Indexing (part 2 – tree structures)

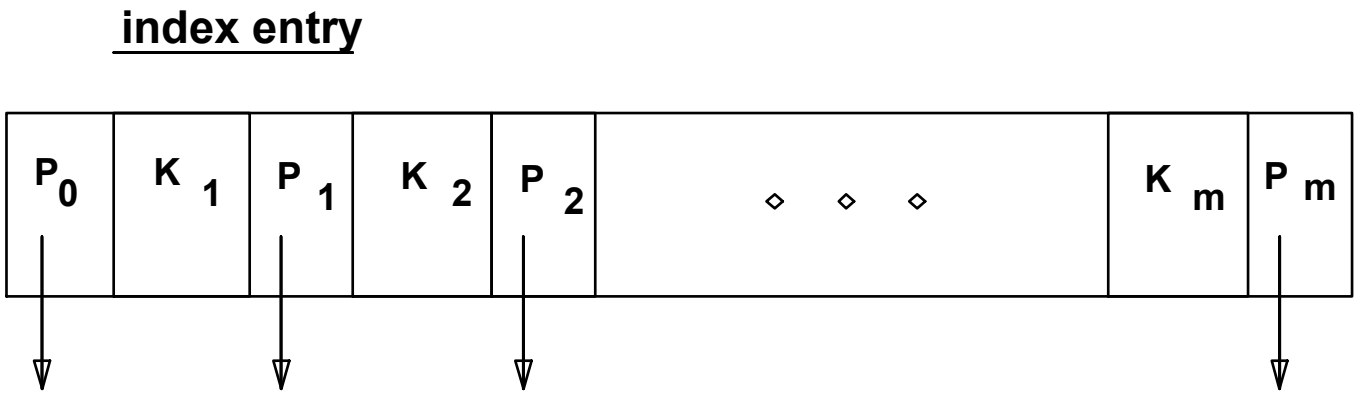
Βάσεις Δεδομένων, 6ο Εξάμηνο, 2023
Εργαστήριο Συστημάτων Βάσεων Γνώσεων και Δεδομένων

Σχολή Ηλεκτρολόγων Μηχ/κών και Μηχ/κών Η/Υ

Database System Concepts, 7th Ed.

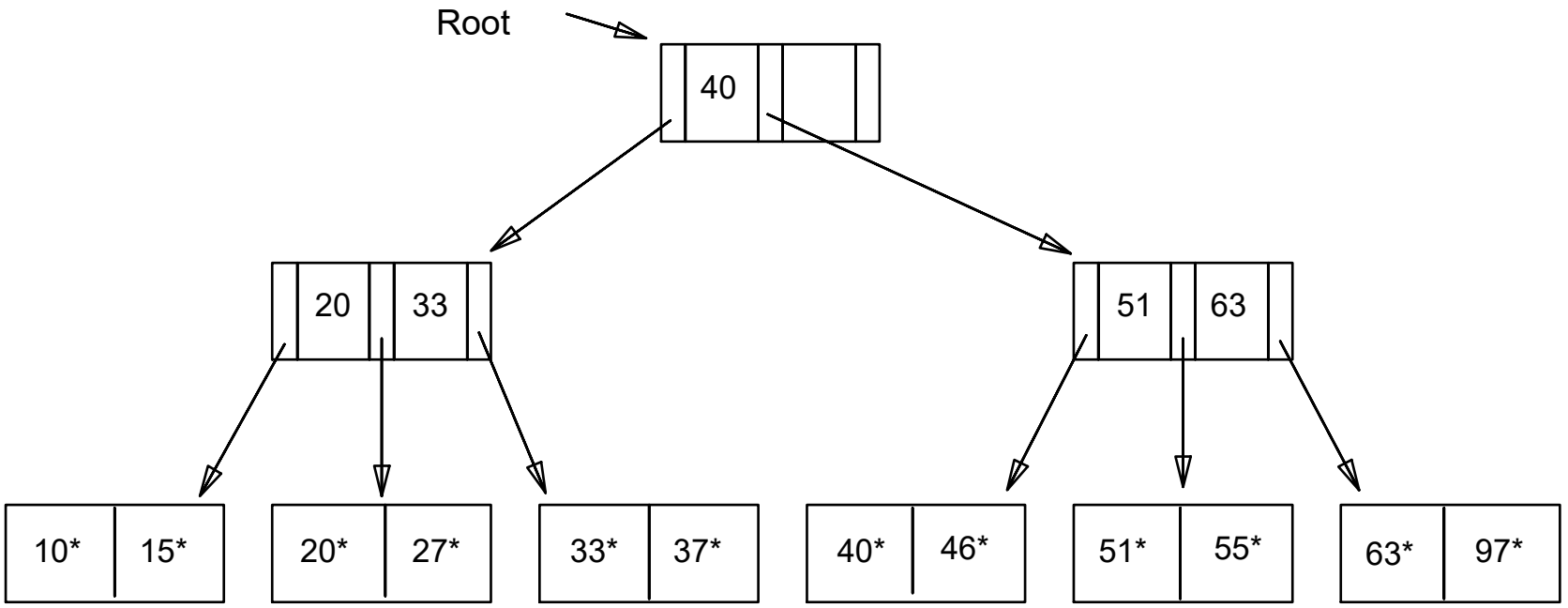
Indexed Sequential Access Method (ISAM)

- ISAM is a multi-level index structure (tree) for accessing records ordered by a key
- Every node of the tree is a disk block
- The nodes keep information **<key value, pointer>**, ordered by key value. The internal nodes point to lower level nodes, whereas the nodes of the last level point to a block of the file (relation). A pointer points to a sub-tree with key values larger or equal to the previous key value and smaller than the value of the next key value



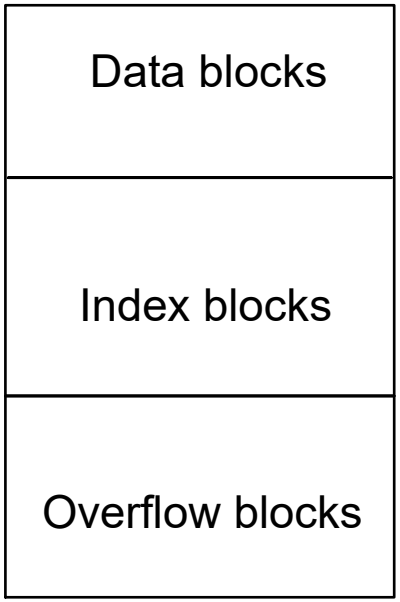
ISAM tree – Example

- Each node can hold 2 entries



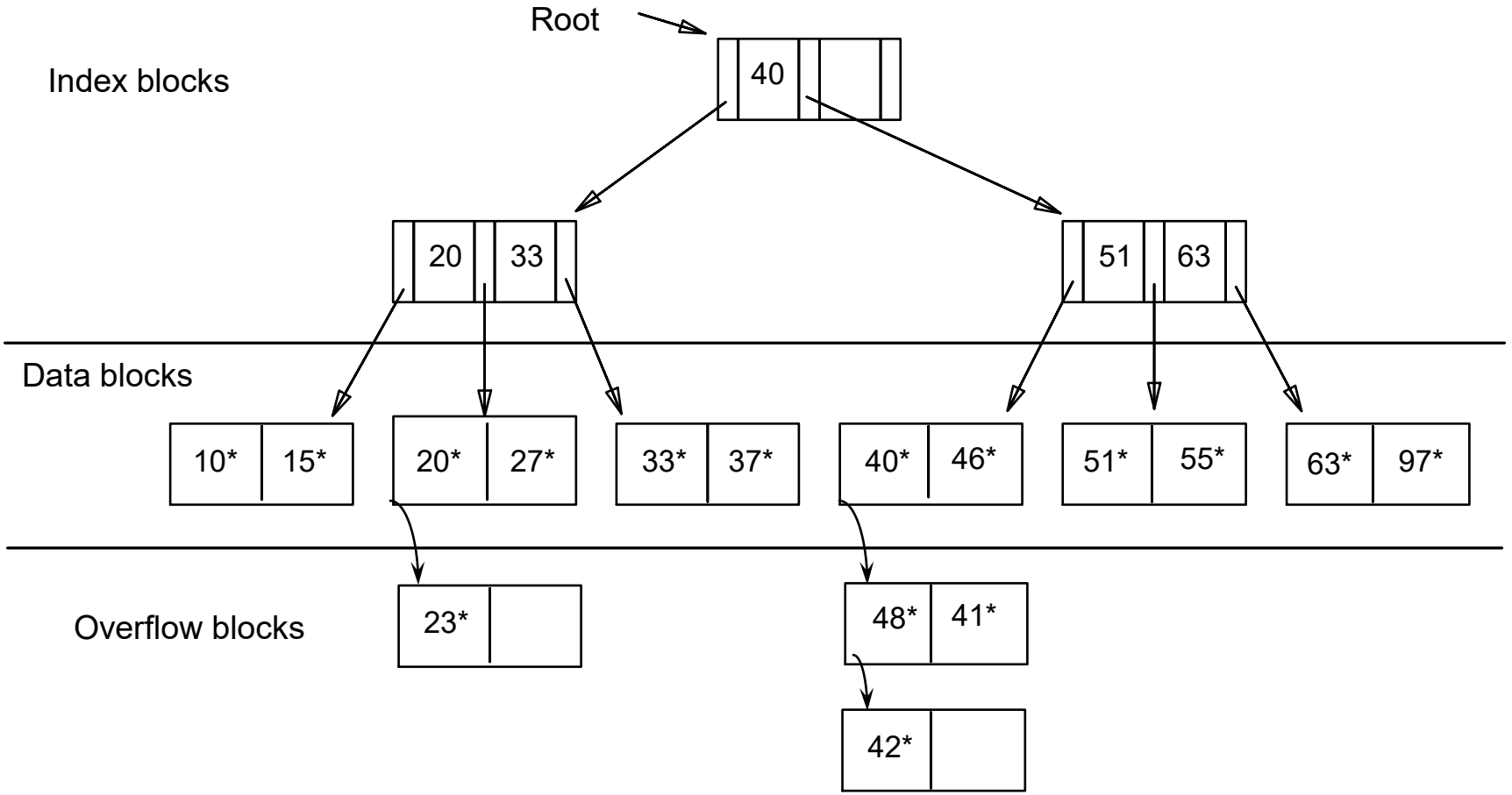
Basic operations

- *Insert*: Leaf (data) blocks allocated sequentially, sorted by search key; then index pages allocated, then space for overflow pages.
- *Index entries*: <search key value, block id>; they 'direct' search for *data entries*, which are in leaf blocks.
- *Search*: Start at root; use key comparisons to go to leaf. Cost $\propto \log_F N$, F = # entries/index block, N = # leaf blocks
- *Insert*: Find leaf the data entry belongs to, and put it there.
- *Delete*: Find and remove from leaf; if empty overflow page, de-allocate.

