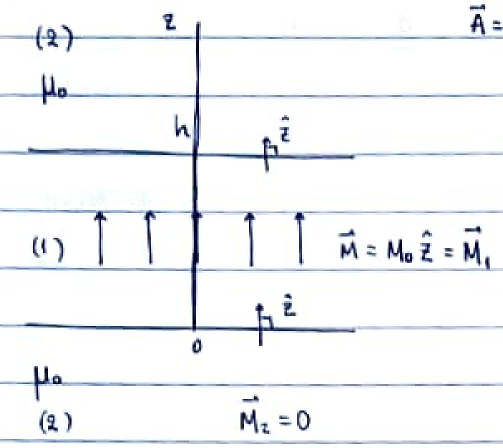
Τρίτη, 20/12/2022

Παράδειγμα 1

$\vec{A} = ;$

Αναφορά : $\vec{A}(z=0)=0$




Επίλυση με εξισώσεις $\vec{B}$

$$\vec{J}_{M_1} = \vec{\nabla} \times \vec{M}_1 = 0$$

$$\vec{J}_{M_2} = 0$$

$$\vec{K}_M(z=0) = \hat{z} \times (\vec{M}_1 - \cancel{\vec{M}_2}^{\,0}) = 0$$

$$\vec{K}_M(z=h) = \hat{z} \times (\vec{M}_2 - \vec{M}_1) = 0$$

Σημειακές σχέσεις $\vec{B}$ : $\nabla \times \vec{B}_{1,2} = \cancel{\mu_0 \vec{J}_{1,2}}^{\,0} + \underbrace{\mu_0 \vec{J}_{M_{1,2}}}_{\nabla \times \vec{M}_{1,2}} = 0$

$$\nabla \cdot \vec{B}_{1,2} = 0$$

$$\hat{n} \times (\vec{B}_2 - \vec{B}_1) = \mu_0 \vec{K} + \mu_0 \vec{K}_M = 0 \quad (\text{για } z=0 \text{ κ' } z=h)$$

$$\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \quad (\text{για } z=0, h)$$

Άρα $\vec{B}_{1,2} = 0$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \Rightarrow \vec{H}_{1,2} = \frac{\vec{B}_{1,2}}{\mu_0} - \vec{M} \Rightarrow \vec{H}_1 = -M_0 \hat{z}, \ \vec{H}_2 = 0$$

$[\vec{M}] = [\vec{H}] = A/m$

$\vec{B} = \nabla \times \vec{A}$ , $\partial/\partial x, \partial/\partial y = 0$

$$\nabla \times \vec{A} = \vec{B} \Rightarrow -\frac{\partial A_y}{\partial z} \hat{x} + \frac{\partial A_x}{\partial z} \hat{y} = 0$$

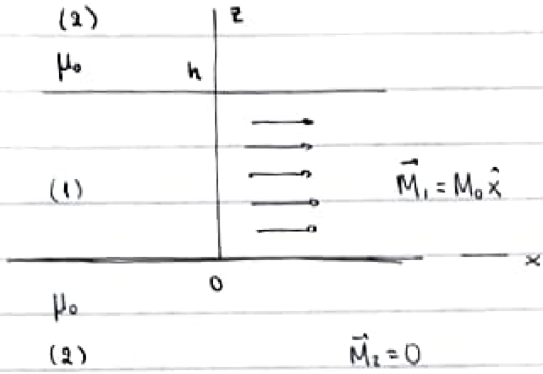
$$\nabla \cdot \vec{A} = 0 \Rightarrow \frac{\partial A_z}{\partial z} = 0$$

(συνθήκη Coulomb)

$$\left\{ \begin{array}{l} \dfrac{\partial A_x}{\partial z} = 0 \\ \dfrac{\partial A_y}{\partial z} = 0 \\ \dfrac{\partial A_z}{\partial z} = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} A_x = C_x \\ A_y = C_y \\ A_z = C_z \end{array} \right.$$

$C_{x_1, y_1, z_1} = 0$ , $\vec{A}_1(z=0) = 0 \Leftrightarrow (\underset{C_{x_1}}{A_{x_1}}, \underset{C_{y_1}}{A_{y_1}}, \underset{C_{z_1}}{A_{z_1}}) = 0$

$\vec{A}_2(z<0) = 0 \Rightarrow C_{x_2, y_2, z_2} = 0$ , $\vec{A}_3(z>h) = 0$




$\vec{B} = ; \rightarrow \vec{A} = ;$

$$\vec{J}_{M_1} = \nabla \times \vec{M}_1 = \frac{\partial M_x}{\partial z}\hat{y} - \frac{\partial M_x}{\partial y}\hat{z} = 0$$

