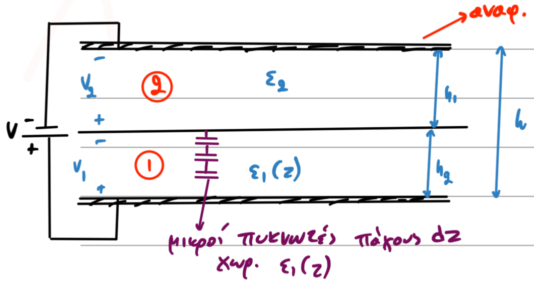
Credits: M. Ρόζος

Παράδειγμα 3



$\vec{E}$, $\vec{D}$, $\vec{P} = \vec{P}(\vec{E})$;

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\rho_P = -\nabla \vec{P}$, σ_P

$\rho = 0$

- $\nabla \vec{D} = 0 \qquad \vec{D} = \sigma\tau\alpha\theta.$

  $\hat{n}(\vec{D}_2 - \vec{D}_1) = 0 \qquad \vec{D} = \sigma \cdot \hat{z} = D\hat{z}$, $\vec{D}_1 = \vec{D}_2$

- $\vec{E}_1 = \dfrac{1}{\varepsilon_1(z)} \vec{D}$, $\vec{E}_2 = \dfrac{1}{\varepsilon_2} D$

- $V_2 = \phi(h_1) - \phi(h) = \displaystyle\int_{h_1}^{h} E_2 \, dz = E_2 h_2$

- $V_t = V_1 + V_2 = D\left(\displaystyle\int_0^{h_1} \frac{dz}{\varepsilon_1(z)} + \frac{h_2}{\varepsilon_2}\right)$

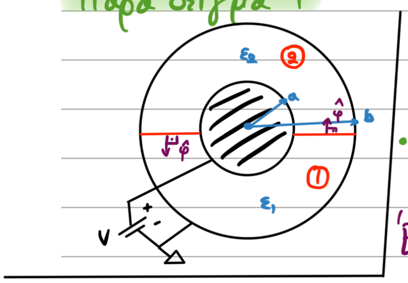
- $\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = (\varepsilon - \varepsilon_0)\vec{E} = \dfrac{\varepsilon - \varepsilon_0}{\varepsilon} \vec{D}$

  $\hookrightarrow \vec{P}_1 = \dfrac{\varepsilon_1(z) - \varepsilon_0}{\varepsilon_1(z)} D\hat{z}$, $\vec{P}_2 = \dfrac{\varepsilon_2 - \varepsilon_0}{\varepsilon_2} D\hat{z}$

- $\rho_{P_1} = -\nabla \vec{P}_1 = \varepsilon_0 D \dfrac{d}{dz}\left(\dfrac{1}{\varepsilon_1(z)}\right)$
- $\rho_{P_2} = -\nabla \vec{P}_2 = 0$
- $\sigma_P(z=0) = -\hat{z}\vec{P}_1 = \dfrac{\varepsilon_1(0) - \varepsilon_0}{\varepsilon_1(0)} D$
- $\sigma_P(z=h_1) = \ldots$
- $\sigma_P(z=h) = \ldots$

- $D = \sigma = Q/S \rightarrow$ εμβαδόν πλάκας
- $V = \sigma(\ldots) = \dfrac{Q}{S}(\ldots)$
- $C = \dfrac{Q}{V} = \dfrac{S}{\int_0^{h_1} \frac{dz}{\varepsilon_1(z)} dz + \frac{h_2}{\varepsilon_2}}$
- $\dfrac{1}{C} = \dfrac{1}{S}\displaystyle\int_0^{h_1} \frac{dz}{\varepsilon_1(z)} + \frac{h_2}{\varepsilon_2 S} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Παράδειγμα 4



$\rho = 0$, $\vec{P}$, ρ_P, σ_P ?

