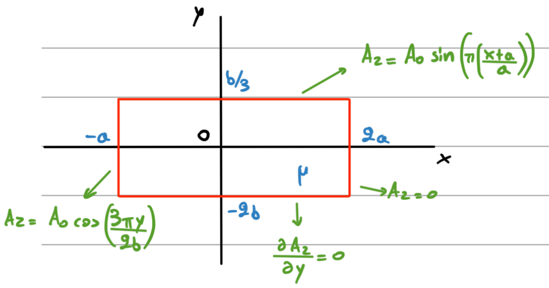
Credits: Χ. Φράγκος

Παράδειγμα 5



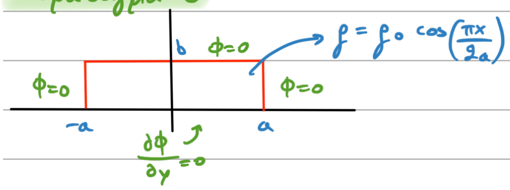
$$\nabla^2 \vec{A} = -\mu \vec{J}$$
$$\nabla^2 A_z = 0$$

- $A_z(y = b/3) = 0$ (οι άλλες οριακές συνθήκες ίδιες)
- $Y_1(y) = \cos(k_1 y)$, $X_1(x) = \sinh(k_1(x-2a))$ $\quad k_1 = \frac{3\pi}{2b}$
- $A_{z_1} = C_1 \cdot \sinh(k_1(x-2a))\cos(k_1 y)$

  $\hookrightarrow A_{z_1}(x=-a, y) = C_1 \sinh(k_1 \cdot 3a)\cos(k_1 y) \Rightarrow C_1 = \frac{-A_0}{\sinh(3k_1 a)}$
- $A_{z_2}(x=-a) = 0 \Rightarrow X_2(x) = \sin(k_2(x+a))$, $k_2 = \pi/4 \Rightarrow Y_2(y) = \cosh(k_2(y+2b))$
- $A_{z_2} = C_2 \sin(k_2(x+a))\cosh(k_2(y+2b))$

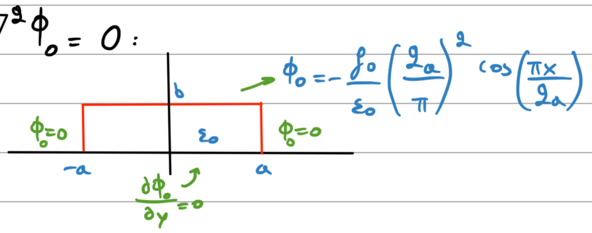
  $\hookrightarrow A_{z_2}(x, y=b/3) = C_2 \sin(k_2(x+a))\cosh(k_2(b/3+2b)) = A_0 \sin(k_2(x+a)) \Rightarrow C_2 = \frac{A_0}{\cosh}$
- $A_z = A_{z_1} + A_2$

Παράδειγμα 6



- $\nabla^2 \phi = -\rho/\varepsilon_0 \Rightarrow \phi = \phi_0 + \phi_\mu$, $\phi_\mu = C_\mu \cos\left(\frac{\pi x}{2a}\right)$

- $\nabla^2 \phi_\mu = -\frac{\rho}{\varepsilon_0} = -\frac{1}{\varepsilon_0}\rho_0 \cos\frac{\pi x}{2a} \Rightarrow C_\mu = \frac{\rho_0}{\varepsilon_0}\left(\frac{2a}{\pi}\right)^2 \Rightarrow \phi_\mu = \frac{\rho_0}{\varepsilon_0}\left(\frac{2a}{\pi}\right)^2 \cos\left(\frac{\pi x}{2a}\right)$
- $\nabla^2 \phi_0 = 0$:



- $\phi_0 = \phi - \phi_r$
- $\phi_0(x=a, y) = \phi(x=a, y) - \phi_\mu(x=a, y) = 0 - 0 = 0$
- $X_0(x) = \cos\left(\frac{\pi x}{2a}\right)$ $\quad$ • $Y_0(y) = \cosh\left(\frac{\pi y}{2a}\right)$

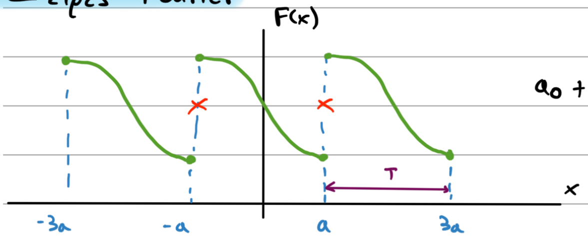
- $\phi_0 = C_0 \cdot \cos\left(\frac{\pi x}{2a}\right)\cosh\left(\frac{\pi y}{2a}\right)$

