Δευτέρα, 09/01/2023

Θέμα 3, 2019

$\dot{x} = Ax(t) + bu(t) + d$, $x \in \mathbb{R}^2$, $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

$y(t) = Cx(t)$

$C = [1 \quad 0]$

A. Αν $d = 0$

$\mathcal{C} = [b \quad Ab] = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow \det\mathcal{C} = -1 \neq 0$ ελέγξιμο

$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \Rightarrow \det\mathcal{O} = 1 \neq 0$ παρατηρήσιμο

ασταθές, διπλή ιδιοτιμή στο $\lambda = 2$

B. $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$, $d = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\dot{x} = Ax + bu(t) + d$

$\begin{cases} \dot{x} = Ax + Bu \\ x(t) = e^{At}x(0) + \int_0^t e^{A(t-s)} Bu(s)ds \end{cases}$

$\Rightarrow x(t) = e^{At}x(0) + \int_0^t e^{A(t-s)} [\underbrace{bu(s) + d}_{b+d}] ds = e^{At}x(0) + \int_0^t \underbrace{e^{A(t-s)}}_{e^{At}e^{-As}} ds\,(b+d)$

$x(t) = e^{At} + e^{At}\int_0^t e^{-As} ds\,(b+d)$

$\hookrightarrow \frac{d}{ds}(-e^{-As}A^{-1})$

$x(t) = e^{At}x(0) + e^{At}(I - e^{-At})A^{-1}(b+d)$

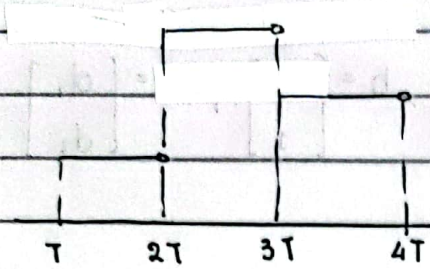
$x(t) = e^{At}x(0) + (e^{At} - I)A^{-1}(b+d)$

$e^{At} = \begin{bmatrix} e^{2t} & te^{2t} \\ 0 & e^{2t} \end{bmatrix}$, $e^{At} = \mathcal{L}^{-1}\{(sI - A)^{-1}\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} s-2 & -1 \\ 0 & s-2 \end{bmatrix}^{-1}\right\} =$

$= \mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\begin{bmatrix} s-2 & 1 \\ 0 & s-2 \end{bmatrix}\right\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{1}{s-2} & 1/(s-2)^2 \\ 0 & 1/s-2 \end{bmatrix}\right\}$

Scanned with CamScanner

Γ. $u(t) = u_d(kT) \quad \forall t \in [kT, (k+1)T)$



$$x((k+1)T) = A_d\, x(kT) + B_d\, u(kT) + d_d$$

$$y(kT) = C_d\, x(kT), \qquad C_d = C$$

$$\dot{x} = Ax + Bu(t) + d$$

$$x((k+1)T) = \underbrace{e^{A[(k+1)T - kT]}}_{A_d} x(kT) + \int_{kT}^{(k+1)T} e^{A[(k+1)T - w]} [bu(w) + d]\, dw$$

$$\left(x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A(t-s)} Bu(s)\, ds \right)$$

$$A_d = e^{AT}$$

$$x((k+1)T) = A_d x(kT) + \int_{kT}^{(k+1)T} e^{A[(k+1)T - w]} dw\, (bu(kT) + 1)$$

$$\int_{kT}^{(k+1)T} e^{A((k+1)T - w)} dw \;\overset{\lambda = w - kT}{\underset{d\lambda = dw}{=\!=}}\; \int_{0}^{T} e^{A(T-\lambda)} d\lambda =$$

$$A \text{ αντιστρ.} : = e^{AT}(I - e^{-AT}) A^{-1} = (e^{AT} - I) A^{-1}$$

$$x((k+1)T) = \underbrace{e^{AT}}_{A_d} x(kT) + \underbrace{(e^{AT} - I) A^{-1} b}_{b_d} u(kT) + \underbrace{(e^{AT} - I) A^{-1} d}_{d_d}$$

$$e^{AT} = \begin{bmatrix} e^{2T} & Te^{2T} \\ 0 & e^{2T} \end{bmatrix}$$

(Για να είναι ευσταθές: $e^{2T} < 1 \Rightarrow$ δεν γίνεται $\Rightarrow$ ασταθές)

