CHAPTER 6

Bipolar Junction Transistors (BJTs)

IN THIS CHAPTER YOU WILL LEARN

1. The physical structure of the bipolar transistor and how it works.
2. How the voltage between two terminals of the transistor controls the current that flows through the third terminal, and the equations that describe these current–voltage characteristics.
3. How to analyze and design circuits that contain bipolar transistors, resistors, and dc sources.

Introduction

In this chapter, we study the other major three-terminal device: the bipolar junction transistor (BJT). The presentation of the material in this chapter parallels but does not rely on that for the MOSFET in Chapter 5; thus, if desired, the BJT can be studied before the MOSFET.

Three-terminal devices are far more useful than two-terminal ones, such as the diodes studied in Chapter 4, because they can be used in a multitude of applications, ranging from signal amplification to the design of digital logic and memory circuits. The basic principle involved is the use of the voltage between two terminals to control the current flowing in the third terminal. In this way, a three-terminal device can be used to realize a controlled source, which as we learned in Chapter 1 is the basis for amplifier design. Also, in the extreme, the control signal can be used to cause the current in the third terminal to change from zero to a large value, thus allowing the device to act as a switch. The switch is the basis for the realization of the logic inverter, the basic element of digital circuits.

The invention of the BJT in 1948 at the Bell Telephone Laboratories ushered in the era of solid-state circuits. The result was not just the replacement of vacuum tubes by transistors in radios and television sets but the eruption of an electronics revolution that led to major changes in the way we work, play, and indeed, live. The invention of the transistor also eventually led to the dominance of information technology and the emergence of the knowledge-based economy.

The bipolar transistor enjoyed nearly three decades as the device of choice in the design of both discrete and integrated circuits. Although the MOSFET had been known very early on, it was not until the 1970s and 1980s that it became a serious competitor to the BJT. By 2014, the MOSFET was undoubtedly the most widely used electronic device, and CMOS technology the technology of choice in the design of integrated circuits. Nevertheless, the BJT remains a significant device that excels in certain applications.

The BJT remains popular in discrete-circuit design, where it is used together with other discrete components such as resistors and capacitors to implement circuits that are assembled on printed-circuit boards (PCBs). Here we note the availability of a very wide selection of BJT types that fit nearly every conceivable application. As well, the BJT is still the preferred device in some very demanding analog and digital integrated-circuit applications. This is especially true in very-high-frequency and high-speed circuits. In particular, a very-high-speed digital logic-circuit family based on bipolar transistors, namely, emitter-coupled logic, is still in use (Chapter 15). Finally, bipolar transistors can be combined with MOSFETs to create innovative circuits that take advantage of the high-input-impedance and low-power operation of MOSFETs and the very-high-frequency operation and high-current-driving capability of bipolar transistors. The resulting technology is known as BiCMOS, and it is finding increasingly larger areas of application (see Chapters 8, 9, 13, and 15).

In this chapter, we shall start with a description of the physical operation of the BJT. Though simple, this physical description provides considerable insight regarding the performance of the transistor as a circuit element. We then quickly move from describing current flow in terms of electrons and holes to a study of the transistor terminal characteristics. Circuit models for transistor operation in different modes will be developed and utilized in the analysis and design of transistor circuits. The main objective of this chapter is to develop in the reader a high degree of familiarity with the BJT. Thus, it lays the foundation for the use of the BJT in amplifier design (Chapter 7).

6.1 Device Structure and Physical Operation

6.1.1 Simplified Structure and Modes of Operation

Figure 6.1 shows a simplified structure for the BJT. A practical transistor structure will be shown later (see also Appendix A, which deals with fabrication technology).

As shown in Fig. 6.1, the BJT consists of three semiconductor regions: the emitter region (*n* type), the base region (*p* type), and the collector region (*n* type). Such a transistor is called an *npn* transistor. Another transistor, a dual of the *npn* as shown in Fig. 6.2, has a *p*-type emitter, an *n*-type base, and a *p*-type collector, and is appropriately called a *pnp* transistor.

A terminal is connected to each of the three semiconductor regions of the transistor, with the terminals labeled **emitter** (E), **base** (B), and **collector** (C).

The transistor consists of two *pn* junctions, the **emitter–base junction** (EBJ) and the **collector–base junction** (CBJ). Depending on the bias condition (forward or reverse) of each of these junctions, different modes of operation of the BJT are obtained, as shown in Table 6.1. The **active mode** is the one used if the transistor is to operate as an amplifier. Switching applications (e.g., logic circuits) utilize both the **cutoff mode** and the **saturation mode**. As the name implies, in the cutoff mode no current flows because both junctions are reverse biased.

As we will see shortly, charge carriers of both polarities—that is, electrons and holes—participate in the current-conduction process in a bipolar transistor, which is the reason for the name *bipolar*.[1]

[1]This should be contrasted with the situation in the MOSFET, where current is conducted by charge carriers of one type only: electrons in *n*-channel devices or holes in *p*-channel devices. In earlier days, some referred to FETs as unipolar devices.

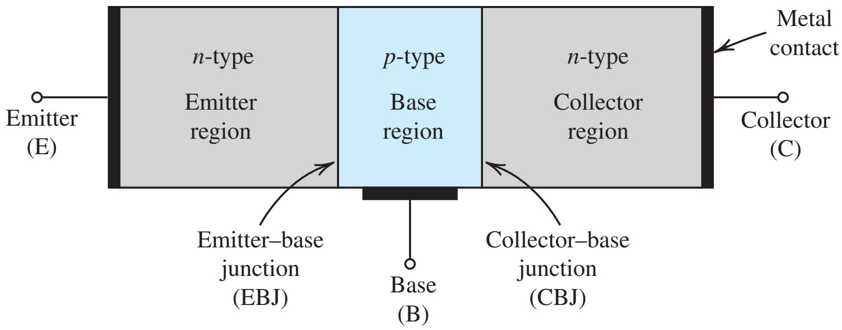


Figure 6.1 A simplified structure of the *npn* transistor.

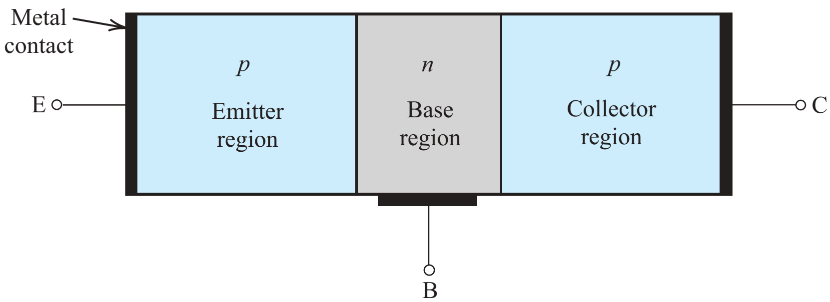


Figure 6.2 A simplified structure of the *pnp* transistor.

Table 6.1 BJT Modes of Operation

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

6.1.2 Operation of the *npn* Transistor in the Active Mode

Of the three modes of operation of the BJT, the active mode is the most important. Therefore, we begin our study of the BJT by considering its physical operation in the active mode.[2] This situation is illustrated in Fig. 6.3 for the *npn* transistor. Two external voltage sources (shown as batteries) are used to establish the required bias conditions for active-mode operation. The voltage V_{BE} causes the *p*-type base to be higher in potential than the *n*-type emitter, thus forward biasing the emitter–base junction. The collector–base voltage V_{CB} causes the *n*-type collector to be at a higher potential than the *p*-type base, thus reverse biasing the collector–base junction.

[2]The material in this section assumes that the reader is familiar with the operation of the *pn* junction under forward-bias conditions (Section 3.5).

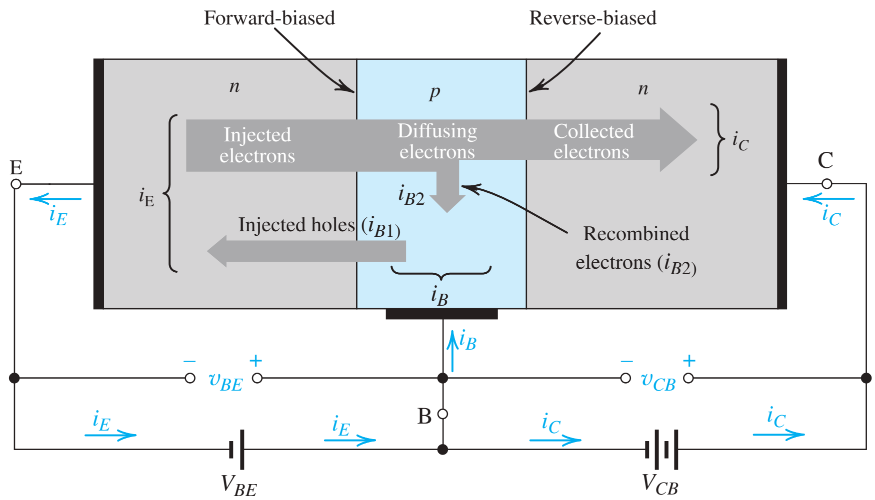


Figure 6.3 Current flow in an *npn* transistor biased to operate in the active mode. (Reverse current components due to drift of thermally generated minority carriers are not shown.)

Current Flow The forward bias on the emitter–base junction will cause current to flow across this junction. Current will consist of two components: electrons injected from the emitter into the base, and holes injected from the base into the emitter. As will become apparent shortly, it is highly desirable to have the first component (electrons from emitter to base) be much larger than the second component (holes from base to emitter). This can be accomplished by fabricating the device with a heavily doped emitter and a lightly doped base; that is, the device is designed to have a high density of electrons in the emitter and a low density of holes in the base.

The current that flows across the emitter–base junction will constitute the emitter current i_E, as indicated in Fig. 6.3. The direction of i_E is "out of" the emitter lead, which, following the usual conventions, is in the direction of the positive-charge flow (hole current) and opposite to the direction of the negative-charge flow (electron current), with the emitter current i_E being equal to the sum of these two components. However, since the electron component is much larger than the hole component, the emitter current will be dominated by the electron component.

From our study in Section 3.5 of the current flow across a forward-biased *pn* junction, we know that the magnitude of both the electron component and the hole component of i_E will be proportional to e^{v_{BE}/V_T}, where v_{BE} is the forward voltage across the base–emitter junction and V_T is the thermal voltage (approximately 25 mV at room temperature).

Let's now focus our attention on the first current component, namely, that carried by electrons injected from the emitter into the base. These electrons will be **minority carriers** in the *p*-type base region. Because their concentration will be highest at the emitter side of the base, the injected electrons will diffuse through the base region toward the collector. In their journey across the base, some of the electrons will combine with holes, which are majority carriers in the base. However, since the base is usually very thin and, as mentioned earlier, lightly doped, the proportion of electrons that are "lost" through this **recombination process** will be quite small. Thus, most of the diffusing electrons will reach the boundary of the collector–base depletion region. Because the collector is more positive than the base (by the reverse-bias voltage v_{CB}), these successful electrons will be swept across the CBJ depletion region into the collector. They will thus get collected and constitute the collector current i_C.

The Collector Current From the foregoing statements, we see that the collector current is carried by the electrons that reach the collector region. Its direction will be opposite to that of the flow of electrons, and thus into the collector terminal. Its magnitude will be proportional to e^{v_{BE}/V_T}, thus

$$i_C = I_S e^{v_{BE}/V_T} \tag{6.1}$$

where the constant of proportionality I_S, as in the case of the diode, is called the **saturation current** and is a transistor parameter. We will have more to say about I_S shortly.

An important observation to make here is that i_C is independent of the value of v_{CB}. That is, as long as the collector is positive with respect to the base, the electrons that reach the collector side of the base region will be swept into the collector and will register as collector current.

The Base Current Reference to Fig. 6.3 shows that the base current i_B is composed of two components. The first component i_{B1} is due to the holes injected from the base region into the emitter region. This current component is proportional to e^{v_{BE}/V_T}. The second component of base current, i_{B2}, is due to holes that have to be supplied by the external circuit in order to replace the holes lost from the base through the recombination process. Because i_{B2} is proportional to the number of electrons injected into the base, it also will be proportional to e^{v_{BE}/V_T}. Thus the total base current, $i_B = i_{B1} + i_{B2}$, will be proportional to e^{v_{BE}/V_T}, and can be expressed as a fraction of the collector current i_C as follows:

$$i_B = \frac{i_C}{\beta} \tag{6.2}$$

That is,

$$i_B = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T} \tag{6.3}$$

where β is a transistor parameter.

For modern *npn* transistors, β is in the range 50 to 200, but it can be as high as 1000 for special devices. For reasons that will become clear later, the parameter β is called the **common-emitter current gain**.

The above description indicates that the value of β is highly influenced by two factors: the width of the base region, W, and the relative dopings of the base region and the emitter region, N_A/N_D. To obtain a high β (which is highly desirable since β represents a gain parameter) the base should be thin (W small) and lightly doped and the emitter heavily doped (making N_A/N_D small). For modern integrated circuit fabrication technologies, W is in the nanometer range.

The Emitter Current Since the current that enters a transistor must leave it, it can be seen from Fig. 6.3 that the emitter current i_E is equal to the sum of the collector current i_C and the base current i_B; that is,

$$i_E = i_C + i_B \tag{6.4}$$

Use of Eqs. (6.2) and (6.4) gives

$$i_E = \frac{\beta + 1}{\beta} i_C \tag{6.5}$$

That is,

$$i_E = \frac{\beta + 1}{\beta} I_S e^{v_{BE}/V_T} \tag{6.6}$$

Alternatively, we can express Eq. (6.5) in the form

$$i_C = \alpha i_E \tag{6.7}$$

where the constant α is related to β by

$$\alpha = \frac{\beta}{\beta + 1} \tag{6.8}$$

Thus the emitter current in Eq. (6.6) can be written

$$i_E = (I_S/\alpha) e^{v_{BE}/V_T} \tag{6.9}$$

Finally, we can use Eq. (6.8) to express β in terms of α, that is,

$$\beta = \frac{\alpha}{1 - \alpha} \tag{6.10}$$

It can be seen from Eq. (6.8) that α is a constant (for a particular transistor) that is less than but very close to unity. For instance, if $\beta = 100$, then $\alpha \simeq 0.99$. Equation (6.10) reveals an important fact: Small changes in α correspond to very large changes in β. This mathematical observation manifests itself physically, with the result that transistors of the same type may have widely different values of β. For reasons that will become apparent later, α is called the **common-base current gain**.

Minority-Carrier Distribution Our understanding of the physical operation of the BJT can be enhanced by considering the distribution of minority charge carriers in the base and the emitter. Figure 6.4 shows the profiles of the concentration of electrons in the base and holes in the emitter of an *npn* transistor operating in the active mode. Observe that since the doping concentration in the emitter, N_D, is much higher than the doping concentration in the base, N_A, the concentration of electrons injected from emitter to base, $n_p(0)$, is much higher than the concentration of holes injected from the base to the emitter, $p_n(0)$. Both quantities are proportional to e^{v_{BE}/V_T}, thus

$$n_p(0) = n_{p0} e^{v_{BE}/V_T} \tag{6.11}$$

where n_{p0} is the thermal-equilibrium value of the minority-carrier (electron) concentration in the base region.

Next, observe that because the base is very thin, the concentration of excess electrons decays almost linearly (as opposed to the usual exponential decay, as observed for the excess holes in the emitter region). Furthermore, the reverse bias on the collector–base junction causes the concentration of excess electrons at the collector side of the base to be zero. (Recall that electrons that reach that point are swept into the collector.)

The tapered minority-carrier concentration profile (Fig. 6.4) causes the electrons injected into the base to diffuse through the base region toward the collector. This electron diffusion

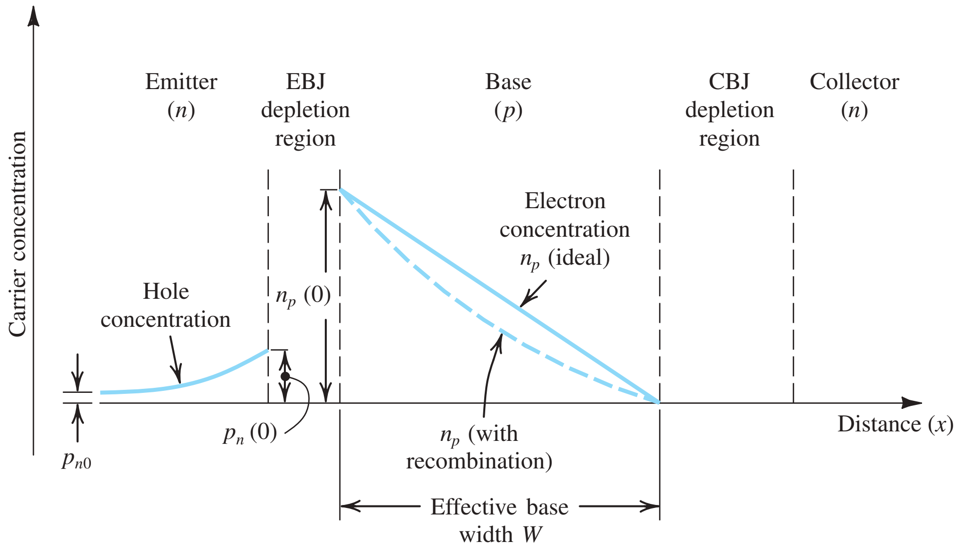


Figure 6.4 Profiles of minority-carrier concentrations in the base and in the emitter of an *npn* transistor operating in the active mode: $v_{BE} > 0$ and $v_{CB} \geq 0$.

current I_n is directly proportional to the slope of the straight-line concentration profile,

$$
\begin{aligned}
I_n &= A_E q D_n \frac{dn_p(x)}{dx} \\
&= A_E q D_n \left(-\frac{n_p(0)}{W}\right)
\end{aligned}
\tag{6.12}
$$

where A_E is the cross-sectional area of the base–emitter junction (in the direction perpendicular to the page), q is the magnitude of the electron charge, D_n is the electron diffusivity in the base, and W is the effective width of the base. Observe that the negative slope of the minority-carrier concentration results in a negative current I_n across the base; that is, I_n flows from right to left (in the negative direction of x), which corresponds to the usual convention, namely, opposite to the direction of electron flow.

The recombination in the base region, though slight, causes the excess minority-carrier concentration profile to deviate from a straight line and take the slightly concave shape indicated by the broken line in Fig. 6.4. The slope of the concentration profile at the EBJ is slightly higher than that at the CBJ, with the difference accounting for the small number of electrons lost in the base region through recombination.

Finally, we have the collector current $i_C = I_n$, which will yield a negative value for i_C, indicating that i_C flows in the negative direction of the x axis (i.e., from right to left). Since we will take this to be the positive direction of i_C, we can drop the negative sign in Eq. (6.12). Doing this and substituting for $n_p(0)$ from Eq. (6.11), we can thus express the collector current i_C as

$$i_C = I_S e^{v_{BE}/V_T}$$

where the saturation current I_S is given by

$$I_S = A_E q D_n n_{p0}/W$$

Substituting $n_{p0} = n_i^2/N_A$, where n_i is the intrinsic carrier density and N_A is the doping concentration in the base, we can express I_S as

$$I_S = \frac{A_E q D_n n_i^2}{N_A W} \tag{6.13}$$

The saturation current I_S is inversely proportional to the base width W and is directly proportional to the area of the EBJ. Typically I_S is in the range of 10^{-12} A to 10^{-18} A (depending on the size of the device). Because I_S is proportional to n_i^2, it is a strong function of temperature, approximately doubling for every 5°C rise in temperature. (For the dependence of n_i^2 on temperature, refer to Eq. 3.2.)

Since I_S is directly proportional to the junction area (i.e., the device size), it will also be referred to as the **scale current**. Two transistors that are identical except that one has an EBJ area, say, twice that of the other will have saturation currents with that same ratio (i.e., 2). Thus for the same value of v_{BE} the larger device will have a collector current twice that in the smaller device. This concept is frequently employed in integrated-circuit design.

Recapitulation and Equivalent-Circuit Models We have presented a first-order model for the operation of the *npn* transistor in the active mode. Basically, the forward-bias voltage v_{BE} causes an exponentially related current i_C to flow in the collector terminal. The collector current i_C is independent of the value of the collector voltage as long as the collector–base junction remains reverse biased; that is, $v_{CB} \geq 0$. Thus in the active mode the collector terminal behaves as an ideal constant-current source where the value of the current is determined by v_{BE}. The base current i_B is a factor $1/\beta$ of the collector current, and the emitter current is equal to the sum of the collector and base currents. Since i_B is much smaller than i_C (i.e., $\beta \gg 1$), $i_E \simeq i_C$. More precisely, the collector current is a fraction α of the emitter current, with α smaller than, but close to, unity.

This first-order model of transistor operation in the active mode can be represented by the equivalent circuit shown in Fig. 6.5(a). Here, diode D_E has a scale current I_{SE} equal to (I_S/α) and thus provides a current i_E related to v_{BE} according to Eq. (6.9). The current of the controlled source, which is equal to the collector current, is controlled by v_{BE} according to the exponential relationship indicated, a restatement of Eq. (6.1). This model is in essence a nonlinear voltage-controlled current source. It can be converted to the current-controlled current-source model shown in Fig. 6.5(b) by expressing the current of the controlled source as αi_E. Note that this model is also nonlinear because of the exponential relationship of the current i_E through diode D_E and the voltage v_{BE}. From this model we observe that if the transistor is used as a two-port network with the input port between E and B and the output port between C and B (i.e., with B as a common terminal), then the current gain observed is equal to α. Thus α is called the common-base current gain.

Two other equivalent-circuit models, shown in Fig. 6.5(c) and (d), may be used to represent the operation of the BJT. The model of Fig. 6.5(c) is essentially a voltage-controlled current source. However, here diode D_B conducts the base current and thus its current scale factor is I_S/β, resulting in the i_B–v_{BE} relationship given in Eq. (6.3). By simply expressing the collector current as βi_B we obtain the current-controlled current-source model shown in Fig. 6.5(d). From this latter model we observe that if the transistor is used as a two-port network with the input port between B and E and the output port between C and E (i.e., with E as the common terminal), then the current gain observed is equal to β. Thus β is called the common-emitter current gain.

