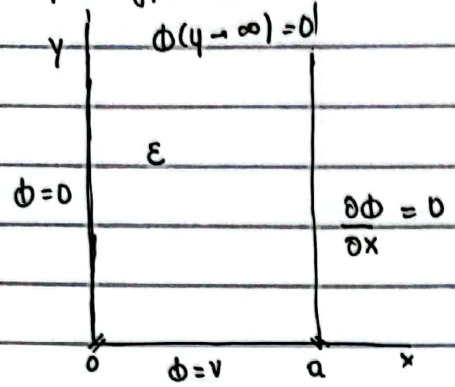
Παράδειγμα 8



$$X_n(x)=\sin(k_n x), \quad k_n=\frac{2n-1}{a}\frac{\pi}{2}$$

$$X'(x)\propto \cos k_n x$$

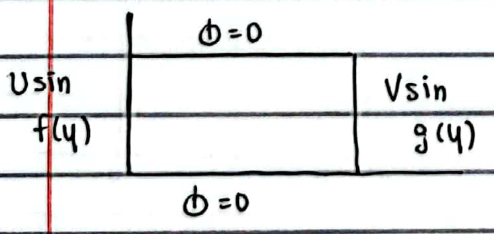
$$Y_n(y)=e^{-k_n y}$$

$$\Phi(x,y)=\sum_{n=1}^{\infty} C_n \sin k_n x \, e^{-k_n y}$$

$$\Phi(x,y=0)=\sum_n C_n \sin(k_n x)=V$$

$$\Rightarrow \sum_n C_n \int_0^{\alpha} \sin(k_n x)\sin(k_m x)dx=\int_0^{\alpha} V\sin(k_m x)dx$$

$$\Rightarrow C_n=\frac{2}{a}\frac{V}{k_n}=\frac{4V}{(2n-1)\pi}$$



(επαλληλία) ή $\Phi=\sin y\,(A\sinh x+B\cosh x)$

3Δ

$\nabla^2 \phi = 0$

$\phi = XYZ$

$K_x + K_y + K_z = 0$

$\Updownarrow$

$\mp k_x^2 \mp k_y^2 \mp k_z^2 = 0$

2Δ: $k_x = k_y = k_z = 0$

$k_x = k_y = k \neq 0$

3Δ: (i) $k_x = k_y = k_z = k = 0$

$\phi(x,y,z) = Axyz + Bxy + \Gamma xz + \Delta yz + Ex + Zy + Hz + \Theta$

(ii) $k_x \neq 0, k_y \neq 0, k_z \neq 0$, $k_x \neq k_y \neq k_z$

$$\phi(x,y,z) = C \underbrace{\left\{\begin{array}{c} e^{\pm} \\ \sinh \\ \cosh \\ \hline \sin \\ \cos \\ e^{\pm i} \end{array}\right\}(k_x x)}_{X(x)} \cdot \underbrace{\left\{\begin{array}{c} \text{»} \\ \hline \text{»} \end{array}\right\}(k_y y)}_{Y(y)} \cdot \underbrace{\left\{\begin{array}{c} \text{»} \\ \hline \text{»} \end{array}\right\}(k_z z)}_{Z(z)}$$

(α) αναγκαστικά μη περιοδική

(β) αναγκ. περιοδική

(γ) μη περιοδική (ή) περιοδική, ανάλογα με τις ΟΣ.

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(iii) $k_i = 0$, $i = x$ ή y ή z

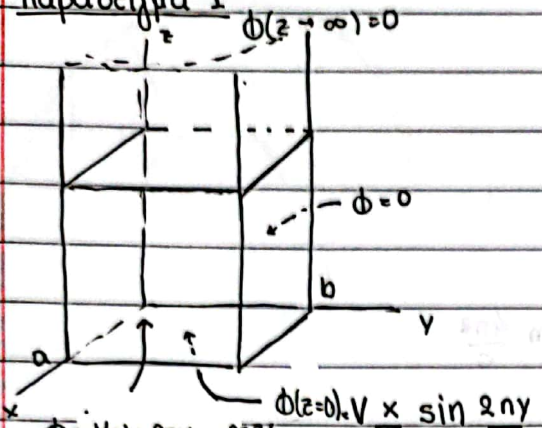
π.χ. $k_z = 0$

$$\Phi(x,y,z) = \cancel{X}\left\{\begin{array}{c} \text{»} \\ \hline \text{»} \end{array}\right\}(k_x x)\left\{\begin{array}{c} \text{»} \\ \hline \text{»} \end{array}\right\}(k_y y)\cdot(Az+B)$$

$$(C_1 z + C_2)$$

$k = k_x = k_y$

Παράδειγμα 1



$\Phi = V\sin\frac{2\pi y}{b}e^{-2\pi z/b}$

$k_x = 0, \quad k_y = \frac{2\pi}{b}, \quad k_z = \frac{2\pi}{b}$

$X(x) = C_1 x + C_2$

$Y(y) = \sin\frac{2\pi y}{b}$

$Z(z) = e^{-\frac{2\pi z}{b}}$

$\Phi(x,y,z) = (C_1 x + C_2)\sin\frac{2\pi y}{b}e^{-\frac{2\pi z}{b}}$

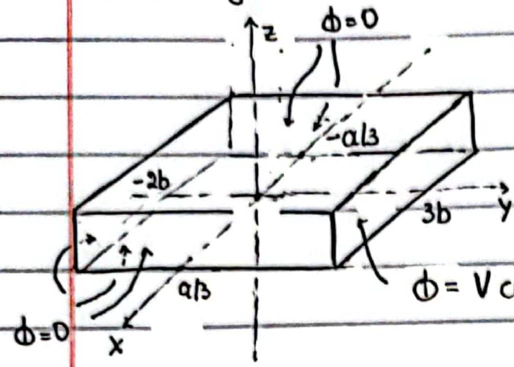
$\Phi(x=0,y,z) = C_2\sin\frac{2\pi y}{b}e^{-\frac{2\pi z}{b}} = 0 \Rightarrow C_2 = 0$