Scanned with CamScanner

Δ. Αν $d=0$ κ' $T=\ln 2$

$u_d(kT) = K x(kT)$, $x(kT) \to 0$ σε πεπερασμένο πλήθος βημάτων

$$A_d = e^{AT} = \begin{bmatrix} e^{2\ln 2} & \ln 2\, e^{2\ln 2} \\ 0 & e^{2\ln 2} \end{bmatrix} = \begin{bmatrix} 4 & 4\ln 2 \\ 0 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ 0 & 1/2 \end{bmatrix}$$

$$B_d = (e^{AT} - \mathbb{I}) A^{-1} b = \begin{bmatrix} 3 & 4\ln 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1/2 & -1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2\ln 2 - 3/4 \\ 3/2 \end{bmatrix}$$

$$\mathcal{C}_d = \begin{bmatrix} 2\ln 2 - 3/4 & 14\ln 2 - 3 \\ 3/2 & 6 \end{bmatrix} \Rightarrow \det \mathcal{C} = 12\ln 2 - \frac{18}{4} - 21\ln 2 + \frac{9}{2} = -9\ln 2 \neq 0$$

(dead-beat απόκριση: πρέπει οι ιδιοτιμές να πάνε στο 0)

$$\det(\lambda \mathbb{I} - (A_d + B_d K_d)) = \lambda^2 \Rightarrow \dots$$

$$K_d = [K_{d1} \quad K_{d2}]$$

$$\left(A_d \text{ stable} \to A \text{ stable}, \quad t \in [kT, (k+1)T), \quad \|x(t)\| = \|e^{\overset{\leq 0}{A_d(t-Tk)}}\| \, \underbrace{\|x(kT)\|}_{\to 0} \right)$$

Ε. $d \neq 0$, $T = \ln 2$, $u_d(kT) = K_1 x(kT) + K_2 \sum_{j=0}^{k-1} (y(jT) - y_d^*)$ (PI ελεγκτής)

$$z(kT) = \sum_{j=0}^{k-1} (y(jT) - y_d^*), \quad x[k] := x(kT)$$

$$x[k+1] = A_d x[k] + B_d (K_1 x[k] + K_2 z[k]) + d_d$$
$$z[k+1] = z[k] + y[k] - y_d^*$$

$$\begin{bmatrix} x[k+1] \\ z[k+1] \end{bmatrix} = \begin{bmatrix} A_d + B_d K_1 & B_d K_2 \\ C_d & \mathbb{I} \end{bmatrix} \begin{bmatrix} x[k] \\ z[k] \end{bmatrix} + \begin{bmatrix} d_d \\ -y_d^* \end{bmatrix}$$

$$\begin{bmatrix} A_d - B_d K & B_d K_2 \\ C_d & \mathbb{I}_p \end{bmatrix} = \begin{bmatrix} A_d & 0 \\ C_d & \mathbb{I}_p \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

Πρέπει $\left(\begin{bmatrix} A_d & 0 \\ C_d & \mathbb{I}_p \end{bmatrix}, \begin{bmatrix} B_d \\ 0 \end{bmatrix} \right)$ ελέγξιμο

$$\text{rank} \begin{bmatrix} A_d - \lambda \mathbb{I}_n & 0 & B_d \\ C_d & (1-\lambda)\mathbb{I}_p & 0 \end{bmatrix} = n+p \quad \forall \lambda \in \mathbb{C}$$

