Introduction to Reinforcement Learning

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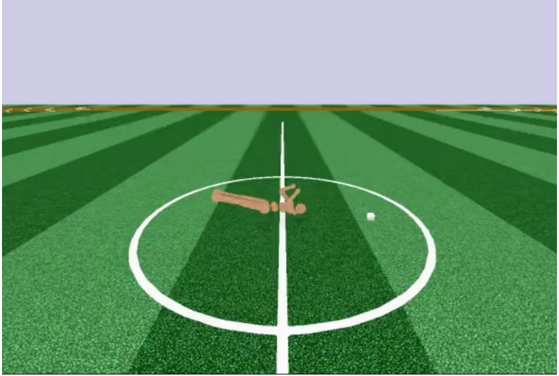
What is RL?

What can RL do?

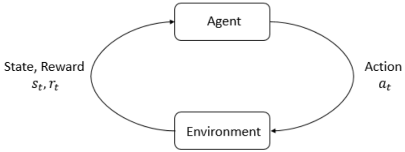
RL can...

- Play video games from raw pixels
- Control robots in simulation and in the real world
- Play Go and Dota 1v1 at superhuman levels



What is RL?

- An *agent* interacts with an *environment*.




```
obs = env.reset()
done = False
while not(done):
  act = agent.get_action(obs)
  next_obs, reward, done, info = env.step(act)
  obs = next_obs
```

- The goal of the agent is to maximize cumulative reward (called *return*).
- Reinforcement learning (RL) is a field of study for algorithms that do that.

Key Concepts in RL

Before we can talk about algorithms, we have to talk about:

- Trajectories
- Return
- Policies
- The RL optimization problem
- Value and Action-Value Functions

Note: For this talk, we will talk about all of these things in the context of *deep* RL, where we use neural networks to represent them.

Trajectories

- A **trajectory** τ is a sequence of states and actions in an environment:

$$\tau = (s_0, a_0, s_1, a_1, ...).$$

  - The initial state s_0 is sampled from a *start state distribution* μ:

$$s_0 \sim \mu(\cdot).$$

  - State transitions depend only on the most recent state and action. They could be deterministic:

$$s_{t+1} = f(s_t, a_t),$$

    or stochastic:

$$s_{t+1} \sim P(\cdot|s_t, a_t).$$

- A trajectory is sometimes also called an **episode** or **rollout**.

Reward and Return

The **reward** function of an environment measures how good state-action pairs are:

$$r_t = R(s_t, a_t).$$

- Example: if you want a robot to run forwards but use minimal energy, $R(s, a) = v - \alpha \|a\|_2^2$.

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The **return** of a trajectory is a measure of cumulative reward along it. There are two main ways to compute return:

- Finite horizon undiscounted sum of rewards::

$$R(\tau) = \sum_{t=0}^{T} r_t$$

- Infinite horizon discounted sum of rewards:

$$R(\tau) = \sum_{t=0}^{\infty} \gamma^t r_t$$

  where $\gamma \in (0, 1)$. This makes rewards less valuable if they are further in the future. (Why would we ever want this? Think about cash: it's valuable to have it sooner rather than later!)

Policies

A **policy** π is a rule for selecting actions. It can be either

- **stochastic**, which means that it gives a probability distribution over actions, and actions are selected randomly based on that distribution ($a_t \sim \pi(\cdot|s_t)$),
- or **deterministic**, which means that π directly maps to an action ($a_t = \pi(s_t)$).

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Examples of policies:

- Stochastic policy over discrete actions:

```
obs = tf.placeholder(shape=(None, obs_dim), dtype=tf.float32)
net = mlp(obs, hidden_dims=(64,64), activation=tf.tanh)
logits = tf.layers.dense(net, units=num_actions, activation=None)
actions = tf.squeeze(tf.multinomial(logits=logits,num_samples=1), axis=1)
```

- Deterministic policy for a vector-valued continuous action:

```
obs = tf.placeholder(shape=(None, obs_dim), dtype=tf.float32)
net = mlp(obs, hidden_dims=(64,64), activation=tf.tanh)
actions = tf.layers.dense(net, units=act_dim, activation=None)
```

The goal in RL is to learn a policy which maximizes expected return. The optimal policy π^* is:

$$\pi^* = \arg\max_{\pi} \mathop{\mathrm{E}}_{\tau \sim \pi} \left[R(\tau) \right],$$

where by $\tau \sim \pi$, we mean

$$s_0 \sim \mu(\cdot), \qquad a_t \sim \pi(\cdot|s_t), \qquad s_{t+1} \sim P(\cdot|s_t, a_t).$$

There are two main approaches for solving this problem:

- policy optimization
- and Q-learning.

