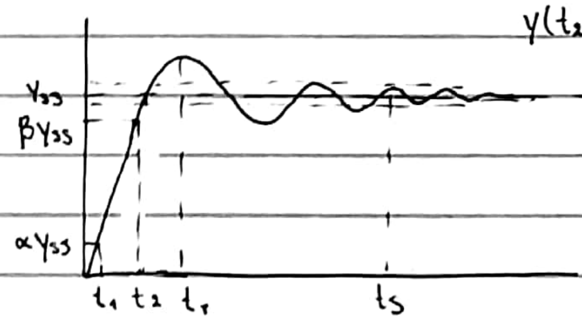
Δευτέρα, 24/10/2022

Απαιτήσεις Απόδοσης

Τελική τιμή y_{ss}
↳ μόνιμη κατάσταση

Χρόνος ανόδου: $t_r = t_2 - t_1$, $y(t_1) = \alpha y_{ss}$ π.χ. $\alpha = 0.1$

$y(t_2) = \beta y_{ss}$ π.χ. $\beta = 0.9$



Χρόνος κορυφής $t_p = \{t \mid y(t) = \max_{s\in[0,+\infty)} y(s)\}$

Ποσοστό υπερύψωσης: $M_p = \dfrac{y_p - y_{ss}}{y_{ss}} \times 100\%$

Χρόνος αποκατάστασης (settling time):

$$t_s = \min\{t \mid |y(t') - y_{ss}| \le \delta_{y_{ss}} \quad \forall t' \ge t\} \qquad \delta = 0.02 \text{ ή } 0.04$$

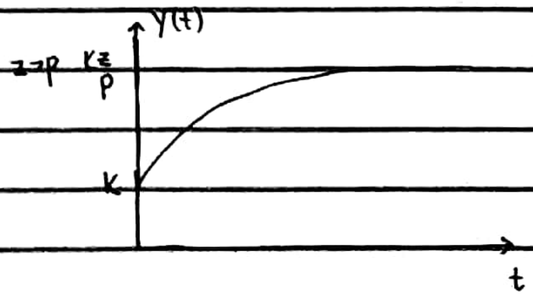
Συστήματα 1ης τάξης

$$G(s) = \frac{K(s+z)}{s+p}, \quad p > 0$$

Για βηματική είσοδο: $Y(s) = G(s)U(s) = \dfrac{K(s+z)}{s(s+p)} = \dfrac{K \cdot z/p}{s} - \dfrac{\frac{K}{p}(z-p)}{s+p}$

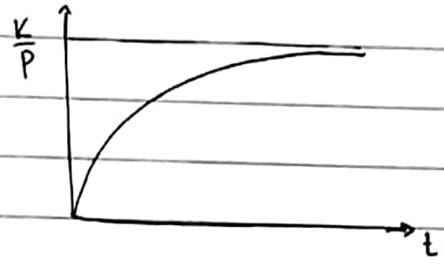
$$y(t) = K\frac{z}{p} - \frac{K}{p}(z-p)e^{-pt}$$

$$y(0) = K, \quad \lim_{t\to\infty} y(t) = y_{ss} = \frac{Kz}{p}$$

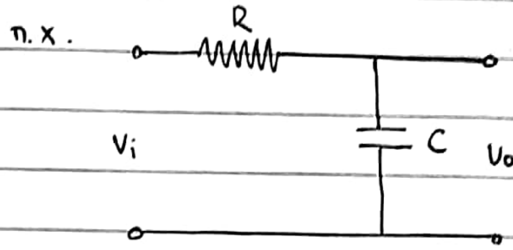


Χωρίς μηδενικό: $G(s)=\dfrac{K}{s+p}$, $Y(s)=\dfrac{K}{s(s+p)}=\dfrac{K/p}{s}-\dfrac{K/p}{s+p}$

$$y(t)=\frac{K}{p}\left(1-e^{-pt}\right)$$



π.χ.



