NTUCSIE ADA 2015
Mini-Homework 12
b03902027 王冠鈞

Answer

Counter value	cost[4]	cost[3]	cost[2]	cost[1]	Total cost of first n operations
0	0	0	0	0	0
1	0	0	0	2	2
2	0	0	4	2	8
3	0	0	0	2	10
4	0	8	4	2	24
5	0	0	0	2	26
6	0	0	4	2	32
7	0	0	0	2	34
8	16	8	4	2	64

We have found that, when we do n operations, the d-th bit will be flipped in a total of $\lfloor \frac{n}{2^{d-1}} \rfloor$ times, and in n there are $\lfloor 1 + \log_2 n \rfloor$ digits when represented in binary. Therefore, in total, it will take:

$$\sum_{i=1}^{\lfloor 1+\log_2 n \rfloor} \lfloor \frac{n}{2^{i-1}} \rfloor \times 2^i$$

$$\begin{aligned}
&= n \times 2 + \lfloor \frac{n}{2} \rfloor \times 2^2 + \lfloor \frac{n}{2^2} \rfloor \times 2^3 + \cdots + \lfloor \frac{n}{2^{\lfloor 1+\log_2 n \rfloor - 1}} \rfloor \times 2^{\lfloor 1+\log_2 n \rfloor} \\
&\leq n \times 2 + \frac{n}{2} \times 2^2 + \frac{n}{2^2} \times 2^3 + \cdots + \frac{n}{2^{\lfloor 1+\log_2 n \rfloor - 1}} \times 2^{\lfloor 1+\log_2 n \rfloor} \\
&= 2n + 2n + 2n + \cdots + 2n \text{ (in total of } \lfloor 1 + \log_2 n \rfloor \text{ terms)} \\
&= 2n \lfloor 1 + \log_2 n \rfloor \leq 2n + 2n \log_2 n = O(n \log n)
\end{aligned}$$

Thus, the amortized time cost is $O(n \log n)$.