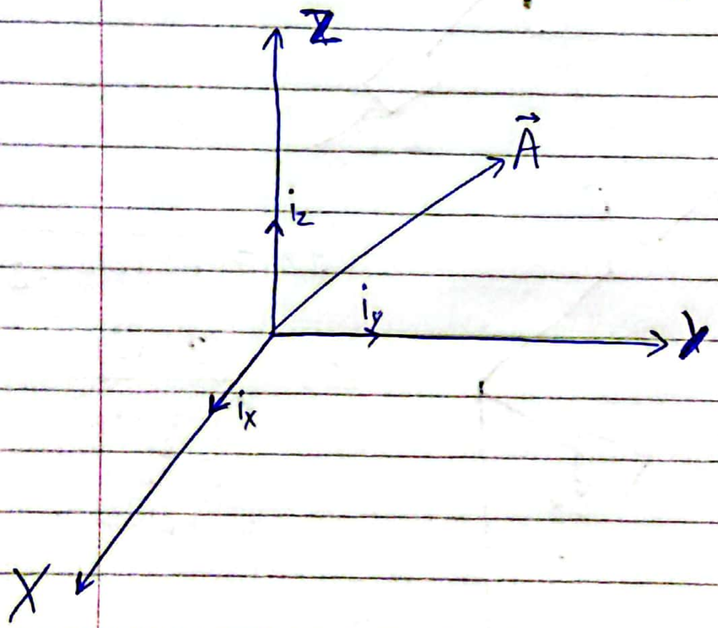
Πεδία Α

Χαρακτηριστικά

Ηλεκτρ. Δύναμη:
- Μεγάλη εμβέλεια
- Δεύτερη πιο ισχυρή
- Ελκτική και απωστική

Θεμελιώδες Ηλ. φορτίο:
- Σε ένα κλειστό κύκλωμα διατηρείται
- $e = 1{,}6 \cdot 10^{-19}\ C$

Καρτεσιανό σύστημα Συντεταγμένων:

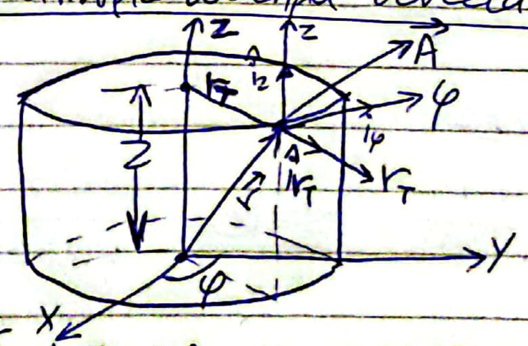


$$\vec{A} = A_x i_x + A_y i_y + A_z i_z$$
$$\vec{r} = x i_x + y i_y + z i_z$$

Κυλινδρικό σύστημα συντεταγμένων:

$$\vec{A} = A_{r_T} \hat{i}_{r_T} + A_\varphi \hat{i}_\varphi + A_z \hat{i}_z$$
$$\vec{r} = r_T \cos\varphi\, \hat{i}_x + r_T \sin\varphi\, \hat{i}_y + z \hat{i}_z = r_T \hat{i}_{r_T} + z \hat{i}_z$$

