Ho Chi Minh City University of Technology
Faculty of Electrical and Electronics Engineering
Department of Telecommunications

CELLULAR CONCEPTS

Lectured by Ha Hoang Kha, Ph.D.
Ho Chi Minh City University of Technology
Email: hahoangkha@gmail.com

Content

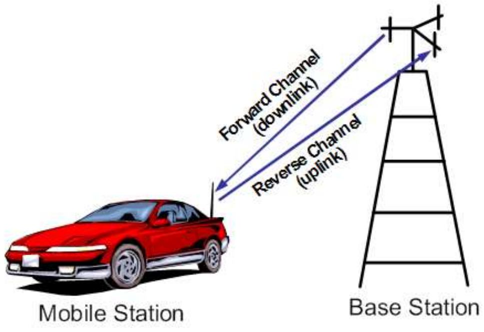
- Introduction to Wireless Systems
- Frequency reuse
- Interference and Grade of Service
- Handoff Strategies
- Improving Coverage & Capacity in Cellular Systems

Reference

- Ta Tri Nghia, Wireless Communications, lecture notes.
- T.S. Rappaport, *Wireless Communications*, Prentice Hall PTR, 1996.

Introduction

- Enable communication to and from mobile users by using radio transmission



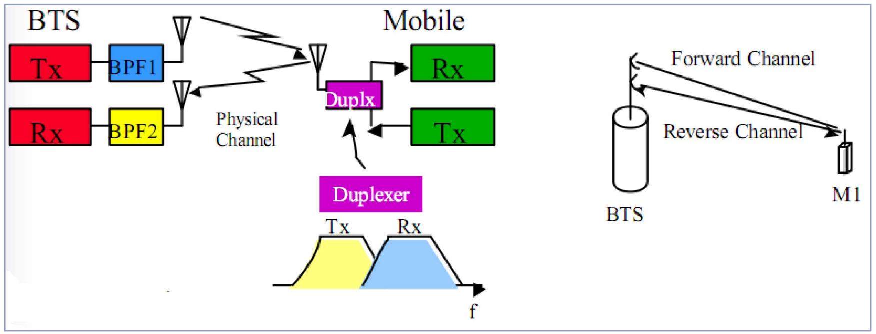
Definitions

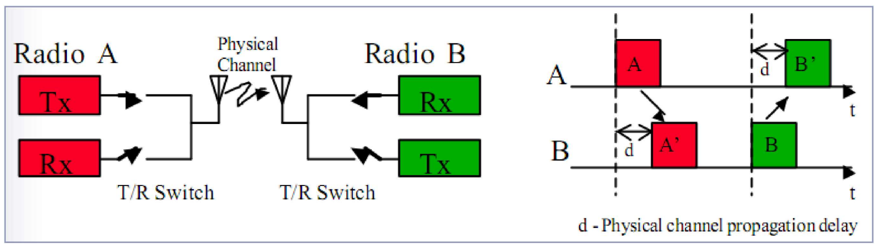
- **Base station**: a fixed station used for radio communication with mobile stations within its coverage region. It consists of several transmitters and receivers which simultaneously handle full duplex communications and generally has a tower which supports several transmitting and receiving antennas.
- **Mobile station:** a radio terminal intended for use while in motion. It contains a transceiver, an antenna, and control circuitry, and may be hand-held units (portables) or mounted in vehicles (mobiles).
- **Forward channel:** radio channel used for transmission of information from the base station to the mobile
- **Reserved channel:** radio channel used for transmission of information from the mobile to the base station
- **Control channel:** radio channel used for transmission of call setup, call request, call initiation, and other beacon or control purposes

Definitions

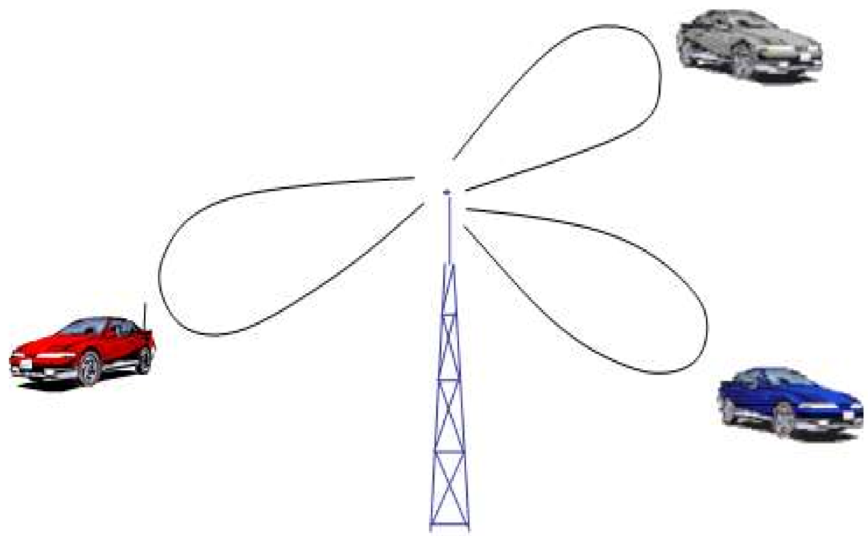
- Simplex
- Half-duplex
- Full-duplex
  - The 2 channels can be separated in frequency – Frequency Division Duplex (FDD)
  - The 2 channels can be separated in time to share a single physical channel – Time Division Duplex (TDD)

FDD vs TDD





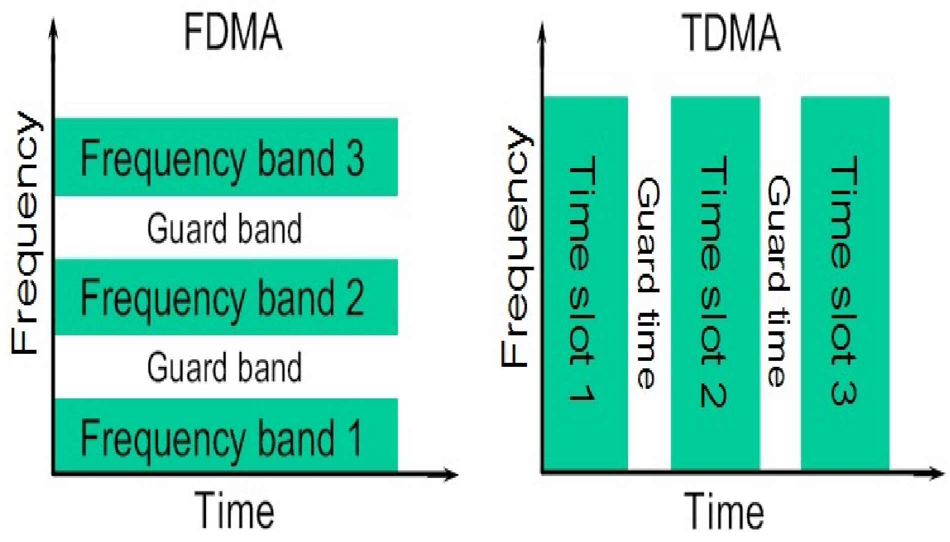
Multiple Access



Multiple Access

- Multiple access
  - FDMA (Frequency Division Multiple Access)
  - TDMA (Time Division Multiple Access)
  - SDMA (Space Division Multiple Access)
  - SSMA (Spread Spectrum Multiple Access)
    - FHMA (Frequency Hopped Multiple Access)
    - CDMA (Code Division Multiple Access)

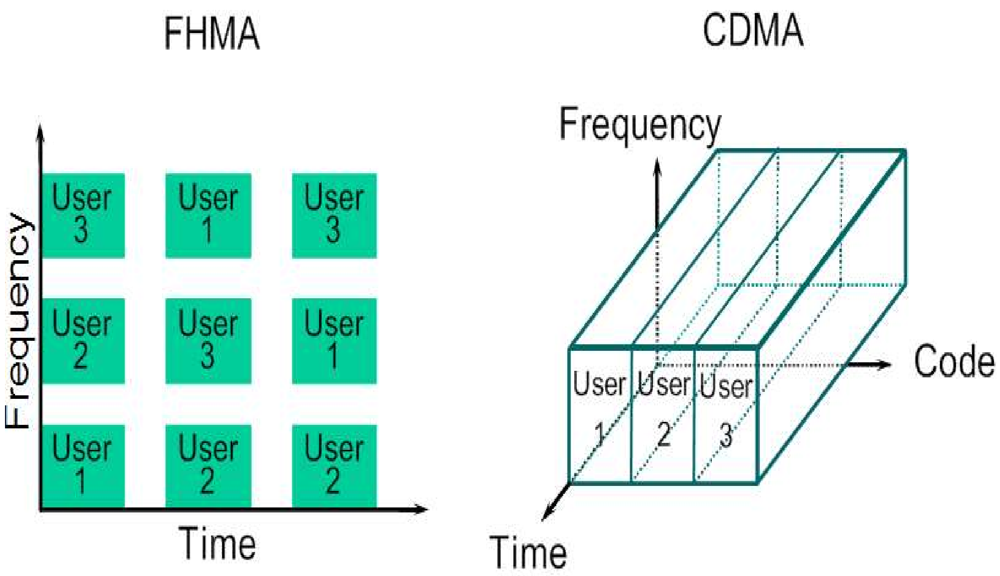
Multiple Access



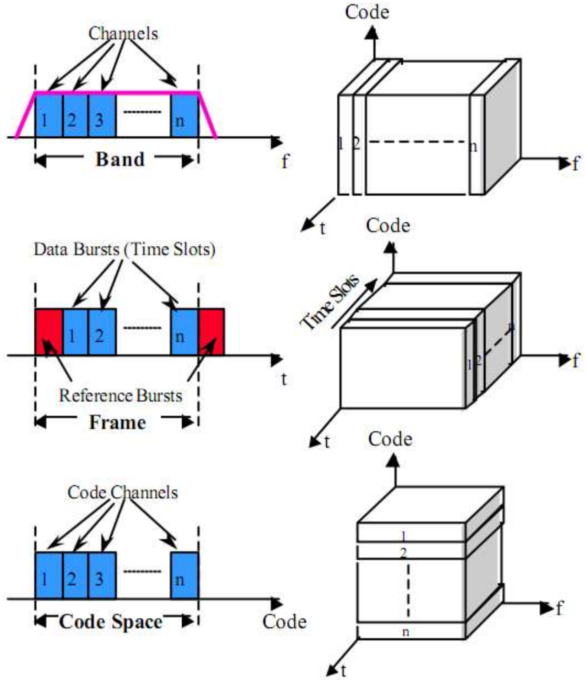
Multiple Access

- **Spread-spectrum multiple access (SSMA):** SSMA uses signals which have a transmission bandwidth that is several orders of magnitude greater than the minimum required RF bandwidth. Each user is assigned a distinct pseudo-noise (PN) code. The users'codes are approximately orthogonal, which allow multiple users share full spectrum of the available bandwidth simultaneously without interfering significantly with each other.
  - **Frequency-hopped multiple access (FHMA):** each user has a different hopping pattern, which is determined by its own distinct PN code.
  - **Code-division multiple access (CDMA):** each user has its own distinct PN sequence. All active users transmit their signals on the same bandwidth and overlap in time. Signal separation is achieved at the receiver by correlation with the proper PN sequence. Therefore, in CDMA each SS signal represents a low interference signal to the others.

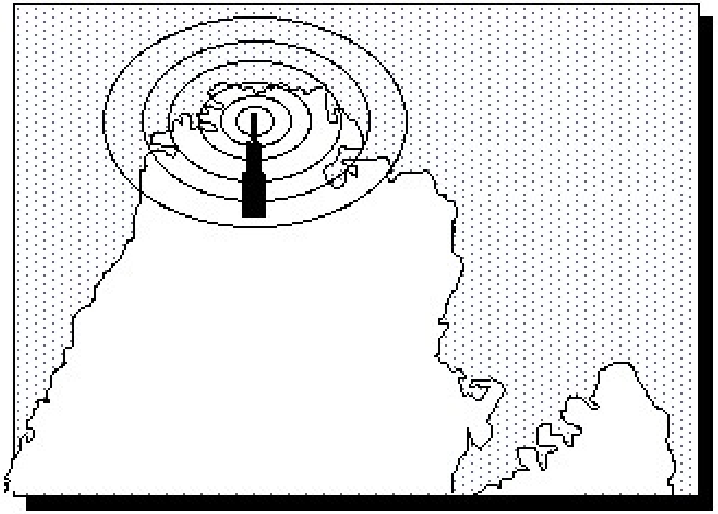
Multiple Access



Multiple Access



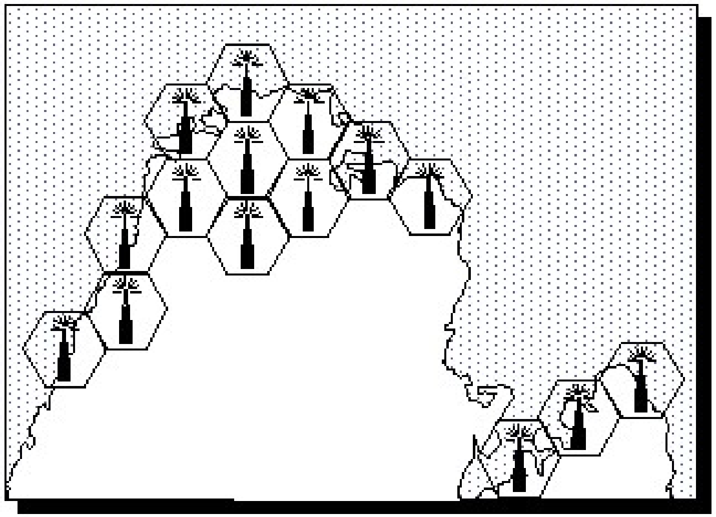
The Cellular Concept



The Cellular Concept

- Why cellular?
  - Radio spectrum is a finite resource.
  - How to accommodate a large number of users over a large geographic area within a limited radio spectrum?
  - The solution is the use of cellular structure which allows frequency reuse.

The Cellular Concept

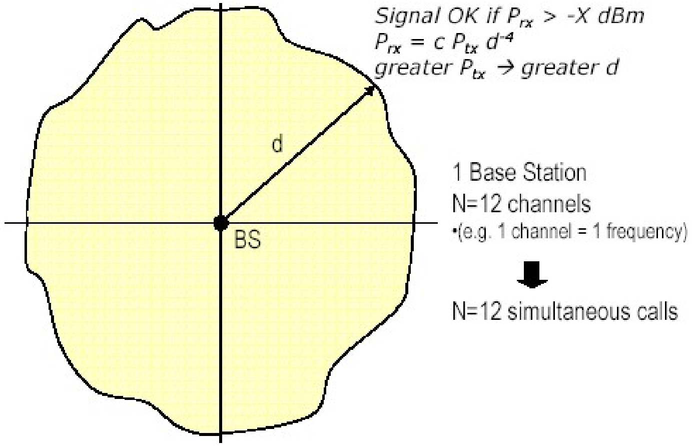


The Cellular Concept

- The large geographic area is divided into smaller areas *cells.*
- Each cell has its own base station providing coverage only for that cell.
- Each base station is allocated a portion of the total number of channels available to the entire system.
- Neighboring base stations are assigned different groups of channels to minimize interference.
- The same group of channels can be reused by another base station located sufficiently far away to keep co-channel *interference levels within tolerable limits.*

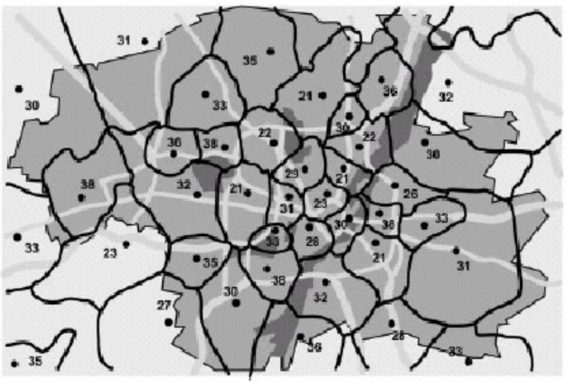
The Cellular Concept

Coverage for a terrestrial zone



The Cellular Concept

Cells in real world



Shaped by terrain, shadowing, etc
Cell border: local threshold, beyond which neighboring BS signal is received stronger than current one

Radio Signal Attenuation

- Average received signal power $\overline{P_r}$ decreases with distance

$$\overline{P_r} \propto d^{-n}$$

where d = distance from transmitter to receiver
n = path loss exponent

- Typical values of n:
  - *n = 2: free space*
  - *n = 2.7 ~ 5: urban cellular radio*
  - *n = 3: open country*
  - *n = 1.6 ~ 1.8: indoor line-of-sight*
- Larger values of n preferred, leading to less interference

Frequency Reuse

- There is a total of S full-duplex channels available for use.
- S channels are divided among N cells into unique and disjoint channel groups which each has k channels.
- Total number of available channels can then be expressed as

$$S = kN$$

- N cells which collectively use the complete set of available channels is called a cluster. N is the cluster size.
- If a cluster is replicated M times, the total number of channels as a measure of capacity is given by

$$C = M\,kN = MS$$

- For a given area, if k is reduced while the cell size is kept constant, more clusters are required to cover the area, and hence more capacity.
- However, a smaller cluster size indicates that co-channel cells are much closer, leading to stronger co-channel interference.

Frequency Reuse

- The smallest possible value of N is desirable for maximizing capacity. This value depends on how much interference a mobile or base station can tolerate while maintaining a sufficient quality of communication.
- Since each cell within a cluster is only assigned 1/N of the total available channels, 1/N is defined as the frequency reuse factor.

Example

A total of 33 MHz of bandwidth is allocated to a cellular telephone system which uses two 25 kHz channels to provide full-duplex voice and control channels, compute the number of channels available per cell if the system uses (a) 4-cell reuse, (b) 7-cell reuse, and (c) 12-cell reuse. If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

Example - Solutions

Total bandwidth = 33 MHz
Channel bandwidth = 25 kHz × 2 = 50 kHz / duplex channels
Total available channels = 33,000/50 = 660 channels
(a) For N = 4, total number of channels per cell = 660/4 = 165
(b) For N = 7, total number of channels per cell = 660/7 ˜ 95
(c) For N = 12, total number of channels per cell = 660/12 = 55

A 1 MHz spectrum for control channels implies that there are 1000/50 = 20 control channels out of the 660 channels available.

(a) For N = 4, we can have 5 control channels and 160 voice channels per cell. However, in practice each cell only needs a single control channel. Thus, 1 control channel and 160 voice channels would be assigned to each cell.

(b) For N = 7, each cell would have 1 control channel, 4 cells would have 91 voice channels each, and 3 cells would have 92 voice channels each. (640-91×7=3)

(c) For N = 12, each cell would have 1 control channel, 8 cells would have 53 voice channels each, and 4 cells would have 54 voice channels each. (640-53×12=4)

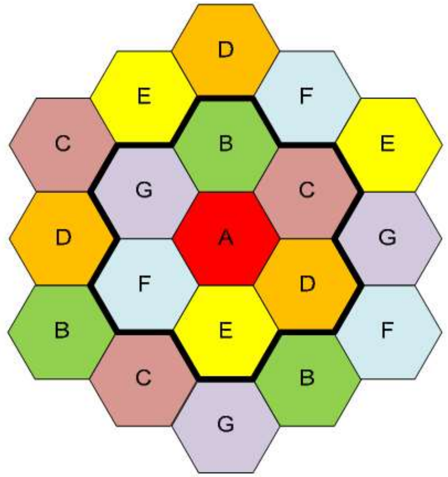
Cell Geometry

- Actual radio coverage of a cell is known as footprint and is determined by environmental conditions.
- Although a real footprint is amorphous in nature, a regular cell shape is needed for systematic system design and analysis.
- From the signal attenuation model, it seems natural to choose a circle to represent the coverage area of a base station.
- However, circles cannot be tessellated, i.e., be overlaid without leaving gaps or overlap.
- There are three possible choices: a square, an equilateral triangle, and a hexagon. Hexagon is used as it is the most circular

R: *cell radius*

Hexagonal Cells

-



Hexagonal Geometry

- In order to tessellate clusters of hexagon cells, the cluster size *N* can only have values which satisfy the following equation

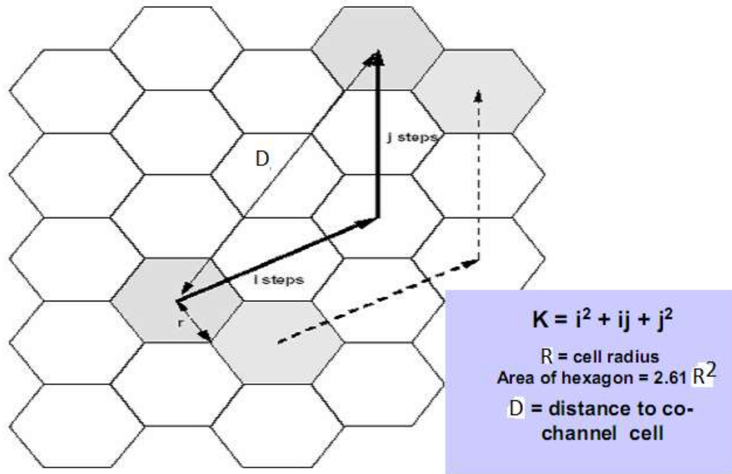
$$N = i^2 + ij + j^2$$

where i and j are non-negative integers. Hence N = 3, 4, 7, 9, 12, etc.

- To find the nearest co-channel neighbors of a cell, one can do the following:
-

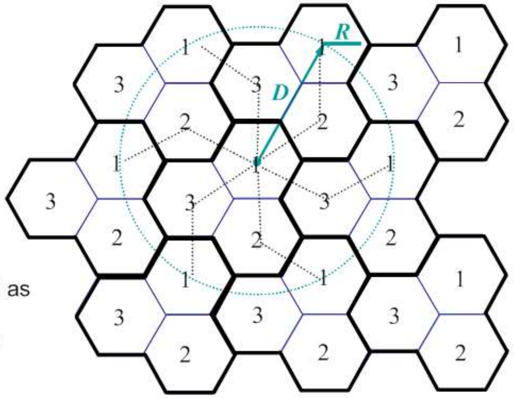
(1) move i cells along any chain of hexagons and then
(2) turn 60 degrees counter-clockwise (or clockwise) and move j cells.
* The roles of i and j can be reversed.

Hexagonal Geometry



Examples

N=3, i=1, j=1

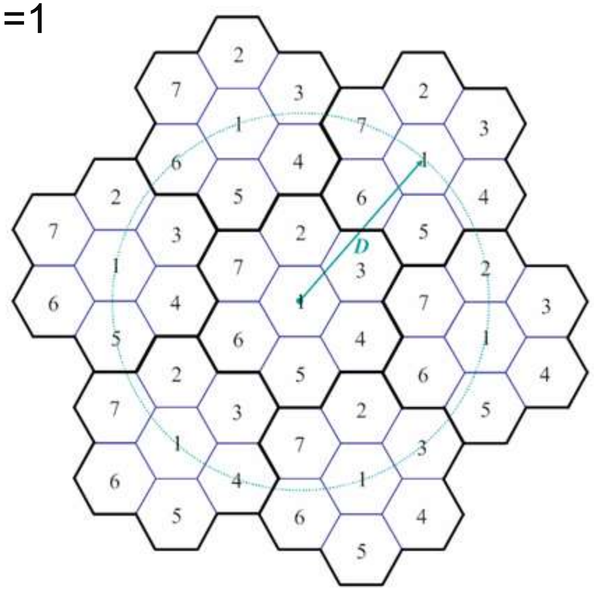


D: *reuse distance* defined as the distance between centers of the nearest co-channels cells.

R: cell radius

Examples

N=7, i=2, j=1

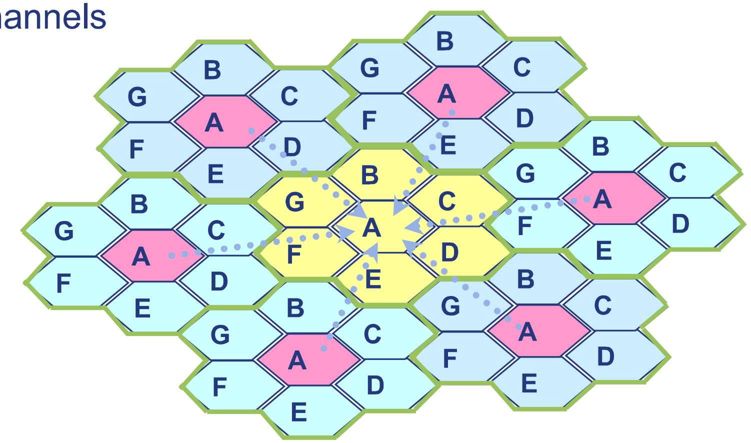


Interference & System Capacity

- The wireless environment constitutes a shared medium
- Interference is the major limiting factor in performance of wireless systems in general
- Types of Interference:
  - Co-channel interference
  - Adjacent channel interference

Co-channel Interference

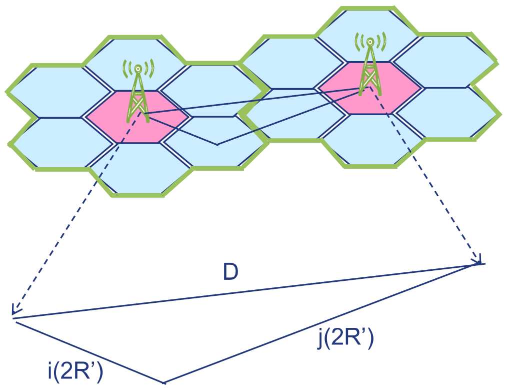
Frequency reuse implies that several cells use the same set of channels



Frequency reuse = 7

- Co-channel interfering cells for cell allocated with channel group A

Distance Separation between Base Stations



$R' = \frac{\sqrt{3}}{2}R$

where R is the Cell Radius

$$N = i^2 + ij + j^2$$

$$D = \sqrt{(2iR')^2 + (2jR')^2 - 2 \times (2iR') \times (2jR')\cos\ 120}$$

$$D = R\sqrt{3(i^2 + ij + j^2)} = R\sqrt{3N}$$

$$\boxed{Q = \frac{D}{R} = \sqrt{3N}}$$

Q: Co-channel reuse Ratio

Co-channel Cells

- For hexagonal geometry, the co-channel reuse ratio Q, defined as the ratio of D to R, is related to the cluster size by

$$Q = D/R = \sqrt{3N}$$

– Smaller Q, larger capacity
– Larger Q, higher transmission quality

	Cluster Size	Co-channel Reuse Ratio
i=1 , j=1	3	3
i=1 , j=2	7	4.58
i=2 , j=2	12	6
i=1 , j=3	13	6.24

Co-channel Interference

- The signal-to-interference ratio (S/I or SIR) for a mobile receiver is given by

$$S/I = \frac{S}{\sum_{i=1}^{i_0} I_i}$$

where S is the received signal power from the desired base station and I_i is the received interference power from the i^{th} co-channel cell base station, and i_0 is the number of co-channel interfering cells.