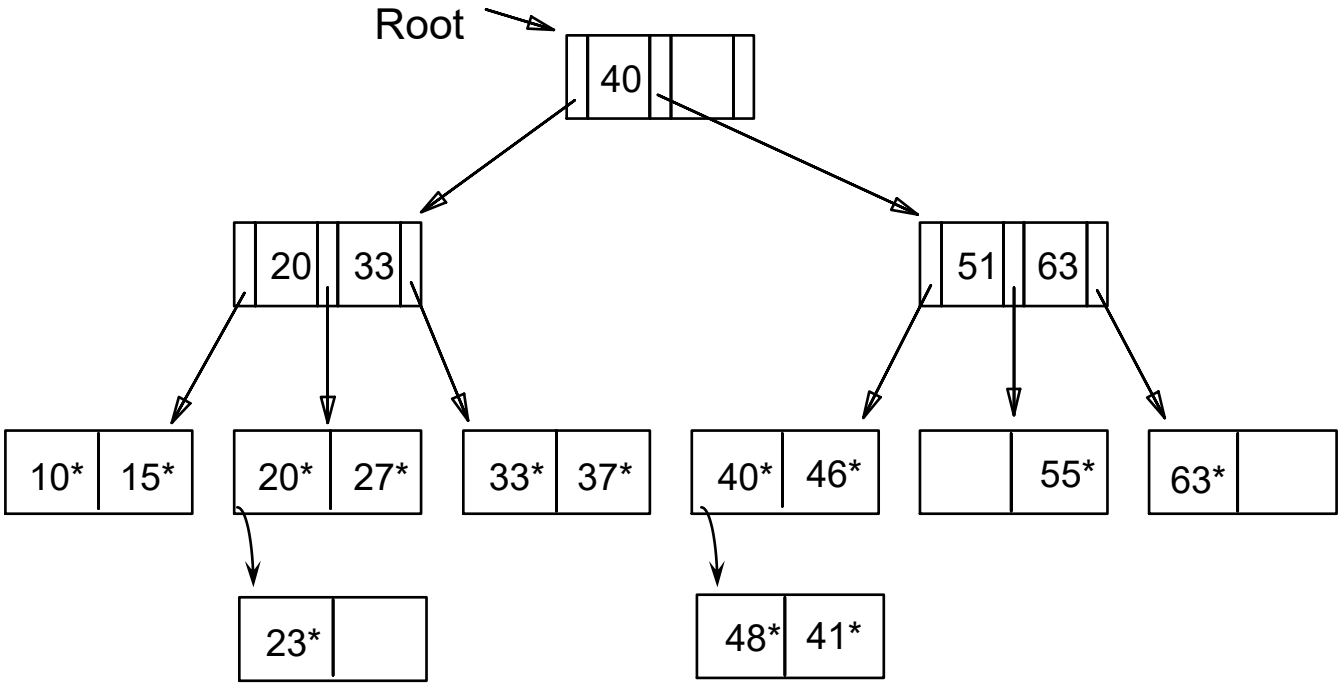


Static tree structure: *inserts/deletes affect only leaf pages*.

After the insertion of 23*, 48*, 41*, 42* ...



After the deletion of 42*, 51*, 97*



Observe that 51 is in the index but not in the data*

Performance of ISAM

- **Performance**

  Assume that we have ***N blocks*** for data (leaves) and ***F*** pointers in every node

  Search process:

Scan:	N
Binary search:	$\log_2 N$
ISAM traversal:	$\log_F N$

ISAM – Comments

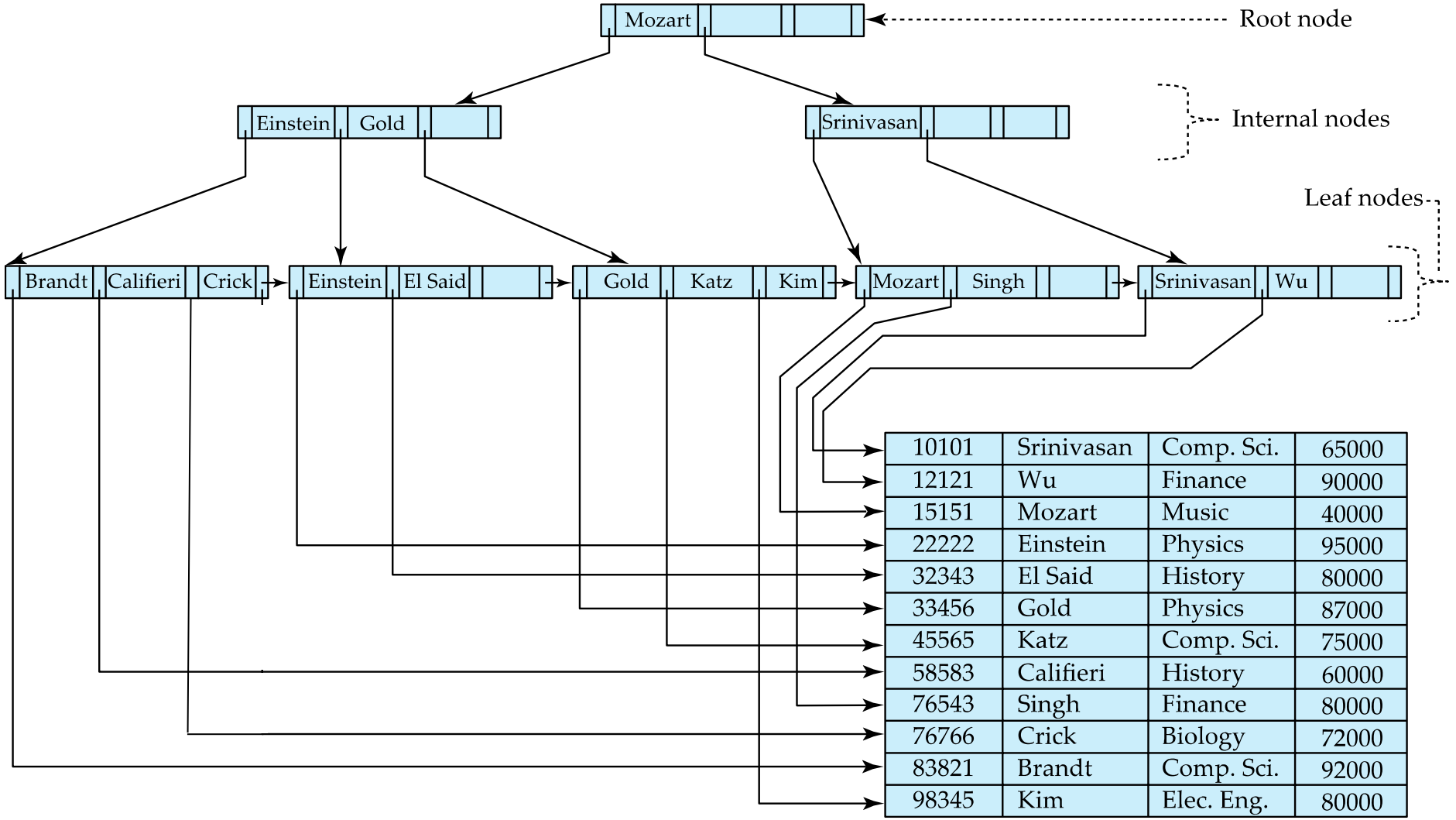
- **Advantages:**
  - It provides an ordered catalog for the file
  - Very good structure for **exact queries,** e.g. *Salary = 400000*
  - ISAM facilitates **range queries,** e.g. *Salary between 350000 and 600000*

- **Disadvantages**
  - It is a **static** structure that becomes easily **unbalanced**
  - If there are a lot of updates in the file then the file can lose its ordering
  - It requires a lot of disk space

B^+-Tree Index Files

- Disadvantage of indexed-sequential files
  - Performance degrades as file grows, since many overflow blocks get created.
  - Periodic reorganization of entire file is required.
- Advantage of B^+-tree index files:
  - Automatically reorganizes itself with small, local changes in the face of insertions and deletions.
  - Reorganization of entire file is not required to maintain performance.
- (Minor) disadvantage of B^+-trees:
  - Extra insertion and deletion overhead, space overhead.
- Advantages of B^+-trees outweigh disadvantages
  - B^+-trees are used extensively

Example of B+-Tree



10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	60000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

B^+-Tree Index Files (Cont.)

A B^+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length (balanced)
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children (aka, *pointers*).
- A leaf node has between $\lceil (n–1)/2 \rceil$ and $n–1$ values (aka, *keys*).
- Special cases:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and $(n–1)$ values.

