RADIO COMMUNICATION CIRCUITS

Chapter 2

Passive Components, Resonators and Impedance Matching

Huynh Phu Minh Cuong
hpmcuong@hcmut.edu.vn

Department of Telecommunications
Faculty of Electrical and Electronics Engineering
Ho Chi Minh city University of Technology

RADIO COMMUNICATION CIRCUITS

Chapter 2

Passive Components, Resonators and Impedance Matching

Textbook:

[1] Steven J. Franke, Wireless Communication Systems, UIUC
Chapter 3, 4, 6

RADIO COMMUNICATION CIRCUITS

Chapter 2

Passive Components, Resonators and Impedance Matching

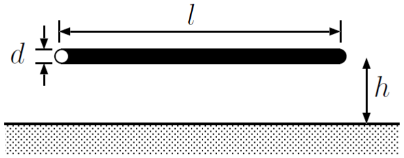
1. **High Frequency Passive Components (Chapter 3)**
2. **RLC Resonators (Chapter 4)**
3. **Impedance Matching Networks (Chapter 6)**

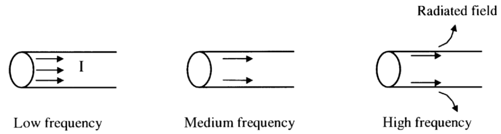
1. High Frequency Characteristics of Passive Components

- Passive electronic components such as resistors, capacitors, and inductors all exhibit:
  - Dielectric and/or ohmic loss
  - Energy storage in electric and magnetic fields within and surrounding the component.
- Accurate models for the impedance of real components must, therefore, include resistance, capacitance, and inductance.
- In addition, the skin effect, and inductance and capacitance associated with the conductors that connect the component to the rest of a circuit cannot be neglected.
- An accurate equivalent circuit model that includes all of these effects may be significantly more complex than the low-frequency circuit model that includes only the basic circuit element.
- It is important to understand the nature of these effects, so that they can be accounted for when designing practical circuits.
- Some of the most useful equivalent circuit models for common components will be discussed in this section.

1. High Frequency Characteristics of Passive Components

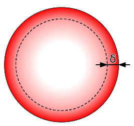
❑ Wire Above a Ground Plane - Resistance of Wires





❑ Resistance of Wires

- Current tends to flow near the external surface of a conductor at high frequencies.
- The current density directed along the axis of a conducting wire is largest at the surface of the conductor, and falls to small values inside the conductor.
- Most of the current flows within the cylindrical shell within one skin depth from the surface. The **skin depth** is denoted by δ [m]:



$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

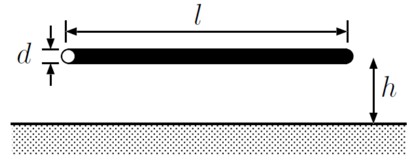
μ, σ are the permeability and conductivity of the conductor $\mu_o = 4\pi \times 10^{-7}\mathrm{H/m}$

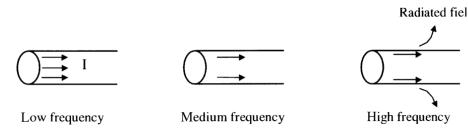
Material	σ S/m
Aluminum	3.5×10^7
Copper	5.8×10^7
Brass	1.5×10^7
Gold	4.1×10^7
Silver	6.1×10^7

- The skin depth in copper at 100 kHz, 10 MHz, and 1 GHz is approximately 0.2 mm, 0.02 mm, and 0.002 mm, respectively

1. High Frequency Characteristics of Components

❑ Wire Above a Ground Plane - Resistance of Wires





- At low frequencies

$$R_{DC} = \frac{l}{\sigma A} = \frac{4l}{\pi d^2 \sigma}$$

- At higher frequencies, where the skin depth is small compared to the wire diameter, The AC resistance can then be written as

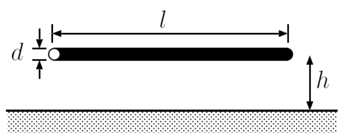
$$R = R_{DC}\frac{\pi(d/2)^2}{\pi d\delta} = R_{DC}\frac{d}{4\delta}, \quad \delta \ll d$$

- Obviously, the wire resistance at high frequency is much greater than its DC resistance.
- The AC resistance of wires will increase as $f^{1/2}$.

High Frequency Characteristics of Components

❑ Wire Above a Ground Plane - Inductance of wires

- The inductance, per unit length, of a wire with length l, diameter d, and distance h from a ground plane is (when d << l and h << l):



$$L = \frac{\mu_0}{2\pi}\cosh^{-1}\frac{2h}{d}$$

- When h/d > 1, then the following approximation is useful: $L \simeq \frac{\mu_0}{2\pi}\ln\frac{4h}{d}$

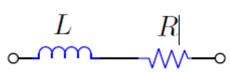
- For values of h/d in the range 1 to 100, above equation predicts that L ranges from 2.8 nH/cm to 12 nH/cm.

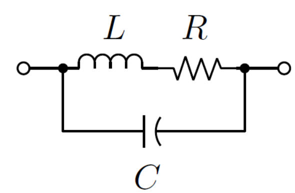
  - At 100 MHz the inductive reactance of such a wire, $X_L = \omega L$, will be approximately 3 Ω/cm, and at 1 GHz it is 30 Ω /cm.
  - For short wires, the resistance is often small enough to be ignored, however **the inductance of the wire is often significant**. In a circuit where impedances are relatively low, the series impedance of even a short connecting lead may have a significant impact on circuit performance.
  - This leads to a fundamental rule of RF circuit design –
    - ✓ At high frequencies it is important to keep the length of interconnecting wires and circuit-board traces short in order to minimize lead inductance.
    - ✓ When components are separated by significant distances, interconnections must be treated as distributed circuit elements, and transmission line models are used to model the conductors that interconnect components.

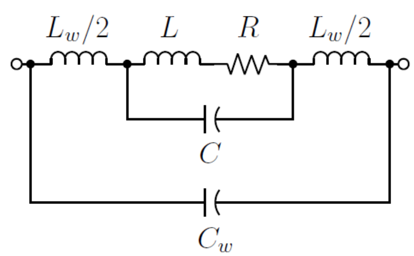
High Frequency Characteristics of Components

❑ Resistors

- Several types of resistors are used in RF circuits, including wire-wound, carbon composition, thick film, and thin film units.
- Resistors can be modeled using the equivalent circuit



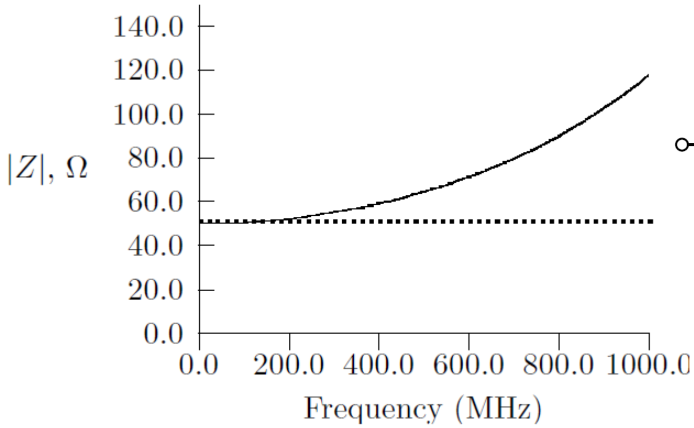


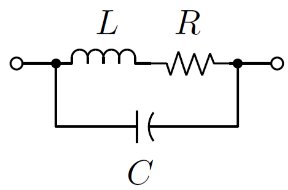


- The inductor in the model represents the inductance associated with the current path through the resistor, and the capacitor represents the capacitance between the two electrodes used to connect the resistor to external circuitry.
- The unavoidable inductance and capacitance associated with the resistors (and other components) are sometimes termed parasitic inductance and capacitance. Both the inductance and capacitance of a resistor depend on the geometry and dimensions of the resistor.

