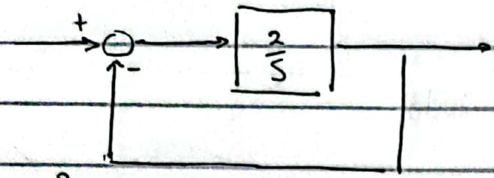
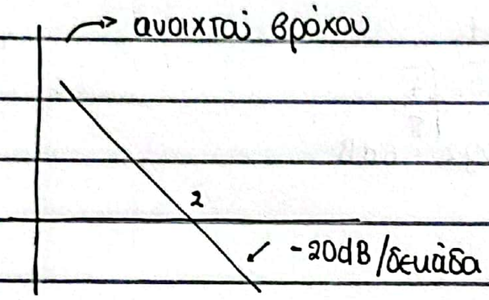
Πέμπτη, 01/12/2022

$G(s) = \frac{2}{s}$



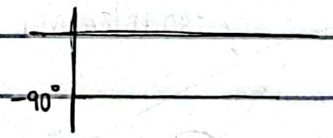

$G_{κλάδου} = \frac{\frac{2}{s}}{1+\frac{2}{s}} = \frac{2}{s+2}$

$20\log 2 - 20\log\omega$




Phase margin: 90°

Gain margin: ∞, (δεν φτάνει ποτέ στις -180°)




$-90° - \omega T$ $\qquad \omega_{gc} T < \frac{\pi}{2} \Rightarrow T < \frac{\pi}{2\omega_{gc}} = \frac{\pi}{4}$

Με καθυστέρηση: $\dfrac{2e^{-Ts}/s}{1+\frac{2e^{-Ts}}{s}}$, $s + 2e^{-Ts} = 0$

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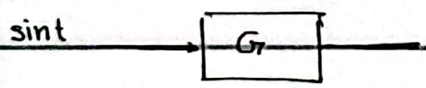
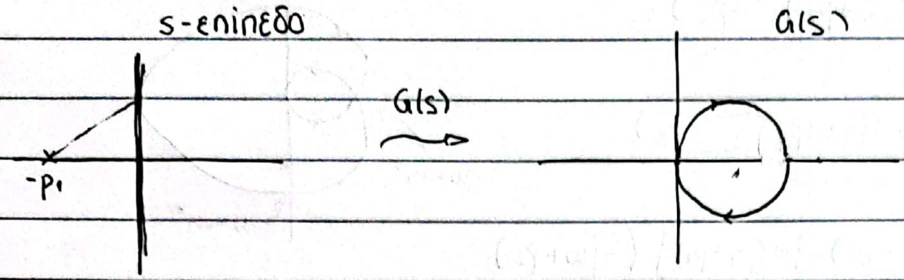
$G(s) = \frac{1}{s+1}$, $|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$, $\arg G(j\omega) = -\tan^{-1}(\omega)$




$\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$

↳ $|G(j\omega)| \sin(t + \arg G(j\omega))$

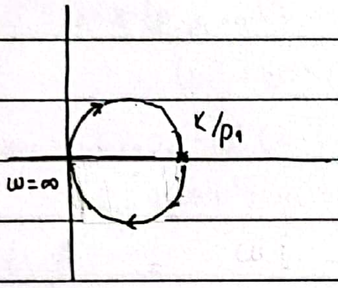
# Πολικά διαγράμματα




$$G(s)=\frac{K}{s+p_1},\quad G(j\omega)=\frac{K}{j\omega+p_1}=\frac{K(-j\omega+p_1)}{p_1^2+\omega^2}=\underbrace{\frac{Kp_1}{p_1^2+\omega^2}}_{X(j\omega)}-j\underbrace{\frac{K\omega}{p_1^2+\omega^2}}_{Y(j\omega)}$$

$K, p_1>0$

$$=\frac{K}{\sqrt{p_1^2+\omega^2}}\,e^{-j\tan(\omega/p_1)}$$




$$X^2(j\omega)+Y^2(\omega)=\frac{K^2p_1^2+K^2\omega^2}{(p_1^2+\omega^2)^2}=\frac{K}{p_1}\underbrace{\left|\frac{Kp_1}{p_1^2+\omega^2}\right|}_{X(j\omega)}$$