B+-Tree Node Structure

- Typical node

P_1	K_1	P_2	...	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).
- The search-keys in a node are ordered

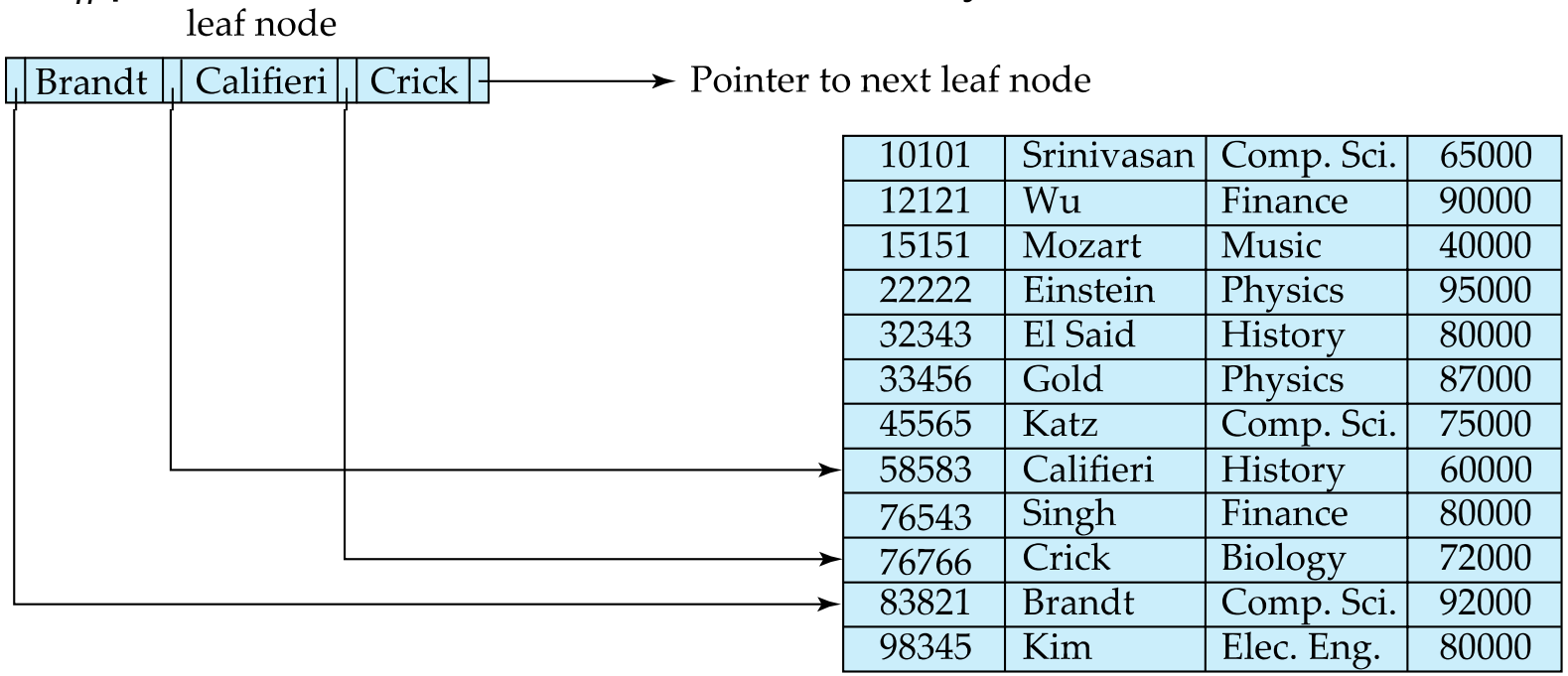
$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

(Initially we assume no duplicate keys, we address duplicates later)

Leaf Nodes in B+-Trees

Properties of a leaf node:

- For $i = 1, 2, \ldots, n-1$, pointer P_i points to a file record with search-key value K_i,
- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than or equal to L_j's search-key values
- P_n points to next leaf node in search-key order



10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	60000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

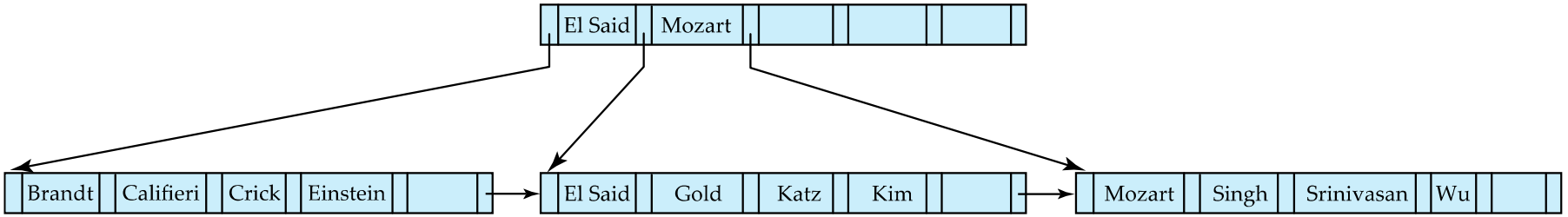
Non-Leaf Nodes in B+-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with m pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \leq i \leq n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}
  - General structure:

P_1	K_1	P_2	$\cdots$	P_{n-1}	K_{n-1}	P_n

Example of B+-tree

- B+-tree for *instructor* file (n = 6)



- Leaf nodes must have between 3 and 5 values ($\lceil (n-1)/2 \rceil$ and $n-1$, with n = 6).
- Non-leaf nodes other than root must have between 3 and 6 children ($\lceil n/2 \rceil$ and n with n =6).
  - So, between 2 and 5 keys
- Root must have at least 2 children.

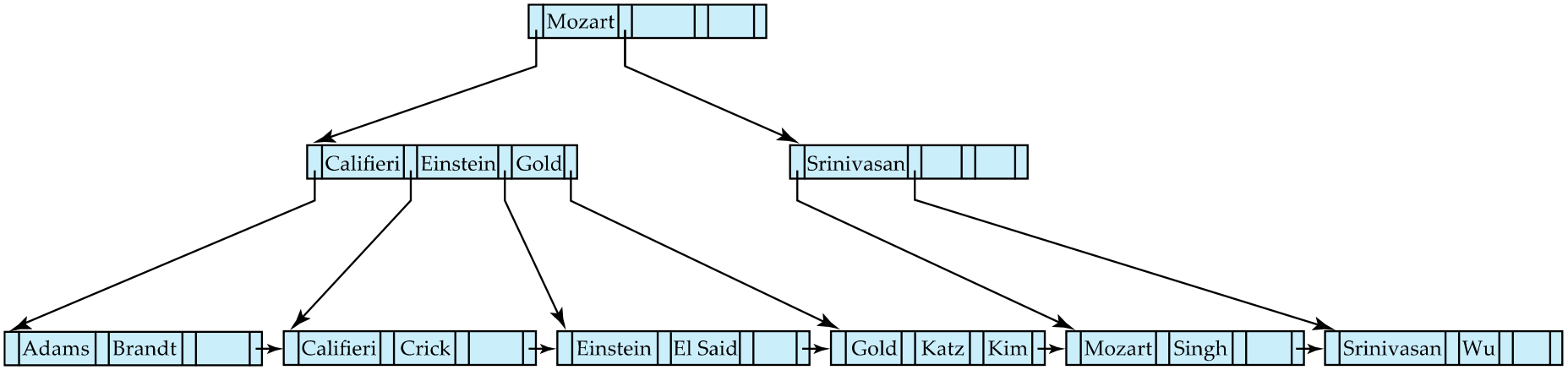
Observations about B^+-trees

- Since the inter-node connections are done by pointers, "logically" close blocks need not be "physically" close.
- The non-leaf levels of the B^+-tree form a hierarchy of sparse indices.
- The B^+-tree contains a relatively small number of levels
  - Level below root has at least $2* \lceil n/2 \rceil$ values
  - Next level has at least $2* \lceil n/2 \rceil * \lceil n/2 \rceil$ values
  - .. etc.
  - If there are K search-key values in the file, the tree height is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
  - thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Queries on B+-Trees

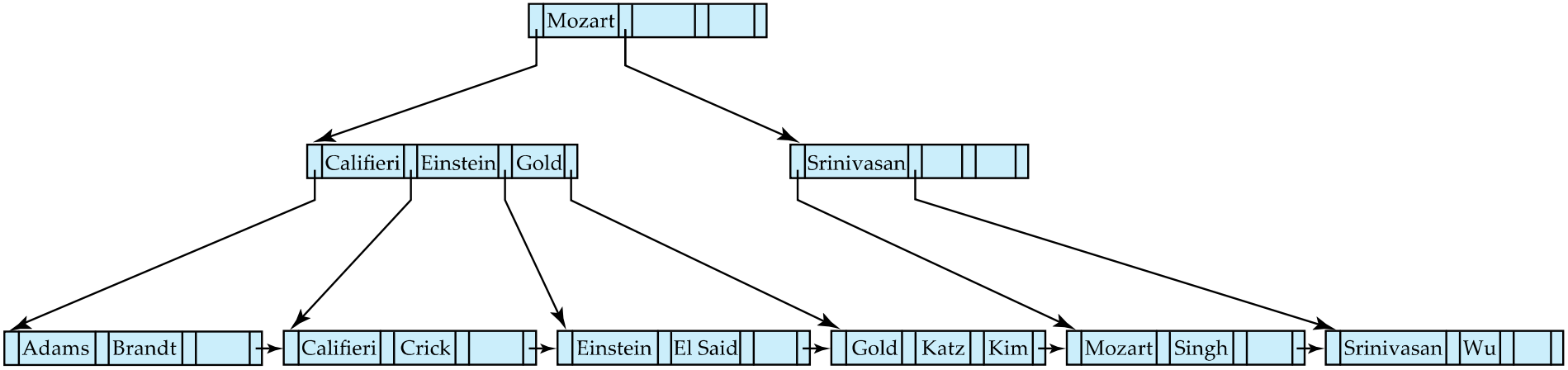
function *find*(*V*)

1. *C=root*
2. **while** (C is not a leaf node)
   1. Let *i* be least number s.t. $V \leq K_i$.
   2. **if** there is no such number *i then*
   3. Set *C* = *last non-null pointer in C*
   4. **else if** (V = C.K_i) Set C = P_{i+1}
   5. **else set** *C* = C.P_i
3. **if** for some *i*, $K_i = V$ **then** return C.P_i
4. **else** return null /* no record with search-key value *V* exists. */



Queries on B+-Trees (Cont.)

- **Range queries** find all records with search key values in a given range
  - See book for details of **function** *findRange*(*lb, ub*) which returns set of all such records
  - Real implementations usually provide an iterator interface to fetch matching records one at a time, using a *next*() function



Queries on B+-Trees (Cont.)

- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes
  - and n is typically around 100 (40 bytes per index entry).
- With 1 million search key values and $n = 100$
  - at most $log_{50}(1{,}000{,}000) = 4$ nodes are accessed in a lookup traversal from root to leaf.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - above difference is significant since every node access may need a disk I/O, costing around 20 milliseconds

Non-Unique Keys

- If a search key a_i is not unique, create instead an index on a composite key (a_i , A_p), which is unique
  - A_p could be a primary key, record ID, or any other attribute that guarantees uniqueness
- Search for $a_i = v$ can be implemented by a range search on composite key, with range (v, $-\infty$) to (v,$+\infty$)
- But more I/O operations are needed to fetch the actual records
  - If the index is clustering, all accesses are sequential
  - If the index is non-clustering, each record access may need an I/O operation

Updates on B+-Trees: Insertion

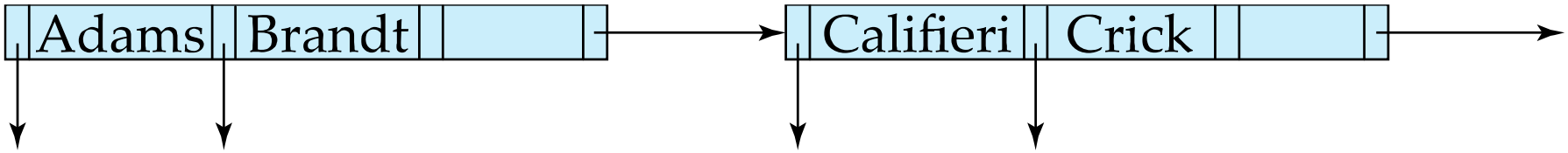
Assume record already added to the file. Let

- *pr* be pointer to the record, and let
- v be the search key value of the record

1. Find the leaf node in which the search-key value would appear
    1. If there is room in the leaf node, insert (v, *pr*) pair in the leaf node
    2. Otherwise, split the node (along with the new (*v, pr*) entry) as discussed in the next slide, and propagate updates to parent nodes.

