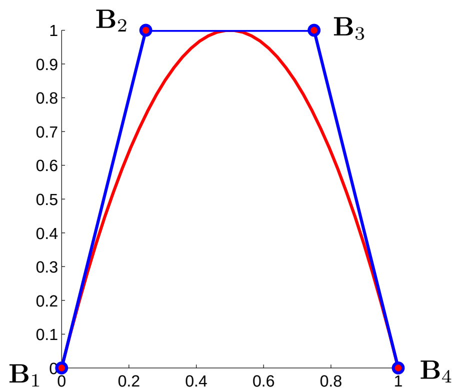
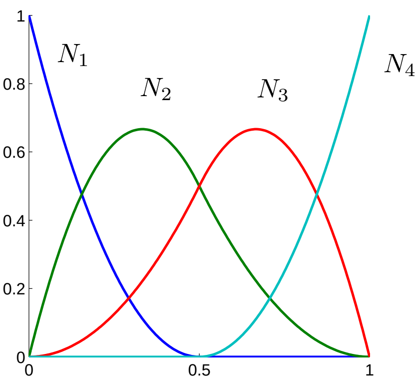
uniformly distributed points on the parameter space (the interval $[0, 1]$) were used. The coordinates and u values at these points were obtained using interpolations from the control points and the nodal unknowns.

In order to assess how high order B-spline elements behave for problems with localized gradients, let us consider the following problem [115]

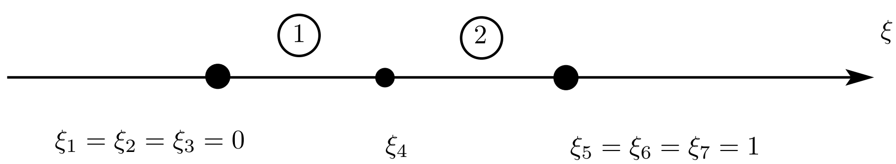
$$u_{,xx}(x) + b(x) = 0 \quad x \in [0, 1]; \quad u(0) = 0, \quad u(1) = 1, \tag{34}$$



A quadratic B-spline curve with 4 control points



4 quadratic basis functions



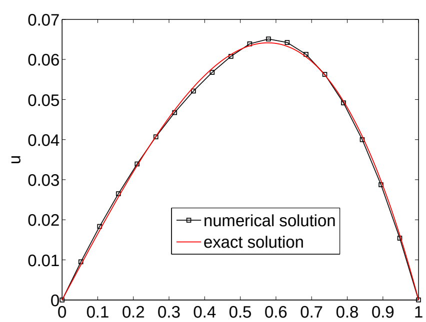
Parametric space

Figure 7: One dimensional isogeometric analysis example: exact geometry, quadratic basis functions and mesh in the parametric space.

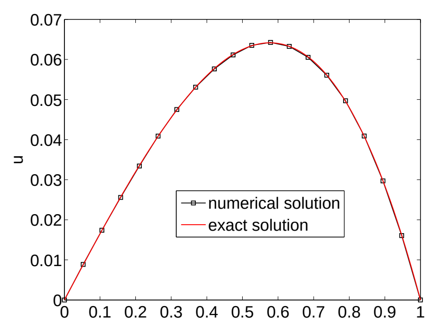
Box 1 Procedure for evaluation of Eq. (32) for element $[\xi_i, \xi_{i+1}]$.

1. $\mathbf{x} = [\mathbf{B}_1; \mathbf{B}_2; \mathbf{B}_3]$
2. $sctr = [1, 2, 3]$
3. Set $\mathbf{K}_e = 0$
4. Loop over Gauss points (GPs) $(\bar{\xi}, w)^*$
   (a) Compute parametric coordinate $\xi = 0.5[(\xi_{i+1} - \xi_i)\bar{\xi} + \xi_{i+1} + \xi_i]$
   (b) Compute derivatives $N_{I,\xi}$ $(I = 1, 2, 3)$ at point ξ
   (c) Define vector $\mathbf{N}_\xi = [N_{1,\xi} \; N_{2,\xi} \; N_{3,\xi}]$
   (d) Compute the first Jacobian $J_\xi = ||\mathbf{N}_\xi \mathbf{x}||$
   (e) Compute the second Jacobian $J_{\bar{\xi}} = 0.5(\xi_{i+1} - \xi_i)$
   (f) Compute shape function derivatives $\mathbf{N}_x = J_\xi^{-1} \mathbf{N}_\xi^{\mathrm{T}}$
   (g) $\mathbf{K}_e = \mathbf{K}_e + J_\xi J_{\bar{\xi}} w \mathbf{N}_x \mathbf{N}_x^{\mathrm{T}}$
5. End loop over GPs
6. Assemble $\mathbf{K}_e$ to the global matrix $\mathbf{K}(sctr, sctr) = \mathbf{K}(sctr, sctr) + \mathbf{K}_e$

* w denotes the weight of a Gauss point.



