10.03.22 Credits: Μάριος Ρόζος

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Άσκηση 1.10

$\frac{\partial}{\partial \varphi} = 0$

• $\vec{J} = -\hat{r}\, J_r(r) = ;$ • $\vec{K}_a = ;$ • $\vec{K}_b = ;$

$$J_r(r) = \frac{2I}{4\pi r^2} = \frac{I}{2\pi r^2} \Rightarrow \vec{J} = -\hat{r}\frac{I}{2\pi r^2} \quad (1)$$

$$K_a(\theta)\cdot 2\pi a \sin\theta = I - \iint |J_r(r)|\cdot dS_r =$$

$$= I - \frac{I}{2\pi}\int_{\theta'=0}^{\theta}\int_{\varphi=0}^{2\pi} \frac{\cancel{r^2}\sin\theta'}{\cancel{r^2}}\, d\theta'\, d\varphi = I - \frac{I}{\cancel{2\pi}}\cancel{2\pi}\int_0^{\theta}\sin\theta'\, d\theta' =$$

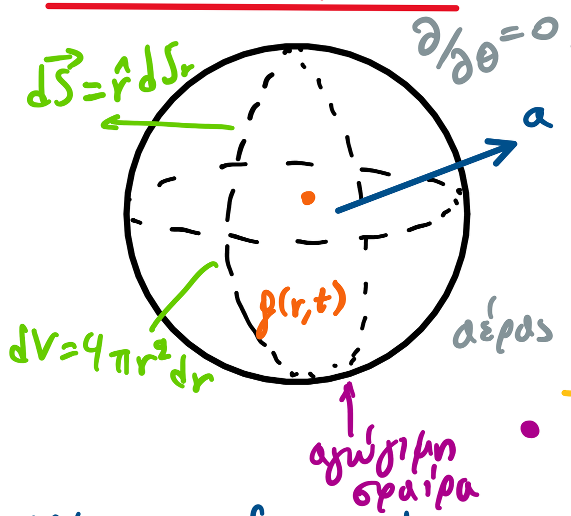
$$= I - I(1-\cos\theta) = I\cos\theta$$

$$K_a(\theta) = \frac{I\cos\theta}{2\pi a}\cot\theta\cdot\hat{\theta}, \quad 0<\theta<\pi$$

Όμοια, προκύπτει:

$$\vec{K}_b = -\frac{I}{2\pi b}\cot\theta\cdot\hat{\theta}, \quad 0<\theta<\pi$$

Άσκηση 1.12



$\partial/\partial\theta = 0$, $\frac{\partial}{\partial\varphi} = 0 \rightarrow$ (σφαιρική συμμετρία)

$\rho(r,t) = \rho\cdot\frac{r}{a}e^{-t/\tau}, \quad 0 \le r \le a$

$\rho(r,t) = 0, \quad r>a,\ t>0$

$\sigma(r=a,\ t=0) = 0$

• $\vec{J}(r,t) = ; \quad 0 \le r < \infty$

Νόμος διατήρηση φορτίου

$$\oint_S \vec{J}\, d\vec{S} = -\frac{d}{dt}\int_V \rho\, dV = -\frac{d}{dt}Q_{\varepsilon\sigma} \quad (1)$$

$$Q_{\varepsilon\sigma}(r,t) = \int_V \rho\, dV = \frac{\rho_0}{a}e^{-t/\tau}\int_{r'=0}^{r} r'\, 4\pi r'^2\, dr' = \frac{\rho_0}{a}e^{-t/\tau}\cdot 4\pi\frac{r^4}{4} = \frac{\rho_0\pi e^{-t/\tau}}{a}r^4 \quad (2)$$

$$(1),(2) \Rightarrow J_r(r,t)\, 4\pi r^2 = -\frac{d}{dt}\left(\frac{\pi\rho_0 r^4}{a}e^{-t/\tau}\right) = \frac{\pi\rho_0}{a\tau}r^4 e^{-t/\tau}$$

$$\Rightarrow J_r(r,t) = \frac{\rho_0}{4a\tau}r^2 e^{-t/\tau} \Rightarrow J_r(r,t) = \frac{\rho_0}{4a\tau}r^2 e^{-t/\tau} \Rightarrow \vec{J}_r(r,t) = \frac{\rho_0}{4a\tau}r^2 e^{-t/\tau}\,\hat{r}, \quad 0 \le r < a$$