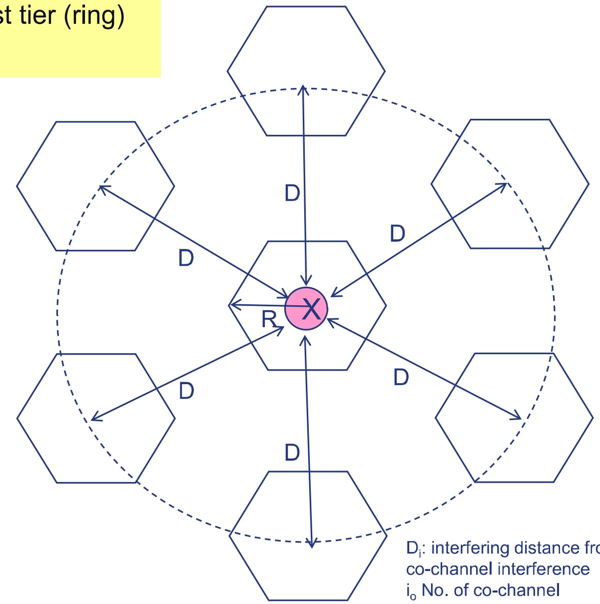
SIR Computations

Assume interference from first tier (ring) of co-channel interferers

$$P_r = P_0\left(\frac{d}{d_0}\right)^{-n}$$

$$SIR = \frac{P_0\left(R/d_0\right)^{-n}}{\sum_{i=1}^{i_0} P_0\left(D_i/d_0\right)^{-n}} = \frac{R^{-n}}{\sum_{i=1}^{i_0} D_i^{-n}}$$

$$\Rightarrow SIR = \frac{\left(\sqrt{3N}\right)^n}{i_0} = \frac{Q^n}{i_0}$$



D_i: interfering distance from i^{th} co-channel interference
i_o No. of co-channel interfering sites

SIR Computations

Assume interference from first tier (ring) of co-channel interferers

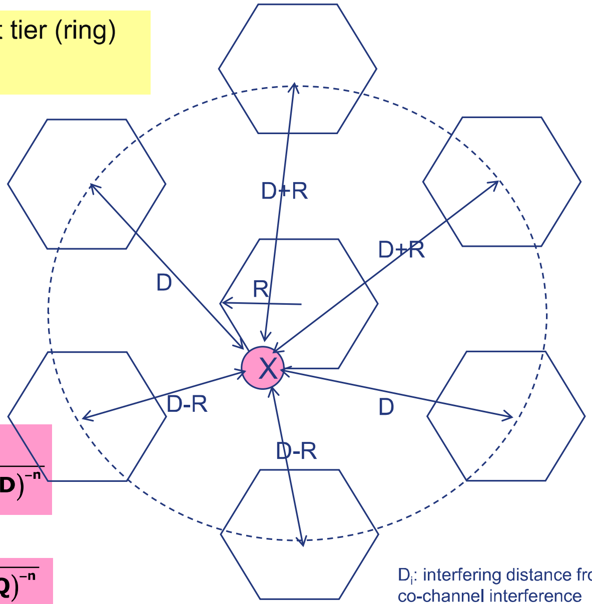
$$P_r = P_0\left(\frac{d}{d_0}\right)^{-n}$$

$$SIR = \frac{P_0\left(R/d_0\right)^{-n}}{\sum_{i=1}^{i_0} P_0\left(D_i/d_0\right)^{-n}} = \frac{R^{-n}}{\sum_{i=1}^{i_0} D_i^{-n}}$$

Worst Case SIR

$$\Rightarrow SIR = \frac{R^{-n}}{2(D-R)^{-n} + 2(D+R)^{-n} + 2(D)^{-n}}$$

$$\Rightarrow SIR = \frac{1}{2(Q-1)^{-n} + 2(Q+1)^{-n} + 2(Q)^{-n}}$$



D_i: interfering distance from i^{t} co-channel interference
i_0 No. of co-channel interfering sites

SIR & System Capacity

$$SIR \propto Q^n, \; Q = D/R = \sqrt{3N}$$

- Improving SIR means increasing cluster size, which corresponds to ***a decrease in system capacity***
- Decreasing the cell size does not affect the SIR as Q=D/R remains constant. A decrease in cell size corresponds to ***an increase in system capacity***

Example

- In First Generation cellular systems, sufficient voice quality is achieved when SIR = 18 dB

$$SIR = \frac{1}{2(Q-1)^{-n} + 2(Q+1)^{-n} + 2(Q)^{-n}}$$

- N=7➔Q=4.6. Worst Case SIR = 49.56 (17 dB)
- To design cellular system with worst performance better than 18 dB, N=9
- ➔ Capacity reduction = 7/9

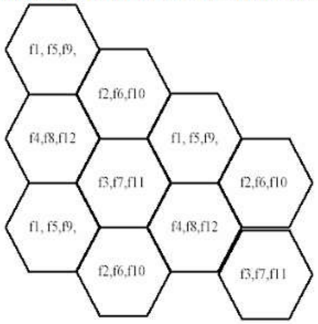
Adjacent Channel Interference

- Interference resulting from signals which are adjacent in frequency to the desired signal
- It is due to imperfect receiver filtering which allow nearby frequencies leak into the passband.
- It is the cause of near-far effect.
- To minimize adjacent channel interference:
  - Use high Q filters
  - Maximize the frequency separation between each channel in a given cell

Frequency Planning

- Typical C/I values used in practice are 13-18 dB.
- Once the frequency reuse cluster size and frequency allocation determined frequencies must be assigned to cells
- Must maintain C/I pattern between clusters.
- Within a cluster – seek to minimize adjacent channel interference
- Adjacent channel interference is interference from frequency adjacent in the spectrum

Example: You are operating a cellular network with 25KHz NMT traffic channels 1 through 12. Labeling the traffic channels as {f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12}
Place the traffic channels in the cells below such that a frequency reuse cluster size of 4 is used and adjacent channel interference is minimized



Example

A cellular system which uses FM and 30 kHz channels requires the S/I to be at least 18 dB for good voice quality. Determine the minimum cluster size assuming a path loss exponent $n = 4$.

- Answer:

the cluster size is 7.

Power Control

- In practice, the power levels transmitted by every mobile are under constant control by the serving base stations.
  - To ensure that each mobile transmits the smallest power necessary to maintain a good quality link on the reverse channel
  - To help prolong battery life
  - To increase dramatically the reverse channel S/I

Trunking and GOS

- Trunking allows a large number of users to share a relatively small number of channels by providing access to each user, on demand, from the pool of available channels.
- Trunking exploits the statistical behavior of users so that a fixed number of channels may accommodate a large, random user community.
  - Trade-off between the number of available channels that are provided and the likelihood of a particular user finding no channels available during the busy hour of the day.
  - efficient use of equipment resources → savings
  - disadvantage is that some probability exists that mobile user will be denied access to a channel
- The grade of service (GOS) is a measure of the ability of a user to access a trunked system during the busiest hour.
- GOS is typically given as the likelihood that a call is blocked.

- "Offered" Traffic Intensity (A)
  - Offered? → not necessarily carried by system (some is blocked or delayed)
  - each user $A_u = \lambda H$ Erlangs (also called ρ in queueing theory)
    - λ = traffic intensity (average arrival rate of new calls, in new requests per time unit, say calls/min).
    - H = average duration of a call (also called 1/ μ in queueing theory)
  - system with U users → $A = UA_u = U\lambda H$ Erlangs
  - capacity = maximum carried traffic = C Erlangs = (equal to total # of available channels that are busy all the time)

- Erlang B formula
  - Calls are either admitted or blocked

$$GOS = \Pr[\text{blocked call}] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}}$$

    - A = total offered traffic
    - C = # channels in **trunking** pool (e.g. a cell)
  - AMPS designed for GOS of 2%
  - blocked call cleared (denied) → BCC

- capacities to support various GOS values

Table 3.4 Capacity of an Erlang B System

Number of Channels C	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

- Note that twice the capacity can support much more than twice the load (twice the number of Erlangs).

Graphical form of Erlang B formulas

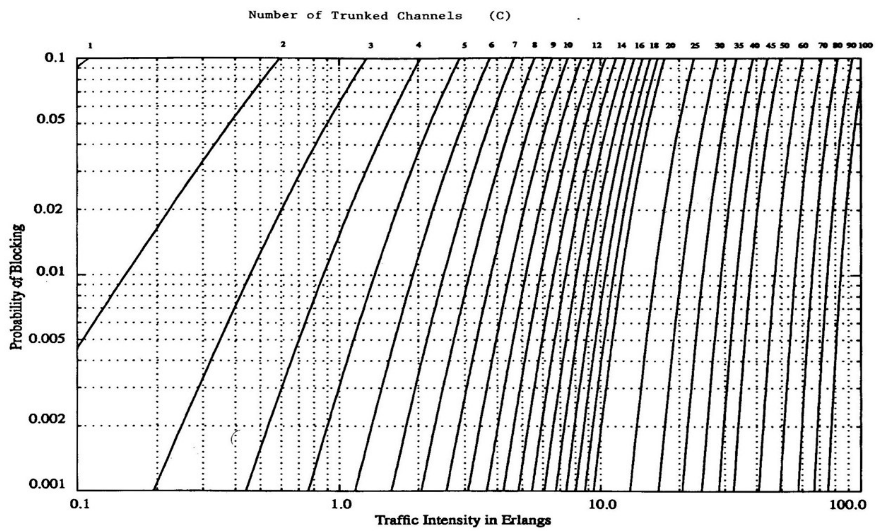


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Example

Example

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 100. Assume each user generates 0.1 Erlangs of traffic.

Solution

From the Erlang B table, we can find the total capacity (the traffic intensity can be supported) in Erlangs for the 0.5% GOS for different number of channels. By using the relation $\rho = U\rho_u$, we can obtain U, the total number of users that can be supported.

Example

(a) Given $N = 1$, $\rho_u = 0.1$, GOS = 0.5%
From the Erlang B table, we obtain $\rho = 0.005$.
Therefore, total number of users $U = \rho/\rho_u = 0.005/0.1 = 0.05$ users
But actually one user can be supported on one channel. So $U = 1$.

(b) Given $N = 5$, $\rho_u = 0.1$, GOS = 0.5%
From the Erlang B table, we obtain $\rho = 1.13$.
Therefore, total number of users $U = \rho/\rho_u = 1.13/0.1 = 11$ users

(c) Given $N = 10$, $\rho_u = 0.1$, GOS = 0.5%
From the Erlang B table, we obtain $\rho = 3.96$.
Therefore, total number of users $U = \rho/\rho_u = 3.96/0.1 = 39$ users

(d) Given $N = 100$, $\rho_u = 0.1$, GOS = 0.5%
From the Erlang B table, we obtain $\rho = 80.91$.
Therefore, total number of users $U = \rho/\rho_u = 80.91/0.1 = 809$ users

Erlang B Table

ρ (Erlangs)

	Blocking Probability (%)									
N	0.01	0.05	0.1	0.5	1	2	5	10	20	40
1	.00010	.00050	.00100	.00503	.01010	.02041	.05263	.11111	.25000	.66667
2	.01425	.03213	.04576	.10540	.15259	.22347	.38132	.59543	1.0000	2.0000
3	.08683	.15170	.19384	.34900	.45549	.60221	.89940	1.2708	1.9299	3.4798
4	.23471	.36236	.43927	.70120	.86942	1.0923	1.5246	2.0454	2.9452	5.0210
5	.45195	.64857	.76212	1.1320	1.3608	1.6571	2.2185	2.8811	4.0104	6.5955
6	.72826	.99567	1.1459	1.6218	1.9090	2.2759	2.9603	3.7584	5.1086	8.1907
7	1.0541	1.3922	1.5786	2.1575	2.5009	2.9354	3.7378	4.6662	6.2302	9.7998
8	1.4219	1.8298	2.0513	2.7299	3.1276	3.6271	4.5430	5.5971	7.3692	11.419
9	1.8256	2.3016	2.5575	3.3326	3.7825	4.3447	5.3702	6.5464	8.5217	13.045
10	2.2601	2.8028	3.0920	3.9607	4.4612	5.0840	6.2157	7.5106	9.6850	14.677
11	2.7216	3.3294	3.6511	4.6104	5.1599	5.8415	7.0764	8.4871	10.857	16.314
12	3.2072	3.8781	4.2314	5.2789	5.8760	6.6147	7.9501	9.4740	12.036	17.954
13	3.7136	4.4465	4.8306	5.9638	6.6072	7.4015	8.8349	10.470	13.222	19.598
14	4.2388	5.0324	5.4464	6.6632	7.3517	8.2003	9.7295	11.473	14.413	21.243
15	4.7812	5.6339	6.0772	7.3755	8.1080	9.0096	10.633	12.484	15.608	22.891
16	5.3390	6.2496	6.7215	8.0995	8.8750	9.8284	11.544	13.500	16.807	24.541
17	5.9110	6.8782	7.3781	8.8340	9.6516	10.656	12.461	14.522	18.010	26.192
18	6.4959	7.5186	8.0459	9.5780	10.437	11.491	13.385	15.548	19.216	27.844
19	7.0927	8.1698	8.7239	10.331	11.230	12.333	14.315	16.579	20.424	29.498
20	7.7005	8.8310	9.4115	11.092	12.031	13.182	15.249	17.613	21.635	31.152

Erlang B Table

ρ (Erlangs)

	Blocking Probability (%)									
N	0.01	0.05	0.1	0.5	1	2	5	10	20	40
21	8.3186	9.5014	10.108	11.860	12.838	14.036	16.189	18.651	22.848	32.808
22	8.9462	10.180	10.812	12.635	13.651	14.896	17.132	19.692	24.064	34.464
23	9.5826	10.868	11.524	13.416	14.470	15.761	18.080	20.737	25.281	36.121
24	10.227	11.562	12.243	14.204	15.295	16.631	19.031	21.784	26.499	37.779
25	10.880	12.264	12.969	14.997	16.125	17.505	19.985	22.833	27.720	39.437
26	11.540	12.972	13.701	15.795	16.959	18.383	20.943	23.885	28.941	41.096
27	12.207	13.686	14.439	16.598	17.797	19.265	21.904	24.939	30.164	42.755
28	12.880	14.406	15.182	17.406	18.640	20.150	22.867	25.995	31.388	44.414
29	13.560	15.132	15.930	18.218	19.487	21.039	23.833	27.053	32.614	46.074
30	14.246	15.863	16.684	19.034	20.337	21.932	24.802	28.113	33.840	47.735
31	14.937	16.599	17.442	19.854	21.191	22.827	25.773	29.174	35.067	49.395
32	15.633	17.340	18.205	20.678	22.048	23.725	26.746	30.237	36.295	51.056
33	16.335	18.085	18.972	21.505	22.909	24.626	27.721	31.301	37.524	52.718
34	17.041	18.835	19.743	22.336	23.772	25.529	28.698	32.367	38.754	54.379
35	17.752	19.589	20.517	23.169	24.638	26.435	29.677	33.434	39.985	56.041
36	18.468	20.347	21.296	24.006	25.507	27.343	30.657	34.503	41.216	57.703
37	19.188	21.108	22.078	24.846	26.378	28.254	31.640	35.572	42.448	59.365
38	19.911	21.873	22.864	25.689	27.252	29.166	32.624	36.643	43.680	61.028
39	20.640	22.642	23.652	26.534	28.129	30.081	33.609	37.715	44.913	62.690
40	21.372	23.414	24.444	27.382	29.007	30.997	34.596	38.787	46.147	64.353

Erlang B Table

ρ (Erlangs)

	Blocking Probability (%)									
N	0.01	0.05	0.1	0.5	1	2	5	10	20	40
41	22.107	24.189	25.239	28.232	29.888	31.916	35.584	39.861	47.381	66.016
42	22.846	24.967	26.037	29.085	30.771	32.836	36.574	40.936	48.616	67.679
43	23.587	25.748	26.837	29.940	31.656	33.758	37.565	42.011	49.851	69.342
44	24.333	26.532	27.641	30.797	32.543	34.682	38.557	43.088	51.086	71.006
45	25.081	27.319	28.447	31.656	33.432	35.607	39.550	44.165	52.322	72.669
46	25.833	28.109	29.255	32.517	34.322	36.534	40.545	45.243	53.559	74.333
47	26.587	28.901	30.066	33.381	35.215	37.462	41.540	46.322	54.796	75.997
48	27.344	29.696	30.879	34.246	36.109	38.392	42.537	47.401	56.033	77.660
49	28.104	30.493	31.694	35.113	37.004	39.323	43.534	48.481	57.270	79.324
50	28.867	31.292	32.512	35.982	37.901	40.255	44.533	49.562	58.508	80.988
51	29.632	32.094	33.332	36.852	38.800	41.189	45.533	50.644	59.746	82.652
52	30.400	32.898	34.153	37.724	39.700	42.124	46.533	51.726	60.985	84.317
53	31.170	33.704	34.977	38.598	40.602	43.060	47.534	52.808	62.224	85.981
54	31.942	34.512	35.803	39.474	41.505	43.997	48.536	53.891	63.463	87.645
55	32.717	35.322	36.631	40.351	42.409	44.936	49.539	54.975	64.702	89.310
56	33.494	36.134	37.460	41.229	43.315	45.875	50.543	56.059	65.942	90.974
57	34.273	36.948	38.291	42.109	44.222	46.816	51.548	57.144	67.181	92.639
58	35.055	37.764	39.124	42.990	45.130	47.758	52.553	58.229	68.421	94.303
59	35.838	38.581	39.959	43.873	46.039	48.700	53.559	59.315	69.662	95.968
60	36.623	39.401	40.795	44.757	46.950	49.644	54.566	60.401	70.902	97.633

Erlang B Table

ρ (Erlangs)

	Blocking Probability (%)									
N	0.01	0.05	0.1	0.5	1	2	5	10	20	40
61	37.411	40.222	41.633	45.642	47.861	50.589	55.573	61.488	72.143	99.297
62	38.200	41.045	42.472	46.528	48.774	51.534	56.581	62.575	73.384	100.96
63	38.991	41.869	43.313	47.416	49.688	52.481	57.590	63.663	74.625	102.63
64	39.784	42.695	44.156	48.305	50.603	53.428	58.599	64.750	75.866	104.29
65	40.579	43.523	45.000	49.195	51.518	54.376	59.609	65.839	77.108	105.96
66	41.375	44.352	45.845	50.086	52.435	55.325	60.619	66.927	78.350	107.62
67	42.173	45.183	46.692	50.978	53.353	56.275	61.630	68.016	79.592	109.29
68	42.973	46.015	47.540	51.872	54.272	57.226	62.642	69.106	80.834	110.95
69	43.775	46.848	48.389	52.766	55.191	58.177	63.654	70.196	82.076	112.62
70	44.578	47.683	49.239	53.662	56.112	59.129	64.667	71.286	83.318	114.28
71	45.382	48.519	50.091	54.558	57.033	60.082	65.680	72.376	84.561	115.95
72	46.188	49.357	50.944	55.455	57.956	61.036	66.694	73.467	85.803	117.61
73	46.996	50.195	51.799	56.354	58.879	61.990	67.708	74.558	87.046	119.28
74	47.805	51.035	52.654	57.253	59.803	62.945	68.723	75.649	88.289	120.94
75	48.615	51.877	53.511	58.153	60.728	63.900	69.738	76.741	89.532	122.61
76	49.427	52.719	54.369	59.054	61.653	64.857	70.753	77.833	90.776	124.27
77	50.240	53.563	55.227	59.956	62.579	65.814	71.769	78.925	92.019	125.94
78	51.054	54.408	56.087	60.859	63.506	66.771	72.786	80.018	93.262	127.61
79	51.870	55.254	56.948	61.763	64.434	67.729	73.803	81.110	94.506	129.27
80	52.687	56.101	57.810	62.668	65.363	68.688	74.820	82.203	95.750	130.94

Erlang B Table

ρ (Erlangs)

	Blocking Probability (%)									
N	0.01	0.05	0.1	0.5	1	2	5	10	20	40
81	53.506	56.949	58.673	63.573	66.292	69.647	75.838	83.297	96.993	132.60
82	54.325	57.798	59.537	64.479	67.222	70.607	76.856	84.390	98.237	134.27
83	55.146	58.649	60.403	65.386	68.152	71.568	77.874	85.484	99.481	135.93
84	55.968	59.500	61.269	66.294	69.084	72.529	78.893	86.578	100.73	137.60
85	56.791	60.352	62.135	67.202	70.016	73.490	79.912	87.672	101.97	139.26
86	57.615	61.206	63.003	68.111	70.948	74.452	80.932	88.767	103.21	140.93
87	58.441	62.060	63.872	69.021	71.881	75.415	81.952	89.861	104.46	142.60
88	59.267	62.915	64.742	69.932	72.815	76.378	82.972	90.956	105.70	144.26
89	60.095	63.772	65.612	70.843	73.749	77.342	83.993	92.051	106.95	145.93
90	60.923	64.629	66.484	71.755	74.684	78.306	85.014	93.146	108.19	147.59
91	61.753	65.487	67.356	72.668	75.620	79.271	86.035	94.242	109.44	149.26
92	62.584	66.346	68.229	73.581	76.556	80.236	87.057	95.338	110.68	150.92
93	63.416	67.206	69.103	74.495	77.493	81.201	88.079	96.434	111.93	152.59
94	64.248	68.067	69.978	75.410	78.430	82.167	89.101	97.530	113.17	154.26
95	65.082	68.928	70.853	76.325	79.368	83.134	90.123	98.626	114.42	155.92
96	65.917	69.791	71.729	77.241	80.306	84.100	91.146	99.722	115.66	157.59
97	66.752	70.654	72.606	78.157	81.245	85.068	92.169	100.82	116.91	159.25
98	67.589	71.518	73.484	79.074	82.184	86.035	93.193	101.92	118.15	160.92
99	68.426	72.383	74.363	79.992	83.124	87.003	94.216	103.01	119.40	162.59
100	69.265	73.248	75.242	80.910	84.064	87.972	95.240	104.11	120.64	164.25

Example

Example 3.5

An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution

System A

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 19$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, A, is obtained as 12 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 12/0.1 = 120$

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280$

System B

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 57$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 45/0.1 = 450$

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44{,}100$

System C

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 100$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 88/0.1 = 880$

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43{,}120$

Therefore, total number of cellular subscribers that can be supported by these three systems are $47{,}280 + 44{,}100 + 43{,}120 = 134{,}500$ users.

Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to

$47{,}280/2{,}000{,}000 = 2.36\%$

Similarly, market penetration of System B is equal to

$44{,}100/2{,}000{,}000 = 2.205\%$

and the market penetration of System C is equal to

$43{,}120/2{,}000{,}000 = 2.156\%$

The market penetration of the three systems combined is equal to

$134{,}500/2{,}000{,}000 = 6.725\%$

Example

Example 3.6

A certain city has an area of 1,300 square miles and is covered by a cellular system using a seven-cell reuse pattern. Each cell has a radius of four miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.

Solution

(a) Given:

Total coverage area = 1300 miles, and cell radius = 4 miles

The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers $2.5981 \times (4)^2$ 41.57 sq. mi.

Hence, the total number of cells are $N_c = 1300/41.57 = 31$ cells.

(b) The total number of channels per cell (C)

= allocated spectrum /(channel width × frequency reuse factor)

= 40,000,000/(60,000 × 7) = 95 channels/cell

(c) Given:

$C = 95$, and $GOS = 0.02$

From the Erlang B chart, we have

traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells × traffic intensity per cell

= 31 × 84 = 2604 Erlangs.

(e) Given traffic per user = 0.03 Erlangs

Total number of users = Total traffic / traffic per user

= 2604 / 0.03 = 86,800 users.

(f) Number of mobiles per channel = number of users/number of channels

= 86,800 / 666 = 130 mobiles/channel.

(g) The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)

$= C \times N_C = 95 \times 31 = 2945$ users, which is 3.4% of the customer base.

Trunking Efficiency

- The number of users can be supported by a channel in a trunked system for a given GOS.
- For the same GOS, the larger the pool, the higher the trunking efficiency.
  - Example: 10 trunked channels at a GOS of 0.01 can support 4.46 Erlangs of traffic, whereas 2 groups of 5 trunked channels can support only 2 × 1.36 Erlangs, or 2.72 Erlangs of traffic.

Blocked Calls Delayed (Erlang C)

- The likelihood of a call not having immediate access to a channel is determined by the Erlang C formula

$$Pr[delay > 0] = \frac{A^C}{A^C + C!\left(1 - \frac{A}{C}\right)\sum_{k=0}^{C-1}\frac{A^k}{k!}}$$

- The probability that a call delayed is forced to wait more than t seconds is given by

$$\begin{aligned} Pr[delay > t] &= Pr[delay > 0]\, Pr[delay > t \mid delay > 0] \\ &= Pr[delay > 0]\exp(-(C-A)t/H) \end{aligned}$$

- The average delay D for all calls in a queued system is given by

$$D = Pr[delay > 0]\frac{H}{C-A}$$

- Where the average delay for those calls which are queued is

$$H/(C-A)$$

Erlang C

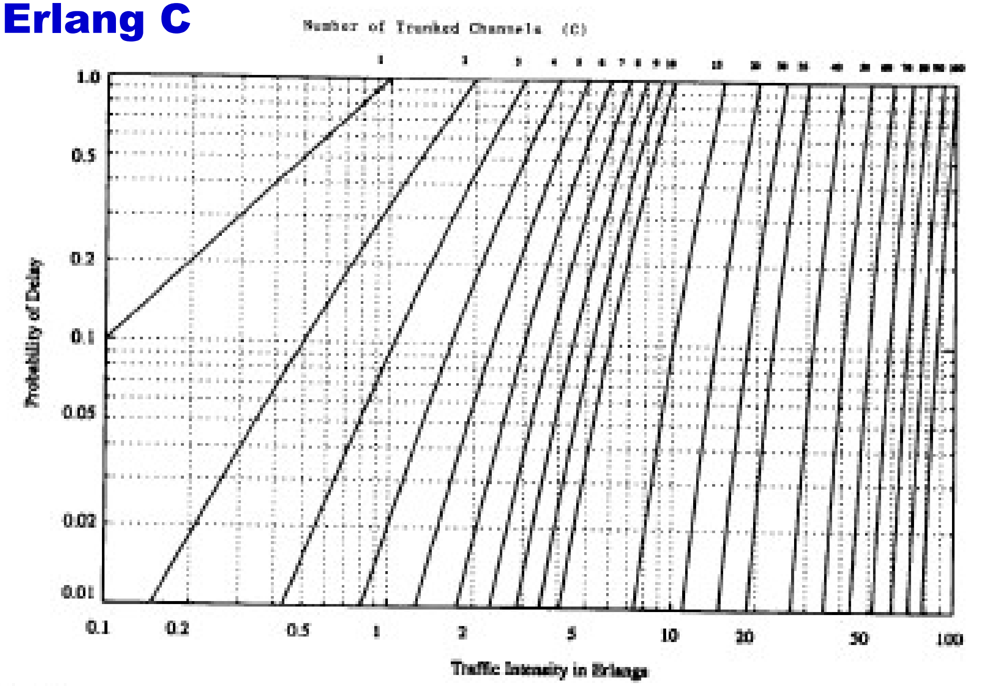


Figure 2.7

The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

Example

A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?

(a) What is the probability that a delayed call will have to wait for more than 10 s?

(c) What is the probability that a call will be delayed for more than 10 seconds?

Example

Given,

Cell radius, $R = 1.387$ km

Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km

Number of cells per cluster = 4

Total number of channels = 60

Therefore, number of channels per cell = 60 / 4 = 15 channels.

(a) From Erlang C chart, for 5% probability of delay with $C = 15$, traffic intensity = 9.0 Erlangs.

Therefore, number of users = total traffic intensity / traffic per user

$= 9.0/0.029 = 310$ users

$= 310$ users$/5$ sq km $= 62$ users/sq km

(b) Given $\lambda = 1$, holding time

$H = A_u/\lambda = 0.029$ hour $= 104.4$ seconds.