(A_d, B_d) ελέγξιμο $\Rightarrow$ $\lambda \neq 1$ rank $= n+p$ ✓

Για $\lambda = 1$, $\text{rank} \begin{bmatrix} A_d - \mathbb{I}_n & B_d \\ C_d & 0 \end{bmatrix} = n+p$

$\Rightarrow \ldots$ ισχύει

οπότε $\exists K_1, K_2$

$$\begin{bmatrix} x^* \\ z^* \end{bmatrix} = \begin{bmatrix} A_d + B_d K_1 & B_d K_2 \\ C_d & \mathbb{I} \end{bmatrix} \begin{bmatrix} x^* \\ z^* \end{bmatrix} + \begin{bmatrix} d_d \\ -y_d^* \end{bmatrix}$$

$$\cancel{z^*} = \underbrace{C_d x^*}_{y^*} + \cancel{z^*} - y_d^* \Rightarrow y^* = y_d^*$$

$$\begin{cases} x^* = (A_d + B_d K_1) x^* + B_d K_2 z^* + d_d \\ C_d x^* = y_d^* \end{cases}$$

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_0$$

$$\underline{a}_i \le a_i \le \bar{a}_i \qquad i = 0, 1, \dots, n$$

είτε πάντα $\underline{a}_i > 0$ είτε πάντα $\bar{a}_i < 0$

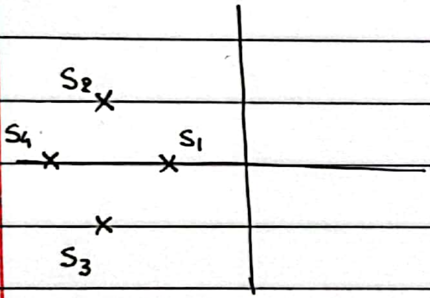
Θεώρημα Kharitonov:

$$P(s; E_1, O_1) = \bar{a}_0 + \bar{a}_1 s + \underline{a}_2 s^2 + \underline{a}_3 s^3 + \bar{a}_4 s^4 + \bar{a}_5 s^5 + \dots$$

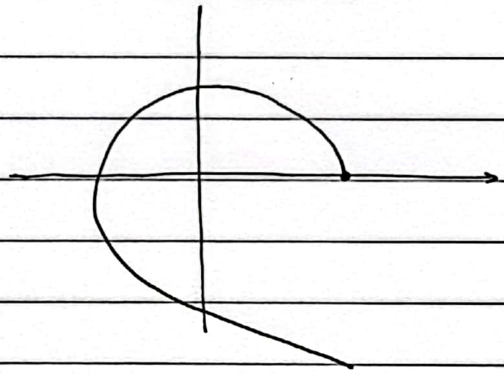
$$P(s; E_1, O_2) = \bar{a}_0 + \underline{a}_1 s + \underline{a}_2 s^2 + \bar{a}_3 s^3 + \bar{a}_4 s^4 + \underline{a}_5 s^5 + \dots$$

$$P(s; E_2, O_1) = \underline{a}_0 + \bar{a}_1 s + \bar{a}_2 s^2 + \underline{a}_3 s^3 + \underline{a}_4 s^4 + \bar{a}_5 s^5 + \dots$$

$$P(s; E_2, O_2) = \underline{a}_0 + \underline{a}_1 s + \bar{a}_2 s^2 + \bar{a}_3 s^3 + \underline{a}_4 s^4 + \underline{a}_5 s^5 + \dots$$



$$P(s) = a_n \prod_{i=1}^{n} (s - s_i) = a_0 + a_1 s + a_2 s^2 + \dots$$



n φορές τέμνει τους άξονες
+
τέμνει εναλλάξ

$$P(j\omega) = (a_0 - a_2\omega^2 + a_4\omega^4 - \dots) + j(a_1\omega - a_3\omega^3 + a_5\omega^5 + \dots)$$

$$\frac{1}{2}\left(P(s) + P(-s)\right) \to \text{even}$$

$$\frac{1}{2}\left(P(s) - P(-s)\right) \to \text{odd}$$