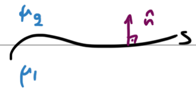
Παρ. 02/12, 15.00 - 17.00 Αναπλήρωση

Κεφάλαιο 4 - ΜΣΠ

- $\oint \vec{H}\, d\vec{l} = \cancel{\frac{d}{dt}\int \vec{D}\cdot d\vec{S}} + \int \vec{J}\, d\vec{S} \longrightarrow \nabla\times\vec{H} = \vec{J}$
- $\oint_S \vec{B}\, d\vec{S} = 0 \longrightarrow \nabla\vec{B} = 0$

Διαπερατότητα • $\mu = \mu_r \cdot \mu_0$ (γραμμικό υλικό)

Ορ. Συνθήκες
- $\hat{n}\cdot(\vec{B}_2 - \vec{B}_1) = 0$
- $\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{K}$



Διανυσματικό δυναμικό ($\vec{A}$)

- $\nabla\vec{B} = 0$
- $\nabla\cdot(\nabla\times\vec{A}) = 0$ — (*) $\nabla\cdot\vec{A} = ?$, Θα πρέπει να ξέρουμε την απόκλιση του $\vec{A}$
- $\vec{B} = \nabla\times\vec{A}$
- $\vec{A}' = \vec{A} + \nabla f$
- $\vec{B}' = \nabla\times\vec{A}' = \nabla\times\vec{A} + \nabla\times\nabla f = \vec{B}$

Θ. Helmholtz

- $\nabla\times\vec{A} = \vec{F}$ (γνωστό)
- $\nabla\cdot\vec{A} = C$ (γνωστό)
- $|\vec{A}| \xrightarrow[r\to\infty]{} 0$
- $|\vec{F}|, C \underset{r\to\infty}{\sim} \frac{1}{r^2}$

$\Rightarrow$ Το $\vec{A}$ ορίζεται μονοσήμαντα

- $\nabla\times\vec{H} = \vec{J}$
- $\vec{B} = \mu\vec{H}$

$\Rightarrow \nabla\times\left(\frac{1}{\mu}\vec{B}\right) = \vec{J}$ (μ ομογ.) $\Rightarrow \nabla\times\nabla\times\vec{A} = \mu\vec{J} \Rightarrow \nabla\times\nabla\times\vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu\vec{J}$

- $\boxed{\nabla\cdot\vec{A} = 0 \quad \text{Συνθήκη Coulomb}}$

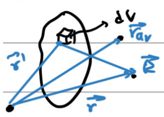
- $\boxed{\nabla^2\vec{A} = -\mu\vec{J} \quad \text{Διανυσματική εξίσωση Poisson}}$

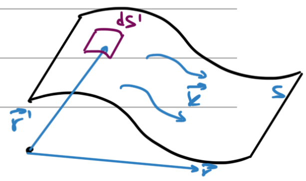
- Καρτεσιανές: $\nabla^2 A_i = -\mu J_i$, $i = x, y, z$
- Κυλινδρικές: $\nabla^2 A_i \neq -\mu J_i$, $i = r, \varphi$
  Σφαιρικές: r, θ, φ

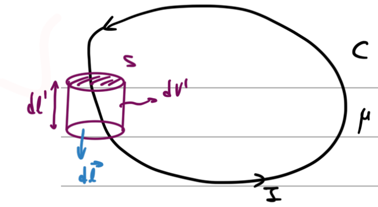
$$\nabla^2\vec{A} = \nabla\nabla\cdot\vec{A} - \nabla\times\nabla\times\vec{A}$$

Αντιστοιχία: $\vec{A} \longleftrightarrow \phi$, $\vec{J} \longleftrightarrow \rho$, $\mu \longleftrightarrow 1/\varepsilon$

Ολοκλ. επαλ.:
- $\phi = \frac{1}{4\pi\varepsilon}\int_V \rho\, dV' \left(\frac{1}{R} - \frac{1}{R_{av}}\right) \longrightarrow \vec{A} = \frac{\mu}{4\pi}\int_V \vec{J}\, dV' \left(\frac{1}{R} - \frac{1}{R_{av}}\right)$
- $\vec{A} = \frac{\mu}{4\pi}\int_S \vec{K}\, dS' \left(\frac{1}{R} - \frac{1}{R_{av}}\right)$, $\vec{K} = \vec{K}(\vec{r}')$