The probability that a delayed call will have to wait for more than 10 s is

$$Pr[delay > t | delay] = \exp(-(C-A)t/H)$$

$$= \exp(-(15-9.0)10/104.4) = 56.29\ \%$$

(c) Given $Pr[delay > 0] = 5\% = 0.05$

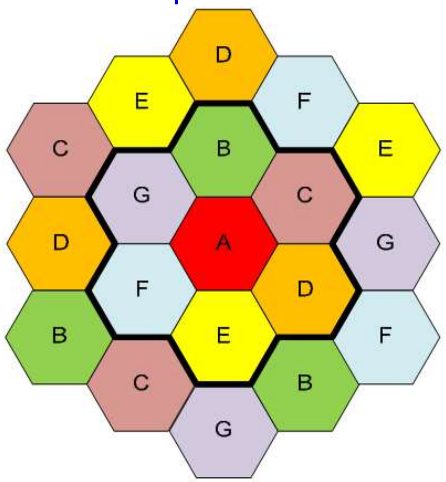
Probability that a call is delayed more than 10 seconds,

$$Pr[delay > 10] = Pr[delay > 0]Pr[delay > t | delay]$$

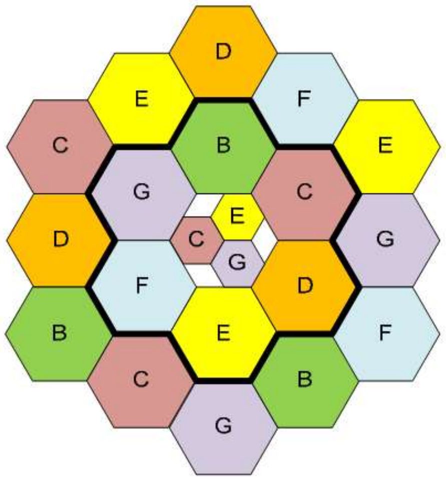
$$= 0.05 \times 0.5629 = 2.81\ \%$$

Cell Splitting

- Subdivides a congested cell into smaller cells, each with its own base station and a corresponding reduction in transmitter power.



Original Cell Distribution



Cell Distribution following the splitting of the cell label A

Improving Coverage and Capacity in Cellular Systems: Cell Splitting

- Subdividing a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmitter power.
- Cell splitting → Increasing system capacity by increasing the number of clusters in a given area
- Decreasing Transmitter Power

$$\mathbf{SIR} \propto \mathbf{Q^n},\ \mathbf{Q = D/R = \sqrt{3N}} \Rightarrow$$

The SIR is independent of transmitted power as long as it is the same for all base stations

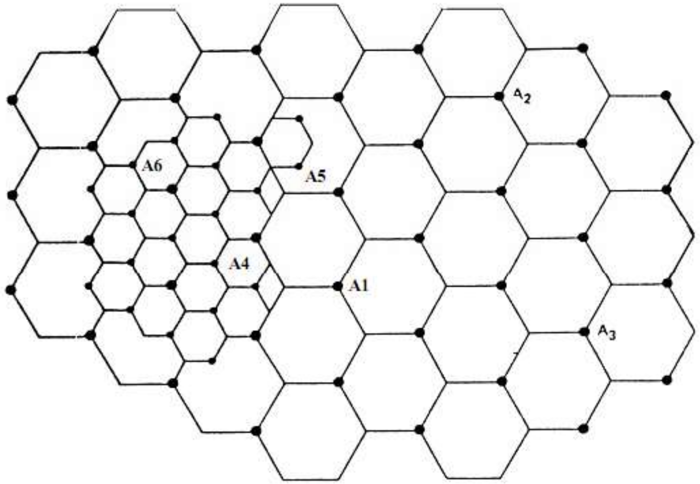
- Why not make Transmitter Power as low as possible?

$$\mathbf{SNR = P_r/Noise} \Rightarrow$$

The SNR must be a above a minimum threshold controlled by P_r

Cell Splitting

- Coexistence of different cell sizes make channel assignments more complicated.
- Need for handoffs increases.

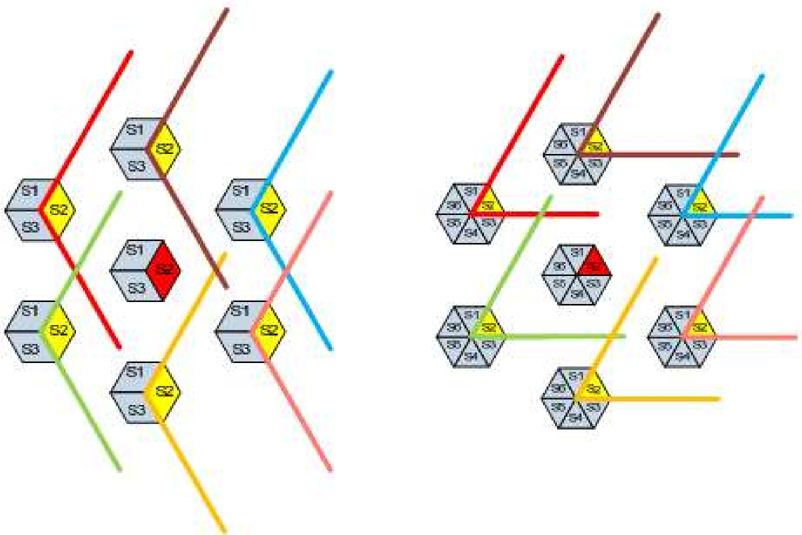


Sectoring

- Use directional antennas to decrease co-channel interference and increase capacity
  - S/I increase → K decrease → Capacity increase
- A cell is normally partitioned into three 120°sectors or six 60°sectors.

- Channels assigned to a cell must be partitioned between the sectors.
  - Requires intra-cell handoff
  - Reduces trunking efficiency of a cell

Sectoring

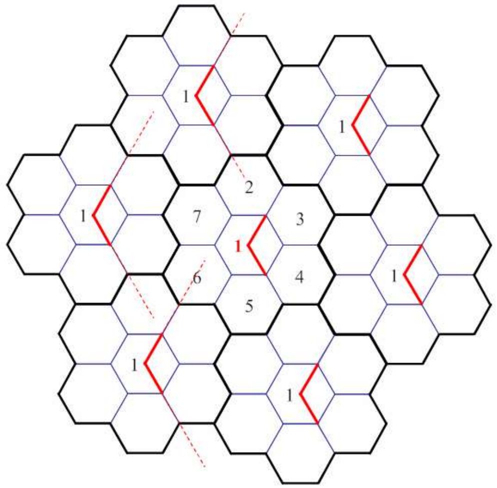


120° Cell Sectoring 60° Cell Sectoring

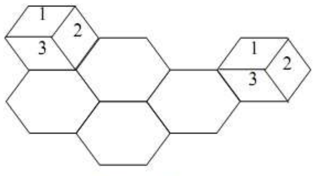
Sectoring

For a 7-cell cluster with 120° sectoring, the number of co-channel cells in the first tier is only $L = 2$ (instead of $L = 6$ for the case of omnidirectional antenna), hence

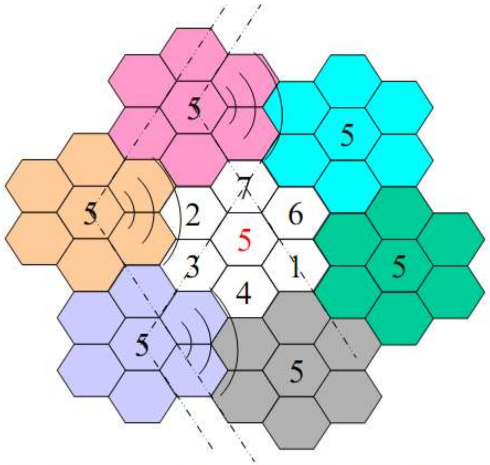
$$S/I \cong \frac{\left(\sqrt{3K}\right)^n}{2}$$



Sectoring

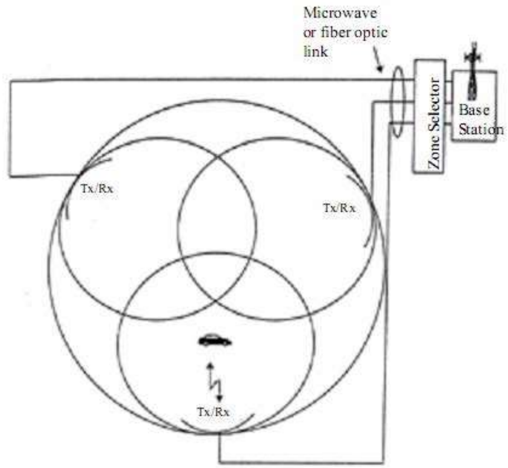


120 sectoring



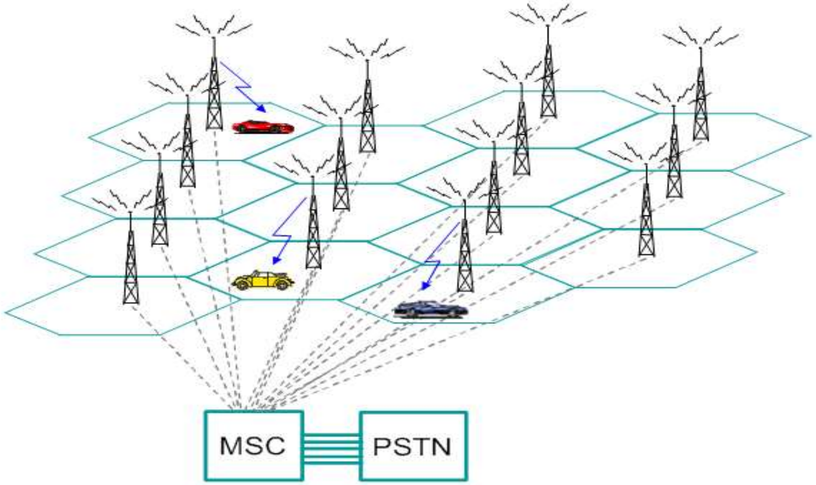
120º sectoring reduces number of interferers from 6 to 2

Sectoring



Cellular System Basics

A basic cellular system



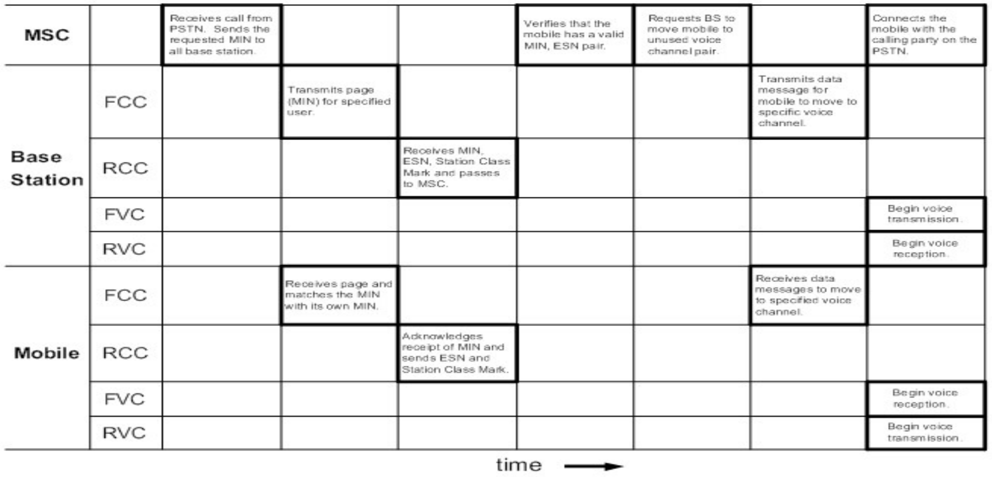
Cellular System Basics

- Cellular system consists of mobile stations, base stations, and mobile-services switching center (MSC)
- All base stations are connected to MSC.
  - A base station serves as a bridge between all mobile users in its cell and connects simultaneous mobile calls to MSC.
- MSC coordinates the activities of all base stations and connects the entire cellular system to the public switched telephone network (PSTN).
  - MSC is sometimes referred to as mobile telephone switching office (MTSO), since it is responsible for connecting all mobiles in a cellular system to the PSTN.

Cellular System Basics

- Communication between the base station and the mobiles is defined by a standard common air interface (CAI) that specifies four different channels.
  - Forward voice channel (FVC): for voice transmission from the base station to mobiles
  - Reverse voice channel (RVC): for voice transmission from mobiles to the base station
  - Forward control channel (FCC) & reverse control channel (RCC): for initiating mobile calls.
    - Control channels are often called setup channels because they are only involved in setting up a call and moving it to an unused voice channel.
    - Control channels transmit and receive data messages that carry call initiation and service requests, and are monitored by mobiles when they do not have a call in progress.
    - Forward control channels also serve as beacons which continually broadcast all of the traffic requests for all mobiles in the system.

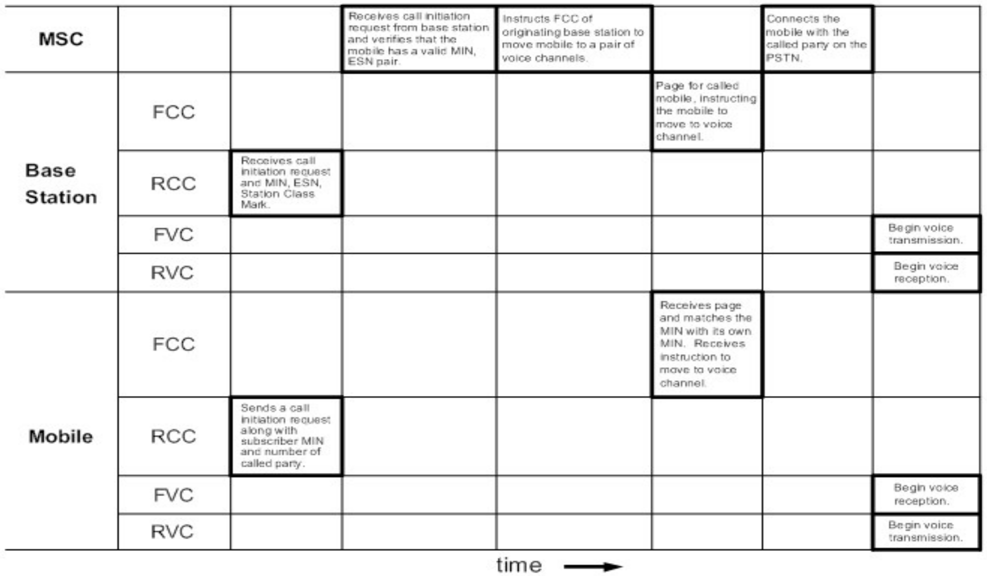
Call Setup



Timing diagram illustrating how a call to a mobile user initiated by a landline subscriber is established

MIN: Mobile Identification Number
ESN: Electronic Serial Number: uniquely identify mobile devices
SCM: Station Class Mark: indicate the maximum transmitter power

Call Setup



Timing diagram illustrating how a call initiated by a mobile is established

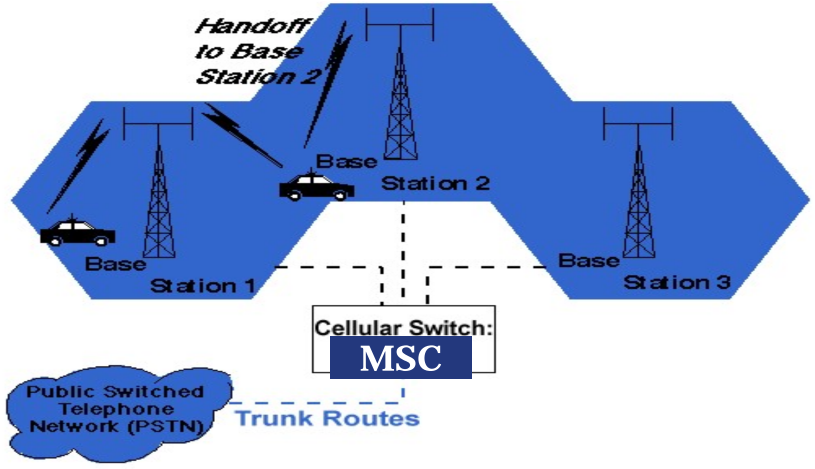
Call Setup

- Call setup is completed within a few seconds and is not noticeable to the user.
- MIN: mobile identification number, which is the subscriber's telephone number
- ESN: electronic serial number
- Station class mark (SCM): indicates what the maximum transmitter power level is for the mobile

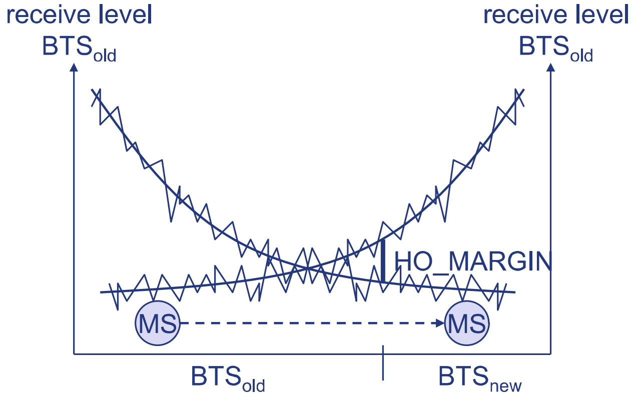
Handoff

- When the mobile moves from one cell to another during a call, the MSC changes the channel of mobile unit and base stations to maintain uninterrupted connection. Special control signaling is applied to the voice channels so that the mobile may be controlled by the base station and the MSC while a call is in progress.

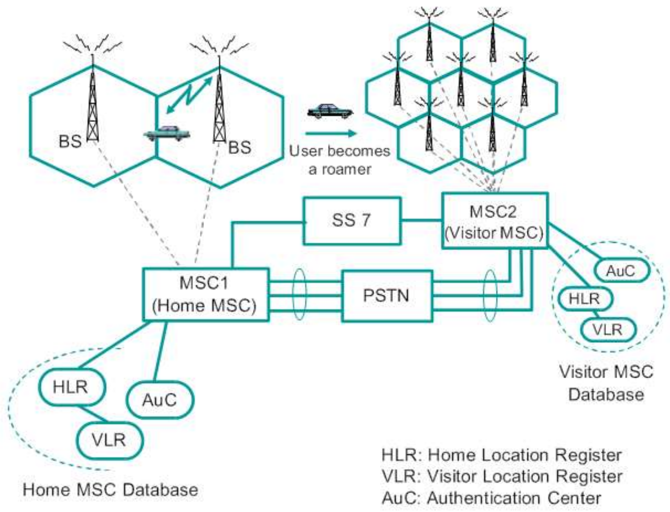
Handover



Handover decision



Cellular Network



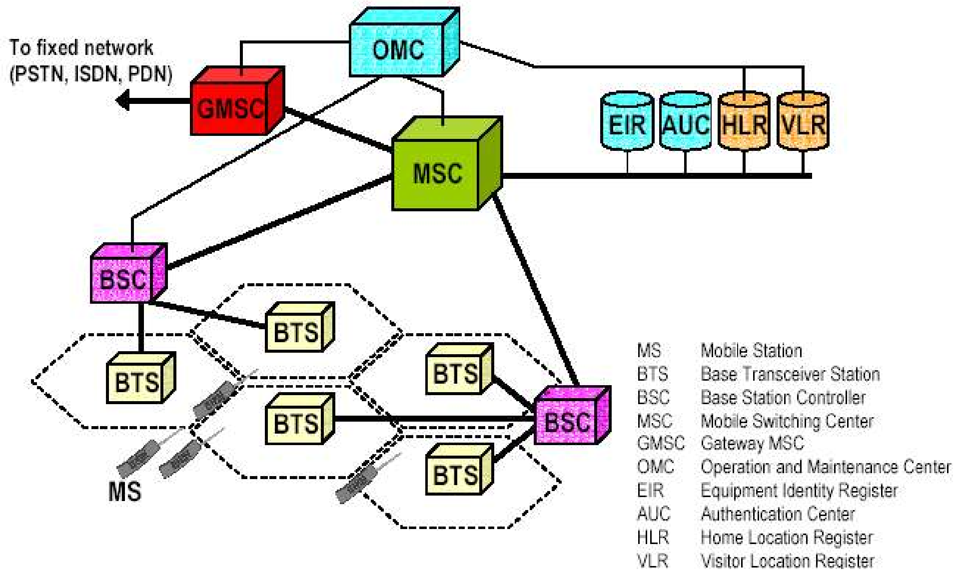
Cellular Network

- A cellular system provides coverage for a particular territory, called *coverage region or market*
- The MSC relies on the following information databases
  - Home location register (HLR): a list of all users (along with their MIN and ESN) who originally subscribed to the cellular system in the coverage region.
  - Visitor location register (VLR): a time-varying list of visiting users (called *romers) in the coverage region who originally subscribed to other cellular* systems.
  - Authentication center (AuC): matches the MIN and ESN of every active mobile in the system with the data stored in the HLR to prevent fraud.
- Interconnection of cellular systems forms a cellular network
  - MSCs are connected via dedicated signaling channels for exchange of location, validation, and call signaling information.
- Cellular network is able to provide service to a mobile subscriber as it moves through different coverage regions. Such a service is referred to as roaming

Roaming

- This allows subscribers to operate in service areas other than the one from which service is subscribed.
- Every several minutes, the MSC issues a global command over each FCC in the system, asking for all mobiles which are previously unregistered to report their MIN and ESN over the RCC.
- By comparing the MIN of a mobile with the MINs contained in its HLR, the MSC is able to quickly identify roamers.
- Once a roamer is identified, the MSC sends a registration request over the landline signaling network to the mobile's home MSC.
- The home MSC validates that the particular mobile has roaming authorization and returns a customer profile to the visited MSC which indicates the availability of features (call waiting, call forwarding, etc.) for the mobile.

GSM essential components



Ho Chi Minh City University of Technology
Faculty of Electrical and Electronics Engineering
Department of Telecommunications

MOBILE PROPAGATION CHANNEL: LARGE-SCALE PATH LOSS

Lectured by Ha Hoang Kha, Ph.D.
Ho Chi Minh City University of Technology
Email: hahoangkha@gmail.com

Content

Reference

- Ta Tri Nghia, Wireless Communications, lecture notes.
- T.S. Rappaport, *Wireless Communications*, Prentice Hall PTR, 1996.

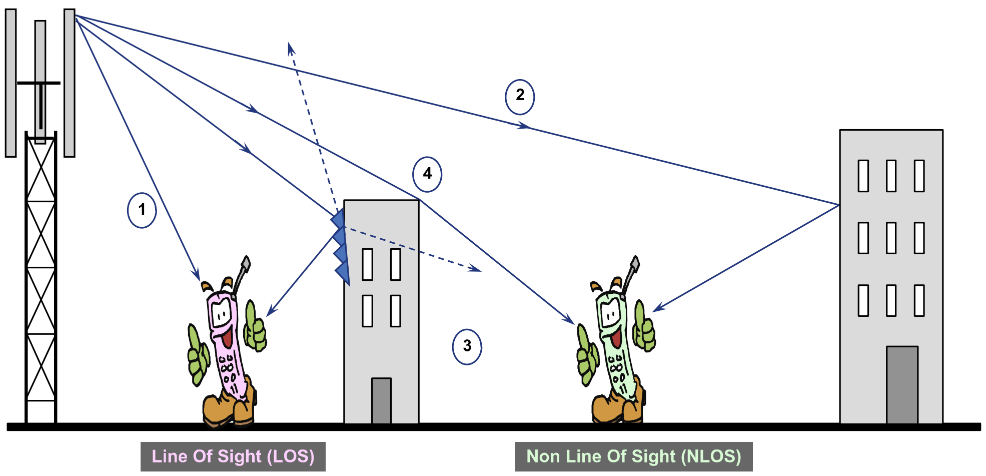
Introduction

- The mobile radio channel places fundamental limitations on the performance of wireless communication systems
- Paths can vary from simple line-of-sight to ones that are severely obstructed by buildings, mountains, and foliage
- Radio channels are extremely random and difficult to analyze
- The speed of motion also impacts how rapidly the signal level fades as a mobile terminals moves about.

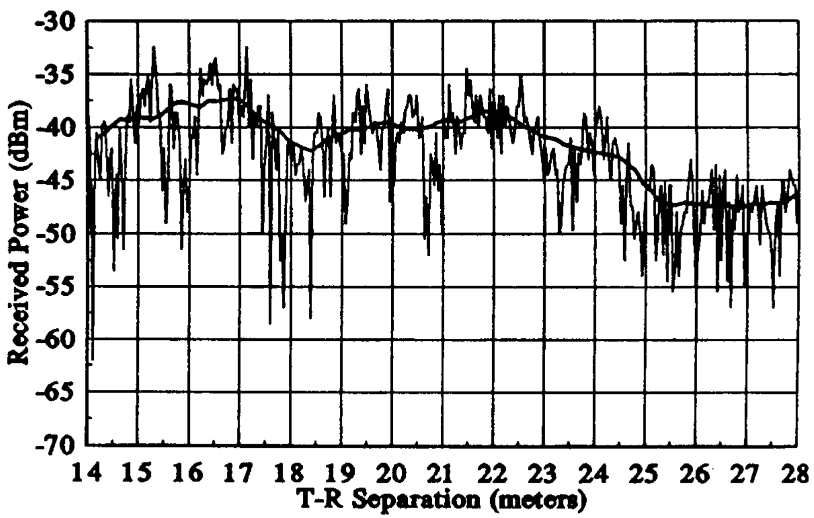
Introduction

- Wired channels are stationary and predictable, radio channels are extremely random and have complex models
- Modeling of radio channels is done in statistical fashion based on measurements for each individual communication system or frequency spectrum

Introduction



Radio Signal Propagation



Radio Signal Propagation

- The smoothed line is the average signal strength. The actual is the more jagged line.
- Actual received signal strength can vary by more than 20 dB over a few centimeters.
- The average signal strength decays with distance from the transmitter, and depends on terrain and obstructions.

