17.03.22

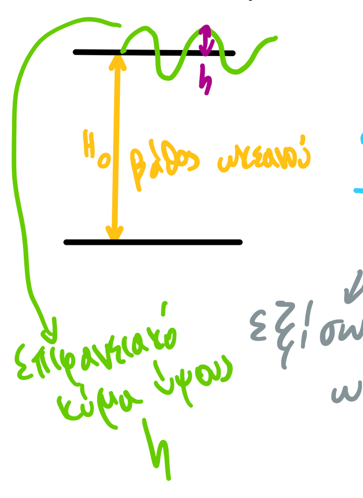
Thursday, 17 March 2022 3:11 PM

1) Εξίσωση Νεύτωνα: $(\rho\, dV)\frac{\partial \vec{v}}{\partial t} = \vec{F}_{ex}$

2) Ασυμπίεστο του νερού: $\vec{\nabla}\cdot\vec{v}=0$

3) Χαμηλό ιξώδες νερού (αστρόβιλο πεδίο): $\vec{\nabla}\times\vec{u}=0$

4) Νόμος Pascal



Συνδυάζοντας τα (1)-(4) καταλήγουμε:

$$\frac{\partial^2 h}{\partial t^2} = \left[\frac{\sigma}{\rho} k \frac{\partial^2 h}{\partial x^2} + g k h\right] \tanh(k H_0)$$

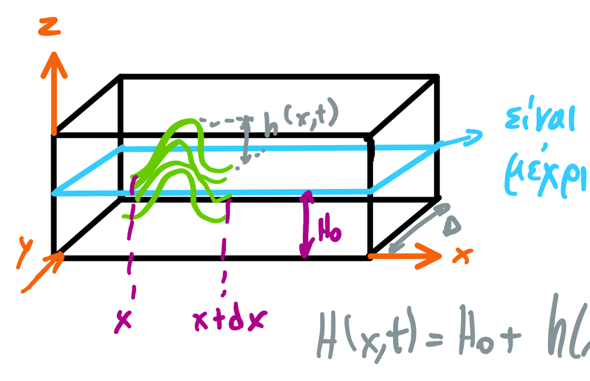
εξίσωση κυμάτων ωκεανού

$\frac{2\pi}{\lambda}$ κυματοδιάνυσμα

Έστω $h(x,t) = h_0 e^{i(kx-\omega t)}$. Αντικαθιστώντας:

$$\omega^2 = \left[\frac{\sigma}{\rho} k^3 - g k\right] \tanh(k H_0)$$

Κύματα σε ρηχό νερό $H_0 \ll \lambda$



είναι γεμάτο μέχρι εδώ

$H(x,t) = H_0 + h(x)$

$$(\rho H_0 D dx)\frac{\partial v}{\partial t} = (H_0 D)[P(x) - P(x+dx)] =$$

$P(x+dx) \to P(x) + \frac{\partial P}{\partial x}dx$

$$= \boxed{\rho\frac{\partial v}{\partial t} = -\frac{\partial P}{\partial x}} \quad (1)$$

Διατήρηση μάζας: $\frac{dm}{dt} = \rho (H_0 D)[v(x) - v(x+dx)]$

$v(x+dx) \to v(x) + \frac{\partial v}{\partial x}dx$

η μεταβολή μάζας εκδηλώνεται ως μεταβολή ύψους

$$\Rightarrow \boxed{\frac{dm}{dt} = -\rho H_0 D \frac{\partial v}{\partial x} dx} \quad (2\alpha)$$

$$\frac{dm}{dt} = \rho D dx \frac{\partial H}{\partial t} = \rho D \frac{\partial h}{\partial t} dx \Rightarrow \frac{dm}{dt} = \frac{D}{g}\frac{\partial(\rho g h)}{\partial t} dx$$

$$\Rightarrow \boxed{\frac{dm}{dt} = \frac{D}{g}\frac{\partial P}{\partial t} dx} \quad (2\beta)$$

