A TIME-DEPENDENT MODEL FOR SEISMIC RISK REDUCTION POLICY ANALYSIS

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Abstract
The seismic risk of cities is constantly changing as cities themselves evolve in time. Key to making informed policy decisions to promote resilient cities is the ability to futurecast the risk of cities based on such potential policy decisions. This work demonstrates a flexible stochastic framework for analysing the potential seismic risk trajectories of cities. The model can be used to better understand how various retrofit policy standard and implementation time-frames affect seismic risk over time. Each policy or combination of policies can be compared with each other, and with the baseline case of "doing nothing." The model therefore serves as a seismic risk reduction policy analysis tool, and also as an advocacy tool, demonstrating that "the risk of doing nothing" is itself a policy decision, usually with the worst of possible consequences. The framework is demonstrated with a hypothetical case-study, which can serve as template for the analysis of real cities.

Introduction
Current earthquake risk assessment methods focus on understanding risk to existing infrastructure in its current state. Yet our ability to make informed and proactive policy decisions to reduce seismic risk depends on our ability to predict risk in dynamic urban development environments, and more importantly the impact of various risk reduction policies on future seismic risk.

In this paper, we propose a flexible stochastic framework for testing and comparing various seismic risk reduction policies, and their sensitivity to various factors including the level of retrofit standard, the time-frame of implementation, the pre-existing quality of building stock, the natural urban development rates and more.

The framework is then applied to a hypothetical case study to demonstrate the broader risk impact as neighborhoods implement seismic retrofits over time amidst varying rates and patterns of urban development. Multiple retrofit policies are compared, both in terms of the level of retrofit standards and the mandatory time frame to retrofit. While it is obvious that retrofit policies lead to decreased seismic risk, this study shows the time-evolution of risk linked to each of the potential retrofit strategies. This methodology is adopted from a framework used to investigate incremental building expansion as the significant driver for increasing vulnerability and risk (Lallemant, 2017).

The main contribution of the paper is to provide a flexible stochastic method to analyze future seismic risk driven by various dynamic processes. The case study serves as a proof of concept rather than prediction of actual risk. Certain simplifying assumptions are made and significant drivers of changing risk are not accounted for such as natural replacement rates of buildings, and structural deterioration over time. The case study can be refined and expanded so it becomes more realistic once actual data for transition rates, vulnerability, and building stock distribution are available.

Methodology
Seismic retrofits occur as discrete processes over time. Such processes can be modeled mathematically using Markov chains. These map the probability of transitioning from one state to another in a given time interval. For instance, in any given year, a vulnerable building can transition into a retrofitted one with a certain probability. This probability of transition therefore represents the annual retrofit rate for this

building type, and is influenced by the retrofit policy (e.g. mandated vs volunteer, etc). Markov models are "memoryless" meaning that any transition to a new state is not dependent on the sequence of events preceding it; thus, it is dependent only on the current state (Agresti, 2003). In the case of numerous building types and retrofit options, all transition probabilities form a transition probability matrix, such as described in Equation 1, which is used to simulate to probability that a building state transitions to another in a given time period. For the purposes of this study, we focus on 5 common building typologies shown in Table 1 as indices (P2 to P6), along with their retrofitted states for a low standard (P7 to P9) and high standard (P10 to P12). Indice P1 is used to represent urban growth, as the probability of an empty site transitioning into a new building. Therefore P1,2 would represent the probability of a green-field site being developed to construct a Concrete Frame Building, while P6,9 represents the probability of transitioning from a vulnerable URM (Unreinforced masonry) building to a URM retrofitted to minimum standards, and so forth.

$$P = \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} & \cdots & p_{1,12} \\ p_{2,1} & p_{2,2} & p_{2,3} & \cdots & p_{2,12} \\ p_{3,1} & p_{3,2} & p_{3,3} & \cdots & p_{3,12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{12,1} & p_{12,2} & p_{12,3} & \cdots & p_{12,12} \end{pmatrix} \tag{1}$$

Where each index is described in Table 1 below.