Propagation Models

- To predict the average received signal strength at a given distance from the transmitter - **large scale propagation models**, hundreds or thousands of meters
  - Characterize received signal strength over distances from 20 m to 20 km
- To predict the variability of the signal strength, at close spatial proximity to a particular location -**Small scale or fading models**
  - Predict magnitude and rate (speed) of received signal strength fluctuations over short distances/time durations
  - "short" → typically a few wavelengths (λ) or seconds
  - Received signal strength can vary drastically by 30 to 40 dB

Free Space Propagation Model

- The free space propagation model is used to predict received signal strength when the transmitter and receiver have a clear, unobstructed line-of-sight path between them
- The free space model predicts that received power decays as function of the transmitter-receiver (T-R) separation distance raised to some power

Free Space Propagation Model

- Free space power received by a receiver antenna which is separated from a radiating transmitting antenna by a distance *d* (Friis free space equation):

$$P_r(d) = \frac{P_t.G_t.G_r.\lambda^2}{(4\pi)^2.d^2.L}$$

P_t is the transmitted power
$P_r(d)$ is the received power
G_t , G_R is the transmitter and receiver antenna gian
d is the T-R separation distance in meters
L is the system loss factor not related to propagation ($L \geq 1$)
λ is the wavelength in meters

Free Space Propagation Model

- The gain of an antenna is related to its effective aperture, A_e

$$G = \frac{4\pi A_e}{\lambda^2}$$

  - A_e is related to the physical size of the antenna
  - λ is related to the carrier frequency
- The effective isotropic radiated power (*EIRP*) represents the maximum radiated power available from a transmitter in the direction of maximum antenna gain, compared to an isotropic radiator

$$EIRP = P_t G_t$$

Free Space Propagation Model

- The path loss for the free space model when antenna gain are included is given by

$$PL(dB) = 10\log\frac{P_t}{P_r} = -10\log\left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right]$$

- When antenna gains are excluded, the antennas are assumed to have unity gain and path loss is given by

$$PL(dB) = 10\log\frac{P_t}{P_r} = -10\log\left[\frac{\lambda^2}{(4\pi)^2 d^2}\right]$$

Free Space Propagation Model

- The Friis free space model is a only valid predictor for P_r for values of which are in the far-field of the transmitting antenna
- The far-field, or Fraunhofer region, of a transmitting antenna is defined as the region beyond the far-field distance d_f, related to the largest linear dimension of the transmitter antenna aperture and the carrier wavelength
- The Fraunhofer distance is given by

$$d_f = \frac{2D^2}{\lambda} \qquad d_f \gg \lambda$$

D is the largest physical linear dimension of the antenna

d_f must satisfy $d_f >> D$, and $d_f >> \lambda$

Free Space Propagation Model

- Large-scale propagation models use a close-in distance, d_0, as a known received power reference point
- The received power, $P_r(d)$, at any distance $d > d_0$, may be related to P_r at P_0
- The reference distance must be chosen such that it lies in the far-field region, that is, $d_0 \geq d_f$, and d_0 is chosen to be smaller than any practical distance used in the mobile communication system
- The received power in free space at a distance greater than d_0 is given by

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2 \qquad , d \geq d_0 \geq d_f$$

Free Space Propagation Model

- Large dynamic range of received power levels, often dBm or dBW units are used to express received power levels

$$P_r(d) = 10\log\left(\frac{P_r(d_o)}{0,001W}\right) + 20\log\left(\frac{d_o}{d}\right) \qquad , d \geq d_o \geq d_f$$

$P_r(d)$ is in dBm

$P_r(d_0)$ is in watts

Example

Given a transmitter produces 50 W of power. If this power is applied to a unity gain antenna with 900 MHz carrier frequency, find the received power at a free space distance of 100 m from the antenna. What is P_r(10 km). Assume unity gain for the receiver antenna

Example

- The received power at 100m

$$P_r(d=100m)=\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}=\frac{50.1.1.(1/3)^2}{(4\pi)^2.100^2.1}=3{,}5.10^{-6}W$$

$$P_r(d=100m)=10\log\frac{3{,}5.10^{-6}}{0{,}001}=-24{,}5dBm$$

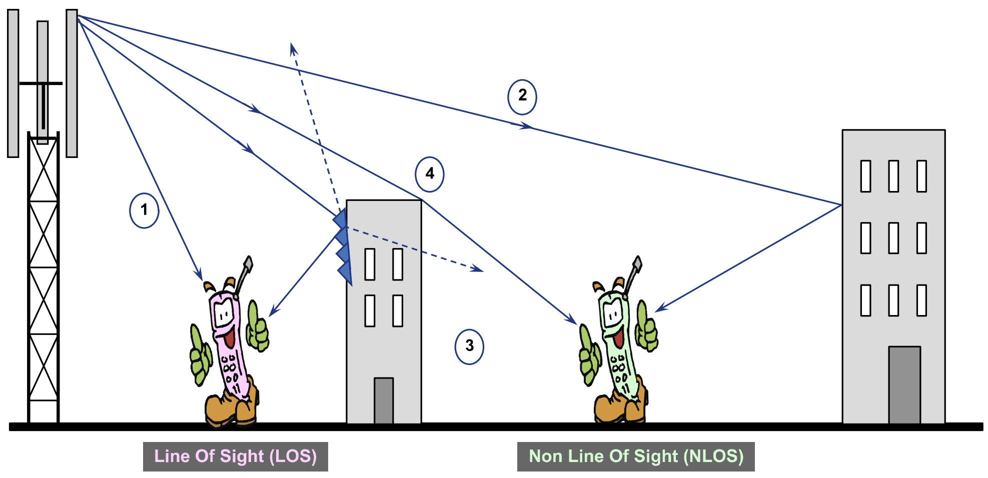
- The received power at 10Km

$$P_r(d=10km)=-24{,}5+20\log\frac{100}{10000}=-64{,}5dBm$$

Propagation Mechanisms

- Receiving power is generally the most important parameter predicted by large-scale propagation model based on the physics of reflection, scattering, and diffraction
- Reflection occurs when a propagating electromagnetic wave impinges upon an object which has very large dimensions when compared to the wavelength of the propagating wave.
- Diffraction occurs when the radio path between the transmitter and receiver is obstructed by a surface that has sharp irregularities
- Scattering occurs when the medium through which the wave travels consists of objects with dimensions that are small compared to the wavelength and where the number of obstacles per unit volume is large

Propagation Mechanisms



Reflection

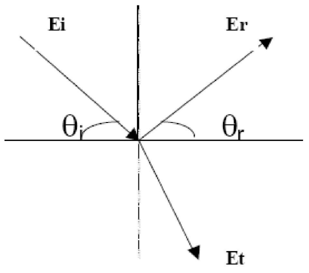
- Reflection occur from the surface of the earth and from buildings and walls
- When a radio wave propagating in one medium impinges upon another medium having different electrical properties, the wave is partially reflected an partially transmitted

Reflection

- The electric field intensity of the reflected and transmitted waves may be related to the incident wave the medium of origin through the Fresnel reflection coefficient (Γ)
- The reflection coefficient is a function of the material properties, and generally depends on the wave polarization, angle of incidence, and the frequency of the propagation wave

Law Of Reflection At The Boundary Between 2 Dielectrics

- Reflection coefficient $\dfrac{E_r}{E_i} = \Gamma$
- Transmission coefficient $\dfrac{E_t}{E_i} = T = 1 + \Gamma$



$\theta_i = \theta_r$

Ground Reflection (2-ray) Model

- In mobile radio channel, the 2-ray ground reflection model is a useful propagation model that is based on geometrics optics, and considers both direct path and a ground reflected propagation path between transmitter and receiver
- This model has been found to be reasonably accurate for predicting the large-scale signal strength over distance of several kilometers for mobile radio systems

Ground Reflection (2-ray) Model

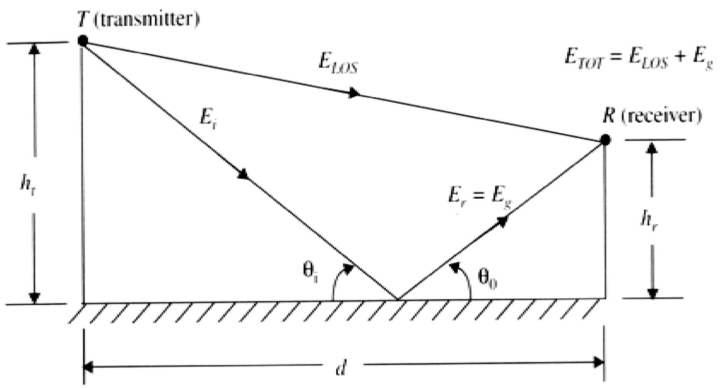


Figure 4.7 Two-ray ground reflection model.

Ground Reflection (2-ray) Model

- In mobile communication systems, the maximum T-R separation distance is at most only a few tens of kilometers, and the earth may be assumed to be flat
- The total received E-field, E_{TOT}, is then a result of the direct line-of-sight component, E_{LOS}, and the ground reflected component, E_g
- If E_0 is the free space E-field (V/m) at the reference distance d0 from the transmitter, then for $d > d_0$, the free space propagating E-field is given by

$$E(d,t) = \frac{E_0 d_0}{d} \cos\left[\omega_c \left(t - \frac{d}{c}\right)\right]$$

- where $|E(d,t)| = E_0 d_0 / d$ represents the envelope of the E-field at *d* distance from the transmitter

Ground Reflection (2-ray) Model

- Two propagating waves arrives at the receiver: the direct wave that travel a distance *d'* ; and the reflected wave that travel a distance *d"*
- The direct LOS component at the receiver can be expressed

$$E_{LOS}(d',t) = \frac{E_0 d_0}{d'} \cos\left[\omega_c \left(t - \frac{d'}{c}\right)\right]$$

- The E-field for the ground reflected wave can be expressed

$$E_g(d'',t) = \Gamma \frac{E_0 d_0}{d''} \cos\left[\omega_c \left(t - \frac{d''}{c}\right)\right]$$

Ground Reflection (2-ray) Model

- According to laws of reflection in dielectrics:

$$\theta_i = \theta_o \qquad E_g = \Gamma E_i \qquad E_t = (1+\Gamma) E_i$$

  - Where Γ is the reflection coefficient for ground
- For small values of θ_i , the reflected wave is equal in magnitude and 180^0 out of phase with the incident wave
- The resultant E-field, assuming perfect ground reflection (i.e. Γ = 1 and $E_t = 0$) is the vector sum of E_{LOS} and E_g

$$|E_{TOT}| = |E_{LOS} + E_g|$$

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'} \cos\left[\omega_c \left(t - \frac{d'}{c}\right)\right] + (-1) \frac{E_0 d_0}{d''} \cos\left[\omega_c \left(t - \frac{d''}{c}\right)\right]$$

Ground Reflection (2-ray) Model

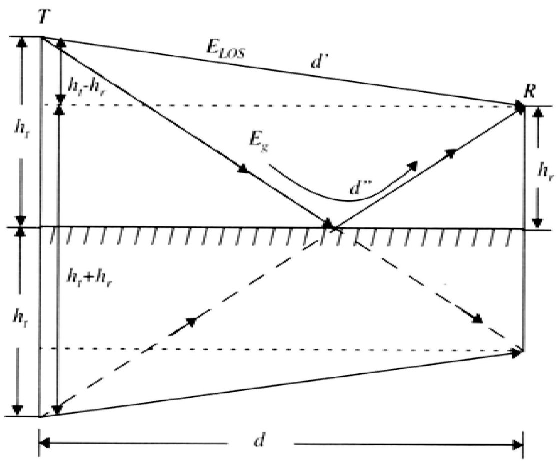


Figure 4.8 The method of images is used to find the path difference between the line-of-sight and the ground reflected paths.

Ground Reflection (2-ray) Model

- The path difference between line-of-sight and the ground reflected paths can be expressed

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$\Delta = d'' - d' \approx \frac{2h_t h_r}{d}$$

- The phase difference θ_Δ between the two E-field component and the time delay τ_d between the arrivals of the components

$$\theta_\Delta = \frac{2\pi}{\lambda}\Delta = \frac{\Delta\omega_c}{c} \qquad \tau_d = \frac{\Delta}{c} = \frac{\theta_\Delta}{2\pi f_c}$$

Ground Reflection (2-ray) Model

- As *d* becomes large, the difference between the distance *d'* and *d''* becomes very small, and the amplitudes of E_{LOS} and E_g is virtually identical and differ only in phase

$$\left|\frac{E_0 d_0}{d}\right| \approx \left|\frac{E_0 d_0}{d'}\right| \approx \left|\frac{E_0 d_0}{d''}\right|$$

- The E-field at the receiver at the distance d from the transmitter can be written as

$$|E_{TOT}| = \frac{E_0 d_0}{d}\sqrt{2 - 2\cos\theta_\Delta} = 2\frac{E_0 d_0}{d}\sin\frac{\theta_\Delta}{2}$$

Ground Reflection (2-ray) Model

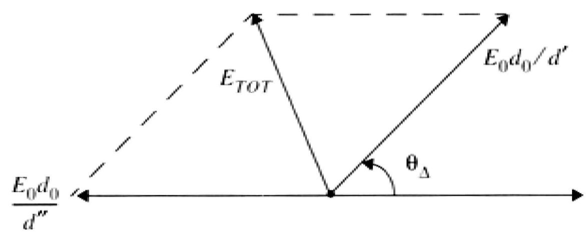


Figure 4.9 Phasor diagram showing the electric field components of the line-of-sight, ground reflected, and total received E-fields, derived from Equation (4.45).

Ground Reflection (2-ray) Model

- If θ_Δ < 0.3 radian

$$\sin\frac{\theta_\Delta}{2} \approx \frac{\theta_\Delta}{2} = \frac{2\pi h_t h_r}{\lambda d} < 0{,}3rad$$

$$d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20 h_t h_r}{\lambda}$$

- E-field can be approximately as

$$E_{TOT} \approx \frac{2E_o d_0}{d} \cdot \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} (V/m)$$

  - *K* is constant related to E_0 , the antenna height and the wavelength

Ground Reflection (2-ray) Model

- The received power at the distance d from the transmitter can be expressed as

$$P_r(d) = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

- Path loss 2-ray model (with antenna gains) can be expressed in dB as

$$PL(dB) = 40\log d - \left(10\log G_t + 10\log G_r + 20\log_t + 20\log h_r\right)$$

  - When $\theta_\Delta = \Pi$, then $d = (4h_t h_r)/\lambda$ is where the ground appears in the first Fresnel zone between the transmitter and receiver

Example

A mobile is located 5 km away from a base station. and uses a vertical $\lambda/4$ monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The E field at 1 km from the transmitter is measured to be 10^{-3} V/m. The carrier frequency used is 900 MHz.

(a) Find the length and effective aperture of the receiving antenna

(b) Find the received power at the mobile using the 2-way ground model assuming the height of the transmitting antenna is 50 m and receiving antenna is 1.5 m above the ground.

Example

- Wavelength: $\lambda = \frac{c}{f} = \frac{3.10^8}{900.10^6} = 0{,}33m$
- Length of the antenna: $L = \lambda/4 = 8.33\ cm$
- Gain of antenna: $G = 2.55\ dB = 1.8$
- Since $d >> \sqrt{h_t h_r}$

$$E_R(d) \approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} (V/m)$$

$$= \frac{2.10^{-3}.1.10^3}{5.10^3} \left[\frac{2\pi.50.1{,}5}{0{,}33.(5.10^3)}\right] = 113{,}1.10^{-6} V/m$$

- Received power

$$P_r(d = 5km) = \frac{(113{,}1.10^{-6})^2}{377} \left[\frac{1{,}8(0{,}33)^2}{4\pi}\right] = 5{,}4.10^{-13} W$$

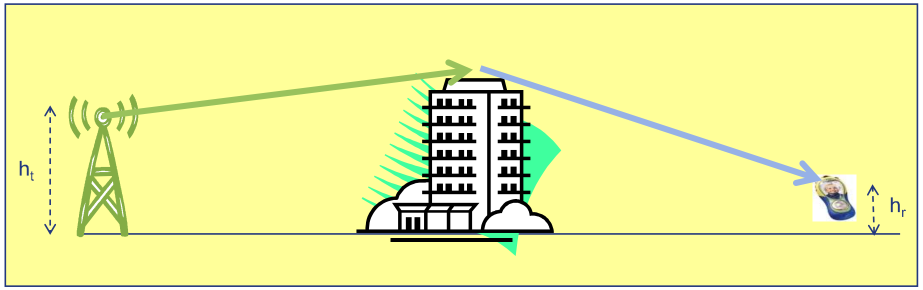
$$= -122{,}68dBW = -92{,}68dBm$$

Diffraction

- Diffraction allows radio signals to propagate around the curved surface of the earth, beyond horizon, and to propagate behind obstructions
- The received field strength decreases rapidly as a receiver moves deeper into obstructed (shadow) region.
- The diffraction field still exists and often has sufficient strength to produce a useful signal

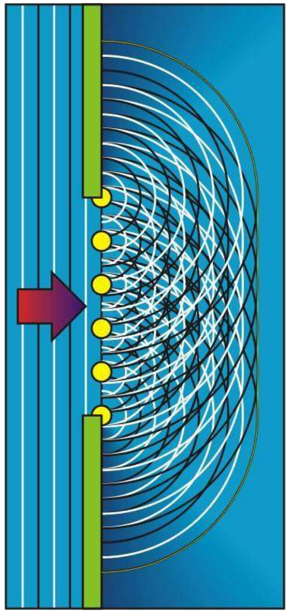
Diffraction

Diffraction allows radio signals to propagate behind obstacles between a transmitter and a receiver

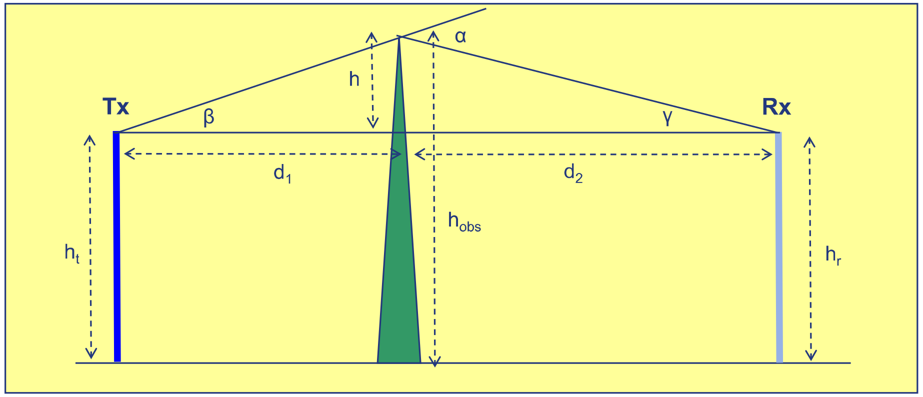


Huygen's Principle & Diffraction

All points on a wavefront can be considered as point sources for the production of secondary wavelets. These wavelets combine to produce a new wavefront in the direction of propagation.



Knife-Edge Diffraction Geometry



$$\Delta = \sqrt{d_1^2 + h^2} + \sqrt{d_2^2 + h^2} - d_1 - d_2 = d_1\sqrt{1+\left(\frac{h}{d_1}\right)^2} + d_2\sqrt{1+\left(\frac{h}{d_2}\right)^2} - d_1 - d_2$$

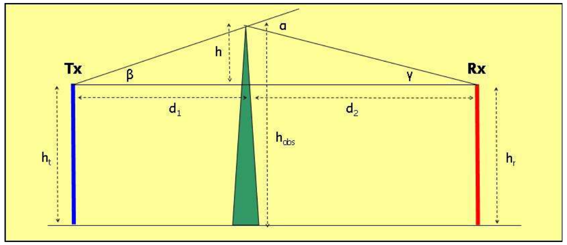
$$\boxed{\Delta \approx \frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)} \quad h << d_1, d_2 \quad \text{where } \sqrt{1+x} \approx 1 + \frac{x}{2} \text{ for } x << 1$$

Δ: Excess Path Length (Difference between Diffracted Path and Direct Path)

Fresnel Zone Diffraction Parameter (v)

Φ: Phase Difference between Diffracted Path and Direct Path)

$$\Phi = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)$$



Assume $\tan\beta \approx \beta \quad \tan\gamma \approx \gamma$

$$\Rightarrow \alpha = \beta + \gamma \approx \frac{h}{d_1} + \frac{h}{d_2} = \frac{h(d_1+d_2)}{d_1 d_2}$$

Fresnel Zone Diffraction Parameter (V)

$$v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}$$

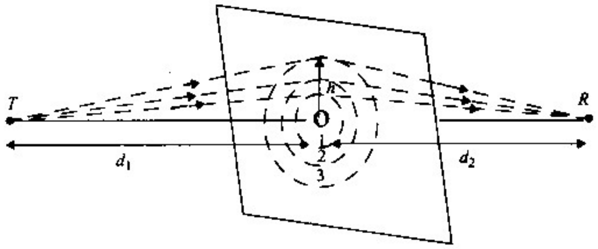
$$\boxed{\Rightarrow \Phi = \frac{\pi}{2}v^2}$$

- v^2=2, 6, 10 … corresponds to ***destructive*** interference between direct and diffracted paths
- v^2=4, 8, 12 … corresponds to ***constructive*** interference between direct and diffracted paths

Fresnel Zones

Fresnel Zones:

Successive regions where secondary waves have a path length from the transmitter to receiver which is nλ/2 greater than the total path length of a line-of-sight path



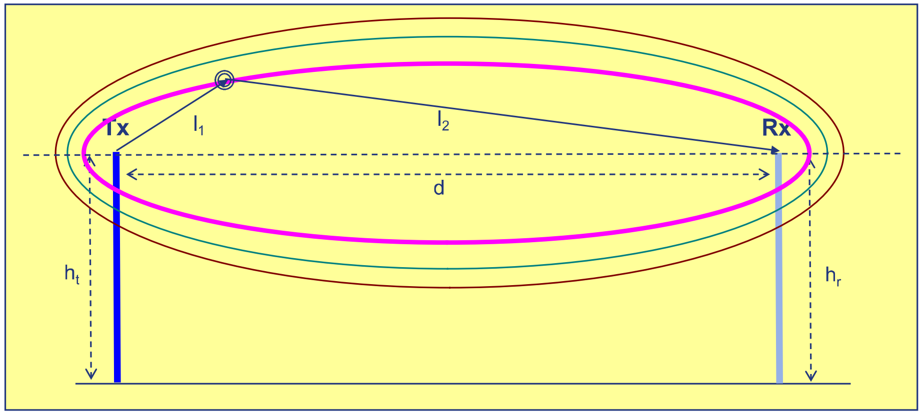
From "Wireless Communications: Principles and Practice" T.S. Rappaport

$$\Delta = \frac{n\lambda}{2} = \frac{r_n^2}{2}\frac{(d_1+d_2)}{d_1 d_2} \Rightarrow r_n = \sqrt{\frac{n\lambda d_1 d_2}{(d_1+d_2)}}$$

r_n: Radius of the n^{th} Fresnel Zone

Diffraction Loss

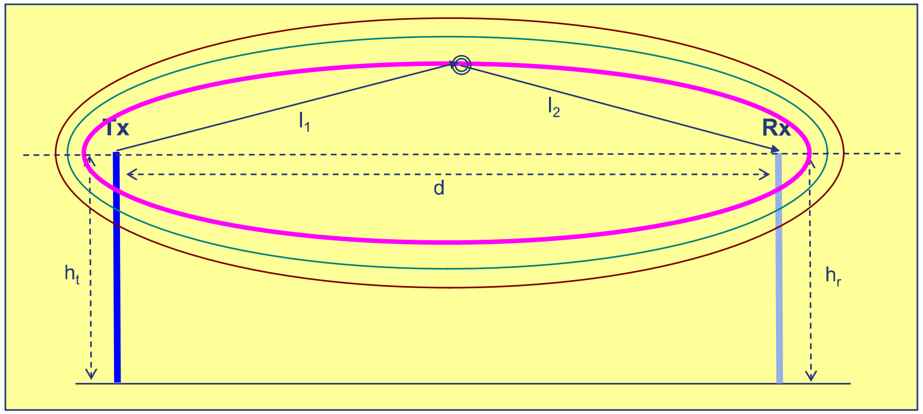
Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



First Fresnel Zone Points ➔ $l_1+l_2-d = (\lambda/2)$

Diffraction Loss

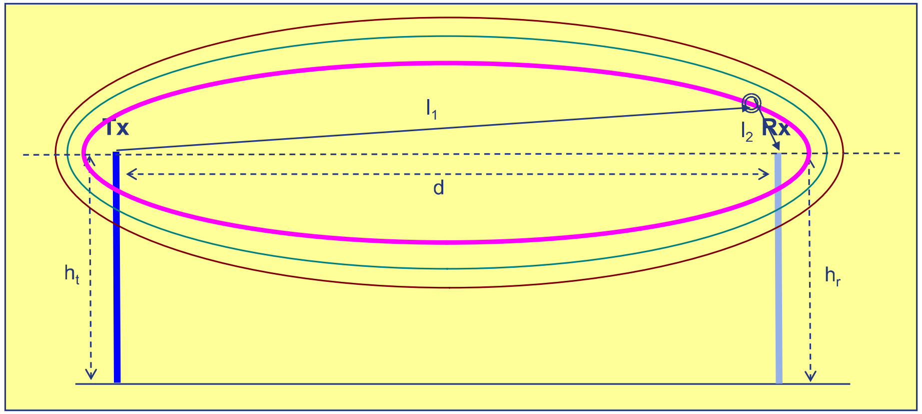
Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



First Fresnel Zone Points ➔ $l_1+l_2-d = (\lambda/2)$

Diffraction Loss

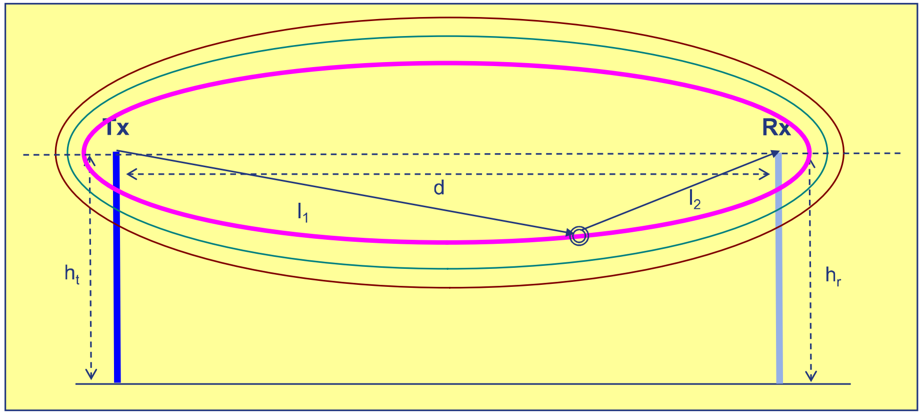
Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



First Fresnel Zone Points ➔ $l_1+l_2-d=(\lambda/2)$

Diffraction Loss

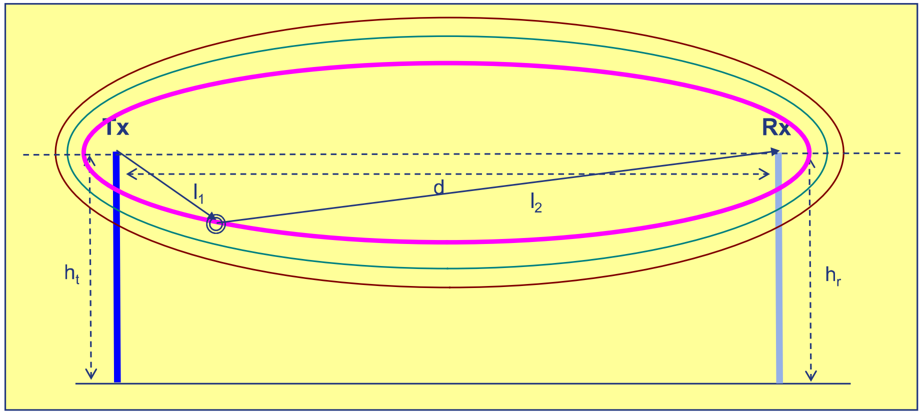
Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



First Fresnel Zone Points ➔ $l_1+l_2-d=(\lambda/2)$

Diffraction Loss

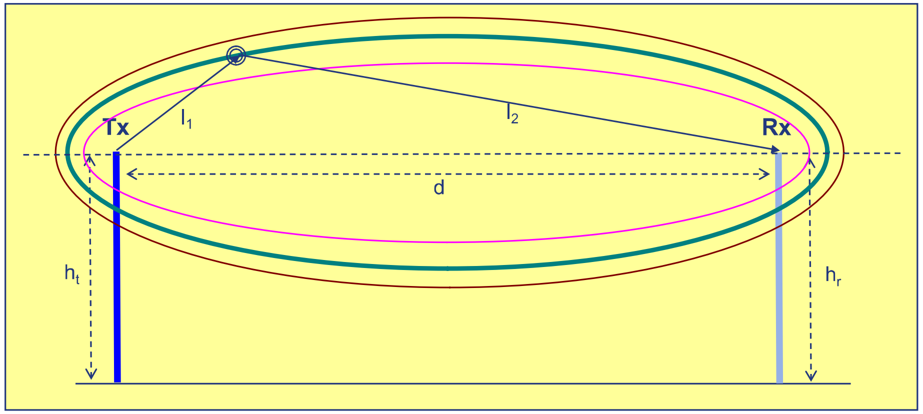
Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



