4η Σειρά Ασκήσεων Φυσικής Ι 1ου Εξαμήνου

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1 α)

$\frac{dr}{dt} = \lambda r \Rightarrow \frac{1}{r}dr = \lambda \cdot dt \Rightarrow \int_a^r \frac{1}{r}dr = \int_0^t \lambda \cdot dt \Rightarrow$

$\ln r - \ln a = \lambda t \Rightarrow \ln\left(\frac{r}{a}\right) = \lambda t \Rightarrow \frac{r}{a} = e^{\lambda t} \Rightarrow$

$r(t) = a e^{\lambda t}$ (1)

Η πυκνότητα της σταγόνας (έστω ρ) είναι σταθερή και ίση με $\rho = \frac{m}{V} \Rightarrow m = \rho \cdot V$

Ο όγκος μιας σφαίρας δίνεται από τον τύπο $V = \frac{4\pi r^3}{3}$, όπου r η ακτίνα της σφαίρας. Από αυτές τις δύο σχέσεις έχουμε πως

$m = \rho \cdot \frac{4\pi}{3} r^3 \overset{(1)}{=} \frac{4\pi\rho a e^{3\lambda t}}{3}$

Για $t=0$, $m(0) = m_0 = \frac{4\pi\rho a}{3}$. Άρα:

$\boxed{m(t) = m_0 e^{3\lambda t}}$

Επιπλέον, $m'(t) = \boxed{\frac{dm}{dt} = 3m_0\lambda e^{3\lambda t}}$

β)

$\frac{dP}{dt} = \Sigma F = mg \Rightarrow d(mv) = mg\,dt \Rightarrow m \cdot dv + v\,dm = mg\,dt \Rightarrow$

$\overset{(a)}{\Longrightarrow} m_0 \cdot e^{3\lambda t} \cdot dv + v \cdot 3m_0\lambda \cdot e^{3\lambda t} dt = m_0 e^{3\lambda t} g\,dt \Rightarrow$

$dv + 3\lambda v \cdot dt = g\,dt \Rightarrow \frac{dv}{dt} + 3\lambda v = g \Rightarrow \frac{dv}{dt} = g - 3\lambda v \Rightarrow$

$$\Rightarrow \frac{1}{g-3\lambda v}\,dv = dt \Rightarrow \int_0^v \frac{1}{g-3\lambda v}\,dv = \int_0^t dt \Rightarrow$$

$$\Rightarrow \frac{\ln(g-3\lambda v) - \ln g}{-3\lambda} = t \Rightarrow \ln\left(1 - \frac{3\lambda}{g}v\right) = -3\lambda t \Rightarrow$$

$$1 - \frac{3\lambda}{g}v = e^{-3\lambda t} \Rightarrow -\frac{3\lambda}{g}v = e^{-3\lambda t} - 1 \Rightarrow$$

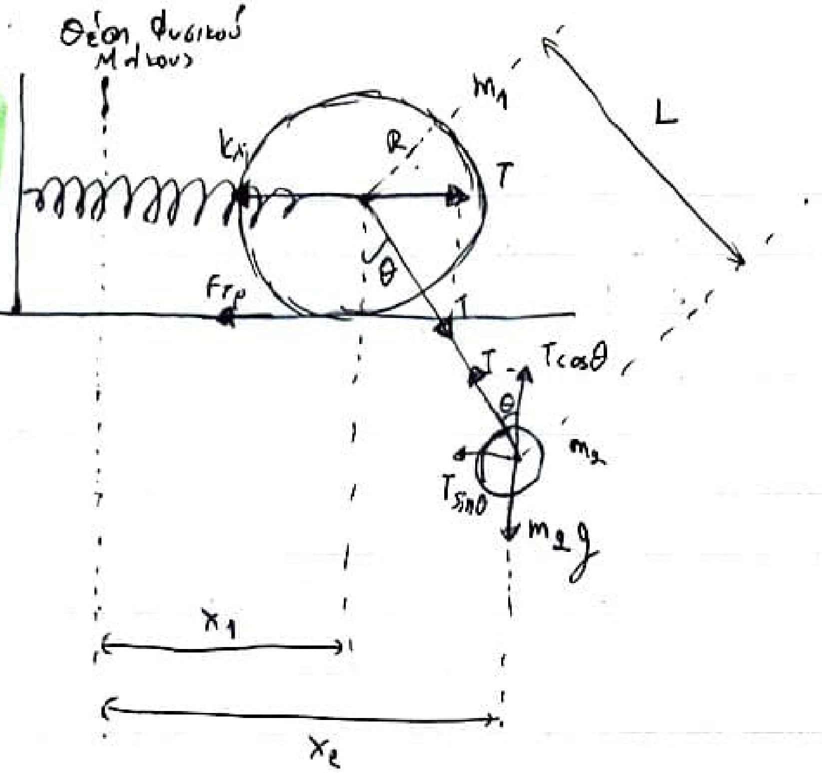
$$\boxed{v(t) = \frac{g}{3\lambda}\left(1 - e^{-3\lambda t}\right)}$$