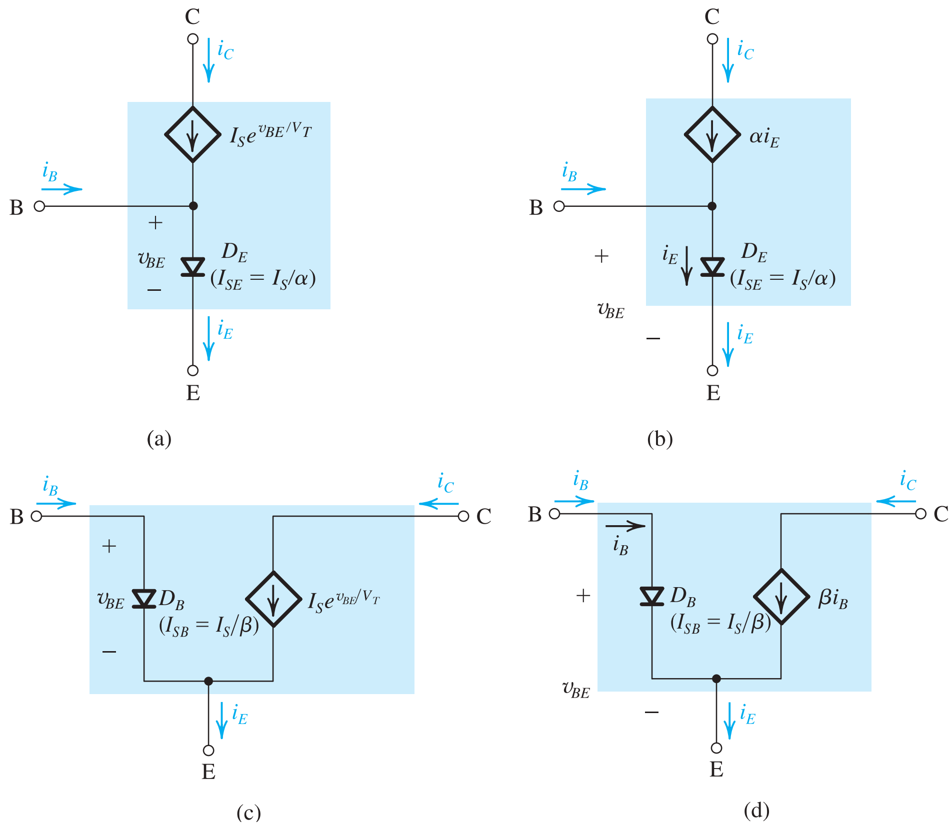


Figure 6.5 Large-signal equivalent-circuit models of the *npn* BJT operating in the forward active mode.

Finally, we note that the models in Fig. 6.5 apply for any positive value of v_{BE}. That is, unlike the models we will be discussing in Chapter 7, here there is no limitation on the size of v_{BE}, and thus these models are referred to as **large-signal models**.

Example 6.1

An *npn* transistor having $I_S = 10^{-15}$ A and $\beta = 100$ is connected as follows: The emitter is grounded, the base is fed with a constant-current source supplying a dc current of 10 μA, and the collector is connected to a 5-V dc supply via a resistance R_C of 3 kΩ. Assuming that the transistor is operating in the active mode, find V_{BE} and V_{CE}. Use these values to verify active-mode operation. Replace the current source with a resistance connected from the base to the 5-V dc supply. What resistance value is needed to result in the same operating conditions?

Example 6.1 *continued*

Solution

If the transistor is operating in the active mode, it can be represented by one of the four possible equivalent-circuit models shown in Fig. 6.5. Because the emitter is grounded, either the model in Fig. 6.5(c) or that in Fig. 6.5(d) would be suitable. Since we know the base current I_B, the model of Fig. 6.5(d) is the most suitable.

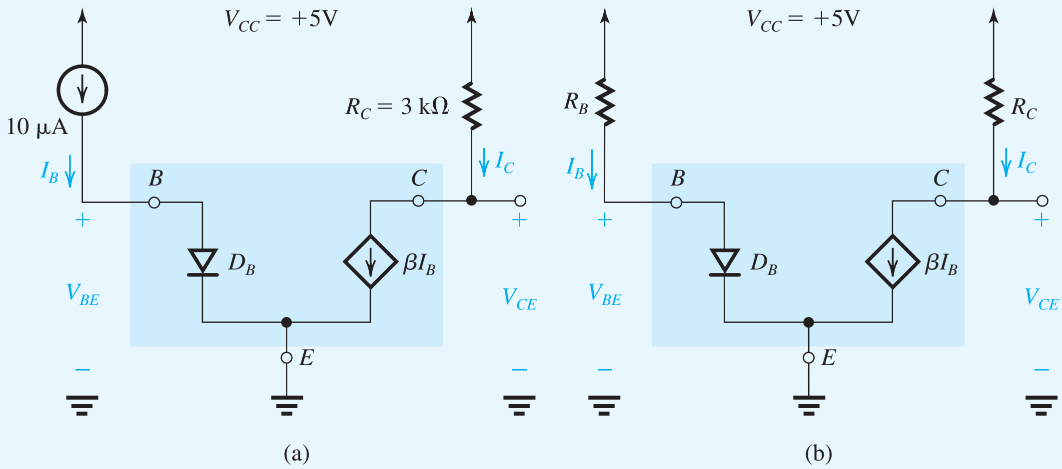


Figure 6.6 Circuits for Example 6.1.

Figure 6.6(a) shows the circuit as described with the transistor represented by the model of Fig. 6.5(d). We can determine V_{BE} from the exponential characteristic of D_B as follows:

$$\begin{aligned} V_{BE} &= V_T \ln \frac{I_B}{I_S/\beta} \\ &= 25 \ln\left(\frac{10 \times 10^{-6}}{10^{-17}}\right) \\ &= 690 \text{ mV} = 0.69 \text{ V} \end{aligned}$$

Next we determine the value of V_{CE} from

$$V_{CE} = V_{CC} - R_C I_C$$

where

$$I_C = \beta I_B = 100 \times 10 \times 10^{-6} = 10^{-3} \text{ A} = 1 \text{ mA}$$

Thus,

$$V_{CE} = 5 - 3 \times 1 = +2 \text{ V}$$

Since V_C at +2 V is higher than V_B at 0.69 V, the transistor is indeed operating in the active mode.

Now, replacing the 10-μA current source with a resistance R_B connected from the base to the 5-V dc supply V_{CC}, as in Fig. 6.6(b), the value of R_B must be

$$\begin{aligned} R_B &= \frac{V_{CC} - V_{BE}}{I_B} \\ &= \frac{5 - 0.69}{10 \ \mu\text{A}} = 431 \text{ k}\Omega \end{aligned}$$

EXERCISES

6.1 Consider an *npn* transistor with $v_{BE} = 0.7$ V at $i_C = 1$ mA. Find v_{BE} at $i_C = 0.1$ mA and 10 mA.
Ans. 0.64 V; 0.76 V

6.2 Transistors of a certain type are specified to have β values in the range of 50 to 150. Find the range of their α values.
Ans. 0.980 to 0.993

6.3 Measurement of an *npn* BJT in a particular circuit shows the base current to be 14.46 μA, the emitter current to be 1.460 mA, and the base–emitter voltage to be 0.7 V. For these conditions, calculate α, β, and I_S.
Ans. 0.99; 100; 10^{-15} A

6.4 Calculate β for two transistors for which $\alpha = 0.99$ and 0.98. For collector currents of 10 mA, find the base current of each transistor.
Ans. 99; 49; 0.1 mA; 0.2 mA

6.5 A transistor for which $I_S = 10^{-16}$ A and $\beta = 100$ is conducting a collector current of 1 mA. Find v_{BE}. Also, find I_{SE} and I_{SB} for this transistor.
Ans. 747.5 mV; 1.01×10^{-16} A; 10^{-18} A

6.6 For the circuit in Fig. 6.6(a) analyzed in Example 6.1, find the maximum value of R_C that will still result in active-mode operation.
Ans. 4.31 kΩ

6.1.3 Structure of Actual Transistors

Figure 6.7 shows a more realistic (but still simplified) cross section of an *npn* BJT. Note that the collector virtually surrounds the emitter region, thus making it difficult for the electrons injected into the thin base to escape being collected. In this way, the resulting α is close to

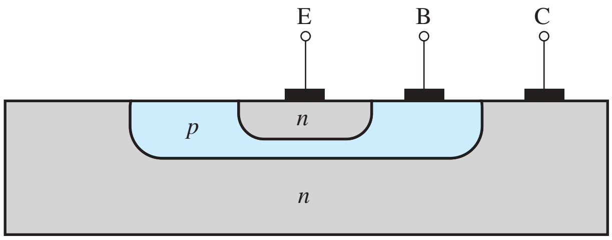


Figure 6.7 Cross section of an *npn* BJT.

unity and β is large. Also, observe that the device is *not* symmetrical, and thus the emitter and collector cannot be interchanged.[3] For more detail on the physical structure of actual devices, the reader is referred to Appendix A.

The structure in Fig. 6.7 indicates also that the CBJ has a much larger area than the EBJ. Thus the CB diode D_C has a saturation current I_{SC} that is much larger than the saturation current of the EB diode D_E. Typically, I_{SC} is 10 to 100 times larger than I_{SE} (recall that $I_{SE} = I_S/\alpha \simeq I_S$).

EXERCISE

6.7 A particular transistor has $I_S = 10^{-15}$ A and $\alpha \simeq 1$. If the CBJ area is 100 times the area of the EBJ, find the collector scale current I_{SC}.
Ans. 10^{-13} A

6.1.4 Operation in the Saturation Mode[4]

As mentioned above, for the BJT to operate in the active mode, the CBJ must be reverse biased. Thus far, we have stated this condition for the *npn* transistor as $v_{CB} \geq 0$. However, we know that a *pn* junction does not effectively become forward biased until the forward voltage across it exceeds approximately 0.4 V. It follows that one can maintain active-mode operation of an *npn* transistor for negative v_{CB} down to approximately −0.4 V. This is illustrated in Fig. 6.8, which is a sketch of i_C versus v_{CB} for an *npn* transistor operated with a constant emitter current I_E. As expected, i_C is independent of v_{CB} in the active mode, a situation that extends

[3] If the emitter and collector are reversed—that is, the CBJ is forward biased and the EBJ is reverse biased—the device operates in a mode called the "reverse-active mode." The resulting values of α and β, denoted α_R and β_R (with R denoting reverse), are much lower than the values of α and β, respectively, obtained in the "forward"-active mode discussed above. Hence, the reverse-active mode has no practical application. The MOSFET, on the other hand, being a perfectly symmetrical device, can operate equally well with its drain and source terminals interchanged.

[4] Saturation means something completely different in a BJT and in a MOSFET. The saturation mode of operation of the BJT is analogous to the triode region of operation of the MOSFET. On the other hand, the saturation region of operation of the MOSFET corresponds to the active mode of BJT operation.

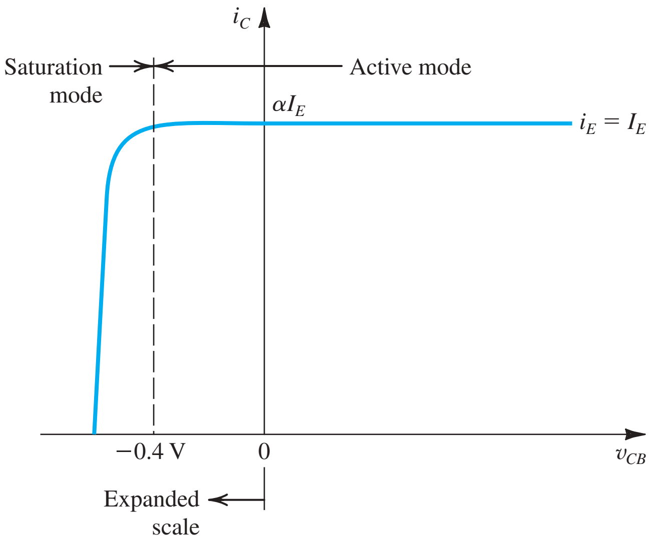


Figure 6.8 The i_C–v_{CB} characteristic of an *npn* transistor fed with a constant emitter current I_E. The transistor enters the saturation mode of operation for $v_{CB} < -0.4$ V, and the collector current diminishes.

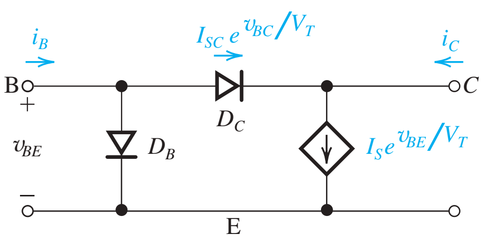


Figure 6.9 Modeling the operation of an *npn* transistor in saturation by augmenting the model of Fig. 6.5(c) with a forward-conducting diode D_C. Note that the current through D_C increases i_B and reduces i_C.

for v_{CB} going negative to approximately −0.4 V. Below this value of v_{CB}, the CBJ begins to conduct sufficiently that the transistor leaves the active mode and enters the saturation mode of operation, where i_C decreases.

To see why i_C decreases in saturation, we can construct a model for the saturated *npn* transistor as follows. We augment the model of Fig. 6.5(c) with the forward-conducting CBJ diode D_C, as shown in Fig. 6.9. Observe that the current i_{BC} will subtract from the controlled-source current, resulting in the reduced collector current i_C given by

$$i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T} \tag{6.14}$$

where I_{SC} is the saturation current for D_C and is related to I_S by the ratio of the areas of the CBJ and the EBJ. The second term in Eq. (6.14) will play an increasing role as v_{BC} exceeds 0.4 V or so, causing i_C to decrease and eventually reach zero.

Figure 6.9 also indicates that in saturation the base current will increase to the value

$$i_B = (I_S/\beta) e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T} \tag{6.15}$$

Equations (6.14) and (6.15) can be combined to obtain the ratio i_C/i_B for a saturated transistor. We observe that this ratio will be *lower* than the value of β. Furthermore, the ratio will decrease as v_{BC} is increased and the transistor is driven deeper into saturation. Because i_C/i_B

of a saturated transistor can be set to any desired value lower than β by adjusting v_{BC}, this ratio is known as **forced** β and denoted β_{forced},

$$\beta_{\text{forced}} = \left.\frac{i_C}{i_B}\right|_{\text{saturation}} \leq \beta \tag{6.16}$$

As will be shown later, in analyzing a circuit we can determine whether the BJT is in the saturation mode by either of the following two tests:

1. Is the CBJ forward biased by more than 0.4 V?
2. Is the ratio i_C/i_B lower than β?

The collector-to-emitter voltage v_{CE} of a saturated transistor can be found from Fig. 6.9 as the difference between the forward-bias voltages of the EBJ and the CBJ,

$$V_{CE\text{sat}} = V_{BE} - V_{BC} \tag{6.17}$$

Recalling that the CBJ has a much larger area than the EBJ, V_{BC} will be smaller than V_{BE} by 0.1 to 0.3 V. Thus,

$$V_{CE\text{sat}} \simeq 0.1 \text{ to } 0.3 \text{ V}$$

Typically we will assume that a transistor at the edge of saturation has $V_{CE\text{sat}} = 0.3$ V, while a transistor deep in saturation has $V_{CE\text{sat}} = 0.2$ V.

EXERCISES

6.8 Use Eq. (6.14) to show that i_C reaches zero at

$$V_{CE} = V_T \ln\left(I_{SC}/I_S\right)$$

Calculate V_{CE} for a transistor whose CBJ has 100 times the area of the EBJ.
Ans. 115 mV

6.9 Use Eqs. (6.14), (6.15), and (6.16) to show that a BJT operating in saturation with $V_{CE} = V_{CE\text{sat}}$ has a forced β given by

$$\beta_{\text{forced}} = \beta \frac{e^{V_{CE\text{sat}}/V_T} - I_{SC}/I_S}{e^{V_{CE\text{sat}}/V_T} + \beta I_{SC}/I_S}$$

Find β_{forced} for $\beta = 100$, $I_{SC}/I_S = 100$, and $V_{CE\text{sat}} = 0.2$ V.
Ans. 22.2

6.1.5 The *pnp* Transistor

The *pnp* transistor operates in a manner similar to that of the *npn* device described above. Figure 6.10 shows a *pnp* transistor biased to operate in the active mode. Here the voltage V_{EB} causes the *p*-type emitter to be higher in potential than the *n*-type base, thus forward biasing the emitter–base junction. The collector–base junction is reverse biased by the voltage V_{BC}, which keeps the *p*-type collector lower in potential than the *n*-type base.

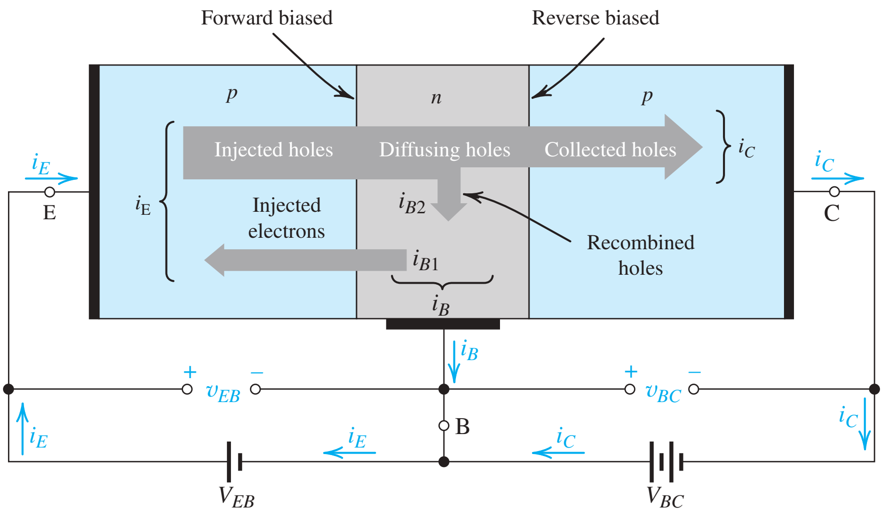


Figure 6.10 Current flow in a *pnp* transistor biased to operate in the active mode.

Unlike the *npn* transistor, current in the *pnp* device is mainly conducted by holes injected from the emitter into the base as a result of the forward-bias voltage V_{EB}. Since the component of emitter current contributed by electrons injected from base to emitter is kept small by using a lightly doped base, most of the emitter current will be due to holes. The electrons injected from base to emitter give rise to the first component of base current, i_{B1}. Also, a number of the holes injected into the base will recombine with the majority carriers in the base (electrons) and will thus be lost. The disappearing base electrons will have to be replaced from the external circuit, giving rise to the second component of base current, i_{B2}. The holes that succeed in reaching the boundary of the depletion region of the collector–base junction will be attracted by the negative voltage on the collector. Thus these holes will be swept across the depletion region into the collector and appear as collector current.

It can easily be seen from the above description that the current–voltage relationship of the *pnp* transistor will be identical to that of the *npn* transistor except that v_{BE} has to be replaced by v_{EB}. Also, the large-signal, active-mode operation of the *pnp* transistor can be modeled by any of four equivalent circuits similar to those for the *npn* transistor in Fig. 6.5. Two of these four circuits are shown in Fig. 6.11. Finally, we note that the *pnp* transistor can operate in the saturation mode in a manner analogous to that described for the *npn* device.

EXERCISES

6.10 Consider the model in Fig. 6.11(a) applied in the case of a *pnp* transistor whose base is grounded, the emitter is fed by a constant-current source that supplies a 2-mA current into the emitter terminal, and the collector is connected to a –10-V dc supply. Find the emitter voltage, the base current, and the collector current if for this transistor $\beta = 50$ and $I_S = 10^{-14}$ A.
Ans. 0.650 V; 39.2 μA; 1.96 mA

6.11 For a *pnp* transistor having $I_S = 10^{-11}$ A and $\beta = 100$, calculate v_{EB} for $i_C = 1.5$ A.
Ans. 0.643 V

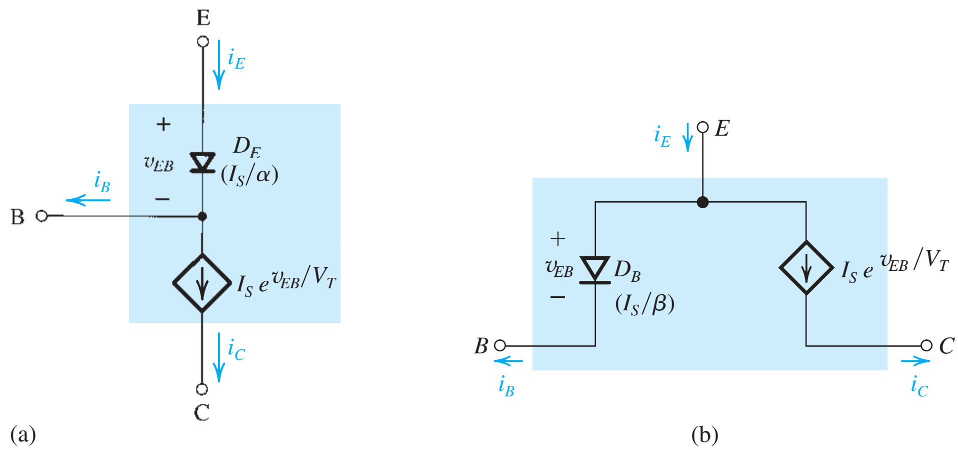


Figure 6.11 Two large-signal models for the *pnp* transistor operating in the active mode.

THE INVENTION OF THE BJT:

The first working transistor was demonstrated at the Bell Labs in late 1947 by John Bardeen and Walter Brattain, who were part of a team led by William Shockley. Made of germanium, the device became known as a point-contact transistor and operated on the field-effect principle. Within a few weeks, however, Shockley wrote a complete description of the bipolar junction transistor (BJT) and filed for a U.S. patent with the title "Circuit Element Utilizing Semiconductor Material."

