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Ηλεκτρομαγνητικό Κύμα στο Κενό (και στα Υλικά)

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad (1) \qquad \vec{\nabla}\cdot\vec{B} = 0 \quad (2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad (3) \qquad \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) \quad (4)$$

Ολοκληρωτικές μορφές Εξ. Maxwell

Θ. Gauss:

$$\oint_{S_{\kappa\lambda}} \vec{E}\cdot d\vec{S} = \frac{Q\ (\text{μέσα στη } S_{\kappa\lambda})}{\varepsilon_0}$$

$$\oint_{S_{\kappa\lambda}} \vec{B}\cdot d\vec{S} = 0$$

Θ. Stokes:

$$\oint_{C_{\kappa\lambda}} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{S}$$

$\forall S$ με σύνορο $C_{\kappa\lambda}$

$$\oint_{C_{\kappa\lambda}} \vec{B}\cdot d\vec{l} = \mu_0\left[\int_{S(C_{\kappa\lambda})} \vec{J}\cdot d\vec{S} + \varepsilon\frac{\partial}{\partial t}\int_{S(C_{\kappa\lambda})} \vec{E}\cdot d\vec{S}\right]$$

Στο απόλυτο κενό ($\rho=0, \vec{J}=0$)

(3): $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$,

(4): $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

$$\frac{\partial}{\partial t}(3) \Rightarrow -\frac{\partial^2 B}{\partial t^2} = \vec{\nabla}\times\left(\frac{\partial\vec{E}}{\partial t}\right) \overset{(4)}{=} \vec{\nabla}\times\left(\frac{1}{\mu_0\varepsilon_0}\vec{\nabla}\times\vec{B}\right) \quad (5\alpha)$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\vec{\nabla}\vec{B}) - \nabla^2\vec{B} \quad (5\beta)$$

$$(5\alpha),(5\beta) \Rightarrow -\frac{\partial^2\vec{B}}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}(-\nabla^2\vec{B}) \Rightarrow \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = \nabla^2\vec{B}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \rightarrow \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \nabla^2 u, \quad \left\{ u \begin{cases} B_x(x,y,z,t) \\ B_y(x,y,z,t) \\ B_z(x,y,z,t) \end{cases} \right\}$$

$$\frac{\partial}{\partial t}(4) \Rightarrow \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \nabla^2 u, \quad \left\{ u \begin{cases} E_x(x,y,z,t) \\ E_y(x,y,z,t) \\ E_z(x,y,z,t) \end{cases} \right\}$$

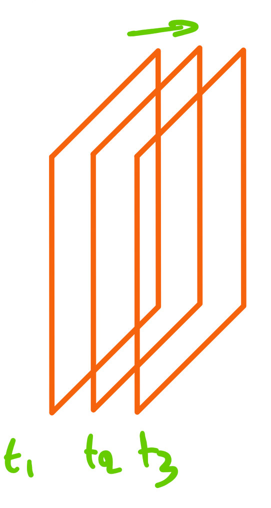
$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

- Έστω επίπεδο Ηλεκτρικό κύμα

$$\vec{E}(\vec{r},t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

φάση $(t=t_0)=$ σταθ. $\Rightarrow \boxed{k_x x + k_y y + k_z z = \text{σταθ.} - \omega t_0 = C}$

Μέτωπο κύματος: επίπεδο $\perp \vec{k}$



το επίπεδο κύμα στο κενό

$$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \vec{\nabla}\cdot\left(E_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right) = 0$$

$$= \vec{\nabla}\cdot\left[(\hat{x}E_{0x} + \hat{y}E_{0y} + \hat{z}E_{0z})\, e^{i(k_x x + k_y y + k_z z - \omega t)}\right]$$