Resistors

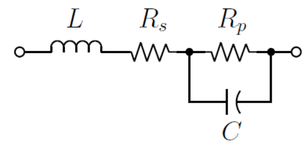
- For example, consider a 50 Ω resistor with lead inductance L = 10 nH, shunt capacitance C = 1 pF. The magnitude of the impedance versus frequency up through 1 GHz calculated using the full model. In this case the series inductance acts to increase the impedance at high frequencies.

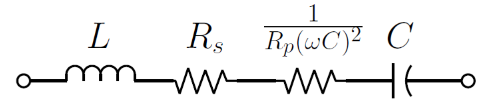




Capacitor

- Capacitors are constructed by separating two conducting electrodes by an insulating medium such as air, or a low-loss dielectric material.
- Loss in the dielectric is modeled as a resistance (Rp) in parallel with the intrinsic capacitance.
- The inductance associated with the current path through the electrodes and any connected leads appears in series as shown in model.
- A resistance in series with the inductor (Rs) models losses in the electrodes and leads.





$R_{esr} = R_s + 1/(R_p\omega^2 C^2)$ equivalent series resistance

- The Q of a capacitor at any frequency is the ratio of the reactance and the ESR

$$Q = \frac{|X|}{R_{esr}} \qquad X = \omega L - 1/(\omega C)$$

- The dissipation factor, d, is the inverse of the Q : $d = \frac{1}{Q} = \frac{R_{esr}}{|X|}$

Capacitor

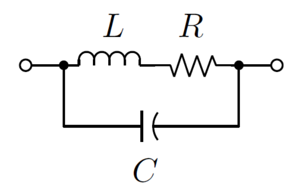
- The dissipation factor is also called the loss tangent because it is the complement of the tangent of the phase angle associated with the capacitor impedance.

$$\tan\delta = \frac{R_{esr}}{|X|} = \omega C R_{esr}$$

- All capacitors will have a series resonant frequency: $f_s = 1/(2\pi\sqrt{LC})$
- Above this frequency, the inductive reactance dominates, and the net reactance is positive. Hence, capacitors are inductive at frequencies above fs! For a given package type the inductance will be roughly independent of the capacitance value, hence the series resonant frequency will be lower for larger values of capacitance.

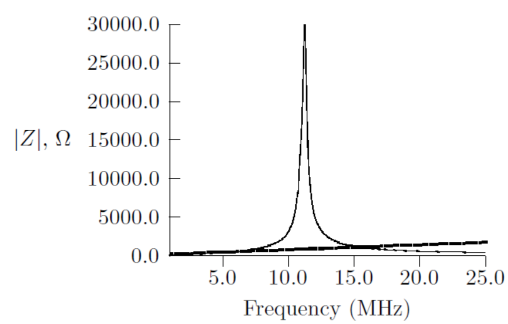
Inductor

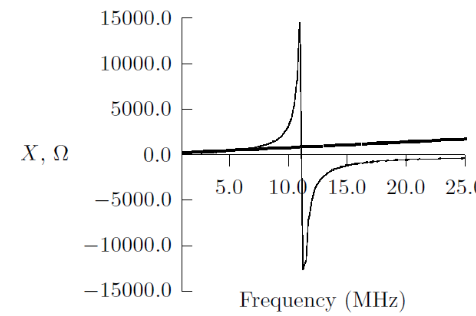
- Equivalent circuit model for an inductor



$$R = r_{DC}\frac{n\pi dD}{4\delta}\ \Omega$$

where r_{DC} is the DC resistance per unit length of the wire, d is the wire diameter, D is the coil diameter, n is the number of turns in the coil, and ' is the skin depth in the wire.





Impedance/Reactance versus frequency for a 10 μH inductor with series resistance of 15 Ω and distributed capacity of 20 pF. The dotted line shows the impedance/reactance for an ideal 10 μH inductor.

High Frequency Characteristics of Passive Components

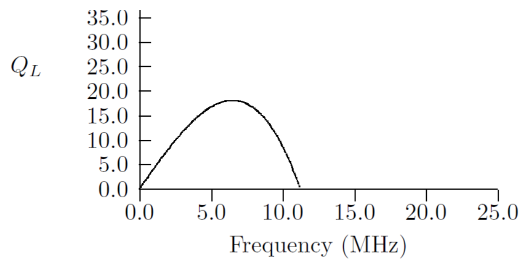
Inductor

- When considering a particular inductor for use in a circuit, the designer needs to be aware of the parallel resonant frequency as well as the "Quality Factor," or Q, of the inductor. The Q of an inductor is defined to be the ratio of inductive reactance and resistance associated with the component, i.e.,

$$Q = \frac{|X_s|}{R_s}$$

where the impedance of the inductor is Z = Rs + jXs.

- The higher the Q, the better the inductor approximates an "ideal" component. The Q is an important parameter if the inductor is to be used in a resonant circuit, filter, or matching network.



RLC Networks and Resonators

Series RLC Resonator

- In RF communication systems, resonant circuits are extensively used to select the wanted signal and reject the unwanted signal.

Consider the series RLC circuit in a filter configuration where the output voltage is taken across the resistor, as shown in Figure 4.1. The voltage transfer function is

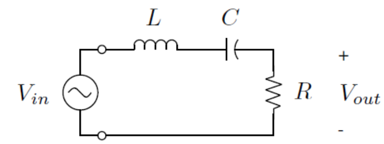


Figure 4.1: Series RLC circuit as a filter.

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + sL + \frac{1}{sC}} \tag{4.1}$$

We will consider sinusoidal excitation under steady-state conditions, in which case we are interested in the frequency response, $H(j\omega)$:

$$H(j\omega) = \frac{R}{R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)} \tag{4.2}$$

RLC Networks and Resonators

Series RLC Resonator

When $\omega = 1/\sqrt{LC}$, the phase shift of the transfer function is zero; this is called the "resonant frequency," ω_o, of the network and is the frequency at which the inductive and capacitive reactances are exactly equal in magnitude and, consequently, cancel each other:

$$\omega_o = \frac{1}{\sqrt{LC}}$$

The transfer function depends on R, L, and C, but only two parameters are necessary to specify the characteristics of the function. Define another quantity Q_s where:

$$Q_s = \frac{\omega_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The frequency response function can be rewritten in terms of only ω_o and Q_s:

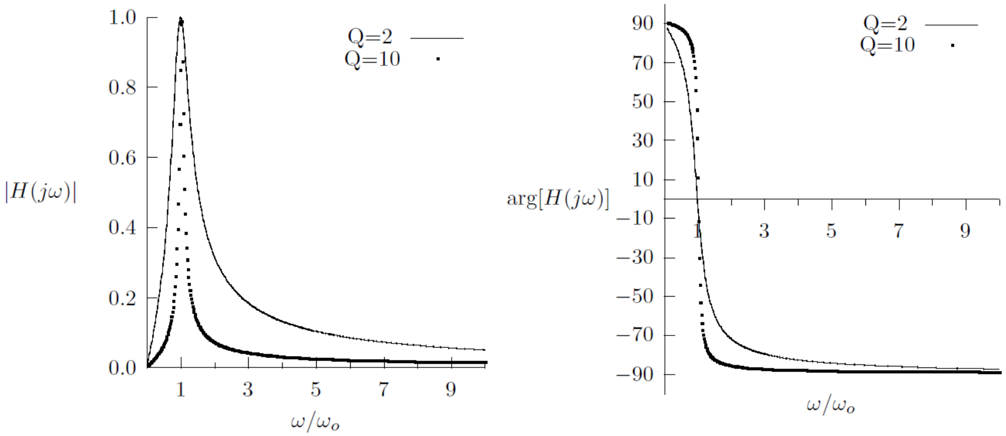
$$H(j\omega) = \frac{1}{1 + jQ_s\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

The parameter Q_s is referred to as the series resonant circuit "Q." In a subsequent section it will be shown that the inverse of this quantity tells us what fraction of the total energy stored in the RLC circuit is dissipated in one complete cycle of the resonant frequency.

RLC Networks and Resonators

Series RLC Resonator

$$H(j\omega) = \frac{1}{1 + jQ_s\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$



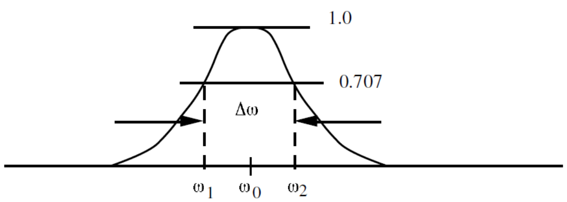
Magnitude and (b) phase of the voltage frequency response for Q=2 (solid line) and Q=10 (dotted line).

Series RLC Resonator

The RLC filter has a "bandpass" characteristic. The separation between the half-power (-3 dB) frequencies is often used to specify the bandwidth of a filter. The 3 dB bandwidth of the filter can be found by determining the difference between the frequencies where $|H(j\omega)| = 0.707$. Denote the lower and upper -3 dB frequencies by ω_1 and ω_2 as shown in Figure 4.4. Then $\Delta\omega = (\omega_2 - \omega_1)$ is referred to as the "3 dB bandwidth" of the filter. It is left as an exercise to show that

$$\Delta\omega = (\omega_2 - \omega_1) = \frac{\omega_o}{Q_s}$$

The bandwidth of a series RLC filter is inversely proportional to the Q_s of the circuit.



- Qs describes the frequency selectivity of the Series RLC Resonator.

Series RLC Resonator

Use a series RLC circuit to couple a voltage source with negligible source resistance to a 50 Ω load as shown in Figure 4.5. The circuit should have a center frequency of 5 MHz and a 3 dB bandwidth of 100 kHz. The bandwidth and center frequency determine Q_s :

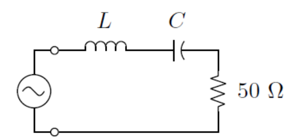


Figure 4.5: Circuit with 5 MHz center frequency and a 3 dB bandwidth of 100 kHz

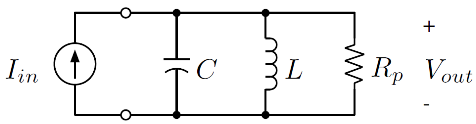
$$Q_s = \frac{f_o}{\Delta f} = \frac{5\,\text{MHz}}{100\,\text{kHz}} = 50 = \frac{\omega_o L}{R} = \frac{2\pi(5\times10^6)L}{50}$$

so

$$L = \frac{50\,50}{2\pi(5\times10^6)} = 79.6\,\mu\text{H}$$

$$C = \frac{1}{\omega_o^2 L} = 12.7\,\text{pF}$$

Parallel RLC Resonator



Parallel RLC as a filter

A parallel RLC circuit being driven by an ideal current source is shown in Figure 4.6. In this application the input current and output voltage are related by an impedance function, i.e.,

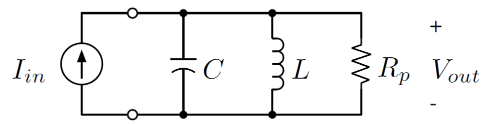
$$\frac{V_{out}(s)}{I_{in}(s)} = Z(s) \quad \text{(impedance)}$$

$$Z(s) = \left[\frac{1}{R_p} + \frac{1}{sL} + sC\right]^{-1}$$

For sinusoidal steady-state excitation

$$Z(j\omega) = \frac{R_p}{1 + jQ_p\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Parallel RLC Resonator



Parallel RLC as a filter

where

$$\omega_o = \frac{1}{\sqrt{LC}}$$

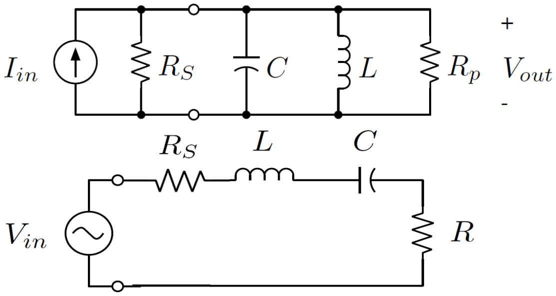
$$Q_p = \frac{R_p}{\omega_0 L} = \sqrt{\frac{C}{L}}R_p$$

This transfer function has exactly the same form as that of the series RLC circuit except for the scaling factor, R_p. Note, however, that the "Q" is defined differently for the parallel RLC. As before, the 3dB bandwidth is

$$\Delta\omega = \frac{\omega_o}{Q_p}$$

- Q_p describes the frequency selectivity of the Parallel RLC Resonator.

❑ Parallel RLC Resonator - Unloaded vs Loaded Q of RLC circuits



$$Q_p' = \frac{R_p \| R_S}{\omega_o L}$$

$$Q_s' = \frac{\omega_o L}{R_S + R}$$

RLC Resonators driven by a source with finite source impedance.

- Compared to the case with infinite source impedance, the finite source impedance causes the Q to be reduced and, hence, the bandwidth to be increased.
- It is common practice to call the Q of the resonant circuit alone (either series or parallel RLC) the unloaded Q, and the Q of the composite circuit, which includes the source resistance and any other resistances that are external to the LC resonator, the loaded Q.
- The loaded Q is always smaller than the unloaded Q.