BJTs dominated the electronics world from the early 1950s to the mid-1970s, when MOSFETs took over the leading position. In 1956, Shockley, Bardeen, and Brattain shared the Nobel Prize in Physics for the discovery of the transistor effect.

6.2 Current–Voltage Characteristics

6.2.1 Circuit Symbols and Conventions

The physical structure used thus far to explain transistor operation is rather cumbersome to employ in drawing the schematic of a multitransistor circuit. Fortunately, a very descriptive and convenient circuit symbol exists for the BJT. Figure 6.12(a) shows the symbol for the *npn* transistor; the *pnp* symbol is given in Fig. 6.12(b). In both symbols the emitter is distinguished by an arrowhead. This distinction is important because, as we have seen in the last section, practical BJTs are not symmetric devices.

The polarity of the device—*npn* or *pnp*—is indicated by the direction of the arrowhead on the emitter. This arrowhead points in the direction of normal current flow in the emitter, which is also the forward direction of the base–emitter junction. Since we have adopted a drawing convention by which currents flow from top to bottom, we will always draw *pnp* transistors in the manner shown in Fig. 6.12(b) (i.e., with their emitters on top).

Figure 6.13 shows *npn* and *pnp* transistors connected to dc sources so as to operate in the active mode. Figure 6.13 also indicates the reference and actual directions of current flow throughout the transistor. Our convention will be to take the reference direction to coincide

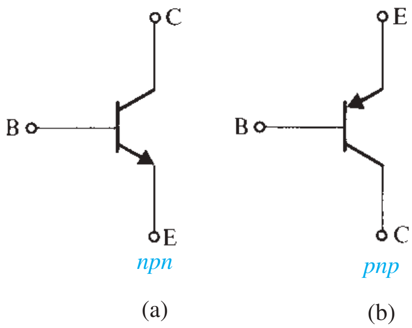


Figure 6.12 Circuit symbols for BJTs.

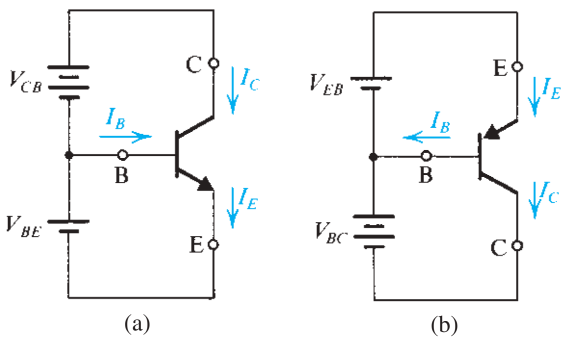


Figure 6.13 Voltage polarities and current flow in transistors operating in the active mode.

with the normal direction of current flow. Hence, normally, we should not encounter a negative value for i_E, i_B, or i_C.

The convenience of the circuit-drawing convention that we have adopted should be obvious from Fig. 6.13. Note that currents flow from top to bottom and that voltages are higher at the top and lower at the bottom. The arrowhead on the emitter also implies the polarity of the emitter–base voltage that should be applied in order to forward bias the emitter–base junction. Just a glance at the circuit symbol of the *pnp* transistor, for example, indicates that we should make the emitter higher in voltage than the base (by v_{EB}) in order to cause current to flow into the emitter (downward). Note that the symbol v_{EB} means the voltage by which the emitter (E) is higher than the base (B). Thus for a *pnp* transistor operating in the active mode v_{EB} is positive, while in an *npn* transistor v_{BE} is positive.

From the discussion of Section 6.1 it follows that an *npn* transistor whose EBJ is forward biased (usually, $V_{BE} \simeq 0.7$ V) will operate in the active mode *as long as the collector voltage does not fall below that of the base by more than approximately 0.4 V*. Otherwise, the transistor leaves the active mode and enters the saturation region of operation.[5]

In a parallel manner, the *pnp* transistor will operate in the active mode *if the EBJ is forward biased (usually,* $V_{EB} \simeq 0.7$ V*) and the collector voltage is not allowed to rise above that of the base by more than 0.4 V or so*. Otherwise, the CBJ becomes forward biased, and the *pnp* transistor enters the saturation region of operation.

[5]It is interesting to contrast the active-mode operation of the BJT with the corresponding mode of operation of the MOSFET: The BJT needs a minimum v_{CE} of about 0.3 V, and the MOSFET needs a minimum v_{DS} equal to V_{OV}, which for modern technologies is in the range of 0.2 V to 0.3 V. Thus we see a great deal of similarity! Also note that reverse biasing the CBJ of the BJT corresponds to pinching off the channel of the MOSFET. This condition results in the collector current (drain current in the MOSFET) being independent of the collector voltage (the drain voltage in the MOSFET).

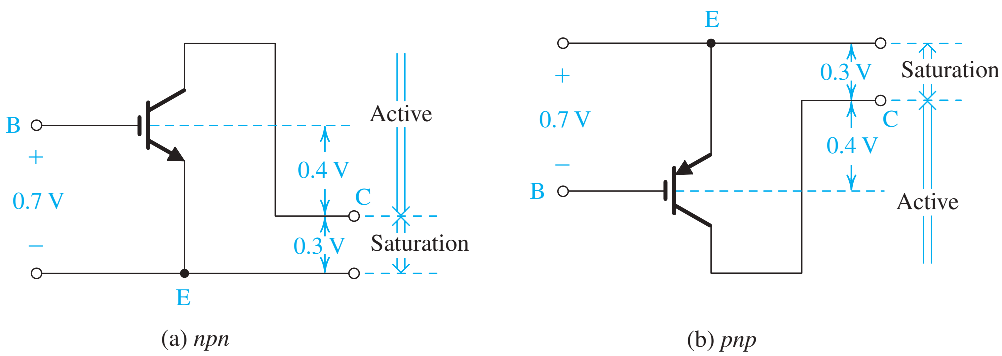


Figure 6.14 Graphical representation of the conditions for operating the BJT in the active mode and in the saturation mode.

Table 6.2 Summary of the BJT Current–Voltage Relationships in the Active Mode

$$i_C = I_S e^{v_{BE}/V_T}$$

$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T}$$

$$i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T}$$

Note: For the *pnp* transistor, replace v_{BE} with v_{EB}.

$i_C = \alpha i_E$	$i_B = (1-\alpha) i_E = \dfrac{i_E}{\beta+1}$
$i_C = \beta i_B$	$i_E = (\beta+1) i_B$
$\beta = \dfrac{\alpha}{1-\alpha}$	$\alpha = \dfrac{\beta}{\beta+1}$

$$V_T = \text{thermal voltage} = \frac{kT}{q} \simeq 25 \text{ mV at room temperature}$$

For greater emphasis, we show in Fig. 6.14 a graphical construction that illustrates the conditions for operating the BJT in the active mode and in the saturation mode. Also, for easy reference, we present in Table 6.2 a summary of the BJT current–voltage relationships in the active mode of operation.

The Collector–Base Reverse Current (I_{CBO}) In our discussion of current flow in transistors we ignored the small reverse currents carried by thermally generated minority carriers. Although such currents can be safely neglected in modern transistors, the reverse current across the collector–base junction deserves some mention. This current, denoted I_{CBO}, is the reverse current flowing from collector to base with the emitter open-circuited (hence the subscript O). This current is usually in the nanoampere range, a value that is many times higher than its theoretically predicted value. As with the diode reverse current, I_{CBO} contains a substantial leakage component, and its value is dependent on v_{CB}. I_{CBO} depends strongly on temperature, approximately doubling for every 10°C rise.[6]

[6]The temperature coefficient of I_{CBO} is different from that of I_S because I_{CBO} contains a substantial leakage component.

Example 6.2

The transistor in the circuit of Fig. 6.15(a) has $\beta = 100$ and exhibits a v_{BE} of 0.7 V at $i_C = 1$ mA. Design the circuit so that a current of 2 mA flows through the collector and a voltage of +5 V appears at the collector.

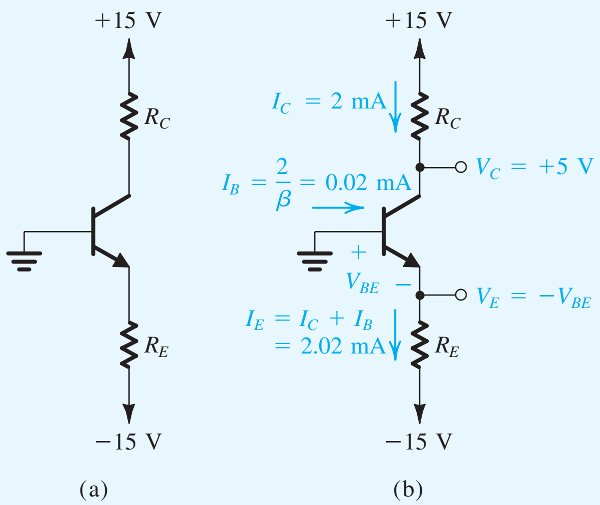


Figure 6.15 Circuit for Example 6.2.

Solution

Refer to Fig. 6.15(b). We note at the outset that since we are required to design for $V_C = +5$ V, the CBJ will be reverse biased and the BJT will be operating in the active mode. To obtain a voltage $V_C = +5$ V, the voltage drop across R_C must be 15 – 5 = 10 V. Now, since $I_C = 2$ mA, the value of R_C should be selected according to

$$R_C = \frac{10\text{ V}}{2\text{ mA}} = 5\text{ k}\Omega$$

Since $v_{BE} = 0.7$ V at $i_C = 1$ mA, the value of v_{BE} at $i_C = 2$ mA is

$$V_{BE} = 0.7 + V_T \ln\left(\frac{2}{1}\right) = 0.717\text{ V}$$

Since the base is at 0 V, the emitter voltage should be

$$V_E = -0.717\text{ V}$$

For $\beta = 100$, $\alpha = 100/101 = 0.99$. Thus the emitter current should be

$$I_E = \frac{I_C}{\alpha} = \frac{2}{0.99} = 2.02\text{ mA}$$

Example 6.2 *continued*

Now the value required for R_E can be determined from

$$R_E = \frac{V_E - (-15)}{I_E}$$
$$= \frac{-0.717 + 15}{2.02} = 7.07 \text{ k}\Omega$$

This completes the design. We should note, however, that the calculations above were made with a degree of precision that is usually neither necessary nor justified in practice in view, for instance, of the expected tolerances of component values. Nevertheless, we chose to do the design precisely in order to illustrate the various steps involved.

EXERCISES

D6.12 Repeat Example 6.2 for a transistor fabricated in a modern integrated-circuit process. Such a process yields devices that exhibit larger v_{BE} at the same i_C because they have much smaller junction areas. The dc power supplies utilized in modern IC technologies fall in the range of 1 V to 3 V. Design a circuit similar to that shown in Fig. 6.15 except that now the power supplies are ± 1.5 V and the BJT has $\beta = 100$ and exhibits v_{BE} of 0.8 V at $i_C = 1$ mA. Design the circuit so that a current of 2 mA flows through the collector and a voltage of $+0.5$ V appears at the collector.

Ans. $R_C = 500\ \Omega$; $R_E = 338\ \Omega$

6.13 In the circuit shown in Fig. E6.13, the voltage at the emitter was measured and found to be –0.7 V. If $\beta = 50$, find I_E, I_B, I_C, and V_C.

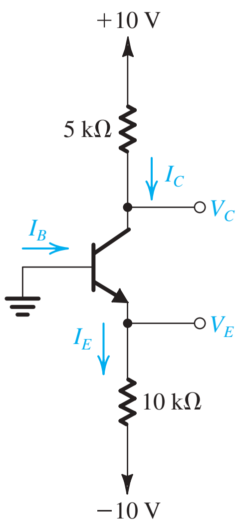


Figure E6.13

Ans. 0.93 mA; 18.2 μA; 0.91 mA; +5.45 V

6.14 In the circuit shown in Fig. E6.14, measurement indicates V_B to be $+1.0$ V and V_E to be $+1.7$ V. What are α and β for this transistor? What voltage V_C do you expect at the collector?

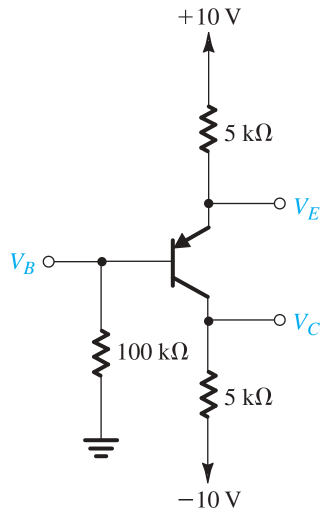


Figure E6.14

Ans. 0.994; 165; –1.75 V

6.2.2 Graphical Representation of Transistor Characteristics

It is sometimes useful to describe the transistor i–v characteristics graphically. Figure 6.16 shows the i_C–v_{BE} characteristic, which is the exponential relationship

$$i_C = I_S e^{v_{BE}/V_T}$$

which is identical to the diode i–v relationship. The $i_E - v_{BE}$ and i_B–v_{BE} characteristics are also exponential but with different scale currents: I_S/α for i_E, and I_S/β for i_B. Since the constant of the exponential characteristic, $1/V_T$, is quite high ($\simeq 40$), the curve rises very sharply. For v_{BE} smaller than about 0.5 V, the current is negligibly small.[7] Also, over most of the normal current range v_{BE} lies in the range of 0.6 V to 0.8 V. In performing rapid first-order dc calculations, we normally will assume that $V_{BE} \simeq 0.7$ V, which is similar to the approach used in the analysis of diode circuits (Chapter 4). For a *pnp* transistor, the $i_C - v_{EB}$ characteristic will look identical to that of Fig. 6.16 with v_{BE} replaced with v_{EB}.

[7]The $i_C - v_{BE}$ characteristic is the BJT's counterpart of the i_D–v_{GS} characteristic of the MOSFET. They share an important attribute: In both cases the voltage has to exceed a "threshold" for the device to conduct appreciably. In the case of the MOSFET, there is a formal threshold voltage, V_t, which lies typically in the range of 0.4 V to 0.8 V. For the BJT, there is an "apparent threshold" of approximately 0.5 V. The i_D–v_{GS} characteristic of the MOSFET is parabolic, and thus is less steep than the i_C–v_{BE} characteristic of the BJT. As will be seen in Chapter 7, this difference has a direct and significant implication for the value of transconductance g_m realized with each device.

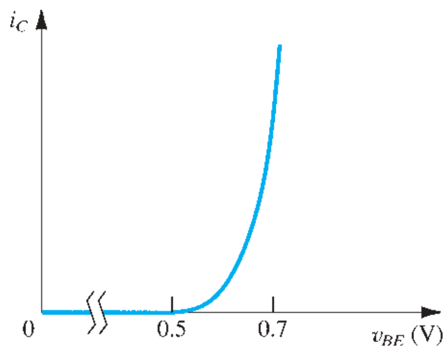


Figure 6.16 The i_C–v_{BE} characteristic for an *npn* transistor.

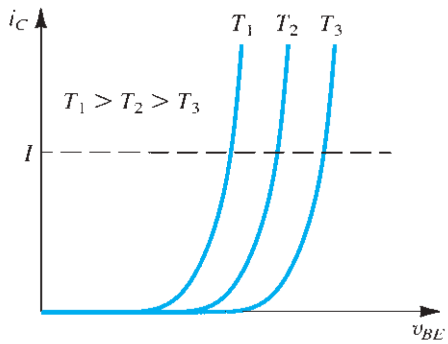


Figure 6.17 Effect of temperature on the i_C–v_{BE} characteristic. At a constant emitter current (broken line), v_{BE} changes by -2 mV/°C.

As in silicon diodes, the voltage across the emitter–base junction decreases by about 2 mV for each rise of 1°C in temperature, provided the junction is operating at a constant current. Figure 6.17 illustrates this temperature dependence by depicting i_C–v_{BE} curves for an *npn* transistor at three different temperatures.

EXERCISE

6.15 Consider a *pnp* transistor with $v_{EB} = 0.7$ V at $i_E = 1$ mA. Let the base be grounded, the emitter be fed by a 2-mA constant-current source, and the collector be connected to a –5-V supply through a 1-kΩ resistance. If the temperature increases by 30°C, find the changes in emitter and collector voltages. Neglect the effect of I_{CBO}.
Ans. –60 mV; 0 V

6.2.3 Dependence of i_C on the Collector Voltage—The Early Effect

When operated in the active region, practical BJTs show some dependence of the collector current on the collector voltage, with the result that, unlike the graph shown in Fig. 6.8, their i_C–v_{CB} characteristics are not perfectly horizontal straight lines. To see this dependence more

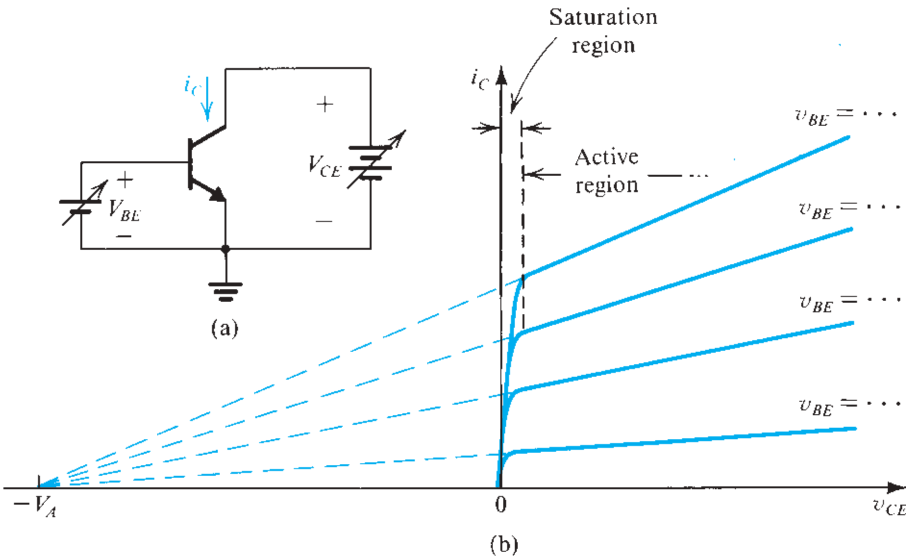


Figure 6.18 (**a**) Conceptual circuit for measuring the i_C–v_{CE} characteristics of the BJT. (**b**) The i_C–v_{CE} characteristics of a practical BJT.

clearly, consider the conceptual circuit shown in Fig. 6.18(a). The transistor is connected in the **common-emitter configuration**; that is, here the emitter serves as a common terminal between the input and output ports. The voltage V_{BE} can be set to any desired value by adjusting the dc source connected between base and emitter. At each value of V_{BE}, the corresponding i_C–v_{CE} characteristic curve can be measured point by point by varying the dc source connected between collector and emitter and measuring the corresponding collector current. The result is the family of i_C–v_{CE} characteristic curves shown in Fig. 6.18(b) and known as **common-emitter characteristics**.

At low values of v_{CE} (lower than about 0.3 V), as the collector voltage goes below that of the base by more than 0.4 V, the collector–base junction becomes forward biased and the transistor leaves the active mode and enters the saturation mode. Shortly, we shall look at the details of the i_C–v_{CE} curves in the saturation region. At this time, however, we wish to examine the characteristic curves in the active region in detail. We observe that the characteristic curves, though still straight lines, have finite slope. In fact, when extrapolated, the characteristic lines meet at a point on the negative v_{CE} axis, at $v_{CE} = -V_A$. The voltage V_A, a positive number, is a parameter for the particular BJT, with typical values in the range of 10 V to 100 V. As noted earlier, it is called the **Early voltage**, after J. M. Early, the engineering scientist who first studied this phenomenon.

At a given value of v_{BE}, increasing v_{CE} increases the reverse-bias voltage on the collector–base junction, and thus increases the width of the depletion region of this junction (refer to Fig. 6.4). This in turn results in a decrease in the **effective base width** W. Recalling that I_S is inversely proportional to W (Eq. 6.13), we see that I_S will increase and that i_C increases proportionally. This is the Early effect. For obvious reasons, it is also known as the **base-width modulation effect**.[8]

[8]Recall that the MOSFET's counterpart is the channel-length modulation effect. These two effects are remarkably similar and have been assigned the same name, Early effect.

The linear dependence of i_C on v_{CE} can be explicitly accounted for by assuming that I_S remains constant and including the factor $(1 + v_{CE}/V_A)$in the equation for i_C as follows:

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right) \tag{6.18}$$

The nonzero slope of the i_C–v_{CE} straight lines indicates that the **output resistance** looking into the collector is not infinite. Rather, it is finite and defined by

$$r_o \equiv \left[\left.\frac{\partial i_C}{\partial v_{CE}}\right|_{v_{BE}\,=\,\text{constant}}\right]^{-1} \tag{6.19}$$

Using Eq. (6.18) we can show that

$$r_o = \frac{V_A + V_{CE}}{I_C} \tag{6.20}$$

where I_C and V_{CE} are the coordinates of the point at which the BJT is operating on the particular i_C–v_{CE} curve (i.e., the curve obtained for v_{BE} equal to constant value V_{BE} at which Eq. (6.19) is evaluated). Alternatively, we can write

$$r_o = \frac{V_A}{I'_C} \tag{6.21}$$

where I'_C is the value of the collector current with the Early effect neglected; that is,

$$I'_C = I_S e^{V_{BE}/V_T} \tag{6.22}$$

It is rarely necessary to include the dependence of i_C on v_{CE} in dc bias design and analysis that is performed by hand. Such an effect, however, can be easily included in the SPICE simulation of circuit operation, which is frequently used to "fine-tune" pencil-and-paper analysis or design.

The finite output resistance r_o can have a significant effect on the gain of transistor amplifiers. This is particularly the case in integrated-circuit amplifiers, as will be shown in Chapter 8. Fortunately, there are many situations in which r_o can be included relatively easily in pencil-and-paper analysis.