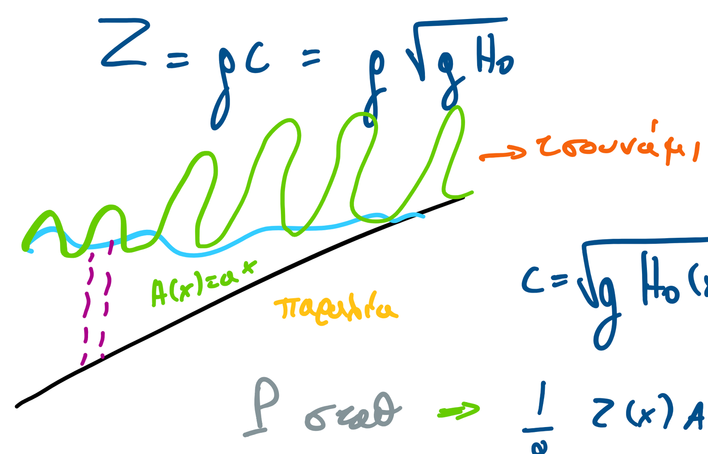
$$(2\alpha),(2\beta) \Rightarrow \frac{\partial v}{\partial x} = -\frac{1}{\rho g H_0}\frac{\partial P}{\partial t} \quad (3)$$

$$\frac{\partial}{\partial t}(3) \Rightarrow \frac{\partial}{\partial t}\frac{\partial v}{\partial x} = -\frac{1}{\rho g H_0}\frac{\partial^2 P}{\partial t^2} \Rightarrow \frac{\partial}{\partial x}\frac{\partial v}{\partial t} = -\frac{1}{\rho g H_0}\frac{\partial^2 P}{\partial t^2} \Rightarrow \frac{\partial}{\partial x}\left(-\frac{1}{\rho}\frac{\partial P}{\partial x}\right) = -\frac{1}{\rho g H_0}\frac{\partial^2 P}{\partial t^2}$$

$$\Rightarrow \boxed{\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = \frac{\partial^2 P}{\partial x^2}}, \quad c = \sqrt{g H_0}$$

Σύνθετη μηχανική αντίσταση του συστήματος

$$Z = \rho c = \rho\sqrt{g H_0}$$



$c = \sqrt{g H_0(x)}$, $z(x) = \rho\sqrt{g H_0(x)}$

P σταθ. $\Rightarrow \frac{1}{2} z(x) A^2(x)\, \omega^2 = P_0 \Rightarrow A \sim x^{1/4}$

Ταλάντωση πλάσματος

$\omega_p^2 = \frac{e^2 n}{\varepsilon_0 m_e}$ n: πυκνότητα φορτισμένων σωματιδίων

$$c^2\frac{\partial^2 E}{\partial z^2} = \omega_p^2 E + \frac{\partial^2 E}{\partial t^2}$$

$$E = E_0 e^{i(kz-\omega t)}$$

$\Rightarrow$ σχέση διασποράς

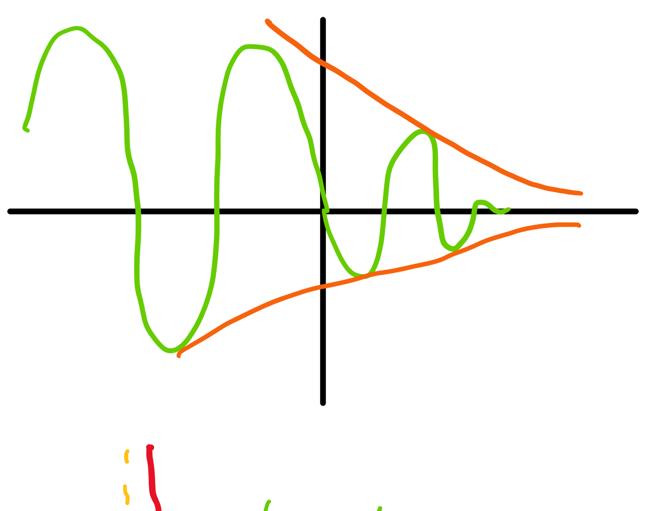
$$\Rightarrow \omega^2 = \omega_p^2 + c^2 k^2 \Rightarrow k = \frac{\sqrt{\omega^2-\omega_p^2}}{c} = \frac{2\pi}{\lambda} \qquad \lambda = \frac{2\pi c}{\sqrt{\omega^2-\omega_p^2}}$$