Επιπλέον, $\lim\limits_{t\to+\infty} v(t) = \lim\limits_{t\to+\infty}\left[\frac{g}{3\lambda}\left(1-e^{-3\lambda t}\right)\right] = \frac{g}{3\lambda}(1-0) \Rightarrow$

$$\boxed{\lim_{t\to+\infty} v(t) = \frac{g}{3\lambda}}$$

2

a)



$$\tan\theta = \frac{x_2 - x_1}{L} \Rightarrow \tan\theta \approx \sin\theta = \frac{x_2 - x_1}{L} \quad (1)$$

$$\Sigma F_1 = m_1 a_1 = T\sin\theta - kx_1 - F_{T\rho} \quad (2)$$

$$\Sigma \tau_1 = I_1 \, a_{\gamma\omega\nu,1} = F_{T\rho} R \Rightarrow m_1 R^2 \cdot a_{\gamma\omega\nu,1} = F_{T\rho} R \xrightarrow{a_1 = a_{\gamma\omega\nu,1} R}$$

$$m_1 R \cdot \frac{a_1}{R} = F_{T\rho} \Rightarrow m_1 a_1 = F_{T\rho} \quad (3)$$

$$(2),(3) \Rightarrow m_1 a_1 = T\overset{(1)}{\sin\theta} - k \cdot x_1 - m_1 a_1 \Rightarrow 2m_1 x_1'' = T\,\frac{(x_1 - x_2)}{L} - kx_1 \Rightarrow$$

$$x_1'' + \frac{T}{2m_1 L} x_1 - \frac{T}{2m_1 L} x_2 + \frac{k}{2m_1} x_1 = 0 \quad (4)$$

$$\left.\begin{aligned} &\Sigma F_{2x} = m_2 a_2 = -T\sin\theta \\ &\Sigma F_{2y} = 0 \Rightarrow T\cos\theta = m_2 g \xrightarrow{\cos\theta \approx 1} T = m_2 g \ (5) \end{aligned}\right\} \Rightarrow m_2 a_2 = -m_2 g \overset{(1)}{\sin\theta} \Rightarrow$$

$$a_2 = -g\,\frac{(x_2 - x_1)}{L} \Rightarrow \boxed{x_2'' + \frac{g}{L} x_2 - \frac{g}{L} x_1 = 0} \quad (6)$$

$$(4),(5) \Rightarrow \boxed{x_1'' + \frac{m_2 g}{2m_1 L} x_1 - \frac{m_2 g}{2m_1 L} x_2 + \frac{k}{2m_1} x_1 = 0} \quad (7)$$

B) • Από (7) και $m_2 = 2m_1$, $\frac{g}{L} = \omega_0^2$, $\frac{k}{2m_1} = \omega_0^2$ έχουμε:

$$x_1'' + \omega_0^2 x_1 - \omega_0^2 x_2 + \omega_0^2 x_1 = 0 \Rightarrow$$

$$x_1'' + 2\omega_0^2 x_1 - \omega_0^2 x_2 = 0 \quad (8)$$

• Από (6) και $\frac{g}{L} = \omega_0^2$ έχουμε:

$$x_2'' + \omega_0^2 x_2 - \omega_0^2 x_1 = 0 \quad (9)$$

• $x_1 = A\cos(\omega t + \varphi) \Rightarrow x_1' = -A\cdot\omega \sin(\omega t + \varphi) \Rightarrow$

$$\left.\begin{aligned} &x_1'' = -A\omega^2 \cos(\omega t + \varphi) \\ &\text{Ομοίως, } x_2'' = -B\omega^2 \cos(\omega t + \varphi) \end{aligned}\right\} (10)$$

