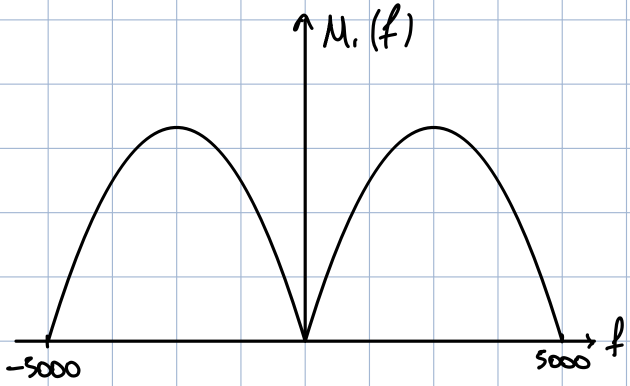
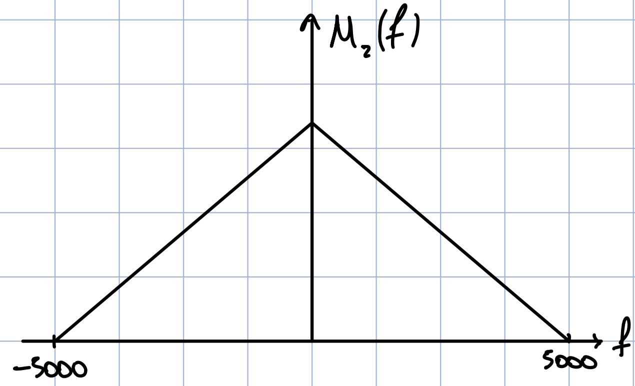
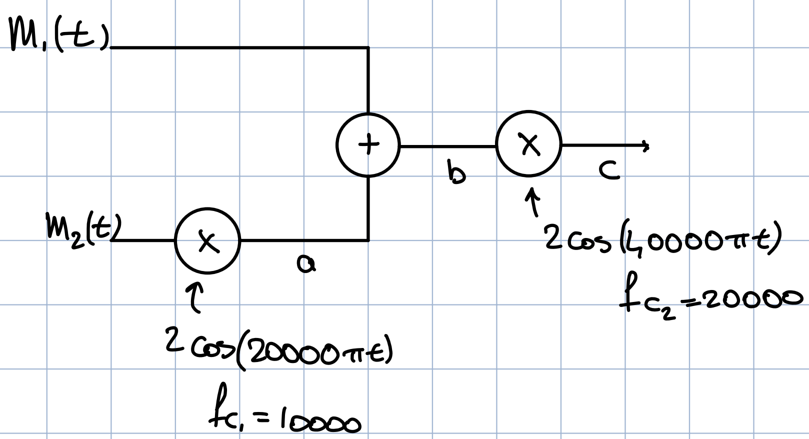
Άσκηση: $m_1(t)$, $m_2(t)$