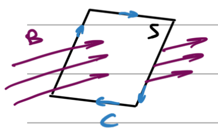




• $\vec J\, dV' = \vec J \cdot S\, d\ell' = I\, d\vec\ell' \Rightarrow \vec A = \frac{\mu}{4\pi}\oint_C I\, d\vec\ell' \left(\frac{1}{R} - \frac{1}{R_{av}}\right)$

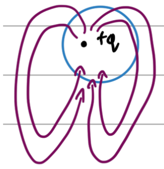
δεν μπορούμε στο άπειρο, γιατί το A θα απειρίζεται

Μαγνητική ροή



• $\Phi_m = \int_S \vec B\, d\vec S = \int_S \nabla\times\vec A\, d\vec S \overset{\text{Θ. Stokes}}{=} \oint_C \vec A\, d\vec\ell$

↳ $\Phi_m = \oint_S \vec B\, d\vec S = 0$



(όσες μπαίνουν τόσες βγαίνουν)

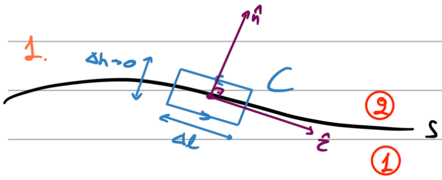
Οριακές Συνθήκες $\vec A$

• Πορεία: $\nabla^2\vec A = -\mu\vec J \rightsquigarrow \vec B = \nabla\times\vec A \rightsquigarrow \vec H = \frac{1}{\mu}\vec B$

1. $\Phi_m = \oint_C \vec A\cdot d\vec\ell$

2. $\nabla\cdot\vec A = 0 \rightsquigarrow \oint_S \vec A\, d\vec S = 0$ (συνθήκη Coulomb)

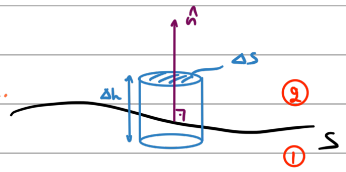
1.



$\Delta h \to 0$, $\Phi_m = 0$

• $A_{1t}\cdot\Delta\ell - A_{2t}\,\Delta\ell = 0 \Rightarrow A_{1t} = A_{2t} \Rightarrow \boxed{\hat n\times(\vec A_2 - \vec A_1) = 0}$

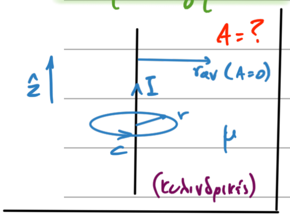
2.



• $A_{2n}\,\Delta S - A_{1n}\,\Delta S = 0 \Rightarrow A_{2n} = A_{1n} \Rightarrow \boxed{\hat n\cdot(\vec A_2 - \vec A_1) = 0}$

3. $\hat n\times(\vec H_2 - \vec H_1) = \vec K \Rightarrow \hat n\times\left(\frac{1}{\mu_2}\vec B_2 - \frac{1}{\mu_1}\vec B_1\right) = \vec K \Rightarrow \boxed{\hat n\times\left(\frac{1}{\mu_2}\nabla\times\vec A_2 - \frac{1}{\mu_1}\nabla\times\vec A_1\right) = \vec K}$

Παράδειγμα 1



$$\left.\begin{aligned} &\bullet\ \nabla^2\vec A = -\mu\vec J \\ &\bullet\ \vec J = \vec J(z) \end{aligned}\right\} \Rightarrow \vec A = \vec A_z(r) \rightsquigarrow \nabla^2 A_z = 0 \xrightarrow{\text{κυλιν.}} \boxed{A_z = C_1\cdot\ln r + C_2}$$

$$\left.\begin{aligned} &\bullet\ \vec B = \nabla\times\vec A = -\frac{dA_z}{dr}\hat\varphi = -\frac{C_1}{r}\hat\varphi \\ &\bullet\ \oint \vec H\, d\vec\ell = I \Rightarrow \vec B = \frac{\mu I}{2\pi r}\hat\varphi \end{aligned}\right\} \Rightarrow \boxed{C_1 = -\frac{\mu I}{2\pi}}$$

• $A_z(r = r_{av}) = 0 \Rightarrow \boxed{C_2 = -C_1 \ln r_{av} = \frac{\mu I}{2\pi}\ln r_{av}}$

• Άρα: $A_z = \frac{\mu I}{2\pi}\ln\left(\frac{r_{av}}{r}\right)$