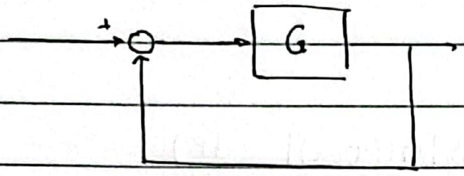
Δευτέρα, 28/11/2022

Απόκριση Συχνότητας

G stable

$A\sin(\omega t) \rightarrow \boxed{G(s)} \rightarrow A|G(j\omega)|\sin(\omega t + \arg G(j\omega))$

διαγράμματα μέτρου και φάσης



$1 + G(s) = 0$

$|G(j\omega_{gc})| = 1$ ← gain crossover frequency

Phase margin: $\Phi_{\pi\varepsilon\rho} = 180° + \arg G(j\omega_{gc})$ → ανοχή στις χρον. καθυστερήσεις στην είσοδο

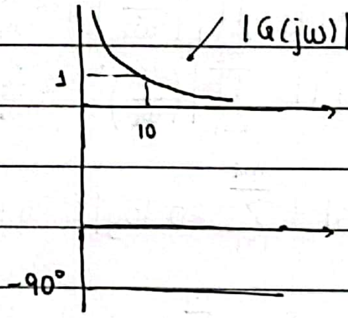
$G(s) = \frac{10}{s}$

$1 + \frac{10}{s} = 0$ πόλος στο -10

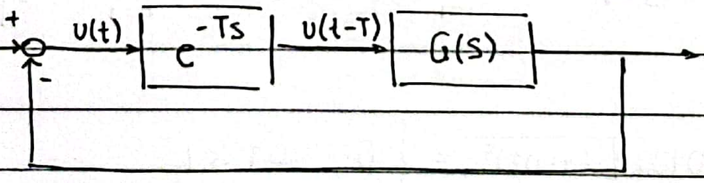
$|G(j\omega_{gc})| = 1 \Rightarrow \omega_{gc} = 10$ rad/sec

$\arg G(j\omega_{gc}) = -90°$

$|G(j\omega)| = \frac{10}{\omega}$



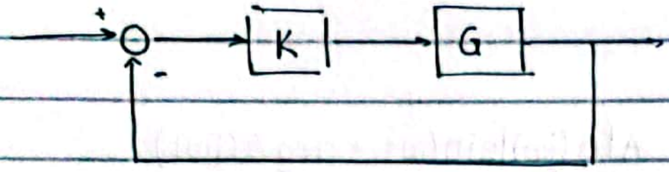
χρονική καθυστέρηση στην είσοδο



$G_{o\lambda} = G(s)e^{-Ts} \Rightarrow |G_{o\lambda}(j\omega)| = |G(j\omega)|$

$\arg G_{o\lambda}(j\omega) = \arg G(j\omega) - \omega T$

$\omega_{gc} T < \Phi_{\pi\varepsilon\rho.} \Rightarrow T < \frac{\Phi_{\pi\varepsilon\rho}}{\omega_{gc}} = \frac{\pi/2}{10} = \frac{\pi}{20}$



Phase crossover frequency : $\arg G(j\omega_{pc}) = -180^\circ$

gain margin : $\dfrac{1}{|G(j\omega_{ph})|}$ ή $-20\log|G(j\omega_{ph})|$ (dB)

Διαγράμματα Bode

$$G(s) = K \frac{\prod_{i=1}^{m_1}(\tilde{\tau}_i s+1)\prod_{k=1}^{m_2}\left(\left(\frac{s}{\tilde{\omega}_k}\right)^2 + 2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}s+1\right)}{s^{n_3}\prod_{i=1}^{n_1}(T_i s+1)\prod_{k=1}^{n_2}\left(\left(\frac{s}{\omega_k}\right)^2 + 2\frac{\zeta_k}{\omega_k}s+1\right)}$$

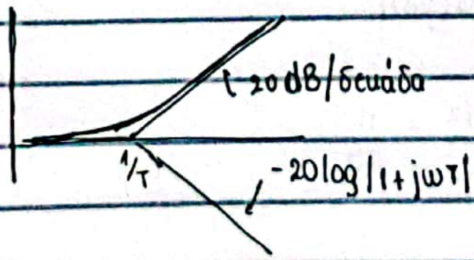
$$G(s) = \frac{s+5}{s\underbrace{(s-1+j)(s-1-j)}_{(s-1)^2+1=s^2-2s+2}(s+2)} = \frac{5}{2\cdot 2}\,\frac{\frac{1}{5}s+1}{s\left(\frac{1}{2}s+1\right)\left(\frac{s^2}{2}-s+1\right)}$$

$$|G(j\omega)| = |K|\frac{\prod_{i=1}^{m_1}|\tilde{\tau}_i j\omega+1|\prod_{k=1}^{m_2}\left|\left(\frac{j\omega}{\tilde{\omega}_k}\right)^2 + 2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}j\omega+1\right|}{|j\omega|^{n_3}\prod_{i=1}^{n_1}|T_i j\omega+1|\cdot\prod_{k=1}^{n_2}\left|\left(\frac{j\omega}{\omega_k}\right)^2 + 2\frac{\zeta_k}{\omega_k}j\omega+1\right|}$$

