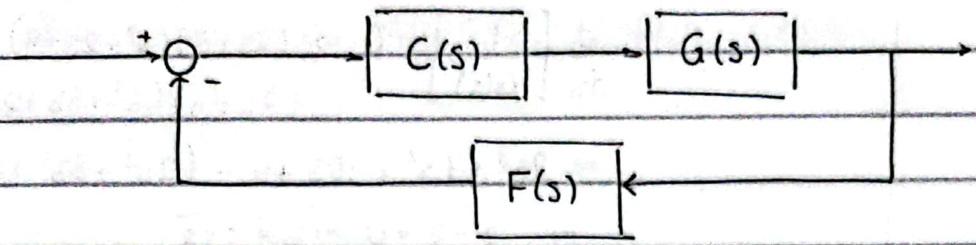
Δευτέρα, 21/11/2022

Παλιό θέμα



$$C(s)=K,\ G(s)=\frac{3(s+1)}{s^2},\ F(s)=\frac{1}{s+K}$$

$$1+C(s)G(s)F(s)=0 \Rightarrow 1+\frac{3K(s+1)}{s^2(s+K)}=0 \Rightarrow s^2(s+K)+3K(s+1)=0 \Leftrightarrow$$

$1+K\tilde{G}(s)=0$

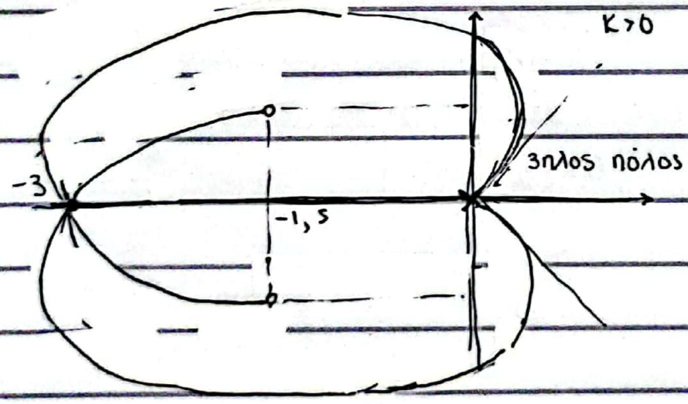
$$\Leftrightarrow s^3+K(s^2+3s+3)=0 \Leftrightarrow$$

$$\Leftrightarrow 1+K\cdot\underbrace{\frac{s^2+3s+3}{s^3}}_{\tilde{G}(s)}=0$$

$$\tilde{G}(s)=\frac{s^2+3s+3}{s^3},\quad \Delta=9-12=-3$$

$$z_{1,2}=-\frac{3}{2}\pm j\frac{\sqrt{3}}{2}$$

$K>0$



$$\frac{d}{ds}\left[\frac{1}{\tilde{G}(s)}\right]=0 \Rightarrow 3s^2(s^2+3s+3)-(2s+3)s^3=0 \Rightarrow$$

$$\Rightarrow s^4+6s^3+9s^2=0 \Rightarrow s^2(s^2+6s+9)=0 \Rightarrow$$

$$\Rightarrow s^2(s+3)^2=0$$

$$\arg(\tilde{G}s_0)=(2k+1)\cdot 180^\circ \Rightarrow \phi_{\alpha\nu}=\begin{cases}60\\180^\circ\\-60^\circ\end{cases}$$

Για ευστάθεια:

s^3	1	3K		
s^2	K	3K	$K>1$	
s^1	3K-3	0	$K=1$	$s^2+3=0 \Rightarrow s=\pm j\sqrt{3}$
s^0	3K			

ΓΤΡ→ Κριτήριο Routh

$$P(s) = a_n s^n + a_{n-1}s^{n-1} + \dots + a_0$$

s^n	$\overset{\to A_{11}}{a_n}$	a_{n-2}
s^{n-1}	$\overset{\nearrow A_{21}}{a_{n-1}}$	a_{n-3}
s^{n-2}	$A_{n-2,1}$	$A_{n-2,2}$
$\vdots$	$\vdots$	$\vdots$
s^{n-i+2}	$A_{i-1,1}$	$A_{i-1,2}$
s^{n-i+1}	$A_{i,1}$	$A_{i,2}$
s^{n-i}	$A_{i+1,1}$	$A_{i+2,2}$

$$A_{i+1,1} = A_{i-1,2} - \frac{A_{i-1,1}}{A_{i,1}} A_{i,2}$$

$$A_{i+1,2} = A_{i-1,3} - \frac{A_{i-1,1}}{A_{i,1}} A_{i,3}$$

$$Q_1(s) = a_n s^n + a_{n-2}s^{n-2} + \dots$$

$$Q_2(s) = a_{n-1}s^{n-1} + a_{n-3}s^{n-3} + \dots$$

$P_1(s) = P(s) = Q_1(s) + Q_2(s)$, $\quad P_1 \to n$ ρίζες

$P_i(s) = Q_i(s) + Q_{i+1}(s)$, $\quad P_2 \to n-1$ ρίζες

$\vdots$

$P_i \to n-i+1$ ρίζες

$P_{i+1} \to n-i$

$P_i(s)$, $P_{i+1}(s)$
έχουν $n-i$ ρίζες στο ίδιο μιγ. ημιεπίπεδο.

$$Q_{i+1}(s) = A_{i+1,1}s^{n-i} + A_{i+1,2}s^{n-i-2} + \dots$$

$$= \Big(A_{i-1,2} - \underbrace{\frac{A_{i-1,1}}{A_{i,1}}}_{q_i} A_{i,2}\Big)s^{n-i} + \Big(A_{i-1,3} - \frac{A_{i-1,1}}{A_{i,1}} A_{i,3}\Big)s^{n-i+2} + \dots$$

$$= \big(A_{i-1,2}s^{n-i} + A_{i-1,3}s^{n-i-2} + \dots\big) - q_i s\big(A_{i,2}s^{n-i-1} + A_{i,3}s^{n-i-3} + \dots\big)$$

