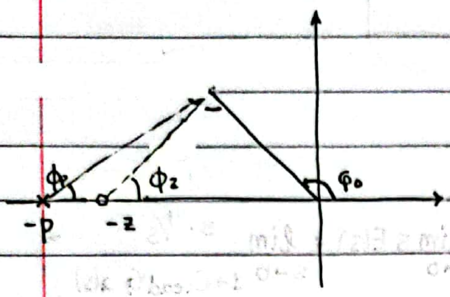
Δευτέρα, 13/03/2023

Lead Controllers

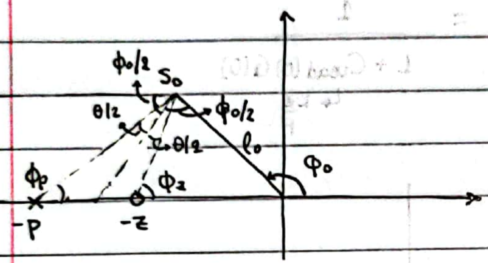
Μέθοδος διχοτόμου



$$C(s) = K\,\frac{s+z}{s+p}$$

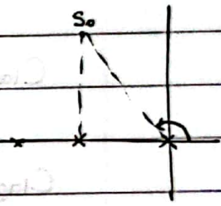
$$\arg(s+z) - \arg(s_0+p) + \arg G(s_0) = (2k+1)\cdot 180^\circ$$

$$\phi_z - \phi_p = \theta$$



πχ. $G(s) = \dfrac{1}{s(s+1)}$, $s_0 = -1 \pm j$, $\arg G(s_0) = -90^\circ - 135^\circ = -225^\circ$

$$\phi_z - \phi_p + (-225^\circ) = -180^\circ \Rightarrow \theta = 45^\circ$$



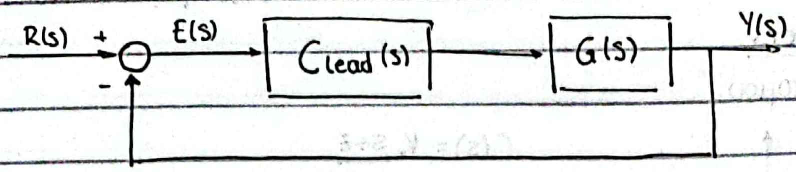
$$\left.\begin{aligned} \overset{\triangle}{PS_0O}:&\quad \frac{\sin\phi_p}{l_0} = \frac{\sin\left(\frac{\phi_0+\theta}{2}\right)}{|p|} \\ \overset{\triangle}{ZS_0O}:&\quad \frac{\sin\left(\frac{\phi_0-\theta}{2}\right)}{|z|} = \frac{\sin\phi_z}{l_0} \end{aligned}\right\} \quad \frac{\sin\phi_p}{\sin\phi_z} = \frac{\sin\left(\frac{\phi_0+\theta}{2}\right)}{\sin\left(\frac{\phi_0-\theta}{2}\right)}\cdot\frac{z}{p} \Rightarrow$$

$$\Rightarrow \frac{z}{p} = \frac{\sin\phi_p\cdot\sin\left(\frac{\phi_0-\theta}{2}\right)}{\sin\phi_z\cdot\sin\left(\frac{\phi_0+\theta}{2}\right)} = \frac{\cos\left(\phi_p - \frac{\phi_0-\theta}{2}\right) - \cos\left(\phi_p + \frac{\phi_0-\theta}{2}\right)}{\cos\left(\phi_z - \frac{\phi_0+\theta}{2}\right) - \cos\left(\phi_z + \frac{\phi_0+\theta}{2}\right)}$$

$$\left[\sin(A)\cdot\sin(B) = \frac{\cos(A-B) - \cos(A+B)}{2}\right]$$

$$\phi_z = \frac{\phi_0+\theta}{2},\ \phi_p = \frac{\phi_0-\theta}{2} \ : \Rightarrow \max\frac{z}{p} = \frac{1-\cos(\phi_0-\theta)}{1+\cos(\phi_0+\theta)}$$

$$z = \begin{cases} \dfrac{Im(s_0)}{\tan\left(\frac{\phi_0+\theta}{2}\right)} - Re(s_0), & αν\ \phi_0+\theta<\pi \\[2ex] -\dfrac{Im(s_0)}{\tan\left(\pi-\frac{\phi_0+\theta}{2}\right)} - Re(s_0), & αν\ \phi_0+\theta>\pi \\[2ex] -Re(s_0), & αν\ \phi_0+\theta=\pi \end{cases}$$



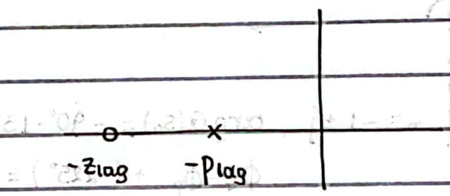
$$\frac{E(s)}{R(s)} = \frac{1}{1 + C_{lead}(s)G(s)}$$

Βηματική είσοδος : $R(s) = \frac{1}{s}$, $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s \cdot 1/s}{1 + C_{lead}(s)G(s)} =$

$$= \frac{1}{1 + \underbrace{C_{lead}(0)}_{\hookrightarrow K\frac{z}{p}}G(0)}$$

Lag Controllers

$$C_{lag}(s) = \frac{s + z_{lag}}{s + p_{lag}}$$



$$C_{lag}(0) = \frac{z_{lag}}{p_{lag}} > 1$$

π.χ. $G(s) = \frac{1}{s(s+1)}$, $s_0 = -2 \pm j2$, $C_{lead}(s) = 20\frac{(s+2)}{s+8}$

