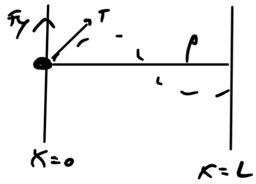
Για ιδανική χορδή (ρ, T): $T\frac{\partial^2 y}{\partial x^2} = \rho \frac{\partial^2 y}{\partial t^2}$, $c = \sqrt{\frac{T}{\rho}} = \frac{k}{\omega}$

↳ Λύση: $y(x,t) = \underbrace{A\sin(kx+\varphi)}_{f(x)}\cos(\omega t)$



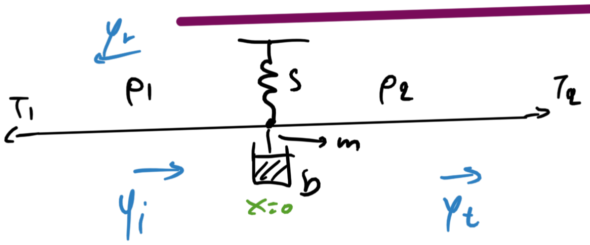
Ακλόνητο σημείο: $y(x_0,t)=0 \Rightarrow f(x_0)=0 \Rightarrow \varphi = -kx_0$

↳ εδώ $x_0 = L$

$F_y = F_0\cos(\omega t)$

2ος ν. N:

$$m\left(\frac{\partial^2 y}{\partial t^2}\right)_{x=0} = T\left(\frac{\partial y}{\partial x}\right)_{x=0} - F_y$$



Προσπίπτον: $y_i = Ae^{i(k_1x-\omega t)}$

Ανακλώμενο: $y_r = Be^{i(-k_1x-\omega t)}$

Διερχόμενο: $y_t = Ce^{i(k_2x-\omega t)}$

Συνθήκη στην ασυνέχεια $x=0$

$(y_i+y_r)_{x=0} = (y_t)_{x=0} \Rightarrow \dots \Rightarrow A+B=C \Rightarrow 1+r=t$

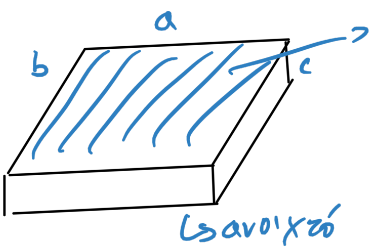
$k_1 = \frac{\omega}{c_1} = \frac{\omega}{\sqrt{\frac{T_1}{\rho_1}}}$

$k_2 = \frac{\omega}{c_2} = \frac{\omega}{\sqrt{\frac{T_2}{\rho_2}}}$

2ος ν. N.

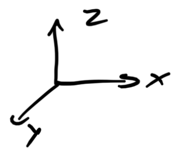
$$m\left(\frac{\partial^2 y_t}{\partial t^2}\right)_{x=0} = T_2\left(\frac{\partial y_t}{\partial x}\right)_{x=0} - T_1\left(\frac{\partial(y_i+y_r)}{\partial x}\right)_{x=0} - s(y_t)_{x=0} - b\left(\frac{\partial y_t}{\partial t}\right)_{x=0}$$

$r = \frac{B}{A}$ $t = \frac{C}{A}$



→ μεμβράνη, m , $\sigma = \frac{m}{a\cdot b}$

↳ ανοιχτό



Μεμβράνη

Θεωρούμε λύση: $z(x,y,t) = \underbrace{A\sin(k_x\cdot x+\theta)}_{X(x)}\cdot\underbrace{B\sin(k_y\cdot y+\varphi)}_{Y(y)}\cos(\omega t)$

Οριακές συνθήκες:

$\begin{cases} X(x=0)=0 \Rightarrow \theta=0 \\ Y(y=0)=0 \Rightarrow \varphi=0 \end{cases}$

$\left.\begin{array}{l} X(x=a)=0 \Rightarrow k_x = \frac{n\pi}{a} \\ Y(y=b)=0 \Rightarrow k_y = m\frac{\pi}{b} \end{array}\right\}$ ακλόνητα άκρα

$$\omega_\mu = c_\mu\sqrt{k_x^2+k_y^2} = c_\mu\pi\sqrt{\frac{n^2}{a^2}+\frac{m^2}{b^2}}$$

