CAB420 Machine Learning - Assignment 1

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Theory

Logistic regression is a method of fitting a probabilistic classifier that gives soft linear thresholds. It is common to use logistic regression with an objective/loss function consisting of the negative log probability of the data plus a L2 regularizer:

$$L(\mathbf{w}) = -\sum_{i=1}^{N} Log\left(\frac{1}{1+e^{y_i\left(\mathbf{w}^T\mathbf{x}_i+b\right)}}\right) + \lambda\|\mathbf{w}\|_2^2$$

(Here w does not include the “extra” weight w0.)

(a) Find the partial derivatives $\frac{\partial L}{\partial w_j}$

Let $u = 1 + e^{y_i\left(\mathbf{w}^T\mathbf{x}+b\right)}$, we have: $L(\mathbf{w}) = -\sum_{i=1}^{N} -log(u) + \lambda\|\mathbf{w}\|_2^2$

Hence,

$$\frac{\partial L}{\partial \mathbf{w}_j} = -\sum_{i=1}^{N} \frac{\partial}{\partial u} - log(u)\frac{\partial u}{\partial \mathbf{w}_j} + \frac{\partial}{\partial \mathbf{w}_j}\lambda\|\mathbf{w}\|_2^2$$

$$\frac{\partial}{\partial w_j}\lambda\|\mathbf{w}\|_2^2 = 2\lambda\mathbf{w}_j$$

$$\frac{\partial}{\partial u} - log(u) = \frac{-1}{u}$$

$$\frac{\partial u}{\partial \mathbf{w}_j} = y_i\mathbf{x}_i e^{y_i(\mathbf{w}^T\mathbf{x}_i+b)}$$

$$\frac{\partial L}{\partial \mathbf{w}_j} = -\sum_{i=1}^{N} 2\lambda\mathbf{w}_j - \frac{y_i\mathbf{x}_i e^{y_i(\mathbf{w}_i^T+b)}}{1+e^{y_i\left(\mathbf{w}^T\mathbf{x}_i+b\right)}}$$

b) Find the partial second derivatives $\frac{\partial L^2}{\partial w_j \partial w_k}$.

We have:

$$\frac{\partial^2 L}{\partial w_j \partial w_k} = \frac{\partial}{\partial w_k}(\sum_{i=1}^{N} \frac{y_i x_i e^{y_i(w^T x_i + b)}}{1+e^{y_i(w^T x_{i,j}+b)}} + 2\lambda w_j)$$

Let:

$$\frac{\partial}{\partial \mathbf{w}_k} \frac{y_i \mathbf{x}_{ij} e^{y_i(\mathbf{w}_i^T + b)}}{1+e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}} = \frac{\partial}{\partial \mathbf{w}_k} \frac{g}{h}$$

while:

$$g = y_i \mathbf{x}_{ij} e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}; h = 1 + e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}$$

Then:

$$\frac{\partial}{\partial \mathbf{w}_j} \frac{g}{h} = \frac{\frac{\partial g}{\partial \mathbf{w}_j} h - g \frac{\partial h}{\partial \mathbf{w}_j}}{h^2}$$

$$\frac{\partial}{\partial \mathbf{w}_k} \frac{g}{h} = \frac{y_i^2 \mathbf{x}_{ij} \mathbf{x}_{ik} e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)} + y_i^2 \mathbf{x}_{ij} \mathbf{x}_{ik} e^{2y_i(\mathbf{w}^T \mathbf{x}_i + b)} - y_i^2 \mathbf{x}_{ij} \mathbf{x}_{ik} e^{2y_i(\mathbf{w}^T \mathbf{x}_i + b)}}{\left(1+e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}\right)^2}$$

$$\frac{\partial}{\partial \mathbf{w}_k} \frac{g}{h} = \frac{y_i^2 \mathbf{x}_{ij} \mathbf{x}_{ik} e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}}{\left(1 + e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}\right)^2}$$

Hence,

$$\text{When } k \neq j; \frac{\partial^2 L}{\partial \mathbf{w}_k \partial \mathbf{w}_j} = \sum_{i=1}^{N} \frac{y_i^2 \mathbf{x}_{ij} \mathbf{x}_{ik} e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}}{\left(1 + e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}\right)^2}$$

$$\text{When } k = j; \frac{\partial^2 L}{\partial \mathbf{w}_k \partial \mathbf{w}_j} = \sum_{i=1}^{N} \frac{y_i^2 \mathbf{x}_{ij} \mathbf{x}_{ik} e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}}{\left(1 + e^{y_i(\mathbf{w}^T \mathbf{x}_i + b)}\right)^2} + 2\lambda$$

(c) From these results, show that $L(w)$ is a convex function.

Function $\mathrm{L}(\mathbf{w})$ is convex because its Hessian H_L is PSD.