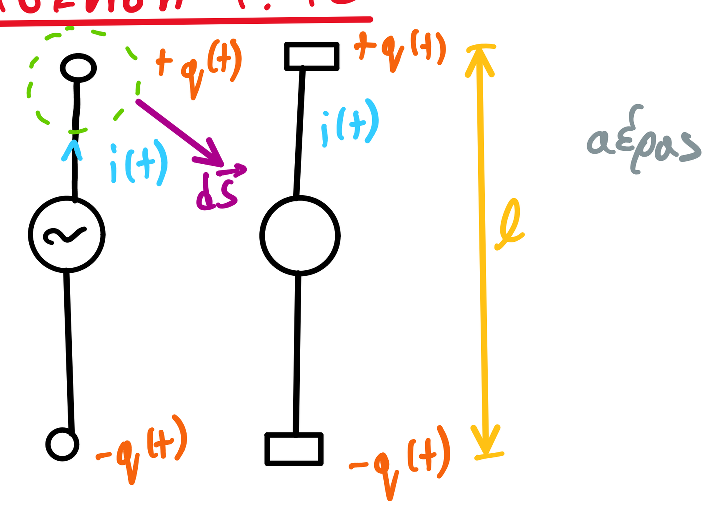
$$\vec{J}_r(r,t) = 0, \quad r>a \quad (\text{αέρας})$$

$$(2) \Rightarrow Q_{\varepsilon\sigma}(r=a,t) = \frac{\rho_0\pi}{a}e^{-t/\tau}a^4 = \pi\rho_0 a^3(1-e^{-t/\tau}), \quad t \ge 0 \quad (5)$$

$$\sigma(r=a,\ t\ge 0) = \frac{\delta Q_{\varepsilon\sigma}(r=a,\ t\ge 0)}{4\pi a^2} = \frac{\rho_0 a}{4}(1-e^{-t/\tau}) \quad (6)$$

Για $t\to\infty$ $\sigma(r=a,\ t\ge 0) \to \frac{\rho_0 a}{4}$

Άσκηση 1.13



$l \ll \lambda \rightarrow$ μήκος κύματος

(δίπολο Hertz)

$i(t) = I_{max}\cos\omega t$

ω: κυκλική συχνότητα

$$\oint \vec{J}\, d\vec{S} = -\frac{d}{dt}q(t)$$

σταθερά ανεξάρτητη από τον χρόνο t

$$\Rightarrow -i(t) = -\frac{d}{dt}q(t) \Rightarrow q(t) = \int i(t)\, dt + C$$

$$= I_{max}\int\cos\omega t\, dt + \sigma\tau\alpha\theta.$$

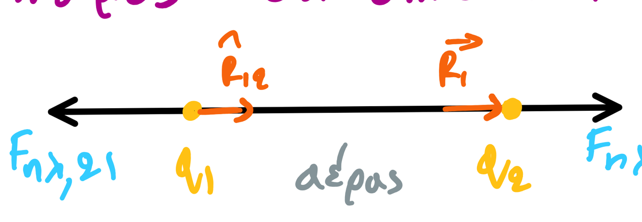
$$= -I_{max}\frac{\sin(\omega t)}{\omega} + \cancel{\sigma\tau\alpha\theta.}^{\,0}$$

αίτιο χρονομεταβλητό άρα σταθ. = 0

$$q(t) = -\frac{I_{max}}{\omega}\sin(\omega t)$$

Σύντομη ανασκόπηση της ιστορικής μεθόδου

Νόμος Coulomb (1785)



