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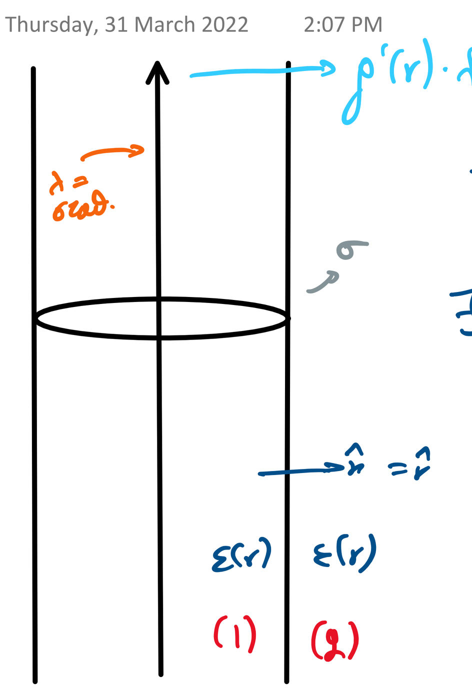


$$\frac{\partial}{\partial\varphi} = 0 = \frac{\partial}{\partial z} \qquad \vec{E}, \vec{D} = ;$$

$\exists$ μόνο $\vec{E}(r), \vec{D}(r)$

B) λύση με σημειακές σχέσεις

$$\left.\begin{array}{l}\vec{\nabla}\times\vec{E}=0\\ \nabla\cdot\vec{D}=\rho(r)\end{array}\right\} \Rightarrow \begin{cases} \frac{1}{r}\cancel{\frac{\partial E_z}{\partial\varphi}}^{0} - \cancel{\frac{\partial E_\varphi}{\partial z}}^{0} = 0 \Rightarrow 0=0 \\ \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = 0 \Rightarrow \frac{dE_z}{dr} = 0 \Rightarrow \begin{cases} E_{z_1} = c_1 & (1\alpha) \\ E_{z_2} = c_2 & (1\beta)\end{cases} \\ \frac{1}{r}\frac{\partial (rE_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial\varphi} = 0 \Rightarrow \frac{d}{dr}(rE_\varphi) = 0 \Rightarrow \begin{cases} E_{\varphi_1} = \frac{d_1}{r} & (2\alpha) \\ E_{\varphi_2} = \frac{d_2}{r} & (2\beta)\end{cases}\end{cases}$$

Οριακές συνθήκες

Για $r=a$: $\hat{r}\times(\vec{E}_2-\vec{E}_1)=0 \Rightarrow E_{\varphi_1} = E_{\varphi_2}$ (3α)
$E_{z_1} = E_{z_2}$ (3β)

Για $r=a$: $\hat{r}\times(\vec{D}_2-\vec{D}_1)=\sigma \Rightarrow D_{r_2} - D_{r_1} = \sigma$ (4)

$$\nabla\cdot\vec{D} = \frac{1}{r}\frac{d}{dr}(rD_r) + \frac{1}{r}\cancel{\frac{d}{d\varphi}D_\varphi}^{0} + \cancel{\frac{dD_z}{dz}}^{0} = \rho(r)$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}(rD_r) = \rho(r) \quad (5)$$

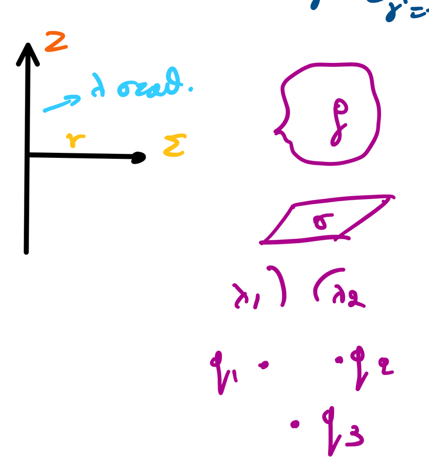
Το σύστημα των (1), (2), (3α), (3β) είναι ομογενές
(δεν συνδέεται με τις πηγές λ, σ, ρ)

Προφανής λύση η $E_{\varphi_{1,2}}=0$, $E_{z_{1,2}}=0$ η οποία είναι μοναδική → θεωρ. μοναδικότητας

(5) $\Rightarrow rD_r(r) = \int r\rho(r)\,dr + c \Rightarrow D_r(r) = \frac{1}{r}\int r\rho(r)\,dr + \frac{c}{r}$ (6)