First Fresnel Zone Points ➔ $l_1+l_2-d=(\lambda/2)$

Diffraction Loss

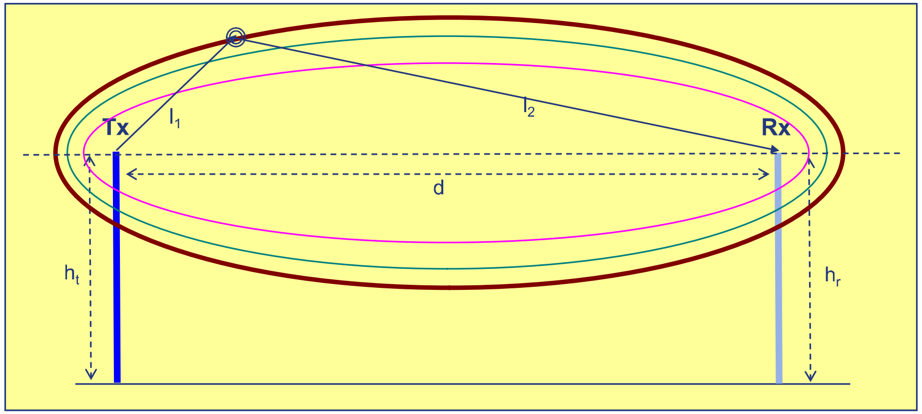
Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



Second Fresnel Zone Points ➔ $l_1+l_2-d=\lambda$

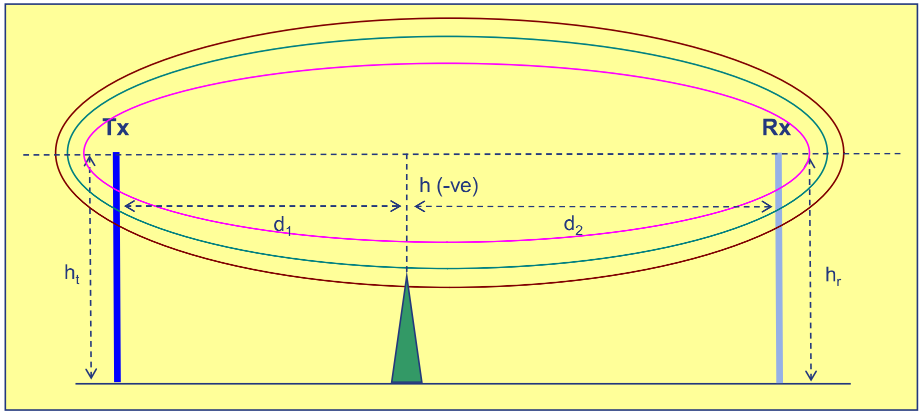
Diffraction Loss

Diffraction Loss occurs from the blockage of secondary waves such that only a portion of the energy is diffracted around the obstacle



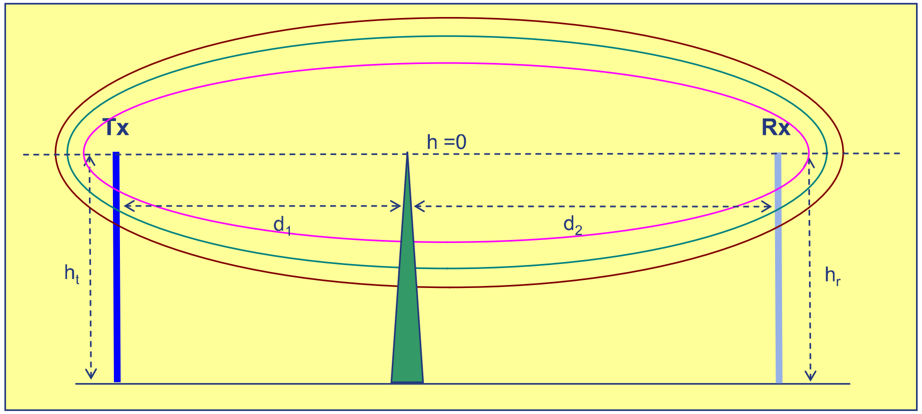
Third Fresnel Zone Points ➔ $l_1+l_2-d = (3\lambda/2)$

Knife-Edge Diffraction Scenarios



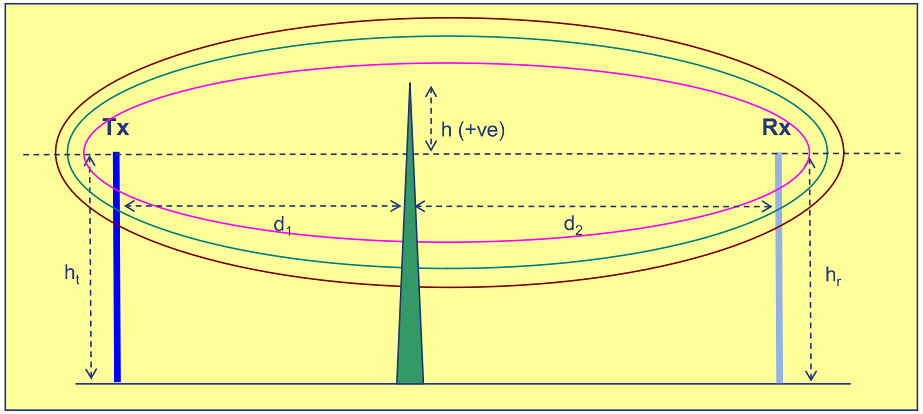
- h & ν are –ve
- Relative Low Diffraction Loss

Knife-Edge Diffraction Scenarios



- h =0
- Diffraction Loss = 0.5

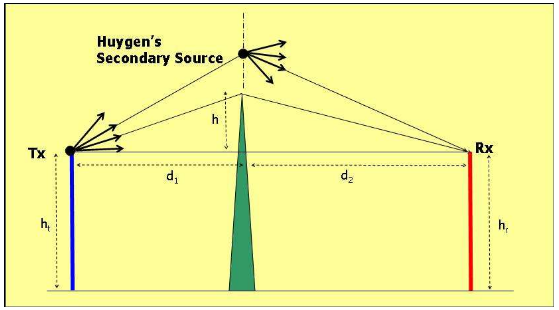
Knife-Edge Diffraction Scenarios



- h & ν are +ve
- Relatively High Diffraction Loss

Knife-Edge Diffraction Model

The field strength at point Rx located in the shadowed region is a vector sum of the fields due to all of the secondary Huygen's sources in the plane above the knife-edge

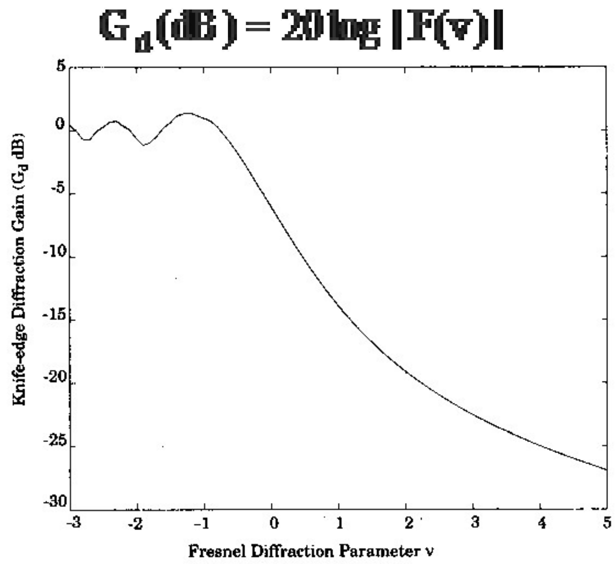


Electric Field Strength, E_d, of a Knife-Edge Diffracted Wave is given By:

$$\frac{E_d}{E_0} = F(v) = \frac{1+j}{2}\int_v^{\infty} \exp\left(-\frac{j\pi t^2}{2}\right)dt$$

E_0: Free-Space Field Strength in absence of Ground Reflection and Knife-Edge Diffraction
F(v) is called the complex Fresnel Integral

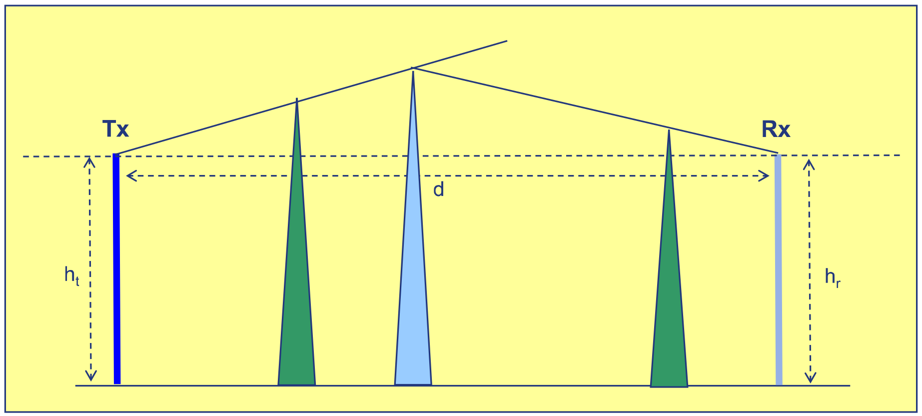
Diffraction Gain



Diffraction Gain Approximation

$G_d(dB) = 0$	$v \le -1$
$G_d(dB) = 20\log(0.5 - 0.62v)$	$-1 \le v \le 0$
$G_d(dB) = 20\log\big(0.5exp(-0.95v)\big)$	$0 \le v \le 1$
$G_d(dB) = 20\log\left(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2}\right)$	$1 \le v \le 2.4$
$G_d(dB) = 20\log\left(\frac{0.225}{v}\right)$	$v > 2.4$

Multiple Knife-Edge Diffraction



- In the practical situations, especially in hilly terrain, the propagation path may consist of more than one obstruction.
- Optimistic solution (by Bullington): The series of obstacles are replaced by a single equivalent obstacle so that the path loss can be obtained using single knife-edge diffraction models.

Example

Compute the diffraction loss between the transmitter and receiver assuming, λ = 1/3 m, d_1 = 1 km, d_2 = 1 km and h = 25m

Example

- Given
  - λ = 1/3 m,
  - d_1 = 1 Km,
  - d_2 = 1 Km
  - h = 25 m
- Fresnel diffraction parameter

$$v = h\sqrt{\frac{2}{\lambda}\cdot\left(\frac{d_1+d_2}{d_1 d_2}\right)} = 25\sqrt{\frac{2}{1/3}\cdot\left(\frac{1000+1000}{1000*1000}\right)} = 2.74$$

$$F(v) = 20\log\left(0.225/2.74\right) = -21.71dB$$

- Diffraction loss is 21.71 dB

Scattering

- The actual received signal in a mobile radio environment is often stronger than what is predicted by reflection and diffraction
- **Reason:** When a radio wave impinges on a rough surface, the reflected energy is spread in all directions due to scattering

Reflection Vs Scattering

- Reflection: Flat surfaces that have much larger dimension than wavelength
- Scattering: When the medium consists of objects with dimensions that are small compared to the wavelength

Testing Surface Roughness using Rayleigh Criterion

$$h_c = \frac{\lambda}{8\sin\theta_i}$$

h_c : Critical Height of Surface Protuberance
Θ_i : Angle of Incidence
λ : Wavelength

Smooth Surface → Minimum to maximum protuberance h is less than h_c

Rough Surface → Minimum to maximum protuberance h is greater than h_c

Reflection Coefficient for Rough Surfaces

$$\Gamma_{rough} = \rho_S \Gamma$$

$$\rho_S = \exp\left(-8\left(\frac{\pi\sigma_h \sin\theta_i}{\lambda}\right)^2\right) \qquad \text{Ament}$$

$$\rho_S = \exp\left(-8\left(\frac{\pi\sigma_h \sin\theta_i}{\lambda}\right)^2\right) I_0\left(8\left(\frac{\pi\sigma_h \sin\theta_i}{\lambda}\right)^2\right) \qquad \text{Boithias}$$

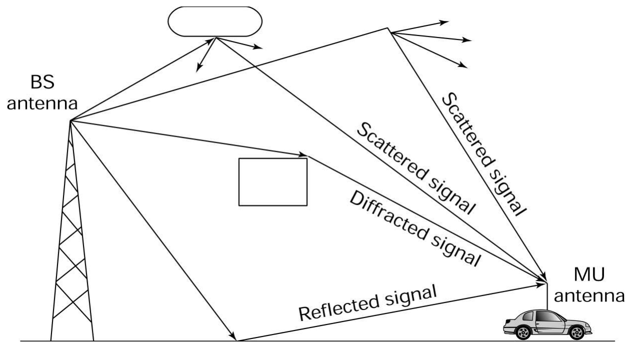
Γ_{rough}: Reflection Coefficient for Rough Surfaces
Γ : Reflection Coefficient for Smooth Surfaces
ρ_S : Scattering Loss Factor
σ_h : Standard deviation of the surface height h about the mean surface height
$I_0(.)$: Bessel Function of the first kind and zero order

Practical Link Budget Design

- Most radio propagation models are derived using a combination of analytical and empirical models
- Empirical approach is based on fitting curves or analytical expressions that recreate a set of measured data
  - Advantages: Takes into account all propagation factors, both known and unknown
  - Disadvantages: New models need to be measured for different environment or frequency
- Over many years, some classical propagation models have been developed, which are used to predict large-scale coverage for mobile communication system design

Path-Loss Models

- The most general case of signal reception might consist of a direct path, reflected paths, diffracted paths, and scattered paths (which makes mathematical analysis cumbersome)
- Path-Loss models are empirical models that are based on fitting curves or analytical expressions that recreate a set of measured data
- Note:
  - A given empirical model might only be valid within the environment where the measurements used to estimate such model have been taken



Log-Distance Path-Loss Model

Theoretical and measurement-based propagation suggest that the average received signal power decreases logarithmically with distance

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n$$

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)$$

$\overline{PL}(d)$: Average path-loss for an arbitrary separation
n : Path-loss exponent

Path-Loss Exponent for Different Environments

Environment	Path-Loss Exponent n
Free-Space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Log-normal Shadowing

- Distance between two nodes alone cannot fully explain the signal strength level at the receiver
- Shadowing has been introduced as a means to model the variation of signal propagation behavior between two different signal paths assuming the same propagation distance

$$PL(d) = \overline{PL}(d) + X_\sigma$$

$$PL(d) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

PL (d): Path-loss model for an arbitrary separation d
X_σ : Shadowing parameter (zero mean Gaussian distributed random variable in dB with standard deviation σ also in dB)

Log-normal Shadowing, *n* and σ

- The log-normal shadowing model indicates the received power at a distance d is normally distributed with a distance dependent mean and with a standard deviation of σ
- In practice the values of n and σ are computed from measured data using linear regression so that the difference between the measured data and estimated path losses are minimized in a mean square error sense.

Example of determining n and σ

- Assume $P_r(d_0)$ = 0dBm and d_0 is 100m
- Assume the receiver power P_r is measured at distances 100m, 500m, 1000m, and 3000m,
- The table gives the measured values of received power

Distance from Transmitter	Received Power
100m	0dBm
500m	-5dBm
1000m	-11dBm
3000m	-16dBm

Example of determining n and σ

- We know the measured values.
- Lets compute the estimates for received power at different distances using long-distance path loss model.

$$\overline{PL}(d)[dB] = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)$$

$$\overline{P_r}(d)[dBm] = P_t[dBm] - \overline{PL}(d)[dB]$$

$$\overline{P_r}(d)[dBm] = P_t[dBm] - \left[\overline{PL}(d_0)[dB] + 10n\log(\frac{d}{d_0})\right]$$

Example of determining n and σ

- The mean square error (MSE) is given with the following formula:

$$MSE = \sum_{i=1}^{k}(p_i - \hat{p}_i)^2$$

p_i is the actual measured value of power at some distance

$\hat{p}_i$ is the estimate of power at that distance

k is the number of measurement samples

Example of determining n and σ

- Since power estimate at some distance depends on *n*, MSE(*n*) is a function of n.
- We would like to find a value of n that will minimize this MSE(n) value. We will call it MMSE: minimum mean square error.
- This can be achieved by writing MSE as a function of n. Then finding the value of n which minimizes this function. This can be done by derivating MSE(n) with respect to n and solving for n which makes the derivative equal to zero.

Example of determining n and σ

Distance	Measured Value of Pr (dBm)	Estimated Value of Pr (dBm)
100m	0	0
500m	-5	-6.99n
1000m	-11	-10n
3000m	-16	-14.77n

MSE = $(0-0)^2 + (-5-(-6.99n))^2 + (-11-(-10n)^2 + (-16-(-14.77n)^2$

MSE = $0 + (6.99n – 5)^2 + (10n – 11)^2 + (14.77n – 16)^2$

- If we open this, we get MSE as a function of n which as second order polynomial.
- We can easily take its derivate and find the value of n which minimizes MSE (MMSE)

Example of determining n and σ

- We are interested in finding the standard deviation about the mean value For this, we will use the following formula

$$\sigma^2 = \frac{\sum_{i=1}^{k}(p_i - \hat{p}_i)^2}{k}$$

p_i is the actual measured value of power at some distance d

$\hat{p}_i$ is the estimate of power at that distance d

k is the number of measurement samples

From the above definition of σ, we can derive that :

$$\sigma^2 = MSE(N)/k$$

$$\sigma^2 = MMSE/k$$

$$\sigma = \sqrt{MMSE/k}$$

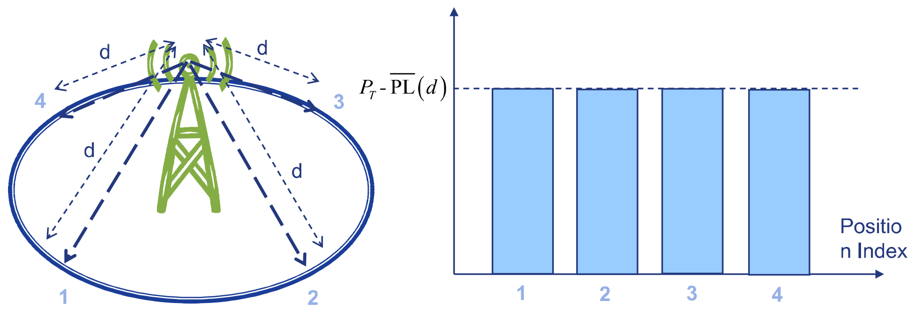
where N is the value that minimizes $MSE(n)$

$MMSE$ is minimum mean square error.

$MSE(n)$ formula is given in the previous slides.

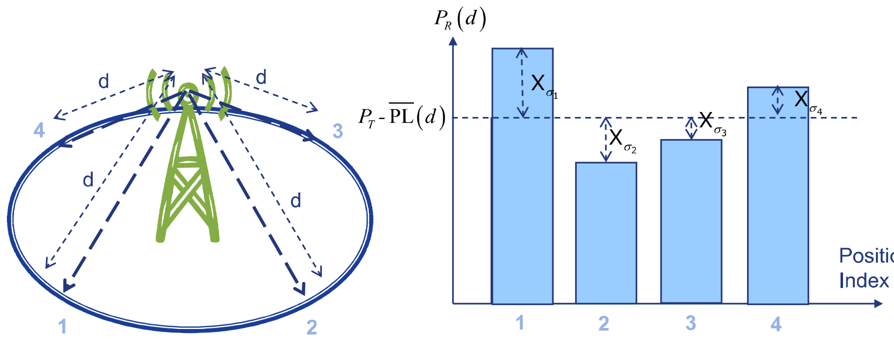
Received Power in Path-Loss Models

$$P_R(d) = P_T - PL(d) = P_T - \overline{PL}(d) - X_\sigma \quad \text{dB}$$



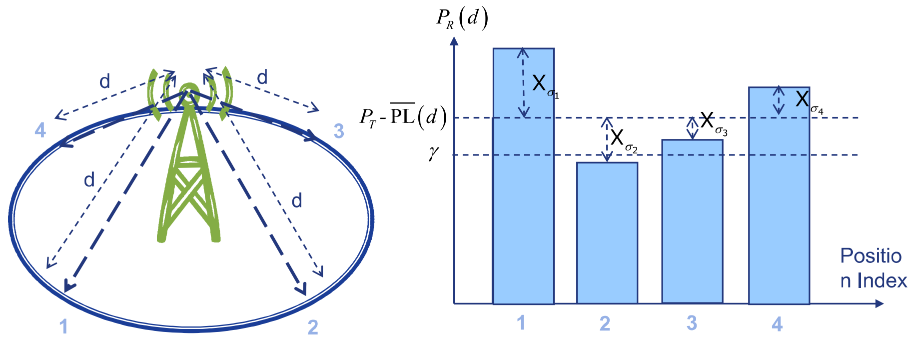
Received Power in Path-Loss Models

$$P_R(d) = P_T - PL(d) = P_T - \overline{PL}(d) - X_\sigma \quad \text{dB}$$



Reception Quality

$$P_R(d) = P_T - PL(d) = P_T - \overline{PL}(d) - X_\sigma \quad \text{dB}$$



γ: Desired received power threshold

$$\Pr\left[P_R(d) < \gamma\right] = \Pr\left[X_\sigma > \left(P_T - \overline{PL}(d)\right) - \gamma\right]$$

Probability of Bad Reception Quality

$$\Pr\left[P_R(d) < \gamma\right] = \Pr\left[X_\sigma > \left(P_T - \overline{\mathrm{PL}}(d)\right) - \gamma\right]$$

$$\Pr[X_\sigma > x_{th}] = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{x_{th}}^{\infty} e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx$$

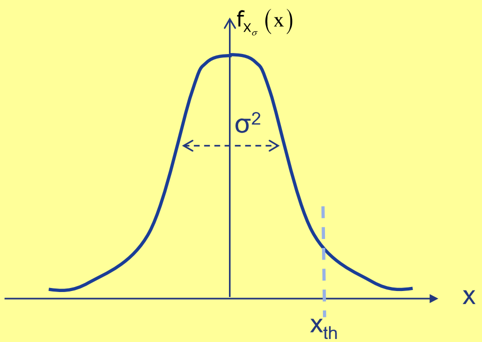
Let $z = \left(\frac{x}{\sigma}\right)$

$$\Pr[X_\sigma > x_{th}] = \frac{1}{\sqrt{2\pi}}\int_{\left(\frac{x_{th}}{\sigma}\right)}^{\infty} e^{-\left(\frac{z^2}{2}\right)}dz$$

$$\Pr[X_\sigma > x_{th}] = Q\left(\frac{x_{th}}{\sigma}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{x_{th}}{\sqrt{2}\sigma}\right)$$

$$\boxed{\Pr\left[P_R(d) < \gamma\right] = Q\left(\frac{\left(P_T - \overline{\mathrm{PL}}(d)\right) - \gamma}{\sigma}\right)}$$

X_σ follows a normal distribution with zero mean and standard deviation σ



Note: $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\left(\frac{z^2}{2}\right)}dz$ $\quad \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}du$

Tabulation of Q-function (0<=z<=3.9)

z	Q(z)	z	Q(z)	z	Q(z)	z	Q(z)
0.0	0.5	1.0	0.15866	2.0	0.02275	3.0	0.00135
0.1	0.46017	1.1	0.13567	2.1	0.01786	3.1	0.00097
0.2	0.42074	1.2	0.11507	2.2	0.01390	3.2	0.00069
0.3	0.38209	1.3	0.09680	2.3	0.01072	3.3	0.00048
0.4	0.34458	1.4	0.08076	2.4	0.00820	3.4	0.00034
0.5	0.30854	1.5	0.06681	2.5	0.00621	3.5	0.00023
0.6	0.27425	1.6	0.05480	2.6	0.00466	3.6	0.00016
0.7	0.24196	1.7	0.04457	2.7	0.00347	3.7	0.00011
0.8	0.21118	1.8	0.03593	2.8	0.00256	3.8	0.00007
0.9	0.18406	1.9	0.02872	2.9	0.00187	3.9	0.00005

Example

Four received power measurements were taken at the distances of 100m, 200m, 1 km and 3 km from a transmitter. These measured values are given in the following table.

Distance from Transmitter (m)	Received Power (dBm)
100 m	0
200 m	-20dBm
1000 m	-35dBm
3000 m	-70dBm

The path loss equation model for other measurements follows log normal shadowing model where d_0 = 100 m.

Example

(a) Find the minimum mean square error (MMSE) estimate for the path loss exponent n.

(b) Calculate the standard deviation about the mean value

(c) Estimate the received power at d = 2 km using the resulting model

(d) Predict the likelihood that the received signal at 2 km will be greater than -60 dBm.

