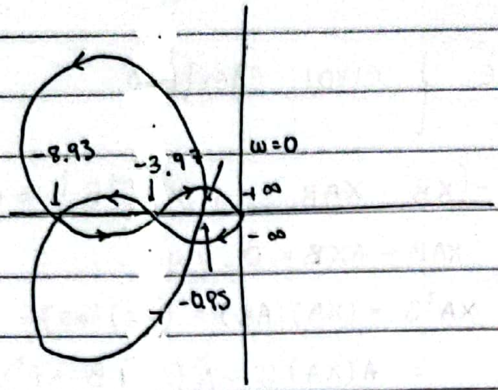
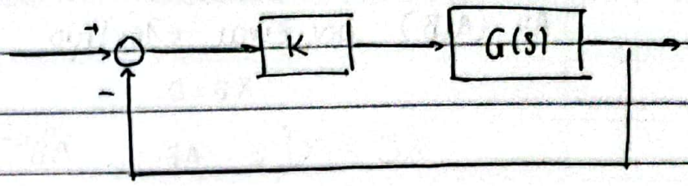
Δευτέρα, 16/01/2023

$$G(s) = \frac{0.1s^2 + 0.35s - 1}{s^4 + 0.2s^3 + 1.97s^2 + 0.2s + 1.05}$$





$$G(0) = -\frac{1}{1.05}$$

1) $-\frac{1}{K} < -8.93 \Rightarrow 0 < K < \frac{1}{8.93}$, $\underline{N=0}$

2) $-8.93 < -1/K < -3.97 \Rightarrow \frac{1}{8.93} < K < \frac{1}{3.97}$, $\underline{N=-2}$ $\Rightarrow Z = N+P = 0$ ευσταθές

3) $-3.97 < -1/K < -0.95 \Leftrightarrow \frac{1}{3.97} < K < \frac{1}{0.95} = 1.05$, $\underline{N=0}$

4) $-0.95 < -1/K < 0 \Rightarrow 1.05 < K$, $\underline{N=+1}$

5) $0 < -1/K \Rightarrow K < 0$, $\underline{N=0}$

$Z = N+P$

4 ρίζες

$P \geq 2$ κ' $P < 4$

↳ (γιατί $N=-2$)

$1.05 > 0 \Rightarrow$ άρτιος αριθμός στο ΔΜΗ $\Rightarrow P=2$

s^4	1	1.97	1.05
s^3	0.2	0.2	
s^2	0.97	1.05	
s^1	$0.2\left(1 - \frac{1.05}{0.97}\right)$	...	
s^0	1.05		

$Z = N+P = 2$ ασταθ. πόλοι

Αν $\begin{cases} AX = XA \\ XB = 0 \end{cases}$ έχει μον. λύση το $X = 0$, τότε (A, B) ελέγξιμο

Αν $\exists\, X \neq 0$ $\quad \begin{matrix} XA = AX \\ XB = 0 \end{matrix} \Rightarrow (A, B)$ δεν είναι ελέγξιμο

$$X\mathcal{C} = X[B \quad AB \ \dots \ A^{n-1}B] = [XB \quad XAB \ \dots \ XA^{n-1}B] = 0$$

$$XAB = AXB = 0$$

$$XA^2B = (XA)(AB) = (AX)(AB) =$$

$$= A(XA)B = A(AX)B = A^2XB = 0$$

$\Rightarrow \mathrm{rank}(\mathcal{C}) < n$ μη ελέγξιμο

Έστω ότι $\begin{cases} XA = AX \\ XB = 0 \end{cases}$ έχει μον. λύση $\overset{X=0}{\wedge}$ κ' (A, B) μη ελέγξιμο

$\cdot \exists \lambda_i$ μη ελέγξιμη

Έστω w_i δεξί ιδιοδιάνυσμα κ' v_i αριστερό

$$Aw_i = \lambda_i w_i$$

$$v_i^T A = \lambda_i v_i^T$$

$\underline{v_i^T B = 0}$ (λ_i μη ελέγξιμη)

