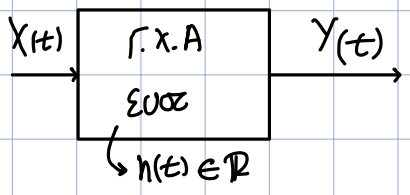
Άσκηση



$S_y(f) = ?$

(πυκνότητα φάσματος ισχύος)

$$S_y(f) = \int_{-\infty}^{\infty} R_y(\tau) e^{-j2\pi f\tau} d\tau$$

$$= \iiint h(\tau_1)\, h(\tau_2)\, R_x(\tau - \tau_1 + \tau_2)\, e^{-j2\pi f\tau}\, d\tau\, d\tau_1\, d\tau_2$$

για το R_x, θέτουμε $\tau = \tau_0 + \tau_1 - \tau_2$

$$\Rightarrow \iiint h(\tau_1)d\tau_1\, h(\tau_2)d\tau_2\, R_x(\tau_0)\, e^{-j2\pi f\tau_0} \cdot e^{-j2\pi f\tau_1} \cdot e^{+j2\pi f\tau_2}\, d\tau_1\, d\tau_2\, d\tau_0$$

$$= \underbrace{\int h(\tau_1)\, e^{-j2\pi f\tau_1}}_{\text{fourier}} \underbrace{\int h(\tau_2)\, e^{j2\pi f\tau_2} d\tau_2}_{} \underbrace{\int R_x(\tau_0)\, e^{-j2\pi f\tau_0} d\tau_0}_{\text{ορισμός}}$$

$$= H(f) \cdot \qquad H(-f) = H^*(f) \qquad S_x(f)$$

$$= H(f) H^*(f) \cdot S_x(f)$$

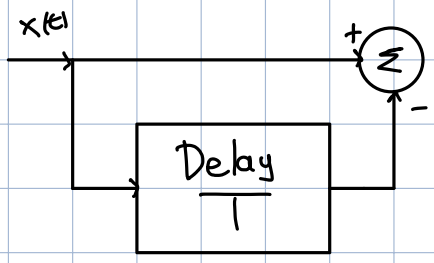
ισχύει διότι $|H(f)|^2 = H(f) \cdot H^*(f)$

$$S_y(f) = |H(f)|^2 \cdot S_x(f)$$

Άσκηση

$S_x(f) = ?$

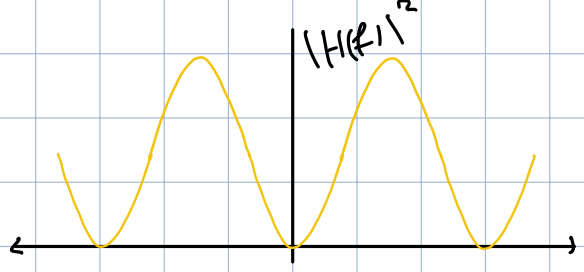
$S_y(f) = ?$



$$Y(t) = X(t) - X(t-T)$$

$$\Rightarrow H(f) = 1 - e^{-j2\pi fT}$$

$$\Rightarrow |H(f)|^2 = (1-\cos(2\pi fT))^2 + \sin^2(2\pi fT) = 2 - 2\cos(2\pi fT) = 4\sin^2(\pi fT)$$



για $\pi fT \ll 1 \Rightarrow \sin(\pi fT) \simeq \pi fT \Rightarrow |H(f)|^2 \simeq 4(\pi fT)^2$

$$S_y(f) \simeq 4(\pi fT)^2 S_x(f)$$

Στοχαστική Ανέλιξη Gauss

Σ.Α. $x(t)$. Ορίζω μία τυχαία μεταβλητή $Y \triangleq \int_0^T g(t)\,x(t)\,dt$

linear functional

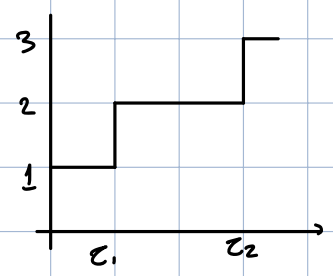
$\Rightarrow$ Αν $\forall\, g(t) : E[Y^2] < \infty \Rightarrow f_y(y) = \frac{1}{\sqrt{2\pi}\,\sigma_y} \exp\left[-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right]$

$\Sigma A\ x(t) \sim N(0,1)$ $\rightarrow$ $f_y(y) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{y^2}{2}\right]$

Θορυβος Βολώς (Shot noise)

$$X(t) = \sum_{k=-\infty}^{\infty} h(t-\tau_k)$$

Σ.Α. $N(t) \rightarrow \#e^-$ που εκπέμπονται στο χρ. διάστ. $(0,t)$



$V = N(t+t_0) - N(t)$

$\hookrightarrow \#e^-$ από t εως $t+t_0$

ακολουθεί κατανομή Poisson

$$P[V=k] = \frac{(\lambda t_0)^k}{k!} e^{-\lambda t_0}, \quad k=0,1,2\cdots$$

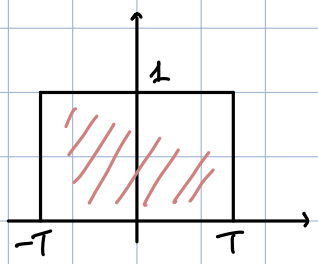
(λ: ρυθμός αφιξης)

$$E[V] = \lambda t_0$$

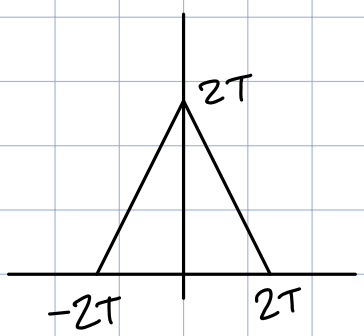
$$E[x(t)] = \mu \int_{-\infty}^{\infty} h(t)\,dt$$

$$\Rightarrow C_x(\tau) = E\left[\left\{x(t+\tau) - E[x(t+\tau)]\right\} * \left\{x(t) - E[x(t)]\right\}\right]$$

$$C_x(\tau) = \mu \int_{-\infty}^{\infty} h(t)\, h(t+\tau)\,dt \quad , \quad h(t) = h(-t) \Rightarrow C_x(\tau) = \mu\, h(\tau) * h(\tau)$$



→ συνέλιξη με τον εαυτό του →



Λευκός Θόρυβος

$$S_w(f) = \frac{N_0}{2}$$

$$R_w(\tau) = \frac{N_0}{2}\delta(\tau)$$