

# INTRODUCTION TO ARTIFICIAL INTELLIGENCE

## EXERCISE 2

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## Introduction:

In this report we show our work on genetic algorithms, when used to optimize functions. Specifically, our algorithm tries to maximize the function  $F(x) = x^T Ax + b^T x + c$ , where  $c$  is a constant,  $b$  is a  $d$ -dimensional vector,  $A$  is a  $d \times d$  matrix, and  $x$  (the free variable) is again a  $d$ -dimensional vector. In our solution, each iteration of the algorithm calls four functions, each one doing a different job as described further in the report. But first, we discuss the user input format, and the population initialization and representation.

## Input:

The user defines the dimensionality, gives the constants of the function  $F$ , the search area, the population size, the crossover and mutation probabilities, as well as the number of the iterations, and the number of individuals to select to keep in each iteration as the best (depending on this number, there may be more individuals that stay the same).

For many of these parameters, some values are suggested, and every parameter is checked that is correct, in case of wrong input (for example negative probability).

## Population initialization and representation:

The initialization is done randomly; as the genetic algorithms uses binary vectors, series of zeros and ones are generated randomly, with equal probability, and stored. The method of representation of the integer numbers we used is the signed binary; the first bit of the number represents the sign (0 for +, 1 for -). This is because we believe it behaves better when it comes to the crossover process (for example if we used the 2 complement representation, and tried to crossover 11111 with 00001, which represent -1 and 1, we could get 11001 and 00111, which represent -7 and 7, which is bad if we want the algorithm to converge for example at 0).

## The algorithm:

As mentioned before, in each iteration of the algorithm, four functions are called. These are discussed below:

### 1. Function fitvalues:

This function takes as a parameter the current population, and returns the values of  $F(x)$ , of the individuals of the population (which are also the fitness values, as we want to maximize the function), and also the scaled-normalized values.

The function also calls function `func` which calculates the value for a given point (as binary vectors), and also `ma2ve` is called to transform the matrix of zeros and ones which represent a number, to a vector of integer numbers.

## **2. Function roulette\_wheel\_selection:**

This function selects some individuals with the roulette wheel selection process, and replaces them in the population at the places of the individuals that have been the longest in the population, because of the FIFO replacement policy (this is done with the use of an offset).

## **3. Function crossover:**

This function takes as parents the individuals that were selected in the roulette wheel selection process, and with a probability given from the user crossovers them, with the single-point procedure. The resulted children are added to the population, again with the FIFO policy. For every parent pair, we do an independent probability test to decide whether we will crossover them.

## **4. Function mutation:**

This function takes as parents again individuals selected in the roulette wheel selection process, and with a probability given from the user creates mutated children, which are again added to the population, with the FIFO policy. For every parent, we do an independent probability test to decide whether we will create a mutated child.

## **Output:**

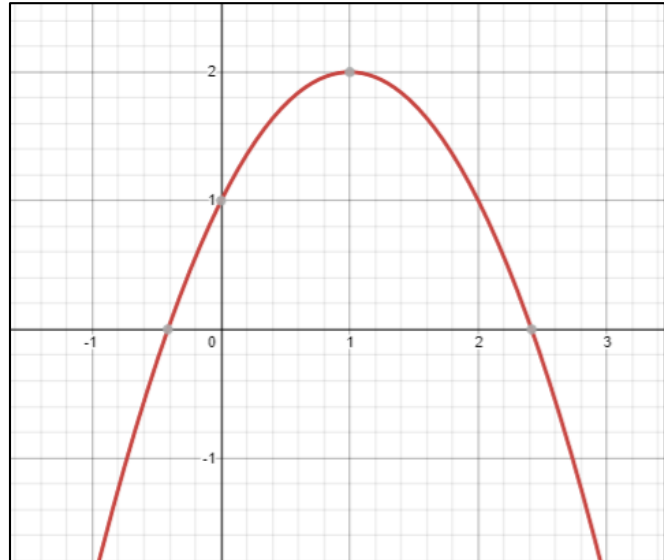
At the end, our program prints the individuals, their  $F(x)$  values, the best value and mean of values of the last population, as well as the best value of all the populations. On the next pages, a couple of test cases are presented.