$$(8),(9),(10) \Big\} \Rightarrow \left.\begin{aligned} &-A\omega^2 \cos(\omega t + \varphi) + 2A\omega_0^2 \cos(\omega t + \varphi) - B\omega_0^2 \cos(\omega t + \varphi) = 0 \\ &\text{και} \\ &-B\omega^2 \cos(\omega t + \varphi) + B\omega_0^2 \cos(\omega t + \varphi) - A\omega_0^2 \cos(\omega t + \varphi) = 0 \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} &-\omega^2 A + 2A\omega_0^2 - B\omega_0^2 = \\ &-B\omega^2 + B\omega_0^2 - A\omega_0^2 = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} &(2\omega_0^2 - \omega^2)A - \omega_0^2 B = 0 \\ &-\omega_0^2 A + (\omega_0^2 - \omega^2)B = 0 \end{aligned}$$

$$\Big\} \Rightarrow \begin{vmatrix} 2\omega_0^2 - \omega^2 & -\omega_0^2 \\ -\omega_0^2 & \omega_0^2 - \omega^2 \end{vmatrix} = 0 \Rightarrow 2\omega_0^4 - \omega_0^2\omega^2 - 2\omega_0^2\omega^2 + \omega^4 - \omega_0^4 = 0 \Rightarrow$$

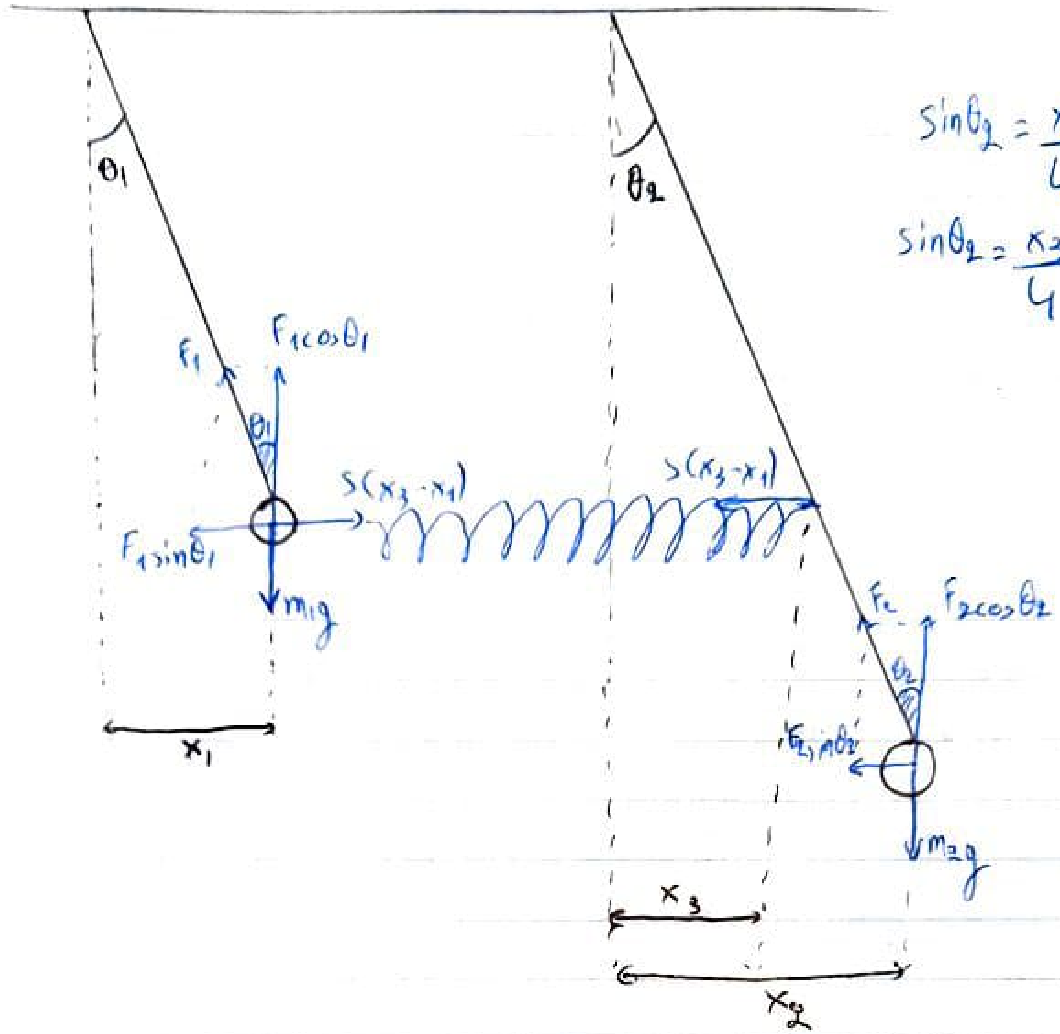
$\Rightarrow \omega^4 - 3\omega_0^2\omega^2 + \omega_0^4 = 0$. Θέτοντας $x = \omega^2$, $y = \omega_0^2$ έχουμε:
$x^2 - 3xy + y^2 = 0$. Αν το αντιμετωπίσουμε ως πολυώνυμο ως προς το:

$\Delta = 9y^2 - 4y^2 = 5y^2 > 0$. $x_{1,2} = \dfrac{3y \pm \sqrt{5}\,y}{2} \Rightarrow$

$\boxed{\omega_{1,2}^2 = \dfrac{3 \pm \sqrt{5}}{2}\omega_0^2} \Rightarrow \boxed{\omega_1 = \sqrt{\dfrac{3+\sqrt{5}}{2}}\,\omega_0}$, $\boxed{\omega_2 = \sqrt{\dfrac{3-\sqrt{5}}{2}}\,\omega_0}$

③

a)



$$\left.\begin{aligned}\sin\theta_2 &= \frac{x_2}{L_2}\\ \sin\theta_2 &= \frac{x_3}{L_1}\end{aligned}\right\} \Rightarrow x_3 = \frac{L_1}{L_2}x_2 \quad (1)$$

■ $\Sigma_{c_1} = I_1\, a_{\gamma w_1} = -m_1 g x_1 + s(x_3 - x_1)\cos\theta_1 L_1 \xrightarrow{\cos\theta_1 \approx 1}$

$$m_1 L_1^2 \frac{a_1}{L_1} = -m_1 g x_1 - s x_1 L_1 + s x_3 L_1 \xrightarrow{(1)}$$

$$m_1 L_1 a_1 = -m_1 g x_1 - s x_1 L_1 + \frac{s L_1^2}{L_2} x_2 \Rightarrow$$

$$\boxed{\ddot{x}_1 + \frac{g}{L_1}x_1 + \frac{s}{m_1}x_1 - \frac{s L_1}{m_1 L_2}x_2 = 0} \quad (2)$$

■ $\Sigma_{c_2} = I_2\, a_{\gamma w_2} = -m_2 g x_2 - s(x_3 - x_1)\cos\theta_2 L_1 \xrightarrow{\cos\theta_2 \approx 1}$

$$m_2 L_2^2 \frac{a_2}{L_2} = -m_2 g x_2 + s x_1 L_1 - s x_3 L_1 \xrightarrow{(1)}$$

$$m_2 L_2 a_2 = -m_2 g x_2 + s L_1 x_1 - \frac{s L_1^2}{L_2} x_2 \Rightarrow$$

$$\boxed{\ddot{x}_2 + \frac{g}{L_2} x_2 - \frac{s L_1}{m_2 L_2} x_1 + \frac{s L_1^2}{m_2 L_2^2} x_2 = 0} \quad (3)$$

β) $x_1 = A\sin(\omega t + \varphi) \Rightarrow \dot{x}_1 = A\omega \sin(\omega t + \varphi) \Rightarrow \ddot{x}_1 = -A\omega^2 \sin(\omega t + \varphi)$

Αντίστοιχα, $\ddot{x}_2 = -B\omega^2 \sin(\omega t + \varphi)$

$$(2) \Rightarrow -A\omega^2 \sin(\omega t + \varphi) + \frac{g}{L_1} A\sin(\omega t + \varphi) + \frac{s}{m_1} A\sin(\omega t + \varphi) -$$

$$- \frac{s L_1}{m_1 L_2} B\sin(\omega t + \varphi) = 0 \Rightarrow$$

$$\boxed{\left(-\omega^2 + \frac{g}{L_1} + \frac{s}{m_1}\right) A - \frac{s L_1}{m_1 L_2} B = 0} \quad (4)$$

$$(3) \Rightarrow -B\omega^2 \sin(\omega t + \varphi) + \frac{g}{L_2} B\sin(\omega t + \varphi) - \frac{s L_1}{m_2 l_2} A\sin(\omega t + \varphi) +$$

$$+ \frac{s L_1^2}{m_2 l_2^2} B\sin(\omega t + \varphi) = 0 \Rightarrow$$

$$\left(-\omega^2 + \frac{g}{L_2} + \frac{s L_1^2}{m_2 L_2^2}\right) B - \frac{s L_1}{m_2 L_2} A = 0 \Rightarrow$$

$$\boxed{-\frac{s L_1}{m_2 L_2} A + \left(-\omega^2 + \frac{g}{L_2} + \frac{s L_1^2}{m_2 L_2^2}\right) B = 0} \quad (5)$$

