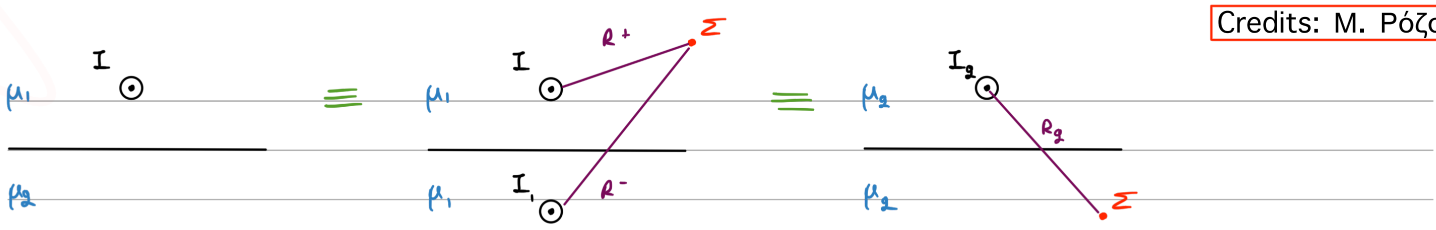
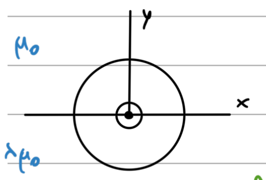
Credits: Μ. Ρόζος

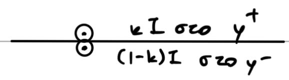


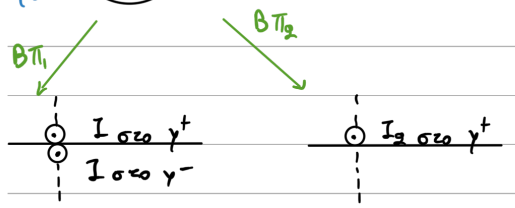
$$I_1 = \frac{\mu_2-\mu_1}{\mu_2+\mu_1} I \quad , \quad I_2 = \frac{2\mu_2}{\mu_2+\mu_1} I$$

Παράδειγμα 1

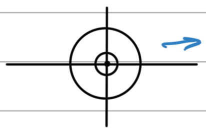


β' τρόπος →





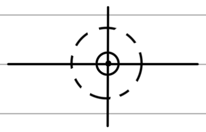
$B\Pi_1$ $\quad I_1 = \frac{\lambda-1}{\lambda+1} I$



$$\rightarrow I_{o\lambda} = I + \frac{\lambda-1}{\lambda+1} I = \frac{2\lambda}{\lambda+1} I$$

$$2\pi r H_1 = \frac{2\lambda}{\lambda+1} \Rightarrow \vec{H}_1 = \frac{2\lambda}{\lambda+1} \cdot \frac{I}{2\pi r} \Longrightarrow \vec{B}_1 = \frac{2\lambda}{\lambda+1} \cdot \frac{\mu_0 I}{2\pi r}$$

$B\Pi 2$ $\quad I_2 = \frac{2}{\lambda+1} I$

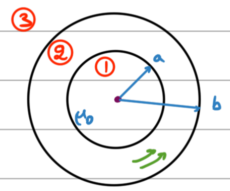


$$\vec{H}_2 = \frac{2}{\lambda+1} \cdot \frac{I}{2\pi r} \Rightarrow \vec{B}_2 = \frac{2}{\lambda+1} \cdot \frac{\lambda\mu_0 I}{2\pi r}$$

(Μπορούμε να πάρουμε $\oint_c \vec{H}\cdot d\vec{l} = I$ ...)

- $\oint_c \vec{B}\, d\vec{l} = \mu_0 (I + I_M) + \mu_0 I$, $\quad$ • $I_M = \lim_{r\to 0} \left[r \int_0^{2\pi} \vec{M}\cdot\hat{\varphi}\, d\varphi \right]$
- $\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \frac{\lambda-1}{\lambda+1} \cdot \frac{I}{\pi r}\, \hat{\varphi}$

Παράδειγμα 3 ¡¡SOS!! (ίδιο παρ. με πόλωση!)



$$\vec{M}_2 = M_0 \frac{r^2}{a^2} \hat{\varphi}$$

$$\vec{A}(r=a) = 0$$

B, H, A ?

- $\vec{J}_{M_1} = \vec{J}_{M_3} = 0$
- $\vec{J}_{M_2} = \nabla\times\vec{M}_2 = \frac{1}{r}\frac{d}{dr}\left(r M_0 \frac{r^2}{a^2}\right) = \frac{3 M_0 r}{a^2} \hat{z}$
- $\vec{K} = \hat{n} \times (\vec{M}_2 - \vec{M}_1)$
- $\vec{K}(r=a) = \hat{r} \times (\vec{M}_2 - \cancel{\vec{M}_1}) = M_0\, \hat{z}$
- $\vec{K}(r=b) = \hat{r} \times (\cancel{\vec{M}_3} - \vec{M}_2) = -M_0 \frac{b^2}{a^2} \hat{z}$

↳ Επίλυση με ολοκληρωτικές για το $\vec{B}$: $\quad \partial/\partial\varphi = 0 = \partial/\partial z$

