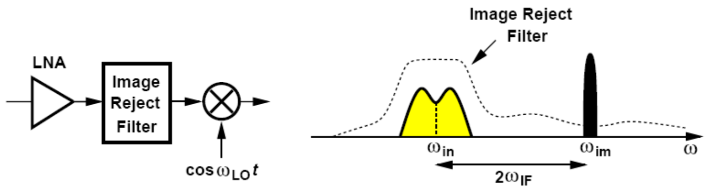


Example - FM Broadcast Receiver

The FM broadcast band covers 88 - 108 MHz. The channels are separated by 200 kHz and are assigned to odd multiples of 100 kHz. In almost all cases the IF frequency is chosen to be 10.7 MHz. As in the previous example "high-side" LO is often used, i.e., the LO tunes from 98.7 to 118.7 MHz.

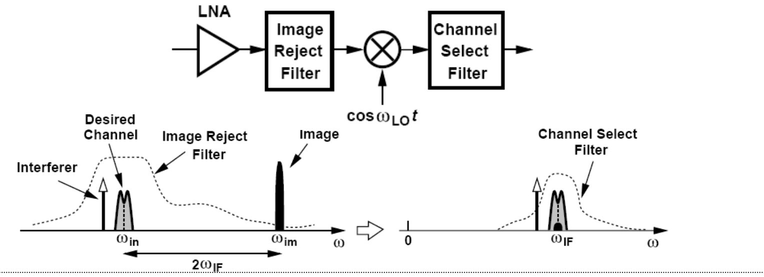
Notice that the IF used for FM broadcast receivers is substantially higher than that used for AM receivers. This choice is motivated by the fact that the image frequency is separated from the desired carrier frequency by 2fIF. Generally speaking, in order tomake the preselector filter relatively easy to build and tune, the IF frequency must be raised as the carrier frequency increases.

❑ Image Rejection versus Channel Selection

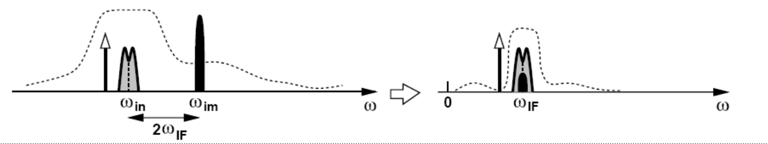


- As noted, the desired channel and the image have a frequency difference of $2\omega_{IF}$. Thus, to maximize image rejection, it is desirable to choose a large value for ω_{IF} i.e., a large difference between ω_{in} and ω_{LO}.
- How large can $2\omega_{IF}$ be? Recall that the premise in a heterodyne architecture is to translate the center frequency to a sufficiently ***low*** value ω_{IF} so that channel selection by means of practical filters becomes feasible.
- **→ Image Rejection - Channel Selection Trade-Off**

❑ Image Rejection versus Channel Selection



- **A high IF allows substantial rejection of the image.**

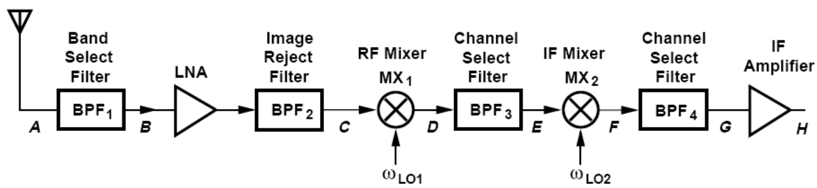


- **A low IF helps with the suppression of in-band interferers.**

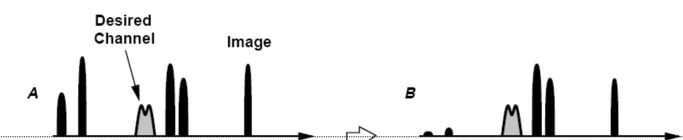
❑ Image Rejection versus Channel Selection

- The trade-off between image rejection and channel selection in the heterodyne architecture often proves quite severe: if the IF is high, the image can be suppressed but complete channel selection is difficult, and vice versa.
- To resolve this issue, the concept of heterodyning can be extended to multiple down-conversions, each followed by filtering and amplification --> **Double-conversion SupperHeterodyne Receiver**

❑ Double-conversion Supper Heterodyne Receivers

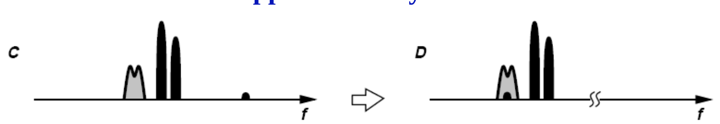


- In a double-conversion receiver the bandwidth of the first IF filter would be chosen to reject "secondary" image frequencies within the first IF bandwidth.
- A secondary image would be an undesired frequency within the passband of the first IF filter that could be mixed into the second IF filter's passband.
- The second IF filter's bandwidth would usually be just wide enough to pass the entire spectrum of the desired signal.