$+ A_{i-1,1}s^{n-i+2}$ $\qquad$ $A_{i,1}s^{n-i+1}$

$$\boxed{Q_{i+1}(s) = Q_{i-1}(s) - q_i s Q_i(s)}$$

$$\left.\begin{aligned} P_i(s) &= Q_i(s) + Q_{i+1}(s) \\ P_{i-1}(s) &= Q_{i-1}(s) + Q_i(s) \end{aligned}\right\} \Rightarrow P_i(s) = P_{i-1}(s) + \underbrace{Q_{i+1}(s) - Q_{i-1}(s)}_{-q_i s Q_i(s)}$$

$$\Rightarrow P_i(s) = P_{i-1}(s) - q_i s Q_i(s)$$

Q_i, Q_{i+1} coprime

$\hat{P}_i(s,q) \triangleq P_{i-1}(s) - q s Q_i(s)$, $\quad q \in [\min\{0, q_i\}, \max\{0, q_i\}]$

$\hat{P}_i(s,0) = P_{i-1}(s)$ $\quad$ πολυώνυμο $n-i+2$ τάξης

$\hat{P}_i(s,q_i) = P_i(s)$ $\leftarrow$ πολυώνυμο τάξης $n-i+1$

Για q^* : $\hat{P}_i(j\beta, q^*) = 0 \Rightarrow \dfrac{P_{i-1}(j\beta)}{Q_{i-1}(j\beta) + Q_i(j\beta)} - q^* j\beta\, Q_i(j\beta) = 0$

$$\left.\begin{aligned} Q_{i-1}(j\beta) - j\beta q^* Q_i(j\beta) = 0 \\ Q_i(j\beta) = 0 \end{aligned}\right\} \Rightarrow Q_i(j\beta) = Q_{i-1}(j\beta) = 0 \quad \text{άτοπο,}$$

οπότε δεν τέμνουν τον φανταστικό άξονα

$$\hat{P}_i(s,q) = A_{i-1,1}s^{n-i+2} + A_{i,1}s^{n-i+1} + \dots - qs\left(A_{i,1}s^{n-i+1} + A_{i,2}s^{n-1-1} + \dots\right)$$
$$= \left(A_{i-1,1} - q\,A_{i,1}\right)s^{n-i+2} + A_{i,1}s^{n-i+1} +$$

$$-\frac{A_{i,1}}{A_{i-1,1} - q\,A_{i,1}} = -\frac{1}{q_i - q}$$

Αν $q_i > 0 \rightarrow -\infty$: ρίζα ευσταθής

Αν $q_i < 0 \rightarrow +\infty$: ρίζα ασταθής

π.χ. $P(s) = s^5 + s^4 - 27s^3 - 13s^2 + 134s + 120$

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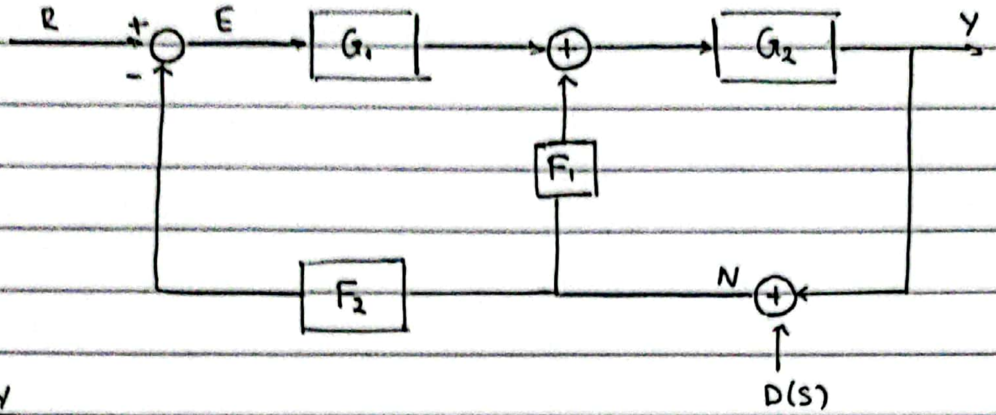
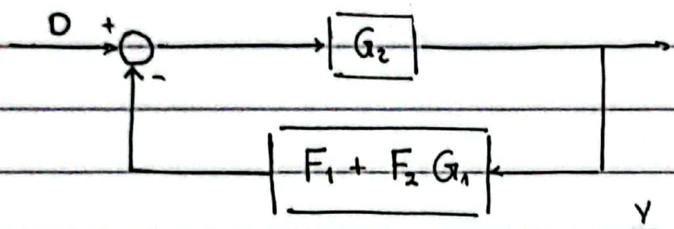


$\frac{Y}{D}, \frac{Y}{R}$

Επαλληλία



$$\frac{Y}{D} = \frac{G_2}{1 + G_2(F_1 + F_2 G_1)}$$

ή $Y = G_2(G_1 E - F_1 N)$
$E = R - F_2 N$
$\Rightarrow Y = G_1 G_2 (R - F_2 N) - G_2 F_1 N$
$= G_2 G_1 - G_2(F_1 + G_1 F_2)(D + Y)$

$$\Rightarrow Y = \frac{G_1 G_2 R}{1 + G_2(F_1 + G_1 F_2)} + \frac{-G_2}{1 + G_2(F_1 + G_1 F_2)} D$$

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