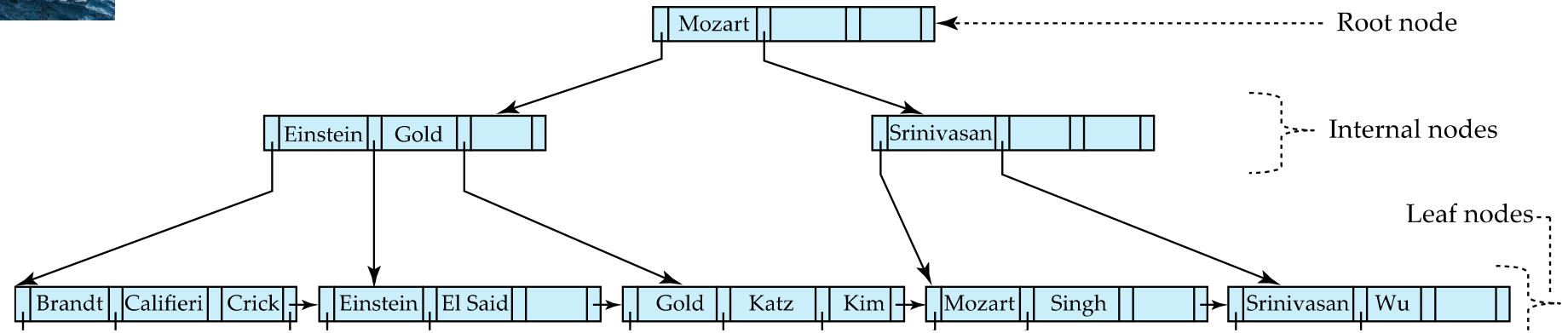
Updates on B+-Trees: Insertion (Cont.)

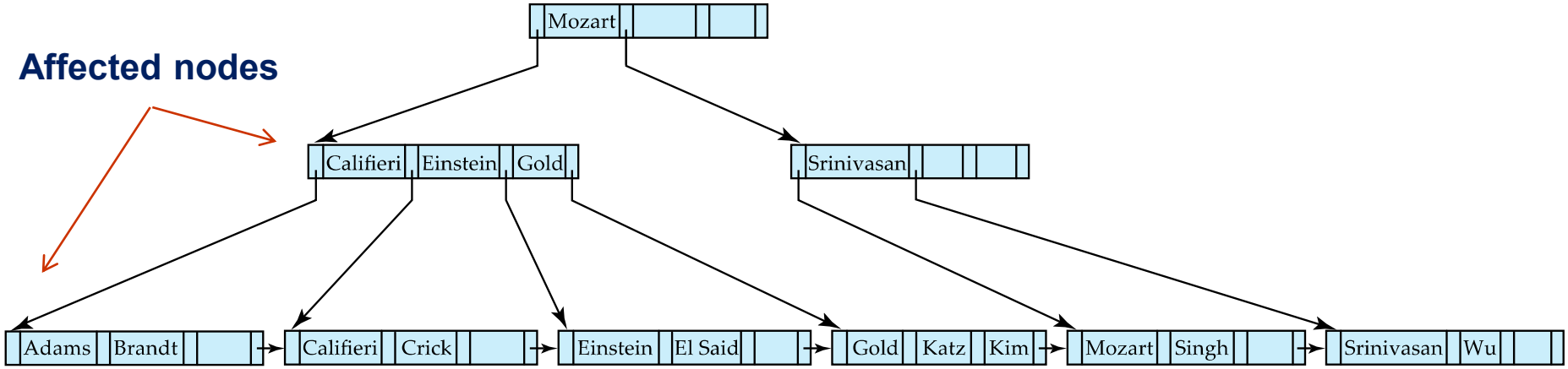
- Splitting a leaf node:
  - take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - let the new node be *p,* and let *k* be the least key value in *p.* Insert (*k,p*) in the parent of the node being split.
  - If the parent is full, split it and **propagate** the split further up.
- Splitting of nodes proceeds upwards till a node that is not full is found.
  - In the worst case the root node may be split increasing the height of the tree by 1.



Result of splitting node containing Brandt, Califieri and Crick on inserting Adams
Next step: insert entry with (Califieri, pointer-to-new-node) into parent

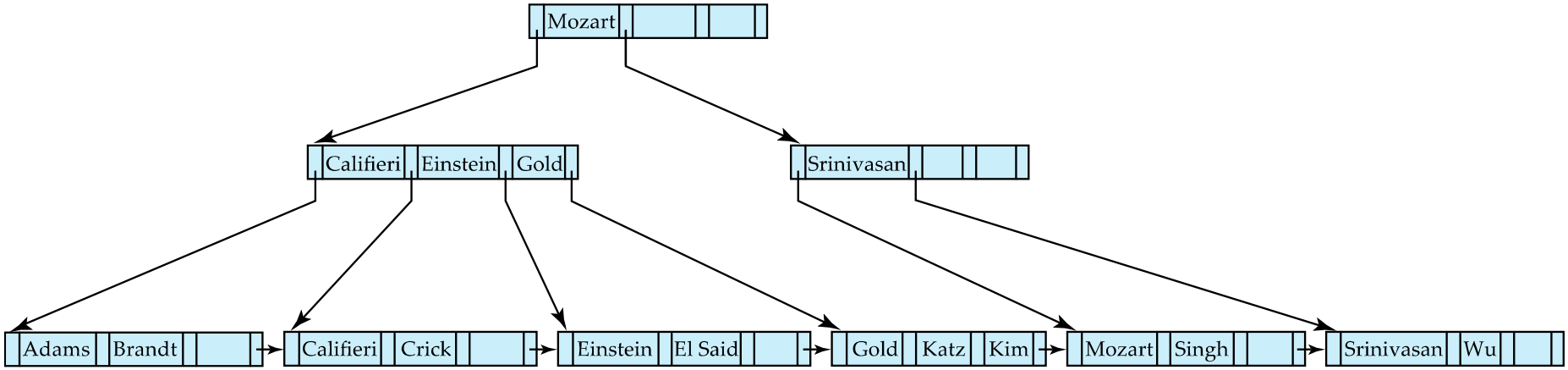
B+-Tree Insertion





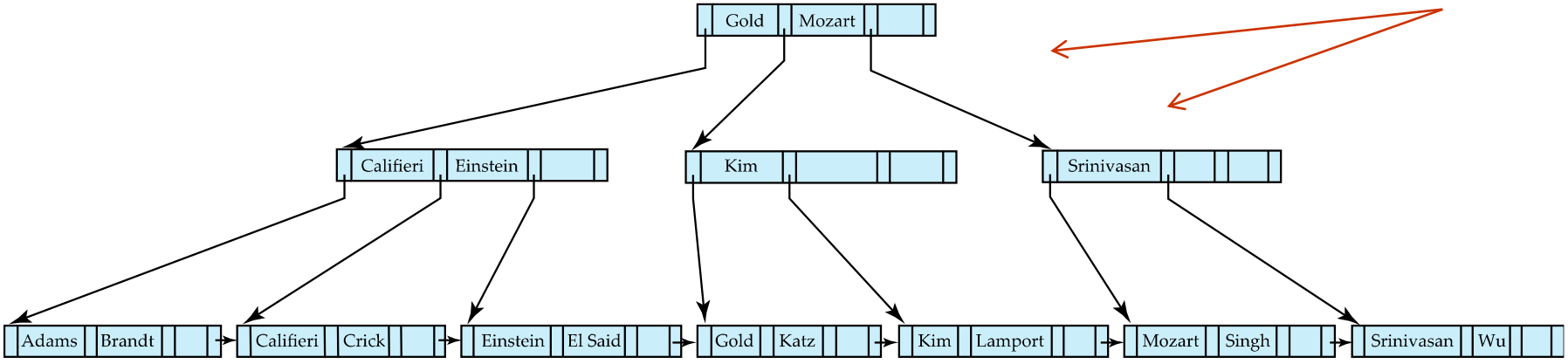
B+-Tree (n=4) before and after insertion of “Adams”

B+-Tree Insertion



B+-Tree (n=4) **before and after insertion of "Lamport"**

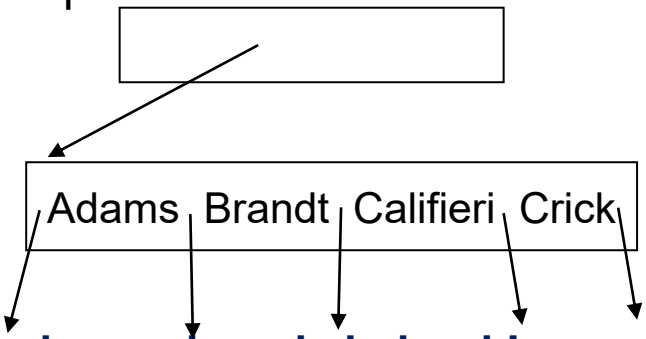
Affected nodes

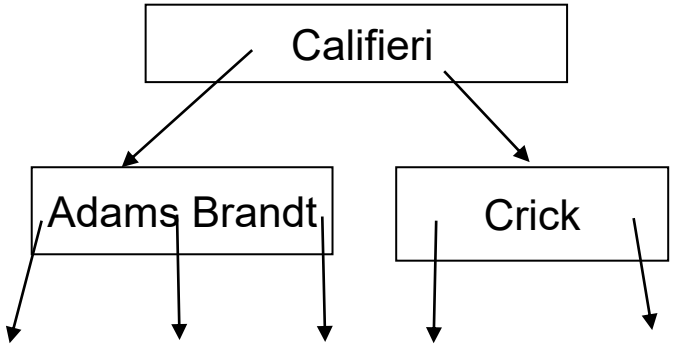


Affected nodes

Insertion in B+-Trees (Cont.)