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} =$$

$$= \frac{\partial}{\partial x}E_{0x}e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \frac{\partial}{\partial y}E_{0y}e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \frac{\partial}{\partial z}E_{0z}e^{i(\vec{k}\cdot\vec{r}-\omega t)} =$$

$$= i k_x E_{0x}e^{i(\vec{k}\cdot\vec{r}-\omega t)} + i k_y E_{0y}e^{i(\vec{k}\cdot\vec{r}-\omega t)} + i k_z E_{0z}e^{i(\vec{k}\cdot\vec{r}-\omega t)} \Rightarrow$$

$$\Rightarrow i(k_x E_{0x} + k_y E_{0y} + k_z E_{0z})\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} = 0 \Rightarrow \vec{k}\cdot\vec{E}_0 = 0 \Rightarrow \vec{E}_0 \perp \vec{k}$$

Αφού στο κενό: $\vec{E}_0 \perp \vec{k}$, ας ορίσουμε $\vec{k} = (0,0,k)$.
και $\vec{E}_0 = (E_0, 0, 0) \Rightarrow \vec{E} = \hat{x}E_0\, e^{i(kz-\omega t)}$

Πραγματική αναπαράσταση: $\vec{E} = \hat{x}E_0\cos(kz-\omega t)$

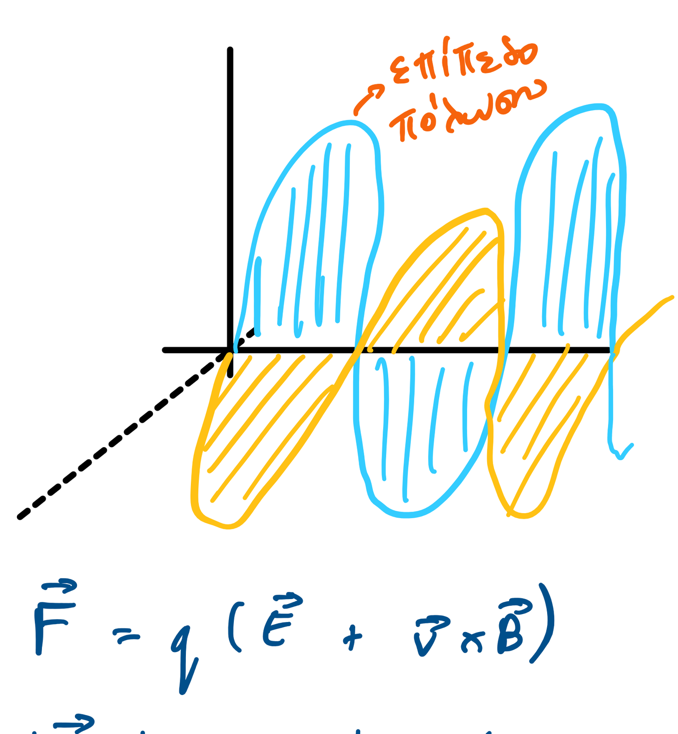
$$(3) \Rightarrow \frac{\partial\vec{B}}{\partial t} = -\vec{\nabla}\times\vec{E} = -\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0\cos(kz-\omega t) & 0 & 0 \end{vmatrix} \Rightarrow$$

$$\Rightarrow \frac{\partial\vec{B}}{\partial t} = -\left[-\hat{y}\left(\frac{\partial 0}{\partial x} - \frac{\partial}{\partial z}E_0\cos(kz-\omega t)\right)\right] \Rightarrow$$

$$\Rightarrow \frac{\partial\vec{B}}{\partial t} = \hat{y}(+kE_0\sin(kz-\omega t)) \Rightarrow \vec{B} = \hat{y}B \Rightarrow$$

$$\Rightarrow \underbrace{\left(\frac{\partial B}{\partial t}\right)dt}_{dB} = kE_0\sin(kz-\omega t)\,dt \Rightarrow \int dB = \int kE_0\sin(kz-\omega t)\,dt$$

