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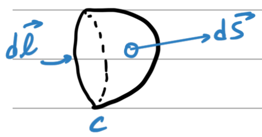
6 σειρές ασκήσεων

Εξισώσεις Maxwell

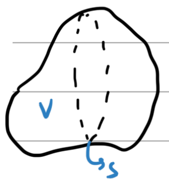


$\rightarrow V_e = -\frac{d}{dt}\Psi_m \Rightarrow \oint_c \vec{E}\,d\vec{l} = -\frac{d}{dt}\int_s \vec{B}\,d\vec{S} \longrightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$[V/m]$ $[Wb/m^2]$ ή $[T]$

$\rightarrow V_m = I + I_D \Rightarrow \oint_c \vec{H}\,d\vec{l} = \int_s \vec{J}\,d\vec{S} + \frac{d}{dt}\int_s \vec{D}\cdot d\vec{S} \longrightarrow \vec{\nabla}\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$

$[A/m]$ $[A/m^2]$ $[C/m^2]$



$\rightarrow \oint_s \vec{D}\,d\vec{S} = \int_V \rho\,dV \Rightarrow \Psi_e = Q_{total} \longrightarrow \vec{\nabla}\vec{D} = \rho$

$\rightarrow \oint_s \vec{B}\,d\vec{S} = 0 \longrightarrow \vec{\nabla}\vec{B} = 0$

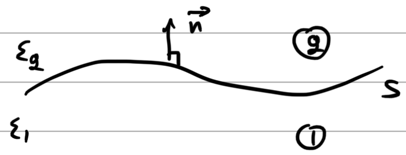
Κεφάλαιο 1: Ηλεκτροστατική

1.1. Εισαγωγή

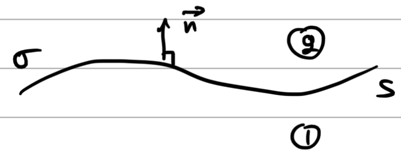
$\frac{d}{dt} = 0$ $\oint_l \vec{E}\,d\vec{l} = 0$ $\oint_l \vec{D}\,d\vec{l} = \int_V \rho\,dV$

$\vec{\nabla}\times\vec{E} = 0$ $\vec{\nabla}\vec{D} = \rho$ $\vec{D} = \varepsilon\vec{E} = (\varepsilon_r\cdot\varepsilon_0)\vec{E}$

Οριακές συνθήκες



$\rightarrow \vec{n}\times(\vec{E}_2 - \vec{E}_1) = 0 \Rightarrow E_{t_2} = E_{t_1}$
↳ tangential

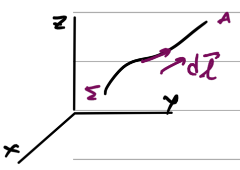


$\rightarrow \vec{n}\cdot(\vec{D}_2 - \vec{D}_1) = \sigma \Rightarrow D_{n_2} = D_{n_1}$

1.2 Ηλεκτροστατικό δυναμικό

$\rightarrow \vec{\nabla}\times\vec{\nabla}\Phi = 0$

$\rightarrow \nabla\times\vec{E} = 0 \Leftrightarrow \vec{E} = -\nabla\vec{\Phi}$



$\rightarrow \Phi_\Sigma - \Phi_A = V_{\Sigma A}$
$[V]$

$\rightarrow \int_\Sigma^A \vec{E}\,d\vec{l} = -\int \nabla\Phi\,d\vec{l} = -\int_\Sigma^A d\Phi = \Phi_\Sigma - \Phi_A = V_{\Sigma A}$

Παράγωγος κατά κατεύθυνση του dl : $\dfrac{d\Phi}{dl} = \hat{n}\cdot\nabla\Phi = \dfrac{d\vec{l}}{|d\vec{l}|}\nabla\Phi \Rightarrow$

$\Rightarrow d\Phi = d\vec{l}\,\nabla\Phi$

$\vec{E}$ συντηρητικό, αστρόβιλο

$\nabla\times\vec{E} = 0$

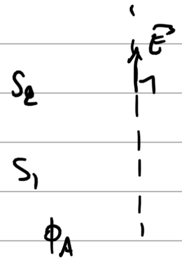
$\vec{E} = -\vec{\nabla}\Phi$

$\oint_C \vec{E}\,d\vec{l} = 0$

Συνήθως $\Phi_A = 0$

Θέση του σημείου A

$\Phi_\Sigma = \int_\Sigma^{\alpha\nu\alpha\varphi.} \vec{E}\,d\vec{l}$



$\Phi_\Sigma = \Phi_2 = \sigma\tau\alpha\vartheta. < \Phi_1$

$\Phi_\Sigma = \Phi_1 = \sigma\tau\alpha\vartheta.$

$\nabla\vec{D} = \rho$, $\vec{D} = \varepsilon\vec{E}$, $\nabla(\varepsilon\vec{E}) = \varepsilon\rho$, $\varepsilon = \varepsilon(\vec{r})$