Θ.δ.ο. ο $X = w_i v_i^T$ ικανοποιεί τις εξισώσεις

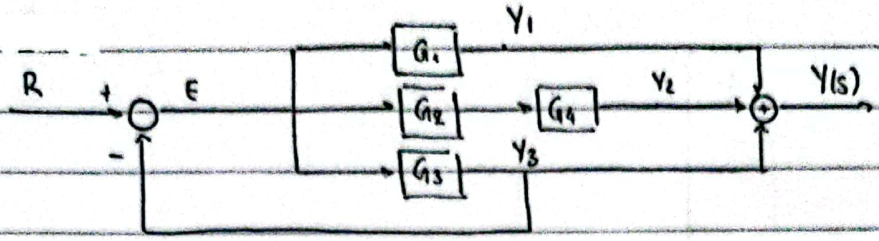
$$AX = Aw_i v_i^T = \lambda_i w_i v_i^T$$

$$XA = w_i v_i^T A = \lambda_i w_i v_i^T = AX$$

$$XB = w_i v_i^T B = 0$$

$$\|X\|_2^2 = \sup_{\|u\|=1} \|Xu\|^2 = \sup_{\|u\|=1} u^T X^T X u = \sup_{\|u\|=1} u^T v_i w_i^T w_i v_i^T v \geq 1$$

(Π2020)



$$Y(s) = Y_1(s) + Y_2(s) + Y_3(s) = G_1 E(s) + G_2 G_4 E(s) + G_3 E(s)$$

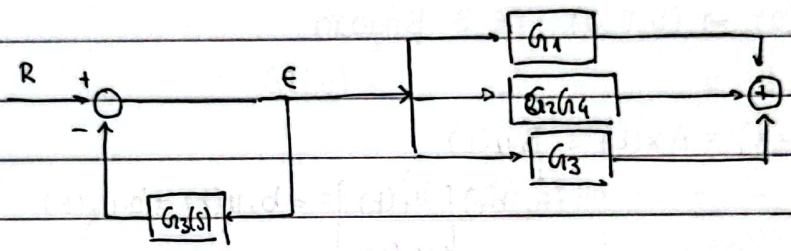
$$E(s) = R(s) - Y_3(s) = R(s) - G_3(s) E(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G_3(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{(G_1 + G_2 G_4 + G_3)}{1 + G_3(s)}$$

β' τρόπος:

ισοδύναμα:



$$\frac{Y(s)}{R(s)} = \frac{1}{1 + G_3(s)} \cdot (G_1 + G_2 G_4 + G_3)$$

$$A = \begin{bmatrix} 1 & \cdots & 1 \\ 1 & & \\ \vdots & & \\ 1 & & 1 \end{bmatrix}_{2023 \times 2023}$$

$e^{At} = ;$

Έχει 2022 φορές ιδιοτιμή το 0

$x_1(k+1) = x_2(k) + u_1(k)$

$x_2(k+1) = x_3(k) + u_1(k)$

$x_3(k+1) = x_1(k) + u_1(k) + u_2(k)$

$x(0) \to (0,0,0)$ σε 3 βήματα

$x(k+1) = Ax(k) + Bu(k)$

$$[b_1 \ b_2]\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = b_1 u_1(k) + b_2 u_2(k)$$

(A, b_1) ή (A, b_2) ελέγξιμο

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \det(\lambda I - A) = \lambda^3 - 1 = (\lambda - 1)(\lambda^2 + \lambda + 1)$$