$\vec{J}_{M_2} = 0$ (διότι $\vec{M}_2 = 0$)

$\vec{K}_M(z=0) = \hat{z} \times (\vec{M}_1 - \vec{M}_2) = M_0 \hat{y}$

$\vec{K}_M(z=h) = \hat{z} \times (\vec{M}_2 - \vec{M}_1) = -M_0 \hat{y}$

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y} = 0$$

$$\nabla \times \vec{B}_{1,2} = 0 \Rightarrow -\frac{\partial B_y}{\partial z}\hat{x} + \frac{\partial B_x}{\partial z}\hat{y} = 0$$

$$\nabla \cdot \vec{B}_{1,2} = 0 \Rightarrow \frac{\partial B_z}{\partial z} = 0$$

$$\left.\begin{array}{l} \dfrac{\partial B_x}{\partial z} = 0 \\[2ex] \dfrac{\partial B_y}{\partial z} = 0 \\[2ex] \dfrac{\partial B_z}{\partial z} = 0 \end{array}\right\} \Rightarrow \vec{B}_{1\,x,y,z} = C_{1\,x,y,z}, \quad \vec{B}_{2\,x,y,z} = C_{2\,x,y,z}$$

$$z=0: \left.\begin{array}{l} \hat{n} \times [\vec{B}_1 - \vec{B}_2] = \mu_0 \cancel{\vec{K}}^{\,0} + \mu_0 \vec{K}_M = \mu_0 M_0 \hat{y} \\ \hat{z} \cdot (\vec{B}_1 - \vec{B}_2) = 0 \end{array}\right\} \Rightarrow \begin{array}{l} B_{1x} - B_{2x} = \mu_0 M_0 \\ -B_{1y} + B_{2y} = 0 \\ B_{1z} - B_{2z} = 0 \end{array}$$

$$z=h: \left.\begin{array}{l} \hat{z} \times [\vec{B}_2 - \vec{B}_1] = -\mu_0 M_0 \hat{y} \\ \hat{z} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \end{array}\right\} \Rightarrow \begin{array}{l} B_{2x} - B_{1x} = -\mu_0 M_0 \\ -B_{2y} + B_{1y} = 0 \\ B_{2z} - B_{1z} = 0 \end{array}$$

z - συνιστώσα : $B_{1z} = C_{1z}$ , $B_{2z} = C_{2z}$

κ' $B_{1,z}$ , $B_{2,z}$ συνεχή στα $z = 0, h$

Άρα $C_{1z} = C_{2z}$

y - συνιστώσα: Όμοιες παρατηρήσεις, άρα $C_{1y} = C_{2y}$

x - συνιστώσα: $B_{1x} = C_{1x}$ , $B_{2x} = C_{2x}$

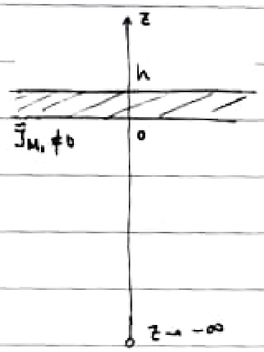
$z = 0$: $B_{1x} - B_{2x} = \mu_0 M_0 \Rightarrow C_{1x} - C_{2x} = \mu_0 M_0$

$z = h$: ίδια εξίσωση

Με $C_{2x}$ ελεύθερη παράμετρο

$B_{2x} = C_{2x}$

$B_{1x} = \mu_0 M_0 + C_{2x}$




Από το $z \to -\infty$

$$\vec{K}_\mu (z \to -\infty) = \underbrace{\vec{K}(z=0) + \vec{K}_\mu (z=h) + \int_0^h \cancel{\vec{J}_{M_1}}^{\,0} \cdot dz}_{\hat{\vec{K}}_M} = 0$$

$\nabla \times \vec{B}_2 = 0$

$\nabla \cdot \vec{B}_2 = 0$

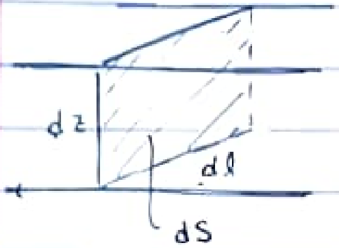
Ο.Σ. $\to$ 0 ρεύμα

$\nexists$ πηγές $\Rightarrow \vec{B}_2(-\infty) = 0 \Rightarrow C_{2x} = 0$

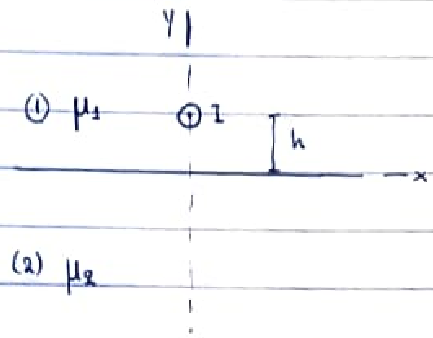
$\vec{B}_2 = 0$, $\vec{B}_1 = \mu_0 M_0 \hat{x}$