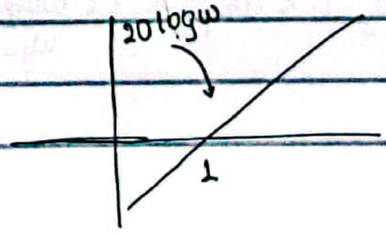
$$20\log_{10}|G(j\omega)| = 20\log|K| + \sum_{i=1}^{m_1}20\log|\tilde{\tau}_i j\omega+1| + \sum_{k=1}^{m_2}20\log\left|\left(\frac{j\omega}{\tilde{\omega}_k}\right)^2+2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}j\omega+1\right|$$

$$-\underbrace{20\log|j\omega|^{n_3}}_{-n_3\log\omega} - \sum_{i=1}^{n_1}20\log|T_i j\omega+1| - \sum_{k=1}^{n_2}20\log\left|\left(\frac{j\omega}{\omega_k}\right)^2+2\frac{\zeta_k}{\omega_k}j\omega+1\right|$$

$$20\log|T\cdot j\omega+1| = 20\log\sqrt{1+(\omega T)^2} = \begin{cases} 0, & \omega T \ll 1 \\ 20\log(\omega T) = 20(\log\omega+\log T), & \omega T \gg 1\end{cases}$$

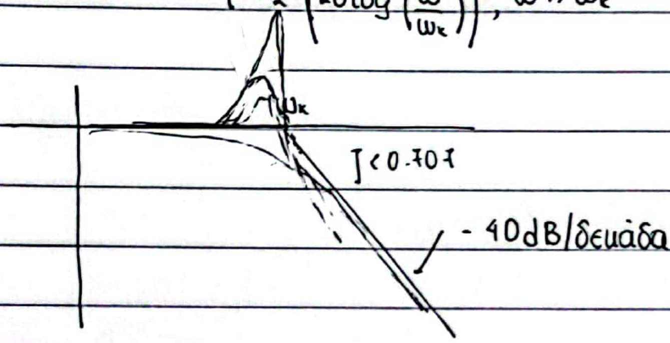


$\omega T = 1 \Rightarrow 20\log\sqrt{2} \simeq 3\,dB$



$$-20\log\left|\left(\frac{j\omega}{\omega_k}\right)^2+2\left(\frac{j\omega}{\omega_k}\zeta_k\right)+1\right| = 20\log\sqrt{\left(1-\left(\frac{\omega}{\omega_k}\right)^2\right)^2+4\zeta_k^2\left(\frac{\omega}{\omega_k}\right)^2} =$$

$$= \begin{cases} 0, & \omega \ll \omega_k \\ -2\cdot\left(20\log\left(\frac{\omega}{\omega_k}\right)\right), & \omega \gg \omega_k \end{cases}$$



$\zeta_k \in (0,1)$

$$M(x) = (1-x)^2 + 4\zeta^2 x, \quad x = \left(\frac{\omega}{\omega_k}\right)^2$$

$$\frac{dM}{dx} = -2(1-x)+4\zeta^2 = 2[x-(1-2\zeta^2)]$$

$$\zeta < \frac{1}{\sqrt{2}} = 0.707$$

$$\omega^* = \omega_k\sqrt{1-2\zeta^2}, \quad x^* = 1-2\zeta^2$$

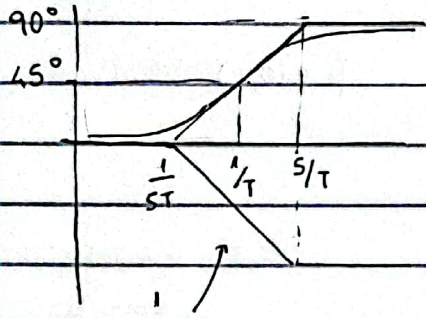
$$M(x^*) = (1-x^*)^2 + 4\zeta^2 x = (2\zeta^2)^2 + 4\zeta^2(1-2\zeta^2) = 4\zeta^2 - 4\zeta^4 = 4\zeta^2(1-\zeta^2)$$

$$\ldots \overset{\omega=\omega^*}{=} -20\log\sqrt{4\zeta^2(1-\zeta^2)}$$

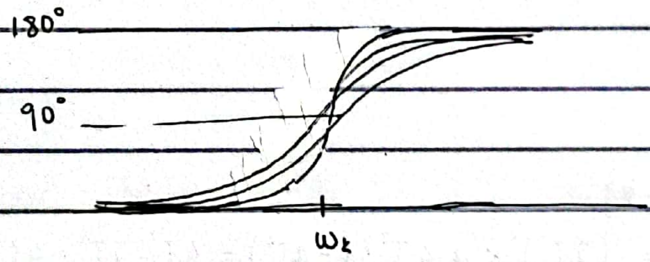
$j\omega$: φάση $= 90^\circ$

$\frac{1}{j\omega}$: φάση $= -90^\circ$

$1 + j\omega T$: φάση $= \tan^{-1}(\omega T) = \begin{cases} 0^\circ, & \omega T \ll 1 \\ 90^\circ, & \omega T \gg 1 \\ 45^\circ, & \text{ενδιάμεσα} \end{cases}$



$$1 + 2j\frac{\omega}{\omega_k}\zeta_k + \left(\frac{\omega}{\omega_k}\right)^2 = \left(1 - \left(\frac{\omega}{\omega_k}\right)^2\right) + j\,\frac{2\omega\zeta_k}{\omega_k}$$



