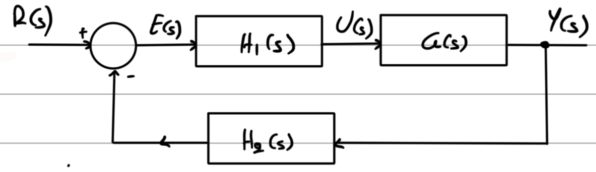



⋮

Συνέχεια από προηγούμενη διάλεξη

⋮

Έστω ότι $z_1=\dots=z_a=0$, $p_1=\dots=p_b=0$. Τότε $a(s)=s^a\prod_{j=b+1}^{m}(s-p_j)$

$$\Pi(s)=s^b\prod_{j=a+1}^{n}(s-z_j)$$

$G_{av}(s)=\hat{k}\dfrac{\bar{a}(s)}{s^{l}\,\bar{\pi}(s)}$, $\bar{a}(s)=\prod_{i=a+1}^{m}(s-z_i)$, $\bar{\Pi}(s)=\prod_{j=b+1}(s-p_j)$, $l=b-a\geq 0$: σύστημα τάξης

Σταθερά σφάλματος για είσοδο $R(s)$: $K=\lim_{s\to 0}\left\{\dfrac{G_{av}(s)}{s\,R(s)}\right\} \Rightarrow e_{ss}=\lim_{s\to 0}\left\{\dfrac{s\,R(s)}{1+G_{av}(s)}\right\}=\lim_{s\to 0}\left\{\dfrac{1}{\frac{1}{sR(s)}+\frac{G_{av}(s)}{sR(s)}}\right\}=$

$$=\frac{1}{k+\lim_{s\to 0}\left\{\frac{1}{sR(s)}\right\}}$$

$$e_{ss}=\lim_{s\to 0}\{s\,E(s)\}=\lim_{s\to 0}\left\{\frac{s\,R(s)}{1+H_1(s)G(s)}\right\} \quad (1)$$

i) Σφάλμα θέσης: $r(t)=u(t)$, $R(s)=1/s$

(1) $\Rightarrow$ $e_{ss,\theta}=\lim_{s\to 0}\left\{\dfrac{1}{1+G_{av}(s)}\right\}=\dfrac{1}{1+G_{av}(0)}=\begin{cases}\dfrac{\bar{\pi}(0)}{\bar{\pi}(0)+\hat{k}\,\bar{a}(0)}, & l=0\\ 0, & l\geq 1\end{cases}$

$K_\theta=\lim_{s\to 0}\{G_{av}(s)\}=G_{av}(0)=\begin{cases}\hat{k}\dfrac{\bar{a}(0)}{\bar{\pi}(0)}, & l=0\\ \infty, & l\geq 1\end{cases}$

Άρα: $e_{ss,\theta}=\dfrac{1}{1+K_\theta}$

ii) Σφάλμα ταχύτητας: $r(t)=t\cdot u(t) \xRightarrow{\mathcal{L}} R(s)=1/s^2$

(1) $\Rightarrow$ $e_{ss,\tau}=\lim_{s\to 0}\left\{\dfrac{1}{s(1+G_{av}(s))}\right\}=\lim_{s\to 0}\left\{\dfrac{1}{s\left(1+\hat{k}\frac{\bar{a}(0)}{\bar{\pi}(0)}\right)}\right\}=\begin{cases}\infty, & l=0\\ \dfrac{1}{\hat{k}}\dfrac{\bar{a}(0)}{\bar{\pi}(0)}, & l=1\\ 0, & l\geq 2\end{cases}$

$K_\tau=\lim_{s\to 0}\{s\,G_{av}(s)\}=\begin{cases}\infty, & l=0\\ \hat{k}\dfrac{\bar{a}(0)}{\bar{\pi}(0)}, & l=1\\ 0, & l\geq 2\end{cases}$

$\Rightarrow e_{ss,\tau}=\dfrac{1}{K_\tau}$

iii) Σφάλμα επιτάχυνσης: $r(t)=\dfrac{t^2}{2}u(t) \xRightarrow{\mathcal{L}} R(s)=1/s^2$

$e_{ss,\varepsilon}=\dots=\begin{cases}\infty, & l=0,1\\ \dfrac{1}{\hat{k}}\dfrac{\bar{a}(0)}{\bar{\pi}(0)}, & l=2\\ 0, & l\geq 3\end{cases}$

$K_\varepsilon=\dots=\begin{cases}\infty, & l=0,1\\ \hat{k}\dfrac{\bar{a}(0)}{\bar{\pi}(0)}, & l=2\\ 0, & l\geq 3\end{cases}$