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

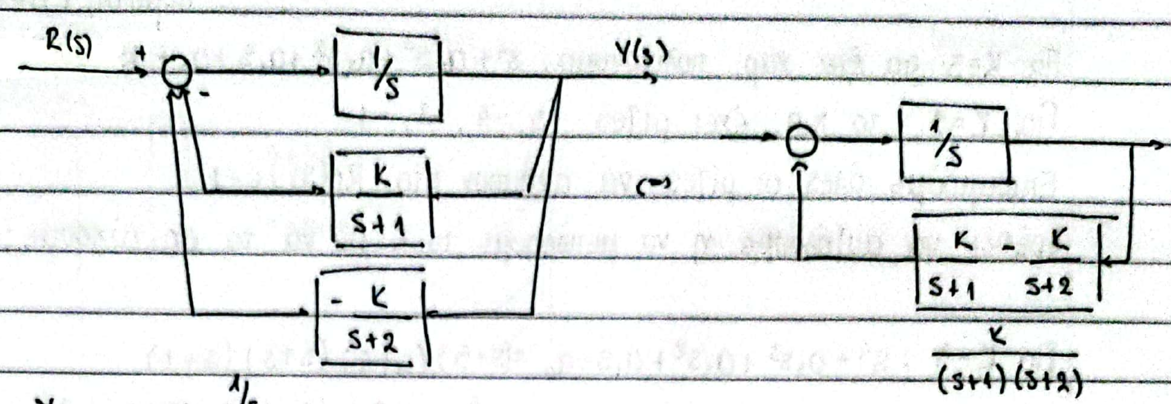
$$\mathcal{C}_1 = [b_1 \ Ab_1 \ A^2 b_1] = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \ \text{rank}\, \mathcal{C}_1 = 1 \Rightarrow \text{όχι ελέγξιμο από } u_1$$

$$\mathcal{C}_2 = [b_2 \ Ab_2 \ A^2 b_2] = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{rank}\, \mathcal{C}_2 = 3 \Rightarrow \text{ελέγξιμο}$$

$$x(3) = Ax(2) + b_2 u_2(2) = A(Ax(1) + b_2 u_2(2)) + b_2 u_2(2) =$$

$$= A^2 x(1) + Ab_2 u_2(2) + b_2 u_2 = A^2(Ax(0) + b_2 u_2(2)) + Ab_2 u_2(1) + b_2 u_2(2)$$

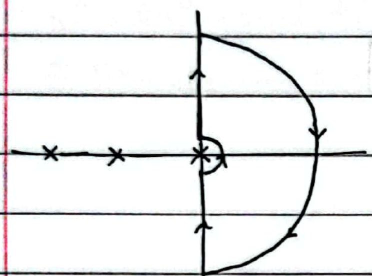
$$x(3) = A^3 x(0) + [b_2 \ Ab_2 \ A^2 b_2]\begin{bmatrix} u_2(2) \\ u_2(1) \\ u_2(0) \end{bmatrix} \to U = -\mathcal{C}^{-1} A^3 x(0)$$



$$\frac{Y}{R} = \frac{1/s}{1+\frac{1}{s}\frac{K}{(s+1)(s+2)}}$$

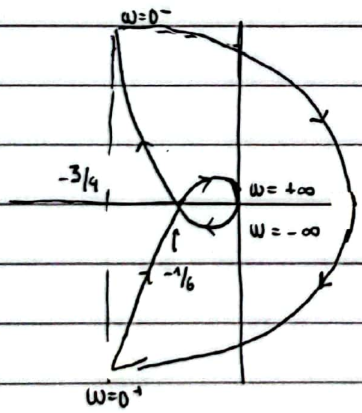
Χαρ. εξίσωση:

$$1 + K\frac{1}{s(s+1)(s+2)} = 0$$



$$G(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)} = \frac{-j}{\omega}\frac{(1-j\omega)(2-j\omega)}{(1+\omega^2)(4+\omega^2)}$$

$$= -j\frac{[(2-\omega^2)-3j\omega]}{[\quad\ldots\quad]} = \frac{-3}{(1+\omega^2)(4+\omega^2)} - j\frac{2-\omega^2}{(1+\omega^2)(4+\omega^2)}$$



$\omega^{+} = \sqrt{2}$ $\quad G(j\omega^{+}) = \frac{-3}{3\cdot 6} = -1/6$

$s = \epsilon e^{j\varphi}$, $-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$, φ αυξάνει

$$G(s) = \frac{1}{\epsilon e^{j\varphi}(1+\epsilon e^{j\varphi})(2+e^{j\varphi})} \simeq \frac{1}{\epsilon}e^{-j\varphi}$$

$P=0$

1) $-\frac{1}{K} < -\frac{1}{6} \Rightarrow 0<K<6$ $\quad N=0$ $\quad \rightarrow Z=0$ ευσταθές

2) $-1/6 < -1/K < 0$ $\quad N=+2$

3) $0 < -1/K$ $\quad N=+1$

s^3	1	2
s^2	3	K
s^1	$2-K/3$	
s^0	K	