Value Functions and Action-Value Functions

Value functions tell you the expected return after a state or state-action pair.

$V^{\pi}(s) = \underset{\tau\sim\pi}{\mathrm{E}}\left[R(\tau)\,\middle\vert\, s_0 = s\right]$	Start in s and then sample from π
$Q^{\pi}(s,a) = \underset{\tau\sim\pi}{\mathrm{E}}\left[R(\tau)\,\middle\vert\, s_0 = s, a_0 = a\right]$	Start in s, take action a, then sample from π
$V^{*}(s) = \max_{\pi} \underset{\tau\sim\pi}{\mathrm{E}}\left[R(\tau)\,\middle\vert\, s_0 = s\right]$	Start in s and then act optimally
$Q^{*}(s,a) = \max_{\pi} \underset{\tau\sim\pi}{\mathrm{E}}\left[R(\tau)\,\middle\vert\, s_0 = s, a_0 = a\right]$	Start in s, take action a, then act optimally

The value functions satisfy recursive **Bellman equations**:

$$V^{\pi}(s) = \underset{\substack{a\sim\pi \\ s'\sim P}}{\mathrm{E}}\left[r(s,a) + \gamma V^{\pi}(s')\right] \qquad Q^{\pi}(s,a) = \underset{s'\sim P}{\mathrm{E}}\left[r(s,a) + \gamma \underset{a'\sim\pi}{\mathrm{E}}\left[Q^{\pi}(s',a')\right]\right]$$

$$V^{*}(s) = \max_{a} \underset{s'\sim P}{\mathrm{E}}\left[r(s,a) + \gamma V^{\pi}(s')\right] \qquad Q^{*}(s,a) = \underset{s'\sim P}{\mathrm{E}}\left[r(s,a) + \gamma \max_{a'} Q^{\pi}(s',a')\right]$$

Q^*

The optimal Q function, Q^*, is especially important because it gives us a policy. In any state s, the optimal action is

$$a^* = \arg\max_a Q^*(s,a).$$

We can measure how good a Q^*-approximator, Q_θ, is by measuring its **mean-squared Bellman error**:

$$\ell(\theta) = \frac{1}{|\mathcal{D}|} \sum_{(s,a,s',r)\in\mathcal{D}} \left(Q_\theta(s,a) - \left(r + \gamma \max_{a'} Q_\theta(s',a') \right) \right)^2.$$

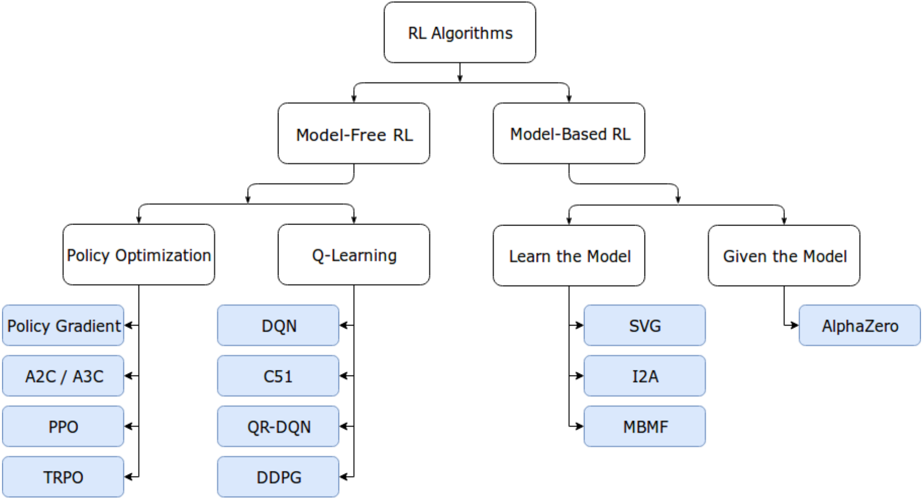
This (roughly) says how well it satisfies the Bellman equation

$$Q^*(s,a) = \mathop{\mathrm{E}}_{s'\sim P}\left[r(s,a) + \gamma \max_{a'} Q^\pi(s',a') \right]$$