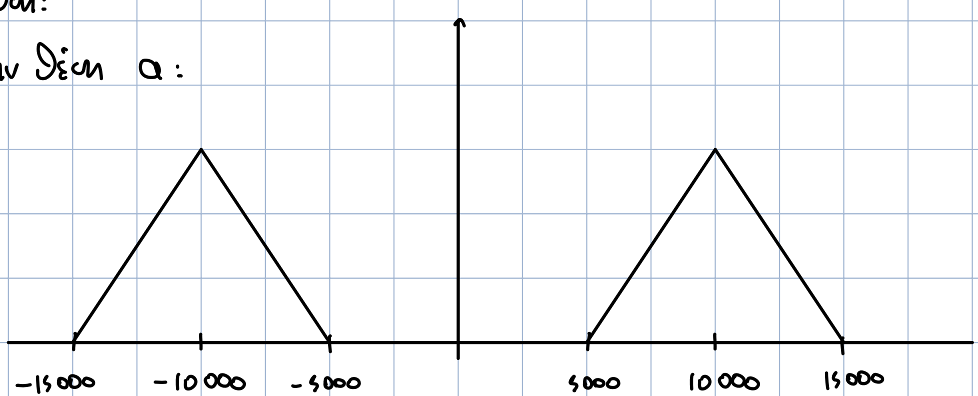




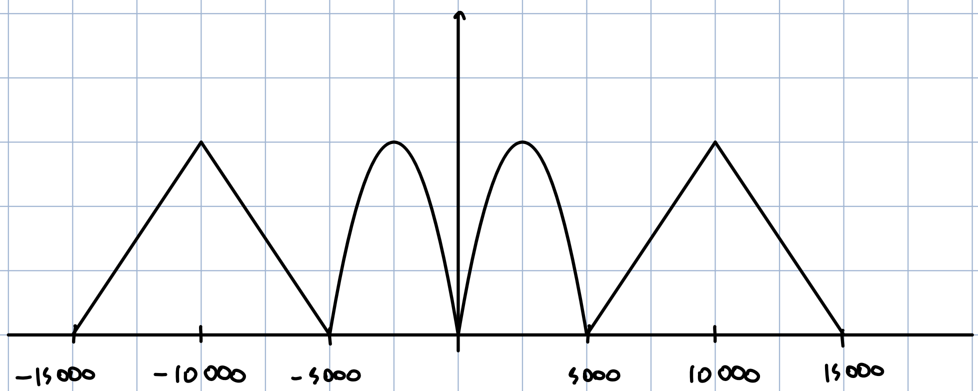


Λύση:

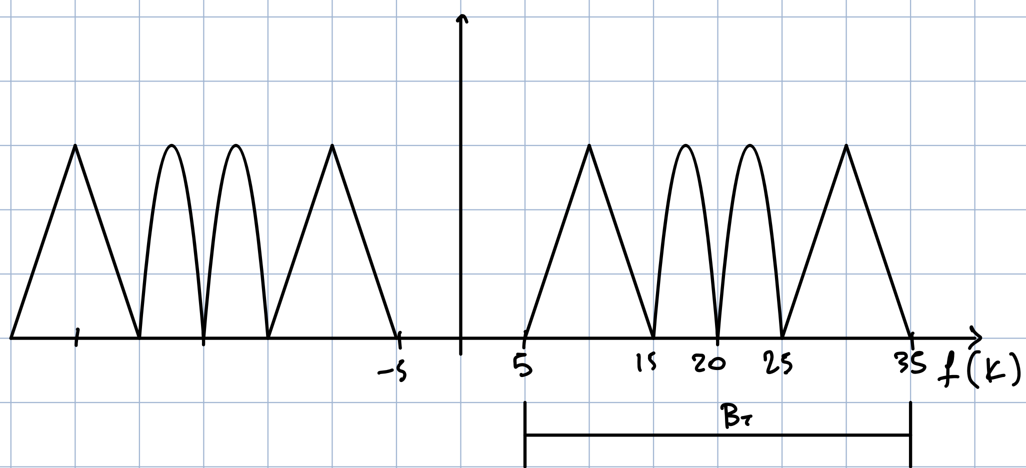
στην θέση a:



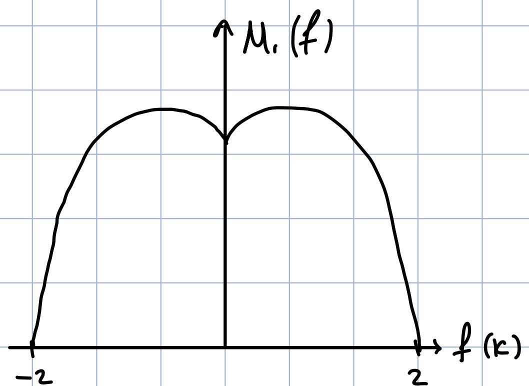
στην θέση b απλά προσθέτουμε το M_1

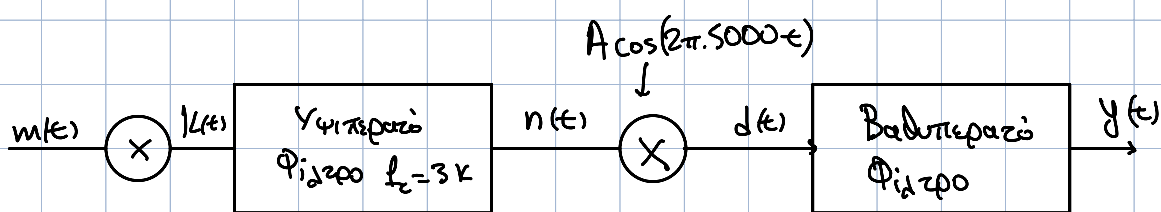


C: , fourier με το $2\cos(40000\pi t)$

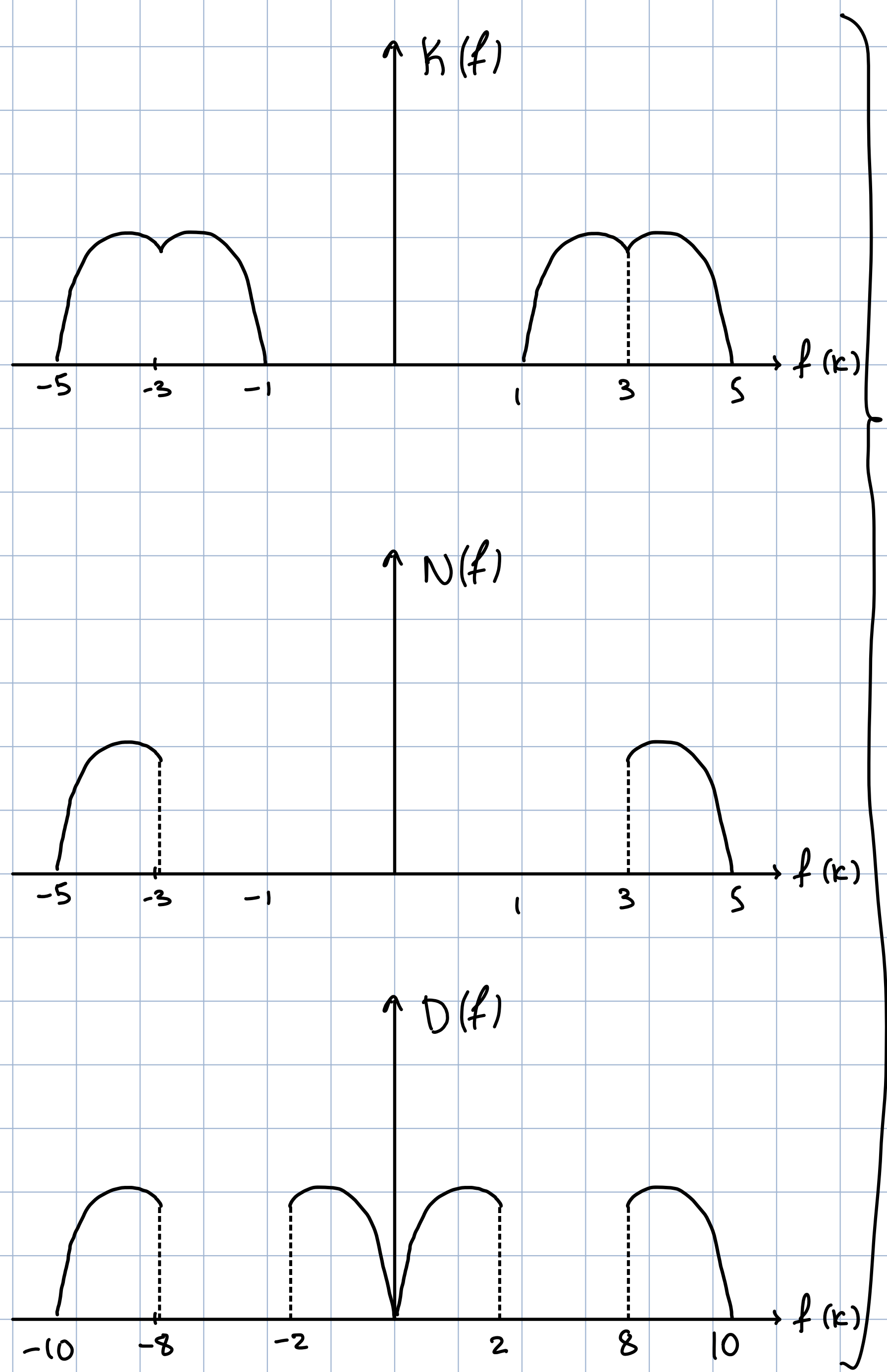


Άσκηση:

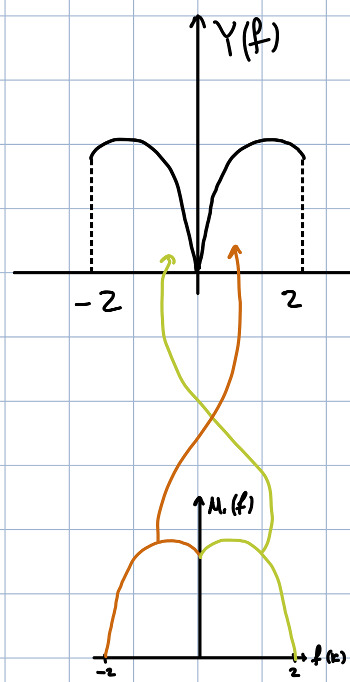




Λύση



Άρα το τελικό θα είναι:



Άσκηση: Έστω στοχαστική διαδικασία $X(t) = A\cos[2\pi F t + \Theta]$

A: σταθερά

F, Θ τ.μ. στατιστικά ανεξάρτητες που είναι ομοιόμορφα κατανεμημένες στο $[F_o, F_h]$ και $[0, 2\pi]$ αντίστοιχα: $R_x(t+\tau, t) = E[x(t+\tau)\,X(t)] =$

$$= A^2 E[\cos(2\pi F(t+\tau)+\Theta) \cdot \cos(2\pi F t + \Theta)]$$

$$= \frac{A^2}{2} E[\cos(2\pi F(2t+\tau) + 2\Theta)] + \frac{A^2}{2} E[\cos 2\pi F\tau]$$

$$= \frac{A^2}{2} E[\cos(2\pi F(2t+\tau))\cos 2\Theta] - \frac{A^2}{2} E[\sin(2\pi F(2t+\tau))\sin 2\Theta] + \frac{A^2}{2} E[\cos(2\pi F\tau)]$$