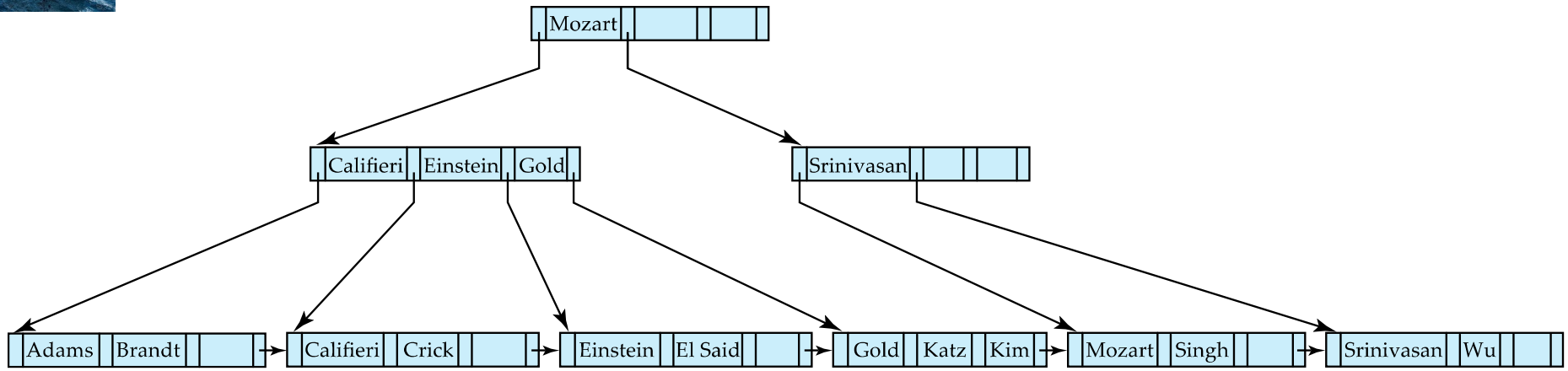
- Splitting a non-leaf node: when inserting (k,p) into an already full internal node N
  - Copy N to an in-memory area M with space for n+1 pointers and n keys
  - Insert (k,p) into M
  - Copy $P_1, K_1, \ldots, K_{\lceil n/2 \rceil - 1}, P_{\lceil n/2 \rceil}$ from M back into node N
  - Copy $P_{\lceil n/2 \rceil + 1}, K_{\lceil n/2 \rceil + 1}, \ldots, K_n, P_{n+1}$ from M into newly allocated node N'
  - Insert $(K_{\lceil n/2 \rceil}, N')$ into parent N
- Example



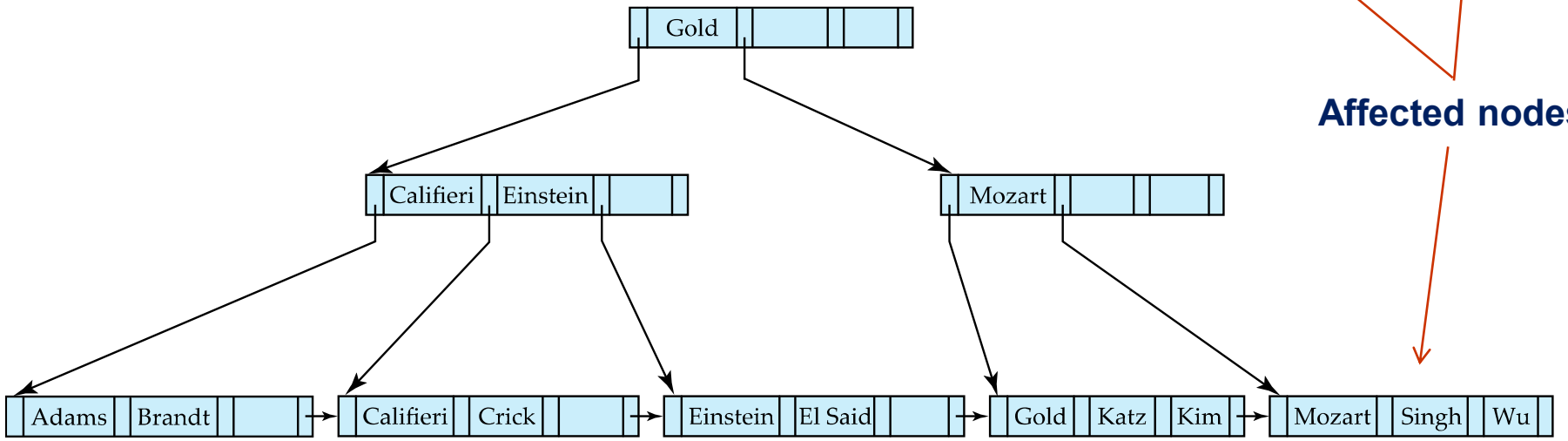


- **Read pseudocode in book!**

Examples of B+-Tree Deletion



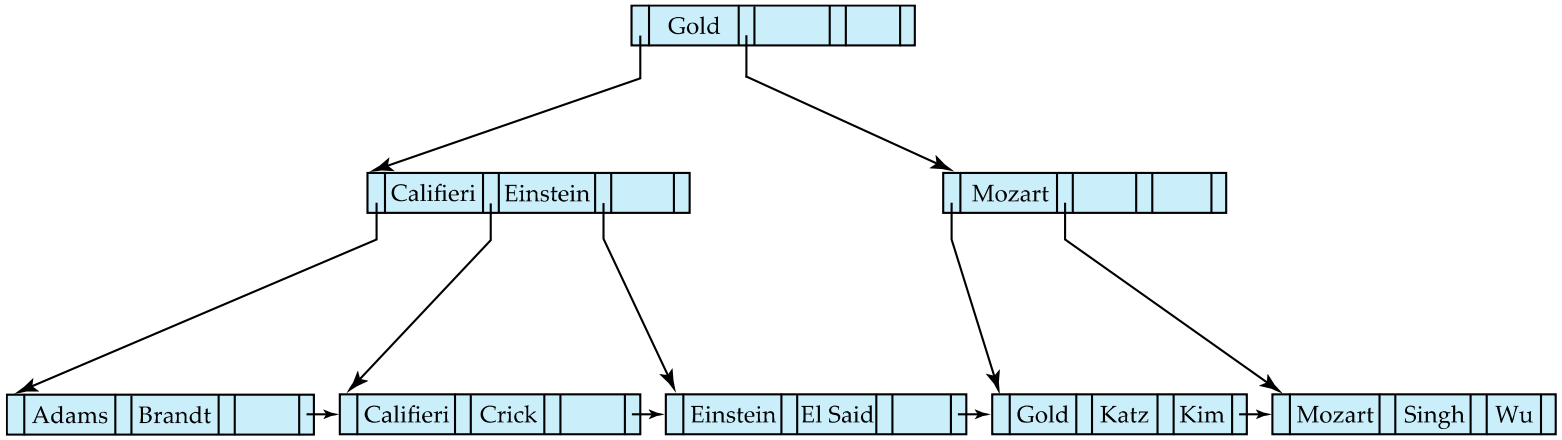
Before and after deleting "Srinivasan"



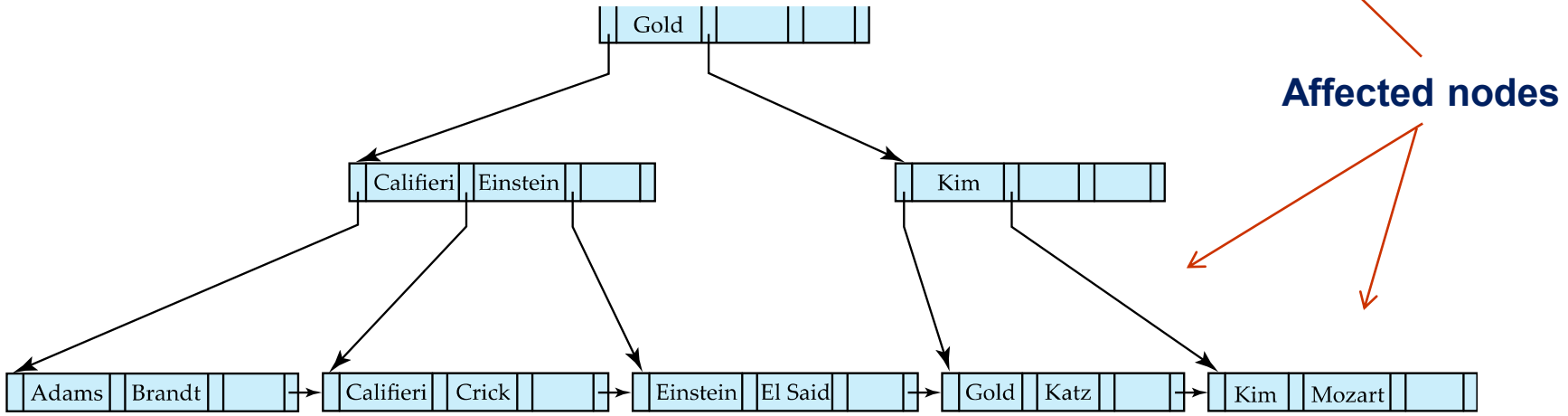
Affected nodes

- Deleting "Srinivasan" causes **merging** of under-full leaves

Examples of B+-Tree Deletion (Cont.)