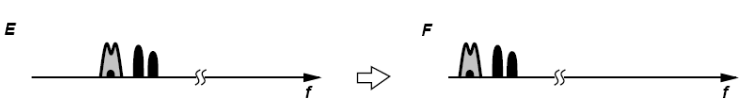


- **The front-end filter selects the band while providing some image rejection as well (Point *B*)**

❑ Double-conversion Supper Heterodyne Receivers

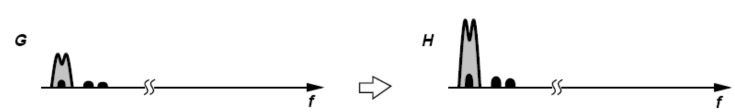


- **After amplification and image-reject filtering, spectrum of *C* obtained**
- **Sufficiently linear mixer translates desired channel and adjacent interferers to first IF (Point *D*)**

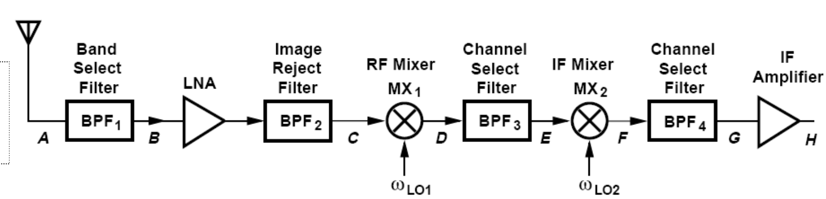


- **Partial channel selection BPF_3 permits the use of a second mixer with reasonable linearity. (Point *E*)**
- **Spectrum is translated to second IF. (Point *F*)**

❑ Double-conversion Supper Heterodyne Receivers



- **BPF_4 suppresses the interferers to acceptably low levels (Point *G*)**
- **An optimum design scales both the noise figure and the IP3 of each stage according to the total gain preceding that stage.**



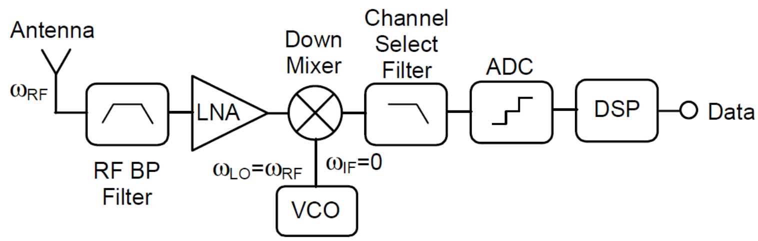
2. RF Transceiver - Receiver

- The problems associated with the image rejection have motivated designers to invent other receiver architecture and smart techniques for the rejection of the image frequency.
- Receiver architectures
  - Heterodyne receiver – image problem
  - Super-heterodyne receiver – more circuits, more power
  - Image-reject receivers
    - ✓ Harley receiver
    - ✓ Weaver architecture
  - Homodyne (direct conversion, zero-IF) – DC offset
  - Low IF
  - Digital IF

2. RF Transceiver - Receiver

Direct-conversion Receiver – (Zero-IF)

- The idea is to translate the RF signal directly to zero-IF frequency.

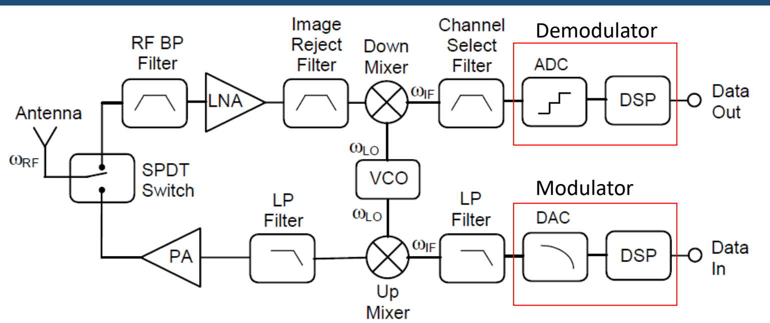


2. RF Transceiver - Receiver

Direct-conversion receivers

- Advantage:
  - The channel filtering can be performed by a low-pass filter.
  - Eliminating the image problem. Hence, no external high-Q image reject filter is required making fully integrated solutions feasible.
- The zero-IF approach seems to be superior compared with other architectures. However, the following issues impede its widespread use in today's radios.
  - The RF carrier and the local oscillator are at the same frequency. Thus, LO leakage to the mixer input can lead to self mixing resulting in a time-varying DC offset at the output of the mixer.
  - This DC offset may corrupt the signal and can lead to a saturation of the following stages thereby significantly degrading the upper boundary of the dynamic range. Consequently, sophisticated offset cancellation techniques are required in practical implementations.
  - The flicker noise of the active devices becomes significant at low frequencies. Thus, low noise amplification and active filtering is difficult for zero-IF topologies degrading the lower limit of the dynamic range.

2. RF Transceiver - Receiver



- Transceivers consist of both a receiver and a transmitter.
- In many cases, a transceiver needs only one multifunctional VCO since it may be used for both the receiver and transmitter.
- SPDT (Single Pole Double Throw) switches can be employed to change between the receive- and transmit- modes.
- These switches exhibits the additional losses of around 0.5–2 dB, which directly add to the overall noise figure in the receiver and reduce the effective PA power.

2. RF Transceiver - Receiver

Example - AM Broadcast Receiver

- A typical AM broadcast receiver covers the frequency range 540 - 1700 kHz.
- AM broadcast stations in the U.S. are assigned frequencies that are integer multiples of 10 kHz, therefore the adjacent channel separation is 10 kHz. The IF frequency is very often chosen to be 455 kHz.