γ) ■ (4) $\Rightarrow (-\omega^2 + \omega_0^2 + \omega_0^2)A - \frac{\omega_0^2}{2}B = 0 \Rightarrow$

$$(-\omega^2 + 2\omega_0^2)A - \frac{\omega_0^2}{2}B = 0 \qquad (6)$$

■ (5) $\Rightarrow \frac{-s l_1}{\frac{m_1}{2} 2l_1} A + \left(-\omega^2 + \frac{g}{2l_1} + \frac{s l_1^2}{\frac{m_1}{2} 4l_1^2}\right) B = 0 \Rightarrow$

$$-\omega_0^2 A + \left(-\omega^2 + \frac{\omega_0^2}{2} + \frac{\omega_0^2}{2}\right) B = 0 \Rightarrow$$

$$-\omega_0^2 A + (-\omega^2 + \omega_0^2) B = 0 \qquad (7)$$

(6), (7) $\Rightarrow \begin{vmatrix} -\omega^2 + 2\omega_0^2 & -\omega_0^2/2 \\ -\omega_0^2 & -\omega^2 + \omega_0^2 \end{vmatrix} = 0 \Rightarrow$

$$\omega^4 - \omega^2\omega_0^2 - 2\omega_0^2\omega^2 + 2\omega_0^4 - \frac{\omega_0^4}{2} = 0 \Rightarrow$$

$$\omega^4 - 3\omega^2\omega_0^2 + \frac{3}{4}\omega_0^4 = 0$$

Αν θέσουμε $x = \omega^2$, $y = \omega_0^2$ τότε $x^2 - 3xy + \frac{3}{4}y^2 = 0$,

$\Delta = 9y^2 - 3y^2 = 6y^2 > 0$, $x_{1,2} = \frac{3y \pm \sqrt{6}\, y}{2} \Rightarrow$

$$\boxed{\omega_1^2 = \frac{3+\sqrt{6}}{2}\omega_0^2}, \quad \boxed{\omega_2^2 = \frac{3-\sqrt{6}}{2}\omega_0^2}$$

④

α) Αν θεωρήσουμε μια ελκτική δύναμη (κεντρική) $\vec{F}(\vec{r}) = -f(r)\hat{r}$, $[f(r) > 0]$ για ένα σώμα μάζας m, όπου $\vec{r}$ η διανυσματική απόσταση από κέντρο έλξης.

$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{r} \times (f(r)\hat{r}) = 0 \Rightarrow$ η στροφορμή διατηρείται σταθερή και είναι ίση με την στροφορμή του δορυφόρου: $L = L_{δορ} = m r_0^2 \omega = m \cdot r_0^2 \cdot \frac{V_0}{r_0} = \boxed{m V_0 \cdot r_0}$

β) Από την έκφραση της ταχύτητας για πολικές συντεταγμένες $\dot{\vec{r}} = \dot{r} \cdot \hat{r} + \dot{\theta} r \cdot \hat{\theta}$ όπου $\hat{r} \perp \hat{\theta}$ έχουμε:

$E_K = \frac{1}{2} m \dot{\vec{r}}^2 = \frac{1}{2} m (\dot{r}^2 + \dot{\theta}^2 r^2)$

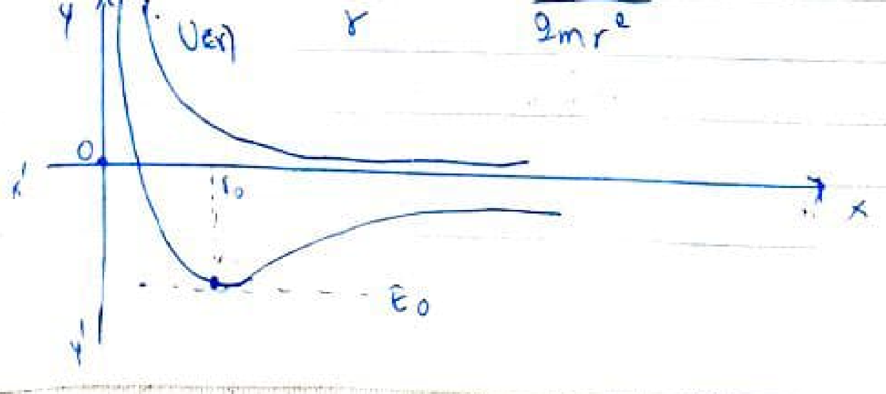
Η ολική ενέργεια είναι $E_{ολ} = U(r) + E_K$ αλλά σε επίπεδες πολικές συντεταγμένες $\vec{L} = \vec{r} \times m \cdot \dot{\vec{r}} = r \cdot \hat{r} \times m(\dot{r}\hat{r} + r \cdot \dot{\theta}\hat{\theta}) = m r^2 \dot{\theta} \hat{z} \Rightarrow \dot{\theta}^2 = \frac{L^2}{m^2 r^4}$ άρα