$\downarrow$

1ης τάξης

$K_v = \lim_{s\to 0} s\,C_{lead}(s)\,G(s) = \frac{20 \cdot 2}{8} = 5$, $e_{ss} = \frac{1}{K_v} = 0.2$

Για να μικρύνω το e_{ss} : $e_{ss} \leq 0.01 \Rightarrow K_v' \geq 100$

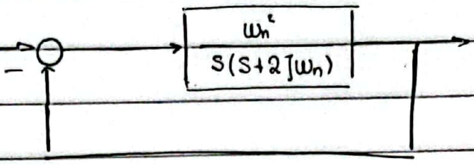
$$K_v' = K_v \cdot \frac{z_{lag}}{p_{lag}} \Rightarrow \frac{z_{lag}}{p_{lag}} = \frac{100}{5} = 20$$

Scanned with CamScanner

$$C_{lead}(s) = K\frac{(s+z)}{s+p} = \frac{Kz}{p}\frac{\left(\frac{s}{z}+1\right)}{\frac{s}{p}+1} = K_{lead}\frac{(a_{lead}T_{lead}s+1)}{T_{lead}s+1}$$

$$T_{lead} = 1/p,\quad a_{lead} = \frac{1}{zT_{lead}} = \frac{P}{Z} > 1$$

$$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$



$$\frac{\frac{\omega_n^2}{s(s+2\zeta\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\zeta\omega_n)}}$$

$$H(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$|H(j\omega)|^2 = 1 \Rightarrow \frac{\omega_n^4}{\omega^2(\omega^2+4\zeta^2\omega_n^2)} = 1 \Rightarrow \omega_{gc}^4 + 4\zeta^2\omega_n^2\omega_{gc}^2 - \omega_n^4 = 0$$

$$\Delta = 16\zeta^4\omega_n^4 + 4\omega_n^4 = 4\omega_n^4(4\zeta^4+1)$$

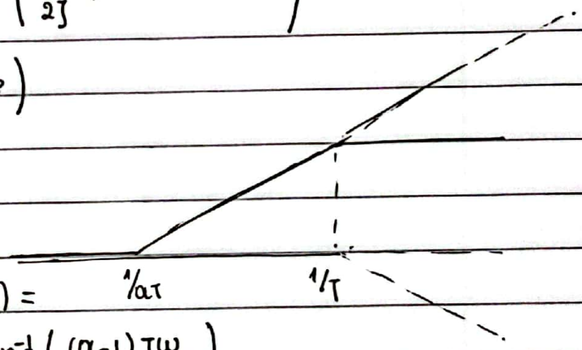
$$\omega_{gc}^2 = \frac{-4\zeta^2\omega_n^2 + 2\omega_n^2\sqrt{1+4\zeta^4}}{2} \Rightarrow \boxed{\omega_{gc} = \omega_n\sqrt{\sqrt{1+4\zeta^4}-2\zeta^2}}$$

$$\arg H(j\omega_{gc}) = -90^\circ - \tan^{-1}\left(\frac{\omega_{gc}}{2\zeta\omega_n}\right)$$

$$\phi_{\pi\varepsilon\rho} = 180^\circ + \arg H(j\omega_{gc}) = 90^\circ - \tan^{-1}\left(\frac{1}{2\zeta}\sqrt{\sqrt{1+4\zeta^4}-2\zeta^2}\right)$$

($M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, το ζ επηρεάζει το M_p)

$$C_{lead}(s) = K_{lead}\frac{a_{lead}T_{lead}+1}{T_{lead}+1}$$



$$\arg C_{lead}(j\omega) = \tan^{-1}(aT\omega) - \tan^{-1}(T\omega) =$$

$$\left(\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right) = \tan^{-1}\left(\frac{(a-1)T\omega}{1+aT^2\omega^2}\right)$$

$$f(x) = \frac{x}{1+ax^2},\quad f'(x) = \frac{1+ax^2-2ax^2}{(1+ax^2)^2} = \frac{1-ax^2}{(1+ax^2)^2},\quad \frac{1}{\sqrt{a}} = \operatorname{argmax} f(x)$$

$$\omega_m T = \frac{1}{\sqrt{a}} \Rightarrow \omega_m = \frac{1}{T\sqrt{a}}$$

$$\arg C_{lead}(j\omega) = \tan^{-1}\left(\frac{\frac{a-1}{\sqrt{a}}}{1+a\cdot\frac{1}{a}}\right) = \tan^{-1}\left(\frac{a-1}{2\sqrt{a}}\right)$$

$$\frac{1}{\cos^2\phi} = 1+\tan^2\phi \Rightarrow \cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}},\quad \boxed{\sin\phi_m = \frac{a-1}{a+1}} \Rightarrow a\sin\phi_m + \sin\phi_m = a-1$$

(Η μέγιστη συνεισφορά φάσης στην $\omega = \frac{1}{T\sqrt{a}}$ θα είναι ϕ_m) $\Rightarrow \boxed{a = \frac{1+\sin\phi_m}{1-\sin\phi_m}}$

$$\left|C_{lead}(j\omega_m)\right| = \sqrt{\frac{1+a^2\tau^2\omega_m^2}{1+\tau^2\omega_m^2}} = \sqrt{\frac{1+a^2\frac{1}{a}}{1+\frac{1}{a}}} = \sqrt{\frac{1+a}{\frac{1+a}{a}}} = \sqrt{a}$$