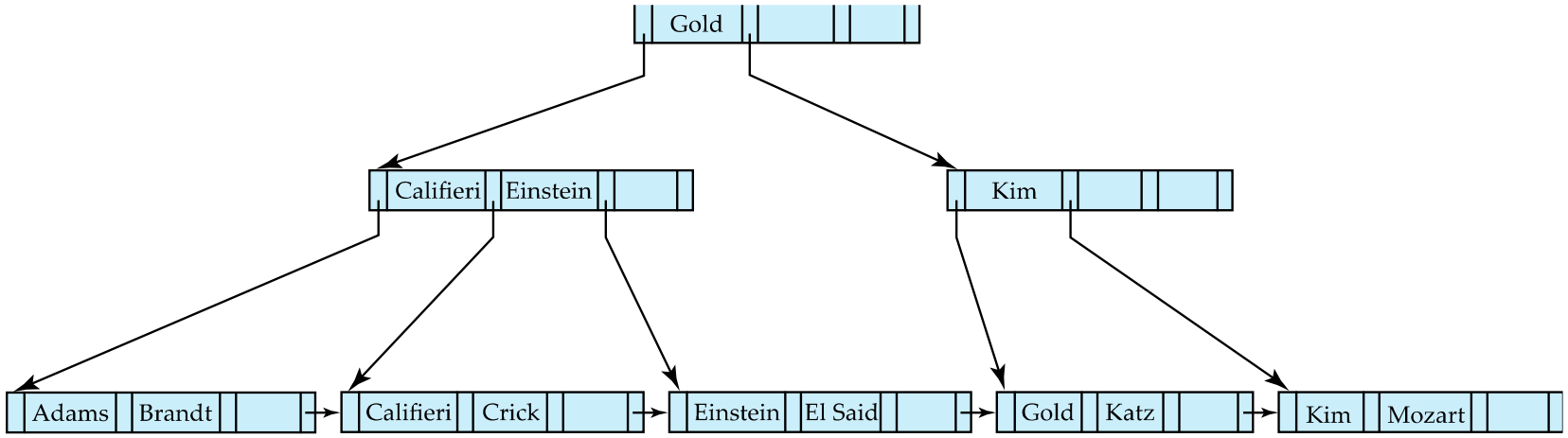


Before and after deleting "Singh" and "Wu"

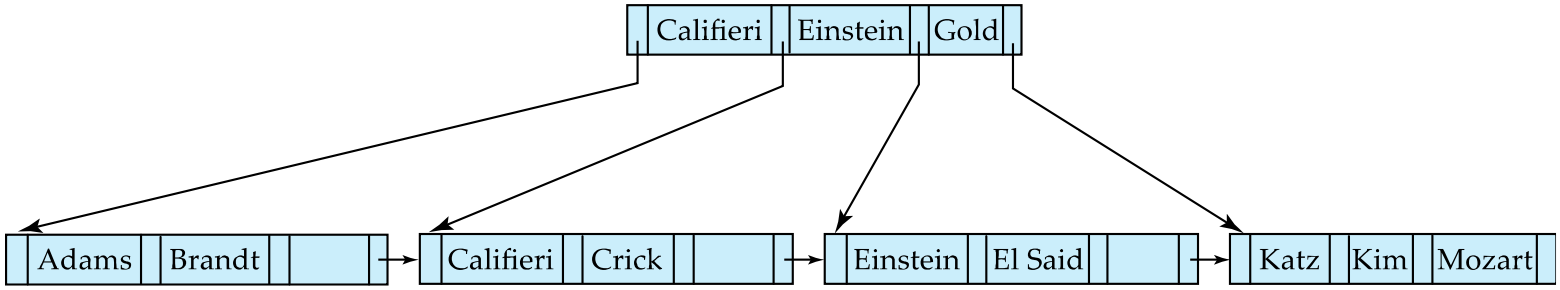


- Leaf containing Singh and Wu became underfull, and **borrowed a value** Kim from its left sibling
- Search-key value in the parent changes as a result

Example of B+-tree Deletion (Cont.)



Before and after deletion of "Gold"



- Node with Gold and Katz became underfull, and was merged with its sibling
- Parent node becomes underfull, and is merged with its sibling
  - Value separating two nodes (at the parent) is pulled down when merging
- Root node then has only one child, and is deleted

Updates on B+-Trees: Deletion

Assume record already deleted from file. Let *V* be the search key value of the record, and *Pr* be the pointer to the record.

- Remove (*Pr, V*) from the leaf node
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then ***merge siblings***:
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B+-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

Complexity of Updates

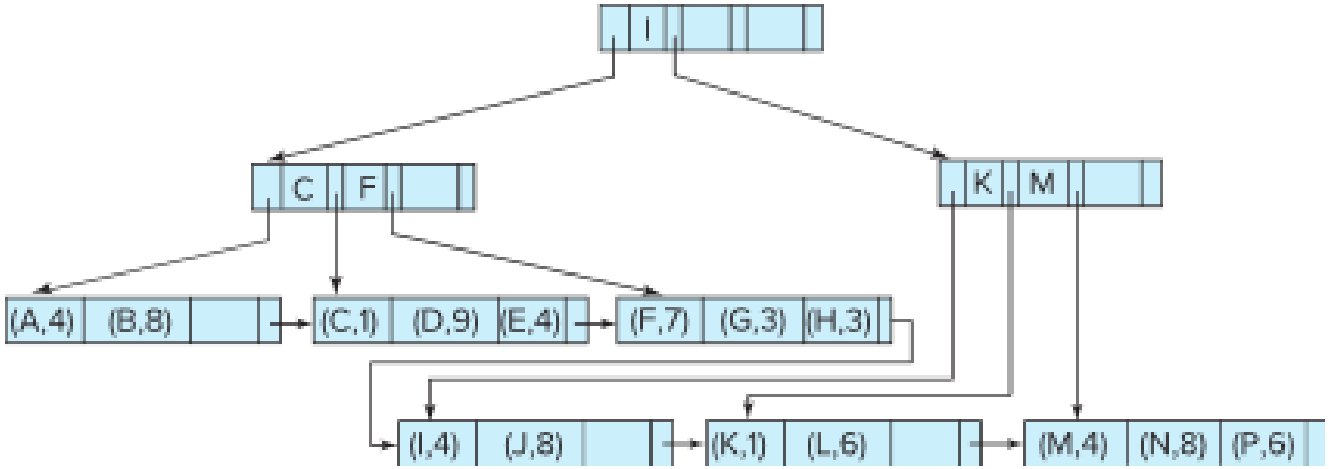
- Cost (in terms of number of I/O operations) of insertion and deletion of a single entry proportional to height of the tree
  - With K entries and maximum fanout of n, worst case complexity of insert/delete of an entry is $O(\log_{\lceil n/2 \rceil}(K))$
- In practice, number of I/O operations is less:
  - Internal nodes tend to be in buffer
  - Splits/merges are rare, most insert/delete operations only affect a leaf node
- Average node occupancy depends on insertion order
  - 2/3rds with random, ½ with insertion in sorted order

B^+-Tree File Organization

- B^+-Tree File Organization:
  - Leaf nodes in a B^+-tree file organization store records, instead of pointers
  - Helps keep data records clustered even when there are insertions/deletions/updates
- Leaf nodes are still required to be half full
  - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B^+-tree index.

B^+-Tree File Organization (Cont.)

- Example of B+-tree File Organization



- Good space utilization important since records use more space than pointers.
- To improve space utilization, involve more sibling nodes in redistribution during splits and merges
  - Involving 2 siblings in redistribution (to avoid split / merge where possible) results in each node having at least $\lfloor 2n/3 \rfloor$ entries

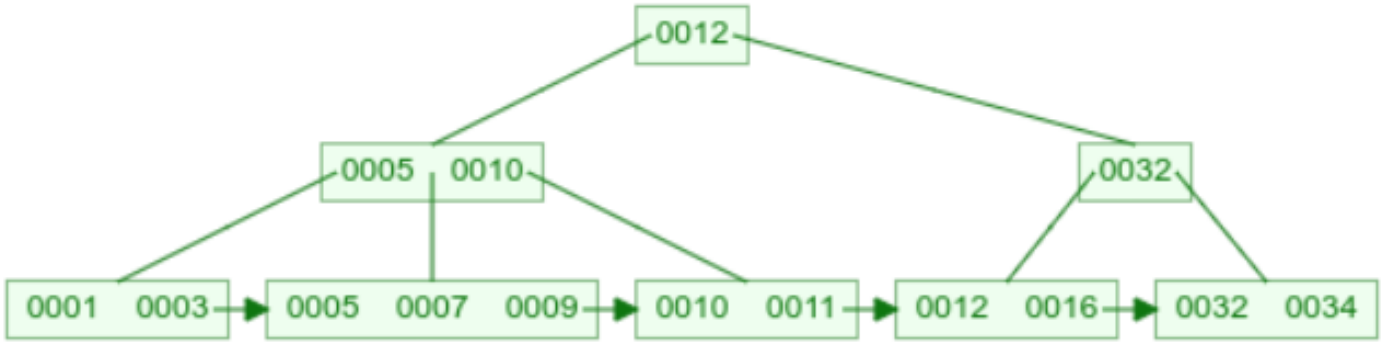
Indexing Strings

- Variable length strings as keys
  - Variable fanout
  - Use space utilization as criterion for splitting, not number of pointers
- **Prefix compression**
  - Key values at internal nodes can be prefixes of full key
    - Keep enough characters to distinguish entries in the subtrees separated by the key value
      - E.g., “Silas” and “Silberschatz” can be separated by “Silb”
  - Keys in leaf node can be compressed by sharing common prefixes

Bulk Loading and Bottom-Up Build

- Inserting entries one-at-a-time into a B^+-tree requires ≥ 1 IO per entry
  - assuming leaf level does not fit in memory
  - can be very inefficient for loading a large number of entries at a time (**bulk loading**)
- Efficient alternative 1:
  - sort entries first (using efficient external-memory sort algorithms discussed later in Section 12.4)
  - insert in sorted order
    - insertion will go to existing page (or cause a split)
    - much improved IO performance, but most leaf nodes half full
- Efficient alternative 2: **Bottom-up B^+-tree construction**
  - As before sort entries
  - And then create tree layer-by-layer, starting with leaf level
    - details as an exercise
  - Implemented as part of bulk-load utility by most database systems

Assume the following B+ tree (n=4):