$$E_{ολ} = U(r) + \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m \frac{L^2}{m^2 r^4} r^2 \Rightarrow$$

$$\boxed{E_{ολ} = U(r) + \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2 m r^2}}$$

Αν ορίσουμε μια συνάρτηση ενεργούς δυναμικής ενέργειας:

$U_{εξ}(r) = -\frac{GMm}{r} + \frac{L^2}{2 m r^2}$

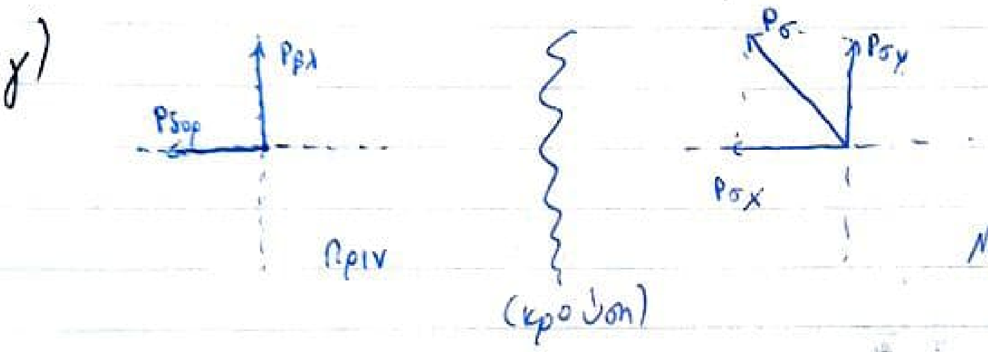


Αν λοιπόν η ακτίνα είναι ίση με r_0, η συνολική τιμή της ενέργειας E_0 είναι ίση με το ελάχιστο της ενεργούς δυναμικής ενέργειας. Άρα το σώμα εκτελεί κυκλική τροχιά.

Αν η ολική ενέργεια $E_{ολ}$ είναι αρνητική αλλά $E_{ολ} > E_0$ υπάρχουν δύο τιμές της ακτίνας r που την ικανοποιούν. Άρα το σώμα εκτελεί ελλειπτική τροχιά.

Αν $E_{ολ} > 0$ το σώμα εκτελεί ανοικτή τροχιά.

γ)



$$ΑΔΟ: \vec{P}_{βλ} + \vec{P}_{δορ} = \vec{P}_σ \Rightarrow (3m V_0)^2 + (mV_0)^2 = (2mV)^2 \Rightarrow$$

$$9m^2V_0^2 + m^2V_0^2 = 4m^2 v^2 \Rightarrow v^2 = \frac{10}{4} v_0^2 \quad (1)$$

Για τον δορυφόρο πριν την κρούση:

$$F_κ = m \cdot a_κ \Rightarrow \frac{GM_Γ m}{r_0^2} = \frac{m V_0^2}{r_0} \Rightarrow r_0 = \frac{GM_Γ}{V_0^2} \quad (2)$$

$$E_{αρχ} = \frac{1}{2} 2m \cdot v^2 + \frac{m^2 V_0^2 r_0^2}{2 \cdot 2m r_0^2} - \frac{GM_Γ 2m}{r_0} \xrightarrow[(2)]{(1)}$$

$$= m \cdot \frac{10}{4} v_0^2 + \frac{mV_0^2}{4} - \frac{GM_Γ \cdot 2m}{\frac{GM_Γ}{V_0^2}} = \frac{11}{4} V_0^2 m - 2mV_0^2 = \frac{3}{4} m V_0^2$$

ΑΔΕ: $E_{\alpha\rho\chi} = \overset{\infty}{E_{\tau\varepsilon\lambda}} \Rightarrow E_{\alpha\rho\chi} = \frac{1}{2}2mV_{\tau\infty}^2 + \cancel{\frac{L}{2\cdot 2mr^2}}^{\,0\,(r\to\infty)} - \cancel{\frac{GM_\Gamma\cdot 2m}{r}}^{\,0\,(r\to\infty)} \Rightarrow$

$\frac{3}{4}mV_0^2 = mV_{\tau\infty}^2 \Rightarrow \boxed{V_{\tau\infty} = \frac{\sqrt{3}}{2}V_0}$

Άρα η ταχύτητα $3v_0$ του βλήματος είναι αρκετή ώστε το συσσωμάτωμα να διαφύγει από το πεδίο βαρύτητας της γης.