Down - conversion: $f_{IF} < f_{cmin} < f_{cmax}$

(2a)	$f_{LO} = f_{IF} + f_c$	$f_{IM} = f_c + 2f_{IF}$	$\lvert f_{IM} - f_c\rvert = 2f_{IF}$	
(2b)	$f_{LO} = f_c - f_{IF}$	$f_{IM} = \lvert f_c - 2f_{IF}\rvert$	$\lvert f_{IM} - f_c\rvert = 2f_{IF}$	if $f_c > 2f_{IF}$
			$\lvert f_{IM} - f_c\rvert = 2f_{LO}$	if $f_c < 2f_{IF}$

Local oscillator tuning range:
1. 995 kHz to 2155 kHz
2. 85 kHz to 1245 kHz

2. RF Transceiver - Receiver

Example - FM Broadcast Receiver

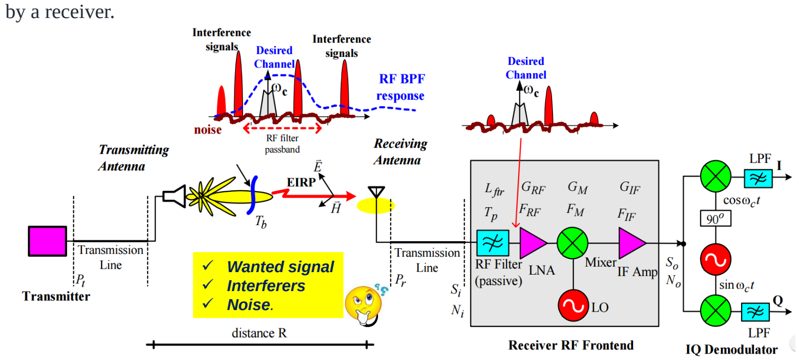
- The FM broadcast band covers 88 - 108 MHz. The channels are separated by 200 kHz and are assigned to odd multiples of 100 kHz.
- In almost all cases the IF frequency is chosen to be 10.7 MHz.
- As in the previous example "high-side" LO is often used, i.e., the LO tunes from 98.7 to 118.7 MHz. Why ?
- Notice that the IF used for FM broadcast receivers is substantially higher than that used for AM receivers.
- This choice is motivated by the fact that the image frequency is separated from the desired carrier frequency by 2fIF.
- Generally speaking, in order to make the preselector filter relatively easy to build and tune, the IF frequency must be raised as the carrier frequency increases.

2. RF Transceiver - Receiver

HW: 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 18 (Image-reject mixer)

3. Noise in RF/Microwave Systems

- The effect of noise is critical to the performance of most RF/Microwave communications, radar, and remote sensing systems.
- Noise ultimately determines the threshold for the minimum signal that can be reliably detected by a receiver.

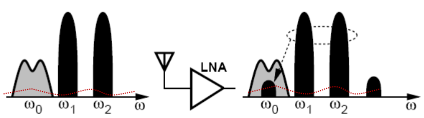


Noise power in a receiver will be introduced from:
- ✓ The external environment through the receiving antenna – External Noise
- ✓ Internal circuits of receivers – Internal Noise

3. Noise in RF/Microwave Systems

External noise may be introduced into a system either by a receiving antenna or by electromagnetic coupling.

Some sources of external RF noise include the following:
- ✓ Thermal noise from the ground
- ✓ Cosmic background noise from the sky
- ✓ Noise from stars (including the sun)
- ✓ Lightning
- ✓ Gas discharge lamps
- ✓ Radio, TV, and cellular stations
- ✓ Wireless devices
- ✓ Microwave ovens
- ✓ Deliberate jamming devices



Noise vs. Interferer

3. Noise in RF/Microwave Systems

Internal Noise generated in a device or component is usually caused by random motions of charges or charge carriers in devices and materials.

Such motions may be due to any of several mechanisms, leading to various types of noise:

- Thermal noise is the most basic type of noise, being caused by thermal vibration of bound charges. It is also known as Johnson or Nyquist noise.
- Shot noise is due to random fluctuations of charge carriers in an electron tube or solid-state device.
- Flicker noise occurs in solid-state components and vacuum tubes. Flicker noise power varies inversely with frequency, and so is often called 1/ f -noise.

3. Noise in RF/Microwave Systems

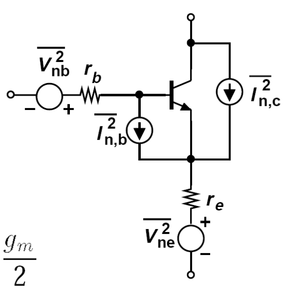
Noise in Bipolar Transistors

- Bipolar transistors contain physical resistances in their base, emitter, and collector regions, all of which generate thermal noise. Moreover, they also suffer from “shot noise” associated with the transport of carriers across the base-emitter junction.

$$\overline{I^2_{n,b}} = 2qI_B = 2q\frac{I_C}{\beta}$$

$$\overline{I^2_{n,c}} = 2qI_C,$$

$$g_m = I_C/(kT/q)$$

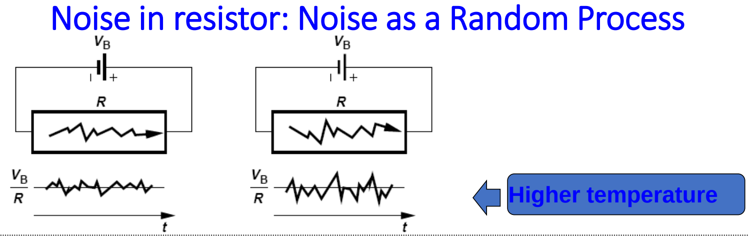


$$\overline{I^2_{n,c}} = 4kT\frac{g_m}{2}$$

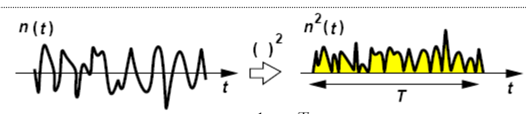
- In low-noise circuits, the base resistance thermal noise and the collector current shot noise become dominant. For this reason, wide transistors biased at high current levels are employed.