Example

- The MMSE estimate may be found using the following method: Let p_i be the received power at a distance d_i and let $\hat{p}_i$ be the estimate for p_i using the $(d/d_0)^n$ path loss model. The sum of squared errors between the measured and estimated is given by

$$J(n) = \sum_{i=1}^{k} (p_i - \hat{p}_i)^2$$

The value of n which minimizes the mean square error can be obtained by equating the derivative of $J(n)$ to zero, and then solving by n

Example

- We find $\hat{p}_i = p_i(d_0) - 10n\log\left(\frac{d_i}{100}\right)$
- Since $P(d_0)$ = 0 dBm, the following estimates for $\hat{p}_i$ in dBm

$$\hat{p}_1 = 0 \qquad \hat{p}_2 = -3n \qquad \hat{p}_3 = -10n \quad \hat{p}_4 = -14.77n$$

- The sum of squared errors is then by

$$J(n) = (0-0)^2 + (20-(-3n))^2 + (-35-(-10n))^2 + (-70-(14.77n))^2$$
$$= 65.25 - 2887.8n + 327.153n^2$$

- Setting

$$\frac{dJ(n)}{dn} = 654.306n - 2887.8 = 0 \rightarrow n = 4.4$$

Example

- The variance $\sigma^2 = \frac{J(n)}{4}$ at n = 4.4 can be obtained

$$J(n) = (0+0)^2 + (20+13.2)^2 + (-35+44)^2 + (-70+64.988)^2 = 152.36$$

$$\sigma^2 = 152.36/4 = 38.09$$

$$\sigma = 6.17dB$$

Example

- The estimate of the received power at d = 2Km is given by

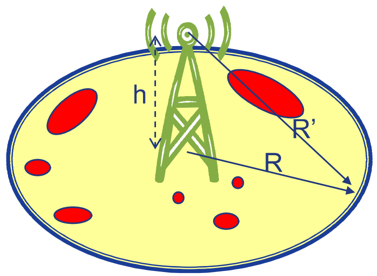
$$\hat{p} = 0 - 10.4,4\log\left(\frac{2000}{100}\right) = 57,24dBm$$

- The probability that the received signal level will be greater than -60dBm is given by

$$\Pr(P_r(d) > -60dBm) = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right)$$
$$= Q\left(\frac{-60-(-57,24)}{6,17}\right) = 67,4\%$$

Percentage of Coverage Area

- Due to the random effects of shadowing some locations within the coverage area will be below a particular desired received signal level
- So, its better to compute how the boundary coverage area relates to the percent of area covered within the boundary



$P_R(d) > \gamma \quad 0 \le d \le R'$ (yellow)

$P_R(d) < \gamma \quad 0 \le d \le R'$ (red)

R: Radius of Coverage Area required for Transmitter

Calculation of Percentage of Coverage Area

Assume h (height of antenna) is negligible, then, U(γ) depicting the percentage of area with received signal strength equal to or exceeding γ may be calculated as follows

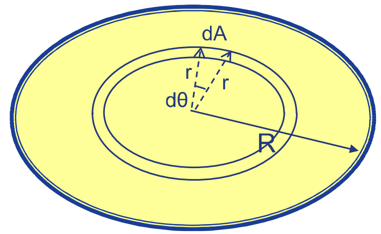
$$U(\gamma) = \frac{1}{\pi R^2}\int \Pr\left[P_R(r) > \gamma\right] dA$$

$$U(\gamma) = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^{R} \Pr\left[P_R(r) > \gamma\right] r\, dr\, d\theta$$

$$\Pr\left[P_R(r) > \gamma\right] = Q\left(\frac{\gamma - \left(P_T - \overline{PL}(r)\right)}{\sigma}\right)$$

$$\Pr\left[P_R(r) > \gamma\right] = \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma - \left(P_T - \overline{PL}(r)\right)}{\sqrt{2}\sigma}\right)$$

$$\Pr\left[P_R(r) > \gamma\right] = \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma - \left[P_T - \left(\overline{PL}(d_0) + 10n\log(r/d_0)\right)\right]}{\sqrt{2}\sigma}\right)$$



R: Radius of Coverage Area required for Transmitter

Calculation of Percentage of Coverage Area

$$\Pr\left[P_R(r) > \gamma\right] = \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma - \left[P_T - \left(\overline{PL}(d_0) + 10n\log(r/d_0)\right)\right]}{\sqrt{2}\sigma}\right)$$

$$\Pr\left[P_R(r) > \gamma\right] = \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma - \left[P_T - \left(\overline{PL}(d_0) + 10n\log(R/d_0) + 10n\log(r/R)\right)\right]}{\sqrt{2}\sigma}\right)$$

$$U(\gamma) = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^{R} \Pr\left[P_R(r) > \gamma\right] r\, dr\, d\theta$$

$$U(\gamma) = \frac{1}{\pi R^2}\int_0^{R} \Pr\left[P_R(r) > \gamma\right] r\, dr \int_0^{2\pi} d\theta = \frac{2}{R^2}\int_0^{R} \Pr\left[P_R(r) > \gamma\right] r\, dr$$

$$U(\gamma) = \frac{2}{R^2}\int_0^{R} r \times \frac{1}{2}\operatorname{erfc}\left(a + b\ln\frac{r}{R}\right) dr$$

$$a = \frac{\gamma - \left[P_T - \left(\overline{PL}(d_0) + 10n\log(R/d_0)\right)\right]}{\sqrt{2}\sigma} \qquad b = \frac{(10n\log e)}{\sqrt{2}\sigma}$$

Calculation of Percentage of Coverage Area

It can be shown that

$$U(\gamma) = \frac{1}{2}\left(1 - \operatorname{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{1-ab}{b}\right)\right]\right)$$

By choosing the signal level such that

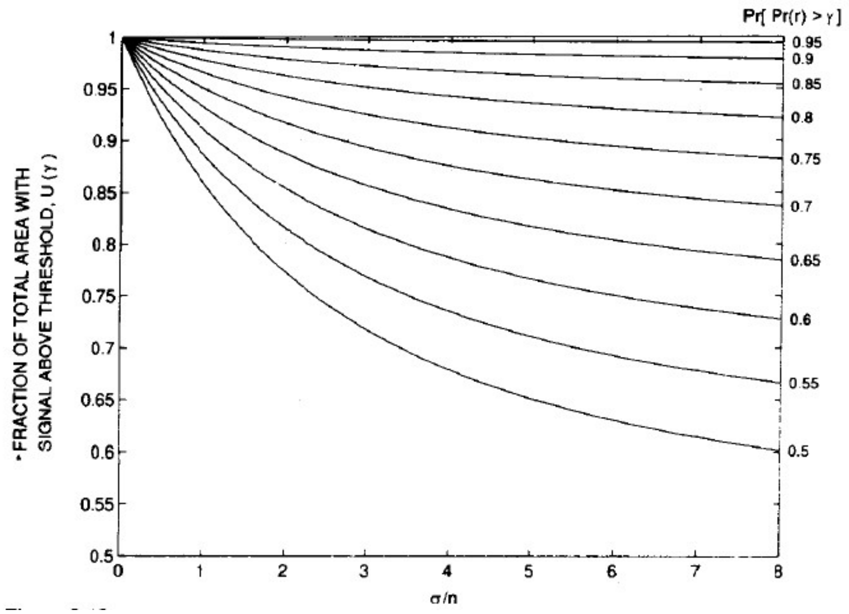
$$\overline{P}_R(R) = \gamma \;\; (i.e., a = 0)$$

Therefore for the case when

Boundary Coverage = 50 %

$$U(\gamma) = \frac{1}{2}\left(1 + \exp\left(\frac{1}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{1}{b}\right)\right]\right)$$

Calculation of Percentage of Coverage Area



Outdoor Propagation Models

- Longley-Rice Model (*Read*)
- Durkin's Model (*Read*)
- Okumura's Model
- Hata Model
- PCS extension to Hata Model
- Walfisch and Bertoni (*Read*)

Okumura's Model

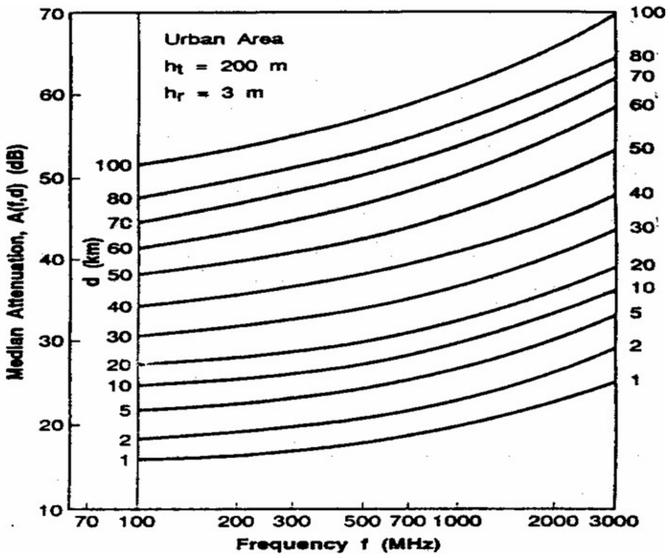
- Okumura's model is one of the most widely used models for signal predictions in urban and sub-urban mobile communication areas
- This model is applicable for frequencies ranging from 150 MHz to 1920 MHz
- It can cover distances from 1 km to 100 km and it can be used for base station heights starting from 30m to 1000m
- The model is based on empirical data collected in detailed propagation tests over various situations of an irregular terrain and environmental clutter

Okumura's Model

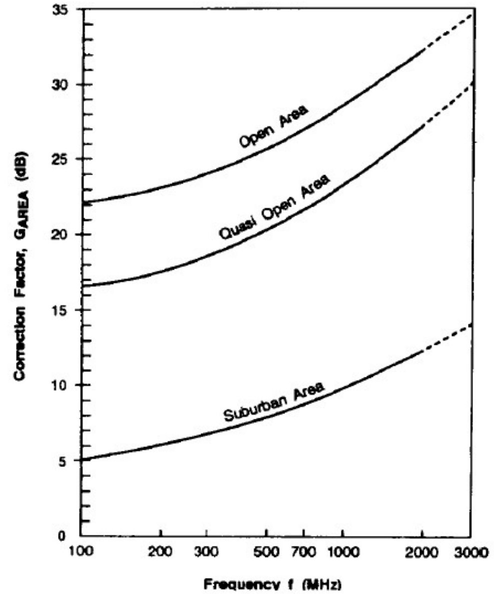
$$L_{50}(dB) = L_F + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{AREA}$$

- L_{50} is the median value or 50th percentile value of the propagation path loss
- L_F is the free space propagation path loss
- A_{mu} is the median attenuation relative to free space
- G_{AREA} is the gain due to the type of environment
- $G(h_{te})$ is the base station antenna height gain factor
- $G(h_{re})$ is the mobile antenna height gain factor

Okumura Model: A_{mu} Curves



Okumura's Model: G_{Area} Curves



Okumura's Model: $G(h_{te})$, $G(h_{re})$

- The empirical model of Okumura assumed h_{te} = 200m, h_{re} = 3m

$$G(h_{te}) = 20\log\left(\frac{h_{te}}{200}\right) \quad 30\text{m} < h_{te} < 1000\text{m}$$

$$G(h_{re}) = 10\log\left(\frac{h_{re}}{3}\right) \quad h_{re} < 3\text{m}$$

$$G(h_{re}) = 20\log\left(\frac{h_{re}}{3}\right) \quad 3\text{m} < h_{re} < 10\text{m}$$

Example

Find the median path loss using Okumura's model for d = 50Km, h_{te} = 100m, h_{re}= 10m in an surban environment. If the base station radiated an EIRP of 1KW at a carrier frequency of 900MHz. Find the power at the receiver (assume a unity gain receiving antenna)

Example

- The free space path loss L_F can be calculated as:

$$L_F = 10\log\frac{\lambda^2}{(4\pi)^2 * d^2} = 10\log\frac{(1/3)^2}{(4\pi)^2 * (50*10^3)^2} = 125.5dB$$

- From the Okumura curves
  - A_{mu} (900MHz, 50Km) = 43 dB
  - G_{area} = 9 dB

$$G(h_{te}) = 20\log\left(\frac{h_{te}}{200}\right) = 20\log\left(\frac{100}{200}\right) = -6dB$$

$$G(h_{re}) = 20\log\left(\frac{h_{re}}{3}\right) = 20\log\left(\frac{10}{3}\right) = 10.46dB$$

Example

- The total mean path loss is

$$L_{50}(dB) = L_F + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{area} = 155.04dB$$

- The average received power is

$$\overline{P_r(d)} = EIRP(dBm) - L_{50}(dB) + G_r(dB)$$
$$= 60dBm - 155.04dB + 0dB = -95.04dBm$$

Hata Model

$$\boxed{\begin{aligned} L_{50}(\text{urban})(\text{dB}) &= 69.55 + 26.26\log(f_c) - 13.82\log(h_{te}) - a(h_{re}) \\ &+ (44.9 - 6.55\log(h_{te}))\log(d) \end{aligned}}$$

- L_{50} is the median value or 50th percentile value of the propagation path loss
- f_c (in MHz) is the frequency (15MHz to 1500MHz)
- h_{te} is the effective transmitter height in meters (30m to 200 m)
- h_{re} is the effective transmitter height in meters (1m to 10 m)
- d is the T-R separation in Km
- $a(h_{re})$ is the correction factor for effective mobile (i.e., receiver) antenna height which is a function of the size of the coverage area

Hata Model: $a(h_{re})$

- For a Medium sized city, correction factor is given by:

$$a(h_{re}) = (1.1\log(f_c) - 0.7)h_{re} - (1.56\log(f_c) - 0.8) \quad \text{dB}$$

- For a Large city, correction factor is given by:

$$a(h_{re}) = 8.29(\log(1.54h_{re}))^2 - 1.1 \quad \text{dB} \quad \text{for } f_c \le 300\text{MHz}$$

$$a(h_{re}) = 3.2(\log(11.75h_{re}))^2 - 4.97 \quad \text{dB} \quad \text{for } f_c \ge 300\text{MHz}$$

Hata Model

- Path loss in suburban area, the equation is modified as

$$L_{50}(dB) = L_{50}(urban) - 2\left[\log(f_c/28)\right]^2 - 5.4$$

- For path loss in open rural areas, the formula is modified as

$$L_{50}(dB) = L_{50}(urban) - 4.78\left[\log(f_c)\right]^2 + 18.33\log(f_c) - 40.94$$

- Hata Model is well-suited for large cell mobile systems

PCS Extension to Hata Model

- An extended version of the Hata model developed by COST-231 working committee for 2 GHz range

$$\boxed{\begin{aligned} L_{50}(urban)(dB) &= 46.3 + 33.9\log(f_c) - 13.82\log(h_{te}) - a(h_{re}) \\ &+ (44.9 - 6.55\log(h_{re}))\log(d) + C_M \end{aligned}}$$

- f_c is the frequency (1500MHz to 2000 MHz)
- h_{te} is the effective transmitter height in meters (30m to 200 m)
- h_{re} is the effective transmitter height in meters (1m to 10 m)
- d is the T-R separation in Km (1 Km to 20 Km)
- C_M=0 dB for medium sized city and suburban areas, C_M=3 dB for metropolitan centers

Indoor Propagation Models

- The indoor radio channel differs from the traditional mobile radio channel in the following aspects
  - Much smaller distances
  - Much greater variability of the environment for a much smaller range of T-R separation distances
  - Difficult to ensure far-field radiation

- Propagation within buildings is strongly influenced by specific features such as
  - Building layout
  - Construction materials
  - Building type
  - Open/Closed doors
  - Locations of antennas

Partition Losses (Same Floor)

Table 3.3 Average Signal Loss Measurements Reported by Various Researchers fc Radio Paths Obstructed by Common Building Material.

Material Type	Loss (dB)	Frequency	Reference
All metal	26	815 MHz	[Cox83b]
Aluminium siding	20.4	815 MHz	[Cox83b]
Foil insulation	3.9	815 MHz	[Cox83b]
Concrete block wall	13	1300 MHz	[Rap91c]
Loss from one floor	20-30	1300 MHz	[Rap91c]
Loss from one floor and one wall	40-50	1300 MHz	[Rap91c]
Fade observed when transmitter turned a right angle corner in a corridor	10-15	1300 MHz	[Rap91c]
Light textile inventory	3-5	1300 MHz	[Rap91c]
Chain-like fenced in area 20 ft high containing tools, inventory, and people	5-12	1300 MHz	[Rap91c]
Metal blanket — 12 sq ft	4-7	1300 MHz	[Rap91c]
Metallic hoppers which hold scrap metal for recycling - 10 sq ft	3-6	1300 MHz	[Rap91c]

Partition Losses between Floors

Table 3.5 Average Floor Attenuation Factor in dB for One, Two, Three, and Four Floors in Two Office Buildings [Sei92b].

Building	FAF (dB)	σ (dB)	Number of locations
Office Building 1:			
Through One Floor	12.9	7.0	52
Through Two Floors	18.7	2.8	9
Through Three Floors	24.4	1.7	9
Through Four Floors	27.0	1.5	9
Office Building 2:			
Through One Floor	16.2	2.9	21
Through Two Floors	27.5	5.4	21
Through Three Floors	31.6	7.2	21

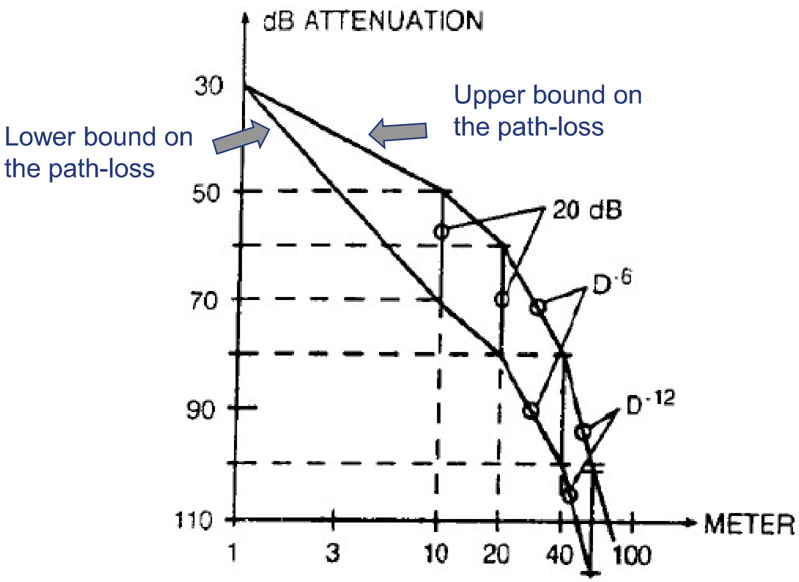
Log-Distance Pathloss Model

- The lognormal shadowing model has been shown to be applicable in indoor environments

$$PL(d) = \overline{PL}(d) + X_\sigma$$

$$PL(d) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

Ericsson Multiple Breakpoint Model



Attenuation Factor Model

- This was described by Seidel S.Y. It is an in-building propagation model that includes
  - Effect of building type
  - Variations caused by obstacles

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n_{SF}\log\left(\frac{d}{d_0}\right) + FAF(dB) + \sum PAF(dB)$$

- n_{SF} represents the path-loss exponent for the same floor measurements
- FAF represents the floor attenuation factor
- PAF represents the partition attenuation factor for a specific obstruction encountered by a ray drawn between the transmitter and receiver

Attenuation Factor Model

- FAF may be replaced by an exponent that accounts for the effects of multiple floor separation

$$\overline{PL}(d)=\overline{PL}(d_0)+10n_{MF}\log\left(\frac{d}{d_0}\right)+\sum PAF(dB)$$

  - n_{MF} represents the path-loss exponent based on measurements through multiple floors

Measured Indoor Path Loss

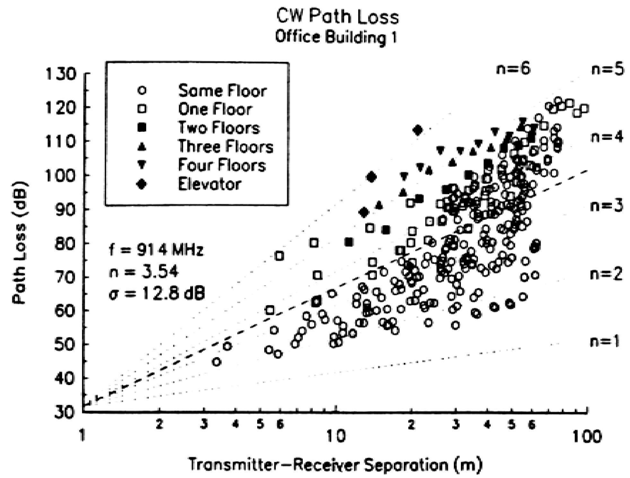


Figure 4.28 Scatter plot of path loss as a function of distance in Office Building 1 [from [Sei92b] © IEEE].

Measured Indoor Path Loss

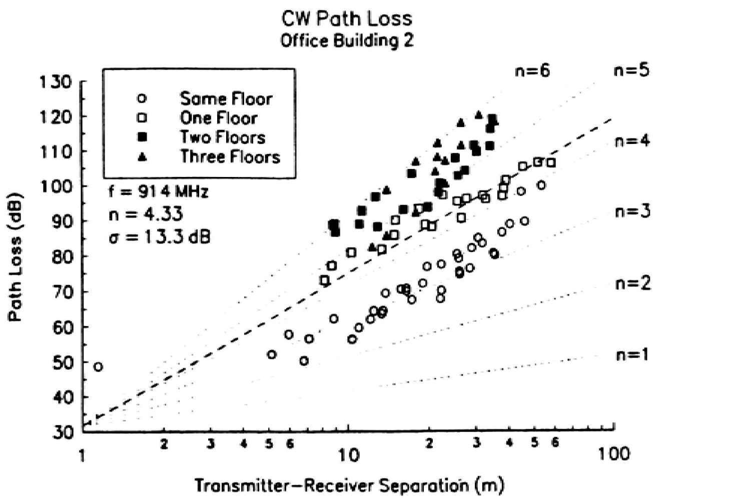


Figure 4.29 Scatter plot of path loss as a function of distance in Office Building 2 [from [Sei92b] © IEEE].

Ho Chi Minh City University of Technology
Faculty of Electrical and Electronics Engineering
Department of Telecommunications

SMALL-SCALE FADING AND MULTI-PATH

Lectured by Ha Hoang Kha, Ph.D.
Ho Chi Minh City University of Technology
Email: hahoangkha@gmail.com

Content

1. Small-scale multipath fading
   - Factors influencing small-scale fading
   - Doppler Shift
2. Impulse response of model of a multipath channel
3. Parameters of mobile multipath channels
   - Time dispersion parameters
   - Coherence bandwidth and coherence time
4. Types of small-scale fading
   - Multipath delay spread: flat and frequency selective fading
   - Doppler spread: Fast and slow fading
5. Rayleigh and Ricean distributions
6. Simulations of wireless channels

References

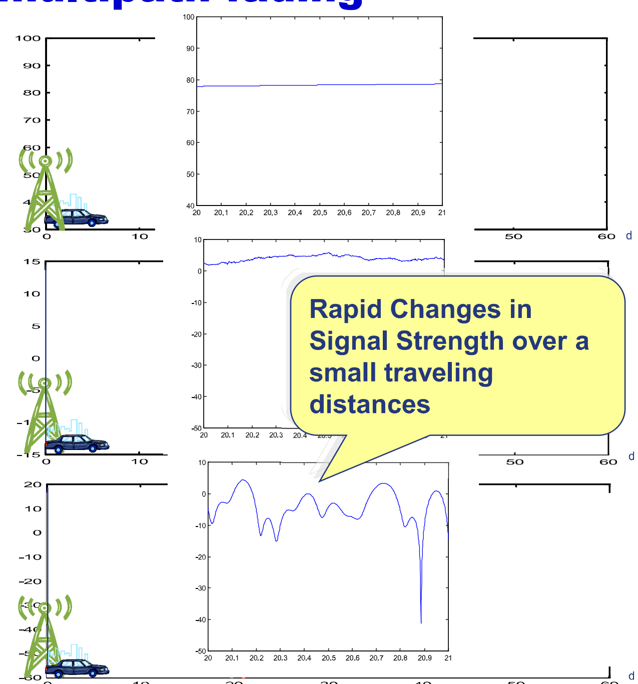
- Ta Tri Nghia, Wireless Communications, lecture notes.
- T.S. Rappaport, *Wireless Communications*, Prentice Hall PTR, 1996.
- Tallal Elshabrawy, wireless Communications, lecture notes.

1. Small-scale multipath fading

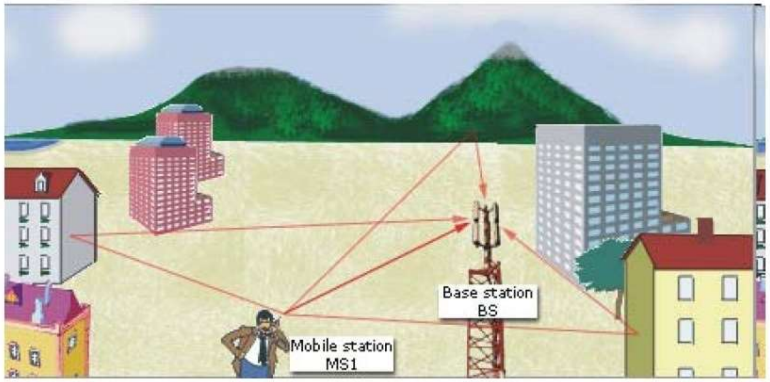
Distance Pathloss
Mobile Speed 3 Km/hr
PL=137.744+ 35.225log$_{10}$(d$_{KM}$)

Lognormal Shadowing
Mobile Speed 3 Km/hr
Shadow Model

Small-Scale Fading
Mobile Speed 3 Km/hr
Jakes's Rayleigh Fading Model



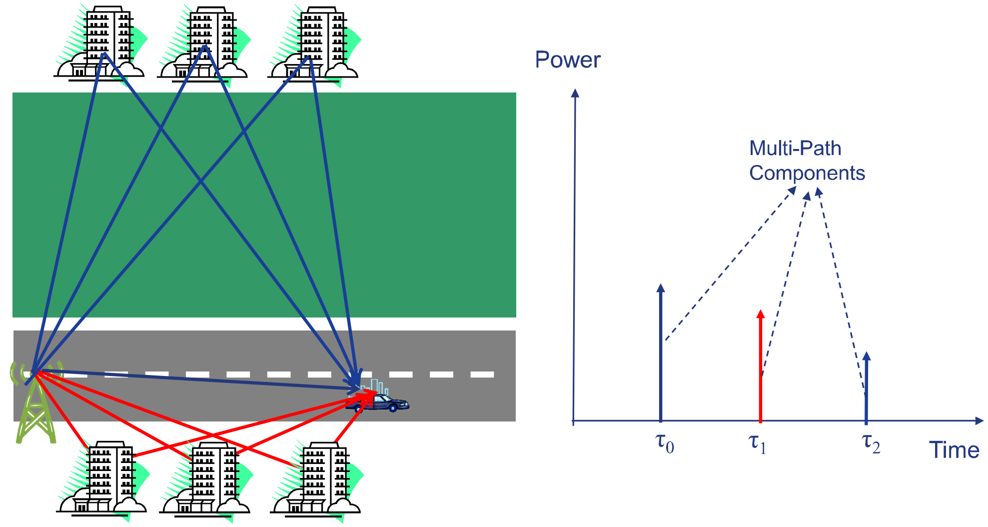
Multi-Path Propagation



Multi-Path in the radio channel creates small-scale fading. The three most important effects are:

- Rapid changes in signal strength over a small travel distance or time interval
- Random frequency modulation due to varying Doppler shifts on different multi-path signals
- Time dispersion (echoes) caused by multi-path propagation delays

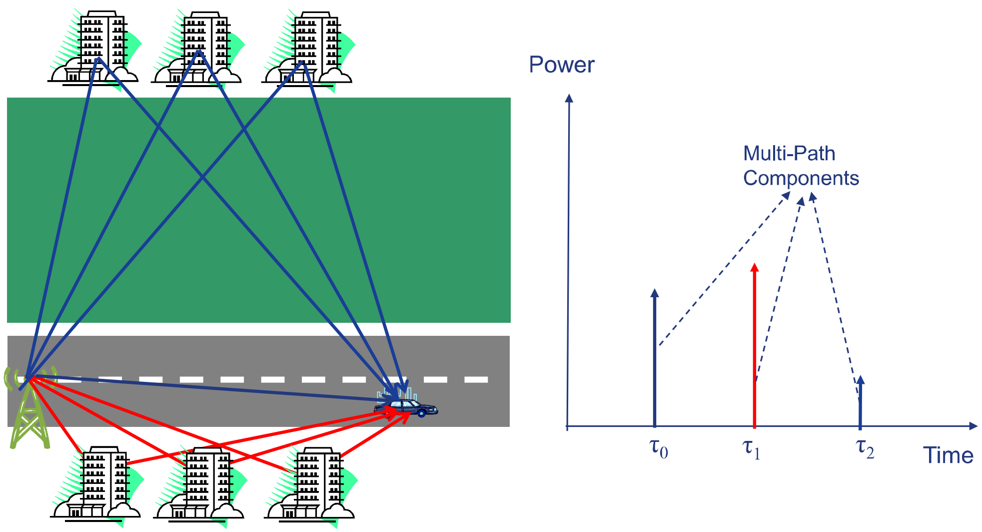
Multi-Path Propagation Modeling



Multi-path results from reflection, diffraction, and scattering off environment surroundings

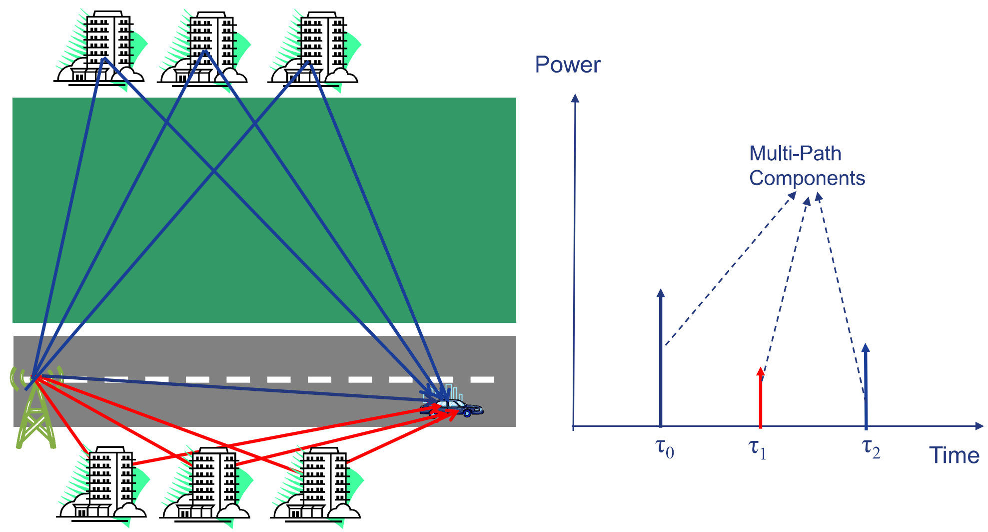
Note: The figure above demonstrates the roles of reflection and scattering only on multi-path

Multi-Path Propagation Modeling



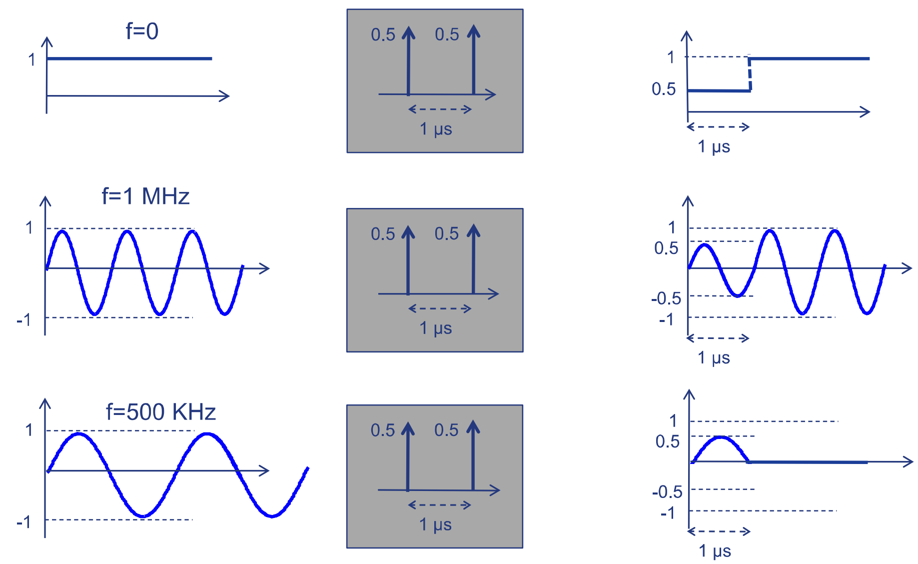
As the mobile receiver (i.e. car) moves in the environment, the strength of each multi-path component varies

Multi-Path Propagation Modeling



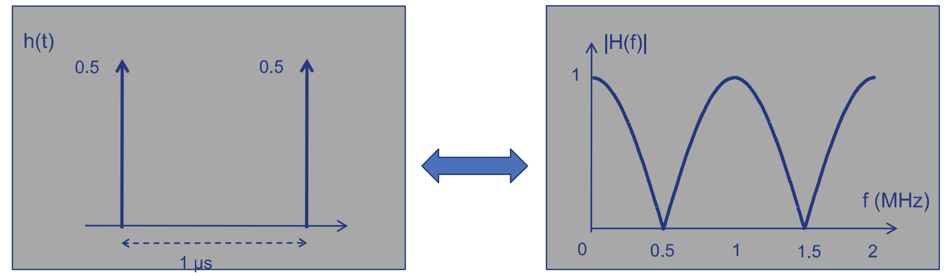
As the mobile receiver (i.e. car) moves in the environment, the strength of each multi-path component varies

Multi-Path = Frequency-Selective



Giải thích các kết quả bên phải?