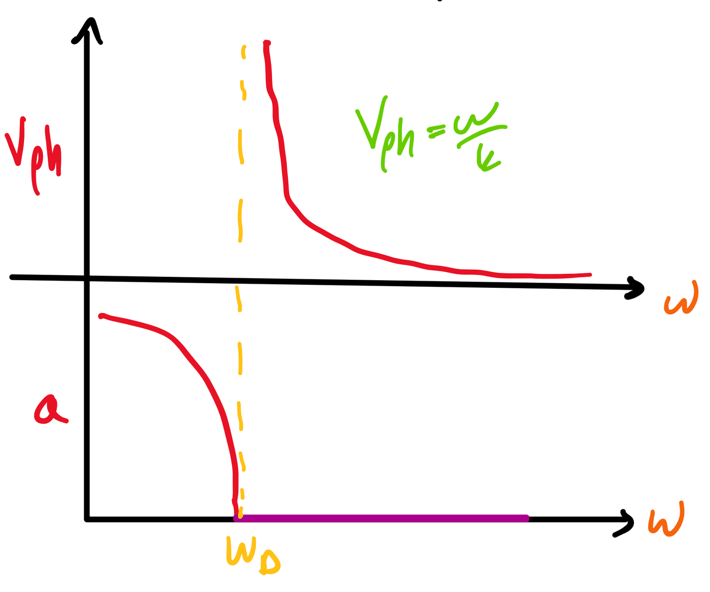
$$V_{ph} = \frac{\omega}{k} = \frac{c\omega}{\sqrt{\omega^2-\omega_p^2}}$$

- Διάδοση: $\omega > \omega_p$

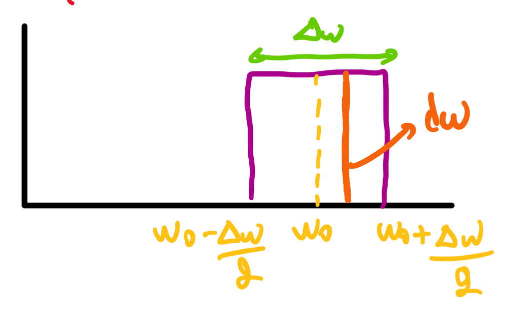
Αν $\omega < \omega_p \Rightarrow k = i\frac{\sqrt{\omega_p^2-\omega^2}}{c} = i a$

$$E(z,t) = E_0 e^{i(iaz-\omega t)} = E_0 e^{-az} e^{-i\omega t}$$





Κυματοομάδες



$$\frac{dy}{d\omega} = y_0 e^{i(kx-\omega t)} \Rightarrow \left(\frac{dy}{d\omega}\right)d\omega = y_0\, d\omega\, e^{i(kx-\omega t)}$$

$$y(x,t) = \int dy = y_0\int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} e^{i(kx-\omega t)}d\omega$$

$k = k(\omega)$

$$\left\{k(\omega) = k(\omega_0) + \left(\frac{dk}{d\omega}\right)_{\omega_0}(\omega-\omega_0)\right\}$$

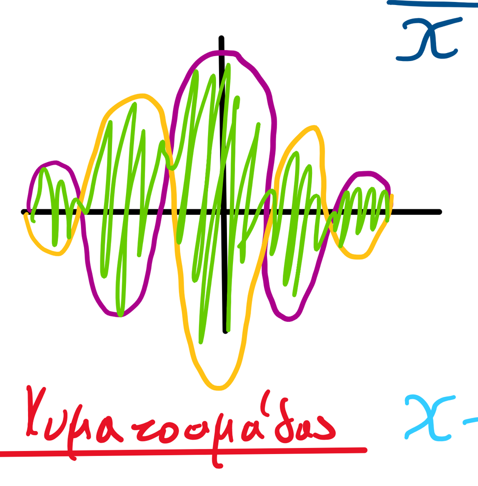
$$y = y_0 e^{i(k_0x-\omega_0 t)}\int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} e^{i\left[(\omega-\omega_0)\left(\frac{dk}{d\omega}\right)_{\omega_0}x - (\omega-\omega_0)t\right]}d\omega$$

$$\left\{\omega-\omega_0 = \xi \Rightarrow d\omega = d\xi \Rightarrow \xi_{max} = \frac{\Delta\omega}{2},\ \xi_{min} = -\frac{\Delta\omega}{2}\right\}$$

$$\Rightarrow y = y_0 e^{i(k_0x-\omega_0 t)}\int_{-\Delta\omega/2}^{\Delta\omega/2} e^{i\xi\left[\left(\frac{dk}{d\omega}\right)_{\omega_0}x - t\right]}d\xi \Rightarrow$$

$$\Rightarrow y = y_0\,\Delta\omega\, e^{i(k_0x-\omega t)}\frac{\sin X}{X}, \quad X = \left[\left(\frac{dk}{d\omega}\right)_{\omega_0}x - t\right]\frac{\Delta\omega}{2}$$

$\frac{\sin X}{X}$:



Μέγιστο Κυματοομάδας $X \to 0$

$$\left(\frac{dk}{d\omega}\right)_{\omega_0}x - t = 0 \Rightarrow \frac{dx}{dt} = \left(\frac{d\omega}{dk}\right)_{k_0} = V_{gr}$$