Παράδειγμα

Σ.Α $X(t)=A\cos(2\pi f_c t+\Theta)+W_1(t)$ με

- Θ τ.μ. ομοιόμορφα κατανεμημένη στο $[0,2\pi]$
- $W(t) \rightarrow$ White noise Gauss, $\mu_W=0$, $S_W(f)=\frac{N_0}{2}$

για την τ.μ. Θ, αμέσως γράφουμε $f_\Theta(\theta)=\begin{cases}\frac{1}{2\pi}, & \theta\in[0,2\pi]\\ 0, & \text{αλλού}\end{cases}$ (Επειδή είναι ο.κ.)

$$E[X(t)]=A\,E[\cancelto{0}{\cos(2\pi f_c t+\Theta)}]+E[\cancelto{0}{W(t)}]=0$$

$$R_X(t,t+\tau)=E[X(t+\tau)X(t)]=$$

$$=\underbrace{A^2E[\cos(2\pi f_c t+2\pi f_c\tau+\Theta)\cdot\cos(2\pi f_c t+\Theta)]}_{\frac{1}{2}[\cos(\,)+\cos(\,)]}+AE[\cos(2\pi f_c t+2\pi f_c\tau+\Theta)W(t)]$$

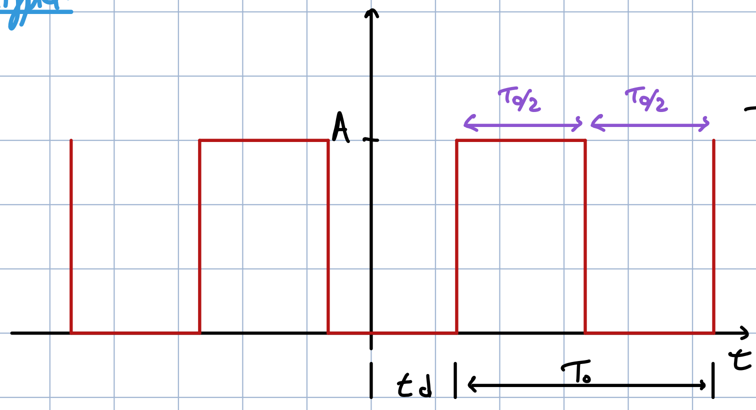
$$+AE[W(t+\tau)\cos(2\pi f_c t+\Theta)]+\underbrace{E[W(t+\tau)W(t)]}\rightarrow R_W(\tau)=\frac{N_0}{2}\delta(\tau)$$

$$=\frac{A^2}{2}E[\cancelto{0}{\cos(4\pi f_c t+2\pi f_c\tau+2\Theta)}+\cos(2\pi f_c\tau)]+\frac{N_0}{2}\delta(\tau)$$

$$=\frac{A^2}{2}E[\cos(2\pi f_c\tau)]+\frac{N_0}{2}\delta(\tau)$$

αυτοί οι δύο όροι βγαίνουν μηδέν γιατί χωρίζοντας την μέση τιμή $\cancelto{0}{E[W(t)]}$, όπως και το $E[\cos(\,)]=0$

Παράδειγμα:



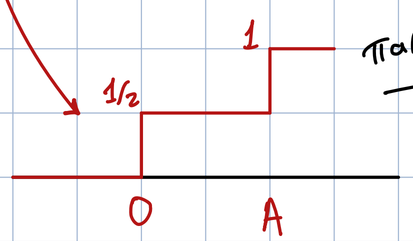
t_d = Τ.Μ. ο.κ. στο $\left[-\frac{T_0}{2}, \frac{T_0}{2}\right]$

συνάρτηση πυκνότητας πιθανότητας

↳ Τ.Μ. $X(t_k)$

συνάρτηση κατανομής π.

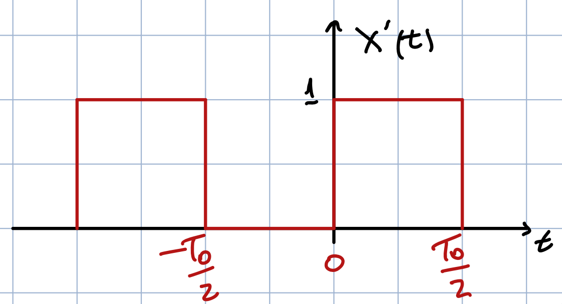
$$F_X(x) \triangleq P[X \le x] = \begin{cases} 0 & , x<0 \\ 1/2 & , 0 \le x < A \\ 1 & , x \ge A \end{cases}$$