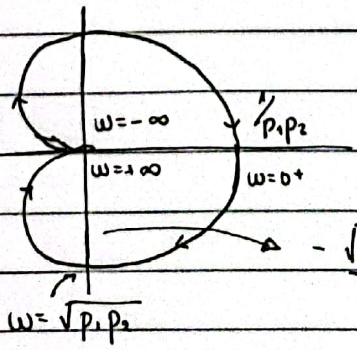
$$X^2+Y^2=\frac{K}{p_1}X \Rightarrow X^2-\frac{2K}{2p_1}X+\frac{K^2}{4p_1^2}+Y^2=\frac{K^2}{4p_1^2} \Rightarrow$$

$$\Rightarrow \left(X-\frac{K}{2p_1}\right)^2+Y^2=\left(\frac{K}{2p_1}\right)^2$$

$$G(s)=\frac{1}{(s+p_1)(s+p_2)},\quad p_1,p_2>0$$

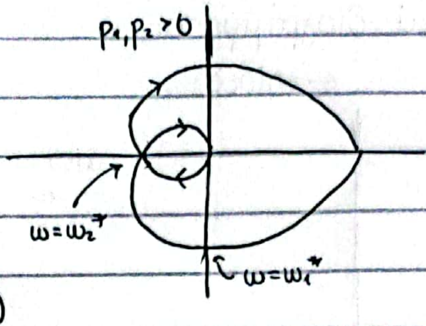
$$G(j\omega)=\frac{1}{(j\omega+p_1)(j\omega+p_2)}=\frac{(-j\omega+p_1)(-j\omega+p_2)}{(\omega^2+p_1^2)(\omega^2+p_2^2)}=\frac{p_1p_2-\omega^2}{(\omega^2+p_1^2)(\omega^2+p_2^2)}-j\omega\frac{p_1+p_2}{(\omega^2+p_1^2)(\omega^2+p_2^2)}$$

$$G(j\omega)=\frac{1}{\sqrt{(\omega^2+p_1^2)(\omega^2+p_2^2)}}\angle -\tan^{-1}\left(\frac{\omega}{p_1}\right)-\tan^{-1}\left(\frac{\omega}{p_2}\right)$$




$$-\frac{\sqrt{p_1p_2}\,(p_1+p_2)}{p_1p_2\,(p_1+p_2)^2}=-\frac{1}{\sqrt{p_1p_2}\,(p_1+p_2)}$$

$$G(s) = \frac{1}{(s+p_1)(s+p_2)(s+p_3)}$$




$$G(j\omega) = \frac{1}{(j\omega+p_1)(j\omega+p_2)(j\omega+p_3)}$$

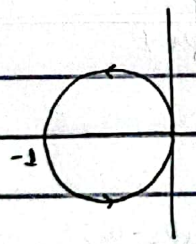
$$= \frac{[(p_1p_2 - \omega^2) - j\omega(p_1+p_2)](-j\omega+p_3)}{(\omega^2+p_1^2)(\omega^2+p_2^2)(\omega^2+p_3^2)}$$

$$= \frac{p_1p_2p_3 - \omega^2(p_1+p_2+p_3)}{(\omega^2+p_1^2)(\omega^2+p_2^2)(\omega^2+p_3^2)} - j\omega\frac{p_1p_3+p_2p_3+p_1p_2 - \omega^2}{(\omega^2+p_1^2)(\omega^2+p_2^2)(\omega^2+p_3^2)}$$

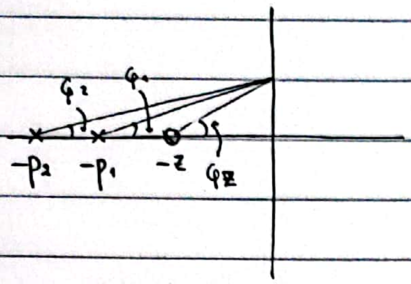
$$\omega_1^* = \sqrt{\frac{p_1p_2p_3}{p_1+p_2+p_3}}, \quad \omega_2 = \sqrt{p_1p_2+p_1p_3+p_2p_3}$$

$$G(s) = \frac{1}{s-1}$$

$$G(j\omega) = \frac{1}{-1+j\omega} = \frac{-1-j\omega}{1+\omega^2} = -\frac{1}{1+\omega^2} - \frac{j\omega}{1+\omega^2} = \frac{1}{\sqrt{1+\omega^2}}$$



$$G(s) = \frac{s+z}{(s+p_1)(s+p_2)}$$




$$\arg G(j\omega) = \phi_z - \phi_1 - \phi_2$$

$$G(j\omega) = \frac{z+j\omega}{(j\omega+p_1)(j\omega+p_2)} = \frac{(z+j\omega)(p_1-j\omega)(p_2-j\omega)}{(p_1^2+\omega^2)(p_2^2+\omega^2)} =$$

$$= \frac{z(p_1p_2-\omega^2)+\omega^2(p_1+p_2)}{(p_1^2+\omega^2)(p_2^2+\omega^2)} + j\omega\,\frac{p_1p_2-\omega^2-z(p_1+p_2)}{(p_1^2+\omega^2)(p_2^2+\omega^2)}$$

$$G(j\omega) = \frac{zp_1p_2-\omega^2(z-p_1-p_2)}{(p_1^2+\omega^2)(p_2^2+\omega^2)} + j\omega\,\frac{p_1p_2-z(p_1+p_2)-\omega^2}{(p_1^2+\omega^2)(p_2^2+\omega^2)}$$

$$z < \frac{p_1p_2}{p_1+p_2}$$

$$z > p_1+p_2$$

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