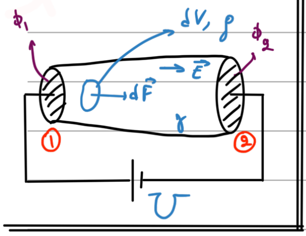
Νόμος Joule



- $d\vec{F} = dq\,\vec{E} = \rho\, dV \cdot \vec{E} = \underbrace{\rho \vec{E}}_{\vec{f}}\, dV = \vec{f}\, dV$ $\quad$ $\vec{f}$: πυκνότητα δύναμης N/m^3
- $dW = d\vec{F} \cdot d\vec{\ell} = (\vec{f}\, dV) \cdot d\vec{\ell}$
- $\langle \vec{v} \rangle = \dfrac{d\vec{\ell}}{dt} \longrightarrow d\vec{\ell} = \langle \vec{v} \rangle\, dt$ $\quad$ Άρα: $dW = (\vec{f}\, dV) \cdot \langle \vec{v} \rangle dt \Rightarrow dW = (\rho \vec{E}) \cdot \langle \vec{v} \rangle dt\, dV$

μέση ταχύτητα e^-

- Πυκνότητα ισχύος: $p = \dfrac{dW}{dt\, dV} = \vec{E} \cdot \boxed{(\rho \langle \vec{v} \rangle)} = \vec{E} \cdot \vec{J}$

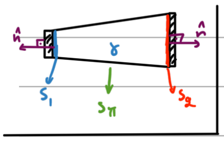
$\vec{J}$: χωρ. πυκν. ρεύματος

$[p] = \dfrac{J}{s \cdot m^3} = W/m^3$

- Συνολική ισχύς: $P = \int_V p\, dV = \int_V \vec{E} \cdot \vec{J}\, dV = -\int_V \nabla\phi \cdot \vec{J}\, dV = -\int_V \nabla(\phi \vec{J})\, dV = -\int_S \phi \vec{J} \cdot d\vec{S}$ *

$\nabla(\phi \vec{J}) = \nabla\phi \cdot \vec{J} + \cancel{\phi \cdot \nabla \vec{J}}^{\,0}$

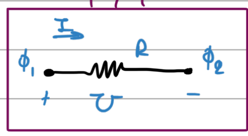
* Όσον αφορά την επιφάνεια S:



$S = S_1 \cup S_2 \cup S_\pi$

- $P = -\int_{S_1} \phi \vec{J} \cdot d\vec{S} - \int_{S_2} \phi \vec{J} \cdot d\vec{S} = \phi_1 \underbrace{\boxed{J \cdot S_1}}_{I} - \phi_2 \underbrace{\boxed{J S_2}}_{I} = (\phi_1 - \phi_2) I = U I$

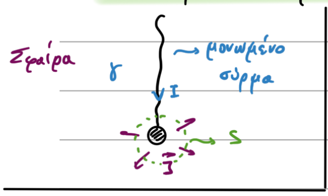
Θυμίζουμε:



- $P = I U = I^2 R = \dfrac{U^2}{R} = \int_V \vec{E} \vec{J}\, dV$
- $R = \dfrac{\int_1^2 \vec{E}\, d\vec{\ell}}{\oint_S \sigma \vec{E} \cdot d\vec{S}} = \dfrac{\int_V \vec{E} \vec{J}\, dV}{I^2} = \dfrac{U^2}{\int_V \vec{E} \cdot \vec{J}\, dV} = \dfrac{P}{I^2} = \dfrac{U^2}{P}$

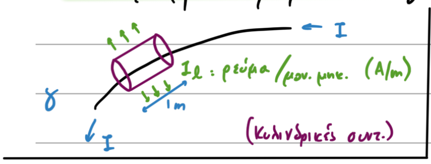
Ρευματικές πηγές στα ΜΗΠ

1. Σημειακή ρευματική πηγή



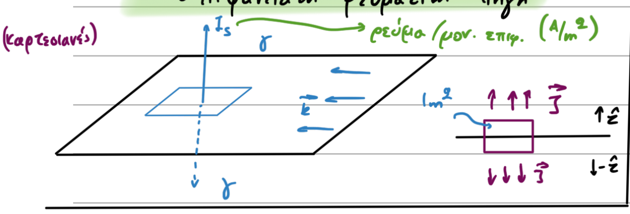
- $I = \oint_S \vec{J}\, d\vec{S} \Rightarrow \vec{J} = \dfrac{I}{4\pi r^2} \hat{r}$
- $\vec{E} = \dfrac{I}{4\pi\gamma r^2} \hat{r}$, $\phi = \dfrac{I}{4\pi\gamma} \left(\dfrac{1}{r} - \dfrac{1}{r_{av}} \right)$

2. Γραμμική ρευματική πηγή



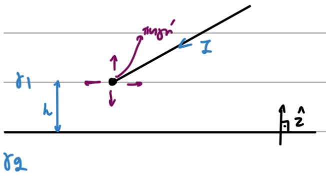
- $\vec{J} = \dfrac{I_\ell}{2\pi r} \hat{r}$, $\vec{E} = \dfrac{I_\ell}{2\pi\gamma r} \hat{r}$