3. Noise in RF/Microwave Systems

Noise in resistor: Noise as a Random Process



- The average current remains equal to V_B/R but the instantaneous current displays random values

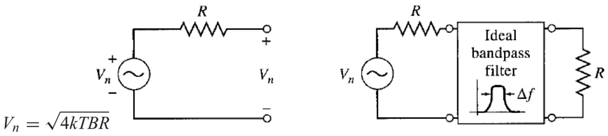


$$P_n = \lim_{T\to\infty}\frac{1}{T}\int_0^T n^2(t)dt$$

- T must be long enough to accommodate several cycles of the lowest frequency.

❑ Noise of Resistor

- The **noisy resistor** can be replaced with a **Thevenin equivalent circuit** of a **noiseless resistor** and a **random-noise voltage generator** v_n



Equivalent circuit of a noisy resistor delivering maximum power to a load resistor through an ideal bandpass filter

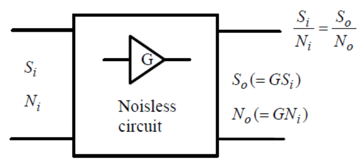
k = 1.380 × 10−23 J/K is Boltzmann's constant.

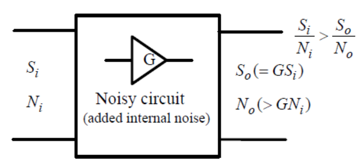
The **maximum power** transfer from the **noisy resistor** to a **load resistance** (Z_L) is when $\boxed{Z_L = R}$

$$P_n = \left(\frac{v_n}{2R}\right)^2 R = \frac{v_n^2}{4R} = \frac{\left(\sqrt{4kTBR}\right)^2}{4R} = kTB \Rightarrow \begin{cases} P_n(W) = 1.38\times10^{-23}\times T(K)\times B(Hz) \\ P_n(\mathbf{dBm}) = 10\log(T_K B_{KHz}) - 108.54 \\ \quad = 10\log(T_K B_{Hz}) - 138.54 \end{cases}$$

$$\text{* when } \begin{cases} \boxed{T = 290^0 K,\ B = 1Hz \Rightarrow P_n = kTB = -174\ (\text{dBm})} \\ T = 290^0 K,\ B = 1kHz \Rightarrow P_n = kTB = -144\ (\text{dBm}) \\ T = 290^0 K,\ B = 1MHz \Rightarrow P_n = kTB = -114\ (\text{dBm}) \end{cases}$$

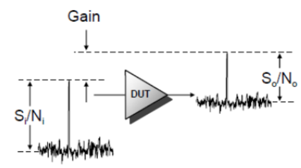
Signal to Noise Ratio $(SNR) = \dfrac{\text{Desired signal power}}{\text{Undesired noise power}}$





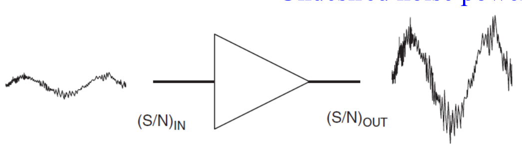
∵ Circuit noise added to input noise

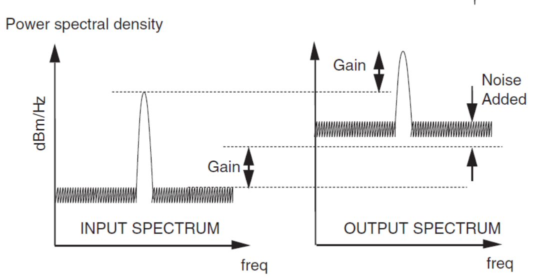
Noise Figure (NF or F): $\boxed{F = \dfrac{(S/N)_i}{(S/N)_o} \geq 1}$



Noise Figure: measures the degradation in the signal-to-noise ratio between the input and output of the component

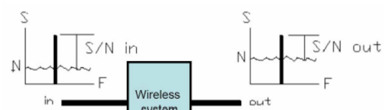
Signal to Noise Ratio $(SNR) = \dfrac{\text{Desired signal power}}{\text{Undesired noise power}}$

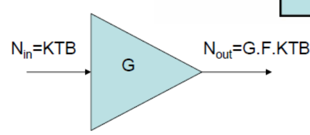




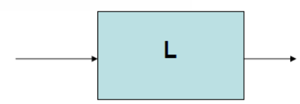
Noise Figure: measures the degradation in the signal-to-noise ratio between the input and output of the component

Noise Figure





Lossy Circuits



NF=L

$$NF = F = \frac{SNR_{in}}{SNR_{out}} = \frac{Si}{So}\cdot\frac{No}{Ni}$$

$No = F.Ni.G,$

$Thermal\ Noise = KTB$

K : Boltzmann Constant

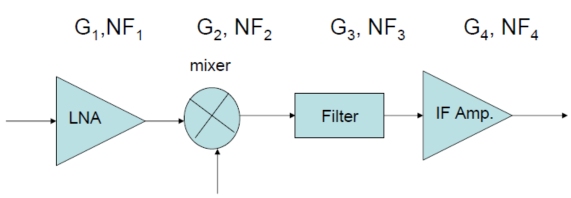
T : Temperature(K)