$\Phi(x=a,y,z) = C_1\cdot a\sin\frac{2\pi y}{b}e^{-\frac{2\pi z}{b}} = V\sin\frac{2\pi y}{b}e^{-2\pi z/b} \Rightarrow C_1 = V/a$

CHECK: $\Phi(x,y,z=0) = \frac{Vx}{a}\sin\frac{2\pi y}{b}$

$\vec{E} = -\nabla\Phi$

Παράδειγμα 2



$\Phi = V \cos \frac{3\pi}{2\alpha} x \sin \frac{4\pi z}{c}$

$k_x = \frac{3\pi}{2\alpha}$, $k_z = \frac{4\pi}{c}$, k_y

$X(x) = \cos \frac{3\pi x}{2\alpha}$, $Z(z) = \sin \frac{4\pi z}{c}$

$Y(y) = \sinh\left(k_y(y+2b)\right)$

$K_x + K_y + K_z = 0$

$\mp k_x^2 \mp k_y^2 \mp k_z^2 = 0 \Rightarrow k_y^2 = k_x^2 + k_z^2 \Rightarrow k_y = \pm\sqrt{\left(\frac{3\pi}{2\alpha}\right)^2 + \left(\frac{4\pi}{c}\right)^2}$

$+ \qquad +$

$\Phi(x,y,z) = C \cos \frac{3\pi x}{2\alpha} \sinh\left(k_y(y+2b)\right) \cdot \sin \frac{4\pi z}{c}$

$\Phi(x, y=3b, z) = C \cdot \cos \frac{3\pi x}{2\alpha} \sinh(k_y(3b+2b)) \sin \frac{4\pi z}{c} = V \cos \frac{3\pi x}{2\alpha} \sin \frac{4\pi z}{c}$

$\Rightarrow C = \frac{V}{\sinh 5b}$

$\Phi(x,y,z) = V \cdot \cos \frac{3\pi x}{2\alpha} \cdot \frac{\sinh(k_y(y+2b))}{\sinh(k_y(3b+2b))} \cdot \sin \frac{4\pi z}{c}$

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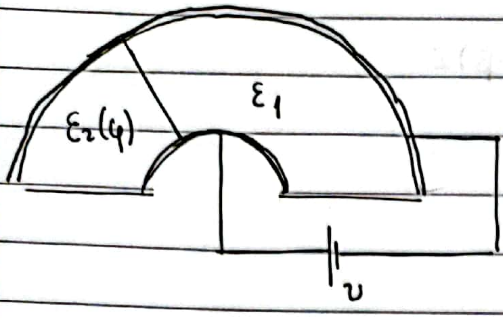
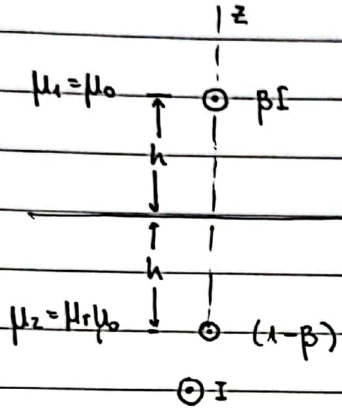
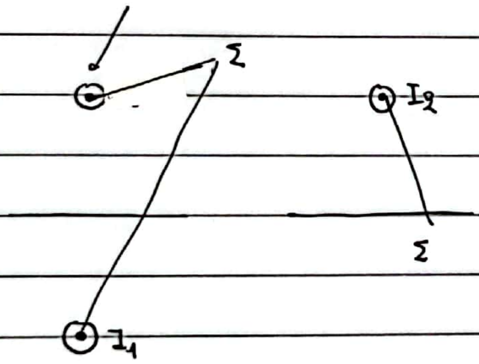


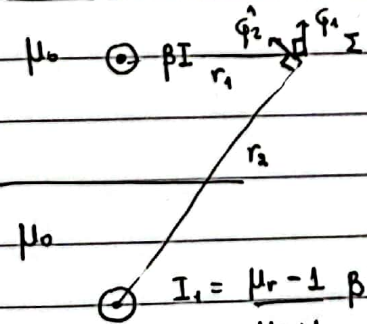




$$I_1 = \frac{\mu_2-\mu_1}{\mu_2+\mu_1} I$$

$$I_2 = \frac{2\mu_1}{\mu_1+\mu_2} I$$

ΒΠ 1



$$I_1 = \frac{\mu_r-1}{\mu_r+1}\beta I + \frac{2\mu_r}{\mu_r+1}(1-\beta) I$$

$$\vec{H}_1 = \frac{\beta I}{2\pi r_1}\hat{\varphi}_1 + \frac{\beta(\mu_r-1)+2\mu_r(1-\beta)}{\mu_r+1}\cdot\frac{I}{2\pi r_2}\hat{\varphi}_2$$

Bn2

$$\otimes \; I_2 = \frac{2}{\mu_r + 1}\beta I + \frac{1-\mu_r}{1+\mu_r}(1-\beta)I$$

$\odot$ $(1-\beta)I$

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