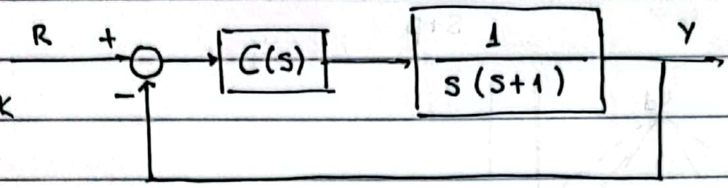
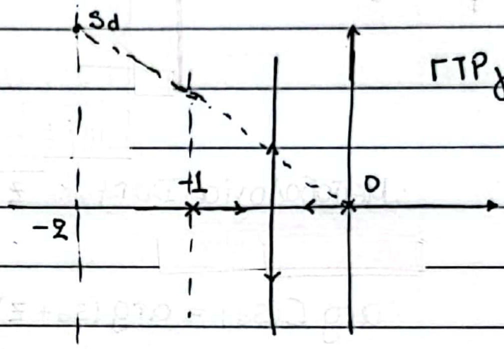
Δευτέρα, 06/03/2023

Δυναμικοί Αντισταθμιστές

$G(s) = \dfrac{1}{s(s+1)}$ $\quad C(s) = K$



$G_{cl}(s) = \dfrac{CG}{1+CG} = \dfrac{K}{s^2+s+K}$ $\quad \Delta = 1-4K$



ΓΤΡ για $K>0$

$K=1$, $M_p = 0.16$, $T_s = 8sec$

$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$, $T_s = \dfrac{4}{\zeta\omega_n}$

$p_{1,2} = -\zeta\omega_n \pm j\omega_d$

$2\zeta\omega_n = 1$

$\omega_n^2 = K \Rightarrow \omega_n = \sqrt{K}$, $\zeta = \dfrac{1}{2\sqrt{K}}$

$T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{1/2} = 8$

$K\uparrow \Rightarrow \zeta\downarrow \Rightarrow$ μειώνεται η απόσβεση $\Rightarrow M_p\uparrow$

$T_p = \dfrac{\pi}{\omega_d}$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\quad$ για $K\uparrow \Rightarrow \zeta\downarrow \Rightarrow T_p\downarrow$

Επιθυμώ $T_s = 2sec$, $M_p = e^{-\pi} \simeq 0.043$

Επιθυμητοί πόλοι: $s_{1,2} = -2 \pm j2$

$\dfrac{\zeta}{\sqrt{1-\zeta^2}} = 1 \Rightarrow \zeta = \dfrac{1}{\sqrt{2}} \Rightarrow \zeta = 0.707$

$\arg(C(s_d)G(s_d)) = -180° \Rightarrow \arg C(s_d) + \arg G(s_d) = -180°$

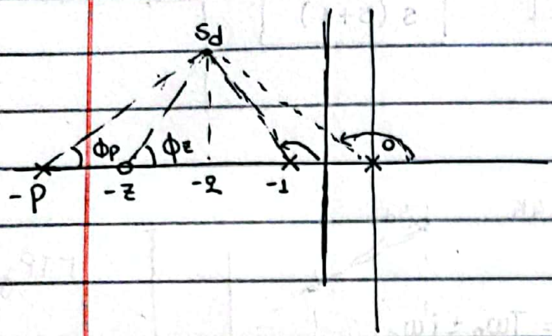
$\arg G(s_d) = -\arg(s_d) - \arg(s_d+1) = -135° - (180° - \tan^{-1} 2) = -251.56°$

$\arg C(s_d) = -180° - \arg G(s_d) = 71.56°$

- Ελεγκτές Πλοήγησης (lead controller)
- Ελεγκτές Καθυστέρησης (lag controller)

Ελεγκτές Προήγησης

$$C_{lead}(s) = K\frac{s+z}{s+p}, \quad p > z > 0$$



Μέθοδος Dorf : $z = -Re(s_d) = \zeta\omega_n = 2$

$$\arg C(s_d) = \arg(s_d+z) - \phi_p = 71.56^\circ \Rightarrow \phi_p = 90^\circ - 71.56^\circ \Rightarrow \phi_p = 18.44^\circ$$

$$\phi_p = \tan^{-1}\left(\frac{2}{p-2}\right) \to \frac{2}{p-2} = \tan(18.43^\circ) \Rightarrow p = 2 + \frac{2}{\tan(18.43^\circ)} \Rightarrow p = 8$$