Table 1. Indices of Transition Probability Matrices

Index	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
Building type	Empty Site	CF	CF- I	CW	TIM	URM	CF	CF-I	URM	CF	CF-I	URM
Level of retrofit	None	None	None	None	None	None	Low	Low	Low	High	High	High

Abbreviations: CF = Concrete Frame; I = Infilled; CW = Concrete Wall; TIM = Timber Framed.

The transition matrices assume that retrofitted buildings do not transition to lower states (from retrofitted to non-retrofitted), and some states are "absorbing states" which are states that cannot be left once entered. The absorbing states represent our simplifying assumption that once buildings are retrofitted, no further modifications are further done. Transition probability rates can either be derived from statistics of building state distribution over time or assumed from reasonable outcomes of building state distribution.

Also, for a building stock with a initial distribution of typologies described by a normalized vector d_o, and a transition probability matrix P, the expected distribution D_t after *t steps* can be calculated as:

$$E\ (D_t \mid D_o = d_o)\ =\ d_o\ \ P^t \tag{2}$$

The vulnerability of each of the building is defined by fragility curves. These curves relate the earthquake intensity and the probability of exceeding or experiencing a certain level of damage. They can be derived analytically (Singhal and Kiremidjian 1996, Lallemant et al. 2015), empirically (Laumann 2005, Noh et al 2014) or heuristically based on expert opinion (Jaiswal et al. 2012).

The annual collapse rate of each building is obtained by integrating the fragility curve over the hazard curve as described in equation 3.

$$\lambda_{Collapse} = \int_{IM_{min}}^{IM_{max}} P(Collapse \mid IM = im) \mid d\lambda_{IM}(im) \mid$$
(3)

Where $\lambda_{IM}(im)$is the ground motion hazard curve and $|d\lambda_{IM}(im)|$is the absolute value of the derivative of the hazard curve.

Since buildings can be in one of several states over time, then collapse rate given an unknown state at time *t* is therefore:

$$\lambda_{Collapse}(t) = \sum_{State\ 1}^{State\ n} \int_{IM_{min}}^{IM_{max}} P(Collapse \mid IM = im \mid State = State_i\ ) P(State = State_i \mid Do = d_0, P = p)\ (t) \mid d\lambda_{IM}(im) \mid$$
(4)

Where $P(Collapse \mid IM = im \mid State = State_i\ )$is the collapse fragility curve for each building type/state, $P(State = State_i \mid Do = d_0, P = p)\ (t)$ is the probability of being in each building state at time *t* given a starting state distribution $d_0$and transition probability matrix *p*.

Equations 3 and 4 can be combined as:

$$\lambda_{Collapse}(t) = \sum_{State\ 1}^{State\ n} \int_{IM_{min}}^{IM_{max}} P(Collapse \mid IM = im \mid State = State_i\ )\ d_0 P^t \mid d\lambda_{IM}(im) \mid$$
(5)

Finally, for a portfolio of buildings described by a vector $d_0 N$ where N is the total number of buildings and $d_0$their initial distribution (% of each building types/states), the annual expected number of buildings collapse at time *t* is described as:

$$\lambda_{Collapse\ Total}(t) = \Lambda_{collapse}\ N d_0 P^t \mid d\lambda_{IM}(im) \mid$$
(6)

Hypothetical Case-Study

Using the methodology described previously, we can test the impact of various policies on regional seismic risk over time. A hypothetical region is used to demonstrate the framework. The region contains four districts. For simplicity, each of the four districts in the region has its own building type distribution and seismic hazard curve, described in Figure 1 and 2 respectively.

Hazard Curves. Four hypothetical hazard curves are developed as idealized power-law hazard curves of the following form:

$$\lambda_{IM}(IM) = k_o\ IM^{-k} \tag{7}$$

Parameters used for the four districts are k0 = 0.0002, 0.0003, 0.00022, 0.00035 and k = 2, 2.1, 2.2, 2.5 for districts 1, 2, 3 and 4 respectively. The resulting fragility curves are shown in Figure 2.