❑ RLC Quality Factor - Q

- Q describes the frequency selectivity of the Resonators and Circuit characteristics.
- The general definition of Q for a system is:

$$Q = 2\pi \frac{\text{Maximum instantaneous stored energy}}{\text{Energy dissipated per cycle}}$$

$$= 2\pi f \frac{\text{Maximum instantaneous stored energy}}{\text{Time} - \text{average power dissipated}}$$

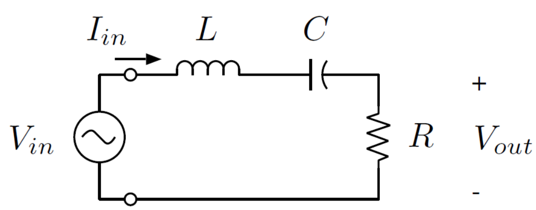
- This definition can be applied to resonant and nonresonant circuits. If this energy based definition is applied to resonant second-order RLC circuits the result is compatible with the (Qs,Qp) defined in the previous section.
- The energy definition is also commonly applied to characterize lossy inductors or capacitors which, by themselves, are non-resonant.

❑ RLC Quality Factor - Q

- We shall first show that the energy definition is consistent with the resonant-circuit Qs that has already been defined for series RLC circuits.
- At resonant frequency:

$$V_{out} = V_{in}$$

$$I_{in} = V_{in}/R$$



- The fact that the voltage across the capacitor is 90 degrees out of phase with the current through the inductor means that the current maximizes at the time when the capacitor voltage is zero.
- At that instant in time, all of the stored energy resides in the inductor and the magnitude of the current phasor (which is the peak current magnitude) can be used to calculate the total stored energy. The stored energy is:

$$E_{max} = \frac{1}{2} L \left| I_{in} \right|^2 = \frac{1}{2} L \frac{\left| V_{in} \right|^2}{R^2}$$

❑ RLC Quality Factor - Q

- Alternatively, at the time instant when the capacitor voltage is maximum then the current in the system is zero and all of the stored energy resides in the capacitor. The magnitude of the capacitor voltage phasor can then be used to calculate the stored energy at that time:

$$E_{max} = \frac{1}{2} C |V_{cap}|^2 = \frac{1}{2} C |\frac{I_{in}}{\omega_o C}|^2 = \frac{1}{2} L |I_{in}|^2$$

- The stored energy comes out the same either way. Actually, it can be shown that the total stored energy in this driven resonant RLC circuit is a constant, so that the maximum instantaneous stored energy is equal to the energy stored at any instant of time.

RLC Quality Factor - Q

- The time-averaged power delivered to (and dissipated in) the network is:

$$P_{avg} = \frac{1}{2}\mathrm{Re}[V_{in}I_{in}^*]$$
$$= \frac{1}{2}|V_{in}|^2/R$$

Then using the energy definition of Q:

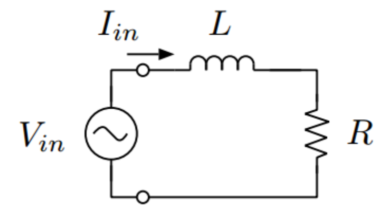
$$Q_s = 2\pi f_o \frac{E_{max}}{P_{avg}} = \frac{\omega_o L}{R}$$

This result is the same as the definition for the series resonant Q that was given earlier.

RLC Quality Factor - Q

- It is also common to apply the definition to a nonresonant circuit, e.g., consider an RL circuit as in:

$$I_{in} = \frac{V_{in}}{R + j\omega L}$$



Nonresonant RL circuit.

- In this case, the inductor is the only energy storage element. Because the current through the inductor varies sinusoidally, the stored energy will oscillate between zero and the maximum value given by:

$$E_{max} = \frac{1}{2}L|I_{in}|^2$$
$$= \frac{1}{2}L|V_{in}|^2\frac{1}{r_s^2 + \omega^2 L^2}$$

RLC Quality Factor - Q

The time-averaged power delivered is

$$P_{avg} = \frac{1}{2}\mathrm{Re}[V_{in}I_{in}^*]$$
$$= \frac{1}{2}|V_{in}|^2 \mathrm{Re}\left[\frac{1}{r_s - j\omega L}\right]$$
$$= \frac{1}{2}|V_{in}|^2 \frac{r_s}{r_s^2 + \omega^2 L^2}$$

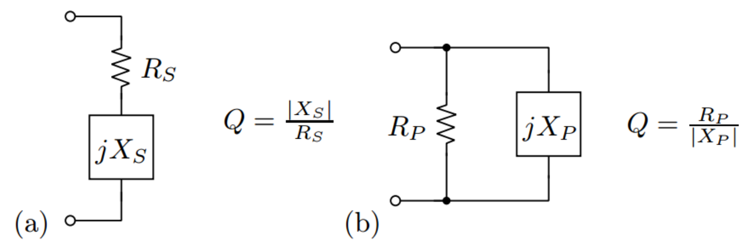
Therefore

$$Q_L = 2\pi f \frac{E_{max}}{P_{avg}} = \frac{\omega L}{r_s}$$

- In this case ω can be any frequency. This type of Q will be called the **component Q**.
- In this example, the RL circuit could represent a model for a lossy inductor.
- The component Q of an inductor evaluated at some frequency ω can be thought of as the resonant Qs or Qp that results if a lossless capacitor is added to form a resonant circuit at the frequency ω.

RLC Quality Factor - Q

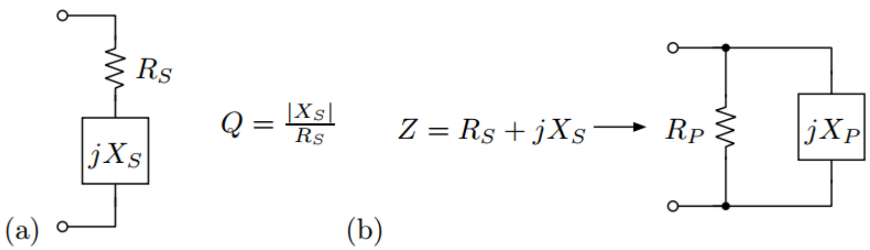
- The concept of component Q is often used to describe the properties of arbitrary circuit elements at a particular frequency.
- For example, if an arbitrary circuit element can be represented by a series impedance Z = Rs + jXs at some frequency, then applying the definition of Q to that component yields Q = |Xs|/Rs as illustrated in Figure (a). For a parallel representation of a circuit branch the component Q is as shown in Figure (b).



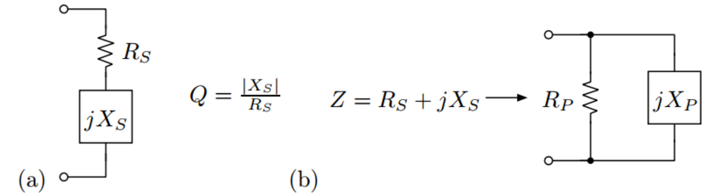
Definition of Q applied to a branch represented in (a) series form and (b) parallel form.

Series-to-Parallel Transformations

- Any circuit element has both a series and a parallel representation.
- Since the energy storage and dissipation properties of the element do not depend on how we represent it, the Q is the same for either representation.
- The component Q concept is useful for series-to-parallel impedance transformations.
- Suppose the series impedance representation for a circuit element is known at a particular frequency as shown in Figure (a)



Series-to-Parallel Transformations



The circuit Q is given by

$$Q = \frac{|X_s|}{R_s}$$

The equivalent parallel representation for the circuit element is shown in Figure 4.11(b). The equivalent parallel resistance and reactance are easily found after equating the impedances of the two models:

$$R_s = \frac{R_p}{1+Q^2} \qquad R_p = R_s(1+Q^2)$$

$$X_s = \frac{X_p}{1+\frac{1}{Q^2}} \qquad X_p = X_s\left(1+\frac{1}{Q^2}\right)$$

- Notice that the Q of the equivalent parallel circuit is Rp/|Xp| which is equal to Q.

