Φυλλάδιο 3

Friday, 7 January 2022 6:33 PM

Άσκηση 2 ✓

(α) $P(A)\,P(B|A)\,P(C|AB) = P(A)\frac{P(AB)}{P(A)}\frac{P(CAB)}{P(AB)} = P(ABC)$.

(β) $P(A|B) = \frac{P(AB)}{P(A)} = \begin{cases} 0, & \emptyset \\ 1, & A \subset B \\ \frac{P(B)}{P(A)}, & B \subset A \end{cases}$

(γ) $P(A|B) > P(A) \Leftrightarrow \frac{P(AB)}{P(B)} > P(A) \Leftrightarrow P(B|A) > P(B)$.

Άσκηση 3 ✓

(α) $P(\text{σφάλμα}) = P(\Delta_0\Pi_1) + P(\Delta_1\Pi_0) = P(\Delta_0|\Pi_1)\,P(\Pi_1) + P(\Delta_1|\Pi_0)\,P(\Pi_0)$

$P(\Delta_0|\Pi_0) + P(\Delta_1|\Pi_0) = 1 \Rightarrow P(\Delta_1|\Pi_0) = \frac{1}{100}$

$P(\Delta_1|\Pi_1) + P(\Delta_0|\Pi_1) = 1 \Rightarrow P(\Delta_0|\Pi_1) = \frac{3}{100}$

(β) $P(\Pi_0|\Delta_1) = \frac{P(\Pi_0\Delta_1)}{P(\Delta_1)} = \frac{P(\Delta_1|\Pi_0)\,P(\Pi_0)}{P(\Delta_1)} = \frac{\frac{1}{100}\cdot\frac{4}{5}}{P(\Delta_1|\Pi_0)\,P(\Pi_0) + P(\Delta_1|\Pi_1)\,P(\Pi_1)} = \dots$

Άσκηση 4 ✓

(α) $P(\Theta|A') = 0{,}1$, $P(\Theta'|A) = 0{,}01$

(θετικός → άρρωστος)

$P(A|\Theta) = \frac{P(\Theta|A)\,P(A)}{P(\Theta)} = \frac{P(\Theta|A)\,P(A)}{P(\Theta|A)\,P(A) + P(\Theta|A')\,P(A')} = \dots$

($P(\Theta|A) = 0{,}99$, $P(A) = 0{,}08$, $P(A') = 0{,}92$)

(β) $P(A|\Theta_1\Theta_2') = \frac{P(\Theta_1\Theta_2'|A)\,P(A)}{P(\Theta_1\Theta_2')} = \frac{P(\Theta_1|A)\,P(\Theta_2'|A)\,P(A)}{P(\Theta_1\Theta_2'|A)\,P(A) + P(\Theta_1\Theta_2'|A')\,P(A')} = \dots$

($P(\Theta_1|A) = 0{,}19$, $P(\Theta_2'|A) = 0{,}08$, $P(A) = 0{,}08$, $P(A') = 0{,}92$)

→ τα στέλνω κτλπ

Άσκηση 5

$P(M_i) = \rho$, $P(A_i) = q$

$P(M) = P(M_1) + P(M_1'A_1'M_2) + \dots + P\left(\bigcap_{i=1}^{k} M_k' \bigcap_{i=1}^{k} A_k' M_{k+1}\right) = \rho + (1-\rho)(1-q)\rho + \dots + (1-\rho)^k(1-q)^k\rho =$

$= \sum_{i=1}^{k}\left[(1-\rho)(1-q)\right]^k + \rho = \rho + \frac{1}{1-(1-\rho)(1-q)}$

Άσκηση 6 ✓ ~ 4ο δύσκολο

- $P(\text{καύκα}) = 1 - P(\text{ακρ. 1}) - P(\text{ακρ. 2}) - \dots = 1 - (\rho_1 + \rho_2 + \dots) = 1 - \sum_{i=1}^{\infty}\rho_i = 1 - \sum_{i=1}^{\infty}\left(\frac{11}{23}\right)^i = 1 - \left(\frac{1}{1-\frac{11}{23}} - 1\right) = 2 - \frac{23}{12} = \frac{1}{12}$
- $p(2\text{ κορ. ακρ.}) = P(\Theta|A_0)\,P(A_0) + P(\Theta|A_1)\,P(A_1) + P(\Theta|A_2)\,P(A_2) + \dots = \sum_{i=2}^{\infty} P(\Theta|A_i)\,P(A_i) = \sum_{i=1}^{\infty}\frac{\binom{i}{2}}{2^i}\left(\frac{11}{23}\right)^i$

($P(\Theta|A_0) = 0$, $P(\Theta|A_1) = 0$)

↓ 2 κορ. ακρ. ↓ 0 ακρ. παιδιά

* Άσκηση 1

Matching problem