- $\nabla \vec{D}_{1,2} = 0 \Rightarrow D_{r_{1,2}} = \dfrac{A_{1,2}}{r}$

Έλεγχος: $\hat{n}(\vec{D}_2 - \vec{D}_1) = 0$, $\vec{E}_{1,2} = \dfrac{\vec{D}_{1,2}}{\varepsilon_{1,2}} = \dfrac{A_{1,2}}{\varepsilon_{1,2} \cdot r} \hat{r}$

- $\overset{=\hat{\varphi}}{\hat{n}} \times (\vec{E}_2 - \vec{E}_1) = 0 \Rightarrow \dfrac{A_1}{\varepsilon_1} = \dfrac{A_2}{\varepsilon_2}$, $\forall r \in (a,b)$

- $V=\int_a^b E_1\,dr=\int_a^b E_2\,dr=\frac{A_1}{\varepsilon_1}\ln\frac{b}{a}=\frac{A_2}{\varepsilon_2}\ln\frac{b}{a} \Rightarrow \frac{A_1}{\varepsilon_1}=\frac{A_2}{\varepsilon_2}=\frac{V}{\ln(b/a)}$

- $\vec{E}_1=\frac{V}{r\ln(b/a)}\hat{r}\,,\qquad \vec{E}_2=\frac{V}{r\ln(b/a)}\hat{r}$

- $\vec{D}_{1,2}=\varepsilon_{1,2}\vec{E}_{1,2}$

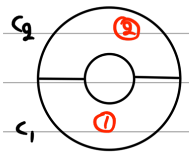
- $\phi_1(r)-\cancelto{0}{\phi_1(b)}=\phi_2(r)-\cancelto{0}{\phi_2(b)}=\int_r^b E_1\,dr=V\frac{\ln(b/r)}{\ln(b/a)}$

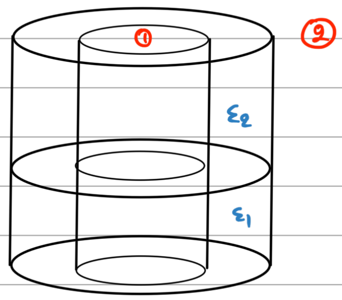
- $\vec{P}_{1,2}=\vec{D}_{1,2}-\varepsilon_0\vec{E}_{1,2}$

$\rho_{\ell 1,2}=-\nabla\cdot\vec{P}_{1,2}$

$\sigma_\ell=\ldots$ (συνολικά 6 σ_ℓ)

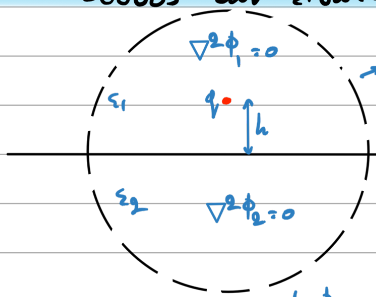
$C=C_1+C_2$ $\qquad\qquad$ $C_1 \,//\, C_2$





2.4 Μέθοδος των ειδώλων σε διηλεκτρικό

Αρχική Διάταξη



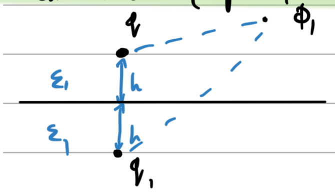
$\phi_1=0$

$\phi_1=\phi_2$

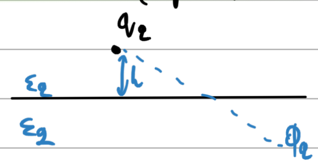
$\varepsilon_1\frac{\partial\phi_1}{\partial n}=\varepsilon_2\frac{\partial\phi_2}{\partial n}$

$\phi_2=0$

Βοηθητικό Πρόβλημα 1



Βοηθητικό Πρόβλημα 2



- $\phi_1=\frac{1}{4\pi\varepsilon_1}\left[\frac{q}{\sqrt{x^2+y^2+(z-h)^2}}+\frac{q_1}{\sqrt{x^2+y^2+(z+h)^2}}\right]$

- $\phi_2=\frac{1}{4\pi\varepsilon_2}\frac{q_2}{\sqrt{x^2+y^2+(z-h)^2}}$

- $\phi_1=\phi_2$ στο $z=0$ $\Rightarrow \frac{q+q_1}{\varepsilon_1}=\frac{q_2}{\varepsilon_2}$

- $\frac{1}{R}=\frac{1}{\sqrt{x^2+y^2+(z\mp h)^2}}$

- $\left.\frac{\partial}{\partial z}\left(\frac{1}{R}\right)\right|_{z=0}=\frac{\pm h}{(x^2+y^2+h^2)^{3/2}}$

