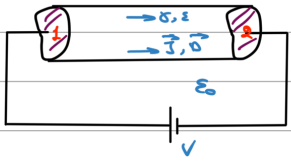
Σχέση R-C

$\vec{D} \to \vec{J}$

$\varepsilon \to \gamma$

$$R = \frac{V}{I} = \frac{\int_1^2 \vec{E}\, d\vec{l}}{\oint_S \vec{J}\, d\vec{S}} \quad , \quad C = \frac{Q}{V} = \frac{\oint_{S_1} \vec{D}\cdot d\vec{S}}{\int_1^2 \vec{E}\, d\vec{l}}$$

$\rho, \sigma = 0$, $\vec{K} = 0$
Δυϊσμός



$1^η$ Περίπτωση: γ: $\vec{E}, \vec{J}$ } Δυϊσμός
$2^η$ Περίπτωση: ε: $\vec{E}, \vec{D}$
$3^η$ Περίπτωση: γ, ε (κ' $\rho, \sigma \neq 0$)

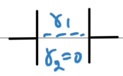
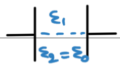
Αν $\rho \neq 0$, δεν ορίζεται η C

Δυϊσμός:

Αντίσταση: $\vec{E}$, $\vec{J}$, γ, $G = 1/R$, $I_{εξ}$, $\tilde{\sigma}$
Πυκνωτής: $\vec{E}$, $\vec{D}$, ε, C, Q, σ

$$\boxed{\begin{aligned} &\hat{n}\cdot(\vec{J}_2 - \vec{J}_1) = -\nabla\vec{K} \\ &\tilde{\sigma} = \hat{n}\cdot\vec{J}_2 = \hat{n}\cdot\vec{J}_1 - \nabla\vec{K} \end{aligned}}$$

γνκ ο δυϊσμός ισχύει εάν έχουμε ίδια γεωμετρία και 1-1 αντιστοιχία στα μεγέθη

Αντ  → Πυκν. 

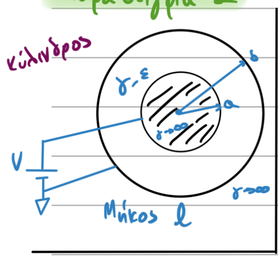
$\gamma_2 << \gamma_1$ $\qquad$ $\varepsilon_2 << \varepsilon_1$

ο Δυϊσμός ισχύει αν ισχύουν και τα 2

εδώ $\gamma_2 = 0 \neq \varepsilon_2$ οπότε δεν θα ισχύει ο δυϊσμός, αλλά επειδή ισχύουν ταυτόχρονα οι $\gamma_2 << \gamma_1$, $\varepsilon_2 << \varepsilon_1$ μπορούμε να τον εφαρμόσουμε*

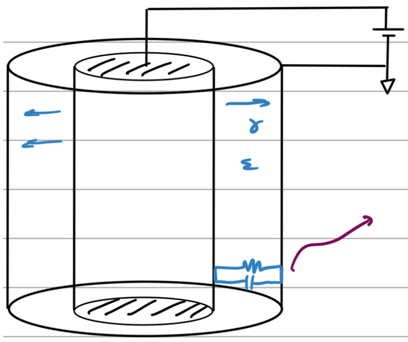
Παράδειγμα 1

κύλινδρος

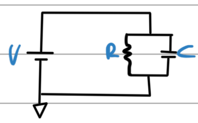


• Είχαμε βρει πως: $R = \dfrac{\ln(b/a)}{2\pi\gamma l} = \dfrac{1}{G} \Rightarrow G = \dfrac{2\pi\gamma l}{\ln(b/a)}$

• Από δυϊσμό: $G \leftrightarrow C$, $\gamma \leftrightarrow \varepsilon$ $\Big\} \Rightarrow C = \dfrac{2\pi\varepsilon l}{\ln(b/a)}$

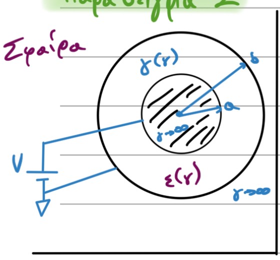


• Υπό την προϋπόθεση: $\rho = \nabla\vec{D} = 0$



Παράδειγμα 2

Σφαίρα

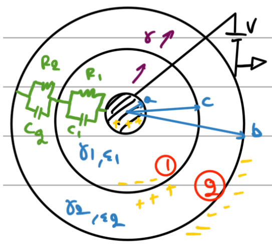


• Είχαμε βρει πως: $R = \dfrac{1}{4\pi}\displaystyle\int_a^b \dfrac{dr}{\gamma(r) r^2} = \dfrac{1}{G} \Rightarrow G = \dfrac{4\pi}{\int_a^b \frac{dr}{\gamma(r) r^2}}$

• Από δυϊσμό: Θεωρούμε $\varepsilon(r)$ ($\leftrightarrow \gamma(r)$): $C = \dfrac{4\pi}{\int_a^b \frac{dr}{\varepsilon(r) r^2}}$

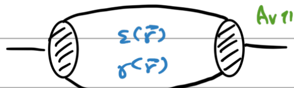
↓

Θα είναι ίδια συνάρτηση! (π.χ. $\gamma(r) = \varepsilon(r) = 1 + r$)



$R = R_1 + R_2$

Ηλεκτρικά φορτία στα Μόνιμα Ηλεκτρικά Πεδία (ΜΗΠ)

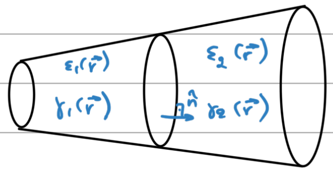


διαν. θέσης
όχι ακτινική συνεταγμένη!

