Πέμπτη, 20/10/2022

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A - \mathbb{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \\ -1 & -1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(A - \mathbb{I})^2 = \begin{bmatrix} & & 0 \\ & & -1 \\ & \text{\huge 0} & -1 \\ & & 0 \\ & & 1 \\ & & 0 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 1 & 0 & & & \\ 0 & 1 & 1 & & & \\ 0 & 0 & 1 & & & \\ & & & 1 & 1 & \\ & & & 0 & 1 & \\ & & & & & 1 \end{bmatrix}$$

$(A-\mathbb{I})u_1 = 0 \Rightarrow u_1 \in Ker(A-\mathbb{I})$

$(A-\mathbb{I})u_2 = u_1 \Rightarrow u_1 \in Im(A-\mathbb{I})$, $(A-\mathbb{I})^2 u_1 = 0 \Rightarrow u_2 \in Ker[(A-\mathbb{I})^2]$

$(A-\mathbb{I})u_3 = u_2 \Rightarrow (A-\mathbb{I})^2 u_2 = 0$

$(A-\mathbb{I})u_4 = 0$

$(A-\mathbb{I})u_5 = u_4$

$(A-\mathbb{I})u_6 = 0$

$Im(Q) = \{y \mid y = Qx\}$

$$u_1 = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad (A-\mathbb{I})x = 0 \Rightarrow \begin{cases} -x_2 - x_5 + x_6 = 0 \\ -x_1 - x_2 + x_4 - x_5 + x_6 = 0 \\ x_2 + x_5 = 0 \end{cases} \Rightarrow \begin{matrix} x_6 = 0 \\ x_1 = x_4 \\ x_2 = -x_5 \end{matrix}$$

$$x = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \alpha \\ -\beta \\ 0 \end{bmatrix}$$

$U_2 \in \mathrm{Im}(A-\mathbb{I}) \cap \mathrm{Ker}\left[(A-\mathbb{I})^2\right]$

$\hookrightarrow [\,*\ *\ *\ *\ *\ 0\,]^T$

$$U_2 = \alpha' \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \beta' \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \gamma' \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} 0 \\ \gamma'-\beta' \\ \alpha'-\beta'+\gamma' \\ 0 \\ \beta' \\ 0 \end{bmatrix}, \quad (A-\mathbb{I})U_2 = U_1 = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow$$

$\Rightarrow \beta' - \gamma' - \beta' = -1 \Rightarrow \gamma = 1$

$$U_2 = \begin{bmatrix} 0 \\ 1-\beta' \\ 1-\beta'+\alpha' \\ 0 \\ \beta' \\ 0 \end{bmatrix}$$

$(A-\mathbb{I})U_3 = U_2 \quad , \quad -U_{31} + U_{34} = \alpha'$

$U_{32}' + V_{35}' = \beta' \quad , \quad U_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} , U_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$V = \left[\begin{array}{ccc|cc|c} 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$(A - \mathbb{I})v_5 = v_4$$

$$\beta' = 0, \ \gamma' = 1, \ a = -1, \quad v_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$A = VJV^{-1}$$

$$f(A) = V f(J) V^{-1}$$

$$J_2(\lambda) = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}, \quad J_2(\lambda, \varepsilon) = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda+\varepsilon \end{pmatrix}$$

$$[\lambda_1 \mathbb{I} - J_2(\lambda, \varepsilon)] v_1 = \begin{bmatrix} 0 & -1 \\ 0 & -\epsilon \end{bmatrix} v_1 = 0, \quad v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$[\lambda_2 \mathbb{I} - J_2(\lambda, \epsilon)] v_2 = \begin{bmatrix} \epsilon & -1 \\ 0 & 0 \end{bmatrix} v_2 = 0 \Rightarrow v_2 = \begin{pmatrix} 1 \\ \epsilon \end{pmatrix}$$

$$T = (v_1 \ \ v_2) = \begin{pmatrix} 1 & 1 \\ 0 & \epsilon \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & -1/\varepsilon \\ 0 & 1/\varepsilon \end{pmatrix}$$