The output resistance r_o can be included in the circuit model of the transistor.[9] This is illustrated in Fig. 6.19, where we show the two large-signal circuit models of a

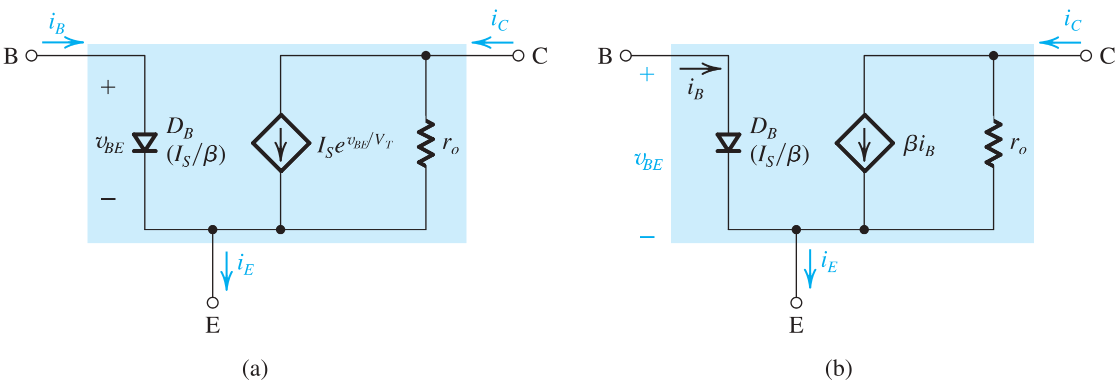


Figure 6.19 Large-signal, equivalent-circuit models of an *npn* BJT operating in the active mode in the common-emitter configuration with the output resistance r_o included.

[9] In applying Eq. (6.21) to determine r_o we will usually drop the prime and simply use $r_o = V_A/I_C$ where I_C is the collector current without the Early effect.

common-emitter *npn* transistor operating in the active mode, those in Fig 6.5(c) and (d), with the resistance r_o connected between the collector and the emitter terminals.

EXERCISES

6.16 Use the circuit model in Fig. 6.19(a) to express i_C in terms of e^{v_{BE}/V_T} and v_{CE} and thus show that this circuit is a direct representation of Eq. (6.18).

6.17 Find the output resistance of a BJT for which $V_A = 100$ V at $I_C = 0.1$, 1, and 10 mA.
Ans. 1 MΩ; 100 kΩ; 10 kΩ

6.18 Consider the circuit in Fig. 6.18(a). At $V_{CE} = 1$ V, V_{BE} is adjusted to yield a collector current of 1 mA. Then, while V_{BE} is kept constant, V_{CE} is raised to 11 V. Find the new value of I_C. For this transistor, $V_A = 100$ V.
Ans. 1.1 mA

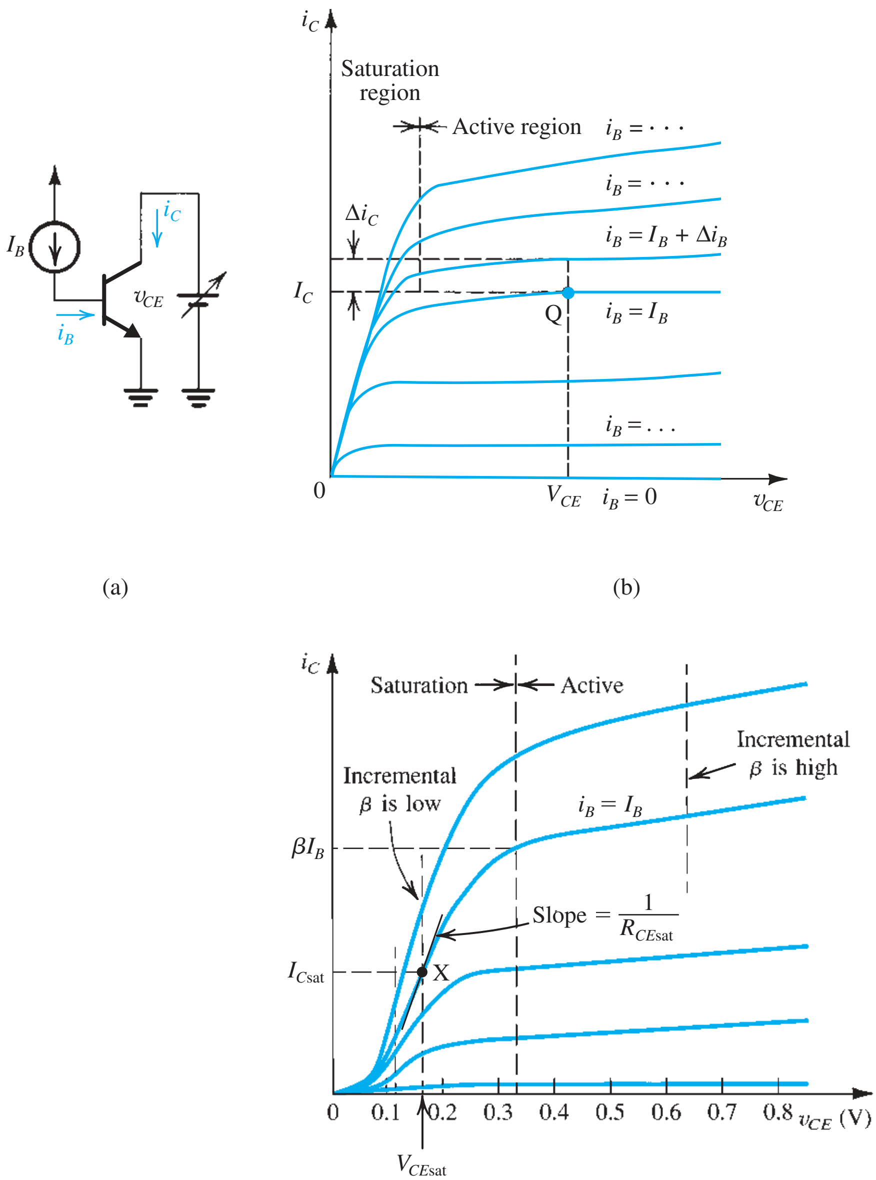
6.2.4 An Alternative Form of the Common-Emitter Characteristics

An alternative way of expressing the transistor common-emitter characteristics is illustrated in Fig. 6.20. Here the base current i_B rather than the base–emitter voltage v_{BE} is used as a parameter. That is, each i_C–v_{CE} curve is measured with the base fed with a constant current I_B. The resulting characteristics, shown in Fig. 6.20(b), look similar to those in Fig. 6.18. Figure 6.20(c) shows an expanded view of the characteristics in the saturation region.

The Common-Emitter Current Gain β In the active region of the characteristics shown in Fig. 6.20(b) we have identified a particular point Q. Note that this operating point for the transistor is characterized by a base current I_B, a collector current I_C, and a collector–emitter voltage V_{CE}. The ratio I_C/I_B is the transistor β. However, there is another way to measure β: change the base current by an increment Δi_B and measure the resulting increment Δi_C, while keeping V_{CE} constant. This is illustrated in Fig. 6.20(b). The ratio $\Delta i_C/\Delta i_B$ should, according to our study thus far, yield an identical value for β. It turns out, however, that the latter value of β (called *incremental*, or ac, β) is a little different from the dc β (i.e., I_C/I_B). Such a distinction, however, is too subtle for our needs in this book. We shall use β to denote both dc and incremental values.[10]

The Saturation Voltage V_{CEsat} and Saturation Resistance R_{CEsat} Refer next to the expanded view of the common-emitter characteristics in the saturation region shown in Fig. 6.20(c). The "bunching together" of the curves in the saturation region implies that the incremental β is lower there than in the active region. A possible operating point in the saturation region is that labeled X. It is characterized by a base current I_B, a collector current I_{Csat}, and a collector–emitter voltage V_{CEsat}. From our previous discussion of saturation, recall that $I_{Csat} = \beta_{\text{forced}} I_B$, where $\beta_{\text{forced}} < \beta$.

[10]Manufacturers of bipolar transistors use h_{FE} to denote the dc value of β and h_{fe} to denote the incremental β. These symbols come from the h-parameter description of two-port networks (see Appendix C), with the subscript $F(f)$ denoting forward and $E(e)$ denoting common emitter.



(c)

Figure 6.20 Common-emitter characteristics. (**a**) Basic CE circuit; note that in (**b**) the horizontal scale is expanded around the origin to show the saturation region in some detail. A much greater expansion of the saturation region is shown in (**c**) .

The i_C–v_{CE} curves in saturation are rather steep, indicating that the saturated transistor exhibits a low collector-to-emitter resistance $R_{CE\text{sat}}$,

$$R_{CE\text{sat}} \equiv \left.\frac{\partial v_{CE}}{\partial i_C}\right|_{\substack{i_B = I_B \\ i_C = I_{C\text{sat}}}} \tag{6.23}$$

Typically, $R_{CE\text{sat}}$ ranges from a few ohms to a few tens of ohms.

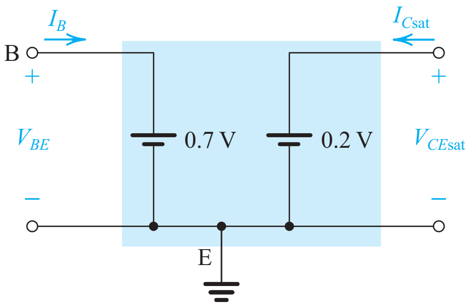


Figure 6.21 A simplified equivalent-circuit model of the saturated transistor.

That the collector-to-emitter resistance of a saturated BJT is small should have been anticipated from the fact that between C and E we now have two forward-conducting diodes in series[11] (see also Fig. 6.9).

A simple model for the saturated BJT is shown in Fig. 6.21. Here V_{BE} is assumed constant (approximately 0.7 V) and V_{CE} also is assumed constant, $V_{CEsat} \simeq 0.2$ V. That is, we have neglected the small saturation resistance R_{CEsat} for the sake of making the model simple for hand calculations.

Example 6.3

For the circuit in Fig. 6.22, it is required to determine the value of the voltage V_{BB} that results in the transistor operating

(a) in the active mode with $V_{CE} = 5$ V
(b) at the edge of saturation
(c) deep in saturation with $\beta_{forced} = 10$

For simplicity, assume that V_{BE} remains constant at 0.7 V. The transistor β is specified to be 50.

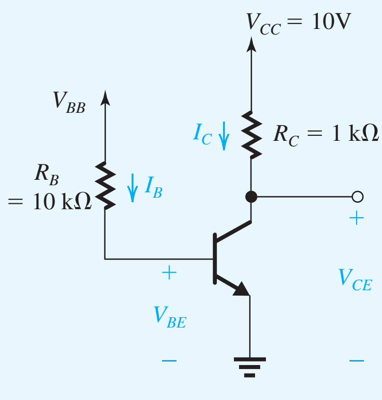


Figure 6.22 Circuit for Example 6.3.

[11]In the corresponding mode of operation for the MOSFET, the triode region, the resistance between drain and source is small because it is the resistance of the continuous (non-pinched-off) channel.

Example 6.3 *continued*

Solution

(a) To operate in the active mode with $V_{CE} = 5$ V,

$$I_C = \frac{V_{CC} - V_{CE}}{R_C}$$
$$= \frac{10 - 5}{1\ \text{k}\Omega} = 5\ \text{mA}$$
$$I_B = \frac{I_C}{\beta} = \frac{5}{50} = 0.1\ \text{mA}$$

Now the required value of V_{BB} can be found as follows:

$$V_{BB} = I_B R_B + V_{BE}$$
$$= 0.1 \times 10 + 0.7 = 1.7\ \text{V}$$

(b) Operation at the edge of saturation is obtained with $V_{CE} = 0.3$ V. Thus

$$I_C = \frac{10 - 0.3}{1} = 9.7\ \text{mA}$$

Since, at the edge of saturation, I_C and I_B are still related by β,

$$I_B = \frac{9.7}{50} = 0.194\ \text{mA}$$

The required value of V_{BB} can be determined as

$$V_{BB} = 0.194 \times 10 + 0.7 = 2.64\ \text{V}$$

(c) To operate deep in saturation,

$$V_{CE} = V_{CE\text{sat}} \simeq 0.2\ \text{V}$$

Thus,

$$I_C = \frac{10 - 0.2}{1} = 9.8\ \text{mA}$$

We then use the value of forced β to determine the required value of I_B as

$$I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{9.8}{10} = 0.98\ \text{mA}$$

and the required V_{BB} can now be found as

$$V_{BB} = 0.98 \times 10 + 0.7 = 10.5\ \text{V}$$

Observe that once the transistor is in saturation, increasing V_{BB} and thus I_B results in negligible change in I_C since $V_{CE\text{sat}}$ will change only slightly. Thus I_C is said to *saturate*, which is the origin of the name "saturation mode of operation."

EXERCISES

6.19 Repeat Example 6.3 for $R_C = 10\ \text{k}\Omega$.
Ans. 0.8 V; 0.894 V; 1.68 V

6.20 For the circuit in Fig. 6.22, find V_{CE} for $V_{BB} = 0$ V.
Ans. $+10$ V

6.21 For the circuit in Fig. 6.22, let V_{BB} be set to the value obtained in Example 6.3, part (a), namely, $V_{BB} = 1.7$ V. Verify that the transistor is indeed operating in the active mode. Now, while keeping V_{BB} constant, find the value to which R_C should be increased in order to obtain (a) operation at the edge of saturation and (b) operation deep in saturation with $\beta_{\text{forced}} = 10$.
Ans. (a) 1.94 kΩ; (b) 9.8 kΩ

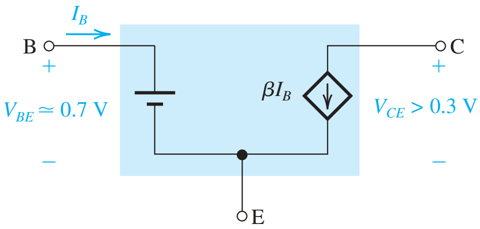
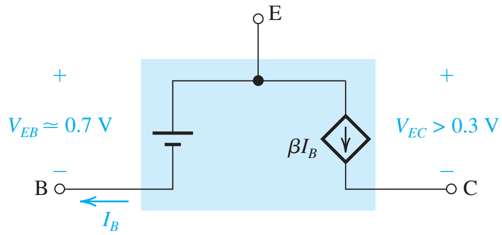
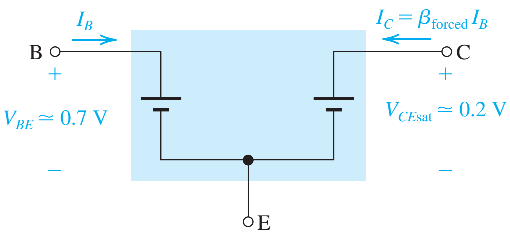
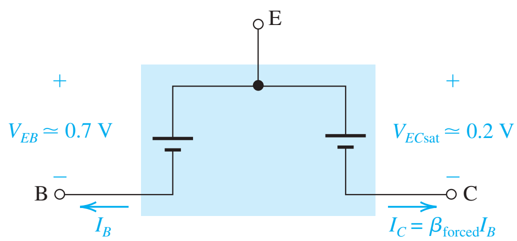
6.3 BJT Circuits at DC

We are now ready to consider the analysis of BJT circuits to which only dc voltages are applied. In the following examples we will use the simple model in which $|V_{BE}|$ of a conducting transistor is 0.7 V and $|V_{CE}|$ of a saturated transistor is 0.2 V, and we will neglect the Early effect. These models are shown in Table 6.3. Better models can, of course, be used to obtain more accurate results. This, however, is usually achieved at the expense of speed of analysis; more importantly, the attendant complexity could impede the circuit designer's ability to gain insight regarding circuit behavior. Accurate results using elaborate models can be obtained using circuit simulation with SPICE. This is almost always done in the final stages of a design and certainly before circuit fabrication. Computer simulation, however, is *not* a substitute for quick pencil-and-paper circuit analysis, an essential ability that aspiring circuit designers must master. The following series of examples is a step in that direction.

As will be seen, in analyzing a circuit the first question that one must answer is: *In which mode is the transistor operating*? In some cases, the answer will be obvious. For instance, a quick check of the terminal voltages will indicate whether the transistor is cut off or conducting. If it is conducting, we have to determine whether it is operating in the active mode or in saturation. In some cases, however, this may not be obvious. Needless to say, as the reader gains practice and experience in transistor circuit analysis and design, the answer will be apparent in a much larger proportion of problems. The answer, however, can always be determined by utilizing the following procedure.

Assume that the transistor is operating in the active mode and, using the active-mode model in Table 6.3, proceed to determine the various voltages and currents that correspond. Then check for consistency of the results with the assumption of active-mode operation; that is, is V_{CB} of an *npn* transistor greater than -0.4 V (or V_{CB} of a *pnp* transistor lower than 0.4 V)? If the answer is yes, then our task is complete. If the answer is no, assume saturation-mode operation and, using the saturation-mode model in Table 6.3, proceed to determine currents and voltages

Table 6.3 Simplified Models for the Operation of the BJT in DC Circuits

	npn	*pnp*
Active EBJ: Forward Biased CBJ: Reverse Biased		
Saturation EBJ: Forward Biased CBJ: Forward Biased		

and then check for consistency of the results with the assumption of saturation-mode operation. Here the test is usually to compute the ratio I_C/I_B and to verify that it is lower than the transistor β (i.e., $\beta_{forced} < \beta$). Since β for a given transistor type varies over a wide range,[12] one must use the lowest specified β for this test. Finally, note that the order of these two assumptions can be reversed.

A Note on Units Except when otherwise specified, throughout this book we use a consistent set of units, namely, volts (V), milliamps (mA), and kilohms (kΩ).

[12]That is, if one buys BJTs of a certain part number, the manufacturer guarantees only that their values of β fall within a certain range, say 50 to 150.

Example 6.4

Consider the circuit shown in Fig. 6.23(a), which is redrawn in Fig. 6.23(b) to remind the reader of the convention employed throughout this book for indicating connections to dc sources. We wish to analyze this circuit to determine all node voltages and branch currents. We will assume that β is specified to be 100.

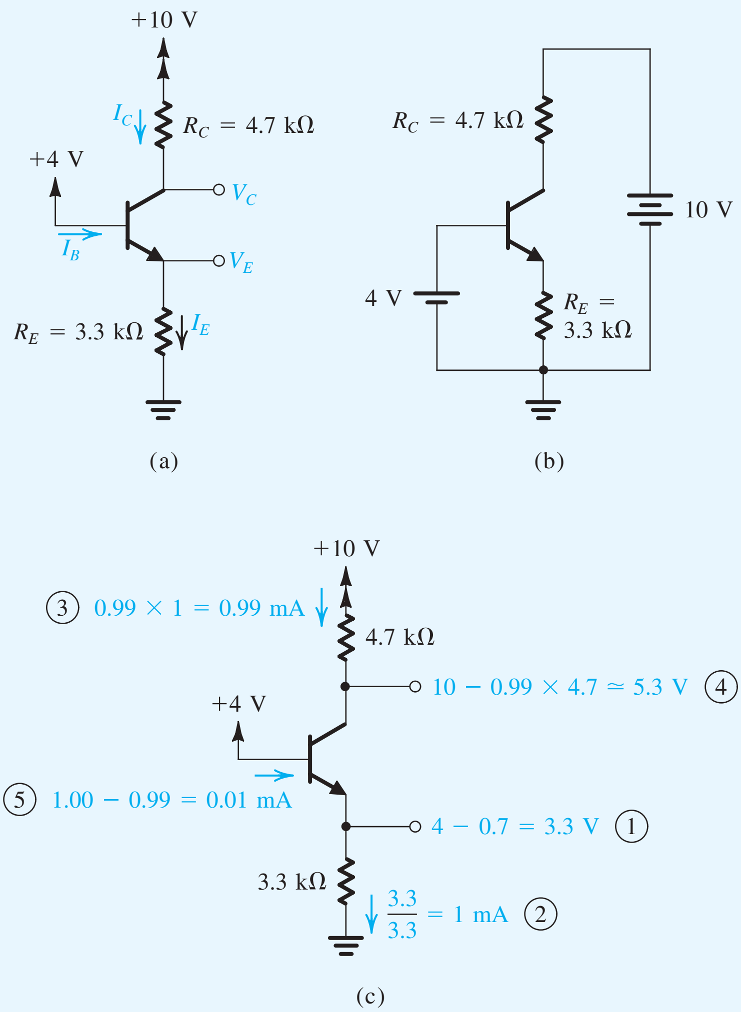


Figure 6.23 Analysis of the circuit for Example 6.4: (**a**) circuit; (**b**) circuit redrawn to remind the reader of the convention used in this book to show connections to the dc sources; (**c**) analysis with the steps numbered.

Example 6.4 *continued*

Solution

Glancing at the circuit in Fig. 6.23(a), we note that the base is connected to +4 V and the emitter is connected to ground through a resistance R_E. Therefore, it is reasonable to conclude that the base–emitter junction will be forward biased. Assuming that this is the case and assuming that V_{BE} is approximately 0.7 V, it follows that the emitter voltage will be

$$V_E = 4 - V_{BE} \simeq 4 - 0.7 = 3.3 \text{ V}$$

We are now in an opportune position; we know the voltages at the two ends of R_E and thus can determine the current I_E through it,

$$I_E = \frac{V_E - 0}{R_E} = \frac{3.3}{3.3} = 1 \text{ mA}$$

Since the collector is connected through R_C to the +10-V power supply, it appears possible that the collector voltage will be higher than the base voltage, which implies active-mode operation. Assuming that this is the case, we can evaluate the collector current from

$$I_C = \alpha I_E$$

The value of α is obtained from

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} \simeq 0.99$$

Thus I_C will be given by

$$I_C = 0.99 \times 1 = 0.99 \text{ mA}$$

We are now in a position to use Ohm's law to determine the collector voltage V_C,

$$V_C = 10 - I_C R_C = 10 - 0.99 \times 4.7 \simeq +5.3 \text{ V}$$

Since the base is at +4 V, the collector–base junction is reverse biased by 1.3 V, and the transistor is indeed in the active mode as assumed.