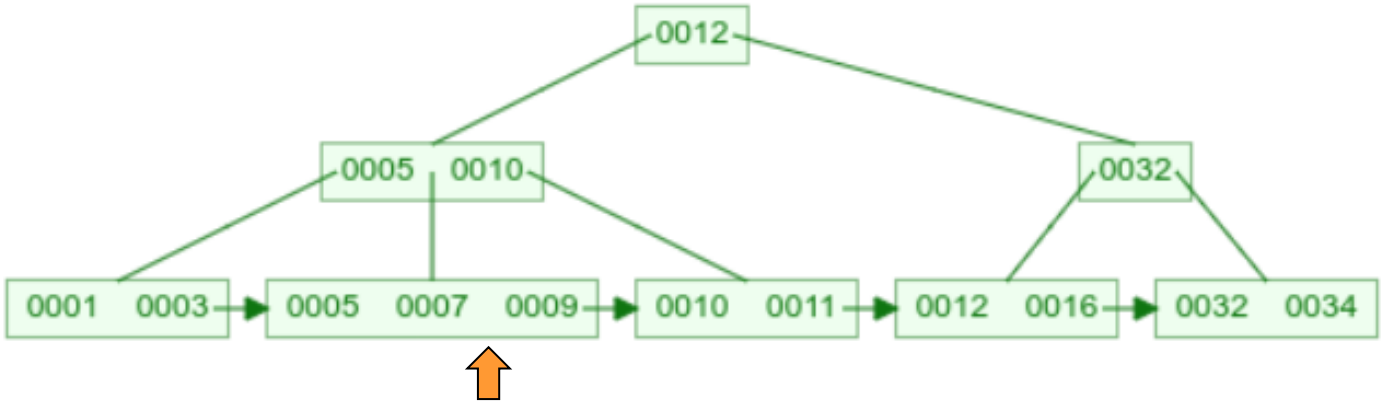


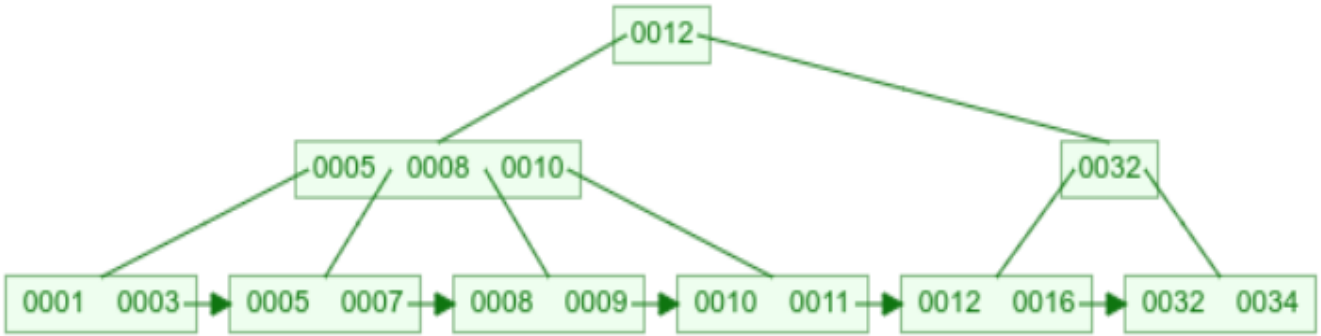
Execute the following actions (one after the other):

- Insert 8
- Insert 36
- Delete 11
- Delete 16

Assume the following B+ tree (n=4):

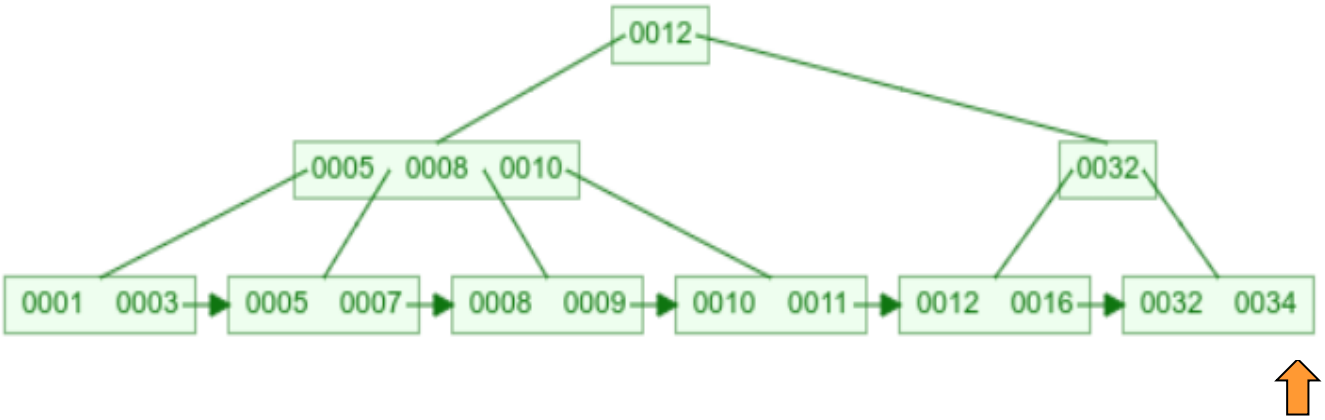
Insert 8

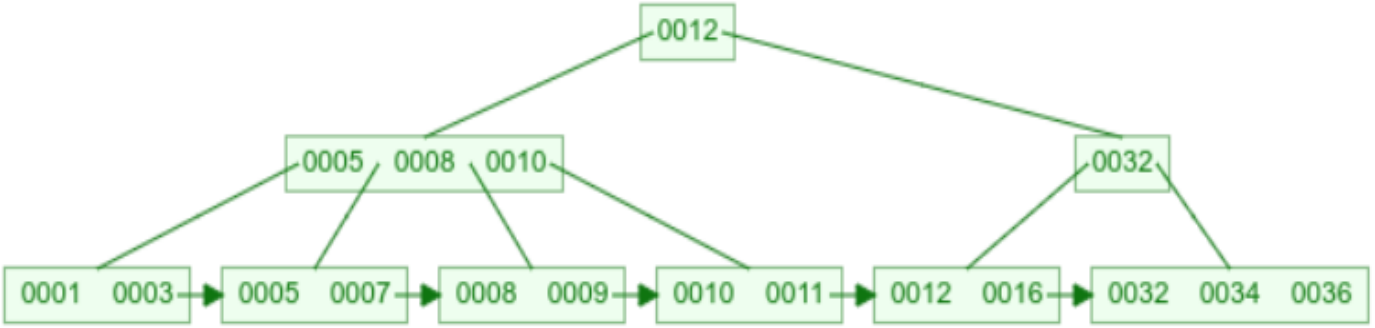




Assume the following B+ tree (n=4):

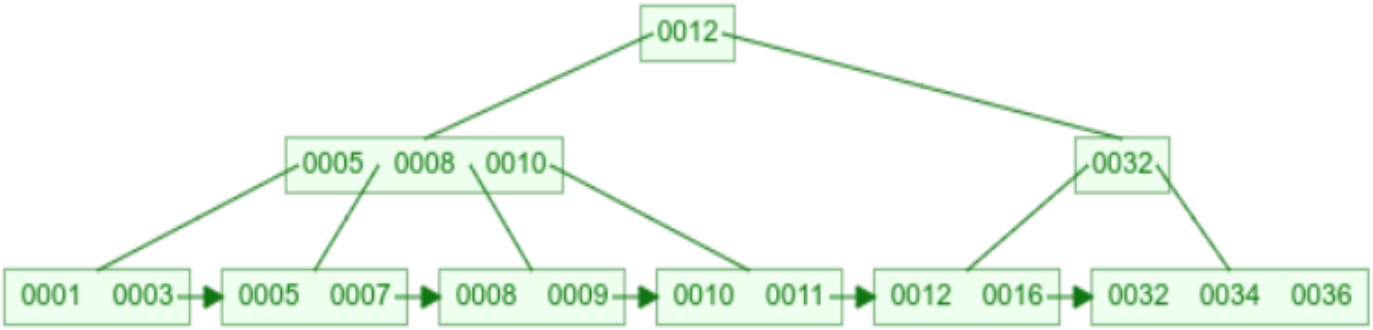
Insert 36

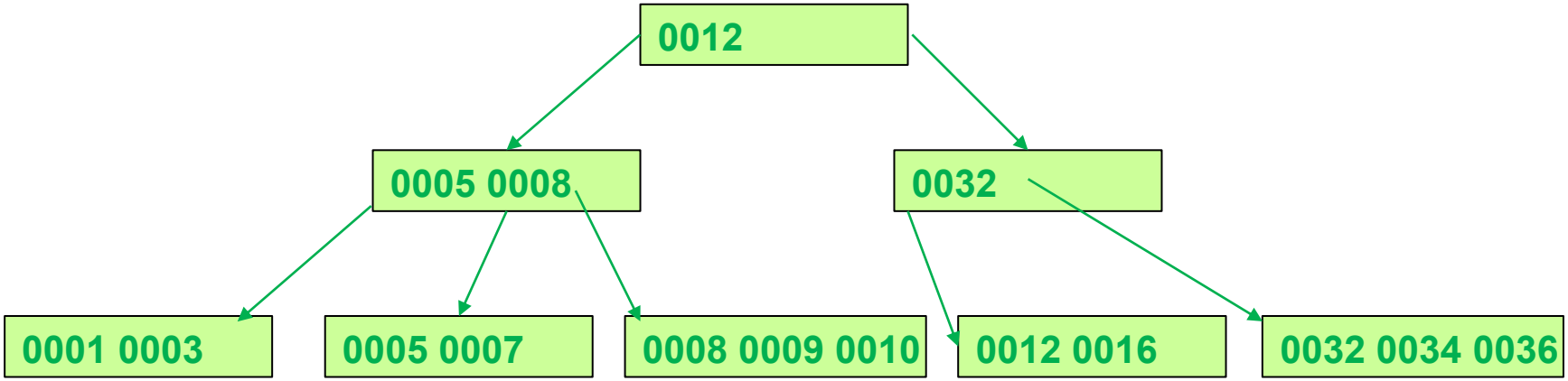




Assume the following B+ tree (n=4):