Series-to-Parallel Transformations

- Thus, the Q of the equivalent parallel representation is the same as that of the original series representation (and vice versa).
- A useful simplification results if Q , 1, in which case

$$R_p \simeq R_s Q^2$$

$$X_p \simeq X_s$$

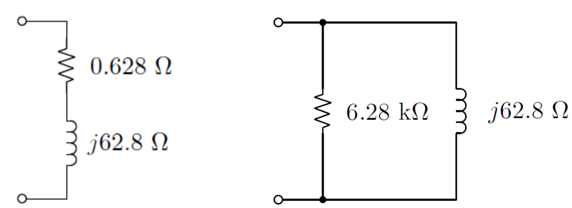
Clearly, a complex impedance that is represented in parallel form, i.e. as $Z_p = R_p || jX_p$ can be transformed to series form, $Z_s = R_s + jX_s$ by defining the Q of the parallel representation ($Q = R_p/|X_p|$) and then calculating R_s and X_s from

$$R_s = \frac{R_p}{1+Q^2}$$

$$X_s = \frac{X_p}{1+\frac{1}{Q^2}}$$

Series-to-Parallel Transformations - Example

A 1 μH inductor has a component Q of 100 at 10 MHz. Find a parallel representation for the inductor. At 10 MHz, $\omega L = 2\pi(10^7)(10^{-6}) = 62.8\,\Omega$. So $Q_L = 100 = \frac{\omega L}{r_s} = \frac{62.8}{}$.



Representation of 1 μH inductor with a component Q of 100 at 10 MHz

Therefore, $r_s = 0.628\,\Omega$. Since $Q_L \gg 1$,

$$X_p \simeq X_s = 62.8\,\Omega$$

$$R_p \simeq Q^2 r_s = 10^4(.628) = 6.28\,k\,\Omega$$

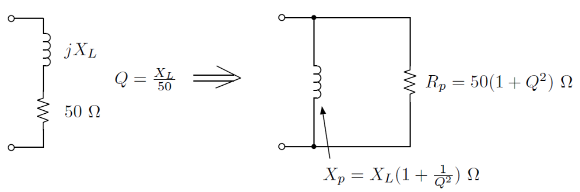
- The equivalent network at 10 MHz is shown in above figure.
- It is important to remember that this equivalent circuit is only valid at 10 MHz.

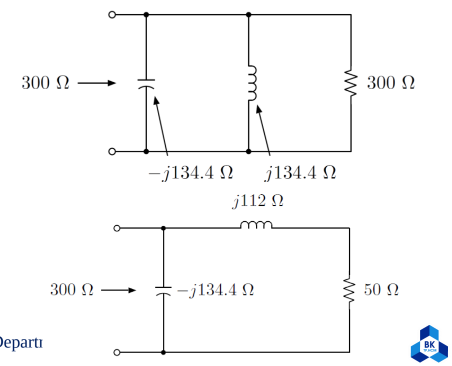
Series-to-Parallel Transformations - Example

Example - Impedance transformation

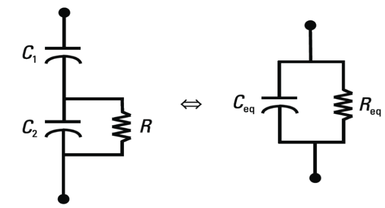
❖ Design a lossless network to transform 50Ω to 300 Ω using a capacitor and an inductor.

- We can start by putting an inductive reactance in series with the 50 Ω resistor.
- The equivalent parallel representation, on the right in Figure shows that the parallel resistance is larger than 50 Ω by the factor $1 + Q^2$.
- Setting $R_p = 300 = 50(1+Q^2)$ yields the Q of the circuit (Q = 2.24) and therefore the value of the series reactance, $X_L = 2.24(50) = 112\ \Omega$.
- Now a shunt capacitor can be added to cancel the inductive reactance. Going back to the original, series, representation yields the final solution for the lossless network that transforms a 50 Ω resistance into a 300 Ω resistance.



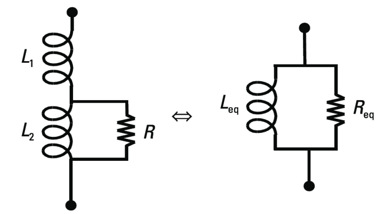


- **Tapped capacitors and inductors**



$$R_{eq} \approx R\left(\frac{C_1 + C_2}{C_1}\right)^2$$

$$C_{eq} \approx \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

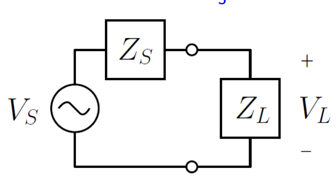


$$R_{eq} \approx R\left(\frac{L_1 + L_2}{L_2}\right)^2$$

$$L_{eq} \approx L_1 + L_2$$

Impedance Matching for Maximum Power Transfer

- In this section we review the motivation for impedance matching and introduce important concepts which will be used in later chapters. Let us first illustrate the basic principles of impedance matching for maximum power transfer.
- A source with impedance $Z_S = R_S + jX_S$ is connected to a load $Z_L = R_L + jX_L$. The peak voltage of the source is V_S:



$$I_L = \frac{V_L}{Z_L}$$

- The problem is to choose R_L and X_L to maximize P_L. The solution is:

$$Z_L = Z_S^*$$

- This maximum power is referred to as the power **av**ailable from the **s**ource, P_{avs}:

$$\boxed{P_{avs} \equiv P_L|_{Z_L=Z_S^*} = \frac{|V_S|^2}{8R_S}}$$

- The time-averaged real power is:

$$P_L = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\}$$

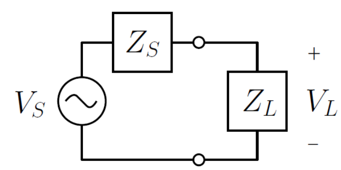
$$P_L = \frac{1}{2}|V_L|^2 Re\{\frac{1}{Z_L{}^*}\}$$

$$= \frac{1}{2}|V_L|^2 \frac{R_L}{|Z_L|^2}$$

$$P_L = \frac{1}{2}|V_S|^2 \frac{R_L}{|Z_L + Z_S|^2}$$

$$= \frac{1}{2}|V_S|^2 \frac{R_L}{(R_L + R_S)^2 + (X_L + X_S)^2}$$

Impedance Matching for Maximum Power Transfer



$$\boxed{P_{avs} \equiv P_L|_{Z_L=Z_S^*} = \frac{|V_S|^2}{8R_S}}$$

- The concept of available power is often used in the radio frequency literature. For example, the output power level of RF signal generators is usually specified by stating the available power in dBm, i.e., "decibels referred to 1 mW." A power level of 6 dBm indicates a power 6 dB higher than 1 mW, or approximately 4 mW.
- It is important to realize that when a signal generator is configured for a given output power level, that power level is the power available from the generator, Pavs, which is the power that the generator will deliver to a conjugately matched load. Signal generators commonly have source impedances of 50Ω (sometimes 75Ω), i.e., the generator will deliver the rated power only to a 50Ω load. With a different load impedance the power delivered to the load will be less than the rated value.