It remains only to determine the base current I_B, as follows:

$$I_B = \frac{I_E}{\beta + 1} = \frac{1}{101} \simeq 0.01 \text{ mA}$$

Before leaving this example, we wish to emphasize strongly the value of carrying out the analysis directly on the circuit diagram. Only in this way will one be able to analyze complex circuits in a reasonable length of time. Figure 6.23(c) illustrates the above analysis on the circuit diagram, with the order of the analysis steps indicated by the circled numbers.

Example 6.5

We wish to analyze the circuit of Fig. 6.24(a) to determine the voltages at all nodes and the currents through all branches. Note that this circuit is identical to that of Fig. 6.23 except that the voltage at the base is now +6 V. Assume that the transistor β is specified to be *at least* 50.

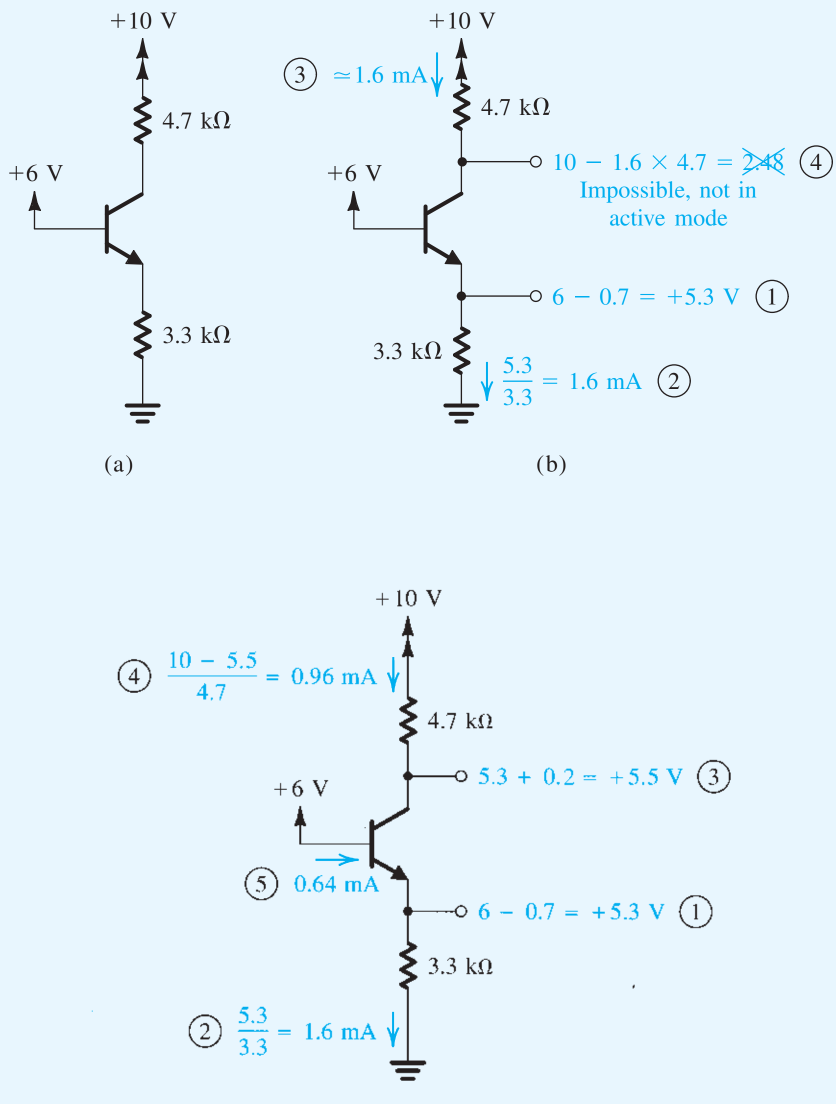


Figure 6.24 Analysis of the circuit for Example 6.5. Note that the circled numbers indicate the order of the analysis steps.

Example 6.5 *continued*

Solution

With +6 V at the base, the base–emitter junction will be forward biased; thus,

$$V_E = +6 - V_{BE} \simeq 6 - 0.7 = 5.3 \text{ V}$$

and

$$I_E = \frac{5.3}{3.3} = 1.6 \text{ mA}$$

Now, assuming active-mode operation, $I_C = \alpha I_E \simeq I_E$; thus,

$$V_C = +10 - 4.7 \times I_C \simeq 10 - 7.52 = 2.48 \text{ V}$$

The details of the analysis performed above are illustrated in Fig. 6.24(b).

Since the collector voltage calculated is less than the base voltage by 3.52 V, it follows that our original assumption of active-mode operation is incorrect. In fact, the transistor has to be in the *saturation* mode. Assuming this to be the case, the values of V_E and I_E will remain unchanged. The collector voltage, however, becomes

$$V_C = V_E + V_{CEsat} \simeq 5.3 + 0.2 = +5.5 \text{ V}$$

from which we can determine I_C as

$$I_C = \frac{10 - 5.5}{4.7} = 0.96 \text{ mA}$$

and I_B can now be found as

$$I_B = I_E - I_C = 1.6 - 0.96 = 0.64 \text{ mA}$$

Thus the transistor is operating at a forced β of

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{0.96}{0.64} = 1.5$$

Since β_{forced} is less than the *minimum* specified value of β, the transistor is indeed saturated. We should emphasize here that in testing for saturation the minimum value of β should be used. By the same token, if we are designing a circuit in which a transistor is to be saturated, the design should be based on the minimum specified β. Obviously, if a transistor with this minimum β is saturated, then transistors with higher values of β will also be saturated. The details of the analysis are shown in Fig. 6.24(c), where the order of the steps used is indicated by the circled numbers.

Example 6.6

We wish to analyze the circuit in Fig. 6.25(a) to determine the voltages at all nodes and the currents through all branches. Note that this circuit is identical to that considered in Examples 6.4 and 6.5 except that now the base voltage is zero.

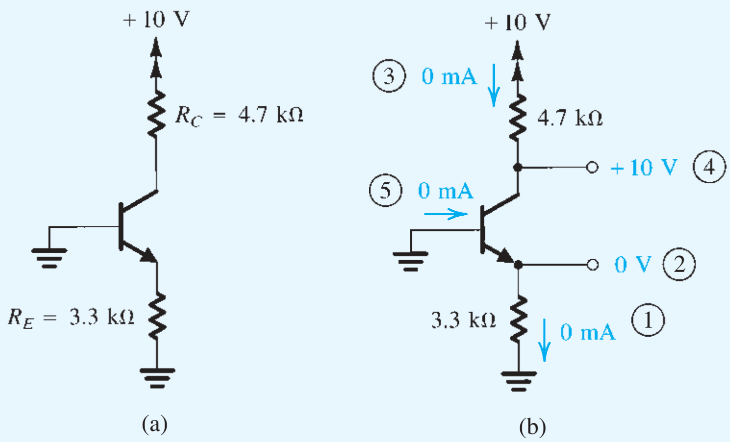


Figure 6.25 Example 6.6: (**a**) circuit; (**b**) analysis, with the order of the analysis steps indicated by circled numbers.

Solution

Since the base is at zero volts and the emitter is connected to ground through R_E, the base–emitter junction cannot conduct and the emitter current is zero. Also, the collector–base junction cannot conduct, since the n-type collector is connected through R_C to the positive power supply while the p-type base is at ground. It follows that the collector current will be zero. The base current will also have to be zero, and the transistor is in the *cutoff* mode of operation.

The emitter voltage will be zero, while the collector voltage will be equal to +10 V, since the voltage drops across R_E and R_C are zero. Figure 6.25(b) shows the analysis details.

EXERCISES

D6.22 For the circuit in Fig. 6.23(a), find the highest voltage to which the base can be raised while the transistor remains in the active mode. Assume $\alpha \simeq 1$.
Ans. +4.7 V

D6.23 Redesign the circuit of Fig. 6.23(a) (i.e., find new values for R_E and R_C) to establish a collector current of 0.5 mA and a reverse-bias voltage on the collector–base junction of 2 V. Assume $\alpha \simeq 1$.
Ans. $R_E = 6.6\ \text{k}\Omega$; $R_C = 8\ \text{k}\Omega$

D6.24 For the circuit in Fig. 6.24(a), find the value to which the base voltage should be changed so that the transistor operates in saturation with a forced β of 5.
Ans. $+5.18$ V

Example 6.7

We want to analyze the circuit of Fig. 6.26(a) to determine the voltages at all nodes and the currents through all branches.

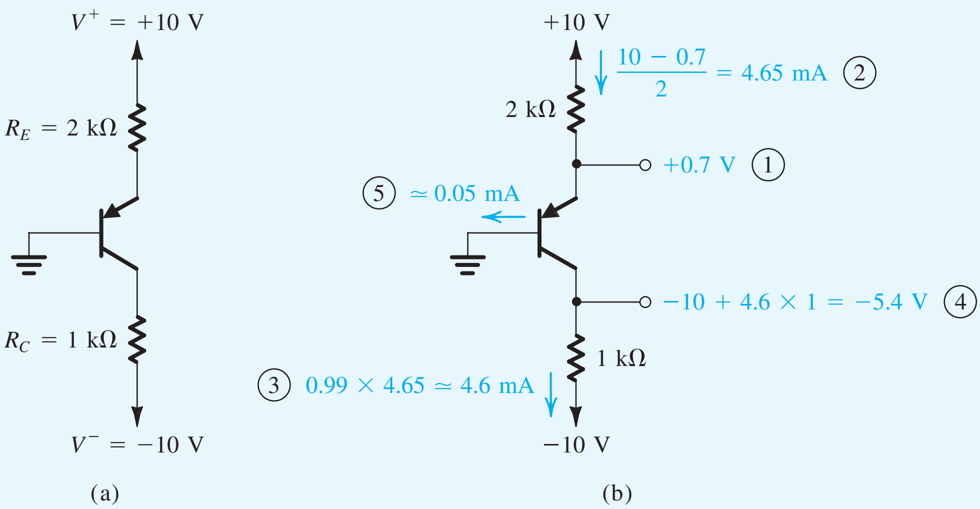


Figure 6.26 Example 6.7: (**a**) circuit; (**b**) analysis, with the steps indicated by circled numbers.

Solution

The base of this *pnp* transistor is grounded, while the emitter is connected to a positive supply ($V^+ = +10$ V) through R_E. It follows that the emitter–base junction will be forward biased with

$$V_E = V_{EB} \simeq 0.7 \text{ V}$$

Thus the emitter current will be given by

$$I_E = \frac{V^+ - V_E}{R_E} = \frac{10 - 0.7}{2} = 4.65 \text{ mA}$$

Since the collector is connected to a negative supply (more negative than the base voltage) through R_C, it is *possible* that this transistor is operating in the active mode. Assuming this to be the case, we obtain

$$I_C = \alpha I_E$$

Since no value for β has been given, we shall assume $\beta = 100$, which results in $\alpha = 0.99$. Since large variations in β result in small differences in α, this assumption will not be critical as far as determining the value of I_C is concerned. Thus,

$$I_C = 0.99 \times 4.65 = 4.6 \text{ mA}$$

The collector voltage will be

$$\begin{aligned} V_C &= V^- + I_C R_C \\ &= -10 + 4.6 \times 1 = -5.4 \text{ V} \end{aligned}$$

Thus the collector–base junction is reverse biased by 5.4 V, and the transistor is indeed in the active mode, which supports our original assumption.

It remains only to calculate the base current,

$$I_B = \frac{I_E}{\beta + 1} = \frac{4.65}{101} \simeq 0.05 \text{ mA}$$

Obviously, the value of β critically affects the base current. Note, however, that in this circuit the value of β will have no effect on the mode of operation of the transistor. Since β is generally an ill-specified parameter, this circuit represents a good design. As a rule, one should strive to *design the circuit such that its performance is as insensitive to the value of β as possible.* The analysis details are illustrated in Fig. 6.26(b).

EXERCISES

D6.25 For the circuit in Fig. 6.26(a), find the largest value to which R_C can be raised while the transistor remains in the active mode.
Ans. 2.26 kΩ

D6.26 Redesign the circuit of Fig. 6.26(a) (i.e., find new values for R_E and R_C) to establish a collector current of 1 mA and a reverse bias on the collector–base junction of 4 V. Assume $\alpha \simeq 1$.
Ans. $R_E = 9.3$ kΩ; $R_C = 6$ kΩ

Example 6.8

We want to analyze the circuit in Fig. 6.27(a) to determine the voltages at all nodes and the currents in all branches. Assume $\beta = 100$.

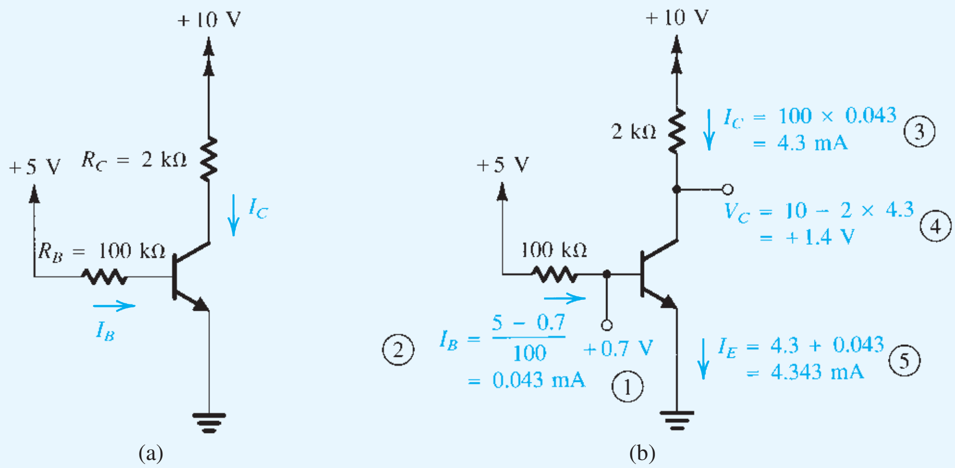


Figure 6.27 Example 6.8: (**a**) circuit; (**b**) analysis, with the steps indicated by the circled numbers.

Solution

The base–emitter junction is clearly forward biased. Thus,

$$I_B = \frac{+5 - V_{BE}}{R_B} \simeq \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

Assume that the transistor is operating in the active mode. We now can write

$$I_C = \beta I_B = 100 \times 0.043 = 4.3 \text{ mA}$$

The collector voltage can now be determined as

$$V_C = 10 - I_C R_C = 10 - 4.3 \times 2 = +1.4 \text{ V}$$

Since the base voltage V_B is

$$V_B = V_{BE} \simeq +0.7 \text{ V}$$

it follows that the collector–base junction is reverse biased by 0.7 V and the transistor is indeed in the active mode. The emitter current will be given by

$$I_E = (\beta + 1) I_B = 101 \times 0.043 \simeq 4.3 \text{ mA}$$

We note from this example that the collector and emitter currents depend critically on the value of β. In fact, if β were 10% higher, the transistor would leave the active mode and enter saturation. Therefore this clearly is a *bad* design. The analysis details are illustrated in Fig. 6.27(b).

EXERCISE

D6.27 The circuit of Fig. 6.27(a) is to be fabricated using a transistor type whose β is specified to be in the range of 50 to 150. That is, individual units of this same transistor type can have β values anywhere in this range. Redesign the circuit by selecting a new value for R_C so that all fabricated circuits are guaranteed to be in the active mode. What is the range of collector voltages that the fabricated circuits may exhibit?

Ans. $R_C = 1.5\ \text{k}\Omega$; $V_C = 0.3$ V to 6.8 V

Example 6.9

We want to analyze the circuit of Fig. 6.28(a) to determine the voltages at all nodes and the currents through all branches. The minimum value of β is specified to be 30.

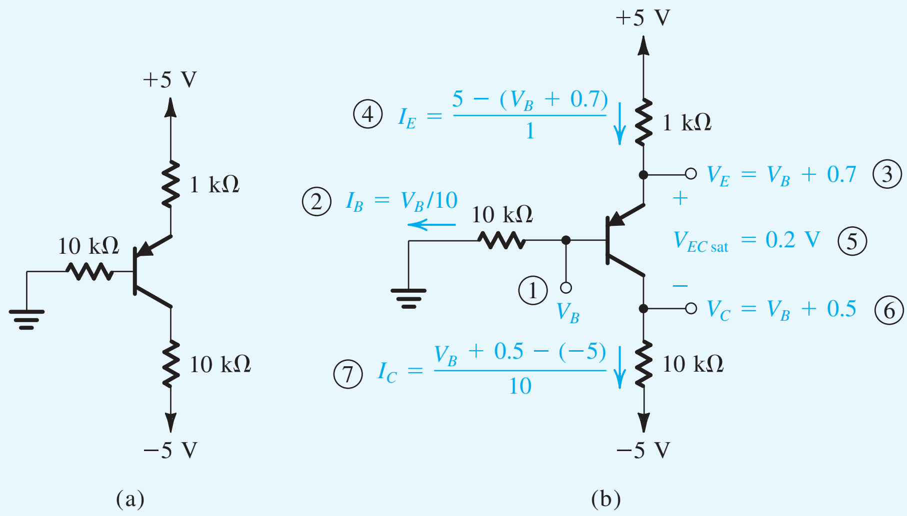


Figure 6.28 Example 6.9: (**a**) circuit; (**b**) analysis with steps numbered.

Solution

A quick glance at this circuit reveals that the transistor will be either active or saturated. Assuming active-mode operation and neglecting the base current, we see that the base voltage will be approximately zero volts, the emitter voltage will be approximately +0.7 V, and the emitter current will be approximately 4.3 mA. Since the maximum current that the collector can support while the transistor remains in the active mode is approximately 0.5 mA, it follows that the transistor is definitely saturated.

Example 6.9 *continued*

Assuming that the transistor is saturated and denoting the voltage at the base by V_B (refer to Fig. 6.28b), it follows that

$$
\begin{aligned}
V_E &= V_B + V_{EB} \simeq V_B + 0.7 \\
V_C &= V_E - V_{EC\text{sat}} \simeq V_B + 0.7 - 0.2 = V_B + 0.5 \\
I_E &= \frac{+5 - V_E}{1} = \frac{5 - V_B - 0.7}{1} = 4.3 - V_B \quad \text{mA} \\
I_B &= \frac{V_B}{10} = 0.1V_B \quad \text{mA} \\
I_C &= \frac{V_C - (-5)}{10} = \frac{V_B + 0.5 + 5}{10} = 0.1V_B + 0.55 \quad \text{mA}
\end{aligned}
$$

Using the relationship $I_E = I_B + I_C$, we obtain

$$4.3 - V_B = 0.1V_B + 0.1V_B + 0.55$$

which results in

$$V_B = \frac{3.75}{1.2} \simeq 3.13 \text{ V}$$

Substituting in the equations above, we obtain

$$
\begin{aligned}
V_E &= 3.83 \text{ V} \\
V_C &= 3.63 \text{ V} \\
I_E &= 1.17 \text{ mA} \\
I_C &= 0.86 \text{ mA} \\
I_B &= 0.31 \text{ mA}
\end{aligned}
$$

from which we see that the transistor is saturated, since the value of forced β is

$$\beta_{\text{forced}} = \frac{0.86}{0.31} \simeq 2.8$$

which is much smaller than the specified minimum β.

Example 6.10

We want to analyze the circuit of Fig. 6.29(a) to determine the voltages at all nodes and the currents through all branches. Assume $\beta = 100$.

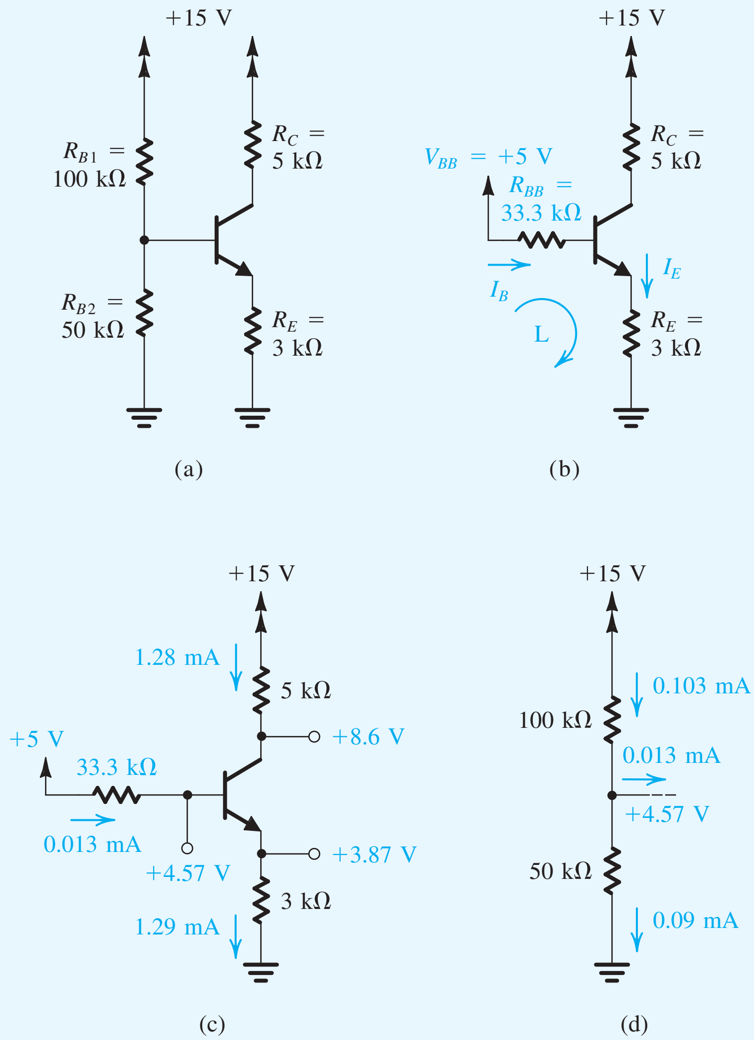


Figure 6.29 Circuits for Example 6.10.

