Άσκηση

Σ.Α. Gauss: $Z(t) = X\cdot\cos(2\pi t) + Y\sin(2\pi t)$

X, Y τ.μ.
$\mu_x = \mu_y = 0$ Gauss, στατιστικά ανεξάρτητες
$\sigma_x^2 = \sigma_y^2 = 1$

$C_z(t_1, t_2) = ?$ $\sigma_z(t) = ?$ στατικ. $Z = ?$

$$\mu_z(t) = E[X\cos() + Y\sin()] = \cancel{E[X]}\cos(\cdot) + \cancel{E[Y]}\sin(\cdot)$$

$\mu_z(t) = 0$

$$C_z(t_1,t_2) = E\{[z(t_1) - \cancel{\mu_z(t_1)}][z(t_2) - \cancel{\mu_z(t_2)}]\} = R_z(t_1,t_2)$$

$$= E[x^2\cos(2\pi t_1)\cos(2\pi t_2) + Y^2\sin(2\pi t_1)\sin(2\pi t_2) + \cancel{XY}\cos 2\pi t_1 \sin 2\pi t_2]$$

$E[X^2] = E[Y^2] = 1$ από τις τιμές των τ.μ.

$$= \cos(2\pi t_1)\cos(2\pi t_2) + \sin(2\pi t_1)\sin(2\pi t_2) = \cos(2\pi \tau)$$

$\tau = t_1 - t_2$

$$\sigma_z^2(t) = E[z^2(t)] = E[x^2\cos^2(2\pi t) + Y^2\sin^2(2\pi t) + 2\cancel{XY}\cos(2\pi t)\sin(2\pi t)] = 1$$

Θερμικός Θόρυβος (thermal noise)

$V_{TN}(t) \rightarrow$ τάση θερμικού θορύβου σε άκρα R υπό θερμοκρασία T και εύρος ζώνης Δf

$$E[V_{TN}^2(t)] = 4kTR\Delta f$$

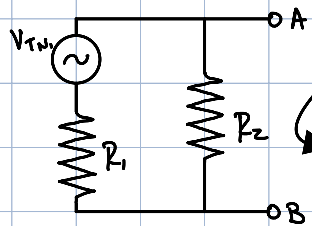
$S_{TN}(f) = 2kTR$ (φασματική πυκνότητα ισχύος του θορύβου)

$$S_{TN}(f) = R_{TN}(\tau) = \int 2kTRe^{j2\pi f\tau}df \Rightarrow \text{σε εύρος } \Delta f \Rightarrow E[V_{TN}^2(t)] = \int_{-\Delta f}^{\Delta f} 2kTR\,df = 4kTR\Delta f$$

Άρα $S_{TN}(f) = \sum_{i=1}^{N} 2kT_iR_i|H_i(f)|^2, \quad |f| \le \Delta f$

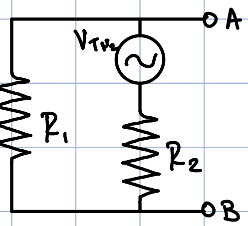
↳ συνάρτηση μεταφοράς

Παράδειγμα:



$H_2(f) = \frac{R_1}{R_1+R_2}$

Θεωρώ $V_{TN_2}(t)=0 \quad \forall t$



$H_2(f) = \frac{R_1}{R_1+R_2}$

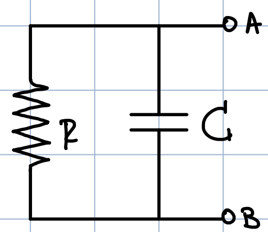
Θεωρώ $V_{TN_1}(t)=0 \quad \forall t$

Αν $T_2 = T_1 = T \Rightarrow S_{TN}(f) = 2kT\left(R_1 \frac{R_2^2}{(R_1+R_2)^2} + R_2 \frac{R_1^2}{(R_1+R_2)^2}\right)$

$$= 2kT \cdot \frac{R_1R_2}{R_1+R_2}$$

$S_{TN}(f) = 2kT.\,Re\{R\}$ → ισοδύναμη αντίσταση κυκλώματος

Παράδειγμα:



$$R_o = \frac{R \cdot \frac{1}{j2\pi fC}}{R + \left(\frac{1}{j2\pi fC}\right)} = \frac{R}{1+j2\pi RC} \cdot \frac{(1-j2\pi RC)}{(1-j2\pi RC)}$$

$$= \frac{R - j2\pi R^2C}{1+(2\pi fRC)^2}$$

$$\Rightarrow S_{TN}(f) = 2KT\frac{R}{1+(2\pi fRC)^2} = 2kT\frac{R}{1+\frac{f^2}{f_0^2}}, \quad (f_0)^{-1} = 2\pi RC$$

$$E[V_{TN}^2(t)] = R_{TN}(0) = \int_{-\infty}^{\infty} S_{TN}(f)\,df = 2kTR\tan^{-1}\left(\frac{f}{f_0}\right)\cdot f_0 = 2kTRf_0\pi = \frac{KT}{C}$$