  $\hookrightarrow \phi_0(x, y=b) = \cos\left(\frac{\pi x}{2a}\right)\cosh\left(\frac{\pi b}{2a}\right) = -\frac{\rho_0}{\varepsilon_0}\left(\frac{2a}{\pi}\right)^2 \cos\left(\frac{\pi x}{2a}\right)$

- $$\phi = \frac{\rho_0}{\varepsilon_0}\left(\frac{2a}{\pi}\right)^2 \cos\left(\frac{\pi x}{2a}\right)\left(1 - \frac{\cosh\left(\frac{\pi y}{2a}\right)}{\cosh\left(\frac{\pi b}{2a}\right)}\right)$$

- $\vec{E} = -\vec{\nabla}\phi$

Σειρές Fourier

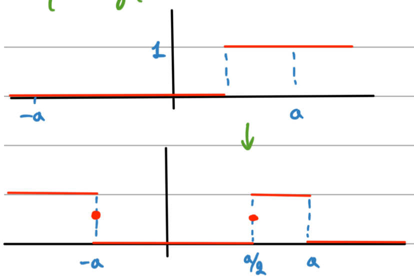


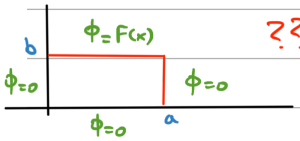
$$a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{a}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{a}\right) = \begin{cases} \text{ΠΕ της } F(x) \text{ αν ΠΕ συνεχής} = F(x), & x \in (-a,a) \\ \dfrac{F(-a^-) + F(-a^+)}{2}, & x = -a \\ \dfrac{F(a^-) + F(a^+)}{2}, & x = a \end{cases}$$

- $F(x)$: τμηματικά λεία
- Περιοδική επέκταση $F(x)$: ΠΕ της F

- $a_0 = \dfrac{1}{2a} \displaystyle\int_{-a}^{a} F(x)\, dx$
- $a_n = \dfrac{1}{a} \displaystyle\int_{-a}^{a} F(x) \cos\left(\frac{n\pi x}{a}\right) dx$
- $b_n = \dfrac{1}{a} \displaystyle\int_{-a}^{a} F(x) \cdot \sin\left(\frac{n\pi x}{a}\right) dx$

Παράδειγμα





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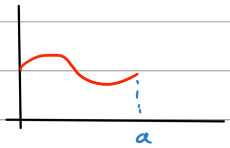
Αν:

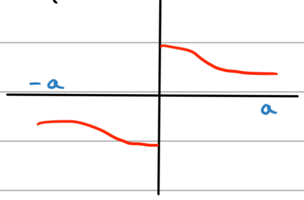


- $\displaystyle\int_{-a}^{a} \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) dx = \begin{cases} 2a, & m = n \\ 0, & m \neq n \end{cases}$
- $\displaystyle\int_{-a}^{a} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = \begin{cases} a, & m = n \neq 0 \\ 0, & m \neq n \text{ ή } m=n=0 \end{cases}$
- $\displaystyle\int_{-a}^{a} \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) dx = 0, \quad \forall\, m,n$

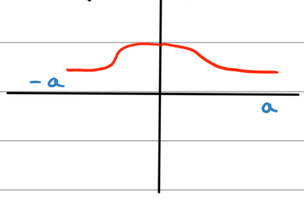
ΣΟΣ $X_n(x) = \sin(k_n x)$, $k_n = \dfrac{n\pi}{a}$ $\Rightarrow$ Περιττή ΠΕ

ΠΕ ← **Περιοδική Επέκταση της $F(x)$**

Περιττή ΠΕ



Άρτια ΠΕ



Η επιλογή ΠΕ επιβάλλεται από τις υπόλοιπες Ορ. Συνθ.

Σειρά Fourier Ημιτόνων (FSS)

$$\sum_{n=1}^{\infty} b_n \cdot \sin\left(\frac{n\pi x}{a}\right) \sim F(x), \quad 0 \leq x \leq a$$

$$b_n = \frac{2}{a} \int_0^a \sin\left(\frac{n\pi x}{a}\right) F(x)\, dx$$

Σειρά Fourier Συνημιτόνων (FCS)

$$a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{a}\right) \sim F(x), \quad 0 \leq x \leq a$$

$$a_n = \frac{2}{a} \int_0^a F(x) \cos\left(\frac{n\pi x}{a}\right) dx$$

- $\displaystyle\int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = \frac{a}{2} \delta_{mn}$
- $\displaystyle\int_0^a \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) dx = \frac{a}{2} (1 + \delta_{mn}) \delta_{mn}$
- $\displaystyle\int_0^a \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) dx \neq 0$

$$\boxed{\delta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}}$$