MIP

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1.1 Parameters and Variables

Sets and Parameters	Definition
n^{I}	Number of stages.
n^{J}	Number of jobs.
n_i^{M}	Number of machines in stage i. $i \in I$.
I	Set of stages. $I = \{1, 2, ..., n^{\mathrm{I}}\}$.
J	Set of jobs. $J = \{1, 2, ..., n^{\mathrm{J}}\}$.
M_i	Set of machines in stage i. $M_i = \{1, 2, ..., n_i^{\mathrm{M}}\}, i \in I$.
Q_{ij}	Queue time limit of job j on stage i. $i \in I \setminus \{1\}, j \in J$.
A_{imj}	Initial production time of job j on machine m of stage i. $i \in I, m \in M_i, j \in J$.
B_{im}	Production time discount after maintenance of machine m of stage i. $i \in I, m \in M_i, B_{im} \in [0, 1]$.
U_{im}	Unfinished production time from the previous day of machine m of stage i. $i \in I, m \in M_i$.
F_{im}	Maintenance lengths of machine m of stage i. $i \in I, m \in M_i$.
D_j	Due time of job j. $j \in J$.
W_j	Tardiness penalties of job j. $j \in J$.
K	A very large positive number.

Table 1: List of sets and parameters and their definitions

Variables	Definition
r_{imj}	1 if job j is completed on machine m of stage i or 0 otherwise. $i \in I, m \in M_i, j \in J$.
v_{im}	1 if machine m of stage i will be maintained or 0 otherwise. $i \in I, m \in M_i$.
z_{im}^{R}	Completion time of maintenance on machine m of stage i. $i \in I, m \in M_i$.
z_{ij}	Completion time of job j on stage i. $i \in I, j \in J$.
p_{imj}	Effective production time of job j on machine m of stage i. $i \in I, m \in M_i, j \in J$.
$x_{imj_1j_2}$	1 if job j_1 precedes job j_2 on machine m of stage i or 0 otherwise. $i \in I, m \in M_i, j_1, j_2 \in J, j_1 \neq j_2$.

y_{imj}^{Before}	1 if maintenance precedes job j on machine m of stage i or 0 otherwise. $i \in I, m \in M_i, j \in J$.
y_{imj}^{After}	1 if job j precedes maintenance on machine m of stage i or 0 otherwise. $i \in I, m \in M_i, j \in J$.
$w_{i_1 m_1 i_2 m_2}$	1 if maintenance timing on machine m_1 of stage i_1 precedes that on machine m_2 of stage i_2 or 0 otherwise. $i_1, i_2 \in I, m_1 \in M_{i_1}, m_2 \in M_{i_2}, (i_1, m_1) \neq (i_2, m_2)$.

Table 2: List of variables used and their definitions

1.2 Model

$$\min \quad \sum_{j \in J} (\max(z_{n_I j} - D_j, 0) W_j)$$

s.t.

$$z_{ij} - (U_{im} + p_{imj}) \geq -K(1 - r_{imj}) \qquad \forall i \in I, m \in M_i, j \in J$$

$$z_{i+1,j} - (z_{ij} + p_{i+1,mj}) \geq -K(1 - r_{i+1,mj}) \qquad \forall i \in I \setminus \{n^{\text{I}}\}, m \in M_i, j \in J$$

$$z_{ij_1} + p_{imj_2} - z_{ij_2} \leq K(1 - x_{imj_1 j_2}) \qquad \forall i \in I, m \in M_i, j_1, j_2 \in J, j_1 \neq j_2$$

$$z_{im}^{\text{R}} \geq U_{im} + F_{im} + K(\sum_{j \in J} y_{imj}^{\text{Before}} - n_J) \qquad \forall i \in I, m \in M_i$$

$$z_{ij} + F_{im} - z_{im}^{\text{R}} \leq K(1 - y_{imj}^{\text{After}}) \qquad \forall i \in I, m \in M_i, j \in J$$

$$z_{im}^{\text{R}} + p_{imj} - z_{ij} \leq K(1 - y_{imj}^{\text{Before}}) \qquad \forall i \in I, m \in M_i, j \in J$$

$$p_{imj} \geq A_{imj} - A_{imj}(1 - B_{im}) y_{imj}^{\text{Before}} \qquad \forall i \in I, m \in M_i, j \in J$$

$$z_{i_1 m_1}^{\text{R}} + F_{i_2 m_2} - z_{i_2 m_2}^{\text{R}} \leq K(1 - w_{i_1 m_1 i_2 m_2}) \qquad \forall i_1, i_2 \in I, m_1 \in M_{i_1}, m_2 \in M_{i_2}, (i_1, m_1) \neq (i_2, m_2)$$

$$\sum_{m \in M_i} r_{imj} = 1 \qquad \forall i \in I, j \in J$$

$$x_{imj_1 j_2} + x_{imj_2 j_1} \geq r_{imj_1} + r_{imj_2} - 1 \qquad \forall i \in I, m \in M_i, j_1, j_2 \in J, j_1 \neq j_2$$

$$2(y_{imj}^{\text{Before}} + y_{imj}^{\text{After}}) \leq r_{imj} + v_{im} \qquad \forall i \in I, m \in M_i, j \in J$$

$$y_{imj}^{\text{Before}} + y_{imj}^{\text{After}} \geq r_{imj} + v_{im} - 1 \qquad \forall i \in I, m \in M_i, j \in J$$

$$w_{i_1 m_1 i_2 m_2} + w_{i_2 m_2 i_1 m_1} \geq v_{i_1 m_1} + v_{i_2 m_2} - 1 \qquad \forall i_1, i_2 \in I, m_1 \in M_{i_1}, m_2 \in M_{i_2}, (i_1, m_1) \neq (i_2, m_2)$$

$$(z_{ij} - p_{imj}) - z_{i-1,j} - Q_{ij} \leq K(1 - r_{imj}) \qquad \forall i \in I \setminus \{1\}, m \in M_i, j \in J$$

$$z_{ij}, z_{im}^{\text{R}}, p_{imj} \geq 0 \qquad \forall i \in I, m \in M_i, j \in J$$

$$y_{imj}^{\text{Before}}, y_{imj}^{\text{After}}, r_{imj}, v_{im} \in \{0, 1\} \qquad \forall i \in I, m \in M_i, j \in J$$

$$x_{imj_1 j_2} \in \{0, 1\} \qquad \forall i \in I, m \in M_i, j_1, j_2 \in J, j_1 \neq j_2$$

$$w_{i_1 m_1 i_2 m_2} \in \{0, 1\} \qquad \forall i_1, i_2 \in I, m_1 \in M_{i_1}, m_2 \in M_{i_2}, (i_1, m_1) \neq (i_2, m_2)$$