Διαγράμματα Bode της $G(s)$

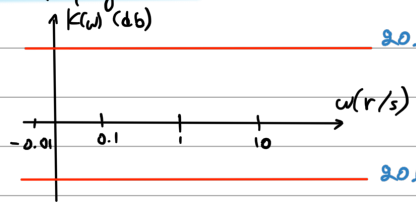
- $$G(s)=K\,\frac{s^{v}\prod_{i=1}^{m_1}(1+s\tilde{T}_i)\cdot\prod_{k=1}^{m_2}\left[1+2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}s+\left(\frac{s}{\tilde{\omega}_k}\right)^2\right]}{s^{\mu}\prod_{i=1}^{n_1}(1+sT_i)\cdot\prod_{k=1}^{n_2}\left[1+2\frac{\zeta_k}{\omega_k}s+\left(\frac{s}{\omega_k}\right)^2\right]} \Rightarrow G(j\omega)=K\,\frac{(j\omega)^{v}\prod_{i=1}^{m_1}(1+j\omega\tilde{T}_i)\cdot\prod_{k=1}^{m_2}\left[1+2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}j\omega+\left(\frac{j\omega}{\tilde{\omega}_k}\right)^2\right]}{(j\omega)^{\mu}\prod_{i=1}^{n_1}(1+j\omega T_i)\cdot\prod_{k=1}^{n_2}\left[1+2\frac{\zeta_k}{\omega_k}j\omega+\left(\frac{j\omega}{\omega_k}\right)^2\right]}$$

- $$K(\omega)=20\log_{10}|G(j\omega)|=20\log_{10}|K|+20v\log_{10}|j\omega|-20\mu\log_{10}|j\omega|+20\sum_{i=1}^{m_1}\log_{10}|1+j\omega\tilde{T}_i|+$$
$$+20\sum_{k=1}^{m_2}\log_{10}\left|1+2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}j\omega+\left(\frac{j\omega}{\tilde{\omega}_k}\right)^2\right|-20\sum_{i=1}^{n_1}\log_{10}|1+j\omega T_i|$$
$$-20\sum_{k=1}^{n_2}\log_{10}\left|1+2\frac{\zeta_k}{\omega_k}j\omega+\left(\frac{j\omega}{\omega_k}\right)^2\right|$$

- $$\varphi(\omega)=\arg\{G(j\omega)\}=\arg\{K\}+v\arg\{j\omega\}-\mu\arg\{j\omega\}+\sum_{i=1}^{m_1}\arg\{1+j\omega\tilde{T}_i\}+\sum_{k=1}^{m_2}\arg\left\{1+2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k}j\omega+\left(\frac{j\omega}{\tilde{\omega}_k}\right)^2\right\}$$
$$-\sum_{i=1}^{n_1}\arg\{1+j\omega T_i\}-\sum_{k=1}^{n_2}\arg\left\{1+2\frac{\zeta_k}{\omega_k}j\omega+\left(\frac{j\omega}{\omega_k}\right)^2\right\}$$

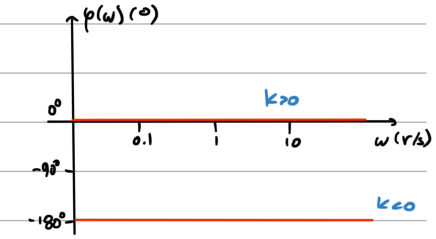
Διαγράμματα Bode παραγόντων

α) Κέρδος K:



$20\log_{10}|K|$, $|K|>1$

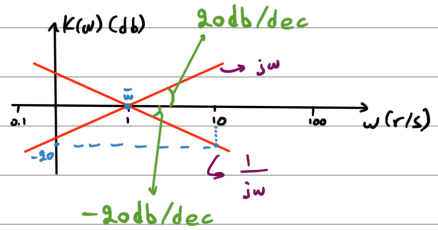
$20\log_{10}|K|$, $|K|<1$



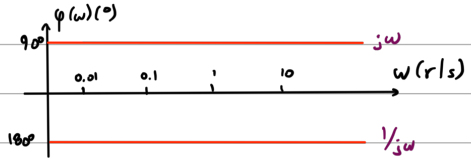
β) Παράγοντας ολοκλήρωσης ή παραγώγισης: $s^{\pm1}\longrightarrow(j\omega)^{\pm1}$

$$K(\omega)=20\log_{10}\left|\frac{1}{j\omega}\right|=20\log_{10}\{1/\omega\}=-20\log_{10}\{\omega\}$$

$$K(\bar{\omega})=0\Rightarrow\log_{10}\{\bar{\omega}\}=0\Rightarrow\bar{\omega}=1$$



$$\varphi(\omega)=\arg\{1/j\omega\}=\arg\{-j/\omega\}=\arctan\{-1/0\}=-90°$$



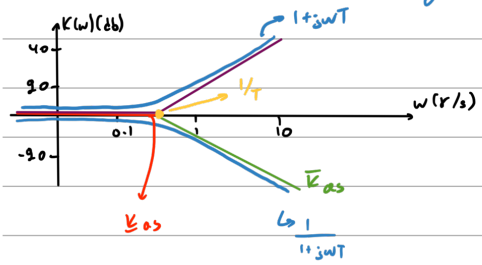
γ) Πρωτοβάθμιος παράγοντας $(1+j\omega T)^{\pm1}$

i) αν $T>0$: $K(\omega)=20\log_{10}\left|\frac{1}{1+j\omega T}\right|=-20\log_{10}\sqrt{1+\omega^2T^2}$