δ) ΑΔΕ: $E_{\alpha\rho\chi} = \cancel{E_{\tau\varepsilon\lambda}}^{\,0} \Rightarrow \frac{1}{2}2mV_\sigma^2 - \frac{GM_\Gamma 2m}{r_0} = 0 \Rightarrow$

$V_\sigma^2 = \frac{GM_\Gamma 2}{r_0} = \frac{GM_\Gamma 2}{\frac{GM_\Gamma}{V_0^2}} = 2V_0^2 \Rightarrow V_\sigma = \sqrt{2}V_0 \quad (1)$

ΑΔΟ: $P_{\alpha\rho\chi} = P_{\tau\varepsilon\lambda} \Rightarrow \sqrt{P'^2_{\beta\lambda} + P^2_{\delta o\rho}} = P_{\sigma\upsilon\sigma} \overset{(1)}{\Rightarrow}$

$(mV'_{\beta\lambda})^2 + (mV_0)^2 = (2m\cdot\sqrt{2}v_0)^2 \Rightarrow m^2V'^2_{\beta\lambda} + m^2V_0^2 = 8m^2V_0^2 \Rightarrow$

$V'_{\beta\lambda} = \sqrt{7}V_0 \quad (2)$

ΑΔΜΕ: $E_{\alpha\rho\chi} = E_{\tau\varepsilon\lambda} \Rightarrow \frac{1}{2}mV_{\alpha\rho\chi}^2 - \frac{GM_\Gamma m}{r_{\alpha\rho\chi}} = \frac{1}{2}mV'^2_{\beta\lambda} - \frac{GM_\Gamma m}{r_0 + r_{\alpha\rho\chi}} \Rightarrow$

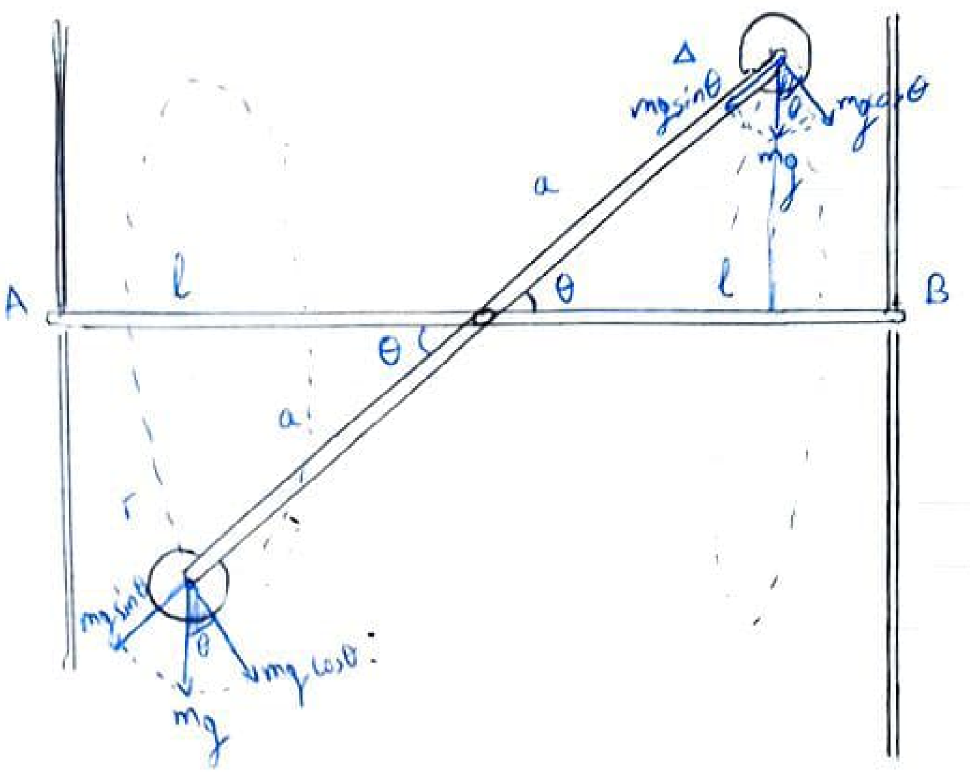
$V_{\alpha\rho\chi}^2 = V'^2_{\beta\lambda} - 2GM_\Gamma\left(\frac{1}{r_0 + r_{\alpha\rho\chi}} - \frac{1}{r_{\alpha\rho\chi}}\right) \Rightarrow$