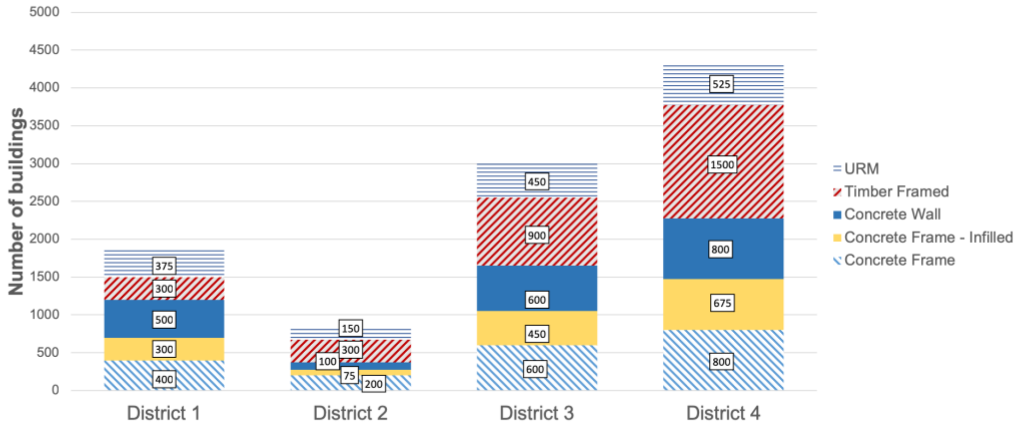

Figure 1. Distribution of building typologies for 4 hypothetical districts

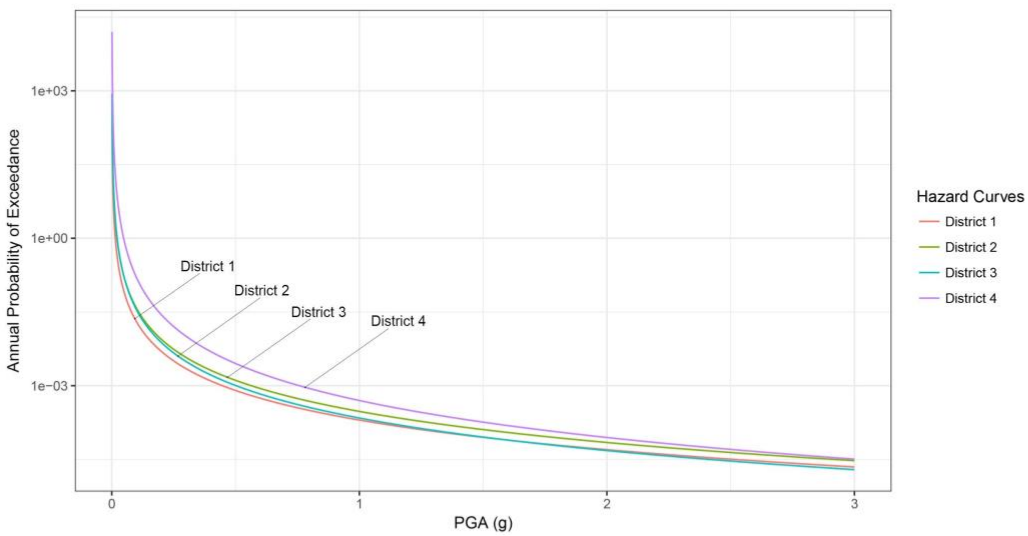

Figure 2. Hypothetical hazard curves for the four districts in the region. Parameters k0 = 0.0002, 0.0003, 0.00022, 0.00035 and k = 2, 2.1, 2.2, 2.5 are used to create these idealized power-law curves from equation 7.

Building Vulnerability Curves. The building vulnerability curves used are the empirical vulnerability curves obtained from damage surveys following the 2010-2011 Canterbury earthquake sequence (Blackbourn & Davey, 2017). For simplicity, we only use fragility curves corresponding to "Extensive / Complete Damage." In addition, hypothetical curves are developed representing Low Retrofit and High Retrofit Standards. While the actual curves do not directly correspond to the 33% and 67% National Building Standard (NBS) for retrofits of earthquake-prone buildings, these are meant to represent such "low" and high" retrofit standard policies.

