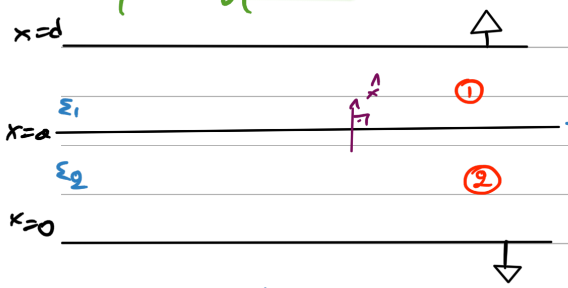
Παράδειγμα 4

$$\frac{\partial}{\partial y} = 0 = \frac{\partial}{\partial z}$$

$\phi_i\,?\,,\ E_i\,?$



$$\rightarrow \nabla^2 \phi_{1,2} = 0 \Rightarrow \frac{\partial^2 \phi_{1,2}}{\partial x_{1,2}^2} = 0 \Rightarrow$$

$$\Rightarrow \phi_{1,2} = A_{1,2}\,x + B_{1,2}$$

$\rightarrow x=a$: $\cdot\ \phi_1 = \phi_2$

$\cdot\ -\varepsilon_1 \left.\frac{\partial \phi_1}{\partial x}\right|_{x=a} + \varepsilon_2 \left.\frac{\partial \phi_2}{\partial x}\right|_{x=a} = \sigma$

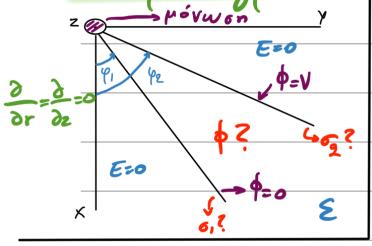
$\rightarrow x=0$: $\phi_2 = 0 \Rightarrow B_2 = 0$

$\rightarrow x=d$: $\phi_1 = 0 \Rightarrow B_1 = -A_1 d$

σύστημα 4x4 =
$$\begin{cases} A_1 = \dfrac{\sigma}{\varepsilon_2 \dfrac{a-d}{a} - \varepsilon_1} \\ A_2 = \dfrac{a-d}{a} A_1 \\ B_1 = -d A_1 \\ B_2 = 0 \end{cases}$$

$$\vec{E}_{1,2} = -\frac{\partial \phi_{1,2}}{\partial x}\hat{x} = -A_{1,2}$$

Παράδειγμα 5



$$\rightarrow \nabla^2\phi = 0 \Rightarrow \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} = 0 \xrightarrow[\text{λόγω μόνωσης στο } 0]{r \neq 0} \frac{\partial^2\phi}{\partial\varphi^2} = 0 \Rightarrow \phi = A\varphi + B$$

$\rightarrow \varphi = \varphi_1$: $\phi = 0$

$\varphi = \varphi_2$: $\phi = V$

2x2 σύστημα
$$\begin{cases} A = V/(\varphi_2 - \varphi_1) \\ B = -\varphi_1 V/(\varphi_2-\varphi_1) \end{cases}$$

$$\rightarrow \phi = \frac{\varphi - \varphi_1}{\varphi_2 - \varphi_1} V, \quad \vec{E} = -\frac{1}{r}\frac{\partial\phi}{\partial r}\hat{\varphi} = \frac{1}{r}\frac{-V}{\varphi_2-\varphi_1}\hat{\varphi}$$

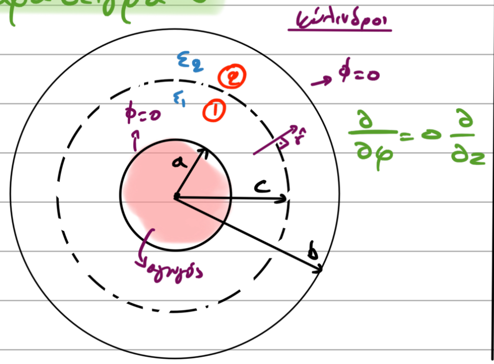
$$\rightarrow \vec{D} = \varepsilon\vec{E} = \frac{1}{r}\frac{-V\varepsilon}{\varphi_2-\varphi_1}\hat{\varphi}$$

$\varphi=\varphi_1$: $\hat{\varphi}\cdot(\vec{D}_2 - \cancel{\vec{D}_1}^{\,0\ (E=0)}) = \sigma_1 \Rightarrow \sigma_1 = \dfrac{-\varepsilon V}{r(\varphi_2-\varphi_1)}$

$\varphi=\varphi_2$: $\hat{\varphi}\cdot(\cancel{\vec{D}_2}^{\,0\ (E=0)} - \vec{D}_1) = \sigma_2 \Rightarrow \sigma_2 = \dfrac{\varepsilon V}{r(\varphi_2-\varphi_1)}$

Παράδειγμα 6

κύλινδροι



$$\frac{\partial}{\partial\varphi} = 0 = \frac{\partial}{\partial z}$$

$$\rightarrow \nabla^2\phi_{1,2} = 0 \Rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial\phi_{1,2}}{\partial r}\right) = 0 \Rightarrow \phi_{1,2} = A_{1,2}\ln r + B_{1,2}$$

