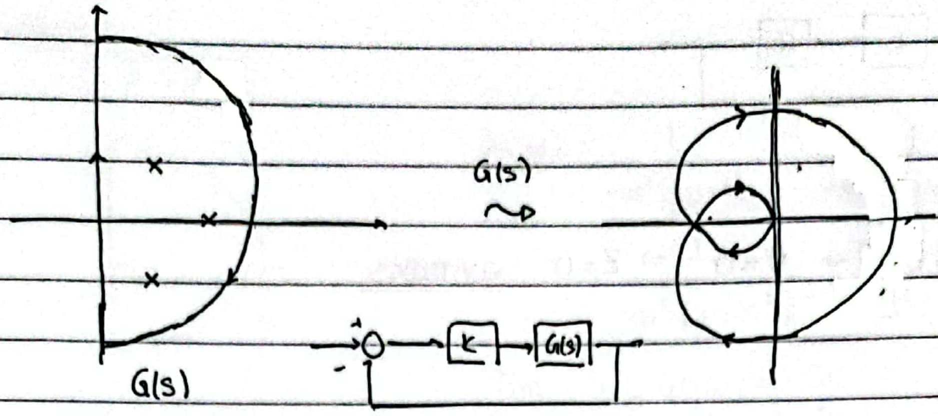
Πέμπτη, 08/12/2022



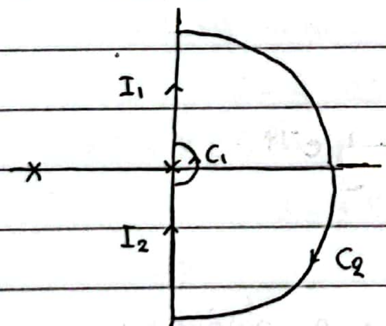
G(s)

P = αρ. πόλων της G(s) στο εσωτερικό της G(s)

N = αριθμός περιελίξεων γύρω από το $\left(-\frac{1}{K}, 0\right)$ (με θετικό πρόσημο για περιέλιξη σύμφωνα με τους δείκτες του ρολογιού, διαφορετικά αρνητικό)

Z = N + P : αρ. πόλων της ΣΜ ΚΛΕΙΣΤΟΥ ΒΡΟΧΟΥ στο εσωτερικό της C

$$G(s) = \frac{1}{s(s+1)}$$



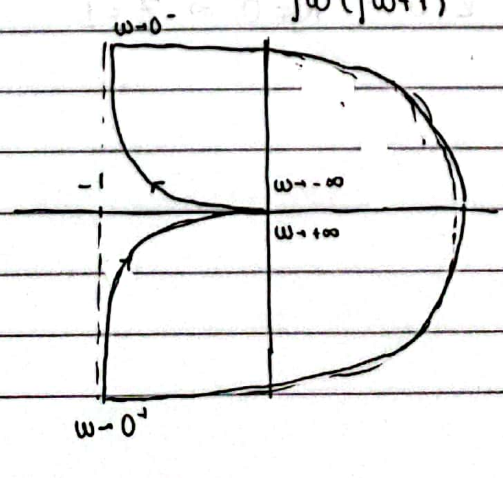
$I_1 \cup I_2 : s = j\omega$

$\omega \in (-\infty, 0) \cup (0, +\infty)$

$C_1 : \underset{\epsilon \to 0}{s = \epsilon e^{j\varphi}}, \ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$ αυξανόμενη

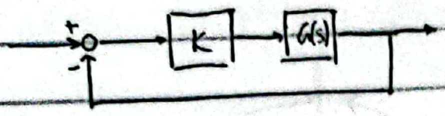
$C_2 : \underset{R \to \infty}{s = Re^{j\varphi}}, \ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$ μειούμενη

$$I_1 \cup I_2 : G(j\omega) = \frac{1}{j\omega(j\omega+1)} = \frac{-j(1-j\omega)}{\omega(1+\omega^2)} = -\frac{1}{1+\omega^2} - \frac{j}{\omega(1+\omega^2)}, \ \omega \in (0, +\infty)$$



$$C_2 : G(Re^{j\varphi}) = \frac{1}{Re^{j\varphi}(Re^{j\varphi}+1)} \to 0$$

$$C_1 : G(\epsilon e^{j\varphi}) = \frac{1}{\epsilon e^{j\varphi}(\epsilon e^{j\varphi}+1)} \simeq \frac{1}{\epsilon} e^{-j\varphi}$$



Περίπτωση 1

$P=0$

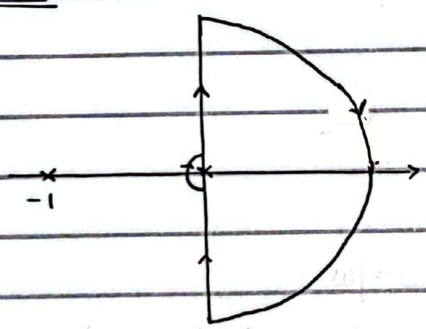
$-\frac{1}{K}<0 \Rightarrow K>0 \Rightarrow N=0 \Rightarrow Z=0$ ευσταθές