Seismic Risk Reduction Policies. The purpose of the framework developed is to compare the impact of various policies (or absence of policies) on seismic risk over time. The types of decisions in the policy-making space includes the level of retrofit standards used, the time-frame for their implementation, the urban development rates (e.g. limits in development to particular regions) and more. Seven policy scenarios are used to demonstrate the diversity of policy options for seismic safety. These are described in Table 2.

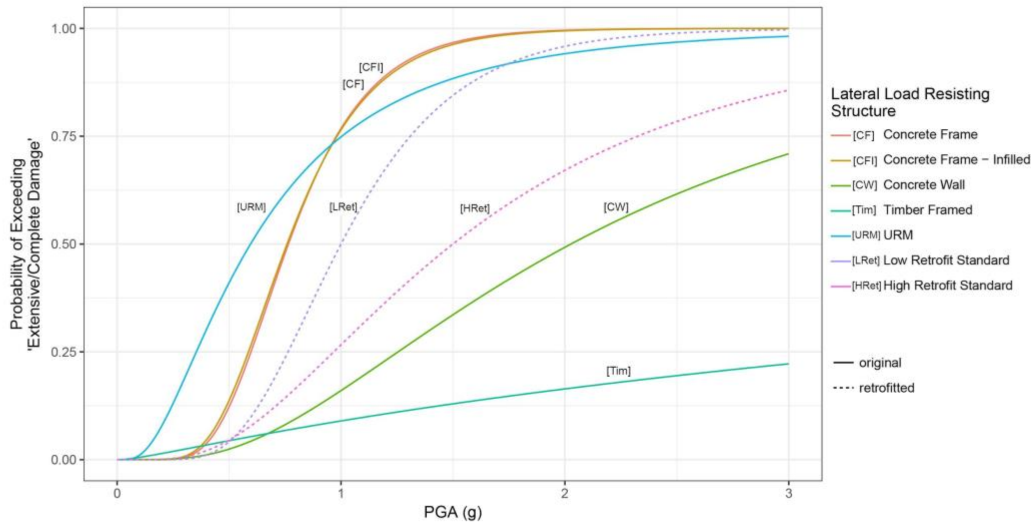

Figure 3. Vulnerability curves for several lateral load resisting structures obtained from empirical damage surveys following the 2010-2011 Canterbury earthquake sequence (Blackbourn & Davey, 2017) and hypothetical low and high retrofit standards.

Table 2. Seismic risk reduction policy scenarios

Policy Scenario	*Type of implementation*	*Retrofit quality*	*Time frame*	*Assumptions*
Policy 1	No retrofits	Not applicable	Not applicable	Natural urban growth drives the construction of new buildings. New buildings are assumed to be either concrete wall or timber framed buildings, both of which have low vulnerability.
Policy 2	Voluntary retrofits	Based on building owner discretion	Long (35 years)	The management of risk is left to the to the discretion of the building owners and tenants, and thus owners are principally responsible for building safety. Information dissemination and promoting voluntary retrofits helps the public make informed decision about earthquake risk. Adapted from previous studies, it is assumed that 59% percent of owners of earthquake-prone buildings have retrofitted their building within 35 years, of which 73% have opted for the minimum retrofit standard and 27% have opted for higher retrofit standards (Egbelakin et al., 2013).
Policy 3	Mandatory	Low standard	Short (15 years)	95% of owners adhere to this policy, of which 95% opted for minimum retrofit standards and 5% opted for higher standards.
Policy 4	Mandatory	Low standard	Long (35 years)	95% of owners adhere to this policy, of which 95% opt for minimum retrofit standards and 5% opt for higher standards.
Policy 5	Mandatory	High standard	Short (15 years)	95% of owners conduct of retrofit, of which 95% adhere to higher retrofit standards while 5% nonetheless opt for minimum retrofit standards.
Policy 6	Mandatory	High standard	Long (35 years)	95% of owners conduct of retrofit, of which 95% adhere to higher retrofit standards while 5% nonetheless opt for minimum retrofit standards.