Σχήμα →

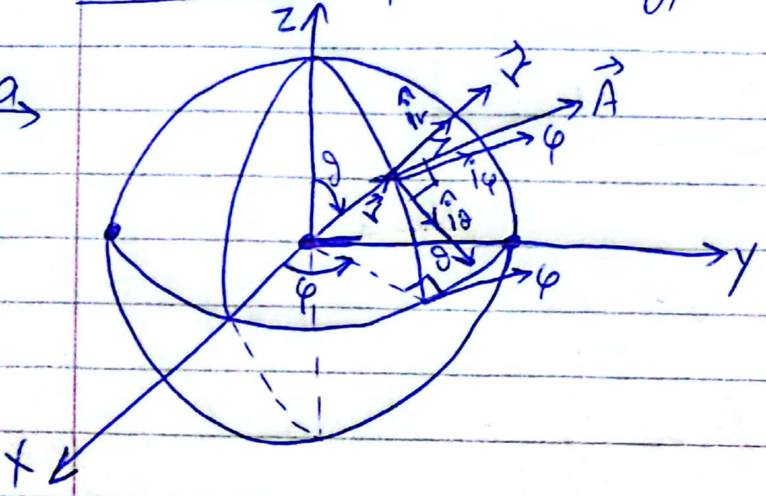


Σχέσεις με το καρτεσιανό:

$$\left.\begin{aligned} x &= r_T \cos\varphi \\ y &= r_T \sin\varphi \\ z &= z \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} r_T &= \sqrt{x^2+y^2}, && r_T \geq 0 \\ \varphi &= \arctan\left(\tfrac{y}{x}\right), && 0 \leq \varphi \leq 2\pi \\ z &= z, && z \in \mathbb{R} \end{aligned}\right.$$

Σφαιρικό σύστημα συντεταγμένων

σχήμα →



$$\vec{A} = A_r \hat{i}_r + A_\vartheta \hat{i}_\vartheta + A_\varphi \hat{i}_\varphi$$

$$\vec{r} = r\hat{i}_r = r\sin\vartheta\cos\varphi\,\hat{i}_x + r\sin\vartheta\sin\varphi\,\hat{i}_y + r\cos\vartheta\,\hat{i}_z$$

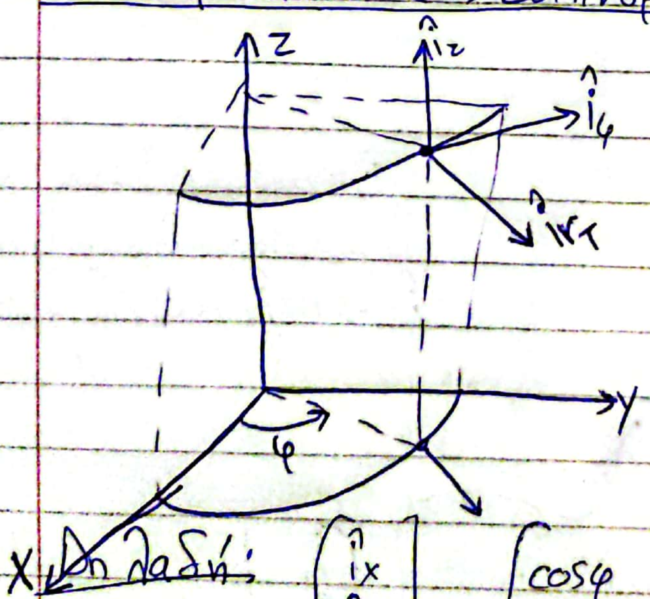
Σχέσεις με το καρτεσιανό σύστημα :

$$\left.\begin{aligned} x &= r\sin\vartheta\cos\varphi \\ y &= r\sin\vartheta\sin\varphi \\ z &= r\cos\vartheta \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} r &= \sqrt{x^2+y^2+z^2}, & r \geq 0 \\ \vartheta &= \arccos\left(\frac{z}{\sqrt{z^2+x^2+y^2}}\right), & 0 \leq \vartheta \leq \pi \\ \varphi &= \arctan\left(\frac{y}{x}\right) & 0 \leq \varphi \leq 2\pi \end{aligned}\right.$$

Μετασχηματισμοί Συστημάτων Συντεταγμένων :

1. Καρτεσιανό ⟷ Κυλινδρικό

σχήμα →



$$\hat{i}_x = \cos\varphi\,\hat{i}_{r_T} - \sin\varphi\,\hat{i}_\varphi$$
$$\hat{i}_y = \sin\varphi\,\hat{i}_{r_T} + \cos\varphi\,\hat{i}_\varphi$$
$$\hat{i}_z = \hat{i}_z$$

