22.03.22

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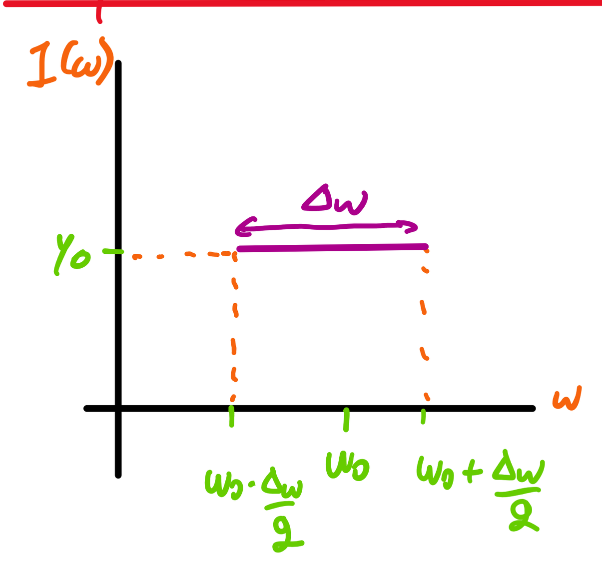
Κυματο-Ομάδες και Θ. Εύρους Ζώνης

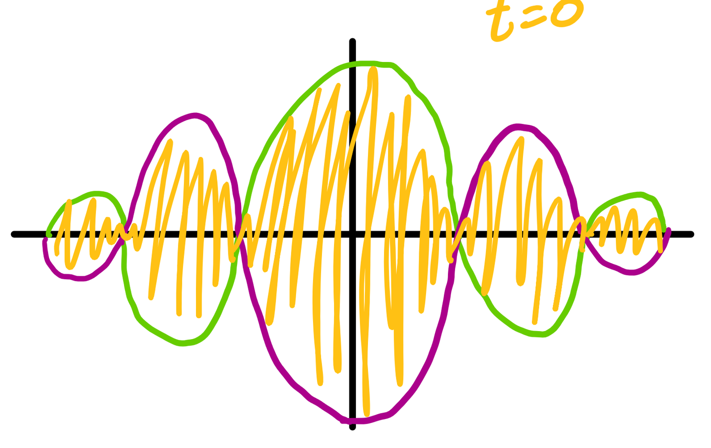


$$y_0 = \int dy = \dots = (y_0 \Delta\omega)\, e^{i(kx - \omega_0 t)} \frac{\sin \mathcal{X}}{\mathcal{X}}$$

$$\mathcal{X} = \left[\left(\frac{dk}{d\omega}\right)_{\omega_0} x - t\right] \frac{\Delta\omega}{2}$$

$\omega_0 \gg \Delta\omega$

$t=0$



Χωρο-Χρονικές Συντεταγμένες Μεγίστου

$$\mathcal{X} = 0 \Rightarrow \left(\frac{dk}{d\omega}\right)_{\omega_0} x - t = 0 \Rightarrow \frac{dx}{dt} \triangleq v_{gr} = \left(\frac{d\omega}{dk}\right)_{k_0}$$

$$V_{ph} = \frac{\omega}{k} = \frac{\omega(k)}{k} = \frac{\omega}{k(\omega)}$$

Τυπικό μέτρο της 'έκτασης' της κυματοομάδας

Η απόσταση "μέγιστου" - "πρώτου μηδενισμού".

$$\Delta \mathcal{X} = \pi \Rightarrow \left[\left(\frac{dk}{d\omega}\right)_{\omega_0} \Delta\omega\, x - t\Delta\omega\right] = 2\pi \Rightarrow \Delta\left[(\Delta k)x - t\Delta\omega\right] = 2\pi$$

(α) $x = x_0$ = σταθ. ⇒ Χρονική έκταση

$$\Rightarrow \Delta[(\Delta k)x_0 - t\Delta\omega] = 2\pi \Rightarrow -(\Delta t)(\Delta\omega) = 2\pi$$

1° Θεωρ. Εύρους Ζώνης: $\boxed{|\Delta t| \cdot |\Delta\omega| = 2\pi}$

$$\left\{ \Delta E = \hbar \Delta\omega = \frac{h}{2\pi}\omega = \frac{h}{2\pi} 2\pi f = hf \right\}$$

$$\left\{ |\Delta t| \frac{|\Delta E|}{\hbar} = 2\pi \Rightarrow |\Delta t| \cdot |\Delta E| = 2\pi \cdot \frac{h}{2\pi} = h \right\}$$