Impedance Matching Networks

❑ Impedance Matching for Maximum Power Transfer

❖ Mismatch Factor

- ➢ The degree to which the actual power delivered to an arbitrary load is smaller than the available power can be quantified in terms of a mismatch factor, **MF**, a quantity that depends on the degree of impedance mismatch between the source and load.
- ➢ For an arbitrary load impedance Z_L the mismatch factor is defined as the ratio of actual delivered power to available power:

$$MF = \frac{P_L}{P_{avs}} = \frac{4R_S R_L}{(R_S+R_L)^2+(X_S+X_L)^2} = \frac{4R_S R_L}{|Z_S+Z_L|^2}$$

- ➢ Mismatch factor is a real number and 0 < MF < 1.
- ➢ The mismatch loss, ML, in decibels is defined as: ML = −10 logMF.

Impedance Matching Networks

❑ Impedance Matching for Maximum Power Transfer

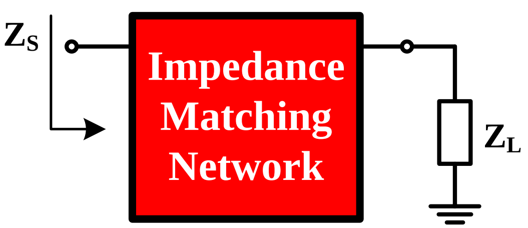
❖ Mismatch Factor

Suppose a $50\,\Omega$ signal generator has available power of 1 mW. The generator is to drive a load impedance of $250 + j100\,\Omega$. What is the power delivered to the load?

The problem could be solved by computing the power delivered to the load using Equation 6.1. Another approach is to compute the mismatch factor from Equation 6.7. This gives a mismatch factor $MF = 0.5$, so $P_L = P_{avs}MF = (1\,\text{mW})0.5 = 0.5\,\text{mW}$. Alternatively, we can express the powers in dBm and use the mismatch loss in dB. The mismatch loss $ML = -10\log(0.5) = 3\,\text{dB}$. The power available from the source is 1 mW, or $10\log\frac{1\,\text{mW}}{1\,\text{mW}} = 0\,\text{dBm}$. The power delivered to the load is $P_L = P_{avs} - ML = 0\,\text{dBm} - 3\,\text{dB} = -3\,\text{dBm}$. Note that a power level of $-3\,\text{dBm}$ is equal to 0.5 mW.

Impedance Matching Networks

Impedance Matching



What are Applications ?

- ➢ Maximum power transfer
- ➢ Transmission Lines
- ➢ Amplifier Design: PA, LNA
- ➢ RF Component Design

Requirement:

- ➢ Simple
- ➢ Use lossless components (L,C, Transmission line, Transformer)
- ➢ Harmonic termination

Design Approaches:

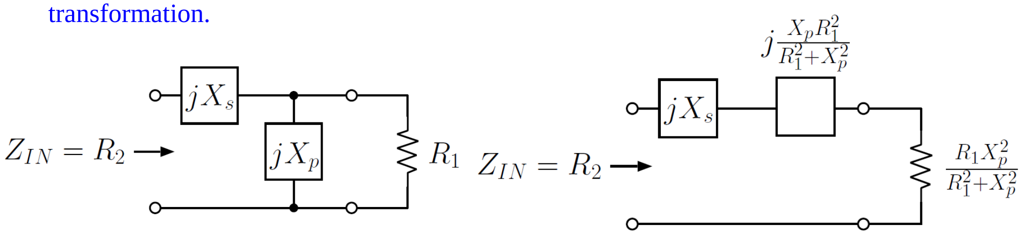
- ➢ Derive analytically component's values
- ➢ Using Smith Chart (ADS Smith Chart tool)

- ❖ Matching with Lumped Elements
- ❖ Single-Stub Matching Networks
- ❖ Double-Stub Matching Networks
- ❖ Quarter-wave Transformer

Impedance Matching Networks - L-Networks

❑ Resistive Terminations: $Z_L = R_1$, $Z_S = R_2$

- ➢ The goal is to transform R1 to R2 at one frequency.
- ➢ The unknown reactances Xs and Xp are easily found with the use of a parallel-to-series transformation.



- ➢ To make the input impedance equal to R_2, we choose

$$X_s = -\frac{X_p R_1^2}{R_1^2+X_p^2} \qquad X_p = \pm R_1\sqrt{\frac{R_2}{R_1-R_2}}$$

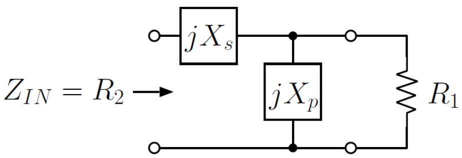
$$R_2 = \frac{R_1 X_p^2}{R_1^2+X_p^2} \qquad X_s = \mp\sqrt{R_2 R_1 - R_2^2}$$

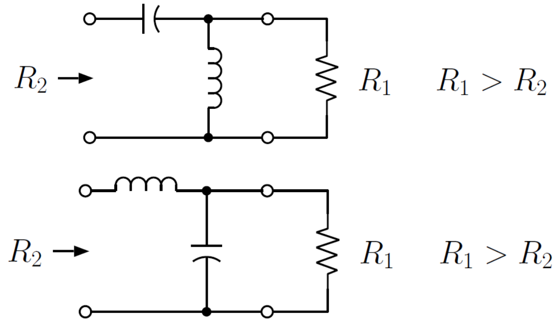
- ➢ The solutions yield real values for Xp and Xs only if $R_1 > R_2$.

Impedance Matching Networks - L-Networks

❑ Resistive Terminations: $Z_L = R_1$, $Z_S = R_2$

➢ Note that there are two possible solutions for the resistive matching problem corresponding to the upper and lower signs





➢ A rule for using a lossless L-network to match two resistances:

The shunt arm of the L-network is connected across the larger of the two resistances.

Impedance Matching Networks - L-Networks

❑ Resistive Terminations: $Z_L = R_1$, $Z_S = R_2$

➢ "Q" of an L-network:

$$Q = \left|\frac{X_p R_1^2/(R_1^2 + X_p^2)}{R_1 X_p^2/(R_1^2 + X_p^{2)})}\right| = \left|\frac{R_1}{X_p}\right| = \sqrt{\frac{R_1}{R_2} - 1}$$

Summary: L-network design equations

The design of an L-network can be summarized as follows. If the terminating resistances are denoted by R_{big} and R_{small} (where $R_{big} > R_{small}$), then the design equations can be written in terms of the network Q,

$$Q = \sqrt{\frac{R_{big}}{R_{small}} - 1},$$

as

$$X_p = \pm R_{big}/Q \qquad X_s = \mp R_{small} Q.$$

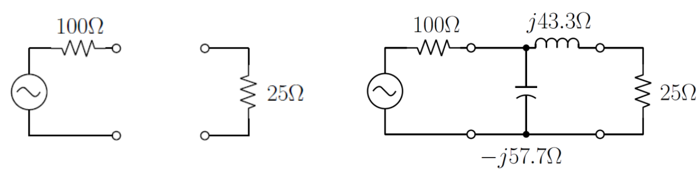
When the upper signs are chosen, the resulting network is of highpass type. The lower signs give a lowpass network. The L-network will be oriented such that the parallel arm is in shunt with R_{big}.