Policy 7	Mandatory	High standard	Short (15 years)	Same as Policy 5 + limited urban development in District 4 95% of owners conduct of retrofit, of which 95% adhere to higher retrofit standards while 5% nonetheless opt for minimum retrofit standards. In addition, no further urban development is allowed in District 4 due to higher seismic hazard. All urban development is concentrated in the three other districts.

Mathematical Representations of Retrofit Policies. Each of the seven policies described above can be converted to transition probability matrices and included in a Markov Chain simulation used in equation 6. Transition probability matrices are mathematical representation of the probability of transition from one state to another for the entire building stock of the region in a given time-step. Indeed, urban growth can be represented as the probability of transitioning from an empty lot to a building in a 1 year period. Retrofit rates can be represented as the probability of transitioning from an earthquake-prone building to a retrofitted building in a 1 year period.

For each policy, a transition probability matrix is calibrated based on retrofit outcomes after 15 or 35 years described previous. We show below an example of the transition probability matrix calibrated for Policy 6 (High-retrofit standard, long frame for implementation).

$$P = \begin{pmatrix} 0.985 & 0 & 0 & 0.0075 & 0.0075 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.819 & 0 & 0.0075 & 0 & 0 & 0.009 & 0 & 0 & 0.172 & 0 & \\ 0 & 0 & 0.819 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.819 & 0 & 0 & 0.009 & 0 & 0 & 0.172 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

Results - Effect of Policy on Regional Seismic Risk

Using Equation 6, we can test the impact of various policies on regional seismic risk over time. Figures 4 and 5 show how the choices described by the 7 policies presented in this study affect the level of seismic risk in our region and scenario. These choices affect not only the level of risk reached, but also how risk changes in time. Mandatory retrofit schemes result in rapid reduction in seismic risk. Since seismic collapse risk is also a factor of the total exposure, Figure 4 shows an eventual increase in risk for all policies, due to natural urban growth.

Conclusion

This study provides a simplified demonstrations for a time-dependent seismic risk analysis framework which enables the investigation of various seismic risk mitigation policies and their impacts over time. Overall, the framework enables stakeholders to analyze future seismic risk and its sensitive to the initial building stock quality, the level of retrofit standard developed, the time-frame for mandatory retrofit, the rate and pattern of urban development and other drivers of changing risk.

The current demonstration has several limitations. Numerous drivers of changing risk are not accounted for, including the natural replacement rate of buildings, and structural deterioration of buildings over time. Other limitations of this demonstration include the simplified hazard model represented as a single hazard curve for each district, the limited number of building types, and the testing of only two retrofit standards. While normalized collapse risk is used as the sole metric for seismic risk, the analysis could easily be extended to expected financial loss, by incorporating the non-collapse damage-state fragility curves and their corresponding loss distributions.

This type of time-dependent analysis can be effectively used to develop and compare novel seismic risk mitigation policies, and be integrated within cost-benefit policy analysis. The Markov chain transition probability matrices can be used to represent numerous discrete or continuous state-change processes affecting seismic risk. As such, the current study provides a flexible framework for further study into drivers of seismic risk, accounting for those drivers of increasing risk along with policies for its reduction.