Αντηχείο

Θεωρούμε λύσεις: $f(x,y,z,t) = \underbrace{A\sin(k_x x+\theta)}_{X(x)}\ \underbrace{B\sin(k_y y+\varphi)}_{Y(y)}\ \underbrace{C\sin(k_z z+\psi)}_{Z(z)}\cos(\omega t)$

Οριακές Συνθήκες:

$X(x=0)=0 \Rightarrow \theta=0$ $\qquad X(x=a)=0 \Rightarrow k_x = n\dfrac{\pi}{a}$

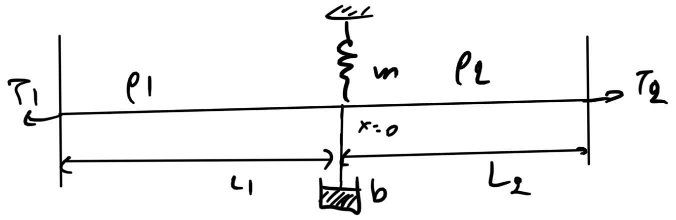
$Y(y=0)=0 \Rightarrow \varphi=0$ $\qquad Y(y=b)=0 \Rightarrow k_y = m\dfrac{\pi}{b}$ $\Big\}$ ακλόνητα άκρα

$Z(z=0)=0 \Rightarrow z=0$

$\left(\dfrac{\partial Z}{\partial z}\right)_{z=c} = 0 \Rightarrow k_z = \dfrac{(2l-1)\pi}{c}$ $\Big\}$ ελεύθερο άκρο

$$\omega_{\alpha\nu\tau} = c_{\eta\chi}\sqrt{k_x^2+k_y^2+k_z^2}$$

Δύναμη ανά μονάδα επιφάνειας: $T = c_\mu^2 \cdot \sigma$



Θεωρούμε λύσεις: $y_{1,2}(x) = f_{1,2}(x)\cos(\omega t)$

$f_{1,2}(x) = A_{1,2}\sin(k_{1,2}x + \varphi_{1,2})$

Οριακές συνθήκες

$f_1(-L_1)=0 \Rightarrow \varphi_1 = k_1 L_1$

$f_2(L_2)=0 \Rightarrow \varphi_2 = k_2 L_2$

$f_1(0)=f_2(0) \Rightarrow A_1\sin\varphi_1 = A_2\sin\varphi_2$

2ος ν.Ν.

$$m\left(\frac{\partial^2 y_1}{\partial t^2}\right)_{x=0} = T_2\left(\frac{\partial y_2}{\partial x}\right)_{x=0} - T_1\left(\frac{\partial y_1}{\partial x}\right)_{x=0} - s(y_1)_{x=0} - b\left(\frac{\partial y_1}{\partial x}\right)_{x=0}$$

$$\vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t} \qquad \vec\nabla\times\vec B = \mu_0\left(\vec j + \varepsilon_0\frac{\partial\vec E}{\partial t}\right) \qquad c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$\vec S = \frac{1}{\mu_0}\vec E\times\vec B \qquad \frac{dW}{dV}\cdot c_0 = |\vec S| \qquad k = \frac{\omega}{c} = \frac{2\pi f}{c}$$

$$\frac{dW}{dV} = \underbrace{\frac{1}{2}\varepsilon_0 E^2}_{\frac{dW_e}{dV}} + \underbrace{\frac{1}{2\mu_0} B^2}_{\frac{dW_m}{dV}}$$

Γραμμική: $\vec{E} = (\hat{x} E_0 \cos\theta + \hat{y} E_0 \sin\theta)\cos(kz-\omega t)$

Κυκλική: $\vec{E} = \hat{x} E_0 \cos(kz-\omega t) + \hat{y} E_0 \sin(kz-\omega t)$

Ελλειπτική: $\vec{E} = \hat{x} E_{0x} \cos(kz-\omega t) + \hat{y} E_{0y} \sin(kz-\omega t)$

Τυχαία: $\vec{E} = \left[\hat{x} E_0 \cos\underbrace{\varphi(t)}_{\text{τυχαία μεταβ. συνάρτ.}} + \hat{y} E_0 \sin\varphi(t)\right]\cos(kz-\omega t)$