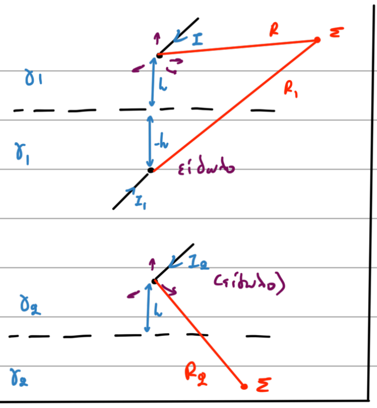
3. Επιφανειακή ρευματική πηγή



- $\vec{J} = \oint_S \vec{J}\, d\vec{S} = \dfrac{I_s}{2} \operatorname{sgn}(z) \cdot \hat{z}$
- $\vec{E} = \dfrac{I_s}{2\gamma} \operatorname{sgn}(z) \cdot \hat{z}$

Μέθοδος ειδώλων στα ΜΗΠ





Θυμίζουμε:

$$\phi = \frac{I}{4\pi\gamma r}$$

- $\phi_1 = \dfrac{1}{4\pi\gamma_1}\left[\dfrac{I}{R} + \dfrac{I_1}{R_1}\right]$, $R = \sqrt{x^2+y^2+(z-h)^2} = R_2$, $R_1 = \sqrt{x^2+y^2+(z+h)^2}$

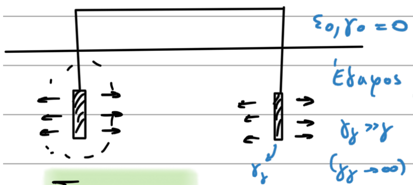
- $\phi_2 = \dfrac{1}{4\pi\gamma_2} \cdot \dfrac{I}{R_2}$

- Οριακές συνθήκες: • $\hat n \times (\vec E_1 - \vec E_2) = 0 \Rightarrow \phi_1(z=0) = \phi_2(z=0)$ $\quad \dfrac{I+I_1}{\gamma_1} = \dfrac{I_2}{\gamma_2}$

  • $\hat n \cdot (\vec J_1 - \vec J_2) = -\nabla \vec K = 0 \Rightarrow \gamma_1 \cdot \hat z \vec E_1 = \gamma_2 \cdot \hat z \vec E_2 \Rightarrow \gamma_1 \left.\dfrac{\partial \phi_1}{\partial z}\right|_{z=0} = \gamma_2 \left.\dfrac{\partial \phi_2}{\partial z}\right|_{z=0} \Rightarrow I = I_1 + I_2$ $\Bigg\} \Rightarrow$

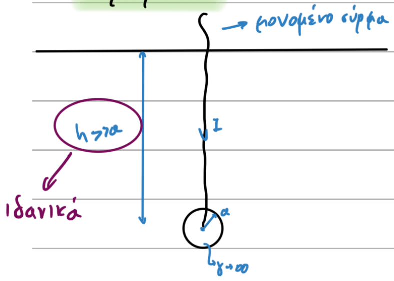
$$\Rightarrow \boxed{I_1 = \frac{\gamma_1 - \gamma_2}{(\gamma_1+\gamma_2)I} \;,\quad I_2 = \frac{2\gamma_2}{(\gamma_1+\gamma_2)I}} \quad (*)$$

Γειωτές (σφαιρικός και ημισφαιρικός)



$$R_\gamma = \frac{U}{I}$$

Σφαιρικός



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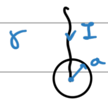
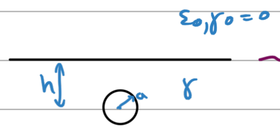


- $\vec J = \dfrac{I}{4\pi r^2}\hat r$ • $\vec E = \dfrac{1}{\gamma}\vec J = \dfrac{1}{4\pi\gamma r^2}\hat r$
- $\phi(r) = \displaystyle\int_r^\infty \vec E\, d\vec l = \dfrac{I}{4\pi\gamma r}$ • $R_\gamma = \dfrac{\phi(a)}{I} = \dfrac{1}{4\pi\gamma a}$

↳ Εάν όμως δεν ισχύει h>>a:



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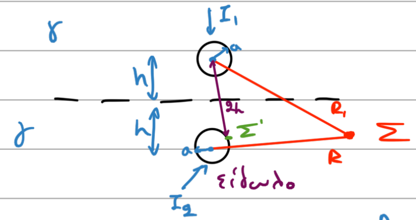
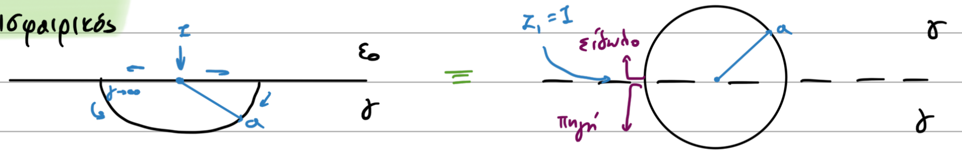
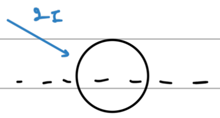


- $\phi = \dfrac{1}{4\pi\gamma}\left(\dfrac{I}{R} + \dfrac{I_1}{R_1}\right)$
- $I_1 = I$ (επειδή $\gamma_0 = 0$) (*)
- $R_\gamma = \dfrac{\phi\,(\text{επιφ. του γειωτή})}{I}$

→ Για $R_1 = 2h$, $\vec R = a$: $R_\gamma = \dfrac{I}{4\pi\gamma}\left(\dfrac{1}{a} + \dfrac{1}{2h}\right)$

Ημισφαιρικός





Άρα για την R_γ, θέλω όπου I, το $2I$ στα αποτελέσματα του σφαιρικού: $R_{ημ} = \frac{1}{2\pi\gamma a} = 2R_{σφ}$

Πρακτικά, ο σφαιρικός είναι 2 παράλληλοι ημισφαιρικοί ίσης αντίστασης, για αυτό: $R_{σφ} = \frac{R_{ημ}}{2}$