$$C_{lead}(s) = K\frac{s+2}{s+8}$$

$$|C(s_d)G(s_d)| = 1 \Rightarrow K\frac{|s_d+2|}{|s_d+8|}\cdot\frac{1}{|s_d||s_d+1|} = 1 \Rightarrow K = \frac{|s_d||s_d+1||s_d+8|}{|s_d+2|} =$$

$$\Rightarrow K = \frac{2\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{40}}{2} \Rightarrow K = 20, \quad \boxed{C_{lead}(s) = 20\cdot\frac{s+2}{s+8}}$$

$$G_{cl}(s) = \frac{20(s+2)}{(s+5)(s^2+4s+8)}$$

(το μηδενικό κάνει το σύστημα πιο γρήγορο, αλλά $M_p\uparrow$)

$$G_{cl}(s) = \frac{\omega_n^2}{(s^2+2\zeta\omega_n + \omega_n^2)}\frac{(s+z)\cdot p'}{(s+p')\cdot z}$$

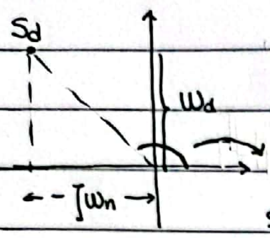
$$Y(s) = \frac{p'\omega_n^2}{z}\cdot\frac{s+z}{(s+p')(s^2+2\zeta\omega_n+\omega_n^2)s}, \quad s_d = -\zeta\omega_n + j\omega_d$$

$$Y(s) = \frac{1}{s} + \frac{A_{p'}}{s+p} + \frac{p'\omega_n^2}{z}\cdot\frac{s_d+z}{s_d(s_d+p')2j\omega_d}\frac{1}{s-s_d} + \frac{p'\omega_n^2}{z}\frac{\bar{s}_d+z}{\bar{s}_d(\bar{s}_d+p')(-2j\omega_d)}\frac{1}{s-\bar{s}_d}$$

$$y(t) = 1 + A_{p'}e^{-p't} + \frac{p'\omega_n^2}{z}\frac{s_d+z}{s_d(s_d+p)2j\omega_d}e^{-\zeta\omega_n t + j\omega_d t} + \frac{p'\omega_n^2}{z}\frac{\bar{s}_d+z}{\bar{s}_d(\bar{s}_d+p')(-2j\omega_d)}e^{-\zeta\omega_n t + j\omega_d t}$$

$$\simeq 1 + \frac{p'\omega_n^2}{z}\frac{|s_d+z|}{|s_d||s_d+p'|\omega_d}e^{-\zeta\omega_n t}\left[\frac{e^{j(\omega_d t + \arg(s_d+z) - \arg(s_d+p') - \arg s_d)}}{2j} - \frac{e^{-j(\omega_d t + \arg(s_d+z) - \arg(s_d+p') - \arg s_d)}}{2j}\right]$$

$$\Rightarrow y(t) \simeq 1 + \frac{p'\cancel{\omega_n^2}}{z}\,\frac{|s_d+z|}{|s_d+p'|\,\cancel{\omega_n}\sqrt{1-\zeta^2}\cdot\cancel{\omega_n}}\,e^{-\zeta\omega_n t}\sin\left(\omega_d t + \arg(s_d+z) - \arg(s_d+p') - \arg s_d\right)$$



$\arg(s_d) = \pi - \phi$

$\sin(x-\pi) = -\sin x$

$$\Rightarrow y(t) \simeq 1 - \frac{p'}{|s_d+p'|}\cdot\frac{|s_d+z|}{z}\,\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \phi + \arg(s_d+z) - \arg(s_d+p')\right)$$

$\pi+\phi$

$$\frac{dy}{dt} = 0 \Rightarrow -\zeta\omega_n \sin(\omega_d t^* + \phi + \underbrace{\arg(s_d+z) - \arg(s_d+p')}_{\phi_c}) + \omega_n\sqrt{1-\zeta^2}\cos(\omega_d t^* + \phi + \phi_c) = 0$$

$$\Rightarrow \tan(\omega_d t^* + \phi + \phi_c) = \frac{\sqrt{1-\zeta^2}}{\zeta} = \tan\phi \rightarrow \omega_d t_k + \phi + \phi_c = k\pi + \phi$$

$T_p = \frac{\pi}{\omega_d}$, $\boxed{T_d = \frac{\pi - \phi_c}{\omega_d}}$

$$M_p = \frac{p'}{|p'+s_d|}\cdot\frac{|s_d+z|}{z}\,e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}(\pi-\phi_c)} \qquad \left(\longleftrightarrow M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}\right)$$