Περίπτωση 2

$K<0 \Rightarrow N=+1 \Rightarrow Z=1$ 1 ασταθής πόλος

$1+\frac{K}{G(s)}=0 \Leftrightarrow s^2+s+K=0$

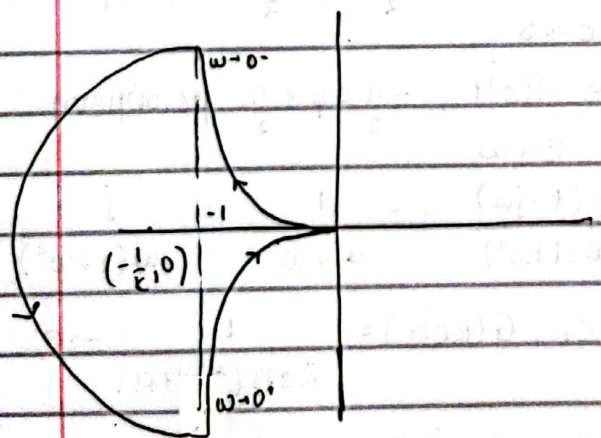
s^2	1	K
s^1	1	0
s^0	K	



$C_1: s=\epsilon e^{j\varphi}$, $\epsilon\to 0$, $\frac{\pi}{2}\le\varphi\le\frac{3\pi}{2}$ μειούμενη

$P=1$

$G(\epsilon e^{j\varphi})\simeq\frac{1}{\epsilon}e^{-j\varphi}$

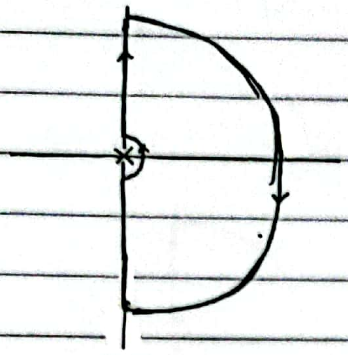


$-\frac{3\pi}{2}\le\varphi\le-\frac{\pi}{2}$ αυξάνεται

$K>0: N=-1 \Rightarrow Z=0$

$K<0: N=0 \Rightarrow Z=1$

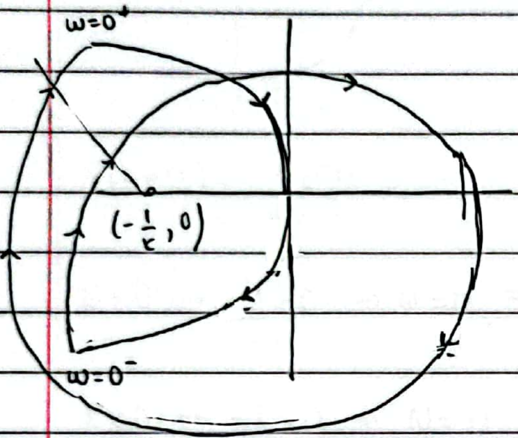
$$G(s) = \frac{1}{s^2(s+1)}$$



$$G(j\omega) = \frac{1}{-\omega^2(j\omega+1)} = \frac{1-j\omega}{-\omega^2(1+\omega^2)} =$$

$$= -\frac{1}{\omega^2(1+\omega^2)} + \frac{j\omega}{\omega^2(1+\omega^2)}, \quad \omega \in (0,+\infty)$$

$$\lim_{\omega\to+\infty} \arg G(j\omega) = -\frac{3\pi}{2}$$



$s = \epsilon e^{j\varphi}$, $-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$ αυξάνεται

$G(\epsilon e^{j\varphi}) \simeq \frac{1}{\epsilon^2} e^{-2j\varphi}$, $-\pi \le -2\varphi \le \pi$

μειώνεται

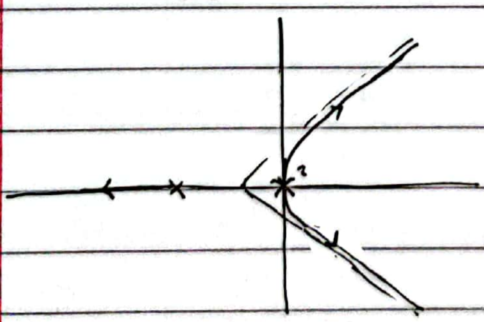
$-\frac{1}{k} < 0 \Leftrightarrow k > 0 \Leftrightarrow N = +2$, $P = 0$

$Z = 2$ ασταθείς πόλοι

$-\frac{1}{k} > 0 \Leftrightarrow k < 0 \Leftrightarrow N = +1$

$Z = +1$, 1 ασταθής πόλος

$k>0$



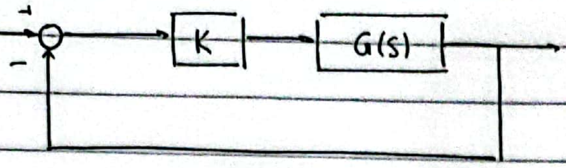
$$\sigma_a = \frac{-1+0+0}{3} = -\frac{1}{3}$$

$$\theta_{ασ} = \frac{(2k+1)\cdot 180^\circ}{3} \begin{cases} 60^\circ \\ 180^\circ \\ -60^\circ \end{cases}$$