Συστήματα 2ης τάξης

$$G(s)=\frac{K(s+z)}{(s+p_1)(s+p_2)}, \quad p_1,p_2>0$$

Για $u(t)=\text{step} \Rightarrow Y(s)=\dfrac{K(s+z)}{s(s+p_1)(s+p_2)}=\dfrac{Kz/p_1p_2}{s}-\dfrac{\frac{K(z-p_1)}{p_1(p_2-p_1)}}{s+p_1}-\dfrac{\frac{K(z-p_2)}{p_2(p_2-p_1)}}{s+p_2}$

$$\Rightarrow y(t)=\frac{Kz}{p_1p_2}-\frac{K(z-p_1)}{p_1(p_2-p_1)}e^{-p_1t}-\frac{K(z-p_2)}{p_2(p_1-p_2)}e^{-p_2t}$$

$$y(0)=\frac{Kz}{p_1p_2}-\frac{K(z-p_1)p_2}{p_1p_2(p_2-p_1)}+\frac{K(z-p_2)p_1}{p_1p_2(p_2-p_1)}=0$$

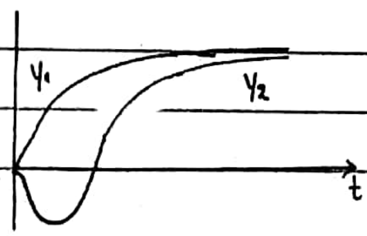
$$\lim_{t\to+\infty}y(t)=y_{ss}=G(0)=\frac{Kz}{p_1p_2}$$

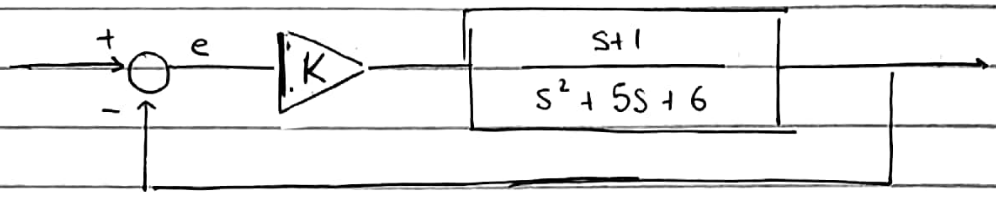
π.χ. $G_1(s)=\dfrac{s+1}{(s+2)(s+3)}$, $G_2(s)=\dfrac{1-s}{(s+2)(s+3)}=\dfrac{(-1)\cdot(s-1)}{(s+2)(s+3)}$

$y_1(t)=\dfrac{1}{6}+\dfrac{1-2}{2\cdot 1}e^{-2t}-\dfrac{1-3}{3(-1)}e^{-3t}$ $\qquad$ $y_2(t)=\dfrac{1}{6}+\dfrac{(-1-2)}{2\cdot 1}e^{-2t}+\dfrac{(-1-3)}{3(-1)}e^{-3t}$

$=\dfrac{1}{6}+\dfrac{1}{2}e^{-2t}-\dfrac{2}{3}e^{-3t}$ $\qquad$ $=\dfrac{1}{6}-\dfrac{3}{2}e^{-2t}+\dfrac{4}{3}e^{-3t}$

$\left.\dfrac{dy_1}{dt}\right|_{t=0}=\left.-e^{-2t}+2e^{-3t}\right|_{t=0}=1$ $\qquad$ $\left.\dfrac{dy_2}{dt}\right|_{t=0}=\left.3e^{-2t}-4e^{-3t}\right|_{t=0}=-1<0$



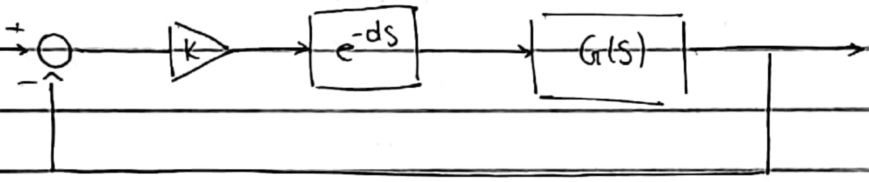


$$1 + kG(s) = 0 \Rightarrow 1 + K\frac{(s+1)}{s^2+5s+6} = 0 \rightarrow s^2 + (5+k)s + (6+k) = 0$$

$$\frac{E}{R} = \frac{1}{1 + KG(s)}$$

$$\frac{1}{1 + KG(0)} = \frac{1}{1+k/6}$$

Αν βάλουμε χρονική καθυστέρηση :



Μπορεί να προκύψει ασταθές σύστημα, αν και το σύστημα ανοιχτού βρόχου είναι ευσταθές