- $\left.\varepsilon_1\frac{\partial\phi_1}{\partial z}\right|_{z=0}=\left.\varepsilon_2\frac{\partial\phi_2}{\partial z}\right|_{z=0}\Rightarrow q=q_1+q_2$

- $q_1=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}q\,,\qquad q_2=2q$

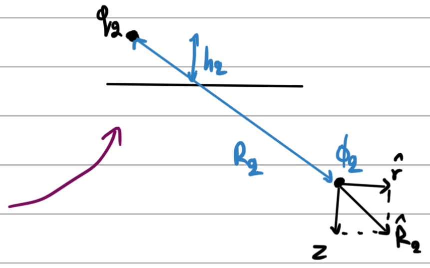
- $\phi_1=\frac{q}{4\pi\varepsilon_1}\left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}}+\frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right]$

- $\vec{E}_1=-\nabla\phi_1$

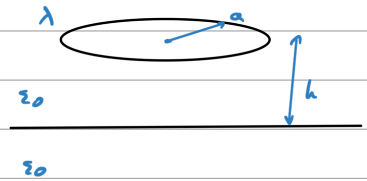
- $\phi_2=0\,,\quad E_2=0$

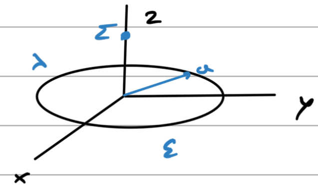
- $\vec{E}_1 = -\nabla\phi_1$, $E_2 = 0$
- $\vec{D}_1 = \varepsilon_1 \vec{E}_1$, $\vec{D}_2 = \varepsilon_2 \vec{E}_2 =$ πεπερ.

Βοηθ. Παρ. 2: $\vec{D}_2 = \dfrac{2q}{4\pi R^2}\hat{R}$



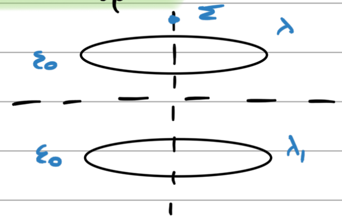
Παράδειγμα 1



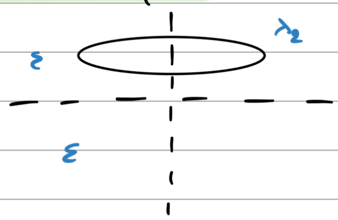


$$\phi = \frac{\lambda a}{2\varepsilon\sqrt{z^2+a^2}}$$

Βοηθ. Παρ. 1



Βοηθ. Παρ. 1



- $\phi_1 = \dfrac{a}{2\varepsilon_0}\left[\dfrac{\lambda}{\sqrt{(z-h)^2+a^2}} + \dfrac{\lambda_1}{\sqrt{(z+h)^2+a^2}}\right]$, $\phi_2 = \dfrac{a}{2\varepsilon}\dfrac{\lambda_2}{\sqrt{(z-h)^2+a^2}}$

$$\left\{\begin{array}{l} \phi_1 = \phi_2 \\ \varepsilon_0 \dfrac{\partial\phi_1}{\partial z} = \varepsilon\dfrac{\partial\phi_2}{\partial z} \end{array}\right\} \Rightarrow \lambda_1 = \frac{\varepsilon_0-\varepsilon}{\varepsilon_0+\varepsilon}\lambda \, , \quad \lambda_2 = \frac{2\varepsilon}{\varepsilon_0+\varepsilon}\lambda$$

- $\vec{E}_1 = -\nabla\phi_1 = \dfrac{\lambda a}{2\varepsilon_0}\left\{\dfrac{z-h}{((z-h)^2+a^2)^{3/2}} + \dfrac{\varepsilon_0-\varepsilon}{\varepsilon_0+\varepsilon}\dfrac{z+h}{((z+h)^2+a^2)^{3/2}}\right\}\hat{z}$

- $\vec{E}_2 = -\nabla\phi_2 = \dfrac{\lambda}{\varepsilon_0+\varepsilon}\dfrac{z-h}{[(z-h)^2+a^2]^{3/2}}\hat{z}$