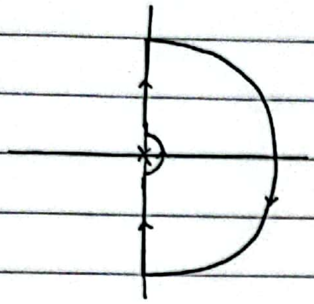
$$-2\varphi_{αν} = (2\nu+1)180^\circ \Rightarrow \varphi_{αν} = \begin{cases} +90^\circ \\ -90^\circ \end{cases}$$

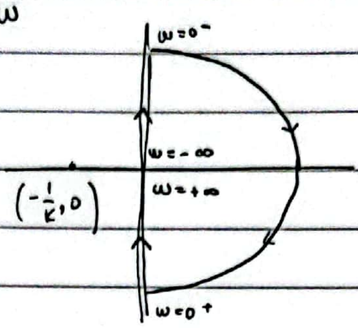
Scanned with CamScanner

Παλιό θέμα (Κ2019)



$G(s) = 1/s \Rightarrow G(j\omega) = \dfrac{1}{j\omega} = -\dfrac{j}{\omega}$





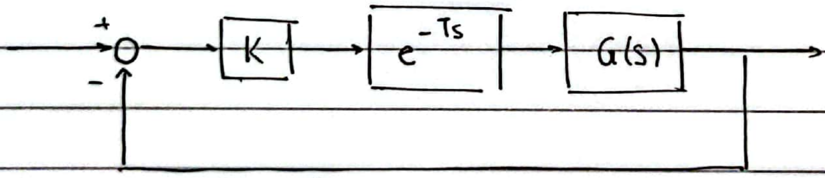
$G(\epsilon e^{j\varphi}) = \dfrac{1}{\epsilon} e^{-j\varphi}$

$K > 0 \Leftrightarrow -\dfrac{1}{K} < 0 \Rightarrow N = 0,\ Z = 0$ ευσταθές

$K < 0 \Leftrightarrow -\dfrac{1}{K} > 0 \Rightarrow N = +1 \Rightarrow Z = 1$, 1 ασταθής πόλος

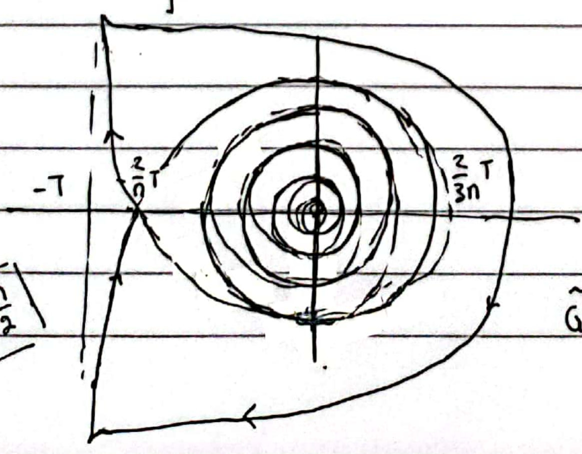
$|G(j\omega_{gc})| = 1 \Rightarrow \boxed{\omega_{gc} = 1 \text{ rad/sec}}$

$\phi_{\pi\epsilon\rho} = 180^\circ + \arg(G(j\omega_{gc})) \Rightarrow \boxed{\phi_{\pi\epsilon\rho} = 90^\circ}$



$\tilde{G}(s) = G(s) e^{-Ts} = \dfrac{e^{-Ts}}{s}$

$\tilde{G}(j\omega) = \dfrac{e^{-j\omega T}}{j\omega} = -\dfrac{j}{\omega}(\cos\omega T - j\sin\omega T) = -T\dfrac{\sin\omega T}{\omega T} - \dfrac{j\cos\omega T}{\omega} = \dfrac{1}{\omega} e^{j(-\frac{\pi}{2} - \omega T)}$



$\cos\omega T \Rightarrow \omega_\nu T = \nu\pi + \dfrac{\pi}{2} \Rightarrow \omega_0 = \dfrac{\pi}{2T}$

$G(j\omega_0) = -\dfrac{1}{\omega_0} = -\dfrac{2T}{\pi}$

$\omega_\nu = (\nu+1)\dfrac{\pi}{T}$

$G(j\omega_1) = \dfrac{1}{\omega_1} = \dfrac{2T}{3\pi}$

$-\dfrac{1}{K} < -\dfrac{2}{\pi}T \Rightarrow \boxed{0 < TK < \dfrac{\pi}{2}}$

$\tilde{G}(\epsilon e^{j\varphi}) = \dfrac{1}{\epsilon} e^{-T\epsilon(\cos\varphi + j\sin\varphi)} e^{-j\varphi}$

$= \dfrac{1}{\epsilon} e^{-T\epsilon\cos\varphi} \cdot e^{j(-\varphi - \epsilon T\sin\varphi)}$