ή

$$\hat{i}_{r_T} = \cos\varphi\,\hat{i}_x + \sin\varphi\,\hat{i}_y$$
$$\hat{i}_\varphi = -\sin\varphi\,\hat{i}_x + \cos\varphi\,\hat{i}_y$$
$$\hat{i}_z = \hat{i}_z$$

Δηλαδή:

$$\begin{bmatrix} \hat{i}_x \\ \hat{i}_y \\ \hat{i}_z \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\tilde{A}_{c\to e}} \begin{bmatrix} \hat{i}_{r_T} \\ \hat{i}_\varphi \\ \hat{i}_z \end{bmatrix}$$

$$\begin{bmatrix}\hat{i}_{r_T}\\ \hat{i}_\varphi\\ \hat{i}_z\end{bmatrix} = \underbrace{\begin{bmatrix}\cos\varphi & \sin\varphi & 0\\ -\sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}}_{A_{R\to C}} \begin{bmatrix}\hat{i}_x\\ \hat{i}_y\\ \hat{i}_z\end{bmatrix}$$

$$\vec{A} = A_x\hat{i}_x + A_y\hat{i}_y + A_z\hat{i}_z = [A_x \ A_y \ A_z]\begin{bmatrix}\hat{i}_x\\ \hat{i}_y\\ \hat{i}_z\end{bmatrix} =$$

$$[A_x \ A_y \ A_z]\,\tilde{A}_{C\to R}\begin{bmatrix}\hat{i}_{r_T}\\ \hat{i}_\varphi\\ \hat{i}_z\end{bmatrix} = [A_{r_T} \ A_\varphi \ A_z]\begin{bmatrix}\hat{i}_{r_T}\\ \hat{i}_\varphi\\ \hat{i}_z\end{bmatrix}$$

και $[A_{r_T} \ A_\varphi \ A_z] = [A_x \ A_y \ A_z]\,\hat{A}_{C\to R} \rightarrow$

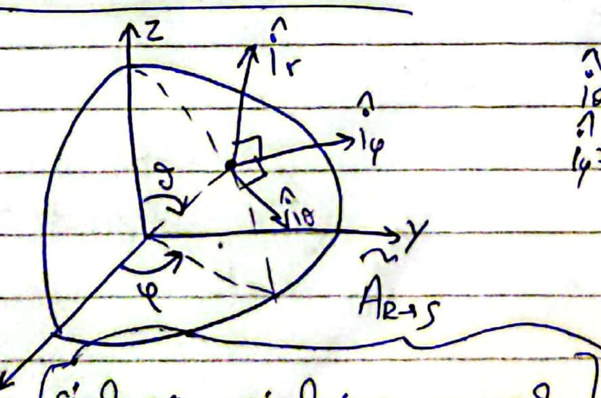
$$\begin{bmatrix}A_{r_T}\\ A_\varphi\\ A_z\end{bmatrix} = \begin{bmatrix}\tilde{A}^T_{C\to R}\end{bmatrix}\begin{bmatrix}A_x\\ A_y\\ A_z\end{bmatrix} = \tilde{A}_{R\to C}\begin{bmatrix}A_x\\ A_y\\ A_z\end{bmatrix}$$

$$\vec{A}_R = A_x\hat{i}_x + A_y\hat{i}_y + A_z\hat{i}_z = A_{r_T}\hat{i}_{r_T} + A_\varphi\hat{i}_\varphi + A_z\hat{i}_z = \vec{A}_C$$

$$\vec{A}_C = \tilde{A}_{R\to C}\vec{A}_R, \quad \vec{A}_R = A_{C\to R}\vec{A}_C$$

<u>Σφαιρικό ⟷ Καρτεσιανό</u>

<u>Σχήμα</u> →



$\hat{i}_r = \sin\vartheta\cos\varphi\,\hat{i}_x + \sin\vartheta\sin\varphi\,\hat{i}_y + \cos\vartheta\,\hat{i}_z$