B : Bandwidth

No=N_{in}G + N_a =KTBG + N_a
N_a is the added noise power generated from internal components

$$NF = \frac{N_o}{GN_{in}} = \boxed{\frac{GN_{in} + N_a}{GN_{in}}} = \boxed{\frac{N_{in} + \frac{N_a}{G}}{N_{in}}}$$

Noise Figure of Cascaded Stages



Friis equation

$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \ldots + \frac{NF_n - 1}{G_1 G_2 \ldots G_{n-1}}$$

NF_n is the noise factor in linear (not in dB) of the n−th stage
G_n is the power gain in linear (not in dB), too.

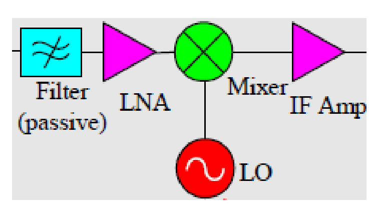
What do we learn from the Friis equation ?

Noise Figure Exercise

1. In a two –stage system, show that :

$$NF = NF_1 + \frac{NF_2 - 1}{G1}$$

2. Using Friis equation, find the total NF of a system having the BPF (with the insertion loss of L) as the first block.
What is your conclusion from the result ?



3. Noise in RF/Microwave Systems

Sensitivity:

Sensitivity Min. detectable signal by the receiver according to a fixed S/N determined by the BER

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{in}}{P_{noise,in}} \cdot \frac{1}{SNR_{out}}$$

$$sensitivity = P_{in,\min} = NF.P_{noise,in}.SNR_{out}$$

$$P_{in,\min} = NF.(KTB).SNR_{out,\min}$$

$$10Log(KTB) = -174 + 10Log(B)$$

$$P_{in,\min}(dBm) = NF|_{dB} - 174\text{dBm/Hz} + 10\text{Log}B + SNR_{out,\min}|_{dB}$$

Noise floor = Total integrated noise of the system

$$\text{F} = NF|_{dB} - 174\text{dBm/Hz} + 10\text{Log}B$$

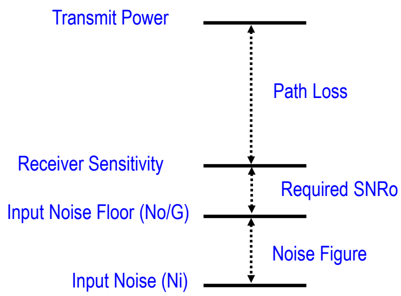
$$\boxed{NF = \frac{N_o}{GN_{in}} = \frac{GN_{in} + N_a}{GN_{in}} = \frac{N_{in} + \frac{N_a}{G}}{N_{in}}}$$

3. Noise in RF/Microwave Systems

Required Receiver Sensitivity – A Qualitative View

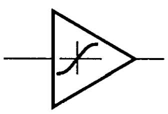
What is the required receiver NF to achieve a certain level of sensitivity?

$$\boxed{NF = \frac{N_o}{GN_{in}} = \frac{GN_{in} + N_a}{GN_{in}} = \frac{N_{in} + \frac{N_a}{G}}{N_{in}}}$$



- To find Receiver NF
  - Transmit Power – FCC regulated
  - Path loss
  - Receiver sensitivity – govern by standards and applications
  - Required SNR – depends on BER requirement and modulation scheme
  - Noise floor – thermal noise or circuit noise limited depending on the modulation schemes

4. Nonlinear Distortion in RF/Microwave Systems

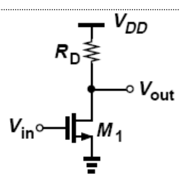


$$y(t) = \alpha x(t).$$ linear

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$ nonlinear

- **The input/output characteristic of a memoryless nonlinear system can be approximated with a polynomial**

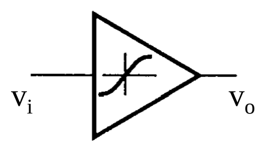
$$\begin{aligned} V_{out} &= V_{DD} - I_D R_D \\ &= V_{DD} - \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{in} - V_{TH})^2 R_D \end{aligned}$$



- **In this idealized case, the circuit displays only second-order nonlinearity**

4. Nonlinear Distortion in RF/Microwave Systems

❑ Nonlinear Distortion



$$v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots,$$

Taylor series: $\mathbf{f(x)=} \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

- **Harmonic generation** (multiples of a fundamental signal)
- **Gain Compression** (gain reduction in an amplifier)
- **Desensitization - Blocking**
- **Inter-modulation Distortion** (products of a two-tone input signal)

4. Nonlinear Distortion in RF/Microwave Systems

Effects of Nonlinearity: Harmonic Distortion

$x(t) = A\cos\omega t$, then

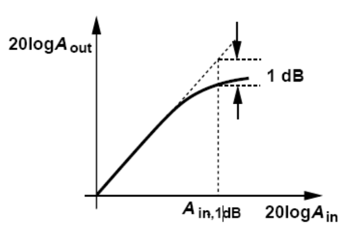
$$\begin{aligned} y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t \\ &= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t. \end{aligned}$$