$$= \frac{A^2}{2} E[\cos(2\pi F(2t+\tau))]\cancelto{0}{E[\cos 2\Theta]} - \frac{A^2}{2} E[\sin(2\pi F(2t+\tau))]\cancelto{0}{E[\sin 2\Theta]} + \frac{A^2}{2} E[\cos(2\pi F\tau)]$$

$$R_x(t+\tau, t) = \frac{A^2}{2} E[\cos 2\pi F\tau] = \frac{A^2}{2}\int_{-\infty}^{\infty} \cos 2\pi F\tau \; P_F(F)\,dF$$

$$= \frac{A^2}{2}\int_{F_L}^{F_h} \frac{1}{F_h - F_L}\cos(2\pi F\tau)\,dF = \dots = \frac{A^2}{4}\int_{-F_h}^{-F_L} \frac{1}{F_h - F_L} e^{j2\pi F\tau}\,dF + \frac{A^2}{4}\int_{F_L}^{F_h} \frac{1}{F_h - F_L} \cdot e^{j2\pi F\tau}\,dF$$

$S_x(F) = ?$

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(F)\,e^{j2\pi F\tau}\,dF$$

$$S_x(F) = \begin{cases} 0 & \text{αλλού} \\ \dfrac{A^2}{4(F_h - F_L)} & F_L \leq |F| \leq F_h \end{cases}$$

