Δευτέρα, 03/04/2023

$\dot{x} = Ax + Bu$

$\frac{1}{2}\int_0^{t_f}(x^T Q x + u^T R u)\,d\tau + \frac{1}{2}x^T(t_f)\,S\,x(t_f)$, $R = R^T > 0$, $Q = Q^T \geq 0$, $S = S^T \geq 0$

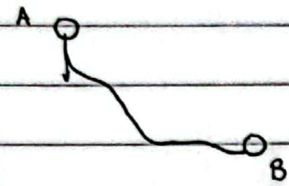
($R > 0$, για να είναι convex η συν. κόστους)

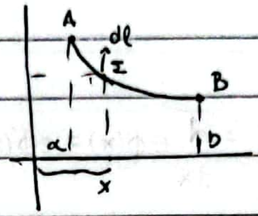
Για $\frac{1}{2}\int_0^{\infty}(x^T Q x + u^T R u)\,dt$ κ' (A, B) σταθεροποιήσιμο

$(Q^{1/2}, A)$ ανιχνεύσιμο

R_0, $R_0^T R_0 = Q$

Βραχυστόχρονο (Bernoulli)

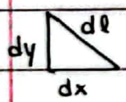




$y(a) = \alpha$, $y(b) = \beta$

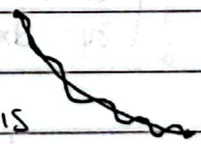
ΑΔΕ : $mga = \frac{1}{2}mv^2 + mgy \Rightarrow v = \sqrt{2g(\alpha - y)}$

$$T_{A\to B} = \int_0^T dt = \int \frac{dl}{v} = \int_\alpha^b \sqrt{\frac{1+(y'(x))^2}{2g(\alpha - y(x))}}\,dx$$



$$dl = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + (y'(x))^2}\,dx$$

$$\min_{y(\cdot)} \int_\alpha^b F(x, y, y')\,dx, \quad y(a) = \alpha, \quad y(b) = \beta$$



Θεωρώ μια βέλτιστη τροχιά κ' εικονικές μετατοπίσεις

$\hookrightarrow \delta$

$$\delta F = \sum_{i=1}^{n} \frac{\partial F}{\partial u_i}\delta u_i, \quad \delta u_i = \epsilon a_i, \quad a_i = \cos\theta_i$$

$$\sum_i \frac{\partial F}{\partial u_i} a_i = 0$$

$$\delta F = \sum \frac{\partial F}{\partial u_i}\delta u_i = 0, \quad f = 0 \Rightarrow \delta f = 0 \Rightarrow \sum \frac{\partial f}{\partial u_i}\delta u_i = 0$$

$$\delta F + \lambda \delta f = 0 \quad \Rightarrow \sum_{k=1}^{n-1}\left(\frac{\partial F}{\partial u_k} + \lambda\frac{\partial f}{\partial u_k}\right)\delta u_k = 0$$

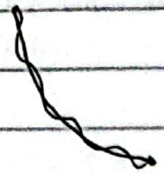
Θέλω να ελαχιστοποιήσω τη $\bar{F} = F + \lambda f$, $\delta\bar{F} = \delta F + \lambda\delta f + \cancel{\delta\lambda f}^{\,0}$

Scanned with CamScanner

(βλ. Calculus of Variations, Lanczos)

$$\min_{y(\cdot)} I = \int_{\alpha}^{b} F(y,y',x)\,dx$$

$$y(a)=\alpha,\ y(b)=\beta$$



$$\bar{f}(x) = f(x) + \epsilon\phi(x)$$

$$\delta y = \epsilon\phi(x)$$

$$\delta y(x)\big|_{x=a} = 0,\ \phi(a)=\phi(b)=0$$

Παράγωγος μεταβολής

$$\frac{d}{dx}\delta y = \frac{d}{dx}\left[\bar{f}(x)-f(x)\right] = \frac{d}{dx}\epsilon\phi(x) = \epsilon\phi'(x)$$

$$\delta I = \int_a^b F(y+\underset{\epsilon\phi}{\delta y},\ y'+\underset{\epsilon\phi'}{\delta y'},\ x)\,dx - \int_a^b F(y,y',x)$$