DC | Fundamental | Second Harmonic | Third Harmonic

- **Even-order harmonics result from α_j with even j**
- **nth harmonic grows in proportion to A^n**

4. Nonlinear Distortion in RF/Microwave Systems

Gain Compression: 1-dB Compression Point



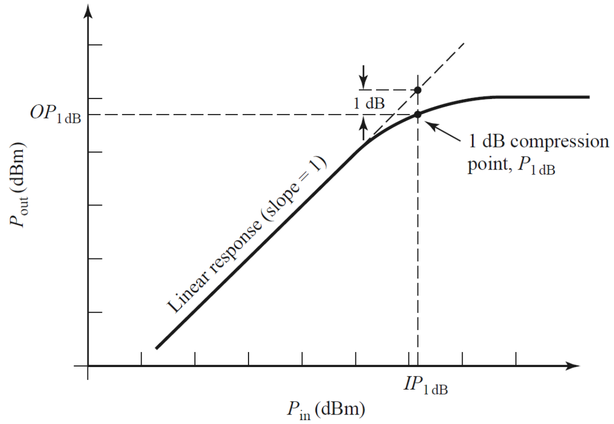
$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2\right| = 20\log|\alpha_1| - 1\text{ dB}.$$

$$A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

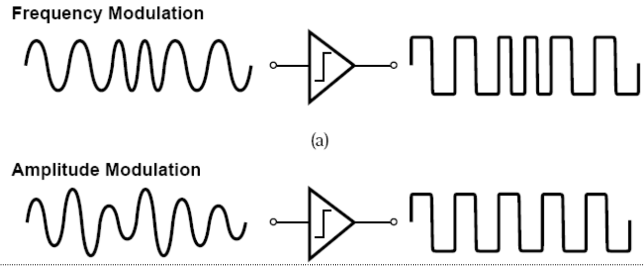
- **Output falls below its ideal value by 1 dB at the 1-dB compression point**
- **Peak value instead of peak-to-peak value**

Pin = ? , Pout = ?
Voltage gain ?
Power Gain ?

Nonlinear Distortion – Harmonic Generation and Gain Compression

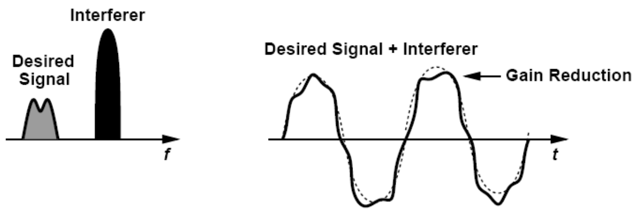


Gain Compression: Effect on FM and AM Waveforms



- FM signal carries no information in its amplitude and hence tolerates compression.
- AM contains information in its amplitude, hence distorted by compression

Gain Compression: Desensitization - Blocking



$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots.$$

For $A_1 << A_2$

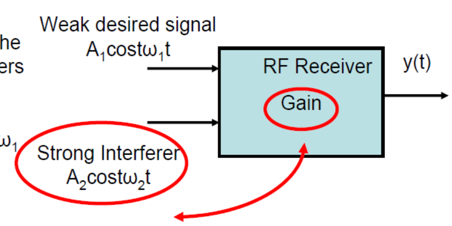
$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

- Desensitization: the receiver gain is reduced by the large excursions produced by the interferer even though the desired signal itself is small.

Nonlinear Distortion – Desensitization and Blocking

Effect of system nonlinearity on the capability of the receiver to extract the weak signal from the strong interferers (Blockers)

By checking the desired frequency ω_1



$$y(t) = (\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2)\cos\omega_1 t, \quad where \quad \alpha_3 < 0$$

$$for \quad A_1 << A_2$$

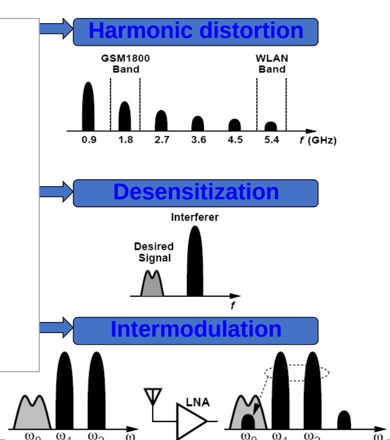
$$y(t) = (\alpha_1 + \frac{3}{2}\alpha_3 A_2^2) A_1 \cos\omega_1 t$$

The gain is compressed now by the effect of the strong interferer. We call this interferer (Blocking signal)

Effects of Nonlinearity: Intermodulation— Recall Previous Discussion

So far we have considered the case of:

- Single Signal
- Signal + one large interferer
- Signal + two large interferers



Effects of Nonlinearity: Inter-modulation Distortion (IMD)

assume $x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$

Thus

$$y(t) = \alpha_1(A_1\cos\omega_1 t + A_2\cos\omega_2 t) + \alpha_2(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^2 + \alpha_3(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^3$$