(a) 2 elements



(b) 4 elements

Figure 8: Comparison of the IGA result against the exact solution for the one dimensional PDE given in Eq. (27).

with

$$b(x) = \begin{cases} \Big\{2\alpha^2 - 4[\alpha^2(x-0.5)]^2\Big\}\exp\Big\{-[\alpha(x-0.5)]^2\Big\} & x \in [0.42, 0.58] \\ 0 & \text{otherwise} \end{cases} \tag{35}$$

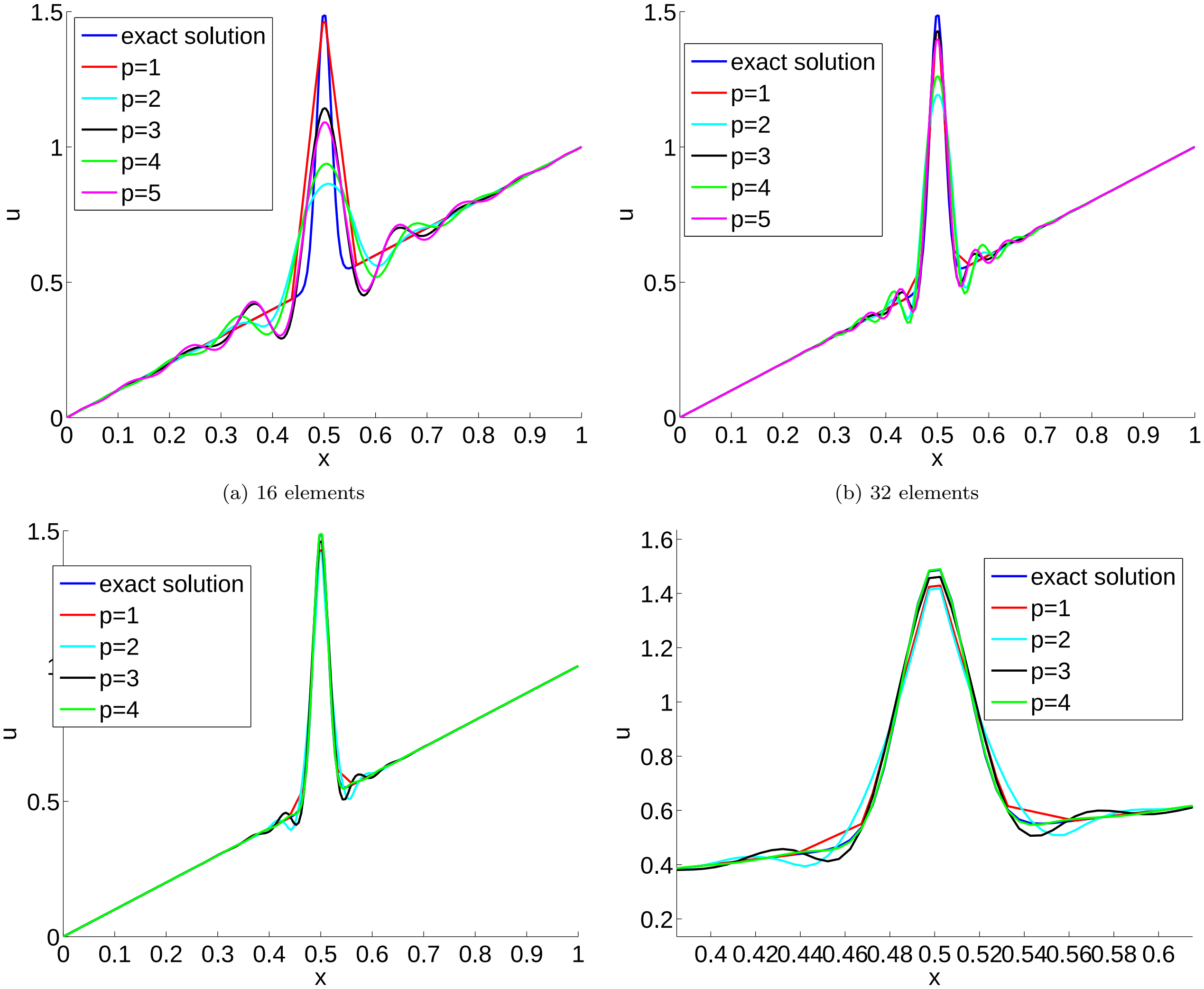
The exact solution of this problem is

$$u(x) = x + \exp\Big\{-[\alpha(x-0.5)]^2\Big\} x \in [0,1] \tag{36}$$

We use a value of 50 for α and the exact solution exhibits a sharp peek at location $x = 0.5$. We are going to solve this problem using elements of order ranging from one (linear elements) to five (quintic elements). To remove error in the numerical integration of the body force term, Eq. (32), 10 GPs were used for each element. We use the k-refinement (to be discussed in detail in Section 4.6) in building meshes of different basis orders. The initial mesh consists of one single linear element with knot vector $\Xi = \{0, 0, 1, 1\}$. The parametrization is thus linear and after performing k-refinement, the parametrization is still linear. Therefore, a point with $x = 0.3$ corresponds to $\xi = 0.3$ in the parameter space. The Matlab file for this problem is **iga1DStrongGradient.m** in the **iga** folder of our source code.