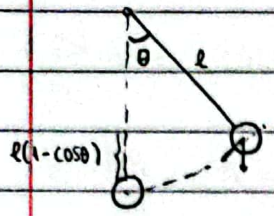
$$= \int_a^b \left(\frac{\partial F}{\partial y}\epsilon\phi + \frac{\partial F}{\partial y'}\epsilon\phi'\right)dx$$

$$\lim_{\epsilon\to 0}\frac{\delta I}{\epsilon} = \int_a^b \left(\frac{\partial F}{\partial y}\phi + \frac{\partial F}{\partial y'}\phi'\right)dx$$

$$\int_a^b \frac{\partial F}{\partial y'}\frac{d}{dx}(\phi)\,dx - \cancel{\frac{\partial F}{\partial y'}\phi\Big|_\alpha^b}^{\,0} - \int_a^b \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)\phi\,dx$$

$$\lim_{\epsilon\to 0}\frac{\delta I}{\epsilon} = \int_a^b \underbrace{\left(\frac{\partial F}{\partial y} + \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)\right)}_{E(x)}\phi(x)\,dx = 0 \quad \forall\phi(x)\ \text{με}\ \phi(a)=\phi(b)=0$$

$$\boxed{\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0}$$ Δ.Ε. Euler-Lagrange



$$I=\int (E_{KIN}-E_{\Delta YN})$$

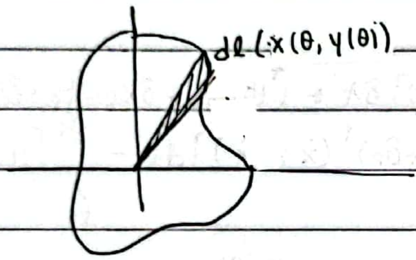
$$E_k=\frac{1}{2}I\dot{\theta}^2,\quad I=m\ell^2$$

$$F=E_{KIN}-E_{\Delta YN}=\frac{1}{2}m\ell^2\dot{\theta}^2-mg\ell(1-\cos\theta)$$

$$\frac{\partial F}{\partial \theta}-\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial \dot{\theta}}\right)=0 \Rightarrow \ddot{\theta}=-\frac{g}{\ell}\sin\theta$$

$$I=\int F(q_1,\dots,q_n;\dot{q}_1,\dot{q}_2,\dots,\dot{q}_n,t)\,dt$$

$$f_i(q_1,\dots,q_n,t)=0$$



$$dE=\frac{1}{2}r\underbrace{d\ell}_{r\,d\theta}=\frac{1}{2}(x^2(\theta)+y^2(\theta))\,d\theta$$

$$\max_{x(\cdot),y(\cdot)} E=\frac{1}{2}\int_0^{2\pi}(x^2(\theta)+y^2(\theta))\,d\theta$$

$$C=\int_0^{2\pi}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$$

$L=\frac{1}{2}(x^2(\theta)+y^2(\theta))$, ακρότατο στη $L+\lambda f$

$\Rightarrow \dots \Rightarrow r=$ σταθ, $2\pi r=C \Rightarrow r=\frac{C}{2\pi}$ κύκλος

$$\dot{x} = f(x,u,t), \quad x \in \mathbb{R}^n, \; u \in \mathbb{R}^m$$

$$J = \int_{t_0}^{t_f} \underbrace{L(x(t), u(t), t)}_{\text{running cost}} dt + \underbrace{\phi(x(t_f), t_f)}_{\text{cost at terminal state}}$$

$$\psi(x(t_f), t_f) = 0, \quad \psi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^p$$

