15.03.22

Tuesday, 15 March 2022 12:51 PM

• **Άσκηση**

$g(t)=te^{-at}u(t)$, $G(f)=;$

$-j2\pi t\, x(t) \rightleftharpoons \frac{d G(f)}{df}$

$e^{-at}u(t) \rightleftharpoons \frac{1}{a+j2\pi f}$ ⇒ $\boxed{G(f)=\frac{1}{(a+j2\pi f)^2}}$

• **Άσκηση**

$y(t)=x(t/4)+x^2(t/3)+x^4(t/2)$

$EZ_x=W$, $x(t)\to LP$ $EZ_y=;$

• $y_1(t)=x(t/4)$

$y_1(t) \rightleftharpoons 4X(4f) \Rightarrow EZ_{y_1}=\frac{W}{4}$

κλιμάκωση

• $y_2^2(t)=x^2(t/3)$

$y_2(t)=x(t/3) \Rightarrow EZ_{y_2}=\frac{W}{3} \Rightarrow EZ_{(y_2)^2}=\frac{2W}{3}$

• $y_3^4(t)=x^4(t/2)$

$y_3(t)=x(t/2)\Rightarrow EZ_{y_3}=\frac{W}{2}\Rightarrow EZ_{(y_3)^2}=2W$

Άρα $\boxed{EZ_y=\max(EZ_{y_1}, EZ_{(y_2)^2}, EZ_{(y_3)^4})=2W}$

• **Άσκηση**

$G(f)=\begin{cases}1, & f>0\\ 1/2, & f=0\\ 0, & f<0\end{cases}$ $g(t)=;$ → λύνεται και με χρήση του ορισμού ή με παραγώγιση

$G(f)=u(f)$

$u(t)\rightleftharpoons \frac12\delta(f)+\frac{1}{j2\pi f}$

$G(f)=g(-f)$

⇒ $\frac12\delta(t)+\frac{1}{j2\pi t}\rightleftharpoons u(-f)=1-u(f)$ ⇒ $\delta(f)$

$-\frac12\delta(t)+\frac{1}{j2\pi t}\rightleftharpoons -u(f)$ ⇒

$\boxed{\frac12\delta(t)-\frac{1}{j2\pi t}\rightleftharpoons u(f)}$

• **Άσκηση**

$g(t)=e^{-t}\cdot \sin(2\pi f_c t)\cdot u(t)$ $G(f)=;$

$e^{-t}u(t)\rightleftharpoons\frac{1}{1+j2\pi f}$

$\sin(2\pi f_c t)\rightleftharpoons \frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$

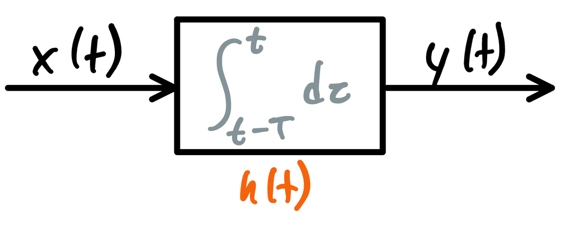
$x\cdot y \rightleftharpoons X * Y$

⇒ $G(f)=\frac{1}{2j}H(f)*[\delta(f-f_c)-\delta(f+f_c)]=$

$=\frac{1}{2j}[H(f-f_c)-H(f+f_c)]$ ⇒

$\boxed{G(f)=\frac{2\pi f}{(1+j2\pi f)^2+(2\pi f_c)^2}}$

• **Άσκηση**



$H(f)=;$

$y(t)=\int_{t-T}^{t}x(\tau)d\tau \Rightarrow Y(f)=F\left\{\int_{-\infty}^{t}x(\tau)d\tau-\int_{-\infty}^{t-T}x(\tau)d\tau\right\}=$

$=\underbrace{\frac{1}{j2\pi f}X(f)+\frac{X(0)}{2}\delta(f)}_{G(f)}-e^{-j2\pi fT}\cdot G(f)=$