$\hat{i}_\theta = \cos\vartheta\cos\varphi\,\hat{i}_x + \cos\vartheta\sin\varphi\,\hat{i}_y - \sin\vartheta\,\hat{i}_z$

$\hat{i}_\varphi = -\sin\varphi\,\hat{i}_x + \cos\varphi\,\hat{i}_y + \emptyset\,\hat{i}_z$

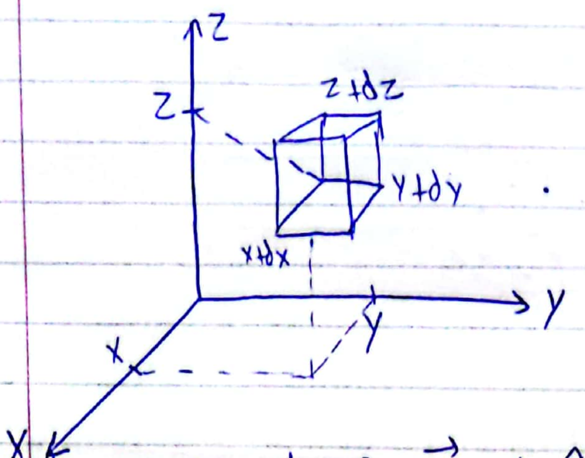
$$\begin{bmatrix}\hat{i}_r\\ \hat{i}_\theta\\ \hat{i}_\varphi\end{bmatrix} = \begin{bmatrix}\sin\vartheta\cos\varphi & \sin\vartheta\sin\varphi & \cos\vartheta\\ \cos\vartheta\cos\varphi & \cos\vartheta\sin\varphi & -\sin\vartheta\\ -\sin\varphi & \cos\varphi & 0\end{bmatrix}\begin{bmatrix}\hat{i}_x\\ \hat{i}_y\\ \hat{i}_z\end{bmatrix}$$

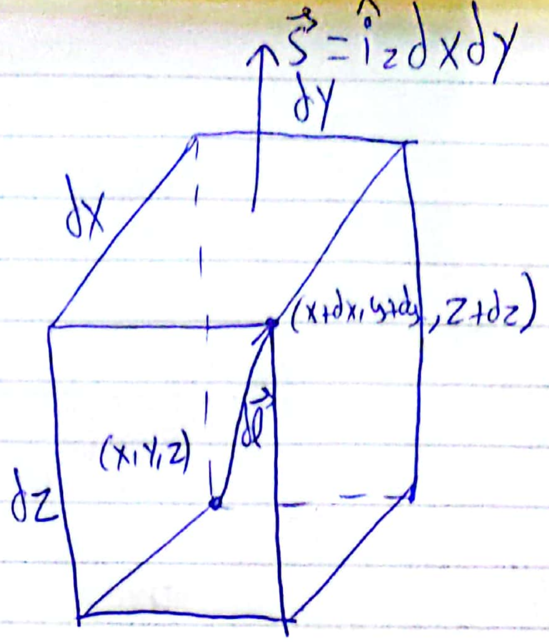
$$\begin{bmatrix}\hat{i}_x\\ \hat{i}_y\\ \hat{i}_z\end{bmatrix} = \begin{bmatrix}\sin\vartheta\cos\varphi & \cos\vartheta\cos\varphi & -\sin\varphi\\ \sin\vartheta\sin\varphi & \cos\vartheta\sin\varphi & \cos\varphi\\ \cos\vartheta & -\sin\vartheta & 0\end{bmatrix}\begin{bmatrix}\hat{i}_r\\ \hat{i}_\theta\\ \hat{i}_\varphi\end{bmatrix}$$

