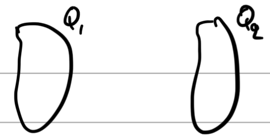
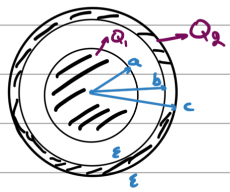
$$W_e = \frac{1}{2} \sum_i \phi_i Q_i$$

$$W_e = \frac{1}{2} p_{11} Q_1^2 + \frac{1}{2} p_{22} Q_2^2 + p_{12} Q_1 Q_2$$



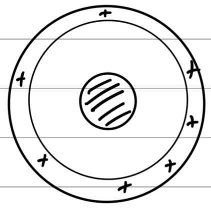
Παράδειγμα 1



Πορεία εργασίας: $\vec{E}, \phi, p_{ij}, c_{ij}, C_{ij}, W_e$

$$\tilde{P}\tilde{Q} = \tilde{\phi} \rightarrow \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

1. $Q_1 = 0 \neq Q_2$



$\vec{E}_1 = 0, \quad a < r < b$

$\vec{E}_2 = \dfrac{Q_2}{4\pi\varepsilon r^2}\hat{r}, \quad r > c$

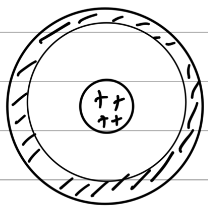
$\phi_2 = \displaystyle\int_c^\infty \vec{E}_2 \, d\vec{r} = \frac{Q_2}{4\pi\varepsilon c}, \quad \phi_1 = \phi_2$

(Αν η κοιλότητα έχει πεδίο 0 εσωτερικά, η κοιλότητα είναι ισοδυναμική)

$\phi_1 = p_{12} Q_2 \Rightarrow p_{12} = \dfrac{\phi_1}{Q_2} = \dfrac{1}{4\pi\varepsilon c}, \quad p_{22} = \dfrac{\phi_2}{Q_2} = p_{12}$

εξαρτώνται από τη γεωμετρία κ' το υλικό

2. $Q_1 \neq 0 = Q_2$



$\vec{E}_1 = \dfrac{Q_1}{4\pi\varepsilon r^2}\hat{r}, \quad a < r < b$

$\vec{E}_2 = \dfrac{Q_2}{4\pi\varepsilon r^2}\hat{r}$

$\phi_1 = \displaystyle\int_a^\infty \dots = \frac{Q_1}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right) \quad (a \to b, \; c \to \infty)$

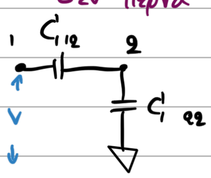
$\phi_2 = \displaystyle\int_c^\infty \dots = \frac{Q_1}{4\pi\varepsilon c}, \quad p_{11} = \frac{1}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right), \quad p_{21} = \frac{1}{4\pi\varepsilon c}$

$$\tilde{C} = \tilde{P}^{-1} = \frac{1}{|\tilde{P}|}\operatorname{adj}(\tilde{P}) \Rightarrow \tilde{C} = \frac{4\pi\varepsilon}{b-a}\begin{bmatrix} ab & -ab \\ -ab & bc - ac + ab \end{bmatrix}$$