Για $\varepsilon = \sigma\tau\alpha\vartheta.$ (ομογενές υλικό)

$\nabla\vec{E} = \rho/\varepsilon \Rightarrow$ $\boxed{\nabla^2\Phi = -\rho/\varepsilon \quad \text{Εξ. Poisson}}$

↳ όταν $\rho = 0$: $\boxed{\nabla^2\Phi = 0 \quad \text{Εξ. Laplace}}$

Καρτεσιανό: $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ $\quad \nabla = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right)$

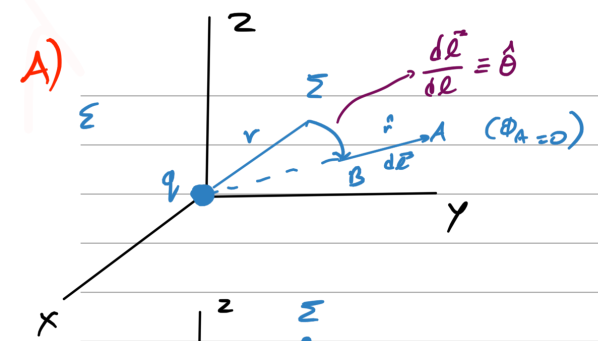
Κυλινδρικό: $\nabla^2 = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \varphi^2} + \dfrac{\partial^2}{\partial z^2}$ $\quad \nabla = \dfrac{\partial}{\partial r}\hat{r} + \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\hat{\varphi} + \dfrac{\partial}{\partial z}\hat{z}$

Σφαιρικό: $\nabla = \dfrac{\partial}{\partial r}\hat{r} + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\hat{\theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \varphi}\hat{\varphi}$

1.3 Υπολογισμός Φ από γνωστά φορτία (q, λ, σ)

$\Phi_\Sigma = \int_\Sigma^{\alpha\nu\alpha\varphi.} \vec{E}\,d\vec{l}$, $\Phi_A = 0$

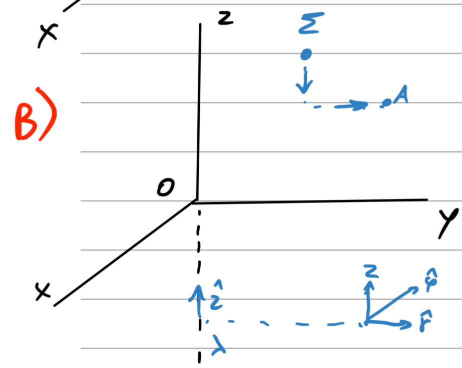
A)



$$\vec{E} = \frac{q}{4\pi\varepsilon r^2}\hat{r}$$

$$\Phi_\Sigma = \int_r^{r_{αν}} \frac{q}{4\pi\varepsilon r^2}\, dr = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{r_{αν}}\right)$$

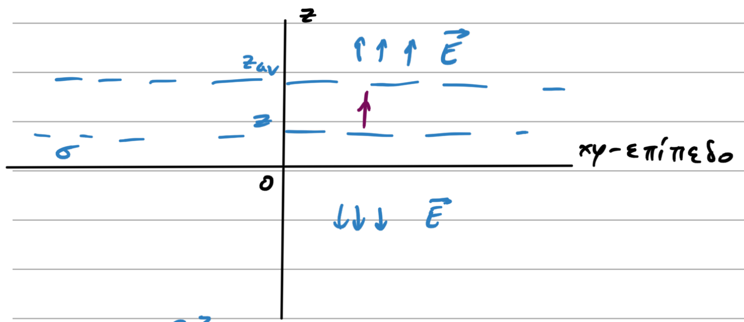
B)



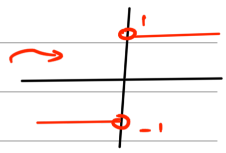
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon r}\hat{r}$$

$$\Phi_\Sigma = \int_r^{r_{αν}} \vec{E}\, d\vec{l} = \int_r^{r_{αν}} \frac{\lambda}{2\pi\varepsilon r}\, dr = -\frac{\lambda}{2\pi\varepsilon}\ln r + \frac{\lambda}{2\pi\varepsilon}\ln r_0$$

Γ)



$$\vec{E} = \frac{\sigma}{2\varepsilon}\, sgn(z)\,\hat{z}$$



$$sgn(z) = \begin{cases} +1, & z > 0 \\ -1, & z < 0 \end{cases} = \frac{z}{|z|}$$

$$\Phi_\Sigma = \int_z^{z_{αν}} \frac{\sigma}{2\varepsilon}\frac{z}{|z|}\, dz = \frac{\sigma}{2\varepsilon}\left(-|z| + |z_{αν}|\right)$$