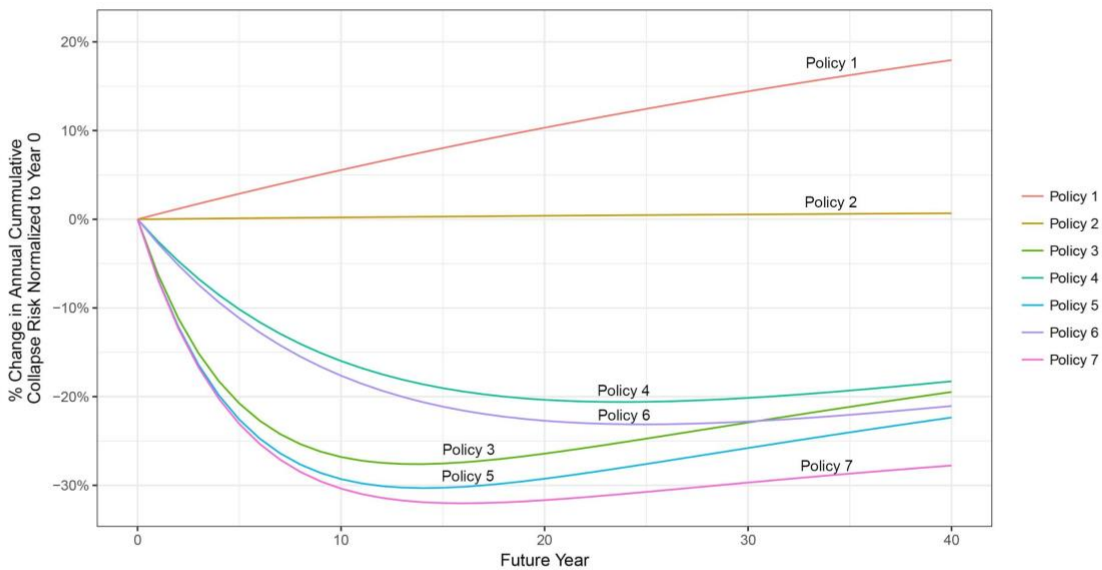

Figure 4. Percent change in annual collapse risk normalized to baseline risk at t=0, for each of the 7 policies and assuming an urban growth rate of 1.5% per year.

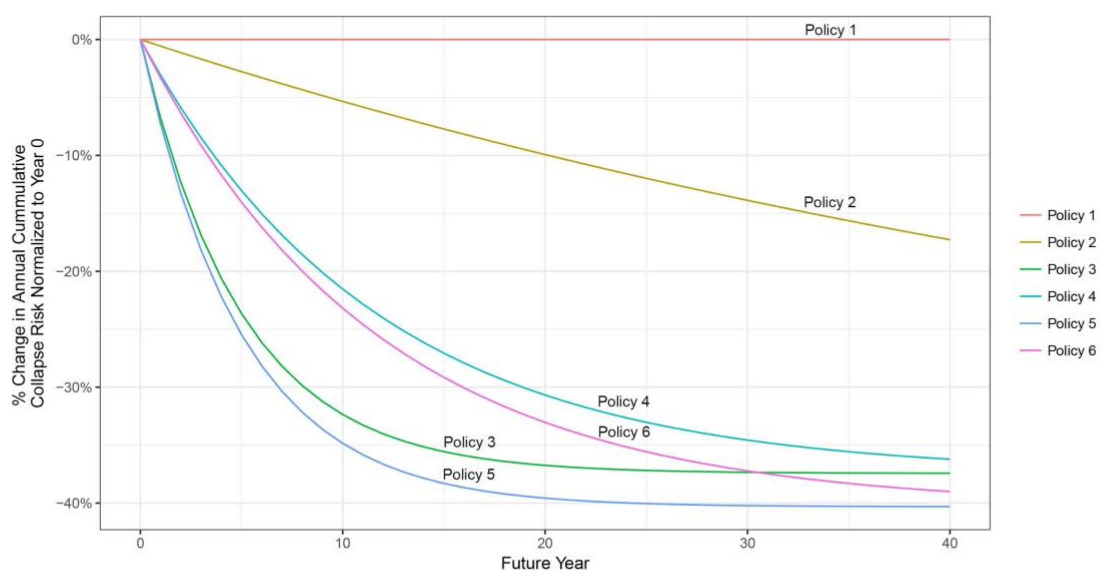

Figure 5. Percent change in annual collapse risk normalized to baseline risk at t=0, for each of the 6 policies and assuming no urban growth (hence policy 7 is not included).

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