→ αρχή απροσδιοριστίας του Heisenberg

(β) $t = t_0$ = σταθ.

$$\Delta[(\Delta k)x - t_0\Delta\omega] = 2\pi \Rightarrow \boxed{(\Delta k)(\Delta x) = 2\pi}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \boxed{\Delta k = -\frac{2\pi}{\lambda^2}\Delta\lambda}$$

$$-\frac{2\pi}{\lambda^2}(\Delta\lambda)(\Delta x) = 2\pi \Rightarrow |(\Delta\lambda)| \cdot |(\Delta x)| = \lambda^2$$

$\lambda = \lambda_0$ (μόνο) $\Rightarrow \Delta\lambda \to 0 \Rightarrow |\Delta x| \to \infty$

* De Broglie $p \to \lambda = \frac{h}{p}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda} p$$

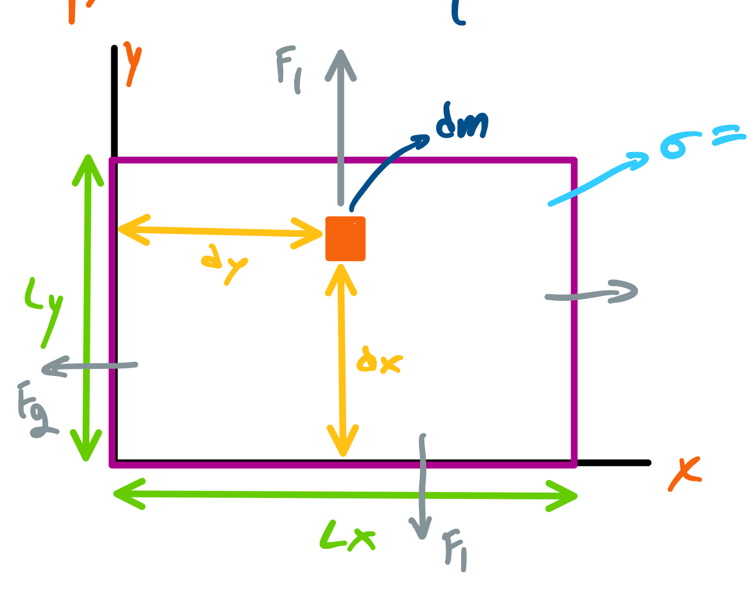
$$(\Delta k)(\Delta x) = 2\pi \Rightarrow \frac{(\Delta p)}{\hbar}(\Delta x) = 2\pi \Rightarrow (\Delta p)(\Delta x) = h$$

$\boxed{p = \frac{h}{2\pi}k = \hbar k}$ → 2η αρχή Heisenberg

Ελαστικά κύματα σε 2-δ

Το Σύστημα: Μεμβράνη "μηδενικού πάχους" με επιφανειακή πυκνότητα μάζας $\sigma = \frac{dm}{dS} = \frac{dm}{dx\,dy}$ = σταθ.

(β) Το σύστημα τείνεται με γνωστή δύναμη ανά μονάδα μήκους.



$\sigma = \frac{dm}{dx\,dy}$ = σταθ.