$\nabla J = -\frac{\partial \rho}{\partial t} = 0$

- $\rho = \nabla \vec D = \nabla[\varepsilon(\vec r)\cdot \vec E(\vec r)] = \nabla\left[\frac{\varepsilon(\vec r)}{\gamma(\vec r)}\vec J(\vec r)\right] = \nabla\left[\frac{\varepsilon(\vec r)}{\gamma(\vec r)}\right]\cdot \vec J(\vec r) + \frac{\varepsilon(\vec r)}{\gamma(\vec r)}\cdot \cancel{\nabla \vec J(\vec r)} = \nabla\left[\frac{\varepsilon(\vec r)}{\gamma(\vec r)}\right]\cdot \vec J(\vec r) \neq 0$ στην γεν. περίπτωση

- $\rho = 0$ αν:
  - $\varepsilon, \gamma =$ σταθ.
  - $\tau = \frac{\varepsilon(\vec r)}{\gamma(\vec r)} =$ σταθ

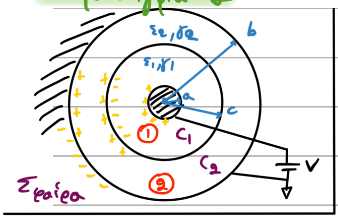
- 

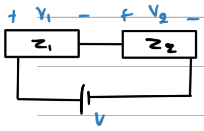
→ $\hat n\cdot(\vec J_2 - \vec J_1) = -\nabla \vec K$

→ $\sigma = \hat n\cdot(\vec D_2 - \vec D_1) = \frac{\varepsilon_2}{\gamma_2}J_{n_2} - \frac{\varepsilon_1}{\gamma_1}J_{n_1} = \left(\frac{\varepsilon_2}{\gamma_2} - \frac{\varepsilon_1}{\gamma_1}\right)J_n \neq 0$

→ Αν $\frac{\varepsilon_2}{\gamma_2} = \frac{\varepsilon_1}{\gamma_1} \Rightarrow \sigma = 0$

Παράδειγμα 1

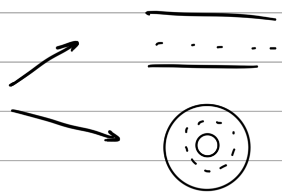




- ①, ② : $\nabla\cdot \vec J_{1,2} = 0 \Rightarrow \vec J_{1,2} = \frac{A_{1,2}}{r^2}\hat r$
- $r = c$: ($\vec K = 0$) $\hat r\cdot(\vec J_2 - \vec J_1) = 0 \Rightarrow A_1 = A_2 = A$
- $\vec E_{1,2} = \frac{1}{\gamma_{1,2}}\vec J_{1,2} = \frac{A}{\gamma_{1,2}\cdot r}\hat r$
- $V = \underbrace{\int_a^b \vec E_1 d\vec \ell}_{V_1} + \underbrace{\int_c^b \vec E_2\cdot d\vec\ell}_{V_2} = \ldots = -A\left(\frac{a-c}{\gamma_1\cdot ac} - \frac{c-b}{\gamma_2\cdot cb}\right) \Rightarrow \boxed{A = \frac{V}{\frac{c-a}{\gamma_1 ac} - \frac{b-c}{\gamma_2 bc}}}$

- $\rho_{1,2} = \nabla \vec D_{1,2} = \nabla\left[\frac{\varepsilon_{1,2}}{\gamma_{1,2}}\right]\cdot \vec J_{1,2} = 0$

↳ Εάν όμως: $\varepsilon_i(\vec r)$ ή /και $\gamma_i(\vec r)$ ⤳ $\rho \neq 0$



- $\sigma(r=a) = \hat r\cdot \vec D_1 = \frac{\varepsilon_1}{\gamma_1}\frac{A}{a^2} \equiv \sigma_a > 0$
- $\sigma(r=b) = -\frac{\varepsilon_2}{\gamma_2}\frac{A}{b^2} \equiv \sigma_b < 0$
- $\sigma(r=c) = \hat r\cdot(\vec D_2 - \vec D_1) = \hat r\cdot\left(\frac{\varepsilon_2}{\gamma_2}\vec J_2 - \frac{\varepsilon_1}{\gamma_1}\vec J_1\right) = \left(\frac{\varepsilon_2}{\gamma_2} - \frac{\varepsilon_1}{\gamma_1}\right)\frac{A}{c^2} = \frac{\varepsilon_2}{\gamma_2}\frac{A}{c^2} - \frac{\varepsilon_1}{\gamma_1}\frac{A}{c^2} \equiv \sigma_c$
- $I_{1,2} = I = \oint_S \vec J\cdot d\vec S = A\cdot 4\pi$

- $C_1 = \frac{Q_1}{V_1} = \frac{\frac{\varepsilon_1}{\gamma_1}\cdot\frac{A}{a^2}\cdot 4\pi a^2}{A\cdot\frac{1}{\gamma_1}\cdot\frac{c-a}{ac}} = \frac{4\pi\varepsilon_1 ac}{c-a}$
- $C_2 = \frac{Q_2}{V_2} = \frac{\sigma_b\cdot 4\pi b^2}{-V_2} = \frac{4\pi\varepsilon_2 bc}{b-c}$

- $R_{o\lambda} = \frac{V}{I} = \underbrace{\frac{1}{\gamma_1}\frac{c-a}{4\pi ac}}_{R_1} + \underbrace{\frac{1}{\gamma_2}\frac{b-c}{4\pi bc}}_{R_2}$