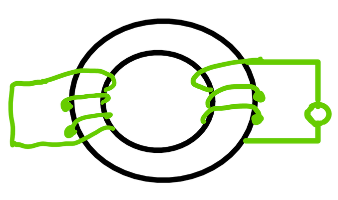
18.03.22

Sunday, 20 March 2022 1:32 AM

Credits: Μάριος Ρόζος

Νόμος επαγωγής του Faraday

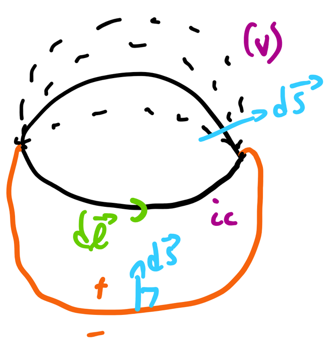
29/08/1831



$$V_\epsilon = -\frac{d\psi_m}{dt} \Leftrightarrow \oint_c \vec{E}\,d\vec{l} = -\frac{d}{dt}\int_s \overbrace{\vec{B}\,d\vec{S}}^{\psi_m} \quad (1)$$

(Νόμος Faraday Maxwell ολοκληρωτική μορφή)

Ηλεκτρεγερτική δύναμη



Νόμος Ampere: $\vec{\nabla}\times\vec{H} = \vec{J}$ (2)

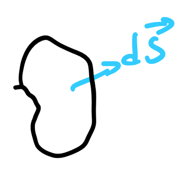
$$\oint_c \vec{H}\,d\vec{l} = \int_s \vec{J}\cdot d\vec{S} \quad (3)$$

$$(1) \xRightarrow{(2)(3)} \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}(x,y,z,t)$$

(Νόμος Faraday-Maxwell σημειακή μορφή)

Γενίκευση του Ν. Ampere - Νόμος Ampere-Maxwell

Νόμος διατήρησης φορτίου: $\oint_s \vec{J}\,d\vec{S} = -\frac{d}{dt}\int_V \rho\, dV = -\frac{dQ_{\epsilon\sigma}}{dt}$



Νόμος διατήρησης φορτίου σε επιφανειακή μορφή: $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}(x,y,z,t)$

(2) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \nabla\vec{J} \Rightarrow \nabla\cdot\vec{J} = 0$

$$\oint_c \vec{H}\,d\vec{l} = \int_s \vec{J}\cdot d\vec{S} + \frac{d}{dt}\int_s \vec{D}\cdot d\vec{S}$$

ρεύμα μετατόπισης

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) \Rightarrow \vec{\nabla}\cdot\vec{J} = -\frac{\partial}{\partial t}\rho$$

Τύπος Lorentz

$$\vec{F}_{em} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\vec{v}\times\vec{B} = q(\vec{E} + \vec{v}\times\vec{B})$$

$q \to \vec{v}$

$$\vec{E} = \frac{\vec{F}_e}{q}\Big|_{J=0?} = \frac{\vec{F}_e}{q} \quad [V/m]$$

$$\vec{F}_m = \vec{F}_{em} - \vec{F}_e = q\vec{v}\times\vec{B}$$

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

1° πείραμα

Έστω $\vec{v} = v_x\hat{x}$ γνωστή

$$F_{m_1} = qv_x\hat{x}\times(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = q(v_x B_y\hat{z} - v_x B_z\hat{y})$$ γνωστό

2° πείραμα

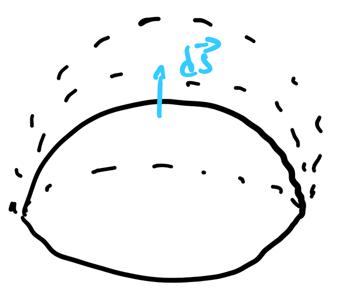
Έστω $\vec{v} = v_z\hat{z}$ γνωστή

$$F_{m_2} = qv_z\hat{z}\times(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = q(v_z B_x\hat{y} - v_z B_y\hat{x})$$ γνωστό

Εξισώσεις Maxwell

Ολοκληρωτικές

(ΗΕΔ) $V_e = -\frac{d}{dt}\psi_m$



$$\oint_c \underbrace{\vec{E}\cdot d\vec{l}}_{V_e\,(\text{ΗΕΔ})} = -\frac{d}{dt}\int_s \underbrace{\vec{B}\cdot d\vec{S}}_{\psi_m}$$ Ν. Faraday-Maxwell

$V_m = I + \frac{d}{dt}\psi_e$

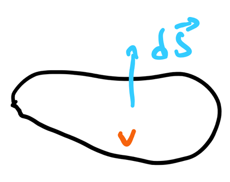
$$\oint_c \underbrace{\vec{H}\cdot d\vec{l}}_{V_m} = \int_s \vec{J}\,d\vec{S} + \frac{d}{dt}\int_s \underbrace{\vec{D}\cdot d\vec{S}}_{\psi_e}$$ Ν. Ampere-Maxwell

V_m: (ΜΕΔ) μαγνητεγερτική δύναμη [A]

James Clerk Maxwell (1831-1879)

A Treatise on Electricity and Magnetism (1873)