Impedance Matching Networks - L-Networks

❑ Resistive Terminations: $Z_L = R_1$, $Z_S = R_2$

Example - Matching resistive source and load with a low-pass L-network

Match a 100 Ω source to a 25 Ω load with a lossless L-network having a low-pass topology. Since the series arm of the L-network connects to the smaller of the two resistances, the L-



Example for L-net matching

network will be oriented as shown in Figure . A capacitor and inductor have been chosen for the shunt and series elements, respectively, because the problem statement specified a low-pass network.

$$Q = \sqrt{\frac{R_{big}}{R_{small}} - 1} = \sqrt{\frac{100}{25} - 1} = \sqrt{3}$$

$$X_p = X_C = -\frac{R_{big}}{Q} = -\frac{100}{\sqrt{3}} = -57.7\,\Omega$$

$$X_s = X_L = Q R_{small} = \sqrt{3}\,25 = 43.3\,\Omega$$

Impedance Matching Networks - L-Networks

❑ Matching Complex Loads with a Lossless L-network

➢ When complex source and loads are involved, there are two basic conceptual approaches that can be used:

1. Absorption - "absorb" the source or load reactance into the matching network.
2. Resonance - series or parallel resonate the source or load reactance at the frequency of interest.

These approaches will be illustrated by example in the following sections.

Impedance Matching Networks - L-Networks

Matching Complex Loads with a Lossless L-network

Example - Absorption

Absorption will be illustrated with an example. Suppose that it is necessary to match the source and load shown in Figure 6.11 at 100 MHz with a lossless L-network.

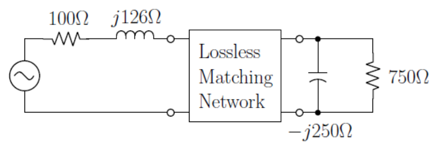
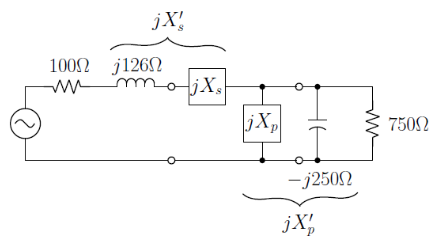


Figure 6.11: L-network with complex source and load

Absorption is applied by lumping the source and load reactances into the series and parallel reactances of the matching network, as shown in Figure 6.12. The lumped reactances



Impedance Matching Networks - L-Networks

Matching Complex Loads with a Lossless L-network

Example - Absorption

X_s' and X_p' can be found by using the design equations for matching between two resistive terminations:

$$\begin{aligned} Q &= \sqrt{750/100 - 1} &= 2.55 \\ X_s' &= \pm 100\,(2.55) &= \pm 255 \\ X_p' &= \mp 750/2.55 &= \mp 294.1 \end{aligned}$$

The lumped reactances can be written in terms of the reactances associated with the source and load, and the L-network reactances:

$$\begin{aligned} X_s' &= X_s + 126, \\ X_p' &= \frac{-250\,(X_p)}{-250 + X_p}. \end{aligned}$$

Impedance Matching Networks - L-Networks

Matching Complex Loads with a Lossless L-network

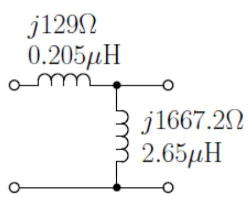
Example - Absorption

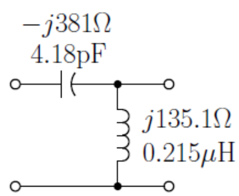
Thus, values of X_s and X_p can be obtained:

$$X_s = X_s' - 126 = \pm 255 - 126 = \begin{cases} 129\Omega \\ -381\Omega \end{cases}$$

$$X_p = \frac{250X_p'}{250+X_p'} = \begin{cases} 1667.2\Omega \\ 135.1\Omega \end{cases}$$

The two solutions found so far are shown in Figure 6.13.





Impedance Matching Networks - L-Networks

Matching Complex Loads with a Lossless L-network

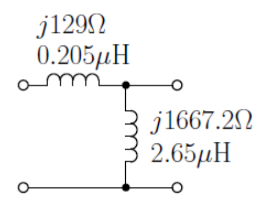
Example - Absorption

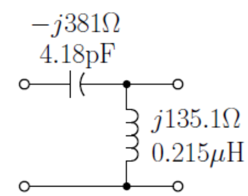
Thus, values of X_s and X_p can be obtained:

$$X_s = X_s' - 126 = \pm 255 - 126 = \begin{cases} 129\Omega \\ -381\Omega \end{cases}$$

$$X_p = \frac{250X_p'}{250+X_p'} = \begin{cases} 1667.2\Omega \\ 135.1\Omega \end{cases}$$

The two solutions found so far are shown in Figure 6.13.





Impedance Matching Networks - L-Networks

❑ Matching Complex Loads with a Lossless L-network

Example - Resonance

Continuing with the example from the previous section, the resonance concept will be employed to find two more L-network solutions for the source and load shown in Figure 6.11. After transforming the source from series to parallel form, and the load from parallel to series form, the source and load are represented as in Figure 6.14. To apply the resonance concept, the source and load are augmented with reactances that resonate with the source and load reactances as shown in Figure 6.16.

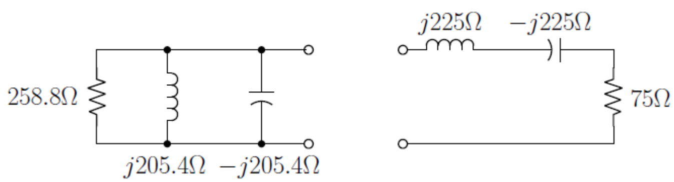


Figure 6.16: Transformed source and load augmented with resonating reactances

Impedance Matching Networks - L-Networks

❑ Matching Complex Loads with a Lossless L-network

Example - Resonance

After resonating the source and load in this manner, the new source and load impedances are purely real (resistive) at the design frequency. An L-network is then designed to match these two resistances, i.e.,

$$
\begin{aligned}
Q &= \sqrt{\frac{258.76}{75} - 1} &&= 1.57 \\
X_s' &= \pm 75\,(1.57) &&= \pm 117.8 \qquad (6.26)\\
X_p' &= \mp 258.76\,/1.57 &&= \mp 164.8
\end{aligned}
$$

Now the circuit can be drawn as shown in Figure 6.17. To complete the design, the resonating

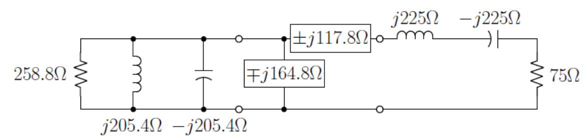


Figure 6.17: L-net with resonating reactances

reactances must be incorporated into the matching network. For example, when the upper signs are chosen, the net parallel reactance will be $-j205.4\Omega || -j164.8\Omega = -j91.4\Omega$. The net series reactance will be $j117.8\Omega + j225\Omega = j342.8\Omega$. The final solutions are shown in Figure 6.18.