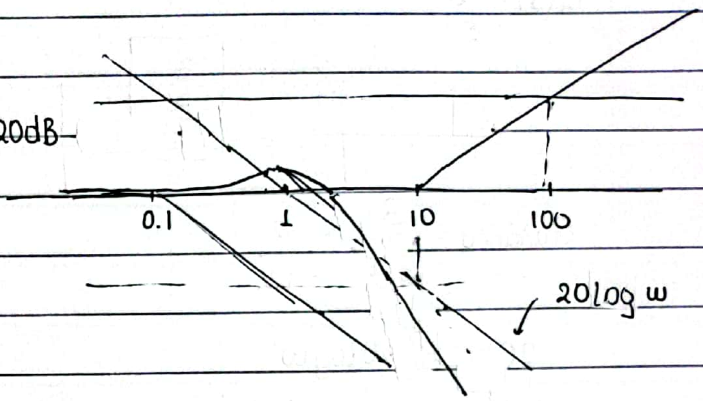
$$\tan^{-1}\left(\frac{\frac{2\omega\zeta_k}{\omega_k}}{1 + \left(\frac{\omega}{\omega_k}\right)^2}\right)$$

(πιο απότομο όσο $\zeta_k \uparrow$)

Για gain margin : κοιτάζω που τέμνει τα 0dB

Για phase —״— : τα -180°

$$G(s) = \frac{10\,(0.1s+1)}{s\left(s^2+\frac{s}{2}+1\right)(10s+1)}$$



$20 \log 10 = 20dB$

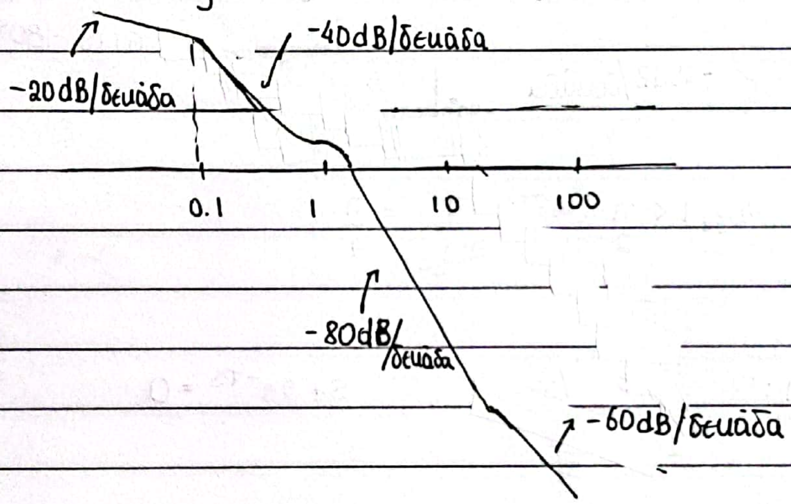
$\frac{1}{s} \rightarrow -20\log\omega$

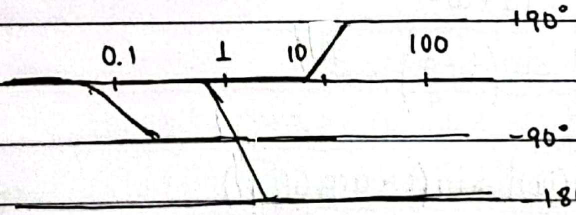
$s^2+\frac{s}{2}+1 \; : \; \omega_k = 1$

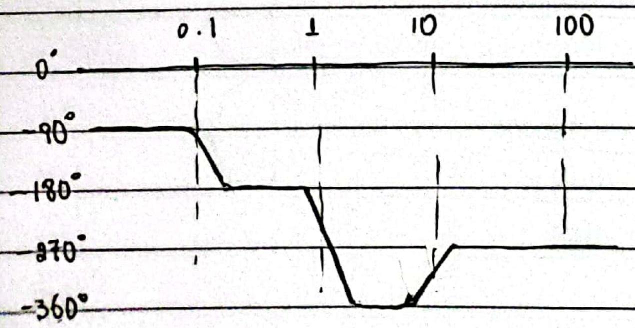
$\frac{2\zeta_k}{\omega_k} = \frac{1}{2} \Rightarrow \zeta_k = \frac{1}{4} = 0.25 < 0.707$

$\omega^* = \omega_k\sqrt{1-2\zeta^2} = \sqrt{\frac{7}{8}}$

$-20\log\sqrt{4\zeta^2(1-\zeta^2)} \approx +20\log 2 = 6dB$







Scanned with CamScanner