$=G(f)\cdot[1-e^{-j2\pi fT}]=$

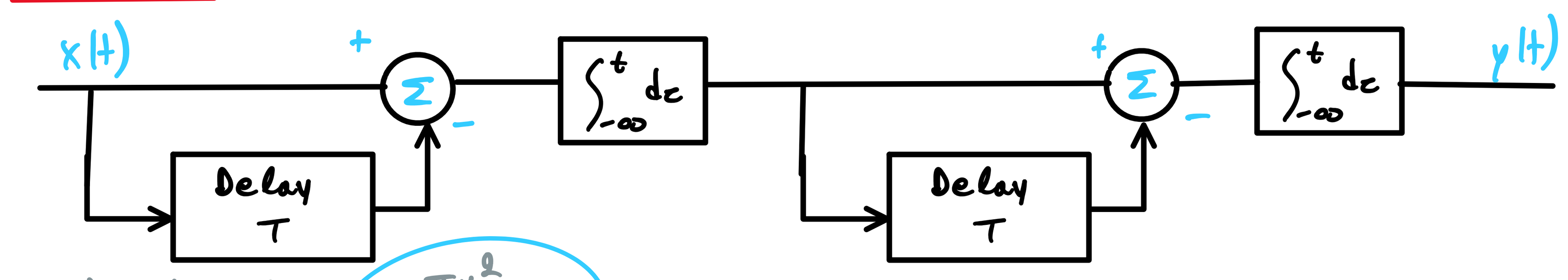
$\left\{1-e^{-j2\pi fT}\xrightarrow{f=0}0\Rightarrow X(0)=0\right\}$

$=\frac{1}{j2\pi f}[1-e^{-j2\pi fT}]X(f) \Rightarrow \frac{Y(f)}{X(f)}=H(f)=\frac{1-e^{-j2\pi fT}}{j2\pi f}$ ⇒

⇒ $H(f)=\frac{e^{-j\pi fT}}{j2\pi f}[e^{j\pi fT}-e^{-j\pi fT}]=\frac{\cancel{2j}\sin(\pi fT)}{\cancel{2j}\,\pi fT}T$ ⇒

⇒ $\boxed{H(f)=T\cdot\text{sinc}(fT)\,e^{-j\pi fT}}$

• **Άσκηση**



$g(t)=\frac{1}{\tau}\int_{t-T}^{t+T} \underbrace{e^{-\frac{\pi u^2}{\tau^2}}}_{x(t)}du$, $\lim_{\tau\to 0}G(f)=0$ $G(f)=?$

$e^{-\pi t^2}\rightleftharpoons e^{-\pi f^2}$

Χρονική κλιμάκωση: $g(at)\rightleftharpoons\frac{1}{|a|}G(f/a)$

$\xrightarrow{a=1/\tau}$ $e^{-\frac{\pi t^2}{\tau^2}}\rightleftharpoons \tau e^{-\pi f^2\tau^2}=X(f)$

$g(t)=\frac1\tau\int_{-\infty}^{t+T}x(u)du-\frac1\tau\int_{-\infty}^{t-T}x(u)du \xrightarrow[\text{+χρ. ολίσθ.}]{\text{γραμμ. + ολοκλ.}} \left[\frac1\tau X(f)\frac{1}{j2\pi f}+\frac1\tau\frac{X(0)}{2}\delta(f)\right]\cdot[e^{j2\pi fT}-e^{-j2\pi fT}]$ ⇒

$\left\{e^{j2\pi fT}-e^{-j2\pi fT}\xrightarrow{f=0}0\Rightarrow X(0)=0\right\}$

⇒ $G(f)=\cancel{2j}\sin(2\pi fT)\cdot\frac{\cancel{\tau}e^{-\pi f^2\tau^2}}{\cancel{j2}\,\pi f}\cdot\frac{1}{\cancel{\tau}}$ ⇒

⇒ $G(f)=\sin(2\pi f)\frac{e^{-\pi f^2\tau^2}}{\pi f}$ ⇒

⇒ $\boxed{G(f)=2T\,\text{sinc}(2fT)\,e^{-\pi f^2\tau^2}}$

• **Άσκηση**

$g(t)=\begin{cases}at, & -T\le t\le T\\0, & \text{αλλού}\end{cases}$ $G(f)=;$

$g(t)=at\cdot\text{rect}\left(\frac{t}{2T}\right)$

rect ⇌ sinc

Παραγωγ. συχν.

⇒ $G(f)=\frac{a}{(2\pi f)^2}\left\{e^{-j2\pi fT}[j2\pi fT+1]+e^{j2\pi fT}[j2\pi fT-1]\right\}$