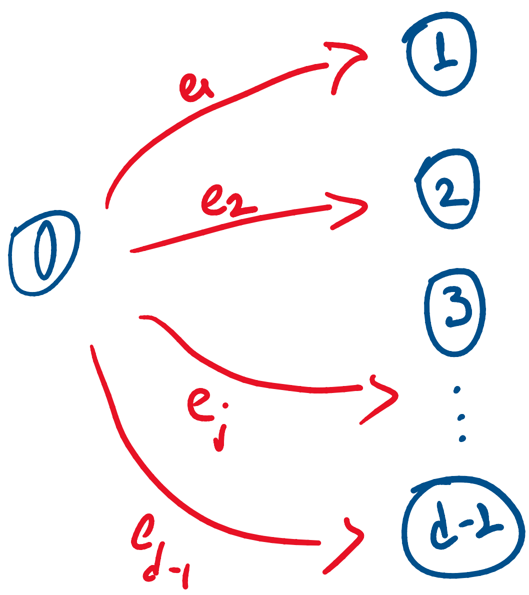
Train Stops

We consider the minimum distance to travel until we can stop, ie $d = \min_j \{d_j\}$.

We start at station 0. We consider initially, stops at any station $\tilde{S}_j = y_j \bmod (d) \equiv d_i \bmod (d)$, for some i
(notice: $j \leq \min(i, d-1)$)



In words, we consider only the stations we can reach from st. 0 with one stop and grouping those where their modulo(d) is the same.

We construct the graph with nodes $\{0, \cup_j \tilde{S}_j\}$ and connect 0 to every $\hat{S}_j$ with an edge $(0, \tilde{S}_j) = e_j$.
The weight of this edge shall be the minimum d_i which can lead us to such a station. Why?

Because for every pair (S_i, S_j) where: $S_i < S_j$ and S_i, S_j correspond to "station" (belong to the class) $\tilde{S}_k = S_i \bmod (d) = S_j \bmod (d)$ then if we are at station S_i we can reach station S_j using only stops of intervals of d.

Proof:
$$\left.\begin{aligned} S_i &= m_i \cdot d + \tilde{S}_k \\ S_j &= m_j \cdot d + \tilde{S}_k \end{aligned}\right\} \Rightarrow S_j - S_i = (m_j - m_i) d$$

But we know that $S_j > S_i \Rightarrow m_j - m_i > 0$

So we can reach S_j from S_i using $m_j - m_i$ stops.

Train Stops (part 2)

For every $\tilde{s}_j$ we consructed we consider all stops of length d_i. For each one we compute where we can stop, ie $\tilde{s}_k$ such that:

$$\tilde{s}_j + d_i \equiv \tilde{s}_k \bmod(d)$$

For each such $\tilde{s}_k$ ($\left|\bigcup_k \{\tilde{s}_k\}\right| \le \left|\bigcup_i \{d_i\}\right|$) we connect $\tilde{s}_j$ to it using edge $(\tilde{s}_j, \tilde{s}_k)$ of weight $e = \min_i \{ d_i \mid \tilde{s}_j + d_i \equiv \tilde{s}_k \bmod(d) \}$

Having completed this graph we can (but won't :)) prove that the weight of the shortest path $(0, \tilde{s}_j)$ is the closest to 0 station s_j, such that $s_j \bmod(d) = \tilde{s}_j$, we can reach.

Then we can answer the desired queries for any station s_i as such:

- We can reach s_i iff we can reach $\tilde{s}_i = s_i \bmod(d)$ and weight $\{SP(0, \tilde{s}_i)\} = s_m \le s_i$

Because we proved that if we can reach s_m, we can reach s_i as well.

Complexity

- Construct graph: $\Theta\left(\{\min d_j\} \cdot \left|\bigcup_j \{d_j\}\right|\right) = \Theta(N \cdot \min\{d_j\})$
- Find SPTs from source 0 (Djikstra): $O(|V|^2) = O(\min^2\{d_j\})$
- Q queries in $\Theta(1)$: $\Theta(Q)$

total: $O((N + \min\{d_j\}) \min\{d_j\} + Q)$