(6) $\Rightarrow D_{r_2} = \frac{d_2}{r}$ (7)

(6) $\Rightarrow D_{r_1} = \frac{1}{r}\int_{r'=0}^{r} r'\rho(r')\,dr' + \frac{d_1'}{r}$ (8)



$\vec{D}_{\Sigma,\sigma\upsilon\nu} = \frac{\partial}{\partial r}\hat{r} + \vec{D}_{\Sigma,\text{υπολοίπων φορτίων}}$

Για $r\to 0$ $\vec{D}_{r,\sigma\upsilon\nu o\lambda} \to \frac{\lambda}{2\pi r}\hat{r}$ (9)

(8) $\Rightarrow 2\pi r D_{r_1} = \int_{r'=0}^{r} 2\pi r'\rho(r')\,dr' + 2\pi d_1'$ (10)

Για $r\to 0$: $2\pi D_{r_1} \to \lambda \Rightarrow 2\pi d_1' \to \lambda \Rightarrow d_1' = \frac{\lambda}{2\pi}$ (11)

(4), (7), (9), (11) $\Rightarrow \frac{d_2'}{a} - \frac{1}{a}\int_0^a r'\rho(r')\,dr' - \frac{\lambda}{2\pi a} = \sigma \Rightarrow d_2' = \dots$

Άλλη απόδειξη ότι $E_\varphi = 0 = E_z$

$E_{z_1} = c_1$, $E_{z_2} = c_2$

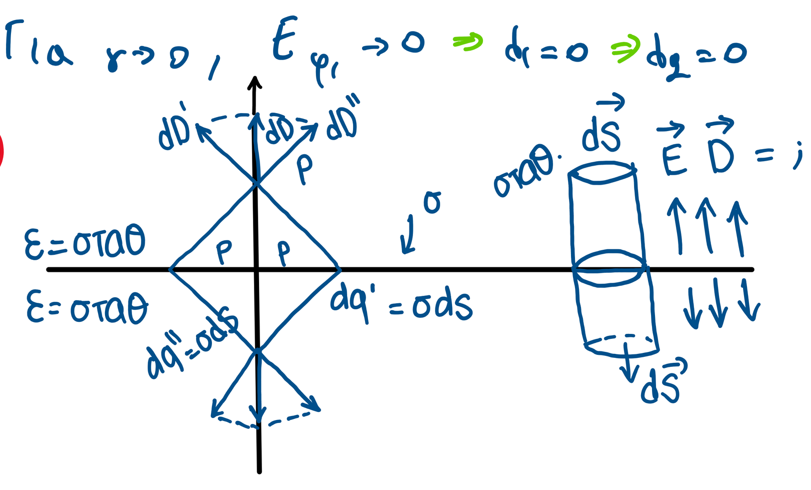
$E_{\varphi_1} = \frac{d_1}{r}$, $E_{\varphi_2} = \frac{d_2}{r}$

(3β) $\Rightarrow c_1 = c_2$. Για $r\to\infty$, $c_2 \to 0$ και $c_1 = c_2 \to 0$
Για $r\to 0$ (9) $\Rightarrow E_{z_1} = 0 \Rightarrow c_1 = 0 \Rightarrow c_2 = 0$

(3α) $\Rightarrow \frac{d_1}{a} = \frac{d_2}{a} \Rightarrow d_1 = d_2$

Για $r\to 0$, $E_{\varphi_1} \to 0 \Rightarrow d_1 = 0 \Rightarrow d_2 = 0$

(3)



$\exists$ μόνο $D_z(z)$, ισχύει ότι $D_z(z) = -D_z(-z)$ [συνθήκη αντισυμμετρίας ή συνθήκη στο άπειρο]