Deep RL Algorithms

Deep RL Algorithms

There are many different kinds of RL algorithms! This is a non-exhaustive taxonomy (with specific algorithms in blue):



We will talk about two of them: Policy Gradient and DQN.

A Few Notes

Using Model-Free RL Algorithms:

Algorithm	a Discrete	a Continuous
Policy optimization	Yes	Yes
DQN / C51 / QR-DQN	Yes	No
DDPG	No	Yes

Using Model-Based RL Algorithms:

- Learning the model means learning to generate next state and/or reward:

$$\hat{s}_{t+1}, \hat{r}_t = \hat{f}_\phi(s_t, a_t)$$

- Some algorithms may only work with an *exact* model of the environment
  - AlphaZero uses the rules of the game to build its search tree

Policy Gradients

- An algorithm for training stochastic policies:
  - Run current policy in the environment to collect rollouts
  - Take stochastic gradient ascent on policy performance using the **policy gradient**:

$$
\begin{aligned}
g &= \nabla_\theta \mathop{\mathrm{E}}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r_t \right] \\
&= \mathop{\mathrm{E}}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \left(\sum_{t'=t}^{T} r_{t'} \right) \right] \\
&\approx \frac{1}{|\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \left(\sum_{t'=t}^{T} r_{t'} \right)
\end{aligned}
$$

- Core idea: push up the probabilities of good actions and push down the probabilities of bad actions
- Definition: sum of rewards after time t is the *reward-to-go* at time t:

$$
\hat{R}_t = \sum_{t'=t}^{T} r_{t'}
$$

Make the model, loss function, and optimizer:

```
# make model
with tf.variable_scope('model'):
    obs_ph = tf.placeholder(shape=(None, obs_dim), dtype=tf.float32)
    net = mlp(obs_ph, hidden_sizes=[hidden_dim]*n_layers)
    logits = tf.layers.dense(net, units=n_acts, activation=None)
    actions = tf.squeeze(tf.multinomial(logits=logits,num_samples=1), axis=1)

# make loss
adv_ph = tf.placeholder(shape=(None,), dtype=tf.float32)
act_ph = tf.placeholder(shape=(None,), dtype=tf.int32)
action_one_hots = tf.one_hot(act_ph, n_acts)
log_probs = tf.reduce_sum(action_one_hots * tf.nn.log_softmax(logits), axis=1)
loss = -tf.reduce_mean(adv_ph * log_probs)

# make train op
train_op = tf.train.AdamOptimizer(learning_rate=lr).minimize(loss)

sess = tf.InteractiveSession()
sess.run(tf.global_variables_initializer())
```

Example Implementation (Continued)

One iteration of training:

```
# train model for one iteration
batch_obs, batch_acts, batch_rtgs, batch_rets, batch_lens = [], [], [], [], []
obs, rew, done, ep_rews = env.reset(), 0, False, []
while True:
    batch_obs.append(obs.copy())
    act = sess.run(actions, {obs_ph: obs.reshape(1,-1)})[0]
    obs, rew, done, _ = env.step(act)
    batch_acts.append(act)
    ep_rews.append(rew)
    if done:
        batch_rets.append(sum(ep_rews))
        batch_lens.append(len(ep_rews))
        batch_rtgs += list(discount_cumsum(ep_rews, gamma))
        obs, rew, done, ep_rews = env.reset(), 0, False, []
        if len(batch_obs) > batch_size:
            break

# normalize advs trick:
batch_advs = np.array(batch_rtgs)
batch_advs = (batch_advs - np.mean(batch_advs))/(np.std(batch_advs) + 1e-8)
batch_loss, _ = sess.run([loss,train_op], feed_dict={obs_ph: np.array(batch_obs),
                                                     act_ph: np.array(batch_acts),
                                                     adv_ph: batch_advs})
```