Solution

The first step in the analysis consists of simplifying the base circuit using Thévenin's theorem. The result is shown in Fig. 6.29(b), where

$$V_{BB} = +15\frac{R_{B2}}{R_{B1}+R_{B2}} = 15\frac{50}{100+50} = +5\text{ V}$$

$$R_{BB} = R_{B1} \parallel R_{B2} = 100 \parallel 50 = 33.3\text{ k}\Omega$$

Example 6.10 *continued*

To evaluate the base or the emitter current, we have to write a loop equation around the loop labeled L in Fig. 6.29(b). Note, however, that the current through R_{BB} is different from the current through R_E. The loop equation will be

$$V_{BB} = I_B R_{BB} + V_{BE} + I_E R_E$$

Now, assuming active-mode operation, we replace I_B with

$$I_B = \frac{I_E}{\beta + 1}$$

and rearrange the equation to obtain

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + [R_{BB}/(\beta + 1)]}$$

For the numerical values given we have

$$I_E = \frac{5 - 0.7}{3 + (33.3/101)} = 1.29 \text{ mA}$$

The base current will be

$$I_B = \frac{1.29}{101} = 0.0128 \text{ mA}$$

The base voltage is given by

$$\begin{aligned} V_B &= V_{BE} + I_E R_E \\ &= 0.7 + 1.29 \times 3 = 4.57 \text{ V} \end{aligned}$$

We can evaluate the collector current as

$$I_C = \alpha I_E = 0.99 \times 1.29 = 1.28 \text{ mA}$$

The collector voltage can now be evaluated as

$$V_C = +15 - I_C R_C = 15 - 1.28 \times 5 = 8.6 \text{ V}$$

It follows that the collector is higher in potential than the base by 4.03 V, which means that the transistor is in the active mode, as had been assumed. The results of the analysis are given in Fig. 6.29(c, d).

EXERCISE

6.28 If the transistor in the circuit of Fig. 6.29(a) is replaced with another having half the value of β (i.e., $\beta = 50$), find the new value of I_C, and express the change in I_C as a percentage.
Ans. $I_C = 1.15$ mA; –10%

Example 6.11

We wish to analyze the circuit in Fig. 6.30(a) to determine the voltages at all nodes and the currents through all branches.

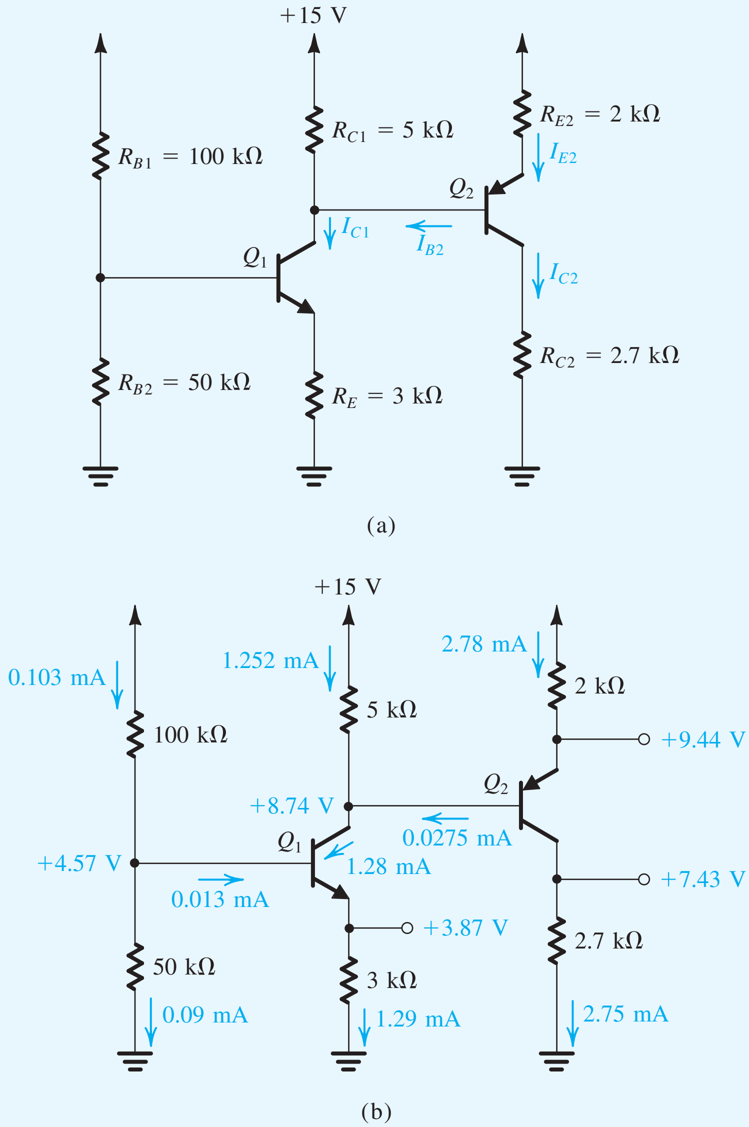


Figure 6.30 Circuits for Example 6.11.

Example 6.11 *continued*

Solution

We first recognize that part of this circuit is identical to the circuit we analyzed in Example 6.10—namely, the circuit of Fig. 6.29(a). The difference, of course, is that in the new circuit we have an additional transistor Q_2 together with its associated resistors R_{E2} and R_{C2}. Assume that Q_1 is still in the active mode. The following values will be identical to those obtained in the previous example:

$$V_{B1} = +4.57 \text{ V} \qquad I_{E1} = 1.29 \text{ mA}$$
$$I_{B1} = 0.0128 \text{ mA} \qquad I_{C1} = 1.28 \text{ mA}$$

However, the collector voltage will be different than previously calculated, since part of the collector current I_{C1} will flow in the base lead of Q_2 (I_{B2}). As a first approximation we may assume that I_{B2} is much smaller than I_{C1}; that is, we may assume that the current through R_{C1} is almost equal to I_{C1}. This will enable us to calculate V_{C1}:

$$\begin{aligned} V_{C1} &\simeq +15 - I_{C1}R_{C1} \\ &= 15 - 1.28 \times 5 = +8.6 \text{ V} \end{aligned}$$

Thus Q_1 is in the active mode, as had been assumed.

As far as Q_2 is concerned, we note that its emitter is connected to +15 V through R_{E2}. It is therefore safe to assume that the emitter–base junction of Q_2 will be forward biased. Thus the emitter of Q_2 will be at a voltage V_{E2} given by

$$V_{E2} = V_{C1} + V_{EB}\big|_{Q_2} \simeq 8.6 + 0.7 = +9.3 \text{ V}$$

The emitter current of Q_2 may now be calculated as

$$I_{E2} = \frac{+15 - V_{E2}}{R_{E2}} = \frac{15 - 9.3}{2} = 2.85 \text{ mA}$$

Since the collector of Q_2 is returned to ground via R_{C2}, it is possible that Q_2 is operating in the active mode. Assume this to be the case. We now find I_{C2} as

$$\begin{aligned} I_{C2} &= \alpha_2 I_{E2} \\ &= 0.99 \times 2.85 = 2.82 \text{ mA} \quad (\text{assuming } \beta_2 = 100) \end{aligned}$$

The collector voltage of Q_2 will be

$$V_{C2} = I_{C2}R_{C2} = 2.82 \times 2.7 = 7.62 \text{ V}$$

which is lower than V_{B2} by 0.98 V. Thus Q_2 is in the active mode, as assumed.

It is important at this stage to find the magnitude of the error incurred in our calculations by the assumption that I_{B2} is negligible. The value of I_{B2} is given by

$$I_{B2} = \frac{I_{E2}}{\beta_2 + 1} = \frac{2.85}{101} = 0.028 \text{ mA}$$

which is indeed much smaller than I_{C1} (1.28 mA). If desired, we can obtain more accurate results by iterating one more time, assuming I_{B2} to be 0.028 mA. The new values will be

$$\text{Current in } R_{C1} = I_{C1} - I_{B2} = 1.28 - 0.028 = 1.252 \text{ mA}$$

$$V_{C1} = 15 - 5 \times 1.252 = 8.74 \text{ V}$$

$$V_{E2} = 8.74 + 0.7 = 9.44 \text{ V}$$

$$I_{E2} = \frac{15 - 9.44}{2} = 2.78 \text{ mA}$$

$$I_{C2} = 0.99 \times 2.78 = 2.75 \text{ mA}$$

$$V_{C2} = 2.75 \times 2.7 = 7.43 \text{ V}$$

$$I_{B2} = \frac{2.78}{101} = 0.0275 \text{ mA}$$

Note that the new value of I_{B2} is very close to the value used in our iteration, and no further iterations are warranted. The final results are indicated in Fig. 6.30(b).

The reader justifiably might be wondering about the necessity for using an iterative scheme in solving a linear (or linearized) problem. Indeed, we can obtain the exact solution (if we can call anything we are doing with a first-order model exact!) by writing appropriate equations. The reader is encouraged to find this solution and then compare the results with those obtained above. It is important to emphasize, however, that in most such problems it is quite sufficient to obtain an approximate solution, provided we can obtain it quickly and, of course, correctly.

In the above examples, we frequently used a precise value of α to calculate the collector current. Since $\alpha \simeq 1$, the error in such calculations will be very small if one assumes $\alpha = 1$ and $I_C = I_E$. Therefore, except in calculations that depend critically on the value of α (e.g., the calculation of base current), one usually assumes $\alpha \simeq 1$.

EXERCISES

6.29 For the circuit in Fig. 6.30, find the total current drawn from the power supply. Hence find the power dissipated in the circuit.
Ans. 4.135 mA; 62 mW

6.30 The circuit in Fig. E6.30 is to be connected to the circuit in Fig. 6.30(a) as indicated; specifically, the base of Q_3 is to be connected to the collector of Q_2. If Q_3 has $\beta = 100$, find the new value of V_{C2} and the values of V_{E3} and I_{C3}.

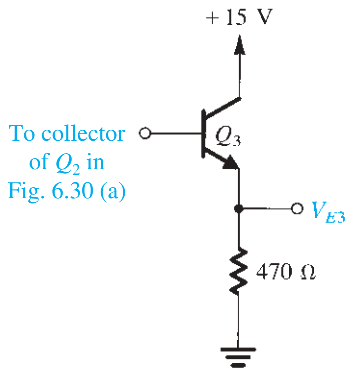


Figure E6.30

Ans. +7.06 V; +6.36 V; 13.4 mA

Example 6.12

We desire to evaluate the voltages at all nodes and the currents through all branches in the circuit of Fig. 6.31(a). Assume $\beta = 100$.

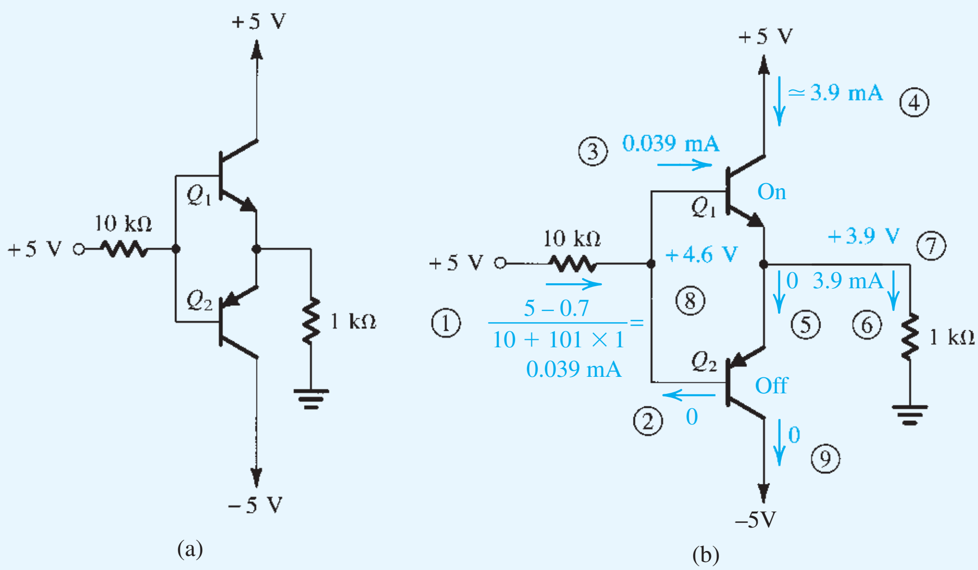


Figure 6.31 Example 6.12: (**a**) circuit; (**b**) analysis with the steps numbered.

Solution

By examining the circuit, we conclude that the two transistors Q_1 and Q_2 cannot be simultaneously conducting. Thus if Q_1 is on, Q_2 will be off, and vice versa. Assume that Q_2 is on. It follows that current will flow from ground through the 1-kΩ resistor into the emitter of Q_2. Thus the base of Q_2 will be at a negative voltage, and base current will be flowing out of the base through the 10-kΩ resistor and into the +5-V supply. This is impossible, since if the base is negative, current in the 10-kΩ resistor will have to flow into the base. Thus we conclude that our original assumption—that Q_2 is on—is incorrect. It follows that Q_2 will be off and Q_1 will be on.

The question now is whether Q_1 is active or saturated. The answer in this case is obvious: Since the base is fed with a +5-V supply and since base current flows into the base of Q_1, it follows that the base of Q_1 will be at a voltage lower than +5 V. Thus the collector–base junction of Q_1 is reverse biased and Q_1 is in the active mode. It remains only to determine the currents and voltages using techniques already described in detail. The results are given in Fig. 6.31(b).

EXERCISES

6.31 Solve the problem in Example 6.12 for the case of a voltage of –5 V feeding the bases. What voltage appears at the emitters?
Ans. –3.9 V

6.32 Solve the problem in Example 6.12 with the voltage feeding the bases changed to +10 V. Assume that $\beta_{min} = 30$, and find V_E, V_B, I_{C1}, and I_{C2}.
Ans. +4.8 V; +5.5 V; 4.35 mA; 0

6.4 Transistor Breakdown and Temperature Effects

A

We conclude this chapter with a brief discussion of two important nonideal effects in the BJT: voltage breakdown, and the dependence of β on I_C and temperature.

6.4.1 Transistor Breakdown

The maximum voltages that can be applied to a BJT are limited by the EBJ and CBJ breakdown effects that follow the avalanche multiplication mechanism described in Section 3.5.3. Consider first the common-base configuration (Fig. 6.32(a)). The i_C–v_{CB} characteristics in Fig. 6.32(b) indicate that for $i_E = 0$ (i.e., with the emitter open-circuited) the collector–base junction breaks down at a voltage denoted by BV_{CBO}. For $i_E > 0$, breakdown occurs at voltages smaller than BV_{CBO}. Typically, for discrete BJTs, BV_{CBO} is greater than 50 V.

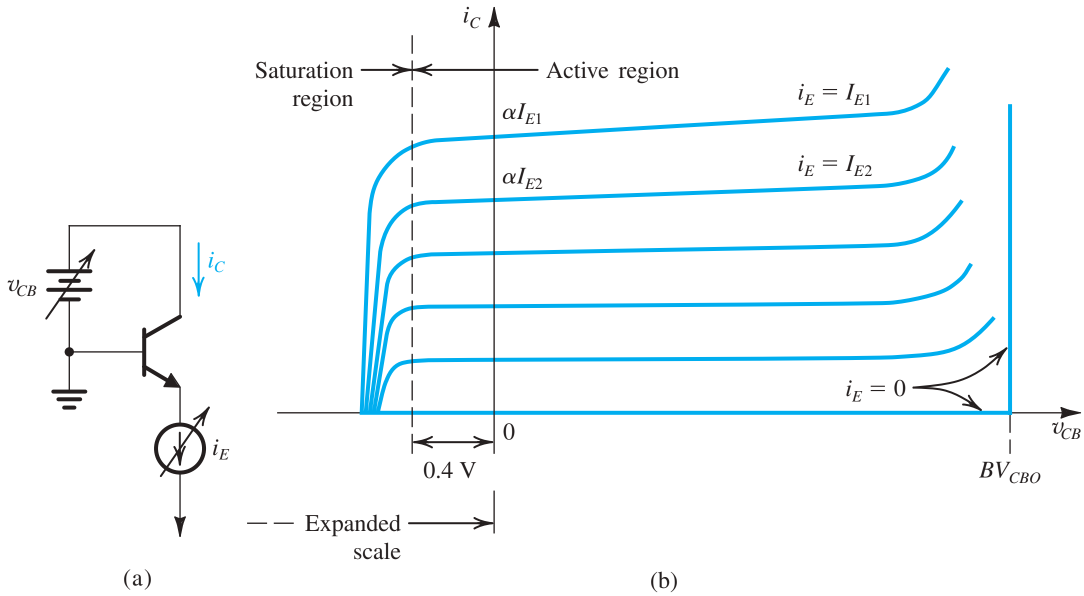


Figure 6.32 The BJT common-base characteristics including the transistor breakdown region.

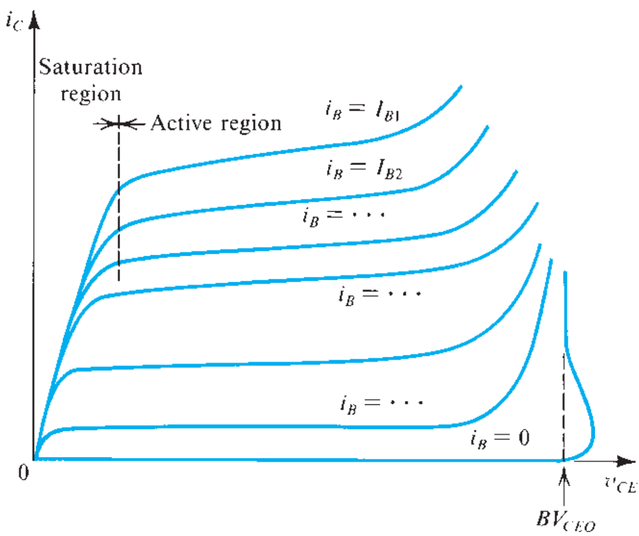


Figure 6.33 The BJT common-emitter characteristics including the breakdown region.

Next consider the common-emitter characteristics of Fig. 6.33, which show breakdown occurring at a voltage BV_{CEO}. Here, although breakdown is still of the avalanche type, the effects on the characteristics are more complex than in the common-base configuration. We will not explain these in detail; it is sufficient to point out that typically BV_{CEO} is about half BV_{CBO}. On transistor data sheets, BV_{CEO} is sometimes referred to as the **sustaining voltage** LV_{CEO}.

Breakdown of the CBJ in either the common-base or common-emitter configuration is not destructive as long as the power dissipation in the device is kept within safe limits. This, however, is not the case with the breakdown of the emitter–base junction. The EBJ breaks down in an avalanche manner at a voltage BV_{EBO} much smaller than BV_{CBO}. Typically, BV_{EBO} is in the range of 6 V to 8 V, and the breakdown is destructive in the sense that the β of the transistor is permanently reduced. This does not prevent use of the EBJ as a zener diode to generate reference voltages in IC design. In such applications one is not concerned with the β-degradation effect. A circuit arrangement to prevent EBJ breakdown in IC amplifiers will be discussed in Chapter 13. Transistor breakdown and the maximum allowable power dissipation are important parameters in the design of power amplifiers (Chapter 12).

EXERCISE

6.33 What is the output voltage of the circuit in Fig. E6.33 if the transistor $BV_{BCO} = 70$ V?

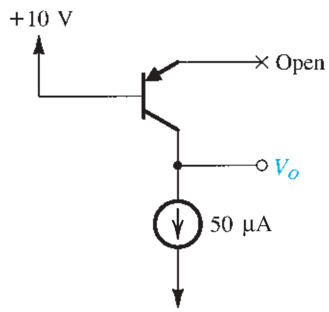


Figure E6.33

Ans. –60 V

6.4.2 Dependence of β on I_C and Temperature

Throughout this chapter we have assumed that the transistor common-emitter dc current gain, β or h_{FE}, is constant for a given transistor. In fact, β depends on the dc current at which the transistor is operating, as shown in Fig. 6.34. The physical processes that give rise to this dependence are beyond the scope of this book. Note, however, that there is a current range over which β is highest. Normally, one arranges to operate the transistor at a current within this range.

Figure 6.34 also shows the dependence of β on temperature. The fact that β increases with temperature can lead to serious problems in transistors that operate at large power levels (see Chapter 12).

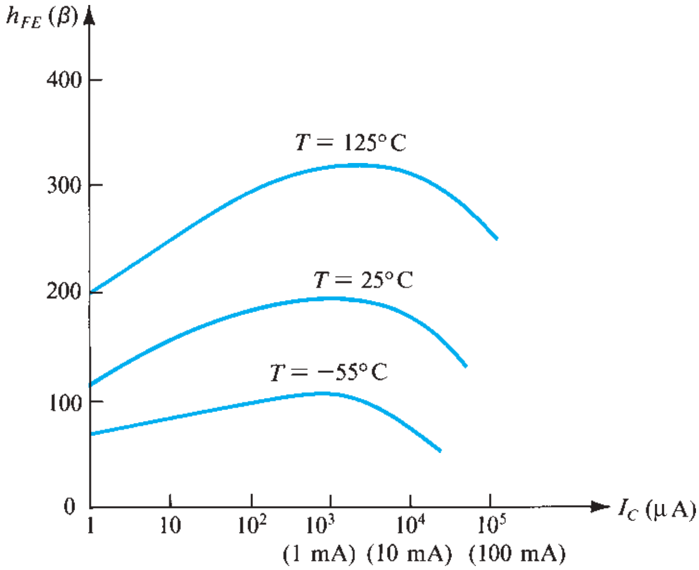


Figure 6.34 Typical dependence of β on I_C and on temperature in an integrated-circuit *npn* silicon transistor intended for operation around 1 mA.