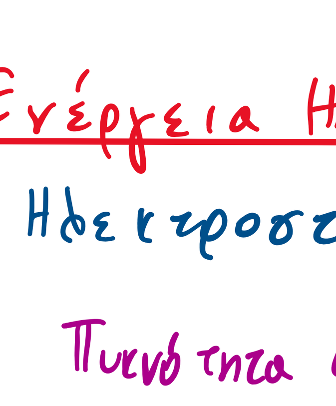
$$\Rightarrow \boxed{\vec{B} = \hat{y}\frac{E_0}{c}\cos(kz-\omega t)} + \cancel{\vec{B}_0} = 0 \qquad c = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$



Στο κενό ($\rho=0, \vec{J}=0$) $\Rightarrow$
$\Rightarrow$ το επίπεδο Η/Μ κύμα: εγκάρσιο $\Rightarrow$
$\Rightarrow \vec{E}$: επίπεδο

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

$$|\vec{F}_{\eta\lambda}| = q|\vec{E}|, \quad |F_{\mu\alpha\gamma}|_{max} = qvB = qE\left(\frac{v}{c}\right) \Rightarrow \frac{F_{\eta\lambda}}{F_{\mu\alpha\gamma}} = \frac{c}{v}$$



$E_{\varepsilon\xi} = E_{\varepsilon\iota\sigma}\cos\theta$

$I \sim E^2 \Rightarrow$

Ν. Malus: $I_{\delta\iota\varepsilon\xi} = I_{\varepsilon\iota\sigma}\cos^2\theta$

Ενέργεια Η/Μ κύματος

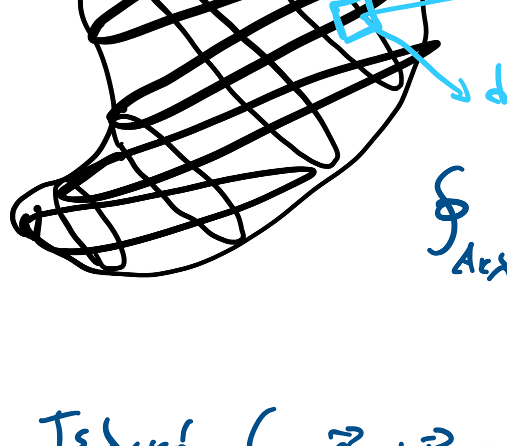
Ηλεκτροστατική και Μαγνητοστατική Ενέργεια

Πυκνότητα Ενέργειας Ηλ. πεδίου: $\rho_{W_E} = \dfrac{\varepsilon_0}{2}|\vec{E}|^2 = \dfrac{dW}{d^3r}$

Πυκνότητα Ενέργειας Μαγν. πεδίου: $\rho_{W_B} = \dfrac{1}{2\mu_0}|B|^2 = \dfrac{dW}{d^3r}$

$B_0 = \dfrac{E_0}{c}$ $\qquad \vec{E}\times\vec{B} \parallel \vec{k}$

$E_0 = B_0 c$ $\qquad \vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$



$\oint_{A_{\kappa\lambda}} \vec{S}\cdot d\vec{A}$ στοιχειώδης επιφάνεια $\qquad$ A = area

$$\oint_{A_{\kappa\lambda}}\left(\frac{1}{\mu_0}\vec{E}\times\vec{B}\right)d\vec{S} \overset{\text{Θ. Gauss}}{=} \int_{\substack{\text{χώρο} \\ \text{με σύνορο } A_{\kappa\lambda}}} \nabla\left(\frac{1}{\mu_0}\vec{E}\times\vec{B}\right)d^3r$$

Τελικά

$$\oint_{A_{\kappa\lambda}} \vec{S}\cdot d\vec{A} = -\frac{\partial}{\partial t}\left[\int_{\substack{\text{όγκος με} \\ \text{σύνορο } A_{\kappa\lambda}}} \rho_{W_E}\, d^3r + \int_{\substack{\text{όγκος με} \\ \text{σύνορο } A_{\kappa\lambda}}} \rho_{W_B}\, d^3r\right]$$