Q-Learning

- Core idea: learn Q^* and use it to get the optimal actions
- Way to do it:
  - Collect experience in the environment using a policy which trades off between acting randomly and acting according to current Q_θ
  - Interleave data collection with updates to Q_θ to minimize Bellman error:

$$\min_\theta \sum_{(s,a,s',r)\in\mathcal{D}} \left(Q_\theta(s,a) - \left(r + \gamma \max_{a'} Q_\theta(s',a') \right) \right)^2$$

...sort of! This actually won't work!

Experience replay:

- Data distribution changes over time: as your Q function gets better and you *exploit* this, you visit different (s, a, s', r) transitions than you did earlier
- Stabilize learning by keeping old transitions in a replay buffer, and taking minibatch gradient descent on mix of old and new transitions

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Target networks:

- Minimizing Bellman error directly is unstable!
- It's *like* regression, but it's not:

$$\min_{\theta} \sum_{(s,a,s',r)\in\mathcal{D}} \left(Q_{\theta}(s,a) - y(s',r)\right)^2,$$

  where the target $y(s', r)$ is

$$y(s',r) = r + \gamma \max_{a'} Q_{\theta}(s',a').$$

  Targets depend on parameters θ—so an update to Q changes the target!
- Stabilize it by *holding the target fixed* for a while: keep a separate target network, $Q_{\theta_{targ}}$, and every k steps update $\theta_{targ} \leftarrow \theta$

DQN Pseudocode

Algorithm 1 Deep Q-Learning

Randomly generate Q-function parameters θ
Set target Q-network parameters $\theta_{targ} \leftarrow \theta$
Make empty replay buffer $\mathcal{D}$
Receive observation s_0 from environment
for $t = 0, 1, 2, \ldots$ **do**
  With probability ϵ, select random action a_t; otherwise select $a_t = \arg\max_a Q_\theta(s_t, a)$
  Step environment to get s_{t+1}, r_t and end-of-episode signal d_t
  Linearly decay ϵ until it reaches final value ϵ_f
  Store $(s_t, a_t, r_t, s_{t+1}, d_t) \rightarrow \mathcal{D}$
  Sample mini-batch of transitions $B = \{(s, a, r, s', d)_i\}$ from $\mathcal{D}$
  For each transition in B, compute

$$y = \begin{cases} r & \text{transition is terminal } (d = \text{True}) \\ r + \gamma \max_{a'} Q_{\theta_{targ}}(s', a') & \text{otherwise} \end{cases}$$

  Update Q by gradient descent on regression loss:

$$\theta \leftarrow \theta - \alpha \nabla_\theta \sum_{(s,a,y) \in B} (Q_\theta(s, a) - y)^2$$

  if $t \% t_{update} = 0$ **then**
    Set $\theta_{targ} \leftarrow \theta$
  end if
end for

Recommended Reading: Deep RL Algorithms

- A2C / A3C: Mnih et al, 2016 (https://arxiv.org/abs/1602.01783)
- PPO: Schulman et al, 2017 (https://arxiv.org/abs/1707.06347)
- TRPO: Schulman et al, 2015 (https://arxiv.org/abs/1502.05477)
- DQN: Mnih et al, 2013 (https://www.cs.toronto.edu/~vmnih/docs/dqn.pdf)
- C51: Bellemare et al, 2017 (https://arxiv.org/abs/1707.06887)
- QR-DQN: Dabney et al, 2017 (https://arxiv.org/abs/1710.10044)
- DDPG: Lillicrap et al, 2015 (https://arxiv.org/abs/1509.02971)
- SVG: Heess et al, 2015 (https://arxiv.org/abs/1510.09142)
- I2A: Weber et al, 2017 (https://arxiv.org/abs/1707.06203)
- MBMF: Nagabandi et al, 2017 (https://sites.google.com/view/mbmf)
- AlphaZero: Silver et al, 2017 (https://arxiv.org/abs/1712.01815)