ομοιογενές

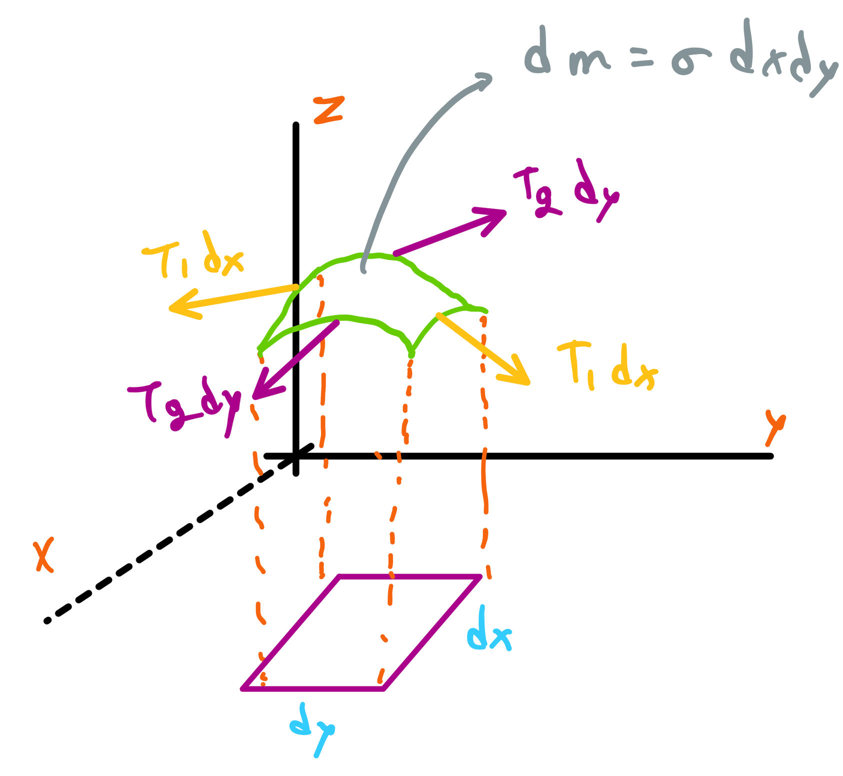
$\frac{F_1}{L_x} = T_1$, $\frac{F_2}{L_y} = T_2$

ανισότροπο

$Z = 0$: κατάσταση ισορροπίας

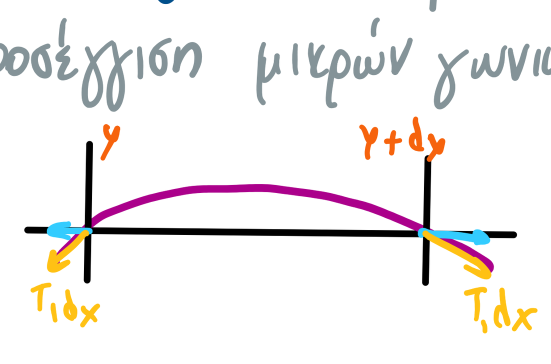
$Z = Z(x, y, t)$

$dm = \sigma\, dx\, dy$



$$(dm)\frac{\partial^2 z}{\partial t^2} = F_{o\lambda, z}$$

Προσέγγιση μικρών γωνιών ⇓



$$F(\perp x)_{,z} = (T_1\, dx)\left[\left(\frac{\partial z}{\partial y}\right)_{y+dy} - \left(\frac{\partial z}{\partial y}\right)_y\right] \Rightarrow F(\perp x)_{,z} = (T_1\, dx)\frac{\partial^2 z}{\partial y^2}\, dy$$

$$\left(\frac{\partial z}{\partial y}\right) = g_1(x,y) \Rightarrow \left[\left(\frac{\partial z}{\partial y}\right)_{y+dy} - \left(\frac{\partial z}{\partial y}\right)_y\right] = \underbrace{\left[g_1(x, y+dy) - g_1(x,y)\right]}_{dg_1(y)} = \frac{\partial g_1}{\partial y}\, dy =$$

$$= \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) dy = \frac{\partial^2 z}{\partial y^2}\, dy$$

Ομοίως για $F(\perp y)_{,z}$:

$$F(\perp y)_{,z} = T_2 \frac{\partial^2 z}{\partial y^2}\, dy\, dx$$

Τελικά:

$$\underbrace{(\sigma\, dx\, dy)}_{dm} \frac{\partial^2 z}{\partial t^2} = \left(T_1 \frac{\partial^2 z}{\partial y^2} + T_2 \frac{\partial^2 z}{\partial x^2}\right)(dx\, dy) \Rightarrow$$

$$\boxed{\sigma \frac{\partial^2 z}{\partial t^2} = T_2 \frac{\partial^2 z}{\partial x^2} + T_1 \frac{\partial^2 z}{\partial y^2}}$$

Για ομοιογενές (σ = σταθ.) και ισότροπο μέσο

$$\left(T_1 = T_2 = T = \frac{\text{Δύναμη}}{\text{Μήκος}} = \text{σταθ.}\right) \Rightarrow \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

$c = \sqrt{T/\sigma}$ = [ταχύτητα] [διάδοσης]