$C_{ii} = \sum_j c_{ij}, \quad C_{ij} = -c_{ij}, \quad C_{ij} = C_{ji}$

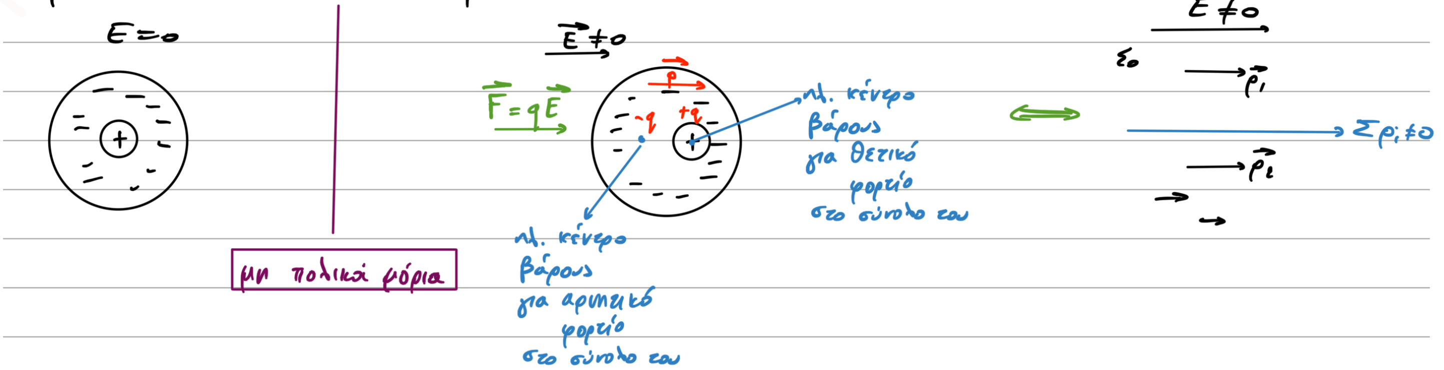
$C_{11} = 0, \quad C_{22} = 4\pi\varepsilon c, \quad C_{12} = C_{21} = \dfrac{4\pi\varepsilon ab}{b-a}$: εξαρτώνται μόνο από γεωμ. κ' υλικό

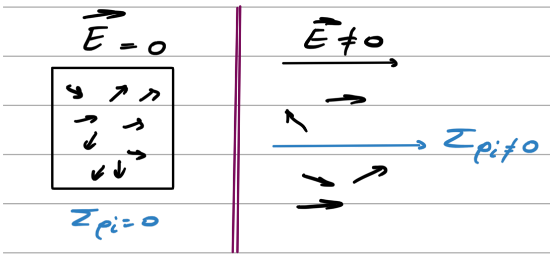
↳ θωρακισμένος, δεν περνά πεδίο, έχουμε ηλ. ροή $a \to b$ κ' $c \to \infty$

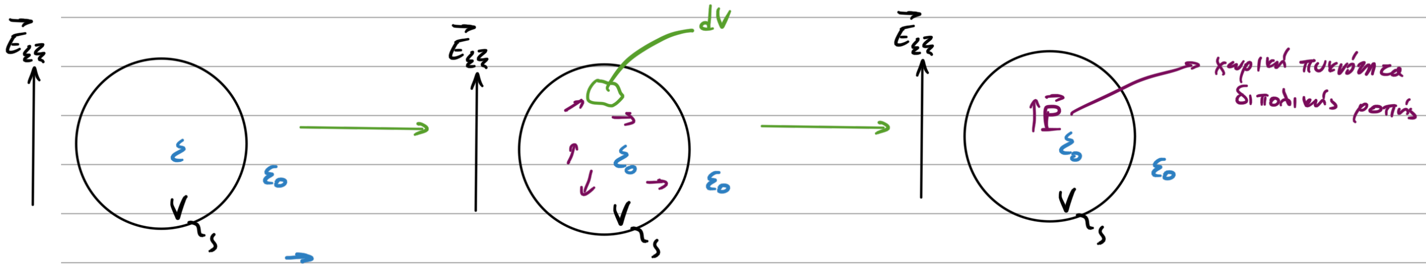


$$C_A = \frac{C_{12} C_{22}}{C_{12} + C_{22}}$$

Κεφάλαιο 2: Διηλεκτρικά υλικά





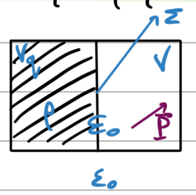


Ισχύει: $dp = \vec P \cdot dV$ — χωρ. πυκν. διπ. ροπής

διπολ. ροπή — $dq = \rho\, dV$

$$[\vec P] = \frac{C}{m^2} = [\vec D]$$

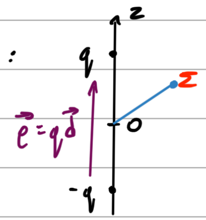
Ελεύθερα φορτία



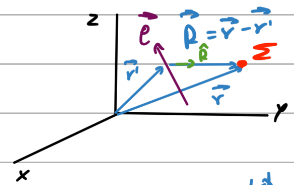
$$\phi = \phi_q + \phi_p \quad , \quad \vec E = -\nabla\phi_q - \nabla\phi_p$$