Multi-Path = Frequency-Selective



- A multi-path channel treats signals with different frequencies differently
- A signal composed of multiple frequencies would be distorted by passing through such channel

Small-scale multipath Propagation

- Small-scale fading, or simply fading, is used to describe the rapid fluctuation of the amplitude of a radio signal over short period of time or travel distance.
- Fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times.
- Multipath waves, combine at the receiver antenna to give a resultant signal which can vary widely in amplitude and phase, depending on distribution of the intensity and relative propagation time of the waves and the bandwidth of the transmitted signal.

1.1. Factors Influencing Small-scale Fading

- Multipath propagation
  - The presence of reflecting objects and scatterers in the channel creates a constantly changing environment that dissipates the signal energy in amplitude, phase, and time
  - The random phase and amplitudes of different multipath components causes fluctuations in signal strength, including small-scale fading, signal distortion or both
- Speed of the mobile
  - The relative motion between the base station and the mobile results in random frequency modulation due to different Doppler shifts on each of the multipath components

Factors Influencing Small-scale Fading

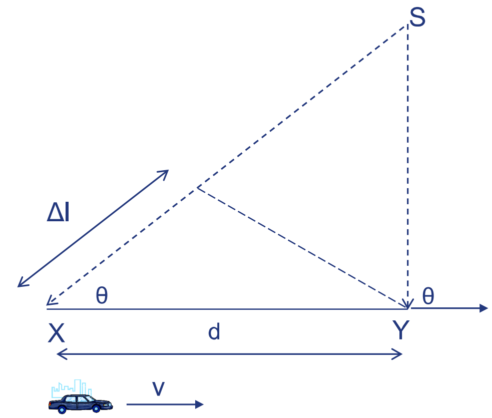
- Speed of surrounding objects
  - Induce a time varying Doppler shift on multipath components
  - If the surrounding objects move at a greater rate than the mobile, this effects dominates the small-scale fading
- The transmission bandwidth of the signal
  - If the transmitted radio signal bandwidth is greater than the "bandwidth" of the multipath channel, the received signal will be distorted
  - The received signal strength will not fade much over a local area (i.e. the small-scale signal fading will not be significant)

1. 2. Doppler Shift

The difference in path lengths traveled by the wave from source S to the mobile at X and Y is Δl

Note: Assume SX, SY >>d such that angle of arrival is nearly equal at X and Y

$$\Delta l = d\cos\theta = v\Delta t\cos\theta$$



Phase Difference due to variation in path lengths

$$\Delta\varphi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$

Doppler Shift is given by

$$\boxed{f_d = \frac{1}{2\pi}\frac{\Delta\varphi}{\Delta t} = \frac{v}{\lambda}\cos\theta}$$

Doppler Shift - Example

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850MHz. For a vehicle moving 60mph, compute the received carrier frequency of the mobile is moving

(a) Directly towards the transmitter

(b) Directly away from the transmitter

(c) In a direction which is perpendicular to the direction of arrival of the transmitted signal

Doppler Shift - Example

- Wavelength of carrier frequency $\lambda = c/f = \frac{3*10^8}{1850*10^6} = 0.162m$
- Vehicle speed

$$v = 60mph = 26.82m/s$$

- The vehicle is moving directly towards the transmitter

$$f = f_c + f_d = 1850*10^6 + \frac{26.82}{0.162} = 1850.00016MHz$$

- The vehicle is moving directly away from the transmitter

$$f = f_c - f_d = 1850*10^6 - \frac{26.82}{0.162} = 1849.999.834MHz$$

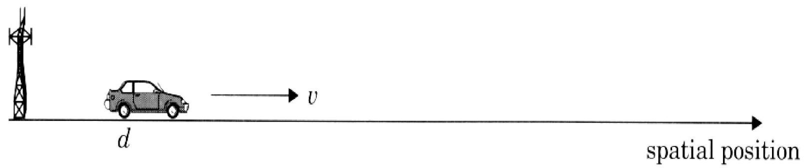
- The vehicle is moving perpendicular to the angle of arrival of the transmitter:

$$\theta = 90^0 \quad , \quad \cos\theta = 0$$

$$f = f_c = 1850MHz$$

2. Impulse Response Model of a Multipath Channel

- Consider the case where the receiver moves along the ground at some constant velocity v.



- For a fixed position *d*, the channel between the transmitter and the receiver can be modeled as a linear time invariant system (LTI).
- Due to the different multipath waves, the impulse response of the linear time invariant channel should be a function of the position of the receiver
  - Channel impulse response can be expressed as *h(d,t)*

Impulse Response Model of a Multipath Channel

- The received signal

$$y(d,t) = x(t) \otimes h(d,t) = \int_{-\infty}^{\infty} x(\tau)h(d,t-\tau)d\tau$$

- For a casual system, $h(d,t) = 0$ for $t < 0$

$$y(d,t) = x(t) \otimes h(d,t) = \int_{-\infty}^{t} x(\tau)h(d,t-\tau)d\tau$$

- Since the receiver moves along the ground at a constant velocity *v*, the position of the receiver can be expressed as

$$d = vt \qquad y(vt,t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau$$

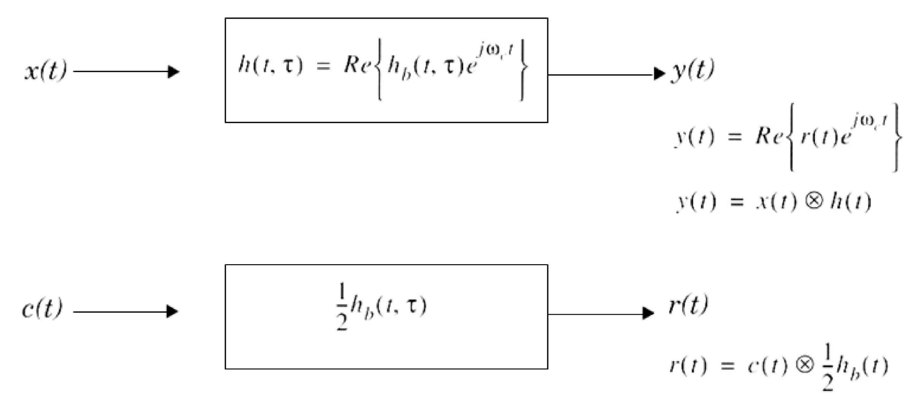
Impulse Response Model of a Multipath Channel

- Since *v* is a constant, $y(vt,t)$ is just a function of *t*, and can be expressed as

$$y(t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau = x(t) \otimes h(d,t) = x(t) \otimes h(vt,t)$$

- Since *v* may be assumed constant over a short time (or distance) interval, let
  - $x(t)$ is transmitted bandpass waveform
  - $y(t)$ is received waveform
  - $h(t, \tau)$ is the impulse response of the time varying multipath radio channel
    - Variable *t* represents the time variations due to motion
    - τ represents the channel multipath delay for a fixed value of *t*

Impulse Response Model of a Multipath Channel



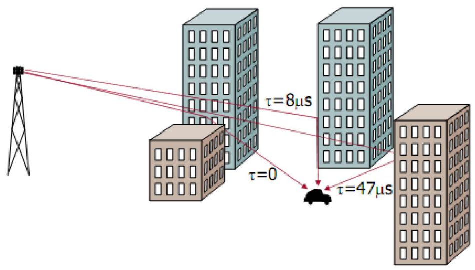
$$x(t) = \mathrm{Re}\left\{c(t)e^{2j\pi f_c t}\right\}$$

$$y(t) = \mathrm{Re}\left\{r(t)e^{2j\pi f_c t}\right\}$$

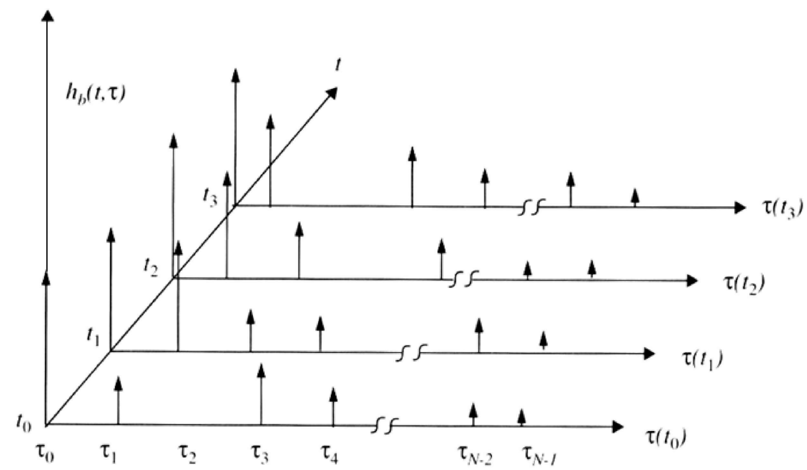
$$r(t) = c(t) \otimes \frac{1}{2}h_b(t,\tau)$$

Small-Scale Multipath Propagation

- Excess delay: the propagation delay relative to that of shortest path
- As the vehicle moves over a short distance, the strength of each path varies because the surface are complex



Excess Delay Concept



Excess Delay Concept

- Discretize the multipath delay axis τ of the impulse response into equal time delay segments called excess delay bins

$$\tau_{i+1} - \tau_i = \Delta\tau$$
$$\tau_0 = 0$$
$$\tau_1 = \Delta\tau$$
$$\tau_2 = 2\,\Delta\tau$$
$$.$$
$$\tau_{N-1} = (N-1)\Delta\tau$$

- Any number of multipath signals received within the i^{th} bin are represented by a single multipath component having delay τ_i
- The maximum excess delay spread of the channel is $N\Delta\tau$
- This model can be used to analyze transmitted signals having bandwidths less than $1/(2\Delta\tau)$

Example

Assume a discrete channel impulse response is used to model urban radio channels with excess delays as large as 100 μs and microcellular channels with excess delays not larger than 4 μs. If the number of multipath bins is fixed at 64 find:

(a) $\Delta\tau$

(b) Maximum bandwidth, which the two models can accurately represent.

Example

- The maximum excess delay of the channel model is given

$$\tau_N = N\Delta\tau$$

- The maximum bandwidth represented accurately by model

$$\Delta\tau = \tau_N / N = \frac{100\mu s}{64} = 1.5625\mu s$$

$$1/(2\Delta\tau) = 1/(2*1.6525\mu s) = 0.32MHz$$

- For urban microcell model

$$\tau_N = 4\mu s, \quad \Delta\tau_N = \tau_N / N = 62.5ns$$

- The maximum bandwidth that can be represented is

$$1/(2\Delta\tau) = 1/(2*62.5ns) = 8MHz$$

Mathematical Model of Base- band Impulse Response

- Since the received signal in a multipath channel consists of a series of attenuated, time-delayed, phase shifted replicas of the transmitted signal, the baseband impulse response of a multipath channel can be expressed

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau) e^{-j[2\pi f_c \tau_i(t) + \phi_i(t,\tau)]} \delta[\tau - \tau_i(t)]$$

  - $a_i(t,\tau)$ and $\tau_i(t)$ is real amplitudes and excess delays of i[th] multipath component at time *t*
  - $\theta(t,\tau) = 2\pi f_c \tau_i(t) + \phi_i(t,\tau)$ represents the phase shift due to free space propagation of the i[th] multipath component, plus any additional phase shifts encountered in the channel

Mathematical Model of Base-band Impulse Response

- If the channel impulse response is assumed to be time invariant, or is at least wide sense stationary over a small-scale time or distance interval, channel impulse response may be simplified as

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i * e^{-j\theta_i} * \delta[\tau - \tau_i]$$

  - When measuring or predicting $h_b(\tau)$, a probing pulse p(t) which approximates a delta function is used at the transmitter

$$p(t) \approx \delta(t-\tau)$$

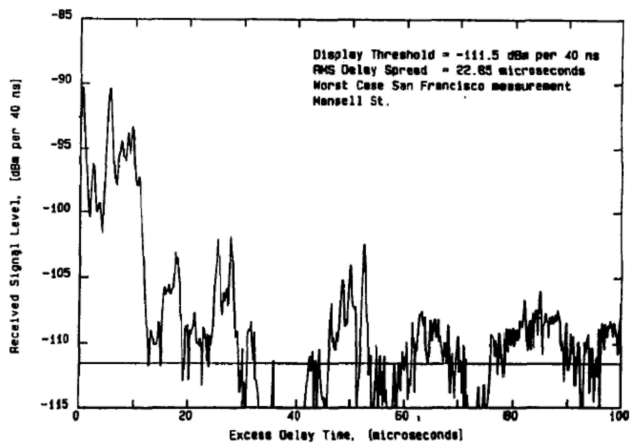
Mathematical Model of Base- band Impulse Response

- For small-scale channel modeling, the power delay profile of the channel is found by taking the spatial average of $|h_b(t,\tau)|^2$ over a local area
- By making several local area measurements of in different locations, it is possible to build an ensemble of power delay profiles, each one representing a possible small-scale multipath channel state $|h_b(t,\tau)|^2$
- The received power delay profile in a local area is given by

$$P(t,\ \tau) \approx k\left|h_b(t,\ \tau)\right|^2$$

Power Delay Profile

The power delay profile depicts the spatial average of received power within the multi-path channel over a radius that is comparable to the signal wavelength



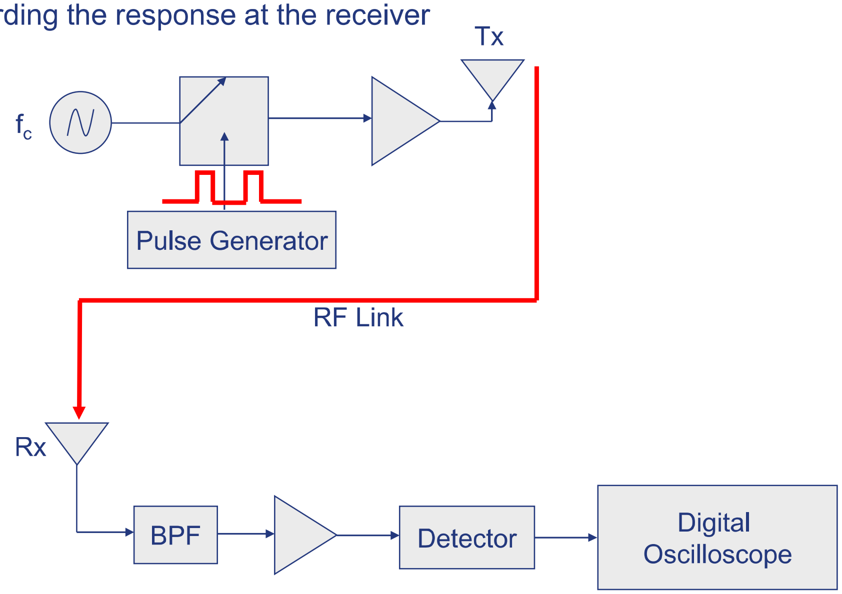
Multi-Path Profile from a 900 MHz cellular system in San Francisco

Small-Scale Multipath Measurements

- Several Methods
  - Direct RF Pulse System
  - Spread Spectrum Sliding Correlator Channel Sounding
  - Frequency Domain Channel Sounding
- These techniques are also called channel sounding techniques

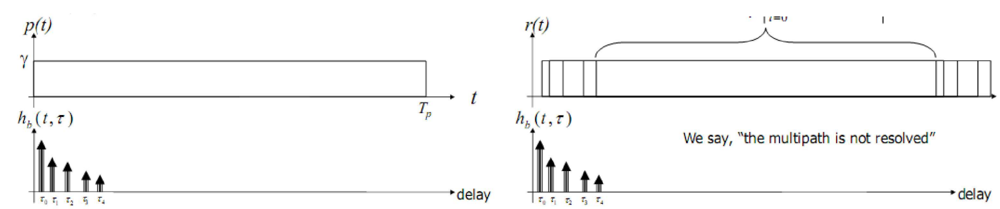
Direct RF Pulse System

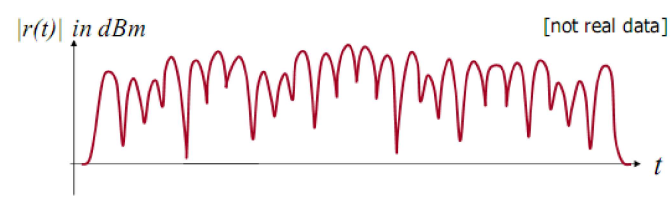
The channel may be probed or "sounded" by transmitting a pulse *p(t)* and recording the response at the receiver



Probing the Channel

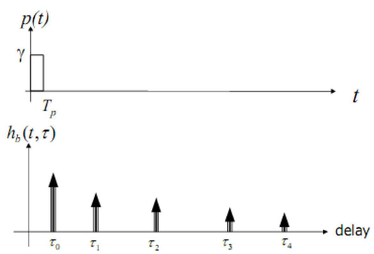
- Narrowband signals

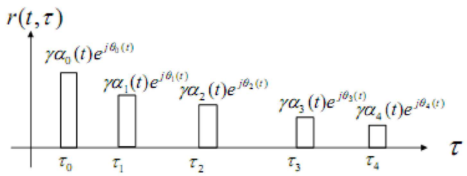




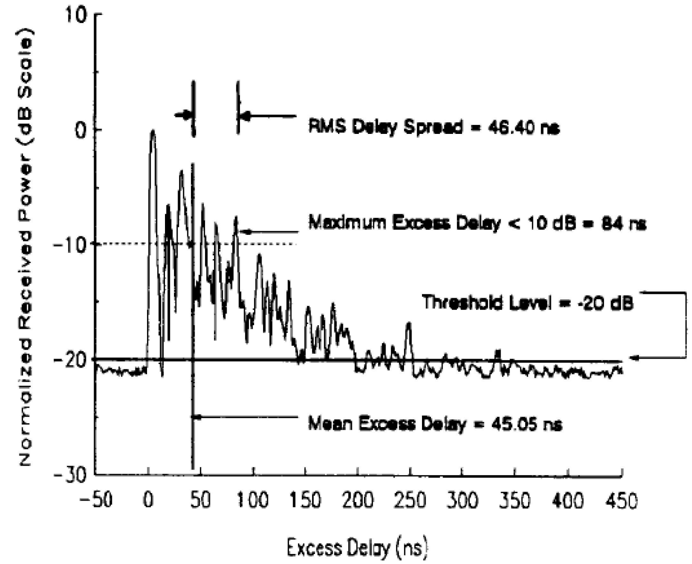
Probing the Channel

- Wideband signals





Example



Example of an Indoor Multi-Path Profile; rms delay spread, mean excess delay, maximum excess delay (10 dB)

3. Parameters of Mobile Multi-Path Channels

- The power delay profile is used to derive some parameters that can help characterize the effect of the wireless channel on signal communication
- Time dispersion parameters
  - Mean excess delay
  - Rms delay spread
  - Excess delay spread (X dB)
- Coherence bandwidth
- Doppler shift: Doppler spread and coherence time

Time Dispersion Parameters

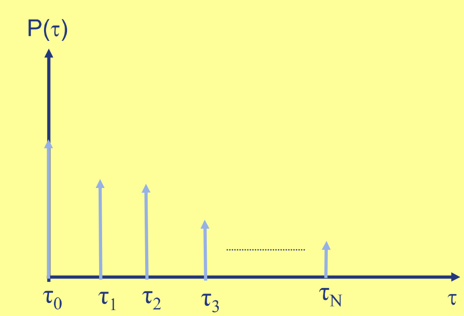
Mean Excess Delay

$$\bar{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

RMS Delay Spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$



Power Delay Profile

Note: **These delays are measured relative to the first detectable signal (multi-path component) arriving at the receiver at $\tau_0=0$**

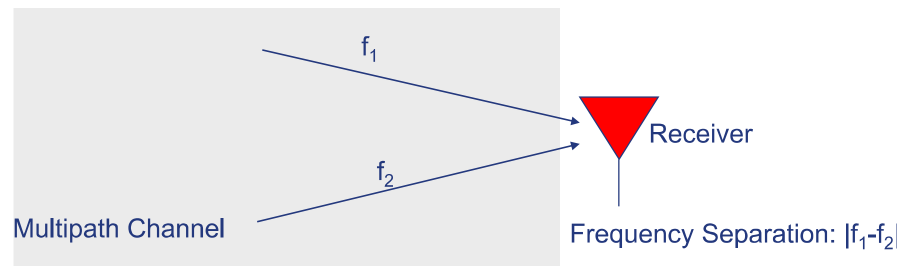
Maximum Excess Delay (XdB) or Excess Delay Spread (XdB):
Time delay during which multi-path energy falls to X dB below the maximum (Note that the strongest component does not necessarily arrive at τ_0)

Measured values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

Coherence Bandwidth (B_C)

- Range of frequencies over which the channel can be considered flat (i.e. channel passes all spectral components with equal gain and linear phase).
  - It is a definition that depends on RMS Delay Spread.
- Two sinusoids with frequency separation greater than B_c are affected quite differently by the channel.



Coherence Bandwidth

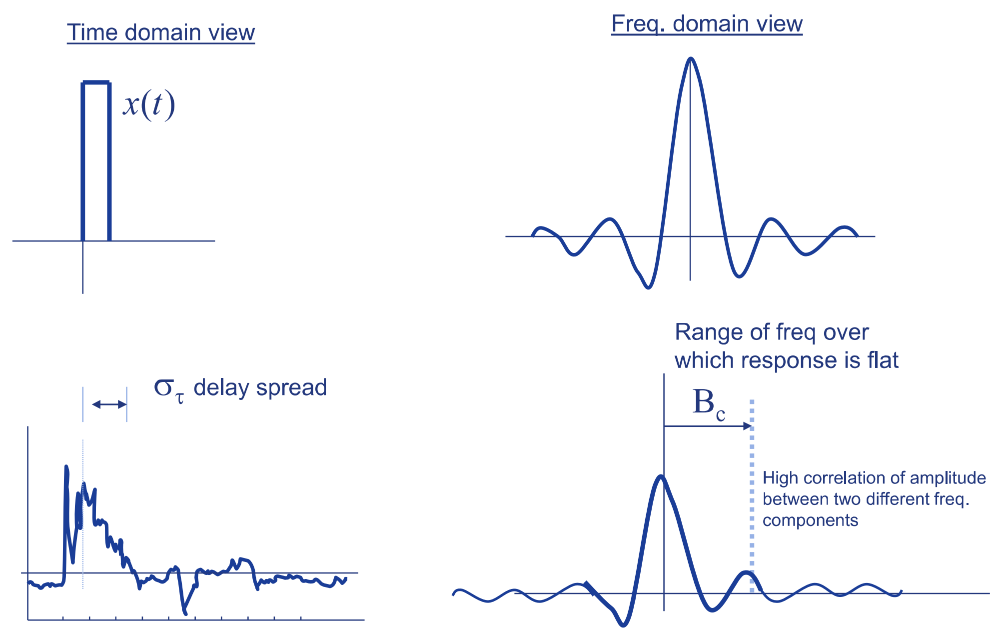
A statistical measure of the range of frequencies over which the channel is can be considered to be "flat" (i.e., a channel which passes all spectral components with approximately equal gain and linear phase)

Coherence Bandwidth over which the frequency correlation function is 0.9

$$B_C = \frac{1}{50\sigma_\tau}$$

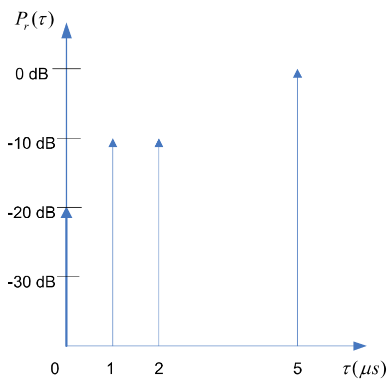
Coherence Bandwidth over which the frequency correlation function is 0.5

$$B_C = \frac{1}{5\sigma_\tau}$$



Example

Calculate the mean excess delay, RMS delay spread, and the maximum excess delay (10dB) for the multipath profile given in the figure below. Estimate 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer ?



Example

- The mean excess delay for the given profile

$$\bar{\tau} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{0.01+0.1+0.1+1} = 4.38\mu s$$

- The second moment for the given power delay profile

$$\overline{\tau^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)}{1.21} = 21.07\mu s^2$$

- The RMS delay spread $\sigma_\tau = \sqrt{21.07-(4.38)^2} = 1.37\mu s$
- The coherence bandwidth is

$$B_C \approx \frac{1}{5\sigma_\tau} = 146KHz$$

- B_c is greater than 30 KHz AMPS will work without an equalizer. However, GSM requires 200 KHz bandwidth which exceeds B_c thus an equalizer would be needed.