$\rightarrow$ $\underline{r=a}$: $\phi_1 = V$

$\rightarrow$ $\underline{r=b}$: $\phi_2 = 0$

$\rightarrow$ $\underline{r=c}$: $\cdot\ \phi_1 = \phi_2$

$\cdot\ -\varepsilon_2 \left.\frac{\partial\phi_2}{\partial r}\right|_{r=c} + \varepsilon_1 \left.\frac{\partial\phi_1}{\partial r}\right|_{r=c} = 0$

4x4 σύστημα $\Rightarrow$

$$\Rightarrow A_1 = -V_0, \quad B_1 = \left(\ln c + \frac{\varepsilon_1}{\varepsilon_2}\ln\frac{b}{c}\right)V_0$$

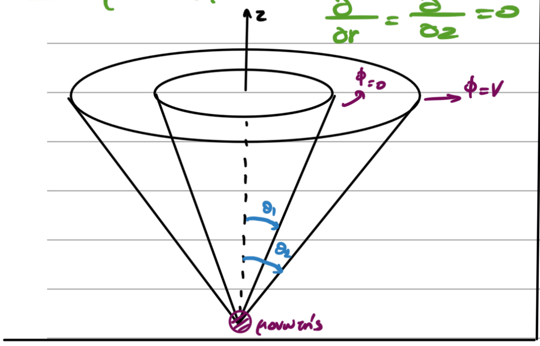
$$A_2 = -\frac{\varepsilon_1}{\varepsilon_2}V_0, \quad B_2 = \frac{\varepsilon_1}{\varepsilon_2}(\ln b)\cdot V_0$$

$$\text{, με: } V_0 = V\left(\ln\frac{c}{a} + \frac{\varepsilon_1}{\varepsilon_2}\ln\frac{b}{c}\right)^{-1}$$

$$\vec E_{1,2} = -\frac{\partial \phi_{1,2}}{\partial r}\hat r \Rightarrow \vec E_1 = \frac{V_0}{r}\hat r \;,\quad \vec E_2 = \frac{\varepsilon_1}{\varepsilon_2}\frac{V_0}{r}\hat r$$

Παράδειγμα 7

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial z} = 0$$



$$\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) = 0 \Rightarrow$$

$r \neq 0$ λόγω μονωτή στο O, $\theta \neq 0$

$$\Rightarrow \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) = 0 \Rightarrow \frac{\partial\phi}{\partial\theta} = \frac{A}{\sin\theta} \Rightarrow \phi = A\int\frac{d\theta}{\sin\theta} =$$

$$= A\int\frac{\frac{1}{2}\,d\theta}{\sin(\theta/2)\cos(\theta/2)} = A\int\frac{\frac{1}{2}\frac{1}{\cos^2\theta/2}\,d\theta}{\tan\theta/2} =$$

$$= A\int\frac{d(\tan\theta/2)}{\tan\theta/2} = A\ln\left(\tan\frac{\theta}{2}\right) + B$$

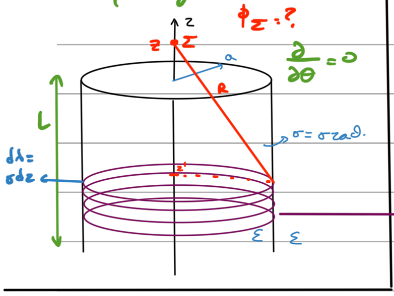
$$\left.\begin{aligned}\theta=\theta_1 &: \phi = 0\\ \theta=\theta_2 &: \phi = V\end{aligned}\right\}\ 2\times2 \text{ σύστημα} \Rightarrow \phi = V\frac{\ln\left(\tan\theta/2 \,/\, \tan\theta_1/2\right)}{\ln\left(\tan\theta_2/2 \,/\, \tan\theta_1/2\right)}$$

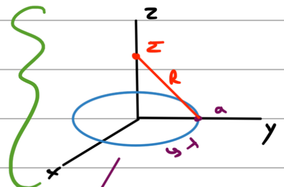
$$\vec E = -\frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat\theta = -\frac{A}{r\sin\theta}\hat\theta$$

Παράδειγμα 8

$\phi_z : ?$

$$\frac{\partial}{\partial\theta} = 0$$





$$\phi_z = \frac{\lambda a}{2\varepsilon\sqrt{z^2+a^2}}$$

$$R = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

Θεωρώ "άπειρα" τέτοια δαχτυλίδια:

$$d\phi = \frac{d\lambda\cdot a}{2\varepsilon\sqrt{(z-z')^2+a^2}} \rightarrow \phi = \int_{-L/2}^{L/2}\frac{\sigma a\,dz'}{2\varepsilon\sqrt{(z-z')^2+a^2}} \Rightarrow \left\{\int\frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)+c\right\} \Rightarrow$$

$$\Rightarrow \phi = \frac{\sigma a}{2\varepsilon}\ln\left(\frac{z+\frac{L}{2}+\sqrt{\left(z+\frac{L}{2}\right)^2+a^2}}{z-\frac{L}{2}+\sqrt{\left(z-\frac{L}{2}\right)^2+a^2}}\right)$$

$$\vec E = -\frac{\partial\phi}{\partial z}\hat z = \hat z\frac{\sigma a}{2\varepsilon}\left[\frac{1}{\sqrt{\left(z-\frac{L}{2}\right)^2+a^2}} - \frac{1}{\sqrt{\left(z+\frac{L}{2}\right)^2+a^2}}\right]$$