Διαφορικά στοιχεία

1. Καρτεσιανό

Σχήμα →



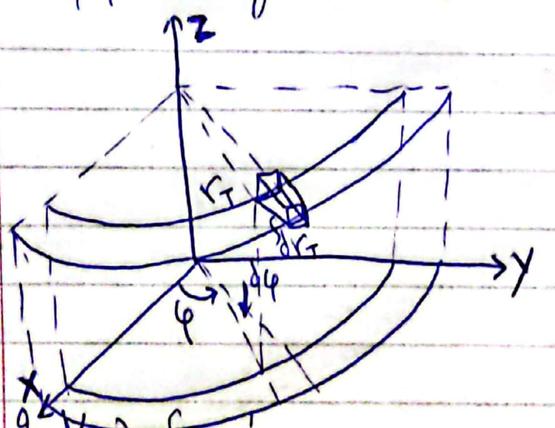


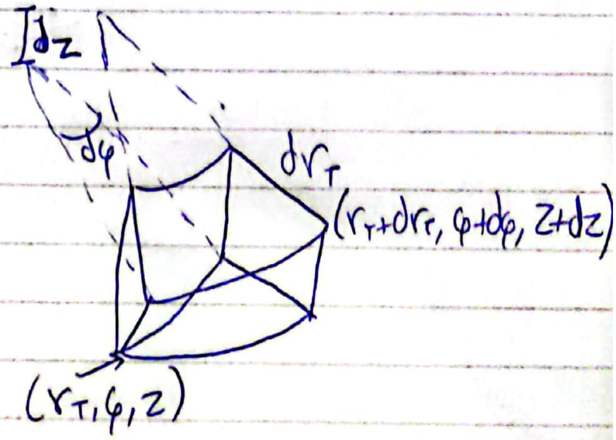
Διαφορικό μήκος: $d\vec{l} = dx\,\hat{i}_x + dy\,\hat{i}_y + dz\,\hat{i}_z$

Το ± για την κατεύθυνση ← Διαφορική επιφάνεια: $d\vec{S} = \pm dx dy\,\hat{i}_z$ ή $\pm dx dz\,\hat{i}_y$ ή $\pm dy dz\,\hat{i}_x$

Διαφορικός όγκος: $dV = dx dy dz$

Σχήμα →





2. Κυλινδρικό

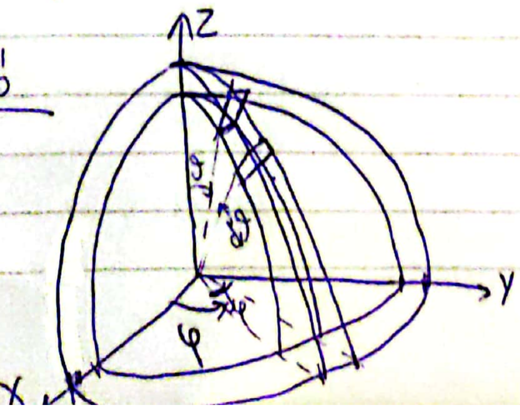
$d\vec{r} = dr_T\,\hat{i}_{r_T} + r_T d\varphi\,\hat{i}_\varphi + dz\,\hat{i}_z$

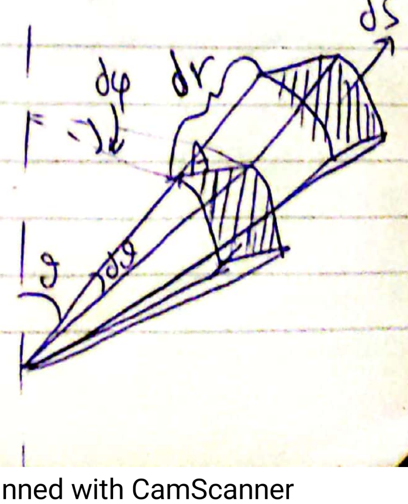
$d\vec{S} = \pm r_T dr_T d\varphi\,\hat{i}_z,\ \pm dr_T dz\,\hat{i}_\varphi,\ \pm r_T d\varphi dz\,\hat{i}_{r_T}$