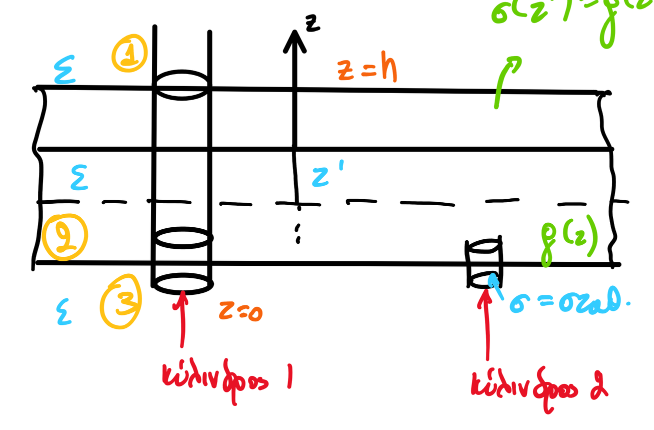
$$\oint_S \vec{D}\cdot d\vec{S} = Q_{\varepsilon\sigma} = \underbrace{D_z(z)\cdot S}_{\text{επάνω βάση}} - \underbrace{D_z(-z)\cdot S}_{\text{κάτω βάση}} + \underbrace{0}_{\text{παράπλευρη επιφάνεια}} = \sigma S$$

$2D_z(z) = \sigma \Rightarrow D_z(z) = \sigma/2 = -D_z(-z)$

$$\vec{D}_z = \begin{cases} \frac{\sigma}{2}\hat{z}, & z>0 \\ -\frac{\sigma}{2}\hat{z}, & z<0 \end{cases} \Rightarrow \vec{D}_z = \hat{z}\frac{\sigma}{2}\,\mathrm{sgn}(z), \quad \mathrm{sgn}(z) = \frac{z}{|z|} = \begin{cases} 1, & z>0 \\ -1, & z<0\end{cases}$$

$\vec{E}(z) = \hat{z}\frac{\sigma}{2\varepsilon}\,\mathrm{sgn}(z)$

(4)



$\vec{E}, \vec{D} = \sigma\tau\alpha\theta.$

$$\frac{\partial}{\partial x} = 0 = \frac{\partial}{\partial y}$$

$\exists$ μόνο $\vec{D}(z)$, $\vec{E}(z)$

$\exists$ μόνο $\vec{D}_z(z)$, $\vec{E}_z(z)$

Ισχύει ότι $D_{z_1} = -D_{z_3}$ (συνθήκη αντισυμμετρίας ή συνθήκη στο άπειρο)

A) Λύση με ολοκληρωτικές εξισώσεις

$$\oint_S \vec{D}\cdot d\vec{S} = Q_{\varepsilon\sigma}$$

Κύλινδρος 1: $\underbrace{D_{z_1}\cdot S}_{\text{επάνω βάση}} - \underbrace{D_{z_3}\cdot S}_{\text{κάτω βάση}} + \underbrace{0}_{\text{παράπλευρη επιφάνεια}} = \sigma S + \int_{z'=0}^{h} \rho(z')\,S\cdot dz'$

$$\Rightarrow 2D_{z_1} = \sigma + \int_{z'=0}^{h}\rho(z')\,dz' \Rightarrow D_{z_1} = \frac{1}{2}\left[\sigma + \int_{z'=0}^{h}\rho(z')\,dz'\right] = -D_{z_3}$$

Κύλινδρος 2: $\underbrace{D_{z_2}(z)\cdot S}_{\text{επάνω βάση}} - \underbrace{D_{z_3} S}_{\text{κάτω βάση}} + \underbrace{0}_{\text{παράπλευρη επιφάνεια}} = \sigma S + \int_{z'=0}^{z}\rho(z')\,S\,dz'$

$$\Rightarrow D_{z_2}(z) = D_{z_3} + \sigma + \int_{z'=0}^{z}\rho(z')\,dz' =$$

$$= -\frac{\sigma}{2} - \frac{1}{2}\int_{z'=0}^{h}\rho(z')\,dz' + \sigma + \int_{z'=0}^{z}\rho(z')\,dz' = \frac{\sigma}{2} - \frac{1}{2}\int_{z'=0}^{z}\rho(z')\,dz' - \frac{1}{2}\int_{z'=z}^{h}\rho(z')\,dz' + \int_{z'=0}^{z}\rho(z')\,dz'$$

$$\Rightarrow D_2(z) = \frac{\sigma}{2} + \frac{1}{2}\int_{z'=0}^{z}\rho(z')\,dz' - \frac{1}{2}\rho(z')\,dz'$$

$$\vec{E}_{1,2,3} = \frac{\vec{D}_{1,2,3}}{2}$$

$$\sigma_{+\infty} = \sigma_{-\infty} = -\frac{1}{2}\left[\sigma + \int_{z'=0}^{h}\rho(z')\,dz'\right]$$