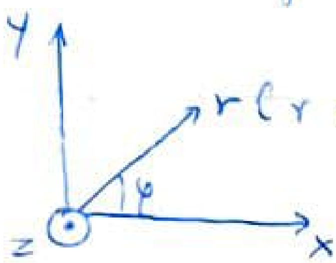
$V_{\alpha\rho\chi}^2 = V'^2_{\beta\lambda} + \frac{2GM_\Gamma r_0}{r_0 + r_{\alpha\rho\chi}} \xrightarrow[r_{\alpha\rho\chi} + r_0 \approx r_0]{r_0 \gg r_{\alpha\rho\chi} \Rightarrow} V_{\alpha\rho\chi}^2 = V'^2_{\beta\lambda} + 2\cdot M_\Gamma \xrightarrow[(2)]{r_0 = \frac{GM_\Gamma}{V_0^2}}$

$V_{\alpha\rho\chi} = \boxed{V = \sqrt{7V_0^2 + 2V_0^2 r_0}}$

⑤

a)





$r(r\cos\varphi, r\sin\varphi, a\sin\theta)$ με $\varphi = \omega t$

$$\vec{r}(t) = (r\cos(\omega t), r\sin(\omega t), a\sin\theta)$$

$$\dot{\vec{r}}(t) = (-r\omega\sin(\omega t), r\omega\cos(\omega t), 0)$$

$$\vec{L}(t) = 2m \cdot \vec{r}(t) \times \dot{\vec{r}}(t) = 2m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r\cos\omega t & r\sin\omega t & a\sin\theta \\ -r\omega\sin\omega t & r\omega\cos\omega t & 0 \end{vmatrix} =$$

$$= 2m \left[\hat{x} \begin{vmatrix} r\sin\omega t & a\sin\theta \\ r\omega\cos\omega t & 0 \end{vmatrix} - \hat{y} \begin{vmatrix} r\cos\omega t & a\sin\theta \\ -r\omega\sin\omega t & 0 \end{vmatrix} + \right.$$

$$\left. + \hat{z} \begin{vmatrix} r\cos\omega t & r\sin\omega t \\ -r\omega\sin\omega t & r\omega\cos\omega t \end{vmatrix} \right] \Rightarrow$$

$$\boxed{\vec{L}(t) = 2m \left[\hat{x}(-r\omega a\sin\theta\cos\omega t) - \hat{y}(r\omega a\sin\theta\sin\omega t) + \hat{z}(r^2\omega) \right]}$$

β) $\frac{d\vec{L}}{dt} = 2m\left[\hat{x}\,(r\omega^2 a \sin\theta \sin\omega t) - \hat{y}\,(r\omega^2 a \sin\theta \cos\omega t)\right]$

$$\vec{\omega} \times \vec{L} = 2m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ -r\omega a \sin\theta \cos\omega t & -r\omega a \sin\theta \sin\omega t & r^2\omega \end{vmatrix} =$$

$$= 2m\left[\hat{x}\,(r\omega^2 a \sin\theta \sin\omega t) - \hat{y}\,(r\omega^2 a \cos\theta \cos\omega t)\right] = \frac{d\vec{L}}{dt}$$

γ) Για το ένα σώμα ισχύει: $\Sigma F = \vec{F}(t) = m \cdot \ddot{\vec{r}}(t)$

$\ddot{\vec{r}}(t) = (-r\omega^2 \cos(\omega t), -r\omega^2 \sin(\omega t), 0)$ $\Rightarrow$

$$\vec{F}(t) = m \cdot (-r\omega^2 \cos(\omega t), -r\omega^2 \sin(\omega t), 0) =$$

$$= (-r\omega^2 m \cos\omega t, -r\omega^2 m \sin\omega t, 0) \Rightarrow$$

$$|\vec{F}| = \sqrt{r^2\omega^4 m^2 \cos^2\omega t + r^2\omega^4 m^2 \sin^2\omega t} = r\omega^2 m \Rightarrow$$

$$\boxed{|\vec{F}| = r\omega^2 m}$$