H_L can be shown to satisfy the criteria $\mathbf{a}^T \mathbf{H} \mathbf{a} \geq 0$ because the output of all functions in the Hessian is always positive.

1. Feature, Classes and Linear Regression

(a) Plot the training data in a scatter plot.

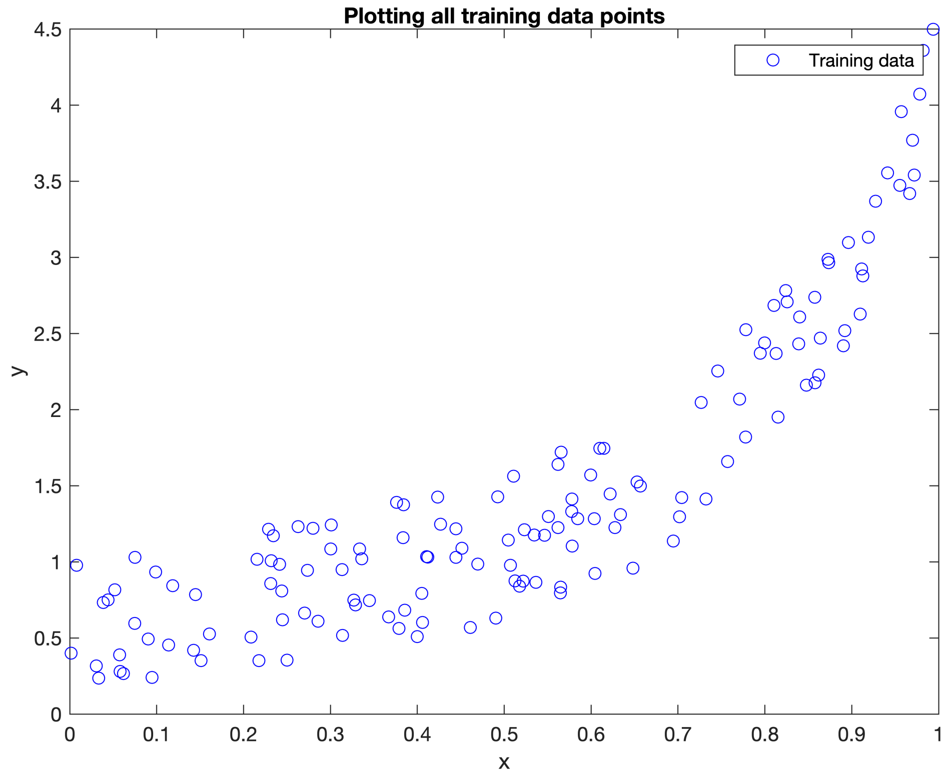
```
% Clean up
clc
clear
close all

disp('1. Features, Classes, and Linear Regression');
```

```
1. Features, Classes, and Linear Regression
```

```

% (a) Plot the training data in a scatter plot.

% Load training dataset
% replace \ to load file on windows
mTrain = load('data/mTrainData.txt');

% Separate features
Xtr = mTrain(: ,1); % X = single feature
Ytr = mTrain(: ,2); % Y = target value
whos
```

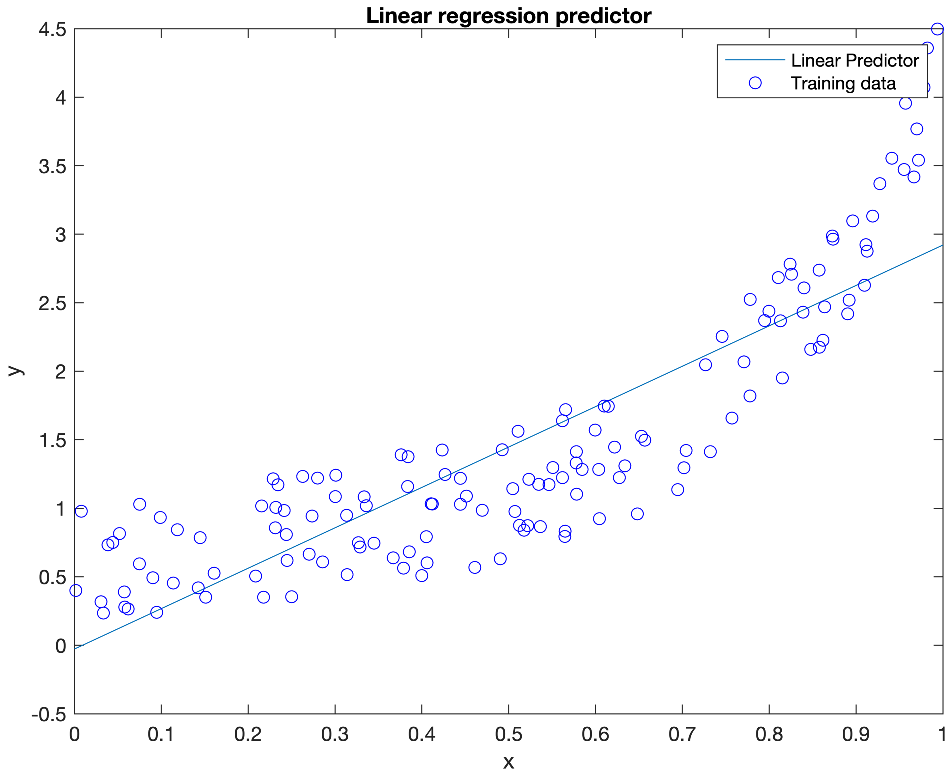
```
  Name          Size            Bytes  Class     Attributes

  Xtr         140x1              1120  double
  Ytr         140x1              1120  double
  mTrain      140x2              2240  double
```

```

% Plot training data
figure('name', 'Training Data');
plot (Xtr, Ytr, 'bo');
xlabel('x');
ylabel('y');
title('Plotting all training data points');
legend('Training data');
```