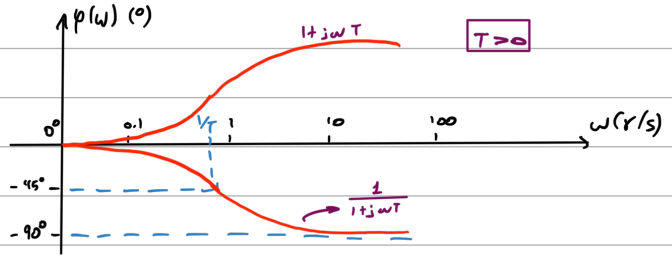
Ασύμπτωτα της $K(\omega)$: αν $\omega\ll 1/T$: $K(\omega)\simeq-20\log_{10}(1)=0\triangleq$ Ασύμπτωτος χαμηλών συχνοτήτων, $K_{as}(\omega)$

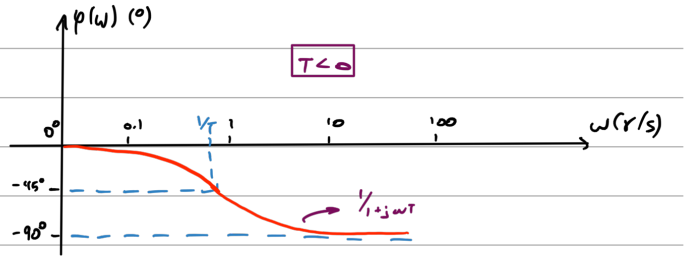
αν $\omega\gg 1/T$: $K(\omega)\simeq-20\log_{10}(\omega T)\triangleq$ Ασύμπτωτος υψηλών συχνοτήτων, $\bar{K}_{as}(\omega)$,

Το σημείο τομής τους: $20\log_{10}(\omega_0T)=0\Rightarrow\omega_0=\frac{1}{T}\rightarrow$ συχν. θλάσης κλίσης -20 db/dec



ii) αν $T<0$ τότε: $\omega_\theta = 1/|T|$



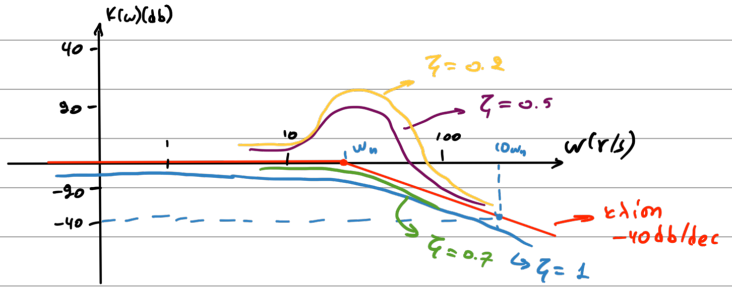


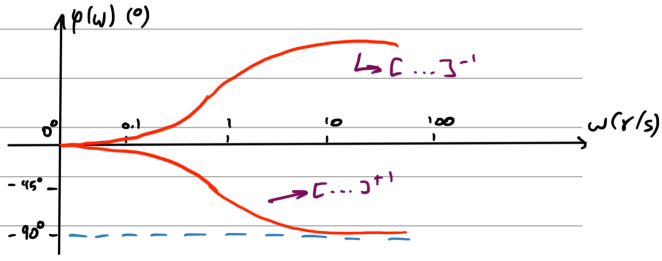
δ) Δευτεροβάθμιος παράγοντας: $\left[1 + 2\frac{\zeta}{\omega_n}(j\omega) + \left(\frac{j\omega}{\omega_n}\right)^2\right]$, $\zeta < 1$

$$K(\omega) = 20\log_{10}\left|\frac{1}{1 + 2\zeta\frac{j\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2}\right| = -20\log_{10}\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

- $\omega \ll \omega_n$: $K(\omega) \simeq -20\log_{10}(1) = 0 \triangleq K_{\alpha\sigma}$
- $\omega \gg \omega_n$: $K(\omega) \simeq -20\log_{10}\left(\frac{\omega}{\omega_n}\right)^2 = -40\log_{10}\left(\frac{\omega}{\omega_n}\right) \triangleq \bar{K}_{\alpha\sigma}$

Το σημείο τομής τους: $K_{\alpha\sigma}(\omega_\theta) = \bar{K}_{\alpha\sigma}(\omega_\theta) \Rightarrow \omega_\theta = \omega_n \rightsquigarrow$ συχν. θλάσης





Παράδειγμα

$$G(s) = \frac{10s-30}{s^4+3s^3+4s^2+2s} = \frac{-30(1-s/3)}{s(s^3+3s^2+4s+2)} = \frac{-30(1-s/3)}{s(s+2)(s^2+s+2)} = \frac{-30(1-s/3)}{4s(1+s/2)(1+s/2+s^2/2)} =$$

$$= \frac{-7.5(1-s/3)}{s(1+s/2)(1+s/2+s^2/2)}$$

$\hookrightarrow \begin{cases} \omega_n = \sqrt{2} \\ \zeta = \sqrt{2}/4 \ (<1) \end{cases}$

$K = -7.5$, $T_1 = -1/3$, $T_2 = 1/2$, $\omega_n = \sqrt{2}$, $\zeta = \sqrt{2}/4 \ (<1)$

Συχνότητα θλάσης: Αριθμητή: $\omega_{\theta_1} = \frac{1}{|T_1|} = 3$ Παρονομαστή: $\omega_{\theta_2} = \frac{1}{T_2} = 2$, $\omega_{\theta_3} = \omega_n = \sqrt{2}$