$\Rightarrow e_{ss,\varepsilon}=\dfrac{1}{K_\varepsilon}$

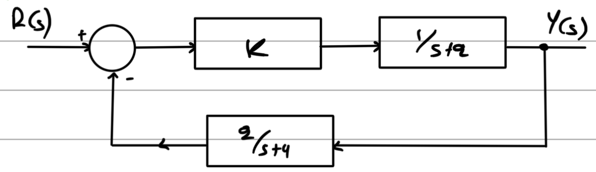
β) Αν $H_2(s) \neq 1$

$$e_{ss} = \lim_{t\to 0}\{\hat{e}(t)\} = \lim_{s\to 0}\{s\hat{E}(s)\} = \lim_{s\to 0}\{[1-G_{κλ}(s)]sR(s)\} \quad (2)$$

**Μηδενισμός σφάλματος θέσης:** $e_{ss,\theta} = \lim_{s\to 0}\{1-G_{κλ}(s)\} = 1 - G_{κλ}(0)$, άρα $e_{ss,\theta}=0 \Rightarrow G_{κλ}(0)=1$

$$G_{κλ}(s) = \frac{Y(s)}{R(s)} = \frac{G_{av}(s)}{H_2(s)[1+G_{av}(s)]} = \frac{H_1(s)G(s)}{1+H_1(s)H_2(s)G(s)} \quad (3)$$

**Παράδειγμα**




$H_1(s)=k$, $H_2(s) = 2/s+4$, $G(s) = 1/s+2$

$$G_{κλ}(s) = \frac{\frac{k}{s+2}}{1+\frac{2k}{(s+2)(s+4)}} = \frac{k(s+4)}{(s+2)(s+4)+2k}$$

Σφάλμα θέσης: $e_{ss,\theta} = \lim_{s\to 0}\left\{[1-G_{κλ}(s)]\frac{s}{s}\right\} = 1 - G_{κλ}(0) = 1-\frac{4k}{8+2k} = \frac{4-k}{4+k}$

Σφάλμα ταχύτητας: $e_{ss,v} = \lim_{s\to 0}\left\{[1-G_{κλ}]\frac{s}{s^2}\right\} = \lim_{s\to 0}\left\{\frac{(s+2-k)(s+4)+2k}{s[(s+2)(s+4)+2k]}\right\} = \infty$

**Χρονική απόκριση διακριτού χρόνου (Δ.Χ.)**

$x((k+1)T) = Ax(kT) + Bu(kT)$, $k \in N$ (1.1)

$y(kT) = Cx(kT) + Du(kT)$, $k \in N$ (1.2)

(1.1) $\Rightarrow$ $x(T) = Ax(0) + Bu(0)$

$x(2T) = Ax(T) + Bu(T) = A^2x(0) + ABu(0) + Bu(T)$

$x(3T) = Ax(2T) + Bu(2T) = A^3x(0) + A^2Bu(0) + ABu(0) + Bu(2T)$

$\vdots \qquad \vdots \qquad \vdots$

$x(kT) = A^k x(0) + \sum_{i=0}^{k-1} A^{k-i-1}Bu(iT)$ (2.1)

(1.2) $\Rightarrow$ $y(kT) = CA^k x(0) + C\sum_{i=0}^{k-1}\left[A^{k-i-1}Bu(iT)\right] + Du(kT)$ (2.2)

**Μετασχηματισμός Z:**

$$Z\{f(kT)\} = \sum_{k=0}^{\infty} f(kT)\cdot z^{-k} \triangleq F(z)$$

**Ιδιότητες μ/σ Z:**

- $Z\{f(k+m)T\} = z^m\left[F(z) - \sum_{j=0}^{m-1} f(jT)z^{-j}\right]$
- $Z\{f(k+1)T\} = z[F(z) - f(0)]$
- $A^k = Z^{-1}\{z[zI-A]^{-1}\}$
- $Z\{x(k+1)T\} = z[X(z) - x(0)]$, όπου $X(z) = Z\{x(kT)\}$, $U(z) = Z\{u(kT)\}$

$(1.1) \xRightarrow{Z} z[X(z) - x(0)] = AX(z) + BU(z) \Rightarrow [zI - A]X(z) = zx(0) + BU(z) \Rightarrow$

$\Rightarrow X(z) = [zI - A]^{-1}x(0) + [zI - A]^{-1}BU(z)$ (3.1)

$(1.2) \xRightarrow{Z} Y(z) = C \cdot X(z) + DU(z) \overset{(3.1)}{=} zC[zI - A]^{-1}x(0) + [C[zI - A]^{-1}B + D]U(z)$ (3.2)

Μήτρα συναρτήσεων μεταφοράς $G(z) = C \cdot [zI - A]^{-1}B + D$ (3.3)

άρα αν $x(0) = 0$ τότε $Y(z) = G(z)U(z)$