$dV = r_T d\varphi dr_T dz$

3. Σφαιρικό

Σχήμα →





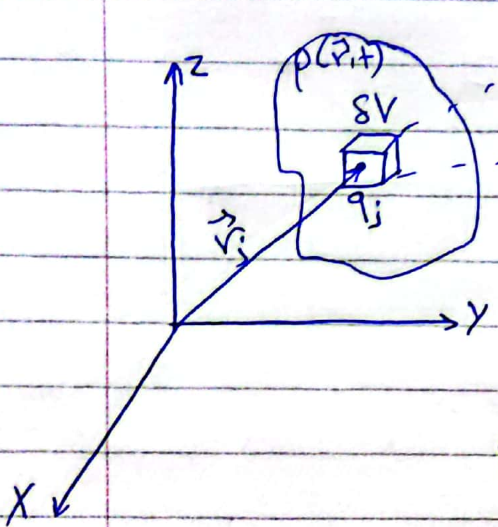
$d\vec{r} = dr\,\hat{i}_r + r\,d\theta\,\hat{i}_\theta + r\sin\theta\,d\varphi\,\hat{i}_\varphi$

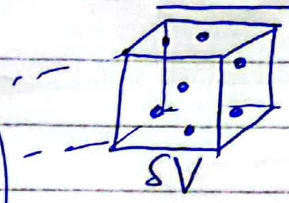
$d\vec{S} = \pm r\,d\theta\,r\sin\theta\,d\varphi\,\hat{i}_r,\ \pm r\sin\theta\,d\varphi\,dr\,\hat{i}_\theta,\ \pm r\,d\theta\,dr\,\hat{i}_\varphi$

$dV = r^2\sin\theta\,d\theta\,d\varphi\,dr$

Πυκνότητες Ηλ. φορτίου: $e = 1{,}6\cdot 10^{-19}\,C$

κάθε ηλ. φορτίο: $\pm Ne$

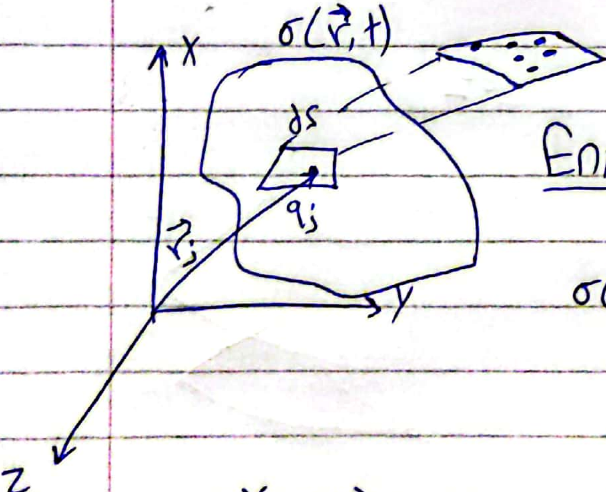




Χωρική πυκνότητα Ηλ. φορτίου

$$\rho(\vec{r},t) = \lim_{\delta V\to 0}\left\{\frac{1}{\delta V}\sum_{q_j\in\delta V} q_j\right\} = \frac{dq}{dV}$$

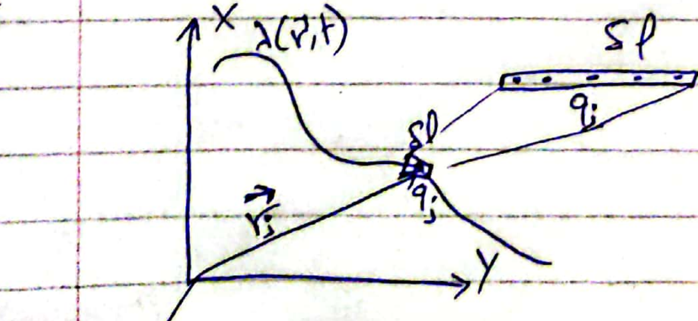
$[C/m^3]$



Επιφανειακή π. ηλ. φ.:

$$\sigma(\vec{r},t) = \lim_{\delta S\to 0}\left\{\frac{1}{\delta S}\sum_{q_j\in\delta S} q_j\right\} = \frac{dq}{dS}$$