Doppler Spread and Coherence Time

- Doppler spread and coherence time are parameters which describe the time varying nature of the channel
- Doppler spread B_D (=f_m) is a measure of spectral broadening due to the Doppler shift associated with mobile motion
- Coherence time is a statistical measure of the time duration over which the channel impulse response is essentially invariant

Coherence time is inversely proportional to Doppler spread

$$T_C \approx \frac{1}{f_m}$$

Coherence time over which the time correlation function is 0.5

$$T_C \approx \frac{9}{16\pi f_m}$$

where f_m is the maximum Doppler shift given by f_m=v/λ

A Common Rule: $$T_C = \sqrt{\frac{9}{16\pi f_m}\frac{1}{f_m}} = \frac{0.423}{f_m}$$

4. Type of Small-scale Fading

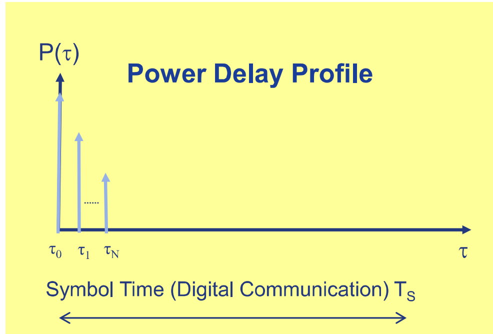
- The time dispersion and frequency dispersion mechanism in a mobile radio channel lead to four possible distinct effects, which are manifested depending on the nature of the transmitted signal, the channel and the velocity
- Multipath delay spread leads to time dispersion and frequency selective fading
- Doppler spread leads to frequency dispersion and time selective fading

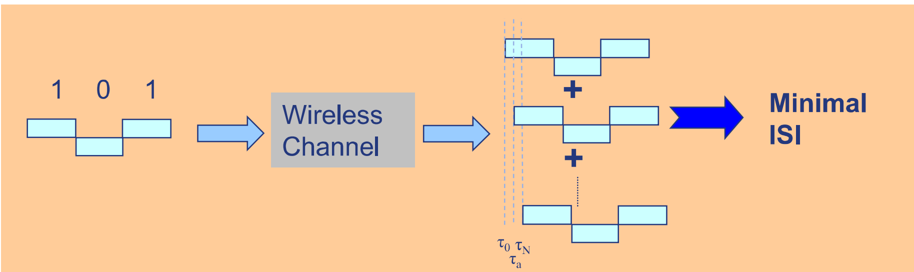
Flat Fading Vs Frequency Selective Fading

Flat Fading

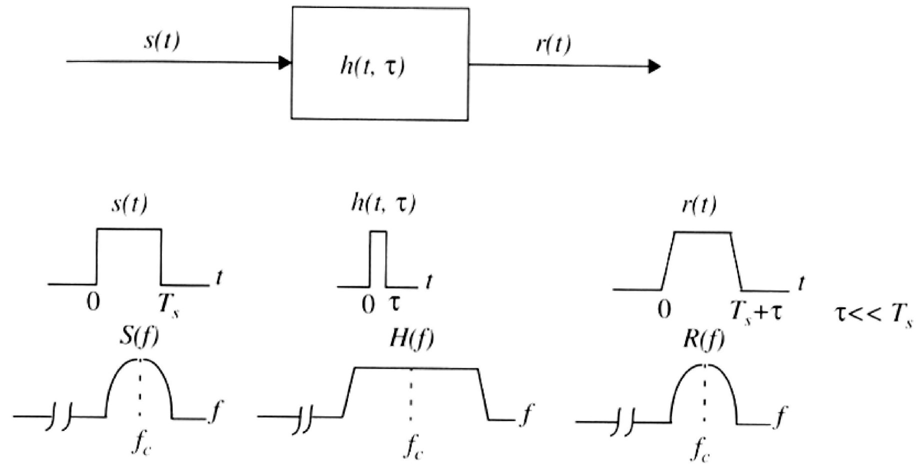
$B_S << B_C \qquad T_S >> \sigma_\tau$

A Common Rule of Thumb:
$T_S > 10\sigma_t$ ➔ Flat fading





Flat Fading



$B_S << B_C$

$T_S >> \delta_\tau$

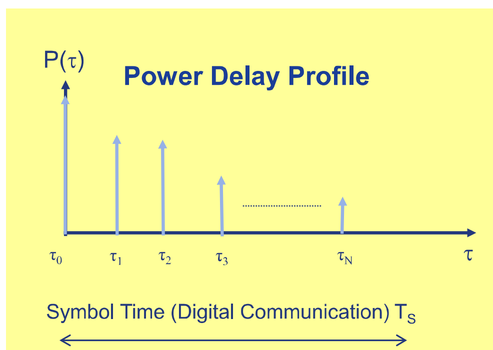
T_S is the reciprocal bandwidth
B_S is the bandwidth

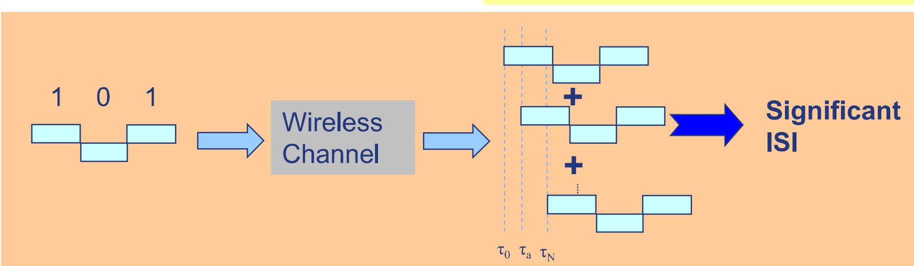
Flat Fading Vs Frequency Selective Fading

Frequency Selective Fading

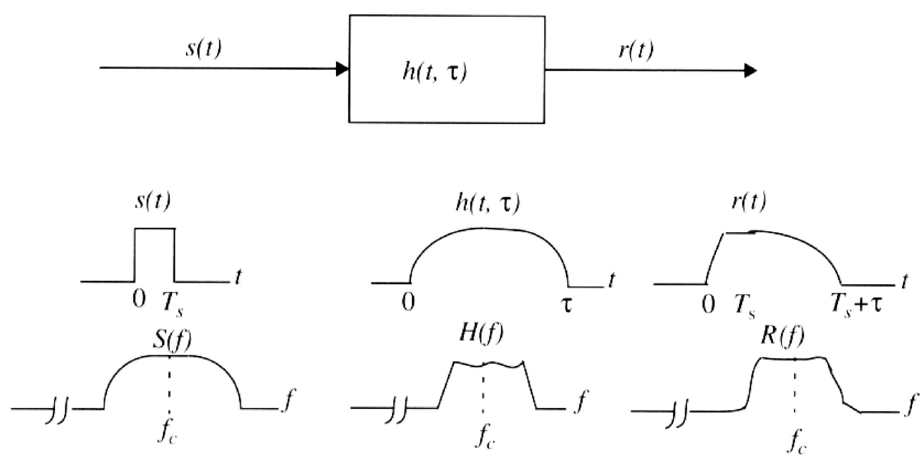
$B_S > B_C \qquad T_S < \sigma_\tau$

A Common Rule of Thumb:
$T_S < 10\sigma_t$ ➔ Frequency Selective Fading





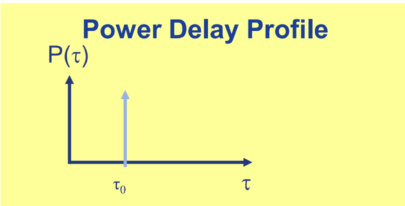
Frequency Selective Fading

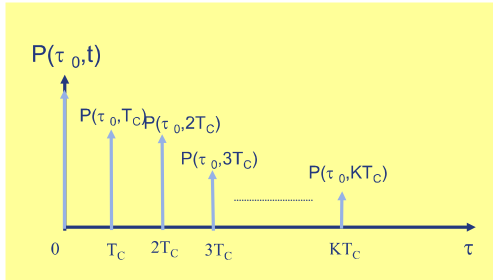


$B_S > B_C \qquad T_S < \sigma_\tau$

A common rule of thumb is that a channel is frequency selective if $\sigma_\tau > 0.1T_S$

Slow Fading Vs Fast Fading





- Consider a wireless channel comprised of a single path component.
- The power delay profile reflects average measurements
- $P(\tau_0)$ shall vary as the mobile moves

Fast Fading

$T_S > T_C \qquad B_S < B_D$

Frequency dispersion (time selective fading)

Slow Fading

$T_S \ll T_C \qquad B_S \gg B_D$

Type of Small-scale Fading

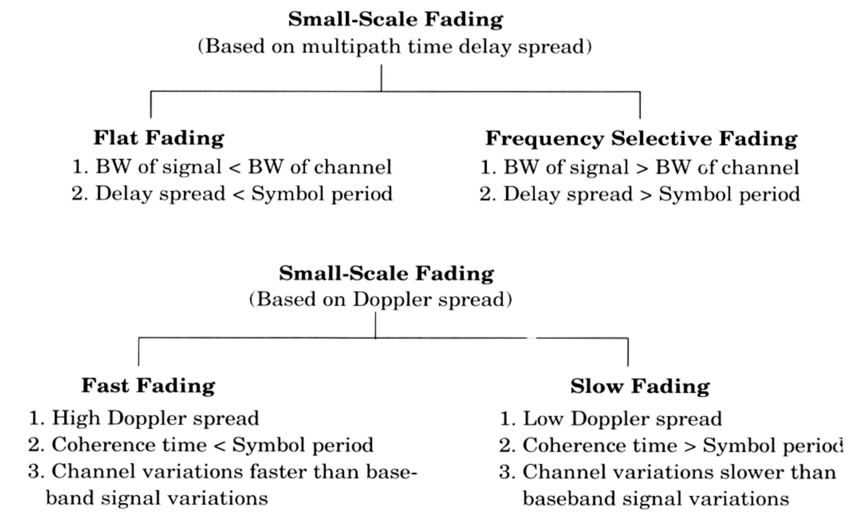
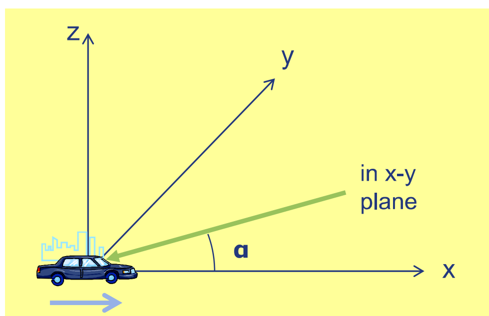


Figure 5.11 Types of small-scale fading.

5. Clarke's Model for Flat Fading

Assumptions:

- Mobile traveling in x direction
- Vertically polarized wave
- Multiple waves in the x-y plane arrive at the mobile antenna at the same time
- Waves arrive at different angles α



For N waves incident at the mobile antenna
Each wave arriving at an angle α_n will experience a different Doppler shift f_n

$$f_n = \frac{v}{\lambda}\cos\alpha_n \quad \Rightarrow \boxed{E_z = E_0\sum_{n=1}^{N} C_n \cos(2\pi f_c t + \theta_n)} \qquad \theta_n = 2\pi f_n t + \varphi_n$$

E_0 amplitude of the local average E-field
C_n random variable representing the amplitude of individual waves
f_c carrier frequency
φ_n random phase shift due to distance traveled by the n^{th} wave

Clarke's Model for Flat Fading

$$E_z(t) = E_0\left(\sum_{n=1}^{N} C_n \cos\theta_n\right)\cos(2\pi f_c t) - E_0\left(\sum_{n=1}^{N} C_n \sin\theta_n\right)\sin(2\pi f_c t)$$

$$E_z(t) = T_c(t)\cos(2\pi f_c t) - T_s(t)\sin(2\pi f_c t)$$

$$T_c(t) = E_0\left(\sum_{n=1}^{N} C_n \cos(2\pi f_n + \varphi_n)\right) \qquad T_s(t) = E_0\left(\sum_{n=1}^{N} C_n \sin(2\pi f_n + \varphi_n)\right)$$

Given that:

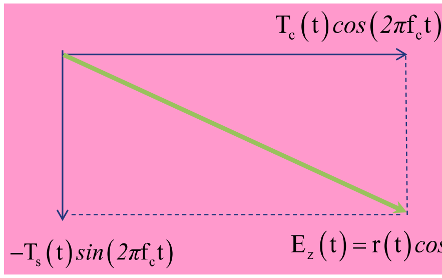
- Φ_n uniformly distributed over 2π
- N is sufficiently large (i.e., the ***central limit theorem*** is applicable)

Therefore:

Both $T_c(t)$ and $T_s(t)$ may be modeled as:
Gaussian Random Processes

Clarke's Model for Flat Fading

$$E_z(t) = T_c(t)\cos(2\pi f_c t) - T_s(t)\sin(2\pi f_c t)$$



$$|E_z(t)| = \sqrt{T_c^2(t) + T_s^2(t)} = r(t)$$

If $\sum_{n=1}^{N} \overline{C_n^2} = 1 \quad \overline{T_c^2} = \overline{T_s^2} = \sigma^2 = E_0^2/2$

$2\sigma^2$: Average received power

Power received at mobile antenna $\propto |E_z(t)|^2 = \overline{r^2}$

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} exp\left(-\dfrac{r^2}{2\sigma^2}\right) & 0 \le r \le \infty \\ 0 & r < 0 \end{cases}$$

⇒ **Rayleigh Distribution**

Rayleigh Fading Distribution

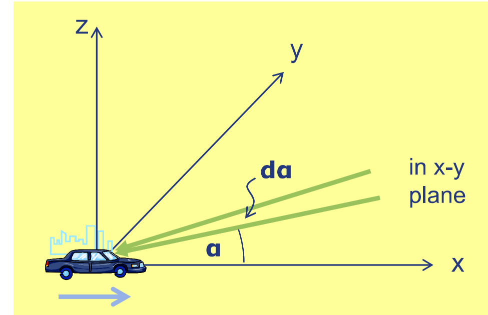
Main Assumption:
- No LOS
- All waves at the mobile receiver experience approximately the same attenuation

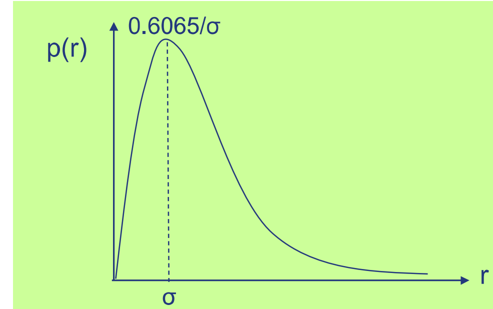
$$\boxed{E_z = E_0 \sum_{n=1}^{N} C_n \cos(2\pi f_c t + \theta_n)}$$

constant $\quad \sum_{n=1}^{N} \overline{C_n^2} = 1$

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} exp\left(-\dfrac{r^2}{2\sigma^2}\right) & 0 \le r \le \infty \\ 0 & r < 0 \end{cases}$$

σ^2: Time average received power before evelope detection
σ : rms value of received voltage before evelope detection





Rayleigh Fading Statistics

Probability the received signal does not exceed a value R

$$Pr(r \le R) = \int_0^R p(r)dr = 1 - exp\left(-\frac{R^2}{2\sigma^2}\right)$$

Mean value of the Rayleigh distribution

$$r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

Variance of the Rayleigh distribution

$$\sigma_r^2 = E[r^2] - E^2[r] = \int_0^\infty r^2 p(r)dr - \sigma^2\frac{\pi}{2}$$

$$\sigma_r^2 = \sigma^2\left(2 - \frac{\pi}{2}\right) = 0.4292\sigma^2$$

Median of the Rayleigh distribution

$$\frac{1}{2} = \int_0^{r_{median}} p(r)dr \Rightarrow r_{median} = 1.177\sigma$$

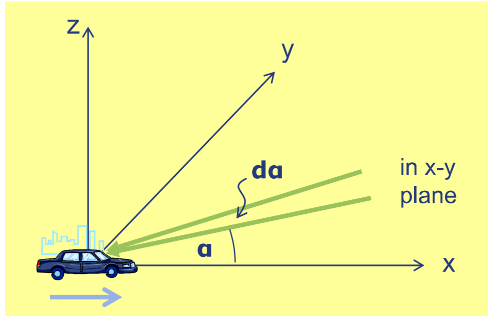
Example

- Consider a channel with Rayleigh fading and average received power Pr=20 dB. Find the probability that the received power is below 10 dB.
- Ans: 0.095

Ricean Fading Distribution

Main Assumption:

- LOS
- There is a dominant wave component at the mobile receiver in addition to experience multiple waves that experience approximately the same attenuation



$$p(r)=\begin{cases}\dfrac{r}{\sigma^2}\exp\left(-\dfrac{r^2+A^2}{2\sigma^2}\right)I_0\left(\dfrac{Ar}{\sigma^2}\right) & A\ge 0, 0\le r\le\infty\\ 0 & r<0\end{cases}$$

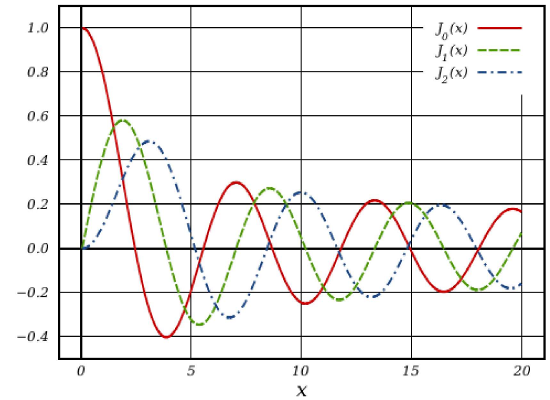
A : Peak amplitude of the dominant signal

$I_0(.)$: Modified Bessel function of the first kind and zero-order

$2\sigma^2$: Time average received power of the non-dominant components

Bessel function

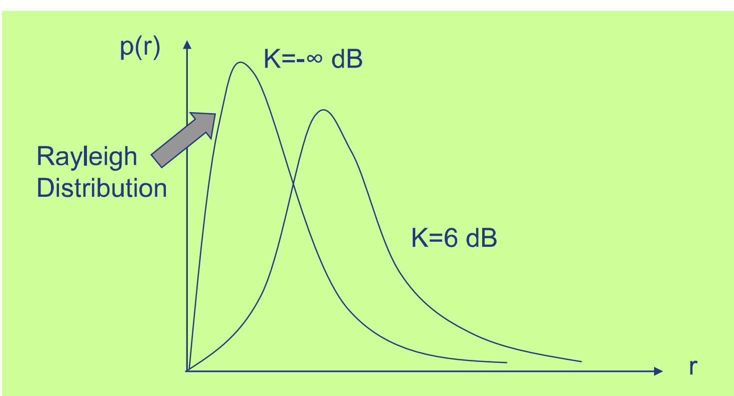
$$J_n(x)=\frac{1}{\pi}\int_0^\pi \cos(n\tau - x\sin(\tau))\,d\tau. \qquad J_n(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i(n\tau - x\sin(\tau))}\,d\tau$$



Ricìean & Rayleigh Fading

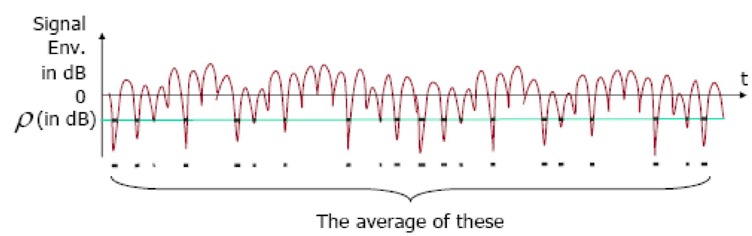
Define K called the *Ricean Factor*:
The ratio between the deterministic signal power and the power of the non-dominant waves

$$K=\frac{A^2}{2\sigma^2}\Rightarrow K(dB)=10\log\frac{A^2}{2\sigma^2}$$



Level Crossing and Fading Statistics

- The average fade duration is the average period of time the normalized envelope is below a level ρ



- Pick a level or threshold $\rho = R/R_{rms}$, where R is the unnormalized threshold and

$$R_{rms}=\sqrt{E\left(\left|h_b(t)\right|^2\right)}$$

Level Crossing and Fading Statistics

- The level crossing rate (LCR) is defined as the expected rate at which the Rayleigh fading envelope, normalized to the local rms signal level, crosses a specified level in a positive-going direction
- The number of level crossing per second is given by

$$N_R = \int_0^\infty \dot{r} p(R,\dot{r}) d\dot{r} = \sqrt{2\pi} f_m \rho e^{-\rho^2} \qquad \rho = R/R_{rms}$$

- Where

$\dot{r}$ is time derivative of $r(t)$ (the slope)

$p(R,\dot{r})$ is the joint density function of r and $\dot{r}$ at $r = R$

$R_{rms} = \sqrt{2\sigma^2}$ rms signal level

Example

- For a Rayleigh fading signal, compute the positive-going level crossing rate for $\rho = 1$ when the maximum Doppler frequency is 20 Hz
- What is the maximum velocity of the mobile for this Doppler frequency if the carrier frequency is 900 MHz?

Solution:

- Use the equation for LCR

$$N_R = \sqrt{2\pi}(20)(1)e^{-1} = 18.44$$

- Use equation of Doppler frequency

$$v = f_D \lambda = 20(1/3) = 6.66 m/s$$

Level Crossing and Fading Statistics

- The average fade duration is defined as the average period of time for which the received signal is below a specified level *R*. $\bar{\tau} = \frac{1}{N_R} \Pr[r \le R]$
- For a Rayleigh fading signal, it is given by

$$\Pr[r \le R] = \frac{1}{T}\sum_i \tau_i = \int_0^R p(r)dr = 1 - \exp(-\rho^2)$$

- So, the average fade duration can be expressed as

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_D \sqrt{2\pi}}$$

Example

- Find the average fade duration for threshold levels $\rho = 0.01$ when the Doppler frequency is 200 Hz

Solution

- Average fade duration is

$$\bar{\tau} = \frac{e^{0.01^2} - 1}{(0.01)200\sqrt{2\pi}} = 19.9 \mu s$$

6. Simulation of Fading Channel

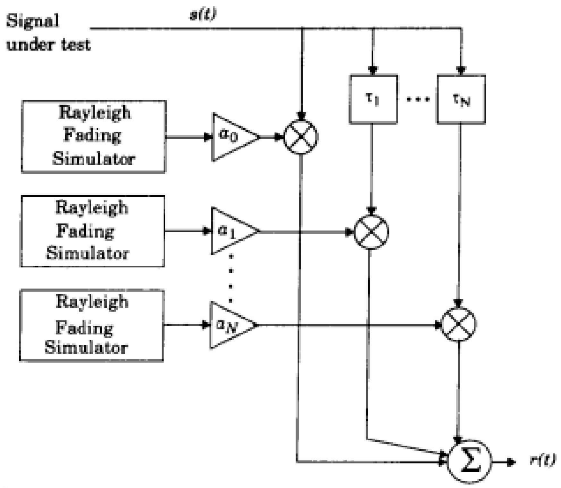
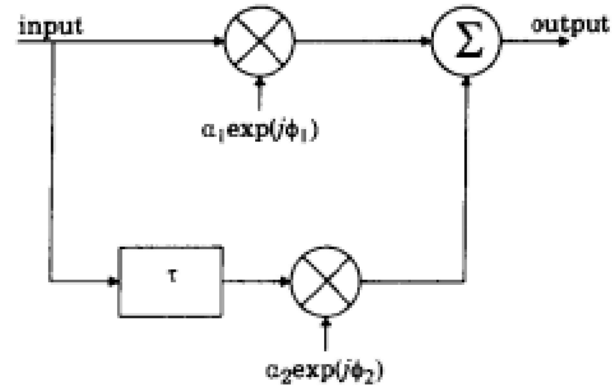
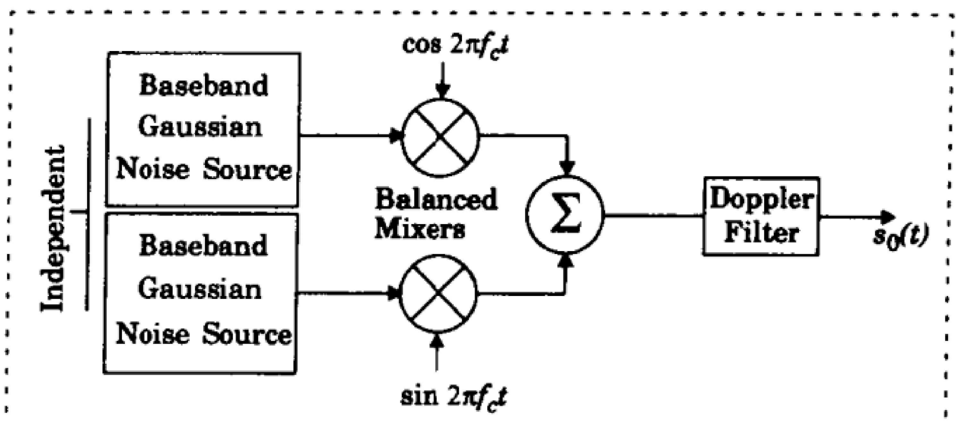


Figure 4.24
A signal may be applied to a Rayleigh fading simulator to determine performance in a wide range of channel conditions. Both flat and frequency selective fading conditions may be simulated, depending on gain and time delay settings.

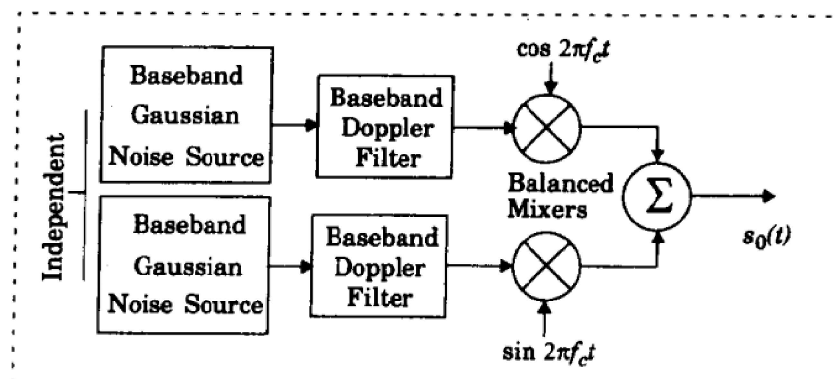
Simulation of Fading Channel



Simulation of Fading Channel



Simulation of Fading Channel



Simulation of Fading Channel

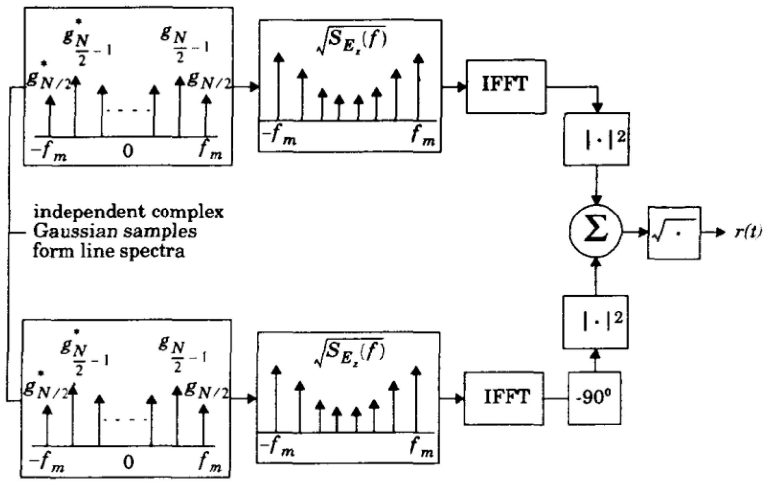


Figure 4.23
Frequency domain implementation of a Rayleigh fading simulator at baseband