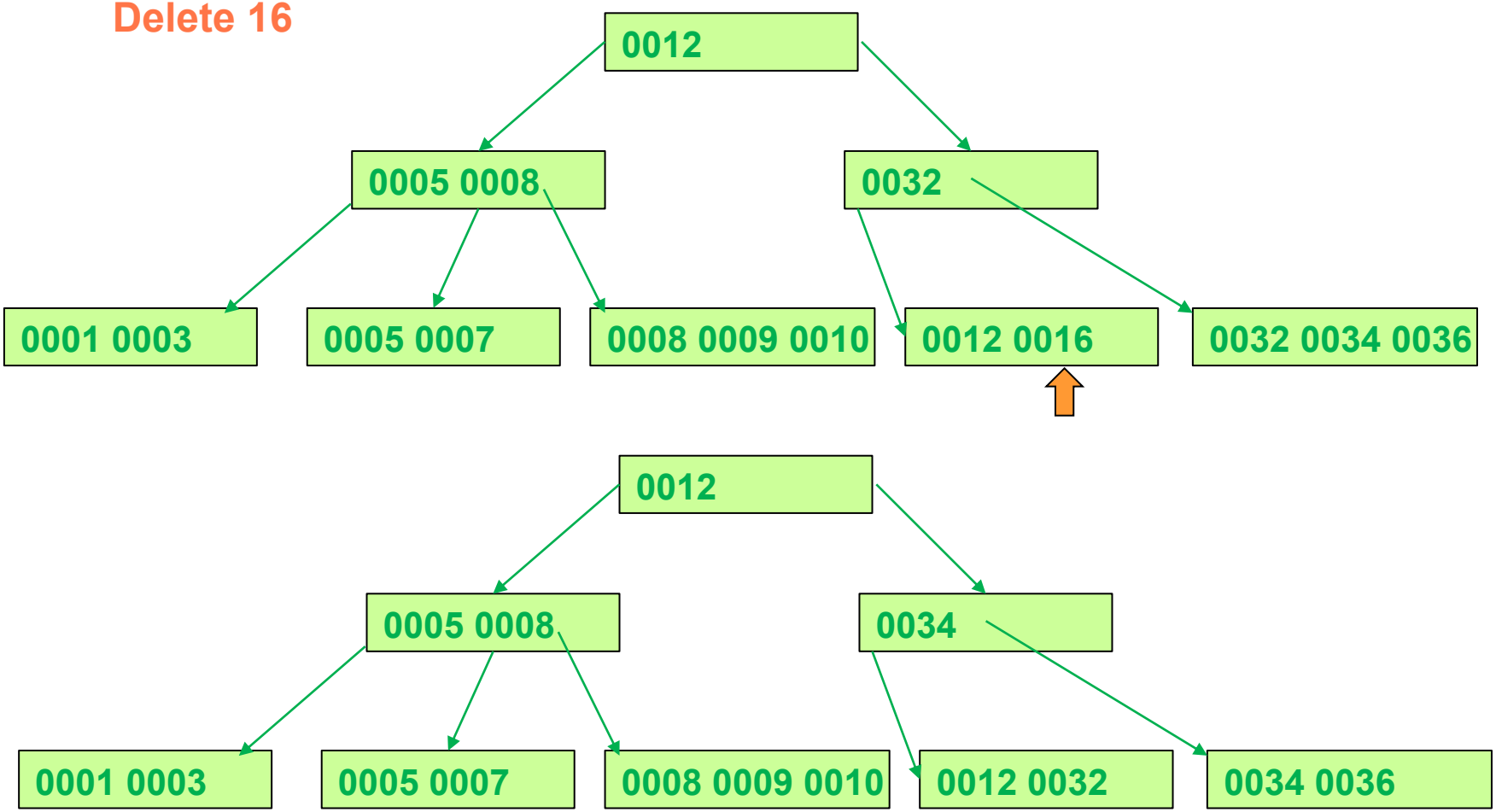
Delete 11





Assume the following B+ tree (n=4):

Delete 16



Indexing in Main Memory

- Random access in memory
  - Much cheaper than on disk/flash
  - But still expensive compared to cache read
  - Data structures that make best use of cache preferable
  - Binary search for a key value within a large B^+-tree node results in many cache misses
- B^+- trees with small nodes that fit in cache line are preferable to reduce cache misses
- Key idea: use large node size to optimize disk access, but structure data within a node using a tree with small node size, instead of using an array.

Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization
- Relative frequency of insertions and deletions
- Is it desirable to optimize average access time at the expense of worst-case access time?
- Expected type of queries:
  - Hashing is generally better at retrieving records having a specified value of the key.
  - If range queries are common, ordered indices are to be preferred
- In practice:
  - PostgreSQL supports hash indices, but discourages use due to poor performance
  - Oracle supports static hash organization, but not hash indices
  - SQLServer supports only B^+-trees

Multiple-Key Access

- Use multiple indices for certain types of queries.
- Example:

  select *ID*

  from *instructor*

  where *dept_name* = “Finance” **and** *salary* = 80000
- Possible strategies for processing query using indices on single attributes:
  1. Use index on *dept_name* to find instructors with department name Finance; test *salary = 80000*
  2. Use index on *salary* to find instructors with a salary of $80000; test *dept_name* = “Finance”.
  3. Use *dept_name* index to find pointers to all records pertaining to the “Finance” department. Similarly use index on *salary*. Take intersection of both sets of pointers obtained.

Indices on Multiple Keys

- **Composite search keys** are search keys containing more than one attribute
  - E.g., (*dept_name, salary*)
- Lexicographic ordering: $(a_1, a_2) < (b_1, b_2)$ if either
  - $a_1 < b_1$, or
  - $a_1 = b_1$ and $a_2 < b_2$

Indices on Multiple Attributes

Suppose we have an index on combined search-key
(*dept_name, salary*).

- With the **where** clause
  where *dept_name* = “Finance” **and** *salary* = 80000
  the index on (*dept_name, salary*) can be used to fetch only records that satisfy both conditions.
  - Using separate indices in less efficient — we may fetch many records (or pointers) that satisfy only one of the conditions.
- Can also efficiently handle
  where *dept_name* = “Finance” **and** *salary* < 80000
- But cannot efficiently handle
  where *dept_name* < “Finance” **and** *balance* = 80000
  - May fetch many records that satisfy the first but not the second condition

Creation of Indices

- Example
  create index *takes_pk* **on** *takes* (*ID,course_ID, year, semester, section*)
  drop index *takes_pk*
- Most database systems allow specification of type of index, and clustering.
- Indices on primary key created automatically by all databases
  - Why?
- Some database also create indices on foreign key attributes
  - Why might such an index be useful for this query:
    - $takes \bowtie \sigma_{name='Shankar'}(student)$
- Indices can greatly speed up lookups, but impose cost on updates
  - Index tuning assistants/wizards supported on several databases to help choose indices, based on query and update workload

Index Definition in SQL

- Create an index

  create index <index-name> **on** <relation-name> (<attribute-list>)

  E.g.,: **create index** *b-index* **on** *branch(branch_name)*

- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key.
  - Not really required if SQL **unique** integrity constraint is supported
- To drop an index

  drop index <index-name>

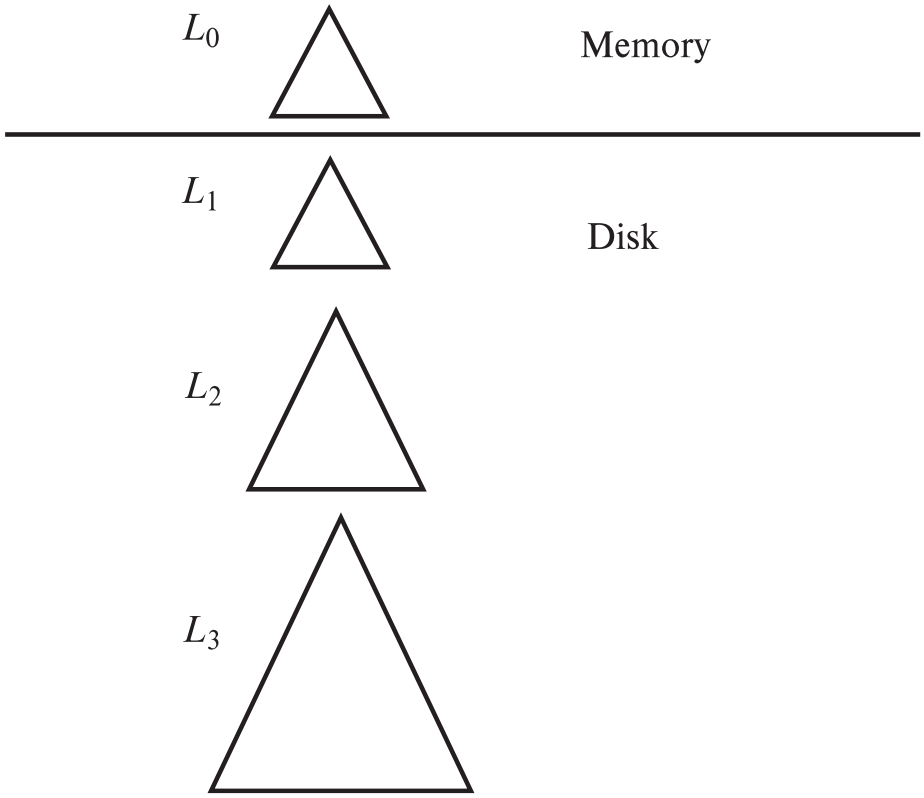
- Most database systems allow specification of type of index, and clustering.

Write Optimized Indices

- Performance of B^+-trees can be poor for write-intensive workloads
  - One I/O per leaf, assuming all internal nodes are in memory
  - With magnetic disks, < 100 inserts per second per disk
  - With flash memory, one page overwrite per insert
- Two approaches to reducing cost of writes
  - **Log-structured merge tree**
  - **Buffer tree**

Log Structured Merge (LSM) Tree

- Consider only inserts/queries for now
- Records inserted first into in-memory tree (L_0 tree)
- When in-memory tree is full, records moved to disk (L_1 tree)
  - B^+-tree constructed using bottom-up build by merging existing L_1 tree with records from L_0 tree
- When L_1 tree exceeds some threshold, merge into L_2 tree
  - And so on for more levels
  - Size threshold for L_{i+1} tree is *k* times size threshold for L_i tree



LSM Tree (Cont.)

- Benefits of LSM approach
  - Inserts are done using only sequential I/O operations
  - Leaves are full, avoiding space wastage
  - Reduced number of I/O operations per record inserted as compared to normal B^+-tree (up to some size)
- Drawback of LSM approach
  - Queries have to search multiple trees
  - Entire content of each level copied multiple times
- Stepped-merge index
  - Variant of LSM tree with multiple trees at each level
  - Reduces write cost compared to LSM tree
  - But queries are even more expensive
    - Bloom filters to avoid lookups in most trees
- Details are covered in Chapter 24

LSM Trees (Cont.)

- Deletion handled by adding special “delete” entries
  - Lookups will find both original entry and the delete entry, and must return only those entries that do not have matching delete entry
  - When trees are merged, if we find a delete entry matching an original entry, both are dropped.
- Update handled using insert+delete
- LSM trees were introduced for disk-based indices
  - But useful to minimize erases with flash-based indices
  - The stepped-merge variant of LSM trees is used in many BigData storage systems
    - Google BigTable, Apache Cassandra, MongoDB
    - And more recently in SQLite4, LevelDB, and MyRocks storage engine of MySQL