$$f(J_2(\lambda, \varepsilon)) = T f(\Lambda) T^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & \varepsilon \end{bmatrix} \begin{bmatrix} f(\lambda) & 0 \\ 0 & f(\lambda+\varepsilon) \end{bmatrix} \begin{bmatrix} 1 & -1/\varepsilon \\ 0 & 1/\varepsilon \end{bmatrix}$$

$$= \begin{bmatrix} f(\lambda) & \frac{f(\lambda+\epsilon) - f(\lambda)}{\varepsilon} \\ 0 & f(\lambda+\varepsilon) \end{bmatrix}$$

$$\varepsilon \to 0 \quad f(J_2(\lambda)) = \lim_{\epsilon \to 0} f(J_2(\lambda, \epsilon)) = \begin{bmatrix} f(\lambda) & f'(\lambda) \\ 0 & f(\lambda) \end{bmatrix}$$

$$f(J_k(\lambda)) = \begin{bmatrix} f(\lambda) & f'(\lambda) & f''(\lambda)/2! & \dots & \frac{1}{(k-1)!} f^{(k-1)}(\lambda) \\ & f(\lambda) & f'(\lambda) & & \\ & & \ddots & & \\ & & & & f'(\lambda) \\ & & & & f(\lambda) \end{bmatrix}$$

$$f(J_2(\lambda,t).t) = T f(\Lambda t) T^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & \varepsilon \end{bmatrix} \begin{bmatrix} f(\lambda t) & 0 \\ 0 & f((\lambda+\varepsilon)t) \end{bmatrix} \begin{bmatrix} 1 & -1/\varepsilon \\ 0 & 1/\varepsilon \end{bmatrix}$$

$(\delta = \varepsilon t)$
$\frac{1}{\varepsilon} = \frac{t}{\delta}$

$$= \begin{bmatrix} f(\lambda t) & \frac{t}{\delta} \cdot \left(f(\lambda t + \delta) - f(\lambda t) \right) \\ 0 & f(\lambda t + \delta) \end{bmatrix}$$

$$f(J_2(\lambda,t,t) = \lim_{\delta \to 0} f(J_2(\lambda,t),t) = \begin{bmatrix} f(\lambda t) & t f'(\lambda t) \\ 0 & f(\lambda t \end{bmatrix}$$

$$f(J_k(\lambda t)) = \begin{bmatrix} f(\lambda t) & t f'(\lambda t) & \frac{t^2}{2!} f''(\lambda t) & \dots & \frac{t^{k-1}}{(k-1)!} f^{(k-1)}(\lambda t) \\ & f(\lambda t) & t f'(\lambda t) & \dots & \frac{t^{k-2}}{(k-2)!} f^{(k-2)}(\lambda t) \\ & & f(\lambda t) & & \\ & & & \ddots & \\ & & & & f(\lambda t) \end{bmatrix}$$

$$e^{J_k(\lambda t)} = \begin{bmatrix} e^{\lambda t} & t e^{\lambda t} & \frac{t^2}{2!} e^{\lambda t} & \dots & \frac{t^{(k-1)}}{(k-1)!} e^{\lambda t} \\ 0 & e^{\lambda t} & t e^{\lambda t} & & \frac{t^{(k-2)}}{(k-2)!} e^{\lambda t} \\ 0 & \vdots & e^{\lambda t} & & \\ \vdots & \vdots & & & \vdots \\ 0 & \cdot & & & e^{\lambda t} \end{bmatrix}$$

Για συστήματα διακριτού χρόνου $x(k+1) = Ax(k) \Rightarrow x(k) = A^k x(0)$

$A^t = V J^t V^{-1}$

Χρονικές Αποκρίσεις

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-s)} Bu(s)\,ds$$

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-s)} Bu(s)\,ds + Du(t)$$

A stable
(Hurwitz)

$x(t) \to x^*$ $\quad Ax^* + Bu^* = 0$

$y(t) \to y^*$ $\quad x^* = -A^{-1}Bu^*$

$$y = [D - CA^{-1}B]u^*$$

$$G(s) = C(s\mathbb{I} - A)^{-1}B + D$$
$$Y(s) = G(s)\,U(s) + C(s\mathbb{I} - A)^{-1}x(0)$$

Θεώρημα Τελικής Τιμής: $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$ (αν το y φτάνει σε μόνιμη κατάστ.)

Για βηματική είσοδο: $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} G(s) = G(0)$$