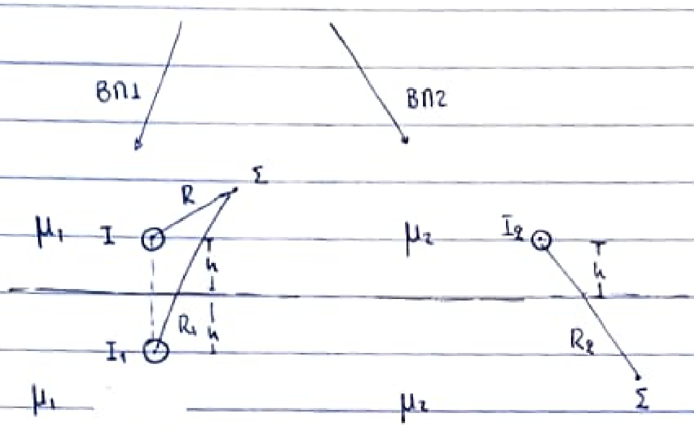
$$I = \int \vec{J}\cdot d\vec{S} = \int \vec{K}\cdot\hat{n}\, dl =$$

$$= \int_l \int_z \vec{J}\cdot\hat{n}\, dz\, dl$$




$$= \int_l \underbrace{\left[\int_z \vec{J}\cdot dz\right]}_{\vec{K}}\cdot\hat{n}\, dl$$




$(1.\vec{A},\ 2.\vec{B})$

<u>BΠ1</u> :


$$\vec{A}' = -\frac{\mu I}{2\pi} \ln r + C$$

$$R = \left[x^2 + (y-h)^2\right]^{1/2}$$

$$R_1 = \left[x^2 + (y+h)^2\right]^{1/2}$$

$$R_2 = R$$

$$\Sigma : A_{1,z} = -\frac{\mu_1 I}{2\pi} \ln R - \frac{\mu_1 I}{2\pi} \ln R_1 + C$$

<u>BΠ2</u> : $$A_{2z} = -\frac{\mu_2 I_2}{2\pi} \ln R_2 + C$$

1. Συνέχεια συνιστωσών του $\vec{A}$ : $\vec{n} \times (\vec{A}_1 - \vec{A}_2) = 0$

$$y=0 : \quad A_{1,z} = A_{2,z} \Rightarrow \boxed{\mu_1 I = \mu_2 I_2 - \mu_1 I_1 \ (1)}$$

2. $$\hat{y} \times [\vec{H}_1 - \vec{H}_2] = 0 \Rightarrow \hat{y} \times \left[\frac{\vec{B}_1}{\mu_1} - \frac{\vec{B}_2}{\mu_2}\right] = 0 \Rightarrow \hat{y} \times \left[\frac{\nabla \times \vec{A}_1}{\mu_1} - \frac{\nabla \times \vec{A}_2}{\mu_2}\right] = 0 \Rightarrow$$

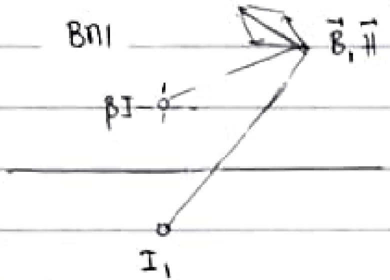
$$\nabla \times \vec{A}_z = \frac{\partial A_z}{\partial y} \hat{x} - \frac{\partial A_z}{\partial x} \hat{y}$$

$$\frac{\partial A_{z1}}{\partial y} = -\frac{\mu_1 I_1}{2\pi} \frac{R'_y}{R} - \frac{\mu_1 I_1}{2\pi} \frac{(R_1)'_y}{R_1}, \quad \frac{\partial A_{z2}}{\partial y} = -\frac{\mu_2 I_2}{2\pi} \frac{(R_2)'_y}{R_2}$$

Προκύπτει ότι $\boxed{I = I_1 + I_2 \quad (2)}$

Τελικά: $I_1 = \dfrac{\mu_2 - \mu_1}{\mu_2 + \mu_1} I$, $\quad I_2 = \dfrac{2\mu_1}{\mu_2 + \mu_1} I$

Υπόδειξη: άσκηση 2: χρησιμοποιώ την παραπάνω άσκηση




Επαλληλία νόμων Ampere

άθροισμα σε $\hat{\varphi}_1, \hat{\varphi}_2$