Figure (9a,b) shows the results obtained with meshes consisting of 16 and 32 elements of C^{p-1} continuity where p denotes the B-spline basis order. It is obvious that smooth basis is not suitable to problems with sharp gradients. Linear elements which have a C^0 continuity at location $x = 0.5$ (precisely at every knots) give better results than high order B-spline elements.

Knot insertion was made to insert the value 0.5 p times to the initial knot vector so that the basis is C^0 continuous at $x = 0.5$. For example for $p = 2$, a vector $\{0.5, 0.5\}$ was inserted to the knot vector. The results are given in Fig. (9c,d) where the peak was captured better. Finally $0.42, 0.5, 0.58$ were added to the knots p times so that the basis are C^0 continuous at locations $x = 0.42, 0.5, 0.58$. The corresponding results are given in Fig. (10). With only 16 cubic elements (25 CPs) the exact solution was well captured. This example showed the flexibility of B-splines/NURBS–high order functions and any level of continuity can be easily achieved.



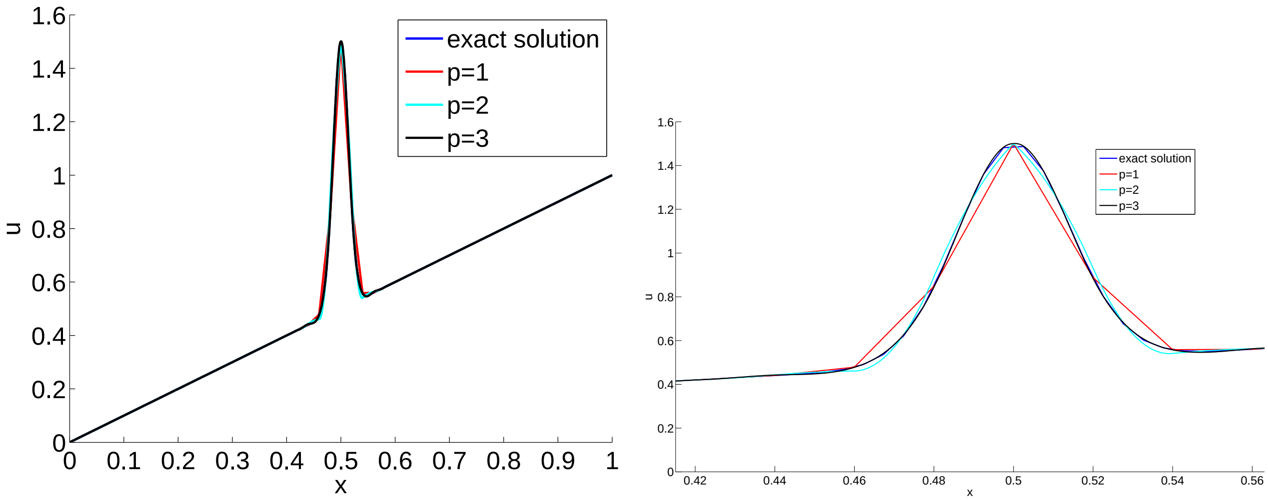
(a) 16 elements

(b) 32 elements

(c) 32 elements with C^0 continuity at $x = 0.5$

(d) 32 elements with C^0 continuity at $x = 0.5$ (close up)

Figure 9: Comparison of the IGA result against the exact solution for the one dimensional PDE given in Eq. (34).



(a) 16 elements with C^0 continuity at $x = 0.42, 0.5, 0.58$

(b) 16 elements with C^0 continuity at $x = 0.42, 0.5, 0.58$ (close up)

Figure 10: Comparison of the IGA result against the exact solution for the one dimensional PDE given in Eq. (34)