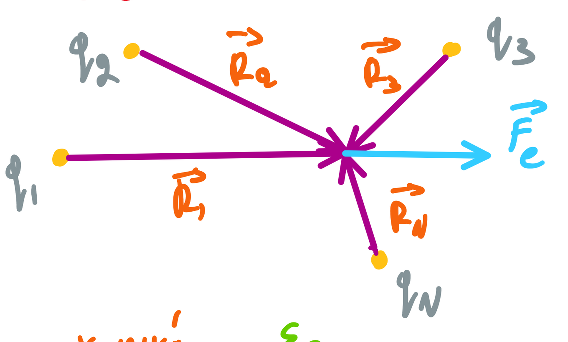
$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 R_{12}^2}\hat{R}_{12}$$

$$\hat{R}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}, \quad |\vec{R}_{12}| = R_{12}$$

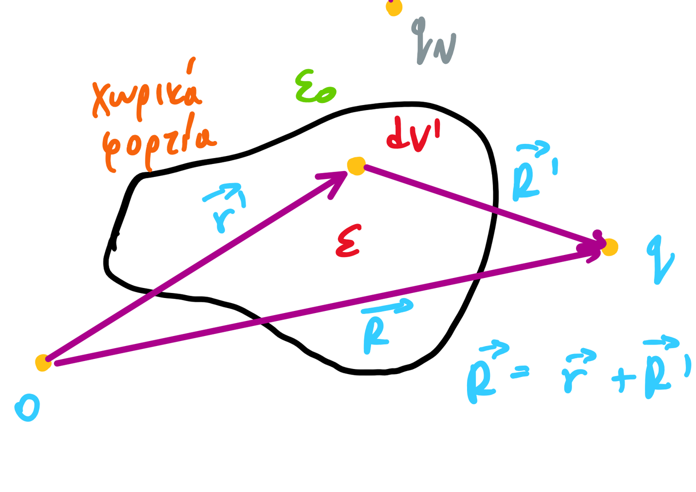
$\varepsilon_0 = 8{,}854$ pF/m διηλεκτρική σταθερά

επιτρεπτότητα του κενού και του αέρα

Επαλληλία



$$\vec{F}_e = \frac{q}{4\pi\varepsilon_0}\sum_{n=1}^{N}\frac{q_n\hat{R}_n}{R_n^2}, \quad \hat{R}_n = \frac{\vec{R}_n}{R_n}$$



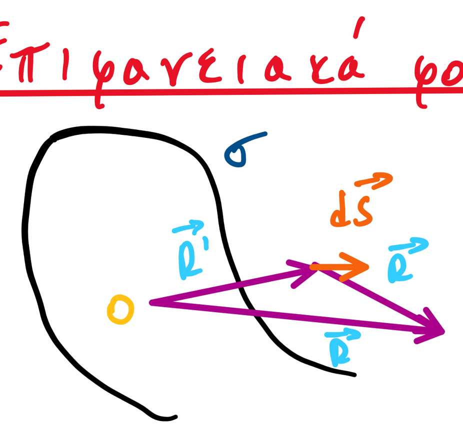
$$\vec{F}_e = \frac{q}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}')\, dV'}{R^2}\hat{R}$$

$$\delta\vec{F}_e = \frac{\delta q}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}')\, dV'}{R^2}$$

ένταση ηλεκτρικού πεδίου (V/m)

$$\vec{E} = \frac{\delta\vec{F}_e}{\delta q} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}')\cdot dV'}{R^2}\hat{R}$$

Επιφανειακά φορτία

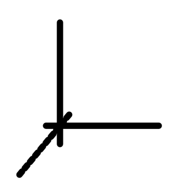


$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(\vec{r}')\, dS'}{R^2}\hat{R}$$

Γραμμικά φορτία

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_l \frac{\lambda(\vec{r}')\, dl'}{R^2}\hat{R}$$

Σημειακό φορτίο



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\ \ldots\ ?$$