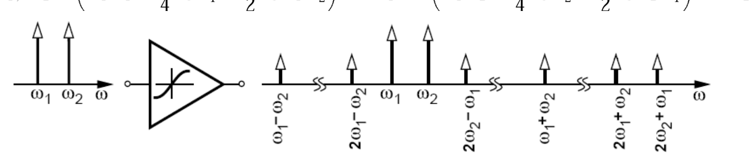
Intermodulation products:

$$\omega = 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t$$

$$\omega = 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 - \omega_1)t$$

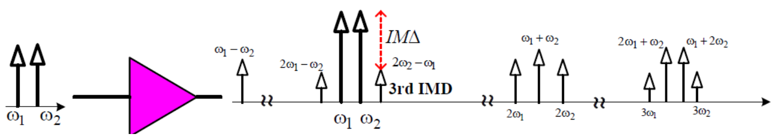
Fundamental components:

$$\omega = \omega_1, \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$$

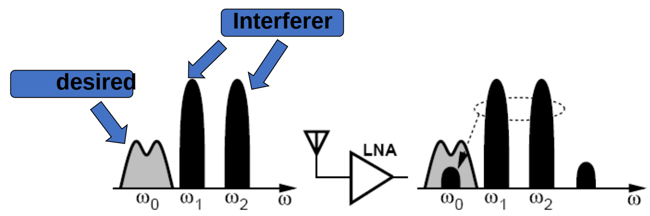


Nonlinear Distortion – Inter-modulation Distortion (IMD)

=> $(m\omega_1 \pm n\omega_2)$ terms : $(m+n)$th order IM products $m, n = 1, 2, 3, 4$.........



Intermodulation Product Falling on Desired Channel



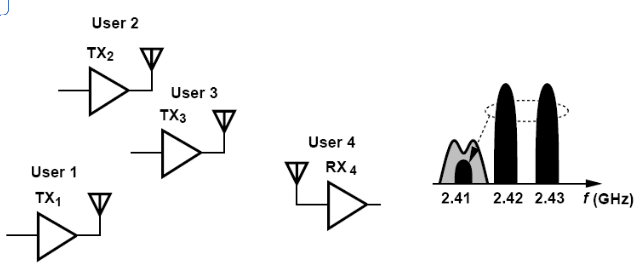
$$2\omega_1 - \omega_2 = \omega_0$$

- **A received small desired signal along with two large interferers**
- **Intermodulation product falls onto the desired channel, corrupts signal.**

Example of Intermodulation

Suppose four Bluetooth users operate in a room as shown in figure below. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz. At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.

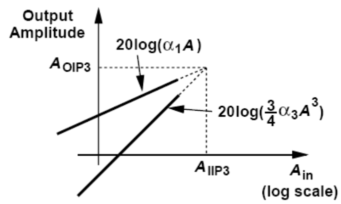
Solution:



Since the frequencies transmitted by Users 1, 2, and 3 happen to be equally spaced, the intermodulation in the LNA of R_{X4} corrupts the desired signal at 2.410 GHz.

Intermodulation: Third Intercept Point

is the point where the output power at ω_1 equals to output power at $(2\omega_1 - \omega_2)$

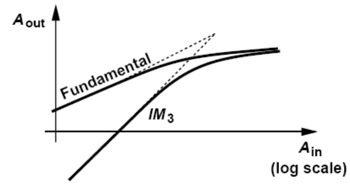


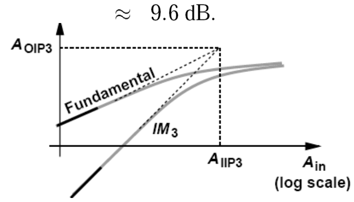
$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}}$$

$$\approx 9.6 \text{ dB}.$$

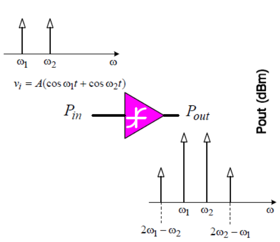


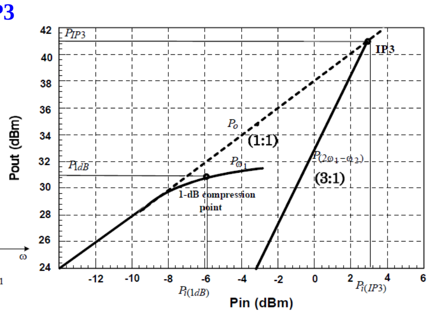


- **IP3 is not a directly measureable quantity, but a point obtained by extrapolation**

Nonlinear Distortion – Inter-modulation Distortion (IMD)

3rd order intercept point : IP3





- linear output power

$$P_o(dBW) = 10\log\left\{\frac{(k_1 A)^2}{2R}\right\} = 10\log\left\{\frac{(A)^2}{2R}\right\} + 10\log(k_1)^2 = P_i + \underbrace{10\log(k_1)^2}_{\text{a constant}}$$

=> $P_o : P_i$ *line* **slope 1:1**

Nonlinear Distortion – Inter-modulation Distortion (IMD)

3rd order intercept point : IP3

- 3rd-order IMD output power

$$P_{(2\omega_1-\omega_2)} = 10\log\left\{\frac{(\frac{3}{4}k_3A^3)^2}{2R}\right\} = 10\log\left[\frac{(A^2)}{2R}\right]^3 + 10\log\left\{[2R]^2(\tfrac{3}{4}k_3)^2\right\}$$

$$= 3\times 10\log\left[\frac{(A^2)}{2R}\right] + 10\log\left\{\tfrac{9}{4}(Rk_3)^2\right\} = 3P_i + \underbrace{10\log\left\{\tfrac{9}{4}(Rk_3)^2\right\}}_{\text{a constant}}$$