## Test 1:

$$F(x) = -x^2 + 2x + 1$$

The function is shown on the picture. As we can see, the maximum is observed at the point  $x=1$ , and its value is  $F(1)=2$ .

Below we can see the terminal, as we run the algorithm. The algorithm managed to find the maximum, but the maximum is not in the final population.



```
Give function dimensionality: 1
Give matrix A (each row you add press Enter) :
-1
Give d-dimensional vector b: 2
Give constant c: 1
Give the range of searched integers as  $d \geq 1$  that for each dimension  $i$ ,  $-2^d < x_i < 2^d$ : 10
Give population size: 100
Give crossover probability (reccomended  $>0.8$ ): 0.85
Give mutation probability ( $0.1 < \text{reccomended} < 0.2$ ): 0.15
Give number of iterations: 1000
Give no of individuals to select to keep in the population, each iteration (rec.  $\sim \text{pop. size}/5$ ): 20
final population: [[0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]], [[0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]], [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]
final population values: [-119, -287, -287, -287, -1442, -1847, -2399, -2302, -2302, -7, -287, -2702, -40]
final population best value: -7
final population value mean: -1519.36
best overall value: 2

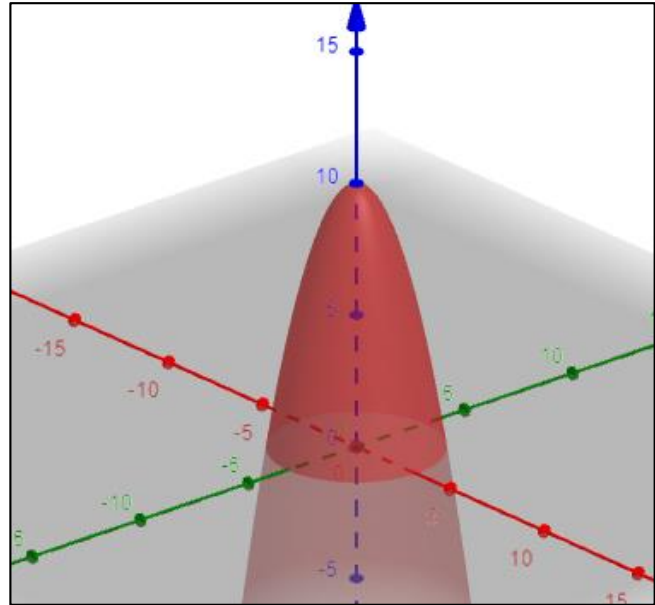
Process finished with exit code 0
|
```

## Test 2:

$$F(x) = 10 - x^2 - y^2 = x^T \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x + [0 \quad 0]x + 10$$

The function is shown on the picture. As we can see, the maximum is observed at the point  $x=(0,0)$ , and its value is  $F(0,0)=10$ .

Below we can see the terminal, as we run the algorithm. The algorithm managed to find the maximum, but again the maximum is not in the final population.



```
Give function dimensionality: 2
Give matrix A (each row you add press Enter) :
-1 0
0 -1
Give d-dimensional vector b: 0 0
Give constant c: 10
Give the range of searched integers as  $d \geq 1$  that for each dimension  $i$ ,  $-2^d < x_i < 2^d$ : 10
Give population size: 100
Give crossover probability (reccomended  $> 0.8$ ): 0.9
Give mutation probability ( $0.1 < \text{reccomended} < 0.2$ ): 0.2
Give number of iterations: 1000
Give no of individuals to select to keep in the population, each iteration (rec.  $\sim \text{pop. size}/5$ ): 18
final population: [[[1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0], [1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0]], [[1, 0, 0, 0,
final population values: [-73438, -7576, -15480, -33864, -1087, -431, -1203, -1480, -2602, -2115, -4670,
final population best value: -31
final population value mean: -10538.14
best overall value: 10
```

```
Process finished with exit code 0
```

```
|
```