Φυλλάδιο 5

Saturday, 8 January 2022 5:55 PM

Άσκηση 1 ✓

• $\int_{R} f_X(x)\,dx = \int_1^{+\infty} cx^{-5}\,dx = \frac{c}{-4}[x^{-4}]_1^{+\infty} = \frac{c}{4} = 1 \Rightarrow c=4$

Άρα $f(x) = \begin{cases} 0, & \text{αλλού} \\ 4x^{-5}, & x \ge 1 \end{cases}$

• $P[X \in [3,5]] = \int_3^5 f(x)\,dx = [-x^{-5}]_3^5 = -5^{-5} + 3^{-5} = \frac{5^5 - 3^5}{15^5}$

• $P[X \ge t] = \int_t^{+\infty} f(x)\,dx = [-x^{-5}]_t^{+\infty} = t^{-5}$

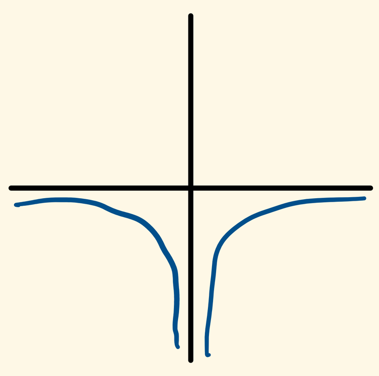
• $F_X(t) = P[X \le t] = \int_{-\infty}^t f(x)\,dx = [-x^{-5}]_{-\infty}^t = -t^{-5}$

Άσκηση 2 ✓

α) $\lim_{x \to +\infty} F(x) = 1 \Rightarrow$ πρέπει $A=0$, $B=1$.

$\Gamma = ;;;$

β) $f(x) = F'(x) = \begin{cases} 0, & \text{αλλού} \\ -\frac{\Gamma}{x^2}, & x \ge 4 \end{cases}$



γ) $P[X<5 \mid X<6] = \frac{P[X<6 \mid X<5]\,P[X<5]}{P[X<6]} = \frac{P[X<5]}{P[X<6]} = \frac{\int_{-\infty}^{5} -\frac{\Gamma}{x^2}\,dx}{\int_{-\infty}^{6} -\frac{\Gamma}{x^2}\,dx} = \frac{\left[\frac{\Gamma}{x}\right]_4^5}{\left[\frac{\Gamma}{x}\right]_4^6} = \dots$

($P[X<6 \mid X<5] = 1$)

Άσκηση 4 ✓

1) $P[Y \le t] = P[aX+b \le t] = P[X \le \frac{t-b}{a}] = \int_{-\infty}^{\frac{t-b}{a}} f_X(x)\,dx = F_X(\frac{t-b}{a})$

2) Ομοίως, ... $= P[X \ge \frac{t-b}{a}] = \int_{\frac{t-b}{a}}^{+\infty} f_X\,dx = -F_X(\frac{t-b}{a})$

3) $P[\max\{0, X\} \le t]$

4) $P[e^X \le t]$

Αν $t \le 0 \Rightarrow P = 0$

Αν $t > 0 \Rightarrow P[X \le \ln t] = F_X(\ln t)$, $t>0$

Άσκηση 5 ✓

α) $P[X \le t] = P[\ln U \ge -t] = P[U \ge e^{-t}] = \int_{e^{-t}}^{+\infty} f_U(x)\,dx$

Αν $e^{-t} < a$, $I=0$

Αν $e^{-t} \ge b$, $I=0$

Αν $e^{-t} \in [a,b)$, $I = \int_{e^{-t}}^{b} f_U(x)\,dx = \int_{e^{-t}}^{b} \frac{1}{b-a}\,dx = \frac{1}{b-a}(b - e^{-t})$, $t \in [-\ln a, -\ln b)$

β) $P[U^2 \le t] = P[-t \le U \le t] = \int_{-t}^{t} f_U(x)\,dx$

Αν $t \le a$, $I=0$

Αν $a \le t \le b$, $I = \int_a^t f_U(x)\,dx = \dots$

Αν $t \ge b$, $I = \int_a^b f_U(x)\,dx = \dots$

γ) $P[\frac{1}{U^2} \le t] = P[t^{-1} \le U^2]$

Αν $t^{-1} \le 0 \Rightarrow P = 1$

Αν $t^{-1} > 0 \Rightarrow P[t^{-1/2} \le U] = \int_{t^{-1/2}}^{+\infty} f_U(x)\,dx$

Αν $t^{-1/2} \ge b$, $I=0$

Αν $a \le t^{-1/2} \le b$, $I = \int_{t^{-1/2}}^{b} f_U(x)\,dx = \dots$

Αν $t^{-1/2} < a$, $I = \int_a^b f_U(x)\,dx = \dots$

Άσκηση 7 ✓

$P[T \le t] = \begin{cases} 1/10, & t \in [0,10] \\ 0, & \text{αλλού} \end{cases}$

$P[X \le t] = \begin{cases} 2/3, & t \le 0 \\ 1/3, & t \in (0,5] \\ 0, & t > 5 \end{cases} \to P[X \le 5 \mid T \le 5] = 1$

αν ξέρω ότι το ταξί θα έρθει σε <5', τότε ο Γιαν σίγουρα θα περιμένει <5'

Άσκηση 8 ✓

$P[F(X) \le t] \overset{F\uparrow}{=} P[X \le F^{-1}(t)] = \int_{-\infty}^{F^{-1}(t)} F(s)\,ds = F(F^{-1}(t)) = t$

Άσκηση 9 ✓

α) $P[X=1] = P[X \le 1] - P[X<1] = P[\frac{1}{X} \ge 1] - P[X<1] = 1 - P[\frac{1}{X} < 1] - P[X<1] = 1 - 2P[X<1] = 1 - 2\int_{-\infty}^{1} F_X\,ds = 1 - 2\left(\frac{x^2}{6}\right)_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$

($P[\frac{1}{X} < 1] = P[X<1]$)

β)

Άσκηση 10

α) $\int_0^1 f_X = c\left(\frac{t^2}{2} - \frac{t^3}{3}\right)_0^1 = c\left(\frac{1}{2} - \frac{1}{3}\right) = 1 \Rightarrow c = 6$

Άρα $f_X = \begin{cases} 6t - 6t^2, & t \in [0,1] \\ 0, & \text{αλλού} \end{cases}$

β) $P[1 - X \le t] = P[X \ge t-1] = \int_{t-1}^{+\infty} f_X\,ds$

Αν $t-1 \ge 1$, $I=0$

Αν $t-1 \le 0$, $I = \int_0^1 f_X\,ds = \dots$

Αν $0 \le t-1 < 1$, $I = \int_{t-1}^{1} f_X\,ds = \dots$

γ)