- $\oint_c \vec{B}\cdot d\vec{l} = \mu_0 (\cancel{I}^{\,0} + I_\mu)$
- $B_1 \cdot 2\pi r = \mu_0 \int_0^r \cancel{J_{M_1}}^{\,0} \cdot 2\pi r'\,dr' \Rightarrow \vec{B}_1 = 0$
- $B_2 \cdot 2\pi r = \mu_0 K_m(r=a)\cdot 2\pi a + \mu_0 \int_a^r J_{M_2}\cdot 2\pi r'\,dr' \Rightarrow \vec{B}_2 = \mu_0 M_0 \frac{r^2}{a^2}\hat{\varphi}$
- $B_3 \cdot 2\pi r = \mu_0 K_m(r=a)\cdot 2\pi a + \mu_0 \int_a^b J_{M_2}\cdot 2\pi r'\,dr' + \mu_0 K_m(r=b)\cdot 2\pi b = 0 \Rightarrow \vec{B}_3 = 0$

- $\vec{B} = \mu_0(\vec{H}+\vec{M})$, $\quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

$\vec{H}_1 = 0$, $\quad \vec{H}_2 = \frac{\vec{B}_2}{\mu_0} - M_2 = 0$, $\quad \vec{H}_3 = 0$

- $\vec{A} = A(r)\,\hat{z}$ (διότι τα ρεύματα ρέουν ως προς z)

↳ $\vec{\nabla}\times\vec{A} = \vec{B} = -\frac{dA}{dr}\hat{\varphi} \Rightarrow A_{1,2,3} = -\int B_{1,2,3}\,dr + c_{1,2,3}$

- $A_1 = c_1$, $\quad$ • $A_2 = -\mu_0 M_0 \frac{r^3}{3a^2} + c_2$, $\quad$ • $A_3 = c_3$

↳ $A_1(r=a) = 0 \Rightarrow c_1 = a$

↳ $A_2(r=a) = -\mu_0 M_0 \frac{a}{3} + c_3 = 0 \Rightarrow c_2 = \mu_0 M_0 \frac{a}{3}$

↳ $A_3(r=b) = A_2(r=b) \Rightarrow c_3 = -\mu_0 M_0 \frac{b^3 - a^3}{3a^2}$

Κεφάλαιο 6: Laplace

$\mathcal{L}\phi = g \qquad \mathcal{L}$: διαφορικός ή ολοκληρωτικός τελεστής

- $a\phi_{xx} + b\phi_{xy} + c\phi_{yy} + d\phi_x + e\phi_y + f\phi = g$, $\quad \phi_{xx} = \frac{\partial^2\phi}{\partial x^2}$

a, b, c: κατηγοριοποίηση των ΜΔΕ

- Γενικώς: $a = a(\phi)$, $b = b(\phi)$, $c = c(\phi)$ μη γραμμική

↳ $b^2 - 4ac$:
  - <0: ελλειπτική
  - >0: υπερβολική
  - $=0$: παραβολική

Η κατηγοριοποίηση αναφέρεται σε 2Δ, a, b, c σταθερές

a, b, c: συνάρτηση της $\vec{r}$

- $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$ Laplace (*)
  ΗΣ προβλήματα
  ΜΣ προβλήματα

  $\boxed{\text{κλειστό χωρίο}}$

  $a=1$, $c=1$, $b=0$, $\Delta<0 \rightarrow$ ελλ. τύπου

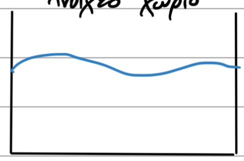
- $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = g(x,y)$ Poisson $\quad g = \frac{\rho}{\varepsilon}$ (*)

↳ Θα δούμε 2Δ, 3Δ → Καρτεσιανές συντεταγμένες

• $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+k^2\phi=0$: Helmholtz

↳ Υπερβολικού τύπου: $k^2\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial t^2}=0$ κυματική εξίσωση

↳ Παραβολικού τύπου: $\frac{\partial\phi}{\partial x}-k\frac{\partial\phi}{\partial t}=0$ εξίσωση διάχυσης



3Δ

$\rho=0 \Rightarrow \nabla^2\phi=0$, $\phi(x,y,z)=X(x)\cdot Y(y)\cdot Z(z) \Rightarrow \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \Rightarrow$

$$\Rightarrow yz\frac{d^2X}{dx^3}+xz\frac{d^2Y}{dy^2}+xy\frac{d^2Z}{dz^2}=0 \underset{\frac{1}{xyz}}{\Rightarrow} \underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{K_x}+\underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{K_y}+\underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{K_z}=0 \Rightarrow K_x+K_y+K_z=0$$

• $X=A_1x+A_2$, $K_x=0$

• $X=A_3\cos k_x x+A_4\sin k_x x$, $K_x<0$, $K_x=-k_x^2<0$
$\quad=\hat{A}_3 e^{jk_x x}+\hat{A}_4 e^{-jk_x x}$

• $X=A_5 e^{k_x x}+A_6 e^{-k_x x}$, $K_x>0$, $K_x=k_x^2>0$
$\quad=\hat{A}_5\cosh(k_x\cdot x)+\hat{A}_6\sinh(k_x\cdot x)$

2Δ

$K_z=0$, $K_x+K_y=0$

$\phi=XY=(A_1x+A_2)(B_1y+B_2)=Axy+Bx+\Gamma y+\Delta$, $K_x=K_y=0$

$$\phi=C\left\{\begin{array}{c} e^{\pm} \\ \sinh \\ \cosh \\ \hline \sin \\ \cos \\ e^{\pm j}\end{array}\right\}(k_x)\left\{\begin{array}{c} e^{\pm} \\ \sinh \\ \cosh \\ \hline \sin \\ \cos \\ e^{\pm j}\end{array}\right\}(k_y),\quad k_x=k_y=k$$