$$E[V_{TN}^2(t)] = \frac{kT}{C}$$

Λευκός Θόρυβος

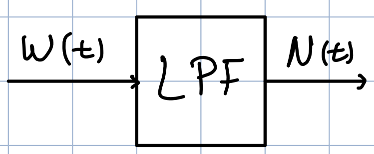
$$S_w(f) = \frac{N_0}{2}$$

Αυτοσυσχέτιση

$$R_w(\tau) = \frac{N_0}{2}\delta(\tau)$$

Πυκνότητα φασματικής ισχύος

Παράδειγμα: Λ.Θ. $\mu_w = 0$, $S_w(f) = \frac{N_0}{2}$



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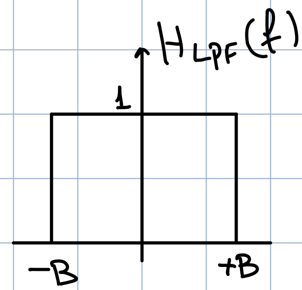


$$S_N(f) = S_w(f)\cdot\left|H_{LPF}(f)\right|^2 = \begin{cases} N_0/2, & |f| \le B \\ 0, & \text{αλλού} \end{cases}$$

$$R_N(\tau) = N_0 B \,\mathrm{sinc}(2B\tau)$$

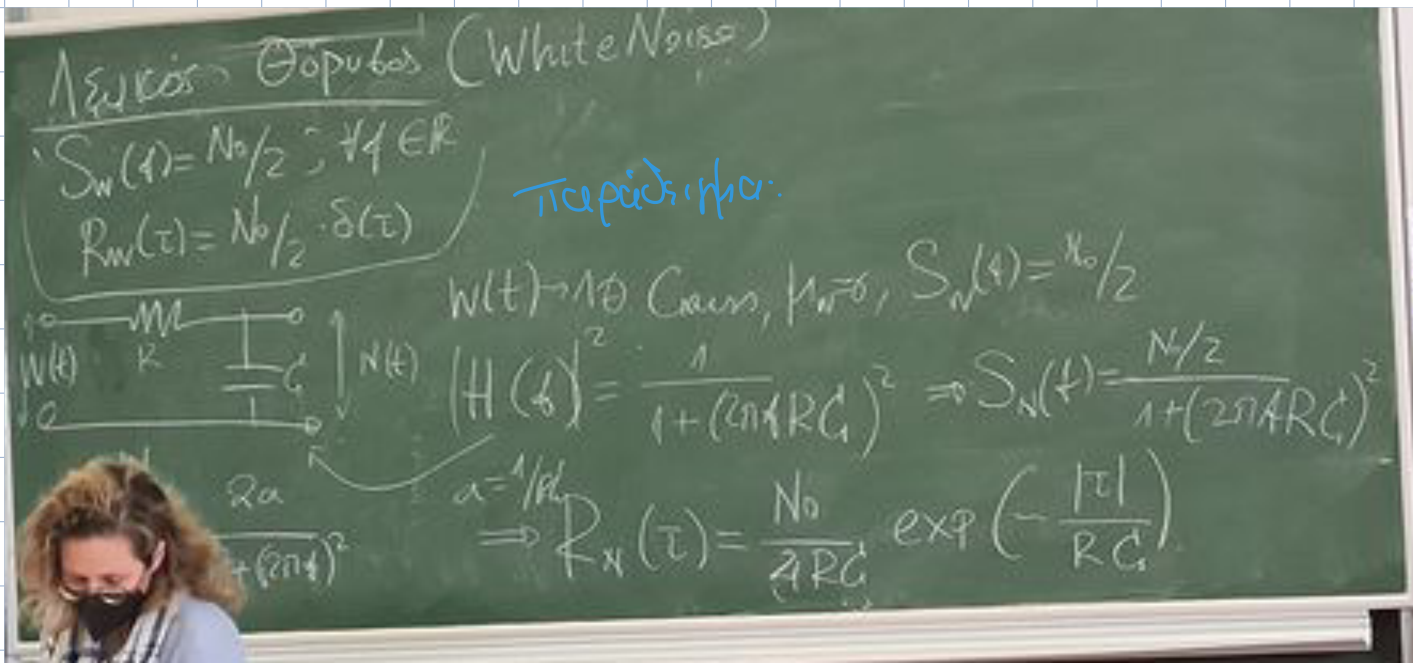
$N(t) \sim$ Gauss

$\mu_N = H(0)\cdot\mu_w = 0$

$$E\left[N\left(t+\frac{k}{2B}\right) - N(t)\right] = 0, \quad \forall k \in \mathbb{Z}$$

$$\sigma_n^2 = E\left[\left(N(t) - \mu_N\right)^2\right] = E\left[N^2(t)\right] \Rightarrow \sigma_N^2 = N_0 B$$

Παράδειγμα:


Λευκός Θόρυβος (White Noise)
$S_W(f) = N_0/2 \,;\ \forall f \in \mathbb{R}$
$R_W(\tau) = N_0/2 \cdot \delta(\tau)$
$W(t)$ R C_1 $N(t)$
$W(t) \to$ ΛΘ Gauss, $\mu_W = 0$, $S_W(f) = N_0/2$
$|H(f)|^2 = \frac{1}{1+(2\pi f R C_1)^2} \Rightarrow S_N(f) = \frac{N_0/2}{1+(2\pi f R C_1)^2}$
$a = 1/RC_1$
$\Rightarrow R_N(\tau) = \frac{N_0}{4RC_1} \exp\left(-\frac{|\tau|}{RC_1}\right)$