παράγωγος

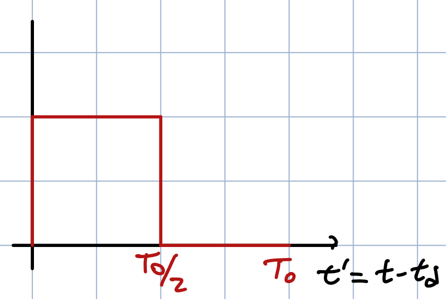
$$f_x(x) = \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x-A)$$

$$\mu_x = E[x(t)] = \int_{-\infty}^{\infty} x f_x(x)\,dx = \frac{A}{2}$$



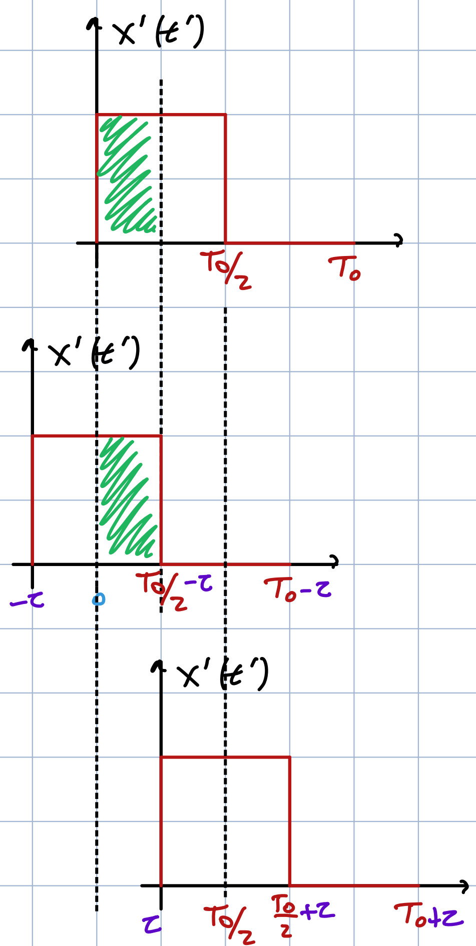
$\Rightarrow \quad x(t) = A \cdot x'(t - t_d)$

$\Rightarrow \quad x(t+\tau) = A\, x'(t+\tau-t_d)$



$$R_x(t,\tau) = E[X(t+\tau)X(t)]$$

$$= E[A^2 X'(t-t_d)\, X'(t+\tau-t_d)]$$



$$\tau \in \left[0, \tfrac{T_0}{2}\right] \Rightarrow R_x(\tau) = \frac{A^2}{T_0}\left(\frac{T_0}{2} - \tau\right)$$

$$R_x(\tau) = \frac{A^2}{T_0}\left(\frac{T_0}{2} - |\tau|\right)$$

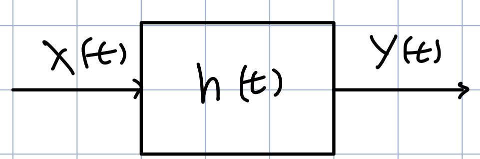
$$\tau \in \left[-\tfrac{T_0}{2}, 0\right] \Rightarrow R_x = \frac{A^2}{T_0}\left(\frac{T_0}{2} + \tau\right)$$

Παράδειγμα: $X(t), Y(t) \rightarrow$ wss Σ.Α.

1) $R_{xy}(-\tau) = R_{yx}(\tau)$

2) $|R_{xy}(\tau)| \leq \frac{1}{2}\left[R_x(0) + R_y(0)\right]$

Παράδειγμα: $X(t) \rightarrow$ wss με $\mu_x = 0$



$$h(t) = \begin{cases} a e^{-at}, & 0 \leq t \leq T \\ 0, & \text{αλλού} \end{cases}$$

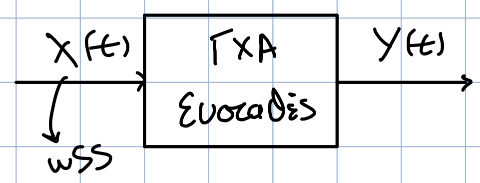
$S_y(f) = ?$

Λύση:

$$h(t) = a e^{-at}\left[u(t-T) - u(t)\right] = \left[-a e^{-a(t-T)} u(t-T)\right] e^{-aT} + a e^{-at} u(t)$$

$$\Rightarrow S_y(f) = |H(f)|^2 S_x(f) = \frac{a^2}{a^2 + (2\pi f)^2}\left\{1 - 2e^{-aT} \cdot \cos(2\pi f T) + e^{-2aT}\right\} S_x(f)$$

Παράδειγμα:



$y''(t) + 3y'(t) + 2y(t) = x(t)$, ψάχνουμε $R_{yx}(\tau)$ συναρτήσει $R_x(\tau)$

γδο:

$$R''_{yx}(\tau) + 3R'_{yx}(\tau) + 2R_{yx}(\tau) = R_x(\tau)$$

$$R_{yx}(\tau) = E\left[Y(t+\tau)X(t)\right] = E\left[\int_{-\infty}^{\infty} h(\lambda)X(t+\tau-\lambda)\,d\lambda \cdot X(t)\right] = \int_{-\infty}^{\infty} h(\lambda)\cdot E\left[x(t+\tau-\lambda)x(t)\right]d\lambda$$

$$R_{yx}(\tau) = \int_{-\infty}^{\infty} h(\lambda)R_x(\tau-\lambda)\,d\lambda = h(\tau) * R_x(\tau)$$

$$H(f) = \frac{1}{(j2\pi f)^2 + 3j2\pi f + 2}$$

$S_{yx}(f) = H(f)\cdot S_x(f)$ και μετά $F^{-1}[\ ]$