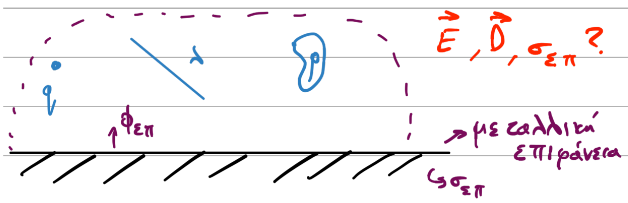
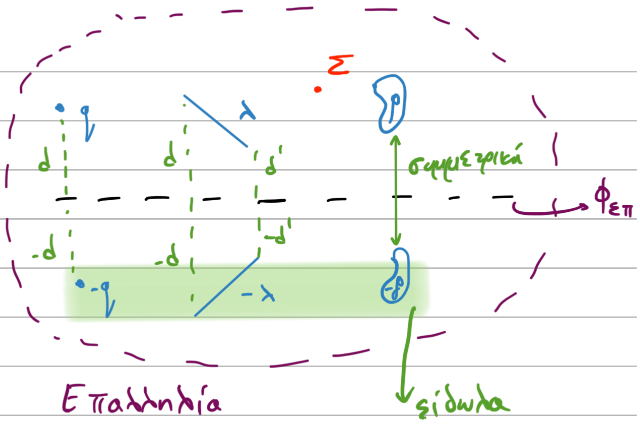
1.8 Μέθοδος ειδώλων

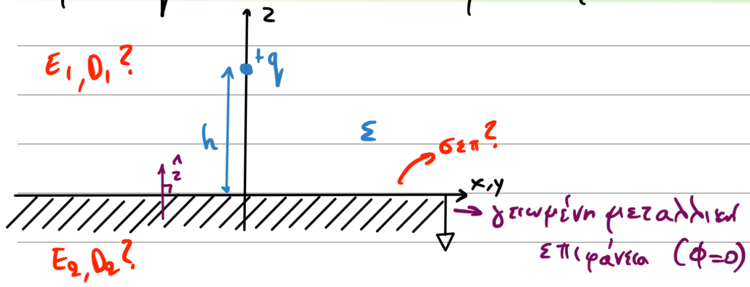


Poisson: $\nabla^2\phi = -\rho/\varepsilon$

$\equiv$



Φορτίο q πάνω από απέραντο μεταλλικό επίπεδο



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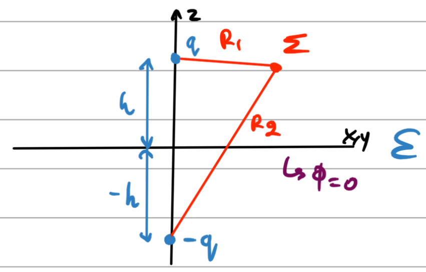
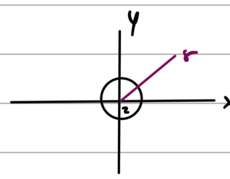


$$\phi_\Sigma = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r} - \cancel{\frac{1}{r_{\alpha\nu}}}^{\,\alpha\nu\to\infty}\right) = \frac{q}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q}{4\pi\varepsilon}\left(\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right)$$

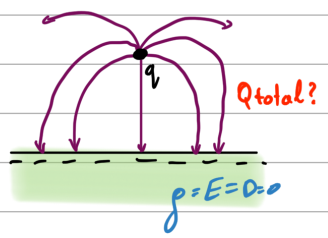
$\phi(z=0) = 0$ ✓ (οπότε εξασφαλίσαμε ορθή λύση)

$$E_1 = -\nabla\phi = \frac{q}{4\pi\varepsilon}\left(\frac{x\hat{x} + y\hat{y} + (z-h)\hat{z}}{\left[x^2+y^2+(z-h)^2\right]^{3/2}} - \frac{x\hat{x} + y\hat{y} + (z+h)\hat{z}}{\left[x^2+y^2+(z+h)^2\right]^{3/2}}\right), \quad \vec{D}_1 = \varepsilon \vec{E}_1$$

$E_2 = D_2 = 0$



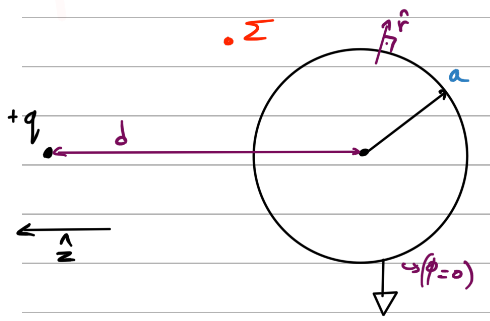
$$\sigma = \hat{z}\cdot\left(\vec{D}_1 - \cancel{\vec{D}_2}^{\,0}\right) = \hat{z}\,\vec{D}_1 = \hat{z}\left(\varepsilon\vec{E}_1\right) = \frac{-hq}{2\pi\big(\underbrace{\boxed{x^2+y^2}}_{r^2}+h^2\big)^{3/2}}$$



$$Q_{tot} = \iint_{x\,y} \sigma\, dx\, dy = \int_{r=0}^{\infty}\int_{\varphi=0}^{2\pi} \sigma(r)\, dr\, r\, d\varphi = -q$$

