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NANYANG TECHNOLOGICAL UNIVERSITY

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING

Math Note

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Contents

Chapter 1

Functional Analysis

1.1 RKHS

1.1.1 Functional Analysis Foundations

Definition 1 (Norm)

Let $\mathcal{F}$ be a vector space over $\mathbb{R}$. A function $\|\cdot\|_{\mathcal{F}} : \mathcal{F} \to [0, +\infty]$ is said to be a `norm` on $\mathcal{F}$ if

1. $\|f\|_{\mathcal{F}} = 0$ iff f $= \mathbf{0}$ (*norm separate points*)
2. $\|\lambda f\|_{\mathcal{F}} = |\lambda| \|f\|_{\mathcal{F}}, \forall \lambda \in \mathbb{R}, \forall f \in \mathcal{F}$ (*positive homogeneity*)
3. $\|f + g\|_{\mathcal{F}} \leq \|f\|_{\mathcal{F}} + \|g\|_{\mathcal{F}}, \forall f, g \in \mathcal{F}$ (*triangle inequality*)

Distance: a notation of distance on $\mathcal{F} : d(f, g) = \|f - g\|_{\mathcal{F}}$

Definition 2 (Convergent Sequence)

A sequence $\{f_n\}_{n=1}^{\infty}$ of elements of a normed vector space $(\mathcal{F}, \cdot)$ is said to *converge* to $f \in \mathcal{F}$ if for every $\epsilon > 0$, there exists $N = N(\epsilon) \in \mathbb{N}$

Definition 3 (Cauchy Sequence)

A sequence $\{f_n\}_{n=1}^{\infty}$ of elements of a normed vector space$(\mathcal{F}, \|\cdot\|_{\mathcal{F}})$ is said to be a *Cauchy(fundamental) Sequence* if for every $\epsilon > 0$, there exists $N = N(\epsilon) \in \mathbb{N}$, such that for all $n \geq N$, $\|f_n - f\| < \epsilon$

Definition 4 (Complete Space)

A space $\mathcal{X}$ is complete if every Cauchy sequence in $\mathcal{X}$ converges: it has a limit, and this limit is in $\mathcal{X}$.

Definition 5 (Banach Space)

Banach space is a complete normed space, i.e, it contains the limits of all its Cauchy sequences.

Bibliography

[1] Armstrong, M.A. *Basic Topology.* England: Editorial Board, 2000.

[2] Coxeter, H.S.M. *Introduction to Geometry.* Toronto: John Wiley and Sons Inc, 1969.

[3] Chronister, James. *Blender Basics.* Estados Unidos: 2004.

[4] Engelking, Ryszard. *General Topology.* Berlin: Hildermann, 1989.

[5] Engelking, Ryszard y Karol Sieklucki. *Topology: A Geometric Approach.* Berlin: Hildermann, 1992.

[6] Hatcher, Allen. *Algebraic Topology.* Cambridge: Cambridge University Press, 2002.

[7] Hoffman, Kenneth y Ray Kunze. *Linear Algebra, second edition.* New Jersey: Prentice Hall. Inc, 1961.

[8] Kackzynski, T., K. Mischainkow y M. Mrozen. *Algebraic Topology: A Computational Approach.* 2000.

[9] Kosniowski, Czes. *TopologÃa Algebraica.* Barcelona: Editorial Revert, 1986.

[10] Lefschetz, Solomon. *Algebraic Topology.* Rhode Island: American Mathematical Society, 1991.

[11] May, J. P. *A Concise Course in Algebraic Topology.* Chicago.

[12] Mill, J. Van. *Infinite-Dimensional Topology.* Amsterdam: Elsevier Science Publications B. V, 1989.

[13] Munkres, James R. *Topology.* New York: Prentice Hall Inc, 2000.

[14] Roseman, Dennis. *Elementary Topology.* New Jersey: Prentice Hall Inc, 1999.

[15] Sato, Hajime. *Algebraic Topology: An Intuituve Approach.* Rhode Island: American Mathematical Society, 1996.

[16] Zomorodian, Afra J. *Topology for Computing*. United States: Cambridge University Press, 2005.