Summary

- Depending on the bias conditions on its two junctions, the BJT can operate in one of three possible modes: cutoff (both junctions reverse biased), active (the EBJ forward biased and the CBJ reverse biased), and saturation (both junctions forward biased). Refer to Table 6.1.
- For amplifier applications, the BJT is operated in the active mode. Switching applications make use of the cutoff and saturation modes.
- A BJT operating in the active mode provides a collector current $i_C = I_S e^{|v_{BE}|/V_T}$. The base current $i_B = i_C/\beta$, and the emitter current $i_E = i_C + i_B$. Also, $i_C = \alpha i_E$, and thus $\beta = \alpha/(1-\alpha)$ and $\alpha = \beta/(\beta+1)$. See Table 6.2.
- To ensure operation in the active mode, the collector voltage of an *npn* transistor must be kept higher than approximately 0.4 V below the base voltage. For a *pnp* transistor, the collector voltage must be lower than approximately 0.4 V above the base voltage. Otherwise, the CBJ becomes forward biased, and the transistor enters the saturation region.
- At a constant collector current, the magnitude of the base–emitter voltage decreases by about 2 mV for every 1°C rise in temperature.
- The BJT will be at the edge of saturation when $|v_{CE}|$ is reduced to about 0.3 V. In saturation, $|v_{CE}| \simeq 0.2$ V, and the ratio of i_C to i_B is lower than β (i.e., $\beta_{\text{forced}} < \beta$).
- In the active mode, i_C shows a slight dependence on v_{CE}. This phenomenon, known as the Early effect, is modeled by ascribing a finite (i.e., noninfinite) output resistance to the BJT: $r_o = |V_A|/I'_C$, where V_A is the Early voltage and I'_C is the dc collector current without the Early effect taken into account. In discrete circuits, r_o plays a minor role and can usually be neglected. This is *not* the case, however, in integrated-circuit design (Chapter 8).
- The dc analysis of transistor circuits is greatly simplified by assuming that $|V_{BE}| \simeq 0.7$ V. Refer to Table 6.3.
- If the BJT is conducting, one assumes it is operating in the active mode and, using the active-mode model, proceeds to determine all currents and voltages. The validity of the initial assumption is then checked by determining whether the CBJ is reverse biased. If it is, the analysis is complete; otherwise, we assume the BJT is operating in saturation and redo the analysis, using the saturation-mode model and checking at the end that $I_C < \beta I_B$.

Computer Simulations Problems

SIM Problems identified by the Multisim/PSpice icon are intended to demonstrate the value of using SPICE simulation to verify hand analysis and design, and to investigate important issues such as allowable signal swing and amplifier nonlinear distortion. Instructions to assist in setting up PSPice and Multisim simulations for all the indicated problems can be found in the corresponding files on the website. Note that if a particular parameter value is not specified in the problem statement, you are to make a reasonable assumption.

Section 6.1: Device Structure and Physical Operation

6.1 The terminal voltages of various *npn* transistors are measured during operation in their respective circuits with the following results:

Case	E	B	C	Mode
1	0	0.7	0.7	
2	0	0.8	0.1	
3	−0.7	0	1.0	
4	−0.7	0	−0.6	
5	1.3	2.0	5.0	
6	0	0	5.0	

In this table, where the entries are in volts, 0 indicates the reference terminal to which the black (negative) probe of the voltmeter is connected. For each case, identify the mode of operation of the transistor.

6.2 Two transistors, fabricated with the same technology but having different junction areas, when operated at a base-emitter voltage of 0.75 V, have collector currents of 0.5 mA and 2 mA. Find I_S for each device. What are the relative junction areas?

6.3 In a particular technology, a small BJT operating at $v_{BE} = 30V_T$ conducts a collector current of 200 μA. What is the corresponding saturation current? For a transistor in the same technology but with an emitter junction that is 32 times larger, what is the saturation current? What current will this transistor conduct at $v_{BE} = 30V_T$? What is the base–emitter voltage of the latter transistor at $i_C = 1$ mA? Assume active-mode operation in all cases.

6.4 Two transistors have EBJ areas as follows: $A_{E1} = 200\ \mu\text{m} \times 200\ \mu\text{m}$ and $A_{E2} = 0.4\ \mu\text{m} \times 0.4\ \mu\text{m}$. If the two transistors are operated in the active mode and conduct equal collector currents, what do you expect the difference in their v_{BE} values to be?

6.5 Find the collector currents that you would expect for operation at $v_{BE} = 700$ mV for transistors for which $I_S = 10^{-13}$ A and $I_S = 10^{-18}$ A. For the transistor with the larger EBJ, what is the v_{BE} required to provide a collector current equal to that provided by the smaller transistor at $v_{BE} =$ 700 mV? Assume active-mode operation in all cases.

6.6 In this problem, we contrast two BJT integrated-circuit fabrication technologies: For the "old" technology, a typical *npn* transistor has $I_S = 2 \times 10^{-15}$ A, and for the "new" technology, a typical *npn* transistor has $I_S = 2 \times 10^{-18}$ A. These typical devices have vastly different junction areas and base width. For our purpose here we wish to determine the v_{BE} required to establish a collector current of 1 mA in each of the two typical devices. Assume active-mode operation.

6.7 Consider an *npn* transistor whose base–emitter drop is 0.76 V at a collector current of 5 mA. What current will it conduct at $v_{BE} = 0.70$ V? What is its base–emitter voltage for $i_C = 5\ \mu$A?

6.8 In a particular BJT, the base current is 10 μA, and the collector current is 800 μA. Find β and α for this device.

6.9 Find the values of β that correspond to α values of 0.5, 0.8, 0.9, 0.95, 0.98, 0.99, 0.995, and 0.999.

6.10 Find the values of α that correspond to β values of 1, 2, 10, 20, 50, 100, 200, 500, and 1000.

***6.11** Show that for a transistor with α close to unity, if α changes by a small per-unit amount $(\Delta\alpha/\alpha)$, the corresponding per-unit change in β is given approximately by

$$\frac{\Delta\beta}{\beta} \simeq \beta\left(\frac{\Delta\alpha}{\alpha}\right)$$

Now, for a transistor whose nominal β is 100, find the percentage change in its α value corresponding to a drop in its β of 10%.

6.12 An *npn* transistor of a type whose β is specified to range from 50 to 300 is connected in a circuit with emitter grounded, collector at +10 V, and a current of 10 μA injected into the base. Calculate the range of collector and emitter currents that can result. What is the maximum power dissipated in the transistor? (*Note:* Perhaps you can see why this is a bad way to establish the operating current in the collector of a BJT.)

6.13 A BJT is specified to have $I_S = 5 \times 10^{-15}$ A and β that falls in the range of 50 to 200. If the transistor is operated in the active mode with v_{BE} set to 0.700 V, find the expected range of i_C, i_B, and i_E.

6.14 Measurements made on a number of transistors operating in the active mode with $i_E = 1$ mA indicate base currents of 10 μA, 20 μA, and 50 μA. For each device, find i_C, β, and α.

6.15 Measurements of V_{BE} and two terminal currents taken on a number of *npn* transistors operating in the active mode are tabulated below. For each, calculate the missing current value as well as α, β, and I_S as indicated by the table.

Transistor	a	b	c	d	e
V_{BE} (mV)	700	690	580	780	820
I_C (mA)	1.000	1.000		10.10	
I_B (μA)	10		5	120	1050
I_E (mA)		1.020	0.235		75.00
α					
β					
I_S					

6.16 When operated in the active mode, a particular *npn* BJT conducts a collector current of 1 mA and has $v_{BE} = 0.70$ V and $i_B = 10$ μA. Use these data to create specific transistor models of the form shown in Fig. 6.5(a) to (d).

6.17 Using the *npn* transistor model of Fig. 6.5(b), consider the case of a transistor for which the base is connected to ground, the collector is connected to a 5-V dc source through a 2-kΩ resistor, and a 2-mA current source is connected to the emitter with the polarity so that current is drawn out of the emitter terminal. If $\beta = 100$ and $I_S = 5 \times 10^{-15}$ A, find the voltages at the emitter and the collector and calculate the base current.

D 6.18 Consider an *npn* transistor operated in the active mode and represented by the model of Fig. 6.5(d). Let the transistor be connected as indicated by the equivalent circuit shown in Fig. 6.6(b). It is required to calculate the values of R_B and R_C that will establish a collector current I_C of 0.5 mA and a collector-to-emitter voltage V_{CE} of 1 V. The BJT is specified to have $\beta = 50$ and $I_S = 5 \times 10^{-15}$ A.

6.19 An *npn* transistor has a CBJ with an area 100 times that of the EBJ. If $I_S = 10^{-15}$ A, find the voltage drop across EBJ and across CBJ when each is forward biased and conducting a current of 1 mA. Also find the forward current each junction would conduct when forward biased with 0.5 V.

***6.20** We wish to investigate the operation of the *npn* transistor in saturation using the model of Fig. 6.9. Let $I_S = 10^{-15}$ A, $v_{BE} = 0.7$ V, $\beta = 100$, and $I_{SC}/I_S = 100$. For each of three values of v_{CE} (namely, 0.4 V, 0.3 V, and 0.2 V), find v_{BC}, i_{BC}, i_{BE}, i_B, i_C, and i_C/i_B. Present your results in tabular form. Also find v_{CE} that results in $i_C = 0$.

***6.21** Use Eqs. (6.14), (6.15), and (6.16) to show that an *npn* transistor operated in saturation exhibits a collector-to-emitter voltage, $V_{CE\text{sat}}$, given by

$$V_{CE\text{sat}} = V_T \ln\left[\left(\frac{I_{SC}}{I_S}\right)\frac{1+\beta_{\text{forced}}}{1-\beta_{\text{forced}}/\beta}\right]$$

Use this relationship to evaluate $V_{CE\text{sat}}$ for $\beta_{\text{forced}} = 50$, 10, 5, and 1 for a transistor with $\beta = 100$ and with a CBJ area 100 times that of the EBJ. Present your results in a table.

6.22 Consider the *pnp* large-signal model of Fig. 6.11(b) applied to a transistor having $I_S = 10^{-14}$ A and $\beta = 50$. If the emitter is connected to ground, the base is connected to a current source that pulls 10 μA out of the base terminal, and the collector is connected to a negative supply of −5 V via a 8.2-kΩ resistor, find the collector voltage, the emitter current, and the base voltage.

6.23 A *pnp* transistor has $v_{EB} = 0.7$ V at a collector current of 1 mA. What do you expect v_{EB} to become at $i_C = 10$ mA? At $i_C = 100$ mA?

6.24 A *pnp* transistor modeled with the circuit in Fig. 6.11 (b) is connected with its base at ground, collector at –2.0 V, and a 1-mA current is injected into its emitter. If the transistor is said to have $\beta = 10$, what are its base and collector currents? In which direction do they flow? If $I_S = 10^{-15}$ A, what voltage results at the emitter? What does the collector current become if a transistor with $\beta = 1000$ is substituted? (*Note:* The fact that the collector current changes by less than 10% for a large change in β illustrates that this is a good way to establish a specific collector current.)

6.25 A *pnp* power transistor operates with an emitter-to-collector voltage of 5 V, an emitter current of 5 A, and $V_{EB} = 0.8$ V. For $\beta = 20$, what base current is required? What is I_S for this transistor? Compare the emitter–base junction area of this transistor with that of a small-signal transistor that conducts $i_C = 1$ mA with $v_{EB} = 0.70$ V. How much larger is it?

6.26 While Fig. 6.5 provides four possible large-signal equivalent circuits for the *npn* transistor, only two equivalent circuits for the *pnp* transistor are provided in Fig. 6.11. Supply the missing two.

6.27 By analogy to the *npn* case shown in Fig. 6.9, give the equivalent circuit of a *pnp* transistor in saturation.

Section 6.2: Current–Voltage Characteristics

6.28 For the circuits in Fig. P6.28, assume that the transistors have very large β. Some measurements have been made on these circuits, with the results indicated in the figure. Find the values of the other labeled voltages and currents.

6.29 Measurements on the circuits of Fig. P6.29 produce labeled voltages as indicated. Find the value of β for each transistor.

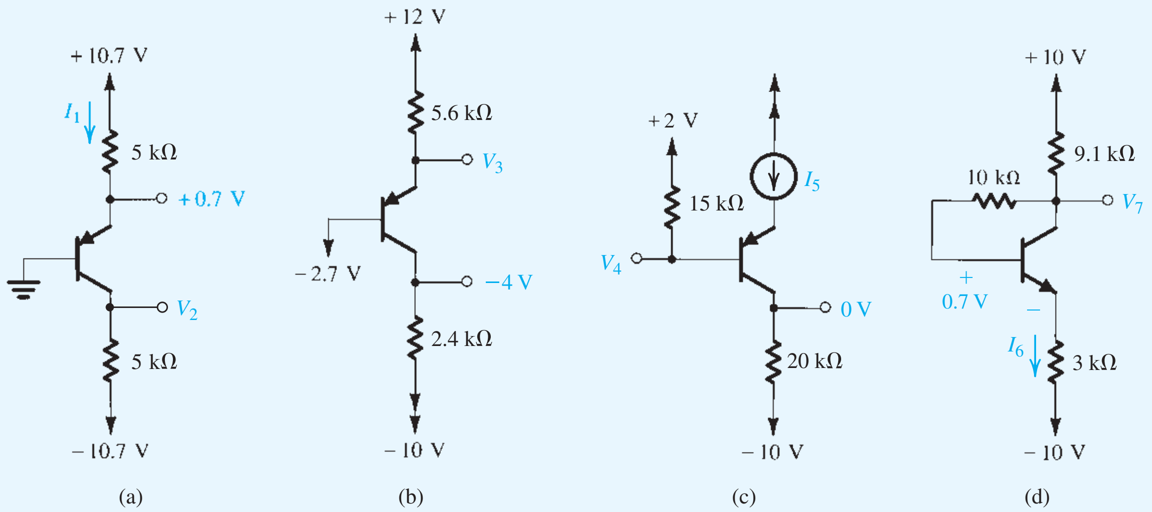


Figure P6.28

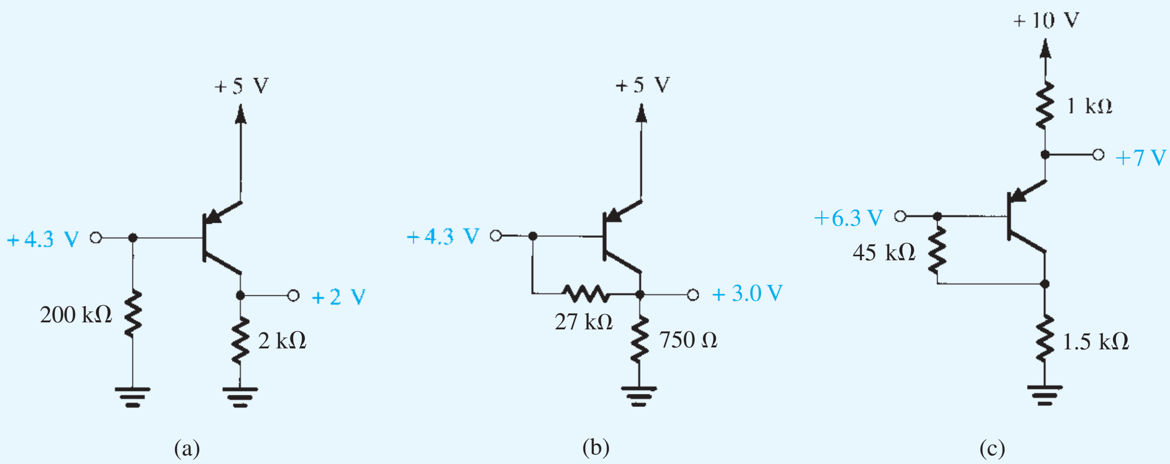


Figure P6.29

6.30 A very simple circuit for measuring β of an *npn* transistor is shown in Fig. P6.30. In a particular design, V_{CC} is provided by a 9-V battery; M is a current meter with a 50-µA full scale and relatively low resistance that you can neglect for our purposes here. Assuming that the transistor has $V_{BE} = 0.7$ V at $I_E = 1$ mA, what value of R_C would establish a resistor current of 1 mA? Now, to what value of β does a meter reading of full scale correspond? What is β if the meter reading is 1/5 of full scale? 1/10 of full scale?

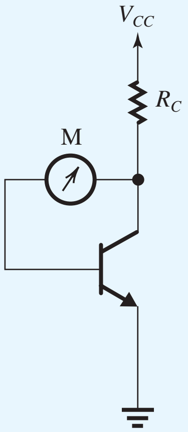


Figure P6.30

6.31 Repeat Exercise 6.13 for the situation in which the power supplies are reduced to ±2.5 V.

D 6.32 Design the circuit in Fig. P6.32 to establish a current of 0.5 mA in the emitter and a voltage of −0.5 V at the collector. The transistor $v_{EB} = 0.64$ V at $I_E = 0.1$ mA, and $\beta = 100$. To what value can R_C be increased while the collector current remains unchanged?

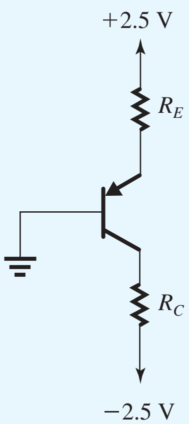


Figure P6.32

D 6.33 Examination of the table of standard values for resistors with 5% tolerance in Appendix J reveals that the closest values to those found in the design of Example 6.2 are 5.1 kΩ and 6.8 kΩ. For these values, use approximate calculations (e.g., $V_{BE} \simeq 0.7$ V and $\alpha \simeq 1$) to determine the values of collector current and collector voltage that are likely to result.

D 6.34 Design the circuit in Fig. P6.34 to establish $I_C = 0.2$ mA and $V_C = 0.5$ V. The transistor exhibits v_{BE} of 0.8 V at $i_C = 1$ mA, and $\beta = 100$.

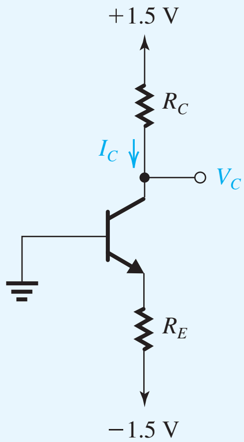


Figure P6.34

6.35 For each of the circuits shown in Fig. P6.35, find the emitter, base, and collector voltages and currents. Use $\beta = 50$, but assume $|V_{BE}| = 0.8$ V independent of current level.

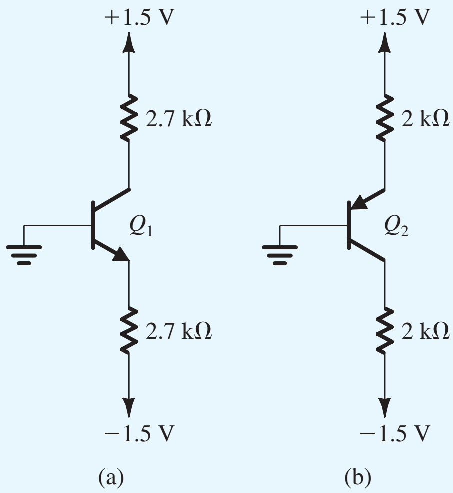


Figure P6.35

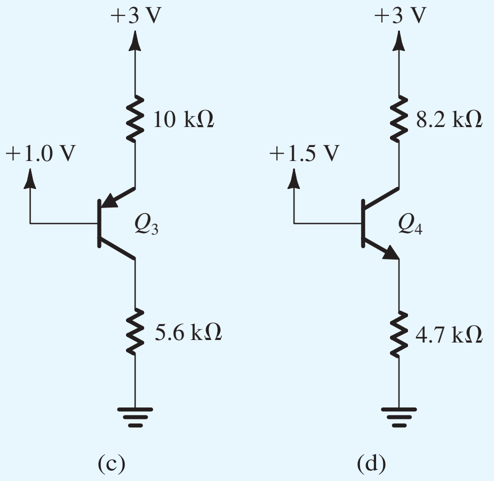


Figure P6.35 *continued*

6.36 The current I_{CBO} of a small transistor is measured to be 10 nA at 25°C. If the temperature of the device is raised to 125°C, what do you expect I_{CBO} to become?

6.37 Augment the model of the *npn* BJT shown in Fig. 6.19(a) by a current source representing I_{CBO}. Assume that r_o is very large and thus can be neglected. In terms of this addition, what do the terminal currents i_B, i_C, and i_E become? If the base lead is open-circuited while the emitter is connected to ground, and the collector is connected to a positive supply, find the emitter and collector currents.

6.38 A BJT whose emitter current is fixed at 1 mA has a base–emitter voltage of 0.70 V at 25°C. What base–emitter voltage would you expect at 0°C? At 100°C?

6.39 A particular *pnp* transistor operating at an emitter current of 0.5 mA at 20°C has an emitter–base voltage of 692 mV.

(a) What does v_{EB} become if the junction temperature rises to 50°C?

(b) If the transistor is operated at a fixed emitter–base voltage of 700 mV, what emitter current flows at 20°C? At 50°C?

6.40 Consider a transistor for which the base–emitter voltage drop is 0.7 V at 10 mA. What current flows for $v_{BE} = 0.5$ V? Evaluate the ratio of the slopes of the i_C–v_{BE} curve at $v_{BE} =$ 700 mV and at $v_{BE} = 500$ mV. The large ratio confirms the point that the BJT has an "apparent threshold" at $v_{BE} \simeq 0.5$ V.

6.41 Use Eq. (6.18) to plot i_C versus v_{CE} for an *npn* transistor having $I_S = 10^{-15}$ A and $V_A = 100$ V. Provide curves for $v_{BE} =$ 0.65, 0.70, 0.72, 0.73, and 0.74 volts. Show the characteristics for v_{CE} up to 15 V.

***6.42** In the circuit shown in Fig. P6.42, current source I is 1.1 mA, and at 25°C $v_{BE} = 680$ mV at $i_E = 1$ mA. At 25°C with $\beta = 100$, what currents flow in R_1 and R_2? What voltage would you expect at node E? Noting that the temperature coefficient of v_{BE} for I_E constant is -2 mV/°C, what is the TC of v_E? For an ambient temperature of 75°C, what voltage would you expect at node E? Clearly state any simplifying assumptions you make.

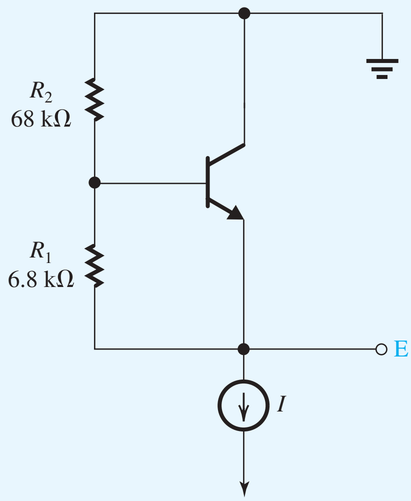


Figure P6.42

6.43 For a particular *npn* transistor operating at a v_{BE} of 680 mV and $I_C = 1$ mA, the i_C–v_{CE} characteristic has a slope of 0.8×10^{-5} ℧. To what value of output resistance does this correspond? What is the value of the Early voltage for this transistor? For operation at 10 mA, what would the output resistance become?