$$\phi_q = \frac{1}{4\pi\varepsilon_0}\int_{\Sigma o\tau\phi_q} \frac{dq'}{R} = \frac{1}{4\pi\varepsilon_0}\int_{V_q} \frac{\rho\, dV'}{R}$$

Θυμίζουμε:



$$\phi = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \xrightarrow{\text{Γενίκευση}}$$



$$\phi = \frac{\vec p \cdot \hat R}{4\pi\varepsilon_0 R^2}$$

$$\downarrow$$

$$d\phi = \frac{d\vec p \cdot \hat R}{4\pi\varepsilon_0 R^2} \to \phi = \int .$$

Άρα: $$\phi_p = \frac{1}{4\pi\varepsilon_0}\int_V \frac{d\vec p \cdot \hat R}{R^2} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec P \cdot \hat R}{R^2}\, dV'$$

Γενικώς: $\vec{p} = \vec{p}(\vec{E})$

Ειδική κατηγορία: Ηλεκτρίτης, $\vec{p}$ ανεξάρτητο του $\vec{E}$

Φορτία πόλωσης

$$\frac{\hat{R}}{R^2} = \nabla' \frac{1}{R} = -\nabla \frac{1}{R} \qquad \frac{\vec{p} \cdot \hat{R}}{R^2} = -\underbrace{\frac{\nabla' \hat{p}}{R}}_{\rightsquigarrow \int_V} + \underbrace{\nabla' \cdot \frac{\vec{p}}{R}}_{\rightsquigarrow \oint_S}$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

Θ. Απόκλισης: $\int_V \nabla \vec{A} \, dV = \oint_S \vec{A} \, d\vec{S}$

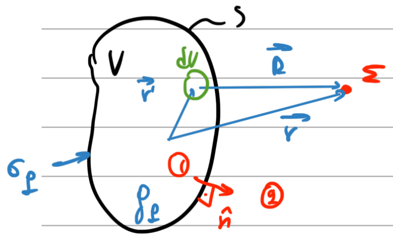
$$\nabla \cdot (f\vec{A}) = \nabla f \cdot \vec{A} + f \cdot \nabla \vec{A}$$

$$\Phi_p = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\boxed{-\nabla' \vec{p}}}{R} dV' + \oint_S \frac{\boxed{\vec{p} \, d\vec{S}}}{R}$$

$-\nabla' \vec{p} \to \rho_p$ χωρ. πυκν. πόλωσης

$\vec{p}\, d\vec{S} \to \boxed{\vec{p}\,\hat{n}}\, ds$

$\hookrightarrow \sigma_p = \hat{n}\,\vec{p}$ επιφ. πυκν. πόλωσης



$$\sigma = \hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = 0$$

$\updownarrow$ $D \rightsquigarrow -\vec{p}$

$$\sigma_p = \hat{n} \cdot (\vec{p}_1 - \vec{p}_2)$$

$$\Phi = \Phi_q + \Phi_p = \frac{1}{4\pi\varepsilon_0} \left[\int_{\text{ΣΟΤΦ}} \frac{dq'}{R} + \int_V \frac{\rho_p \, dV'}{R} + \oint_S \frac{\sigma_p \, dS'}{R} \right] \to \vec{E} = -\nabla\Phi$$

(ελ. φορτία)

$$Q_{p,tot} = \int_V \rho_p \, dV + \oint_S \sigma_p \, dV = 0$$