Άσκηση: X_n : ανεξ. τ.μ. μηδενικής μέσης τιμής και μεταβλητότητας σ^2

$$Y_n = \frac{X_n + X_{n+1}}{2}, \quad E[Y_n]=0 \quad (E[X_i]=0)$$

$$C_y(i,j) \underset{\text{Επειδή } E[Y_i]=0}{=} E[Y_i \cdot Y_j] = \frac{1}{4} E[(X_i + X_{i-1})(X_j + X_{j-1})]$$

$$= \frac{1}{4}\left\{E[X_i X_j] + E[X_i X_{j-1}] + E[X_{i-1} X_j] + E[X_{i-1} X_{j-1}]\right\}$$

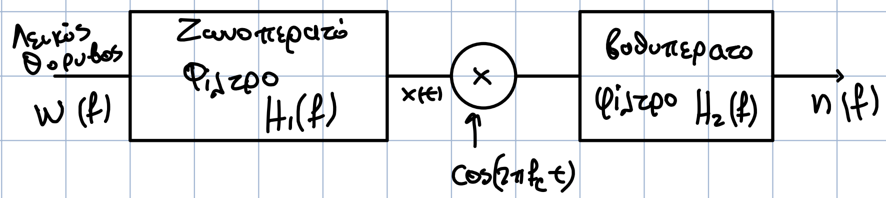
$$= \begin{cases} i=j, \ E[\cancel{X_i^2}]^{\sigma^2} + E[\cancel{X_{i-1}^2}]^{\sigma^2}, \ (E[\cancel{X_i}]^{0} E[X_{i-1}]) \ \text{άρα} \ \frac{1}{4}\cdot 2\sigma^2 = \frac{\sigma^2}{2}, \ i=j \\ \quad \left(\substack{X_i \ X_{i-1} \\ \text{σ. ανεξ}}\right) \\ i+j=1, \ E[\cancel{X_{j+1} X_j}]^{0} + E[\cancel{X_{j+1} X_{j-1}}]^{0} + E[X_j^2]^{\to\sigma^2} + E[\cancel{X_j X_{j-1}}]^{0} \ \text{άρα} \ \frac{\sigma^2}{4}, \ |i-j|=1 \\ \text{αλλού}, \ 0 \end{cases}$$

Άσκηση: $x(t) = A\sin(2\pi F t)$, F τ.μ.

$$P_F(F) = \begin{cases} 1/m, & 0 \le F \le m \\ 0, & \text{αλλού} \end{cases}$$

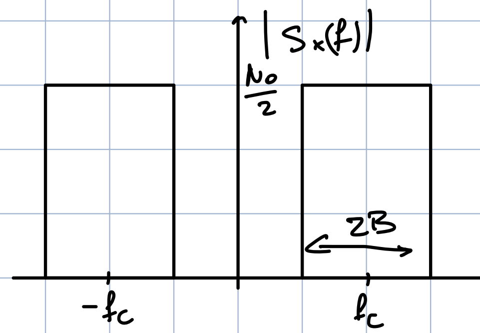
$$E[X(t)] = E[A\sin 2\pi F t] = \ !!$$

Άσκηση: Λευκός Θόρυβος Gauss μηδενικής μέσης τιμής φασματική πυκνότητα ισχύος $\frac{N_0}{2}$



$$S_w(f)=\frac{N_0}{2}$$

$$S_x(f)=\begin{cases}\frac{N_0}{2} & -f_c-B\leq f\leq -f_c+B\\ 0 & \text{αλλού}\\ \frac{N_0}{2} & f_c-B\leq f\leq f_c+B\end{cases}$$



$$Y(t)=X(t)\cos 2\pi f_c t$$

$$S_y(f)=\int_{-\infty}^{\infty} R_y(\tau)\exp(-j2\pi f\tau)\,d\tau$$

$$R_y(\tau)=E\left[Y(t+\tau)\,Y(t)\right]=\dots=\frac{1}{2}R_x(\tau)\cos 2\pi f_c\tau$$

$$S_y(f)=\dots=\frac{1}{4}\left[S_x(f-f_c)+S_x(f+f_c)\right]$$