$[C/m^2]$



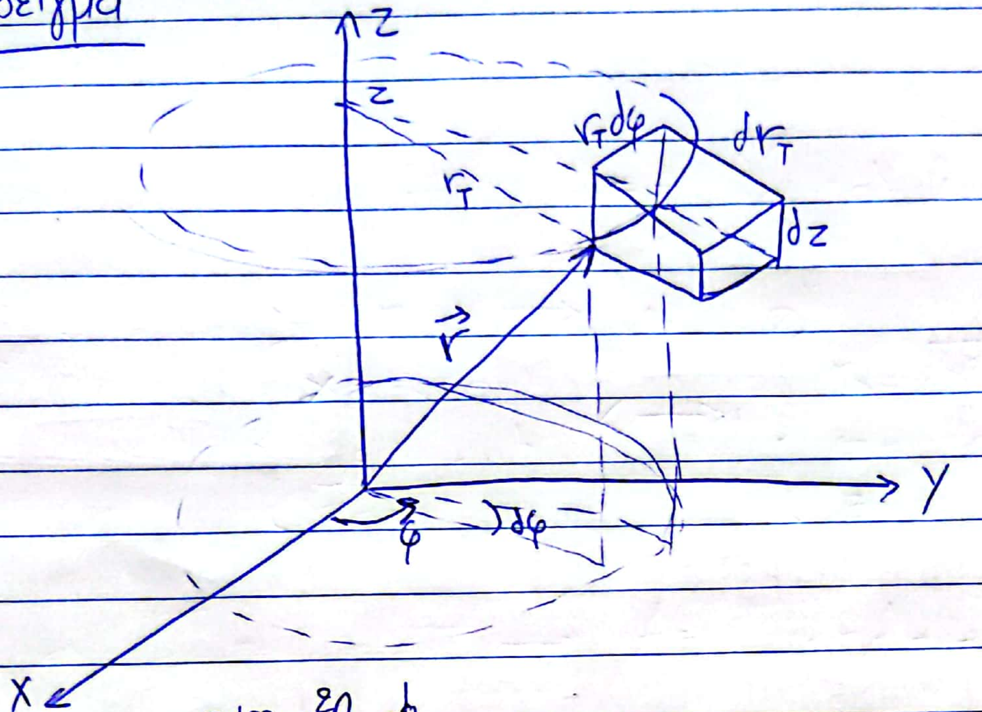
$$\lambda(\vec{r},t) = \lim_{\delta l\to 0}\left\{\frac{1}{\delta l}\sum_{q_j\in\delta l} q_j\right\} = \frac{dq}{dl}$$

Στοιχειώδες Ηλ. φορτίο

$$dq = \{\rho\,dV,\ \sigma\,dS,\ \lambda\,dl,\ q_j\}$$

Παράδειγμα

Σχήμα



$$Q = \int_V \rho \, dV = \int_{z=-\infty}^{+\infty} \int_{\varphi=0}^{2\pi} \int_{r_T=0}^{b} \rho_0 \frac{a^2}{z^2+a^2} r_T \, dr_T \, d\varphi \, dz$$

$$= \rho_0 \int_{z=-\infty}^{+\infty} \frac{a^2}{z^2+a^2} dz \int_{\varphi=0}^{2\pi} d\varphi \int_{r_T=0}^{b} r_T \, dr_T = \rho_0 \left\{ a \cdot \arctan\left(\frac{a}{z}\right) \Big|_{-\infty}^{+\infty} \right\} \left\{ 2\pi \right\} \left\{ \frac{b^2}{2} \right\}$$

$$= \rho_0 \pi^2 a b^2$$