(b) Create a linear regression learner using the above functions. Plot it on the same plot as the training data.

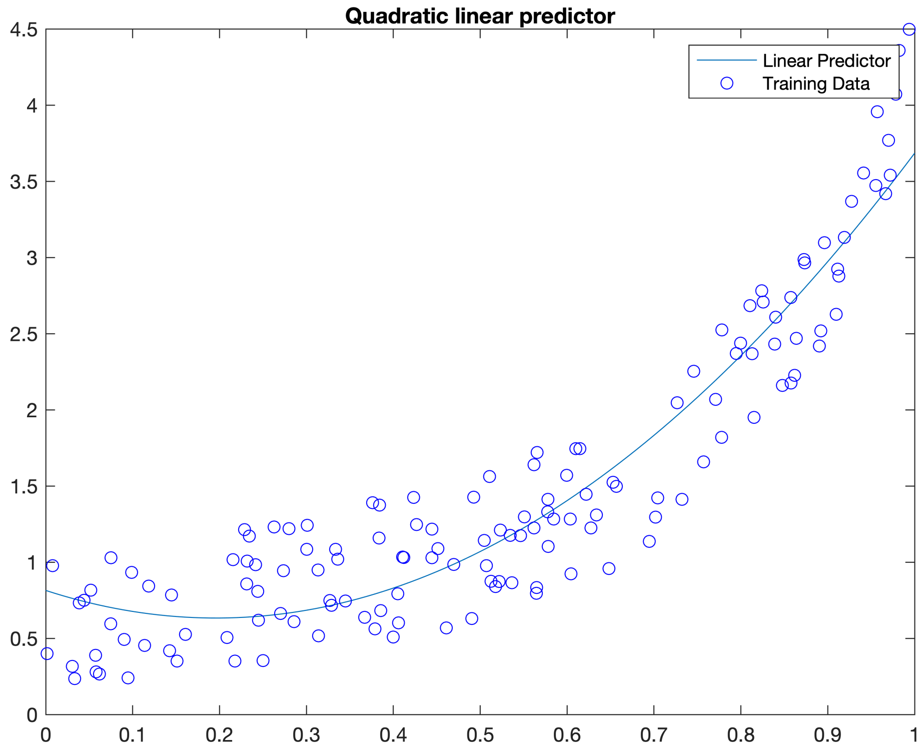
```
linXtr = polyx(Xtr, 1);
learner_linear = linearReg(linXtr, Ytr);
xline = [0:.01:1]'; % Transpose
yline = predict(learner_linear, polyx(xline, 1));

figure('name', 'Linear regression predictor');
plot(xline, yline);
hold on % Plot training data and label figure.
plot (Xtr, Ytr, 'bo');
legend('Linear Predictor','Training data');
xlabel('x');
ylabel('y');
title('Linear regression predictor');
```



(c) Create plots with the data and a higher-order polynomial (3, 5, 7, 9, 11, 13).