=> $P_{(2\omega_1-\omega_2)} : P_i$ *line* **slope 3:1**

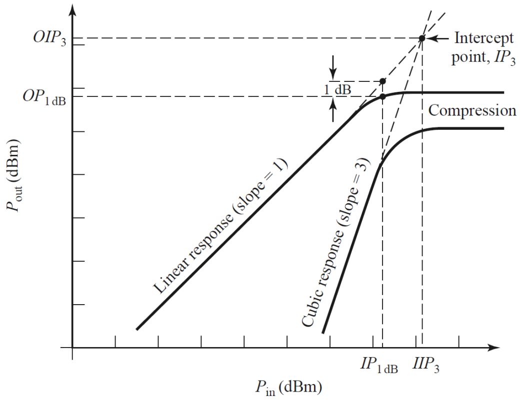
=> **3rd-order intercept point** (IP3) is the ***intersection point*** of

$P_o : P_i$ **line** & $P_{(2\omega_1-\omega_2)} : P_i$ **line**

- **Output power at IP3 (OIP3)** : P_{IP3}
- **Input power at IP3 (IIP3)** : $P_{i(IP3)}$

Nonlinear Distortion – Inter-modulation Distortion (IMD)

3rd order intercept point : IP3 Input IP is the point where the output power at ω_1 equals to output power at $(2\omega_1 - \omega_2)$



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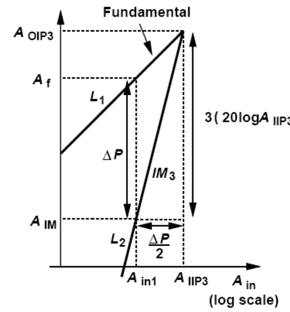
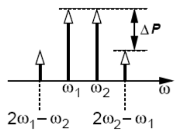


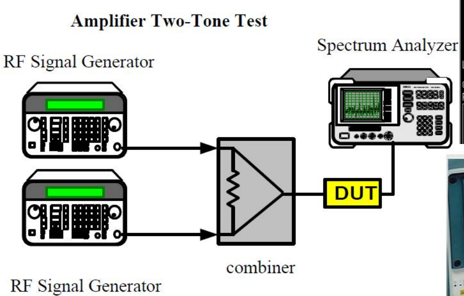


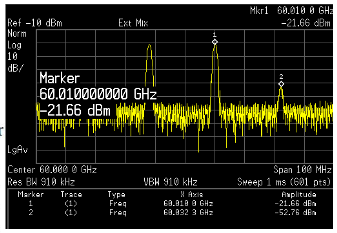
$$IIP_3|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}|_{dBm}$$

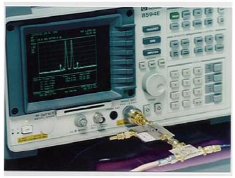
- For a given input level (well below P_{1dB}), the IIP_3 can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level, where all values are expressed as logarithmic quantities.

Nonlinear Distortion – Inter-modulation Distortion (IMD)

Determine IP3 by Spectrum Measurement







(Wilkinson power divider can be used as a combiner)

Example of Third Intercept Point

A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What IIP_3 is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.

Solution:

G_1 G_2

$$\frac{1}{IIP_3} = \frac{1}{IIP_{3,1}} + \frac{G_1}{IIP_{3,2}} + \frac{G_1 . G_2}{IIP_{3,3}} + \ldots$$

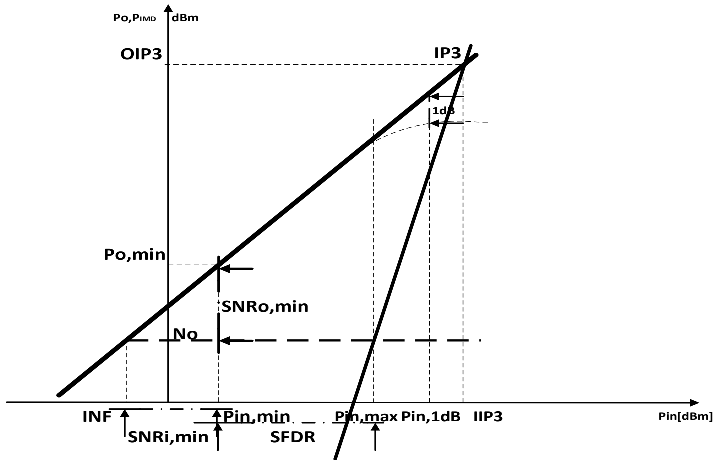
IIP₃: Power quantity
G: Power gain

Nonlinear Distortion – SFDR)

Spurious Free Dynamic Range SFDR

1. Dynamic Range (DR): Ratio of the maximum input level that the circuit can tolerate to the minimum input level at which the circuit provides reasonable signal quality.
2. Upper-end of the dynamic range (DR) depends on the intermodulation behavior
3. The lower end depends on sensitivity and NF

Nonlinear Distortion – SFDR)



4. Nonlinear Distortion in RF/Microwave Systems

Nonlinear Distortion – SFDR)

Spurious Free Dynamic Range SFDR

$$SFDR = P_{in,\max} - P_{in,\min} = P_{in,\max} - (-174 + 10\log B + SNR_{out,\min} + NF)$$

$P_{in,\max}$ = maximum input level in a two-tone test, where IM3 ≤ noise floor

$$SFDR = \frac{2(\mathrm{IIP_3} - \mathrm{F})}{3} - SNR_{out,\min}$$

Prove ?