Impedance Matching Networks - L-Networks

❑ Matching Complex Loads with a Lossless L-network

Example - Resonance

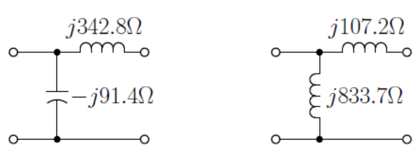


Figure 6.18: Resonating reactances incorporated into matching network

Thus, we have found four possible solutions that can be obtained using a lossless L-network.

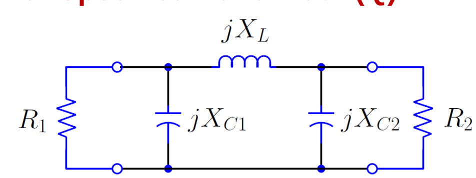
The example considered here allowed 4 possible L-network solutions because transforming the source and load caused R_{big} and R_{small} to swap positions. This will not always be the case. Thus, for some source and load combinations, there will be only two L-network solutions, and in other cases there will be four solutions.

3. Impedance Matching Networks – 3-Element MN

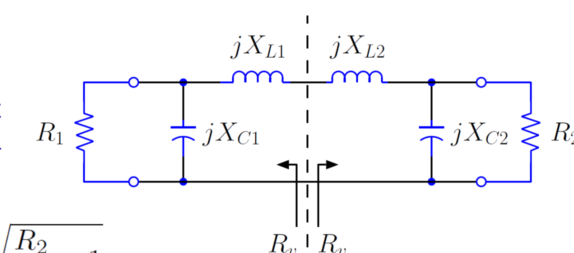
- The L-network does not give the designer freedom to choose the Q (bandwidth) of the matching network.
- The addition of a third matching element makes it possible to design for a match and a specified Q.

Design of Pi- and T-networks for Specified Bandwidth (Q)

- Matching two resistive terminations where $R_2 > R_1$



- The Pi-network can be thought of as two back-to-back L-networks that act to match both R_1 and R_2 to a “virtual resistance” R_v, $R_v < R_1, R_2$.



$$Q_1 = \sqrt{\frac{R_1}{R_v} - 1}, \quad \text{and} \quad Q_2 = \sqrt{\frac{R_2}{R_v} - 1}$$

Design of Pi- and T-networks for Specified Bandwidth (Q)

- We are assuming that $R_2 > R_1$, and therefore Q_2 will be larger than Q_1.
- Formost practical purposes the Q of the Pi-network can be approximated by Q2. This is especially true if R2 >> R1. If R2 is only slightly larger than R1, then the overall Q of the network will be somewhat larger than Q2.
- The design procedure follows:

1. Determine the required Q of the matching network by considering the required bandwidth, BW, and center frequency, f_o ($Q = f_o/BW$). This Q is taken to be equal to Q_2, and thus the virtual resistance, R_v, is determined. **Note that R_v must be smaller than R_1, and therefore the Pi-network can only be used to obtain a larger Q than would have been provided by the simpler L-network.** Also note that the relationship $BW = f_o/Q$ is only exactly true for a simple parallel or series RLC circuit. Thus, the actual bandwidth of your circuit may be different from the specified design value. If a particular design requires that the bandwidth be precisely determined, it is a good idea to simulate the performance of the matching network using a computer-aided design program in order to verify that the performance will be satisfactory.

2. Once R_v is found, the values of X_{C1}, X_{L1}, X_{C2}, and X_{L2} can be calculated using the previously derived formulas for L-network matching.

Design of Pi- and T-networks for Specified Bandwidth (Q)

- **The design procedure can be summarized by Equations:**
  Q_2 = Desired Q

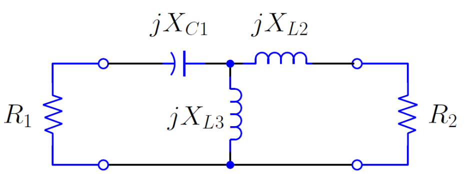
$$X_{C2} = -\frac{R_2}{Q_2}$$

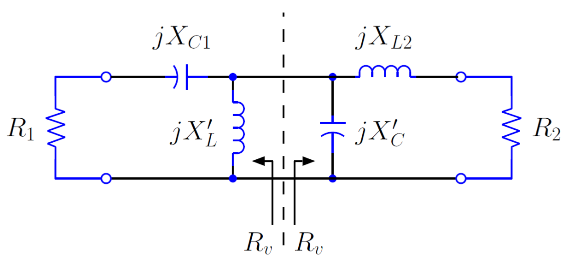
$$X_{C1} = -\sqrt{\frac{R_1 R_2}{(Q_2^2+1) - \frac{R_2}{R_1}}}$$

$$X_L = \frac{R_2 Q_2 + R_2\sqrt{\frac{R_1}{R_2}(Q_2^2+1) - 1}}{Q_2^2+1}$$

Design of Pi- and T-networks for Specified Bandwidth (Q)

- The Pi-network is most useful for matching when the values of R1 and R2 are not too small.
- If R1 and R2 are small, the virtual resistance will be even smaller, and the capacitor values will turn out to be impractically large.
- If either terminating resistance is significantly less than 50 Ω, the T-network will usually be a more practical choice.
- We can think of this network as two back-to-back L-networks

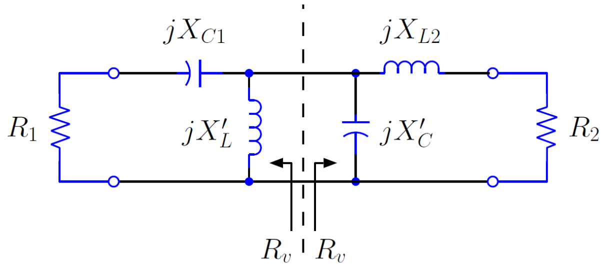




- Note that since $R_v > R_1$ and $R_v > R_2$, this network will have a larger Q than a single L-network that matches R_1 to R_2.
- The overall Q of the network is set by Q1 since we assume R2 > R1.

$$Q_1 = \sqrt{\frac{R_v}{R_1} - 1} \quad \text{and} \quad Q_2 = \sqrt{\frac{R_v}{R_2} - 1}$$

Design of Pi- and T-networks for Specified Bandwidth (Q)



- The overall Q of the network is set by Q1 since we assume R2 > R1.

$$Q_1 = \sqrt{\frac{R_v}{R_1} - 1} \quad \text{and} \quad Q_2 = \sqrt{\frac{R_v}{R_2} - 1}$$

- Two elements X'_L and X'_C that appear in the back-to-back L-networks are combined into a single element, X_{L3} in the T-network.

$$\boxed{\begin{aligned} X_{C1} &= -R_1 Q_1 \\ X_{L2} &= R_2\sqrt{\frac{R_1}{R_2}(Q_1^2+1) - 1} \\ X_{L3} &= \frac{R_1(Q_1^2+1)}{Q_1 - \sqrt{\frac{R_1}{R_2}(Q_1^2+1) - 1}} \end{aligned}}$$

Home Work

Chapter 6:
1, 2, 3, 4, 5, 6, 10, 11, 13, 22 ,23, 24