Φορτίο δίπλα σε αγώγιμη μεταλ. σφαίρα

$$\frac{\partial}{\partial \varphi} = 0$$



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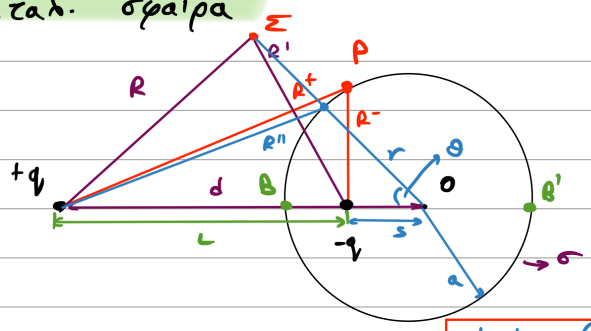


$\boxed{\begin{array}{l} \text{distance}(q-O) = d \\ \text{distance}(q'-O) = s \\ \text{distance}(q,-q') = L = d-s \end{array}}$

$$\phi_P = \frac{1}{4\pi\varepsilon}\left(\frac{q}{R^+} + \frac{q'}{R^-}\right) = 0 \Rightarrow \frac{R}{R'} = -\frac{q}{q'} = \sigma\tau\alpha\vartheta. = c$$

→ Λόγος $\frac{R^+}{R^-}$ στο σημείο B: $\frac{d-a}{a-s}$

→ Λόγος $\frac{R^+}{R^-}$ στο σημείο B': $\frac{d+a}{s+a}$

$$\Rightarrow \frac{d-a}{a-s} = \frac{d+a}{s+a} = -\frac{q}{q'} = c \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} c = \dfrac{\frac{d}{a}-1}{1-\frac{s}{a}} \\[2ex] c = \dfrac{\frac{d}{a}+1}{1+s/a} \end{array}\right\} \Rightarrow c = \frac{d}{a} = \frac{a}{s}$$

, δηλαδή: $\boxed{q' = -q\,\frac{a}{d}\,, \quad s = \frac{a^2}{d}}$

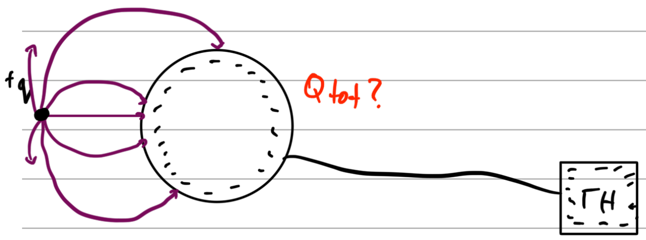
→ $s = \frac{a^2}{d} = \frac{a^2}{L+s} \Rightarrow a = \frac{c^2 L}{c^2-1}\,, \quad s = \frac{L}{c^2-1}$

→ $$\Phi_\Sigma = \frac{q}{4\pi\varepsilon}\left(\frac{1}{R} - \frac{a/d}{R'}\right)$$

$$R = \sqrt{r^2+d^2-2rd\cos\theta}\,, \quad R' = \sqrt{r^2+s^2-2rs\cos\theta}$$

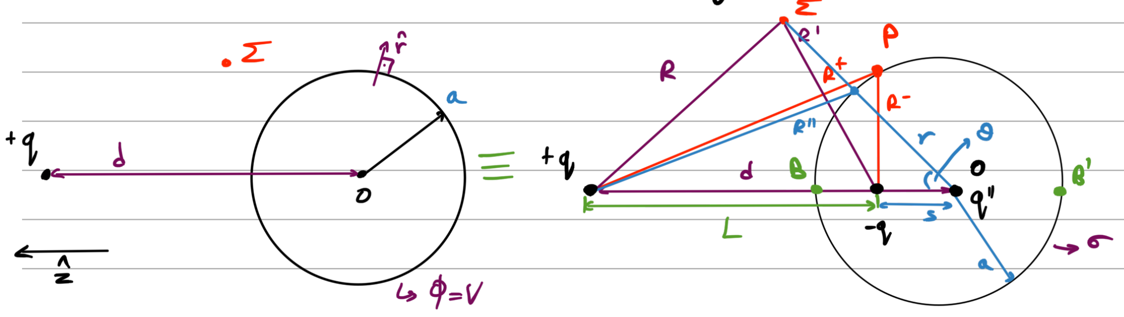
→ $\vec{E} = -\nabla\Phi_\Sigma$

$$\sigma = \varepsilon E_r(r=a) = \frac{-q(d^2-a^2)}{4\pi a\,(\underbrace{a^2+d^2-2ad\cos\theta}_{R''})^{3/2}} = \frac{-q(d^2-a^2)}{4\pi a\,(R'')^{3/2}}$$



$$Q_{total} = \iint_S \sigma\, dS = q' = -q\,\frac{a}{d}$$

→ Ίδιο, αλλά η σφαίρα δεν είναι γειωμένη



→ $\phi_{q''} = \dfrac{q''}{4\pi\varepsilon r}$, $\phi_{q''}(r=a) = \dfrac{q''}{4\pi\varepsilon a} \equiv V \Rightarrow q'' = 4\pi\varepsilon a V$

→ $\phi_\Sigma = \dfrac{q}{4\pi\varepsilon}\left(\dfrac{1}{R} - \dfrac{a/d}{R'}\right) + \dfrac{q''}{4\pi\varepsilon r} = \dfrac{q}{4\pi\varepsilon}\left(\dfrac{1}{R} - \dfrac{a/d}{R'}\right) + V\dfrac{a}{r}$

→ Πάνω στην επιφάνεια: $\phi = \dfrac{q}{4\pi\varepsilon}\left(\dfrac{1}{R^+} - \dfrac{a/d}{R^-}\right) + V$

→ $\sigma = -\dfrac{q\,(d^2 - a^2)}{4\pi a\,(R'')^{3/2}} + \varepsilon\dfrac{V}{a}$