6.44 For a BJT having an Early voltage of 50 V, what is its output resistance at 1 mA? At 100 μA?

6.45 Measurements of the i_C–v_{CE} characteristic of a small-signal transistor operating at $v_{BE} = 710$ mV show that $i_C = 1.1$ mA at $v_{CE} = 5$ V and that $i_C = 1.3$ mA at $v_{CE} = 15$ V. What is the corresponding value of i_C near saturation? At what value of v_{CE} is $i_C = 1.2$ mA? What is the value of the Early voltage for this transistor? What is the output resistance that corresponds to operation at $v_{BE} = 710$ mV?

6.46 Give the *pnp* equivalent circuit models that correspond to those shown in Fig. 6.19 for the *npn* case.

6.47 A BJT operating at $i_B = 10\ \mu A$ and $i_C = 1.0$ mA undergoes a reduction in base current of 1.0 μA. It is found that when v_{CE} is held constant, the corresponding reduction in collector current is 0.08 mA. What are the values of β and the incremental β or β_{ac} that apply? If the base current is increased from 10 μA to 12 μA and v_{CE} is increased from 8 V to 10 V, what collector current results? Assume $V_A = 100$ V.

6.48 For the circuit in Fig. P6.48 let $V_{CC} = 10$ V, $R_C = 1$ kΩ, and $R_B = 10$ kΩ. The BJT has $\beta = 50$. Find the value of V_{BB} that results in the transistor operating

(a) in the active mode with $V_C = 2$ V;
(b) at the edge of saturation;
(c) deep in saturation with $\beta_{\text{forced}} = 10$.

Assume $V_{BE} \simeq 0.7$ V.

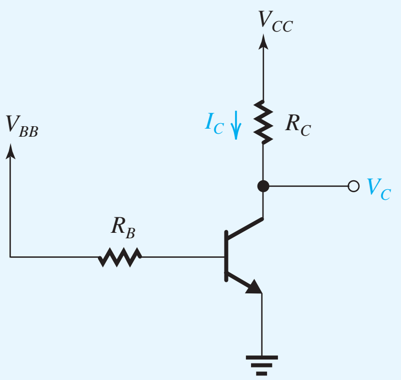


Figure P6.48

SIM D ***6.49** Consider the circuit of Fig. P6.48 for the case $V_{BB} = V_{CC}$. If the BJT is saturated, use the equivalent circuit of Fig. 6.21 to derive an expression for β_{forced} in terms of V_{CC} and (R_B/R_C). Also derive an expression for the total power dissipated in the circuit. For $V_{CC} = 5$ V, design the circuit to obtain operation at a forced β as close to 10 as possible while limiting the power dissipation to no larger than 20 mW. Use 1% resistors (see Appendix J).

6.50 The *pnp* transistor in the circuit in Fig. P6.50 has $\beta = 50$. Show that the BJT is operating in the saturation mode and find β_{forced} and V_C. To what value should R_B be increased in order for the transistor to operate at the edge of saturation?

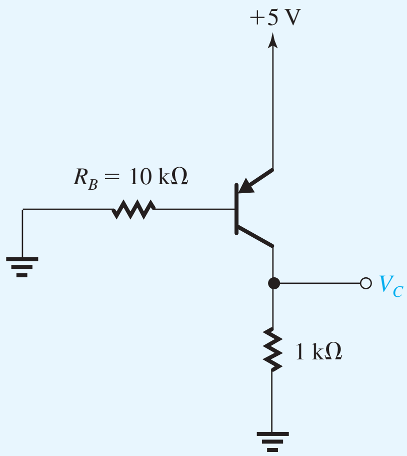


Figure P6.50

Section 6.3: BJT Circuits at DC

6.51 The transistor in the circuit of Fig. P6.51 has a very high β. Find V_E and V_C for V_B (a) +2.0 V, (b) +1.7 V, and (c) 0 V.

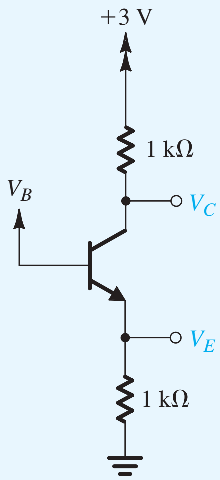


Figure P6.51

6.52 The transistor in the circuit of Fig. P6.51 has a very high β. Find the highest value of V_B for which the transistor still operates in the active mode. Also, find the value of V_B for which the transistor operates in saturation with a forced β of 2.

6.53 Consider the operation of the circuit shown in Fig. P6.53 for V_B at –1 V, 0 V, and +1 V. Assume that β is very high. What values of V_E and V_C result? At what value of V_B does the emitter current reduce to one-tenth of

SIM = Multisim/PSpice; * = difficult problem; ** = more difficult; *** = very challenging; D = design problem its value for $V_B = 0$ V? For what value of V_B is the transistor just at the edge of conduction? ($v_{BE} = 0.5$ V) What values of V_E and V_C correspond? For what value of V_B does the transistor reach the edge of saturation? What values of V_C and V_E correspond? Find the value of V_B for which the transistor operates in saturation with a forced β of 2.

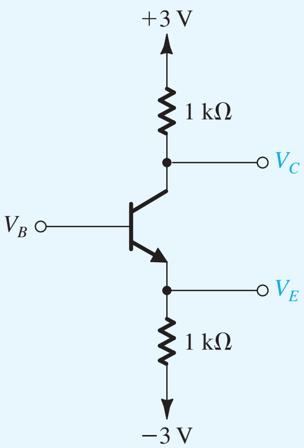


Figure P6.53

6.54 For the transistor shown in Fig. P6.54, assume $\alpha \simeq 1$ and $v_{BE} = 0.5$ V at the edge of conduction. What are the values of V_E and V_C for $V_B = 0$ V? For what value of V_B does the transistor cut off? Saturate? In each case, what values of V_E and V_C result?

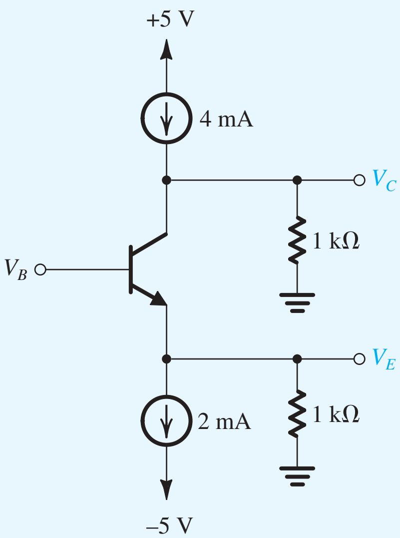


Figure P6.54

D 6.55 Consider the circuit in Fig. P6.51 with the base voltage V_B obtained using a voltage divider across the 3-V supply. Assuming the transistor β to be very large (i.e., ignoring the base current), design the voltage divider to obtain $V_B = 1.2$ V. Design for a 0.1-mA current in the voltage divider. Now, if the BJT $\beta = 100$, analyze the circuit to determine the collector current and the collector voltage.

6.56 A single measurement indicates the emitter voltage of the transistor in the circuit of Fig. P5.56 to be 1.0 V. Under the assumption that $|V_{BE}| = 0.7$ V, what are V_B, I_B, I_E, I_C, V_C, β, and α ? (*Note:* Isn't it surprising what a little measurement can lead to?)

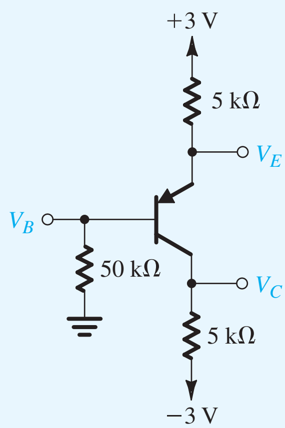


Figure P6.56

D 6.57 Design a circuit using a *pnp* transistor for which $\alpha \simeq 1$ using two resistors connected appropriately to ±3 V so that $I_E = 0.5$ mA and $V_{BC} = 1$ V. What exact values of R_E and R_C would be needed? Now, consult a table of standard 5% resistor values (e.g., that provided in Appendix J) to select suitable practical values. What values of resistors have you chosen? What are the values of I_E and V_{BC} that result?

6.58 In the circuit shown in Fig. P6.58, the transistor has $\beta = 40$. Find the values of V_B, V_E, and V_C. If R_B is raised to 100 kΩ, what voltages result? With $R_B = 100$ kΩ, what value of β would return the voltages to the values first calculated?

SIM = Multisim/PSpice; * = difficult problem; ** = more difficult; *** = very challenging; D = design problem

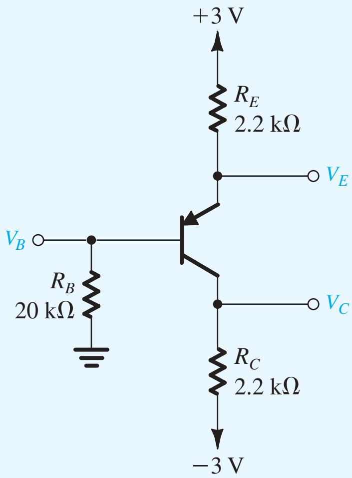


Figure P6.58

6.59 In the circuit shown in Fig. P6.58, the transistor has $\beta = 50$. Find the values of V_B, V_E, and V_C, and verify that the transistor is operating in the active mode. What is the largest value that R_C can have while the transistor remains in the active mode?

SIM 6.60 For the circuit in Fig. P6.60, find V_B, V_E, and V_C for $R_B = 100$ kΩ, 10 kΩ, and 1 kΩ. Let $\beta = 100$.

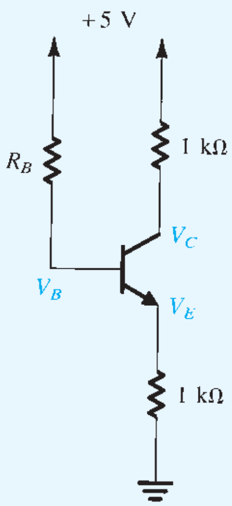


Figure P6.60

6.61 For the circuits in Fig. P6.61, find values for the labeled node voltages and branch currents. Assume β to be very high.

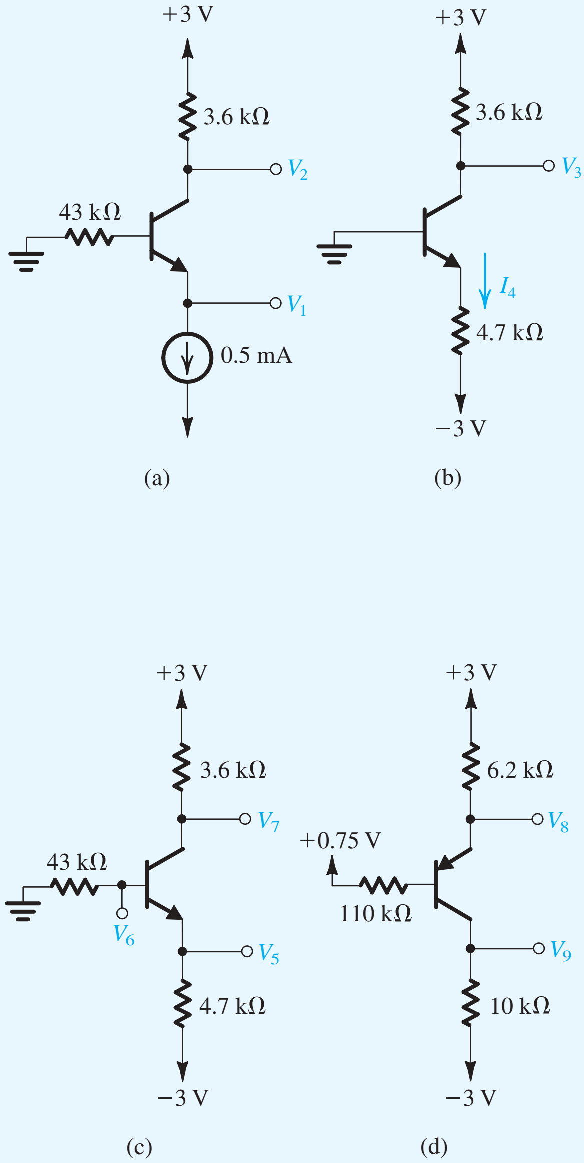
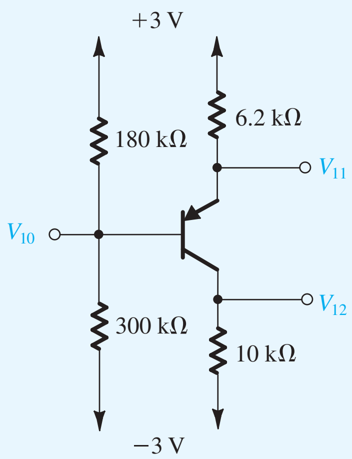


Figure P6.61



(e)

Figure P6.61 *continued*

***6.62** Repeat the analysis of the circuits in Problem 6.61 using $\beta = 100$. Find all the labeled node voltages and branch currents.

D **6.63 It is required to design the circuit in Fig. P6.63 so that a current of 1 mA is established in the emitter and a voltage of −1 V appears at the collector. The transistor type used has a nominal β of 100. However, the β value can be as low as 50 and as high as 150. Your design should ensure that the specified emitter current is obtained when $\beta = 100$ and that at the extreme values of β the emitter current does not change by more than 10% of its nominal value. Also, design for as large a value for R_B as possible. Give the values of R_B, R_E, and R_C to the nearest kilohm. What is the expected range of collector current and collector voltage corresponding to the full range of β values?

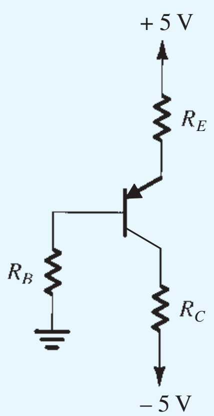


Figure P6.63

D 6.64 The *pnp* transistor in the circuit of Fig. P6.64 has $\beta = 50$. Find the value for R_C to obtain $V_C = +2$ V. What happens if the transistor is replaced with another having $\beta = 100$? Give the value of V_C in the latter case.

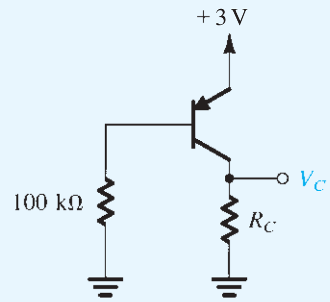


Figure P6.64

*****6.65** Consider the circuit shown in Fig. P6.65. It resembles that in Fig. 6.30 but includes other features. First, note diodes D_1 and D_2 are included to make design (and analysis) easier and to provide temperature compensation for the emitter–base voltages of Q_1 and Q_2. Second, note resistor R, whose purpose is to provide negative feedback (more on this later in the book!). Using $|V_{BE}|$ and $V_D = 0.7$ V independent of current, and $\beta = \infty$, find the voltages V_{B1}, V_{E1}, V_{C1}, V_{B2}, V_{E2}, and V_{C2}, initially with R open-circuited and then with R connected. Repeat for $\beta = 100$, with R open-circuited initially, then connected.

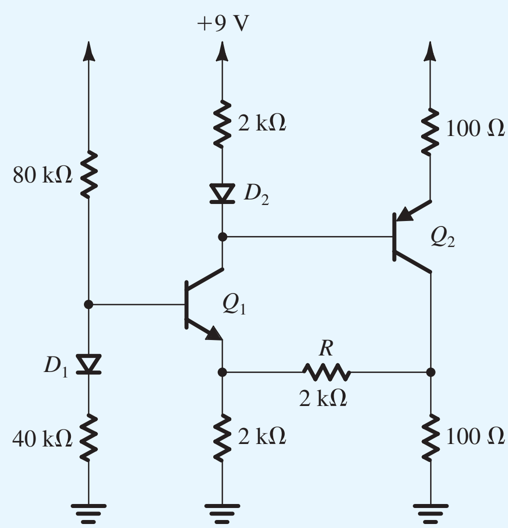


Figure P6.65

***6.66** For the circuit shown in Fig. P6.66, find the labeled node voltages for:

(a) $\beta = \infty$
(b) $\beta = 100$

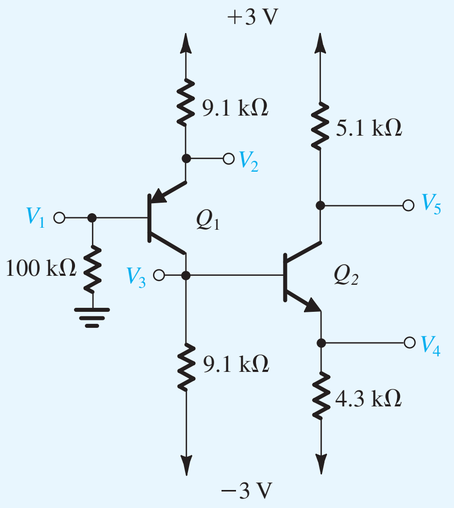


Figure P6.66

D *6.67 Using $\beta = \infty$, design the circuit shown in Fig. P6.67 so that the emitter currents of Q_1, Q_2, and Q_3 are 0.5 mA, 0.5 mA, and 1 mA, respectively, and $V_3 = 0$, $V_5 = -2$ V, and $V_7 = 1$ V. For each resistor, select the nearest standard value utilizing the table of standard values for 5% resistors in Appendix J. Now, for $\beta = 100$, find the values of V_3, V_4, V_5, V_6, and V_7.

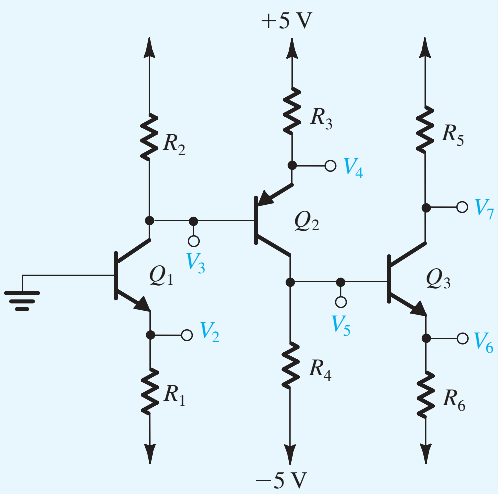


Figure P6.67

***6.68** For the circuit in Fig. P6.68, find V_B and V_E for v_I = 0 V, +2 V, –2.5 V, and –5 V. The BJTs have $\beta = 50$.

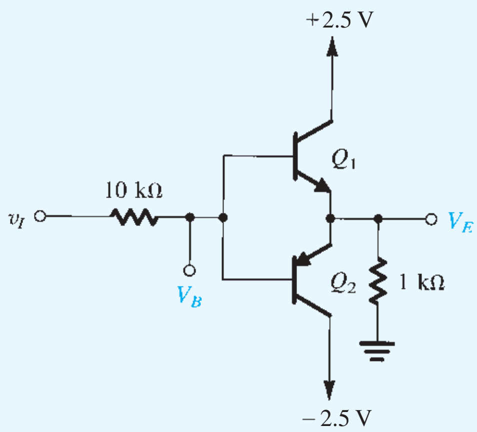


Figure P6.68

****6.69** All the transistors in the circuits of Fig. P6.69 are specified to have a minimum β of 50. Find approximate values for the collector voltages and calculate forced β for each of the transistors. (*Hint:* Initially, assume all transistors are operating in saturation, and verify the assumption.)

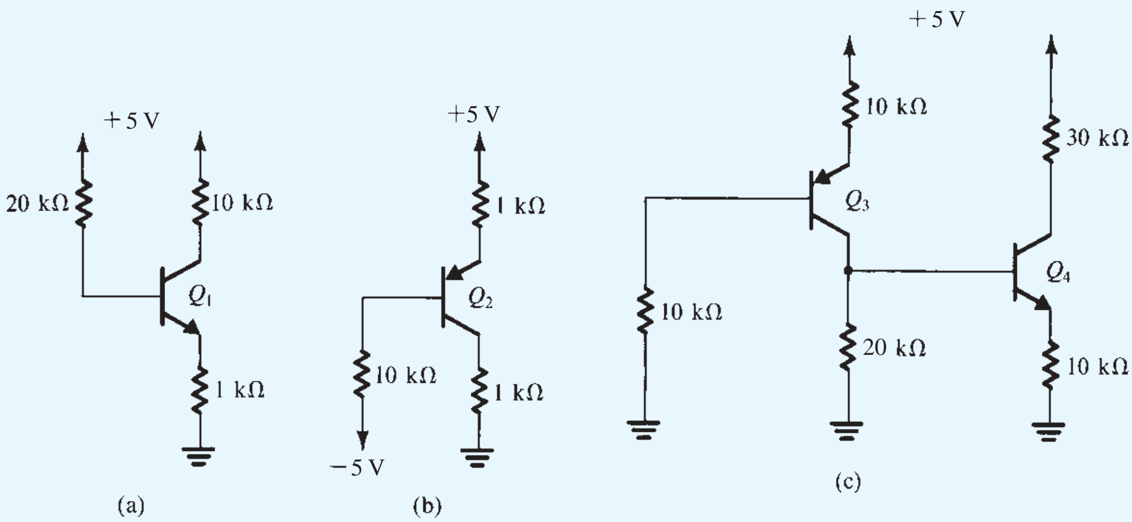

+5 V
20 kΩ
10 kΩ
Q_1
1 kΩ
(a)
+5 V
1 kΩ
Q_2
10 kΩ
1 kΩ
−5 V
(b)
+5 V
10 kΩ
30 kΩ
Q_3
Q_4
10 kΩ
20 kΩ
10 kΩ
(c)

Figure P6.69