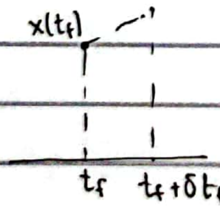
$$\tilde{J} = \int_{t_0}^{t_f} \Big[L(x,u,t) + \underbrace{p^T (f(x,u,t) - \dot{x})}_{\sum p_i (f_i - \dot{x}_i)} \Big] dt + \phi(x(t_f), t_f) + \underbrace{\lambda^T \psi(x(t_f), t_f)}_{\sum \lambda_i \psi_i}$$

$$H(x,p,u,t) := L(x,u,t) + p^T f(x,u,t)$$

$$\delta\tilde{J} = \phi(x_f + \delta x_f, t_f + \delta t_f) - \phi(x_f, t_f) + \psi^T \delta\lambda + \lambda^T \psi(x_f + \delta x_f, t_f + \delta t_f) - \lambda^T \psi(x_f, T_f)$$
$$+ \int_{t_0}^{t_f + \delta t_f} \left[H(x + \delta x, u + \delta u, p + \delta p, t) - (p + \delta p)^T (\dot{x} + \delta \dot{x}) \right] dt - \int_{t_0}^{t_f} \left[H(x,p,u,t) - p^T \dot{x} \right] dt$$

$$\delta\tilde{J} = \delta t_f \left(\frac{\partial \phi}{\partial t} + \sum \lambda_i \frac{\partial \psi_i}{\partial t} + H - p^T \dot{x} \right) + \left(\frac{\partial \phi}{\partial x} + \sum \lambda_i \frac{\partial \psi_i}{\partial x} \right)^T \delta x_f$$
$$+ \int_0^{t_f} \left[\left(\frac{\partial H}{\partial x} \right)^T \delta x + \left(\frac{\partial H}{\partial u} \right)^T \delta u + \left(\frac{\partial H}{\partial p} \right)^T \delta p - \underbrace{p^T \delta \dot{x}}_{(*)} - \delta p^T \dot{x} \right] dt$$

$$-\int_{t_0}^{t_f} p^T \delta \dot{x} \, dt = -\int_{t_0}^{t_f} p^T \frac{d}{dt} (\delta x) dt = -p^T(t_f) \delta x(t_f) + \int_{t_0}^{t_f} \dot{p}^T \delta x \, dt$$



$$\delta x_f = \delta x(t_f) + \dot{x}(t_f) \delta t_f$$

$$(*) = \int_{t_0}^{t_f} \left[\left(\frac{\partial H}{\partial x} + \dot{p} \right)^T \delta x + \left(\frac{\partial H}{\partial u} \right)^T \delta u + \left(\frac{\partial H}{\partial p} - \dot{x} \right)^T \delta p \right] dt$$
$$+ \delta t_f \left[\frac{\partial \phi}{\partial t} + \sum \lambda_i \frac{\partial \psi_i}{\partial t} + H \right] \Bigg|_{t = t_f}$$
$$+ (\delta x_f)^T \left(\frac{\partial \phi}{\partial x} + \sum \lambda_i \frac{\partial \psi_i}{\partial x} - p \right) \Bigg|_{t = t_f}$$

$$\begin{cases} \dfrac{\partial H}{\partial u} = 0 & \text{(δίνει βέλτιστη είσοδο)} \\[2ex] \dot{x} = \dfrac{\partial H}{\partial p} = f(x,u,t) & \\[2ex] \dot{p} = -\dfrac{\partial H}{\partial x} & \text{(co-state equation, εξίσωση συμπληρωματικών καταστάσεων)} \end{cases}$$

<u>Οριακές συνθήκες</u>: $\delta x_f: \left. p - \dfrac{\partial \phi}{\partial x} - \sum \lambda_i \dfrac{\partial \psi_i}{\partial x} \right|_{t_f} = 0$

$$\delta t_f: \left. H + \dfrac{\partial \phi}{\partial t} + \sum \lambda_i \dfrac{\partial \psi_i}{\partial t} \right|_{t_f} = 0$$

↳ Αναγκαίες συνθήκες για βέλτιστο νόμο ελέγχου