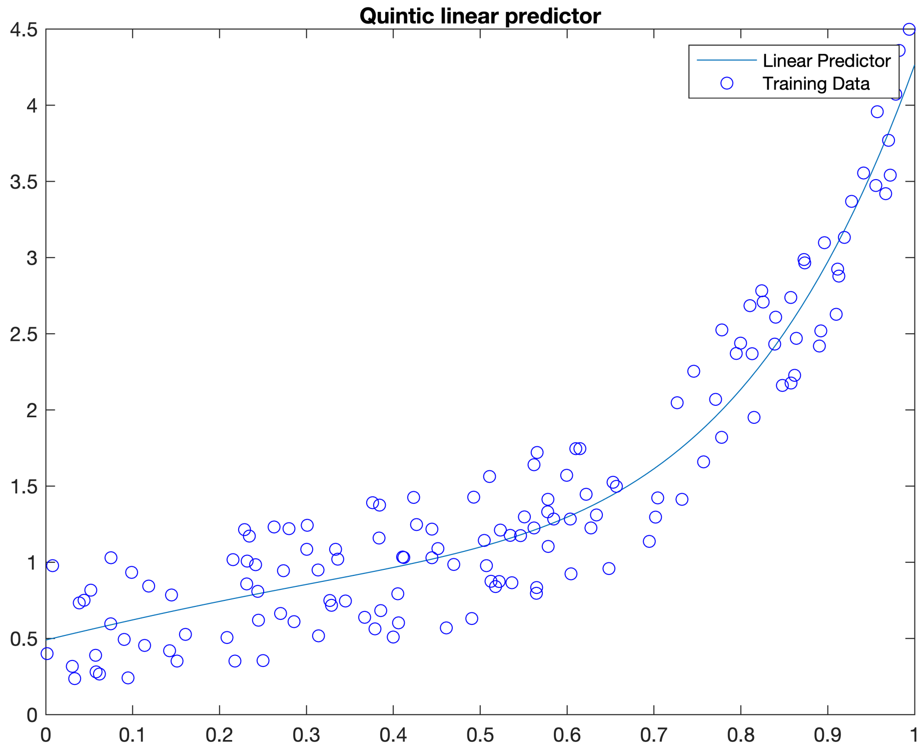
Quadric:

```
quadXtr = polyx(Xtr, 2);
learner_quadratic = linearReg(quadXtr, Ytr); % Create and learn a regression predictor
xline = [0:.01:1]' ; % Transpose
yline = predict(learner_quadratic, polyx(xline, 2)); % Assuming quadratic features
figure('name', 'Quadratic linear predictor');
plot(xline, yline);
hold on % Plot training data and label figure.
plot (Xtr, Ytr, 'bo');
legend('Linear Predictor', 'Training Data');
title('Quadratic linear predictor');
```



Quintic:

```
quinXtr = polyx(Xtr, 5);
learner_quintic = linearReg (quinXtr , Ytr);
yline = predict (learner_quintic , polyx(xline ,5)); % assuming quintic features
figure('name', 'Quintic linear predictor');
plot ( xline , yline );
hold on
plot (Xtr, Ytr, 'bo');
legend('Linear Predictor', 'Training Data');
title('Quintic linear predictor');
```



(d) Calculate the mean squared error associated with each of your learned models on the training data.

```
% Linear
yhat = predict(learner_linear, linXtr);
mseLinTrain = immse(yhat, Ytr);
fprintf('The MSE for the linear predictor on training data was: %.4f\n', mseLinTrain);
```

```
The MSE for the linear predictor on training data was: 0.2366
```

```
% Quadratic
yhat = predict(learner_quadratic, quadXtr);
mseQuadTrain = immse(yhat, Ytr);
fprintf('The MSE for the quadratic linear predictor on training data was: %.4f\n', mse
```

```
The MSE for the quadratic linear predictor on training data was: 0.1092
```

```
% Quintic
yhat = predict(learner_quintic, quinXtr);
mseQinTrain = immse(yhat, Ytr);
fprintf('The MSE for the quintic linear predictor on training data was: %.4f\n', mseQi
```

```
The MSE for the quintic linear predictor on training data was: 0.0813
```

(e,f,g) Calculate the MSE for each model on the test data (in mTestData.txt). Compare the obtained MAE values with the MSE values obtained above.

```
mTest = load('data/mTestData.txt');
xtest = mTest(: ,1); ytest = mTest(: ,2);
% Linear
Xtest = polyx(xtest, 1);
yhat = predict(learner_linear, Xtest);
mseLinTest = immse(yhat, ytest);
fprintf('The MSE for the linear predictor on test data was: %.4f\n', mseLinTest);
```

```
The MSE for the linear predictor on test data was: 0.2353
```

```
% Quadratic
Xtest = polyx(xtest, 2);
yhat = predict(learner_quadratic, Xtest);
mseQuadTest = immse(yhat, ytest);
fprintf('The MSE for the quadratic linear predictor on test data was: %.4f\n', mseQuadT
```

```
The MSE for the quadratic linear predictor on test data was: 0.0972
```

```
% Quintic
Xtest = polyx(xtest, 5);
yhat = predict(learner_quintic, Xtest);
mseQuinTest = immse(yhat, ytest);
fprintf('The MSE for the quintic linear predictor on test data was: %.4f\n', mseQuinTes
```

```
The MSE for the quintic linear predictor on test data was: 0.0959
```

Compare: Above value show that MSE for each model on test data is not varied with the MSE on Train Data.