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \omega_n, \zeta > 0$$

$$\Delta = 4\zeta^2\omega_n^2 - 4\omega_n^2 = 4\omega_n^2(\zeta^2 - 1)$$

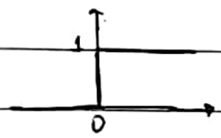
Για να εμφανιστούν μιγαδικές ρίζες, $\zeta \in (0,1)$

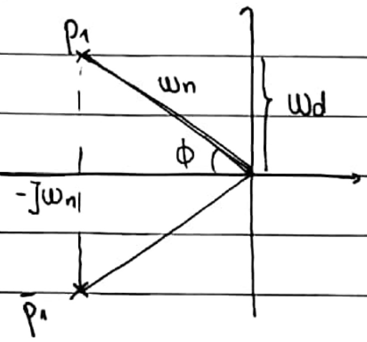
$$p_{1,2} = -\zeta\omega_n \pm j\,\underline{\omega_n\sqrt{1-\zeta^2}}$$

ω_d : συχνότητα αποσβενυμένων ταλαντώσεων

$u(t) = \text{step} \longrightarrow Y(s) = \dfrac{\omega_n^2}{s(s-p_1)(s-\bar{p}_1)}$

$U(s) = 1/s$





$$Y(s) = \frac{\omega_n^2}{|p_1|^2}\cdot\frac{1}{s} + \frac{\omega_n^2}{p_1(p_1-\bar{p}_1)}\cdot\frac{1}{s-p_1} + \frac{\omega_n^2}{\bar{p}_1(p_1-\bar{p}_1)}\cdot\frac{1}{s-\bar{p}_1}$$

$$y(t) = 1 + \frac{\omega_n^2}{p_1(p_1-\bar{p}_1)}e^{p_1 t} + \frac{\omega_n^2}{\bar{p}_1(p_1-\bar{p}_1)}e^{\bar{p}_1 t}$$

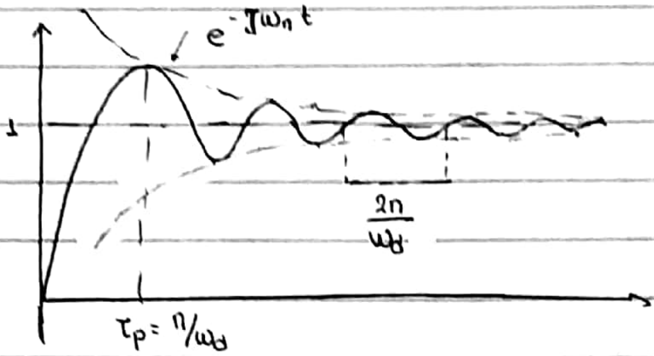
$$p_1 - \bar{p}_1 = 2j\omega_d : \quad \frac{\omega_n^2}{2j\omega_d\omega_n e^{j\phi_{p_1}}}e^{-\zeta\omega_n t}e^{j\omega_d t} - \frac{\omega_n^2}{2j\omega_d\omega_n e^{-j\phi_{p_1}}}e^{-\zeta\omega_n t}e^{-j\omega_d t}$$

$$y(t) = 1 + \frac{\omega_n^2}{2j\,\omega_n^2\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\left[e^{j(\omega_d t + \phi_{p_1})} - e^{-j(\omega_d t + \phi_{p_1})}\right]$$

$$= 1 + \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t + \underbrace{\phi_{p_1}}_{\pi-\phi})$$

$$\sin(\pi + x) = -\sin x$$

$$\Rightarrow \boxed{y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)}$$



$$\frac{dy}{dt} = 0 \Rightarrow -\zeta\omega_n e^{-\zeta\omega_n t}\sin(\omega_d t+\phi) + e^{-\zeta\omega_n t}\omega_d\cos(\omega_d t+\phi) = 0$$

$$\Rightarrow \tan(\omega_d t+\phi) = \frac{\omega_d}{\zeta\omega_n} = \frac{\sqrt{1-\zeta^2}}{\zeta} = \tan\phi$$

$$\omega_d t_p + \phi = \pi + \phi \Rightarrow \boxed{t_p = \frac{\pi}{\omega_d}}$$

$$y(t_p) = 1 - \frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}}\sin(\underbrace{\omega_d t_p}_{\pi} + \phi)$$

$$-\sin\phi = -\frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n}$$

$$\Rightarrow y(t_p) = 1 + e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$