$\oint \vec{D}\cdot d\vec{S} = \int_V \rho\, dV$ Ν. Gauss για το ηλ. πεδίο



$\oint_s \vec{B}\cdot d\vec{S} = 0$ Ν. Gauss για το μαγν. πεδίο

Αλληλεξάρτηση των εξ. Maxwell

A) Χρονοσταθερά πεδία $\left(\frac{\partial}{\partial t} = 0\right)$

$$\oint_c \vec{E}\,d\vec{l} = 0$$

$$\oint_s \vec{D}\cdot d\vec{S} = \int_V \rho\, dV = Q_{\epsilon\sigma}$$

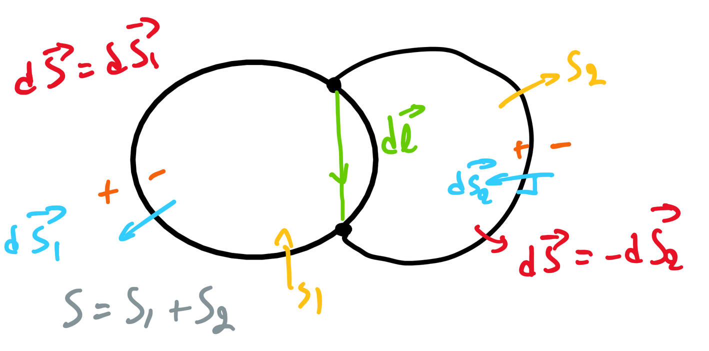
$$\oint_c \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{S}$$

$$\oint_s \vec{B}\cdot d\vec{S} = 0$$

$$\oint_s \vec{J}\cdot d\vec{S} = 0$$

B) Χρονομεταβλητά πεδία $\left(\frac{\partial}{\partial t} \neq 0\right)$

$$\oint_c \vec{E}\,d\vec{l} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{S}$$



$$\left.\begin{aligned}\oint_c \vec{E}\cdot d\vec{l} &= -\frac{d}{dt}\int_{s_1}\vec{B}\cdot d\vec{S}_1\\ \oint_c \vec{E}\cdot d\vec{l} &= -\frac{d}{dt}\int_{s_2}\vec{B}\,d\vec{S}_2\end{aligned}\right\} \Rightarrow \frac{d}{dt}\int_{s_1}\vec{B}\cdot d\vec{S} = \frac{d}{dt}\int_{s_2}\vec{B}\cdot d\vec{S}_2 \Rightarrow \frac{d}{dt}\left[\int_{s_1}\vec{B}\cdot d\vec{S} + \int_{s_2}\vec{B}\cdot d\vec{S}\right] = \frac{d}{dt}\left[\oint_s \vec{B}\cdot d\vec{S}\right] = 0$$

$\oint_s \vec{B}\cdot d\vec{S} \rightarrow$ σταθερά ανεξάρτητη από τον χρόνο

$$\text{to} \Rightarrow \oint_s \vec{B}\cdot d\vec{S} = 0$$

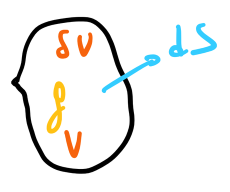
$$\left.\begin{aligned}\oint_c \vec{H}\cdot d\vec{l} &= \int_{s_1}\vec{J}\cdot d\vec{S}_1 + \frac{d}{dt}\int_{s_1}\vec{D}\cdot d\vec{S}_1\\ \oint_c \vec{H}\cdot d\vec{l} &= \int_{s_2}\vec{J}\cdot d\vec{S}_2 + \frac{d}{dt}\int_{s_2}\vec{D}\,d\vec{S}_2\end{aligned}\right\} \Rightarrow \int_{s_1}\vec{J}\,d\vec{S}_1 + \frac{d}{dt}\int_{s_1}\vec{D}\cdot d\vec{S}_1 = \int_{s_2}\vec{J}\cdot d\vec{S}_2 + \frac{d}{dt}\int_{s_2}\vec{D}\cdot d\vec{S}_2$$

$$\Rightarrow \oint_s \vec{J}\cdot d\vec{S} + \frac{d}{dt}\oint_s \vec{D}\cdot d\vec{S} = 0 \Rightarrow -\frac{d}{dt}\int_V \rho\, dV + \frac{d}{dt}\int_V \vec{\nabla}\cdot\vec{D}\cdot dV = 0$$

$$\Rightarrow \frac{d}{dt}\left[\int_V(\vec{\nabla}\cdot\vec{D} - \rho)\,dV\right] = 0$$

Εξισώσεις Maxwell σε διαφορική μορφή

$$\oint_s \vec{D}\cdot d\vec{S} = \int_V \rho\, dV$$



$$\oint_s \vec{D}\cdot d\vec{S} = \rho(x,y,z)\,\delta V$$

$$\lim_{\delta V\to 0}\frac{\oint_s \vec{D}\cdot d\vec{S}}{\delta V} = \boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

Όμοια προκύπτει από $\oint_s \vec{B}\cdot d\vec{S} = 0$ ότι $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$

Όμοια, $\oint_s \vec{J}\cdot d\vec{S} = -\frac{d}{dt}\rho\, dV \Rightarrow \boxed{\vec{\nabla}\,\vec{J} = -\frac{\partial\rho}{\partial t}}$